

MEASURES OF SPECIALISATION

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1 Notation

- Sectors, industries:
 $i \in [1 \dots I]$
 $j \in [1 \dots J]$
- Countries, regions:
 $c \in [1 \dots C]$
 $d \in [1 \dots D]$
- Plants: $p \in [1 \dots P_{ic}]$
- Exports: X
- Imports: M
- Production (output, value added, employment): O

2 Specialisation in Trade

2.1 The Balassa-Hoover Index of Revealed Comparative Advantage

- $B_{ic} = \frac{X_{ic}}{\sum_i X_{ic}} \bigg/ \frac{\sum_c X_{ic}}{\sum_i \sum_c X_{ic}}, \quad B_{ic} \geq 0$
- $B_{ic}^{SYMM} = \frac{B_{ic} - 1}{B_{ic} + 1}, \quad -1 < B_{ic}^{SYMM} < 1$

2.2 The Michaely Index of Inter-Industry Trade Specialisation

- $M_{ic} = \frac{X_{ic}}{\sum_i X_{ic}} - \frac{M_{ic}}{\sum_i M_{ic}}, \quad -1 < M_{ic} < 1$

2.3 The Grubel-Lloyd Index of Intra-Industry Trade

- $GL_{ic} = 1 - \frac{|X_{ic} - M_{ic}|}{X_{ic} + M_{ic}} = \frac{2 \min(X_{ic}, M_{ic})}{X_{ic} + M_{ic}}, \quad 0 \leq GL_{ic} \leq 1$
- $GL_c = \frac{2 \sum_i \min(X_{ic}, M_{ic})}{\sum_i (X_{ic} + M_{ic})}, \quad 0 \leq GL_c \leq 1$

- $GL_{ic}^{AQUINO} = 1 - \frac{|X_{ic}^* - M_{ic}^*|}{X_{ic}^* + M_{ic}^*},$

where $X_{ic}^* = X_{ic} * 0.5([\sum_i (X_{ic} + M_{ic})] / \sum_i X_{ic}),$

$$M_{ic}^* = M_{ic} * 0.5([\sum_i (X_{ic} + M_{ic})] / \sum_i M_{ic})$$

- $GL_c = GL_c^{VIIT} + GL_c^{HIIT}$

$$= \frac{2\sum_j \min(X_{jc}, M_{jc})}{\underbrace{\sum_i (X_{ic} + M_{ic}) + \sum_j (X_{jc} + M_{jc})}_{GL_c^{VIIT}}} + \frac{2\sum_i \min(X_{ic}, M_{ic})}{\underbrace{\sum_i (X_{ic} + M_{ic}) + \sum_j (X_{jc} + M_{jc})}_{GL_c^{HIIT}}},$$

where $0.85 \leq \frac{UV_i^X}{UV_i^M} \leq 1.15$, and $\frac{UV_j^X}{UV_j^M} < 0.85$ or $\frac{UV_j^X}{UV_j^M} > 1.15$

- $GL_{ic}^{MIIT} = 1 - \frac{|\Delta X_{ic} - \Delta M_{ic}|}{|\Delta X_{ic}| + |\Delta M_{ic}|}$

3 Specialisation in Production: Industry Concentration / Country Specialisation

3.1 Concentration Ratios

- $CR_i^N = \sum_c \left(\frac{O_{ic}}{\sum_c O_{ic}} \right)$ for N c 's with the largest ratio $\frac{O_{ic}}{\sum_c O_{ic}}$
- $CR_c^N = \sum_i \left(\frac{O_{ic}}{\sum_i O_{ic}} \right)$ for N i 's with the largest ratio $\frac{O_{ic}}{\sum_i O_{ic}}$
- $0 < CR^N \leq 1$

3.2 The Herfindahl Index

- $H_i = \sum_c \left(\frac{O_{ic}}{\sum_c O_{ic}} \right)^2$
- $H_c = \sum_i \left(\frac{O_{ic}}{\sum_i O_{ic}} \right)^2$
- $0 < H \leq 1$

3.3 Standard Deviations of Shares

- $SD_i = \text{std} \left(\frac{O_{ic}}{\sum_c O_{ic}} \right)_i$
- $SD_c = \text{std} \left(\frac{O_{ic}}{\sum_i O_{ic}} \right)_c$
- $0 \leq SD < 1$

3.4 The Finger-Kreinin Bilateral Index (similarity)

- $F_{cd} = \sum_i \min \left(\frac{O_{ic}}{\sum_i O_{ic}}, \frac{O_{id}}{\sum_i O_{id}} \right), \quad 0 < F < 1$

