Tacit Collusion under Fairness and Reciprocity*

Doruk İriş and Luís Santos-Pinto†

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Abstract

This paper studies the impact of fairness and reciprocity on cooperation in infinitely repeated games. A reciprocal player responds to unkind behavior of rivals with unkind actions (negative reciprocity), while at the same time, it responds to kind behavior of rivals with kind actions (positive reciprocity). The paper shows that when players are reciprocal, collusive action profiles (prices or quantities) are easier to sustain. Thus, fairness concerns among producers with reciprocal preferences who interact repeatedly can have adverse welfare consequences for consumers.

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†Doruk İriş is a graduate student at Universidade Nova de Lisboa. Luís Santos-Pinto is a professor at University of Lausanne. Corresponding author: Luís Santos-Pinto, University of Lausanne, Faculty of Business and Economics, Internef 535, CH-1015, Lausanne, Switzerland. Ph: 41-216923658. E-mail: LuisPedro.SantosPinto@unil.ch.

"*If they see me planting too much cocoa, they'll do things to my land and my family, and they won't bear fruit; really bad things; puripuri and other witchcraft.*"

1 Introduction

The quote is taken from a farmer in Papua New Guinea when he was explaining to Keir Martin, a social anthropologist, why he had only cultivated half of his three-hectare block and why, like him, none of his fellow villagers planted the whole of their blocks of land. According to Martin (2009): "Such an avoidance of profit maximization might appear economically irrational. But from the perspective of those villagers, putting that extra work just to make oneself target for jealousy of one's neighbors would be highly irrational behavior."

The assumption that individuals behave as if maximizing their material payoffs, despite its central role in economic analysis, is at odds with a large body of evidence from psychology and from experimental economics. Economic agents often pursue objectives other than actual payoff maximization. Many observed departures from material payoff maximizing behavior arise through actions that favor fairness or reciprocity.

Rabin (1993) argues that the parties of a transaction care about fairness in the sense that they "like to help those who are helping them, and hurt those who are hurting them" (pp. 1281). Fairness and reciprocity have been shown to explain behavior in bargaining games and in trust games. For example, in ultimatum games offers are usually much more generous than predicted by equilibrium and low offers are often rejected. These offers are consistent with an equilibrium in which players make offers knowing that other players may reject allocations that appear unfair.[1]

Motivated by this evidence, we ask: "can fairness and reciprocity facilitate cooperation?" Since this is a very broad question we focus on infinitely repeated games. This important class of games tells us how cooperative outcomes can be sustained when players interact repeatedly. This literature also tell us what are the factors that help or hinder collusion. For example, it is now well known that concentration, barriers to entry, cross-ownership, symmetry and multi-market contracts facilitate collusion–see Feuerstein (2005).

To model reciprocal preferences we follow Segal and Sobel's (2007) and assume that players in a strategic environment have preferences not only over the outcomes but also the strategies. A player's utility is additively separable in monetary and fairness payoffs. Monetary payoffs are revenues minus costs and fairness payoffs are a weighted average of the rivals' monetary payoffs where the weights depend on how the rivals' choices are expected to differ from the fair ones. If a player expects a rival to play a kind (mean) strategy, then he places a positive (negative) weight on that rival's monetary payoff. If a player expects a

[1] Sobel (2005) argues that models of interdependent preferences such as reciprocity can provide clearer and more intuitive explanations of interesting economic phenomena.

rival to play a fair strategy then he places zero weight on that rivals' monetary payoff.

We start by showing that the impact of fairness and reciprocity on static price competition is ambiguous, that is, it can lead to more or less cooperative outcomes. The intuition as follows. If reciprocal players think that the fair prices of the rivals are at most the smallest equilibrium prices of the rivals in the game with self-interested players, then the equilibrium attained is a positive reciprocity state. In this case equilibrium prices are higher than those self-interested players would set since reciprocators want to reward the rivals for having set prices higher than the fair ones. In contrast, if reciprocal players believe that the fair prices of the rivals are at least the largest equilibrium prices of the rivals in the game with self-interested players, then the equilibrium attained is a negative reciprocity state. In this case equilibrium prices are lower than those self-interested players would set since reciprocators want to punish the rivals for having set prices lower than the fair ones.

The main result of the paper shows that, under a reasonable assumption on the concept of fair prices, collusive prices are easier to sustain when players are reciprocal. This happens because (i) the incentive to deviate from the collusive scheme is less when players are reciprocal and/or (ii) the possible punishment phases that can be sustained are harsher. The intuition as follows.

If players think that the fair prices of the rivals are less than the collusive prices, then collusion becomes a positive reciprocity state. In this case players' monetary payoffs from collusion are the same as the ones obtained in the game with self-interested players but in addition there are fairness payoff gains since players think that their rivals are being kind. This effect makes collusion *more* attractive to reciprocal players. Additionally, if players think that the fair prices of the rivals are at least the largest equilibrium prices of the static game with self-interested players, then Nash reversion becomes a negative reciprocity state. This implies that the punishment imposed after cheating occurs is more severe when players are reciprocal. This effect also makes collusion *more* attractive to reciprocal players. However, the unilateral single period deviation payoff is higher with reciprocal players because it includes the benefit a player derives from being treated kindly by the rivals (the rivals are playing their collusive prices). This effect makes collusion *less* attractive to reciprocal players. The assumption that monetary payoffs are large by comparison with fairness payoffs implies that the increase in collusive payoff is of first-order whereas the increase in the unilateral single period deviation payoff is of second-order.

Finally, we show that our main result also holds under quantity competition and when players use optimal punishments rather than grim trigger punishments.

The main policy implication of this paper is that fairness concerns among producers with reciprocal preferences who interact repeatedly can have adverse welfare consequences for consumers. Our work stands in contrast with findings in Rabin (1993) and Rotemberg (2006) which show that fairness concerns by the part of consumers can increase consumer welfare.[2] Thus, social preferences in imperfectly competitive markets might lead to different outcomes depending on who has such preferences (producers or consumers) and what is the comparison group.

The rest of the paper proceeds as follows. Section 2 sets-up the model. Section 3 analyzes the impact that fairness and reciprocity have on incentives for collusion when players use grim trigger strategies. Section 4 discusses the results. Section 5 concludes the paper. Appendix A contains the proofs of all results in the main text. Appendix B states and proves results when players use optimal punishments.

2 Set-up

The existing theories of social preferences can be classified into three broad categories. The first one is the distributional preference approach where social preferences only depend on the distribution of material payoffs. This includes Fehr and Schmidt (1999) and Bolton and Ockenfels (2000). These models are highly tractable and capture a wide range of phenomena but fail to explain the fact that preferences depend on more than outcomes, namely, intentions also matter.

The second category consists of intention-based models and includes Rabin (1993), Dufwenberg and Kirchsteiger (2004), and Falk and Fischbacher (2006), among others. These models assume that reciprocity depends on overall strategies and beliefs (and beliefs about beliefs) building on Geanakoplos et al. (1989) theory of psychological games. In Rabin (1993) utility is additively separable in monetary and fairness payoffs and the weight a player places on rivals' monetary payoffs depends on his perception of the rivals' intentions, which are evaluated using (i) beliefs about the rivals' strategy choices, and (ii) beliefs about the rivals' beliefs about his strategy. Dufwenberg and Kirchsteiger (2004) develop a theory of reciprocity for extensive form games where players update beliefs about intentions as the game unfolds and make a choice accordingly. Falk and Fischbacher (2006) model reciprocity in incomplete information games. Intention-based models have two major weaknesses: they use specific functional forms and are highly intractable.[3]

The third category explores the axiomatic foundations that generate utility functions that display social preferences. Nielson (2006) proposes a preference axiom which leads to a foundation of Fehr and Schmidt (1999) inequity aversion model. Segal and Sobel (2007) provide an axiomatic foundation for interdependent preferences that can reflect reciprocity, inequity aversion, altruism as well as spitefulness. The key innovation of their approach is that, in addition to conventional preferences over outcomes, players in a strategic environment also

[2] For example, Rabin (1993) shows that a monopolist ought to set price lower than "the monopoly price" if consumers have concerns about fairness.

[3] Sobel (2005) points out some of the drawbacks of the distributional-preferences and intention-based approaches to reciprocity.

have preferences over strategy profiles. Assuming that the players have preferences over strategies allows one to study situations where a player's preference is affected by the behavior of other players.

