Positive Self-Image in Tournaments*

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This paper analyzes the implications of worker overestimation of productivity for firms in which incentives take the form of tournaments. Each worker overestimates his productivity but is aware of the bias in his opponent's self-assessment. The manager of the firm, on the other hand, correctly assesses workers' productivities and self-beliefs when setting tournament prizes. The paper shows that, under a variety of circumstances, firms can benefit from worker positive self-image. The paper also shows that worker positive self-image can improve welfare in tournaments. In contrast, workers' utility declines due to their own misguided choices.

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1 Introduction

This paper is related to a recent strand of literature in economics that studies the welfare consequences of behavioral biases. The paper focuses on the welfare implications of worker overestimation of skill when firms use tournaments to provide incentives.[1]

The paper derives two main results. First, it shows that, under a variety of circumstances, firms can benefit from worker positive self-image if they wisely structure prizes in tournaments. The paper argues that in order to do that, firms should take into account how self-image changes workers' incentives to exert effort. This finding is consistent with the idea that some parties involved in a contract might gain when other parties are not fully rational.

Second, the paper finds that moderate levels of worker positive self-

[1] Tournaments are one of many forms of providing incentives in firms. Managers are involved in promotion tournaments: vice-presidents compete to be promoted to president and senior executives compete to become CEO. Salespeople are often paid bonuses that depend on their sales relative to those of the other salespeople in the firm.

image can improve welfare in tournaments. This happens when: (i) workers have increasing absolute risk aversion, and (ii) positive self-image lowers the prizes that the firm needs to pay for workers' effort. This result is consistent with the theory of the second-best.

In this paper a worker with a positive self-image overestimates his productivity of effort but has an accurate assessment of his cost of effort and his outside option. The firm correctly assesses workers' productivities and self-beliefs. Each worker is aware that his opponent's perception of ability is mistaken but thinks that his own perception is correct. Thus, the firm and each worker hold divergent beliefs about the worker's productivity.[2]

Worker positive self-image has two effects in tournaments for fixed prizes. First, it makes participation in tournaments more attractive to workers than it should actually be. Since in a tournament higher prizes are paid to workers who produce higher output, a worker with a positive self-image will overestimate the probability that he will attain a high prize. This effect of worker positive self-image is favorable to the firm.[3]

Second, worker positive self-image can change workers' incentives to ex-

[2] In the standard tournament literature all parties are assumed to hold identical and accurate beliefs regarding the distribution of output induced by workers' effort choices.

[3] Santos-Pinto (2008), shows that positive self-image workers place more value in a contract with a wage-incentive scheme that is nondecreasing in output than accurate workers. By definition a tournament is a nondecreasing incentive scheme.

ert effort. A worker who thinks that he is more able than others may think that if he works harder the increase in utility associated with a higher probability of success more than compensates the increase in disutility associated with higher effort. It could also be argued that positive self-image reduces effort provision by workers. A worker who overestimates his probability of winning the tournament may think that by reducing effort the decrease in disutility of effort more than compensates the decrease in utility steaming from a lower probability of success.[4]

The first main finding of the paper is that firms can be better off with a positive self-image workforce if they wisely structure prizes in tournaments. This result holds under the following circumstances: (1) worker risk neutrality, (2) worker risk aversion and complementarity between self-image and effort, and (3) worker risk aversion, substitutability between self-image and effort, and moderate impact of self-image on effort.

When workers are *risk neutral* the firm can counter any impact of positive self-image on effort by changing the prize spread (the difference between the winner's prize and the loser's prize) while keeping total prizes fixed. Since a positive self-image worker overestimates the probability of winning the

[4] There could also be a non-monotonic relation between self-image and effort. For example, positive self-image may increase effort when a worker's effort level is lower than that of his coworker but reduce it when it is higher. I also considers this possibility.

tournament, the firm can change the prize spread and reduce total prizes.

When workers are *risk averse* and self-image and effort provision are *complements (*higher self-image increases workers' perceived marginal probability of winning the tournament for all effort levels*)* the firm can get more effort for a fixed prize structure or the same amount of effort with lower prizes. This result is valid under very general conditions.

Matters are not so straightforward when workers are *risk averse* and self-image and effort provision are *substitutes*. In this case the firm may not be able to counter the unfavorable impact of positive self-image on effort by raising the prize spread while simultaneously reducing total prizes. The reason is worker risk aversion implies that workers must be compensated for an increase in the prize spread. However, the paper shows that the firm is better off with a positive self-image workforce if workers are risk averse and higher self-image only leads to a moderate reduction in the perceived marginal probability of winning the tournament for all effort levels.

The paper also shows that firms can be better off with a positive self-image workforce when workers are risk averse and the relation between self-image and effort is non-monotonic. This happens when output is either exponentially or normally distributed. These two examples also illustrate how the relation between effort and self-image depends on effort levels, perceptions of skill, and technology.

The second main finding of the paper is that moderate levels of positive self-image can improve welfare in tournaments. This happens when workers have increasing absolute risk aversion and positive self-image lowers the prizes that the firm needs to pay for workers' effort.

