Education, human capital accumulation and economic growth

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ABSTRACT

This paper explores in a very simple way what is the relationship between human capital accumulation, measured by education, and the economic growth rate; and what is the role of the determinants of such accumulation. The results presented here are consistent with those obtained by other authors; in general, we emphasized the existing differences in the countries’ educational systems and the different priorities they should consider. We conclude that ignoring such differences could result in inefficiencies, which we can see as negative impacts in the economic growth rate and in the accumulation of qualities by the labour market.

1 I want to recognize the excellent role of Alexander Mihailov as course assistant, thank you very much for your help and time. All theoretical mistakes and problems of interpretation are, of course, of my responsibility.
I. INTRODUCTION

Most of the papers focused on directly exploring the relationship between education and the economic growth rate directly have found that there is enough evidence that such a relation does not exist (except for the primary educational level). However, this limitation could be avoided by studying the indirect impact via human capital accumulation.

In this paper, we regress two simple linear equations for 15 representative countries around the world for the period 1960-1990 (considering observations in five-year periods). The first regression is the equation of GDP per capita determination taking into account the initial period and other related variables. The principal variable of interest determining GDP per capita is the level of labour supply, which accounts for all the different educational levels (i.e. primary education, secondary education, and higher education).

The second equation is that of human capital determination, which is basically determined by public expenditure in education. However, one first problem arises here: we do not know what the best indicator of human capital is; hence, we build an indicator using all the information on education we have, and we use it as the dependent variable.

So, we explore what is the relationship between educational attainment and the average growth rate; and we study what the role of the determinants of the accumulation of human capital is. The results of running the first and second regressions, whose specification is introduced in section II, are displayed in sections III and IV respectively, where we also do some comparisons between them. The relationship between these results is next evidenced exploring the Korean case in section V. Finally, section VI concludes.

\(^2\) In fact, we calculate the average growth rate for each period we consider.
II. SPECIFYING THE MODEL

As noted by Appleton (2000), a general model which explores both the determinants of the educational inputs and outputs and their consequences can be specified in the following way:

(1) \[ U = U(H, L, S, C) \]

where \( S \) represents the educational inputs required to generate productive human capital \( H \), which could be interpreted, in the spirit of Appleton (2000), as knowledge and skills. \( L \) is labour supply, and \( C \) represents consumption of all other goods that do not contribute to human capital\(^3\).

Our interest is to see how human capital accumulation \((H)\) can contribute to utility maximization\(^4\). In general, we can consider at least three ways to do so. The first one, as can be seen in (1), is a direct contribution in the utility function; this is because the more human capital each individual can accumulate (in terms of knowledge and skills) the more happiness will be obtained (for each one)\(^5\).

The human capital production function could be specified in the following way:

(2) \[ H = H(S) \]

We can consider a lot of factors being educational inputs, \( S \), however, in this paper, we are going to consider only measurable factors. In general, one can take into account a lot of unobservable variables too (which usually are difficult to measure) such as individual characteristics, the quality of the services provided, the community characteristics, and the interest of the country in accumulating human capital.

On the other hand, human capital characteristics, \( H \), could affect the utility function, as we can see again in the function (1), via consumption, \( C \), which is, at the same time, determined by wages.

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\(^3\) The study made by Appleton (2000) focus on both education and health as determinants of human capital accumulation by modelling capital as something produced by the household.

\(^4\) We are going to refer to educational level as human capital accumulation, human capital characteristics, or human capital capacity indifferently throughout the paper.

\(^5\) Or at least, we cannot imagine the contrary.
The consumption function could be specified as follows:

\( C = C(W) \) \hspace{1cm} (3)

And the wages function:

\( W = W(H) \) \hspace{1cm} (4)

This *indirect* process of affecting the utility function considers increases in wages as a result of increases in the human capital capacity, but this last *sub-process* (human capital affecting wages) could not be direct. As we are going to see especially in sections III and V, this sub-process can be the result of an increasing in or a high economic growth rate. As far as a country is able to assimilate new technology as a result of its *well-educated* society (for example in developing countries), firms are encouraged to adopt the advanced technologies developed in high income countries. The last is the third way in which human capital accumulation can affect the maximization problem, and it is, in fact, the relationship in which we are interested in.

We can summarize the above discussion using the following figure.

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6 Appleton (2000) specifies a more general model to explore the effects of education on income generation, and provides the evidence for sub-Saharan countries considering models which use agricultural production functions, manufacturing production functions, wage earnings functions, and income and consumption functions.

7 Of course, this figure represents a very simple panoramic of the relationship in which we are interested, and, as a result, it presents some limitations. For example, it is possible to find a relationship between wages and human capital in the opposite way we are considering. Empirically, the “typical” problem of causality may arise considering these two variables.
Normally however, the challenge of specifying a utility function to prove empirically is not so easy to surpass because we do not have a happiness measure; that is, we do not have any way to say if the individuals, at the aggregate or at individual level, are happier in general terms and, hence, a way to say if more human capital could increase the general or the individual utility.

For that reason, in empirical applications one can consider the consumption component of utility as the best happiness measure and, hence, just consider indirect contributions of human capital in the utility function measured by the consumption function.

To avoid such kind of complications, we will consider, as we said before, only the indirect via of affecting the utility function and in the indirect sub-process of affecting wages. And besides, for simplicity, we are going to consider the relation after the Economic Growth block in the scheme as invariably right (and so, we do not have to prove it).

