

# Infrequent Portfolio Decisions: A Solution to the Forward Discount Puzzle<sup>1</sup>

Philippe Bacchetta  
University of Lausanne  
Swiss Finance Institute  
CEPR

Eric van Wincoop  
University of Virginia  
NBER

February 25, 2009

<sup>1</sup>An earlier version of the paper was distributed under the title “Incomplete Information Processing: A Solution to the Forward Discount Puzzle”. We would like to thank the editor, three referees, as well as Gianluca Benigno, Eric Fisher, Pierre-Olivier Gourinchas, Robert Kollmann, Richard Lyons, Nelson Mark, Michael Melvin, Michael Moore, H el ene Rey, Jaume Ventura, and participants at various conferences and seminars. We also thank Elmar Mertens for outstanding research assistance and both Michael Sager and Michael Melvin for many useful discussions about the operation of the active currency management industry. van Wincoop gratefully acknowledges financial support from the Bankard Fund for Political Economy, the National Science Foundation (grant SES-0649442) and the Hong Kong Institute for Monetary Research. Bacchetta acknowledges financial support by the National Centre of Competence in Research “Financial Valuation and Risk Management” (NCCR FINRISK).

# Infrequent Portfolio Decisions: A Solution to the Forward Discount Puzzle

## Abstract

The uncovered interest rate parity (UIP) equation is the cornerstone of most models in international macro. However, this equation does not hold empirically since the forward discount, or interest rate differential, is negatively related to the subsequent change in the exchange rate. This forward discount puzzle implies that excess returns on foreign currency investments are predictable. Motivated by the fact that even today only a tiny fraction of foreign currency holdings are actively managed, we investigate to what extent infrequent portfolio decisions can explain the puzzle. We calibrate a two-country model in which agents make infrequent foreign currency portfolio decisions. We show that the model can account for large deviations from UIP as seen in the data. It can also account for several related empirical phenomena, including that of “delayed overshooting”. We also show that making infrequent portfolio decisions is optimal as the welfare gain from active currency management is smaller than the corresponding fees. The results hold up under a variety of extensions: carry trade (expectations conditioned on current interest rate differentials only), small fraction of actively managed currency positions, multiple currencies, and additional assets.

# 1 Introduction

One of the best established and most resilient puzzles in international finance is the forward discount puzzle.<sup>1</sup> Fama (1984) illuminated the problem with a regression of the monthly change in the exchange rate on the preceding one-month forward premium. The uncovered interest rate parity (UIP) equation, which is the cornerstone of many models in international macro, implies a coefficient of one. But surprisingly Fama found a negative coefficient for each of nine different currencies. A currency whose interest rate is high tends to appreciate. This implies that high interest rate currencies have predictably positive excess returns. The literature following Fama (1984) has continued to report deviations from UIP that are large and statistically significant. This is confirmed in Table 1, which reports regression coefficients of excess returns for five foreign currencies on the difference between U.S. and foreign interest rates. In each case, the excess return predictability coefficient is negative and significantly different from zero. UIP is therefore clearly rejected. The average excess return predictability coefficient is -2.5.<sup>2</sup>

Most models assume that investors incorporate instantaneously all new information in their portfolio decisions. To explain the forward premium puzzle, we depart from this assumption. Portfolio decisions are usually not made on a continuous basis. While there now exists an industry that actively manages foreign exchange positions of investors, it only developed in the late 1980s and still manages only a tiny fraction of cross border financial holdings.<sup>3</sup> Outside this industry

---

<sup>1</sup>For surveys see Lewis (1995), Engel (1996), or Sarno (2005). Some of the more recent contributions include Backus, Foresi and Telmer (2001), Beakert, Hodrick and Marshall (1997), Burnside et al. (2006), Burnside et al. (2007), Chaboud and Wright (2005), Chinn (2006), Chinn and Meredith (2005), Chinn and Frankel (2002), Fisher (2006), Flood and Rose (2002), Gourinchas and Tornell (2004), Lustig and Verdelhan (2007), Mark and Wu (1998), Sarno, Valente and Leon (2006) and Verdelhan (2007).

<sup>2</sup>The reported predictability in the literature may be overstated due to small sample bias and bias caused by the persistence of the forward discount. However, these problems usually can only explain a part of the total bias. See, for example, Stambaugh (1999), Campbell and Yogo (2006), or Liu and Maynard (2005).

<sup>3</sup>It consists of hedge funds exploiting forward discount bias and financial institutions that provide such services to individual clients. The latter include currency overlay managers, commodity trading advisors and leveraged funds offered by established asset management firms. See Sager and Taylor (2006) for a recent description of the foreign exchange market.

there is little active currency management over horizons relevant to medium-term excess return predictability. Banks conduct extensive intraday trade, but hold virtually no overnight positions.<sup>4</sup> Mutual funds do not actively exploit excess returns on foreign investment since they only trade within a certain asset class and cannot freely reallocate between domestic and foreign assets. Finally, Lyons (2001) points out that most large financial institutions do not even devote their own proprietary capital to currency strategies based on the forward discount bias.

We examine the impact of infrequent portfolio decisions in a simple two-country general equilibrium model that is calibrated to data for the five currencies in Table 1. Agents have the choice between actively managing their foreign exchange positions, at a cost, and making infrequent portfolio decisions. We measure the cost of active currency management as the fees charged by the active currency management industry. For the purpose of this paper we take these fees as given and do not model what accounts for them.<sup>5</sup> We find that all or most investors do not find it in their interest to actively manage their foreign exchange positions as the resulting welfare gain does not outweigh the cost.

There are two distinct features that are surprising about the forward discount anomaly. The first aspect is the consistent sign of the bias. Why would the excess return be high for currencies whose interest rate is relatively high? Infrequent portfolio decisions by investors provides a natural explanation. Froot and Thaler (1990) and Lyons (2001) have informally argued that models where some agents are slow in responding to new information lead to predictability in the right direction. The argument is simple. An increase in the interest rate of a particular currency will lead to an increase in demand for that currency and therefore an appreciation of the currency. But when investors make infrequent portfolio decisions, they

---

<sup>4</sup>Two thirds of trade in the foreign exchange market is done among banks that are foreign exchange dealers (BIS, 2004). But since they hold little foreign exchange overnight, the huge intraday trading volume in the forex market is mostly irrelevant for medium-term excess return predictability. Any positions that they take during the day are reversed later in the day.

<sup>5</sup>The fees are likely to reflect at least three elements: (i) the costs associated with collecting and processing information, computing the optimal portfolio, and attracting and distributing funds, (ii) profit margins due to financial expertise and product differentiation and (iii) a profit sharing component intended to deflect agency and monitoring costs. There exists a substantial literature investigating the compensation of portfolio managers. See for example Berk and Green (2005) or Dybvig, Farnsworth and Carpenter (2004) and references therein.

will continue to buy the currency as time goes on.<sup>6</sup> This can cause a continued appreciation of the currency, consistent with the evidence documented by Fama (1984) that an increase in the interest rate leads to a subsequent appreciation. It also implies that a higher interest rate raises the expected excess return of the currency.

Infrequent portfolio decisions can also explain the dynamic response of currency depreciation, or excess returns, to changes in interest rates. The forward discount at time  $t$  also predicts excess returns at future dates. This feature is typically overlooked in the literature. Consider a regression of a future three-month excess return  $q_{t+k}$ , from  $t+k-1$  to  $t+k$ , on the current interest rate differential  $i_t - i_t^*$ . Figure 1 shows the evidence for the five currencies in Table 1, where  $k$  increases from 1 to 30. There is significant predictability with a negative sign for five to ten quarters. Over longer horizons, however, the slope coefficient becomes insignificant or even positive. This is consistent with findings that uncovered interest parity holds better at longer horizons.<sup>7</sup> The persistence in the predictability of excess returns is related to the phenomenon of delayed overshooting. Eichenbaum and Evans (1995) first documented that after an interest rate increase, a currency continues to appreciate for another 8 to 12 quarters before it starts to depreciate.<sup>8</sup> As pointed out above, this is exactly what one expects to happen when investors make infrequent portfolio decisions.

The second surprising aspect of the forward premium puzzle is that investors do not exploit the predictability of excess returns. The standard explanation is that an excess return reflects a risk premium. But many surveys written on the forward discount puzzle have concluded that explanations for the forward discount puzzle related to time-varying risk premia have all fallen short.<sup>9</sup> Our analysis shows that, given the high risk involved, a small asset management cost discourages investors

---

<sup>6</sup>This is consistent with the evidence in Froot, O’Connell, and Seasholes (2001), who show that cross-country equity flows react with lags to a change in returns, while the contemporaneous reaction is muted.

<sup>7</sup>See for example Chinn and Meredith (2005), Boudoukh et al. (2005), or Chinn (2006).

<sup>8</sup>Gourinchas and Tornell (2004) explain both predictability and delayed overshooting with distorted beliefs on the interest rate process.

<sup>9</sup>See Lewis (1995) or Engel (1996). Recently Verdelhan (2006) has more success based on a model with time-varying risk aversion due to habit formation. On the other hand, Burnside et al. (2006) find that excess returns are uncorrelated with a broad range of risk factors.

from actively exploiting the predictability. This risk is illustrated in Figure 2, which shows for one currency, the DM/\$, a scatter plot of the excess return on DM against the U.S. minus German interest rate differential. The negative slope of the regression line represents predictability. It is clear though that predictability is largely overshadowed by risk.<sup>10</sup> This means that for many investors it is not worthwhile to actively trade on excess return predictability. Even for those who do actively trade on predictability, the high risk limits the positions they will take. We show in the context of the model that a small fraction of financial wealth actively devoted to forward bias trade does not unravel the impact of infrequent decision making.

We show that excess return predictability resulting from infrequent portfolio decisions is even stronger when agents condition exchange rate expectations on a limited set of variables. Even in the active currency management industry exchange rate expectations are conditioned on only a small subset of the information space. For example, the most common active currency management strategy is carry trade, which is mostly based on current interest rate differentials. We show that when exchange rate expectations are based on either current interest rate differentials alone or random-walk expectations, excess return predictability is larger than in the case where expectations are conditioned on the entire information set. We will argue that this common practice is not necessarily irrational, particularly in the presence of information processing costs, finite data samples and time-varying model parameters.

Our theoretical analysis is related to recent developments in the stock market literature.<sup>11</sup> On the one hand, several studies show how asset allocation is affected by predictability.<sup>12</sup> On the other hand, some recent papers examine the impact of infrequent portfolio decisions when asset returns are exogenous and there is no predictability.<sup>13</sup> However, the literature has not linked predictability with infre-

---

<sup>10</sup>More formally, this is reflected in the low  $R^2$  for excess return regressions in Table 1, which is on average 0.09.

<sup>11</sup>Evidence of excess return predictability has been extensively documented for stock and bond markets (e.g. see Cochrane, 1999).

<sup>12</sup>See for example Kandel and Stambaugh (1996), Campbell and Viceira (1999), or Barberis (2000).

<sup>13</sup>Duffie and Sun (1990), Lynch (1996), and Gabaix and Laibson (2002) have all developed models where investors make infrequent portfolio decisions because of a fixed cost of information

quent portfolio decisions: those papers that examine the impact of predictability assume it exogenous, while papers that examine infrequent portfolio decisions do not examine its impact on asset prices. Our paper departs from the existing literature by incorporating both predictability and infrequent portfolio decisions and by showing that the latter can cause the former.

The remainder of the paper is organized as follows. Section 2 describes a two-country general equilibrium model where all investors make infrequent portfolio decisions, which is calibrated to data for the five currencies in Table 1. Section 3 discusses the implications of the model for the forward discount and delayed overshooting puzzles. It also considers extensions of the model to the case where agents condition exchange rate expectations on a limited set of variables and to investors that actively manage their portfolio each period. Section 4 considers trade in multiple currencies and in an asset whose return is uncorrelated with exchange rates. Section 5 relates our analysis to other aspects of the existing literature on the forward discount puzzle. Section 6 concludes.

## 2 A Model of Infrequent Decision Making

This section presents a model of the foreign exchange market where investors make infrequent portfolio decisions. First, we describe the basic structure of the model, the basic mechanism, and the solution method. We then discuss under what cost of active portfolio management it is optimal for all investors to make infrequent portfolio decisions. Some technical details are covered in the Appendix, with a Technical Appendix available on request providing full technical detail.

### 2.1 Model's Description

#### 2.1.1 Basic Setup

We develop a one good, two-country, dynamic general equilibrium model. The overall approach is to keep the model as simple as possible while retaining the key ingredients needed to highlight the role of infrequent decision making. There are overlapping generations (OLG) of investors who each live  $T + 1$  periods and derive

---

collection and decision making.

utility from end-of-life wealth. Each period a total of  $n$  new investors are born, endowed with one unit of the good that can be invested in assets described below. The infrequent decision making is modeled by assuming that investors make only one portfolio decision when born for the next  $T$  periods. The threshold portfolio management cost under which it is indeed optimal to make infrequent portfolio decisions is discussed below.

This OLG setup is easier to work with than alternatives where agents have infinite horizons and either make portfolio decisions every  $T$  periods or each period have a constant probability of making a portfolio decision. In that case optimal saving-consumption decisions have to be solved for as well and will depend on the frequency of portfolio decisions. We have abstracted from saving decisions by assuming that agents derive utility from end-of-life wealth. This allows us to focus squarely on portfolio decisions. We want to emphasize though that while an infinite horizon setup is more complicated, the mechanisms at work would be similar to those in our simpler OLG framework. The crucial element is that information is incorporated gradually into portfolio decisions because only a limited fraction of agents make new portfolio decisions each period. It is of little relevance for what follows whether this new information is incorporated by a new generation, as in the OLG model, or by a subset of infinitely-lived investors.

The model contains one good and three assets. In the goods market purchasing power parity (PPP) holds:  $p_t = s_t + p_t^*$ , where  $p_t$  is the log-price level of the good in the Home country and  $s_t$  the log of the nominal exchange rate. Foreign country variables are indicated with a star. The three assets are one-period nominal bonds in both currencies issued by the respective governments and a risk-free technology with real return  $\bar{r}$ .<sup>14</sup> Bonds are in fixed supply in the respective currencies. This is implemented through a balanced budget fiscal policy, by adjusting government spending.<sup>15</sup> This is made explicit in the Technical Appendix but is not relevant for anything that follows.<sup>16</sup>

---

<sup>14</sup>This is necessary to tie down the real interest rate since the model does not contain saving and investment decisions.

<sup>15</sup>Government spending is equal to the earnings on a constant holding of the riskfree technology minus interest payments.

<sup>16</sup>Our setup implies that monetary shocks have real effects even though prices are flexible. The reason is that Ricardian equivalence does not hold as agents have finite lives. Moreover, it is government spending, and not lump-sum taxes, that adjusts to satisfy the budget constraint.

In addition to the agents described above, there are two other sets of agents that play an entirely auxiliary role and are not responsible for the excess return predictability in the model. The first is a set of agents in each country that can hold money or domestic bonds. They play no role other than to generate a standard money demand equation. The second group is a set of liquidity traders. Their noisy demand for Foreign bonds is modeled exogenously. Their behavior allows us to match the observed exchange rate volatility in the data, but they do not directly contribute to excess return predictability.

We first describe the monetary policy rules adopted by central banks, then optimal portfolio choice, and finally asset market clearing.

### 2.1.2 Monetary Policy

We model monetary policy of the Home and Foreign central bank asymmetrically. This allows us to capture in a simple way that investors in currency markets face the choice between an essentially risk-free bond and a bond whose return depends on nominal exchange rate risk. We do so by assuming that the Home country commits to a constant price level, while the Foreign central bank chooses a time-varying interest rate (in a way described below). Investors then have a choice between a risk-free Home bond and a risky Foreign bond whose return depends on nominal exchange rate fluctuations. Since PPP is assumed to hold, Home and Foreign investors face the same real return.<sup>17</sup>

For money demand, we follow Bacchetta and van Wincoop (2006) and assume that money facilitates the production process with a simple functional form. In Appendix A.1 we show that money demand is simply equal to  $m_t - p_t = -\alpha i_t$  and  $m_t^* - p_t^* = -\alpha i_t^*$  for the Home and Foreign country.

