Direct Preference for Wealth in Aggregate Household Portfolios*

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Abstract

According to standard theory, wealth should have no intrinsic value. Yet, conventional wisdom, alternative theories, and data suggest it might. We verify whether or not households have direct preferences over wealth in selecting assets. The fully structural econometric model focuses on a multivariate Brownian motion in optimal consumption, portfolios and wealth. Using aggregate portfolio data, we find that wealth (i) is directly valued, (ii) reduces marginal utility of consumption and (iii) reduces consumption risk aversion, while we reject the HARA, and CRRA restrictions. Consequently, wealth-dependent utility generates a larger IMRS risk, justifying a larger, more predictable risk premium and a lower risk-free rate.

JEL classification: G11, G12.

Keywords: Portfolio choice, Wealth-dependent preferences, Preference for status, Asset pricing, Equity premium, Risk-free rate, Predictability.

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Introduction

Recent research on preference-based explanations of asset market dynamics focuses on the minimum admissible (also known as subsistence, reference or marginal–utility bliss) consumption level. Standard representations of preferences such as the widely-used Constant Relative Risk Aversion (CRRA) utility, restrict the minimum consumption level to zero. Hyperbolic Absolute Risk Aversion (HARA) utility is more general in allowing for a constant non-zero bliss level. Conversely, the more recent habit literature allows for bliss to be determined by conditional state variables, such as lagged individual or aggregate consumption.

Focusing on minimum admissible consumption instead of curvature or impatience indices is sensible. Indeed, bliss and attitudes toward risk are closely related. The closer consumption is to bliss, the steeper is the marginal utility schedule. This implies that small movements in consumption cause larger fluctuations in marginal utility. To the extent that the agent selects his portfolio so as to hedge away these risks, in HARA utility, the distance between consumption and bliss directly determines risk aversion and portfolio. Moreover, the CRRA restriction of zero bliss is not innocuous. Under the more general HARA utility, non-zero bliss implies that both consumption and asset holding schedules have a state-independent non-zero intercept. This is important to the extent that it is a sufficient condition for predicted consumption and portfolio shares of wealth to be time-varying, a feature that is also found in the data (see Figure 1).\(^1\)

[ Insert Figure 1 here ]

Furthermore, allowing for time-varying instead of constant bliss is intuitively appealing. In both CRRA and HARA, this level is taken as a deep parameter. This restriction appears excessive. Subsistence may be interpreted as a subjective, as well as physiological, measure. It seems more
realistic to allow for our basic needs to evolve with age, habits or wealth levels. What is considered benchmark when young or poor need not be the same at an older age or when richer. Reference consumption should be allowed to evolve with time and/or economic conditions, as is the case under reference-dependent utility where preferences are determined over deviations from a consumption– or endowment–determined reference point. Prominent examples of the former are internal and external consumption–based habit models.

Unfortunately, the empirical gains of consumption–based habit preferences has been mitigated. On one hand, the higher savings rate required to maintain habits helps to understand the low observed returns on risk-less assets. However, ultimately, the only source of inter-temporal marginal rates of substitution (IMRS) risk remains consumption. This risk is the sole determinant of excess returns in preference-based models of asset returns. Since aggregate consumption is a despairingly smooth series, it is weakly correlated with returns. The quantity of consumption risk is consequently too low to justify the high observed premia on risky assets. Finally, the cyclical properties of the surplus consumption to habit ratio are apparently insufficient to fully capture the predictability in observed excess returns. Counter-cyclical premia points out to counter-cyclical risk aversion; the latter cannot be generated in sufficient levels by standard habit models with i.i.d. consumption growth alone.

These elements suggest that bliss consumption level should be (i) nonzero, and (ii) state-dependent. Since consumption was found to be inadequate, a natural alternative as a determinant of bliss is wealth. Wealth, as primary state variable in dynamic programs, is a good proxy for economic conditions. Furthermore, direct preference for wealth can be explained through a status-seeking, ‘capitalistic spirit’, a preference for non-marketed goods, as well as a preference for durable goods argument. We therefore introduce a wealth-dependent utility (WDU) function in which the
Bernoulli concave transform is applied to an affine function of consumption and wealth. This utility simplifies to a HARA class under wealth independence, otherwise bliss is a linear function of wealth.

Under *ratchet* preferences, higher wealth increases the bliss level, causing clockwise rotations in the marginal utility (MU) schedule, and increasing consumption risk aversion, i.e. consumption risk aversion is pro-cyclical. Heuristically, a ratchet investor who is richer has a higher minimal consumption. As wealth increases, any given level of consumption is thus closer to this reference, and the ratchet investor is consequently more risk averse.

Conversely, under *blasé* preferences, higher wealth lowers the bliss level, causing counter-clockwise rotations in the MU schedule. As a result, consumption risk aversion decreases i.e. consumption risk aversion is counter-cyclical. In intuitive terms, a blasé investor values less the same unit of consumption as he becomes richer. Because he is further away from bliss, a richer blasé agent suffers less in marginal utility from small movements around that same level of consumption, and is consequently less risk averse.

Contrary to consumption–based habit models, bliss is thus only indirectly related to past consumption. Movements in the reference point can be caused by factors that are independent of the agent’s decisions, such as the individual returns that compose the total wealth portfolio. Therefore, the agent has only partial control over movements in bliss through his savings and portfolio decisions. We show that this will have important consequences for the MU risk, and therefore the optimal rules and pricing implications.

Our application focuses on the inter-temporal consumption–portfolio problem. We start by discussing closed-form solutions for optimal consumption and portfolio rules. These rules are obtained by adapting an iso-morphism result of Schroder and Skiadas (2002) for linear habit models.
We find that both consumption, and the value invested in assets are affine functions of wealth. This implies that neither the average propensity to consume, nor the portfolio shares are constant, but move depending on the wealth level. This result is useful to the extent that it predicts cyclical movements in the value invested in assets relative to one another, as highlighted in Figure 1. Moreover, time variation in the consumption-wealth ratio accords with findings that this variable is counter-cyclical.

Our main contribution is however empirical. Estimation of the model focuses on the closed-form consumption and portfolio rules. More precisely, we estimate a fully structural multivariate system composed of instantaneous changes in consumption, asset holdings, wealth, and asset returns. The first three elements incorporate the full theoretical restrictions, both on the conditional first, and conditional second moments. Asset returns are unrestricted and included to correct inference for the uncertainty regarding the distributional parameters used in the closed-form expressions. We innovate from standard approaches which typically treat consumption growth as exogenous and estimate the model applying the theoretical restrictions on returns instead.

The resulting empirical model is a multivariate Brownian motion that presents estimation challenges as both drifts and diffusions are affine functions of the state variable. These functions do not admit closed-form expressions for the transition density. We therefore resort to a homoscedasticity-inducing transformation which also stationarizes the drift term. The transformed model can conveniently be estimated using a discrete time differencing approach without inducing any time discretization biases.

Using aggregate asset holdings data from the Flow of Funds, we estimate the model by maximum likelihood for three utility functions: CRRA, HARA, and WDU. Our first step is to test the theoretical restrictions that guarantee monotonicity and positive discounting. We next discuss the
estimated parameters, inference, and derived variables of interest, followed by formal specifications tests. Under a suitable identification strategy, we find that our estimates are (i) theoretically acceptable, and (ii) intuitively realistic. In particular, both the curvature parameter and risk aversion index are within reasonable bounds when the subjective discount rate is calibrated to a realistic value. Our results are also indicative of a significant blasé behavior in portfolio choices. Moreover, risk aversion is found to be counter-cyclical, increasing in downturns and falling during recoveries. This result is consistent with findings using returns that allow for time-varying risk aversion. Finally we find that the null of CRRA preferences is rejected when tested against HARA or WDU utility.

We discuss these results in light of the asset pricing implications of our model. Wealth-dependent utility generates a linear multi-factor premia in which both consumption, and total wealth (i.e. market) risks are theoretically valued by investors. Moreover, the price of consumption risk remains the Arrow-Pratt coefficient of relative consumption risk aversion, whereas the price of market risk has the intuitive interpretation of being the Arrow-Pratt coefficient, evaluated over relative wealth risk aversion. Hence, our pricing kernel can be interpreted as a weighted average between a standard, wealth-independent C-CAPM, and a static CAPM, where the weights are given by the relative importance of consumption, versus wealth risk aversion. Two-factor pricing kernels are a salient feature of WDU models, but also of non-expected utility frameworks. In contrast to the latter, our approach is derived under Von Neumann–Morgenstern (VNM) preferences; a test that the market risk is valued simplifies to a test of wealth dependence, rather than a joint test of state- and time- non-separability.

Our framework has the potential to explain the three main empirical anomalies of the C-CAPM. First, the additional risk contributed by covariances of individual returns with total wealth is likely
to help explain the high observed premia on risky assets. Secondly, since both consumption and wealth risk aversion follow cyclical movements, this model can address the predictability found in excess returns. In particular, the wealth–to–consumption ratio, a known predictor of excess returns, is explicit in our predicted premia. Finally, we show that our model may explain the low rate of return on a risk-free asset through the effect on the mean and variance of the inter-temporal marginal rate of substitution.

The rest of this paper is organized as follows. After discussing the relevant literature in Section I, we outline the model, and the closed-form solutions in Section II. Next, we introduce the empirical methods in Section III, and present the estimation results in Section IV. We discuss the pricing implications of these results in Section V, before concluding.

I Relevant literature

Our modelling approach for preferences can be related to the literature on state-dependent preferences. These preferences assume that the agent’s within-period utility is a function of consumption, as well as one (or many) state variable(s). These variables are usually restricted to be conditional states, i.e. they do not belong to the control set at the time of the decision. The models essentially differ in (i) their choice of the state, and (ii) the functional form for the utility. Table 1 describes a sample of state-dependent models.

