The Impact of Incorporating the Cost of Errors into Bankruptcy Prediction Models

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Abstract

The current methodology to evaluate default and bankruptcy prediction models is to determine their precision - the percentage of firms predicted correctly. In this study we develop a framework for incorporating Type I (the amount lost from lending to a firm which goes bankrupt) and Type II (the opportunity cost of not lending to a firm which does not go bankrupt) error costs into the evaluation of prediction models. We then test this new framework by comparing the prediction model with a naive model of lending to all firms in the population based on the net profit each would generate. Our results indicate that prediction models can outperform naive models or other models only under certain conditions. This supports our hypothesis that the usefulness of prediction models cannot be fully assessed independently of the costs of forecast errors.

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1 Introduction

The use of a model, based on financial ratios, which yields probability estimates of the likelihood of firms defaulting on their obligations has obvious benefits to a wide variety of groups and individuals. It is also used in a wide variety of research in finance and accounting. Empirical attempts to utilize financial information to predict bankruptcy began with studies by Beaver (1966)[6] and Altman (1968)[1] which used discriminant analysis. This research demonstrated the ability of accounting variables to discriminate between bankrupt and non-bankrupt firms. Ohlson (1980)[17] and Zmijewski (1984)[22] extended this work with logit and probit models. Most of the subsequent research in this area has focused on the appropriate statistical methods used to develop the model, finding the variables which best discriminate between the bankrupt and non-bankrupt firms, and demonstrating the performance of the prediction model by examining the percentage of firms predicted correctly.[1] Credit risk models, starting with Merton (1974)[15], examine both the probability of default and loss given default.[2] Begley et al. (1997)[8], Barniv et al. (2002)[4], Hillegeist et al. (2004)[11] and Beaver (2005)[7] demonstrate the continued interest in and debate over this topic, with the focus on the best variables and statistical methodology. With the emergence of credit derivatives, extensive research has been done on credit default swaps and other credit derivatives.[3]

Currently, bankruptcy prediction studies assess a model's ability to predict by counting the total errors and generally correctly classify 95% or more of a sample into bankrupt and non-bankrupt categories. However, as noted by Palepu (1986)[18] in a study on predicting takeover targets, Type I and Type II errors are likely to be quite different. A Type I error cost is the cost (amount lost) of lending to a firm which defaults or goes bankrupt. A Type II error cost is the opportunity cost (or the profit not made) from not lending to a firm which does not default. Taking into account these differential costs, it may be more profitable to lend to all firms (or no firms) than to base lending decisions on a bankruptcy prediction model. Altman et al. (1977)[2] argue that a reasonable approximate cost for Type I errors

[1]For a review of the literature see Zavgren (1983)[21] and Jones (1987)[14].

[2]For a review of the literature see Crouchy et al (2000)[10]. For an overview of the treatment of the probability of default and loss given default in credit risk models see Altman et al. (2004)[3].

[3]For a review of the literature see Batten and Hogan (2002)[5].

is 70% of the amount lent, and the cost for Type II errors is 2% of the amount that could have been lent, and then concludes that 35 loans to firms which do not go bankrupt can be foregone for each loan to a bankrupt firm.[4] However, this excludes the impact of the heterogeneity of firm size among the sample population. Incorrectly foregoing loans to 35 large firms which do not go bankrupt is not equivalent to incorrectly lending to one small firm which does go bankrupt. Conversely, many loans to firms that do not go bankrupt may be foregone if a loan to one Enron is avoided. The size of the firm (or loan amount) and the relative cost of errors need to be included in the evaluation of bankruptcy prediction models.

The purpose of this paper is to demonstrate the necessity of incorporating the costs of errors to properly assess a prediction model's ability to add value to a potential user as well as in evaluating one model against another. We use the decision context of a lender to show how incorporating both Type I and Type II error costs impact the model's evaluation. It may seem obvious to some that the usefulness of a prediction model can not be fully assessed without considering these costs. However, the extensive body of research examining and using the ability of financial statement information to predict bankruptcy does not yet include this assessment. This paper fills this void.

In the next section we develop a framework to compare different models with constant cost of errors, with a non-constant cost of errors, and discuss the probability of default and loss given default. We then set out our methodology, model, and data. This is followed by an empirical test using a decision context of a bank lending funds, with sample proportions close to those in the real world, using costs of bankruptcy and testing the model developed in one period on a following period. Finally, we present our results and a conclusion.

2 A Framework to Compare Different Models

Bankruptcy and default prediction models provide probabilities of default for firms in a given sample. To maximize profit a lender will take this information and choose a specific probability of default, or

[4] These estimates appear reasonable. However, the empirical evidence to support these costs is limited.

cut-off point, and not lend to firms with a greater probability of bankruptcy. This chosen probability of default yields the precision of a model which is defined as the percentage of correctly classified firms.

We define P as the percentage of firms that did not declare bankruptcy in the given time period t in the sample of firms from which we draw the cut-off probability. Consequently, $1-P$ is the percentage of firms which declared bankruptcy. If firms are ranked based on a prediction model with a probability of bankruptcy from 0 to 1, each cut-off (or probability) level will yield a different level of precision for the model. We define C, C_n and C_b as the levels of precision with respect to all, non-bankrupt and bankrupt firms. C is the overall percentage of correctly classified firms at time t with respect to the population. C_n is the percentage of correctly classified non-bankrupt firms with respect to P. C_b is the percentage of correctly classified bankrupt firms at time t with respect to the $1-P$.

The overall precision of the model is:

$$C = P * C_n + (1 - P) * C_b \tag{1}$$

For example, assume we have a sample of 1000 firms of which 900 ($P = 90\%$) do not file for bankruptcy and 100 (1 - P = 10%) file for bankruptcy. Next, assume all but 90 of the non-bankrupts and 5 of the bankrupts are correctly classified by the prediction model. In this case $C_n = (900 - 90)/900 = 90\%$, $C_b = (100 - 5)/100 = 95\%$, and $C = P * C_n + (1 - P)C_b = 90\% * 90\% + 10\% * 95\% = 90.5\%$ (or you can do the calculation simply as (1000-95)/1000)).

2.1 A Better Way to Evaluate the Value of a Prediction Model

As noted above, prior research assesses the value of a prediction model by comparing the percentage of firms, bankrupt and non-bankrupt, predicted correctly by the model. We argue this fails to capture the impact of the nature of the costs of errors.

We define a Type I error cost as the percentage loss from lending to a bankrupt firm, and a Type II error cost as the opportunity loss from not lending to a non-bankrupt firm (or a gain from lending to a non-bankrupt firm). T is the ratio of Type I to Type II error costs. Historical Type I and Type II error costs are measurable at time t. We begin with a constant T ratio across all firms. Then, instead of looking at the precision of the prediction model defined as a percentage of correctly classified firms, we incorporate the costs of errors ratio (T) into the equation and compute the profit that the lender makes if he uses the prediction model. We replace Type I and Type II error costs with a single T ratio in order to get the following equation:

$$G = P * C_n - (1 - P) * (1 - C_b) * T \tag{2}$$

G is the profit or loss the lender makes when he incorporates the costs of errors (T).