The Home country central bank commits to a constant price level  $p_t = 0$  by setting the log money supply constant at  $m = -\alpha \bar{r}$ . In the absence of Home inflation, risk-free arbitrage implies that the Home nominal interest rate is the same as that on the risk-free technology:  $i_t = \bar{r}$ . It follows that the Home money market is in equilibrium when  $m = -\alpha \bar{r}$ . The Foreign central bank follows the

---

<sup>17</sup>In reality the risky asset differs from the point of view of investors in different countries. One could model this by introducing nominal rigidities that give rise to real exchange rate fluctuations, but this generates additional complexities that we aim to avoid. Subsection 2.2 provides a more detailed discussion of alternative modeling assumptions, including nominal rigidities.

following Wicksellian interest rate rule:

$$i_t^* = \psi(p_t^* - \bar{p}_t^*) - u_t \quad (1)$$

where

$$u_t = \rho u_{t-1} + \varepsilon_t^u \quad \varepsilon_t^u \sim N(0, \sigma_u^2) \quad (2)$$

and  $\bar{p}_t^*$  is the target log price level, which we will simply set at zero. The error term captures Foreign monetary policy innovations. Since  $p_t = 0$ , we have  $p_t^* = -s_t$ . The forward discount is then:

$$fd_t \equiv i_t - i_t^* = \psi s_t + u_t + \bar{r} \quad (3)$$

Given the chosen interest rate, the Foreign money supply accommodates money demand changes:  $m_t^* = p_t^* - \alpha i_t^* = -s_t - \alpha i_t^*$ .

These assumptions imply that there are in essence only two assets, one with a risk-free real return  $\bar{r}$  and one with a stochastic real return. The latter is the Foreign bond, which has a real return of  $s_{t+1} - s_t + i_t^*$ .

### 2.1.3 Portfolio Choice

Since PPP holds, Foreign and Home investors choose the same portfolio. They have constant relative risk-aversion preferences over end-of-life consumption, with a rate of relative risk-aversion of  $\gamma$ . Investors born at time  $t$  maximize  $E_t W_{t+T}^{1-\gamma} / (1-\gamma)$ , where  $W_{t+T}$  is end-of-life financial wealth that will be consumed. Investors make only one portfolio decision when born, investing a fraction  $b_t^I$  in Foreign bonds.<sup>18</sup> End of life wealth is then

$$W_{t+T} = \prod_{k=1}^T R_{t+k}^p \quad (4)$$

where  $R_{t+k}^p$  is the gross investment return from  $t+k-1$  to  $t+k$ ,

$$R_{t+k}^p = (1 - b_t^I) e^{i_{t+k-1}} + b_t^I e^{s_{t+k} - s_{t+k-1} + i_{t+k-1}^*} \quad (5)$$

In order to solve for optimal portfolios, a second order approximation of log portfolio returns is adopted. Define  $q_{t+k} = s_{t+k} - s_{t+k-1} + i_{t+k-1}^* - i_{t+k-1}$  as the

---

<sup>18</sup>The portfolio share is held constant for  $T$  periods, which fits reality better than investors deciding on an entire path of portfolio shares for the next  $T$  periods.

excess return on Foreign bonds from  $t + k - 1$  to  $t + k$  and  $q_{t,t+T} = q_{t+1} + \dots + q_{t+T}$  as the cumulative excess return from  $t$  to  $t + T$ . Appendix A.1 shows that the optimal portfolio rule is

$$b_t^I = b^I + \frac{E_t q_{t,t+T}}{\gamma \sigma_I^2} \quad (6)$$

where  $b^I$  is a constant and  $\sigma_I^2$  is defined as

$$\sigma_I^2 = \left(1 - \frac{1}{\gamma}\right) \text{var}_t(q_{t,t+T}) + \frac{1}{\gamma} \sum_{k=1}^T \text{var}_t(q_{t+k}) \quad (7)$$

The optimal portfolio therefore depends on the expected excess return over the next  $T$  periods, with less aggressive portfolio choices made when either agents are more risk averse or there is more uncertainty about future returns.

#### 2.1.4 Liquidity Traders

There is another group of investors referred to as liquidity traders. They are introduced in order to match two key features of exchange rate data. First, it is important to match the observed exchange rate volatility since it affects optimal portfolios through uncertainty about future excess returns. Interest rate shocks alone are not sufficient in this regard and it would violate extensive evidence that observed exchange rate volatility is largely disconnected from observed macro fundamentals.<sup>19</sup> Second, it is important to match the well-known stylized fact that exchange rates behave close to a random walk. This is of relevance in the decision about whether to actively manage the portfolio or not. If there were large predictable components to exchange rate changes, the gain from active portfolio management would be larger. Interest rate shocks alone do not necessarily generate an exchange rate that is close to a random walk.

Liquidity traders start with zero wealth. Their investment behavior is modeled exogenously.<sup>20</sup> At time  $t$  they invest  $X_t$  in Foreign bonds and  $-X_t$  in Home

---

<sup>19</sup>A substantial literature has confirmed the initial findings by Meese and Rogoff (1983) that observed macro fundamentals explain very little of exchange rate volatility for horizons up to 1 or 2 years. Lyons (2001) has called this the exchange rate determination puzzle. Bacchetta and van Wincoop (2004, 2006) show that in the presence of heterogeneous information even small liquidity shocks can have a large effect on exchange rates movements, so that exchange rates are disconnected from macroeconomic fundamentals.

<sup>20</sup>It can also be modeled endogenously. See for example Bacchetta and van Wincoop (2006).

bonds, both measured in terms of Home currency. The next period they consume the return on their investment. We assume that  $X_t = (\bar{x} + x_t)\bar{W}$ , where  $\bar{W}$  is aggregate steady state financial wealth,  $\bar{x}$  is a constant and  $x_t$  follows the process:

$$x_t = C(L)\varepsilon_t^x = (c_1 + c_2L + c_3L^2 + \dots)\varepsilon_t^x \quad \varepsilon_t^x \sim N(0, \sigma_x^2) \quad (8)$$

The magnitude of the shocks is chosen to match observed exchange rate volatility and the polynomial  $C(L)$  such that in equilibrium the exchange rate is close to a random walk. We will return to this below when discussing the solution method.

### 2.1.5 Market Clearing

The last model equation is the Foreign bond market clearing condition. We can abstract from the Home bond market clearing condition because the Home bond is a perfect substitute for the risk-free technology, which is in infinitely elastic supply. Moreover, the goods market will automatically clear by Walras' law. The Technical Appendix discusses all market equilibrium conditions.

There is a fixed supply  $B$  of Foreign bonds in Foreign currency, which is  $Be^{st}$  in terms of Home currency. Investors are born with an endowment of one, but their wealth accumulates over time. Let  $W_{t-k,t}^I$  be the wealth in Home currency at time  $t$  for an investor born at  $t - k$ . This is equal to the product of total returns over the past  $k$  periods,  $W_{t-k,t}^I = \prod_{j=1}^k R_{t-k+j}^p$ . The market clearing condition for Foreign bonds is then

$$n \sum_{k=1}^T b_{t-k+1}^I W_{t-k+1,t}^I + X_t = Be^{st} \quad (9)$$

The equilibrium exchange rate can be solved from the Foreign bond market equilibrium condition (9). However, there is no simple closed form solution. Even after linearizing (9) it involves a complicated difference equation that can only be solved numerically. Nonetheless, in the next subsection we provide some intuition behind the key ingredients of the Foreign bond market clearing condition that drive the behavior of the exchange rate and lead to delayed overshooting.

## 2.2 The Key Features of the Model

Now that the model has been laid out, we can analyze the mechanism that could explain the forward discount puzzle. We first describe the basic intuition and the

key features leading to this mechanism. Then we explain that the mechanism is robust to different modeling assumptions and is not dependent on the specific stylized framework used in this paper.

### 2.2.1 The Basic Mechanism

First let us describe the mechanism in intuitive terms. Consider an increase in the Foreign interest rate that makes Foreign bonds more attractive. The new generation of investors will have a larger demand for Foreign bonds, which puts upward pressure on the Foreign currency. As the Foreign currency appreciates, older investors, who want to keep their initial allocation, rebalance their portfolio by selling Foreign bonds. This implies that in equilibrium new investors buy Foreign bonds from existing investors. Subsequently agents continue to shift towards Foreign bonds, so that the Foreign currency continues to appreciate. This creates an excess return in Foreign currency since it appreciates and still offers a higher interest rate. Eventually demand for the Foreign currency will drop as the Foreign interest rate declines, leading to a depreciation of the Foreign currency. This is the phenomenon of delayed overshooting.

The central aspects of the model can be illustrated more formally by considering a linearized form of the Foreign bond market clearing condition. Let us abstract from liquidity traders and define the steady state supply of Foreign bonds relative to total financial wealth as  $b = Be^{\bar{s}}/\bar{W}$ . The steady state fraction invested in Foreign bonds by all investors is then equal to  $b$ . Linearizing (9) with respect to steady state portfolio shares and wealth and expressing it in Foreign currency, we have

$$\frac{1}{T}\bar{W}\sum_{k=1}^T b_{t-k+1}^I + b\left(\frac{W_t}{S_t} - \bar{W}\right) = B \quad (10)$$

where  $W_t = n\sum_{k=1}^T W_{t-k+1,t}^I$  is total wealth in terms of Home currency. The left hand side captures the two sources of demand for Foreign bonds. The first is portfolio reallocation, associated with changes in portfolio shares. The second is portfolio growth, capturing changes in demand due to changes in wealth.

Now consider a shock that makes Foreign bonds more attractive. The first key ingredient of the model is that agents make infrequent and staggered portfolio decisions. Thus, if the shock is sufficiently persistent, optimal portfolios continue

to shift towards Foreign bonds for some period of time. The first term on the left hand side of (10) then rises over time.

The second key ingredient is passive portfolio rebalancing, which is captured by the second term on the left hand side of (10). Since the supply of Foreign bonds is constant, there must be investors willing to take the other side when there is an increased demand for Foreign bonds. Otherwise portfolio shares could never change and the expected excess return would have to be constant in equilibrium. In our model the other side takes the form of passive portfolio rebalancing. An appreciation of the Foreign currency (rise in  $S_t$ ) leads to a sale of Foreign bonds to rebalance portfolios: the second term on the left hand side of (10) decreases.

While this describes the essence of the model, the equilibrium is actually more complicated. First, the wealth  $W_t$  in Home currency is not constant. It is affected by interest rate shocks as well as the exchange rate. However, as long as some of the wealth is held in Home bonds, its value declines when measured in Foreign currency. This leads to the portfolio rebalancing described above. Another key simplification in the discussion above is that we abstracted from the fact that optimal portfolio shares depend on expected changes in the exchange rate. The numerical solution will solve for the full dynamics of the exchange rate and determine quantitatively if this mechanism can explain the forward premium puzzle.<sup>21</sup>

### 2.2.2 Robustness

The model makes a number of simplifying assumptions in order to improve transparency and tractability. But the basic mechanism continues to hold when considering a variety of extensions. Several extensions, such as the presence of traders that actively manage their portfolio, as well as multiple currencies and assets, will be discussed in subsequent sections. Here we will more briefly discuss some other extensions, leaving algebraic details to the Technical Appendix.

We assumed that prices are perfectly flexible and that PPP holds. When instead local currency prices are sticky,  $P$  and  $P^*$  change gradually in response to shocks, so that the real exchange rate  $SP^*/P$  is closely correlated with the nom-

---

<sup>21</sup>Notice that risk aversion is also an important ingredient of the model as it prevents infinite portfolio positions in response to changes in expected excess returns. Under risk-neutrality  $E_t q_{t,t+T} = 0$  in equilibrium for each period  $t$ . This can only be the case when the expected one-period excess return is always zero, which implies UIP.

inal exchange rate. Real exchange rate movements imply two differences with respect to the flexible price case. First, Home and Foreign investors have a different constant term in their optimal portfolio expressions that captures a hedge against real exchange rate fluctuations. Importantly though, portfolios depend in the same way on the expected excess return as before. Second, the relative value of investors' initial endowments moves with the real exchange rate when we assume that endowments are constant in real terms in both countries. We show in the Technical Appendix that both of these changes affect the market clearing condition (which becomes more complicated), but do not fundamentally alter the basic mechanism.<sup>22</sup>

As discussed above, a key aspect of the model is that investors changing their portfolio find an "other side" to allow their adjustment. This is realistic as in practice there are always outstanding limit orders that reflect how much others are willing to buy and sell at different prices. We have modeled this "other side" through portfolio rebalancing arising from preset portfolio shares combined with the assumption of a positive supply of outside government bonds. Such portfolio rebalancing is a realistic feature of the data, as documented for example by Hau and Rey (2008) for mutual funds.

Two points are worth making in this regard. First, any other elements generating a passive limit order schedule as a negative function of the exchange rate will work as well. Hau and Rey (2006) provide a variety of other motivations for such a passive limit order schedule, for example associated with exports and imports. Second, the presence of outside bonds, while realistic, is not critical for the portfolio rebalancing mechanism to work. All that is needed is that there are agents that sell bonds for reasons other than portfolio allocation. This is natural with a government, but any other institution (e.g. firms) that sells bonds for the purpose of financing their operations will do.

To illustrate the latter point, we can consider a real model with equity in which there is naturally a positive supply of capital as in a Lucas tree model. In the Technical Appendix we show that such a model can be made virtually identical to the bond model in the paper, with the nominal interest rate shock replaced by a dividend shock and the exchange rate replaced by the relative price of Foreign

---

<sup>22</sup>It is also worth noting that when prices are sticky, it makes little differences whether bond supplies are constant in nominal or real terms as price indices move only gradually.

equity. A positive Foreign dividend shock leads to a gradual portfolio reallocation towards Foreign equity. This leads to a gradual increase in the price of Foreign equity, and therefore again excess return predictability. The higher price of Foreign equity causes a passive sale of Foreign equity through portfolio rebalancing, which takes the “other side” of the market.

Finally, we have adopted a rather simple fiscal policy, with government spending adjusting each period to balance the budget and no taxation. One can introduce more realistic government policies that involve taxation. In the Technical Appendix we consider a variety of tax policies where taxes adjust to keep the budget balanced. This affects a bit the wealth of the newborn agents when they are taxed and introduces an additional portfolio hedge term (against future tax changes) when the exiting generation is taxed. But these complications again do not fundamentally change the mechanism that drives our key results.

### 2.3 Solving the Model

We now briefly outline the solution method, leaving details to Appendix A.3 and the Technical Appendix. The first step is to linearize the market clearing condition for Foreign bonds around steady state values of the exchange rate, asset returns and portfolio shares. After substituting the optimal portfolios (6) into the market equilibrium condition, the equilibrium exchange rate can be derived. Start with the following conjecture for the equilibrium exchange rate:

$$s_t = A(L)\varepsilon_t^u + B(L)\varepsilon_t^x \quad (11)$$

where  $A(L) = a_1 + a_2L + \dots$  and  $B(L) = b_1 + b_2L + \dots$  are infinite lag polynomials. Conditional on this conjectured exchange rate equation, compute excess returns as well as their first and second moments that enter into the optimal portfolios. One can then solve for the parameters of the polynomials by imposing the linearized bond market equilibrium condition.

But rather than solving for  $A(L)$  and  $B(L)$  given the process for interest rate and liquidity demand shocks, we solve instead for  $A(L)$ ,  $b_1$  and  $C(L)$  such that (i) the Foreign bond market equilibrium condition is satisfied and (ii)  $\hat{x}_t \equiv B(L)\varepsilon_t^x$  follows an AR process:

$$\hat{x}_t = \rho_x \hat{x}_{t-1} + b_1 \varepsilon_t^x \quad (12)$$

The latter implies  $b_k = \rho_x^{k-1} b_1$  for  $k > 1$ . Rather than taking the process of liquidity demand shocks as given, it is chosen such that the impact of these shocks on the exchange rate follows an AR process. By setting the AR coefficient  $\rho_x$  close to 1, the exchange rate then becomes close to a random walk.

After jointly solving  $b_1$  and  $A(L)$ , the parameters of  $C(L)$  follow immediately from the market clearing condition. Since the polynomial  $A(L)$  has an infinite number of parameters, and solving it jointly with  $b_1$  therefore requires solving an infinite number of non-linear equations, the polynomial  $A(L)$  is truncated after  $\bar{T}$  lags. We set  $a_k = 0$  for  $k > \bar{T}$  and solve  $b_1, a_1, \dots, a_{\bar{T}}$  from  $\bar{T}+1$  non-linear equations. Since interest rate shocks are temporary, their impact on the exchange rate dies out anyway, making this approximation very precise for large  $\bar{T}$ . In practice we set  $\bar{T}$  so large that increasing it any further has no effect on the results.