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Sundaresan (1989); Constantinides (1990); Ferson and Constantinides (1991); Detemple and Zapatero (1991) are early examples of (internal) habit models. Specifically, the time-varying bliss
factor $-\eta_t$ is a function of past consumption profiles which represent the state. If $\eta_t' < 0$, then high current consumption imply larger future bliss levels. Similarly, Campbell and Cochrane (1999) allow for bliss to be related to consumption. However, they restrict consumption to be aggregate consumption profile $\bar{C}_t$, i.e. both a conditional and an unconditional state variable (external habit).

Unfortunately, empirical analysis is detrimental to both the internal and external consumption–based habit models. Ferson and Constantinides (1991) find that internal habit preferences are able to explain the low risk-free rate, but not the high equity premia. Tallarini and Zhang (2005) find that the estimated Campbell and Cochrane (1999) external habit model (i) is rejected at the 1% level, produces (ii) subjective factors greater than one, and (iii) excessive risk aversion parameters and levels. Whereas the model can reproduce the unconditional premia on stock, it cannot match the higher moments, including the high variance, and large spread between premia in recession and in recoveries. Moreover, the model predicts a large, negative return on risk-free asset. Li (2001, 2005) confirms that when compared to other predictors, such as the dividend yield, or the wealth–to–consumption ratio, surplus consumption over consumption–based habit explains only a modest fraction of the predictability in returns.

The intuition for these mitigated results is straightforward. A habit investor needs to save more in order to maintain his future habits. Consequently, he is willing to pay a higher, and possibly excessive, price for the risk-less asset. However, the only source of MU risk for habit preferences remains consumption. Since this series is quite smooth, its covariance with returns is too low to justify the high premia, unless risk aversion is excessive (see also Barberis et al., 2001, for a discussion). Moreover, consumption–based habit models can only partially address the predictability of excess returns. In particular, the counter–cyclical properties of equity premia can
only be explained in part by the counter-cyclicality of the price of consumption risk, as captured by the pro-cyclical surplus consumption over habit.

Our model parallels habit in terms of functional form, but differs in the choice of state variable. Similar to habit models, we also focus on the bliss point but instead relate the state to the investor’s total wealth. As Munro and Sugden (2003) argue, habit models capture reference-dependent preferences (Kahneman and Tversky, 1979). Past decisions determine the value of a reference point, and deviations about this point determine utility. The reference can be a function of past consumption (as is the case under habit), or endowment (as is the case in this paper). Munro and Sugden (2003, p. 420) contend that the second case is more appropriate in a dynamic framework, and in particular, when buying assets. Habit related to lagged consumption on the other hand is relevant when the budget constraint is a function of contemporary variables only.

Because consumption risk is minimal, we resort to total wealth risk. This is achieved by introducing wealth as the preference state variable. If this second source of risk is large, and if this risk is positively valued by the market, then its presence should help resolving the high equity premium puzzle. Second, by keeping the habit perspective, we hope to preserve the model’s favorable results with respect to the predicted risk-free rate. Third, compared to consumption, cyclical movements in wealth hold more promise in generating corresponding movements in risk aversion and, consequently, in predicted premia. To see this, consider (de-trended) consumption, and financial wealth and (untransformed) consumption shares of wealth plotted in Figure 2, again with the XRI index.² The data suggests that (i) wealth is strongly pro-cyclical whereas (ii) consumption has comparatively negligible cyclical component, such that (iii) the consumption share of wealth is strongly counter-cyclical. A high likelihood of a recession is associated with marked declines in wealth, and almost no reduction in consumption, pointing out to considerable smoothing of consumption by
agents. Unsurprisingly, the consumption share increases during downturns and is a good predictor of the counter-cyclical patterns in excess returns (Lettau and Ludvigson, 2001a,b; Li, 2005).

[ Insert Figure 2 here ]

B Wealth-dependent utility

According to standard theory, wealth should have no intrinsic value to investors. Wealth is a mere instrument with which current and future non-durable consumer goods are acquired. Utility is defined over these goods only, not over the means through which they are obtained.³ In this light, wealth can only have an indirect effect on portfolio decisions. Given current endowment, consumption paths are chosen optimally; portfolios are a mean to support this optimal path. All the main determinants of asset holdings, including risk aversion or time substitutability, depend on wealth only through its indirect impact on consumption.

Yet, conventional wisdom suggests that net worth carries additional virtues (see Smith, 1759; Veblen, 1899, for early discussions). Wealth – particularly tangible wealth – encompasses undeniable conspicuous consumption characteristics. People do enjoy nice cars, houses, furniture and jewelry in part for the services they yield, but also for the social status that is attached to their ownership. Formally, a direct preference for wealth can be rationalized as a reduced-form representation of a deeper concern for other elements. In particular, if (i) non-marketed goods are valuable, if (ii) status is a ranking mechanism that determines the allocation of these goods, and if (iii) wealth is a metric that determines status, then wealth-dependent utility can be thought of as resulting from these un-modelled effects. Examples of such goods include country club memberships, invitations to charity events, or even the quality of mates in couples.
For instance, Cole et al. (1992, 1995) consider the case of a matching game where agents have preferences over the quality of their mate. This quality is important to the extent that it determines the amount of joint consumption of the couple, and bequests to descendants whose utility is also valued. However, the quality of a match is a non-marketed good. Status is then a signalling device that projects the agent’s quality and attracts high-quality potential mates. To the extent that agents use relative wealth as an observable description of status, a matching equilibrium exists where agents accumulate more wealth than otherwise. Cole et al. (1992) show that this outcome is tantamount to a setup with direct preference for wealth but without matching, even though preferences in the matching game are only over marketed consumption goods. A similar concern for wealth can endogenously be generated in a matching model studied by Corneo and Jeanne (1999) where matches allow agents to consume non-marketed goods.

Following this reduced-form approach, Bakshi and Chen (1996) incorporate direct preference for total wealth to the standard consumption utility. They rationalize wealth dependency through a preference-for-status argument put forward by Robson (1992). Agents care about their social position, relative to their reference group. As a result, the pricing kernel incorporates both consumption and total wealth risk. They show that this increment in MU risk can successfully address the main pricing anomalies. This is confirmed by Gong and Zou (2002) who extend the analysis to a stochastic growth setting, and by Smith (2001) who extends it to a non-expected utility setting. They find that preference for wealth results in a higher premia, more investment in risky assets, a lower consumption-wealth ratio and consequently a larger growth rate. This last result is confirmed in a deterministic growth setup with homogenous agents by Futagami and Shibata (1998) and by Corneo and Jeanne (2001). Kuznitz and Kandel (2003) consider a portfolio selection problem with a time-varying investment set and additively separable preferences for wealth. They show that
the share of wealth allocated to risky assets is much lower, reflecting the additional risk from the covariance of wealth and returns. Similarly, Falato (2003) introduces direct preference for wealth in the utility function. Imposing a pro-cyclical effect of wealth (‘happiness maintenance’), he derives the pricing implications, and shows that mild pro-cyclicality generates a larger volatility in the price-dividends ratio and consequently, a higher premia.

Our approach is qualitatively similar. In our discussion of results in Section V, we also derive pricing implications and closed-form solutions. However, our specification for within-period utility bears closer resemblance to the habit literature. Our focus is on the role of wealth in determining the bliss factor, rather than using a multiplicative specification. As already mentioned, an advantage of our approach is on the intuitive interpretation of the price of total wealth risk as the Arrow-Pratt level of relative risk aversion. Moreover, contrary to Bakshi and Chen (1996) Gong and Zou (2002), or Kuznitz and Kandel (2003), our structure explicitly generates time-varying consumption risk aversion. Movements in bliss cause rotations, rather than shifts, in the MU schedule and corresponding movements in risk aversion. Finally, contrary to Falato (2003), we do not impose any *ex-ante* pro- or counter-cyclicality on attitudes toward risk, but let this issue be determined by the data.

Other WDU models restrict direct preferences to specific elements of total wealth. First, Barberis et al. (2001) consider a model where utility is subject to (i) loss aversion, and (ii) wealth dependence. Their framework uses the prospect theory of Kahneman and Tversky (1979) to allow for an asymmetric utility effect from losses and gains, whether current ($X_{t+1}$), or past ($z_t$) in addition of consumption utility. Moreover, the level of *risky financial* wealth $W_{fin,t}$ affects consumption utility. They find an additional financial risk in the pricing equation which reduces the emphasis placed upon consumption risk, and allows them to solve the main pricing anomalies.
Secondly, other models restrict direct preferences over only certain elements of tangible wealth (i.e., real estate and durable goods). Grossman and Larocque (1990); Detemple and Giannikos (1996); Chetty and Szeidl (2004) focus on preferences over illiquid and pre-committed durables, such as housing. Grossman and Larocque (1990) omit preferences over non-durables altogether, whereas Detemple and Giannikos (1996) allows for both a service \( g_{\text{serv}}(W_{\text{tan},t}) \), and a status \( g_{\text{status}}(W_{\text{tan},t}) \) utility flow from holding tangible wealth. Aït-Sahalia et al. (2004); Yogo (2005) focus instead on direct preferences over durable goods, such as luxury items. The main findings of these models can be summarized as follows. The quantity of risk of financial or tangible assets is much more important than that of consumption risk. Moreover its price is positive, thereby alleviating the C-CAPM problems in explaining the equity premium. Second, these models generate much more predictability, either through their time-varying price of wealth risk, or through time-varying betas, thereby addressing the predictability puzzle better.