Their representation theorem shows that the payoff function of a player with such preferences is of the form

$$V_i(\sigma_i, \sigma_{-i}^*) = u_i(\sigma_i, \sigma_{-i}^*) + \sum_{j \neq i} w_{ij}(\sigma^*) u_j(\sigma_i, \sigma_{-i}^*), \quad (1)$$

where σ_i is the strategy of player i, σ^* is how the game is expected to be played, u_i is the utility from outcomes of player i, u_j is the utility from outcomes of player $j \neq i$, and $w_{ij}(\sigma^*)$ is a coefficient that measures the weight player i gives to player j's utility, which is a function of the entire strategy profile. Positive values of the coefficient mean that player i is willing to sacrifice his utility from outcomes in order to increase the payoff of player j. Negative values mean that player i is willing to sacrifice his utility from outcomes in order to lower player j's payoff. Since the coefficient depends on the strategy chosen by player j, there is scope to model reciprocity.[4]

We apply Segal and Sobel's (2007) approach to a dynamic game where n players, $n > 2$, play the same stage game over an infinite horizon $t = 0, 1, 2, \ldots$. The repeated game monetary payoff of player i of choosing strategy $s_i = (a_i^1, a_i^2, ...)$ when rivals play strategies s_{-i} is given by

$$\Pi_i(s_i, s_{-i}) = \sum_{t=1}^{\infty} \delta^{t-1} \pi_i(a_i^t, a_{-i}^t),$$

where $\pi_i(a_i^t, a_{-i}^t)$ represents player i's monetary payoff at stage t, a function of player i's action at t, a_i^t, and the actions of the rivals at t, a_{-i}^t. Players discount the future at rate $\delta \in (0, 1)$. To model reciprocity we assume that the weight player i places on player j's repeated game monetary payoff depends only on player j's strategy and on player i's perception of what is the fair strategy of player j, s_{ij}^f. We also assume throughout that players' preferences as well as their exogenous perceptions of the fair strategies of the rivals are common knowledge. The repeated game payoff of reciprocal player i of choosing strategy $s_i = (a_i^1, a_i^2, ...)$ when rivals play strategies s_{-i} is given by

$$U_i(s_i, s_{-i}, s_{-i}^f) = \sum_{t=1}^{\infty} \delta^{t-1} \pi_i(a_i^t, a_{-i}^t) + \alpha \sum_{j \neq i} \sum_{t=1}^{\infty} \delta^{t-1} w_{ij}(a_j^t, a_{ij}^f) \pi_j(a_i^t, a_{-i}^t)$$

where $\alpha > 0$ is a normalization. The central behavioral feature of these preferences is the assumption that players care about the intentions of the rivals. If player i expects player j to treat him kindly, then w_{ij} will be positive, and player i will wish to treat player j kindly. If player i expects player j to treat

[4] The underlying preferences in (1) are defined over outcomes. If an outcome specifies a material payoff to each player, then it is permissible for u_i to depend on other players' material payoffs. Thus, this approach also generalizes the inequity aversion approach.

him badly, then w_{ij} will be negative, and player i will wish to treat player j badly. If player i expects player j to be fair, then w_{ij} will be zero, and there is no issue of reciprocity.

Denote the dynamic game with reciprocal players by $\Gamma^r_\infty(u,s)$, where $u = (u_1, ..., u_n)$ and $s = (s_1, ..., s_n)$ and the dynamic game with self-interested players by $\Gamma^s_\infty(\pi, s)$, where $\pi = (\pi_1, ..., \pi_n)$. Players are able to sustain a collusive outcome when the payoff from collusion is no less than the payoff from deviation. To understand how fairness and reciprocity influence collusion we will compare the incentive compatibility condition of self-interested players in $\Gamma^s_\infty(\pi, x)$ to that of reciprocal players in $\Gamma^r_\infty(u, x)$ assuming that these two games are identical in all respects (monetary payoffs and the number of players) with the exception of players' preferences.

To perform this analysis we focus on infinitely repeated games. More specifically, we consider the cases where players' actions are strategic complements (e.g., price competition with products that are imperfect substitutes) and strategic substitutes (e.g., quantity competition with products that are perfect substitutes). We also consider two alternative modes of punishments after deviations: grim trigger and optimal punishments**.**

3 Grim Trigger Punishments

The standard model used to study collusion in infinitely repeated games assumes that players use grim trigger strategies to punish any deviation from collusion, that is, following a deviation players switch to a Nash equilibrium of the stage game forever after. Thus, when self-interested player uses grim trigger punishments in $\Gamma^s_\infty(\pi, x)$, each player i will prefer to play his collusive strategy $s^c_i = (a^c_i, a^c_i, ...)$ if the payoff from collusion, $\pi_i(a^c)/(1-\delta)$, is no less than the payoff from defection which consists of the one period gain from deviating $\pi_i(BR^s_i(a^c_{-i}), a^c_{-i})$ plus the discounted payoff of inducing Nash reversion forever $\delta\pi_i(a^{ns})/(1-\delta)$, that is,

$$\pi_i(BR^s_i(a^c_{-i}), a^c_{-i}) + \frac{\delta}{1-\delta}\pi_i(a^{ns}) \leq \frac{1}{1-\delta}\pi_i(a^c).$$

Solving for δ we obtain

$$\delta^s_{a^c} = \frac{\pi_i(BR^s_i(a^c_{-i}), a^c_{-i}) - \pi_i(a^c)}{\pi_i(BR^s_i(a^c_{-i}), a^c_{-i}) - \pi_i(a^{ns})} \leq \delta. \tag{2}$$

The collusion strategy profile s^c can be sustained by self-interested players who are patient enough such that $\delta^s_{a^c} \leq \delta$ where $\delta^s_{a^c}$ is the critical discount factor above which s^c can be sustained by self-interested players.

The same reasoning applies when players have reciprocal preferences. A reciprocal player i plays the collusive strategy s^c_i in $\Gamma^r_\infty(u, x)$ using a grim trigger strategy as long as the following condition holds

$$u_i(BR^r_i(a^c_{-i}), a^c_{-i}, a^f_{-i}) + \frac{\delta}{1-\delta}u_i(a^{nr}, a^f_{-i}) \leq \frac{1}{1-\delta}u_i(a^c, a^f_{-i}),$$

where u_i denotes the stage game payoff of a reciprocal player, a function of the actions played and perceptions of the fair actions of the rivals. Solving for δ we obtain

$$\delta^r_{a^c} = \frac{u_i(BR^r_i(a^c_{-i}), a^c_{-i}, a^f_{-i}) - u_i(a^c, a^f_{-i})}{u_i(BR^r_i(a^c_{-i}), a^c_{-i}, a^f_{-i}) - u_i(a^{nr}, a^f_{-i})} \leq \delta. \tag{3}$$

When players have reciprocal preferences it follows that the collusive strategy profile s^c can be sustained if players are patient enough such that $\delta^r_{a^c} \leq \delta$ where $\delta^r_{a^c}$ is the critical discount factor above which s^c can be sustained by reciprocal players.

We will use (2) and (3) to characterize the impact that fairness and reciprocity have on collusion when players use grim trigger strategies. To perform this analysis we compare the critical discount factor above which the collusive strategy profile can be sustained when players are self-interested to the critical discount factor when players are reciprocal. We say that fairness and reciprocity facilitate collusion when the collusive strategy profile can be sustained at a lower critical discount factor when players are reciprocal than when they are self-interested. If the opposite happens we say that fairness and reciprocity make collusion harder.

3.1 Strategic Complements

We now specialize the model by assuming that players' actions are strategic complements. This assumption means that a player's incremental returns from increasing his own action are increasing in the rivals' actions. The canonical market game where players' actions are strategic complements is price competition with imperfect substitutes. We use this game to study the impact of fairness and reciprocity on collusion when players' actions are strategic complements.

In each stage player i chooses price, p_i, and his payoff in that stage is given by

$$u_i(p_i, p_{-i}, p^f_{-i}) = \pi_i(p_i, p_{-i}) + \alpha \sum_{j \neq i} w_{ij}(p_j, p^f_{ij}) \pi_j(p_i, p_{-i}), \tag{4}$$

where $\pi_i(p_i, p_{-i})$ is the monetary payoff and $\alpha \sum_{j \neq i} w_{ij}(p_j, p^f_{ij}) \pi_j(p_i, p_{-i})$ the fairness payoff. The monetary payoff is the difference between revenue and cost, that is,

$$\begin{aligned} \pi_i(p_i, p_{-i}) &= R_i(p_i, p_{-i}) - C_i(D_i(p_i, p_{-i})) \\ &= p_i D_i(p_i, p_{-i}) - C_i(D_i(p_i, p_{-i})), \end{aligned}$$

where $R_i(p_i, p_{-i})$ is revenue, $C_i(D_i(\cdot))$ is the cost of production, and $D_i(p_i, p_{-i})$ is the demand faced by player i. We assume that $D_i(\cdot)$ is decreasing with p_i, increasing with p_{-i}, and $C_i(\cdot)$ is increasing with $D_i(\cdot)$. Furthermore, we assume that

$$w_{ij}(p_j, p^f_{ij}) \begin{cases} > 0 \text{ if } p_j > p^f_{ij} \\ = 0 \text{ if } p_j = p^f \\ < 0 \text{ otherwise} \end{cases}. \tag{5}$$

The assumptions on $w_{ij}(p_j, p_{ij}^f)$ capture the fact that a reciprocal player cares about the intentions of the rivals. The first condition expresses positive or constructive reciprocity. If a player expects one of her rivals to charge a price higher than the fair price, then she puts a positive weight on that rival's profit and she is willing to sacrifice some of her profit to increase that rival's profit. The second condition says that if a player expects one of her rivals to choose the fair price, then she places no weight on that rival's profit. The third condition expresses negative or destructive reciprocity. If player a expects one of her rivals to undercut her perception of fair price, then she puts a negative weight on that rival's profit and she is willing to sacrifice some of her profit to reduce that rival's profit.