The intuition behind this result is as follows. If workers have increasing absolute risk aversion and hold accurate perceptions of skill, there is undersupply of effort, that is, the effort level chosen by the firm will be less than the effort level that maximizes welfare. If positive self-image lowers the prizes that the firm needs to pay for workers' effort, then the firm prefers to select a higher effort level with a positive self-image workforce than with an accurate one. However, if worker positive self-image is too high welfare might decrease since either workers might shirk or the firm might decide to select an effort level that is greater than the one that maximizes welfare. Thus, moderate levels of worker positive self-image improve welfare because they reduce the undersupply of effort problem caused by increasing absolute risk aversion.

This result is consistent with the theory of the second best. According to this theory introducing a new distortion–worker positive self-image–in an environment where another distortion is already present–worker increasing absolute risk aversion–, can increase welfare. Of course, welfare does not always rise when workers' have biased beliefs. It was clear from the previous paragraph that high levels of positive self-image might reduce welfare when workers have increasing absolute risk aversion. The paper also shows that positive self-image always reduces welfare when workers are risk neutral.

Evidence from psychology and economics shows that most individuals hold overly favorable views of their skills.[5] This tendency is also present in workers' self-assessments of performance in their jobs. Myers (1996) cites a study according to which: "In Australia, 86 percent of people rate their job performance as above average, 1 percent as below average." Baker et al. (1998) cite a survey of General Electric Company employees according to which: "58 percent of a sample of white-collar clerical and technical workers rated their own performance as falling within the top 10 percent of their peers in similar jobs, 81 percent rated themselves as falling in the top 20 percent. Only about 1 percent rated themselves below the median."

Entrepreneurs, currency traders, and fund managers have also been shown to overestimate their skills. Oberlechner and Osler (2004) find that 75 percent of currency traders in foreign exchange markets think they are bet-

[5] Positive self-image is a staple finding in psychology. According to Myers (1996), a textbook in social psychology, "(...) on nearly any dimension that is both *subjective* and *socially desirable*, most people see themselves as better than average."

ter than average. Similarly, Brozynski et al. (2006) find that fund managers' hold overly positive views of their relative performance.[6]

Two experiments provide direct support for the notion that tournaments attract individuals who overestimate their skills. Camerer and Lovallo (1999) consider a market entry game where subjects payoffs are based on rank, which is determined either randomly or through a test of skill. They find that there is more entry when relative skill determines payoffs, which suggests that individuals overestimated their ability to do well on the test relative to others. Park and Santos-Pinto (2005) show that players in real world poker and chess tournaments overestimate their performance and are willing to bet on their overly positive perceptions of skill.

My paper is an additional contribution to the growing literature on the impact of behavioral biases on markets and organizations. The paper is closely related to papers in that literature that study the impact of biased beliefs on the employment relationship.[7]

Hvide (2002) shows that a worker can gain from overestimating his skill if that improves his bargaining position against the firm (the outside option).

[6] Overconfidence and positive self-image can persist and survive in the long run in financial markets–see Kyle and Wang (1997). Theoretical models of financial markets also predict that these biases lead to increased trading activity. Deaves et al. (2003) confirm this prediction using an asset market experiment.

[7] I will focus on papers that use the principal-agent with unobservable effort approach to model the worker-firm relationship.

The firm is made worse off by the worker's positive self-image. Bénabou and Tirole (2003) show that if a firm is better informed about a worker's skill than the worker, effort and self-image are complements, then the firm has an incentive to boost the worker's self-image by offering low-powered incentives that signal trust to the worker and increase motivation. Gervais and Goldstein (2007) find that a firm is better off with a team of workers who overestimate their skill when there are complementarities between workers' efforts.

My paper shows how firms can design prize structures in tournaments to take advantage of workers' inflated self-perceptions of skill. This finding stands in contrast to those in Hvide (2002) and does not rely on the assumption of complementarity between workers' efforts present in Gervais and Goldstein (2007). My paper also shows that moderate levels of worker positive self-image can improve welfare in tournaments. This result is in line with findings in Waldman (1994) and Bénabou and Tirole (2002).

2 Set-up

In this section I incorporate worker positive self-image into a generalized version of Nalebuff and Stiglitz's (1983) rank-order tournament model. The timing of the model is as follows: (1) the firm chooses the optimal prizes;

(2) workers observe the realization of a common "environmental" shock (this may be interpreted as an uncertain factor specific to one activity but that affects all workers within that activity similarly); (3) workers choose simultaneously the optimal level of effort after observing the realization of the common shock and the prizes chosen by the firm; (4) the output of each worker is determined by the worker's effort choice, the common shock, and an idiosyncratic shock specific to each agent and distributed independently across agents; (5) the firm observes the workers' ranking in terms of output; and (6) the firm awards the prizes to the workers according to their ranking.[8]

Throughout the paper attention is restricted to tournaments played between two workers.[9] Let $U^i\left(y^i, a^i\right)$ denote worker i's von Neumann-Morgenstern utility function, which is assumed to be increasing in income, y^i and decreasing in effort, a^i, with $a^i \in \mathcal{A}^i = [0, \infty)$, $i = 1, 2$. Let $\bar{U}$ represent the utility of an outside option. Worker i's output, q^i, is a stochastic function of his effort, in the sense that each level of effort induces a distribution

[8] By assumption, in a tournament, the firm is not able to observe workers' effort choices. This introduces the element of moral hazard to this multi-agent setting. The firm is also not able to observe the realization of the common shock and the realization of the idiosyncratic shocks.