Contrary to these models, our theoretical model does not restrict preference for status to specific components of total wealth.\(^4\) Instead, preference for total wealth is preference for a composite good which is perfectly substitutable in individual elements such as tangible or financial wealth. Moreover, our framework does not require a differential treatment of losses and gains, i.e. we abstract completely from loss aversion as in Barberis et al. (2001). Our approach is therefore considerably simpler to implement; we require identification of a single additional parameter. Importantly, this parameter is estimated, rather than calibrated. Its reasonableness can be established by (i) formally testing the theoretical restrictions and (ii) deriving the asset pricing implications of the model evaluated at its estimated parameters to compare it with known facts characterizing returns.
II Model

This section outlines the model. We subsequently characterize optimal consumption and asset holdings. Finally, we obtain the closed-form expressions for the differential equations governing consumption, asset values and wealth.

A Economic environment and preferences

In order to emphasize the role of our alternative preference specification, we consider a complete-markets and representative-agent framework similar that studied by Merton (1971) or by Lucas (1978). The stochastic environment is characterized by continuous information with filtration on $Z_t \in \mathbb{R}^n$, a standard Brownian motion. The investment set consists of $n$ risky securities and one risk-less asset. Denote by $\boldsymbol{\mu}_{p,t} \in \mathbb{R}^n$ and by $\boldsymbol{\sigma}_{p,t} \in \mathbb{R}^{n \times n}$ the adapted drift and diffusion processes for the risky returns, and by $r_t \in \mathbb{R}$ the short rate process. We start by imposing a constant set restriction, i.e. $\boldsymbol{\mu}_{p,t} = \boldsymbol{\mu}_p, \boldsymbol{\sigma}_{i,t} = \boldsymbol{\sigma}_i$, and, $r_t = r, \forall i, t$. This assumption is relaxed later when we discuss pricing implications.

The representative agent’s objective is to select consumption $C_t$ and portfolio weights $v_t \in \mathbb{R}^n$ so as to maximize VNM utility characterized by direct preference over wealth $W_t$:

$$\max_{\{C_t, v_t\}, t} \mathbb{E}_0 \int_0^\infty \exp(-\rho t) U(C_t, W_t) dt,$$  

subject to

$$dW_t = \{[v'_t(\boldsymbol{\mu}_p - r) + r]W_t - C_t\} dt + W_t v'_t \boldsymbol{\sigma}_p dZ_t,$$
where \( E_0 \) is a conditional expectations operator, and \( \rho > 0 \) is a subjective discount rate. The agent’s within-period utility is given by:

\[
U(C, W) = \begin{cases} 
\frac{(\eta_c C + \eta_0 + \eta_w W)^{1-\gamma}}{1-\gamma}, & \text{if } \gamma \neq 1; \\
\log(\eta_c C + \eta_0 + \eta_w W), & \text{if } \gamma = 1.
\end{cases}
\] (3)

Utility (3) belongs to the HARA class advocated by Rubinstein (1974), modified to allow for wealth dependence. Following Merton (1990, p. 137), the necessary HARA restrictions are:

\[
\eta_c > 0, \quad \eta_c C + \eta_0 + \eta_w W > 0, \quad \gamma \geq 0.
\] (4)

These are required to guarantee monotonicity and concavity. We add a further theoretical restriction that bounds below and above the term \( \eta_w \):

\[
-1 < \eta_w / \eta_c < \rho.
\] (5)

This condition allows for negative or positive values of the loading of wealth in the utility function, \( \eta_w \), but limits the size of the effect.\(^5\)

Subject to restrictions (4) and (5), the utility function (3) has interesting properties. The expression \( C_{\text{bliss},t} \equiv -(\eta_0 + \eta_w W_t) / \eta_c \) has the interpretation of a fictitious reference, or bliss level of consumption, where bliss is defined with respect to marginal, as opposed to the level of, utility.\(^6\)

More precisely, as consumption falls toward bliss, marginal utility goes to infinity, such that \( C_{\text{bliss},t} \) is the minimum admissible consumption level. In addition, under WDU, the bliss level changes because of changes in wealth. In contrast, slow-moving habit or durability models let the reference
level be a function of past consumption, whether individual, or aggregate. Finally, CRRA utility and HARA utility fix the bliss consumption to 0 and $-\eta_0/\eta_c$ respectively, both state-independent levels.

The marginal utility, and the Arrow-Pratt coefficient of absolute risk aversion (calculated with respect to consumption and wealth) are respectively:

$$U_x = \frac{\eta_x}{(\eta_c C + \eta_0 + \eta_w W)^\gamma},$$  \hspace{1cm} (6)

$$R^{ax} = \frac{-U_{xx}}{U_x} = \frac{\gamma \eta_x}{(\eta_c C + \eta_0 + \eta_w W)^\gamma}, \hspace{1cm} x = c, w. \hspace{1cm} (7)$$

To understand the direct impact of wealth on these variables, consider the effect on marginal utility of consumption following an increase in $W$. As shown in Figure 3, when $\eta_w < 0$, as wealth increases, so does the minimum level of consumption from $-\eta_w W_0$, to $-\eta_w W_1$. Hence, a negative wealth dependence involves a ratchet effect whereby the bliss consumption level increases in wealth. This leads to a clockwise rotation in the marginal utility schedule from $U_{c,0}$ to $U_{c,1}$, and increases marginal utility from $a$ to $b$. Put differently, as wealth increases, the agent approaches his reference consumption, and becomes more averse toward consumption risk.

[ Insert Figure 3 here ]

Next, these movements in marginal utility and risk aversion are reversed for $\eta_w > 0$, as shown in Figure 4. An increase in wealth now reduces minimum admissible consumption from $-\eta_w W_0$, to $-\eta_w W_1$. This causes a counter-clockwise rotation in the marginal utility schedule from $U_{c,0}$ to $U_{c,1}$, and a reduction in the marginal utility of consuming the same level of nondurable good falls from $a$ to $b$. This effect could be related to blasé behavior; as the investor becomes richer, for a given level of consumption, both marginal utility and consumption risk aversion fall.
The utility function (3) can also be thought of as a linear habit model where the habit stock is defined to be a function of current wealth:

$$W_t = \frac{1}{\pi_t} E_t \int_t^\infty \pi_s C_s ds,$$

where $\pi_t$ is a state-price density. In this perspective, instantaneous utility is not only a function of current consumption, but also of the future consumption paths that current wealth could support (see Kuznitz and Kandel, 2003, for a discussion). These paths determine the benchmark through which current consumption scenarios are evaluated. As is shown next, this habit stock interpretation of (3) considerably simplifies the solution to the agent’s problem.

B Optimal Consumption and Portfolio Rules

The agent’s problem (1) could be solved using standard dynamic programming methods. It turns out however that a simpler alternative is available. We mentioned earlier that the preferences (3) belong to the linear habit class where the habit stock is defined to be current wealth. Schroder and Skiadas (2002) show that closed-form expressions for linear habit models (the primal problem) are conveniently obtained by simple modifications to the standard solutions in models without habit (the dual problem). Their analysis is cast in terms of consumption-based habit, but it can be readily extended to our wealth-based habit setup. First, by appropriately redefining the state-price density, expressions for the dual short rate and dual risk premia can be obtained. Second, these expressions are then substituted back into the known solutions to the dual problem. Third, the
solutions to the primal problem are obtained by adding in the wealth-in-the-utility term to the second-step solutions.

In what follows let \( X_t \) refer to a variable in the primal problem and let \( \hat{X}_t \) refer to its dual problem counterpart. We start by defining the dual variables as follows:

\[
\hat{C}_t \equiv C_t + \frac{\eta_w}{\eta_c} W_t, \quad (9)
\]

\[
\hat{U}(\hat{C}_t) \equiv \frac{(\hat{C}_t + \frac{\eta_0}{\eta_c})^{1-\gamma}}{1-\gamma} = U(C_t, W_t), \quad (10)
\]

where \( U(C_t, W_t) \) is as in (3), since expected utility is defined only up to an affine transformation.

Next, replace for \( C_t \) in budget constraint (2) by using (9) to obtain:

\[
W_t = \left\{ \left[ v'_t (\mu_p - r) + r \right] W_t - \hat{C}_t + \eta_w/\eta_c W_t \right\} dt + W_t v'_t \sigma_p dZ_t,
\]

\[
= \left\{ \left[ v'_t (\mu_p - r) + (r + \eta_w/\eta_c) \right] W_t - \hat{C}_t \right\} dt + W_t v'_t \sigma_p dZ_t,
\]

\[
= \left\{ \left[ v'_t (\mu_p - r) + \hat{r} \right] W_t - \hat{C}_t \right\} dt + W_t v'_t \sigma_p dZ_t, \quad (11)
\]

where \( \hat{r} \equiv r + \eta_w/\eta_c \). Observe that wealth, portfolio, and the risk premia \( (\mu_p - r) \) remain unchanged.

Second, let \( \pi_t \) be the (primal) state-price density. The previous analysis suggests that its dual analog must satisfy:

\[
\hat{\pi}_t \equiv e^{-(\eta_w/\eta_c)t} \pi_t, \quad (12)
\]
from which,

\[ d\hat{\pi}_t/\hat{\pi}_t = -(\eta_w/\eta_c)dt + d\pi_t/\pi_t. \]  

(13)

To see that (12) is the appropriate dual state price density, note that a standard no-arbitrage argument establishes that the risk-free rate and risk premia process for the state-price density \( \hat{\pi}_t \) in the dual market must satisfy:

\[ \hat{r} = -\mu_{\hat{\pi}}/\hat{\pi}_t \]  

(14)

\[ = (\eta_w/\eta_c) + r \]  

(15)

\[ \mu_p - \hat{r} = -(1/\hat{\pi}_t) \sigma_p \sigma'_{\hat{\pi}} \]  

(16)

\[ = \mu'_p - r, \]  

(17)

as was required under (11).