This is then compared to the profit a lender would make if he used a naive lending policy (lending to everyone in the sample) which can be written as:

$$N = P - (1 - P) * T \tag{3}$$

Again, in this equation we replace Type I and Type II error costs with a single T to obtain the profit or loss the lender makes when using the naive model (N). In this case, C_n equals 100% and C_b equals 0%, (the percentage of correctly classified bankrupt firms equals 0 because we lend to every firm).

In order for the naive model to beat the prediction model in the sense of increasing profit ($N > G$), from (2) and (3) the following must hold:

$$T < \frac{P * (1 - C_n)}{(1 - P) * C_b} \tag{4}$$

Under the condition that the model's precision is higher than the overall proportions of non-bankrupt and bankrupt in the sample ($C > P$), we can see from (1) and (4) that T (the ratio of the cost of errors) must be smaller than 1 for the naive lending policy to beat the prediction model:

$$T < \frac{P * (1 - C_n)}{(1 - P) * C_b} < 1 \tag{5}$$

For the prediction model to beat the naive lending policy the following condition must be fulfilled:

$$T > \frac{P * (1 - C_n)}{(1 - P) * C_b} \tag{6}$$

Every C_n and C_b combination corresponds to a unique cut-off probability. In order to properly assess the prediction model one must check all the cut-offs against the naive lending model at the given cost of errors ratio T.

Figure 1 gives an example of minimum T ratios required for a bankruptcy prediction model to beat a naive model. We set the P at 99%, we vary C from 90 to 99% and C_b from 15 to 95% (implicitly we vary C_n as well). If the values of the T ratio are higher than the shown values, the bankruptcy prediction model will beat the naive lending policy. As can be seen from Figure 1, the ability of a model to outperform a naive model depends on the specific value of T - or the Type I / Type II error cost relationship.

2.2 Comparing Models with a Constant Cost of Errors

When comparing across models, both the cut-off and the error costs will impact the percentage value added of the particular model. If we define:

- $C_{n1,2}$=percentage of non-bankrupt firms correctly predicted according to models 1 and 2
- $C_{b1,2}$=percentage of bankrupt firms correctly predicted according to models 1 and 2
- $G_{1,2}$=profit or loss when we incorporate the cost of errors into models 1 and 2

We find:

$$G_1 = P * C_{n1} - (1 - P) * (1 - C_{b1}) * T \tag{7}$$

$$G_2 = P * C_{n2} - (1 - P) * (1 - C_{b2}) * T \tag{8}$$

In order for $G_1 > G_2$, the following has to hold:

if $C_{b2} > C_{b1}$:

$$\frac{P * (C_{n1} - C_{n2})}{(1 - P) * (C_{b2} - C_{b1})} > T \tag{9}$$

if $C_{b2} < C_{b1}$:

$$\frac{P * (C_{n1} - C_{n2})}{(1 - P) * (C_{b2} - C_{b1})} < T \tag{10}$$

From the above we see that the choice of model depends on the samples used, which relates to the population targeted. For example, for a bank to maximize it's profit, the choice of model depends on unique population characteristics of the bank's lending opportunities, the best bankruptcy prediction model for the specific population, and a unique cut-off probability given the population and model.

2.3 Non-Uniform Loans

Above we assumed uniform loans - all loans were the same size. We now relax this assumption. If loans are in different amounts (i.e. a function of firm size) the bankruptcy prediction model will yield the following profit of loss:

$$G = \sum_{i=1}^{C_n * PN} L_i - T * \sum_{j=1}^{(1-C_b)*(F-PN)} L_j \tag{11}$$

If the lender decides to lend to all firms, this will yield the following profit or loss:

$$N = \sum_{i=1}^{PN} L_i - T * \sum_{j=1}^{F-PN} L_j \tag{12}$$

PN is a number of non-bankrupt firms (instead of percentage of non-bankrupt firms previously used as P). F presents the overall number of firms in the population. L_i and L_j are the loans (amounts) granted to non-bankrupt and bankrupt firms respectively.

From the two equations above follows the condition that the cost of errors ratio (T) has to satisfy for prediction model to beat the naive model:

$$T > \frac{\sum_{i=1}^{PN} L_i - \sum_{i=i}^{Cn*PN} L_i}{\sum_{j=1}^{F-PN} L_j - \sum_{j=1}^{(1-C_b)(F-PN)} L_j} \tag{13}$$

In order for one prediction model to beat another prediction model, the following has to hold (if $C_{b2} > C_{b1}$):

$$T > \frac{\sum_{j=1}^{(1-C_{b1})*(F-PN)} L_j - \sum_{j=1}^{(1-C_{b2})*(F-PN)} L_j}{\sum_{i=1}^{C_{n2}*PN} L_i - \sum_{i=1}^{C_{n1}*PN} L_i} \tag{14}$$

2.4 Variable Cost of Errors

Next we relax our assumption of a constant cost of errors ratio (T) to examine the impact of variable cost of errors. We define α and β as Type I and Type II error costs respectively.

The profit or loss to the lender when using the prediction model is: [5]

$$G = \sum_{i=1}^{C_n*P} \beta_i * L_i - \sum_{j=1}^{(1-C_b)*(F-P)} \alpha_j * L_j \tag{15}$$

[5] Unlike in the previous cases, G and N here present total profit or loss, and not profit or loss per unit of Type II error.

The profit or loss to the lender when using the naive model is:

$$N = \sum_{i=1}^{P} \beta_i * L_i - \sum_{j=1}^{F-P} \alpha_j * L_j \tag{16}$$

Instead of using the observed Type I and Type II error costs at time t, as we used above, it is possible to estimate expected costs for each company. These expected costs are used in combination with estimated likelihoods of default to determine an optimal cut-off to maximize the lender's expected profit in the next period.

As the probability of default/bankruptcy and expected loss given default increases, the lender will charge a higher lending rate. Expected Type I and Type II error costs will vary across firms depending on their expected probability of default and loss given default. The link between the probability of default and loss given default is present in the Merton (1974)[15] model as both depend on the same factors. Murphy(2003)[16] provides an overview of yields and spreads across different currencies and rating. He finds evidence that credit risk premiums (spreads) vary across currencies. Hull et al. (2004)[12] explore the relationship between credit default swap (CDS) spreads, bond yields and credit rating announcements. They find a significant positive relationship between CDS spreads and bond yields.

