

Human Capital and Employment Risks Diversification*

Pascal St-Amour

Faculty of Business and Economics (HEC)

University of Lausanne

Swiss Finance Institute

June 23, 2015

*I am indebted to Julien Hugonnier for extremely helpful discussions and suggestions. I have also benefited from useful comments from Marnix Amand, Tony Berrada, and Florian Pelgrin. Financial support from the Swiss Finance Institute is gratefully acknowledged. The usual disclaimer applies.

Abstract

Educational expenditures and attainment have increased over the last decades despite rising prices of education, and stagnating income returns. This paper appends partial diversification against Poisson job displacement, and re-employment risks to a human capital investment problem. Numerical solutions are calculated, simulated, and shown to be consistent with unemployment duration dependence (stigma), post re-employment income loss (scarring), counter-cyclical properties of education demand, as well as the moral hazard responses to changes in public insurance policies. The relative importance of diversification vs income motives is evaluated, and the effects of observed changes in the returns to human capital are investigated.

Keywords— Demand for education; unemployment duration dependence; unemployment stigma; income scarring; work displacement; re-employment probability.

JEL classification— I26, J24, J64, J65

1 Introduction

Educational attainment has increased sharply in the US, as well as in most countries over the last decades.¹ This remarkable growth in human capital cannot be solely explained by better educational technology; education expenditure shares of income and of personal expenditures as well as education real prices have increased over the same period. Similarly, the growth in disposable resources cannot be relied upon as sole explanatory factor; education expenditures shares tend to be counter-cyclical, and increase when income falls in recessions.

The previous elements suggest that strong, and improving expected returns must have justified the households' decision to invest more in human capital. These rewards can be separated between education's aptitude in generating higher labor revenues, and its capacity to protect these gains from adverse labor market shocks. The former is partly verified by the income returns to education. Median real weekly earnings data suggests that the education income premia are significant, and have increased up to 2000, but have stagnated afterwards.

The employment risk diversification is also a plausible explanation for several reasons. First, incomplete coverage and progressivity of unemployment insurance (UI) programs entail that the income losses associated with unemployment are substantial, and the more so for the higher revenues (and therefore more educated) workers.² Moreover, the recent evolution in UI programs has been detrimental to the high incomes. Put differently, the value at risk is much more important, and increasingly so for the more educated. Second, education fortunately offers proven insurance against employment hazards. While unconditional unemployment rates are much lower, so is the conditional job loss (i.e. displacement) risk, whereas post-unemployment job finding (re-employment) is much faster the higher the educational attainment. Since all employment risks are strongly

¹For example, see the special report of *The Economist* on tertiary education, "The whole world is going to university", March 28th, 2015.

²Progressivity obtains through the taxation of UI benefits which entails that net replacement rates (i.e. after-tax replacement of after-tax income) are lower for the higher incomes (OECD, 2015).

cyclical, education thus provides significant protection against otherwise undiversifiable macro shocks.

Motivated by these data features, our objective is to devise a tractable and parsimonious framework³ that allows for employment risks diversification to supplement the income premia motivation for human capital demand. We show that this (relatively) simple model is capable of reproducing a number of key facts, such as unemployment duration dependence (i.e. stigma), fall in income if re-employed after prolonged unemployment spells (i.e. scarring), the cyclical properties of education demand, as well as the moral hazard responses to changes in UI characteristics. This model is relied upon to gauge the contribution of employment risks diversification to total human capital demand. We then investigate whether changes in the income and diversification returns are consistent with the observed rise in attainment.

Including employment risks diversification motives in the demand for human capital is warranted for both normative and positive reasons. First, the uncertainty associated with unemployment is largely viewed as undiversifiable, both through market-provided and through self-insurance. Allowing for the latter imposes a reassessment of the optimality of UI policies (Pavoni, 2009). Second, the design of UI programs is also likely to be affected by self-insuring through education. For example, imposing a flatter UI replacement schedule decreases the motivations for the lower-educated workers to invest in education, but raises the stakes for the better educated ones, and therefore their demand for additional diversification through human capital. Positive education gradients in income implies that the resulting changes in the optimal investment will affect the income distribution, therefore raising additional equity issues.

Third, the private benefits to education are likely to be seriously under-estimated if diversification components are abstracted from. These gains justify a reassessment of the optimality of investing public funds in *ex-post* income security measures – such as UI programs – versus in *ex-ante* preventive actions – such as education – in order

³Indeed, the model is constructed around only four primitive equations: A law of motion for capital, employment hazard and income functions, and the agent's objective function.

to counter the social costs associated with unemployment. More generally, appending the capacity to self-ensure against employment risks directly affects the welfare costs of macroeconomic fluctuations.

From a theoretical perspective, when compared to income premia motivations, employment risks diversification through human capital has received little attention in academic research. Self-insurance against conditional employment risks is certainly present in the Search and Matching (SM) literature (e.g. Burdett and Mortensen, 1980; Mortensen and Pissarides, 1994), but this is achieved through a static, and often mechanic setup. Indeed, employment risks stem from either TFP (Shimer, 2005; Moscarini and Postel-Vinay, 2013) or idiosyncratic shocks (Bagger et al., 2014) against which re-employment is accelerated through costly, yet static efforts by the unemployed. Furthermore, displacement risks are diversified by experience which is acquired mechanically through work tenure, or by firm-decided investment in training. These models do not involve dynamic choices by agents facing such elements as economic depreciation (i.e. obsolescence) or path-dependent adjustment costs of the human capital stock, and none studies continuous decisions that are made across the employment statuses.⁴ Put bluntly, the SM literature does not incorporate lifetime human capital decisions made by households.

Conversely, the Human Capital (HC) literature seldom ventures in the conditional risk arena that SM research specializes in. Whereas most of HC contributions incorporate the usual income enhancement motives (Ben-Porath, 1967; Heckman, 1976; Huggett et al., 2006; Polachek et al., 2013; Kredler, 2014), the employment risks remain either absent or undiversifiable (Rogerson and Schindler, 2002; Krebs, 2003; Huggett et al., 2011; Cervellati and Sunde, 2013). None consider the human-capital dependence of displacement, scarring, re-employment, or duration dependence concerns that should affect the attractiveness of education.

This paper innovates by combining elements of both the SM and HC strands. More precisely, we consider dynamic human capital accumulation that is non-firm-specific, and

⁴For example, Charlot and Malherbet (2013); Decreuse and Granier (2013) allow for one-shot education choices made during younger age only.

where conditional employment risks diversification motives featured in the SM frameworks are appended to the more traditional income premia motivations studied by the HC literature. As for SM models, displacement and re-employment can be affected by workers' decisions. Unlike SM however, these decisions are dynamic, and made across age and employment statuses. We also depart from SM and follow the HC tradition in limiting our analysis to partial, rather than general equilibrium in that the optimal allocation is derived taking the employment hazard and income functions as given. As such, our analysis can be considered as a first step in bringing *bona fide* human capital into a full-fledged dynamic general equilibrium favored by SM methods.

Our modeling strategy features infinitely-lived, risk-neutral agents that are heterogeneous in both their human capital attainment, and their employment status (employed, unemployed). Access to financial markets is limited, and inter-temporal consumption smoothing is achieved by investing in their human capital exclusively. The latter is depreciable, and continuously adjusted through deliberate (non-mechanic) household investment decisions that are subject to diminishing returns. Human capital is assumed to be observable and valued by employers as reflected in higher wages, lower displacement, and higher re-employment probabilities for the better educated. As mentioned earlier, this part of the equation is taken as given, and not solved endogenously.

Under the assumption of Poisson arrival rates for changes in employment statuses, the continuous-time agent's problem is iso-morphic to endogenous discounting, which complicates the solution of the Hamilton-Jacobi-Bellman joint system of first-order differential equations. We therefore resort to numerical projection methods based on Chebyshev polynomials, with shape-preserving constraints guaranteeing the monotonicity and curvature of the pair of status-dependent value functions. The model is parametrized, solved and simulated with the calibration chosen so as to reproduce the observed unemployment, displacement, and re-employment rates as well as the mean human capital investment share of income.

Our baseline parametrization yields optimal allocation rules that are consistent with both intuition, and empirical regularities. First, as expected, welfare is increasing in human capital, and is always higher for the employed. Second, we find that optimal investment level is higher for the employed, whereas its corresponding share of revenues is higher for the unemployed. We also highlight non-monotone effects of human capital on investment that stem from two opposing forces. On the one hand, progressive UI programs imply that the expected income loss is more important for the unemployed better educated. This income gap effect is dominant at high education levels. On the other hand, higher human capital diversifies the employment risks faced by the agents, and thereby lowers the attractiveness of investing further resources in education. This effects is dominant at low education levels. It follows that investment levels and shares increase (resp. fall) in human capital for high (resp. low) education levels.

Importantly, our calibration yields two unique, and dynamically stable steady-state levels of human capital (i.e. one per status) with the latter being higher for the employed. Coupled with our earlier findings, these dynamic features play a crucial role in ensuring that the model predictions accord with the empirical facts. First, displacement of a long-tenured worker entails an optimal decline in human capital for the duration of the unemployment spell until a new (lower) steady-state level is attained. As the re-employment probabilities increase in human capital, it follows that re-employment probabilities fall endogenously with duration, consistent with the observed duration dependence (stigma effects). Similarly, the optimal fall in human capital following displacement of long-tenured workers triggers a corresponding decline in income. Any re-employment necessarily occurs at a lower level of human capital, and therefore lower wages, consistent with the evidence on scarring effects of unemployment. Finally, because optimal investment shares increase in human capital, and are higher for the unemployed, displacement in recessions entails an immediate rise in education expenditures shares, followed by a graduate fall as human capital drops towards its lower steady-state value, consistent with the cyclical properties that are observed in the data.

Having confirmed the model's concordance with the empirical regularities, we can compute the marginal contribution of diversification motives to human capital accumulation. Towards that aim, we (i) shut down parametrically the ability to diversify displacement and re-employment risks, (ii) re-adjust exogenous intensities to maintain mean hazard rates, and (iii) re-compute the optimal rules when accumulation is governed by income rewards only. This mean-preserving re-adjustment produces non-neutral effects as the high-displacement and low-re-employment low-educated agents see their risks exposure adjusted downwards, whereas high-educated agents see theirs revised upwards. Our simulated results show that both level and investment in human capital fall by about one fifth when diversification is not possible.

Our next objective is to assess the effects of permanent changes in the returns to human capital. For that purpose, we consider a comparative statics exercise that involves two modifications in the income returns, and two for the employment risks motivations. In particular, we look at the effects of an increase in the human capital premia in employment revenues (as observed in the data before 2000), as well as a decrease in net UI coverage that is more important for the better-educated (as observed in the data after 2000). We then consider a deterioration in re-employment probabilities, as well as an increase in displacement rates that is more unfavorable for the less-educated (as observed in the data after 2000). We show that the first three of the four changes could justify the rise in educational expenditures and attainment. The effects of changes in UI coverage in particular highlight the moral hazard responses in the form of more self-insurance when publicly-provided employment risk insurance deteriorates.

The rest of the paper is organized as follows. We first review the main stylized facts linking human capital and labor market conditions in Section 2. Next, the model is outlined in Section 3. The optimal allocations are detailed in Section 4, and contrasted with the empirical regularities, starting first with the baseline calibration, followed by a comparative exercise in which the key parameters are altered. A conclusion in Section 5

briefly reviews the main findings, while discussing limitations and proposing avenues for future research.

2 Some stylized facts on human capital and labor market conditions

Table 1.A shows the remarkable increase in US educational attainment over the last 50 years. Indeed, the share of population aged 25 and over with at most a high school degree was cut in half from 83.6% in 1960 to 41.6% in 2013. During the same period, the share of people with at least some college education increased 3.5 times, from 16.5% to 58.4%.

Such an increase in educational attainment could have been achieved through constant inputs and better technology. Panel B shows that this is apparently not the case. Both the total (i.e. public + private) education expenditure share of GDP, and the share of private consumption expenditures allocated to education increased over the same period, despite rising education real prices. Increased demand for education could also be fueled by higher income. However, Figure 1 show that this is only partly verified as education shares of total personal expenditures are counter-cyclical and increase in downturns. More disposable resources alone is therefore not a convincing argument for more education demand.

Taken together, these elements suggest instead that increased expected rewards rather than better technology, falling costs, or richer households could explain the increase in the demand for human capital. Evidence and introspection both agree that higher wages, and better protection against adverse revenue shocks should be key determinants of these returns. Figure 2 displays the real (base year 1984) median weekly earnings by education attainment level. Both the levels in Panel A, and deviations from high school earnings in B indicate a substantial income premia associated with education, with workers with Bachelor degrees earning between 242\$ and 274\$ more per week than those with less than high school. Moreover, this premia is relatively a-cyclical, and increases continuously up

to early 2000, before stagnating afterwards. All in all, the existence of a positive income return to human capital would be consistent with the rise in educational attainment only up to a certain point; the stagnation in the premia in the second half of our sample is hinting that other elements are at play.

Unemployment costs are mainly associated with the drop in revenue that job loss entails. This is reflected in part in Figure 3 which plots the evolution in UI net replacement rates (NRR), i.e. the ratio of after-tax UI revenue to after-tax pre-unemployment income. Since UI benefits are taxable income, NRR is a more appropriate measure of the income gap between employed and unemployed. Three elements stand out. First, the income costs of unemployment are substantial, with NRR percentages ranging between 30 and 50% in 2012. Moreover, the UI system is quite progressive; higher incomes (which are also the more educated) suffer proportionally more than lower incomes. Third, the UI evolution is not favorable to the unemployed as the NRR are falling continuously after 2003. This analysis suggests that the unemployment income gap is larger and increasing for the more educated workers. However, if education also lowers the unconditional unemployment probability, as well as reduces the conditional risks of displacement and/or increases the probability of re-employment, then higher value at risk for income combined with better protection against adverse employment shocks would induce a strong diversification motive for investing in human capital.

Figure 4 reports the unemployment rates by education levels. It clearly indicates that human capital is a powerful self-insurance instrument against unconditional unemployment risk. Indeed, the difference between the unemployment rates of workers with less than a high school diploma and those with at least a Bachelor degree is very counter-cyclical and varies between 3.9 % in recoveries, and 11.1% at the through of the 2008 recession, with a median value of 6.1%. Furthermore, the unemployment spread is also falling up to 2005, and increasing thereafter. Unconditional unemployment diversification is therefore a likely, but incomplete explanation for the increased demand in education.