Hence, the (exogenous) short rate process used by the agent in the dual problem is simply the short rate process in the primal problem plus the wealth dependency parameter; the risk premium process used by the agent is the same in both the dual and the primal problem. Under the isomorphism result of Schroder and Skiadas (2002), we can:

1. use the known solutions of Merton for the dual problem \((\hat{C}_t, \hat{v}_t)\) as functions of \(W_t, \hat{r}, \mu'_p - r,\)

2. correct the short rate in these solutions using (15),

3. get back the expression for \(C_t\) by inverting (9); the expression for \(v_t\) is the same as that for \(\hat{v}_t.\)
Following this iso-morphism approach reveals that the indirect utility $J(W_t)$, the optimal consumption $C_t$ and the value of risky assets $V_t ≡ v_tW_t$ are respectively given by:

$$J(W_t) = \frac{(G + FW_t)^{1-\gamma}}{1 - \gamma},$$ \hfill (18)

$$C_t = \frac{\eta_0}{\eta_c} \left\{ \left( \frac{\gamma - 1}{\gamma} \right) \left( \frac{\rho}{\gamma - 1} + \frac{0.5Q}{\gamma} \right) - \frac{1}{\gamma} \right\} W_t,$$

$$V_t = \left( \frac{\eta_0}{\eta_c} \right) \frac{\Sigma_{pp}^{-1}(\mu_p - r)}{r + \eta_w/\eta_c} + \frac{\Sigma_{pp}^{-1}(\mu_p - r)}{\gamma} W_t,$$ \hfill (20)

where,

$$F ≡ \eta_c \left\{ \left( \frac{\gamma - 1}{\gamma} \right) \left( r + \eta_w/\eta_c + \frac{\rho}{\gamma - 1} + 0.5Q/\gamma \right) \right\}^{\gamma/(\gamma - 1)},$$

$$G ≡ \frac{\eta_0}{\eta_c} \left\{ \left( \frac{\gamma}{\gamma - 1} \right) \frac{1}{\eta_c} \left( \frac{F}{\eta_c} \right)^{-1} - \frac{\rho}{F(\gamma - 1)} - \frac{0.5Q}{\gamma F} \right\}^{-1},$$

$$Q ≡ (\mu_p - r)^t \Sigma_{pp}^{-1}(\mu_p - r) \geq 0,$$

$$\Sigma_{ij} ≡ E[\sigma_iZ_t, dZ'_t, \sigma'_j]$$

It can be shown that these solutions correspond exactly to those obtained using the more traditional dynamic programming approach.

Equation (18) highlights interesting characteristics of the value function. First, we find that $J(W_t)$ is iso-morphic to the instantaneous utility function $U(C_t, W_t)$ in (3). The particular form of wealth dependence that we are considering supposes that the Bernoulli transform is applied to an affine function of wealth. This functional has the property that the value function is also in the
HARA class. Note in particular that $\eta_0 = 0$ implies $G = 0$, such that the value function becomes iso-elastic despite the wealth dependence.

Second, following our previous discussion, we can analyze risk aversion using the marginal utility of wealth schedule, $J_{w,t}$, and the distance of an arbitrary wealth level $W_t$ from minimum admissible bliss level. In particular, straightforward manipulations reveal that:

\begin{align}
W_{\text{bliss}} &\equiv -\frac{G}{F} = \frac{-\eta_0}{\eta_c r + \eta_w}, \\
&= \frac{-\eta_0}{\eta_c r + \eta_w}, \quad (21) \\
-\frac{W_t J_{w,t}}{J_{w,t}} &= \frac{\gamma W_t}{W_t - W_{\text{bliss}}}. \quad (22)
\end{align}

The constant bliss level of wealth (21) can take on negative or positive values depending on the parameters $\eta_i, i = 0, c, w$ and on the interest rate $r$. In particular, since $\eta_c, r > 0$, $\eta_w < 0$ pushes the bliss level away from zero, $\eta_w > 0$ pushes it toward zero. For $W_{\text{bliss}} < 0$, a positive $\eta_w$ (blasé) moves the bliss asymptote to the right (see Figure 5). Given any wealth level, the agent is closer to bliss, and therefore characterized by a higher degree of absolute risk aversion. A negative $\eta_w$ (ratchet) decreases absolute risk aversion for the opposite reason. When $W_{\text{bliss}} > 0$, movements in bliss are reversed, and we find that a blasé investor has lower absolute risk aversion than a ratchet investor. With respect to relative risk aversion (22), a (negative) positive bliss implies that risk aversion is (pro-) counter-cyclical.

[ Insert Figure 5 here ]

Next, the optimal rules (19) and (20) are affine in wealth. As for the standard HARA utility, imposing $\eta_0 = 0$ results in the iso-elastic case of both rules being proportional to net worth. Otherwise, wealth dependence affects both the intercept $(C_t, V_t)$ and the slope $(C_t)$ of the closed-form solutions. To isolate these effects, it is useful to resort to our previous analysis of the value
function. Note that we can indeed rewrite the optimal rules as:

\[
C_t = -\left\{ \left( \frac{\gamma - 1}{\gamma} \right) W_{\text{bliss}} \left[ \frac{\rho}{\gamma - 1} + 0.5Q/\gamma \right] + \frac{\eta_0}{\gamma \eta_c} \right\} \\
+ \left\{ \left( \frac{\gamma - 1}{\gamma} \right) \left[ r + \frac{\rho}{\gamma - 1} + 0.5Q/\gamma \right] - \frac{\eta_w}{\gamma \eta_c} \right\} W_t,
\]

(23)

\[
(\mu_p - r) V_t = \frac{Q}{\gamma} \{ -W_{\text{bliss}} + W_t \},
\]

(24)

where \(W_{\text{bliss}}\) is given by (21). For the rest of this section’s analysis, assume that the investor is at least moderately risk averse, i.e \(\gamma > 1\).

First, turning to consumption, we obtain the intuitive result that for positive \(W_{\text{bliss}}\), minimum consumption, i.e the intercept in (23), is negative (or less positive), and positive (or less negative) otherwise. \textit{Ceteris paribus}, \(W_{\text{bliss}} > 0\) implies a steeper marginal utility of wealth at the optimum, and consequently, greater \(J_w\) risk. The risk-averse investor reacts to this by increasing wealth away from bliss. This is achieved by decreasing consumption and increasing savings. Secondly, regardless of \(W_{\text{bliss}}\), a blasé investor always has a lower marginal propensity to consume out of wealth than a ratchet investor. This result is again intuitively appealing. Since a blasé investor positively values status, he always saves more at the margin.

Third, (24) expresses the expected excess return (in $ terms) on the optimal total wealth portfolio. As usual, higher curvature \(\gamma\) results in more conservative positions. Again, bliss levels of wealth influence the intercept terms. A positive \(W_{\text{bliss}}\) implies more MU risk at any wealth levels. The risk-averse agent hedges away these risks by selecting more conservative portfolios. Negative bliss values however reduce risk aversion and increase the asset values held in risky assets.

Fourth, for reasons discussed earlier, when bliss is negative, \(\eta_w < 0\) shifts bliss to the left and decreases risk aversion; the ratchet investor therefore selects a more risky portfolio, the blasé a
more conservative one. These positions are reversed for \( W_{bliss} > 0 \); the blasé investor’s portfolio is more risky compared to the ratchet’s.

Clearly linearity for the optimal rules (19), and (20) implies that the change in consumption and portfolio are \( dC_t = c_w dW_t \), and \( dV_t = v_w dW_t \), where \( c_w, v_w \) are constants defined by (19) and (20). We can also substitute the solutions in the budget constraint (2) to obtain the closed-form expression for instantaneous changes in wealth. Consequently the instantaneous changes in consumption, the value invested in assets, and wealth are:

\[
\begin{align*}
    dC_t &= \left\{ \left( \frac{\gamma - 1}{\gamma} \right) \left[ (r + \rho/\gamma) + 0.5Q/\gamma \right] - \frac{1}{\gamma} \frac{\eta_w}{\eta_c} \right\} dW_t, \\
    dV_t &= \frac{\Sigma_{pp}^{-1}(\mu_p - r)}{\gamma} dW_t, \\
    dW_t &= \left[ \left( \frac{\eta_0/\eta_c + (r + \eta_w/\eta_c) W_t}{\gamma (r + \eta_w/\eta_c)} \right) \left\{ \left( \frac{1}{\gamma} \right) 0.5Q + r + \eta_w/\eta_c - \rho \right\} dt \\
    &+ (\mu_p - r) \Sigma_{pp}^{-1} \sigma_p dZ_t \right\}.
\end{align*}
\]

III Estimation

A Econometric Model

Estimation focuses on the multivariate Brownian motion given by (25)–(27), which can be written as:

\[
\begin{align*}
    dC_t &= c_w dW_t, \\
    dV_t &= v_w dW_t, \\
    dW_t &= [\mu_0 + \mu_w W_t] dt + [\sigma_0 + \sigma_w W_t] dZ_t,
\end{align*}
\]
where $c_w, v_w, \mu_0, \mu_w, \sigma_0, \sigma_w$ are constant loadings that depend only on the deep parameters. In principle, estimation the model could be undertaken over returns or over quantities data. We select the second approach for a number of reasons.

First, estimating optimal allocations imposes considerably more theoretical restrictions that are related to the deep parameters on the joint first and second moments. With respect to deep parameters, standard analyses of returns treat the equilibrium quantities in the pricing kernels as exogenous; the theoretical restrictions are imposed on the prices of risk exclusively, with conditional second moments left unrestricted. In comparison, the allocations analysis produces theoretical restrictions on both first and second moments of changes in consumption, asset holdings and wealth (through the restrictions on $c_w, v_w, \mu_0, \mu_w, \sigma_0, \sigma_w$), while returns are treated as exogenous. In the absence of prior information on $\eta_w$ in particular, these additional restrictions will be useful in identifying the preference parameters of interest. Second, empirical studies of aggregate optimal consumption and asset holdings are much less frequent than asset pricing studies. We believe that focusing on quantities rather than on returns thus provides another perspective that complements existing results (Lo and Wang, 2001, also argue in favor of using the informational content of quantities more thoroughly).