## 2.4 On the Optimality of Infrequent Decision Making

Under what circumstances is the passive portfolio management strategy followed by all traders in the model optimal? There is a trade-off between the higher expected returns under active portfolio management and the cost involved. Assume that the cost of active portfolio management is a fraction  $\tau$  of wealth per period. The question then is how large  $\tau$  needs to be for it to be optimal for all traders to make decisions infrequently. We will refer to the level of  $\tau$  where expected utility is the same under active and passive portfolio management strategies as the *threshold cost*. As long as the actual  $\tau$  is above this threshold, it is optimal for traders to make infrequent portfolio decisions.

In order to determine the threshold cost, we must consider the alternative where traders make portfolio decisions each period.<sup>23</sup> An investor with an actively managed portfolio must solve a more complicated multi-period portfolio decision problem. Since equilibrium expected returns are time varying, the optimal dynamic portfolio contains a hedge against changes in future expected returns. In Appendix A.2 we solve the optimal portfolio problem for an investor who makes portfolio decisions each period and faces the portfolio management cost  $\tau$ . We then compute expected utility under both active and passive portfolio manage-

---

<sup>23</sup>We will abstract from scenarios where agents make portfolio decisions at intervals between one and  $T$ .

ment and derive an expression for the threshold costs where utility breaks even. The Technical Appendix provides additional details.

## 2.5 Parameterization

The model is calibrated to data for the five currencies on which Table 1 and Figure 1 are based, with a period set equal to one quarter. The AR process for the forward discount is estimated for the countries and sample period corresponding to the excess return regression reported in Table 1.<sup>24</sup> We set  $\psi = 0$  and  $\bar{r} = 0$  so that  $u_t = fd_t$ . The parameters  $\rho_u$  and  $\sigma_u$  are set equal to the average across the countries of the estimated processes. This yields  $\rho_u = 0.8$  and  $\sigma_u = 0.0038$ .

A couple of comments are in order with regards to the assumption that  $\psi = 0$ . It is consistent with very low estimates of  $\psi$  reported for example in Engel and West (2005). Results do not change much when  $\psi$  is a very small positive number. Setting  $\psi$  exactly equal to zero has the advantage that it rules out additional excess return predictability associated with liquidity shocks. When  $\psi = 0$ , exchange rate fluctuations resulting from liquidity shocks do not affect the interest differential and therefore cannot contribute to excess return predictability.<sup>25</sup> Even though  $\psi = 0$  implies an exogenous interest rate rule, we show in the Technical Appendix that there is nonetheless a unique stochastic equilibrium for the exchange rate.

The process for the supply  $x_t = C(L)\varepsilon_t^x$  cannot be observed directly. As discussed above, this process is chosen to match observed exchange rate volatility and the near-random walk behavior of exchange rates. To be precise,  $\sigma_x$  is set such that the standard deviation of  $s_{t+1} - s_t$  in the model is equal to the average standard deviation of the one quarter change in the log exchange rate for the five currencies and time period reported in Table 1. The average standard deviation is 0.057. The polynomial  $C(L)$  is chosen such that  $\hat{x}_t$  follows an AR process as in (12) with AR coefficient  $\rho_x = 0.99$ . This means that the exchange rate is close to a random walk since liquidity demand shocks dominate exchange rate volatility.

In the benchmark parameterization we set  $T = 8$ . This implies that agents make one portfolio decision in two years, so that half of the agents change their

---

<sup>24</sup>We use three-month Euro-market interest rates from Datastream between December 1978 and December 2005.

<sup>25</sup>When  $\psi > 0$  liquidity shocks can lead to further excess return predictability. This mechanism is emphasized by McCallum (1994).

portfolio during a particular year. In order to get some sense of the magnitude of  $T$  it is useful to realize that trade in the foreign exchange market is closely tied to international trade in stocks, bonds and other assets. A value of  $T = 8$  corresponds well to some evidence for the stock market. The Investment Company Institute (2002) reports that only 40% of U.S. investors change their stock or mutual fund portfolios during any particular year.<sup>26</sup> Setting  $T = 8$  also corresponds well to evidence reported by Parker and Julliard (2005) and Jagannathan and Wang (2005) that Euler equations for asset pricing better fit the data when returns are measured over longer horizons of one to three years.

The rate of relative risk aversion  $\gamma$  is set at 10. This is in the upper range of what Mehra and Prescott (1985) found to be consistent with estimates from micro studies, but consistent with more recent estimates by Bansal and Yaron (2004) and Vissing-Jorgensen and Attanasio (2003).<sup>27</sup> A risk-aversion of 10 also reduces the well known extreme sensitivity of portfolios to expected excess returns in this type of model.<sup>28</sup> Since both  $\gamma$  and  $T$  are key parameters and hard to precisely calibrate to the data, the next section will also conduct sensitivity analysis over a broad range of values of these parameters.

The final set of parameters are related to the steady state of the model.<sup>29</sup> We set  $\bar{x}$  such that  $b = 0.5$ . This corresponds to a two-country setup with half of the assets supplied by the US and the other half by the rest of the world. Without loss of generality, the nominal supply  $B$  is set equal to  $b\bar{W}$  such that the steady state log exchange rate  $\bar{s}$  is zero.

---

<sup>26</sup>For a discussion of evidence on infrequent trading see Biliias et al. (2005) and Vissing-Jorgensen (2004).

<sup>27</sup>The estimates in Bansal and Yaron (2004) are based on a general equilibrium model that can explain several well known asset pricing puzzles. The estimates in Vissing-Jorgensen and Attanasio (2003) are based on estimating Euler equations using consumption data for stock market participants.

<sup>28</sup>Other ways to improve this feature include loss aversion preferences, habit formation preferences, parameter uncertainty, transaction costs, and portfolio benchmarking.

<sup>29</sup>There is also the truncation parameter  $\bar{T}$  used in the solution method, which is set at 60 quarters. Increasing it further does not affect the results.

### 3 Explaining the Forward Premium Puzzle

We now examine the model’s quantitative implications for excess return predictability. We will show that the model indeed generates such predictability. We first present the results in our benchmark case and provide the intuition on the mechanism leading to predictability. We also find that the threshold cost of active portfolio management is below the fees charged by the active portfolio management industry, so that it is indeed optimal for all investors to make infrequent portfolio decisions. We then consider additional moments that the model sheds light on and alternative parameterizations. We finally consider extensions where agents condition exchange rate expectations on a limited set of variables and where some agents actively manage their portfolio each period.

#### 3.1 Benchmark Results

Panel A of Figure 3 reports results when regressing excess returns  $q_{t+k}$  on the forward discount  $fd_t$ , similar to Figure 1. While standard models predict coefficients around the zero line, the model is able to generate negative coefficients for small values of  $k$ , followed by positive coefficients for larger  $k$ . The usual one-period ahead coefficient is equal to -0.95.

In order to allow for better comparison to results based on the data reported in Table 1, we have simulated the model over a 25-year period. Panel B reports the frequency distribution of one-period ahead predictability coefficients based on 1000 simulations of the model over a 25-year period. The average excess return predictability is very close to the population moment of -0.95. However, the excess return predictability varies considerably across simulations. This is consistent with empirical evidence that the predictability coefficient is unstable over time (e.g., see Chinn and Meredith, 2005). The excess return predictability coefficient is less than -1 in 48% of the simulations and less than -2 in 12% of simulations. This means that the findings in the data are well within reach of the model. This can be compared to the case where investors make portfolio decisions each period. In that case the excess return predictability coefficient is close to zero (-0.014) and is never less than -1 in 1000 simulations of the model over a 25-year period.<sup>30</sup>

---

<sup>30</sup>The fact that it is not exactly zero is because the change in the exchange rate changes the

It is important to emphasize that we obtain these results even though we have tied our hands in many ways to match other aspects of the data. In particular, we constrain the volatility of exchange rates to be the same as in the data and we replicate the near-random walk behavior of exchange rates. We also match the volatility and persistence of interest rate differentials in the data. We will now give some intuition for why substantial excess return predictability endogenously develops in the model.

### *Delayed Overshooting*

Panel C of Figure 3 provides the key intuition behind our findings. It shows the impulse response of the exchange rate to a one standard deviation decrease in the Foreign interest rate. It compares the benchmark model to the case where all investors make portfolio decisions each period. In the latter there is standard overshooting, i.e., the lower Foreign interest rate causes an immediate appreciation of the Home currency, followed by a gradual depreciation. With infrequent portfolio decisions, however, there is delayed overshooting, consistent with the empirical findings of Eichenbaum and Evans (1995). The initial appreciation of the Home currency is now smaller, followed by two subsequent quarters of appreciation and then a gradual depreciation.

The continued appreciation for the first couple of quarters is a result of the delayed portfolio response of investors. Investors making portfolio decisions at the time the shock occurs sell Foreign bonds in response to the news of a lower Foreign interest rate. The next period a different set of investors adjust their portfolio. They too will sell Foreign bonds in response to the lower interest rate, leading to a continued appreciation of the Home currency.

The currency continues to appreciate for three quarters. The reason why the delayed overshooting does not last longer than three quarters is that at that point investors start buying Foreign bonds again. Investors know that the Foreign interest rate will continue to be lower than the Home interest rate, but they also realize that eventually the Home currency will depreciate. This is because investors who sold Foreign bonds at the time the shock happened will increase their holdings of Foreign bonds 8 quarters later when they adjust their portfolio again.<sup>31</sup> After

---

real supply of the foreign asset,  $Be^{-s_t}$ , which has a small risk-premium effect.

<sup>31</sup>More precisely, and leading to the same outcome, they are replaced by a new generation that

all, the interest rate differential in favor of Home bonds is expected to be much smaller 8 quarters later. Three periods after the shock the expected depreciation of the Home currency over the next 8 quarters is sufficient to more than offset the expected interest differentials in favor of the Home bonds. Investors will then start buying Foreign bonds again, causing the Home currency to gradually depreciate.

This of course assumes very careful forward looking behavior on the part of investors, which requires a full understanding of future portfolio choices of other investors and full processing of all available information to predict the exchange rate two years into the future. This extent of knowledge may be unrealistic, an issue to which we will turn below.

### *Threshold Cost*

It is optimal for all agents to follow a passive portfolio management strategy when the threshold cost  $\tau$  is below the actual cost of active portfolio management. In comparing the actual cost to the theoretical threshold cost it is important to scale both in terms of the portfolio risk. In practice the fees charged by institutions that actively manage FX positions are linear in the risk of the fund. To illustrate this, consider two funds, A and B. Assume that the portfolio share invested in Foreign bonds is always twice as high for fund A as for fund B, so that the risk (standard deviation of return) is twice as high for fund A. Since the excess return generated by fund A is also twice as high, it must be that the fee is twice as high for fund A. Otherwise there is an arbitrage opportunity. This explains why the fees charged by the active currency management industry are linear in the level of risk.

At 20% risk, a typical fee for a currency fund is a 1% management fee plus 20% of profits.<sup>32</sup> In practice this implies a total fee of about 4%. At 2% risk the fee is then 40 basis points. When comparing the threshold cost in the model to fees charged by these FX funds it is therefore important to compare them at the same level of risk. We will report both the threshold cost and the actual cost (the

---

chooses a new portfolio.

<sup>32</sup>One can check the fees on Bloomberg. For example, in early 2008 the Goldman Sachs Global Currency Portfolio has a 1% management fee and 20% incentive fee. These numbers are 1.55% and 20% for the JP Morgan Managed Currency Fund; 0.75% and 20% for the Morgan Stanley FX Alpha Plus RC400 fund; or 1.5% and 20% for the ABN AMRO Alternative Investments Currency Fund. Other funds have similar numbers.

fees) at 5% risk. The fee is then 1%. In order to compute the threshold cost  $\tau$  in the model we first compute the annualized cost  $\tau$  such that agents are indifferent between active and passive portfolio management (as described in section 2.2). We then simulate the model 10,000 times to compute the standard deviation of the annual return (return over 4 quarters). We then scale the threshold cost by the ratio of 0.05 to the standard deviation of the return in order to express the threshold cost at 5% risk.

The resulting threshold cost is 0.70%, which is below the 1% fee charged by active fund managers. Given the fees it is therefore optimal for all investors to adopt a passive portfolio strategy.<sup>33</sup> The reason that the threshold cost is small is that there is so much uncertainty about future returns. Panel D of Figure 3 illustrates that the predictability of excess returns by interest differentials is simply overwhelmed by uncertainty, as is the case in the data. This uncertainty reduces the welfare gain from active portfolio management.<sup>34</sup>

#### *Additional Moments and Parameterizations*

Table 2 presents results on sensitivity analysis with regard to the parameters  $\gamma$  and  $T$ . We vary both over a wide range, showing results for  $\gamma = 1$  and  $\gamma = 50$  and for  $T = 4$  and  $T = 12$ . The table also shows some additional moments, particularly the first-order autocorrelation of quarterly log-exchange rate changes and the  $R^2$  of the excess return predictability regression. Under the benchmark parameterization the first-order autocorrelation is 0.004, consistent with the near-random walk behavior of exchange rates. In the data the average first-order autocorrelation is slightly higher at 0.055, but a value of 0 (random walk) cannot be rejected at the 10% confidence level for any of the currencies. The  $R^2$  is 0.011 under the benchmark parameterization, lower than the average 0.09 in the data.

---

<sup>33</sup>We should also note that the fees represent only the amount paid to a currency fund and do not include other costs like the selection of the fund, its monitoring and agency costs.

<sup>34</sup>We do not use Sharpe ratios because they are neither a welfare metric nor a number that can be related to the cost of active portfolio management. It is therefore hard to judge whether a particular Sharpe ratio is large or small. Nonetheless, in line with our findings, Lyons (2001) reports that interviews with proprietary traders and desk managers shows that Sharpe ratios for currency strategies are below their cutoff for capital allocation. He argues that therefore “as an empirical matter, most large financial institutions do not devote their proprietary capital to currency strategies.”

The sensitivity analysis leads to some key insights. First, the model’s findings are robust over a wide range of parameters. An excess return predictability coefficient of less than -2 over a 25-year period is consistent with the model under all parameterizations at a 5% confidence level (and less than -1 at a 28% confidence level). Moreover, the threshold cost is remarkably insensitive to the choice of parameters and is always below observed fees. Second, excess return predictability is larger the higher the rate of risk aversion and the less frequent agents make portfolio decisions (higher  $T$ ). When the rate of risk aversion is very small ( $\gamma = 1$ ) agents choose very large portfolio positions in response to non-zero expected excess returns. Equilibrium expected excess returns will then be smaller and excess return predictability more limited.

### 3.2 Conditioning Forecasts on a Limited Set of Variables

Although investors in the model make infrequent portfolio decisions, they fully know the model and condition exchange rate expectations on the infinite information space available to them (current and lagged interest rates and liquidity demand shocks). In other words, investors have rational expectations and are able to determine the future behavior of other investors and the full path of future returns in response to shocks that have already occurred. As explained above, it is this forward looking behavior that leads investors to start buying Foreign bonds after three periods, which limits the extent of delayed overshooting.

However, the actual behavior of investors is at odds with this description. Many large financial institutions do not bother to try to outperform the random walk when forming expectations of the exchange rate one month or more into the future. This may not be surprising given the well-known difficulty to beat the random walk in predicting exchange rates (e.g., Meese and Rogoff, 1983). To the extent that FX portfolios are based on exchange rate forecasts, investors tend to use very simple forecasting rules, even in the active currency management industry. The widespread use of carry trade strategies, focusing on current interest rate differentials, clearly illustrates this point.

Consequently, we consider two strategies where investors condition exchange rate forecasts on limited information. In most of the analysis we will focus on the case where agents make optimal forecasts of future exchange rate changes

conditional on the current interest rate differential only, as with carry trade. They therefore have full knowledge of the Fama regression coefficient, and at all possible horizons. The second strategy assumes random walk expectations, so that investors expect future spot rates to be equal to the current spot rate. In either case they also fully understand the AR(1) process of the interest rate differential.<sup>35</sup>

There are various ways to rationalize the commonly observed practice of conditioning expectations of exchange rate changes on limited information. First, there are information processing costs.<sup>36</sup> This is particularly relevant for agents that do not have their FX portfolios actively managed by professionals (all agents in our model so far). If agents had access to an infinite amount of data and if the model and parameters would never change, they could perfectly learn about the model. But the cost of processing an infinite amount of data would be large. This cost can be avoided by either simply adopting random walk expectations or by using easily available information about Fama regression coefficients.