Figure 5 displays the evolution of the monthly labor market flows from the employed and unemployed statuses. It shows that displacement and re-employment are strongly cyclical series. Moreover, we witness an important deterioration after 2000, with displacement (resp. re-employment) increasing (resp. decreasing). Figure 6 relies on Displaced Worker Survey data to show that these macro effects are not distributed evenly across human capital levels. In Panel A, one-year displacement rates are plotted against educational attainment for three sample years. Regardless of the period, a clear education gradient is apparent, with less educated workers suffering a much higher displacement risk. Moreover, both the displacement level and the education gradient increase sharply in 1994 and 2010, i.e. after the recessions of 1991, and 2008, when compared to 2000, i.e. a pre-2001 recession period. Similarly, Panel B shows that one-year re-employment probabilities are associated with a strong positive education gradient, with more educated workers facing a higher reentry probability. Again, both levels and slope are cyclical; re-employment falls after a recession, and the more so for the less-educated workers. Panel C confirms the findings in Figure 2 that earnings are strongly and positively related to education, yet are relatively a-cyclical. Finally, Panel D shows that the income loss associated with unemployment is not only related to low NRR. Indeed, those workers who are re-employed following an unemployment spell usually suffer from a substantial income loss (scarring), which increases importantly in recessions, and also tends to be worse for the low human capital workers.

Overall, these stylized facts show that the increased human capital attainment cannot be explained only by improved education technology, falling prices, or richer households. Furthermore, the income premia is certainly a valid motive for education, but only up to a point. On the other hand, the income loss associated with unemployment is substantial, is higher for the more educated, and is increasing over time. Fortunately, education provides very good insurance against the high unconditional unemployment, high displacement and low re-employment risks, as well as higher scarring risk that are associated with recessions.

Taken together, these elements warrant appending partially diversifiable employment risks to the more traditional income motive in the decision to invest in human capital.

3 Model

Consider an economy where workers are characterized by two sources of heterogeneity:⁵ Human capital, and labor market status (i.e. employed, unemployed). Following the HC literature, the former is defined as the publicly measurable set of skills accumulated by workers over their lifetime. We assume that investment in human capital is decided by agents, and takes place both within (e.g. through experience or voluntary training), and outside (e.g. through formal and informal education) employment. The direct (e.g. tuition fees, books, software, ...), and indirect (e.g. opportunity cost of time spent acquiring skills) investment costs are borne by individuals. Human capital provides no direct utility to the agent, but is valued by employers, as reflected in more favorable conditions with respect to wages, firing and hiring for agents with higher skill levels. From this perspective, human capital is therefore productive, rather than cultural, and general, rather than firm-specific. Labor market statuses are stochastic and the transition matrix between employment and unemployment spells is agent-specific, in that it depends on the accumulated level of human capital. Agents thus select optimal investment paths taking into account its joint benefits in terms of income premia and employment risk diversification.

Human capital accumulation Each agent has a level of human capital $H \in \mathbb{R}_+$, whose law of motion is deterministic, status-independent, and given by:

$$\dot{H}_t \equiv \frac{dH_t}{dt} = -\delta H_t + AI_t^\alpha H_t^{1-\alpha}, \quad \alpha, \delta \in (0, 1) \quad (1)$$

⁵For tractability, we abstract from additional sources of heterogeneity, such as differences in family background, preferences, or ability, and that are discussed in Heckman (2008); Polachek et al. (2013) in the context of HC models.

The human capital accumulation (1) is standard in the HC literature, (e.g. Ben-Porath, 1967; Heckman, 1976; Huggett et al., 2006; Kredler, 2014) and captures continuous, as opposed to period-specific (e.g. young age only) investment. The gross investment function $I^g(I_t, H_t) \equiv \dot{H}_t + \delta H_t = AI_t^\alpha H_t^{1-\alpha}$ is monotone increasing and concave in its arguments. Depreciation can be interpreted as technological obsolescence in the sense that depreciated capital adversely affects labor market opportunities, as will be discussed next.

Employment statuses A person's labor market status follows a Poisson stochastic process with each agent being either employed ($i = e$), or unemployed ($i = u$). Importantly, the arrival intensity is assumed to be dependent of the human capital level. More specifically, let T^i , be the random time of job displacement ($i = u$) from current employment, or re-employment ($i = e$) from current unemployment, with Poisson arrival intensities:

$$\lambda^i(H_t) = \lim_{\tau \rightarrow 0} \frac{1}{\tau} \Pr [t < T^i < t + \tau \mid H_t], \quad i = u, e \quad (2)$$

The time t probability of remaining employed ($k = u$) or remaining unemployed ($k = e$) up to $t + s$ is therefore:

$$P_t [T^k > t + s] = \exp \left[- \int_t^{t+s} \lambda^k(H_\tau) d\tau \right], \quad k = u, e. \quad (3)$$

The intensity functions $\lambda^i : \mathbb{R}_+ \rightarrow \mathbb{R}_{++}$ satisfy

$$\lambda_H^u \leq 0; \quad \lambda_{HH}^u \geq 0; \quad \lambda_H^e \geq 0; \quad \lambda_{HH}^e \leq 0;$$

and are assumed to be bounded below by:

$$\lambda_0^e = \lim_{H \rightarrow 0} \lambda^e(H); \quad \lambda_0^u = \lim_{H \rightarrow \infty} \lambda^u(H).$$

An agent can thus reduce his exposure to conditional employment risks by decreasing his displacement intensity λ^u , and/or increasing his re-employment intensity λ^e by augmenting his human capital through investment, subject to diminishing returns.

Income process The income process $Y^i : \mathbb{R}_+ \rightarrow \mathbb{R}_{++}$ is status-, and human-capital-dependent, and is monotone increasing, concave in human capital, subject to:

$$\begin{aligned} 0 &< Y^u(H) < Y^e(H) \\ 0 &\leq Y_H^u(H) < Y_H^e(H) \\ 0 &\geq Y_{HH}^u(H) > Y_{HH}^e(H) \end{aligned} \tag{4}$$

Higher levels of human capital thus yield higher employment income $Y^e(H)$, subject again to diminishing returns. Unemployment income $Y^u(H)$ is lower, also increasing and concave, but subject to more important decreasing returns. The latter reflects the imperfect coverage, and progressivity of UI programs highlighted in Figure 3.

Preferences All agents are infinitely-lived, risk-neutral, and select dynamic investment in human capital I to maximize expected discounted (at rate ρ) utility, subject to a budget constraint, and taking arrival rate (2) and income (4) functions as given. We assume that agents do not have access to financial assets, but achieve inter-temporal smoothing through human capital only.⁶ Let $V^i(H_0)$ denote the time-0 value function conditional upon status i . Under the Law of Iterated Expectations, for statuses $i, j = e, u; i \neq j$, the agent's problem iso-morphic to endogenous discounting:

$$V^i(H_0) = \max_{\{I\}} \int_0^\infty \exp \left[- \int_0^t (\rho + \lambda^j(H_s)) ds \right] [Y^i(H_t) - I_t + \lambda^j(H_t)V^j(H_t)] dt \tag{5}$$

⁶Note that no access to financial markets entails that all investment plans must be self-financing, i.e. $I_t \leq Y_t^i, \forall i, t$. This constraint is not imposed in the solution method; instead calibration ensures that consumption $C_t = Y_t^i - I_t$ is non-negative.

subject to human capital accumulation (1). The corresponding Hamilton-Jacobi-Bellman (HJB) representation of (5) is given by :

$$0 = \max_{\{I\}} - \rho V^i(H) - \lambda^j(H) [V^i(H) - V^j(H)] + Y^i(H) - I - \delta H V_H^i(H) + A I^\alpha H^{1-\alpha} V_H^i(H). \quad (6)$$

Remark 1 (Convex cost equivalence) One could argue that the cost of human capital acquisition should depend on the size of the investment as well as on the level of accumulated capital, e.g. be less costly for smaller adjustments and/or for higher level of educational attainment. Both features are implicit in our model since the formulation with diminishing returns (6) is equivalent to assuming linear diffusion, and convex adjustment costs:

$$0 = \max_{\{I^g\}} - \rho V^i(H) - \lambda^j(H) [V^i(H) - V^j(H)] + Y^i(H) - I^g P^g(I^g, H) - \delta H V_H^i(H) + I^g V_H^i(H),$$

where I^g is gross investment and where the agent-specific cost of education satisfies:

$$P^g(I^g, H) = A^{-(1+\psi)} \left(\frac{I^g}{H} \right)^\psi, \quad \psi \equiv \frac{1-\alpha}{\alpha},$$

an increasing function of the gross investment to human capital ratio I^g/H .⁷ This suggests that personal education costs $P^g(I^g, H)$ are higher at high gross investment and/or low human capital levels.

Remark 2 (Status-dependent technology and costs) One could further argue that the specification of the agent's problem (6) is too restrictive in assuming that both the human capital acquisition cost and the accumulation technology are status-independent. Alternatively, the opportunity cost of time spent on education could be lower for the

⁷The calibration presented in Table 2 has $\alpha = 0.25 \implies \psi = 3$, so that education costs are increasing and convex in the gross investment to capital ratio.

unemployed, whereas their human capital might depreciate faster.⁸ Both arguments can be captured by the following variant of the agent’s problem (6):

$$0 = \max_{\{I\}} -\rho V^i(H) - \lambda^j(H) [V^i(H) - V^j(H)] + Y^i(H) - P^i I - \delta^i H V_H^i(H) + AI^\alpha H^{1-\alpha} V_H^i(H), \quad (7)$$

where $\delta^u > \delta^e$, $P^u < P^e$, and where our baseline formulation (6) imposes the restriction $\delta^i = \delta$, $P^i = 1$, $i = e, u$. For completeness, we have also calculated solutions with status-dependent technology (7), and the results we obtain are qualitatively similar to the ones under the more parsimonious formulation in (6), and discussed below.⁹ Importantly, we can generate *optimal* depreciation of human capital, and higher education shares of income for the unemployed without requiring a higher depreciation *rate*, or a lower price for the latter. As will become clear shortly, endogenous depreciation plays a key role in reproducing duration dependence for the unemployed, a challenging requirement for human capital models.¹⁰

Remark 3 (Risk aversion) The specification of the agent’s problem (6) assumes risk-neutral preferences with respect to employment status risk. In a different context, Hugonnier et al. (2013) show how the HJB can be modified to allow for aversion to Poisson risk. Adapting their framework to our setting reveals that the agent’s problem is now:

$$0 = \max_{\{I\}} -\rho V^i(H) - \frac{\lambda^j(H) V^i(H)}{1-\gamma} \left[1 - \left(\frac{V^j(H)}{V^i(H)} \right)^{1-\gamma} \right] + Y^i(H) - I - \delta H V_H^i(H) + AI^\alpha H^{1-\alpha} V_H^i(H). \quad (8)$$

⁸See Kroft et al. (2013); Krebs (2007); Keane and Wolpin (1997) for discussion of skill depreciation during unemployment spells.

⁹The alternative (7) preserves the properties of the investment and welfare functions, as well the dynamic properties obtained under the benchmark case (see Figure 7). The complete set of results can be obtained from the author upon request.

¹⁰Indeed, Kroft et al. (2013) note that duration dependence:

“[...] is not easily generated by a model of human capital depreciation when the rate of human capital depreciation is steady and the same across labor markets.” (p. 1161)

where $\gamma \geq 0$ is the coefficient of risk aversion; imposing risk neutrality $\gamma = 0$ returns our baseline formulation (6). For completeness, we also recalculated the optimal rules allowing for risk aversion using the agent's problem (8). As could be expected, risk-averse agents increase their demand for human capital due to its self-insurance properties compared to risk-neutral individuals. The rest of the results however remain qualitatively similar, and we therefore retain our simpler formulation (6) which allows for better convergence properties.¹¹

Returning to our baseline model with status-independent technology, and risk-neutral preferences, the first-order condition to the agent's problem (6) is given as:

$$I^i = H [A\alpha V_H^i(H)]^{\frac{1}{1-\alpha}}$$

Substituting back into the objective function reveals that the joint HJB system simplifies to:

$$\begin{aligned} 0 &= -\rho V^e(H) - \lambda^u(H) [V^e(H) - V^u(H)] + Y^e(H) \\ &\quad - \delta H V_H^e(H) + \kappa H [A V_H^e(H)]^{\frac{1}{1-\alpha}}, \\ 0 &= -\rho V^u(H) - \lambda^e(H) [V^u(H) - V^e(H)] + Y^u(H) \\ &\quad - \delta H V_H^u(H) + \kappa H [A V_H^u(H)]^{\frac{1}{1-\alpha}}, \end{aligned} \tag{9}$$

for $\kappa \equiv (1 - \alpha)\alpha^{\frac{\alpha}{1-\alpha}}$.

The solution to the first-order differential system (9) is nontrivial for two reasons. First, the two value functions $[V^e(H), V^u(H)]$ must be solved jointly. Second, and more importantly, the Poisson hypothesis entails that the discount rates are not constant, but depend on the current human capital state via its effect on the intensities $\lambda^i(H)$. It

¹¹Using a realistic risk aversion coefficient $\gamma = 3.0$ and adapting the intensity function parameters so as to maintain constant the baseline unemployment, displacement, and re-employment rates reported in Table 4 increases average investment and human capital by more than 45%, and investment shares by 22% compared to our benchmark case. Aside from this, the shape of the investment and welfare functions, as well the dynamic properties again remain as under the benchmark case (see Figure 7). The full set of results can be obtained upon request.

follows that analytical solutions are impractical, and that numerical approaches must be used.

Projection method The computational solution we resort to relies on projection methods, and is based on Chebyshev polynomials (Dangl and Wirl, 2004; Judd, 1992), subject to shape-preserving restrictions (Cai and Judd, 2013). More precisely, it uses the following steps:

1. Set state space $H \in [a, b]$, which is normalized to $x(H) \in [-1, 1]$
2. Approximate $V^e(H), V^u(H)$ by Chebyshev polynomials:

$$V^i(H) = \sum_{j=0}^{M-1} c_j^i T_j(x), \text{ where } T_j(x) = \cos[j \arccos(x)]; \quad (10)$$

3. Compute (analytically) $V_H^i(H), V_{HH}^i(H)$ from the Chebyshev polynomial (10). Substitute the former, along with $V^i(H)$ in HJB's (9).
4. Find $(c^e, c^u) \in \mathbb{R}^{2M}$ that minimize the L^2 norm over (9) which are evaluated at M nodes in $[a, b]$, subject to monotonicity, and concavity restrictions:

$$V_H^i(H) \geq 0, \quad V_{HH}^i(H) \leq 0$$

that are evaluated at $N > M$ nodes in $[a, b]$.