**Transformation** One major problem in estimating (28)–(30) is that there exists no closed-form transition density for multi-variate Brownian motions with affine drifts and diffusions. Indeed, analytical expressions for the likelihood function exist only for a limited class of Itô processes (Melino, 1996). Unfortunately, our multi-variate process does not belong to this class. Alternative solutions include discrete (Euler) approximations, and simulating the continuous-time paths between the
discretely-sampled data, either through classical (Durham and Gallant, 2002) or through Bayesian (Eraker, 2001) approaches.

Our solution to this problem is different and considerably simpler to implement. It is based on a homoscedasticity-inducing transformation for general Brownian motions. It will be shown that this approach also stationarizes the drift term. Consequently, a standard discretized approximation is appropriate, efficient, and unbiased. In particular, a straightforward application of Itô’s lemma reveals the following.

Lemma 1 Let $X_t \in \{C_t, V_t\}$ be defined as follows:

\[ X_t = x_0 + x_w W_t, \]  \hspace{1cm} (31)

\[ dW_t = (\mu_0 + \mu_w W_t)dt + (\sigma_0 + \sigma_w W_t)dZ_t, \]  \hspace{1cm} (32)

where $x, \mu, \sigma$ are constants defined in (19) and (20), and in (27), and consider the following transformation:

\[ \tilde{X}_t = \log \left[ x_w \sigma_0 + \sigma_w (X_t - x_0) \right] / \sigma_w, \]  \hspace{1cm} (33)

Then, $\tilde{X}_t$ has constant drift and diffusion given by:

\[ d\tilde{X}_t = \left[ \frac{\mu_w}{\sigma_w} - 0.5 \sigma_w \right] dt + dZ_t. \]  \hspace{1cm} (34)

Proof. First, (31) and (32) reveal that:

\[ dX_t = \left[ x_w \mu_0 + \mu_w (X_t - x_0) \right] dt + \left[ x_w \sigma_0 + \sigma_w (X_t - x_0) \right] dZ_t \]  \hspace{1cm} (35)

\[ = \mu(X_t) dt + \sigma(X_t) dZ_t. \]  \hspace{1cm} (36)
Next, by Itô’s lemma, we have for $\tilde{X}_t = \tilde{X}(X_t)$:

\[ d\tilde{X}_{j,t} = \left[ \mu(X_t)\tilde{X}'(X_t) + 0.5\sigma(X_t)^2 \tilde{X}''(X_t) \right] dt + \sigma(X_t)\tilde{X}'(X_t)dZ_t \]  

(37)

Observe that $\mu_0/\mu_w = \sigma_0/\sigma_w$ to substitute in (37) to obtain (34).

The transformation (33) requires that its first derivative with respect to the Itô process $X_t$ is the inverse of the diffusion. It is usually introduced in order to stationarize the diffusion (Shoji and Ozaki, 1998; Aït-Sahalia, 2002; Durham and Gallant, 2002). In our case, both drift and diffusion are affine and have intercept and slope coefficients that are closely inter-related. Consequently the theoretical restrictions implied by the model are such that the transformation *also* stationarizes the drift term. This is fortunate since the resulting transformed model is easily estimated by maximum likelihood. In particular, the discretization of (34):

\[ \Delta \tilde{X}_t = \left[ \frac{\mu_w}{\sigma_w} - 0.5\sigma_w \right] + \epsilon_t \]  

(38)

where $\epsilon_t$ is a standard Gaussian term, can be consistently and efficiently estimated by MLE (e.g. Gourieroux and Jasiak, 2001, pp. 287–288).

**Likelihood function** The optimal rules in (19) and (20) take the moments of the returns’ distribution $\mu_p, \Sigma_{pp}$, as well as the risk-free rate $r$ as given. These moments could be estimated in an external round, using a two-step method, and substituted back into the optimal rules to obtain the predicted rules. Instead, we perform a single-step procedure and incorporate the mean and covariance matrix of the risky returns into the calculation of the likelihood function. This approach has the advantage of factoring in the parametric uncertainty concerning $\mu_p, \Sigma_{pp}$ into the calculation of
the standard errors of the deep parameters. Specifically, denote by $\tilde{X}_t \equiv [\tilde{C}_t, \tilde{V}_t, \tilde{W}_t]'$ the $n + 2$
vector of transformed variables, the model to be estimated is the following:

$$
\begin{pmatrix}
\Delta \tilde{X}_t \\
\Delta P_t / P_{t-1}
\end{pmatrix}
= 
\begin{pmatrix} 
\mu_x \\
\mu_p
\end{pmatrix}
+ 
\begin{pmatrix}
\epsilon_x \\
\epsilon_p
\end{pmatrix},
\sim \text{N.I.D.}
\begin{pmatrix}
0_x \\
0_p
\end{pmatrix},
\begin{pmatrix}
I_x & 0 \\
0 & \Sigma_{pp}
\end{pmatrix},
\tag{39}
$$

where $\mu_x$ is given by (38), and $I_x$ is an $n + 2$ identity matrix.

First, in accordance with the maintained assumption of the model, all the innovations are
Gaussian. Second, as mentioned earlier, the transformation in Lemma 1 implies that the quantities
innovations are standardized white noise. Third, consistent with the model, the covariance matrix
is block diagonal, i.e. we impose the absence of cross-correlations between innovations in quantities
and returns. Any potential covariance between the two is fully taken into account in the closed-form
solutions; allowing for additional correlations is not theoretically justified.

With these elements in mind, the contributions to the likelihood function (with constant term
omitted) are given by:

$$
f_t = -0.5 \log[\det(\Sigma)] + \log[\det(K_t)] - 0.5 \epsilon_t' \Sigma^{-1} \epsilon_t
\tag{40}
$$

where $\Sigma$ is defined implicitly in (39), while $K_t \equiv \text{Diag}([K_{c,t}, K_{v,t}, K_{w,t}, 1, \ldots])$ and $K_{x,t} = 1/[x_w \sigma_0 + \sigma_w (X_t - x_0)]$ is a Jacobian correction term associated with the transformation (38).

The parameter vector is then $\theta \equiv \{\gamma, \rho, \eta_0, \eta_c, \eta_w, \mu_p, Q_{pp}\}$, where $Q_{pp} \equiv \text{Chol}(\Sigma_{pp})$ is the $n$-dimensional triangular Cholesky root of the returns covariance matrix.

**Hypothesis tests** It will be recalled that theoretical restrictions for HARA and WDU utility
are necessary to guarantee that marginal utility is non-negative. In particular, for both models,
restriction (4) is required for monotone preferences, whereas for WDU utility, (5) verifies that the agent has a positive effective discount rate.

We also consider two benchmarks in assessing the performance of the WDU model. As mentioned earlier, CRRA utility is obtained by imposing that $\eta_0, \eta_w = 0$, whereas HARA utility imposes $\eta_w = 0$. To the extent that it has been studied extensively in asset pricing models, CRRA utility constitutes a natural benchmark. HARA utility, although less popular, has the advantage of optimal rules which are not proportional to wealth (see the previous discussion). Both the theoretical restrictions and the model selection tests will be performed and discussed below.

B Data

Our data set consists of post-war U.S. quarterly observations on aggregate consumption, asset holdings and corresponding returns indices. The time period covered ranges from 1952:II to 2000:IV, for a total of 195 observations. All quantities are expressed in real, per-capita terms, where the aggregate price index is taken to be the implicit GDP deflator. Similarly, all returns are converted in real terms by subtracting the inflation index.

Consumption  The consumption series is the aggregate expenditure on Non-Durables and Services. The source of the data is the Bureau of Economic Analysis NIPA series. This series has been used in most asset pricing studies.

Assets  The aggregate portfolio holdings are defined as follows:

$$V_t = [V_{0,t}, V_{1,t}, V_{2,t}]$$

$$= [\text{Deposits}, \text{Bonds}, \text{Stocks}].$$
Each asset holdings are obtained from the Flow of Funds Accounts made available by the Board of Governors of the Federal Reserve (Table L.100). They correspond to the level values of asset holdings by households and non-profit organizations (see also Lettau and Ludvigson, 2003). More precisely the individual assets (mnemonic) and financial wealth are:

- Deposits (FL15400005): Includes foreign, checkable, time, savings deposits and money market fund shares.
- Stocks (FL153064105): Corporate equities directly held by households.

- Wealth: Deposits + Bonds + Stocks ($W_t = V_{0,t} + V_{1,t} + V_{2,t}$).

Deposits will thus be taken to represent the risk-free asset, whereas both long-term government bonds and corporate equity are proxies for the risky assets.

The choice of specific portfolio holdings was dictated by a number of practical elements. First, these assets correspond to some of the largest asset holdings for U.S. households, and their returns have been studied extensively in the asset pricing literature, thus providing useful benchmarks for our analysis. In particular, we are interested in verifying whether the pricing anomalies associated with cash and stock returns have dual analogs in optimal allocations. Second and related, these assets have corresponding returns series. Those returns are required to evaluate the distributional parameters $\mu_p, \Sigma_{pp}$ that are used to compute the theoretical asset holdings. Other portfolio holdings such as pension and life insurance reserves are also important in relative size. However, no clear returns indices are available for these assets.\footnote{9}

Our definition of wealth has been used in theoretical models of portfolio choice (e.g. Campbell et al., 2003). Its main advantage is that wealth is thus observable and the definition provides
more structure on the econometric model since one of the theoretical asset holding is defined residually.\textsuperscript{10} However, the definition is narrow in the sense that it abstracts from tangible (real) and human wealth. Unfortunately, real returns indices on durable goods are difficult to evaluate, and these assets were omitted from our selected holdings series $V_t$. Moreover, human wealth is not observable, whether in levels or in rates of returns and thus also eliminated.