Using the probability of default and loss given default, we can compute the spread over the risk free rate in the following way (following Hull and White (2000)[13]:

$$1 - e^{-x} = PD * LGD \tag{17}$$

$$x = -\ln(1 - PD * LGD) \tag{18}$$

or if we look at total return:

$$TypeIIerrorcost = \beta = \frac{PD * LGD * (1+r)}{1 - PD * LGD} \qquad (19)$$

where x and β are the spreads over the risk free rate, PD is the probability of default, LGD is the loss given default, and r is the discount rate.

To adjust the loss given default to a Type I error cost we make the following adjustments:

$$TypeIerrorcost = \alpha = \frac{LGD * (1+r)}{1 - PD * LGD} \qquad (20)$$

The LGD is computed based on the nominal value of the debt at default and is computed without the interests that would to be payed at maturity.

$\frac{1+r}{1-PD*LGD}$ represents the initial investment the bank makes, and can be replaced by any kind of investment. The above shows the expected percentage return above the risk free rate if no default occurs and the expected percentage loss if default occurs under the no arbitrage assumption.

From the equations 13 and 14, the expected Type I to Type II cost of errors ratio T is equal to:

$$T = \frac{1}{PD} \qquad (21)$$

2.5 Probability of Default and Loss Given Default

One of the examples of estimating probability of bankruptcy and loss given default is the Merton model. The research done by Altman et al.(2004)[3] explains a link between the probability of default and the loss given default in a Merton model. In order to show PD, LGD and costs of errors, we follow on Merton (1974)[15], Altman (2004)[3] and Crosbie(2003)[9] to show an example of changes of probability of default and loss given default in the Merton model, as well as changes in Type I and Type II costs of errors:

$$PD = \Phi(-\frac{\ln(\frac{V_A}{X_t}) + (\mu - \frac{\sigma_A^2}{2})t}{\sigma_A\sqrt{t}}) = \Phi(-d_2) \tag{22}$$

$$LGD = 1 - e^{\mu t}\frac{V_A}{X_t}\frac{\Phi(-\frac{\ln(\frac{V_A}{X_t})+(\mu+\frac{\sigma_A^2}{2})t}{\sigma_A\sqrt{t}})}{\Phi(-\frac{\ln(\frac{V_A}{X_t})+(\mu-\frac{\sigma_A^2}{2})t}{\sigma_A\sqrt{t}})} = LGD = 1 - e^{\mu t}\frac{V_A}{X_t}\frac{\Phi(-d_1)}{\Phi(-d_2)} \tag{23}$$

$$LGD = 1 - e^{d_2\sigma_A\sqrt{t}+\frac{\sigma_A^2}{2}t\sqrt{t}}\frac{\Phi(-d_2 - \sigma_A\sqrt{t})}{\Phi(-d_2)} \tag{24}$$

Where V_A is the market value of firm's assets estimated from the market price of equity and debt, X_t is the default point that can be total debt, short term debt or a combination of the two (i.e. KMV model) depending on the underlying assumptions of the model.[6] Asset drift is μ and volatility is σ. Default occurs if the value of firm's assets reaches the default point. d_2 is defined in the KMV model as the distance to default (in this case it is derived assuming log normal distribution of the firm's assets). If d_2 changes the probability of default and LGD change. Volatility is the second factor that can be varied. It it thus possible to rank firms in KMV (or Merton) model by these two factors and incorporate costs of errors into that model.

Figure 2 shows the increase of Type I and Type II error costs with the decrease of $\frac{V_A}{X_t}$ ratio. We compute Type I and Type II error costs by using the PD and LGD from the Merton model, under the assumption of constant drift of 3% and volatility of 20%. We use the equations 13 and 14 from the section 2.4.

In this setting, the expected Type I and Type II costs of errors vary across firms depending on the prediction of their future value. After estimating the costs of errors per firm, it is possible to incorporate the per firm results into the prediction model, and compare the profit or loss that using that model would bring with some other prediction model. Choosing the best cut-off probability is another advantage of this approach, since in this case, the cut-off probability will be the result of

[6] For an explanation of the KMV model see Crosbie and Bohn (2003)[9] and Vasicek (1997)[20]

estimated profit or loss, and not the result of the model precision C (the number of correctly classified firms).

3 Methodology

To empirically assess how Type I and Type II error costs impact the usefulness of bankruptcy prediction models we examine the costs of errors in a specific decision making context using a sample that is representative of the true population proportions. The decision making context is a bank faced with the choice of whether to make a loan for a fixed percentage of the customer's total assets (book value). The bank estimates a probability of bankruptcy using a bankruptcy prediction model.[7]

Our evaluation methodology is to:

1. Develop parameter estimates for the predictive model based on a sample of bankrupt and non-bankrupt firms in a given year.
2. Compute the probability of bankruptcy for each firm in the sample (by applying the model with its estimated parameters to the specific values of each firm in the sample).
3. Rank firms in the sample by their probability of bankruptcy from lowest to highest.
4. Determine the expected profit, at each probability level, if loans were made only to firms with a lower probability of bankruptcy. This step requires estimates of Type I and Type II error costs. Specifically, at each probability whether the firm filed for bankruptcy or not is known. If the firm did not file for bankruptcy then a profit is made in the amount of the loan times the Type II error cost. If the firm filed for bankruptcy then a loss is made in the amount of the loan times the Type I error cost. The cumulative profit is calculated at each probability level using the total profit to the prior level plus the profit or minus the loss at each new level. The profit a lender would make by lending to all firms is the cumulative profit and loss across all firms.

[7] Assuming that the loan will be a fixed percentage of the firm's assets is made in the belief that the size of a loan is positively correlated with firm size. Discussions with lending officers indicated that a prime consideration in determining the amount a bank is willing to lend is the size of a firm's assets.

5. Select the probability level that maximizes expected profit at the cut-off probability. The appropriate cut-off point is determined in two different ways. The first is to empirically find the probability that maximized the lender's profit in a prior period. The second, which we call the theoretical cut-off, is to calculate the point where the lender no longer expects to make a profit on additional loans given specific profit and loss percentages, which can be determined for any level of Type I and Type II error costs as follows:

   - $PD^* = \beta/(\beta+\alpha)$

     Proof: The expected % profit on any individual loan is:

     (a) $(1-PD)*\beta - PD*\alpha = \beta - PD*\beta - PD*\alpha = \beta - PD(\beta+\alpha)$, where $PD =$ the probability that a firm will declare bankruptcy. If loans are made when the expected value is positive (a rational assumption), then they are made where

     (b) $\beta - PD(\beta+\alpha) > 0$ Our cut-off may then be written as

     (c) $\beta - PD(\beta+\alpha) = 0$, which is restated as

     (d) $\beta = PD(\beta+\alpha)$ and

     (e) $PD = \beta/(\beta+\alpha)$

     where

   - $PD^* =$ the cut-off point where no further loans should be made

   - $\alpha =$ the percentage of the loan that is expected to be lost if the borrower declares bankruptcy-Type I error cost

   - $\beta =$ the percentage profit lost (opportunity cost) if the loan is not given and the borrower does not declare bankruptcy-Type II error cost

6. Compare the lender's ex-post profit using the cut-off point with the profit that the lender would have realized if loans had been made to all firms. Specifically, the profit used at the cut-off point is subtracted from the profit obtained by lending to all firms. This is the value added from using the model. This total is then divided by the profit obtained by lending to all firms to obtain

the percentage value added. We compare the percentage value added at different cut-offs (each Type I / Type II relationship has its own cut-off) and for different time periods.

To overcome the lack of empirical evidence on the magnitude of Type I and Type II error costs, we varied these costs from 100/1 to 1/1.

The ratio of Type I to Type II error costs can be used in the place of varying Type I and Type II error costs individually (assuming that they are constant in % term across firms of different sizes and probabilities of bankruptcy). This reduces the number of runs required and simplifies the paper. The algebra is as follows:

$$ArgMax[\beta * L_i(p) - \alpha * L_j(p)] = ArgMax[L_i(p) - (\alpha/\beta) * L_j(p)]$$

where:

- α = the Type I error cost, β = the Type II error cost,
- L_j = LoanB = the loan to a bankrupt firm, and
- L_i = LoanNB = the loan to a non-bankrupt firm.

As the ratio of Type I to Type II cost of errors decreases, the optimal lending cut-off point will increase. To illustrate, if a Type I cost is 100 times the size of a Type II cost (i.e. if a firm declares bankruptcy the percentage of the loan lost is 100%, while if the firm does not declare bankruptcy the amount of profit made on the loan is 1%) the optimal calculated cut-off is 0.0099 (i.e. loans should be made to all firms with a probability of bankruptcy less than 0.99%), at 50 to 1 (i.e. 50% and 1% respectively) the optimal cut-off is 0.0196, while at 1 to 1 the optimal cut-off is 0.5[8].

To examine any changes in the model across different time periods, the above procedure was repeated for 6 different time periods. Each estimated version was tested on the year following the estimation sample period. The periods used were as follows:

[8] Most prior studies use a cut-off of 0.50 which implies a Type I/Type II error cost ratio of 1/1.

	Estimation Period	Prediction Period
i)	1998	1999
ii)	1999	2000
iii)	2000	2001
iv)	2001	2002
v)	2002	2003
vi)	1998-2002	2003

One limitation of our empirical methodology is that it assumes fixed error costs; i.e. the potential loss/profit on a loan does not change as the probability of bankruptcy increases. As noted in section 2.4 above, the interest rate charged to a borrower is likely to increase as the probability of bankruptcy increases thus changing the Type I and Type II error cost relation as the credit rating of the firm changes. As noted by Altman et al. [3], the loss given default is likely to increase as the probability of bankruptcy increases. These two factors could cancel each other out or they may alter the predictive ability of the model. In principle, armed with a bankruptcy prediction model, lenders could adjust the rate to equalize expected profit across all loans. They do not appear to do this. Stiglitz and Weiss (1981)[19] suggest that banks resort to credit rationing (as assumed in this model) to avoid problems of adverse selection that arise when borrowers know more about their own ability to repay than lenders do.

4 Sample Selection

The sample of firms that declared bankruptcy, and the sample of firms that did not declare bankruptcy were collected separately. All financial and service firms were excluded, by dropping firms with SIC codes greater than 6,000, to avoid any reduction in the predictive ability of the model occurring due to differences between the financial and service firm statements and those of industrial firms.

4.1 The Bankrupt Firm Sample

The bankrupt firm sample consists of all U.S. firms that went bankrupt between January 1, 1998 and December 31, 2003. The 6 year period was chosen because of the data availability and quantity of bankrupt firms to allow a year by year analysis. We define firms belonging to our sample as those meeting the following conditions:

- Incorporated in the United States and filed for bankruptcy protection under the Chapter 11 of US Bankruptcy Code.
- Not a financial institution
- Published annual or quarterly report for the accounting period ending at most 365 days before filing for Chapter 11
- Total liabilities according to the last report filed before the bankruptcy were greater than US$ 100 million.

The list of firms that declared bankruptcy over the period studied was compiled from Edward Altman's database of corporate bankruptcies. Each firm was traced to it's filing with the SEC to confirm the date of the bankruptcy filing, the date of the last financial statements and the date of filing of the last financial statements prior to bankruptcy. This allows the timing of the release of the financial information to be explicitly considered, and reduces any overstatement of the predictive ability of the model by using information released after the bankruptcy filing (as noted by Ohlson (1980)[17]). Financial data were then obtained from the Compustat and Thomson Financial databases.

4.2 The Non-Bankrupt Firm Sample

The sample of non-bankrupt firms and their related financial information was obtained from Standard and Poor's Compustat database. If a firm listed on Compustat did not file for bankruptcy in a given year or in the prior year, financial information on the firm from its previous year end was included in the sample. A separate listing was obtained for each year, resulting in most of the non-bankrupt firms being included repeatedly. Our study has a higher sample proportion of bankrupt firms (though closer than most previous studies) than the true population proportions because the Compustat database does not include every U.S. firm. A bias may exist if inclusion of the missing firms would alter either the parameters or the cut-off point. If the excluded firms are similar in nature and size to the included firms, then the model can easily be adjusted to reflect the true population proportions as demonstrated by Palepu (1986)[18][9]. To the extent the excluded firms are different from the included firms, the impact on the model is unknown. The higher proportion of bankrupt firms in the sample versus the population may be partially mitigated because the omitted non bankrupt firms are smaller, on average, than those included (i.e. Compustat, on average, includes the larger firms). Finally, our sample proportions may be representative for firms of the size included in our study (total liabilities before the bankruptcy greater than US$ 100 million).

5 The Model

This study uses the Logit model, and is estimated using the maximum likelihood procedure in the statistical package SPSS. The probability estimates can be readily interpreted as the probability of the firm's entering bankruptcy. The model has the following formulation:

$$PD_i = F(Z_i) = 1/[1 + exp(-Z_i)], where$$

$$Z_i = B_1 + B_2 * Variablel + B_3 * Variable2...$$

In this context, the explanatory variables were chosen from the prior literature based on their ability to consistently discriminate between bankrupt and non bankrupt firm.[10] Included are:

[9]Our conclusions are unchanged if we adjust our model as suggested by Palepu (1986)[18]

[10]See Ohlson (1980)[17] and Zmijewski (1984)[22]. Naturally other variables could be found which would improve the

1. Return on assets (ROA) = Net Income/Total Assets. ROA proxies for the true profitability of a firm. As profitability increases, the firm will have less difficulties meeting its obligations, and it is predicted that the probability of bankruptcy will decrease.

2. Liquidity (LIQ) = Current Assets/Current Liabilities. LIQ proxies for the ability of a firm to meet liabilities as they come due. As liquidity increases, the ability of the firm to meet its-current obligations- increases, and the probability of bankruptcy is predicted to decrease.

3. Leverage (LEV) = Total Debt/Total Assets. LEV proxies for the indebtedness of a firm. As the percentage of the firm's financing by debt increases, its ability to meet the interest and capital repayments becomes less likely, and the probability of bankruptcy is predicted to increase.

4. Log of Total Assets (LTA). LTA is included to control for firm size. As the size of a firm increases, lenders may be more willing to renegotiate lending terms, and it is predicted that the probability of bankruptcy will decrease. Standardizing Total Assets by its log is done to normalize the distribution.

The variables and the predicted signs of their coefficients are the following:

Variable Predicted	Sign
ROA (return on assets)	-
LIQ (liquidity)	-
LEV (financial leverage)	+
LTA (log of total assets)	-

A profile analysis of the data indicates marked differences between the sample of bankrupt and non-bankrupt firms, as shown in Table 1. As expected, ROA is negative in all periods for the bankrupt sample while it is positive across all periods for the non-bankrupt sample. The mean of LIQ in the non-bankrupt sample is more than twice the level of the bankrupt sample in all years except 1998

model. Finding the best variables and model is beyond the scope of this paper.

where it is 76% higher. LEV for the bankrupt sample is close to or more than double the levels of the non-bankrupt sample (with an average difference across the 6 years of 113%). Finally, the average size of a bankrupt firm at $1.61 billion is just under half the average size of the non-bankrupt firms at $3.19 billion, and the median size of a bankrupt firms at $381 million compared to $621 million for the non-bankrupts (note - the table reports the log of total assets). It is also interesting to note the dramatic changes in the variables across years in the mean, median, and the standard deviations. A direct comparison of these variables to previous studies cannot be made because the earlier studies either do not provide a profile analysis, or include other types of firms in their sample. Additionally, any differences may reflect changes due to the time period under study rather than the impact on the nature of bankrupt versus non-bankrupt firms.

6 Results

Table 2 lists the estimated model parameters. The signs of the coefficients are as predicted over all periods. The variables ROA and LEV are statistically different from zero[11] in all time periods, but not LIQ (for which the null can not be rejected in 1 of the 5 time periods) or LTA (for which the null can not be rejected in all time periods). All parameters fluctuate widely (more than 50%) from time period to time period. The Pseudo R-squared range from 0.071 to 0.125 which is consistent with findings of Hillegeist et al. [11] for accounting-based models.

The optimal cut-offs, calculated by expected values and determined empirically for each time period, are presented in Table 3. The large variations between time periods illustrate how sensitive the empirically determined cut-offs are to the time period. This is especially true for 2002. The results for 2002 are driven by one unexpected (low probability) very large bankruptcy - Worldcom. If this firm is removed from the sample, the results are in line with the other periods.

Table 4 and Table 5 contrast the profit under the naive model (lending to all firms) with the profit obtained at different Type I and Type II error cost relationships using cut-offs determined empirically

[11] The null hypothesis that the estimated coefficient is equal to zero can be rejected at the 0.05 level.

and theoretically respectively. The value added by using the model is calculated as the profit determined at a given cut-off less the profit under the naive model for varying Type I and Type II error cost relationships. The percentage value added from using the model is this net amount saved divided by the profit under the naive model. As expected, there is a positive correlation between the ratio of Type I to Type II error costs and the value added from using the model. The percentage saved by using the prediction model versus a naive model increases as the error ratio increases. Note some very high percentages occur when the profit from lending to all firms is close to 0. The prediction model outperforms the naive lending policy in three of the five subsequent periods (all but 2002 and 2003) using the empirically determined cut-off and four of the five subsequent periods (all but 2002) using the theoretically determined cut-off (for T from 1 to 100).[12] The failure in 2002 is a result of the Worldcom bankruptcy. Worldcom could be described as an unexpected bankruptcy in the sense it was the result of a major fraud which was not apparent in the financials. The size of the firm created a major impact on the profit at the cut-off. The empirically determined results in 2003 are also affected by Worldcom and a few other unexpected bankruptcies (e.g. US Airways and Adelphia). However, here the result is due to the impact of those bankruptcies on the cut-offs used in 2003. As discussed above, the empirically determined cut-offs in 2003 are those which would have maximized profit for different Type I /Type II error cost relationship in 2002. The Worldcom bankruptcy dramatically altered (lowered) the cut-offs and in this way reduced the profit at the given cut-off. This also explains why the results using the theoretically determined cut-off were not affected in 2003, as they were not impacted by the Worldcom bankruptcy which occurred in 2002. If Worldcom and others are removed from the sample, then the results are consistent across all years. Also, using a longer time period eliminates the impact of a Worldcom type bankruptcy.

Table 6 shows the precision (number of correctly classified firms as a percentage of overall number of firms in the sample) when we use theoretical and empirical cut-offs. The precision of the prediction model in both cases decreases with the increase in T error cost ratio. This contrasts with the added value from using the prediction model that includes cost of errors increases with the increase of the

[12] In 2002 and 2003 the prediction model outperforms or under-performs the naive model depending on the used cost or errors ratio T.

error cost ratio T (Tables 4 and 5). Table 6 also shows that when models evaluated by precision using empirical versus theoretical cut-offs are compared it is not always the one with the higher precision that adds the most value to the lender. For example, in the year 2000 the empirical cut-offs yield higher prediction model precision then the theoretical cut-offs when the cost of error ratio T varies from 5 to 100. But the value added from using the theoretical cut-offs is higher than the value added from using the empirical cut-offs in 2000 when the T ratio is varied from 5 to 100.

Table 7 shows the added value of prediction model using the empirically determined cut-offs when compared to the precision approach. In the precision approach, cut-offs are computed in the prior periods to maximize the number of correctly classified firms into non-bankrupt and bankrupt. This cut-off is then used in our framework and contrasted with our prior results. In four of the five periods including costs of errors adds value to the lender. The disturbance caused by Worldcom is again mitigated if Worldcom bankruptcy was removed or a longer estimation period is used.

