Tacit Collusion under Fairness and Reciprocity*

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This version: May 25, 2011

Abstract

This paper departs from the standard profit-maximizing model of firm behavior by assuming that managers are motivated in part by personal animosity–or respect–towards their rivals. A reciprocal manager responds to unkind behavior of rivals with unkind actions (negative reciprocity), while at the same time, it responds to kind behavior of rivals with kind actions (positive reciprocity). We provide conditions on managers' perceptions of fair actions of their rivals under which collusive action profiles (prices or quantities) are easier to sustain. Thus, fairness and reciprocity concerns among managers can have adverse welfare consequences for consumers.

JEL Classification Numbers: D43, D63, L13, L21.
Keywords: Fairness; Reciprocity; Collusion; Repeated Games.

*We gratefully acknowledge comments from Matthew Rabin, Joel Sobel, and seminar audiences at University of Copenhagen, University of Cergy-Pontoise, Free University of Amsterdam, University of Geneva, and University of Lausanne.
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1 Introduction

The assumption that individuals behave as if maximizing their material payoffs, despite its central role in economic analysis, is at odds with a large body of evidence from psychology and from experimental economics. Economic agents often pursue objectives other than actual payoff maximization. Many observed departures from material payoff maximizing behavior arise through actions that favor fairness or reciprocity.

Fairness and reciprocity have been shown to explain behavior in bargaining games and in trust games. For example, in ultimatum games offers are usually much more generous than predicted by equilibrium and low offers are often rejected. These offers are consistent with an equilibrium in which proposers make offers knowing that responders may reject allocations that appear unfair.

The impact of fairness and reciprocity on market outcomes is an active area of research. Rabin (1993) and Rotemberg (forthcoming) show that fairness concerns by the part of consumers can improve consumer welfare. For example, Rabin (1993) finds that a monopolist ought to set price lower than "the monopoly price" if consumers have concerns about fairness.

In this paper we ask whether reciprocity may help to sustain collusive behavior. For instance, if a collusive agreement is seen by the parties as a fair outcome, then if one party reneges on the agreement and undercuts the price (or boosts its output), its rivals may be offended and hence punish the deviator aggressively (even at extra cost to themselves).

To perform this analysis we rule out fairness concerns by the part of consumers with respect to firms and vice-versa. This allows us to focus on the impact of fairness concerns among firms on collusive outcomes. The assumption that firms have fairness concerns and behave reciprocally towards their rivals finds support on experimental evidence where subjects play the role of firms.

In Lehman (2001), individuals placed in the role of a manager were asked to report satisfaction with various combinations of sales figures for their own firm, as well as for a competing firm. Attention to fairness was found to be a significant factor.

Huck et al. (2001) show that a Stackelberg leader finds it hard to exploit that advantage in experimental markets. The reason is that the Stackelberg follower acts more aggressively than predicted by the subgame perfect equilibrium of these market games. In fact, followers punish the leader by supplying a higher quantity than their most profitable response to the leader's quantity. This behavior is in line with the observed negative reciprocity of responders in the ultimatum game when the proposer tries to exploit his first-mover advantage.

Armstrong and Huck (2010) argue that sometimes managers are motivated in part by personal animosity–or respect–towards a rival. Thus, firms might punish rivals who behave "unfairly" towards them. For example, firms might sometimes care when their rivals obtain an "unfair" share of industry profits, for instance by cheating on a collusive agreement.

To model reciprocity we follow Segal and Sobel (2007) and assume that players in a strategic environment have preferences not only over the outcomes but also the strategies. A player's utility is additively separable in monetary and fairness payoffs. Monetary payoffs are revenues minus costs and fairness payoffs are a weighted average of the rivals' monetary payoffs where the weights depend on how the rivals' choices are expected to differ from the fair ones. If a player expects a rival to play a kind (mean) strategy, then he places a positive (negative) weight on that rival's monetary payoff. If a player expects a rival to play a fair strategy then he places zero weight on that rivals' monetary payoff.

In a standard setting, collusion can be sustained as an equilibrium by self-interested players if they interact infinitely often and are sufficiently patient. A player is said to be patient if his discount factor is sufficiently close to one. In order to determine whether collusion is or is not facilitated by reciprocity we compare the minimal discount factor that allows the same collusive outcome to be sustained when players are reciprocal and when they are self-interested.

We find that reciprocity facilitates collusion under price competition if each player considers that each rival's fair price is greater than the rival's self-interested Nash price but less than the rival's collusive price. However, collusion under price competition might be harder to sustain when each player considers that each rival's fair price is greater than the rival's collusive price.

These two results also extend to quantity competition. Reciprocity facilitates collusion under quantity competition when each player considers that each rival's fair output is greater than that rival's collusive output but smaller than that rival's self-interested Nash output. In contrast, collusion under quantity competition might be harder to sustain when each player considers that each rival's fair quantity is smaller than the rival's collusive quantity.

The analysis is conducted assuming that players play Nash reversion punishments. In the appendix we extend the analysis to the case where players play credible punishments using penal codes à la Abreu (1988).

Our paper is an additional contribution to the literature on the factors that help or hinder collusion. It is now well known that concentration, barriers to entry, cross-ownership, symmetry and multi-market contracts facilitate collusion–see Feuerstein (2005). We provide conditions under which reciprocity can facilitate collusion.

The main policy implication of our paper is that fairness concerns by firms with reciprocal managers can have adverse welfare consequences for consumers. In contrast, Rabin (1993) and Rotemberg (forthcoming) find that fairness concerns by the part of consumers can increase consumer welfare. Thus, social preferences in imperfectly competitive markets might lead to different outcomes depending on who has such preferences (producers or consumers) and what is the comparison group.

The rest of the paper proceeds as follows. Section 2 sets-up the model. Section 3 considers the case where players' choices are strategic complements. Section 4 considers the case of strategic substitutes. Section 5 discusses the findings. Section 6 concludes the paper. Appendix A contains the proofs of all results in the main text. Appendix B states and proves results when players use optimal punishments.

2 Set-up

The existing theories of social preferences can be classified into three broad categories. The first one is the distributional preference approach where social preferences only depend on the distribution of material payoffs. This includes Fehr and Schmidt (1999) and Bolton and Ockenfels (2000). These models are highly tractable and capture a wide range of phenomena but fail to explain the fact that preferences depend on more than outcomes, namely, intentions also matter.

The second category consists of intention-based models and includes Rabin (1993), Dufwenberg and Kirchsteiger (2004), and Falk and Fischbacher (2006), among others. These models assume that reciprocity depends on overall strategies and beliefs (and beliefs about beliefs) building on Geanakoplos et al. (1989) theory of psychological games. In Rabin (1993) utility is additively separable in monetary and fairness payoffs and the weight a player places on rivals' monetary payoffs depends on his perception of the rivals' intentions, which are evaluated using (i) beliefs about the rivals' strategy choices, and (ii) beliefs about the rivals' beliefs about his strategy. Dufwenberg and Kirchsteiger (2004) develop a theory of reciprocity for extensive form games where players update beliefs about intentions as the game unfolds and make a choice accordingly. Falk and Fischbacher (2006) model reciprocity in incomplete information games. Intention-based models have two major weaknesses: they use specific functional forms and are highly intractable (see Sobel, 2005).

The third category explores the axiomatic foundations that generate utility functions that display social preferences. Nielson (2006) proposes a preference axiom which leads to a foundation of Fehr and Schmidt (1999) inequity aversion model. Segal and Sobel (2007) provide an axiomatic foundation for interdependent preferences that can reflect reciprocity, inequity aversion, altruism as well as spitefulness. The key innovation of their approach is that, in addition to conventional preferences over outcomes, players in a strategic environment also have preferences over strategy profiles. This allows one to study situations where a player's preference is affected by the behavior of other players.