3.5 The Krugman Bilateral Index (dissimilarity)

- $K_{cd} = \sum_i \left| \frac{O_{ic}}{\sum_i O_{ic}} - \frac{O_{id}}{\sum_i O_{id}} \right|, \quad 0 < K < 2$

3.6 The Locational Gini Index

- $G_i = 0.5 * \sum_c (CS_{ic}^i [O_{ic}] + CS_{i,c-1}^i [O_{ic}]) B_{ic} - 0.5, \quad 0 \leq G_i \leq 1$
 where $CS_{ic}^i =$ cumulated sum of $\frac{O_{ic}}{\sum_c O_{ic}}$, ranked in ascending order.
- $G_c = 0.5 * \sum_i (CS_{ic}^c [O_{ic}] + CS_{i-1,c}^c [O_{ic}]) B_{ic} - 0.5, \quad 0 \leq G_c \leq 1,$
 where $CS_{ic}^c =$ cumulated sum of $\frac{O_{ic}}{\sum_i O_{ic}}$, ranked in ascending order.

3.7 The Amity Index

- $A_i = \sqrt{\frac{1}{C} \sum_c \left(\frac{O_{ic}}{\sum_c O_{ic}} - \frac{\sum_i O_{ic}}{\sum_i \sum_c O_{ic}} \right)^2}$

- $A_c = \sqrt{\frac{1}{I} \sum_i \left(\frac{O_{ic}}{\sum_i O_{ic}} - \frac{\sum_c O_{ic}}{\sum_i \sum_c O_{ic}} \right)^2}$
- $A \geq 0$

3.8 The Theil Index (Entropy)

- $T_i = \frac{1}{C} \sum_c \left(\frac{O_{ic}}{\frac{1}{C} \sum_c O_{ic}} * \ln \left[\frac{O_{ic}}{\frac{1}{C} \sum_c O_{ic}} \right] \right), 0 \leq T_i \leq \ln C$
- $T_c = \frac{1}{I} \sum_i \left(\frac{O_{ic}}{\frac{1}{I} \sum_i O_{ic}} * \ln \left[\frac{O_{ic}}{\frac{1}{I} \sum_i O_{ic}} \right] \right), 0 \leq T_c \leq \ln I$
- Additive decomposability: $T_c = T_{c1} + T_{c2}$ if $c1$ and $c2$ are orthogonal

4 Industry Concentration and the “Lumpiness” of Production

4.1 The Devereux-Griffith-Simpson Measure

- Herfindahl of geographical concentration (i subscripts are implied):

$$H^{GEO} = \sum_c \left(\frac{\sum_p O_{pc}}{\sum_c \sum_p O_{pc}} \right)^2, \frac{1}{C} \leq H^{GEO} \leq 1$$

- Herfindahl of industrial concentration:

$$H^{IND} = \sum_c \sum_p \left(\frac{O_{pc}}{\sum_c \sum_p O_{pc}} \right)^2, \frac{1}{\sum_c P_c} \leq H^{IND} \leq 1$$

- $H^{GEO} - H^{IND} = ?$

→ suppose $\sum_c P_c = 2C$ (i.e. there are more plants than regions)

and $O_{pc} = K \quad \forall p, c$,

and $P_c = 2 \quad \forall c$,

then $H^{GEO} = \frac{1}{C}$ and $H^{IND} = \frac{1}{2C}$

∴ $H^{GEO} - H^{IND} > 0$ even though there is no geographical concentration!

- Define:

$$\hat{H}^{IND} = H^{IND} - \frac{1}{\sum_c P_c}, \quad 0 \leq \hat{H}^{IND} \leq 1$$

$$\hat{H}^{GEO} = H^{GEO} - \frac{1}{\min\left(\sum_c P_c, C\right)}, \quad 0 \leq \hat{H}^{GEO} \leq 1$$

- $DGS = \hat{H}^{GEO} - \hat{H}^{IND}, \quad -1 \leq DGS \leq 1$

(“absolute” measure: does not scale regions by their size)

4.2 The Ellison-Glaeser Measure

- Define:

$$\tilde{H}_i^{GEO} = \sum_c \left(\frac{\sum_p O_{pci}}{\sum_p \sum_c O_{pci}} - \frac{\sum_p \sum_i O_{pci}}{\sum_p \sum_c \sum_i O_{pci}} \right)^2$$

$$\tilde{\tilde{H}}_i^{GEO} = \sum_c \left(\frac{\sum_p \sum_i O_{pci}}{\sum_p \sum_c \sum_i O_{pci}} \right)^2 \quad (\text{“regional Herfindahl”})$$

- $EG_i = \frac{\frac{\tilde{H}_i^{GEO}}{1 - \tilde{\tilde{H}}_i^{GEO}} - H_i^{IND}}{1 - H_i^{IND}}, \quad -1 \leq EG_i \leq 1$