Then, which we hoped to find are positive responses of the economic growth to increases in the "amount" of capacities the population can accumulate (the latter acting in response to increases in the general level of education). In doing this, we can consider that the final result will be increases in wages, in the level of consumption and, of course, in the individual utility.

III. HUMAN CAPITAL ACCUMULATION AND ECONOMIC GROWTH.
THE EVIDENCE

III.1. The procedure

In this section, we consider the educational inputs as given, and so we take into account only the effects of human capital accumulation on economic growth. In doing so, we use the data constructed by Barro and Lee (2000), and Summers, Heston, Aten and Nuxoll (1995). We specify and test the most simple linear regression model; we use data from 1960 to 1990 in 5-year periods and we consider 15 countries from a sample around the world. Of course, by

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8 There exist however some theoretical measures of happiness constructed using variables related with health and education. If we use such variables, the effects of human capital on the utility "function" would be trivial.
construction, all this data are compatible and comparable through time and countries\(^\text{10}\).

The simple model we will test is specified by the equation:

\[
\frac{1}{5} \sum_{\tau=1}^{3} (\ln GDP_{\tau+1} - \ln GDP_{\tau}) = c + \beta DGR_{i,t} + \varepsilon_{i,t}
\]

where \(GDP\) is the real gross domestic product per capita in constant dollars (which is expressed in international 1985 prices), and \(DGR\) represents a set of matrices (all of them of the same size \(i, t\)) determining the average growth rate. \(\varepsilon\) is a disturbance term; it captures all the other factors that could affect the economic growth rate, and is supposed to satisfy \(E(\varepsilon)=0\), to have identical variances, and to be uncorrelated across time and countries.

The index \(i\) represents each country under consideration and \(t\) is the period of reference; \(\tau\) represents each one of the years corresponding to each period \(t\) and, of course, \(\tau\) and \(t\) have the same value each five years (i.e. \(t+1=\tau+5\)).

We consider the following 15 countries:

<table>
<thead>
<tr>
<th>Table 1. List of countries considered</th>
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<tbody>
<tr>
<td>Central African Rep.</td>
</tr>
<tr>
<td>Canada</td>
</tr>
<tr>
<td>Costa Rica</td>
</tr>
<tr>
<td>Mexico</td>
</tr>
<tr>
<td>United States</td>
</tr>
<tr>
<td>Venezuela</td>
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<tr>
<td>Korean Rep.</td>
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<tr>
<td>Syria</td>
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</tbody>
</table>

Besides, we consider the following variables explaining the average growth rate (the set of matrices \(DGR\)): 1) the average population growth rate, 2) real private

\(^{9}\) There exist some estimations (Bloom et al. 2002) showing that each year of education raises subsequent wages by about 9%. Besides, refer again to Appleton (2000) to have a more detailed description of the effects after the block of Human Capital in the scheme.

\(^{10}\) For details about the construction of the data base, refer to Lee-Barro (1996) and Heston-Summers (1991).
investment as a percentage of GDP (expressed in 1985 international prices), 3) real government consumption as a percentage of GDP (expressed too in 1985 international prices), and, most importantly, 4) the indicator(s) of human capital accumulation.

As we said before, we are going to use the data constructed by Barro-Lee (2000). This data contains educational attainment at various levels: no schooling, primary school, secondary school and higher education for the total population and male and female population separately. All data is expressed in percentages.

Because an indicator of the degree of education in a given society is, at the same time, an indicator of the qualification of the labour force, we consider the educational attainment levels of the population of 25 years and above. Such a population fraction seems to be more appropriate in our case than that of 15 years and above (we think qualification is captured in a more strong way by population of 25 years, population with 15 years possibly will continue studying).

In general, the structure of the labour supply market could be graphically shown as follows:

Some studies have used the average years of schooling as indicator of human capital accumulation; and other studies, such as Bairam-Kulkolkarn (2001), use several indicators, but only one at each time. In our case, we are going to
consider all the labour market at the same time, and we will try to find the most relevant model to do comparisons with the results obtained by other authors.

The procedure used to estimate equation (4) is, initially, pooled least squares. A good motivation to use this kind of procedure is suggested in Vinod and Ullah (1981), and is reproduced in Bairam-Kulkolkarn (2001): "When dealing with cross section and time series data, where each individual cross-section sample is small so that sharp inferences about the coefficients are not possible, it is a common practice in applied work to pool data together and estimate common regressions... The basic motivation for pooling time series and cross-section data is that, if the model is properly specified, pooling provides more efficient estimation, inference, and possibly prediction."

However, it is possible to improve the results obtained using some other estimation methods. For example, we can assume that the intercept values of the various countries’ economic growth rate are different. In general, we can introduce dummy variables that allow the intercept term to vary over time and over cross-section units; but in our case, we will introduce dummy variables in order to accept inherent differences between the countries considered, this is equivalent to run a model with fixed-individual effects.

We have defined one dummy variable as follows:

<table>
<thead>
<tr>
<th>Table 2. Definition of Dummy Variables</th>
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<tbody>
<tr>
<td>ADVANCED COUNTRIES</td>
</tr>
<tr>
<td>Canada</td>
</tr>
<tr>
<td>United States</td>
</tr>
<tr>
<td>France</td>
</tr>
<tr>
<td>Germany, West</td>
</tr>
<tr>
<td>Netherlands</td>
</tr>
<tr>
<td>New Zealand</td>
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<tr>
<td>Spain</td>
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</tbody>
</table>

But, there could exists at least one more way of (probably) obtaining more representative results. One of them consists in considering the variance-covariance matrix to estimate the model, i.e. we can use the generalized least squares methodology. We cannot forget however that this methodology is
sometimes equivalent to the ordinary least squares procedure, particularly if the error variances are equal among countries and time.