Their representation theorem shows that the payoff function of a player with such preferences is of the form

$$V_i(\sigma_i, \sigma_{-i}^*) = u_i(\sigma_i, \sigma_{-i}^*) + \sum_{j \neq i} w_{ij}(\sigma^*) u_j(\sigma_i, \sigma_{-i}^*), \tag{1}$$

where σ_i is the strategy of player i, $\sigma^* \equiv (\sigma_1, ..., \sigma_n)$ is how the game is expected to be played, u_i is the utility from outcomes of player i, u_j is the utility from outcomes of player $j \neq i$, and $w_{ij}(\sigma^*)$ is a coefficient that measures the weight player i gives to player j's utility, which is a function of the entire strategy profile. Positive values of the coefficient mean that player i is willing to sacrifice his utility from outcomes in order to increase the payoff of player j. Negative values mean that player i is willing to sacrifice his utility from outcomes in order to lower player j's payoff. Since the coefficient depends on the strategy chosen by player j, there is scope to model reciprocity.

We apply Segal and Sobel's (2007) approach to a dynamic game where n players, $n > 2$, play the same stage game over an infinite horizon $t = 0, 1, 2, \ldots$. The repeated game monetary payoff of player i of choosing strategy $s_i = (a_i^1, a_i^2, ...)$ when rivals play strategies s_{-i} is given by

$$\Pi_i(s_i, s_{-i}) = \sum_{t=1}^{\infty} \delta^{t-1} \pi_i(a_i^t, a_{-i}^t),$$

where $\pi_i(a_i^t, a_{-i}^t)$ represents player i's monetary payoff at stage t, a function of player i's action at t, a_i^t, and the actions of the rivals at t, a_{-i}^t. Players discount the future at rate $\delta \in (0, 1)$. To model reciprocity we assume that the weight player i places on player j's repeated game monetary payoff depends only on player j's strategy and on player i's perception of what is the fair strategy of player j, s_{ij}^f. We also assume throughout that players' preferences as well as their exogenous perceptions of the fair strategies of the rivals are common knowledge. The repeated game payoff of reciprocal player i of choosing strategy $s_i = (a_i^1, a_i^2, ...)$ when rivals play strategies s_{-i} is given by

$$U_i(s_i, s_{-i}, s_{-i}^f) = \sum_{t=1}^{\infty} \delta^{t-1} \pi_i(a_i^t, a_{-i}^t) + \alpha \sum_{j \neq i} \sum_{t=1}^{\infty} \delta^{t-1} w_{ij}(a_j^t, a_{ij}^f) \pi_j(a_i^t, a_{-i}^t)$$

where $\alpha > 0$ is a normalization. The central behavioral feature of these preferences is the assumption that players care about the intentions of the rivals. If player i expects player j to treat him kindly, then w_{ij} will be positive, and player i will wish to treat player j kindly. If player i expects player j to treat him badly, then w_{ij} will be negative, and player i will wish to treat player j badly. If player i expects player j to be fair, then w_{ij} will be zero, and there is no issue of reciprocity.

Denote the dynamic game with reciprocal players by $\Gamma_{\infty}^r(u, s)$, where $u \equiv (u_1, ..., u_n)$ and $s \equiv (s_1, ..., s_n)$ and the dynamic game with self-interested players by $\Gamma_{\infty}^s(\pi, s)$, where $\pi \equiv (\pi_1, ..., \pi_n)$. Players are able to sustain a collusive outcome when the payoff from collusion is no less than the payoff from deviation. To understand how fairness and reciprocity influence collusion we will compare the incentive compatibility condition of self-interested players in $\Gamma_{\infty}^s(\pi, s)$ to that of reciprocal players in $\Gamma_{\infty}^r(u, s)$ assuming that these two games are identical in all respects (monetary payoffs and the number of players) with the exception of players' preferences.

To perform this analysis we consider the cases where players' actions are strategic complements (e.g., price competition with products that are imperfect substitutes) and strategic substitutes (e.g., quantity competition with products that are perfect substitutes). We also consider two alternative modes of punishments after deviations: Nash reversion and optimal punishments.

The standard approach to study collusion in infinitely repeated games assumes that players use grim trigger strategies to punish any deviation from collusion, that is, following a deviation players switch to a Nash equilibrium of the stage game forever after. Thus, when self-interested player uses grim trigger punishments in $\Gamma^s_\infty(\pi, s)$, each player i will prefer to play his collusive strategy $s_i^c \equiv (a_i^c, a_i^c, ...)$ if the payoff from collusion, $\pi_i(a^c)/(1-\delta)$, is no less than the payoff from defection which consists of the one period gain from deviating $\pi_i(BR_i^s(a_{-i}^c), a_{-i}^c)$ plus the discounted payoff of inducing Nash reversion forever $\delta\pi_i(a^{ns})/(1-\delta)$, that is,

$$\pi_i(BR_i^s(a_{-i}^c), a_{-i}^c) + \frac{\delta}{1-\delta}\pi_i(a^{ns}) \leq \frac{1}{1-\delta}\pi_i(a^c).$$

Solving for δ we obtain

$$\delta_{a^c}^s = \frac{\pi_i(BR_i^s(a_{-i}^c), a_{-i}^c) - \pi_i(a^c)}{\pi_i(BR_i^s(a_{-i}^c), a_{-i}^c) - \pi_i(a^{ns})} \leq \delta. \tag{2}$$

The collusion strategy profile s^c can be sustained by self-interested players who are patient enough such that $\delta_{a^c}^s \leq \delta$ where $\delta_{a^c}^s$ is the critical discount factor above which s^c can be sustained by self-interested players.

The same reasoning applies when players have reciprocal preferences. A reciprocal player i plays the collusive strategy s_i^c in $\Gamma^r_\infty(u, x)$ using a grim trigger strategy as long as the following condition holds

$$u_i(BR_i^r(a_{-i}^c), a_{-i}^c, a_{-i}^f) + \frac{\delta}{1-\delta}u_i(a^{nr}, a_{-i}^f) \leq \frac{1}{1-\delta}u_i(a^c, a_{-i}^f),$$

where u_i denotes the stage game payoff of a reciprocal player, a function of the actions played and perceptions of the fair actions of the rivals. Solving for δ we obtain

$$\delta_{a^c}^r = \frac{u_i(BR_i^r(a_{-i}^c), a_{-i}^c, a_{-i}^f) - u_i(a^c, a_{-i}^f)}{u_i(BR_i^r(a_{-i}^c), a_{-i}^c, a_{-i}^f) - u_i(a^{nr}, a_{-i}^f)} \leq \delta. \tag{3}$$

When players have reciprocal preferences it follows that the collusive strategy profile s^c can be sustained if players are patient enough such that $\delta_{a^c}^r \leq \delta$ where $\delta_{a^c}^r$ is the critical discount factor above which s^c can be sustained by reciprocal players.

We will use (2) and (3) to characterize the impact that fairness and reciprocity have on collusion when players use grim trigger strategies. To perform this analysis we compare the critical discount factor above which the collusive strategy profile can be sustained when players are self-interested to the critical discount factor when players are reciprocal. We say that fairness and reciprocity facilitate collusion when the collusive strategy profile can be sustained at a lower critical discount factor when players are reciprocal than when they are self-interested. If the opposite happens we say that fairness and reciprocity make collusion harder.

3 Strategic Complements

We now study the impact of fairness and reciprocity on collusion in a specialized model, where players' actions are strategic complements. This assumption means that a player's incremental returns from increasing his own action are increasing in the rivals' actions. The canonical market game where players' actions are strategic complements is price competition with imperfect substitutes.

In each stage player i chooses price, p_i, and his payoff in that stage is

$$u_i(p_i, p_{-i}, p^f_{-i}) = \pi_i(p_i, p_{-i}) + \alpha \sum_{j\neq i} w_{ij}(p_j, p^f_{ij})\pi_j(p_i, p_{-i}), \tag{4}$$

where $\pi_i(p_i, p_{-i})$ is the monetary payoff and $\alpha \sum_{j\neq i} w_{ij}(p_j, p^f_{ij})\pi_j(p_i, p_{-i})$ the fairness payoff. The monetary payoff is the difference between revenue and cost, that is,

$$\begin{aligned}\pi_i(p_i, p_{-i}) &= R_i(p_i, p_{-i}) - C_i(D_i(p_i, p_{-i})) \\ &= p_i D_i(p_i, p_{-i}) - C_i(D_i(p_i, p_{-i})),\end{aligned}$$

where $R_i(p_i, p_{-i})$ is revenue, $C_i(D_i(\cdot))$ is the cost of production, and $D_i(p_i, p_{-i})$ is the demand faced by player i. We assume that $D_i(\cdot)$ is decreasing with p_i, increasing with p_{-i}, and $C_i(\cdot)$ is increasing with $D_i(\cdot)$. Furthermore, we define the weight function as follows:

$$w_{ij}(p_j, p^f_{ij}) \begin{cases} > 0 \text{ if } p_j > p^f_{ij} \\ = 0 \text{ if } p_j = p^f \\ < 0 \text{ otherwise} \end{cases}. \tag{5}$$

The assumptions on weight function capture the fact that a reciprocal player cares about the intentions of the rivals. The first condition expresses positive reciprocity. If a player expects one of her rivals to charge a price higher than the fair price, then she puts a positive weight on that rival's profit and she is willing to sacrifice some of her profit to increase that rival's profit. The second condition says that if a player expects one of her rivals to choose the fair price, then she places no weight on that rival's profit. The third condition expresses negative reciprocity. If player a expects one of her rivals to undercut her perception of fair price, then she puts a negative weight on that rival's profit and she is willing to sacrifice some of her profit to reduce that rival's profit.

