MEASURES OF SPECIALISATION

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1 Notation

- Sectors, industries:
  $i \in [1...I]$
  $j \in [1...J]$
- Countries, regions:
  $c \in [1...C]$
  $d \in [1...D]$
- Plants: $p \in [1...P_{ic}]$
- Exports: X
- Imports: M
- Production (output, value added, employment): O

2 Specialisation in Trade

2.1 *The Balassa-Hoover Index of Revealed Comparative Advantage*

- $B_{ic} = \dfrac{X_{ic}}{\sum_i X_{ic}} \Bigg/ \dfrac{\sum_c X_{ic}}{\sum_i \sum_c X_{ic}}, \quad B_{ic} \geq 0$

- $B_{ic}^{SYMM} = \dfrac{B_{ic} - 1}{B_{ic} + 1}, \quad -1 < B_{ic}^{SYMM} < 1$

2.2 *The Michaely Index of Inter-Industry Trade Specialisation*

- $M_{ic} = \dfrac{X_{ic}}{\sum_i X_{ic}} - \dfrac{M_{ic}}{\sum_i M_{ic}}, \quad -1 < M_{ic} < 1$

2.3 *The Grubel-Lloyd Index of Intra-Industry Trade*

- $GL_{ic} = 1 - \dfrac{|X_{ic} - M_{ic}|}{X_{ic} + M_{ic}} = \dfrac{2\min(X_{ic}, M_{ic})}{X_{ic} + M_{ic}}, \quad 0 \leq GL_{ic} \leq 1$

- $GL_c = \dfrac{2\sum_i \min(X_{ic}, M_{ic})}{\sum_i (X_{ic} + M_{ic})}, \quad 0 \leq GL_c \leq 1$

- $GL_{ic}^{AQUINO} = 1 - \frac{|X_{ic}^* - M_{ic}^*|}{X_{ic}^* + M_{ic}^*}$,

  where $X_{ic}^* = X_{ic} * 0.5([\sum_i (X_{ic} + M_{ic})] / \sum_i X_{ic})$,

  $M_{ic}^* = M_{ic} * 0.5([\sum_i (X_{ic} + M_{ic})] / \sum_i M_{ic})$

- $GL_c = GL_c^{VIIT} + GL_c^{HIIT}$

$$= \underbrace{\frac{2\sum_j \min(X_{jc}, M_{jc})}{\sum_i (X_{ic} + M_{ic}) + \sum_j (X_{jc} + M_{jc})}}_{GL_c^{VIIT}} + \underbrace{\frac{2\sum_i \min(X_{ic}, M_{ic})}{\sum_i (X_{ic} + M_{ic}) + \sum_j (X_{jc} + M_{jc})}}_{GL_c^{HIIT}},$$

  where $0.85 \le \frac{UV_i^X}{UV_i^M} \le 1.15$, and $\frac{UV_j^X}{UV_j^M} < 0.85$ or $\frac{UV_j^X}{UV_j^M} > 1.15$

- $GL_{ic}^{MIIT} = 1 - \frac{|\Delta X_{ic} - \Delta M_{ic}|}{|\Delta X_{ic}| + |\Delta M_{ic}|}$

3 Specialisation in Production: Industry Concentration / Country Specialisation

3.1 *Concentration Ratios*

- $CR_i^N = \sum_c \left(\frac{O_{ic}}{\sum_c O_{ic}} \right)$ for N c's with the largest ratio $\frac{O_{ic}}{\sum_c O_{ic}}$
- $CR_c^N = \sum_i \left(\frac{O_{ic}}{\sum_i O_{ic}} \right)$ for N i's with the largest ratio $\frac{O_{ic}}{\sum_i O_{ic}}$
- $0 < CR^N \le 1$

3.2 *The Herfindahl Index*

- $H_i = \sum_c \left(\frac{O_{ic}}{\sum_c O_{ic}} \right)^2$
- $H_c = \sum_i \left(\frac{O_{ic}}{\sum_i O_{ic}} \right)^2$
- $0 < H \le 1$

3.3 *Standard Deviations of Shares*

- $SD_i = \text{std}\left(\frac{O_{ic}}{\sum_c O_{ic}}\right)_i$
- $SD_c = \text{std}\left(\frac{O_{ic}}{\sum_i O_{ic}}\right)_c$
- $0 \leq SD < 1$

3.4 *The Finger-Kreinin Bilateral Index (similarity)*

- $F_{cd} = \sum_i \min\left(\frac{O_{ic}}{\sum_i O_{ic}}, \frac{O_{id}}{\sum_i O_{id}}\right), \quad 0 < F < 1$

3.5 *The Krugman Bilateral Index (dissimilarity)*

- $K_{cd} = \sum_i \left|\frac{O_{ic}}{\sum_i O_{ic}} - \frac{O_{id}}{\sum_i O_{id}}\right|, \quad 0 < K < 2$

3.6 *The Locational Gini Index*

- $G_i = 0.5 * \sum_c \left(CS^i_{ic}[O_{ic}] + CS^i_{i,c-1}[O_{ic}]\right)B_{ic} - 0.5, \quad 0 \leq G_i \leq 1$

  where $CS^i_{ic} =$ cumulated sum of $\frac{O_{ic}}{\sum_c O_{ic}}$, ranked in ascending order.