### III. 2. The results

Tables A1 and A2 in appendix A show the best 6 models we could find mixing our available data. We present only the variables which were statistically significant different from zero at least once (even when we present in the definition of variables all the data we considered at the beginning). Each regression was run twice, the upper-results are the estimated coefficients, $c$ and $\beta$'s, using the pooling least squares method (with fixed effects), and the lower-results are those obtained running equation (4) using the GLS method with weighted cross-section coefficients.

We show in the parentheses below the estimated coefficient the resulting $p$-value. It is important to notice that a variable was excluded if its $p$-value was larger than 15%. Besides, the tables show the $R^2$ and the Adjusted $R^2$ for each case. In the case of GLS estimation procedure, the $R^2$ out of parentheses is the weighted $R^2$; the other one (within parentheses) is the unweighted one (simply GLS).

As we can see, no matter the two different estimation methods, as we can note in tables A1 and A2, the results obtained for the coefficients $c$ and $\beta$ are only slightly different and not all the levels of labour supply are significantly different from zero at the same time.

It is just to notice that the results obtained differ a little bit with the original results presented in the preliminary version of this paper; nevertheless (and fortunately), the stronger results are sustained.

#### III.2.1. The whole market

Before to analyze the results obtained for the variables related with education, let us talk rapidly about the other variables we included. Not surprising results are the positive impacts of the lagged growth rate and the real government consumption on the (actual) economic growth rate; not surprising too is the

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11 Nevertheless, it is important to mention that the standard deviations (which are not shown here) are always smaller in the case of the estimation by GLS than by PLS, which implies a gain in efficiency.
strong and negative impact of the population growth rate. But maybe surprising are the strong and negative impact of the (actual) real private investment, and the opposite directions the impacts of the real government consumption and the same variable lagged have on growth.

First, why a negative impact of real private investment? We can give an intuitive answer to this question through another one: if I invest today, when I will obtain results?. Of course, the answer is "not today, but today I am poorer than yesterday" (because today I spent some money).

However, maybe governments are spending in projects with very short-term results (for example, constructing a highway benefiting the trade between two cities). That explains why (actual) real government consumption has positive impacts on (actual) growth. But, at the same time, if the projects in which the government is spending have a really short productive life, then what it spent yesterday would be seen tomorrow as a waste of money (and maybe tomorrow government will need more resources -the resources it spent yesterday-). The last explains the opposite signs of actual and past real government consumption.

Now, let us focus exclusively in the variables on education. Models in table A1 show the results obtained when the dummy variable for advanced countries was included. In that case, we could not do distinctions between the male and female educational levels, i.e. we had to consider the labour market as a whole. The first level of education, even when its impact is in all cases of less than 1%, seems to be always important determining the economic growth rate (only the first level completed!!!); it appears in the three regressions considered, and their significance is always at less than 5%-level.

The result that only the primary education is important to growth (not just attainment) is not surprising. We can consider workers without primary education completed as a very unqualified labour force. Even when people with some primary educational level can be incorporated into the labour market, their equivalent production will not be large enough to affect in some way the economies’ average growth rate.

The conclusion above can be extended for the case of secondary education; this is a really good new in relation with the last version of this paper. In this case (when we consider differences between advanced and developing countries), we can see that secondary education completed is important determining the
economic growth rate, and the direction of movements of it is the same (and with the same intensity) as movements in the variable for primary education\textsuperscript{12}. In the latter version of this paper, we could not say what kind of secondary studies are important to growth (in that case, we could not distinguish between secondary completed and some attainment at this level).

However, some similar effect to what we mention before is related with the effects of higher educational levels. In regression 1, (where secondary education seems to be not important) it is some post secondary attainment what is important to growth; whereas in regression 2, where secondary completed is important determining the economic growth rate, some level of higher education completed (for example, to finish under-graduate studies) is important too\textsuperscript{13}. Both effects act with the same magnitude and in the same direction, this is (strangely and very sadly) negative whichever the case!!!.

This strange conclusion may be due to the absence of other variables explaining concert differences between the countries considered. Some countries could have benefited from increasing higher educational levels, and the others not. For example, industrialized countries, in which innovation is very important in determining GDP and growth, need to have higher levels of technological knowledge and so, investing in specialized education necessarily has a positive impact on the growth rate.

In the developing countries, in which the process of catch-up of advanced technology is actually more important in determining income, maybe do not need so much specialized labour force in comparison with the advanced countries. Investing in very specialized education could be inefficient, but not so inefficient if they invest in education specialized enough to absorb and adapt the knowledge generated by the advanced countries.

On the other hand, investing in high education in poor countries could be very inefficient if their systems of primary and secondary education do not properly work; poor countries need to have efficient systems of elementary education first to be able to incorporate more sophisticated technology in their economic market.

\textsuperscript{12}However regression 1 indicates that there is not evidence to say that secondary level of education has some importance or influence on growth, the impact of that variable was the same as in the other cases, but the result is not presented because in that case its p-value exceeded 15%.