Let

$$A_i(p_i, p_{-i}, p^f_{-i}) = \underset{p_i \in P_i}{\arg\max}\ \pi_i(p_i, p_{-i}) + \alpha \sum_{j\neq i} w_{ij}(p_j, p^f_{ij})\pi_j(p_i, p_{-i}),$$

denote the set of maximizers of player i's stage game problem. The players will never choose an infinite price for finite quantities, implying the players' price choice set to be compact set in $\mathcal{R}$. We assume that u_i is order upper semi-continuous in p_i. This assumption together with the choice set being compact guarantee the set of maximizers $A_i(p_i, p_{-i}, p^f_{-i})$ to be nonempty.

We also assume that u_i has increasing differences in (p_i, p_{-i}), that is, for any fixed p^f_{-i}, $u_i(p_i, p'_{-i}, p^f_{-i}) - u_i(p_i, p''_{-i}, p^f_{-i})$ is increasing in p_i for all $p'_{-i} \geq p''_{-i}$. This assumption implies that fairness payoffs are small by comparison with monetary payoffs which guarantees that prices are strategic complements. As we show in Appendix A, both $\Gamma^r(u,p)$ and $\Gamma^s(\pi,p)$ admit largest and smallest pure-strategy Nash equilibrium profiles, $\underline{p}$ and $\overline{p}$. We further assume that u_i has decreasing differences in (p_i, p^f_{-i}). The following result shows how players' perceptions of the fair prices of the rivals influence the extremal equilibrium prices of this game.

Lemma 1: *The smallest and the largest pure-strategy Nash equilibria of* $\Gamma^r(u,p)$, $\underline{p}^{nr}$ *and* $\overline{p}^{nr}$, *are nonincreasing functions of* $p^f = (p^f_{-1}, ..., p^f_{-n})$.

Lemma 1 is a comparative statics result that characterizes the impact that players' perceptions of the fair prices of their rivals have on the Nash equilibrium prices of the stage game. This result says that the higher are players' perceptions of what the fair prices of the rivals should be, the lower will the equilibrium prices be. This happens because an increase in p^f_{-i} shifts the best reply of a reciprocal player i towards origin. In other words, the higher player i perceives the fair price for the other players to be, the more he would like to set a smaller price for any price of the other players. The critical assumption that drives this result is that u_i has decreasing differences in (p_i, p^f_{-i}), that is, the marginal returns from increasing prices are decreasing with a player's perception of the fair prices of the rivals.

It turns out that all our results hold independently of whether we consider the largest or the smallest pure-strategy Nash equilibria of $\Gamma^r(u,p)$ and $\Gamma^s(\pi,p)$. Therefore, without loss of generality, we drop the "bar" notation from now on, and simply write $p^{nr} \equiv (p^{nr}_1, ..., p^{nr}_n)$ and $p^{ns} \equiv (p^{ns}_1, ..., p^{ns}_n)$, referring to either $\overline{p}^{nr}$ and $\overline{p}^{ns}$ or $\underline{p}^{nr}$ and $\underline{p}^{ns}$, respectively. In addition, we assume the following: $p^f_{-i} \geq p^{ns}_{-i}$ for all i, i.e., too low prices of the rivals are considered unfair, which is reasonable assumption given the focus of the paper is tacit collusion among players.

Next we show how preferences for fairness and reciprocity change the outcome of static price competition. To do that we compare the Nash equilibria of the stage game with self-interested players to that of the stage game with reciprocal players.

Proposition 1: *If* $p^f_{-i} \geq p^{ns}_{-i}$ *for all i, then (i)* $p^{nr} \leq p^{ns}$; *and (ii)* $u_i(p^{nr}, p^f_{-i}) \leq \pi_i(p^{ns})$.

This result tells us how fairness and reciprocity change the nature of static price competition. If reciprocal players believe that the fair prices of the rivals are higher than the equilibrium prices of the rivals in the game with self-interested players, then prices set by reciprocators will be lower than those set by self-interested players. In this case, fairness and reciprocity lead to a more competitive outcome.

The intuition behind Proposition 1 is as follows. When reciprocal players believe that the fair prices of the rivals are higher than the equilibrium prices of the rivals in the game with self-interested players, the Nash equilibrium of the game with reciprocal players are negative reciprocity state: reciprocal players expect their rivals to set unfair prices. This implies that reciprocal players wish to punish their rivals. They do it by setting a price lower than the price a self-interested player would set. The lower equilibrium prices reduce players' monetary payoffs and in addition lead to payoff losses due to the unkind behavior of the rivals.

We now turn our attention to how fairness and reciprocity change the nature of dynamic price competition. The repeated game payoff of strategy $p_i = (p_i^1, p_i^2, ...)$ when rivals play strategies p_{-i} is given by

$$U_i(p_i, p_{-i}, p_{-i}^f) = \sum_{t=1}^{\infty} \delta^{t-1} \pi_i(p_i^t, p_{-i}^t) + \alpha \sum_{j \neq i} \sum_{t=1}^{\infty} \delta^{t-1} w_{ij}(p_j^t, p_{ij}^f) \pi_j(p_i^t, p_{-i}^t)$$

When players use stationary strategies the repeated game payoff becomes

$$\begin{aligned} U_i(p_i, p_{-i}, p_{-i}^f) &= \sum_{t=1}^{\infty} \delta^{t-1} \pi_i(p_i, p_{-i}) + \alpha \sum_{j \neq i} w_{ij}(p_j, p_{ij}^f) \sum_{t=1}^{\infty} \delta^{t-1} \pi_j(p_i, p_{-i}) \\ &= \frac{1}{1-\delta} \left[\pi_i(p_i, p_{-i}) + \alpha \sum_{j \neq i} w_{ij}(p_j, p_{ij}^f) \pi_j(p_i, p_{-i}) \right] \\ &= \frac{1}{1-\delta} u_i(p, p_{-i}, p_{-i}^f). \end{aligned}$$

From now on we assume that Nash punishments in $\Gamma_{\infty}^r(u, p)$ and in $\Gamma_{\infty}^s(\pi, p)$ are the pure strategy Nash equilibria of $\Gamma^r(u, p)$ and $\Gamma^s(\pi, p)$, respectively. We are now ready to state the main result of the paper.

Proposition 2: *Let $\pi_i(p_i, p_{-i}) = (p_i - c_i) D_i(p_i, p_{-i})$ and $p_{ij}^f \in [p_j^{ns}, p_j^c]$ for all i and $j \neq i$. Then the critical (minimum) discount factor needed to sustain collusion at p^c is lower in $\Gamma_{\infty}^r(u, p)$ than in $\Gamma_{\infty}^s(\pi, p)$, that is $\delta_{p^c}^r < \delta_{p^c}^s$.*

The intuition behind this result is as follows. If each player considers that each rival's fair price is greater than the Nash price of that rival when all players are self-interested, then Nash reversion becomes a negative reciprocity state since players expect the rivals to set unfair prices. This implies that reciprocal players wish to punish their rivals for their expected unkindness. They do it by setting a price lower than the price a self-interested player would set. This reduces players' monetary payoffs and in addition leads to fairness payoff losses due to the unkind behavior of the rivals. Hence, the punishment imposed after cheating occurs is more severe. This effect makes collusion *more* attractive.