Functional forms and calibration Solving the model implies setting the functional forms for the arrival intensities (2), as well as for the income process (4). We therefore parametrize the two functions:

$$\lambda^i(H) = \lambda_0^i + \lambda_1^i H^{-\xi^i}, \quad \lambda_0^i, \lambda_1^i \geq 0; \quad \xi^i > -1, \quad (11)$$

and

$$Y^i(H) = \beta_0^i + \beta_1^i H^{\beta_2^i}, \quad \beta_0^i, \beta_1^i \geq 0; \quad \beta_2^i \in (0, 1). \quad (12)$$

The model's parameters $\boldsymbol{\theta} = (\alpha, \boldsymbol{\beta}, \delta, \boldsymbol{\lambda}, \boldsymbol{\xi}, \rho)$ are calibrated to satisfy the monotonicity and curvature conditions,¹² and to replicate the observed unconditional unemployment, displacement, and re-employment probabilities, as well as the investment share of income. The selected parameters' values are reported in Table 2.

Simulation Further insights on the model's performance can be obtained through its simulation. Towards that aim, we run the following procedure. For agents $j = 1, 2, \dots, J$ we initialize the dynamic process by drawing the initial status $S_{j,0} \sim \{e, u\}$ using the observed unconditional unemployment rate, and by drawing $H_{j,0} \sim [a, b]$ for the initial capital level. Then, for periods $t = 0, 1, 2, \dots, T$, and for each agent j :

1. Set $i = S_{j,t}$, and interpolate the optimal rules in order to compute investment, and welfare,

$$\begin{aligned} I_{j,t} &= I^i(H_{j,t}), \\ V_{j,t} &= V^i(H_{j,t}), \end{aligned}$$

and use the intensity function (11) and income function (12) to compute:

$$\begin{aligned} \lambda_{j,t} &= \lambda^i(H_{j,t}), \\ Y_{j,t} &= Y^i(H_{j,t}). \end{aligned}$$

¹²In particular, setting $\xi^e \in (-1, 0)$, and $\xi^u > 0$ ensures increasing and concave (resp. decreasing and convex) re-employment (resp. displacement) probabilities.

2. Use the human capital accumulation (1), and the status probability (3) to update capital and status as:

$$H_{j,t+1} = H_{t+1}(I_{j,t}, H_{j,t}),$$

$$S_{j,t+1} \sim \{e, u\} \mid \exp[-\lambda_{j,t}].$$

This procedure is repeated over $T = 200$ periods, for a population of $J = 10'000$ individuals. The first burn-in period of $t \leq 50$ is discarded, and the sample means are computed over the remaining 150 periods. A subset of the resulting simulated moments are reported in Table 3 and contrasted with the US data in 2013. It confirms that the observed statistics (1st column) with respect to the unemployment rate $u = 0.076$, the displacement $\Pr[u' \mid e] = 0.056$, and re-employment $\Pr[e' \mid u] = 0.610$, as well as the education share of income $I/Y \in [0.017, 0.066]$ (depending on whether or not public contributions to education expenditures are included) are reproduced quite well by the baseline calibration (2nd column).

4 Results

4.1 Baseline results

Figure 7 plots our baseline results. First, in Panel A, the displacement risks are lower, decreasing, and convex whereas the re-employment probabilities are higher, increasing and concave in health. Both functions correctly reproduce the observed range, and shapes found in the data (Figure 6.A and B). Next, the income levels in Panel B indicate a higher, increasing, and steeper revenue function for the employed, compared to the unemployed, consistent with the progressive UI net replacement rates (Figure 3).

The optimal investment indicates that the level of education expenditures is always higher for the employed (Panel C), whereas the share of labor revenues allocated to education (Panel D) is higher for the unemployed. Since the latter is strictly less than one,

this confirms that investment plans are always feasible, i.e. $C^i(H) = Y^i(H) - I^i(H) > 0, \forall i, H$. Due to the opposing forces of employment risks diversification and income gaps motives, both investment levels and shares are non-monotone in human capital level.

To understand these forces we can define the net protection from employment risk $\Delta\Lambda(H)$ as re-employment minus displacement probabilities. Similarly, the net income gap $\Delta Y(H)$ can be defined as employment income minus unemployment revenues:

$$\begin{aligned}\Delta\Lambda(H) &\equiv \exp[-\lambda^u(H)] - \exp[-\lambda^e(H)], \\ \Delta Y(H) &\equiv Y^e(H) - Y^u(H).\end{aligned}$$

An exogenous increase in $\Delta\Lambda(H)$ therefore reduces the incentives to invest since protection from employment risk is improved, whereas an increase in $\Delta Y(H)$ raises the income value at risk, and therefore the attractiveness of investment. Since Panels A and B confirm that both protection and income gaps are increasing in H , the net effect on investment motives is therefore uncertain. Further analysis however reveals that the marginal gains in protection, $\Delta_H\Lambda(H)$ outweigh the marginal gains of income gap $\Delta_H Y(H)$ at low H , whereas the effect is reversed for the better-educated. It follows that dominating protection effects reduce the attractiveness of investment, such that I^i, I_s^i fall in H for low education levels, regardless of employment status. After a certain threshold however, income gap effects dominate and both investment levels and shares are increasing in H .

Panel E shows that the model yields two unique (one per employment status), and dynamically stable steady-state levels of human capital. The steady-state level for the employed ($H_{ss}^e = 1.18$) is more than 40% higher than that for the unemployed ($H_{ss}^u = 0.82$), consistent with the unconditional unemployment distribution by education levels outlined in Figure 4. Finally, Panel F confirms that welfare is unsurprisingly increasing and concave in H , and is always higher for the employed.

Figure 8 plots a representative sample of the 10'000 simulated optimal dynamic trajectories for human capital $H_{j,t}$. At time $t = 0$, the initial state is drawn from

$H_{j,0} \sim [0.05, 2.0]$. As expected from the previous discussion, the human capital paths rapidly converge towards the range delimited by the two steady-state values H_{ss}^e, H_{ss}^u (dotted lines). Since the unconditional probability of unemployment is low, most of the distribution is concentrated around the employed, rather than unemployed steady-state. Because the employment risks are never entirely diversifiable, the converging distribution is not degenerate. Indeed, each dip in the trajectories correspond to an unemployment spell, where the optimal strategy calls for a reduction in human capital, and a break from the dynamic path towards the steady-state illustrated by the prolonged increases.

Importantly, these characteristics accord with many features observed in the data, and outlined by the literature. First, a number of researchers have identified duration dependence in unemployment whereby unemployed workers face lower re-employment probabilities the longer their unemployment spells (e.g. Kroft et al., 2014; Eriksson and Rooth, 2014; Kroft et al., 2013; Fujita and Moscarini, 2013). This unemployment stigma effect is often interpreted as indicating that employers rely on unemployment duration as a sorting mechanism. Figure 9 suggests that such discrimination is potentially justifiable when human capital is factored in. Indeed, suppose a long-tenured employed worker with steady-state human capital H_{ss}^e loses his job and moves from ① to ② in Panel A. Dynamic stability implies that human capital starts falling optimally towards H_{ss}^u in ③. In Panel B, the newly unemployed agent immediately moves from ① to the re-employment probability ②. The optimal reduction in human capital in Panel A induces a corresponding fall in re-employment probabilities in Panel B from ② towards ③. This fall will be more important the longer the duration of the unemployment spell. Put differently, any unemployment duration stigma from employers would be consistent with the decline in human capital optimally decided by displaced agents.

Moreover, as illustrated in Figure 6.D, a large literature has pointed out that displaced workers are usually re-employed at much less favorable conditions (Carrington and Fallick, 2014; Huckfeldt, 2014; Fang and Silos, 2012; Krebs, 2007; Farber, 2005; Rogerson and Schindler, 2002; Ruhm, 1991; Stern, 1990). These scarring effects are again reproduced

endogenously by the model. In Figure 9.C, a displaced long-tenured worker sees his income fall from point ① to ②, and then a gradual fall towards ③ that is induced by the optimal decline in the level of human capital towards point ③ in Panel A. Following re-employment, his income moves back to $Y^e(H)$ at point ④, and will remain lower than his previous income, until the previous steady-state level H_{ss}^e is again reached in the long-run.

Finally, our discussion of the cyclical properties of education demand in Figure 1 showed that the education shares are counter-cyclical; they tend to increase in recessions, and fall in the subsequent recoveries. The model's predictions are once more consistent with these properties. In Panel D, long-tenured displaced workers' income share spent on education jump up from ① to ②, since, as discussed earlier, education shares of income are higher for the unemployed (Figure 7.D). Shares then fall as optimal dynamic path follows a decline in human capital from ② to ③. Re-employment induces an upward shift in human capital growth from ③ to ④ in Panel A, and an immediate fall in education shares from point ③ to ④ in Panel D, and a subsequent, slower growth in expenditures shares as human capital increases back to its steady-state value H_{ss}^e from point ④ to ①.

4.2 Comparative statics

We now turn to a comparative statics exercise. More specifically, we start from the optimal allocation, say $\mathbf{X} = \mathbf{X}(H; \boldsymbol{\theta})$, that is obtained under the baseline parametrization $\boldsymbol{\theta}$ listed in Table 2. We next modify the model's deep parameters $\boldsymbol{\theta} \rightarrow \tilde{\boldsymbol{\theta}}$ in a manner that is either counter-factual or consistent with specific changes observed in the data. We then re-compute, and re-simulate the optimal rules $\tilde{\mathbf{X}} = \tilde{\mathbf{X}}(H; \tilde{\boldsymbol{\theta}})$ under the assumption that these parametric modifications are permanent, and calculate the percentage changes from the baseline calibration $\%(\tilde{\mathbf{X}} - \mathbf{X})$.

4.2.1 Gauging the diversification

We first start with a counter-factual exercise in order to measure the importance of the employment risks diversification motives in the households' decision to invest in human capital. For that purpose, we remove the ability to adjust exposition to displacement and re-employment risks, and re-calculate the associated optimal allocations. The differences with our baseline allocation thus measure the marginal contribution of diversification to total investment demand.

We consider three alternative specifications obtained by shutting down the parameter λ_1^i which controls the endogeneity in the intensity process, and resetting the base parameter λ_0^i so as to maintain constant the baseline mean displacement and/or re-employment rates in Table 4. Such a change is thus unconditionally neutral in its effect on the intensities, yet is not homogeneous and will affect agents differently depending on their human capital level. In particular, removing the endogeneity of the intensity function and adjusting the base parameter so as to maintain average hazard rates entails that the changes are always more favorable for the low-educated agents (*ex-post* lower displacement, higher re-employment), than for the better educated ones (*ex-post* higher displacement, lower re-employment). Consequently, the net effect $\%(\tilde{\mathbf{X}} - \mathbf{X})$ depends on the resulting change in the distributions between types $i = e, u$, which is determined endogenously by the model.

Exogenous displacement This case is obtained by imposing $\lambda_1^u = 0$, and by setting $\lambda_0^u = -\ln(1 - 0.054) = 0.056$ from the benchmark statistics. Figure 10.A confirms that this results in a fall (resp. increase) in displacement for the low (resp. high) education agents. The other results in Figure 10, as well as in column B.2 of Table 4 both confirm a strong fall in the incentive to invest (-15.3%), which induces a corresponding fall in human capital. The disinvestment is particularly large for the low-education employed (Figures 10.C). Because re-employment remains endogenous, it falls accordingly, inducing unemployment increases. Note finally that the mean fall in income is less important (-

7.3%) than that of investment. Consequently, consumption increases and, when combined with the lower displacement risk at low H , leads to a corresponding increase in welfare.

Exogenous re-employment This case is obtained by imposing $\lambda_1^e = 0$, and by setting $\lambda_0^e = -\ln(1 - 0.628) = 0.989$ from the benchmark statistics. Figure 11.A shows that this results in an increase (resp. fall) in re-employment for the low (resp. high) education agents. The other results in Figure 11, as well as in column B.3 of Table 4 indicate that removing the capacity to diversify re-employment risks reduces incentives to invest (-16.6%), especially for the high-education unemployed (Figure 11.C, D). Since displacement remains endogenous, the fall in human capital increases displacement, and consequently also unemployment. Again the fall in income is less important (-8.1%), justifying a gain in consumption that complements the improvement in re-employment for low H agents; welfare consequently increases.

Exogenous displacement and re-employment This case is obtained by imposing $\lambda_1^u, \lambda_1^e = 0$, and by setting $\lambda_0^u = 0.056$, and $\lambda_0^e = 0.989$ from the baseline statistics. Combining both elements results, unsurprisingly, in the most important changes, as can be inferred from Figure 12, as well as in column B.4 of Table 4. The loss in the employment risks diversification capacity leads to a large disinvestment (-21.4%) and corresponding drop in human capital, especially for the highly educated unemployed (Figure 12.C, D). Unemployment, displacement and re-employment are (by construction) all constant. Welfare again improves as less resources are devoted to diversification, and more to consumption while employment risks are lower for low H workers.

Overall, our simulation results indicate that employment risks diversification accounts for about 21% of both the investment level, and the accumulated human capital stock. Removing the capacity to diversify displacement and re-employment hazards leads to a deterioration in the employment and income situation. However, it also frees up resources that can be used for consumption purposes that compensate the loss in the employment risks diversification capacity, justifying the increase in welfare.

4.2.2 Structural changes in human capital returns

A second comparative statics exercise identifies four potential changes in the primitives that could be consistent with the increase in human capital discussed earlier. The results are plotted in Figures 13–16, while the changes relative to the baseline calibration in the moments from the simulated output are reported in columns Alt. 1–Alt. 4 of Table 5.

Alt. 1 Increasing income spreads up to 2000 Figure 2.B showed that the income premia of the more educated workers relative to the less educated increased steadily up to the early 2000’s. We model this change by lowering the fixed employed income parameter β_0^e , and increasing the income sensitivity to human capital β_1^e in (12). First, in Figure 13.B, the employed income schedule is clearly rotated counter-clockwise, thereby increasing the income return incentive for human capital accumulation. In Panels C, and D, the investment levels and shares are higher for both employed and unemployed, except when the education level for the latter are low, in which case the drop in income forces a reduction of investment. Despite the higher marginal return on education, the important deterioration in base income results in welfare deterioration for both employed and unemployed agents in Panel F.

The simulated moments in column B.1 of Table 5 confirms our findings. Both the unemployment and displacement rates fall, whereas the increase in re-employment is more important. The improvement in employment risks can be traced to the important increase in investment (+42.1%) which results in large increases in human capital (+41.8%) and income (+28%). Despite this, welfare falls (-8.6%) due to the important fall in disposable income $Y(H) - I(H)$.