Let

$$A_i(p_i, p_{-i}, p_{-i}^f) = \underset{p_i \in P_i}{\arg\max}\ \pi_i(p_i, p_{-i}) + \alpha \sum_{j \neq i} w_{ij}(p_j, p_{ij}^f)\pi_j(p_i, p_{-i}),$$

denote the set of maximizers of player i's stage game problem as a function of p_i, p_{-i} and p_{-i}^f. For finite quantities, the players will never choose an infinite price. Hence, the players' price choice set is compact set in $\mathcal{R}$. We assume that u_i is order upper semi-continuous in p_i. The choice set being compact with this assumption guarantees that the set of maximizers $A_i(p_i, p_{-i}, p_{-i}^f)$ is nonempty.

We also assume that u_i has increasing differences in (p_i, p_{-i}), that is, for any fixed p_{-i}^f, $u_i(p_i, p'_{-i}, p_{-i}^f) - u_i(p_i, p''_{-i}, p_{-i}^f)$ is increasing in p_i for all $p'_{-i} \geq p''_{-i}$. This assumption guarantees that prices are strategic complements.[5] Together, these assumptions imply that $\Gamma^r(u, p)$ is a supermodular game.[6] By Milgrom and Roberts (1990) we know that if $\Gamma^r(u, p)$ is a supermodular game, then there exist largest and smallest serially undominated strategies for each player, $\overline{p}_i$ and $\underline{p_i}$. Moreover, the strategy profiles $\underline{p}$ and $\overline{p}$ are pure-strategy Nash equilibrium profiles. Thus, the existence of a Nash equilibrium of the stage game is guaranteed. Let us further assume that u_i has decreasing differences in (p_i, p_{-i}^f). The following result shows how players' perceptions of the fair prices of the rivals influence the extremal equilibrium prices of this game.

Lemma 1: *The smallest and the largest pure-strategy Nash equilibria of* $\Gamma^r(u, p)$*, i.e.,* $\underline{p}^{nr}$ *and* $\overline{p}^{nr}$*, are nonincreasing functions of* $p^f = (p_{-1}^f, ..., p_{-n}^f)$*.*

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[5] If the payoff function is differentiable, then u_i having increasing differences in (p_i, p_{-i}), is equivalent to the assumption that the cross partial derivatives of u_i with respect to p_i and p_j for any player j, is non-negative, that is,

$$\frac{\partial^2 u_i}{\partial^2 p_i p_j} = \underbrace{\frac{\partial^2 \pi_i}{\partial^2 p_i p_j}}_{\geq 0} + \alpha w_{ij}(p_j, p_{ij}^f) \underbrace{\frac{\partial^2 \pi_j}{\partial^2 p_i p_j}}_{\geq 0} + \alpha \frac{\partial w_{ij}(p_j, p_{ij}^f)}{\partial p_j} \frac{\partial \pi_j}{\partial p_i} \geq 0.$$

Note that if a player cares only about monetary payoffs and if the payoff function is differentiable, then the increasing differences assumption boils down to $\frac{\partial^2 \pi_i}{\partial^2 p_i p_j} > 0$. In the game with reciprocal players and differentiable payoff functions, the assumption will be satisfied if $\frac{\partial^2 \pi_i}{\partial^2 p_i p_j} > 0$ and α is sufficiently small.

[6] The result, Lemma A, is stated and proved formally in the appendix.

Lemma 1 is a comparative statics result that characterizes the impact that players' perceptions of the fair prices of their rivals have on the Nash equilibrium prices of the stage game. This result says that the higher are players' perceptions of what the fair prices of the rivals should be, the lower will the equilibrium prices be. The critical condition that drives this result is the assumption that the payoff function u_i has decreasing differences in (p_i, p^f_{-i}).[7] This assumption says that the marginal returns from increasing prices are decreasing with a player's perception of the fair prices of the rivals. This implies that an increase in p^f_{-i} shifts the best reply of a reciprocal player i towards origin. In other words, the higher player i perceives the fair price for the other players to be, the more it would like to set a smaller price for any price of the other players.

Our first result shows how preferences for fairness and reciprocity change the outcome of static price competition. To do that we compare the Nash equilibria of the stage game with self-interested players to that of the stage game with reciprocal players. The findings are summarized in Proposition 1.

Proposition 1:
(i) If reciprocal players believe that the fair prices of the rivals are at least the largest equilibrium prices of the rivals in $\Gamma^s(\pi, p)$, that is, $p^f_{-i} \geq \overline{p}^{ns}_{-i}$, for all i, then (a) $\overline{p}^{nr} \leq \overline{p}^{ns}$, and $u_i(\overline{p}^{nr}, p^f_{-i}) \leq \pi_i(\overline{p}^{ns})$, and (b) $\underline{p}^{nr} \leq \underline{p}^{ns}$ and $u_i(\underline{p}^{nr}, p^f_{-i}) \leq \pi_i(\underline{p}^{ns})$.
(ii) If reciprocal players think that the fair prices of the rivals are at most the smallest equilibrium prices of the rivals in $\Gamma^s(\pi, p)$, that is, $p^f_{-i} \leq \underline{p}^{ns}_{-i}$, for all i, then (c) $\overline{p}^{nr} \geq \overline{p}^{ns}$, and $u_i(\overline{p}^{nr}, p^f_{-i}) \geq \pi_i(\overline{p}^{ns})$, and (d) $\underline{p}^{nr} \geq \underline{p}^{ns}$ and $u_i(\underline{p}^{nr}, p^f_{-i}) \geq \pi_i(\underline{p}^{ns})$.

This result tells us that the impact of fairness and reciprocity on static price competition is ambiguous, that is, it can lead to more or less cooperative outcomes. Part (i) tells us that if reciprocal players believe that the fair prices of the rivals are at least the largest equilibrium prices of the rivals in the game with self-interested players, then prices set by reciprocators will be lower than those set by self-interested players. In contrast, part (ii) tells us that if reciprocal players think that the fair prices of the rivals are at most the smallest equilibrium prices of the rivals in the game with self-interested players, then prices set by reciprocators will be higher than those set by self-interested players.

The intuition behind Proposition 1 is as follows. When reciprocal players believe that the fair prices of the rivals are at least the largest equilibrium prices of the rivals in the game with self-interested players, the smallest and the largest Nash equilibria of the game with reciprocal players are negative reciprocity states: reciprocal players expect their rivals to set unfair prices. This implies that reciprocal players wish to punish their rivals. They do it by setting a price lower than the price a self-interested player would set. The lower equilibrium prices reduce players' monetary payoffs and in addition lead

[7] If u_i is differentiable this assumption is equivalent to $\frac{\partial w_{ij}(p_j, p^f_{ij})}{\partial p^f_{ij}} \frac{\partial \pi_j}{\partial p_i} < 0$ for all j.

to payoff losses due to the unkind behavior of the rivals. In contrast, when reciprocal players think that the fair prices of the rivals are at most the smallest equilibrium prices of the rivals in the game with self-interested players, the smallest and the largest Nash equilibria of the game with reciprocal players are positive reciprocity states: reciprocal players expect their rivals to set kind prices. This implies that reciprocal players wish to reward their rivals. They do it by setting a higher price than the price a self-interested player would set. The higher equilibrium prices increase players' monetary payoffs and in addition lead to payoff gains due to the kind behavior of the rivals.