Additionally, if each player considers that each rival's fair price is less than that rival's collusive price, then collusion becomes a positive reciprocity state given that players view the prices set by their rivals as fair. In this case players' monetary payoffs from collusion are the same as the ones obtained in the game with self-interested players but in addition there are fairness payoff gains since players consider rivals to be kind. This effect also makes collusion *more* attractive.

In contrast, the short-run benefit to deviating is larger with reciprocal players than with self-interested ones. This happens because the short-run deviation payoff of a reciprocal player also includes the benefit that player derives from being treated kindly by the rivals (the rivals are playing their collusive prices). This effect makes collusion *less* attractive. However, if monetary payoffs are large by comparison with fairness payoffs, which is a reasonable assumption, the increase in the short-run benefit to deviating is of second-order.

Our next result shows fairness and reciprocity might no longer facilitate collusion when the conditions on players' perceptions of rivals' fair prices in Proposition 2 are not satisfied.

Proposition 3: *Let $\pi_i(p_i, p_{-i}) = (p_i - c_i)D_i(p_i, p_{-i})$ and $p_{ij}^f > p_j^c$ for all i and $j \neq i$. Then the effect of fairness and reciprocity on the critical (minimum) discount factor needed to sustain collusion at p^c is ambiguous.*

If players think that the fair prices of the rivals are strictly higher than the collusive prices, then collusion becomes a negative reciprocity state given that players view the rivals' collusive prices as unfair. In this case players' monetary payoffs from collusion are the same as the ones obtained in the game with self-interested players but in addition there are fairness payoff losses since players think that their rivals are being unkind. This effect makes collusion *less* attractive. On the other hand, the punishment imposed after cheating occurs is still more severe when players are reciprocal than when they are self-interested which makes collusion *more* attractive. If the first effect dominates, then reciprocity makes collusion harder to sustain.

4 Strategic Substitutes

We now study the impact of fairness and reciprocity on collusion in dynamic quantity-setting games, where players' actions are strategic substitutes. Player's actions being strategic substitutes means that a player's incremental returns from increasing his own action are decreasing in the rivals' actions. The canonical market game where players' actions are strategic substitutes is quantity competition with products that are perfect substitutes.

Assume that in each period player i chooses quantity, q_i, and his payoff in that period is given by

$$u_i(q_i, Q_{-i}) = \pi_i(q_i, Q_{-i}) + \alpha w_i(Q_{-i}, Q_{-i}^f) \sum_{j \neq i} \pi_j(q_i, Q_{-i}),$$

where $\pi_i(q_i, Q_{-i})$ is the monetary payoff and $\alpha w_i(Q_{-i}, Q_{-i}^f) \sum_{j \neq i} \pi_j(q_i, Q_{-i})$ is the fairness payoff, with $\alpha > 0$. Player i's monetary payoff, $\pi_i(q_i, q_{-i})$, is the difference between revenue and cost, that is,

$$\begin{aligned} \pi_i(q_i, Q_{-i}) &= R_i(q_i, Q_{-i}) - C_i(q_i) \\ &= P(Q)q_i - C_i(q_i), \end{aligned}$$

where $R_i(q_i, Q_{-i})$ is revenue, $C_i(q_i)$ is the cost of production, and $P(Q)$ is the inverse market demand with $Q = \sum q_i$. We assume that $P(Q)$ is strictly positive on some bounded interval $(0, \bar{Q})$ with $P(Q) = 0$ for $Q \geq \bar{Q}$. We also assume that $P(Q)$ is twice continuously differentiable with $P'(Q) < 0$ (in the interval for which $P(Q) > 0$). Players' costs of production are assumed to be twice continuously differentiable with $C_i'(q_i) \geq 0$. It is also assumed that the decreasing marginal revenue property holds, that is, $P'(Q) + P''(Q)\, q_i \leq 0$, and $P'(Q) - C_i''(q_i) < 0$. Furthermore, we assume that the weight that player i places on the rivals' aggregate monetary payoffs depends on player i's perception of the fair aggregate output of the rivals, Q_{-i}^f, and on the actual aggregate output of the rivals such that

$$w_i(Q_{-i}, Q_{-i}^f) \begin{cases} > 0 \text{ if } Q_{-i} < Q_{-i}^f \\ = 0 \text{ if } Q_{-i} = Q_{-i}^f \\ < 0 \text{ otherwise} \end{cases}, \tag{6}$$

where $w_i(Q_{-i}, Q_{-i}^f)$ is assumed to be differentiable in both arguments with $\partial w_i / \partial Q_{-i} < 0$ and $\partial w_i / \partial Q_{-i}^f > 0$. The first condition in (6) expresses positive reciprocity. If a player expects her rivals to produce less than her perception of fair output, then she is willing to sacrifice some of her profit to increase the rivals' profits. The third condition in (6) expresses negative reciprocity. If a player expects her rivals to produce more than her perception of fair output, then she is willing to sacrifice some of her profit to reduce the rivals' profits.

We assume that monetary payoffs are large by comparison with fairness payoffs otherwise best replies of reciprocal players in a static Cournot oligopoly might no longer have a negative slope across all quantities. Moreover, we assume analogous assumption: $Q_{-i}^f \leq Q_{-i}^{ns}$ for all i, i.e., too high quantities of the rivals are considered unfair.

Proposition 4: *If $\Gamma^r(u, p)$ and $\Gamma^s(\pi, p)$ satisfy the conditions stated and $Q_{-i}^f \in [Q_{-i}^c, Q_{-i}^{ns}]$ for all i, then the critical (minimum) discount factor needed to sustain collusion at q^c is lower in $\Gamma_\infty^r(u, q)$ than in $\Gamma_\infty^s(\pi, q)$, that is, $\delta_{q^c}^r < \delta_{q^c}^s$.*

Proposition 4 provides conditions under which fairness and reciprocity facilitate collusion when players' choices are strategic substitutes. If reciprocal players think that the fair output of their rivals is greater than the rivals' collusive output, then playing the collusive output is more attractive in the dynamic quantity-setting game with reciprocal players than in the game with self-interested players.

Additionally, the assumption that reciprocal players think that the fair output of their rivals is smaller than the equilibrium output of the rivals in static game with self-interested players implies that the Nash equilibrium of the stage game with reciprocal players is a negative reciprocity state. Hence, the punishment imposed after cheating occurs becomes more severe with reciprocal players than with self-interested players.

In contrast, the single period deviation payoff in the game with reciprocal players is larger than the single period deviation payoff in the game with self-interested players. This happens because the unilateral single period deviation payoff of a reciprocal player also includes the benefit that player derives from being treated kindly by the rivals (the rivals are playing their collusive outputs). However, this effect is of second-order when monetary payoffs are large by comparison with fairness payoffs.

Our last result shows fairness and reciprocity might no longer facilitate collusion when the conditions on players' perceptions of rivals' fair quantities in Proposition 4 are not satisfied.

Proposition 5: *If $\Gamma^r(u,p)$ and $\Gamma^s(\pi,p)$ satisfy the conditions stated and $Q^f_{-i} > Q^c_{-i}$ for all i, then the effect of fairness and reciprocity on the critical (minimum) discount factor needed to sustain collusion at p^c is ambiguous.*

If players think that each rival's fair output is greater than that rival's collusive output, then collusion becomes a negative reciprocity state and this might make collusion harder to sustain.

5 Discussion

Our results hold provided certain conditions are met. For example, we rule out fairness concerns by the part of consumers. This assumption was made on methodological grounds, to better isolate the effect of fairness and reciprocity among firms on collusive outcomes.

We also rule out that firms have fairness considerations with respect to consumers. Contrary to this assumption, Engel (2007) reports that when subjects know that they are playing against human buyers (instead of simulated demand), collusion rates decrease substantially. This might undermine the effects predicted by the model.

We study the impact of fairness and reciprocity on collusion when players use Nash reversion to punish deviations. We have chosen to conduct the main analysis under Nash reversion punishments because simple strategies are more appealing since it is not very realistic that economic agents play complex strategies. However, Abreu's (1988) theory of optimal punishments can be an alternative framework of analysis.