Table 2 reports the sample moments for the consumption and asset holdings in percentage of wealth (those series were plotted in Figures 1–2). A first observation is that the shares of wealth allocated to consumption, deposits and stocks are roughly of the same order of magnitude, and similarly volatile. Bonds on the other hand represent a much lower share of wealth and are smoother.

[ Insert Table 2 here ]

**Returns** We follow Campbell et al. (2003) in constructing the returns series that correspond to our assets definition. The return on cash is taken to be the real return on 3-months Treasury Bills. The return on bonds is proxied by the real return on 5-years T-Bills. Finally, stock returns are evaluated as the value-weighted returns on the NYSE, NASDAQ and AMEX markets. Bond and stock returns were obtained from the CRSP data file. Again, the inflation series is computed from the GDP deflator.

Table 3 presents sample moments of the real returns. These series have been widely discussed in the asset pricing literature, so we only briefly outline their main features. First, we observe that both bonds and stocks warrant a positive premia. Equity returns however are clearly larger, and much more volatile. Next, we find that both cash and bonds as well as bonds and stocks are positively, and similarly correlated. Cash and stocks on the other hand display no covariance.
IV Results

A Estimation details

Identifiability  The theoretical model in (25)–(27) presents some important challenges for identification. Indeed, the parameters are often expressed as ratios of one another which usually results in poor identifiability. These problems affect both the HARA and WDU models, but not the CRRA model. As is well known, utility is defined only up to an affine transformation such that the parameter $\eta_c$ plays no role in the optimal rules when $\eta_0, \eta_w = 0$. In preliminary estimation rounds, we experimented with numerous identification strategies which we briefly discuss.

A first approach was to fix the subjective discount rate $\rho$ to a realistic value, and to let $\eta_c$ be flexible. We found that both HARA and WDU models were then poorly identifiable; results were highly dependent on starting values, and convergence problems were noticed. Second, we let $\rho$ be flexible, and fixed $\eta_c$. Whereas HARA utility was well identified and yielded realistic $\rho$ estimates, the WDU model was not. In particular, we found that we could not identify $\rho$ and $\eta_w$ separately. Nonetheless, the effective discount rate $\rho - \eta_w/\eta_c$ was uniquely identifiable, and realistic. Finally, we fixed both $\rho, \eta_c$, and found this approach to be the most satisfactory. Both models were then clearly identified, with robustness to starting values and rapid convergence. We found that the curvature parameter $\gamma$ was completely independent from the choice of calibration for $\eta_c$. Moreover, changing $\eta_c$ resulted in changing the estimated $\eta_0, \eta_w$ in the same proportions, such that the T-statistics were always unaffected by the calibrated value of $\eta_c$. This again indicates that although the ratios $\eta_0/\eta_c, \eta_w/\eta_c$ are well identified, the separate parameters are not. We therefore
fix $\rho = (1 + 0.035)^{1/4} - 1$, a realistic value, and follow the asset pricing literature in arbitrarily imposing $\eta_c = 1$ to estimate $\gamma, \eta_0, \eta_w$. The vector of free parameters is then $\{\gamma, \eta_0, \eta_w, \mu_p, Q_{pp}\}$.

B Parameter estimates

Table 4 presents the estimated parameters for model (39). Panel A imposes the CRRA restrictions that $\eta_0 = \eta_w = 0$; Panel B imposes the HARA restriction that $\eta_w = 0$. Panel C relaxes these restrictions altogether for the WDU model.

[ Insert Table 4 here ]

Theoretical restrictions First, regarding the monotonicity restriction, CRRA utility trivially respects non-negative marginal utility. In the case of HARA and WDU, this condition needs to be verified. We test that monotonicity is always maintained by evaluating (4) at the minimum consumption and wealth levels:

$$\eta_c \min(C_t) + \eta_0 + \eta_w \min(W_t) > 0$$

Since $\eta_c \equiv 1$ and $\eta_w$ is estimated positive, this approach is sufficient to guarantee monotonicity throughout our sample. For HARA utility, the statistic (standard error) is 6,474 (0.35); for WDU, it is 6,6157 (87.95). We thus conclude that monotonicity condition (4) is verified for both HARA and WDU.

Second, we verify that the effective discount rate for WDU preferences is non-negative as in (5). Since $\eta_c \equiv 1$, this is obtained by testing

$$H_0 : \rho - \eta_w = 0,$$
against the alternative of negative discounting. Evaluated at our parameter estimates in Panel C, the effective discount rate is -0.0099 (0.0121), a negative but low value that is not statistically significant, such that the null is not rejected. We therefore conclude that all three models satisfy the theoretical restrictions and proceed with the analysis of the point estimates.

**Individual estimates** The estimates for the curvature parameter $\gamma$ in Table 4 are positive, significant and realistic for all three preference specifications. Indeed, it is widely recognized that this parameter should be positive, but less than 10 for iso-elastic utility (e.g. Mehra and Prescott, 1985). Moreover, the point estimates are lower for WDU. Whether or not this translates into a lower level of risk aversion for these functionals will be addressed below.

Next, we find that the bliss parameter $\eta_0$ is negative and very significant for HARA utility, and even more negative, but less significant under WDU. This implies that the reference consumption level is positive under HARA preferences. Under WDU, the fictitious bliss level ranges between -100 and -800 and remains negative throughout. Third, the wealth dependence parameter $\eta_w$ is positive, and significant (based on a Wald test), thereby rejecting the null of HARA utility when tested against the WDU alternative. Our results are therefore consistent with a statistically significant blasé behavior with respect to financial wealth. A test of the joint CRRA restrictions ($\eta_0 = \eta_w = 0$) reveals that the null of iso-elastic preferences is strongly rejected when tested against either HARA or WDU.

**C  Relative Risk Aversion Estimates**

Figure 6 plots the risk aversion estimates for the three utility functions. Panel A plots the consumption risk aversion, $-C_t U_{cc,t}/U_{c,t}$, panel B the wealth risk aversion $-W_t U_{ww,t}/U_{w,t}$, and panel
The indirect utility function risk aversion is given by $-W_t J_{w,t}/J_{w,t}$. The dotted line corresponds to CRRA utility, the dashed line to HARA, and the thick, solid line to WDU preferences.

We find in Panel A that CRRA utility generates the highest, and WDU the lowest level of consumption risk aversion. Moreover, consumption risk aversion under HARA is almost flat compared to that obtained under WDU, i.e. HARA generates no perceptible cyclical variation in attitudes towards consumption risk. Clearly, in panel B, the wealth risk aversion index is zero for both CRRA and HARA. The level for WDU is positive, generally lower, and less volatile compared to consumption risk aversion.

A reduced-form interpretation of the representative agent’s risk aversion can be obtained from the indirect utility function $J_t$, and its corresponding relative risk aversion index $-W_t J_{w,t}/J_{w,t}$ in (22). This variable is plotted in panel C. Because the indirect utility is iso-morphic to the instantaneous utility, the CRRA function has a constant index equal to $\gamma$. For most of our sample, this level is lower than that obtained under HARA and WDU. Note finally that the risk aversion under WDU is lower than that obtained under HARA, with parallel, although more volatile, time paths.

We can explore the issue of cyclical movements of attitudes toward risk by comparing risk aversion series with indices of the state of the economy. For that purpose, we use the University of Michigan Consumer Confidence Index, a pro-cyclical subjective state measure, which we plot against the various measures of risk aversion obtained under WDU. Panel A of Figure 7 plots the consumption risk aversion against the confidence index. Clear counter-cyclical patterns emerge. Risk aversion is initially decreasing until the late 60’s, when the confidence index is stable. Then,
risk aversion increases as the index falls in the early and mid 70's. The gradual recovery in consumer sentiment is associated with a smooth decline in consumption risk aversion.

[ Insert Figure 7 here ]

The correspondence between attitudes toward risk and consumer confidence is even more striking for wealth risk aversion in panel B. Pro-cyclical movements in wealth risk aversion mimic almost exactly those in confidence, particularly up until the mid 70's. After that period, the gradual increase in confidence is associated with a smooth increase in wealth risk aversion.

We therefore find strong counter-cyclical movements in consumption risk aversion, and strong pro-cyclical movements in wealth risk aversion. To verify which one of those two conflicting influences dominates, we plot the indirect risk aversion (22) against the confidence index in panel C. Again, a counter-cyclical movement clearly emerges. To understand this result, our estimates reveal that the bliss level of wealth (21) is \( W_{\text{bliss}} = 4126.1 \), a positive value, whereas wealth in our sample ranges between 11 thousand and 48 thousand $. As wealth increases above bliss, movements in marginal utility are reduced and risk aversion falls. This accords with our previous discussion of the value function in (18) that for positive bliss, a blasé investor has lower and counter-cyclical risk aversion.

V Discussion: Implications for Asset Returns

Our empirical results obtained from the estimation of optimal consumption and portfolio rules using only financial assets and financial wealth can be summarized as follows:

1. The intercept parameter \( \eta_0 \) is non-zero and significant.

2. The wealth-dependence term \( \eta_w \) is positive and significant.
3. The curvature parameter $\gamma$ is positive, realistic, and lower under WDU.

4. Risk aversion is counter-cyclical.

Because there is a relative paucity of empirical results for these allocations models, it is difficult to establish whether or not our findings make sense on a comparative basis. Nonetheless, we can use the implied theoretical restrictions on returns to obtain further perspective on our optimal allocations results.