Alt. 2 Deterioration in UI net replacement rates after 2000 We saw that the net replacement rates in unemployment insurance regimes fell after 2000, especially for the higher income, and therefore more educated (see Figure 3). We model this change by lowering the unemployed income sensitivity to human capital parameter β_1^u in (12). As

seen in Figure 14.B, this results in a downward shift of the unemployment income profile that is larger for the high H agents. Since the income loss under unemployment $\Delta Y(H)$ is now more important, the income return motive for human capital acquisition increases, and Panels C, and D indicate that investment levels and shares increase for both types of agents, especially for the unemployed. This substitution from public towards self-insurance is costly to the agent, and Panel F shows that the unemployment income loss lowers welfare, especially for the unemployed.

Column B.2 of Table 5 confirms that the net effects on employment risks are modest when calculated under our alternative parametrization. Indeed, the optimal response in self-insurance offsets the deterioration in UI benefits with more investment (+2.4%) and human capital (+2.4%), such that unemployment, displacement and re-employment, as well as income remain virtually unchanged. This feature highlights the moral hazard inefficiency when self-insurance is allowed; private diversification through human capital increases when publicly-provided insurance conditions become unfavorable. Again this costly substitution towards more self-insurance when public coverage diminishes lowers welfare.

Alt. 3 Falling re-employment after 2000 Figure 5 displayed the fall in re-employment rates after 2000, while Figure 6.B showed that such shifts tend to be more severe for the lower education levels. These elements are captured by lowering the constant parameter λ_0^e in the re-employment intensity function (11). In Figure 15.A, lower re-employment is more important for the low H . Since expected unemployment duration increases, this creates a risk diversification motive for additional education, and investment levels and shares increase in Panels C and D, especially for the less educated unemployed agents. More investment and additional employment risks are unambiguously welfare reducing in Panel F.

Indeed, column B.3 of Table 5 indicates that the increase in investment and human capital (+7.9%) is important, but not enough to offset the deterioration in the returns to

diversification; the re-employment probability falls (-1.3%), whereas unemployment and displacement are not affected. It follows that welfare drops (-7.5%), despite the rise in income (+3.6%).

Alt. 4 Increasing displacement after 2000 Figure 5 also showed that displacement rates increased after 2000, while Figure 6.A indicated that these increases were more severe for the less educated. We incorporate such effects by increasing the human capital sensitivity λ_1^u in the unemployment intensity function (11). This results in more acute displacement for all, especially for the lower levels of human capital in Figure 16.A. Panels C, and D show that, contrary to previous cases, the investment levels and shares are adjusted differently across employment statuses. Indeed, whereas the employed increase moderately their investment, the unemployed agents lower both levels and shares, especially at low education levels. This reduction in investment expenditures can be explained by the fall of the human capital's diversification capacity against displacement risk. It is also apparent in column B.4 of Table 5 which highlights the deterioration in employment risks, as well as the fall in both investment, human capital (-1.1% each) and income (-2.7%); unsurprisingly, welfare is negatively affected.

Overall, the simulated levels of human capital as well as the investment levels and shares in Table 5 increase for three of the four comparative statics scenarios. Put differently, the model predicts that increasing income spreads for the more educated prior to 2000, and/or deteriorating UI net replacement rates, as well as falling re-employment rates after 2000 are consistent with higher educational attainment in Table 1. Conversely, the decline in the diversification ability to ward off displacement results in a lowering of the attractiveness of investing in human capital; both investment and stock fall.

5 Conclusion

Not everyone is equal in the face of employment risks. This paper has reviewed compelling evidence that the better educated not only earn higher incomes, but they also protect

it better from adverse macro fluctuations. Indeed, both unconditional unemployment, and conditional displacement risks are much lower, whereas conditional re-employment is much higher for the better educated. We have argued that such diversification considerations should condition the demand for human capital. Surprisingly, they are mainly absent from existing literature. Search and Matching models specialize in diversifiable conditional risks yet abstract from dynamic human capital accumulation by households, whereas Human Capital models specialize in dynamic allocations, yet abstract from diversifiable conditional employment risks.

We have proposed a simple, and tractable human capital accumulation model that relies upon only four primitive equations, and that bridges this gap by allowing conditional employment risks to supplement the usual income premia motivation for the demand for education. This model was solved numerically and shown to replicate a number of empirical regularities, such as observed unemployment, displacement and re-employment rates, human capital expenditures share of income, unemployment stigma, income scarring, cyclical properties of education demand, and moral hazard effects of UI coverage. Our simulation exercise evaluated the diversification motivation to 21% of the total investment and accumulated human capital levels. We next showed how observed structural changes affecting the returns to human capital could generate the strong rise in educational expenditures and attainment.

Adopting such a streamlined setup admittedly comes at a price. The first drawback is that we resort to a partial, rather than a general equilibrium analysis. We have followed the agent's perspective in taking the income and hazard functions as given. However, these are endogenous objects that should be solved in equilibrium, as is done under SM frameworks. Put differently, we have focused exclusively on the supply side of the labor market equation. It remains to be shown that our functional forms for the returns to education are consistent with a fully-fledged demand side. Despite this limitation, we nonetheless feel that our results are sufficiently promising to warrant future effort towards that aim.

Second, our solution method is numerical rather than analytical and therefore remains tributary to a specific calibration. Simpler parametrizations might be able to yield closed forms, although the endogeneity of discounting under the Poisson assumption makes this highly hypothetical. Alternatively, a *bona fide* estimation (e.g. based on Simulated Moments) might improve upon our simple calibration, and allow for more thorough hypothesis testing. Finally, we have not fully explored the normative implications of our results. As mentioned earlier, allowing for self-insurance alters the optimality and design of public intervention such as UI and/or education programs. Hopefully, future research could fruitfully be applied to these issues, relying on the insight provided by this type of models.

References

- Bagger, Jesper, Francois Fontaine, Fabien Postel-Vinay, and Jean-Marc Robin (2014) ‘Tenure, experience, human capital, and wages: A tractable equilibrium search model of wage dynamics.’ *American Economic Review* 104(6), 1551 – 1596
- Ben-Porath, Y. (1967) ‘The production of human capital and the life cycle of earnings.’ *Journal of Political Economy* 75(4), 352 – 365
- Burdett, Kenneth, and Dale T. Mortensen (1980) ‘Search, layoffs, and labor market equilibrium.’ *Journal of Political Economy* 88(4), 652 – 672
- Cai, Yongyang, and Kenneth L. Judd (2013) ‘Shape-preserving dynamic programming.’ *Mathematical Methods of Operations Research* 77(3), 407 – 421
- Carrington, William J., and Bruce C. Fallick (2014) ‘Why do earnings fall with job displacement?’ Working Paper 14-05, Federal Reserve Bank of Cleveland, June
- Cervellati, Matteo, and Uwe Sunde (2013) ‘Life expectancy, schooling, and lifetime labor supply: Theory and evidence revisited.’ *Econometrica* 81(5), 2055 – 2086

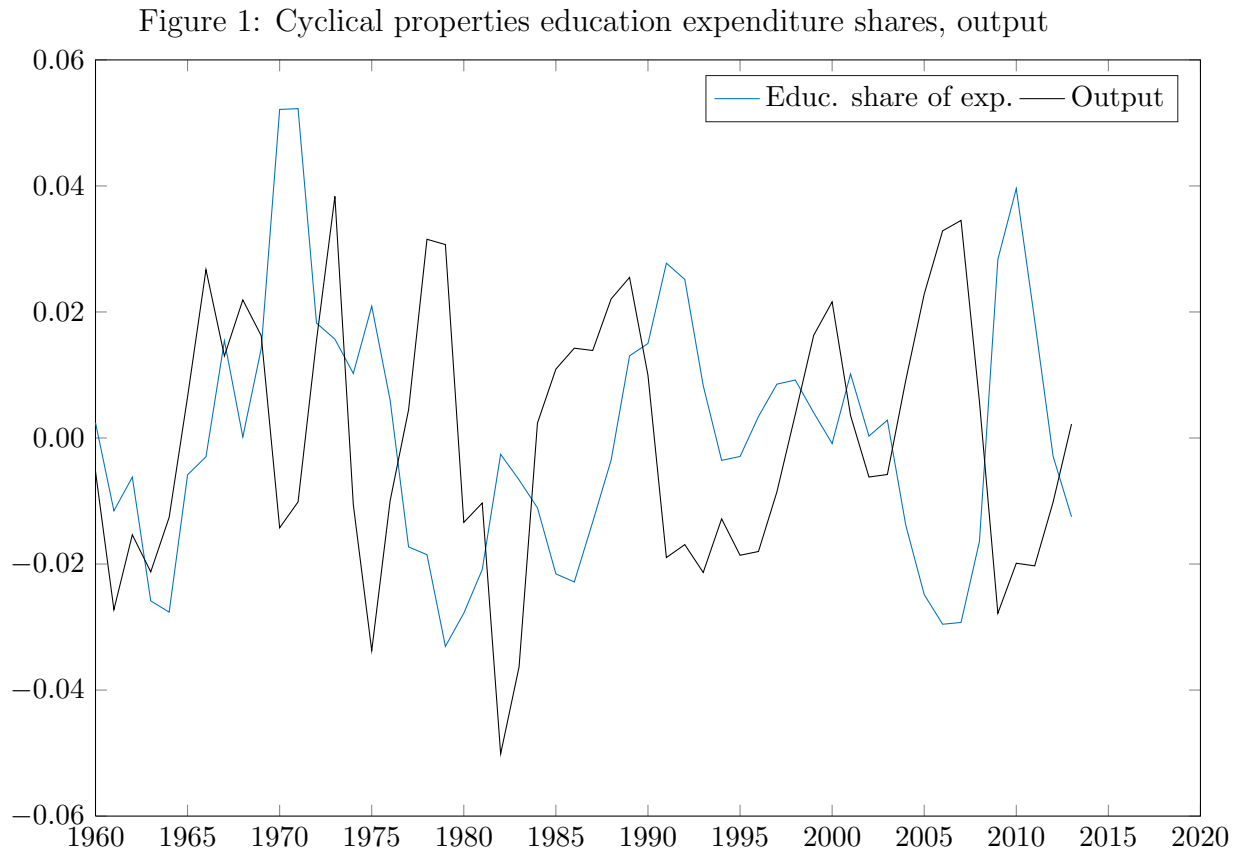
- Charlot, Olivier, and Franck Malherbet (2013) 'Education and employment protection.' *Labour Economics* 20, 3 – 23
- Dangl, Thomas, and Franz Wirl (2004) 'Investment under uncertainty: Calculating the value function when the Bellman equation cannot be solved analytically.' *Journal of Economic Dynamics and Control* 28(7), 1437 – 1460
- Decreuse, Bruno, and Pierre Granier (2013) 'Unemployment benefits, job protection, and the nature of educational investment.' *Labour Economics* 23, 20 – 29
- Eriksson, Stefan, and Dan-Olof Rooth (2014) 'Do employers use unemployment as a sorting criterion when hiring? evidence from a field experiment.' *American Economic Review* 104(3), 1014 – 1039
- Fang, Lei, and Pedro Silos (2012) 'Wages and unemployment across business cycles: A high-frequency investigation.' Working Paper 2012-16, Federal Reserve Bank of Atlanta, October
- Farber, Henry S. (2005) 'What do we know about job loss in the United States? evidence from the displaced workers survey, 1984-2004.' *Federal Reserve Bank of Chicago Economic Perspectives* 29(2), 13 – 28
- Fujita, Shigeru, and Giuseppe Moscarini (2013) 'Recall and unemployment.' NBER Working Paper 19640, National Bureau of Economic Research
- Heckman, James J. (1976) 'A life-cycle model of earnings, learning, and consumption.' *Journal of Political Economy* 84(4), S11 – 44
- Heckman, James J. (2008) 'School, skill, and synapses.' Working paper 14064, National Bureau of Economic Research, June
- Huckfeldt, Christopher (2014) 'The scarring effect of recessions: A quantitative analysis.' Unpubl. manuscript, Cornell University

- Huggett, Mark, Gustavo Ventura, and Amir Yaron (2006) ‘Human capital and earnings distribution dynamics.’ *Journal of Monetary Economics* 53(2), 265 – 290
- (2011) ‘Sources of lifetime inequality.’ *American Economic Review* 101(7), 2923 – 2954
- Hugonnier, Julien, Florian Pelgrin, and Pascal St-Amour (2013) ‘Health and (other) asset holdings.’ *The Review of Economic Studies* 80(2), 663–710
- Judd, Kenneth L. (1992) ‘Projection methods for solving aggregate growth model.’ *Journal of Economic Theory* 58(2), 410 – 452
- Keane, Michael P., and Kenneth I. Wolpin (1997) ‘The career decisions of young men.’ *Journal of Political Economy* 105(3), 473 – 522
- Krebs, Tom (2003) ‘Human capital risk and economic growth.’ *Quarterly Journal of Economics* 118(2), 709 – 744
- (2007) ‘Job displacement risk and the cost of business cycles.’ *American Economic Review* 97(3), 664 – 686
- Kredler, Matthias (2014) ‘Experience vs. obsolescence: A vintage-human-capital model.’ *Journal of Economic Theory* 150, 709 – 739
- Kroft, Kory, Fabian Lange, and Matthew J. Notowidigdo (2013) ‘Duration dependence and labor market conditions: Evidence from a field experiment.’ *Quarterly Journal of Economics* 128(3), 1123 – 1167
- Kroft, Kory, Fabian Lange, Matthew J. Notowidigdo, and Lawrence F. Katz (2014) ‘Long-term unemployment and the great recession: The role of composition, duration dependence, and non-participation.’ NBER Working Paper 20273, National Bureau of Economic Research
- Mortensen, Dale T., and Christopher A. Pissarides (1994) ‘Job creation and job destruction in the theory of unemployment.’ *Review of Economic Studies* 61(3), 397 – 415

- Moscarini, Giuseppe, and Fabien Postel-Vinay (2013) ‘Stochastic search equilibrium.’ *Review of Economic Studies* 80(4), 1545 – 1581
- OECD (2015) ‘Tax and benefit systems: OECD indicators.’ <http://www.oecd.org/social/benefits-and-wages.htm>
- Pavoni, Nicola (2009) ‘Optimal unemployment insurance, with human capital depreciation, and duration dependence.’ *International Economic Review* 50(2), 323 – 362
- Polachek, Solomon W., Tirthatanmoy Das, and Rewat Thamma-Apiroam (2013) ‘Heterogeneity in the production of human capital.’ Discussion paper 7335, IZA, April. forthcoming *Journal of Political Economy*
- Rogerson, Richard, and Martin Schindler (2002) ‘The welfare costs of worker displacement.’ *Journal of Monetary Economics* 49(6), 1213 – 1234
- Ruhm, Christopher J. (1991) ‘Are workers permanently scarred by job displacements?.’ *American Economic Review* 81(1), 319 – 324
- Shimer, Robert (2005) ‘The cyclical behavior of equilibrium unemployment and vacancies.’ *American Economic Review* 95(1), 25 – 49
- Stern, Steven (1990) ‘The effects of firm optimizing behaviour in matching models.’ *Review of Economic Studies* 57(4), 647 – 660