[9] This symplifies the algebra. The results generalize to tournaments with more than two workers.

over output

$$G^i\left(q^i|e^i(a^i,\omega)\right), \tag{1}$$

$i=1,2$, where $e^i\left(a^i,\omega\right)$ is a measure of worker i's productivity and ω is the common shock. A worker's productivity is assumed to strictly increasing in effort but marginal productivity is subject to diminishing returns to effort. The cumulative distribution $G^i\left(q^i|e^i\right)$ is assumed to satisfy the monotone likelihood ratio condition, that is, for $q_2^i > q_1^i$ and $e_2^i > e_1^i$

$$\frac{g^i\left(q_2^i|e_2^i\right)}{g^i\left(q_2^i|e_1^i\right)} > \frac{g^i\left(q_1^i|e_2^i\right)}{g^i\left(q_1^i|e_1^i\right)}, \tag{2}$$

$i=1,2$, where $g^i\left(q^i|e^i\right)$ is the density function of $G^i\left(q^i|e^i\right)$.[10] Worker i's perception of his productivity is given by $e^i\left(a^i,\lambda^i,\omega\right)$, where $\lambda^i \in \mathbf{R}^+$ parameterizes worker i's degree of positive self-image. So, from worker i's perspective, each level of effort induces a distribution over output

$$G^i\left(q^i|e^i\left(a^i,\lambda^i,\omega\right)\right), \tag{3}$$

$i=1,2$. Worker i has a positive self-image if $G^i\left(q^i|e^i\left(a^i,\lambda^i,\omega\right)\right)$ first-order stochastically dominates $G^i\left(q^i|e^i\left(a^i,\omega\right)\right)$ for all $a^i \in \mathcal{A}^i$. That is, for any

[10] It is a well known result that this condition implies that $F^i(q^i|e_2^i)$ first-order stochastically dominates $F^i(q^i|e_1^i)$, for $e_2^i > e_1^i$.

effort level of worker i, worker i thinks that he is more likely to produce higher levels of output than he actually is. When $G^i\left(q^i|e^i\left(a^i,\lambda^i,\omega\right)\right) \equiv G^i\left(q^i|e^i\left(a^i,\omega\right)\right)$ we say that worker i has an accurate self-image and let $\lambda^i = \gamma^i$.

Worker i's mistaken beliefs of productivity influence behavior through worker i's perceived probability of winning the tournament. For a given output level of worker j, say $\bar{q}^j$, worker i perceived probability of winning the tournament is given by

$$\begin{aligned}\Pr\left(Q^i \geq q^j\right) &= 1-\Pr\left(Q^i \leq q^j\right) \\ &= 1-G^i\left(q^j|e^i\right).\end{aligned}$$

Thus, worker i's (unconditional) perceived probability of winning the tournament is given by

$$P^i\left(a^i,a^j,\lambda^i\right) = \int\left[1-G^i\left(q^j|e^i\left(a^i,\lambda^i,\omega\right)\right)\right]g^j\left(q^j|e^j\left(a^j,\omega\right)\right)dq^j, \qquad (4)$$

$j \neq i$, $i = 1,2$. We see that (1), (2), (3), and (4) imply that worker i's perceived probability of winning the tournament is increasing in worker i's effort choice, decreasing in worker j's effort choice, and increasing in worker i's self-image.

To be able to compute equilibria when workers' hold mistaken beliefs I follow Squintani's (2006) approach and assume that: (1) the manager of the firm correctly assesses workers' abilities and self-beliefs, (2) each worker is aware that his opponent's perception of ability is mistaken, and (3) each worker thinks that his own perception of ability is correct.

Worker i's ex post monetary income from taking part in the tournament is given by

$$y^i = \begin{cases} y_L & \text{if } q^i < q^j \\ y_W & \text{otherwise} \end{cases},$$

$j \neq i$, $i = 1, 2$, where y_L is the loser's prize and y_W the winner's prize, with $y_L < y_W$. Worker i's interim perceived expected utility (the utility after having observed the realization of the common shock but before the realization of the idiosyncratic shocks) is given by

$$V^i\left(a^i, a^j, \lambda^i, y_L, y_W\right) = U^i\left(y_L, a^i\right) + P^i\left(a^i, a^j, \lambda^i\right)\left[U^i\left(y_W, a^i\right) - U^i\left(y_L, a^i\right)\right]$$

$j \neq i$, $i = 1, 2$, and worker i's ex ante perceived expected utility is given by

$$E\left[V^i\left(a^i, a^j, \lambda^i, y_L, y_W\right)\right],$$

$j \neq i$, $i = 1, 2$, where the expectation is taken with respect to the common shock.