We now turn our attention to how fairness and reciprocity change the nature of dynamic price competition. The repeated game payoff of strategy $p_i = (p_i^1, p_i^2, ...)$ when rivals play strategies p_{-i} is given by

$$U_i(p_i, p_{-i}, p_{-i}^f) = \sum_{t=1}^{\infty} \delta^{t-1} \pi_i(p_i^t, p_{-i}^t) + \alpha \sum_{j \neq i} \sum_{t=1}^{\infty} \delta^{t-1} w_{ij}(p_j^t, p_{ij}^f) \pi_j(p_i^t, p_{-i}^t)$$

When players use stationary strategies the repeated game payoff becomes

$$\begin{aligned} U_i(p_i, p_{-i}, p_{-i}^f) &= \sum_{t=1}^{\infty} \delta^{t-1} \pi_i(p_i, p_{-i}) + \alpha \sum_{j \neq i} w_{ij}(p_j, p_{ij}^f) \sum_{t=1}^{\infty} \delta^{t-1} \pi_j(p_i, p_{-i}) \\ &= \frac{1}{1-\delta} \left[\pi_i(p_i, p_{-i}) + \alpha \sum_{j \neq i} w_{ij}(p_j, p_{ij}^f) \pi_j(p_i, p_{-i}) \right] \\ &= \frac{1}{1-\delta} u_i(p, p_{-i}, p_{-i}^f). \end{aligned}$$

For the dynamic game with self-interested players, $\Gamma_{\infty}^s(\pi, p)$, we know from Friedman (1971) that for a sufficiently high discount factor, there is a subgame-perfect Nash equilibrium of $\Gamma_{\infty}^s(\pi, p)$ at a collusive price p^c with payoff $\pi(p^c)$ where $\pi(p^c)$ is any payoff which gives every player strictly more than the payoff of the largest Nash equilibrium of $\Gamma^s(\pi, p)$, that is, $\pi_i(p^c) > \pi_i(\overline{p}^{ns})$, for all i. Lemma 3 applies this result to the dynamic game with reciprocal players, $\Gamma_{\infty}^r(u, p)$.

Lemma 2: *If $p_{ij}^f \in [\overline{p}_j^{ns}, p_j^c]$ for all i and $j \neq i$, then there is a sufficiently high discount factor such that there exists a subgame-perfect Nash equilibrium of $\Gamma_{\infty}^r(u, p)$ at p^c.*

This result states that given the fair prices profile, p^f, for any p^c such that the players' payoffs at the collusive prices are higher than their payoffs at the largest Nash equilibrium of the stage game, collusion can be sustained by reciprocal players at the strategy profile p^c. We are now ready to state our first result about the impact of fairness and reciprocity on collusion.

A natural reference point in a collusive framework is to assume that players believe that a rival is fair when the rival charges exactly its agreed upon collusive price. Let us also consider that Nash punishments in $\Gamma_{\infty}^r(u, p)$ and in $\Gamma_{\infty}^s(\pi, p)$

be either at the smallest or largest pure strategy Nash equilibria of $\Gamma^r(u,p)$ and $\Gamma^s(\pi,p)$, respectively. Proposition 2 shows that in this case reciprocal preferences facilitate collusion.

Proposition 2: *If $p_{ij}^f = p_j^c$ for all i and $j \neq i$, then the critical (minimum) discount factor needed to sustain collusion at p^c is lower in $\Gamma_\infty^r(u,p)$ than in $\Gamma_\infty^s(\pi,p)$, that is $\delta_{p^c}^r < \delta_{p^c}^s$.*

The intuition behind this result is straightforward. If reciprocal players think that the fair price of each of their rivals is the rival's collusive price, then the prices set under grim trigger punishments will be lower than those that self-interested players would set. This happens because when players are reciprocators and expect their rivals to set unkind prices in the punishment phase, they have an incentive to lower their own price since they derive pleasure from hurting the rivals for their nasty behavior. If the grim trigger punishments prices of reciprocators are lower than those of self-interested players, then Nash punishments are harsher with reciprocal players than with self-interested ones. Additionally, since the collusive prices are considered to be fair, there is no impact of fairness and reciprocity on the collusive payoffs nor on the single period deviation payoffs. Thus, the critical discount factor needed to sustain a collusive outcome must be lower with reciprocal players than with self-interested ones.

What if players have more general perceptions of what the fair prices of their rivals should be? Our second result shows that if players think that fair prices of the rivals are between the largest Nash prices of the stage game with self-interested players and the collusive prices, and marginal costs are constant, then it is easier to sustain collusion when players are reciprocal than when they are self-interested.

Proposition 3: *Let $\pi_i(p_i, p_{-i}) = (p_i - c_i)D_i(p_i, p_{-i})$, and $p_{ij}^f \in [\overline{p}_j^{ns}, p_j^c]$ for all i and $j \neq i$. Then the critical (minimum) discount factor needed to sustain collusion at p^c is lower in $\Gamma_\infty^r(u,p)$ than in $\Gamma_\infty^s(\pi,p)$, that is $\delta_{p^c}^r < \delta_{p^c}^s$.*

The intuition for this result is as follows. If players think that the fair prices of the rivals are less than the collusive prices, then collusion becomes a positive reciprocity state. In this case players' monetary payoffs from collusion are the same as the ones obtained in the game with self-interested players but in addition there are fairness payoff gains since players think that their rivals are being kind. This effect makes collusion more attractive when players are reciprocal than when they are self-interested.

Additionally, if players think that the fair prices of the rivals are greater than the largest Nash equilibrium prices of the stage game with self-interested players, then Nash reversion becomes a negative reciprocity state. This implies that the punishment imposed after cheating occurs is more severe when players are reciprocal than when they are self-interested. This happens because monetary payoffs are lower than the payoffs of self-interested players and in addition there are fairness payoff loses since players think that the rivals are being unkind.

This effect also makes collusion more attractive when players are reciprocal than when they are self-interested.

Clearly, these two effects make collusion *more* attractive to reciprocal players than to self-interested ones. However, the unilateral single period deviation payoff is higher with reciprocal players than with self-interested ones. This happens because the unilateral single period deviation payoff of a reciprocal player also includes the benefit that player derives from being treated kindly by the rivals (the rivals are playing their collusive prices). This effect of fairness and reciprocity makes collusion *less* attractive to reciprocal players than to self-interested ones. The assumption that monetary payoffs are large by comparison with fairness payoffs implies that the increase in collusive payoff is of first-order whereas the increase in the unilateral single period deviation payoff is of second-order.[8]

3.2 Strategic Substitutes

We now show that fairness and reciprocity facilitate collusion in dynamic quantity-setting games with grim trigger punishments. Thus, when players are reciprocal, collusive action profiles are easier to sustain not only when players' actions are strategic complements but also when they are strategic substitutes.

When players' actions are strategic substitutes a player's incremental returns from increasing his own action are decreasing in the rivals' actions. The canonical market game where players' actions are strategic substitutes is quantity competition with products that are perfect substitutes. We use this game to study the impact of fairness and reciprocity on collusion when players' actions are strategic substitutes.

Assume that in each period player i chooses quantity, q_i, and his payoff in that period is given by

$$u_i(q_i, Q_{-i}) = \pi_i(q_i, Q_{-i}) + \alpha w_i(Q_{-i}, Q^f_{-i}) \sum_{j \neq i} \pi_j(q_i, Q_{-i}),$$

where $\pi_i(q_i, Q_{-i})$ is the monetary payoff and $\alpha w_i(Q_{-i}, Q^f_{-i}) \sum_{j \neq i} \pi_j(q_i, Q_{-i})$ is the fairness payoff, with $\alpha > 0$. Player i's monetary payoff, $\pi_i(q_i, q_{-i})$, is the difference between revenue and cost, that is,

$$\begin{aligned} \pi_i(q_i, Q_{-i}) &= R_i(q_i, Q_{-i}) - C_i(q_i) \\ &= P(Q)q_i - C_i(q_i), \end{aligned}$$

where $R_i(q_i, Q_{-i})$ is revenue, $C_i(q_i)$ is the cost of production, and $P(Q)$ is the inverse market demand with $Q = \sum q_i$. We assume that $P(Q)$ is strictly positive on some bounded interval $(0, \bar{Q})$ with $P(Q) = 0$ for $Q \geq \bar{Q}$. We also assume that $P(Q)$ is twice continuously differentiable with $P'(Q) < 0$ (in the interval for which $P(Q) > 0$). Players' costs of production are assumed to be

[8] The assumption that u_i has increasing differences in (p_i, p_{-i}) implies that monetary payoffs are large by comparison with fairness payoffs.

twice continuously differentiable with $C_i'(q_i) \geq 0$. It is also assumed that the decreasing marginal revenue property holds, that is, $P'(Q) + P''(Q)\, q_i \leq 0$, and $P'(Q) - C_i''(q_i) < 0$. Furthermore, we assume that the weight that player i places on the rivals' aggregate monetary payoffs depends on player i's perception of the fair aggregate output of the rivals, Q_{-i}^{f}, and on the actual aggregate output of the rivals such that

$$w_i(Q_{-i}, Q_{-i}^{f}) \begin{cases} > 0 \text{ if } Q_{-i} < Q_{-i}^{f} \\ = 0 \text{ if } Q_{-i} = Q_{-i}^{f} \\ < 0 \text{ otherwise} \end{cases}, \tag{6}$$

where $w_i(Q_{-i}, Q_{-i}^{F})$ is assumed to be differentiable in both arguments with $\partial w_i / \partial Q_{-i} < 0$ and $\partial w_i / \partial Q_{-i}^{F} > 0$. The first condition in (6) expresses positive reciprocity. If a player expects her rivals to produce less than her perception of fair output, then she is willing to sacrifice some of her profit to increase the rivals' profits. The third condition in (6) expresses negative reciprocity. If a player expects her rivals to produce more than her perception of fair output, then she is willing to sacrifice some of her profit to reduce the rivals' profits.

Finally, we assume that monetary payoffs are large by comparison with fairness payoffs otherwise best replies of reciprocal players in a static Cournot oligopoly might no longer have a negative slope across all quantities.

Proposition 4: *If $\Gamma^r(u,p)$ and $\Gamma^s(\pi,p)$ satisfy the conditions stated and $Q_{-i}^{f} \in [Q_{-i}^{c}, Q_{-i}^{ns}]$ for all i, then the critical (minimum) discount factor needed to sustain collusion at q^c is lower in $\Gamma_\infty^r(u,q)$ than in $\Gamma_\infty^s(\pi,q)$, that is, $\delta_{q^c}^r < \delta_{q^c}^s$.*

Proposition 4 shows that fairness and reciprocity also facilitate collusion when players' choices are strategic substitutes. It says that if players think that the fair aggregate output of the rivals is greater than or equal to the collusive output but less than or equal to the aggregate output of the rivals in the Nash equilibrium of the stage game with self-interested players, then it is easier to sustain collusion when players are reciprocal than when they are self-interested.

The intuition is similar to that of Proposition 3. If reciprocal players think that the fair output of their rivals is greater than the joint self-interested collusive output of the rivals, then playing the collusive output is more attractive in the dynamic quantity-setting game with reciprocal players than in the game with self-interested players. This happens because the collusive monetary payoffs are the same as the ones obtained in the game with self-interested players but in addition there are payoff gains from positive reciprocity since reciprocal players think that their rivals are being kind.

Additionally, if reciprocal players perceive that the fair output of their rivals is smaller than the aggregate output of the rivals in the Nash equilibrium of the stage game with self-interested players, then the punishment imposed after cheating occurs becomes more severe in the dynamic game with reciprocal players than in the dynamic game with self-interested players. This happens because, the Nash equilibrium of the stage game with reciprocal players becomes a negative reciprocity state. This is bad for players since it reduces monetary payoffs (by comparison with the monetary payoffs of self-interested players) and leads to payoff loses from negative reciprocity since reciprocal players think that the rivals are being mean.

In contrast, the single period deviation payoff in the game with reciprocal players is larger than the single period deviation payoff in the game with self-interested players. This happens because the unilateral single period deviation payoff of a reciprocal player also includes the benefit that player derives from being treated kindly by the rivals (the rivals are playing their collusive outputs). However, this effect is of second-order since monetary payoffs are larger by comparison with fairness payoffs.

4 Discussion

Our main results hold provided certain conditions are met. Clearly, the most important condition is the one about players' perceptions of fair prices of the rivals. As we mentioned before, we think that it is natural to assume that players would think that if the rivals play their collusive prices the rivals are being fair. However, if players' perceptions of fair prices of the rivals are higher than the rivals' collusive prices, then the impact of fairness and reciprocity on cooperation is ambiguous. The reason is that collusion may become a negative reciprocity state. In that case players' monetary payoffs from collusion are the same as the ones obtained in when players are self-interested but in addition there are fairness payoff losses since players think that their rivals are being unkind. This effect makes collusion less attractive when players are reciprocal than when they are self-interested.

The assumption that Nash punishments are either at the smallest or largest pure strategy Nash equilibria is essentially a technical condition. This condition is necessary when the stage game has multiple equilibria since in a supermodular game we can state unambiguous comparative static results for the largest and the smallest Nash equilibria but not for other Nash equilibria.

So far the paper has indicated that fairness and reciprocity facilitate collusion when players use Nash reversion to punish deviations. However, Abreu's (1988) theory of optimal punishments can be an alternative framework of analysis. In appendix B we show that our findings also extends to the optimal punishments framework (we only analyze the dynamic price-setting market game since the quantity-setting case is similar).

The intuition of this result is as follows. First, the benefit of deviating today (the unilateral single period deviation payoff minus the collusive payoff) when players use optimal punishments is the same as when they use grim trigger punishments. We already know from Proposition 2 that if monetary payoffs are large by comparison with fairness payoffs, then the increase in the collusive payoff due to fairness considerations is of first-order whereas the increase in the unilateral single period deviation payoff is of second-order. Thus, the benefit of deviating is smaller for reciprocators than for self-interested players no matter if players use optimal punishments or grim trigger punishments.

Second, if reciprocal players think that the fair prices are smaller than the collusive prices, then the prices set on the initial path are perceived as kind behavior by the other players and lead to positive fairness payoffs. Therefore, when the prices of the initial path are set, the payoffs for reciprocal players are higher than those for self-interested players.

Third, it is well known that punishments are more severe when players use optimal punishments than when they use Nash reversion strategies. If reciprocal players think that the fair prices of the rivals are greater than the largest Nash prices of the stage game with self-interested players, then seeing the rivals setting punishment prices lower than Nash prices will be perceived as nastier behavior than seeing the rivals setting Nash prices. Therefore, reciprocal players will set lower prices than self-interested players during the punishment phase under optimal punishments.

The second and the third effects imply that the cost of deviating (the collusive payoff minus the payoff of entering a punishment stage) is larger for reciprocal players than for self-interested players when players use optimal punishments.

5 Conclusion

This paper shows that fairness and reciprocity can facilitate collusion in infinitely repeated games. This result is valid not only when players' choices are strategic complements but also when they are strategic substitutes. The result also holds no matter if players use grim trigger punishments or optimal punishments. Thus, fairness concerns among producers with reciprocal preferences who interact repeatedly can have adverse welfare consequences for consumers.

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6 Appendix A

Lemma A: *If (i) P_i is a compact interval in $\mathcal{R}$, (ii) $u_i(p_i, p_{-i}, p^f_{-i})$ is order upper semi-continuous in p_i for fixed p_{-i} and order continuous in p_{-i} for a fixed p_i, and $u_i(p_i, p_{-i}, p^f_{-i})$ has a finite upper bound, (iii) u_i is supermodular in p_i for fixed p_{-i}, and (iv) $u_i(p_i, p_{-i}, p^f_{-i})$ has increasing differences in (p_i, p_{-i}), then $\Gamma^r(u, p)$ is a supermodular game.*

Proof of Lemma A: According to Milgrom and Roberts (1990), a game $\Gamma(u, x)$ is supermodular if (*i*) the choice set is a compact interval in $\mathcal{R}$, (*ii*) u_i is order upper semi-continuous in x_i for x_{-i} and order continuous in x_{-i} for a fixed x_i, and it has a finite upper bound, (*iii*) u_i is supermodular in x_i for fixed x_{-i}, and (*iv*) u_i has increasing differences in (x_i, x_{-i}).
The price stage game with reciprocal players $\Gamma^r(u, p)$ satisfies condition (*i*) since it is never optimal for players to choose an infinite price for any finite quantity. We have assumed that u_i also satisfies all the requirements of condition (*ii*). Condition (*iii*) is satisfied since the choice variables of players are scalars. Condition (*iv*) is satisfied if for any two aggregate actions of the others p'_{-i}, p''_{-i} with $p'_{-i} \geq p''_{-i}$ (product order) the difference $u_i(p_i, p'_{-i}, P^f_i) - u_i(p_i, p''_{-i}, P^f_i)$ is increasing (or non-decreasing) in p_i, which is assumed as well. Therefore $\Gamma^r(u, p)$ is supermodular game. *Q.E.D.*

Proof of Lemma 1: It is an application of Theorem 6 in Milgrom and Roberts (1990) with a slight difference. In their setting, the smallest and largest pure strategy equilibria of the game depends on a scalar, but in our model it depends on a vector. Nevertheless, the proof is immediate since we propose the smallest and largest equilibria is nonincreasing with the fair price perception for any player j, which is a scalar. As a result, if the vector increases in every component, then the smallest and largest equilibria do not increase. *Q.E.D.*

Proof of Proposition 1: The stage game $\Gamma^s(\pi, p)$ is obtained from the stage game $\Gamma^r(u, p)$ by setting $\alpha = 0$. Thus, if $\Gamma^r(u, p)$ is a supermodular game so is $\Gamma^s(\pi, p)$. This means that $\Gamma^s(\pi, p)$ also has a smallest and a largest Nash equilibria in pure-strategies. Denote these two equilibria by $\underline{p}^{ns}$ and $\overline{p}^{ns}$, respectively. Observe that if $p^f_{-i} = \overline{p}^{ns}_{-i}$ then $\overline{p}^{nr} = \overline{p}^{ns} = \overline{p}^n$ and $u_i(\overline{p}^n, p^f_{-i}) = \pi_i(\overline{p}^n)$ since $w_{ij}(\overline{p}^n_j, p^f_{ij}) = 0$ for all j. (i) If $\overline{p}^{ns}_{-i} < p^f_{-i}$, then $\overline{p}^{nr} \leq \overline{p}^{ns}$ by Lemma 1. These two inequalities imply $\overline{p}^{nr}_{-i} < p^f_{-i}$ which together with (5) imply $w_{ij}(\overline{p}^{nr}_j, p^f_{ij}) < 0$ for all j. But then it follows that $u_i(\overline{p}^{nr}, p^f_{-i}) < \pi_i(\overline{p}^{ns})$ by the fact that $w_{ij}(\overline{p}^{nr}_j, p^f_{ij}) < 0$ for all j and $\pi_i(\overline{p}^{nr}) \leq \pi_i(\overline{p}^{ns})$ for all i. Similarly, (ii) if $p^f_{-i} < \underline{p}^{ns}_{-i}$, then $\overline{p}^{nr} \geq \overline{p}^{ns}$ by Lemma 1. These two inequalities imply $p^f_{-i} < \underline{p}^{nr}_{-i}$ which together with (5) imply $w_{ij}(\overline{p}^{nr}_j, p^f_{ij}) > 0$ for all j. But then it follows that $u_i(\overline{p}^{nr}, p^f_{-i}) > \pi_i(\overline{p}^{ns})$ by the fact that $w_{ij}(\overline{p}^{nr}_j, p^f_{ij}) > 0$ for all j and $\pi_i(\overline{p}^{nr}) \geq \pi_i(\overline{p}^{ns})$ for all i. *Q.E.D.*

Proof of Lemma 2: If $p^f_{ij} \in [\overline{p}^{ns}_j, p^c_j]$ for all i and $j \neq i$, then by (5) $w_{ij}(p^c_j, p^f_{ij}) \geq 0$ and $w_{ij}(\overline{p}^{nr}_j, p^f_{ij}) \leq 0$, for all i and $j \neq i$. This in turn implies that

$$u_i(p^c, p^f_{-i}) \geq \pi_i(p^c). \tag{7}$$

We also know that

$$\pi_i(p^c) > \pi_i(\overline{p}^{ns}) > \pi_i(\underline{p}^{ns}). \tag{8}$$

If $p^f_{ij} \geq \overline{p}^{ns}_j$ for all i and $j \neq i$, then we know from Proposition 1 that

$$u_i(\overline{p}^{nr}, p^f_{-i}) \leq \pi_i(\overline{p}^{ns}), \text{ and } u_i(\underline{p}^{nr}, p^f_{-i}) \leq \pi_i(\underline{p}^{ns}) \tag{9}$$

for all i. From (7), (8) and (9) we obtain

$$u_i(p^c, p^f_{-i}) > u_i(\overline{p}^{nr}, p^f_{-i}) \text{ and } u_i(p^c, p^f_{-i}) > u_i(\underline{p}^{nr}, p^f_{-i})$$

for all i, which by Friedman (1971) implies that there exists a sufficiently high discount factor such that p^c is a subgame-perfect Nash equilibrium of $\Gamma^r(u, p)$. *Q.E.D.*

Proof of Proposition 2: By Friedman (1971) and Lemma 2, the assumptions made imply that p^c is a subgame-perfect Nash equilibrium of $\Gamma^s(\pi, p)$ and of $\Gamma^r(u, p)$. Next we show that the critical discount factor at which p^c can be sustained using grim trigger punishments in $\Gamma^r_\infty(u, p)$ is lower than the critical discount factor at which p^c can be sustained using grim trigger punishments in $\Gamma^s_\infty(\pi, p)$, that is, $\delta^r_{p^c} < \delta^s_{p^c}$. From (2) and (3) sufficient conditions are that

$$u_i(BR^r_i(p^c_{-i}), p^c_{-i}, p^f_{-i}) - u_i(p^c, p^f_{-i}) \leq \pi_i(BR^s_i(p^c_{-i}), p^c_{-i}) - \pi_i(p^c), \tag{10}$$

and

$$u_i(BR^r_i(p^c_{-i}), p^c_{-i}, p^f_{-i}) - u_i(p^{nr}, p^f_{-i}) \geq \pi_i(BR^s_i(p^c_{-i}), p^c_{-i}) - \pi_i(p^{ns}), \tag{11}$$

where $p^{nr} = \underline{p}^{nr}$ and $p^{ns} = \underline{p}^{ns}$ or $p^{nr} = \overline{p}^{nr}$ and $p^{ns} = \overline{p}^{ns}$. If $p^f_{ij} = p^c_j$ for all i and $j \neq i$, then $w_{ij}(p^c_j, p^f_{ij}) = 0$ for all i and $j \neq i$ which implies that

$$u_i(p^c, p^f_{-i}) = \pi_i(p^c), \tag{12}$$

for all i, and

$$u_i(BR^r_i(p^c_{-i}), p^c_{-i}, p^f_{-i}) = \pi_i(BR^s_i(p^c_{-i}), p^c_{-i}), \tag{13}$$

for all i. It follows from (12) and (13) that (10) is satisfied as an equality. If $p^f_{ij} = p^c_j > \overline{p}^{ns}_j$ for all i and $j \neq i$, then the inequalities in (9) hold strictly for all i. It follows from this and (13) that (11) is satisfied as a strict inequality. Thus $\delta^r_{p^c} < \delta^s_{p^c}$. *Q.E.D.*

Proof of Proposition 3: By Friedman (1971) and Lemma 2, the assumptions made imply that p^c is a subgame-perfect Nash equilibrium of $\Gamma^s(\pi, p)$ and of

$\Gamma^r(u,p)$. We want to show that the critical discount factor at which p^c can be sustained using grim trigger punishments in $\Gamma^r_\infty(u,p)$ is lower than the critical discount factor at which p^c can be sustained using grim trigger punishments in $\Gamma^s_\infty(\pi,p)$, that is, $\delta^r_{p^c} < \delta^s_{p^c}$. From (2) and (3) sufficient conditions are that (10) and (11) are satisfied.

We start by showing that $\pi_i(p_i,p_{-i}) = (p_i - c_i)D_i(p_i,p_{-i})$ and $p^f_{ij} \leq p^c_j$ for all $j \neq i$ imply that (10) is satisfied as a strict inequality. We have that

$$\begin{aligned} u_i(BR^r_i(p^c_{-i}), p^c_{-i}, p^f_{-i}) - u_i(p^c, p^f_{-i}) &= \pi_i(BR^r_i(p^c_{-i}), p^c_{-i}) - \pi_i(p^c) \\ &+ \alpha \sum_{j\neq i} w_{ij}(p^c_j, p^f_{ij})(p^c_j - c_j)[D_j(BR^r_i(p^c_{-i}), p^c_{-i}) - D_j(p^c)] \\ &\leq \pi_i(BR^r_i(p^c_{-i}), p^c_{-i}) - \pi_i(p^c) < \pi_i(BR^s_i(p^c_{-i}), p^c_{-i}) - \pi_i(p^c) \end{aligned}$$

The equality is obtained from (4) and from the assumption $\pi_i(p_i,p_{-i}) = (p_i - c_i)D_i(p_i,p_{-i})$. The weak inequality comes from the assumption that $p^f_{ij} \leq p^c_j$ which implies $w_{ij}(p^c_j,p^f_{ij}) \geq 0$, and the assumption that D_j is increasing with p_i which together with $p^{dr}_i < p^c_i$ imply $D_j(BR^r_i(p^c_{-i}), p^c_{-i}) - D_j(p^c) < 0$. The strict inequality comes from the fact that $BR^s_i(p^c_{-i})$ is the best-reply to p^c_{-i} by a self-interested player.

We now show that if $\overline{p}^{ns}_j \leq p^f_{ij}$ for all $j \neq i$ and α is sufficiently small, then (11) is satisfied. Rewrite (11) as

$$[u_i(BR^r_i(p^c_{-i}), p^c_{-i}, p^f_{-i}) - \pi_i(BR^s_i(p^c_{-i}), p^c_{-i})] + [\pi_i(p^{ns}) - u_i(p^{nr}, p^f_{-i})] \geq 0.$$

From Proposition 1 we have that

$$\pi_i(p^{ns}) \geq u_i(p^{nr}, p^f_{-i}).$$

If $\overline{p}^{ns}_j \leq p^f_{ij}$ for all $j \neq i$, then $w_{ij}(p_j, p^f_{ij}) \geq 0$ for all $j \neq i$. Taking a first-order Taylor series expansion of $u_i(BR^r_i(p^c_{-i}), p^c_{-i}, p^f_{-i})$ around $\alpha = 0$ we obtain

$$\begin{aligned} u_i(BR^r_i(p^c_{-i}), p^c_{-i}, p^f_{-i}) &\approx \pi_i(BR^s_i(p^c_{-i}), p^c_{-i})) \\ &+ \alpha[\sum_{j\neq i} w_{ij}(p_j, p^f_{ij})\pi_j(BR^s_i(p^c_{-i}), p^c_{-i})], \end{aligned}$$

which is equivalent to

$$\begin{aligned} u_i(BR^r_i(p^c_{-i}), p^c_{-i}, p^f_{-i}) - \pi_i(BR^s_i(p^c_{-i}), p^c_{-i}) &\approx \\ \alpha[\sum_{j\neq i} w_{ij}(p_j, p^f_{ij})\pi_j(BR^s_i(p^c_{-i}), p^c_{-i})] &\geq 0 \end{aligned}$$

Thus $\delta^r_{p^c} < \delta^s_{p^c}$. *Q.E.D.*

Proof of Proposition 4: We need to show that $Q^f_{-i} \in [Q^c_{-i}, Q^{ns}_{-i}]$ for all i, implies $\delta^r_{q^c} < \delta^s_{q^c}$, where $\delta^r_{q^c}$ is the critical discount factor above which q^c can be sustained in $\Gamma^r_\infty(u,q)$ and $\delta^s_{q^c}$ is the critical discount factor above which q^c can be sustained in $\Gamma^s_\infty(\pi,q)$. From (2) and (3) sufficient conditions are that

$$u_i(BR^r_i(Q^c_{-i}),Q^c_{-i}) - u_i(q^c) \le \pi_i(BR^s_i(Q^c_{-i}),Q^c_{-i}) - \pi_i(q^c) \tag{14}$$

and

$$u_i(BR^r_i(Q^c_{-i}),Q^{cs}_{-i}) - u_i(q^{nr}) \ge \pi_i(BR^s_i(Q^c_{-i}),Q^{cs}_{-i}) - \pi_i(q^{ns}). \tag{15}$$

(i) We start by showing that $Q^f_{-i} \in [Q^c_{-i}, Q^{ns}_{-i}]$ implies (14) is satisfied as a strict inequality. We have that

$$\begin{aligned} u_i(BR^r_i(Q^c_{-i}),Q^c_{-i}) - u_i(q^c) &= \pi_i(BR^r_i(Q^c_{-i}),Q^c_{-i}) - \pi_i(q^c) \\ &+ \alpha w_i(Q^c_{-i},Q^f_{-i})\left[P(BR^r_i(Q^c_{-i})+Q^c_{-i}) - P(Q^c)\right]Q^c_{-i} \\ &\le \pi_i(BR^r_i(Q^c_{-i}),Q^c_{-i}) - \pi_i(q^c) < \pi_i(BR^s_i(Q^c_{-i}),Q^c_{-i}) - \pi_i(q^c) \end{aligned}$$

The strict inequality follows from the fact that $BR^s_i(Q^c_{-i})$ is the best reply to Q^c_{-i} for self-interested players. If $Q^c_{-i} \le Q^f_{-i}$ then $w_i(Q^c_{-i},Q^f_{-i}) \ge 0$. Furthermore, $Q^f_{-i} \le Q^{ns}_{-i}$ implies $BR^r_i(Q^c_{-i}) > q^c_i$ which in turn implies $P(BR^r_i(Q^c_{-i})+Q^c_{-i}) < P(Q^c)$, since $P'(\cdot) < 0$.

(ii) We now show that $Q^f_{-i} \in [Q^c_{-i}, Q^{ns}_{-i}]$ implies that (15) is satisfied. Rewrite (15) as

$$[u_i(BR^r_i(Q^c_{-i}),Q^c_{-i}) - \pi_i(BR^s_i(Q^c_{-i}),Q^c_{-i})] + [\pi_i(q^{ns}) - u_i(q^{nr})] \ge 0.$$

We have that

$$u_i(q^{nr}) = \pi_i(q^{nr}) + \alpha w_i(Q^{nr}_{-i},Q^f_{-i})\sum_{j\ne i}\pi_j(q^{nr}) \le \pi_i(q^{ns}).$$

The inequality follows from $w_i(Q^{nr}_{-i},Q^f_{-i}) \le 0$ and Proposition 3 in Santos-Pinto (2006) which shows that $Q^f_{-i} \le Q^{ns}_{-i}$ implies $q^{ns}_i \le q^{nr}_i$ and $\pi_i(q^{nr}) \le \pi_i(q^{ns})$, for all i. Taking a first-order Taylor series expansion of $u_i(BR^r_i(Q^c_{-i}),Q^{cs}_{-i})$ around $\alpha = 0$ we have that

$$\begin{aligned} u_i(BR^r_i(Q^c_{-i}),Q^c_{-i}) \quad &\approx \quad \pi_i(BR^s_i(Q^c_{-i}),Q^c_{-i}) \\ &\qquad + \alpha[w_i(Q^c_{-i},Q^f_{-i})\sum_{j\ne i}\pi_j(BR^s_i(Q^c_{-i}),Q^c_{-i})]. \end{aligned}$$

which is equivalent to

$$\begin{aligned} &u_i(BR^r_i(Q^c_{-i}),Q^c_{-i}) - \pi_i(BR^s_i(Q^c_{-i}),Q^c_{-i}) \approx \\ &\qquad \alpha[w_i(Q^c_{-i},Q^f_{-i})\sum_{j\ne i}\pi_j(BR^s_i(Q^c_{-i}),Q^c_{-i})] \ge 0 \end{aligned}$$

since $Q^c_{-i} \le Q^f_{-i}$ implies that $w_i(Q^c_{-i},Q^f_{-i}) \ge 0$. Thus, $Q^f_{-i} \in [Q^c_{-i}, Q^{ns}_{-i}]$ for all i, implies $\delta^r_{q^c} < \delta^s_{q^c}$. *Q.E.D.*

7 Appendix B

Abreu (1988) introduces a rule which consists of an initial path (that is an infinite stream of one period action profiles) and punishments (that are also infinite streams for any deviation from the initial path or from a prescribed punishment). He introduces the notion of *simple* strategy profile in which a specific punishment takes place after any deviation for each particular player. Thus, the simple strategy profiles have a description of $(n+1)$ paths for an n-player game. On the other hand, an arbitrary strategy profile may consist of infinite amount of punishments and depends on complex history-dependent formulas.

We begin by introducing additional notations and definitions, after we show an optimal simple penal code exists. Finally, we state conditions under which it is easier to sustain collusion with reciprocal players than with self-interested ones under optimal punishments.

A pure strategy of player i is denoted σ_i. Each σ_i is a sequence of functions, $\sigma_i(1), \sigma_i(2), ..., \sigma_i(t), ...$, one for each t. The function for all periods t determines player i's action at t as a function of the actions of all players in previous periods. Formally, at $t = 1, \sigma_i(1) \in P_i$ and for $t = 2, 3, ...,$ $\sigma_i(t) : P^{t-1} \to P_i$. Player i's strategy set is denoted Σ_i, and the set of strategy profiles is denoted $\Sigma \equiv \Sigma_1 \times \Sigma_2 \times ... \times \Sigma_n$.

A path (or punishment), $\widetilde{P}$, is a stream of action profiles $\{p(t)\}_{t=1}^{\infty}$ and let $\Omega \equiv P^{\infty}$ be the set of punishments. Any strategy profile $\sigma \in \Sigma$ generates a path denoted $\widetilde{P}(\sigma) = \{p(\sigma)(t)\}_{t=1}^{\infty}$, and it is defined as follows:

$$p(\sigma)(1) = \sigma(1) \text{ and}$$
$$p(\sigma)(t) = \sigma(t)(p(\sigma)(1), ..., (p(\sigma)(t)).$$

Player i's payoff from path $\widetilde{P} \in \Omega$ is given by $v_i^x : \Omega \to \mathcal{R}$ for $x = \{r, s\}$ such that

$$v_i^x(\widetilde{P}) = \begin{cases} \sum_{t=1}^{\infty} \delta^t u_i(p(t)) \text{ if } x = r \\ \sum_{t=1}^{\infty} \delta^t \pi_i(p(t)) \text{ if } x = s \end{cases}$$

where u_i is given by (4) and (5). Player i's payoff function is given by $\widetilde{v}_i^x : \Sigma \to \mathcal{R}$ such that $\widetilde{v}_i^x(\sigma) = v_i(\widetilde{P}(\sigma))$.