A Figures

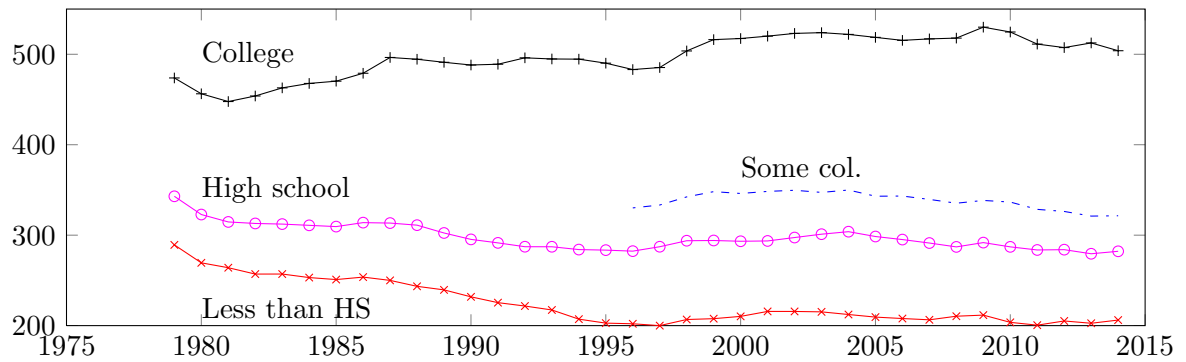
A.1 Data



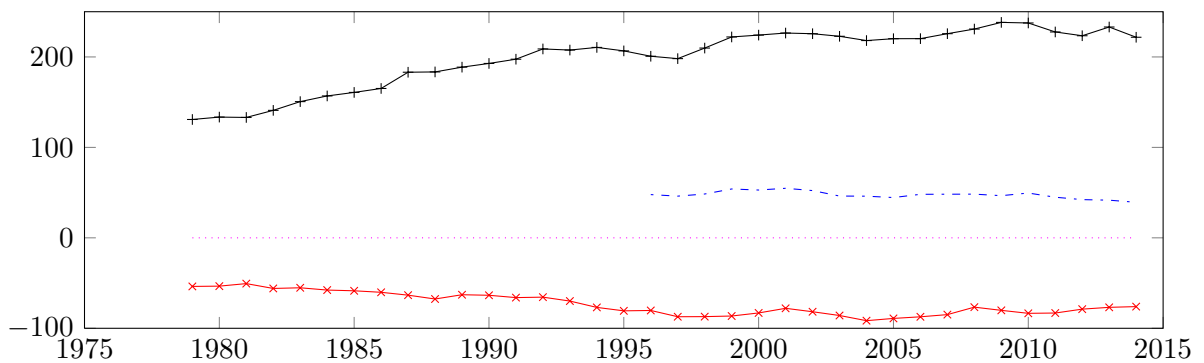
Notes: Data from Bureau Economic Analysis. HP-filtered log data. Personal education expenditures share of total consumption expenditure. Output is real GDP.

Figure 2: Median real weekly earnings

A. Real weekly earnings

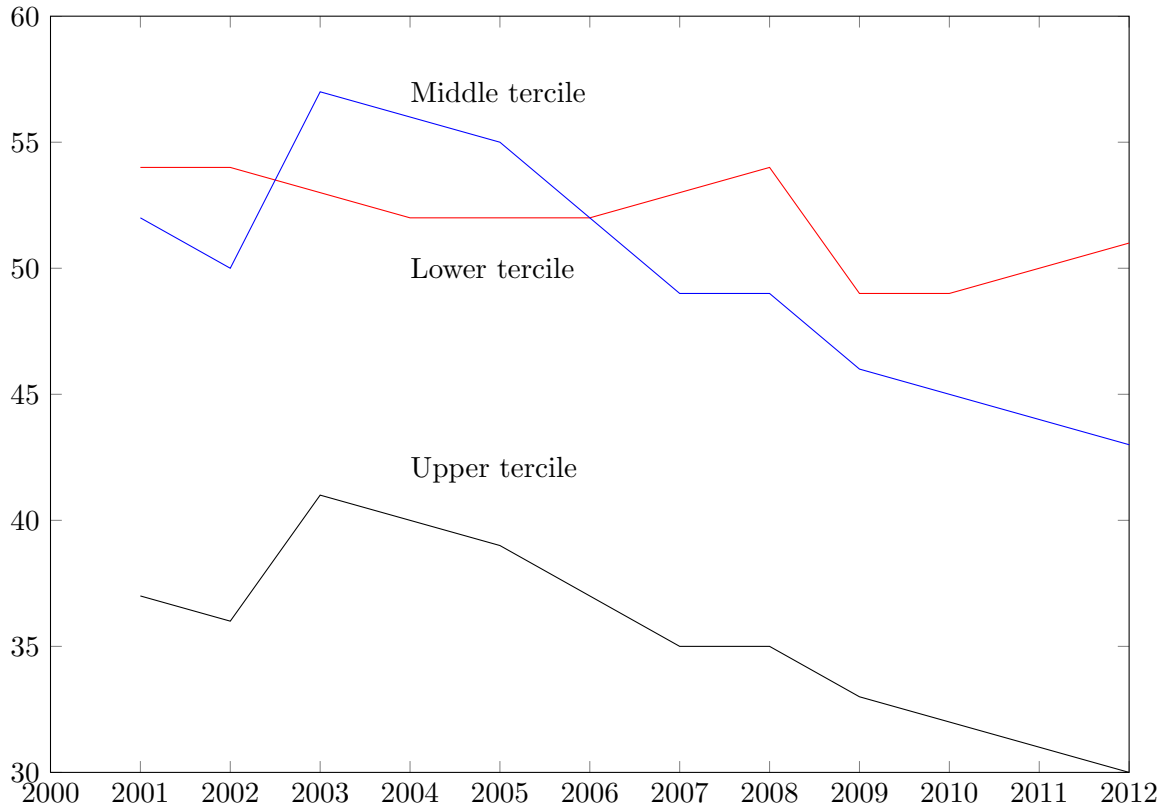


B. Relative to high school



Notes: Data from Current Population Survey. Median weekly earnings in real (base 1982-84) \$. Legend: Less than high school (-x-); High school (-o-); Some college (- . -); Bachelor or more (+).

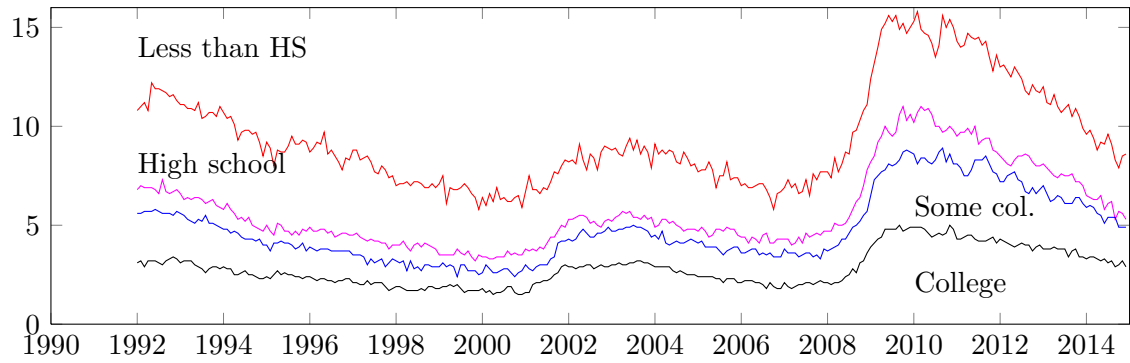
Figure 3: UI net replacement rates



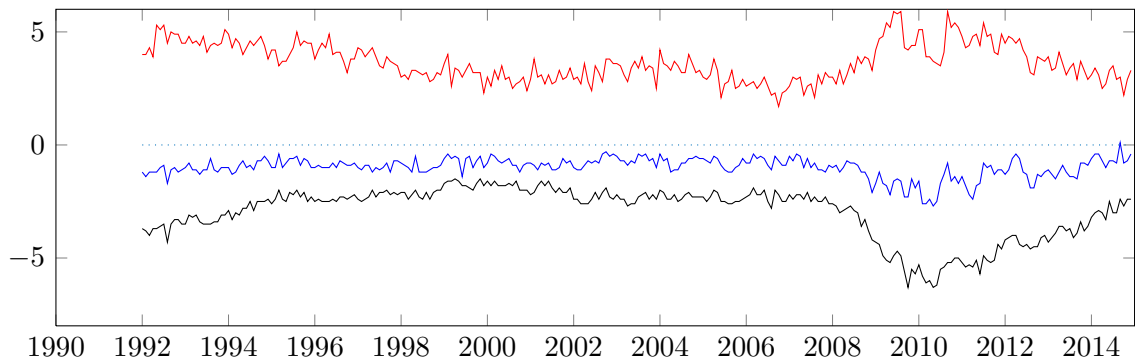
Notes: Data from OECD (2015), corresponding to US. Initial unemployment period. Family with 3 children, single earner. In percentage of pre-displacement earnings. Legend: 67% of AW (—); 100% of AW (—); 150% of AW (—).

Figure 4: Unemployment rates by education levels

A. Unemployment rates

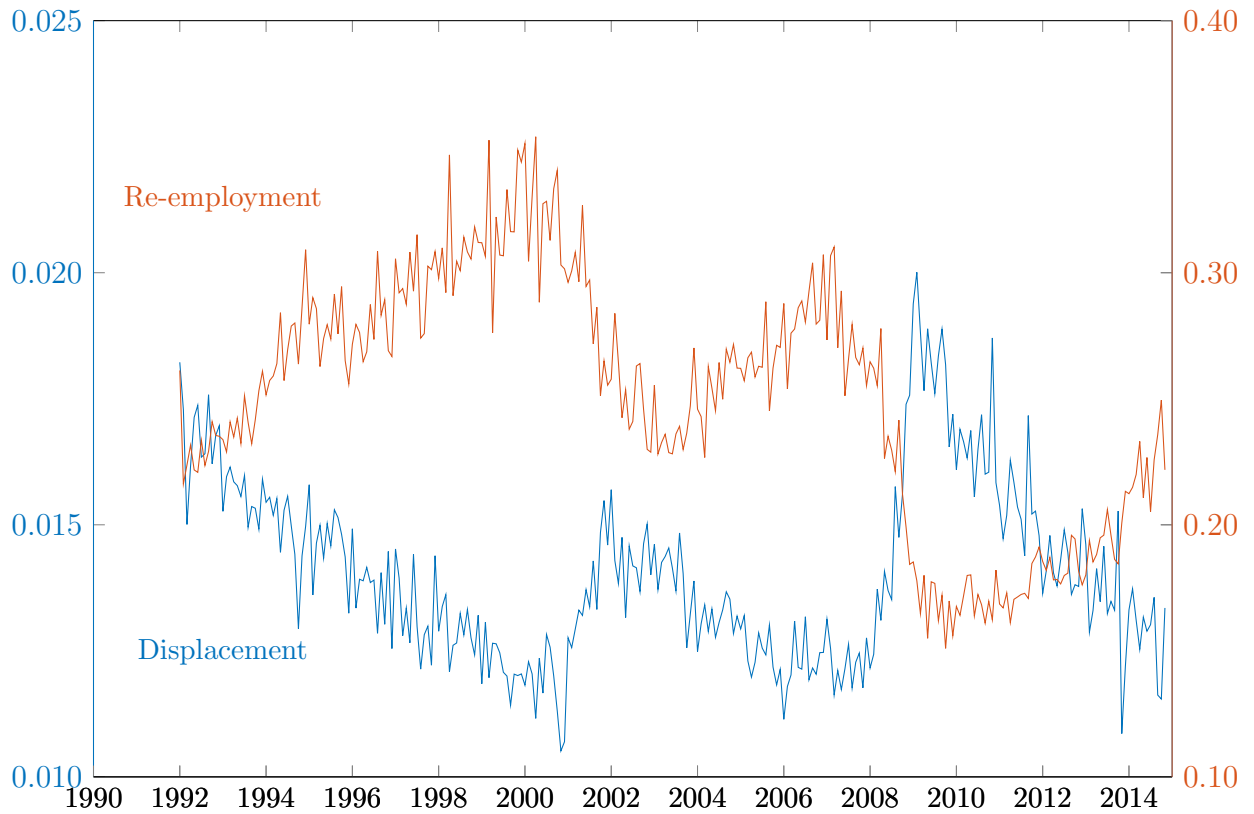


B. Relative to high school



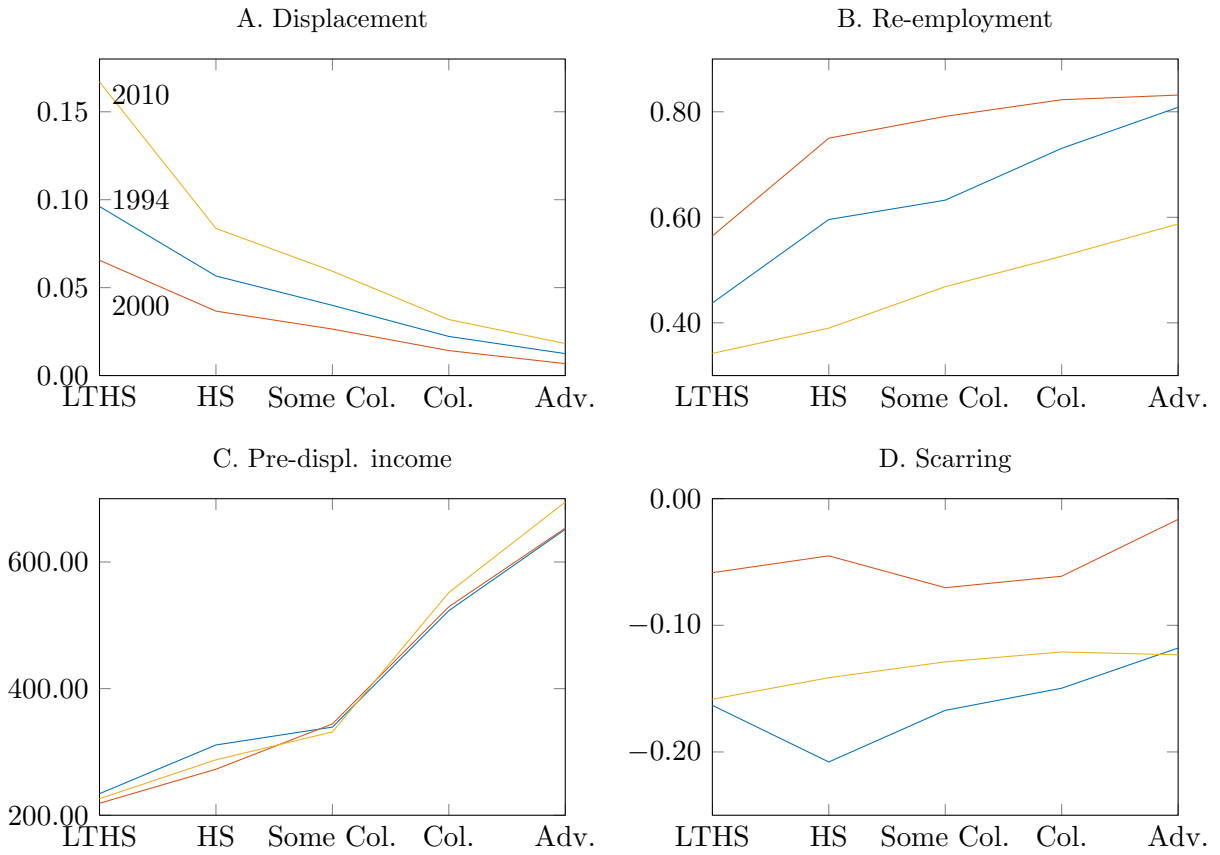
Notes: Data from Current Population Survey. Legend: Less than high school (—); High school (—); Some college (—); Bachelor or more (—).