The existence of penal code punishments gives necessary and sufficient conditions for an outcome to be a subgame perfect equilibrium. On the contrary, Nash reversion punishments give only sufficient conditions. This is a problem since sufficient conditions do not prevent the existence of a harsher punishment in the self-interested case, which is not a Nash reversion punishment, such that the target payoff is a subgame perfect equilibrium for a smaller discount factor in the self-interested case than in the reciprocity case.

We show in Appendix B that our main finding is also valid in the optimal punishments framework. The intuition behind this result is as follows. First, the benefit of deviating today (the unilateral single period deviation payoff minus the collusive payoff) when players use optimal punishments is the same as when they use grim trigger punishments. We already know from Proposition 2 that if monetary payoffs are large by comparison with fairness payoffs, then unilateral single period deviation payoff is of second-order. Thus, the benefit of deviating is smaller for reciprocators than for self-interested players no matter if players use optimal punishments or grim trigger punishments.

Second, if reciprocal players think that the fair prices are smaller than the collusive prices, then the prices set on the initial path are perceived as kind behavior by the other players and lead to positive fairness payoffs. Therefore, when the prices of the initial path are set, the payoffs for reciprocal players are higher than those for self-interested players.

Third, it is well known that punishments are more severe when players use optimal punishments than when they use Nash reversion strategies. If reciprocal players think that the fair prices of the rivals are greater than the largest Nash prices of the stage game with self-interested players, then seeing the rivals setting punishment prices lower than Nash prices will be perceived as nastier behavior than seeing the rivals setting Nash prices. Therefore, reciprocal players will set lower prices than self-interested players during the punishment phase under optimal punishments.

The second and the third effects imply that the cost of deviating (the collusive payoff minus the payoff of entering a punishment stage) is larger for reciprocal players than for self-interested players when players use optimal punishments.

Throughout the paper we assume fair price perceptions to be exogenous and constant between periods. This assumption is in the line with Fehr and Falk (1999), who find practically no change in either behavior or perception of fairness over time in a wage-setting context. However, fair price perceptions could be formed endogenously.

There is no common agreement in the literature on how fairness perceptions evolve in time. Kahneman et al. (1986, pp.730-1) write: "Psychological studies of adaptation suggest that any stable state of affairs tends to become accepted eventually, at least in the sense that alternatives to it no longer readily come to mind. Terms of exchange that are initially seen as unfair may in time acquire the status of a reference transaction...[people] adapt their views of fairness to the norms of actual behavior." Franciosi et al. (1995) provide experimental support for these ideas in a price-setting context.

One approach, widely used in the habit formation literature (Ryder and Heal, 1973; Carroll et al., 2000; Fuhrer, 2000), consists in assuming that current prices affect future fair price perceptions as follows:

$$p_{ij}^{f,t} = \alpha p_j^{t-1} + (1-\alpha)\, p_{ij}^{f,t-1}, \text{ for any } i \text{ and } j \neq i, \tag{7}$$

where $\alpha \in (0,1)$. According to (7) player i's fair price perception today is a linear combination of her last period's fair price perception and of the actual price set by player j in that period. In other words, if player j sets a price today higher than what player i considers to be fair, then player i's fair price perception increases in the next period, (i.e., p_{ij}^f does up). On the other hand, if player j sets a price today lower than what player i considers to be fair, then player i's fair price perception decreases in the next period, (i.e., p_{ij}^f goes down). As the game unfolds, player i's fair price perception for any rival j converges to that rival's actual price decision.

We are now ready to investigate whether our predictions affected in any fundamental way by (7). Let us assume that α is not too high, that is, players do not adjust their reference levels too quickly. Letting fair price perceptions be determined by habit formation implies that prices p^{nr} will vary over time. However, for any given initial fair price perceptions we can derive future fair price perceptions, and therefore prices $p^{nr,t}$ at any period t.

If fair prices are below the collusive prices for any player i and for all rivals of player i, then the critical discount factor needed to sustain collusion at p^c in the game with reciprocal players remains below the one in the game with self-interested players along the equilibrium path, which is along the lines of Proposition 2. Intuitively, as the game progresses, $p_{ij}^{f,t}$ converges to p^c for any i and $j \neq i$, and so $\delta_{p^c}^r$ converges to $\delta_{p^c}^s$ since w_{ij} converges to zero. But for an α not too high, fair prices never exceeds the collusive prices, implying that $\delta_{p^c}^r$ never exceeds $\delta_{p^c}^s$. Hence, allowing for fair price perceptions to be determined by habit formation does not affect the qualitative nature of our findings.

6 Conclusion

This paper contributes to the literature on how fairness and reciprocity affect market outcomes. Most of this literature has focused on the impact of fairness concerns by consumers on welfare. Here we take a complementary approach and focus on the role played by fairness concerns of firms on collusive behavior.

Our main departure from the standard model of firm behavior is the assumption that managers in firms are motivated in part by personal animosity–or respect–towards a rival. Hence, firms might punish rivals who behave "unfairly" towards them and reward rivals who behave "fairly."

We provide conditions on players' perceptions of the rivals' fair actions under which reciprocity facilitates collusion. If these conditions do not hold, reciprocity can make collusion harder to sustain. We show that these results are valid no matter if players' choices are strategic complements or substitutes. The results also hold no matter if players use grim trigger punishments or optimal punishments.

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Appendix A

Lemma 0 (Existence): For the stage game with reciprocal players $\Gamma^r(u, p)$, there exist smallest and largest pure-strategy Nash equilibria, $\underline{p}^{nr}$ and $\overline{p}^{nr}$. Similarly, for the stage game with self-interested players $\Gamma^s(\pi, p)$, there exist smallest and largest pure-strategy Nash equilibria $\underline{p}^{ns}$ and $\overline{p}^{ns}$.

Proof of Lemma 0: According to Theorem 4 in Milgrom and Roberts (1990), a game $\Gamma(u, x)$ is supermodular if (i) the choice set is a compact interval in $\mathcal{R}$, (ii) u_i is order upper semi-continuous in x_i for x_{-i} and order continuous in x_{-i} for a fixed x_i, and it has a finite upper bound, (iii) u_i is supermodular in x_i for fixed x_{-i}, and (iv) u_i has increasing differences in (x_i, x_{-i}).
The price stage game with reciprocal players $\Gamma^r(u, p)$ satisfies condition (i) since it is never optimal for players to choose an infinite price for any finite quantity. We have assumed that u_i also satisfies all the requirements of condition (ii). Condition (iii) is satisfied since the choice variables of players are scalars. Condition (iv) is satisfied if for any two aggregate actions of the others p'_{-i}, p''_{-i} with $p'_{-i} \geq p''_{-i}$ (product order) the difference $u_i(p_i, p'_{-i}, P_i^f) - u_i(p_i, p''_{-i}, P_i^f)$ is increasing (or non-decreasing) in p_i, which is assumed as well. Therefore $\Gamma^r(u, p)$ is supermodular game. It then follows from Theorem 5 in Milgrom and Roberts (1990) that (i) there exist smallest and largest serially undominated strategies for each player i, $\underline{p_i}$ and $\overline{p_i}$; and (ii) the strategy profiles $\underline{p} \equiv \left(\underline{p_1}, ..., \underline{p_n}\right)$ and $\overline{p} \equiv (\overline{p_1}, ...\overline{p_n})$ are pure Nash equilibrium profiles. The stage game with self-interested players is straightforward, since the stage game $\Gamma^s(\pi, p)$ is obtained from the stage game $\Gamma^r(u, p)$ by setting $\alpha = 0$, meaning that $\Gamma^s(\pi, p)$ is also a supermodular game. *Q.E.D.*

Proof of Lemma 1: It is an application of Theorem 6 in Milgrom and Roberts (1990) with a slight difference. In their setting, the smallest and largest pure strategy equilibria of the game depends on a scalar, but in our model it depends on a vector. Nevertheless, the proof is immediate since we propose the smallest and largest equilibria is nonincreasing with the fair price perception for any player j, which is a scalar. As a result, if the vector increases in every component, then the smallest and largest equilibria do not increase. *Q.E.D.*

Proof of Proposition 1: Observe that if $p^f_{-i} = p^{ns}_{-i}$ for all i, then $p^{nr} = p^{ns} = p^n$ and $u_i(p^n, p^f_{-i}) = \pi_i(p^n)$ since $w_{ij}(p^n_j, p^f_{ij}) = 0$ for all i and $j \neq i$. (i) If $p^{ns}_{-i} < p^f_{-i}$ for all i, then $p^{nr} \leq p^{ns}$ by Lemma 1. These two inequalities imply $p^{nr}_{-i} < p^f_{-i}$ for all i, which together with (5) imply $w_{ij}(\overline{p}^{nr}_j, p^f_{ij}) < 0$ for all i and $j \neq i$. But then it follows that $u_i(p^{nr}, p^f_{-i}) < \pi_i(p^{ns})$ by the fact that $w_{ij}(p^{nr}_j, p^f_{ij}) < 0$ for all i and $j \neq i$ and $\pi_i(p^{nr}) \leq \pi_i(p^{ns})$ for all i and $j \neq i$. *Q.E.D.*

Lemma 2: *If $p^f_{ij} \leq p^c_j$ for all i and $j \neq i$, then there is a sufficiently high discount factor such that there exists a subgame-perfect Nash equilibrium of*

$\Gamma^r_\infty(u,p)$ *at* p^c.