\textsuperscript{13}In these cases, however, the results are less significant than those of the primary and secondary educational levels.
One important variable which considers differences between countries is the average years of schooling in the labour supply market. While this variable was around 9 years in the advanced countries in 1960, it was less than 1 year in the African countries in the same year. However, the total growth rate of this variable was more than 400% for the poor countries, while it was of around 25% for the industrialized ones. In general, this variable is significantly different from zero in regressions 1 and 2 in table A1, and its impact in each case is very close: An increase of one-year of schooling in the labour supply market will lead to an increase of around 0.6% in the average growth rate.

The latter results obtained are, in general, consistent with those obtained by other authors. Nelson and Phelps (1966) and Bairam and Kulkolkarn (2001) explain the importance of the composition of the workers’ level of education for economic growth: “Different levels of education are complements and not perfectly substitutes...There are required inputs for education production. Higher education is for innovation while lower education is for developing or utilizing new innovation...Training some workers to very high standards but ignoring the rest may be inefficient because the poorly trained workers are unable to fully develop or even utilise new innovation. This may slow down the return to innovators and the growth rate. On the other hand, only emphasising basic knowledge and ignoring higher education may reduce a country’s ability to innovate and hence the growth rate will be low... Thus, the design of educational policy needs to take into account the complementary nature of different forms of human capital and not concentrate too heavily on one form alone”.

III.2.2. Men vs. Women

We will turn now to table A2 which shows the results obtained from models which do not consider the dummy variable. When it was the case, we could not consider the labour market as a whole, but in this case, we can distinguish between the male and female labour markets.

In this case, the interpretation of results is a little bit more complicated for the case of primary education level. The most important thing to be noticed is that primary education is important either in the case of men and women. (We have different combinations of results: some attainment to primary school for men and primary education completed for women, and some attainment to primary

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14 This observation justifies the conclusions obtained above: poor countries are investing more in primary level than...
education for women and primary education completed for men), the result for some attainment to primary education is however really small (and almost equal in both equations -4 and 6), it justifies why it does not appear when we consider the whole labour market in table A1.

In the case of secondary level, either education for men and women is always important determining the economic growth, however, their effects act in opposite way!!!. Because normally both men and women have the same treatment and rights in schools15, the final impact (resulted from two forces acting in opposite directions at the same time) is equal to what we obtained for secondary education in equations 2 and 3 in table A1.

The conclusion obtained above is very surprising, and the only explanation we can give is that of the discrimination. When such discrimination exists, there exist a lot of well-educated women in the labour market without possibilities to find a job (maybe women with less qualification can be hired for doing works in where the effort required is very small. That justifies why primary education for women has some positive impacts determining growth). In that case, we can consider expenditure (public or private) in education for women (at least at secondary level) as a waste of resources, and hence spending in educating women as inefficient.

Obviously, discrimination is not the case for all the countries, but the simple model we are using here does not have any variable explaining and determining such a difference.

IV. THE DETERMINANTS OF HUMAN CAPITAL ACCUMULATION

IV.1. Looking for a good indicator of human capital

In section III, we explained what the relationship between human capital accumulation and economic growth is, but we do not know yet what determines that accumulation.

We said in the last section that there are required inputs for education production and that countries should not train workers to very high standards. But, what

15 Women cannot be excluded of education.
happens if countries invest in all levels of education in the same way?; what is the effect of an increase in the public expenditure in education for the accumulation of human capital and hence for growth?\(^{16}\). The objective in this section is to try to answer these two questions.

We will use the data constructed by Lee-Barro (1997), which includes the educational inputs in five-year periods; we will use the same estimation methods described in the last section and, of course, we are going to consider the same 15 countries as before.

However, here we have to deal with a first problem: what is the best indicator of the human capital accumulation? As we said before, many of the authors use the average years of schooling as the best one, but such an indicator is not really general. Instead of it, we are going to build an indicator which aggregates all the educational levels we considered in the last section.

To do so, we are going to use the principal components method, method which is usually used when there are too many regressors relative to little data, and when there is multicollinearity between regressors. But, in our case, the reason to use this method is more technical: we can capture the variations of all the educational levels at the same time with only one variable.

The principal components method is a technique for re-expressing the original explanatory variables \(x\)'s with a smaller number of new explanatory variables \(y\)'s that are linear combinations of the \(x\)'s. These \(y\)'s will be orthogonal (and without multicollinearity problems) and will capture as much as possible of the variation in the \(x\)'s.

The first \(y\) (the first component, say \(y_1\)) of a set of variables \(x\)'s is a weighted average of that variables. The weights are chosen to make the composite variable reflect the maximum possible proportion of the total variation in the set, and it is, in fact, the component we are going to use here. But, additional principal components can be calculated; however, the first component usually captures

\(^{16}\) In the last section we saw that the general public expenditure is important in determining the growth rate, but we did not say anything about what happens if such expenditure is decomposed in different kinds of expenditure. We tried to include public spending in education instead of total public spending in equation (4), but this variable was not statistically different from zero. Then, we can say that a direct relationship between public expenditure in education and growth does not exist, but we can still find an indirect one via human capital accumulation.
enough of the variation in the set to be an adequate representative of that set in its own\textsuperscript{17}.

We built and indicator named \textit{HCA}, which is the principal component (the first one) of all the educational variables (educational levels). In this case, our principal component captures 50\% of the total variation in the whole set of the variables we are considering\textsuperscript{18}.

