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Recursive measures of total wealth and portfolio return

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This paper presents and assesses a procedure to generate recursive measures of aggregate total wealth and portfolio return. The procedure is more flexible and yields more realistic measures, compared to the classical replacement cost and present value methods.

I. Introduction

By definition, total wealth corresponds to the sum of tangible, financial, and human asset values. Theoretically, current (future) total wealth corresponds to the main state (control) variable in many dynamic programming problems. Thus, it is not surprising that total wealth plays a central role in fundamental economic and financial dynamic theories. For example, consumption theories reveal that total wealth is intimately related to permanent income, and as such is a prime determinant of consumption. Also, asset-pricing theories teach that return on total wealth, or market portfolio return, is closely linked to the price of market risk.

However, there is an important gap between these dynamic theories and their empirical applications. That is, the theories are ahead of measurements because no official, unique, or well-accepted measures of total wealth and portfolio return, exists. In fact, there exist various measures obtained from conceptually different methods. For example, a classical method to measure total wealth consists in evaluating the costs required to replace tangible, financial, and human assets (e.g. Kendrick, 1976; Eisner, 1989). Unfortunately, exhaustive assessments of the replacement costs are most challenging. An alternative classical method to measure total wealth consists in evaluating the present value of expected future net incomes generated by the various assets (e.g. Jorgenson and Fraumeni, 1989; Campbell, 1996). Unfortunately, tractable assessments of the present value impose strong restrictions on future income growth and discount rate.

This paper presents and assesses a procedure to generate recursive measures of aggregate total wealth and portfolio return. The procedure offers the considerable advantage of being easy to implement. It solves recursively a bivariate system, which possesses a clear economic interpretation under few assumptions frequently invoked in the consumption and asset pricing literatures. The procedure also offers important advantages, relative to the classical methods. It does not only yield measures of total wealth, but also of portfolio return, and it does not require the assessment of any replacement cost nor the imposition of any restriction on future income growth and discount rate.

The procedure is illustrated for the post-war US economy. The recursive measure of total wealth is substantially larger than national income, and is mainly composed of human capital in comparison to tangible and financial assets. The recursive measure of portfolio return is sizeably larger than the risk free rate, is similar to the return on human assets, and is smaller and smoother than the returns on tangible and financial assets.

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In addition, the recursive measure of total wealth lies between those obtained from the replacement cost and present value methods, and it is consistent with cross-sectional measures. The recursive measure of portfolio excess return exhibits reasonable movements relative to alternative measures, and accords with the well accepted notion that the portfolio displays a much stronger degree of diversification than disaggregated assets. Finally, the recursive measures of excess returns on home and corporate equities track remarkably well the historical measures, compared to those obtained from the classical methods.

II. The Procedure

The procedure relies on the bivariate system:

$$W_{t+1} = \left(1 + r_{p,t+1}\right)(W_t - C_t), \quad (1)$$

$$Y_t = \frac{r_{p,t+1}}{\left(1 + r_{p,t+1}\right)} W_t. \quad (2)$$

The variable W_t is aggregate total wealth in period t, C_t is aggregate private consumption, Y_t is national income, and $r_{p,t+1} = \Sigma_{i=1}^{n} \alpha_{i,t} r_{i,t+1}$ is the (net) portfolio return. Also, $r_{i,t+1}, \alpha_{i,t} = W_{i,t}/W_t$, and $W_{i,t}$ are the return, portfolio share, and value of asset i, where $\Sigma_{i=1}^{n} \alpha_{i,t} = 1$.

Equations 1 and 2 rely on few assumptions, which are frequently invoked in the aggregate consumption and asset pricing literatures. First, Equation 1 is interpreted as the inter-temporal budget constraint of a representative agent. The assumption that the constraint is binding follows directly from the usual agent's non-satiety implied by monotone preferences. Second, Equation 2 is an income-wealth relation that defines national income (including profits and wages) as the income from all productive assets. This is consistent with the common assumption that all assets (including tangible and financial stocks and human capital) are traded, as implied by market completeness. Third, rewriting Equation 1 as $(W_{t+1} - W_t)/(1 + r_{p,t+1}) = [r_{p,t+1}/(1 + r_{p,t+1})] W_t - C_t$ and using Equation 2 yield $S_t = Y_t - C_t$, where $S_t = (W_{t+1} - W_t)/(1 + r_{p,t+1})$. This is the fundamental saving identity implied by the popular benchmark of a closed and private economy.

In practice, the procedure offers the considerable advantage of being easy to implement. Specifically, it consists in the following simple initialization and recursion.

Initialization

Initial values of total wealth and portfolio return are determined from steady-state values. The initial value of total wealth $W_1 = [r_{p,2}/(1 + r_{p,2})] Y_1$ is obtained from Equation 2, and from the initial observation of a historical measure of national income and the initial value of portfolio return. The initial value of portfolio return $r_{p,2} = (g_y - 1)/(1 - g_c)$ is obtained from Equation 1, and from the steady-state values associated with the stationary-inducing transformations for growing variables: $g_{y,t} = Y_t/Y_{t-1}, g_{c,t} = C_t/Y_t$, and $g_{w,t} = W_t/Y_t = (1 + r_{p,t+1})/r_{p,t+1}$. The steady-state values for g_y and g_c are fixed to the sample means, computed from historical measures of national income and aggregate consumption.

Recursion

Subsequent values of total wealth and portfolio return are generated recursively for $t = 2, \ldots, T$. The recursive measure of total wealth $W_{t+1} = (1 + r_{p,t+1})(W_t - C_t)$ is obtained from Equation 1, and from a historical measure of aggregate consumption and the recursive measure of portfolio return. The recursive measure of portfolio return $r_{p,t+1} = Y_t/(W_t - Y_t)$ is obtained from Equation 2, and from a historical measure of national income and the recursive measure of total wealth.

The procedure also offers important advantages, relative to the classical methods. For example, the procedure is obviously not based on replacement costs. Thus, it allows one to avoid the difficult task of evaluating accurately and exhaustively these costs. This contrasts with the basic concepts underlying the replacement cost method. In addition, the procedure is consistent with both time-varying income growth and discount rate. To see this, consider the present value $W_t = E_t \Sigma_{i=0}^{\infty} \Pi_{j=0}^{i} z_{t+j} Y_{t+i} = Y_t E_t \Sigma_{i=0}^{\infty} \Pi_{j=0}^{i} z_{t+j} g_{y,t+j}$ to obtain an alternative interpretation of Equation 2 with $r_{p,t+1}/(1 + r_{p,t+1}) = [E_t \Sigma_{i=0}^{\infty} \Pi_{j=0}^{i} z_{t+j} g_{y,t+j}]^{-1}$ – where E_t is the expectation operator conditional on information available in period $t, g_{y,t}$ is the time-varying gross growth rate of income, and z_t is a variable stochastic discount factor. Thus, the procedure allows one to avoid the imposition of restrictions on income growth and discount rate. This contrasts with the tractable applications of the present value method. Finally, the procedure jointly yields recursive measures of total wealth and portfolio return. This contrasts with the replacement cost and present value methods, which only provide measures of total wealth.

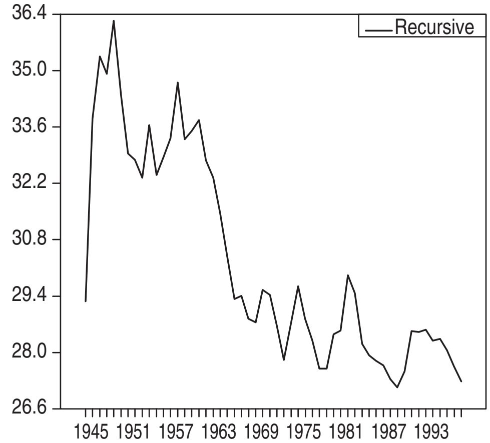

Fig. 1. Wealth-Income Ratio

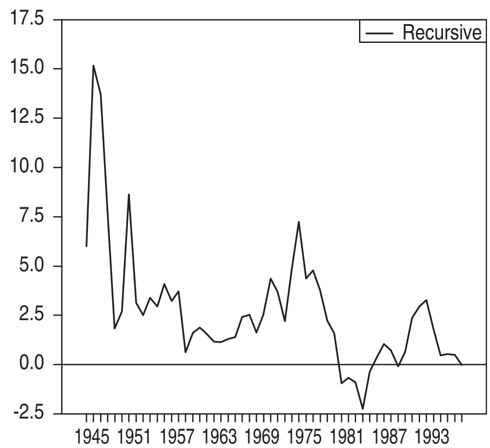

Fig. 2. Portfolio Excess Return (%)

III. The Recursive Measures

The procedure is applied for the US economy over the 1945–1998 period. For this purpose, the historical measures of national income and consumption correspond to the nominal gross domestic product and nominal private aggregate expenditures on nondurable goods and services (source: National Income and Product Accounts), normalized by the gross domestic product implicit deflator (base year 1992) and by the total population (source: Census Bureau). Note that the selected historical measure of national income excludes the value of time allocated to non-market activities (e.g. volunteer work, commuting, and leisure). However, the contribution of these activities to total wealth may be debatable. Additionally, the gross domestic product excludes capital gains. However, these gains are marginal compared to total revenues. For instance, in 1999 the net capital gains for the median household were a meagre $552, out of annual money income of over $40 000 (source: Census Bureau, Current Population Survey, March 1999 and 2000).

Figures 1 and 2 display the ratio of total wealth to national income and the portfolio excess return. The wealth-income ratio assesses the importance of total wealth relative to a natural benchmark. The excess return assesses the importance of the portfolio return relative to the risk free rate, which is proxied by the return on three-month US T-Bills. Empirically, the wealth-income ratio is initially 29.27 in 1945, reaches a maximum of 36.24 in 1949, attains a minimum of 27.14 in 1989, and stabilizes to 27.28 in 1998. This suggests that total wealth is always substantially larger than national income. To get an idea on the scale of these variables, in 1998 real, per capita, total wealth was $762 359, while real, per capita, national income was $27 939. In addition, the portfolio excess return is initially 6.01% in 1945, reaches a maximum of 15.18% in 1946, attains a minimum of −2.24% in 1984, and stabilizes to −0.01% in 1998. This suggests that the portfolio return is often sizeably larger than the risk free rate.

Table 1 reports descriptive statistics associated with the main components of total wealth. To do so, total wealth is measured recursively, tangible and financial wealth are taken from published aggregate data (source: Flow of Funds Accounts of the US, Balance Sheet of Households and Non-profit Organizations), while the unobserved human wealth is constructed residually. Also, the portfolio return is generated recursively, while the implicit asset returns are computed as $r_{i,t+1} = W_{i,t+1}/[\alpha_{i,t}(W_t - C_t)] - 1$. Note that this expression can be interpreted in terms of Equation 1, by isolating $W_{i,t+1}$ and summing over all assets to yield $\Sigma_{i=1}^{n} W_{i,t+1} = [\Sigma_{i=1}^{n}\alpha_{i,t}(1 + r_{i,t+1})](W_t - C_t)$, or equivalently, $W_{t+1} = (1 + r_{p,t+1})(W_t - C_t)$.

The total wealth-income ratio on average, is closely followed by the human wealth-income ratio, which in turn greatly exceeds the tangible and financial wealth-income ratios. This indicates that total wealth is mainly composed of human capital. In 1998 for

Table 1. Recursive measures

	Wealth-income ratio		Portfolio share (%)		Excess return (%)	
Asset	Mean	SD	Mean	SD	Mean	SD
Total	30.26	2.60	–	–	2.66	3.15
Non-human	3.95	0.33	13.18	1.88	3.65	3.83
Tangible	1.40	0.16	4.67	0.82	3.90	4.52
Financial	2.56	0.27	8.52	1.27	3.60	4.61
Human	26.31	2.75	86.82	1.88	2.50	3.33

Note: The sample covers the 1945–1998 period.

Table 2. Alternative measures

Measure	Wealth-income ratio		Portfolio excess return (%)	
	Mean	SD	Mean	SD
RC	6.50	0.61	12.08	2.84
RM	30.99	2.46	2.88	1.87
PV_1	34.85	1.03	3.21	2.94
PV_2	59.52	2.43	2.34	2.92

Note: For RC, total wealth is total capital, year-end totals (Source: Eisner, 1989, Table 13, pp. 263–6). For RM, total wealth is measured recursively. For PV_1, total wealth is the sum of the values of tangible and financial assets (source: Flow of Funds Accounts of the US, Balance Sheet of Households and Non-profit Organizations) and of the expected present value of labour incomes, constructed from a first-order autoregressive process for total compensation of employees (Source: National Income and Product Accounts) and a discount rate of 3.5%. For PV_2, total wealth is full private national wealth (Source: Jorgenson and Fraumeni, 1989, Table 5.32, p. 271). The sample covers the 1949–1981 common period.

example, real, per capita, total wealth was $762 359, while real, per capita, tangible, financial and human wealth were $42 523, $99 350, and $620 486.

The portfolio excess return is similar to the excess return on human capital, and is smaller and smoother than the excess returns on tangible and financial assets. This indicates that the portfolio return is mainly determined by the return on human assets, and is weakly related to tangible and financial assets. This also accords with Roll's (1977) critique stating that the market portfolio return is poorly proxied by stock-market indices.

IV. Alternative Measures

The recursive measures (RM) are compared to classical replacement cost measures (RC) and present value measures (PV_1 and PV_2) over the 1949–1981 common period. To this end, Table 2 reports summary statistics associated with the various measures. Total wealth is measured recursively for RM; is evaluated at the costs required to replace tangible, financial, and human assets for RC (source: Eisner, 1989); is the present value of marketable assets for PV_1; and is the present value including both market and non-market activities for PV_2 (source: Jorgenson and Fraumeni, 1989). In addition, the portfolio return is generated recursively for RM and is computed from the implicit return $r_{p,t+1} = W_{t+1}/(W_t - C_t) - 1$ for RC, PV_1, and PV_2. Note that this expression is straightforwardly derived from Equation 1.

The wealth-income ratio obtained from RM is, on average, larger than that of RC, similar to that of PV_1, and smaller than that of PV_2. This indicates that the recursive measure is not an outlier: it yields a mean wealth-income ratio that is very close to the midpoint. In 1981 for example, real, per capita, total wealth was evaluated at $156 792 for RC, $587 166 for RM, $674 518 for PV_1, and $1 105 667 for PV_2. In addition, the recursive measure is consistent with cross-sectional measures. In 1992 for example, real, per capita, total wealth was $698 004 for RM, while it was on average $1 000 000 per household (Heaton and Lucas, 2000) or $417 688 per capita, given that there are 2.63 individuals per median household (source: Census Bureau, 1990 Census).

The portfolio excess return obtained from RM is similar to those of PV_1 and PV_2. As intuition suggests, the return on non-market activities is negligible – given that the measures extracted from both RM and PV_1 include only market activities, whereas the measure based on PV_2 also incorporates non-market activities. Moreover, the portfolio excess return generated from RM is, on average, much smaller than that of RC, but is as volatile. This suggests that the recursive measure is more reasonable than that of RC – given that the latter is larger, on average, and smoother than returns on most disaggregated assets. For instance, the frequently used S&P500 stock-market index return displays a mean of 7.92% and a standard deviation of 16.57%.

The recursive and alternative measures are further confronted to historical measures of excess returns on home and corporate equities. Note that home and corporate equities are important components: they represent, on average, 47% of tangible wealth and 19% of financial wealth (source: Flow of Funds Accounts of the US, Balance Sheet of Households and Non-profit Organizations). The historical measures of returns on home and corporate equities are obtained from the HPI index of repeat sales of single-family houses (source: Office of Federal Housing Enterprise Oversight) and from the S&P500 stock-market index (source: Board of Governors of the Federal Reserve Bank). As above, the implicit asset returns are generated from $r_{i,t+1} = W_{i,t+1}/[\alpha_{i,t}(W_t - C_t)] - 1$.

The excess return on home equities obtained from RM displays a correlation of 0.98 with the historical excess return, while the measures of RC, PV_1, and PV_2 feature correlations of 0.69, 0.78, and 0.78 with the data for the 1976–1981 common period. In addition, the excess return on corporate equities generated from RM exhibits a correlation of 0.89 with the historical excess return, whereas the measures of RC, PV_1, and PV_2 feature correlations of

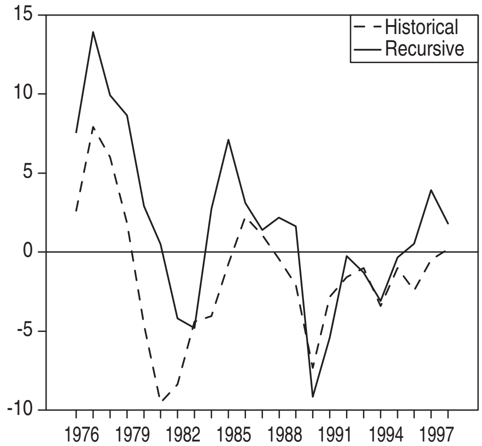

Fig. 3. Excess Return on Home Equity (%)

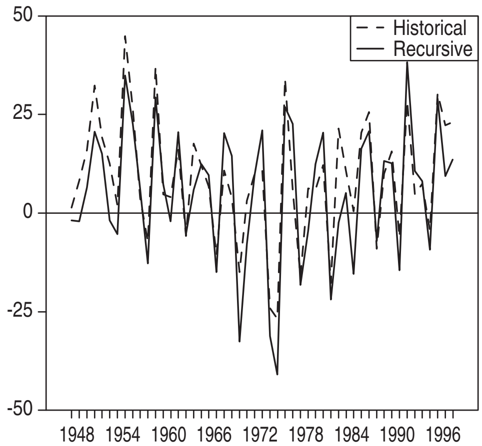

Fig. 4. Excess Return on Corporate Equity (%)

0.21, 0.19, and 0.19 for the 1949–1981 common period. This indicates that the recursive measures of excess returns display, by far, the strongest correlations with the historical excess returns, for both home and corporate equities. Figures 3 and 4 confirm that the recursive measures track remarkably well the data over the 1976–1998 and 1948–1998 common periods for home and corporate equities.

Overall, the comparison exercise reveals that the recursive measures of total wealth, portfolio return, and disaggregated asset returns outperform those obtained from the classical methods. Moreover, the recursive measure of total wealth is consistent with cross-sectional measures. The recursive measure of portfolio excess return is in line with the concept that the portfolio is more diversified than disaggregated assets. Finally, the recursive measures of excess returns on disaggregated assets are almost identical to the historical measures.

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