Proof of Lemma 2: If $p^f_{ij} \leq p^c_j$ for all i and $j \neq i$, then by (5) $w_{ij}(p^c_j, p^f_{ij}) \geq 0$ and $w_{ij}(p^{nr}_j, p^f_{ij}) \leq 0$, for all i and $j \neq i$. This in turn implies that

$$u_i(p^c, p^f_{-i}) \geq \pi_i(p^c). \tag{8}$$

We also know that

$$\pi_i(p^c) > \pi_i(p^{ns}). \tag{9a}$$

If $p^f_{ij} \geq p^{ns}_j$ for all i and $j \neq i$, then we know from Proposition 1 that

$$u_i(p^{nr}, p^f_{-i}) \leq \pi_i(p^{ns}) \tag{10}$$

for all i. From (8), (9a) and (10) we obtain

$$u_i(p^c, p^f_{-i}) > u_i(p^{nr}, p^f_{-i})$$

for all i, which by Friedman (1971) implies that there exists a sufficiently high discount factor such that p^c is a subgame-perfect Nash equilibrium of $\Gamma^r(u,p)$. *Q.E.D.*

Proof of Proposition 2: By Friedman (1971) and Lemma 2, the assumptions made imply that p^c is a subgame-perfect Nash equilibrium of $\Gamma^s(\pi,p)$ and of $\Gamma^r(u,p)$. We want to show that the critical discount factor at which p^c can be sustained using grim trigger punishments in $\Gamma^r_\infty(u,p)$ is lower than the critical discount factor at which p^c can be sustained using grim trigger punishments in $\Gamma^s_\infty(\pi,p)$, that is, $\delta^r_{p^c} < \delta^s_{p^c}$. From (2) and (3) sufficient conditions for $\delta^r_{p^c} < \delta^s_{p^c}$ are

$$u_i(BR^r_i(p^c_{-i}), p^c_{-i}, p^f_{-i}) - u_i(p^c, p^f_{-i}) \leq \pi_i(BR^s_i(p^c_{-i}), p^c_{-i}) - \pi_i(p^c), \tag{11}$$

and

$$u_i(BR^r_i(p^c_{-i}), p^c_{-i}, p^f_{-i}) - u_i(p^{nr}, p^f_{-i}) \geq \pi_i(BR^s_i(p^c_{-i}), p^c_{-i}) - \pi_i(p^{ns}), \tag{12}$$

and at least one inequality holds strictly.

We start by showing that $\pi_i(p_i, p_{-i}) = (p_i - c_i)D_i(p_i, p_{-i})$ and $p^f_{ij} \leq p^c_j$ for all $j \neq i$ imply that (11) is satisfied as a strict inequality. We have that

$$\begin{aligned} u_i(BR^r_i(p^c_{-i}), p^c_{-i}, p^f_{-i}) - u_i(p^c, p^f_{-i}) &= \pi_i(BR^r_i(p^c_{-i}), p^c_{-i}) - \pi_i(p^c) \\ &+ \alpha \sum_{j \neq i} w_{ij}(p^c_j, p^f_{ij})(p^c_j - c_j)[D_j(BR^r_i(p^c_{-i}), p^c_{-i}) - D_j(p^c)] \\ &\leq \pi_i(BR^r_i(p^c_{-i}), p^c_{-i}) - \pi_i(p^c) < \pi_i(BR^s_i(p^c_{-i}), p^c_{-i}) - \pi_i(p^c) \end{aligned}$$

The equality is obtained from (4) and from the assumption $\pi_i(p_i, p_{-i}) = (p_i - c_i)D_i(p_i, p_{-i})$. The weak inequality comes from the assumption that $p^f_{ij} \leq p^c_j$

which implies $w_{ij}(p^c_j, p^f_{ij}) \geq 0$, and the assumption that D_j is increasing with p_i which together with $p^{dr}_i < p^c_i$ imply $D_j(BR^r_i(p^c_{-i}), p^c_{-i}) - D_j(p^c) < 0$. The strict inequality comes from the fact that $BR^s_i(p^c_{-i})$ is the best-reply to p^c_{-i} by a self-interested player.
We now show that if $p^{ns}_j \leq p^f_{ij}$ for all $j \neq i$ and α is sufficiently small, then (12) is satisfied. Rewrite (12) as

$$[u_i(BR^r_i(p^c_{-i}), p^c_{-i}, p^f_{-i}) - \pi_i(BR^s_i(p^c_{-i}), p^c_{-i})] + [\pi_i(p^{ns}) - u_i(p^{nr}, p^f_{-i})] \geq 0.$$

From Proposition 1 we have that

$$\pi_i(p^{ns}) \geq u_i(p^{nr}, p^f_{-i}).$$

If $p^{ns}_j \leq p^f_{ij}$ for all $j \neq i$, then $w_{ij}(p_j, p^f_{ij}) \geq 0$ for all $j \neq i$. Taking a first-order Taylor series expansion of $u_i(BR^r_i(p^c_{-i}), p^c_{-i}, p^f_{-i})$ around $\alpha = 0$ we obtain

$$u_i(BR^r_i(p^c_{-i}), p^c_{-i}, p^f_{-i}) \approx \pi_i(BR^s_i(p^c_{-i}), p^c_{-i})) + \alpha[\sum_{j \neq i} w_{ij}(p_j, p^f_{ij})\pi_j(BR^s_i(p^c_{-i}), p^c_{-i})],$$

which is equivalent to

$$u_i(BR^r_i(p^c_{-i}), p^c_{-i}, p^f_{-i}) - \pi_i(BR^s_i(p^c_{-i}), p^c_{-i}) \approx \alpha[\sum_{j \neq i} w_{ij}(p_j, p^f_{ij})\pi_j(BR^s_i(p^c_{-i}), p^c_{-i})] \geq 0.$$

Thus $\delta^r_{p^c} < \delta^s_{p^c}$. *Q.E.D.*

Proof of Proposition 3: If $p^f_{ij} > p^c_j$ for all i and $j \neq i$, then the following term

$$\alpha \sum_{j \neq i} w_{ij}(p^c_j, p^f_{ij})(p^c_j - c_j)[D_j(BR^r_i(p^c_{-i}), p^c_{-i}) - D_j(p^c)] \quad (13)$$

we used to show (11) in the proof of proposition 2 might become negative and might lead the reverse condition to hold,

$$u_i(BR^r_i(p^c_{-i}), p^c_{-i}, p^f_{-i}) - u_i(p^c, p^f_{-i}) > \pi_i(BR^s_i(p^c_{-i}), p^c_{-i}) - \pi_i(p^c) \quad (14)$$

Therefore, the effect of fairness and reciprocity on the critical (minimum) discount factor needed to sustain collusion at p^c is ambiguous, i.e., either $\delta^r_{p^c} < \delta^s_{p^c}$ or $\delta^r_{p^c} \geq \delta^s_{p^c}$. *Q.E.D.*