Abreu (1988) introduces the simple strategy profile, which is defined by $(n+1)$-vector of paths $(\widetilde{P}^0, \widetilde{P}^1, ..., \widetilde{P}^n)$ and a rule. The initial path is $\widetilde{P}^0$, and for each player $i \in \{1, ..., n\}$, $\widetilde{P}^i$ is the punishment for player i. Any unilateral deviation of player i from the ongoing path is responded by imposing $\widetilde{P}^i$. If more than one player deviate, the ongoing path continues to be followed and deviators will not be punished. Formally:

Let $\widetilde{P}^i \in \Omega$, $i = 0, 1, ..., n$. The *simple strategy profile* $\sigma(\widetilde{P}^0, \widetilde{P}^1, ..., \widetilde{P}^n)$ specifies: (i) play $\widetilde{P}^0$ until some player deviates unilaterally from $\widetilde{P}^0$; (ii) for any $j \in \{1, ..., n\}$, play $\widetilde{P}^j$ if the jth player deviates unilaterally from $\widetilde{P}^i$, $i = 0, 1, ..., n$, where $\widetilde{P}^i$ is an ongoing previously specified path; continue with $\widetilde{P}^i$ if no deviations occur or if two or more players deviate simultaneously.

A simple strategy $\sigma(\widetilde{P}^0, \widetilde{P}^1, ..., \widetilde{P}^n)$ profile is *perfect* if and only if no one-shot deviation by any player $j \in \{1, ..., n\}$ from $\widetilde{P}^i$, $i = 0, 1, ..., n$, yields player j a higher payoff, when all players conform with $\widetilde{P}^j$ after the deviation.[9] Let Σ^p denote the set of perfect equilibrium strategy profiles of $\Gamma_\infty(\delta)$. The perfect equilibrium paths $\Omega^p = \{\widetilde{P}(\sigma)|\sigma \in \Sigma^p\}$, and payoffs $V = \{v(\widetilde{P})|\widetilde{P} \in \Omega^p\}$.

We introduce three more definitions from Abreu (1988) before stating the existence result. An *optimal penal code* is an n-vector of the strategy profiles $\{\underline{\sigma}^1, ..., \underline{\sigma}^n\}$ such that for all i,

$$\underline{\sigma}^i \in \Sigma^p \text{ and } \widetilde{v}_i(\underline{\sigma}^i) = \min\{\widetilde{v}_i(\sigma)|\sigma \in \Sigma^p\}.$$

Let $\sigma^i(\widetilde{P}^1, ..., \widetilde{P}^n) = \sigma(\widetilde{P}^i, \widetilde{P}^1, ..., \widetilde{P}^n)$. The *simple penal code* $(\widetilde{P}^1, ..., \widetilde{P}^n)$ is the n-vector of the strategy profiles $\sigma^1(\widetilde{P}^1, ..., \widetilde{P}^n), ..., \sigma^n(\widetilde{P}^1, ..., \widetilde{P}^n)$. Finally, a simple penal code $(\widetilde{P}^1, ..., \widetilde{P}^n)$ is an *optimal simple penal code* if it is an optimal penal code.

Lemma 3: *If Σ^p is non-empty, P is a compact topological space and given p^f, $u : P \times p^f \to R^n$ is continuous, then an optimal simple penal code exists.*

Proof of Lemma 3: The lemma follows from Abreu (1988) under the assumptions of $u(.)$. *Q.E.D.*

Similarly, an optimal simple penal code exists for a continuous payoff function $\pi : P \to \mathcal{R}^n$. Let present discounted value of player i's payoffs from the period $t + 1$ to ∞ along the path $\widetilde{P}$ be

$$v_i^x(\widetilde{P}; t + 1) = \begin{cases} \sum_{k=1}^{\infty} \delta^k u_i(p(t + k)) \text{ if } x = r \\ \sum_{k=1}^{\infty} \delta^k \pi_i(p(t + k)) \text{ if } x = s \end{cases},$$

and player i's payoff under her optimal penal code, $\underline{v}_i^x = \widetilde{v}_i^x(\underline{\sigma}^i)$. The following result indicates the use of optimal penal code to characterize the set of perfect equilibrium paths.

Lemma 4: If an optimal penal code exists, then $\widetilde{P}^0 \in \Omega^p$ if and only if

$$u_i(p_i^{dr}, p_{-i}^0(t)) - u_i(p^0) \leq v_i^r(\widetilde{P}^0; t + 1) - \underline{v}_i^r \quad (16)$$

$$\pi_i(p_i^{ds}, p_{-i}^0(t)) - \pi_i(p^0) \leq v_i^s(\widetilde{P}^0; t + 1) - \underline{v}_i^s \quad (17)$$

Proof of Lemma 4: The lemma follows from Abreu (1988). *Q.E.D.*

The left-hand-side of inequalities (16) and (17) are the benefit of deviating today for reciprocators and self interested players, respectively. The right-hand-side is the cost of deviating. Observe that the prices in each period of the initial path can be considered as any collusive prices.

[9] This relation holds if the set of payoffs of the stage game is bounded (i.e. $\{u(p)|p \in P\}$ is bounded).

Since the existence of optimal simple penal code is guaranteed under the given assumptions, our final result shows that fairness and reciprocity facilitate collusion when players use optimal simple penal codes.

Proposition 5: *Assume (i) u_i has decreasing differences in (p_i, p^f_{-i}), for all i, (ii) $\pi_i(p_i, p_{-i}) = (p_i - c_i)D_i(p_i, p_{-i})$, and (iii) $p^f_{ij} \in [\overline{p}^{ns}_j, p^c_j]$ for all i and $j \neq i$. Let p^0 satisfy $\pi_i(p^o) > \pi_i(\overline{p}^{ns})$ for all i. If an optimal simple penal code exist, then the critical (minimum) discount level to sustain collusion at $\widetilde{P}^0$is lower in the game with reciprocal players $\Gamma^r_\infty(n, u, p, P^f)$ than in the game with self-interested players $\Gamma^s_\infty(n, \pi, p)$, that is $\delta^r_{p^0} < \delta^s_{p^0}$.*

Proof of Proposition 5: The minimum critical discount factor will be obtained if the inequality (16) and (17) hold with equality respectively for reciprocators and self-interested players, otherwise the discount factor can be decreased by a small amount without violating the inequality. In Proposition 3, we proved the LHS of the equations being smaller for reciprocators, hence a smaller discount level is possible for the reciprocators. In addition, the following condition is immediate

$$v^r_i(\widetilde{P}^0; t+1) \geq v^s_i(\widetilde{P}^0; t+1)$$

considering the initial path where each player i sets at least the collusive price p^c_i for each stage, until one deviates. Hence for any fair price perception $p^f_{ij} \in [\overline{p}^{ns}_j, p^c_j]$ for all i and $j \neq i$, the prices set at the initial path will be perceived as kind behavior, thus the condition holds. Note that, if the prices set at the initial path are equal to collusive prices p^c_i for each player i and $p^f_{ij} = p^c_j$ for all i and $i \neq i$, then the condition holds with equality. Finally, to complete the proof we need to compare the payoff of any player i in the optimal penal code $\underline{v}^x_i$. In the optimal penal code, the players punish the deviated player i via playing a pure strategy profile $\underline{\sigma}^i \in \Sigma^p$, which gives the lowest possible payoff to player i. Let ${}^{nx}\underline{\sigma}$ denote the strategy profile where in each stage players set Nash prices. Since ${}^{nx}\underline{\sigma} \in \Sigma^p$, in each stage the optimal penal code for player i, $\underline{v}^x_i$, is at least as severe as ${}^{nx}\underline{\sigma}$, which means each player j sets $\underline{p}_j \leq p^{ns}_j$. Note that, if the set prices in the penal code is such that $\underline{p}_j = p^{ns}_j = p^f_{ji}$ for all j and $i \neq j$, then the payoffs from the penal code are equal for self interested and reciprocal player i, $\underline{v}^r_i = \underline{v}^s_i$. Otherwise, the reciprocal players perceive the unkind behavior and negative reciprocity implies the payoff under optimal penal code is harsher for reciprocal players than self-interested players, that is $\underline{v}^r_i < \underline{v}^s_i$. Since all these conditions leads the reciprocators to have a smaller critical discount factor than self-interested players, that is $\delta^r_{p^0} < \delta^s_{p^0}$, we are done. *Q.E.D.*