We consequently consider the implications of our model and of our results for asset returns. For that purpose, we relax the assumption that the investment set is constant. Again, we can resort to the iso-morphism result of Schroder and Skiadas (2002) to map the result from the dual problem to the primal problem. A standard argument establishes that equilibrium state-price deflator in the dual market is given by the marginal utility of consumption:

$$\hat{\pi}_t = e^{-\rho t} \hat{U}_{c,t}. \tag{41}$$

Based on this we can:

1. compute the process for $d\hat{\pi}_t/\hat{\pi}_t$ using dual utility (10) and dual consumption (9),

2. compute the dual short rate and risk premia processes $\hat{r}_t, \mu_{p,t} - \hat{r}_t$ using (14) and (16),

3. map those dual expressions back into their primal counterparts $r_t, \mu_{p,t} - r_t$ by inverting (15) and (17).

These calculations reveal that the risk premia is:

$$\mu_{p,t} - r_t = R^c_t \text{Cov}_t \left( \frac{dC_t}{C_t}, \frac{dP_t}{P_t} \right) + R^w_t \text{Cov}_t \left( \frac{dW_t}{W_t}, \frac{dP_t}{P_t} \right). \tag{42}$$
The risk-free rate is given by:

\[ r_t = \rho - \frac{\eta_w}{\eta_c} + R_t^{rc} \mathbb{E}_t \left( \frac{dC_t}{C_t} \right) + R_t^{rw} \mathbb{E}_t \left( \frac{dW_t}{W_t} \right) - 0.5 \left( \frac{\gamma + 1}{\gamma} \right) \times \left[ (R_t^{rc})^2 \text{Var}_t \left( \frac{dC_t}{C_t} \right) + 2R_t^{rc} R_t^{rw} \text{Cov}_t \left( \frac{dC_t}{C_t}, \frac{dW_t}{W_t} \right) + (R_t^{rw})^2 \text{Var}_t \left( \frac{dW_t}{W_t} \right) \right] \]  

(43)

where:

\[ R_t^{rx} = -\frac{X_t U_{xx,t}}{U_{x,t}} = \left( \frac{\gamma \eta_e X_t}{\eta_c C_t + \eta_0 + \eta_w W_t} \right), \quad X_t \in \{C_t, W_t\} \]

are the consumption and wealth relative risk aversion indices.

**Risk premia** The premia (42) is a two-factor pricing model, with the C-CAPM consumption beta supplemented by the CAPM total wealth return beta. In particular, (7) reveals that the quantity of consumption risk, i.e. \( \text{Cov}_t (dC_t/C_t, dP_t/P_t) \), is priced by the Arrow-Pratt risk aversion level, measured with respect to consumption, i.e. \( R_t^{rc} \). Similarly, the quantity of the market risk, i.e. \( \text{Cov}_t (dW_t/W_t, dP_t/P_t) \), is priced by the corresponding Arrow-Pratt risk aversion, measured with respect to wealth, i.e. \( R_t^{rw} \). The model can thus be interpreted as a linear combination of a static CAPM (\( \eta_c = 0 \)), and a standard C-CAPM (\( \eta_w = 0 \)), where the weights depend on the relative contributions of consumption and wealth to the agent’s utility.

Duffie and Epstein (1992) also obtain a two-factor model, although their model is derived under non-expected utility, rather than VNM preferences. In addition, the relative weights depend on the distance between risk aversion, and the inverse of the elasticity of inter-temporal substitution. Hence, an expected-utility maximizer (risk aversion inversely equal to elasticity of inter-temporal substitution) does not price market risk. In contrast, our agent maximizes expected utility but, for \( \eta_w \neq 0 \), nonetheless values market risks. Moreover, the relative weights under WDU reflect
the importance of consumption versus wealth risk aversion. This is fortunate to the extent that it provides an intuitively appealing interpretation where each risk is being priced by its corresponding risk aversion measure.

Our estimates indicate that optimal consumption is not proportional to wealth (finding 1). This has important consequences for the pricing equations. To see this consider the case where \( \eta_0 = 0 \) in (19). Then, the consumption/wealth ratio is constant, and the growth rates on consumption and wealth are equal: \( \frac{dC_t}{C_t} = \frac{dW_t}{W_t} \). Consequently, so are the covariance terms. Substitute in the premium (42) to obtain that:

\[
\mu_{p,t} - r_t|_{\eta_0=0} = \left( \frac{\gamma \eta_c C_t}{\eta_c C_t + \eta_w W_t} + \frac{\gamma \eta_w W_t}{\eta_c C_t + \eta_w W_t} \right) \text{Cov}_t \left( \frac{dC_t}{C_t}, \frac{dP_t}{P_t} \right),
\]

which is simply the standard C-CAPM with CRRA preferences, in which wealth dependence plays no role. Hence, our finding 1 that \( \eta_0 \neq 0 \) is important to allow for wealth dependence to impact asset returns. In this perspective, our unequivocal rejection of the CRRA model can be interpreted as the dual in optimal allocations of its empirical returns anomalies.

Second, the presence of a second source of IMRS risk is a welcomed addition in finding a solution for the equity risk premium puzzle. Finding 2 establishes that \( \eta_w > 0 \) such that the price of the market risk, \( R_{t}^{m} \), is positive. This result is consistent with the multi-factor empirical literature which finds that market risk is positively valued by the market (Chen et al., 1986; Ferson and Harvey, 1991). If the quantity of market risk is also positive, then a high equity premia need not be explained by consumption risk alone. This is also confirmed in our data set. Table 5 establishes that the total wealth risk of corporate stocks is much larger (by a ratio of 91:1) than consumption risk.
A consequence of estimating $\eta_w > 0$ is that this larger market risk can justify the high observed premia at a lower level of risk aversion. This is consistent with our finding 3 that the curvature parameter $\gamma$, and the risk aversion estimates in general, are lower under WDU.

[ Insert Table 5 here ]

Third, the prices of both consumption and market risks are time-varying, rather than constant as in other models of wealth-dependent preferences (Bakshi and Chen, 1996; Smith, 2001). Our results indicate that relative risk aversion with respect to consumption (wealth) was counter- (pro-) cyclical, with the overall indirect utility risk aversion being counter-cyclical (finding 4). This result would be consistent with the predictability puzzle whereby the conditional premia are observed to fall during booms, and pick up during recessions (Cochrane, 1997; Guvenen, 2003). In the absence of strong conditional heteroscedasticity effects in the quantities of consumption or market risks, predictability would be explained in our model by cyclical movements in risk aversion. Indeed, the conditional premia in (42) explicitly incorporate the wealth–to–consumption ratio through the prices of consumption $R_t^{rc}$ and market $R_t^{rw}$ risks. This ratio has been found to be a strong predictor of returns in multi-factor pricing models (Lettau and Ludvigson, 2001a,b), as discussed in Section I.

**Risk-free rate**  As is well known, the risk-free rate puzzle is a by-product of the equity premium puzzle (Weil, 1989; Kocherlakota, 1996). A high risk aversion implies a low elasticity of inter-temporal substitution, and a high risk-free rate to induce savings. We have already mentioned that wealth dependence result in lower curvature indices (finding 3), thereby potentially addressing the risk-free rate puzzle.

Nonetheless, it is interesting to study the impact of WDU for the predicted risk-free rate. As in the standard case, the risk-free rate (43) captures a first-order and a second-order effect...
reflecting the mean and the variance of the IMRS. In our model however, marginal utility depends on movements in both consumption and in wealth.

Our empirical findings would be consistent with a low risk-free for a number of reasons. First, the effective discount rate in (43) is now \( \rho - \eta_w / \eta_c \); a positive \( \eta_w \) consistent with blasé behavior (finding 2) reduces it and consequently helps in reproducing the low observed \( r_t \). Second, a low \( r_t \) is achieved if the second-order effect on IMRS is stronger than the first-order one. More precisely, allowing for wealth dependence affects both the mean (through the conditional mean terms for consumption and wealth growth) and the variance of the IMRS (through their conditional variance and covariance terms).

In particular, regardless of the sign of \( \eta_w \), the variance of innovations in wealth enters negatively and reduces the risk-free rate. Table 5 shows that the volatility of wealth growth is more than 60 times larger than that of consumption growth (Lettau and Ludvigson, 2003, p. 2, also find that measured wealth growth is much more volatile than consumption growth over short horizons). This effect should therefore be important towards reducing the predicted rates. Moreover, the sample moments indicate that consumption growth is positively correlated with wealth growth. Since \( \eta_w \) was estimated to be positive this covariance in (43) tends to reduce further the predicted rate. Note however that \( \eta_w > 0 \) implies that the mean growth rate of wealth affects positively the predicted risk-free rate. Since the empirical moments in Table 5 indicate that mean consumption and wealth growth rates are roughly equal, this first-order effect could be important in increasing the predicted rate.

**Internal vs external WDU** Our WDU model has been derived under an *internal* wealth-dependence effect, whereby the agent’s *own* wealth affects his utility. In contrast, Campbell and
Cochrane (1999) consider an external habit where bliss is determined by the other agents’ consumption levels. It seems relevant therefore to ask how our results are modified if we substitute the aggregate wealth level, say $\bar{W}_t$, instead of the personal wealth $W_t$ in the preferences (3).

It can be shown that the premium (42) is unaffected when wealth preferences are external. Since aggregate wealth is beyond the agent’s control in the latter case, external wealth is simply an exogenous state variable that conditions preferences. Following Cox et al. (1985), if this variable is valued by the investor, it is priced, if in addition it covaries with other assets, then it warrants a premium. However, under external WDU, the risk-free rate (43) is modified compared to internal wealth preferences. In particular, the use of Envelope theorem under internal wealth preferences implies that the effective discount rate is $\rho - \eta_w/\eta_c$; under external wealth preferences, this rate is simply $\rho$. The agent internalizes the fact that he can (partially) control future wealth, and therefore future bliss. Consequently, at the optimum, the agent’s subjective rate of time preference is affected. A ratchet investor ($\eta_w < 0$) is more impatient since higher future wealth raises bliss and its associated marginal utility risk; a blasé investor ($\eta_w > 0$) is more patient for the opposite reasons.