Proof of Proposition 4: We need to show that $Q^f_{-i} \geq Q^c_{-i}$ for all i, implies $\delta^r_{q^c} < \delta^s_{q^c}$, where $\delta^r_{q^c}$ is the critical discount factor above which q^c can be sustained in $\Gamma^r_\infty(u, q)$ and $\delta^s_{q^c}$ is the critical discount factor above which q^c can be sustained in $\Gamma^s_\infty(\pi, q)$. From (2) and (3) sufficient conditions are that

$$u_i(BR^r_i(Q^c_{-i}), Q^c_{-i}) - u_i(q^c) \leq \pi_i(BR^s_i(Q^c_{-i}), Q^c_{-i}) - \pi_i(q^c) \quad (15)$$

and

$$u_i(BR_i^r(Q_{-i}^c), Q_{-i}^{cs}) - u_i(q^{nr}) \geq \pi_i(BR_i^s(Q_{-i}^c), Q_{-i}^{cs}) - \pi_i(q^{ns}). \qquad (16)$$

(i) We start by showing that $Q_{-i}^f \geq Q_{-i}^c$ implies (15) is satisfied as a strict inequality. We have that

$$\begin{aligned}
u_i(BR_i^r(Q_{-i}^c), Q_{-i}^c) - u_i(q^c) &= \pi_i(BR_i^r(Q_{-i}^c), Q_{-i}^c) - \pi_i(q^c) \\
&+ \alpha w_i(Q_{-i}^c, Q_{-i}^f)\left[P(BR_i^r(Q_{-i}^c) + Q_{-i}^c) - P(Q^c)\right] Q_{-i}^c \\
&\leq \pi_i(BR_i^r(Q_{-i}^c), Q_{-i}^c) - \pi_i(q^c) < \pi_i(BR_i^s(Q_{-i}^c), Q_{-i}^c) - \pi_i(q^c)
\end{aligned}$$

The strict inequality follows from the fact that $BR_i^s(Q_{-i}^c)$ is the best reply to Q_{-i}^c for self-interested players. If $Q_{-i}^c \leq Q_{-i}^f$ then $w_i(Q_{-i}^c, Q_{-i}^f) \geq 0$. Furthermore, $Q_{-i}^f \leq Q_{-i}^{ns}$ implies $BR_i^r(Q_{-i}^c) > q_i^c$ which in turn implies $P(BR_i^r(Q_{-i}^c)+Q_{-i}^c) < P(Q^c)$, since $P'(\cdot) < 0$.

(ii) We now show that $Q_{-i}^f \leq Q_{-i}^{ns}$ implies that (16) is satisfied. Rewrite (16) as

$$[u_i(BR_i^r(Q_{-i}^c), Q_{-i}^c) - \pi_i(BR_i^s(Q_{-i}^c), Q_{-i}^c)] + [\pi_i(q^{ns}) - u_i(q^{nr})] \geq 0.$$

We have that

$$u_i(q^{nr}) = \pi_i(q^{nr}) + \alpha w_i(Q_{-i}^{nr}, Q_{-i}^f) \sum_{j \neq i} \pi_j(q^{nr}) \leq \pi_i(q^{ns}).$$

The inequality follows from $w_i(Q_{-i}^{nr}, Q_{-i}^f) \leq 0$ and the fact that $Q_{-i}^f \leq Q_{-i}^{ns}$ implies $q_i^{ns} \leq q_i^{nr}$ and $\pi_i(q^{nr}) \leq \pi_i(q^{ns})$, for all i. Taking a first-order Taylor series expansion of $u_i(BR_i^r(Q_{-i}^c), Q_{-i}^{cs})$ around $\alpha = 0$ we have

$$\begin{aligned}
&u_i(BR_i^r(Q_{-i}^c), Q_{-i}^c) \approx \\
&\qquad \pi_i(BR_i^s(Q_{-i}^c), Q_{-i}^c) + \alpha[w_i(Q_{-i}^c, Q_{-i}^f) \sum_{j \neq i} \pi_j(BR_i^s(Q_{-i}^c), Q_{-i}^c)].
\end{aligned}$$

which is equivalent to

$$\begin{aligned}
&u_i(BR_i^r(Q_{-i}^c), Q_{-i}^c) - \pi_i(BR_i^s(Q_{-i}^c), Q_{-i}^c) \approx \\
&\qquad \alpha[w_i(Q_{-i}^c, Q_{-i}^f) \sum_{j \neq i} \pi_j(BR_i^s(Q_{-i}^c), Q_{-i}^c)] \geq 0
\end{aligned}$$

since $Q_{-i}^c \leq Q_{-i}^f$ implies that $w_i(Q_{-i}^c, Q_{-i}^f) \geq 0$. Thus, $Q_{-i}^f \geq Q_{-i}^c$ for all i, implies $\delta_{q^c}^r < \delta_{q^c}^s$. *Q.E.D.*

Proof of Proposition 5: If $Q_{-i}^f < Q_{-i}^c$ for all i, then the following term

$$\alpha w_i(Q_{-i}^c, Q_{-i}^f)\left[P(BR_i^r(Q_{-i}^c) + Q_{-i}^c) - P(Q^c)\right] Q_{-i}^c \qquad (17)$$

we used to show (11) in the proof of proposition 2 might become negative, and might lead the reverse condition to hold,

$$u_i(BR_i^r(Q_{-i}^c), Q_{-i}^c) - u_i(q^c) > \pi_i(BR_i^s(Q_{-i}^c), Q_{-i}^c) - \pi_i(q^c) \qquad (18)$$

Therefore, the effect of fairness and reciprocity on the critical (minimum) discount factor needed to sustain collusion at Q^c is ambiguous, i.e., either $\delta^r_{p^c} < \delta^s_{p^c}$ or $\delta^r_{p^c} \geq \delta^s_{p^c}$. *Q.E.D.*

Appendix B

Abreu (1988) introduces a rule which consists of an initial path (that is an infinite stream of one period action profiles) and punishments (that are also infinite streams for any deviation from the initial path or from a prescribed punishment). He introduces the notion of *simple* strategy profile in which a specific punishment takes place after any deviation for each particular player. Thus, the simple strategy profiles have a description of $(n+1)$ paths for an n-player game. On the other hand, an arbitrary strategy profile may consist of infinite amount of punishments and depends on complex history-dependent formulas.

We begin by introducing additional notations and definitions, after we show an optimal simple penal code exists. Finally, we state conditions under which it is easier to sustain collusion with reciprocal players than with self-interested ones under optimal punishments.

A pure strategy of player i is denoted σ_i. Each σ_i is a sequence of functions, $\sigma_i(1), \sigma_i(2), ..., \sigma_i(t), ...$, one for each t. The function for all periods t determines player i's action at t as a function of the actions of all players in previous periods. Formally, at $t = 1, \sigma_i(1) \in P_i$ and for $t = 2, 3, ...,$ $\sigma_i(t) : P^{t-1} \to P_i$. Player i's strategy set is denoted Σ_i, and the set of strategy profiles is denoted $\Sigma \equiv \Sigma_1 \times \Sigma_2 \times ... \times \Sigma_n$.

A path (or punishment), $\widetilde{P}$, is a stream of action profiles $\{p(t)\}_{t=1}^{\infty}$ and let $\Omega \equiv P^{\infty}$ be the set of punishments. Any strategy profile $\sigma \in \Sigma$ generates a path denoted $\widetilde{P}(\sigma) = \{p(\sigma)(t)\}_{t=1}^{\infty}$, and it is defined as follows:

$$p(\sigma)(1) = \sigma(1) \text{ and}$$
$$p(\sigma)(t) = \sigma(t)(p(\sigma)(1), ..., (p(\sigma)(t)).$$

Player i's payoff from path $\widetilde{P} \in \Omega$ is given by $v_i^x : \Omega \to \mathcal{R}$ for $x = \{r, s\}$ such that

$$v_i^x(\widetilde{P}) = \begin{cases} \sum_{t=1}^{\infty} \delta^t u_i(p(t)) \text{ if } x = r \\ \sum_{t=1}^{\infty} \delta^t \pi_i(p(t)) \text{ if } x = s \end{cases}$$

where u_i is given by (4) and (5). Player i's payoff function is given by $\widetilde{v}_i^x : \Sigma \to \mathcal{R}$ such that $\widetilde{v}_i^x(\sigma) = v_i(\widetilde{P}(\sigma))$.