Stepping outside the specifics of our model for a moment, there are at least two additional reasons for conditioning forecasts on limited information. These apply even when agents optimally exploit all available information to form expectations. First, in practice agents do not have access to an infinite amount of data to derive exchange rate forecasts. The best they can do is select a set of variables with the best predictive power based on a particular model selection criterion. Forecasts will then be conditioned on a limited number of variables as more variables eat up degrees of freedom.<sup>37</sup> Second, reality is far more complex, with time-varying model parameters and uncertainty about the nature of the model itself. Sarno and Valente (2009) show that as a result of changes in the model or parameters the

---

<sup>35</sup>The expected excess return over the next  $T$  periods is the sum of the expected interest rate differentials and the expected exchange rate changes. Expected interest differentials are computed based on knowledge of the AR(1) process of the interest differential. Expected exchange rate changes are either zero (random walk expectations) or equal to the best forecast conditional on current interest rate differentials.

<sup>36</sup>Consistent with that, Fama (1991) suggests that “a weaker and economically more sensible version of the efficient market hypothesis says that prices reflect information to the point where the marginal benefits of acting on information do not exceed the marginal cost”.

<sup>37</sup>One common criterion to select variables is the adjusted  $R^2$ . In our example with 25 years of data for 5 currencies, we find that the adjusted  $R^2$  drops when we add a one quarter lagged interest differential to a Fama regression. Based on this selection criterion agents would condition exchange rate expectations on current interest differentials only.

best that agents can do in terms of out-of-sample forecasts is to adopt random walk expectations. This is because agents do not know which variables will be most important in future periods even if they can determine this ex post. A richer model may therefore provide a more solid foundation for the observed practice of conditioning forecasts on limited information. However, the mechanism that gives rise to excess return predictability in our model would be similarly relevant in a more complex model as long as there is infrequent FX portfolio management.

#### *More Predictability*

Figure 4 shows the main results for the first strategy, where investors use the Fama regression (in the model) to predict exchange rates. All the parameters are as in the benchmark parameterization. The usual one-period ahead regression coefficient of the excess return on the forward discount is now -2.1. This is close to the average regression coefficient found in the data and reported in Table 1. Panel A of Figure 4 shows that the coefficient continues to be negative for 5 quarters, declining in absolute size, then turns positive and eventually back to zero for very long lags. This closely matches the data reported in Figure 1.<sup>38</sup> In 41% of simulations of the model over a 25-year period the coefficient is now less than -2.5.

These results are important for several reasons. First, as discussed above, the assumption of conditioning on a limited information set more closely captures reality than the benchmark model. Second, while the excess return predictability coefficient of -0.95 in the benchmark model cannot be statistically rejected by the data, it is still a long way off from the average predictability coefficient of -2.5 reported in Table 1. When conditioning expectations only on the interest rate differential the predictability coefficient is close to the point estimate in the data.

#### *More Delayed Overshooting*

The more negative regression coefficient than under the benchmark model can be explained by more delayed overshooting. Panel C of Figure 4 shows that after a drop in the Foreign interest rate, the Home currency appreciates for eight quarters. In contrast to the benchmark model, investors continue to sell Foreign bonds for

---

<sup>38</sup>In the data this coefficient continues to be negative for about 10 quarters, but its coefficient is insignificantly different from zero after about 5 quarters. Also, the decline of this coefficient back to zero in the data happens after 30 quarters, not reported in Figure 1.

eight quarters. The expected excess return over 8 quarters is now proportional to the interest rate differential, with a coefficient of  $\beta_1 + \dots + \beta_8$ , where  $\beta_k$  is the regression coefficient in  $q_{t+k} = \alpha_k + \beta_k f d_t$ . The sum of the first eight coefficients is -2.6. This means that the expected excess return over the next eight quarters is -2.6 times the current forward discount. Investors therefore continue to sell Foreign bonds during the first eight quarters when the lower Foreign interest rate raises the forward discount. After eight quarters investors start buying Foreign bonds again because the first group of investors selling Foreign bonds when the shock happened is replaced by another generation. Foreign bonds are by then more attractive than they were eight quarters earlier since the interest rate on Foreign bonds has gradually increased over time.

#### *Threshold Cost*

The threshold cost of active portfolio management, again measured at 5% risk, is now 0.87%. This is still below the fee of 1% charged for active currency management. It therefore remains optimal for all agents to make infrequent portfolio decisions. Panel D of Figure 4 shows that a scatter plot of excess return observations versus the forward discount, based on a 25-year simulation of the model, is again very similar to what we found in the data reported in Figure 2. Excess return predictability remains overwhelmed by uncertainty, so that the gain from active portfolio management remains small.

#### *Additional Moments and Parameterizations*

Table 3 is the analogue of Table 2 for the case where exchange rate expectations are only conditioned on current interest rate differentials, reporting additional moments and sensitivity analysis. The first-order autocorrelation of quarterly exchange rate changes is now 0.050, virtually the same as in the data (0.055). The  $R^2$  of the excess return predictability regression is larger than under the benchmark model (0.053) but still somewhat lower than in the data (0.09).

Sensitivity analysis again shows that the results are robust for a very wide range of the parameters  $\gamma$  and  $T$ . An excess return predictability coefficient of less than -2 over a 25-year period is consistent with the model under all parameterizations at a 25% confidence level. The threshold cost is also quite insensitive to the choice of parameters and below observed fees. In contrast to Table 2, we now see that

excess return predictability is larger the lower the rate of risk aversion. The lower the rate of risk aversion, the larger the switch to Home bonds after a drop in the Foreign interest rate, and therefore the larger the appreciation of the Home currency in subsequent periods. This leads to more predictability.<sup>39</sup>

### *Two Final Comments*

Two final comments are in order. First, when investors adopt random walk expectations, the one-period ahead excess return predictability coefficient is even more negative, -2.54. In that case investors continue to sell Foreign bonds to an even greater extent over the first eight periods because they do not expect the domestic currency to depreciate at any time in the future. There continues to be delayed overshooting for eight periods in this case.<sup>40</sup> Second, the assumption that agents condition exchange rate expectations on current interest rate differentials does not by itself explain excess return predictability. Infrequent portfolio decisions are key to the results. If all investors make portfolio decisions each period, using only current interest rates to forecast future excess returns, the one-period ahead excess return predictability coefficient would be -0.08.

### **3.3 Investors with Actively Managed Portfolios**

We now introduce investors with actively managed portfolios into the model. The industry that actively manages foreign exchange positions has only recently developed (it did not exist until the late 1980s) and is still quite small. The assumption that we have made so far, that no investors actively manage their currency positions, is therefore currently (and certainly over the past 25 years) a good approximation. Nonetheless this market does exist and has been growing. A natural question is therefore how large this market needs to become in order for it to start eroding excess return predictability.

---

<sup>39</sup>Predictability is now remarkably insensitive to  $T$ . On the one hand a larger  $T$  leads to longer delayed overshooting, which increases predictability. On the other hand, portfolios become less aggressive for larger  $T$  as they are based on expected interest differentials further into the future. This limits predictability. The predictability becomes smaller though when  $T$  is less than 4. The predictability coefficient is -1 for  $T = 2$  and -0.08 for  $T = 1$ .

<sup>40</sup>In Bacchetta and van Wincoop (2007), we examine the case of random walk expectations in more detail.

We assume that the cost of active portfolio management is lower than the threshold cost for a fraction  $f$  of agents and above the threshold cost for all other agents. A fraction  $f$  of investors therefore actively manage their portfolio. Of the  $n$  agents that are born each period,  $n_F = fn$  will then manage their portfolio actively, while  $n_I = (1 - f)n$  make infrequent portfolio decisions.

Appendix A.2 derives the optimal portfolio of agents that actively manage their portfolio, which depends on the expected excess return over the next period. The new Foreign bonds market equilibrium condition is

$$n_F \sum_{k=1}^T b_{t-k+1,t}^F W_{t-k+1,t}^F + n_I \sum_{k=1}^T b_{t-k+1,t}^I W_{t-k+1,t}^I + X_t = B e^{st} \quad (13)$$

where  $W_{t-k+1,t}^F$  is the wealth at time  $t$  of agents born at time  $t - k + 1$  who actively manage their portfolio and  $b_{t-k+1,t}^F$  the portfolio share these agents choose at time  $t$ . The threshold cost  $\tau$  is now computed as the level of  $\tau$  such that agents are indifferent between active and passive portfolio management when a fraction  $f$  of agents manages their wealth actively.

In evaluating the impact of active management, we need a metric for the extent of active portfolio management that can be compared to what we know about the size of the existing industry. While there are no publicly available numbers on the size of the active currency management industry, those familiar with the industry have indicated to us that the total wealth managed by the industry in 2006 was about \$1.5 trillion at 2% risk. We will again report results at 5% risk, as we have done for the threshold cost. In that case the size of the industry is \$600 billion. We will normalize this by total global external positions, which are almost entirely claims on foreign currencies. In 2004 world external wealth was \$56.6 trillion (see Lane and Milesi-Ferretti, 2006), so that about 1.1% of global FX positions are actively managed at 5% risk.

In the model a fraction  $f$  of wealth is actively managed. This is a fraction  $2f$  of steady state external wealth. We scale this by multiplying  $2f$  with the ratio of the standard deviation of the annual portfolio return under active management and 0.05. This gives us the wealth that is actively managed at 5% risk, divided by steady state external positions, which can be compared to the approximately 1.1% of global FX positions that are currently actively managed by the industry.

Figure 5 shows both the excess return predictability coefficient and the threshold cost (at 5% risk) as a function of the ratio of actively managed wealth to exter-

nal positions. The vertical line measures the current estimated size of the industry. It is clear from panel A that excess return predictability drops as the extent of actively managed wealth increases. This is not surprising as active currency management aims to exploit the profits from the existing excess return predictability, which therefore erodes the predictability. However, since the proportion of active currency management is currently estimated at 1.1%, the impact on predictability remains small. Under the benchmark parameterization, the coefficient changes from -0.95 to -0.78. When exchange rate expectations are conditioned on current interest rate differentials, so that active portfolio management takes the form of carry trade, there is a change from -2.12 to -1.87.

There is a natural limit to the size of the industry that actively manages currency positions. This is illustrated in Panel B of Figure 5. It shows that the threshold cost declines substantially as the fraction of actively managed wealth increases. This is not surprising because of the reduction in excess return predictability. The profit opportunities left unexploited go down with the increase in actively managed portfolios. It would therefore not be optimal for too many investors to actively manage their currency positions.<sup>41</sup>

## 4 Multiple Currencies and Other Assets

So far we assumed that the only assets traded are nominal bonds of two countries. In this section we briefly discuss two extensions. The first is trade in multiple currencies (nominal bonds of multiple countries). The second is the introduction of another asset whose return is uncorrelated with that on the nominal bonds.

First consider the multiple currency case. We assume that there are now  $N > 2$  countries. Country  $N$  is the Home country, which in the data is the United States. We continue to assume that half of the steady state world bond supply is from the Home country, with the other countries each contributing an equal fraction of the remaining global bond supply. Portfolio demand now depends on the entire correlation structure of asset returns. Apart from matching the observed standard deviation of exchange rates, we now also match the observed correlation

---

<sup>41</sup>A possible counterweight to this, which our model is not set up to address, is that the fees charged for active portfolio management may decline when more foreign exchange positions become actively managed. This can be the result of fixed cost components of portfolio management.

of exchange rates across countries. We calibrate the model to the 5 currencies in Table 1, setting  $N = 6$ . Here we simply summarize the key results and describe the model and its calibration in Appendix A.4.

First consider the case where expectations are conditioned on the entire information space, which consists of current and past interest rate shocks and liquidity demand innovations of all countries. We continue to assume that  $\gamma = 10$  and  $T = 8$ . The excess return predictability coefficient declines to -0.74, while the threshold cost rises to 0.96%. We obtain predictability similar to the two-country benchmark case if we raise either  $T$  or  $\gamma$ . For example, if we raise  $T$  to 12, the excess return predictability coefficient becomes -0.93. The threshold cost is now 1.03%, close to the observed fee.

Next consider the case where agents use only current interest rate differentials for all currencies to form exchange rate expectations. The excess return predictability coefficient becomes -2.61, while the threshold cost becomes 1.9%. This threshold is somewhat above the fee charged by the active currency management industry. This result is attractive as it becomes endogenously optimal for some investors to actively manage their FX positions, as seen in the data. When we introduce enough active traders such that at 5% risk a fraction 1.1% of steady state external wealth is actively managed, as in the data, the threshold cost decreases to 1.2% while the excess return predictability declines to -1.73. This case is remarkable in that the model can now account for the observed size of the active currency management industry given the observed fee, while at the same time accounting for excess return predictability and matching the volatility, persistence and cross-country correlation of interest rates and exchange rates.

The increase in the threshold cost is simply explained by the increased diversification benefits of currency management. The impact on predictability can be explained by a decrease in the relative supply of each Foreign bond as the number of Foreign bonds increases. This can be seen most clearly in the case where the correlation across different currencies is zero, since optimal portfolio shares for infrequent traders in each Foreign bond remain the same as in the two-country case. On the other hand, the extent of portfolio rebalancing by passive traders is more limited as a fraction of total wealth since the relative supply of each Foreign bond is lower.

The impact on predictability depends on how expectations are formed. When

all information is used, the smaller passive portfolio demand leads to smaller equilibrium portfolios shares for agents making new portfolio decisions. Equilibrium expected excess returns must then be smaller, which explains a smaller excess return predictability. On the other hand, when expectations are based on interest differentials only, smaller passive portfolio demand leads to larger exchange rate changes to clear the bonds market. A higher interest on a particular Foreign bond then implies larger subsequent appreciation of that Foreign currency, leading to more predictability.

Another way to extend the asset menu is to introduce an asset other than nominal bonds. Some have argued that foreign exchange risk can be largely diversified away because the returns on other assets (particularly equity) are not much correlated with foreign exchange returns (e.g. Lyons, 2001, p. 213). In order to address the extent to which diversification affects the previous analysis, we add a third asset to the benchmark two-country model. Its return is assumed to be uncorrelated with the excess return on Foreign bonds and its expected return is assumed constant. We leave all details to the Technical Appendix and discuss only the key implications and intuition.

The additional asset only affects the equilibrium through reduced portfolio rebalancing, which results from the smaller share of Foreign bonds in total wealth. For reasons discussed above, excess return predictability is slightly smaller when expectations are based on all information and somewhat higher when expectations are conditioned on current interest rates only. The threshold cost declines in the former case while rising in the latter. Other than the impact on limit orders through portfolio rebalancing the third asset plays no role. If we kept passive portfolio demand unchanged as a fraction of total wealth (e.g. by introducing limit orders in other ways), the threshold cost and excess return predictability would remain unchanged.

The important result is that foreign exchange risk is as important as in the model without the third asset. It is true that the risk of Foreign bonds can be diversified away when a large fraction of wealth is invested in the uncorrelated third asset. But if the expected excess return on Foreign bonds is positive, active traders wish to invest a large amount in Foreign bonds and currency risk does matter. In the optimum the actively managed portfolio position is such that the expected excess return exactly compensates for the foreign exchange risk exposure.

## 5 Discussion

In this section we relate the previous analysis to five distinct aspects of the existing literature on the forward premium puzzle. First, how does the model connect to risk-premium based explanations of the forward discount puzzle? Second, is the model consistent with evidence of excess return predictability at very short horizons? Third, how does the model relate to survey evidence of predictable expectational errors? Fourth, how can the model shed light on a variety of other stylized facts associated with excess return predictability in the foreign exchange market? Finally, can the infrequent portfolio decision explanation also account for excess return predictability in other financial markets?

### *Connection to Risk Premium Explanations*

The standard assumption in finance is that expected excess returns reflect a risk premium. This assumes that agents continuously rethink the optimality of their portfolios. In this paper we have deviated from this by considering the implications of infrequent decisions about portfolios due to a cost of making such decisions. However, this does not mean that the model is entirely disconnected from risk-premium explanations. First, in subsection 3.3 we have introduced investors who do make decisions each period. From the perspective of these investors the expected excess return is identical to a risk premium. It should be emphasized though that it is the infrequent decision making by the great majority of investors that generates this time varying risk premium. If all investors manage their currency positions actively, the equilibrium expected excess return would be much smaller.