To conclude, our wealth-dependent framework has the theoretical potential to successfully address the three main pricing anomalies of the C-CAPM. Our empirical findings in optimal allocations are consistent with a WDU explanation of empirical asset returns puzzles. Whether or not similar estimation results with returns are obtained will require further analysis which we leave on the research agenda.
References


Notes

1 The data is obtained from the Flow of Funds, is made available by the Board of Governors of the Federal Reserve Bank, and is discussed in further details below. The identified cyclical patterns are robust to the choice of the wealth series in computing consumption and portfolio shares. Replacing ‘Financial wealth’ by the more comprehensive ‘Net worth’ (i.e. including tangible assets, mutual funds, pension plans, ...net of liabilities) in the denominator in order to derive the consumption and portfolio shares has no qualitative incidence on the patterns identified in Figure 1.

2 The de-trended series are measured as deviations from a quadratic deterministic trend for log consumption and log wealth respectively.

3 Static problems define utility over terminal wealth, but implicitly assume that this wealth is entirely consumed.

4 Although, our empirical implementation relies on elements of financial wealth for practical reasons which are discussed below.

5 The theoretical restriction (5) stems from the financial problem we are analyzing. Consider the discrete-time analog of maximizing (1) subject to (2). First-order and Envelope conditions yield the following:

\[ U_{c,t} = \exp(-\rho)E_t\{[U_{c,t+1} + U_{w,t+1}]R_{i,t+1}\}, \]
or,

\[
1 = \exp(-\rho)(1 + \eta_w/\eta_c)E_t \left\{ \left( \frac{\eta_cC_{t+1} + \eta_0 + \eta_wW_{t+1}}{\eta_cC_t + \eta_0 + \eta_wW_t} \right)^{-\gamma} R_{i,t+1} \right\}.
\]

This Euler equation has a familiar representation, with the exception that the subjective discount factor is now modified to allow for wealth dependence. It is reasonable to expect that the effective discount factor, \( \exp(-\rho)(1 + \eta_w/\eta_c) \in (0, 1) \). Restriction (5) follows immediately.

\[\text{6}\] The bliss level is fictitious in the sense of being a subjective, possibly negative, reference level.

\[\text{7}\] In comparison, typical habit models define the benchmark habit stock in terms of cumulated lagged consumption (e.g. Constantinides, 1990, p. 522):

\[
U = U(C_t, X_t), \text{ where } X_t \equiv \int_0^t \exp[-a(t - s)]C_s ds + \exp(-at)X_0.
\]

\[\text{8}\] Following standard practices, the risk-free rate \( r \) is calibrated to its mean value.

\[\text{9}\] For example, pension reserves are typically invested differently by fund managers whether they are defined benefit or defined contribution. Finding a unique pricing index for this series in the absence of detailed information on the funds’ composition is impractical.

\[\text{10}\] In particular, (20) reveals that, for the risk-free asset:

\[
V_{0,t} = v_{00} + v_{w,0}W_t
\] (44)
where, 

\[ v_{00} = -v_{10} - v_{20}, \quad v_{w0} = 1 - v_{1w} - v_{2w}. \] (45)

11 Clearly, modifying the model so that the agent’s bliss now depend on the difference between own and aggregate wealth, i.e. \( \eta_w(W_t - \bar{W}_t) \) would eliminate the market risk factor in the premia (42). In equilibrium, a representative agent’s wealth is also the aggregate wealth: \( W_t = \bar{W}_t \).
### Tables

#### Table 1: Related Preference Models

<table>
<thead>
<tr>
<th>Authors</th>
<th>Functional form</th>
</tr>
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<tbody>
<tr>
<td><strong>A. Habit models</strong></td>
<td></td>
</tr>
<tr>
<td>Sundaresan (1989); Constantinides (1990), Ferson and Constantinides (1991); Detemple and Zapatero (1991)</td>
<td>$[C_t + \eta(C_s)]^{1-\gamma}, s \leq t$</td>
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<td>Li (2001, 2005)</td>
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<tr>
<td>Campbell and Cochrane (1999); Li (2001, 2005)</td>
<td>$[C_t + \eta(C_t)]^{1-\gamma}$</td>
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<td>Li (2001, 2005); Tallarini and Zhang (2005)</td>
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<tr>
<td><strong>B. Wealth-dependent utility models</strong></td>
<td></td>
</tr>
<tr>
<td><strong>B.1 Preference for total wealth</strong></td>
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<tr>
<td>Bakshi and Chen (1996); Futagami and Shibata (1998), Gong and Zou (2002)</td>
<td>$C_t^{1-\gamma}g(W_t, \bar{W}_t)$</td>
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<tr>
<td>Smith (2001)</td>
<td>${[C_t^{\alpha}W_t^{\beta}]^{\gamma} + \beta[EtU_{t+1}]^{\theta(1-\gamma)/\gamma}}^{1-\gamma}$</td>
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<td>Corneo and Jeanne (2001)</td>
<td>$C_t^{1-\gamma} + g(W_t, \bar{W}_t)$</td>
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<td>Kuznitz and Kandel (2003)</td>
<td>$C_t^{1-\gamma} + g(W_t)$</td>
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<td>Falato (2003)</td>
<td>$[h(C_t, W_t)]^{1-\gamma}$</td>
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<td>This paper</td>
<td>$[C_t + \eta(W_t)]^{1-\gamma}$</td>
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<tr>
<td><strong>B.2 Preference for financial or tangible wealth</strong></td>
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<tr>
<td>Barberis et al. (2001)</td>
<td>$C_t^{1-\gamma}[1 + v(X_{t+1}, W_{fin,t}, \nu_t)/C_t]^{1-\gamma}$</td>
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<td>Grossman and Larocque (1990)</td>
<td>$U(C_t, g_{serv}(W_{tan,t}) + g_{status}(W_{tan,t}))$</td>
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<tr>
<td>Detemple and Giannikos (1996)</td>
<td>$C_t^{1-\gamma} + \mu W_{tan,t}^{1-\gamma}$</td>
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<tr>
<td>Chetty and Szeidl (2004)</td>
<td>$(C_t - \eta_c)^{1-\gamma} + (W_{tan,t} + \eta_w)^{1-\gamma}$</td>
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<tr>
<td>Ait-Sahalia et al. (2004)</td>
<td>${(1 - \alpha)C_t^{\rho} + \alpha W_{tan,t}^{\rho}}^{1-\gamma}$</td>
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<td>Yogo (2005)</td>
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Table 2: Sample moments: Shares of wealth

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<th>correlation</th>
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<td>Consumption</td>
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<td>0.096</td>
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<tr>
<td>Deposits</td>
<td>0.473</td>
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<td>Bonds</td>
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<tr>
<td>Stocks</td>
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<td>0.110</td>
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Table 3: Sample moments: Real Returns

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<tr>
<td>Deposits</td>
<td>0.017</td>
<td>0.022</td>
<td>1.000</td>
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<tr>
<td>Bonds</td>
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<td>Stocks</td>
<td>0.137</td>
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Table 4: Parameter Estimates

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<tr>
<td></td>
<td>A. CRRA</td>
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<td>B. HARA</td>
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<td>C. WDU</td>
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<tr>
<td>$\gamma$</td>
<td>6.187</td>
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<td>5.341</td>
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<td>$\eta_0$</td>
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<td>-49.462</td>
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<tr>
<td>$\eta_w$</td>
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<td>—</td>
<td>0</td>
<td>—</td>
<td>0.019</td>
<td>3.215</td>
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<tr>
<td>$\mu_1$</td>
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<td>3.329</td>
<td>0.007</td>
<td>2.395</td>
<td>0.008</td>
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<tr>
<td>$\mu_2$</td>
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<td>0.028</td>
<td>6.129</td>
<td>0.023</td>
<td>3.846</td>
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<td>$Q(1,1)$</td>
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<td>19.369</td>
<td>0.031</td>
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<td>$Q(1,2)$</td>
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<td>2.573</td>
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Note: Estimated model (39). Assets: [Deposits, Bonds, Stocks]. Wealth: $W_t = V_{0,t} + V_{1,t} + V_{2,t}$. Sample period: 1952:II–2000:IV. Fixed parameters $\rho = (1 + 0.035)^{1/4} - 1$, and $\eta_c = 1$. $\mu_p$ are the drift parameters, $Q_{pp} \equiv \text{Chol}(\Sigma_{pp})$ is the Cholesky root of the covariance matrix of the returns process.

Table 5: Sample moments: Consumption, wealth growth and stock returns

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<td>Consumption growth</td>
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<td>Wealth growth</td>
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<td>Stock returns</td>
<td>0.13811</td>
<td>0.34218</td>
<td>0.11709</td>
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Figure Legends

Figure 1 Shares of wealth: Cash, bonds and stocks.

Figure 2 De-trended log wealth and log consumption and untransformed consumption share.

Figure 3 Effects of increase in wealth on marginal utility, Ratchet Investors ($\eta_w < 0$).

Figure 4 Effects of increase in wealth on marginal utility, Blasé Investors ($\eta_w > 0$).

Figure 5 Marginal utility of wealth at the optimum.

Figure 6 Arrow-Pratt Measures of Risk Aversion.

Figure 7 Risk Aversion and University of Michigan Consumer Confidence Index.
Figures

Figure 1:
Figure 2:

\[ U_c = \eta_c (\eta_c C + \eta_0 + \eta_w W)^{-\gamma} \]

Figure 3:
\[ U_c = \eta_c (\eta_c C + \eta_0 + \eta_w W)^{-\gamma} \]

Figure 4:

\[ J_w = F^{1-\gamma} (W - W_{bliss})^{-\gamma} \]

Figure 5:
Figure 6:

A. Consumption risk aversion

B. Wealth risk aversion

C. Indirect utility risk aversion
Figure 7: