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An empirical analysis of aggregate household portfolios

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Abstract

This paper analyzes the important time variation in US aggregate household portfolios. To do so, we first use flexible descriptions of preferences and investment opportunities to derive household optimal decision rules that nest static, myopic, and non-myopic portfolio allocations. We then compare these rules to the data through formal statistical analysis. Our main results reveal that: (i) static and myopic investment behaviors are rejected, (ii) non-myopic portfolio allocations are supported, and (iii) the Fama–French factors best explain empirical portfolio shares.

JEL classification: G11; G12

Keywords: Dynamic hedging positions; Generalized recursive preferences; Static, Myopic, and Non-myopic portfolio allocations; Time-varying investment opportunity set

1. Introduction

One striking feature of US aggregate household portfolios is that holdings of cash, bonds, and stocks relative to wealth exhibit pronounced fluctuations through time. Specifically, from the mid 1970s to the late 1980s the empirical share of cash drastically increased, holdings of stocks substantially decreased, while the demand of bonds mildly declined (see Fig. 1). This seems at odds with a prediction associated with *static* portfolio allocations, namely that portfolio rules are time-invariant. These decision rules are optimal regardless of the investors' risk aversion, as long as the investment opportunity set is constant. Under such an environment, investors do not perform dynamic hedging because shocks to state variables have no effect on the distribution of future asset returns. Thus, investors act as if their planning horizon is only one period. This reflects the behavior of short-term investors.

Another important characteristic of US aggregate household portfolios is that the empirical shares display different dynamic properties. In particular, the ratio of the empirical share of bonds to that of stocks falls dramatically between the early 1950s and 1970s, and displays strong upward movements afterwards (see Fig. 2). This seems inconsistent with a prediction derived from the two-fund-separation theorem that the mix of risky assets (such as bonds and stocks) is time-invariant. These rules are optimal, for example, when the relative risk aversion is unity, even if the investment opportunity set is not constant. Under this case, investors never take dynamic hedging positions since they ignore the effects of shocks on future asset returns. This reflects the behavior of *myopic* investors.

These observations suggest that US aggregate household portfolios may be in line with the predictions related to time-varying investment opportunity set and *non-myopic* portfolio allocations, which state that portfolio rules are time-varying and the mix of risky assets also varies. These rules are optimal, for instance, when the relative risk

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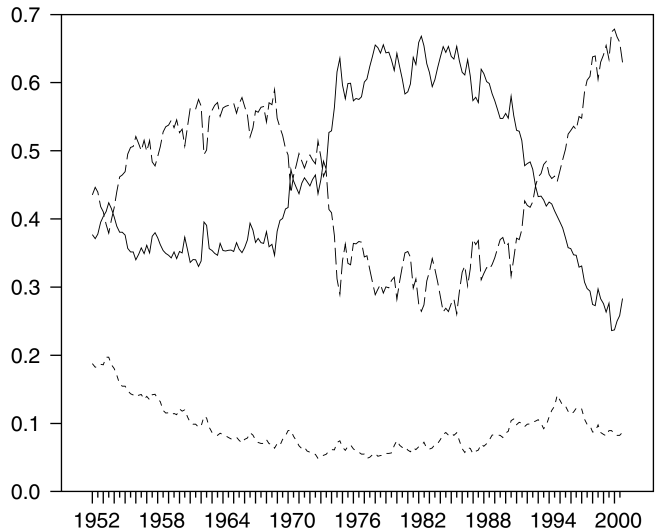

Fig. 1. The solid (dashed) [dotted] lines represent the empirical household portfolio shares of cash (stocks) [bonds]. Each empirical share is measured as the value of assets hold by households relative to wealth, where wealth is the sum of the values of holdings of cash, bonds, and stocks (See the Data appendix).

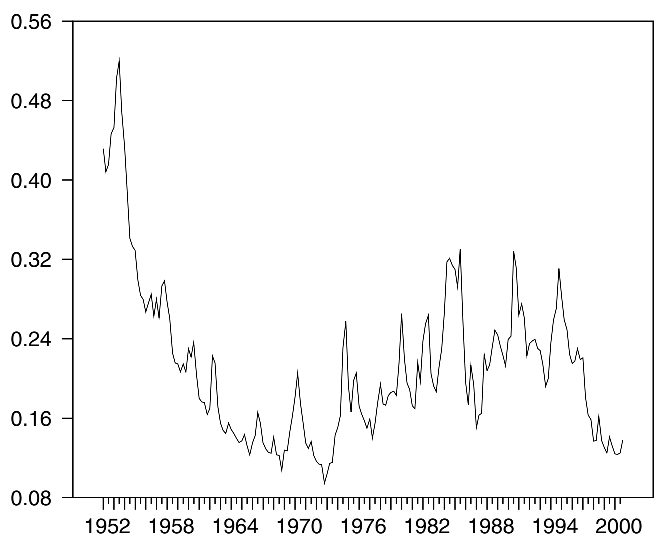

Fig. 2. The solid line represents the ratio of the empirical household share of bonds to that of stocks.

aversion exceeds one, and when the investment opportunity set is not constant. In this context, investors have a multi-period planning horizon, and have dynamic hedging demands to account for the effects of shocks on future asset returns. This reflects the behavior of long-term investors.

The objective of this paper is to analyze a flexible framework's ability in reproducing US aggregate household portfolio shares. In particular, we verify: (i) whether these portfolios are best characterized as being static, myopic or non-myopic, and (ii) which (if any) factors are used by households in selecting asset holdings. To do so, we study the household asset positions from a partial equilibrium environment where the asset supply is perfectly elastic, rather than the pricing implications from a general equilibrium perspective where the asset supply is perfectly inelastic. For our application, the partial equilibrium approach is relevant given that we focus exclusively on asset holdings of the household sector, rather than those of all sectors (which also include businesses, governments, and non-residents). In fact, the household asset holdings represent only a fraction of those hold by all sectors, and as such does not correspond to the aggregate wealth of the entire economy. For example, in 2005 the financial assets directly held by US households was less than one third of those held by all sectors (Source: Board of Governors of the Federal Reserve Bank, Balance Sheet of All Sectors).

Specifically, we consider a general setting that involves time- and state-non-separable preferences (i.e. non-expected utility) as well as various specifications of investment opportunity sets. These preferences are useful in disentangling the investors' attitudes towards risk and inter-temporal substitution. Also, changes in investment opportunities are described from unrestricted vector autoregression (VAR) processes involving asset returns and factor variables. Similar theoretical environments are analyzed for the cases of single risky asset and state variable (Campbell and Viceira, 1999), many risky assets and a single state variable (Normandin and St-Amour, 2002), and several risky assets and state variables (Campbell et al., 2003). Importantly, these environments are attractive since they yield optimal portfolio rules that nest static, myopic, and non-myopic portfolio allocations. The theoretical environment is presented in Section 2.

Also, we consider various wide-ranging specifications of the VAR for the return process. A first specification simply relates the return variables associated with cash, bonds, and stocks to constant terms. This baseline case ensures that investment opportunities are constant, so that portfolio allocations are static. The other specifications link current return variables on their own lagged values as well as past values of factor variables. These alternative cases imply that investment opportunities are time-varying, such that portfolio allocations may be non-myopic. Also, the selected sets of factors include the seminal Fama and French (1993) factors, the well-known Chen et al. (1986) macroeconomic factors, as well as the Campbell et al. (2003) factors. The estimation results, reported in Section 3, for the quarterly post-war US data reveal important implications for portfolio allocations. First, the baseline specification is rejected. This finding refutes the hypothesis of a constant investment set. Second, the conventional criteria of fit are very close across the various alternative factor sets. Consequently, it is difficult at this point to identify the most influential factor set actually used for portfolio allocations.

Next, we apply formal statistical tests to verify whether the empirical and predicted portfolio shares exhibit identical means, volatilities, and co-movements. The empirical portfolio shares are constructed for cash, bonds, and stocks from quarterly aggregate US household data for the post-war period. The predicted portfolio shares, elaborated in Section 4, are evaluated from the optimal rules and the

VAR processes associated with the various sets of factors. The test results highlight two key implications for the assessment of portfolio allocations. First, the various moments of the empirical portfolio shares are *never* replicated from the baseline specification, nor from the combinations of any alternative factor sets with a relative risk aversion of one. This empirical evidence refutes both static and myopic investment behaviors. Second, the properties of the empirical portfolio shares are best explained by combining the Fama–French factors with reasonable values of relative risk aversion larger than unity. This provides empirical support for non-myopic portfolio allocations.

For completeness, we perform statistical tests to check whether the empirical and predicted consumption shares display the same means and volatilities. The empirical consumption share is constructed as the consumption-wealth ratio from quarterly aggregate US data for the post-war period. The predicted consumption shares, explained in Section 5, are evaluated from the optimal consumption rule and the VAR processes associated with the baseline and Fama–French specifications. The test results reveal two important implications for the evaluation of portfolio allocations. First, the moments of the empirical consumption share are never reproduced from the baseline specification, nor from the combination of the Fama–French factors with a relative risk aversion of one. This provides additional evidence against static and myopic portfolio allocations. Second, the properties of the empirical consumption share can be recovered by combining the Fama–French specification with reasonable values of relative risk aversion larger than unity. This provides additional evidence in favor of non-myopic portfolio allocations.

Altogether, the return, portfolio, and consumption analyses lead to key conclusions for US aggregate household portfolio allocations. First, there is a clear rejection of static and myopic investment behaviors. Second, there is an empirical support for non-myopic portfolio allocations. Third, there is some evidence suggesting that the Fama–French factors are those which best explain empirical portfolio shares.

2. Theoretical environment

This section presents the household's problem, specifies the dynamics of the state variables, and explains the approximate consumption and portfolio decision rules.

2.1. Household's problem

We consider the following household's problem:

$$u_t = \max_{\{c_t,\alpha_{i,t}\}_{i=2}^{n_r}} \left[(1-\delta)c_t^{\frac{\psi-1}{\psi}} + \delta(E_t u_{t+1}^{1-\gamma})^{\frac{\psi-1}{\psi(1-\gamma)}}\right]^{\frac{\psi}{\psi-1}}, \tag{1}$$

$$\text{s.t.} \quad w_{t+1} = (1+r_{p,t+1})(w_t - c_t), \tag{2}$$

$$r_{p,t+1} = r_{1,t+1} + \sum_{i=2}^{n_r} \alpha_{i,t} xr_{i,t+1}. \tag{3}$$

The term E_t denotes the expectation operator conditional on information available in period t, u_t is the within-period utility, c_t is real consumption, w_t is real wealth, $r_{p,t+1}$ is the real (net) return on the wealth portfolio, $r_{1,t+1}$ is the real (net) return for a benchmark risky asset, $r_{i,t+1}$ is the real (net) return for an alternative risky asset i, while $xr_{i,t+1} = (r_{i,t+1} - r_{1,t+1})$ and $\alpha_{i,t}$ are the associated excess return (relative to the benchmark return) and portfolio share, with asset $i = 2, \ldots, n_r$. Also, $0 < \delta < 1$ is a time discount factor, $\gamma > 0$ is the relative risk aversion, $\psi > 0$ is the elasticity of inter-temporal substitution, and n_r is the number of risky assets.

Eq. (1) describes the preferences of an infinitely-lived representative household from a generalized recursive utility function (Epstein and Zin, 1989; Weil, 1990). The restriction $\gamma = \psi^{-1}$ yields the standard state- and time-separable Von Neumann-Morgenstern preferences. This implies that the household is indifferent to the timing of the resolution of uncertainty (of the temporal lottery over consumption). Conversely, $\gamma \neq \psi^{-1}$ yields non-separable preferences. In particular, the agent prefers an early (late) resolution of uncertainty when $\gamma > \psi^{-1} (\gamma < \psi^{-1})$. Eq. (2) represents the usual inter-temporal budget constraint. Eq. (3) defines the wealth portfolio return from the benchmark and excess returns and from the restriction that the portfolio shares of all assets sum to unity.

The problem (1)–(3) stipulates that the current consumption and contemporaneous portfolio shares correspond to the household's choice variables. Also, the current wealth is a predetermined variable, while future benchmark and excess returns are exogenous variables.

2.2. Dynamics of the state variables

We specify the law of motion describing the dynamics of the state variables as:

$$\begin{aligned} &\mathbf{s}_{t+1} = \mathbf{\Phi}_0 + \mathbf{\Phi}_1 \mathbf{s}_t + \mathbf{v}_{t+1}, \\ &\mathbf{v}_{t+1} \sim \text{NID}(\mathbf{0}, \mathbf{\Sigma}). \end{aligned} \tag{4}$$

Here, $\mathbf{s}_{t+1} = \begin{pmatrix}\mathbf{r}'_{t+1} & \mathbf{f}'_{t+1}\end{pmatrix}'$ is the $(n_s \times 1)$ vector of state variables, $\mathbf{r}_{t+1}$ is the $(n_r \times 1)$ vector of return variables which includes the benchmark and excess returns, $\mathbf{f}_{t+1}$ is the $(n_f \times 1)$ vector of factor variables which contains additional exogenous variables, and $\mathbf{v}_{t+1}$ is the $(n_s \times 1)$ vector of state-variable innovations which are assumed to follow a normal distribution with zero means and a homoscedastic structure. Also, $\mathbf{\Phi}_0$ is the $(n_s \times 1)$ vector of intercepts and $\mathbf{\Phi}_1$ is the $(n_s \times n_s)$ matrix of slope coefficients.

Eq. (4) is a first-order vector autoregressive (VAR) process. A baseline specification of this process imposes the absence of dynamic feedbacks between future and current state variables ($\mathbf{\Phi}_1 = \mathbf{0}$). These restrictions yield a constant investment opportunity set; i.e. the return variables are independently and identically distributed. The baseline specification will be useful shortly to study static portfolio allocations.

The alternative specifications of the VAR process capture the dependence of future return variables on their present values as well as on current factor variables ($\mathbf{\Phi}_1 \neq \mathbf{0}$). Our selected specifications will involve different factors, which are among the most widely used in the empirical asset-pricing literature, and which are well-known to have predictive power on returns. These specifications lead to time-varying investment opportunity sets. Again, this will prove to be useful below to study myopic and non-myopic portfolio allocations.

2.3. Decision rules

The shares are defined as the ratios of consumption and portfolio relative to wealth:

$$\alpha_{c,t} \equiv \frac{c_t}{w_t}, \tag{5}$$

$$\alpha_{i,t} \equiv \frac{w_{i,t}}{w_t}, \tag{6}$$

where $\alpha_{c,t}$ is the consumption share of wealth, $w_t = \sum_{i=1}^{n_r} w_{i,t}$, while $\alpha_{i,t}$ and $w_{i,t}$ are the portfolio share and the value of asset i, with $i = 1, \ldots, n_r$.

The optimal decision rules associated with the household's problem (1)–(3) and the VAR process (4) relate the shares (5) and (6) to the contemporaneous state variables. Unfortunately, the exact analytical solution is unknown for general values of the parameters describing the preferences of the household and the dynamics of the state variables. We circumvent this problem by approximating the decision rules from the numerical procedure developed in Campbell et al. (2003).

In brief, the approximation of the decision rules first relates the portfolio return to asset returns; this holds exactly in continuous time. The approximation also log-linearizes the budget constraint around the unconditional expectation of the log consumption share; this holds exactly for constant consumption shares. The approximation then relies on a second-order Taylor expansion of the Euler equations around the conditional expectations of consumption growth and returns; this holds exactly when the variables are conditionally normally distributed. The approximation is finally obtained by solving a recursive non-linear equation system whose coefficients are complex functions of the parameters involved in the preferences (1) and the VAR process (4). Whereas the system can be solved analytically when there is a unique state variable affecting a single risky asset return (Campbell and Viceira, 1999) or many risky asset returns (Normandin and St-Amour, 2002), it must be solved numerically in our multiple states and risky assets environment.

The solution establishes that the logarithm of the consumption share and the portfolio shares are respectively quadratic and affine in the state variables:

$$\log(\alpha_{c,t}) = b_0 + \mathbf{B}_1'\mathbf{s}_t + \mathbf{s}_t'\mathbf{B}_2\mathbf{s}_t, \tag{7}$$

$$\alpha_{i,t} = a_{0i} + \mathbf{A}_{1i}'\mathbf{s}_t, \tag{8}$$

$$\alpha_{1,t} = 1 - \sum_{i=2}^{n_r} \alpha_{i,t}. \tag{9}$$

The terms b_0 and a_{0i} are scalars, $\mathbf{B}_2$ is a $(n_s \times n_s)$ lower triangular matrix, whereas $\mathbf{B}_1$ and $\mathbf{A}_{1i}$ are $(n_s \times 1)$ vectors, with $i = 2, \ldots, n_r$. As mentioned above, these terms depend on the preference parameters δ, ψ, and γ in (1) and on the VAR parameters $\mathbf{\Phi}_0$, $\mathbf{\Phi}_1$, and $\mathbf{\Sigma}$ in (4), and are solved numerically.

Campbell and Koo (1997) show for the case of a single state variable that the approximation (7)–(9) is very precise, especially when the consumption share is not excessively volatile. Furthermore, the approximation nests the known exact analytical solutions obtained under myopic consumption behavior ($\psi = 1$) where the consumption share is constant, myopic portfolio allocation ($\gamma = 1$) where the demand for dynamic hedging portfolios is zero, and constant investment opportunity sets ($\mathbf{\Phi}_1 = \mathbf{0}$) (Giovannini and Weil, 1989).

It is worth stressing that the myopic consumption and portfolio rules become optimal for all values of ψ and γ when the investment opportunities are constant ($\mathbf{\Phi}_1 = \mathbf{0}$). That is, the household chooses consumption and the portfolio as if the planning horizon is only one period. Consequently, the household selects a fixed consumption share, regardless of its elasticity of inter-temporal substitution. Furthermore, the household never takes dynamic hedging positions, whatever its risk aversion. This behavior reflects the static portfolio allocation of a short-term investor.

In contrast, non-myopic rules are optimal for $\psi \neq 1$ and $\gamma \neq 1$, and when the investment opportunities are time-varying ($\mathbf{\Phi}_1 \neq \mathbf{0}$). In this case, the household forms decisions from a planning horizon that exceeds one period. As a result, the household chooses a consumption share that varies through time, as long as its elasticity of inter-temporal substitution differs from unity. Moreover, the household performs inter-temporal hedging to reduce its exposure to adverse changes in investment opportunities, as long as its risk aversion is larger than unity. This hedging behavior reflects the non-myopic portfolio allocation of a long-term investor.

In our analysis, we compare the decision rules (8) and (9) associated with the baseline and various alternative specifications of the VAR process (4) in order to verify (i) whether actual portfolios are static, myopic or non-myopic, and (ii) which (if any) factors are used by households in selecting asset holdings.

3. Return analysis

In this section, we estimate various specifications of the VAR process (4). We concentrate our attention on cash, bonds, and stocks. Given our objective, the specifications always include observable measures of return variables related to these assets. In particular, the return on cash is defined as the benchmark, $r_{1,t+1}$, and is measured as the real (net) ex post return on short-term Treasury bills, $r_{tb,t+1}$. Moreover, the excess returns, $xr_{i,t+1}$, are measured for bonds, $xr_{b,t+1}$, and for stocks, $xr_{s,t+1}$.

We also include different combinations of factor variables. We focus on the factors that are the most frequently invoked in the empirical asset-pricing literature. For example, the usual goods-market variables are the growth of production, $prod_t$, and the rate of inflation, inf_t. These variables seek to capture macroeconomic, and thus undiversifiable, risks (Chen et al., 1986). The conventional stock-market variables incorporate the (logarithm of the) dividend-price ratio, $(d_t - p_t)$, to improve the predictability of returns (Campbell et al., 2003), as well as the excess returns smb_t (Small Minus Big) and hml_t (High Minus Low) to capture unobserved economic risk exposure (Fama and French, 1993; Ferson and Harvey, 1999). Finally, the common bond-market variables $term_t$ and def_t measure the term-structure and default risks (Chen et al., 1986; Fama and French, 1993).

We define the various specifications from different mixes of return and factor variables. We estimate these specifications from post-war US quarterly data (see the Data Appendix). As is standard practice, Table 1 reports the OLS parameter estimates and the adjusted $\overline{R}^2$ statistics for the return equations, while Table 2 presents the cross-correlations between the innovations of return variables and other state variables. The specification and estimation results are the following.

BASIC. This is our baseline (or *BASIC*) specification. It relies on $\mathbf{s}_{t+1} = \mathbf{r}_{t+1}$ and $\mathbf{r}_{t+1} = (\,r_{tb,t+1} \quad xr_{b,t+1} \quad xr_{s,t+1}\,)'$, so that it includes all the return variables, but no factor variables. It also imposes the restrictions $\mathbf{\Phi}_1 = \mathbf{0}$ on the VAR process (4), so that investment opportunities are constant and portfolio allocations are therefore static.

The empirical results indicate that the intercept estimates of the VAR process correspond to the means of the (real, annualized) return variables. Also, the innovations of excess returns on stocks and bonds are statistically positively correlated. Finally, the $\overline{R}^2$ statistics are, by construction, null for all return equations.

FF. This specification captures the Fama–French (or *FF*) factors, which are smb_{t+1}, hml_{t+1}, and $xr_{s,t+1}$ for the excess returns on various stock portfolios and $term_{t+1}$ and def_{t+1} for the excess returns on corporate and government bonds. Our specification is $\mathbf{s}_{t+1} = (\,\mathbf{r}'_{t+1} \quad \mathbf{f}'_{t+1}\,)'$, where $\mathbf{r}_{t+1} = (\,r_{tb,t+1} \quad xr_{b,t+1} \quad xr_{s,t+1}\,)'$ and $\mathbf{f}_{t+1} = (\,smb_{t+1} \quad hml_{t+1} \quad term_{t+1} \quad def_{t+1}\,)'$. Thus, the selected factors include no goods-market variables, some stock-market variables, and all the bond-market variables. Moreover, our specification does not impose any restrictions on the dynamic feedbacks, $\mathbf{\Phi}_1 \neq \mathbf{0}$, so that investment opportunities are time-varying and portfolio allocations may be non-myopic.

Empirically, the return variables, stock-market variables, and bond-market variables all jointly statistically influence the returns on short-term Treasury bills as well as the excess returns on bonds, while the stock-market variables is the only group that significantly affects the excess returns on stocks. In addition, the correlations between the unexpected component of short-term returns or excess bond returns and the innovations of return variables as well as bond-market variables are jointly significant, while the correlations between unanticipated excess stock returns and the innovations of return variables as well as stock-market variables are jointly significant. Finally, the $\overline{R}^2$ is by far the largest for the benchmark-return equation, and is smaller for excess bond returns, and particularly, for excess stock returns.

CRR. This specification takes into account the Chen-Roll-Ross (or *CRR*) macroeconomic factors, which are $prod_{t+1}$, inf_{t+1} (decomposed in unexpected and expected terms), and $xr_{s,t+1}$, $term_{t+1}$, and def_{t+1} for excess returns on several stock portfolios. Our specification is $\mathbf{s}_{t+1} = (\,\mathbf{r}'_{t+1} \quad \mathbf{f}'_{t+1}\,)'$, where $\mathbf{r}_{t+1} = (\,r_{tb,t+1} \quad xr_{b,t+1} \quad xr_{s,t+1}\,)'$ and $\mathbf{f}_{t+1} = (\,prod_{t+1} \quad inf_{t+1} \quad term_{t+1} \quad def_{t+1}\,)'$. These factors include all the goods-market and bond-market variables, but no stock-market variables. Also, $\mathbf{\Phi}_1 \neq \mathbf{0}$, so that investment opportunities are time-varying and portfolio allocations may be non-myopic.

The estimates indicate that the return variables, goods-market variables, and bond-market variables jointly statistically affect short-term returns as well as excess stock returns, while only the return variables and bond-market variables jointly significantly predict excess bond returns. The correlations between the unexpected component of short-term returns or excess bond returns and the innovations of return variables, goods-market variables, as well as bond-market variables are jointly significant, whereas only the correlations between unanticipated excess stock returns and the innovations of return variables are jointly statistically different from zero. The $\overline{R}^2$ statistics reveal that short-term returns are in large part predictable, excess bond returns are less so, and excess stock returns are the least predictable.

CCV. This specification is based on the Campbell-Chan-Viceira (or *CCV*) factors. These factors include the three return variables as well as the dividend-price ratio, the term structure of interest rates, and the nominal (net) ex post return on short-term Treasury bills. Given that both the real and nominal ex post returns on short-term Treasury bills are included, an identical specification is obtained through Fisher's law by omitting the nominal return and incorporating the inflation rate. Our specification exploits this notion to yield $\mathbf{s}_{t+1} = (\,\mathbf{r}'_{t+1} \quad \mathbf{f}'_{t+1}\,)'$, where $\mathbf{r}_{t+1} = (\,r_{tb,t+1} \quad xr_{b,t+1} \quad xr_{s,t+1}\,)'$ and $\mathbf{f}_{t+1} = (\,inf_{t+1} \quad d_{t+1} - p_{t+1} \quad term_{t+1}\,)'$. Hence, this set of factors includes a single goods-market variable, one stock-market variable, and one bond-market variable. Again, $\mathbf{\Phi}_1 \neq \mathbf{0}$, so that investment opportunities are time-varying and portfolio allocations may be non-myopic.

The estimates indicate that all variable groups statistically influence the benchmark return, only the return variables and bond-market variable jointly significantly affect excess bond returns, whereas only the goods-market

Table 1
Return analysis: Estimates of the parameters

	BASIC			*FF*			*CRR*			*CCV*			*ALL*		
	$r_{tb,t+1}$	$xr_{b,t+1}$	$xr_{s,t+1}$	$r_{tb,t+1}$	$xr_{b,t+1}$	$xr_{s,t+1}$	$r_{tb,t+1}$	$xr_{b,t+1}$	$xr_{s,t+1}$	$r_{tb,t+1}$	$xr_{b,t+1}$	$xr_{s,t+1}$	$r_{tb,t+1}$	$xr_{b,t+1}$	$xr_{s,t+1}$
cst	0.017[a]	0.011	0.062[a]	−0.001	−0.021	0.011	−0.003	−0.001	0.115[b]	−0.024[a]	0.076	0.715[a]	−0.019[c]	0.113	0.850[a]
Return variables															
$r_{tb,t}$				0.705[a]	1.382[a]	−0.028	0.823[a]	1.573[a]	−3.833[a]	0.799[a]	1.333[a]	−0.305	0.821[a]	1.545[a]	−4.013[a]
$xr_{b,t}$				−0.013	−0.067	0.384[a]	−0.002	−0.076	0.016	−0.002	−0.066	0.439[a]	−0.001	−0.092	−0.053
$xr_{s,t}$				0.003	−0.050[b]	0.065	0.006[b]	−0.063[a]	−0.058	0.007[a]	−0.070[a]	−0.037	0.006[a]	−0.053[b]	−0.057
				[71.83[a]]	[5.998[a]]	[1.452]	[51.92[a]]	[7.698[a]]	[2.174[b]]	[89.42[a]]	[7.531[a]]	[1.595]	[52.28[a]]	[6.508[a]]	[2.240[b]]
Goods-market variables															
$prod_t$							−0.013	−0.382[b]	0.032				−0.007	−0.328[c]	0.262
inf_t							0.147[b]	−0.150	−5.654[a]	0.162[a]	−0.332	−2.877[a]	0.180[a]	−0.197	−6.516[a]
							[2.065[c]]	[1.558]	[5.375[a]]	[10.88[a]]	[0.698]	[6.419[a]]	[2.682[b]]	[1.158]	[6.958[a]]
Stock-market variables															
smb_t				−0.013	−0.289[b]	−1.013[a]							−0.017	−0.298[b]	−0.918[b]
hml_t				−0.040[a]	−0.122	−0.323							−0.033[b]	−0.133	−0.641[c]
$d_t - p_t$										−0.006[b]	0.026	0.163[a]	−0.004	0.033	0.210[a]
				[2.501[b]]	[1.929[c]]	[2.590[b]]				[3.112[b]]	[1.037]	[5.062[a]]	[2.281[b]]	[1.632]	[4.083[a]]
Bond-market variables															
$term_t$				0.114	2.420[a]	2.987	0.268[b]	2.427[a]	−4.343	0.248[a]	1.897[a]	0.844	0.317[a]	2.614[a]	−4.174
def_t				0.223[a]	−0.308	0.948	−0.057	−0.579	10.40[a]				−0.063	−0.555	10.93[a]
				[3.731[a]]	[3.760[a]]	[0.950]	[2.369[b]]	[2.950[b]]	[3.722[a]]	[5.181[a]]	[4.608[a]]	[0.112]	[3.313[a]]	[3.434[a]]	[4.375[a]]
$\overline{R}^2$	0.000	0.000	0.000	0.590	0.116	0.036	0.589	0.112	0.063	0.597	0.107	0.057	0.597	0.121	0.107

Note: Entries are the OLS parameter estimates of the return equations for the different specifications of the VAR process. Numbers in brackets are the *F* statistics of the test that the estimates associated with the return variables, the goods-market variables, the stock-market variables, or the bond-market variables are jointly null. *a*, *b*, and *c* indicate that the estimates are individually or jointly significant at the 5%, 10%, and 15% levels.

Table 2
Return analysis: Cross-correlations of the state-variable innovations

	BASIC			*FF*			*CRR*			*CCV*			*ALL*		
	r_{tb}	xr_b	xr_s	r_{tb}	xr_b	xr_s	r_{tb}	xr_b	xr_s	r_{tb}	xr_b	xr_s	r_{tb}	xr_b	xr_s
Return variables															
r_{tb}		−0.020	−0.020		−0.399[a]	−0.096		−0.385[a]	−0.046		−0.371[a]	−0.029		−0.404[a]	−0.047
xr_b	−0.020		0.197[a]	−0.399[a]		0.191[a]	−0.385[a]		0.216[a]	−0.371[a]		0.197[a]	−0.404[a]		0.186[a]
xr_s	−0.020	0.197[a]		−0.096	0.191[a]		−0.046	0.216[a]		−0.029	0.197[a]		−0.047	0.186[a]	
	[0.155]	[7.607[a]]	[7.607[a]]	[32.67[a]]	[37.96[a]]	[8.865[a]]	[29.17[a]]	[37.81[a]]	[9.462[a]]	[26.87[a]]	[34.23[a]]	[7.692[a]]	[32.09[a]]	[38.38[a]]	[7.140[a]]
Goods-market variables															
prod							0.389[a]	−0.222[a]	0.009				0.405[a]	−0.217[a]	0.035
inf							−0.767[a]	−0.119[b]	−0.072	−0.761[a]	−0.132[b]	−0.098	−0.766[a]	−0.096	−0.046
							[143.5[a]]	[12.31[a]]	[1.021]	[112.3[a]]	[3.380[b]]	[1.863]	[145.7[a]]	[10.923[a]]	[0.648]
Stock-market variables															
smb				−0.086	−0.015	0.416[a]							−0.080	−0.020	0.420[a]
hml				0.046	0.048	−0.374[a]							0.044	0.056	−0.387[a]
$d - p$										0.124[b]	0.020	0.059	0.097	−0.038	0.026
				[1.845]	[0.491]	[60.71[a]]				[2.983[b]]	[0.077]	[0.675]	[3.443]	[0.966]	[63.41[a]]
Bond-market variables															
term				−0.330[a]	0.147[a]	0.080	−0.312[a]	0.147[a]	0.025	−0.290[a]	0.126[b]	0.063	−0.297[a]	0.140[b]	−0.001
def				−0.271[a]	0.811[a]	0.056	−0.287[a]	0.824[a]	0.126[b]				−0.316[a]	0.825[a]	0.105[c]
				[35.37[a]]	[131.8[a]]	[1.850]	[34.86[a]]	[141.7[a]]	[3.201]	[16.32[a]]	[3.080[b]]	[0.770]	[36.48[a]]	[135.8[a]]	[2.139]

Note: Entries are the cross-correlations between the innovations of return variables and other state variables for the different specifications of the VAR process. Numbers in brackets are the χ^2 statistics of the Box-Pierce test that the cross-correlations between the innovations of each return variable and the other return variables, the goods-market variables, the stock-market variables, or the bond-market variables are jointly null. *a*, *b*, and *c* indicate that the cross-correlations are individually or jointly significant at the 5%, 10%, and 15% levels.

variable and stock-market variable statistically affect excess stock returns. The correlations between the unanticipated benchmark return and the innovations of all variable groups are jointly statistically different from zero, the correlations between unexpected excess bond returns and the innovations of return variables, goods-market variable, and bond-market variable are significant, while only the correlations between unanticipated excess stock returns and the innovations of return variables are jointly significant. Again, the $\overline{R}^2$ is the largest for the benchmark-return equation, is smaller for excess bond returns, and is the smallest for excess stock returns.

ALL. This specification nests every (or *ALL*) factors of the previous specifications. That is, $\mathbf{s}_{t+1} = (\mathbf{r}'_{t+1} \quad \mathbf{f}'_{t+1})'$, where $\mathbf{r}_{t+1} = (r_{tb,t+1} \quad xr_{b,t+1} \quad xr_{s,t+1})'$ and $\mathbf{f}_{t+1} = (prod_{t+1} \quad inf_{t+1} \quad smb_{t+1} \quad hml_{t+1} \quad d_{t+1} - p_{t+1} \quad term_{t+1} \quad def_{t+1})'$. Thus, this set includes all the goods-market, stock-market, and bond-market variables. Furthermore, $\mathbf{\Phi}_1 \neq \mathbf{0}$.

Empirically, every variable group statistically affects short-term returns and excess stock returns, while only the return variables and bond-market variables jointly significantly alter excess bond returns. Moreover, the correlations between the unexpected benchmark return or excess bond returns and the innovations of return variables, goods-market variables, as well as bond-market variables are jointly statistically different from zero, and the correlations between unexpected excess stock returns and the innovations of return variables and stock-market variables are jointly significant. Finally, the $\overline{R}^2$ statistics indicate that short-term returns are largely predictable, excess bond returns are more difficult to forecast, and excess stock returns are even more difficult to predict.

Overall, the return analysis reveals four important implications for portfolio allocations. First, both the significance levels of dynamic feedbacks and $\overline{R}^2$ statistics indicate that the baseline specification is rejected. This finding refutes the notion that investment opportunities are constant, and thus, that portfolio allocations should be static. Second, the significance levels of dynamic feedbacks show that certain variable groups affect the return variables for all alternative factor sets. This result accords with the empirical asset-pricing literature documenting the predictability of returns, and confirms that investment opportunities are time-varying. Third, the significance levels of innovation correlations highlight several co-movements between the return and factor variables. These co-movements are necessary conditions for dynamic hedging strategies, which accords with the fact that the empirical shares of cash, bonds, and stocks feature pronounced fluctuations through time (see Figs. 1 and 2). Fourth, the $\overline{R}^2$ statistics are very close across the various alternative factor sets. Consequently, the identification of the most influential factor set for portfolio allocations remains an open question. For this reason, rather than focusing on a single factorial specification, we next perform our portfolio analysis for every factor sets.

4. Portfolio analysis

This section compares the empirical and predicted household portfolio shares. For this purpose, the empirical shares are constructed for cash, bonds, and stocks by evaluating the definition (6) from a measure of (financial) wealth corresponding to the sum of the aggregate values of the three assets hold by households and from quarterly US data covering the post-war period (see the Data Appendix). In comparison, the values of cash, treasury securities, corporate equity, and total financial assets directly held by households in 2005 correspond to 64.1%, 11.7%, 30.0%, and 32.9% of those for all sectors, which also include businesses, governments, and non-residents (Source: Board of Governors of the Federal Reserve Bank, Balance Sheet of All Sectors).

Tabel 3 reports descriptive statistics for empirical portfolio shares computed from official and adjusted data. The official data record the market values of cash and stocks, but the book values of bonds. The adjusted data approximate the market values of bonds by discounting the principal amount outstanding of Treasury bonds from the Treasury constant maturity rate (Board of Governors of the Federal Reserve Bank) under the assumptions that

Table 3
Portfolio analysis: Empirical shares

	Mean			*Volatility*			*Comovement*		
	cash	*bond*	*stock*	*cash*	*bond*	*stock*	*cash − bond*	*cash − stock*	*bond − stock*
OfficialData	0.458	0.092	0.450	0.122	0.034	0.109	−0.497	−0.963	0.246
AdjustedData									
1 *year*	0.460	0.088	0.452	0.123	0.034	0.110	−0.513	−0.964	0.265
3 *years*	0.465	0.079	0.456	0.125	0.034	0.110	−0.539	−0.965	0.300
5 *years*	0.469	0.071	0.460	0.126	0.034	0.111	−0.557	−0.967	0.328
10 *years*	0.477	0.055	0.468	0.128	0.032	0.112	−0.578	−0.972	0.370
20 *years*	0.479	0.035	0.479	0.128	0.027	0.116	−0.534	−0.980	0.358

Note: *OfficialData*: Entries are the means, standard deviations, and cross-correlations of the empirical household portfolio shares of cash, bonds, and stocks computed from official data. These data record the market values of cash and stocks, but the book values of bonds. *AdjustedData*: Entries are the means, standard deviations, and cross-correlations of the empirical household portfolio shares of cash, bonds, and stocks computed from adjusted data. These data approximate the market values of bonds under the assumptions that the maturity of the debt is 1 year, 3 years, 5 years, 10 years, and 20 years.

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Importantly, the empirical portfolio shares computed from official and adjusted data exhibit similar features. In particular, both the empirical shares of cash and stocks display large means and volatilities, whereas the empirical share of bonds has much lower average and standard deviation. Also, the empirical share of cash exhibits negative co-movements with the empirical share of bonds as well as the one for stocks, while the empirical shares of bonds and stocks are positively correlated. Given the robustness of the results, the remaining of the paper uses the empirical portfolio shares computed from official data.

The predicted portfolio shares are constructed by evaluating the decision rules (8) and (9). To this end, we evaluate the coefficients of the decision rules from specific values for the parameters of the VAR process (4) and investor's preferences (1). For the VAR parameters, we use the OLS estimates presented previously for the various specifications. The baseline (alternative) specification permits us to test constant (time-varying) investment opportunity sets obtained from $\mathbf{\Phi}_1 = \mathbf{0}(\mathbf{\Phi}_1 \neq \mathbf{0})$ from a portfolio, rather than returns' perspective.

For the preference parameters, we calibrate the time discount factor to the standard value of $\delta = 0.979$, which corresponds to a quarterly (net) discount rate of 2.1%. We also set the elasticity of inter-temporal substitution to the single value $\psi = 1/2$, given that alternative values have no effect on the predicted portfolio shares. A similar invariance result is obtained for the cases of single risky asset and state variable (Campbell and Viceira, 1999), multiple risky assets and unique state variable (Normandin and St-Amour, 2002), and several risky assets and state variables (Campbell et al., 2003). We further fix the relative risk aversion to the values $\gamma = 1, 2, 5$, and 10. These calibrations are reasonable given the widely accepted beliefs about attitudes towards risk (Mehra and Prescott, 1985). Moreover, the different calibrations allow us to test myopic (non-myopic) portfolio rules induced by $\gamma = 1(\gamma > 1)$.

Finally, we perform formal statistical tests to confront the empirical and predicted portfolio shares by focusing on their means, volatilities, and co-movements. These tests rely on $\chi^2(1)$-distributed Wald statistics that take into account the uncertainty related to the estimates of the VAR parameters, using the δ-method. Table 4 presents the averages, Table 5 reports the standard deviations, and Table 6 shows the correlations of the predicted portfolio shares.

BASIC. A non-conservative investor ($\gamma = 1$) takes, on average, a short position on cash, and long positions on bonds and stocks. Thus, the investor borrows a sizable share of his wealth in the less risky asset to finance large holdings in bonds and stocks. This occurs because the Sharpe ratios associated with excess returns are positive for both bonds and stocks. In addition, the share invested in stocks is always larger than that held in bonds. This is due to the notion that stocks display the largest Sharpe ratio. More precisely, the Sharpe ratios are 0.152 for bonds and 0.355 for stocks. These ratios are computed as the sum of the mean and half of the variance of the quarterly (annualized) excess return, normalized by its standard deviation.

Table 4
Portfolio analysis: Mean

	BASIC			*FF*			*CRR*		
γ	*cash*	*bond*	*stock*	*cash*	*bond*	*stock*	*cash*	*bond*	*stock*
1	−0.774[b]	0.735	1.039[a]	−0.915	0.834	1.080[a]	−0.862	0.752	1.110[a]
	(0.662)	(0.660)	(0.233)	(2.181)	(2.337)	(0.278)	(3.015)	(4.002)	(0.100)
2	0.111	0.369	0.520	0.232	0.191	0.577	0.394	−0.048	0.654
	(0.331)	(0.330)	(0.116)	(82.14)	(143.8)	(61.77)	(967.1)	(967.5)	(15.69)
5	0.642[c]	0.149	0.209[a]	0.688	0.058	0.255	0.904	−0.177	0.273
	(0.125)	(0.132)	(0.047)	(164.5)	(250.6)	(86.22)	(2754)	(2707)	(47.86)
10	0.819[a]	0.076	0.105[a]	0.796	0.058	0.146	0.694	0.167	0.139
	(0.067)	(0.067)	(0.024)	(210.8)	(308.3)	(97.63)	(3704)	(3625)	(79.31)

	CCV			*ALL*		
γ	*cash*	*bond*	*stock*	*cash*	*bond*	*stock*
1	−0.883	0.796	1.087[c]	−0.416	−0.562	1.977
	(7.871)	(12.77)	(0.423)	(367.3)	(374.05)	(324.24)
2	3.726	−3.469	0.743	5.411	−6.022	1.611
	(1080)	(1058)	(23.28)	(244.3)	(224.0)	(256.3)
5	3.823	−3.109	0.285	7.391	−7.111	0.720
	(2881)	(2847)	(35.17)	(561.0)	(653.1)	(99.97)
10	2.539	−1.657	0.119	5.748	−5.078	0.330
	(3884)	(3830)	(54.83)	(1632)	(1634)	(15.50)

Note: Entries are the means of the predicted household portfolio shares. The means of the empirical household portfolio shares are 0.458 for cash, 0.092 for bonds, and 0.450 for stocks. Numbers in parentheses are the standard errors of the predicted means. *a*, *b*, and *c* indicate that the difference between the predicted and empirical means is significant at the 5%, 10%, and 15% levels. These tests use the variance of the difference, which is computed as $D'\Xi D$ – where D is the vector of numerical derivatives of the difference with respect to the parameters of the VAR process, and Ξ is the covariance matrix of these parameters.

Table 5
Portfolio analysis: Volatility

	BASIC			*FF*			*CRR*		
γ	*cash*	*bond*	*stock*	*cash*	*bond*	*stock*	*cash*	*bond*	*stock*
1	0.000[a]	0.000[a]	0.000[a]	3.856	3.889	0.880	3.736	4.005	1.147
	(0.000)	(0.000)	(0.000)	(4.293)	(5.524)	(0.9677)	(7.003)	(8.110)	(1.108)
2	0.000[a]	0.000[a]	0.000[a]	1.950	1.955	0.436	1.904	2.122	0.577
	(0.000)	(0.000)	(0.000)	(2.368)	(3.229)	(0.834)	(15.81)	(17.27)	(3.661)
5	0.000[a]	0.000[a]	0.000[a]	0.788	0.787	0.173	0.826	0.919	0.232
	(0.000)	(0.000)	(0.000)	(1.615)	(1.871)	(0.465)	(9.180)	(10.53)	(2.307)
10	0.000[a]	0.000[a]	0.000[a]	0.395	0.394	0.087	0.429	0.475	0.117
	(0.000)	(0.000)	(0.000)	(2.519)	(2.443)	(0.288)	(5.095)	(5.978)	(1.313)

	CCV			*ALL*		
γ	*cash*	*bond*	*stock*	*cash*	*bond*	*stock*
1	3.647	3.717	1.016	4.036	4.238	1.211
	(4.918)	(6.435)	(1.063)	(8.231)	(5.727)	(4.213)
2	1.921	1.993	0.503	2.862	3.045	0.726
	(2.313)	(2.747)	(1.618)	(10.62)	(4.207)	(2.413)
5	0.796	0.829	0.201	1.356	1.440	0.290
	(0.958)	(1.080)	(0.941)	(4.903)	(2.745)	(0.808)
10	0.403	0.421	0.100	0.723	0.766	0.145
	(0.498)	(0.543)	(0.521)	(3.012)	(2.249)	(0.405)

Note: Entries are the standard deviations of the predicted household portfolio shares. The standard deviations of the empirical household portfolio shares are 0.122 for cash, 0.034 for bonds, and 0.109 for stocks. Numbers in parentheses are the standard errors of the predicted standard deviations. *a*, *b*, and *c* indicate that the difference between the predicted and empirical standard deviations is significant at the 5%, 10%, and 15% levels. These tests use the variance of the difference, which is computed as $D'\Xi D$ – where D is the vector of numerical derivatives of the difference with respect to the parameters of the VAR process, and Ξ is the covariance matrix of these parameters.

Table 6
Portfolio analysis: Co-movement

	BASIC			*FF*			*CRR*		
γ	*cash − bond*	*cash − stock*	*bond − stock*	*cash − bond*	*cash − stock*	*bond − stock*	*cash − bond*	*cash − stock*	*bond − stock*
1	0.000[a]	0.000[a]	0.000[a]	−0.974[a]	−0.076	−0.151	−0.959[a]	0.089	−0.369
	(0.000)	(0.000)	(0.000)	(0.056)	(1.508)	(1.387)	(0.056)	(2.001)	(1.203)
2	0.000[a]	0.000[a]	0.000[a]	−0.975[a]	−0.098	−0.125	−0.965[a]	0.248	−0.494
	(0.000)	(0.000)	(0.000)	(0.123)	(2.473)	(2.059)	(0.206)	(4.465)	(3.641)
5	0.000[a]	0.000[a]	0.000[a]	−0.976[a]	−0.114	−0.106	−0.970[b]	0.283	−0.507
	(0.000)	(0.000)	(0.000)	(0.182)	(3.770)	(3.192)	(0.284)	(9.042)	(7.223)
10	0.000[a]	0.000[a]	0.000[a]	−0.976	−0.120	−0.100	−0.972[c]	0.286	−0.503
	(0.000)	(0.000)	(0.000)	(0.410)	(4.252)	(4.289)	(0.319)	(11.20)	(9.000)

	CCV			*ALL*		
γ	*cash − bond*	*cash − stock*	*bond − stock*	*cash − bond*	*cash − stock*	*bond − stock*
1	−0.962[a]	−0.070	−0.204	−0.958[a]	0.021	−0.306
	(0.115)	(1.661)	(1.353)	(0.229)	(7.898)	(8.180)
2	−0.968[a]	0.015	−0.267	−0.972[a]	0.133	−0.364
	(0.230)	(1.614)	(1.182)	(0.171)	(10.35)	(10.22)
5	−0.970[c]	0.042	−0.283	−0.980[a]	0.193	−0.383
	(0.313)	(1.854)	(1.378)	(0.104)	(10.03)	(9.601)
10	−0.971[c]	0.051	−0.287	−0.983[a]	0.210	−0.387
	(0.325)	(1.884)	(1.466)	(0.124)	(9.400)	(8.991)

Note: Entries are the correlations between the predicted household portfolio shares. The correlations of the empirical household portfolio shares are −0.497 between cash and bonds, −0.963 between cash and stocks, and 0.246 between bonds and stocks. Numbers in parentheses are the standard errors of the predicted correlations. *a*, *b*, and *c* indicate that the difference between the predicted and empirical correlations is significant at the 5%, 10%, and 15% levels. These tests use the variance of the difference, which is computed as $D'\Xi D$ – where D is the vector of numerical derivatives of the difference with respect to the parameters of the VAR process, and Ξ is the covariance matrix of these parameters.

In contrast, a conservative investor ($\gamma > 1$) takes long positions in all assets, as the share of cash increases and those in bonds and stocks decrease when the relative risk aversion increases. This is explained by the static portfolio allocation: a larger demand for cash allows the investor to diversify away the risk, given that the unexpected benchmark return is negatively correlated with unanticipated excess returns on bonds and on stocks (see Table 2).

However, there is no such increase in the demand for bonds, since unanticipated excess returns on bonds and stocks are positively correlated.

For almost all reasonable values of relative risk aversion, the averages of the predicted portfolio shares of cash and stocks are significantly different from those of the empirical shares. Furthermore, the zero standard deviations and correlations of the predicted shares are always statistically different from their empirical counterparts. In sum, these test results indicate that the investor's static behavior is rejected by the data.

FF. A myopic investor ($\gamma = 1$) takes, on average, a short position on cash, and long positions on bonds and stocks. This myopic behavior is similar than the static behavior just explained for the *BASIC* case. This arises because both the myopic and static portfolio allocations abstract from hedging strategies.

From a statistical perspective, the myopic behavior predicts a mean for the share of stocks and a correlation between the shares of cash and bonds that are significantly different from their empirical counterparts. In addition, all the means (in absolute values) and volatilities, as well as most correlations (in absolute values) of the predicted shares largely numerically over-state those of the empirical shares. These findings are not in favor of the myopic behavior.

A non-myopic investor ($\gamma > 1$) takes long positions in all assets, as the share of cash increases and those in bonds and stocks decrease when the relative risk aversion increases. Interestingly, the non-myopic behavior implies that the decrease in the share of stocks (bonds) is less (more) pronounced, relative to that found from the static behavior. This suggests that stocks are better dynamic hedges against adverse changes in investment opportunities.

Statistically, the non-myopic behavior predicts means and volatilities that are never significantly different from those found in the data. This behavior further predicts correlations for the shares of cash and bonds with that of stocks that are never statistically different from the empirical ones, whereas most of the predicted correlations between the shares of cash and bonds are significantly different from the data. Admittedly, most of these inference results reflect the fact that the predicted portfolio shares are very imprecisely estimated. As a result, the confidence intervals around the point estimates of the various moments are very large, and as such typically include the values of the moments computed from the empirical portfolio shares. This inefficiency problem is especially important when the investment opportunity set is time variant. Also, this problem becomes even more severe as the risk aversion increases.

Nevertheless, it is worth stressing that the predicted shares exhibit the appropriate signs and magnitudes for the means when $\gamma = 2$ and 5 for all assets, and adequate volatilities when $\gamma = 5$ and 10 for stocks. In addition, the predicted correlations for the share of cash with those of bonds and stocks display the correct signs, regardless of the value of γ. Overall, these results provide empirical support for the non-myopic portfolio allocation obtained by combining reasonable degrees of risk aversions with the *FF* set of factors.

CRR. As above, a myopic investor ($\gamma = 1$) takes, on average, a short position on cash, and long positions on bonds and stocks. Also, the predicted mean for the share of stocks and correlation between the shares of cash and bonds are statistically at odds with the data. Finally, all the predicted means (in absolute values) and volatilities over-estimate the empirical ones, while the predicted correlations often display the wrong signs. Again, these findings indicate that the myopic behavior is refuted.

A non-myopic investor ($\gamma > 1$) usually takes long positions in cash and stocks, but short positions in bonds. Also, the non-myopic behavior induced by the *CRR* factor set implies that the fall in demand for bonds is so pronounced that it drives the share of bonds to be negative, in contrast to that obtained from the *FF* case.

The test results reveal that the non-myopic behavior associated with the *CRR* specification predicts means and volatilities that are never statistically different from the empirical ones. Also, the predicted correlations for the shares of cash and bonds with that of stocks are never statistically different from the empirical ones, whereas the predicted correlations between the shares of cash and bonds are significantly different from the data. Again, most of these inference results reflect the fact that the predicted portfolio shares are very imprecisely estimated.

However, the predicted means for the share of bonds is almost always negative, whereas the empirical counterpart is positive. Moreover, the correlations between the shares of cash and stocks exhibit the wrong sign, for almost all reasonable values of γ. For these reasons, the non-myopic portfolio allocation derived from the *CRR* specification is performing worse than the one related to the *FF* factor set.

CCV. A myopic investor ($\gamma = 1$) takes, on average, identical asset positions as those explained previously. Also, this myopic behavior features similar statistical and numerical properties as those described above. Consequently, the myopic behavior is once again inconsistent with the data.

A non-myopic investor ($\gamma > 1$) takes similar asset positions as those explained for the *CRR* specification. However, the predicted means for the portfolio shares seem economically rather implausible, as they over-state the empirical ones by a large order of magnitude. Moreover, the predicted means for the share of bonds and correlations between the shares of cash and stocks systematically display the wrong signs. Hence, the non-myopic portfolio allocation derived from the *CCV* specification is also less attractive than the one associated with the *FF* factor set.

ALL. A myopic investor ($\gamma = 1$) is characterized by a behavior that is numerically and statistically close to that documented above. As a result, the myopic behavior is once more time at odds with the data.

A non-myopic investor ($\gamma > 1$) has a behavior that numerically and statistically parallels those discussed for the *CCV* specification. For this reason, the non-myopic portfolio allocation derived from the *ALL* factor set is less appropriate than the one associated with the *FF* case.

So far, our portfolio analysis highlights two key findings for the assessment of portfolio allocations. First, the various moments of the empirical portfolio shares are never replicated from the baseline specification, nor from the combinations of any alternative factor sets with a relative risk aversion of one. This empirical evidence refutes both static and myopic investment behaviors. Second, the properties of the empirical portfolio shares are best explained by the non-myopic allocations obtained by combining the *FF* factors with reasonable values of relative risk aversion larger than unity. For example, this case yields predictions that are almost always statistically appropriate, and that exhibit the correct signs and numerical magnitudes. In particular, this is the only case for which the predicted means for the share of bonds and correlations between the shares of cash and stocks display the adequate signs. For these reasons, the *FF* case is our preferred specification.

Finally, we compare the empirical shares to those predicted from various subsets of the *FF* factors to detect which variables of our preferred specification are the most important to explain portfolio allocations. (This exercise is performed by setting the relative risk aversion to $\gamma = 5$. The results are similar when $\gamma = 2$ and $\gamma = 10$.) Table 7 reports the cross-correlations between the empirical shares and those computed from all or some variables of the *FF* factors, whereas Fig. 3 plots the empirical shares and a selection of the predicted shares. The correlations between the empirical shares and those predicted from all the *FF* factors are positive, but small and insignificantly different than zero. Also, the correlations between the empirical share of cash and those obtained from the return variables, the stock-market variables, and the bond-market variables are never significant. However, the correlation is positive, numerically large, and statistically significant when def_t is used. In fact, the drastic increase of the empirical share of cash from the mid 1970s to the late 1980s is well predicted by the share constructed from def_t. Likewise, the correlations between the empirical share of stocks and that predicted from the return variables, the stock-market variables, and the bond-market variables are never significant. However, the correlations are significantly positive and about the same size when $r_{tb,t}$ and hml_t are invoked. The empirical share of stocks displays positive comovements with those related to hml_t from the beginning of the 1960s to the mid 1970s and to $r_{tb,t}$ from the mid 1970s to the late 1990s. In contrast, the correlations between the empirical share of bonds and that computed from the stock-market variables and the bond-market variables are positively significant. In particular, the correlation attains the largest positive value when def_t is used. The mild increase of the empirical share of bonds during the post-1980 period is better captured than the slow decline of the pre-1980 episode by the predicted share associated with def_t. Overall, this analysis suggests that def_t is the key variable of the *FF* factors to explain the shares of cash and bonds, whereas $r_{tb,t}$ and hml_t are the prime determinants of the share of stocks.

Table 7
Portfolio analysis: Cross-correlations of empirical and predicted shares

	cash	*bond*	*stock*
FF factors	0.136	0.010	0.126
	(0.098)	(0.065)	(0.089)
Return variables	−0.162	−0.121	0.015
	(0.117)	(0.085)	(0.080)
r_{tb}	−0.145	−0.098	0.132[b]
	(0.101)	(0.069)	(0.078)
xr_b	0.057	−0.042	−0.077
	(0.082)	(0.055)	(0.079)
xr_s	−0.107	−0.068	0.099
	(0.079)	(0.059)	(0.074)
Stock-market variables	0.094	0.157[a]	0.048
	(0.070)	(0.056)	(0.072)
smb	0.049	0.133[a]	0.014
	(0.696)	(0.057)	(0.072)
hml	0.113	0.066	0.106[c]
	(0.084)	(0.053)	(0.069)
Bond-market variables	−0.094	0.115[a]	−0.293
	(0.083)	(0.052)	(0.258)
term	−0.209	−0.012	−0.229
	(0.154)	(0.210)	(0.161)
def	0.652[a]	0.420[a]	−0.201
	(0.040)	(0.041)	(0.113)

Note: Entries are the cross-correlations between the empirical and predicted household portfolio shares. The predicted portfolio shares are computed by combining a relative risk aversion of $\gamma = 5$ with all or some variables involved in the *FF* factors. Numbers in parentheses are the standard errors computed by the generalized method of moments. *a*, *b*, and *c* indicate that the cross-correlation is significant at the 5%, 10%, and 15% levels.

5. Consumption analysis

In this section, we verify for completeness whether the predicted consumption behavior accords with the data. The empirical consumption share is constructed by evaluating the definition (5) from quarterly aggregate US data for the post-war period (see the Data Appendix). This share exhibits a mean of 0.135 and a standard deviation of 0.026.

The predicted consumption shares are constructed by evaluating the decision rule (7). To do so, we fix the VAR parameters to their OLS estimates. The baseline (alternative) specification enables us to test constant (time-varying) investment opportunity sets derived from $\mathbf{\Phi}_1 = \mathbf{0}(\mathbf{\Phi}_1 \neq \mathbf{0})$, where the consumption share is fixed (variable). For briefness, we limit our analysis of the alternative VAR processes to the *FF* case, i.e. our preferred specification for portfolio shares.

As before, we use the standard calibration $\delta = 0.979$ and the reasonable values $\gamma = 1, 2, 5$, and 10. This time, however, we consider the following calibrations $\psi = 1, 1/2$,

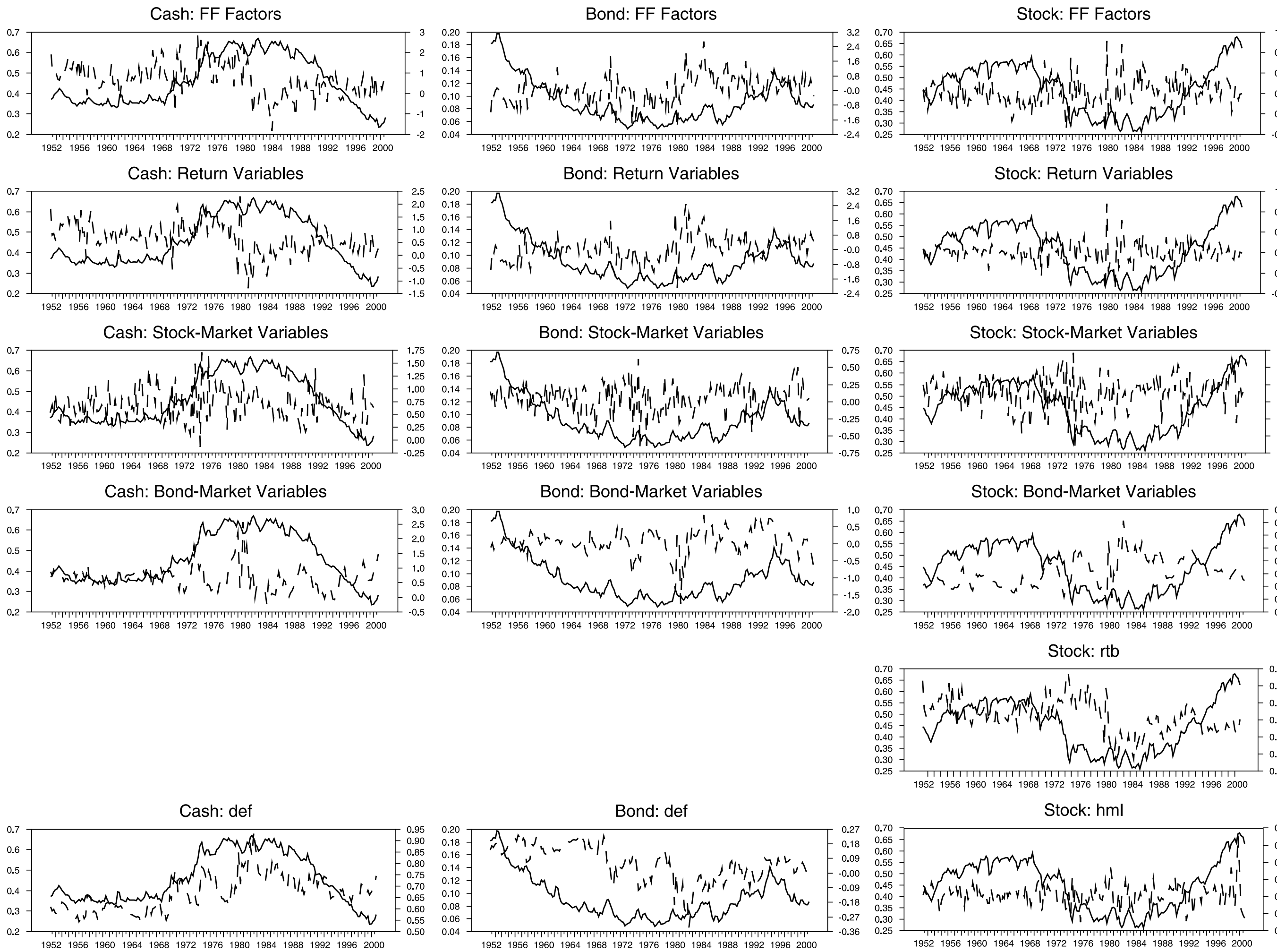

Fig. 3. The solid (dashed) lines represent the empirical (predicted) household portfolio shares. The left-hand (right-hand) scale is for the empirical (predicted) shares. The predicted shares are computed by combining a relative risk aversion of $\gamma = 5$ with all or some variables involved in the *FF* factors. To ease comparisons, all the predicted shares are normalized to have identical means than those obtained from the full set of *FF* factors.

1/5 and 1/10, given that the elasticity of inter-temporal substitution is known to affect the predicted consumption shares (Campbell and Viceira, 1999; Normandin and St-Amour, 2002). The different calibrations permit us to study consumption shares predicted by separable (non-separable) preferences obtained from $\gamma = \psi^{-1}$ ($\gamma \neq \psi^{-1}$). In addition, we can test myopic (non-myopic) consumption rules induced by $\psi = 1 (\psi < 1)$, where the consumption share is fixed (variable).

Again, we apply formal statistical tests to confront the empirical and predicted consumption shares by focusing on their means and volatilities. Table 8 presents the averages and the standard deviations of the predicted consumption shares.

BASIC. The consumer always spends a constant fraction of his wealth, where this proportion is equal to the reasonable calibration for the quarterly (net) discount rate. From an economic perspective, this is a consequence of the absence of changes in investment opportunities, imposed by the *BASIC* specification. From a statistical perspective, this implies that there is no uncertainty related to the estimates of consumption shares. Hence, the predicted means and volatilities are always significantly different from the empirical counterparts.

FF. A myopic consumer ($\psi = 1$) has an identical spending pattern than that observed for the *BASIC* specification. This time, however, this is explained by the exact cancellation of the inter-temporal substitution and income effects associated with changes in investment opportunities. Statistically, the predicted means and volatilities are significantly at odds with the data.

A non-myopic consumer ($\psi < 1$) has consumption shares that always exhibit positive means and volatilities. In addition, these predicted means and volatilities decrease in relative risk aversion. This suggests that a highly risk-averse agent prefers to reduce his consumption exposure to changes in the states.

Statistically, the non-myopic behavior predicts means and volatilities that are never significantly different from those found in the data. These inference results reflect the fact that the predicted consumption shares are imprecisely estimated. Also, the predicted means and volatilities greatly

Table 8
Consumption analysis: Mean and volatility

		BASIC		*FF*	
γ	$1/\psi$	*Mean*	*Volatility*	*Mean*	*Volatility*
1	1	0.021[a]	0.000[a]	0.021[a]	0.000[a]
		(0.000)	(0.000)	(0.000)	(0.000)
1	2	0.021[a]	0.000[a]	1.190	0.667
		(0.000)	(0.000)	(2.221)	(1.245)
1	5	0.021[a]	0.000[a]	18.88	22.00
		(0.000)	(0.000)	(32.06)	(37.37)
1	10	0.021[a]	0.000[a]	55.25	79.40
		(0.000)	(0.000)	(463.8)	(666.8)
2	1	0.021[a]	0.000[a]	0.021[a]	0.000[a]
		(0.000)	(0.000)	(0.000)	(0.000)
2	2	0.021[a]	0.000[a]	0.137	0.035
		(0.000)	(0.000)	(1.034)	(0.268)
2	5	0.021[a]	0.000[a]	0.456	0.212
		(0.000)	(0.000)	(4.563)	(2.122)
2	10	0.021[a]	0.000[a]	0.751	0.410
		(0.000)	(0.000)	(4.035)	(2.201)
5	1	0.021[a]	0.000[a]	0.021[a]	0.000[a]
		(0.000)	(0.000)	(0.000)	(0.000)
5	2	0.021[a]	0.000[a]	0.037	0.005
		(0.000)	(0.000)	(1.245)	(0.168)
5	5	0.021[a]	0.000[a]	0.055	0.013
		(0.000)	(0.000)	(2.167)	(0.490)
5	10	0.021[a]	0.000[a]	0.065	0.017
		(0.000)	(0.000)	(1.468)	(0.380)
10	1	0.021[a]	0.000[a]	0.021[a]	0.000[a]
		(0.000)	(0.000)	(0.000)	(0.000)
10	2	0.021[a]	0.000[a]	0.023	0.002
		(0.000)	(0.000)	(2.968)	(0.295)
10	5	0.021[a]	0.000[a]	0.023	0.004
		(0.000)	(0.000)	(5.651)	(0.911)
10	10	0.021[a]	0.000[a]	0.024	0.005
		(0.000)	(0.000)	(1.770)	(0.313)

Note: Entries are means and standard deviations of the predicted household consumption share. The mean and standard deviation of the empirical household consumption share are 0.135 and 0.026. Numbers in parentheses are the standard errors of the predicted means and standard deviations. *a*, *b*, and *c* indicate that the difference between the predicted and empirical moments is significant at the 5%, 10%, and 15% levels. These tests use the variance of the difference, which is computed as $D'\Xi D$ – where D is the vector of numerical derivatives of the difference with respect to the parameters of the VAR process, and Ξ is the covariance matrix of these parameters.

numerically under-state the empirical ones, as long as $\gamma = 1$. In contrast, the predicted means and volatilities exhibit the appropriate magnitudes for the separable preferences $\gamma = \psi^{-1} = 2$ and non-separable preferences $\gamma = 5$ and $\psi^{-1} = 10$. Thus, combining these reasonable calibrations for the preference parameters with the estimates for the *FF* specification parameters yields predictions that accord with the data.

In brief, the consumption analysis reveals two important results for the evaluation of portfolio allocations. First, the moments of the empirical consumption share are never reproduced from the *BASIC* specification, nor from the combination of the *FF* factors with a relative risk aversion of one. These facts confirm the rejection of both static portfolio allocations and myopic investment behaviors. Second, the properties of the empirical consumption share can be recovered by combining the *FF* specification with reasonable values of relative risk aversion larger than unity. Interestingly, this accords with our earlier findings in favor of non-myopic portfolio allocations.

Overall, this paper has highlighted the important time variation in US aggregate household portfolio allocations. Given this evidence, we asked: (i) whether these portfolios are best described as static, myopic or non-myopic, and (ii) which set of factors (if any) are used by households in selecting asset holdings. To analyze this, we first used flexible descriptions of preferences and investment opportunities to derive optimal decision rules that nest static, myopic, and non-myopic portfolio allocations. We then compared these rules to the data through formal statistical analysis. Our main results revealed that (i) static and myopic investment behaviors are rejected, (ii) non-myopic portfolio allocations are supported, and (iii) the Fama–French factors best explain empirical portfolio shares.

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Data appendix

This appendix describes the quarterly US data covering the 1952:II-2000:IV period.

Portfolio and consumption variables

cash: Portfolio share of cash. This corresponds to the value of cash hold by households (and non-profit organizations) relative to wealth. The value of cash is measured by the seasonally unadjusted US nominal checkable deposits and currency plus nominal time and saving deposits (Source: Board of Governors of the Federal Reserve Bank, Balance Sheet of Households and Non-profit Organizations).

bond: portfolio share of bonds. This is the value of bonds hold by households (and non-profit organizations) divided by wealth. The value of bonds is captured by the seasonally unadjusted nominal US government securities (Source: Board of Governors of the Federal Reserve Bank, Balance Sheet of Households and Non-profit Organizations).

stock: portfolio share of stocks. This is the value of stocks hold by households (and non-profit organizations) normalized by wealth. The value of stocks corresponds to the seasonally unadjusted US nominal corporate equities (Source: Board of Governors of the Federal Reserve Bank, Balance Sheet of Households and Non-profit Organizations).

cons: consumption share. This is the value of consumption of households divided by wealth. The value of consumption is measured by the seasonally adjusted US nominal private consumption expenditures on non-durable goods and services (Source: US Department of Commerce, Bureau of Economic Analysis).

wealth. This is the sum of the values of cash, bonds, and stocks.

Return variables

$r_{tb,t}$: Ex post real Treasury bill rate. This is the difference between the quarterly (annualized) nominal return on 90-day US Treasury bill (Source: Center for Research in Security Prices) and the inflation rate.

$xr_{b,t}$: excess bond return. This is the difference between the quarterly (annualized) nominal return on five-year US Treasury bonds (Source: Center for Research in Security Prices) and the quarterly (annualized) nominal return on 90-day US Treasury bill.

$xr_{s,t}$: excess stock return. This is the difference between the quarterly (annualized) nominal value-weighted return (including dividends) on the NYSE, NASDAQ, and AMEX markets (Source: Center for Research in Security Prices) and the quarterly (annualized) nominal return on 90-day US Treasury bill.

Goods-market variables

inf_t: Inflation rate. This is the quarterly (annualized) growth rate of the seasonally adjusted US gross domestic product implicit deflator (Source: US Department of Commerce, Bureau of Economic Analysis).

$prod_t$: production growth. This is the difference between the quarterly (annualized) growth rate of the seasonally adjusted US nominal gross domestic product (Source: US Department of Commerce, Bureau of Economic Analysis) and the inflation rate.

Equity-market variables

smb_t (Small Minus Big): Excess small-portfolio return. This is the difference between the quarterly (annualized) average return on three small US portfolios and the quarterly (annualized) average return on three big US portfolios (Source: http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html).

hml_t (High Minus Low): excess value-portfolio return. This is the difference between the quarterly (annualized) average return on two value US portfolios and the quarterly (annualized) average return on two growth US portfolios (Source: http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html).

$d_t - p_t$: dividend-price ratio. This is the difference between the logarithm of the dividend payout and the logarithm of the price index. The dividend payout and the price index are calculated from the value-weighted returns (including and excluding dividends) on the NYSE, NASDAQ, and AMEX markets (Source: Center for Research in Security Prices).

Bond-market variables

$term_t$: Excess long-term government-bond return. This term structure of interest rates is the difference between the quarterly (annualized) interest rate on five-year zero-coupon US government bonds (Source: Center for Research in Security Prices) and the quarterly (annualized) nominal return on 90-day US Treasury bill.

def_t: excess long-term corporate-bond return. This bond default premium is the difference between the quarterly (annualized) nominal yield on Baa US corporate bonds (Source: Moody's Investors Service) and the quarterly (annualized) nominal return on 90-day US Treasury bill.

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