The firm is assumed to be risk neutral and to be concerned exclusively with the maximization of profits, that is, the difference between expected benefits and compensation costs:

$$\pi(q, y) = E\left[Q^1 + Q^2\right] - (y_L + y_W).$$

My analysis will focus on the monopsonistic firm, that is, a firm selling its product in a competitive output market but that has considerable influence in the input (labor) market.[11] The firm's problem is to find the optimal wages for the winning and the losing parties, (y_L, y_W) and the optimal effort choice for the workers, $\left(a^1, a^2\right)$, subject to the constraint that the latter be implemented as a Nash equilibrium between the workers by the chosen incentive scheme and the constraint that each worker receives an ex-ante perceived expected utility that is at least his reservation utility. Thus, the

[11] This is the dual of the firm's problem in Lazear and Rosen (1981) and Nalebuff and Stiglitz (1983).

firm solves

$$\max_{y_L, y_W} E\left[\sum_{i=1,2} \int q^i g^i \left(q^i | e^i\right) dq^i\right] - (y_L + y_W)$$

$$\text{s.t.} \quad a^i \in \arg\max_{a^i \in \mathcal{A}^i} V^i \left(a^i, a^j, \lambda^i, y_L, y_W\right), \ i = 1, 2,$$

$$E\left[V^i \left(a^i, a^j, \lambda^i, y_L, y_W\right)\right] \geq \bar{U}, \ i = 1, 2.$$

We know from Grossman and Hart (1983) and from Mookherjee (1984), that this problem can be decomposed into two parts: the *implementation problem* and the *effort selection problem*. In the implementation problem the firm, for any arbitrary effort pair (a_1, a_2), chooses the pair (y_L, y_W) that minimizes the firm's implementation cost, $C\left(a^1, a^2, \lambda^1, \lambda^2\right)$, subject to the incentive compatibility and the participation constraints. In the effort selection problem the firm chooses the effort pair $\left(a^1, a^2\right)$ that maximizes the difference between expected benefits and implementation cost. I will use this decomposition to characterize the impact of positive self-image on tournament outcomes.

3 The Specialized Model

This section specializes the model and shows that an equilibrium exists. I consider a special case of Nalebuff and Stiglitz's (1983) model.

First, I assume that workers are weakly risk averse and have identical von Neumann-Morgenstern utility functions additively separable in income and effort, that is

$$U^i\left(y^i, a^i\right) = u(y^i) - c(a^i),$$

where u and c are twice differentiable with $u' > 0$, $u'' \leq 0$, $c' > 0$, $c'' > 0$, and $c(0) \geq 0$. Second, I assume that workers have the same degree of positive self-image, that is $\lambda^1 = \lambda^2 = \lambda$. Third, I assume that there is no common shock to simplify the analysis. Fourth, worker's perceived stochastic production function is given by

$$Q^i = e^i(a^i, \lambda) + \varepsilon^i, \; i = 1, 2.$$

where G^i is the distribution function of ε^i, g^i its density, with g^i symmetric, $E(\varepsilon^i) = 0$, and $E(\varepsilon^i \varepsilon^j) = 0$ for $i \neq j$. I also assume that workers' perceived stochastic production functions are identical. Worker i's perceived probability of winning the tournament function $P\left(a^i, a^j, \lambda\right)$ is given by

$$P\left(a^i, a^j, \lambda\right) = \int \left[1 - G^i\left(e^j(a^j) - e^i(a^i, \lambda) + \varepsilon^j\right)\right] g^j(\varepsilon^j) d\varepsilon^j$$

Fifth, I assume that $P\left(a^i, a^j, \lambda\right)$ is twice differentiable in a^i, and differentiable in a^j and λ. Thus, worker i's perceived expected utility is given by

$$V^i\left(a^i, a^j, \lambda^i, y_L, y_W\right) = u\left(y_L\right) + P^i\left(a^i, a^j, \lambda\right) \triangle u - c(a^i), \tag{5}$$

where $\triangle u = u\left(y_W\right) - u\left(y_L\right)$. Worker i maximizes his perceived expected utility by choosing an optimal effort level, taking worker j's effort level, and prizes as given. Notice that, for each effort level selected by worker j worker i may either choose a positive effort level or a zero effort level (shirk). Thus, worker i solves

$$\max\left\{\max_{a^i>0}\left[P\left(a^i, a^j, \lambda\right) \triangle u - c\left(a^i\right)\right], P\left(0, a^j, \lambda\right) \triangle u - c(0)\right\}.$$

The global incentive compatibility condition is satisfied if both the level of positive self-image as well as the variance of the idiosyncratic shocks are not excessively high.[12] If the level of positive self-image is very high, the worker

[12] When the variance of the tournament is very high luck becomes much more important than effort (or self-image) and workers prefer not exert any effort. This is a standard

may think that his probability of winning the tournament is so high that he is better off by shirking.

Assuming that the global incentive compatibility condition is satisfied we can study the relaxed optimization problem

$$\max_{a^i>0} \; P\left(a^i, a^j, \lambda\right) \triangle u - c\left(a^i\right).$$

The first-order condition for this problem is given by

$$P_{a^i}\left(a^i, a^j, \lambda\right) \triangle u = c'\left(a^i\right),$$

and the second-order condition by

$$P_{a^i a^i}\left(a^i, a^j, \lambda\right) \triangle u - c''\left(a^i\right) < 0.$$

The second-order condition can be satisfied under a variety of conditions. For example, it is satisfied when the perceived probability of winning is increasing and concave in own effort, that is, $P_{a^i a^i} < 0$.[13] The second-order condition is also satisfied if $P_{a^i a^i}\left(a^i, a^j, \lambda\right) \triangle u < \min_{a^i} c''(a^i)$.[14]

feature of tournament models.

[13] This condition is satisfied if $G^{ii}_{ee}(q|e) > 0$, that is, if there are stochastically diminishing returns to effort. See Koh (1992).

[14] This condition ensures that workers' expected utility function is concave in own effort

Let $\Gamma_e(\lambda, y_L, y_W)$ denote the workers' simultaneous effort choice subgame.