The results support several observations. First, they demonstrate how incorporating the costs of errors impacts the assessment of prediction models. Second, one large (or a few small) unanticipated bankruptcy (or the economics of a particular year) can create the situation where a predictive model cannot beat the naive model at certain or even any Type I / Type II error cost relationship(s). Third, it underlines the importance of testing the model on a subsequent time period (the way a model could be used in the real world) as opposed to using a hold-out sample as done in prior research. The evidence does not indicate whether either method for determining the cut-off (empirical or theoretical) is superior. Using a theoretically determined cut-off would limit the impact of a Worldcom like bankruptcy to one year. However, this result can also be achieved by using more than one year to do the analysis. [13]

[13] We ran the analysis using data from 2 years and 3 years onto the following year (i.e. build the model on data from 2001 and 2002 and testing on 2003, and building the model on data from 2000 and 2001 and 2002 and testing it on data from 2003) and this removed the impact of Worldcom on 2002. and 2003.

7 Conclusion

This study examines the impact of incorporating the cost of errors into bankruptcy and default prediction models. We find that the percentage increase in profit from using a prediction model versus a naive decision model of lending to all firms varies with the ratio of Type I to Type II error costs, the time period, and the method of selecting the cut-off. This illustrates the importance of determining the relationship of Type I to Type II error costs before making any statements about a model's value added ability. It also underlines the sensitivity of the model to the underlying assumptions.

Clearly, incorporating the costs of errors into bankruptcy prediction models is essential to properly assess the true usefulness, in the sense of value added, of the model. It also provides a means to properly compare two different models as the superiority of one model over another cannot be fully measured unless the costs of errors are taken into account. The possibility of incorporating constant and variable costs of errors into a bankruptcy prediction model adds value to an investor and should be used in research to properly assess the superiority of one model over another.

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Table 1: Financial Ratios of Bankrupt and Non-Bankrupt Firms from 1998 to 2003

Our population was all public US non-financial firms with total assets greater than $100 million. The bankrupt sample was taken from Altman's bankruptcy database. The non-bankrupt sample was taken from Compustat database. ROA is the Return on Assets, LIQ Current Ratio, LEV Leverage, and LTA is the Log of Total Assets.

Bankrupt sample							
Year		**1998**	**1999**	**2000**	**2001**	**2002**	**2003**
Number of firms		36	31	74	86	59	41
ROA							
	Mean	-25.56	-33.57	-23.56	-28.03	-27.80	-19.73
	Median	-21.72	-16.86	-15.09	-21.44	-22.15	-16.00
	Std. Dev.	19.84	52.20	24.22	25.10	26.66	17.64
	Interquartile range	34.04	41.60	22.63	32.07	29.19	27.36
LIQ							
	Mean	1.31	0.78	0.99	1.01	0.78	0.91
	Median	0.78	0.42	0.72	0.46	0.46	0.55
	Std. Dev.	1.63	0.84	0.86	1.75	0.99	0.92
	Interquartile range	1.33	0.87	1.05	0.73	0.81	0.92
LEV							
	Mean	1.08	1.24	1.07	1.21	1.45	1.34
	Median	0.95	1.12	1.02	0.98	1.09	1.16
	Std. Dev.	0.43	0.59	0.43	0.69	0.98	0.62
	Interquartile range	0.45	0.63	0.39	0.58	0.62	0.73
LTA							
	Mean	5.73	6.03	6.34	6.27	6.26	5.83
	Median	5.49	5.72	6.31	6.13	6.06	5.65
	Std. Dev.	0.89	1.29	1.06	1.36	1.55	1.08
	Interquartile range	0.91	2.24	1.55	1.80	1.55	1.28

Nonbankrupt sample							
Year		**1998**	**1999**	**2000**	**2001**	**2002**	**2003**
Number of firms		1729	2082	2455	2568	2643	2595
ROA							
	Mean	4.76	4.18	2.67	0.35	1.18	1.87
	Median	4.91	4.50	3.91	2.30	2.68	2.95
	Std. Dev.	7.91	8.23	10.54	9.21	8.17	7.37
	Interquartile range	7.68	7.55	8.19	8.59	7.45	7.42
LIQ							
	Mean	2.31	2.19	2.30	2.55	2.44	2.39
	Median	1.80	1.70	1.67	1.69	1.67	1.71
	Std. Dev.	2.09	2.31	2.68	3.66	3.29	2.66
	Interquartile range	1.61	1.39	1.55	1.66	1.60	1.50
LEV							
	Mean	0.56	0.58	0.58	0.58	0.58	0.58
	Median	0.57	0.58	0.59	0.58	0.58	0.57
	Std. Dev.	0.28	0.26	0.28	0.31	0.30	0.30
	Interquartile range	0.33	0.29	0.32	0.35	0.36	0.35
LTA							
	Mean	6.34	6.56	6.68	6.69	6.70	6.76
	Median	6.17	6.38	6.48	6.45	6.44	6.53
	Std. Dev.	1.65	1.56	1.54	1.52	1.51	1.51
	Interquartile range	2.28	2.11	2.11	2.11	2.18	2.17

Table 2: Estimated Model Parameters

Estimated regression coefficients when all the variables are used in the regression. Coefficient is the regression coefficient, S.E. is the standard error, Wald is the Wald statistic and Significance is the significance level of the coefficient. ROA is the Return on Assets, LIQ Current Ratio, LEV Leverage, and LTA is the Log of Total Assets.

Period		ROA	LIQ	LEV	LTA	Constant
1998	**Coefficient**	-0.175	-0.087	1.480	-0.165	-4.634
	S.E.	0.022	0.134	0.439	0.152	1.168
	Wald	64.109	0.419	11.349	1.176	15.732
	Significance	0.000	0.518	0.001	0.278	0.000
1999	**Coefficient**	-0.126	-1.142	2.086	-0.098	-4.309
	S.E.	0.021	0.397	0.589	0.173	1.400
	Wald	34.519	8.284	12.539	0.324	9.466
	Significance	0.000	0.004	0.000	0.569	0.002
2000	**Coefficient**	-0.094	-0.904	1.820	-0.184	-2.991
	S.E.	0.009	0.212	0.347	0.110	0.907
	Wald	97.218	18.162	27.510	2.781	10.864
	Significance	0.000	0.000	0.000	0.095	0.001
2001	**Coefficient**	-0.124	-0.803	0.997	-0.043	-3.749
	S.E.	0.011	0.169	0.267	0.101	0.815
	Wald	120.579	22.635	13.981	0.179	21.135
	Significance	0.000	0.000	0.000	0.672	0.000
2002	**Coefficient**	-0.129	-1.100	1.301	-0.600	-3.974
	S.E.	0.015	0.283	0.427	0.114	1.063
	Wald	71.147	15.133	9.278	0.273	13.986
	Significance	0.000	0.000	0.002	0.602	0.000
2003	**Coefficient**	-0.148	-0.825	1.303	-0.241	-3.273
	S.E.	0.021	0.279	0.399	0.153	1.189
	Wald	47.503	8.759	10.684	2.486	7.573
	Significance	0.000	0.003	0.001	0.115	0.006

Estimated regression coefficients when only the significant variables are used (10% significance level). Coefficient is the regression coefficient, S.E. is the standard error, Wald is the Wald statistic and Significance is the significance level of the coefficient.

Period		ROA	LIQ	LEV	LTA	Constant
1998	**Coefficient**	-0.178		1.601		-5.937
	S.E.	0.022		0.411		0.478
	Wald	67.445		15.196		154.474
	Significance	0.000		0.000		0.000
1999	**Coefficient**	-0.130	-1.125	2.077		-4.985
	S.E.	0.021	0.398	0.597		0.796
	Wald	39.045	7.989	12.097		39.198
	Significance	0.000	0.005	0.001		0.000
2000	**Coefficient**	-0.094	-0.904	1.820	-0.184	-2.991
	S.E.	0.009	0.212	0.347	0.110	0.907
	Wald	97.218	18.162	27.510	2.781	10.864
	Significance	0.000	0.000	0.000	0.095	0.001
2001	**Coefficient**	-0.125	-0.798	1.005		-4.057
	S.E.	0.011	0.169	0.266		0.376
	Wald	128.784	22.335	14.219		116.584
	Significance	0.000	0.000	0.000		0.000
2002	**Coefficient**	-0.130	-1.074	1.335		-4.441
	S.E.	0.015	0.277	0.423		0.588
	Wald	74.145	15.014	9.983		56.956
	Significance	0.000	0.000	0.002		0.000
2003	**Coefficient**	-0.156	-0.808	1.367		-4.947
	S.E.	0.021	0.283	0.406		0.629
	Wald	55.442	8.175	11.351		61.808
	Significance	0.000	0.004	0.001		0.000

Table 3: Probability Cut-offs

These are the cut-off points, or the probability levels, where a lender would stop making loans (lend to firms up to this probability). The theoretically determined cut-off is the point where a lender no longer expects to make a profit on additional loans given the specific profit and loss percentages. These cut-offs will not vary across time. The empirically determined cut-off is the one which maximized the lender's profit in the prior period, and so these will vary across time. T is the ratio of Type I (the cost of lending to a firm which then goes bankrupt) to Type II (the opportunity cost of not lending to a firm which does not go bankrupt). See Section 3 for a more detailed explanation.

		Empirical						
T	Expected	1998	1999	2000	2001	2002	2003	1998-2002
100	0.010	0.010	0.028	0.034	0.011	0.002	0.022	0.014
50	0.020	0.010	0.055	0.034	0.014	0.002	0.022	0.014
25	0.038	0.072	0.055	0.034	0.042	0.003	0.022	0.038
10	0.091	0.252	0.099	0.090	0.072	0.003	0.118	0.072
5	0.167	0.295	0.376	0.090	0.120	0.003	0.134	0.12
2	0.333	0.295	0.376	0.229	0.292	0.003	0.134	0.287
1	0.500	0.295	0.376	0.566	0.780	0.006	0.134	0.783

Table 4: Value added from prediction model using empirically determined cut-offs compared to the naive model (lending to all firms)

At each probability whether the firm filed for bankruptcy or not is known. If the firm did not file for bankruptcy then a profit is made in the amount of the loan times the Type II error cost. If the firm filed for bankruptcy then a loss is made in the amount of the loan times the Type I error cost. Firms are ranked in probability of bankruptcy from 0 to 1. A cumulative profit is calculated at each probability level using the total profit to the prior level plus the profit or minus the loss at each new level. The profit a lender would make by lending to all firms is the cumulative profit and loss across all firms. The cumulative profit at a given cut-off minus the profit from lending to all firms is the net value added. The net value added divided by the profit from lending to all firms is the percentage value added. T is the ratio of Type I (the cost of lending to a firm which then goes bankrupt) to Type II (the opportunity cost of not lending to a firm which does not go bankrupt). See Section 3 for a more detailed explanation. The cut-off is determined empirically by using the cut-off which maximized profit in the prior period and is provided in Table 3. Years in brackets represent the prior period from which empirical cut-offs were determined. 2002w is the year 2002 sample without the Worldcom bankruptcy.

T	1999	2000	2001	2002	2002w	2003	2003 (1998-2002)
100	63.74%	784.13%	96.10%	3.23%	219.19%	15.19%	9.06%
50	17.66%	56.94%	127921.60%	-26.49%	239.54%	5.64%	3.45%
25	7.46%	18.83%	41.14%	1.71%	50.72%	2.17%	1.04%
10	1.73%	4.45%	9.71%	-5.12%	9.78%	0.68%	0.32%
5	0.69%	1.24%	3.80%	-2.86%	3.93%	0.22%	0.10%
2	0.11%	0.37%	0.27%	-1.09%	2.22%	-0.05%	0.03%
1	-0.08%	0.10%	0.09%	-0.59%	1.06%	-0.03%	0.01%

Table 5: Value added from prediction model using the theoretically determined cut-offs compared to the naive model (lending to all firms)

At each probability whether the firm filed for bankruptcy or not is known. If the firm did not file for bankruptcy then a profit is made in the amount of the loan times the Type II error cost. If the firm filed for bankruptcy then a loss is made in the amount of the loan times the Type I error cost. A cumulative profit is calculated at each probability level using the total profit to the prior level plus the profit or minus the loss at each new level. The profit a lender would make by lending to all firms is the cumulative profit and loss across all firms. The cumulative profit at a given cut-off minus the profit from lending to all firms is the net value added. The net value added divided by the profit from lending to all firms is the percentage value added. T is the ratio of Type I (the cost of lending to a firm which then goes bankrupt) to Type II (the opportunity cost of not lending to a firm which does not go bankrupt). See Section 3 for a more detailed explanation. The cut-off is the probability where the lender no longer expects to make a profit on additional loans given the specific profit and loss percentages. These cut-offs are provided in Table 3. Cut-Offs are theoretically determined cut-offs. 2002w is the year 2002 sample without the Worldcom bankruptcy.

T	1999	2000	2001	2002	2002w	2003
100	63.74%	853.66%	132.85%	0.62%	208.57%	9.04%
50	17.11%	71.06%	108018.72%	31.61%	287.85%	2.46%
25	6.36%	19.24%	42.16%	3.31%	50.19%	1.04%
10	1.37%	4.50%	9.73%	-2.73%	10.82%	0.32%
5	0.62%	1.77%	3.54%	-0.73%	5.47%	0.09%
2	0.14%	0.45%	0.33%	0.02%	2.38%	0.03%
1	0.02%	0.09%	0.07%	-0.01%	1.15%	0.02%

Table 6: Precision of the prediction model when theoretical and empirical cut-offs are used

Precision of the prediction model when the theoretical cut-offs were used. Every cut-off will give a number of correctly classified firms. We calculate the precision of the model as the number of correctly classified firms divided by the overall number of firms in the sample. See Section 2 for more detailed explanation. Theoretical (expected) cut-offs are presented in the Table 3. For more detailed explanation of the theoretical cut-offs see Section 3.