Figure 5: Monthly labor flows



Notes: Data from Labor Force Statistics from the Current Population Survey. Displacement (in blue, left-hand scale) defined as migration from Employed to Unemployed, divided by total employed. Re-employment (in red, right-hand scale) defined by migration from Unemployed to Employed, divided by total unemployed.

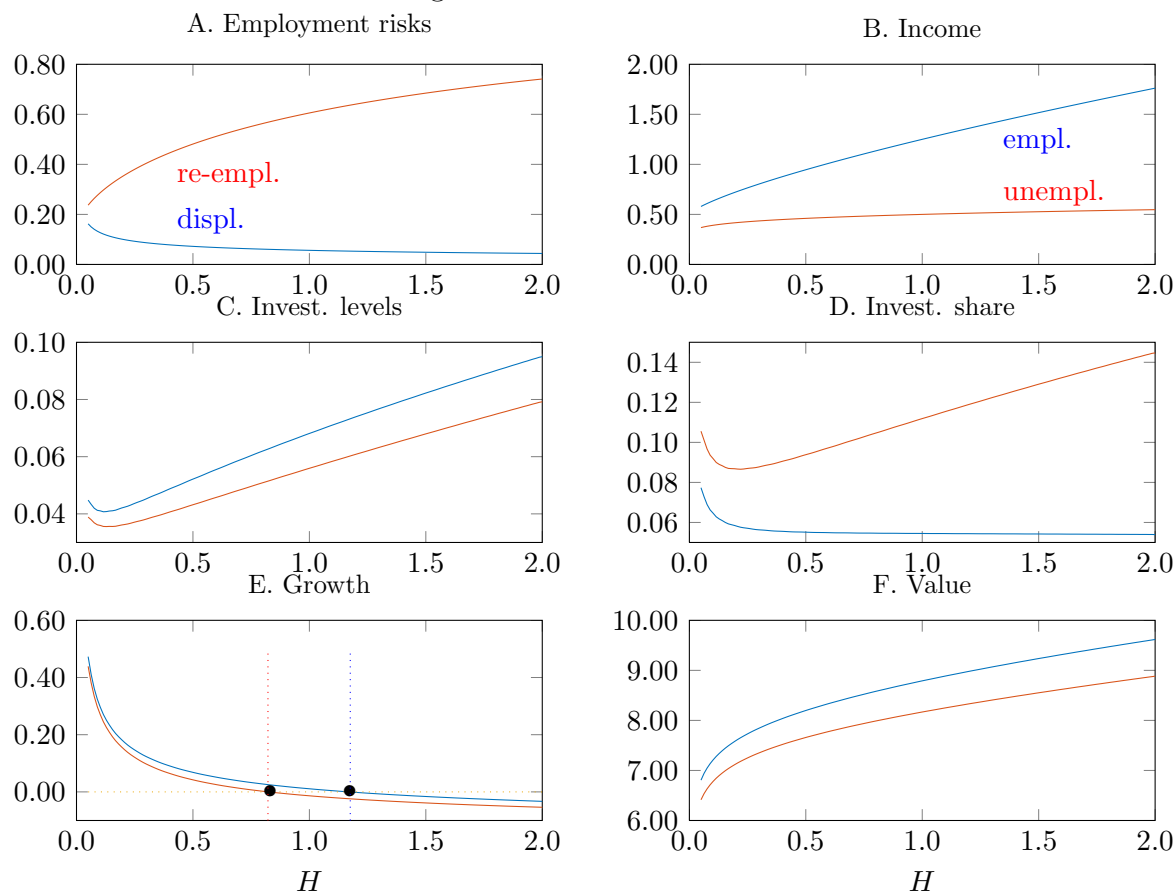
Figure 6: Conditional risks



Notes: Data from Displaced Workers Survey. Legend: 1994 (blue); 2000 (red); 2010 (yellow). A. Displacement $P[u_1 | e_0, H_0]$; B. Re-employment $P[e_1 | u_0, H_0]$; C. Pre-displacement income $[Y_0 | e_0, H_0]$; D. Scarring $\%[Y_1 - Y_{-1} | e_1, u_0, e_{-1}]$.

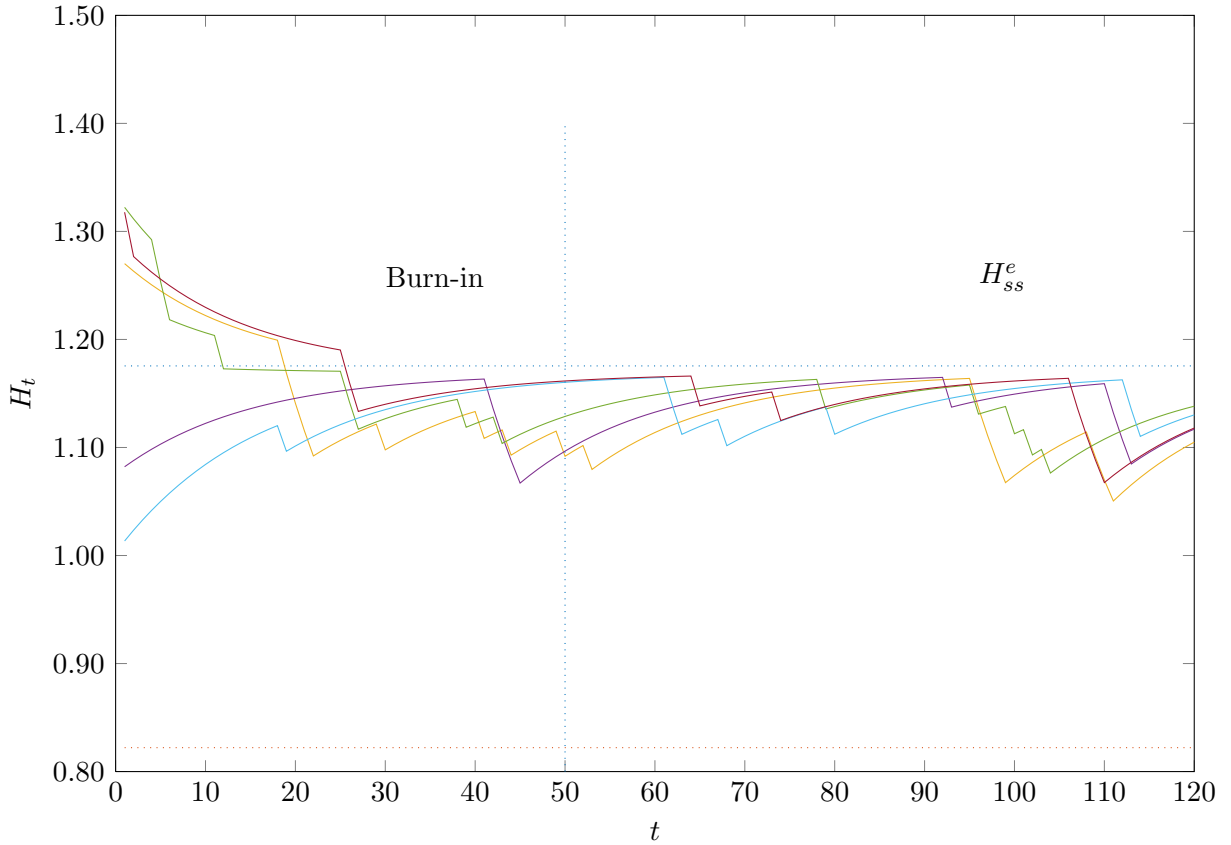
A.2 Baseline results

Figure 7: Baseline results



Notes: Calibrated employment risks and income (Panels A, B) and calculated optimal investment levels and shares (Panels C, D), as well as human capital growth and welfare (Panels E, F) using baseline calibration in Table 2. Blue (resp. red) lines in Panels C–F correspond to employed (resp. unemployed) statuses.

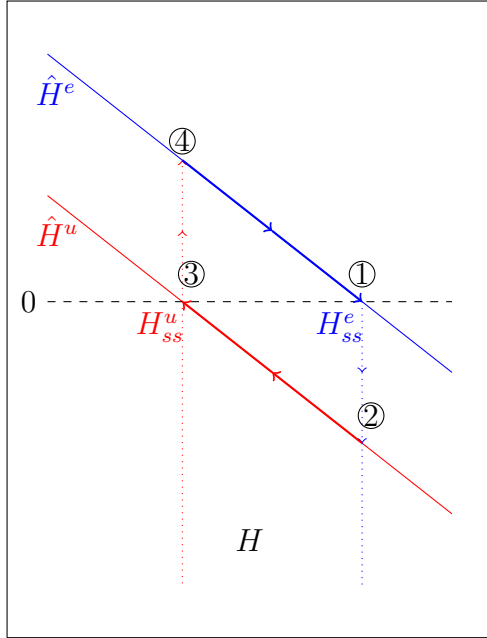
Figure 8: Simulated human capital trajectories



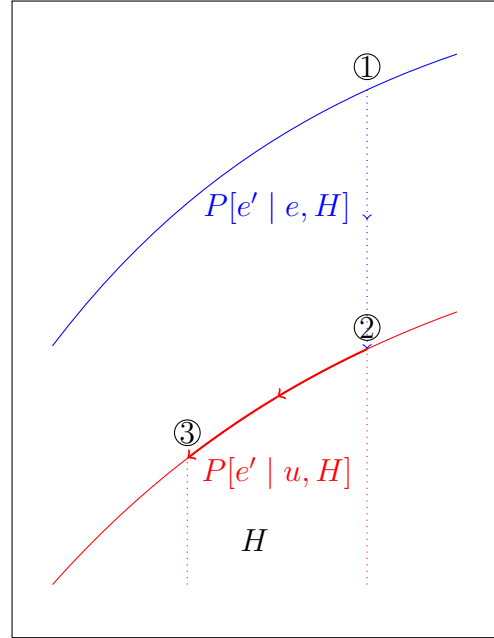
Notes: Sample from 10'000 simulated optimal paths for human capital where dotted lines are employed (resp. unemployed) steady-state values $H_{ss}^e = 1.18$ (resp. $H_{ss}^u = 0.82$). Simulated moments reported in Table 3 calculated omitting the burn-in period values.