Second, there is also a risk premium for investors making infrequent portfolio decisions. For those investors a  $T$ -period Euler equation applies:

$$E_t(c_{t+T})^{-\gamma} q_{t,t+T} = 0 \tag{14}$$

where  $c_{t+T}$  is consumption at  $t + T$ . The risk premium for infrequent investors applies over  $T$  periods and is equal to the rate of risk aversion times the covariance of the excess return over  $T$  periods and consumption in  $T$  periods. For stock returns there is indeed evidence that long-horizon Euler equations fit the data better. Jagannathan and Wang (2005) show that the Euler equation fits the data substantially better at a one-year horizon than a monthly horizon. They argue that

infrequent portfolio and consumption decisions can account for this. Parker and Julliard (2005) find that the Euler equation fits the data best with consumption growth measured over three years. They argue that one reason for this may be the “presence of constraints on information flow” and refer to a literature where agents make infrequent portfolio decisions.

### *Short Horizons*

Chaboud and Wright (2005) report evidence that uncovered interest parity holds for a narrow window of two hours around 5pm New York time. At first sight this evidence may appear inconsistent with our framework. However, their evidence is implied by the absence of intraday interest payment and a one-time interest payment at 5pm. As there is a fixed interest payment at 5pm, the interest rate differential approaches infinity per unit of time for a shrinking interval around 5pm. There must then be a discrete change in the exchange rate at that time, corresponding to the interest rate differential, in order to avoid infinite arbitrage positions. UIP will then hold almost exactly. This is similar to the arbitrage that would take place with a stock going ex dividend. If we extend the window to more than a few hours, the interest differential per unit of time is much smaller and the combination of exchange rate risk and risk aversion prevents investors from taking infinite positions. Chaboud and Wright show that with daily observations there are again large deviations from UIP. Thus, our model is consistent with their results as long as there is a small share of traders with actively managed FX positions.

### *Survey Evidence of Predictable Expectational Errors*

Many papers on the forward discount puzzle argue that the bias must be the result of either time varying risk premia or predictable expectational errors (e.g. Froot and Frankel, 1989). The logic of this argument is based on the assumption that all agents make active portfolio decisions each period. In that case the expected excess return is equal to a risk premium and the actual excess return is equal to a risk premium plus expectational error. The bias therefore results from either the risk premium or the expectational error being negatively correlated with the forward discount. This decomposition is no longer valid in our model since the Euler equation does not apply on a periodic basis for investors making infrequent portfolio decisions.

There is extensive evidence of predictable expectational errors based on survey data on exchange rate expectations.<sup>42</sup> This has led to a number of papers that explain the forward discount puzzle by explicitly introducing irrational agents. For example, Mark and Wu (1998) account for the forward discount puzzle by introducing an exogenous expectational error that is negatively correlated with the interest rate differential. Gourinchas and Tornell (2004) explain the puzzle by assuming that agents incorrectly perceive the interest rate process and never learn. However, another interpretation of the evidence is that agents are fully rational but either do not find it in their interest to reevaluate exchange rate expectations on a continuous basis when they make infrequent portfolio decisions or rationally condition expectations on a limited information set for the various reasons discussed in section 3.2. That is the route we have taken in this paper.

### *Extensions*

Several other stylized facts related to the forward discount puzzle have been documented in the literature. The model proposed in this paper certainly cannot account for all of them. However, the analysis can be extended to deal with several of the additional features. We briefly mention three of them.

First, we could introduce long-term bonds. The model would then replicate the empirical evidence showing that the forward discount puzzle tends to go away over long horizons. Chinn and Meredith (2005) provide such evidence using regressions of the change in the exchange rate over a long horizon of 5 or 10 years on the interest rate differential for long-term bonds with corresponding maturity. They find coefficients of respectively 0.67 and 0.68. Without introducing long-term bonds we can conduct a closely related exercise of regressing the average excess return on foreign currency investments over the next  $K$  periods on the forward discount at time  $t$ . The resulting coefficient is the average of the coefficients  $\beta_k$  of the excess return regressions  $q_{t+k} = \alpha_k + \beta_k f d_t + \epsilon_{t+k}$ , for  $k$  from 1 to  $K$ . Both in the model and in the data these average predictability coefficients gradually decline in absolute size as  $K$  increases and are close to zero when  $K = 20$  (5 years).

A second extension is to modify the monetary policy rules in order to introduce persistent inflation shocks. This will allow the model to account for evidence by

---

<sup>42</sup>See Bacchetta, Mertens, and van Wincoop (2009) for a recent review of the evidence, which holds in other financial markets as well.

Bansal and Dahlquist (2000) that there is less excess return predictability for developing countries. Consider for example a change in Home country's monetary policy from a zero inflation target to a 10% inflation target. The only change that this generates in the model is in the steady state. There will now be a constant 10% steady state depreciation and the Home interest rate will be 10% higher. In deviation from this steady state the solution is the same as before. Such a change in policy therefore raises both  $s_{t+1} - s_t$  and  $fd_t$  by the same large amounts. One can therefore expect that persistent inflation shocks in the model will lead to a much higher coefficient in a regression of  $s_{t+1} - s_t$  on  $fd_t$ .

A third extension is to introduce transaction costs. Burnside et al. (2006) show that transaction costs are non-trivial relative to the size of profits from strategies exploiting excess return predictability. Sarno, Valente and Leon (2006) argue that transaction costs can account for non-linearities in the relationship between excess return predictability and the size of the interest rate differential. This is because transaction costs lead to a band of inaction.<sup>43</sup> When interest rate differentials are small, the gains from trading on the expected excess return may not outweigh the transaction cost, so that the excess return remains predictable. But when the interest rate differential gets large enough, active traders will take aggressive positions to exploit excess return predictability. Since introducing transaction costs will further reduce the welfare gain from active portfolio management, it provides a reinforcing motive for making infrequent portfolio decisions.

#### *Predictability in Other Financial Markets*

While this paper has focused on predictability in the foreign exchange market, excess returns are also predictable in other markets (see Cochrane, 1999). The explanation of infrequent portfolio decisions would be similarly relevant in those other financial markets. For example, for stock and bond markets there is plenty of evidence that most investors make infrequent decisions. In Section 2 we already reported evidence by the Investment Company Institute that only

---

<sup>43</sup>See Baldwin (1990) and the discussion in Lyons (2001, 206-220). A transaction cost of exchanging home bonds for foreign bonds is quite different from limited participation models where there is a transaction cost of exchanging bonds for money, the latter used for consumption. Alvarez, Atkeson, and Kehoe (2009) use such a model to shed light on the forward discount puzzle. In their model all agents can exchange all bonds at no cost.

40% of investors change their stock or mutual fund holdings during a particular year. Vissing-Jorgensen (2004) provides similar evidence based on the Survey of Consumer Finances. Agnew et al. (2003) and Ameriks and Zeldes (2001) find that pension fund reallocation is even far less frequent. Mutual funds themselves cannot freely arbitrage between stocks and bonds. Hedge funds can conduct such arbitrage, but still account for only a very small fraction of financial wealth.

In parallel to the delayed overshooting evidence for the foreign exchange market, it is widely documented that stock prices respond with delay to new publicly available information. Stock prices continue to move in the same direction six to twelve months after public events such as earnings announcements, stock issues and repurchases and dividend initiations and omissions.<sup>44</sup>

## 6 Conclusion

The model of infrequent portfolio decisions developed in the paper can shed light on many key empirical stylized facts related to the forward premium puzzle. First, it can explain why little of foreign exchange exposure is actively managed. The welfare gain from active management of currency positions is small since exchange rates are notoriously hard to predict. These welfare gains are generally below fees charged for active portfolio management. Second, infrequent portfolio decisions lead to a delayed impact of interest rate shocks on exchange rates. This can explain the phenomenon of “delayed overshooting”, whereby the exchange rate continues to appreciate over time after a rise in the interest rate. Third, the delayed overshooting leads to substantial excess return predictability in the direction seen in the data. Fourth, even future excess returns continue to be predictable by the current forward discount, with the magnitude of the predictability declining as time goes on.

We should stress that the model with infrequent decision making can explain the forward premium puzzle while matching other aspects of the data, in particular various univariate properties of exchange rates and interest rates (volatility and persistence). This reinforces the credibility of the explanation. In the multi-

---

<sup>44</sup>See Hong and Stein (1999) for references. The literature is most extensive regarding continued stock price appreciation subsequent to a positive earnings announcement, which has become known as “post earnings announcement drift.”

currency framework we found that the model can additionally account for the size of the active currency management industry while matching the correlation across countries of interest rates and currencies.

There are two natural directions for future research. First, we have seen that the magnitude of excess return predictability is even larger when agents condition expectations only on current interest rate differentials. While this is consistent with what we see in the FX market (e.g., carry trade), some of the most plausible explanations that we gave for this phenomenon (short samples, time-varying model parameters) fall outside the model that we have employed in this paper. It is therefore natural to introduce such features to the model in order to develop a better theoretical foundation for this phenomenon. Second, we have argued that there is extensive evidence of infrequent portfolio decisions in other financial markets. A natural direction for future work will be to evaluate to what extent infrequent portfolio decisions can quantitatively account for the extent of excess return predictability in other financial markets.

## A Appendix

This Appendix provides some of the technical background for the paper. Full technical details can be found in a Technical Appendix available upon request.

### A.1 Money Demand

Agents holding money live for two periods. Consider the Home agents (the description for the Foreign country is analogous). At time  $t$  they receive a transfer of Home money, which they invest in Home bonds and Home money. At  $t + 1$  they receive income from production and assets, return the money transfer they received at time  $t$  in the form of a tax, and consume the remainder. They derive utility from expected consumption at  $t + 1$ . Production is assumed to depend on real money balances as in Bacchetta and van Wincoop (2006):

$$y_{t+1} = \bar{y} - \tilde{m}_t (\ln(\tilde{m}_t) - 1) / \alpha \quad (15)$$

where  $\tilde{m}_t$  are real money balances and  $\bar{y}$  is a constant. Agents receive a money transfer of  $M_t$ , which they return to the government at  $t + 1$  through a tax. There-

fore

$$c_{t+1} = y_{t+1} + \left( \frac{M_t}{P_t} - \tilde{m}_t \right) (e^{i_t} - 1) \frac{P_t}{P_{t+1}} \quad (16)$$

The first-order condition with respect to real money balances is

$$-\frac{1}{\alpha} \ln(\tilde{m}_t) = E_t \left( e^{i_t} - 1 \right) \frac{P_t}{P_{t+1}} \quad (17)$$

Linearizing the right hand side (around  $i_t = p_t = 0$ ) gives

$$\ln(\tilde{m}_t) = -\alpha i_t \quad (18)$$

Imposing money market equilibrium we then have

$$m_t - p_t = -\alpha i_t \quad (19)$$

where  $m_t$  is the log money supply. Analogously,  $m_t^* - p_t^* = -\alpha i_t^*$  for the Foreign country. Notice that these agents do not impact the bond and goods market equilibria. Their demand for bonds is zero as  $M_t/P_t = \tilde{m}_t$  in equilibrium, while in period  $t + 1$  they simply consume their own production.

## A.2 Optimal Portfolios

We first describe how we derive the optimal portfolio (6) of investors making infrequent portfolio decisions. For investors born at time  $t$  the value function is:

$$V_t = E_t e^{(1-\gamma)(r_{t+1}^p + \dots + r_{t+T}^p)} / (1 - \gamma) \quad (20)$$

We adopt a second order approximation for the log return:

$$r_{t+k}^p = \bar{r} + b_t^I q_{t+k} + 0.5 b_t^I (1 - b_t^I) \text{var}_t(q_{t+k}) \quad (21)$$

Substituting this into the value function, maximization with respect to  $b_t^I$  yields

$$b_t^I = b^I + \frac{E_t q_{t,t+T}}{\gamma \sigma_I^2} \quad (22)$$

where

$$b^I = \frac{0.5 \sum_{k=1}^T \text{var}_t(q_{t+k})}{\gamma \sigma_I^2} \quad (23)$$

and  $\sigma_I^2$  is defined in (7). Notice that  $\sigma_I^2$  and  $b^I$  are constants because the conditional second moments entering these expressions are not time-varying.

For investors making frequent portfolio decisions the optimal portfolio is more complex since it involves a hedge against changes in future investment opportunities. Consider an agent born at time  $t$ . We will compute the optimal portfolio and value function at  $t+k$  for  $k=0, \dots, T-1$ . We make the following guess for the value function:

$$V_{t+k} = e^{Y'_{t+k} H_k Y_{t+k}} (1-\tau)^{(1-\gamma)(T-k)} W_{t+k}^{1-\gamma} / (1-\gamma) \quad (24)$$

where  $W_{t+k}$  is wealth at  $t+k$ ,  $H_k$  is a square matrix of size  $\bar{T}+2$  and  $Y_{t+k}$  is the state space. The latter consists of  $Y_{t+k} = (\varepsilon_{t+k}^u, \dots, \varepsilon_{t+k+1-\bar{T}}^u, \hat{x}_t, 1)'$ . Since in principle the state space is infinitely long, for tractability reasons it is truncated after  $\bar{T}$  periods (with  $\bar{T}$  very large), similar to the exchange rate solution. The key conjecture is that the term in the exponential of the value function is quadratic in the state space.

We know that

$$W_{t+k+1} = (1-\tau)W_{t+k}e^{r_{t+k+1}^p} \quad (25)$$

We again adopt a second order approximation for the log return:

$$r_{t+k+1}^p = \bar{r} + b_{t,t+k}^F q_{t+k+1} + 0.5b_{t,t+k}^F(1-b_{t,t+k}^F)\sigma_F^2 \quad (26)$$

where  $\sigma_F^2$  is the conditional variance of next period's excess return. After substituting (25) and (26) into the Bellman equation  $V_{t+k} = E_{t+k}V_{t+k+1}$ , we have

$$V_{t+k} = E_{t+k}e^{v_{t+k+1}}(1-\tau)^{(1-\gamma)(T-k)}W_{t+k}^{1-\gamma}/(1-\gamma) \quad (27)$$

where

$$v_{t+k+1} = (1-\gamma)\bar{r} + (1-\gamma)b_{t,t+k}^F q_{t+k+1} + (1-\gamma)0.5b_{t,t+k}^F(1-b_{t,t+k}^F)\sigma_F^2 + Y'_{t+k+1} H_{k+1} Y_{t+k+1} \quad (28)$$

It is useful to write

$$q_{t+k+1} = M_1^k Y_{t+k} + M_2^k \epsilon_{t+k+1} \quad (29)$$

and

$$Y_{t+k+1} = N_1^k Y_{t+k} + N_2^k \epsilon_{t+k+1} \quad (30)$$

where

$$\epsilon_{t+k+1} = \begin{pmatrix} \epsilon_{t+k+1}^u \\ \epsilon_{t+k+1}^x \end{pmatrix} \quad (31)$$

After substituting (29)-(30) into (28) we can compute  $E_{t+k}e^{v_{t+k+1}}$ . Maximizing the resulting time  $t+k$  value function with respect to  $b_{t,t+k}^F$  yields the optimal portfolio

$$b_{t,t+k}^F = \bar{b}^F(k) + \frac{E_{t+k}(q_{t+k+1})}{(\gamma-1)\hat{\sigma}_F^2(k) + \sigma_F^2} + D^k Y_{t+k} \quad (32)$$

where

$$\bar{b}^F(k) = \frac{0.5\sigma_F^2}{(\gamma-1)\hat{\sigma}_F^2(k) + \sigma_F^2} \quad (33)$$

is a constant and

$$\hat{\sigma}_F^2(k) = M_2^k \Omega^k (M_2^k)' \quad (34)$$

$$\Omega^k = (\Sigma^{-1} - 2C_2^k)^{-1} \quad (35)$$

$$\Sigma = \text{var}(\epsilon_{t+k+1}) \quad (36)$$

$$C_2^k = (N_2^k)' H_{k+1} N_2^k \quad (37)$$

$$D^k = 2M_2^k \Omega^k (N_2^k)' H_{k+1} N_1^k / [(\gamma-1)\hat{\sigma}_F^2(k) + \sigma_F^2] \quad (38)$$

The second term in the optimal portfolio depends on the expected excess return over the next period. In the denominator  $\sigma_F^2 = \text{var}_t(q_{t+1})$ . The term  $\hat{\sigma}_F^2(k)$  is in practice very close to  $\text{var}_t(q_{t+1})$  as well, so that the denominator is close to  $\gamma \text{var}_t(q_{t+1})$ . The third term captures a hedge against changes in future expected returns.  $D^k$  is a vector of constant terms, so this term is linear in the state space.