T	1999	2000	2001	2002	2003
100	81.30%	84.90%	61.61%	68.95%	98.14%
50	87.59%	91.54%	80.60%	85.94%	98.63%
25	91.67%	93.83%	89.71%	92.12%	98.75%
10	95.07%	96.24%	95.18%	96.23%	98.79%
5	96.71%	96.96%	96.68%	98.00%	98.71%
2	97.90%	97.39%	97.17%	98.59%	98.71%
1	98.53%	97.43%	97.40%	98.52%	98.71%

Precision of the prediction model when the empirical cut-offs were used. Every cut-off will give a number of correctly classified firms. We calculate the precision of the model as the number of correctly classified firms divided by the overall number of firms in the sample. See Section 2 for more detailed explanation. Empirical cut-offs are presented in the Table 3. For more detailed explanation of the empirical cut-offs see Section 3.

T	1999	2000	2001	2002	2003
100	81.30%	92.88%	88.51%	72.69%	96.74%
50	81.30%	95.22%	88.51%	79.61%	96.74%
25	94.50%	95.22%	88.51%	93.15%	97.34%
10	97.34%	96.24%	95.10%	95.60%	97.34%
5	97.56%	97.31%	95.10%	96.93%	97.34%
2	97.56%	97.31%	96.95%	98.00%	97.34%
1	97.56%	97.31%	97.59%	97.71%	98.03%

Table 7: Value added from prediction model using empirically determined cut-offs compared to the precision model (the smallest number of misclassified firms)

At each probability whether the firm filed for bankruptcy or not is known. If the firm did not file for bankruptcy then a profit is made in the amount of the loan times the Type II error cost (the opportunity cost of not lending to a firm which does not go bankrupt). If the firm filed for bankruptcy then a loss is made in the amount of the loan times the Type I error cost (the cost of lending to a firm which then goes bankrupt). A cumulative profit is calculated at each probability level using the total profit to the prior level plus the profit or minus the loss at each new level. The profit a lender would make by lending to all firms is the cumulative profit and loss across all firms. The cumulative profit at a given cut-off minus the profit from lending to all firms is the net value added. The net value added divided by the profit from lending to all firms is the percentage value added. T is the ratio of Type I to Type II error costs. See Section 3 for a more detailed explanation. The empirically determined cut-offs are provided in Table 3. The precision model cut-offs are those which maximize the percentage of correctly classified firms in the prior period. Years in brackets represent the prior period from which empirical cut-offs were determined. 2002w is the year 2002 sample without the Worldcom bankruptcy.

T	1999	2000	2001	2002	2002w	2003	2003 (1998-2002)
100	20.61%	123.87%	94.88%	43.35%	220.89%	8.21%	6.35%
50	5.00%	25.97%	285.92%	405.39%	25.39%	3.10%	2.45%
25	2.29%	9.46%	26.39%	19.67%	14.30%	1.07%	0.60%
10	0.03%	1.61%	6.72%	-1.12%	0.26%	0.27%	0.16%
5	0.00%	0.00%	2.60%	-1.25%	-0.61%	0.03%	0.02%
2	0.00%	0.00%	-0.08%	-0.73%	0.23%	-0.12%	0.00%
1	0.00%	0.00%	-0.01%	-0.48%	0.02%	-0.05%	0.00%
Precision cut-offs	0.295	0.376	0.630	0.547	0.547	0.024	0.590

Figure 1: Changes in maximal T ratio with model precision

If T ratio is lower than this point, the lend-to-all policy outperforms the bankruptcy prediction model. C is the overall precision of a bankruptcy/default prediction model. C_b is the precision of a model for bankrupt firms. T is the Type I (percentage loss from lending to a bankrupt firm) to Type II (opportunity loss from not lending to a non-bankrupt firm) error cost ratio. For the purpose of this example, the proportion of non-bankrupt firms in the sample is set to 99%.

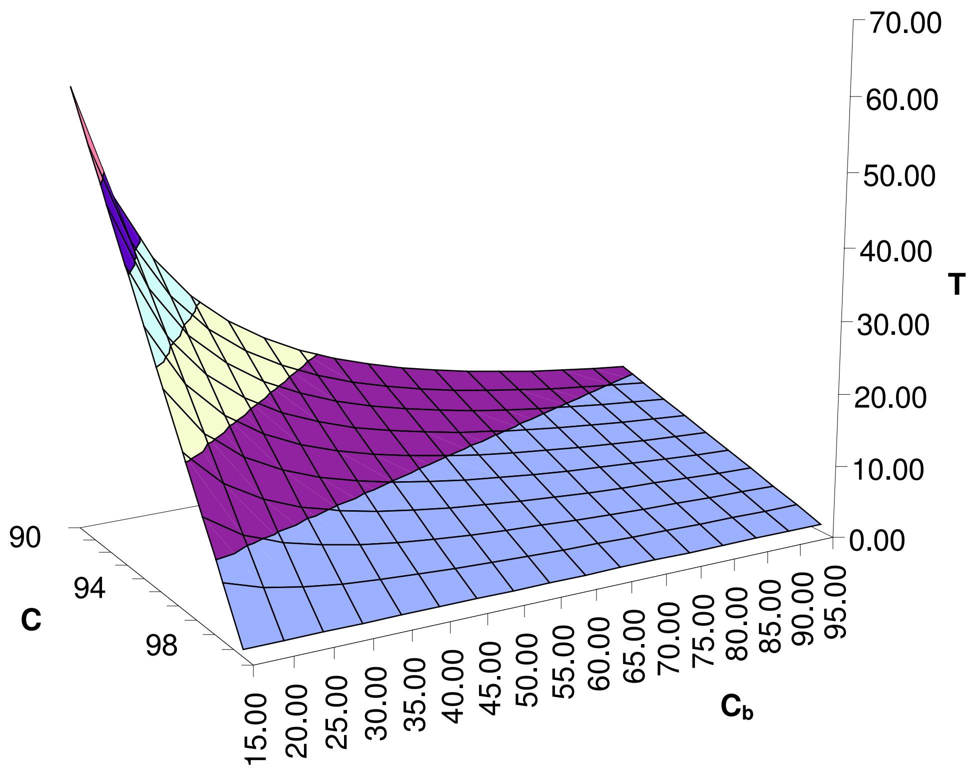

Figure 2: Changes in Type I and Type II error costs with change in $\frac{V_A}{X_t}$

The volatility is set at 20% and the drift at 3%. As the ratio between V_A - The value of the assets and X_t - The default point change, the Type I error cost (percentage loss from lending to a bankrupt firm) and Type II error cost (opportunity loss from not lending to a non-bankrupt firm) change.