Figure 9: Endogenous stigma, scarring, cyclicity

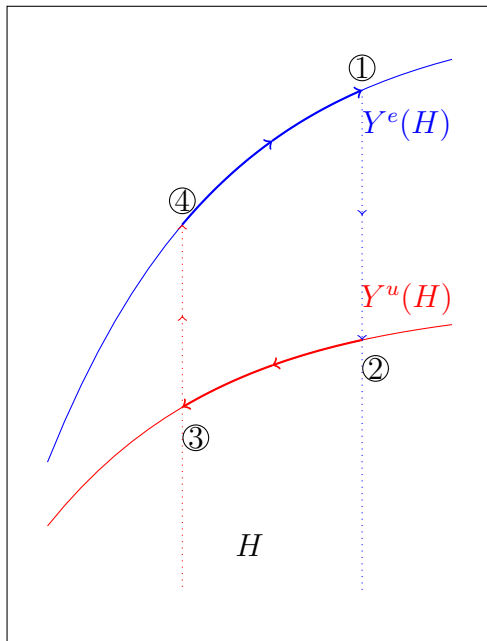
A. Growth



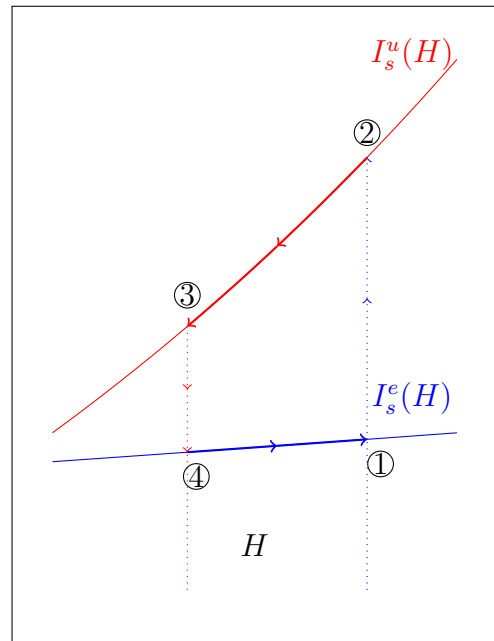
B. Unempl. stigma



C. Income scarring

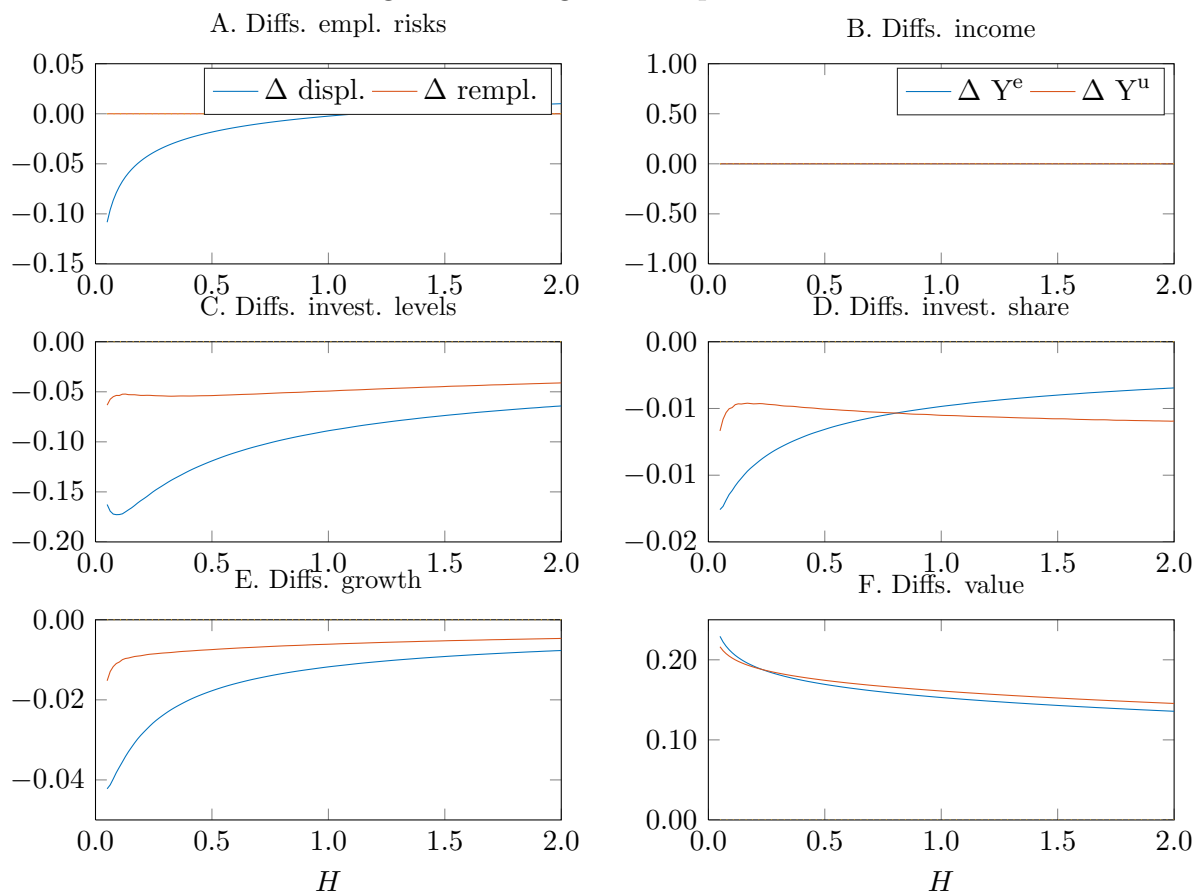


D. Educ shares cycl.



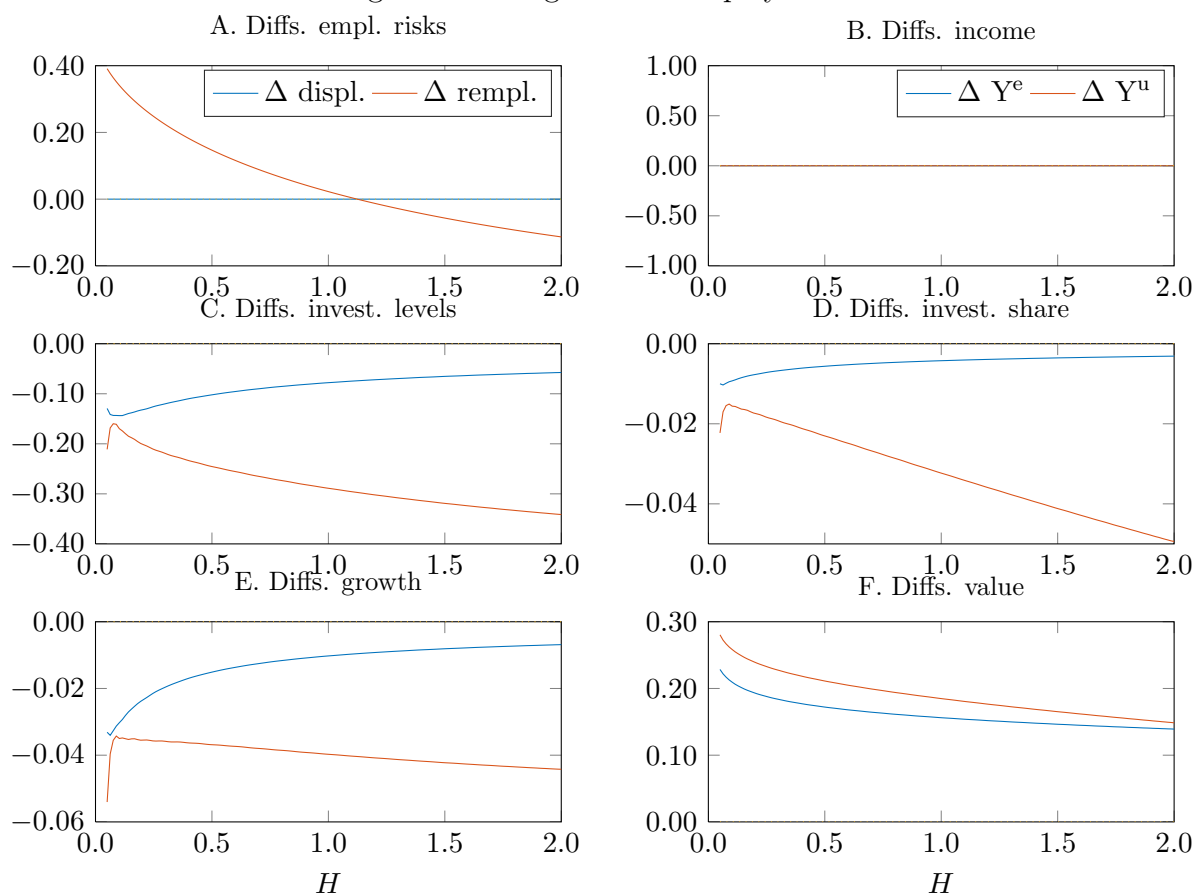
A.3 Comparative statics

Figure 10: Exogenous displacement



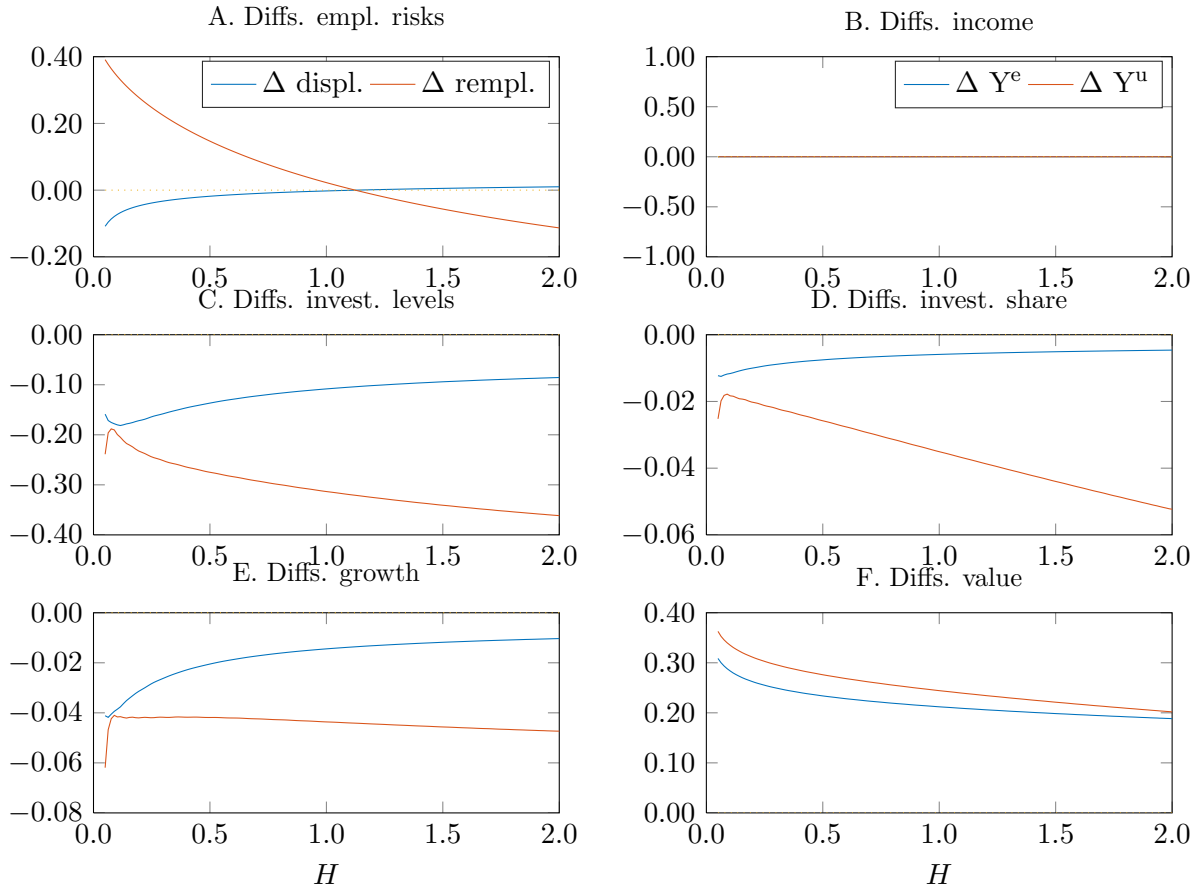
Notes: Alternative parametrization detailed in Table 4.A. Plots correspond to percentage changes in employed (blue), and unemployed (red) variables, relative to baseline parametrization.

Figure 11: Exogenous re-employment



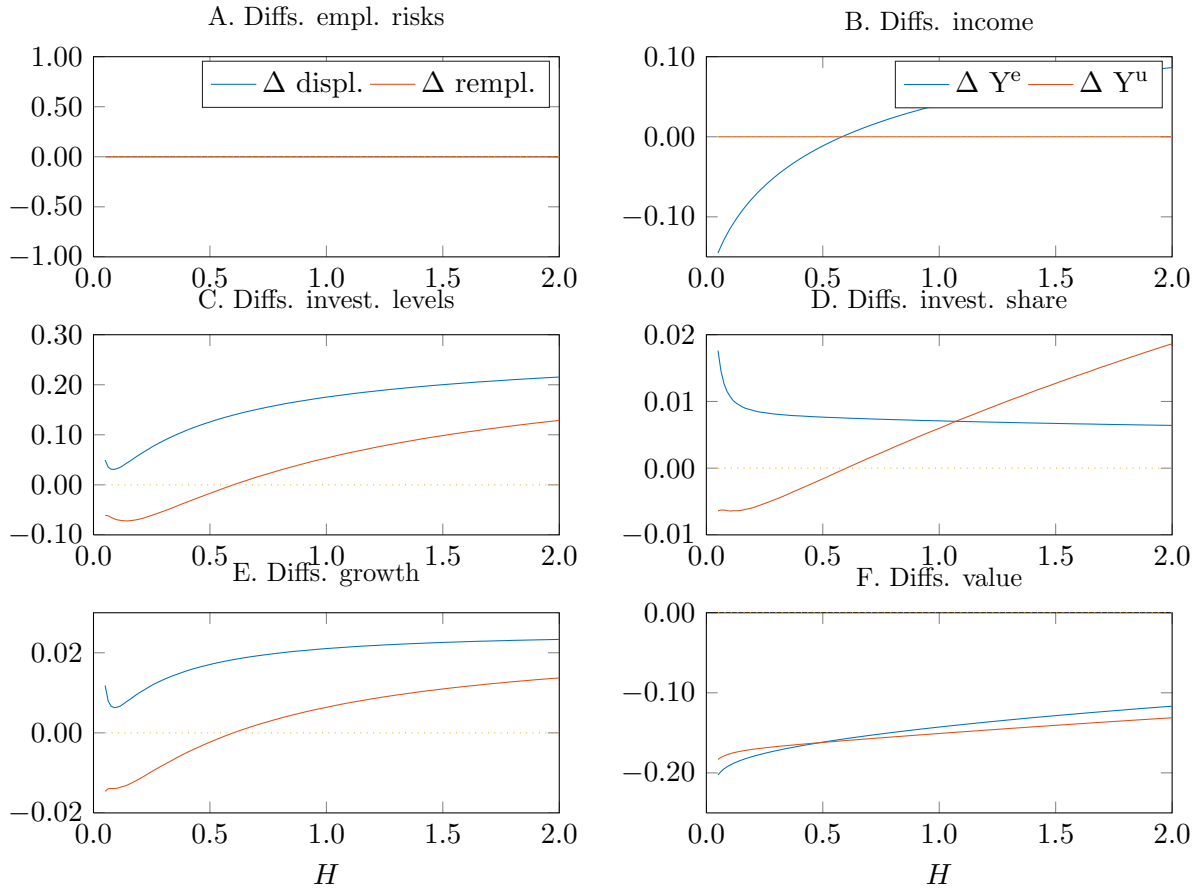
Notes: Alternative parametrization detailed in Table 4.A. Plots correspond to percentage changes in employed (blue), and unemployed (red) variables, relative to baseline parametrization.

Figure 12: Exogenous displ. and re-empl.