Then, the equation to be estimated is:

\begin{equation}
HCA_{i,t} = \alpha + \gamma EI_{i,t} + \eta_{i,t}
\end{equation}

As before, \( i \) is the cross-section observation and \( t \) is the period considered. \( EI \) is a set of matrices (all of them of the same size \( i, t \)) determining the human capital accumulation indicator, \textit{HCA}. \( \eta \) is a disturbance term, which captures all the other factors that could affect the economic growth rate, and is supposed to satisfy \( E(\eta)=0 \), to have \textit{identical variances}, and to be \textit{uncorrelated} across time and countries. Once again, we can improve our results using the same techniques we described if the last section. We will use the same dummies as before and we are going to include them in (5) when it is possible.

As before, we have run several models in order to find the best one and to justify our conclusions. Again, we can see that running the model by PLS give us results only slightly different to those obtained by the GLS estimation method\textsuperscript{19}.

\textbf{IV.2. The results}

\textit{IV.2.1. All the countries and the whole period}

Refer again to tables in appendix A. In the first two regressions of Table A3 (regressions 7 and 8) we did not use dummy variables, but we found models explaining \textit{HCA} by over than 50\%. The difference between 7 and 8 is that, in 7, we used real government \textit{current} educational expenditure \textit{per- pupil} (PPP-
adjusted 1985 international dollars) instead of the same variable as percentage of GDP. We can see that, whichever the case, public expenditure in education is important determining the accumulation of human capital\textsuperscript{[20]}. 

Another important variable explaining HCA in models 7 and 8 is the pupil-teacher ratio, but it is important only at the secondary level. The importance of the ratio pupil-teacher at secondary level, not the first one, could be due to the fact that in most of the countries considered here, maybe there are enough teachers to satisfy the "actual" demand for education at primary level. But more important could be the fact that, if the educational system is not efficient then countries are not taking advantage of the stock of teachers they have and then, the inclusion of more of them will not have any impact on accumulating human capital\textsuperscript{[21]}. In fact, we can see that the ratio of the average real salary of primary school teachers (PPP-adjusted 1985 international dollars) has a negative impact on HCA. At secondary level, there are not enough teachers to satisfy the demand for schooling (then one more is more valuable than at primary level), or the system allows the country to take advantage of an increasing stock of teachers, or both.

The next step is now to accept inherent differences between the countries considered (i.e. we are going to include the dummy variable). In this case (regression 9 in table A3), the pupil-teacher ratio is not important neither at primary nor at secondary level, and the role of the real government current educational expenditure seems to be less important than before. Besides, in this case public expenditure in education at secondary level is now important determining HCA, but, as we can see, this impact is \textit{negative}.

The last results can be due to that we said in the last section: \textit{for many countries, to spend in the secondary educational level could be inefficient}. Even when we said that, in general, secondary teachers are more valuable than those of the primary level (i.e. there are positive responses to increasing the number of teachers at secondary school), if the actual activities of the countries do not require "high" qualification, then invest on it could be a waste of resources (for example, to invest in secondary education when it is more primary education what is needed).

\textsuperscript{20} However, when we consider real government current expenditure per pupil as a percentage of GDP, public spending in secondary education is not significant; and when we consider only real government current expenditure, its significance at the same level is very low.

\textsuperscript{21} We can think as relevant one of the answers, or the other one, or the two at the same time; but, in reality, if the educational system is efficient, we can think in positive effects of increasing the stock of teachers even when it seems to be enough.
IV.2.2 And...

We cannot generalize the results obtained until now because actually we do not know if they are sensitive or not to changes across time and countries. To explore that possibility, we are going to do the same exercise as before but now considering different time periods and cross-sectional observations.

First we can take a look at table A4. This table shows models 7, 8 and 9, but now everyone was run in *periods*. In any case, as before, the pupil-teacher ratio at primary school is significant; and, in some cases, the public expenditure at primary school has a positive and significant impact on *HCA*. Besides, the public expenditure in secondary education has the same performance as in the previous case\(^{22}\), and the ratio of the average real salary of primary school teachers to real per capita GDP is always significant and negative.

Then, even when some coefficients tend to be non-significant, most of them tend to be constant through time and so, *the same conclusions as before are sustained*.

And now, because the data allows us to do it, we ran some models considering the advanced countries and the developing countries separately. The results can be viewed in tables A5 and A6 respectively.

As (probably) we hoped, in the case of only advanced countries (table A5), all the variables for primary and secondary levels (including the pupil-teacher ratio at primary and secondary levels) are significant and positive. So, if the educational system is efficient (as we can expect for advanced countries) then, an increasing in the stock of teachers (relative to the number of students at any level) always has a positive impact on the capacities of workers in the labour market.

Even more, government expenditure either in primary and secondary school is always positive. Hence, because in advanced countries *the system works in the way it should, investing in education is always a good idea*.

However, we can see that the salary of primary school teachers still has a negative impact as before. The only reason for that result could be that (despite

\(^{22}\) I.e. positive without including the dummy variables and negative otherwise.
that it is just a speculation), if in advanced countries teachers are over-paid, then increasing their salaries could be considered, again, as a waste of resources.

Now, we are going to consider the developing countries (table A6). In this case, again, expenditures in secondary education and the salary of primary school teachers have a negative impact on HCA. Besides, neither the pupil-teacher ratio at primary school nor the pupil-teacher ratio at secondary school are significant. (Even when public expenditure in primary and secondary education is important for developing countries, probably are not more teachers, principally at primary level, what they need.)