We can now also compute the threshold cost. We solve the value function at time  $t$  with backwards induction, starting with the known value function at  $t+T$ ,  $V_{t+T} = W_{t+T}^{1-\gamma}/(1-\gamma)$ , which corresponds to  $H_T = 0$ . Since each investor starts with wealth equal to 1, the value function at birth for an investor making frequent portfolio decisions is  $e^{Y_t' H_0 Y_t} (1-\tau)^{(1-\gamma)T}/(1-\gamma)$ . For an investor making only one portfolio decision for  $T$  periods, the time  $t$  value function is  $V_t = E_t W_{t+T}^{1-\gamma}/(1-\gamma)$ . After substituting  $W_{t+T} = e^{r_{t+1}^p + \dots + r_{t+T}^p}$ , maximization with respect to  $b_t^I$  yields the optimal portfolio (32) and a time  $t$  value function that takes the form  $e^{Y_t' H Y_t}/(1-\gamma)$ . When born, investors need to decide whether to actively manage their portfolio before observing the state  $Y_t$ . In a more realistic framework where agents have infinite lives and make portfolio decisions every  $T$  periods, this corresponds to agents

deciding on the frequency of portfolio decisions before observing future states when portfolio decisions are actually made. We therefore compare the unconditional expectation of the time  $t$  value functions for the two strategies, where the expectation is with respect to the unconditional distribution of  $Y_t$ . The threshold cost is the level  $\tau$  such that expected utility is the same under both strategies.

### A.3 Solving the Equilibrium Exchange Rate

Consider the market equilibrium condition (13). The case where all investors make infrequent portfolio decisions (eq. (9)) is easily found by setting  $n_F = 0$  and  $n_I = n$ . A first order Taylor approximation of (13) gives:

$$\begin{aligned} n_F \sum_{k=1}^T b_{t-k+1,t}^F + n_I \sum_{k=1}^T b_{t-k+1}^I + n_F \bar{k}^F + n_I \bar{k}^I + \\ \sum_{k=1}^{T-1} (n_F k^F(k) + n_I k^I(k)) q_{t-k+1} + (\bar{x} + x_t) \bar{W} = B + B s_t \end{aligned} \quad (39)$$

where

$$\begin{aligned} \bar{k}^F &= \sum_{k=1}^{T-1} \bar{b}^F(k) k (\bar{r} - \tau) \\ k^F(k) &= \sum_{j=1}^{T-k} \bar{b}^F(j-1) \bar{b}^F(k+j-1) \\ \bar{k}^I &= \sum_{k=1}^{T-1} \bar{b}^I k \bar{r} \\ k^I(k) &= (T-k) (\bar{b}^I)^2 \end{aligned}$$

Steady state financial wealth is defined as total financial wealth when the returns on Home and Foreign bonds are equal to their steady state levels ( $\bar{r}$  for Home bonds and 0 for Foreign bonds),  $\tau = 0$  and the fraction invested in Foreign bonds is  $b$ . Based on that definition we have

$$\bar{W} = w n T \quad (40)$$

where

$$w = \sum_{k=1}^T (\bar{R}^p)^{k-1} / T \quad (41)$$

$$\bar{R}^p = (1-b)e^{\bar{r}} + b \quad (42)$$

The constant term in the portfolio of liquidity traders,  $\bar{x}$ , is set such that the market clearing condition holds in steady state for a given real interest rate  $\bar{r}$ . Finally, we subtract the steady state from both sides of (39), divide it by  $nT$ , and use the expressions for optimal portfolio shares to get an expression in deviation from steady state:

$$f \frac{E_t \tilde{q}_{t+1}}{\gamma \sigma^2} + f DY_t + (1-f) \frac{1}{T} \sum_{k=1}^T \frac{E_{t-k+1} \tilde{q}_{t-k+1, t-k+1+T}}{\gamma \sigma_I^2} + \sum_{k=1}^{T-1} \frac{1}{T} (f k^F(k) + (1-f) k^I(k)) \tilde{q}_{t-j+1} + w x_t = w b s_t \quad (43)$$

where  $f = n_F/n$  is the fraction of agents making frequent portfolio decisions, the tilde denotes excess returns in deviation from their steady state and

$$D = \frac{1}{T} \sum_{k=1}^T D^{k-1}$$

$$\frac{1}{\sigma^2} = \frac{1}{T} \sum_{k=1}^T \frac{\gamma}{(\gamma-1) \hat{\sigma}_F^2(k-1) + \sigma_F^2}$$

We conjecture (11) with  $A(L) = a_1 + a_2 L + a_3 L^2 + \dots$  and  $B(L) = b_1 + b_2 L + b_3 L^2 + \dots$ . Substituting (11) into the market equilibrium condition (43), we obtain an equilibrium exchange rate equation. We then need to equate the conjectured to the equilibrium exchange rate equation. We choose the process

$$x_t = C(L) \epsilon_t^x = (c_1 + c_2 L + c_3 L^2 + \dots) \epsilon_t^x \quad (44)$$

such that  $\hat{x}_t = B(L) \epsilon_t^x$  follows the AR process (12). We normalize such that  $c_1 = 1$ .

We therefore choose  $A(L)$ ,  $b_1$  and  $C(L)$  such that (i) the Foreign bond market equilibrium condition (43) is satisfied and (ii)  $\hat{x}_t = B(L) \epsilon_t^x$  follows the AR process in (12). The latter implies imposing  $b_{k+1} = \rho_x b_k$  for  $k \geq 1$ . Imposing the market equilibrium condition involves computing first and second moments of excess returns based on the conjectured exchange rate process. After that is done both sides of the market equilibrium equation can be written as a linear function of the underlying innovations at time  $t$  and earlier. We then need to equate the coefficients multiplying these innovations on the right and left side of the equation, which involves solving a fixed point problem. The overall approach is rather straightforward, but the algebra is lengthy and can be found in the Technical Appendix.

## A.4 Multiple Currencies

There are  $N$  countries, with  $N > 2$ . Country  $N$  is the Home country, which in the data is the United States. We continue to assume that half of the steady state world bond supply is from the Home country, with the other countries each contributing an equal fraction of the remaining global bond supply. The Home country again follows a zero-inflation policy, so that its nominal interest rate is equal to  $i_t^N = \bar{r}$ . For the remaining  $N - 1$  countries the interest rate again follows an AR process. For country  $n$  we assume  $i_t^n = -u_t^n$  with

$$u_t^n = \rho u_{t-1}^n + \varepsilon_t^n \quad \varepsilon_t^n \sim N(0, \sigma_u^2) \quad (45)$$

The interest rate innovations  $\varepsilon_t^n$  have a common correlation across countries. Calibrating the model to the 5 currencies of Table 1, we set  $N = 6$  and set the correlation across countries of shocks to the AR process for the forward discount equal to the average of that in the data, which is 0.69. Liquidity demand for each of the Foreign bonds is modeled and calibrated as before, with the correlation of liquidity demand innovations across currencies chosen to match the average correlation of one-quarter exchange rate changes across countries, which is 0.56.

Apart from becoming more complex, the computation of optimal portfolios of both infrequent and frequent traders is similar to the two-country case. If we now define  $\mathbf{q}_{t,t+T}$  as the entire vector of excess returns of the  $N - 1$  Foreign bonds from  $t$  to  $t + T$ , and analogously define  $\mathbf{q}_{t+k}$  as the vector of excess returns from  $t + k - 1$  to  $t + k$ , the optimal vector of portfolio shares invested in the  $N - 1$  Foreign bonds by an infrequent trader born at  $t$  is

$$\mathbf{b}_t^I = \mathbf{b}^I + (\boldsymbol{\sigma}_I^2)^{-1} \frac{E_t \mathbf{q}_{t,t+T}}{\gamma} \quad (46)$$

where  $\mathbf{b}^I$  is a vector of constants and

$$\boldsymbol{\sigma}_I^2 = \left(1 - \frac{1}{\gamma}\right) \text{var}_t(\mathbf{q}_{t,t+T}) + \frac{1}{\gamma} \sum_{k=1}^T \text{var}_t(\mathbf{q}_{t+k}) \quad (47)$$

Demand for a particular Foreign currency now depends also on the expected excess returns of other currencies. This is the result of the correlation of excess returns across currencies, captured by  $\boldsymbol{\sigma}_I^2$ .

The solution of the model is analogous to before, although the equilibrium exchange rates now depend on interest rate and liquidity demand innovations for each of the  $N - 1$  Foreign bonds. In particular

$$\mathbf{s}_t = \mathbf{A}(L)\boldsymbol{\varepsilon}_t^u + \mathbf{B}(L)\boldsymbol{\varepsilon}_t^x \quad (48)$$

where  $\boldsymbol{\varepsilon}_t^u$  and  $\boldsymbol{\varepsilon}_t^x$  are vectors:  $\boldsymbol{\varepsilon}_t^u = (\varepsilon_t^1, \dots, \varepsilon_t^{N-1})'$  and  $\boldsymbol{\varepsilon}_t^x = (\varepsilon_t^{1,x}, \dots, \varepsilon_t^{N-1,x})'$  and the  $\mathbf{a}_i$ ,  $\mathbf{b}_i$  in the lag polynomials  $\mathbf{A}(L) = \mathbf{a}_1 + \mathbf{a}_2L + \mathbf{a}_3L^2 + \dots$ ,  $\mathbf{B}(L) = \mathbf{b}_1 + \mathbf{b}_2L + \mathbf{b}_3L^2 + \dots$ , are matrices. As a result of the symmetry of the model with regard to the  $N - 1$  Foreign countries, we can use the fact that all off-diagonal elements of the  $\mathbf{a}_i$  and  $\mathbf{b}_i$  matrices must be the same. Otherwise the solution proceeds as before following the method of undetermined coefficients described in Section 3. Full details of the analysis can be found in the Technical Appendix.

## References

- [1] Agnew, J., P. Balduzzi, and A. Sundén (2003), "Portfolio Choice and Trading in a Large 401(k) Plan," *American Economic Review* 93, 193-215.
- [2] Alvarez, Fernando, Andrew Atkeson, and Patrick J. Kehoe (2009), "Time-Varying Risk, Interest Rates, and Exchange Rates in General Equilibrium," *Review of Economic Studies*, forthcoming.
- [3] Ameriks, John and Stephen P. Zeldes (2001), "How Do Household Portfolio Shares Vary with Age?" Working Paper 6-120101, TIAA-CREF Institute.
- [4] Bacchetta, Philippe, and Eric van Wincoop (2004), "A Scapegoat Model of Exchange Rate Determination," *American Economic Review* 94, 114-118.
- [5] Bacchetta, Philippe and Eric van Wincoop (2006), "Can Information Heterogeneity Explain the Exchange Rate Determination Puzzle?," *American Economic Review* 96, 552-576.
- [6] Bacchetta, Philippe and Eric van Wincoop (2007), "Random Walk Expectations and the Forward Discount Puzzle," *American Economic Review, Papers and Proceedings*, 97, 346-350, May.
- [7] Bacchetta, Philippe, Elmar Mertens, and Eric van Wincoop (2009), "Predictability in Financial Markets: What Does Survey Expectations Tell Us?," *Journal of International Money and Finance*, forthcoming.
- [8] Backus, David K., Silverio Foresi and Chris I. Telmer (2001), "Affine Term Structure Models and the Forward Premium Anomaly," *Journal of Finance* 56(1), 279-304.
- [9] Baldwin, Richard E. (1990), "Re-interpreting the Failure of Foreign Exchange Market Efficiency Tests: Small Transaction Costs, Big Hysteresis Bands," NBER Working Paper 3319.
- [10] Bank for International Settlements (BIS) (2004), "Triennial Central Bank Survey of Foreign Exchange and Derivative Market Activity in April 2004".

- [11] Barberis, Nicholas (2000), "Investing for the Long Run when Returns Are Predictable," *Journal of Finance* LV, 225-265.
- [12] Bansal, Ravi and Amir Yaron (2004), "Risks for the Long Run: A Potential Resolution of Asset Pricing Puzzles," *Journal of Finance* 59, 1481-1509.
- [13] Bansal, Ravi and Magnus Dahlquist (2000), "The Forward Discount Puzzle: Different Tales for Developed and Emerging Economies," *Journal of International Economics* 51, 115-144.
- [14] Bekaert, Geert, Robert J. Hodrick and David Marshall (1997), "The Implications of First Order Risk Aversion for Asset Market Risk Premiums," *Journal of Monetary Economics* 40, 3-39.
- [15] Berk, Jonathan B. and Richard C. Green (2004), "Mutual Fund Flows and Performance in Rational Markets," *Journal of Political Economy*, 112 (6), 1269-1295.
- [16] Biliass, Yannis, Dimitris Georgarakos and Michael Haliassos (2005), "Portfolio Inertia and Stock Market Fluctuations," working paper, Goethe University of Frankfurt.
- [17] Boudoukh, Jacob, Matthew Richardson, and Robert F. Whitelaw (2005), "The Information in Long-Maturity Forward Rates: Implications for Exchange Rates and the Forward Premium Anomaly," mimeo.
- [18] Burnside, Craig, Martin Eichenbaum, Isaac Kleshchelski, and Sergio Rebelo (2006), "The Returns to Currency Speculation," working paper, Northwestern University.
- [19] Burnside, Craig, Martin Eichenbaum, and Sergio Rebelo (2007), "Understanding the Forward Premium Puzzle: A Microstructure Approach " working paper, Northwestern University.
- [20] Campbell, John Y. and Motohiro Yogo (2006), "Efficient Tests of Stock Return Predictability," *Journal of Financial Economics* 81, 27-60.

- [21] Campbell, John. Y. and Luis M. Viceira (1999), "Consumption and Portfolio Decisions when Expected Returns are Time Varying," *Quarterly Journal of Economics* May, 433-495.
- [22] Campbell, Rachel, Kees Koedijk, James Lothian and Ronald Mathieu (2007), "Irving Fisher, the UIP Puzzle and the Peso Problem," working paper, Erasmus University.
- [23] Chaboud, Alain P. and Jonathan H. Wright (2005), "Uncovered Interest Rate Parity: It Works, But Not For Long," *Journal of International Economics* 66(2), 349-362.
- [24] Chinn, Menzie D. (2006), "The (Partial) Rehabilitation of Interest Rate Parity in the Floating Rate Era: Longer Horizons, Alternative Expectations, and Emerging Markets," *Journal of International Money and Finance* 25, 7-21.
- [25] Chinn, Menzie D. and Guy Meredith (2005), "Testing Uncovered Interest Parity at Short and Long Horizons During the post-Bretton Woods Era," NBER Working paper 11077.
- [26] Chinn, Menzie and Jeffrey Frankel (2002), "Survey Data on Exchange Rate Expectations: More Currencies, More Horizons, More Tests," in W. Allen and D. Dickinson, eds., *Monetary Policy, Capital Flows and Financial Market Developments in the Era of Financial Globalization: Essays in Honor of Max Fry*, London: Routledge, 145-167.
- [27] Cochrane, John (1999), "New Facts in Finance," *Economic Perspectives*, Federal Reserve Bank of Chicago, 23 (3), 36-58.
- [28] Duffie, Darrell and Tong-sheng Sun (1990), "Transactions Costs and Portfolio Choice in a Discrete-Continuous-Time Setting," *Journal of Economic Dynamics and Control*," 14, 35-51.
- [29] Dybvig, Philip H., Heber K. Farnsworth and Jennifer N. Carpenter (2004), "Portfolio Performance and Agency," New York University, Law and Economics Research Paper No. 04-010.

- [30] Eichenbaum, Martin and Charles L. Evans (1995), "Some Empirical Evidence on the Effects of Shocks to Monetary Policy on Exchange Rates," *Quarterly Journal of Economics* 110, 975-1009.
- [31] Engel, Charles (1996), "The Forward Discount Anomaly and the Risk Premium: A Survey of Recent Evidence," *Journal of Empirical Finance* 3, 123-192.
- [32] Engel, Charles and Akito Matsumoto (2008), "Portfolio Choice in a Monetary Open-Economy DSGE Model," working paper, University of Wisconsin.
- [33] Engel, Charles and Kenneth D. West (2005), "Exchange Rates and Fundamentals," *Journal of Political Economy* 113(3), 485-517.
- [34] Fama, Eugene F. (1984), "Forward and Spot Exchange Rates," *Journal of Monetary Economics* 14, 319-338.
- [35] Fama, Eugene F. (1991), "Efficient Capital Markets: II," *Journal of Finance*, 46 (5), 1575-1617.
- [36] Fisher, Eric O'N. (2006), "The Forward Premium in a Model with Heterogeneous Prior Beliefs," *Journal of International Money and Finance*, forthcoming.
- [37] Flood, Robert and Andrew K. Rose (2002), "Uncovered Interest Parity in Crisis," *IMF Staff Papers* 49(2), 252-266.
- [38] Froot, Kenneth A. and Jeffrey A. Frankel (1989), "Forward Discount Bias: Is it an Exchange Risk Premium?" *Quarterly Journal of Economics* 104, February, 139-161.
- [39] Froot, Kenneth A., Paul G.J. O'Connell, and Mark S. Seasholes (2001), "The Portfolio Flows of International Investors" *Journal of Financial Economics* 59, 151-193.
- [40] Froot, Kenneth A. and Richard H. Thaler (1990), "Anomalies: Foreign Exchange," *Journal of Economic Perspectives* 4(3), 179-192.

- [41] Gabaix, Xavier and David Laibson (2002), “The 6D Bias and the Equity Premium Puzzle,” in Ben Bernanke and Kenneth Rogoff, eds, *NBER Macroeconomics Annual*, 16, 257-312.
- [42] Gourinchas, Pierre-Olivier and Aaron Tornell (2004), “Exchange Rate Puzzles and Distorted Beliefs,” *Journal of International Economics* 64, 303-333.
- [43] Hau, Harald and Hélène Rey (2006), “Exchange Rates, Equity Prices and Capital Flows,” *Review of Financial Studies* 19, 273-317.
- [44] Hau, Harald and Hélène Rey (2008), “Global Portfolio Rebalancing under the Microscope,” mimeo.
- [45] Hong, Harrison and Jeremy C. Stein (1999), “A Unified Theory of Underreaction, Momentum Trading and Overreaction in Asset Markets,” *Journal of Finance*, 54(6), 2143-2184.
- [46] Investment Company Institute (ICI) (2002), “Equity Ownership in America, 2002”.
- [47] Jagannathan, Ravi and Yong Wang (2005), “Consumption Risk and the Cost of Equity Capital,” NBER working paper 11026.
- [48] Kandel, Shmuel, and Robert F. Stambaugh (1996), “On the Predictability of Stock Returns: An Asset-Allocation Perspective,” *Journal of Finance* LI, 385-424.
- [49] Lane, Philip R. and Gian Maria Milesi-Ferretti (2006), “The External Wealth of Nations Mark II: Revised and Extended Estimates of Foreign Assets and Liabilities, 1970-2004,” IMF Working Paper WP/06/69.
- [50] Lewis, Karen K. (1995), “Puzzles in International Financial Markets,” in Gene M. Grossman and Kenneth Rogoff (eds), *Handbook of International Economics* (Amsterdam, Elsevier Science), 1913-1971.
- [51] Liu, Wei, and Alex Maynard (2005), “Testing Forward Rate Unbiasedness Allowing for Persistent Regressors,” *Journal of Empirical Finance*, forthcoming.

- [52] Lustig, Hanno and Adrien Verdelhan (2007), “The Cross-Section of Foreign Currency Risk Premia and US Consumption Growth Risk,” *American Economic Review*, 97(1), 89-117.
- [53] Lynch, Anthony W. (1996), “Decision Frequency and Synchronization Across Agents: Implications for Aggregate Consumption and Equity Return,” *The Journal of Finance* 51(4), 1479-1497.
- [54] Lyons, Richard K. (2001), *The Microstructure Approach to Exchange Rates*, MIT Press, (Cambridge, Massachusetts).
- [55] Mark, Nelson and Yangru Wu (1998), “Rethinking Deviations from Uncovered Interest Parity: The Role of Covariance Risk and Noise,” *The Economic Journal* 108, 1686-1706.
- [56] McCallum, Bennett T. (1994), “A Reconsideration of the Uncovered Interest Rate Parity Relationship,” *Journal of Monetary Economics* 33, 105-132.
- [57] Meese, Richard A. and Kenneth Rogoff (1983), “Empirical Exchange Rate Models of the Seventies: Do They Fit Out of Sample?” *Journal of International Economics* 14, 345-373.
- [58] Mehra, Rajnish and Edward C. Prescott (1985), “The Equity Premium: A Puzzle,” *Journal of Monetary Economics*, 15, 145-161.
- [59] Parker, Jonathan A. and Christian Julliard (2005), “Consumption Risk and the Cross Section of Expected Returns,” *Journal of Political Economy* 113, 185-222.
- [60] Sager, Michael J. and Mark P. Taylor (2006), “Under the Microscope: The Structure of the Foreign Exchange Market,” *International Journal of Finance and Economics* 11, 81-95.
- [61] Sarno, Lucio, Giorgio Valente and Hyginus Leon (2006), “The Forward Discount Puzzle and Nonlinearity in Deviations from Uncovered Interest Parity: A New Perspective,” *Review of Finance* 10, 443-482.

- [62] Sarno, Lucio and Giorgio Valente (2009), "Exchange Rates and Fundamentals: Footloose or Evolving Relationship?," *Journal of the European Economic Association*, forthcoming.
- [63] Sarno, Lucio (2005), "Towards a Solution to the Puzzles in Exchange Rate Economics: Where do we Stand?," *Canadian Journal of Economics* 38(3), 673-708.
- [64] Stambaugh, Robert F. (1999), "Predictive Regressions," *Journal of Financial Economics* 54, 375-421.
- [65] Verdelhan, Adrien (2007), "A Habit-Based Explanation of the Exchange Rate Risk Premium," mimeo, Boston University.
- [66] Vissing-Jorgensen, Annette (2004), "Perspectives on Behavioral Finance: Does "Irrationality" Disappear with Wealth? Evidence from Expectations and Actions," in *2003 NBER Macroeconomics Annual*, 139-194.
- [67] Vissing-Jorgensen, Annette and Orazio P. Attanasio (2003), "Stock-Market Participation, Intertemporal Substitution, and Risk-Aversion," *American Economic Review Papers and Proceedings*, 383-391.

Table 1: Predictable Excess Returns

$q_{t+1} = \alpha + \beta(i_t - i_t^*) + \epsilon_{t+1}$			
Currencies	$\beta$	$\sigma(\beta)$	$R^2$
DEM	-1.8344**	0.8189	0.05
GBP	-2.9537***	1.1214	0.10
JPY	-4.0626***	0.7438	0.16
CND	-1.5467***	0.5305	0.05
CHF	-2.3815***	0.8068	0.09
Average	-2.5558***	0.6192	0.09

Note:  $q_{t+1} = \Delta s_{t+1} - (i_t - i_t^*)$ .  $\Delta s_{t+1}$  refers to the 3-month change in the log exchange rate. The exchange rate is measured as net-of-period rate from IFS. Interest rates are 3-month rates as quoted in the London Euromarket and were obtained from Datastream (Thomson Financial). \*\*\* and \*\* denote significance at respectively the 1% and 5% level. SUR system estimated from 109 quarterly observations over sample from December 1978 to December 2005. Newey-West standard errors with 1 lag. "Average" refers to the equally weighted average of the regression coefficients.

Table 2: Sensitivity Analysis

parameters	predictability coefficient $\beta$ in $q_{t+1} = \alpha + \beta f d_t$	prob. $\beta < -1$	prob. $\beta < -2$	delayed over-shooting	auto-correlation $s_t - s_{t-1}$	$R^2$	threshold cost (5% risk)
benchmark ( $\gamma = 10, T = 8$ )	-0.95	48	12	3	0.004	0.011	0.70
$\gamma = 1$	-0.49	28	5	2	0.002	0.014	0.60
$\gamma = 50$	-1.16	56	19	5	-0.002	0.016	0.60
$T = 4$	-0.56	29	6	2	0.001	0.004	0.62
$T = 12$	-1.12	52	16	3	0.001	0.015	0.69

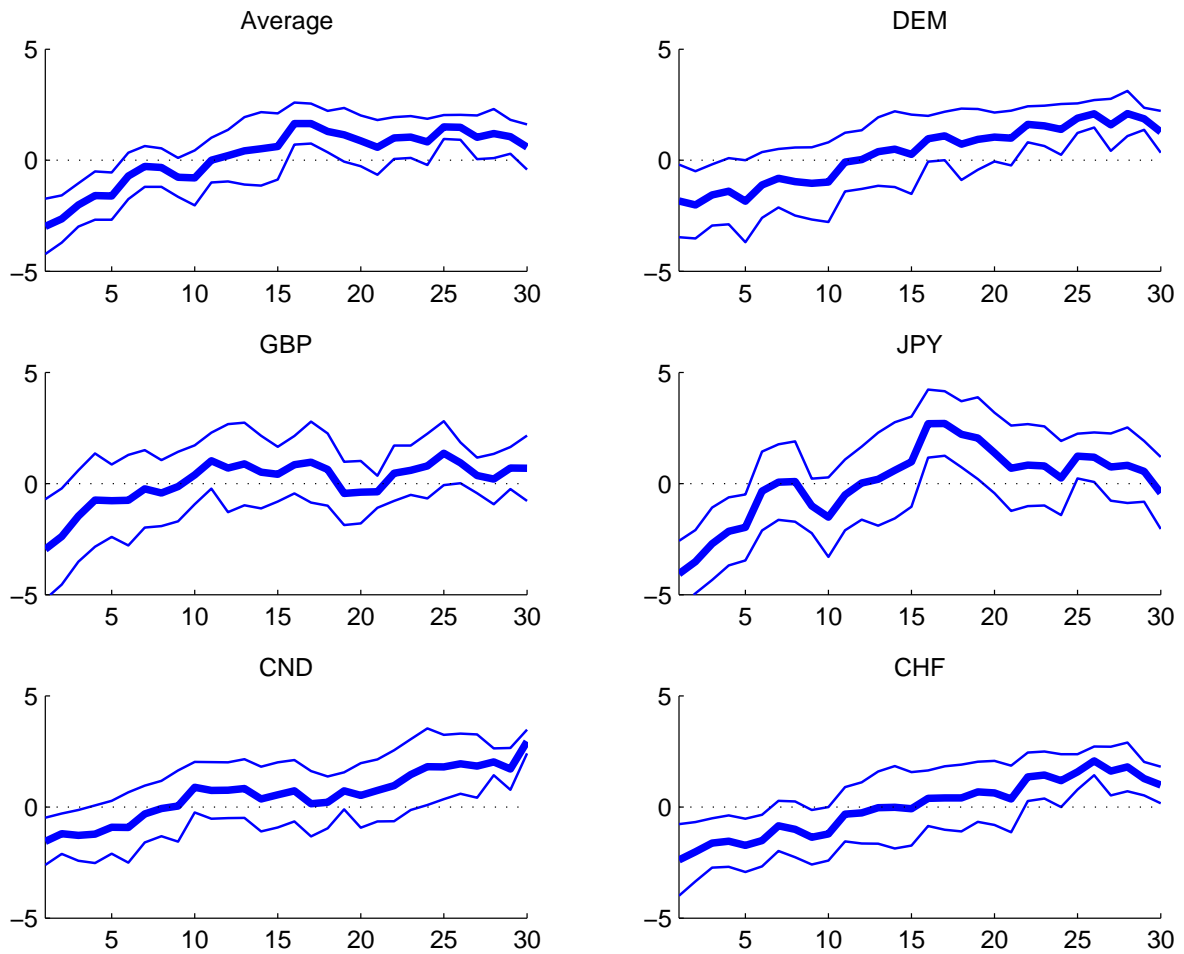
Note: The third and fourth column show the percentage of times that the excess return predictability coefficient is less than respectively -1 and -2 based on 1000 simulations of the model over a 25-year period. The fifth column shows the number of subsequent periods of appreciation of the Home currency after a drop in the Foreign interest rate. It is a measure of delayed overshooting. The sixth column is the first-order autocorrelation of quarterly log exchange rate change. The seventh column reports the  $R^2$  of the excess return predictability regression. The final column reports the threshold cost  $\tau$  at 5% risk.

Table 3: Sensitivity Analysis–Currency Forecasts Conditioned on Current Interest Rates Only

parameters	predictability coefficient $\beta$ in $q_{t+1} = \alpha + \beta f d_t$	prob. $\beta < -1$	prob. $\beta < -2$	delayed over- shooting	auto- correlation $s_t - s_{t-1}$	$R^2$	threshold cost (5% risk)
benchmark ( $\gamma = 10, T = 8$ )	-2.12	90	54	8	0.050	0.053	0.87
$\gamma = 1$	-2.41	94	69	7	0.101	0.068	0.63
$\gamma = 50$	-1.38	68	25	8	0.000	0.023	0.64
$T = 4$	-1.93	87	48	4	0.092	0.044	0.84
$T = 12$	-1.87	84	43	12	0.020	0.042	0.82

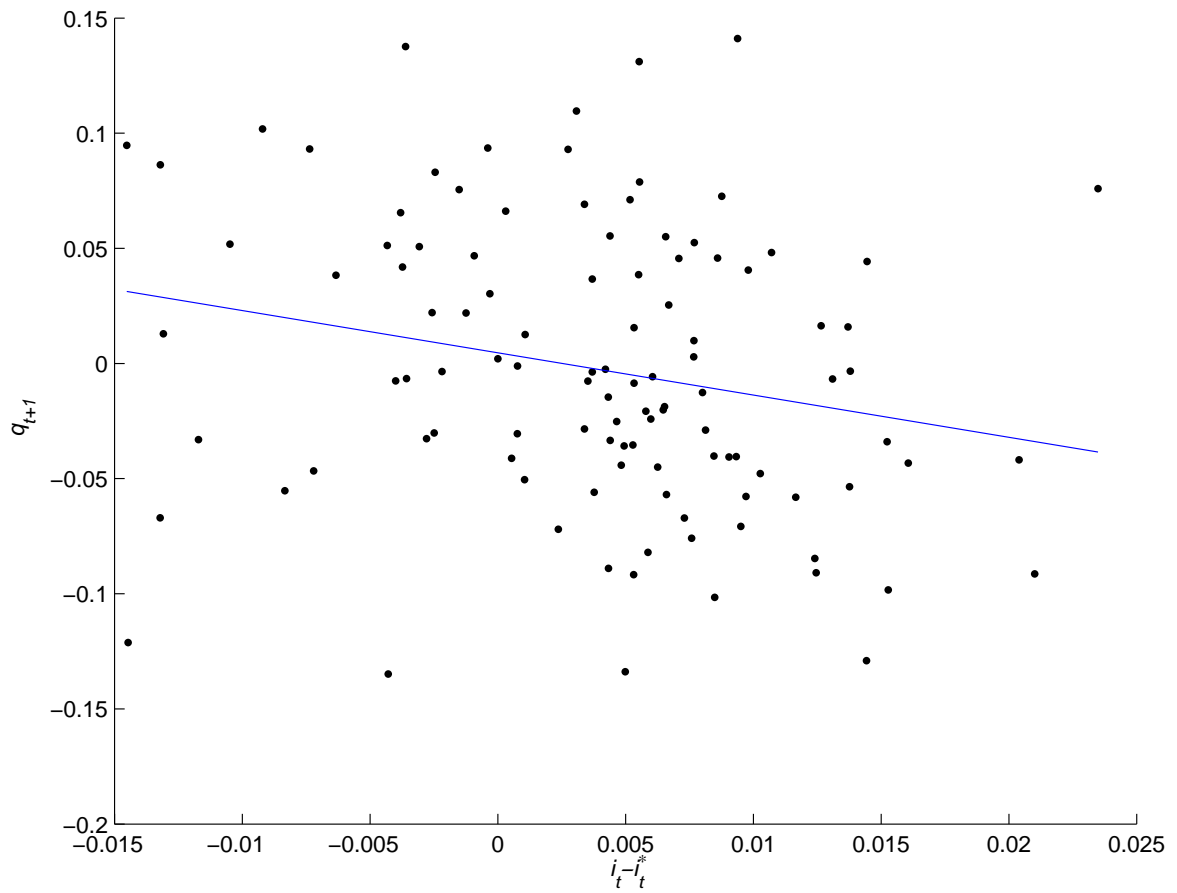
Note: The third and fourth column show the percentage of times that the excess return predictability coefficient is less than respectively -1 and -2 based on 1000 simulations of the model over a 25-year period. The fifth column shows the number of subsequent appreciations of the Home currency after a drop in the Foreign interest rate. It is a measure of delayed overshooting. The sixth column is the first-order autocorrelation of quarterly log exchange rate change. The seventh column reports the  $R^2$  of the excess return predictability regression. The final column reports the threshold cost  $\tau$  at 5% risk.