Proposition 1 *If the symmetry and differentiability assumptions hold, the global incentive compatibility condition is satisfied, and V^i is strictly concave in a^i, then there exists a unique pure-strategy symmetric Nash equilibrium of $\Gamma_e(\lambda, y_L, y_W)$ with $a^i > 0$, $i = 1, 2$.*

The proof that a Nash equilibrium exists relies on the classical existence result due to Debreu (1952), Glicksberg (1952), and Fan (1952). The assumptions that the workers' expected utility is differentiable and strictly concave in own effort guarantee that there exists a unique equilibrium in pure strategies. The assumption that the global incentive compatibility condition is satisfied rules out a pure-strategy equilibrium where workers shirk. The symmetry assumptions guarantee that the equilibrium is symmetric.

In the unique symmetric pure-strategy Nash equilibrium the first-order condition of the representative worker's optimization problem becomes

$$P_a(a, \lambda) \triangle u = c'(a). \tag{6}$$

Equation (6) is the analogue of Nalebuff and Stiglitz's "cornerstone equa-

by requiring that the cost function is sufficiently convex. See the discussion in Lazer and Rosen (1981, p.845, fn. 2), or Nalebuff and Stiglitz (1983).

tion of tournaments" but now modified to take into account the presence of worker mistaken beliefs of ability. It tells us that in equilibrium workers should increase their effort level up to the point where the perceived marginal benefit of doing so–the perceived marginal probability of winning the tournament times the utility differential between winning and losing–equals its incremental cost–the marginal disutility of effort.

The relation between self-image and effort can be obtained from (6).[15] Effort and self-image are complements (substitutes) if, for a fixed prize structure, higher self-image increases (reduces) workers' effort. Differentiation of (6) with respect to self-image gives us

$$\frac{\partial a}{\partial \lambda} = -\frac{\partial^2 V/\partial a \partial \lambda}{\partial^2 V/\partial a^2} = -\frac{P_{a\lambda}(a,\lambda)\triangle u}{P_{aa}(a,\lambda)\triangle u - c''(a)}. \tag{7}$$

The denominator in (7) is the second-order condition and is negative. Since the utility prize spread is always positive the relation between effort and self-image only depends on the sign of $P_{a\lambda}(a,\lambda)$, that is, how self-image influences workers' perceived marginal probability of winning the tournament.

Thus, self-image and effort are complements when $P_{a\lambda}(a,\lambda) > 0$ for all a, that is, when a higher self-image increases workers' perceived marginal

[15] We can also see from (5) that the interaction between self-image and effort in the workers' perceived expected utility comes only through the perceived probability of winning the tournament.

probability of winning the tournament for all effort levels. In contrast, if a higher self-image decreases workers' perceived marginal probability of winning the tournament for all effort levels, $P_{a\lambda}(a,\lambda) < 0$ for all a, self-image and effort are substitutes. Since the sign of $P_{a\lambda}(a,\lambda)$ is jointly determined by effort levels, perceptions of skill, and technology (the distribution of output given effort), so is the relation between self-image and effort.

4 Risk Neutral Workers

If workers are risk neutral, we have (up to an affine transformation) $u(y) = y$ for all y. In this case the firm's per worker implementation problem is given by

$$\begin{aligned} \min_{y_L, y_W} \quad & \frac{1}{2}(y_L + y_W) \\ \text{s.t.} \quad & P_a(a,\lambda) \triangle y = c'(a) \\ & y_L + P(\lambda) \triangle y - c(a) \geq \bar{U}, \end{aligned}$$

where $\triangle y = y_W - y_L$. In the solution to this problem the participation constraint is always binding, otherwise it would be possible to implement the same effort level at a lower cost (by reducing $y_L + y_W$ while leaving $y_W - y_L$ unchanged). Solving the incentive compatibility constraint and the participation constraint for the optimal losing and winning prizes we obtain

$$y_L = \bar{U} + c(a) - \frac{P(\lambda)}{P_a(a,\lambda)} c'(a), \tag{8}$$

$$y_W = \bar{U} + c(a) + \frac{1 - P(\lambda)}{P_a(a,\lambda)} c'(a). \tag{9}$$

Adding up (8) and (9) and diving by 1/2 we have that per worker implementation cost is given by

$$C(a,\lambda) = \bar{U} + c(a) - \frac{P(\lambda) - \frac{1}{2}}{P_a(a,\lambda)} c'(a). \tag{10}$$

Let $T(\lambda)$ denote the tournament game with positive self-image workers and $T(\gamma)$ denote the tournament game with workers who have accurate perceptions of skill. Let $\hat{\lambda}$ denote the level of positive self-image that makes the worker indifferent between shirking and exerting effort. I use (10) to prove my next result.

Proposition 2 *If workers are risk neutral, then the firm's profits are higher in* $T(\lambda)$ *than in* $T(\gamma)$, *with* $\gamma < \lambda < \hat{\lambda}$.

This result shows that if workers are risk neutral, then the firm's cost of implementing any effort level is lower with a positive self-image workforce than with an accurate workforce. The intuition for this result is that workers' risk neutrality together with their positive self-image allow the firm to alter the prize spread while simultaneously reducing prizes. The change in the prize spread allows the firm to neutralize any impact of positive self-image on incentives. If self-image and effort are complements (substitutes), then the firm should reduce (increase) the prize spread to reduce (increase) effort. The fact that positive self-image makes participation in the tournament seem more attractive to workers than it actually should be allows the firm to reduce prizes.[16]

5 Risk Averse Workers

If workers are risk averse the firm's implementation problem is given by

$$\begin{aligned} \min_{u_L, u_W} \quad & \frac{1}{2}\left[h\left(u_L\right) + h\left(u_W\right)\right] \\ \text{s.t.} \quad & P_a\left(a, \lambda\right) \triangle u = c'\left(a\right) \\ & u_L + P\left(\lambda\right) \triangle u - c\left(a\right) \geq \bar{U}, \end{aligned}$$

[16] Positive self-image may be good or bad for the firm when workers are risk neutral but are protected by a limited liability constraint ($y_L \geq L \geq 0$). If higher self-image reduces effort and the limited liability contraint is binding, then positive self-image is bad for the firm. This happens because the firm needs to raise the prize spread to implement a given effort level. But, since the loser's prize cannot decrease due to limited liability, the only way for the firm to increase the prize spread is to raise the winner's prize.

where the firm's control variables are utility payments (u_L, u_W), with $u_L = u(y_L)$ and $u_W = u(y_W)$, rather than monetary payments (y_L, y_W), and $h = u^{-1}$. I will use this problem to study the impact of worker positive self-image on the firm's profits when workers are risk averse for (i) complementarity, (ii) substitutability, and (iii) non-monotonic relation between self-image and effort.

5.1 Self-Image and Effort Complements

If self-image and effort are complements, then, for a fixed prize structure, positive self-image leads workers to exert more effort than they would exert if they had accurate perceptions of skill. Furthermore, positive self-image relaxes the workers' participation constraint. This implies that the firm can implement the same actions with lower prizes or obtain more output for the same prizes. Thus, the firm's profits are higher in a tournament where workers overestimate their abilities and self-image and effort are complements, than in a tournament with accurate workers.

Proposition 3 *If workers are risk averse and $P_{a\lambda} \geq 0$, then the firm's profits are higher in $T(\lambda)$ than in $T(\gamma)$, with $\gamma < \lambda < \hat{\lambda}$.*

This result can be proved under very general conditions and does not depend on the particular assumptions of the specialized model. Appendix 2 shows that one can use the theory of supermodular games to show that if self-image and effort are weak complements and there is a weak complementarity in workers' effort choices, then the firm's profits are higher with a positive self-image workforce than with an accurate workforce.

It is easy to find perceptions of skill and production functions that lead to a complementarity between self-image and effort. For example, if output is uniformly distributed with support on $[\gamma a^i - \sigma, \gamma a^i + \sigma]$, with $\sigma > 0$, each worker perceives his own output to be uniformly distributed with support on $[\lambda a^i - \sigma, \lambda a^i + \sigma]$, with $\gamma < \lambda < \hat{\lambda}$, and the cost of effort function is sufficiently convex, then effort and self-image are complements.

5.2 Self-Image and Effort Substitutes

When self-image and effort are substitutes and workers are risk averse the firm may not be able to neutralize the unfavorable impact of positive self-image on effort by raising the prize spread while simultaneously reducing prizes. This happens because risk aversion implies that workers must be compensated for increases in the prize spread. However, we also know that, for fixed prizes, positive self-image workers find the tournament more attractive than accurate workers.

Now, consider the firm's implementation problem. The firm selects prizes to induce a desired level of effort subject to the individual rationality and incentive compatibility constraints. This opens the possibility that the firm, aware of workers' positive self-image and of its unfavorable effect in effort, may be able to choose a prize structure that implements the same effort level that the firm would like to implement if workers had accurate self-images and do it at a smaller cost. My next result provides a condition under which the firm can do that.

Proposition 4 *If workers are risk averse and* $-\frac{P'(\lambda)P_a(a,\lambda)}{1-P(\lambda)} < P_{a\lambda}(a,\lambda) < 0$ *for all* a, *then the firm's profits are higher in* $T(\lambda)$ *than in* $T(\gamma)$, *with* $\gamma < \lambda < \hat{\lambda}$.

This result says that if workers are risk averse, effort and self-image are substitutes, and higher self-image leads to a moderate reduction in workers' perceived marginal probability of winning the tournament for all effort levels, then the firm is still better off with a positive self-image workforce than with an accurate workforce.

When workers are risk averse they dislike increases in the prize spread. This makes it costly for the firm to counter the unfavorable impact that positive self-image has on effort. However, if higher self-image leads to a moderate reduction in workers' perceived marginal probability of winning the tournament for all effort levels, $-\frac{P'(\lambda)P_a(a,\lambda)}{1-P(\lambda)} < P_{a\lambda}(a,\lambda)$, the firm can increase effort by raising the utility prize spread while reducing prizes. The firm does this by reducing both the loser's and the winner's prize in a way such that the reduction in the utility of the loser's prize is larger than the reduction in the utility of the winner's prize. The reduction in prizes increases firm's profits.

If workers are risk averse and higher self-image leads to a large reduction in workers' perceived marginal probability of winning the tournament for all effort levels, $P_{a\lambda} < -\frac{P'(\lambda)P_a(a,\lambda)}{1-P(\lambda)}$, then the impact of positive self-image on implementation cost is ambiguous. The only thing we can say, without making further assumptions, is that higher worker risk aversion makes it harder for the firm to benefit from positive self-image for sufficiently large impact on effort. To see this suppose that the firm faces two or more workforces that only differ in their degree of risk aversion (ordered by the concavity of u). The cost of implementing an arbitrary effort level will be higher for the most risk averse workforce since the higher is risk aversion, the more expensive it becomes for the firm to increase the prize spread to counter a large unfavorable impact of positive self-image on effort.

5.3 Non-Monotonic Relation between Self-Image and Effort

Effort and self-image may be substitutes over some effort levels but complements over others. This case is of interest since plausible specifications of workers' perceptions of skill and technology imply a non-monotonic relation between effort and self-image.

Opening up this possibility complicates the analysis substantially and I am no longer able to state general results that link worker self-image to firm profits. So, I specialize the model even further and assume that output is either exponentially or normally distributed.[17] Appendix 3 shows that both production functions imply a non-monotonic relation between self-image and effort and contains the proofs. Here, I summarize the findings and give the intuition.

I find that positive self-image is always good for the firm when output is exponentially distributed and workers are risk averse. In this case positive self-image and effort are substitutes at symmetric effort levels so the firm must raise the prize spread in order to implement the same effort level as with accurate workers. The firm is able to do that while simultaneously decreasing both the winning and losing prizes.

[17] These are two stochastic production functions that are commonly used in the tournament literature. See Lazear and Rosen (1981) and Nalebuff and Stiglitz (1983).

When output is normally distributed and workers are risk averse, worker positive self-image is beneficial to the firm for low levels of bias. This happens because for a low level of bias, positive self-image and effort are complements at symmetric effort levels. In this case the firm is able to implement the same level of effort as with accurate workers by lowering the prize spread and reducing prizes. When positive self-image is high it is no longer clear if the firm is better off with a positive self-image or with an accurate workforce.

These two results show that even if there is a non-monotonic relation between effort and self-image the firm can be better off with a positive self-image workforce. They also show that if firms want to take advantage of worker positive self-image, they should care whether effort and self-image are substitutes or complements at symmetric effort levels.

6 Welfare

This section presents the other main finding of the paper, that welfare can be enhanced by moderate levels of worker positive self-image.

We know from Nalebuff and Stiglitz (1983) that welfare in a tournament is maximized at the first-best contract. The first-best contract specifies a level of effort and a reward that is independent of outcome. The first-best level of effort, a^{FB}, and the first-best reward, y^{FB}, are the solution to $\max_{a,y}$

$B(a)-y$ subject to $u(y)-c(a) \geq \bar{U}$, where $B(a)$ is the firm's expected benefit from effort. In the first-best contract effort is supplied until the marginal utility from income multiplied by the expected marginal benefit of effort is just equal to the marginal disutility of effort.

Let $y = 0.5(y_L + y_W)$ and $x = 0.5\triangle y$. If effort is not observable and the representative worker has self-image λ, the second-best contract is the vector $(a^{SB}(\lambda), x^{SB}(\lambda), y^{SB}(\lambda))$ that solves

$$\max_{a,x,y} \quad B(a) - y$$

$$\text{s.t.} \quad P_a(a,\lambda)\left[u(y+x) - u(y-x)\right] = c'(a)$$

$$u(y-x) + P(\lambda)\left[u(y+x) - u(y-x)\right] - c(a) \geq \bar{U},$$

where $\gamma \leq \lambda$. To show that moderate worker overestimation of skill can increase welfare in tournaments I only need to state conditions under which worker positive self-image makes the firm move the second-best level of effort closer to the first-best. Proposition 5 provides conditions for this to happen.

Proposition 5 *If workers are risk averse with u' concave, $C(a,\lambda) < C(a,\gamma)$, and $C_{aa}(a,\lambda) > 0$, for all a, then $a^{SB}(\gamma) < a^{SB}(\lambda) \leq a^{FB}$ and welfare is higher in $T(\lambda)$ than in $T(\gamma)$, with $\gamma < \lambda < \min(\hat{\lambda}, \tilde{\lambda})$, where $a^{SB}(\tilde{\lambda}) = a^{FB}$.*

This result shows that if positive self-image lowers the prizes that the firm needs to pay for workers' effort, then moderate levels of positive self-image reduce the undersupply of effort caused by increasing absolute risk aversion of workers.

The intuition for the result is as follows. If workers have increasing absolute risk aversion, then the second-best level of effort with accurate workers is smaller than the first-best.[18] If positive self-image reduces the firm's cost of implementing effort, then the second-best level of effort with positive self-image workers will be greater than the second-best level of effort with accurate workers. This improves welfare if worker positive self-image is moderate. Large levels of worker positive self-image might reduce welfare since they can either lead the firm to select a second-best level of effort greater than the first-best or make shirking overly attractive to workers. The assumption that $\lambda < \min(\hat{\lambda}, \tilde{\lambda})$ rules out these two possibilities.

The assumption of increasing absolute risk aversion is critical for this result. If workers have decreasing absolute risk aversion the impact of positive self-image on welfare is ambiguous since the second-best effort level with accurate workers can be greater than, equal to, or smaller than the first-best.[19]

[18] Nalebuff (1982) shows that this result holds for a firm that hires workers from a competitive labor market.

[19] See Nalebuff (1982) or Nalebuff and Stiglitz (1983).