So, we have seen that different conclusions can be obtained if different sets of countries are taken into account. We have seen that the general results (i.e. considering the whole set of countries) are "biased" by the "power" of the developing countries. However, our experiment of considering two different set of countries allow us to conclude that only when the educational system is efficient, increasing the educational expenditures in education, whatever the level is, will lead to an increase in the capacities of the labour force, which finally will lead to an increase in the economic growth rate.

Besides, we have shown that countries cannot invest in education all of them in the same way. Different countries have different priorities and so, to invest a lot in a non-prioritary or in an actually non-efficient level of education will lead to negative impacts on the accumulation of human capital (i.e. while advanced countries can invest in any level of education, investing in secondary levels is "bad" for developing countries!).

V. THE KOREAN CASE\(^\text{23}\)

In order to have a clearer idea of how the indirect impact of the human capital accumulation and its determinants on the economic growth rate works (the connection between sections III and IV), and what a country can do to increase its economic growth rate via human capital accumulation, we are going to take a look at the case of Korea, which is, without doubt, one of the most successful cases around the world\(^\text{24}\).

\text{\textsuperscript{23}} We are going to follow Lee, Jong-Wha (1997), refer to it for an extensive and more detailed analysis of the topic.

\text{\textsuperscript{24}} Maybe the reader is asking why we do not use HCA in equation (4) (instead of the educational variables we used) in order to relate the results obtained in sections III and IV. The reason is that this variable is not
Since the early 1960’s, Korea has grown by more than 8 percent each year, making it the fastest growing economy in the world. Its success has been attributed to the accumulation of a stock of well-educated work-force. For example, in 1960 the educational attainments of Koreans far exceeded those of the populations of developing countries. Nonetheless, Korea’s educational attainment in 1960 was not at all near to that of the OECD countries, nor higher than that of many Latin American Countries. The average years of schooling of the population aged 25 and above (used in this paper) almost tripled from 3.23 years in 1960 to 9.25 years in 1990, exceeding the average of almost all the considered countries and, in general, the average of the OECD countries as a whole.

However, the evolution in the educational attainment is not the only important feature impacting the economic growth rate in Korea but one can also consider their superb quality of education. In this paper, the quality of education was not considered and, in fact, it is a very difficult characteristic to measure. Nevertheless, there exist some studies, such as Barro-Lee (1996), which allow us to do some comparisons between countries considering the scores of internationally comparable tests of pupils’ achievement in cognitive skills such as numeracy, literacy and scientific reasoning. Korean students are the best performers in the science tests and second to Chinese students in math tests.

But, what did the Koreans do to improve their quality of education? Some literature argues that public spending is important in improving quality of education. There was no evidence, however, that Korean government has spent more than any other government in order to promote quantity and quality of education.

Government expenditure on education as a percentage of GDP has been relatively low in Korea, compared with other developing countries. Then, high quality of education in Korea is normally viewed as an outcome of high quality of teachers, long school days and low repetition and drop-out rates, rather than of high public expenditures.
Besides, we can distinguish several historical factors that encouraged strong demand for and supply of education before the "big boom". The strong demand for education was attributable to strong social and economic motivations. Korean society considered education as a very important way to obtain higher social levels and for the strong desire for social mobility. The desire for social mobility was also closely associated with economic motivation. Education was considered as a key factor for people to get into good occupations that often guaranteed both economic rewards and social status.

But, of course, without expanding the supply of education, it would not have been possible for everyone to be educated. The expansion in the supply of education is generally associated with the contribution of Japanese colonial rule to education in Korea. The most important contribution to education made by Japan was the construction of physical facilities and the large number of Japanese-educated Korean teachers. However, the contribution of the US Army Military government is recognized too in the expansion of educational opportunities, which is estimated to have contributed with about two-thirds of the operating costs of running the primary schools during 1945-48. Besides, the government contributed to the decline of illiteracy among adults by initiating a "national campaign for literacy", which very effectively raised the adult literacy rate from 22 percent in 1945 to approximately 80 percent in 1960.

There exist special features in Korea that have contributed to the growth of education. The export-oriented development strategy (1962-1973), which has encouraged workers to acquire more education and training. The presence of competitive labour markets brings efficient allocation of labour resources, and, in turn, provides more incentives for higher education during the successive transformation of the economy along with technological sophistication. Also, global competition imposed by outward orientation forced the government and firms to invest actively in the accumulation of human capital for the economy.

In addition to the equitable distribution initially inherited in 1960 (which was originated from historical conditions, such as the two land reforms and the Korean War), Korea did not deteriorate its income distribution during its rapid growth period. This achievement of growth with equity is attributed to many factors, including strong growth of employment and low unemployment, as well as such government social welfare policies as the livelihood protection programs and medical assistance program. As common in developing countries, poor households are unable to pay for education of their children. But in Korea, due to
equal income distribution, the majority of Korean parents were able to pay the substantial costs of schooling.

Finally, *what has been the contribution of human development to economic growth?* Some estimations show that, in Korea, human capital has made an additional contribution to income growth by encouraging capital investment, considering that rates of return to investment are high due to well-educated labour force. Due to the high capacity of absorbing new technology information, firms are, in general, encouraged to adopt the advanced technologies developed in high income countries.

The imports of high-technology products (in particular intermediate inputs) were subsidized by the government for a long period. The adaptation of the inflows of the advanced foreign technology was not an automatic process at all, but was only possible by a well-educated labour force that Korea had at the beginning of its export-oriented industrialization.