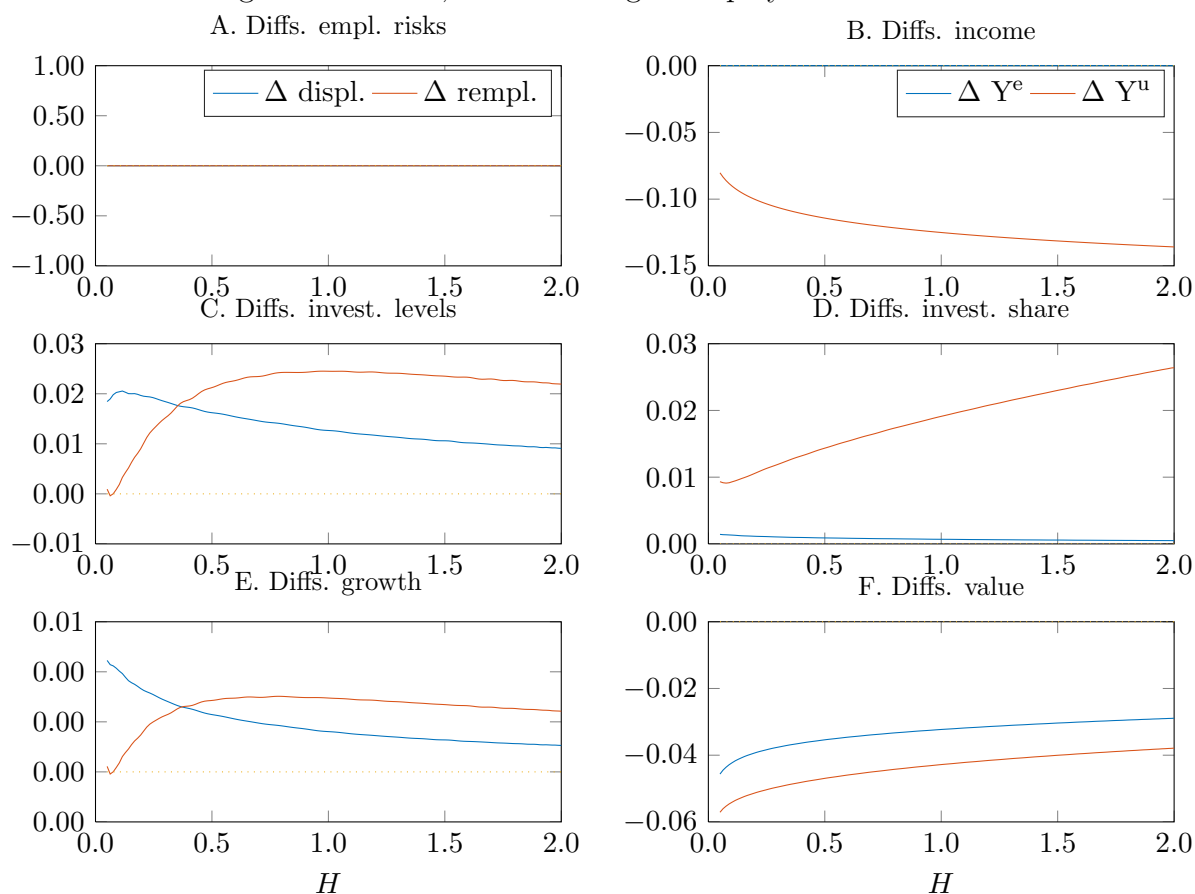
Notes: Alternative parametrization detailed in Table 4.A. Plots correspond to percentage changes in employed (blue), and unemployed (red) variables, relative to baseline parametrization.

Figure 13: Alt. 1, Increasing income spreads



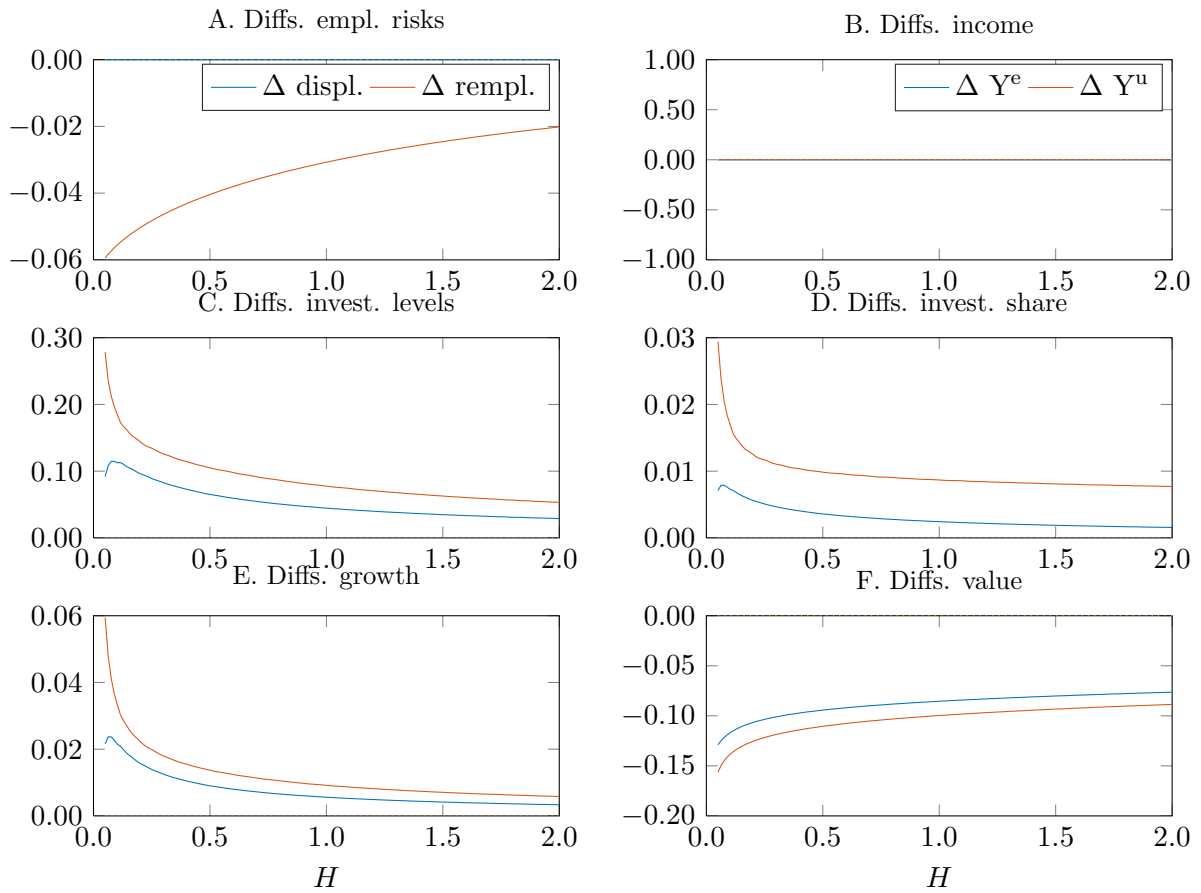
Notes: Alternative parametrization detailed in Table 5.A. Plots correspond to percentage changes in employed (blue), and unemployed (red) variables, relative to baseline parametrization.

Figure 14: Alt. 2, Deteriorating unemployment revenues



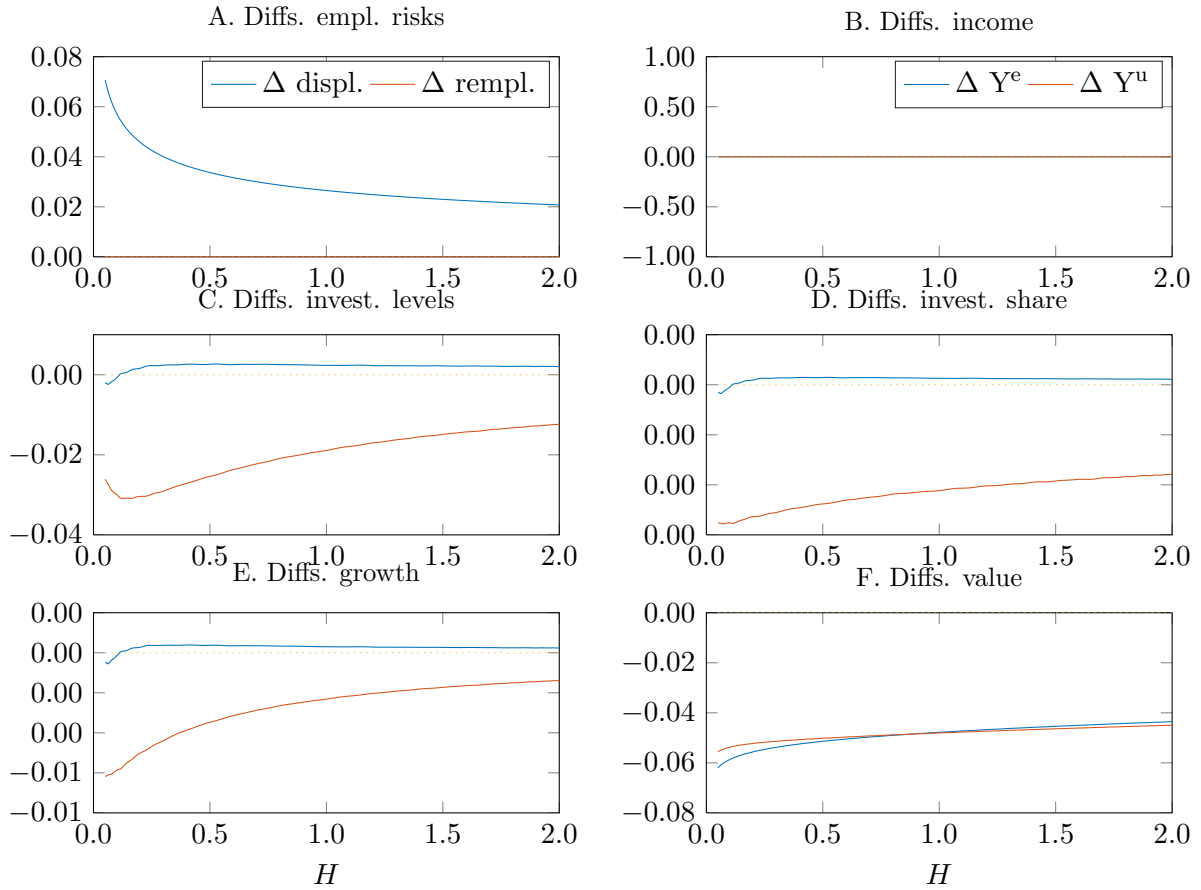
Notes: Alternative parametrization detailed in Table 5.A. Plots correspond to percentage changes in employed (blue), and unemployed (red) variables, relative to baseline parametrization.

Figure 15: Alt. 3, Falling re-employment



Notes: Alternative parametrization detailed in Table 5.A. Plots correspond to percentage changes in employed (blue), and unemployed (red) variables, relative to baseline parametrization.

Figure 16: Alt. 4, Increasing displacement



Notes: Alternative parametrization detailed in Table 5.A. Plots correspond to percentage changes in employed (blue), and unemployed (red) variables, relative to baseline parametrization.

B Tables

Table 1: US educational attainment, shares, and prices

A. Educational attainment				
Year	Less than HS	High school	Some Coll	Coll
1960	59.0%	24.6%	8.8%	7.7%
1980	31.4%	36.8%	14.9%	17.0%
2000	15.9%	33.1%	25.4%	25.6%
2013	11.8%	29.8%	26.7%	31.7%

B. Education shares, and real prices			
Year	Income share	Consumption share	Real price
1960	3.6%	1.0%	0.266
1980	5.5%	1.4%	0.325
2000	6.1%	2.0%	0.682
2013	6.6%	2.4%	1.110

Notes: Panel A: Attainment in % of US population aged 25 years and older. Data from Current Population Survey. Less than high school defined as less than 12 years of formal schooling, college defined as 4 additional years. Panel B: Income share of education defined as total government plus personal consumption expenditures on education divided by GDP. Consumption share of education defined as personal consumption expenditures on education divided by total personal consumption expenditures. Real price defined as Personal expenditures Education price index divided by CPI (base 2008). Data from Bureau Economic Analysis.

Table 2: Calibrated parameters

parameter	value	parameter	value	parameter	value
α	0.25	δ	0.50	ρ	0.04
λ_0^e	0.150	λ_1^e	0.780	ξ^e	-0.624
λ_0^u	0.001	λ_1^u	0.057	ξ^u	0.377
β_0^e	0.50	β_1^e	0.75	β_2^e	0.75
β_0^u	0.25	β_1^u	0.25	β_2^u	0.25
a	0.05	b	2.0	$M(N)$	60 (150)

Notes: Parameters α, δ for investment function in (1), where we have set $A = 1$; discount rate ρ in (5); employment intensity parameters λ^i, ξ^i in (11); income function parameters β^i in (12); dimension of the state space $H \in (a, b)$, with number of Chebyshev nodes M , and shape-preserving nodes N .

Table 3: Simulated model: Matching the moments

	Data (2013)	Baseline
u	0.076	0.080
$\Pr[u' e]$	0.056	0.054
$\Pr[e' u]$	0.610	0.628
I/Y	0.066 [†]	0.059
	0.017 [‡]	

Notes: Unemployment u , displacement $\Pr[u' | e]$, and re-employment $\Pr[e' | u]$ data are from BLS, Current Population Survey, and Displaced Workers Summary; education consumption expenditures share of GDP, I/Y , †: Total (private + public), ‡: private only, data from BEA. Baseline results from calibration in Table 2. Simulated results from simulation over 10'000 individuals over 200-year period, with 50-year burn-in period.

Table 4: Simulated results: Gauging the diversification motives

	Base	Exo. Displ.	Exo. Re-empl.	Exo. Displ & Re-empl.
A. Alternative calibrations				
λ_0^e	0.150		0.989	0.989
λ_1^e	0.780		0.000	0.000
λ_0^u	0.001	0.056		0.056
λ_1^u	0.057	0.000		0.000
B. Simulated moments				
u	0.080	0.4%	0.5%	-0.1%
$\Pr[u' e]$	0.054	0.0%	0.4%	0.0%
$\Pr[e' u]$	0.628	-3.2%	0.0%	0.0%
H	1.136	-15.2%	-16.9%	-21.6%
Y	1.261	-7.3%	-8.1%	-10.1%
I	0.071	-15.3%	-16.6%	-21.4%
I/Y	0.059	-0.5%	-0.7%	-0.9%
V	8.869	13.3%	13.5%	18.5%

Notes: Baseline results from calibration in Table 2. Panel B: Simulated results from simulation over 10'000 individuals over 200-year period, with 50-year burn-in period. Results in columns 2–4 correspond to percentage deviation from baseline simulated results.

Table 5: Simulated results: Alternatives

	Base	Alt. 1	Alt. 2	Alt. 3	Alt. 4
A. Alternative calibrations					
λ_0^e	0.150			0.075	
λ_1^u	0.057				0.086
β_0^e	0.500	0.400			
β_1^e	0.750	0.900			
β_1^u	0.250		0.188		
B. Simulated moments					
u	0.080	-1.6%	-0.2%	0.0%	3.4%
$\Pr[u' e]$	0.054	-0.6%	0.0%	-0.1%	2.6%
$\Pr[e' u]$	0.628	6.8%	0.5%	-1.3%	-0.2%
H	1.136	41.8%	2.4%	7.9%	-1.2%
Y	1.261	28.0%	0.8%	3.6%	-2.7%
I	0.071	42.1%	2.4%	7.9%	-1.1%
I/Y	0.059	0.7%	0.2%	0.3%	0.2%
V	8.869	-8.6%	-2.9%	-7.5%	-5.1%

Notes: Baseline results from calibration in Table 2. Alternative parametrization are Alt. 1 (Fig. 13): Increasing income spreads up to 2000; Alt. 2 (Fig. 14): Deterioration in UI net replacement rates after 2000; Alt. 3 (Fig. 15): Falling re-employment after 2000; Alt. 4 (Fig. 16): Increasing displacement after 2000. Panel B: Simulated results from simulation over 10'000 individuals over 200-year period, with 50-year burn-in period. Results in columns 2–5 correspond to percentage deviation from baseline simulated results.