Figure 1: Excess Return Predictability



Note: Excess return predictability coefficients  $\beta_k$  of regressions  $q_{t+k} = \alpha + \beta_k(i_t - i_t^*) + \varepsilon_{t+k}$  for each currency. Thin lines are standard error bands ( $\pm 2$  s.e.). Same quarterly data as in Table 1. The average refers to the GDP-weighted average of the excess return predictability coefficients.

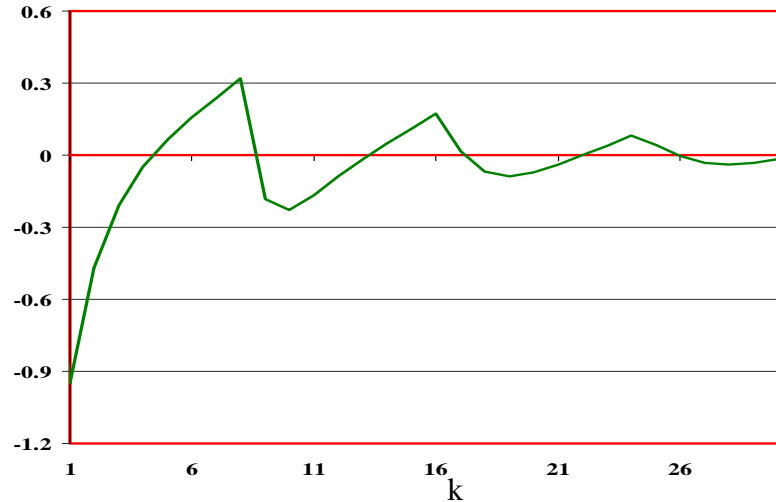
Figure 2: Excess Return Predictability for DEM



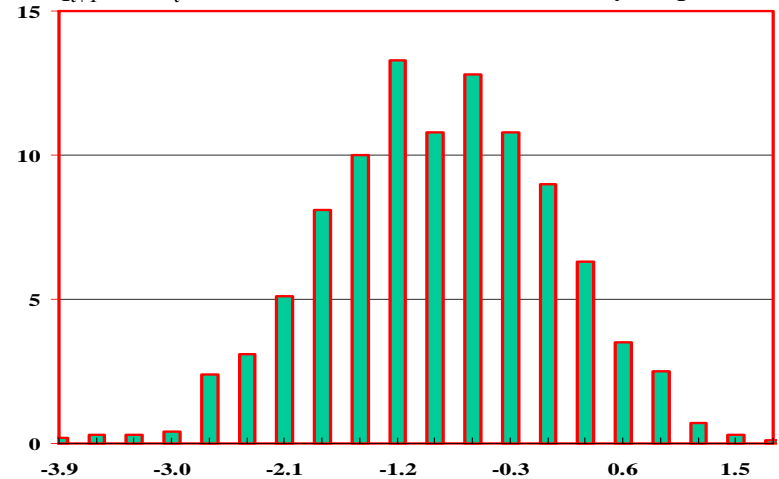
Note: Same quarterly data as in Table 1. OLS Slope = -1.8344 (s.e. = 0.8189, computed with 1 Newey-West lag).

Figure 3 Excess Return Predictability - Benchmark Parameterization

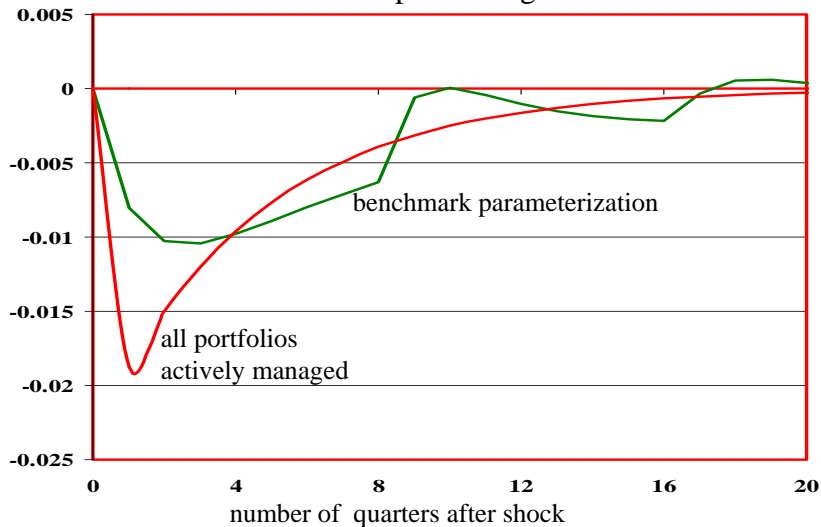
Panel A: Regression coefficient of  $q_{t+k}$  on  $fd_t$



Panel B: Frequency distribution of regression coefficient of  $q_{t+1}$  on  $fd_t$  based on 1000 simulations of 25-year period



Panel C: Impulse response exchange rate after one standard deviation drop in Foreign interest rate



Panel D: Simulation of 25-year period: excess return and forward discount

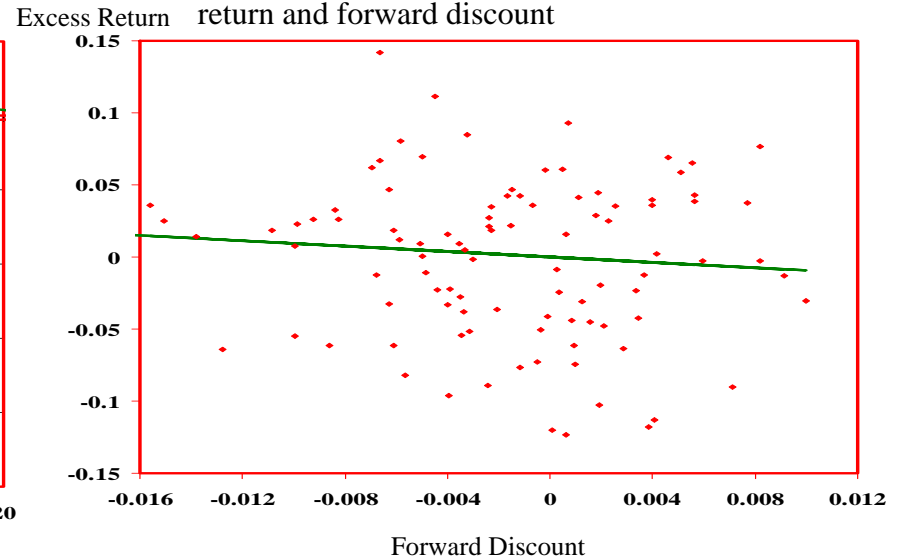
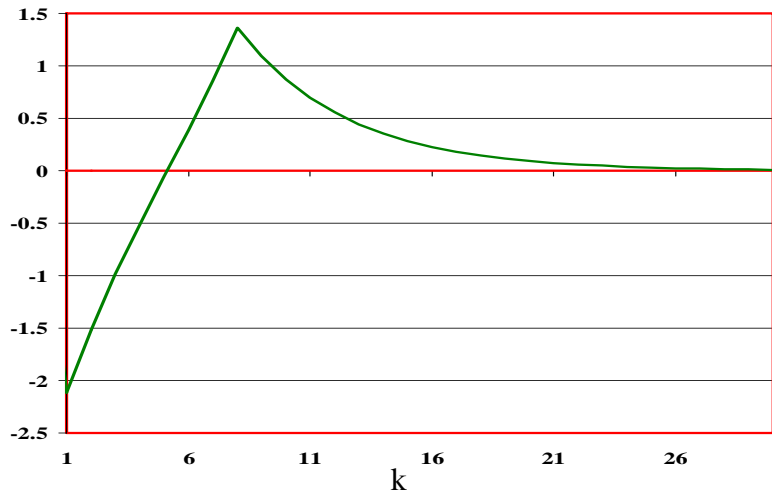
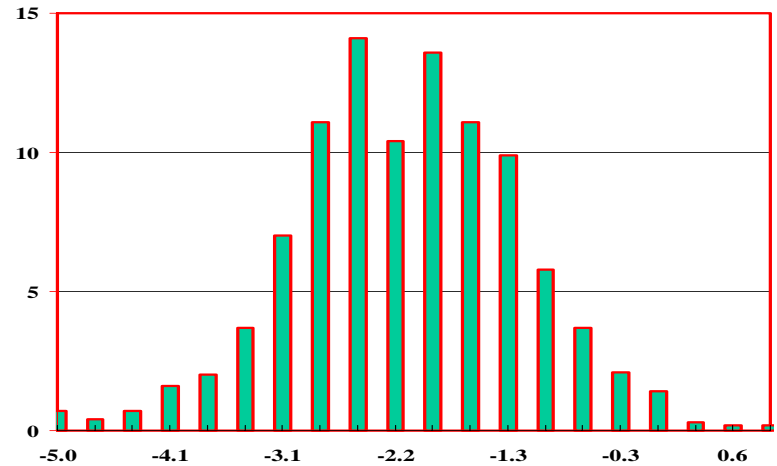


Figure 4 Excess Return Predictability—Currency Forecasts Conditioned on Current Interest Rates Only

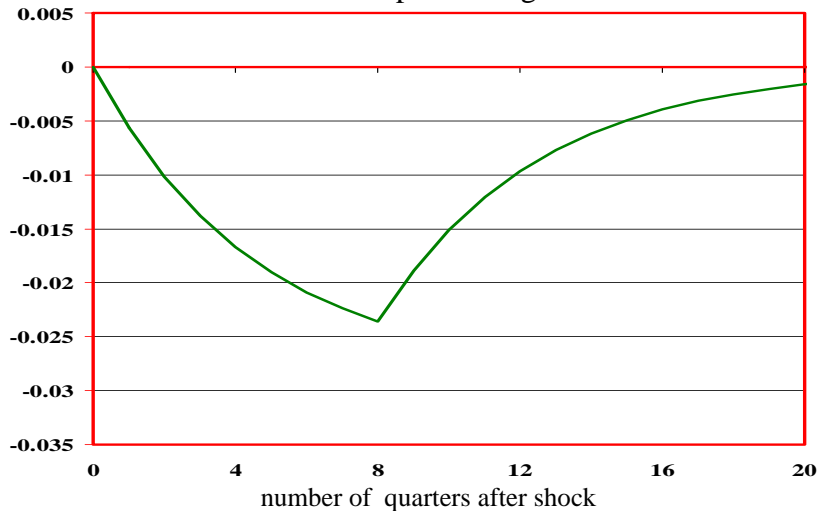
Panel A: Regression coefficient of  $q_{t+k}$  on  $fd_t$



Panel B: Frequency distribution of regression coefficient of  $q_{t+1}$  on  $fd_t$  based on 1000 simulations of 25-year period



Panel C: Impulse response exchange rate after one standard deviation drop in Foreign interest rate



Panel D: Simulation of 25-year period: excess return and forward discount

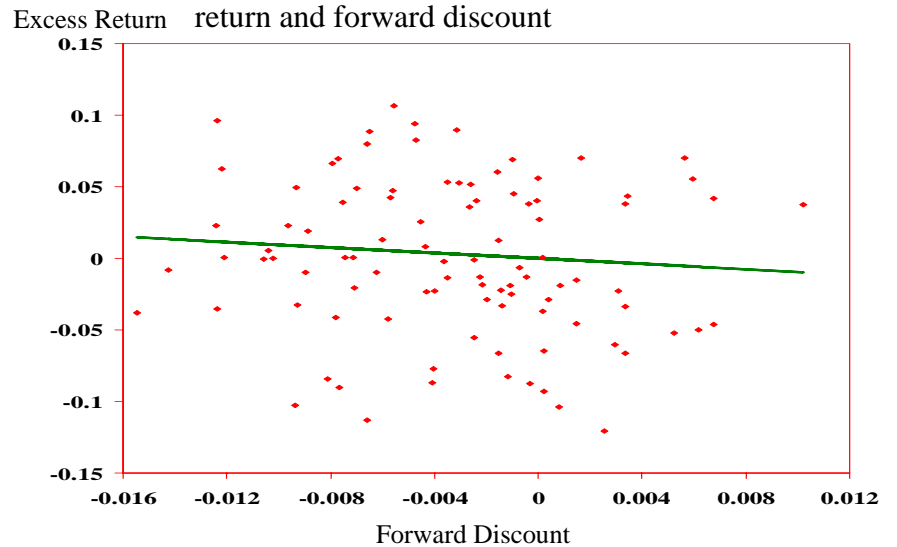
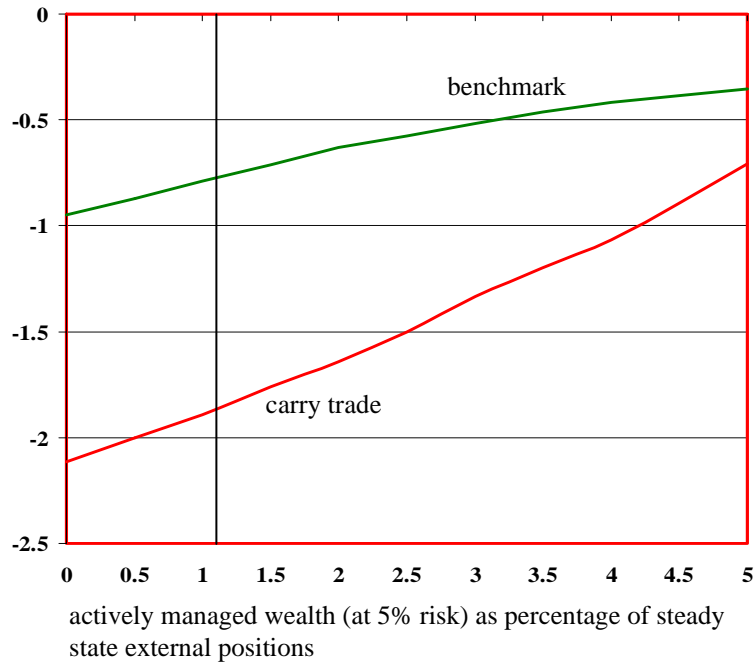


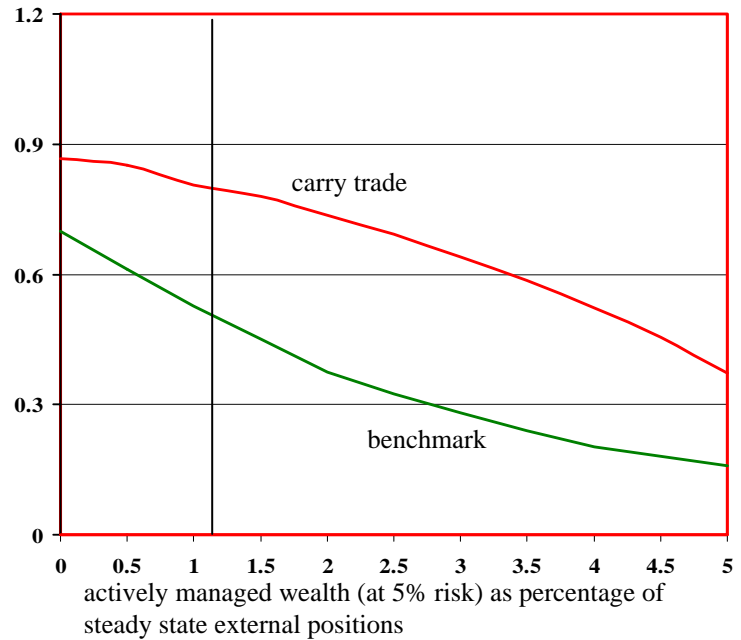
Figure 5 Actively Managed Portfolios: Impact on Predictability and Threshold Cost\*

Panel A: Predictability coefficient  $\beta$  of regression

$$q_{t+1} = \alpha + \beta f d_t$$



Panel B: Threshold Cost



\* Vertical line represents actual size of active currency management industry