**VI. CONCLUSIONS**

In this paper, we ran two simple linear regression models to estimate, first, the impact of human capital accumulation on economic growth considering the educational inputs as given and, next, to estimate what is the relationship between human capital and its inputs.

We found that workers with primary education completed have always a significant and positive impact on the average growth rate. And, in fact, we saw that workers with secondary school level have the same impact as the workers with only primary school. However, the latter impact is the result of two contrary forces acting at the same time: the male labour market with a positive impact, and the female labour market impacting the economic growth rate in a negative way. We attributed this conclusion to the existence of some discriminatory procedures in the labour market to female workers in some countries.

Exploring what happens with the workers with higher levels of schooling, we found that its impact is negative in determining the average growth. We said that this conclusion is possible because of the differences between the countries considered: *some countries could have benefited from increasing higher educational*
levels, but the others not, and, in fact, we saw that poor countries are investing more in primary education than in higher educational levels.

The last conclusion was corroborated when we explored the determinants of the human capital accumulation. We saw that when we can consider different sets of countries, only the educational market of the industrialized countries seems to be efficient. Advanced countries can invest efficiently in any level of education (even in the most specialized level).

The rest of the countries have to note what their priorities are. Developing countries, which actually benefit more from absorbing and adapting the technologies generated in countries with high technologies (the countries with high rates of innovation), could invest more efficiently in less specialized educational levels in comparison with the advanced countries; but, at the same time, they cannot forget the other educational levels.

Finally, we took a look at the case of Korea. We said that most of the success of this country is due to the high quality of its human resources, but we said too that the high quality of education in Korea can be considered more as an outcome of high quality of teachers, long school days, and low repetition and drop-out rates, than of high public expenditures.

Korea has benefited from some historical factors which we saw in section V, the more important are: its equitable income distribution across time, and the strong desire of the Korean society to be well-educated.

We saw that the output (effectively, a well-educated society) has made an additional contribution to income growth: because the high capacity of absorbing new technology information, firms are encouraged to adopt the advanced technologies developed in high income countries.

That conclusion for Korea is consistent with the principal results we obtained here. Only when the educational system is efficient we can go into the best of the worlds: spending in education we will get a high economic growth with human development, one of the most difficult dreams to be reached, but with the highest reward.
APPENDIX A. Tables

Definition of variables (used in determining growth)

GR: Growth Rate
POPGR: Population Growth Rate
REALINV: Real investment /share of GDP (1985 international prices)
REALGOV: Real government consumption /share of GDP (1985 international prices)
1st LC: First level of education completed
2nd LC: Second level of education completed
POSTSEC: Post-Secondary level
POSTSECC: Post-Secondary level completed
AYEAR: Average years of Schooling
DUMADV: Dummy (Advanced Countries)
DUMMY: Variable for males
DUMF: Variable for females
(-1): variable one period lagged

Remark. All the coefficients are jointly statistically significant different from zero at 1%-level in all the regressions considered using the F-statistic criteria.

Table A1. Education as determinant of Economic Growth (models including the dummy variable)

<table>
<thead>
<tr>
<th>No.</th>
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<th>GR</th>
<th>POP</th>
<th>REAL</th>
<th>REAL</th>
<th>REAL</th>
<th>DUM</th>
<th>2nd</th>
<th>POST</th>
<th>POST</th>
<th>AYEA</th>
<th>DUM</th>
<th>Est. Method</th>
<th>R squared</th>
<th>Adj. R squared</th>
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</thead>
<tbody>
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<td>0.002</td>
<td>-0.003</td>
<td>0.001</td>
<td>-0.001</td>
<td>-0.001</td>
<td>-0.001</td>
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### Table A2. Education as determinant of Economic Growth (models without dummy variable)

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<th>No.</th>
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<th>POP GR (-1)</th>
<th>REAL INV</th>
<th>REAL GOV</th>
<th>REAL GOV (-1)</th>
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<th>1st LF</th>
<th>1st LCM</th>
<th>1st LCF</th>
<th>2nd LCM</th>
<th>2nd LCF</th>
<th>AV YEARM</th>
<th>Est. Method</th>
<th>R squared</th>
<th>Adj. R squared</th>
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<td>0.005 (0.047)</td>
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<td>-0.002 (0.000)</td>
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<td>-</td>
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</table>
Definition of variables (used in determining HCA)

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
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<tbody>
<tr>
<td>TEAPRI</td>
<td>Pupil-Teacher ratio at primary school</td>
</tr>
<tr>
<td>TEASEC</td>
<td>Pupil-Teacher ratio at secondary school</td>
</tr>
<tr>
<td>GEEPRI</td>
<td>Real government current educational expenditure per pupil at primary school (PPP-adjusted, 1985 international dollars)</td>
</tr>
<tr>
<td>GEESEC</td>
<td>Real government current educational expenditure per pupil at secondary school (PPP-adjusted, 1985 international dollars)</td>
</tr>
<tr>
<td>SHPUPP</td>
<td>Ratio of GEEPRI to real per capita GDP (in percentage)</td>
</tr>
<tr>
<td>SSPUPS</td>
<td>Ratio of GEESEC to real per capita GDP (in percentage)</td>
</tr>
<tr>
<td>SALARP</td>
<td>Average real salary of primary school teachers (PPP-adjusted 1985 international dollars)</td>
</tr>
<tr>
<td>SHSALP</td>
<td>Ratio of SALARP to real per capita GDP (in percentage)</td>
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<td>DUMADVANCED</td>
<td>Dummy (Advanced countries)</td>
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Table A3. Determining HCA (all countries included, 1960-1990)

