THE “CORE-PERIPHERY” MODEL OF THE NEW ECONOMIC GEOGRAPHY

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1 Basic setup

- 2 regions/countries: 1, 2
- 2 sectors: \( M, F \) ("manufacturing", "farming")
- \( M \): monopolistically competitive (horizontally differentiated products: "varieties")
- \( F \): perfectly competitive (homogenous product)
- \( M \): iceberg trade costs: for \( T > 1 \) units of the good shipped, 1 unit arrives
- \( F \): freely traded
- 2 sector-specific production factors: \( L_M, L_F \)
  \[
  L = L_M + L_F = L_{M1} + L_{M2} + L_{F1} + L_{F2}
  \]
  \[
  \frac{L_M}{L_F} = \gamma
  \]
  \[
  \frac{L_{M1}}{L_{M2}} = \lambda_1, \quad \frac{L_{M2}}{L_{M1}} = \lambda_2
  \]
  \[
  \frac{L_{F1}}{L_{F2}} = \phi_1, \quad \frac{L_{F2}}{L_{F1}} = \phi_2
  \]
- \( L_M \) mobile across regions \( \implies \lambda_1, \lambda_2 \) endogenous (main variable of interest!)
- \( L_F \) immobile across regions \( \implies \phi_1, \phi_2 \) exogenous
- \( F \) assumed as numéraire sector: price per unit = wage rate = 1
2 Demand

Utility of the representative consumer:

\[ U = F^{1-\delta} M^\delta, \quad 0 < \delta < 1, \]

\[ M = \left( \sum_{i=1}^{N} c_i^\rho \right)^{\frac{\rho}{\rho-1}}, \quad 0 < \rho < 1, \tag{1} \]

where \( c \) represents the quantity consumed and \( i \in \{1, \ldots, N\} \) denotes varieties of the differentiated good \( M \).

Hence, with symmetric varieties/firms, there are external benefits to the size of the \( M \) sector:

\[ M = (Ne^\rho)^\frac{1}{\rho} = N^{\frac{1}{\rho} - 1}(Ne), \]

i.e. utility \( M \) increases faster than the claims on real resources from expansion of the sector, \( Ne \) (as \( N^{\frac{1}{\rho} - 1} > 1 \)).

The representative consumer’s budget constraint for \( M \):

\[ \sum_{i=1}^{N} \rho_i c_i = \delta Y. \tag{2} \]

Demand for variety \( j \) by a representative consumer (maximise eq.(1) s.t. eq.(2), see Brakman et al., 2001, pp. 70f.):

\[ c_j = p_j^{-\epsilon} \left( I^{-1} \delta Y \right), \]

where:

\[ I \equiv \left( \sum_{i=1}^{N} p_i^{1-\epsilon} \right)^{\frac{1}{1-\epsilon}}; \quad M = \frac{\delta Y}{I}; \quad \epsilon = \frac{1}{1-\rho} > 1. \]

Real wage:

\[ w = \frac{W}{F}, \]

where \( W \) is the “nominal” wage, in terms of \( F \).
3 Supply

Production function for $F$:

$$F = (1 - \gamma)L.$$  

Firms’ production function in the $M$ sector:

$$L_{M_1} = \alpha + \beta x_i.$$  

Firms’ profit function in the $M$ sector:

$$\pi_i = p_i x_i - W(\alpha + \beta x_i).$$

Constant price-elasticity of demand $\epsilon \implies$ mark-up pricing (MR=MC):

$$p \left( 1 - \frac{1}{\epsilon} \right) = \beta W, \text{ or } p = \beta \frac{W}{\rho}.$$  

Free entry $\implies$ zero-profits $\implies$

- firm scale:

$$x = \frac{\alpha (\epsilon - 1)}{\beta},$$

- per-firm labour requirement:

$$l = \alpha \epsilon,$$

- no. of varieties $i$:

$$N = \frac{\gamma L}{l} = \frac{\gamma L}{\alpha \epsilon}.$$
4 Equilibrium

Six equilibrium equations, six endogenous variables:

- **Incomes:**
  \[ Y_1 = \lambda_1 W_1 \gamma L + \phi_1 (1 - \gamma)L \]
  \[ Y_2 = \lambda_2 W_2 \gamma L + \phi_2 (1 - \gamma)L \]

- **Price indices:**
  \[ I_1 = \left( \frac{\beta}{\rho} \right) \left( \frac{\gamma L}{\alpha e} \right)^{\frac{1}{1-\epsilon}} \left( \lambda_1 W_1^{1-\epsilon} + \lambda_2 T^{1-\epsilon} W_2^{1-\epsilon} \right)^{\frac{1}{1-\epsilon}} \]
  \[ I_2 = \left( \frac{\beta}{\rho} \right) \left( \frac{\gamma L}{\alpha e} \right)^{\frac{1}{1-\epsilon}} \left( \lambda_2 W_2^{1-\epsilon} + \lambda_1 T^{1-\epsilon} W_1^{1-\epsilon} \right)^{\frac{1}{1-\epsilon}} \]

- **Wages:**
  \[ W_1 = \rho^{3-\rho} \left( \frac{\delta}{(e-1)\alpha} \right)^{\frac{1}{2}} (Y_1 I_1^{-1} + Y_2 T^{1-\epsilon} I_2^{1-\epsilon})^{\frac{1}{2}} \]
  \[ W_2 = \rho^{3-\rho} \left( \frac{\delta}{(e-1)\alpha} \right)^{\frac{1}{2}} (Y_2 I_2^{-1} + Y_1 T^{1-\epsilon} I_1^{1-\epsilon})^{\frac{1}{2}} \]

endogenous variables: \( Y_1, Y_2, I_1, I_2, W_1, W_2 \)
exogenous variables: \( L, \alpha, \beta, \gamma, \delta, \phi, \rho, \epsilon, \tau \)
short-run exogenous, long-run endogenous: \( \lambda_1, \lambda_2 \)

→ highly non-linear system \( \Rightarrow \) use simulations

Long-run equilibrium:

- \( w_1 = w_2 \) ("interior solution", symmetric equilibrium if regions 1 and 2 are identical), or
- \( \lambda_1 = 1, w_1 \geq w_2 \) (full agglomeration in region 1, "core-periphery outcome"), or
- \( \lambda_2 = 1, w_1 \leq w_2 \) (full agglomeration in region 2, "core-periphery outcome").

- Law of motion: \( \Delta \lambda_1 = -\Delta \lambda_2 = \eta \left( \frac{w_1}{w_2} \right) \), where \( \eta \) represents the “speed of adjustment”