Abreu (1988) introduces the simple strategy profile, which is defined by $(n+1)$-vector of paths $(\widetilde{P}^0, \widetilde{P}^1, ..., \widetilde{P}^n)$ and a rule. The initial path is $\widetilde{P}^0$, and for each player $i \in \{1, ..., n\}$, $\widetilde{P}^i$ is the punishment for player i. Any unilateral deviation of player i from the ongoing path is responded by imposing $\widetilde{P}^i$. If more than one player deviate, the ongoing path continues to be followed and deviators will not be punished. Formally:

Let $\widetilde{P}^i \in \Omega$, $i = 0, 1, ..., n$. The *simple strategy profile* $\sigma(\widetilde{P}^0, \widetilde{P}^1, ..., \widetilde{P}^n)$ specifies: (i) play $\widetilde{P}^0$ until some player deviates unilaterally from $\widetilde{P}^0$; (ii) for any $j \in \{1, ..., n\}$, play $\widetilde{P}^j$ if the jth player deviates unilaterally from $\widetilde{P}^i$, $i = 0, 1, ..., n$, where $\widetilde{P}^i$ is an ongoing previously specified path; continue with $\widetilde{P}^i$ if no deviations occur or if two or more players deviate simultaneously.

A simple strategy $\sigma(\widetilde{P}^0, \widetilde{P}^1, ..., \widetilde{P}^n)$ profile is *perfect* if and only if no one-shot deviation by any player $j \in \{1, ..., n\}$ from $\widetilde{P}^i$, $i = 0, 1, ..., n$, yields player j a higher payoff, when all players conform with $\widetilde{P}^j$ after the deviation.[1] Let Σ^p denote the set of perfect equilibrium strategy profiles of $\Gamma_\infty(\delta)$. The perfect equilibrium paths $\Omega^p = \{\widetilde{P}(\sigma) | \sigma \in \Sigma^p\}$, and payoffs $V = \{v(\widetilde{P}) | \widetilde{P} \in \Omega^p\}$.

We introduce three more definitions from Abreu (1988) before stating the existence result. An *optimal penal code* is an n-vector of the strategy profiles $\{\underline{\sigma}^1, ..., \underline{\sigma}^n\}$ such that for all i,

$$\underline{\sigma}^i \in \Sigma^p \text{ and } \widetilde{v}_i(\underline{\sigma}^i) = \min\{\widetilde{v}_i(\sigma) | \sigma \in \Sigma^p\}.$$

Let $\sigma^i(\widetilde{P}^1, ..., \widetilde{P}^n) = \sigma(\widetilde{P}^i, \widetilde{P}^1, ..., \widetilde{P}^n)$. The *simple penal code* $(\widetilde{P}^1, ..., \widetilde{P}^n)$ is the n-vector of the strategy profiles $\sigma^1(\widetilde{P}^1, ..., \widetilde{P}^n), ..., \sigma^n(\widetilde{P}^1, ..., \widetilde{P}^n)$. Finally, a simple penal code $(\widetilde{P}^1, ..., \widetilde{P}^n)$ is an *optimal simple penal code* if it is an optimal penal code.

Lemma 3: *If Σ^p is non-empty, P is a compact topological space and given p^f, $u : P \times p^f \to R^n$ is continuous, then an optimal simple penal code exists.*

Proof of Lemma 3: The lemma follows from Abreu (1988) under the assumptions of $u(.)$. *Q.E.D.*

Similarly, an optimal simple penal code exists for a continuous payoff function $\pi : P \to \mathcal{R}^n$. Let present discounted value of player i's payoffs from the period $t + 1$ to ∞ along the path $\widetilde{P}$ be

$$v_i^x(\widetilde{P}; t+1) = \begin{cases} \sum_{k=1}^{\infty} \delta^k u_i(p(t+k)) \text{ if } x = r \\ \sum_{k=1}^{\infty} \delta^k \pi_i(p(t+k)) \text{ if } x = s \end{cases},$$

and player i's payoff under her optimal penal code, $\underline{v}_i^x = \widetilde{v}_i^x(\underline{\sigma}^i)$. The following result indicates the use of optimal penal code to characterize the set of perfect equilibrium paths.

Lemma 4: If an optimal penal code exists, then $\widetilde{P}^0 \in \Omega^p$ if and only if

$$u_i(p_i^{dr}, p_{-i}^0(t)) - u_i(p^0) \leq v_i^r(\widetilde{P}^0; t+1) - \underline{v}_i^r \quad (19)$$

$$\pi_i(p_i^{ds}, p_{-i}^0(t)) - \pi_i(p^0) \leq v_i^s(\widetilde{P}^0; t+1) - \underline{v}_i^s \quad (20)$$

Proof of Lemma 4: The lemma follows from Abreu (1988). *Q.E.D.*

[1] This relation holds if the set of payoffs of the stage game is bounded (i.e. $\{u(p) | p \in P\}$ is bounded).

The left-hand-side of inequalities (19) and (20) are the benefit of deviating today for reciprocators and self interested players, respectively. The right-hand-side is the cost of deviating. Observe that the prices in each period of the initial path can be considered as any collusive prices.

Since the existence of optimal simple penal code is guaranteed under the given assumptions, our final result shows that fairness and reciprocity facilitate collusion when players use optimal simple penal codes.

Proposition 6: *Assume (i) u_i has decreasing differences in (p_i, p^f_{-i}), for all i, (ii) $\pi_i(p_i, p_{-i}) = (p_i - c_i)D_i(p_i, p_{-i})$, and (iii) $p^f_{ij} \in [\overline{p}^{ns}_j, p^c_j]$ for all i and $j \neq i$. Let p^0 satisfy $\pi_i(p^o) > \pi_i(\overline{p}^{ns})$ for all i. If an optimal simple penal code exist, then the critical (minimum) discount level to sustain collusion at $\widetilde{P}^0$ is lower in the game with reciprocal players $\Gamma^r_\infty(n, u, p, P^f)$ than in the game with self-interested players $\Gamma^s_\infty(n, \pi, p)$, that is $\delta^r_{p^0} < \delta^s_{p^0}$.*

Proof of Proposition 6: The minimum critical discount factor will be obtained if the inequality (19) and (20) hold with equality respectively for reciprocators and self-interested players, otherwise the discount factor can be decreased by a small amount without violating the inequality. In Proposition 3, we proved the LHS of the equations being smaller for reciprocators, hence a smaller discount level is possible for the reciprocators. In addition, the following condition is immediate

$$v^r_i(\widetilde{P}^0; t+1) \geq v^s_i(\widetilde{P}^0; t+1)$$

considering the initial path where each player i sets at least the collusive price p^c_i for each stage, until one deviates. Hence for any fair price perception $p^f_{ij} \in [\overline{p}^{ns}_j, p^c_j]$ for all i and $j \neq i$, the prices set at the initial path will be perceived as kind behavior, thus the condition holds. Note that, if the prices set at the initial path are equal to collusive prices p^c_i for each player i and $p^f_{ij} = p^c_j$ for all i and $i \neq i$, then the condition holds with equality. Finally, to complete the proof we need to compare the payoff of any player i in the optimal penal code $\underline{v}^x_i$. In the optimal penal code, the players punish the deviated player i via playing a pure strategy profile $\underline{\sigma}^i \in \Sigma^p$, which gives the lowest possible payoff to player i. Let ${}^{nx}\underline{\sigma}$ denote the strategy profile where in each stage players set Nash prices. Since ${}^{nx}\underline{\sigma} \in \Sigma^p$, in each stage the optimal penal code for player i, $\underline{v}^x_i$, is at least as severe as ${}^{nx}\underline{\sigma}$, which means that the optimal punishment of player j in the reciprocity case, $\underline{p}_j$, satisfies $\underline{p}_j \leq p^{ns}_j$. Note that, if the prices set in the penal code are such that $\underline{p}_j = p^{ns}_j = p^f_{ji}$ for all j and $i \neq j$, then the payoffs from the penal code are equal for reciprocal and self-interested players, that is, $\underline{v}^r_i = \underline{v}^s_i$. Otherwise, the reciprocal players perceive the unkind behavior of their rivals and negative reciprocity implies the payoff under the optimal penal code is harsher for reciprocal players than self-interested players, that is $\underline{v}^r_i < \underline{v}^s_i$. Hence $\delta^r_{p^0} < \delta^s_{p^0}$. *Q.E.D.*