<table>
<thead>
<tr>
<th>Regression</th>
<th>Estimation method</th>
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<th>Adjusted R squared</th>
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### Table A4. Determining HCA (all countries included per periods)

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<th>GEFPRI</th>
<th>GESEC</th>
<th>SHUPP</th>
<th>SHALP</th>
<th>DUMADVANCE</th>
<th>Period</th>
<th>Estimation method</th>
<th>R squared</th>
<th>Adjusted R squared</th>
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<td>1975-1990</td>
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<td>1960-1975</td>
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<td>1975-1990</td>
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<th>TEASEC</th>
<th>GEESPRI</th>
<th>GEESEC</th>
<th>SHPUPP</th>
<th>SHPUPS</th>
<th>SALARP</th>
<th>SHSALP</th>
<th>Estimation method</th>
<th>R squared</th>
<th>Adjusted R squared</th>
</tr>
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</tr>
<tr>
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<td>0.0005 (0.000)</td>
<td>0.0008 (0.003)</td>
<td>-0.0011 (0.000)</td>
<td>GLS (CSW)</td>
<td>0.75 (0.50)</td>
<td>0.72 (0.54)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Table A6. Determining HCA (developing countries)

<table>
<thead>
<tr>
<th>Regression</th>
<th>z2</th>
<th>GEESPRI</th>
<th>GEESEC</th>
<th>SHSALP</th>
<th>Estimation method</th>
<th>R squared</th>
<th>Adjusted R squared</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>-1.303 (0.000)</td>
<td>0.003 (0.000)</td>
<td>-0.001 (0.004)</td>
<td>-0.001 (0.005)</td>
<td>PLS</td>
<td>0.48</td>
<td>0.45</td>
</tr>
<tr>
<td>15</td>
<td>-1.352 (0.000)</td>
<td>0.003 (0.000)</td>
<td>-0.001 (0.015)</td>
<td>-0.001 (0.009)</td>
<td>GLS (CSW)</td>
<td>0.76 (0.47)</td>
<td>0.75 (0.43)</td>
</tr>
</tbody>
</table>
APPENDIX B. THE PRINCIPAL COMPONENTS ANALYSIS.

We talked about the intuition behind the principal components method in section IV. Let us now give an introduction on the algebraic way to obtain the principal components.

Let \( x_1, x_2, \ldots, x_p \) be the elements of a \( p \)-component (random) vector \( x \), with mean 0 and variance-covariance matrix \( \Sigma = (\sigma_{ij}) \). \( \Sigma \) is a real positive semidefinite matrix. Let its eigenvalues be \( \delta_1 \geq \delta_2 \geq \ldots \geq \delta_p \geq 0 \). From the theory of matrices, we know that there exists a \( p \times p \) orthogonal matrix

\[
\Gamma = \begin{bmatrix} \gamma_1 & \gamma_2 & \cdots & \gamma_p \end{bmatrix}
\]

such that

\[
\Sigma \Gamma = \Gamma \Delta
\]

or

\[
\Sigma = \Gamma \Delta \Gamma'
\]

where \( \Delta = \text{diag} (\delta_1, \delta_2, \ldots, \delta_p) \). Consider the orthogonal transformation

\[
\nu = \Gamma' x
\]

Then \( \nu_1, \nu_2, \ldots, \nu_p \), the elements of \( \nu \), are called the principal components of \( x \). \( \nu_1 \), which corresponds to the maximum eigenvalue \( \delta_1 \), is called the first principal component, \( \nu_2 \) is the second, and so on. If the eigenvalues of \( \Sigma \) are repeated, and only \( t \) of them are distinct, the matrix \( \Gamma \) is not uniquely determined. It could be multiplied by a post-factor

\[
A = \text{diag} (A_1, A_2, \ldots, A_t)
\]

where \( A_i \) is any orthogonal matrix of order \( m_i \), \( m_i \)'s being the multiplicities of the eigenvalues. The principal components of \( x \) are therefore unique except for a pre-factor \( A' \) of the type of (B.4) applied to \( \nu \). From (B.3) and (B.2) we obtain
The principal components are thus uncorrelated and their variances are \( \delta_1, \delta_2, \ldots, \delta_p \). An overall measure of the variability of \( x \) can be taken as \( tr \Sigma \) or the generalized variance \( |\Sigma| \) if \( \Sigma > 0 \).

On account of (B.2)

\[
tr \Sigma = tr \Gamma \Delta \Gamma = tr \Gamma \Delta
\]

(B.6)

\[
= tr \Delta = \sum_{i=1}^{p} \hat{\delta}_i
\]

(as \( \Gamma \Gamma^T = I \), \( \Gamma \) being orthogonal), and

(B.7)

\[
|\Sigma| = |\Gamma \Delta \Gamma^T| = |\Delta| = \prod_{i=1}^{p} \delta_i
\]

(if all \( \delta \)'s are \( >0 \))

But \( tr \Delta, |\Delta| \) are the corresponding measures of variability for the principal \( \nu \).
This shows that the total variation remains the same, even after transforming from \( x \) to the principal components.
VII. REFERENCES


**Summers**, Heston, Aten and **Nuxoll** (1995). *Penn world table Mark 5.6a*. Center for international comparisons at the University of Pennsylvania, Department of economics.