- $G_c = 0.5 * \sum_i \left(CS^c_{ic}[O_{ic}] + CS^c_{i-1,c}[O_{ic}]\right)B_{ic} - 0.5, \quad 0 \leq G_c \leq 1,$

  where $CS^c_{ic} =$ cumulated sum of $\frac{O_{ic}}{\sum_i O_{ic}}$, ranked in ascending order.

3.7 *The Amiti Index*

- $A_i = \sqrt{\frac{1}{C}\sum_c \left(\frac{O_{ic}}{\sum_c O_{ic}} - \frac{\sum_i O_{ic}}{\sum_i \sum_c O_{ic}}\right)^2}$

- $A_c = \sqrt{\frac{1}{I}\sum_i \left(\frac{O_{ic}}{\sum_i O_{ic}} - \frac{\sum_c O_{ic}}{\sum_i \sum_c O_{ic}} \right)^2}$
- $A \geq 0$

3.8 *The Theil Index (Entropy)*

- $T_i = \frac{1}{C}\sum_c \left(\frac{O_{ic}}{\frac{1}{C}\sum_c O_{ic}} * \ln\left[\frac{O_{ic}}{\frac{1}{C}\sum_c O_{ic}} \right] \right), \quad 0 \leq T_i \leq \ln C$
- $T_c = \frac{1}{I}\sum_c \left(\frac{O_{ic}}{\frac{1}{I}\sum_i O_{ic}} * \ln\left[\frac{O_{ic}}{\frac{1}{I}\sum_i O_{ic}} \right] \right), \quad 0 \leq T_c \leq \ln I$
- Additive decomposability: $T_c = T_{c1} + T_{c2}$ if $c1$ and $c2$ are orthogonal

4 Industry Concentration and the "Lumpiness" of Production

4.1 *The Devereux-Griffith-Simpson Measure*

- Herfindahl of geographical concentration (i subscripts are implied):

$$H^{GEO} = \sum_c \left(\frac{\sum_p O_{pc}}{\sum_c \sum_p O_{pc}} \right)^2, \quad \frac{1}{C} \leq H^{GEO} \leq 1$$

- Herfindahl of industrial concentration:

$$H^{IND} = \sum_c \sum_p \left(\frac{O_{pc}}{\sum_c \sum_p O_{pc}} \right)^2, \quad \frac{1}{\sum_c P_c} \leq H^{IND} \leq 1$$

- $H^{GEO} - H^{IND} = ?$

  $\rightarrow$ suppose $\sum_c P_c = 2C$ (i.e. there are more plants than regions)

  and $O_{pc} = K \quad \forall \; p, c$,

  and $P_c = 2 \quad \forall \; c$,

  then $H^{GEO} = \frac{1}{C}$ and $H^{IND} = \frac{1}{2C}$

  $\therefore$ $H^{GEO} - H^{IND} > 0$ even though there is no geographical concentration!

- Define:

$$\hat{H}^{IND} = H^{IND} - \frac{1}{\sum_c P_c} \ , \quad 0 \le \hat{H}^{IND} \le 1$$

$$\hat{H}^{GEO} = H^{GEO} - \frac{1}{\min\left(\sum_c P_c, C\right)} \ , \quad 0 \le \hat{H}^{GEO} \le 1$$

- $DGS = \hat{H}^{GEO} - \hat{H}^{IND}, \quad -1 \le DGS \le 1$

  ("absolute" measure: does not scale regions by their size)

4.2 *The Ellison-Glaeser Measure*

- Define:

$$\tilde{H}_i^{GEO} = \sum_c \left(\frac{\sum_p O_{pci}}{\sum_p \sum_c O_{pci}} - \frac{\sum_p \sum_i O_{pci}}{\sum_p \sum_c \sum_i O_{pci}} \right)^2$$

$$\tilde{\tilde{H}}_i^{GEO} = \sum_c \left(\frac{\sum_p \sum_i O_{pci}}{\sum_p \sum_c \sum_i O_{pci}} \right)^2 \quad \text{("regional Herfindahl")}$$

- $$EG_i = \frac{\dfrac{\tilde{H}_i^{GEO}}{1 - \tilde{\tilde{H}}_i^{GEO}} - H_i^{IND}}{1 - H_i^{IND}}, \quad -1 \le EG_i \le 1$$