Strict convexity of $C(a, \lambda)$ in a guarantees that the firm's per worker effort selection problem $\max_{a \geq 0} B(a) - C(a, \lambda)$ has a unique solution given that $B(a)$ is concave. The assumption that positive self-image reduces the firm's cost of implementing effort guarantees that worker positive self-image moves the second-best level of effort closer to the first-best. If workers have increasing absolute risk aversion and positive self-image increases the firm's cost of implementing effort the undersupply of effort problem is worsened and welfare decreases.[20]

This result is consistent with the theory of the second best.[21] This theory tells us that (i) in a world without distortions introducing a distortion reduces welfare and (ii) in a world where at least one distortion is present, introducing a new distortion might improve welfare (or reduce it). In a tournament where workers are risk neutral and have accurate perceptions of skill there are no distortions and the firm will select the first-best effort level. Introducing a distortion–worker increasing absolute risk aversion–in this setting generates undersupply of effort which reduces welfare. Proposition 5 shows us that introducing another distortion–worker positive self-image–can improve welfare.

[20] Propositions 3 and 4 provide conditions under which implementation cost goes down with positive self-image.

[21] See Lipsey and Lancaster (1956).

Proposition 5 is in line with results in Waldman (1994) and Bénabou and Tirole (2002). Waldman (1994) considers an environment where individuals compete in wealth accumulation, utility depends on wealth and disutility from effort, and males can overestimate or underestimate their own abilities. He finds that if there is sexual inheritance of the traits disutility from effort and perception of ability, then males exhibiting both disutility from effort and overestimation of abilities can be an evolutionary stable strategy. This happens because overestimation partially offsets the individual's incentives to choose low effort level by exaggerating the monetary returns to additional effort.

Bénabou and Tirole (2002) show that individuals with an imperfect knowledge of their skills and time-inconsistent preferences may prefer not to receive information about their abilities in order to preserve their self-confidence. This happens because maintaining a positive self-image improves motivation when ability and effort are complements. They also find that while positive thinking can improve welfare, it can also be self-defeating (and nonetheless pursued).

Of course, welfare in tournaments does not always improve with worker positive self-image. As we have just seen, when workers have increasing absolute risk aversion and positive self-image is very high welfare might decrease. I will now show that worker positive self-image always reduces welfare when workers are risk neutral.

It is a well know result in the tournament literature that if workers are risk neutral, then the firm can achieve the first-best level of effort by setting a prize spread equal to $\triangle y^* = c'(a^{FB})/P_a(a^{FB}, \gamma)$, where a^{FB} is the first-best level of effort, the solution to $\max_{a \geq 0} B(a) - [\bar{U} + c(a)]$. If workers are risk neutral and have positive self-image, then the firm can implement effort level $a^*(\lambda)$ by setting a prize spread equal to $\triangle y^*(\lambda) = c'(a^*(\lambda))/P_a(a^*(\lambda), \lambda)$, where $a^*(\lambda)$ is the solution to $\max_{a \geq 0} B(a) - C(a, \lambda)$, with $C(a, \lambda)$ given by (10). Recall from Proposition 2 that $C(a, \lambda) < \bar{U} + c(a)$. The reduction in implementation cost, concavity of $B(a)$, strict convexity of $C(a, \lambda)$ in a, and the first-order condition of each effort selection problem imply that $a^{FB} < a^*(\lambda)$ for all $\lambda \in (\gamma, \hat{\lambda})$.[22] Thus, positive self-image reduces welfare under worker risk neutrality since it leads to oversupply of effort.

This result is also consistent with the theory of the second best. Introducing a distortion–worker positive self-image–in an environment where there are no distortions–workers are risk neutral and have accurate self-images–reduces welfare.

[22] A sufficient condition for $C(a, \lambda)$ to be strictly convex in a when workers are risk neutral is $c'''(a) \leq 0$.

7 Workers' Utility

From the point of view of an outside observer who knows the worker's actual productivity, if a worker's beliefs are mistaken, then the worker's ex-ante actual expected utility will differ from his reservation utility. For example, the ex-ante actual expected utility of a risk neutral worker who overestimates his productivity is given by

$$V(a,\lambda,y_L,y_W) = \frac{y_L + y_W}{2} - c(a) = \bar{U} - \frac{P(\lambda) - \frac{1}{2}}{P_a(a,\lambda)} c'(a),$$

where the second equality is obtained by replacing $y_L + y_W$ by $C(a,\lambda)$. We see that if the worker overestimates his productivity, then $P(\lambda) > 1/2$, and his ex-ante actual expected utility is smaller than his reservation utility.

The example illustrates a general result, that does not require a formal proof. From the perspective of an outside observer, positive self-image workers are worse off by comparison with accurate workers since the firm will pay them less than their reservation utility.[23]

[23] This is consistent field and experimental data that shows that mistaken perceptions of risk lead to financial losses. See Camerer and Lovallo (1991), Simon and Houghton (2003), and Malmendier and Tate (2008).

8 Discussion

The main implication of this paper for hiring decision by firms is that, everything else equal, firms should have a preference for hiring workers who overestimate skill when they use tournaments to provide incentives. In other words, if two job applicants have the same productivities, preferences towards risk, cost of effort, and outside options, then the firm should hire the one who holds the most positive view of his skill.

In settings where performance depends on ability positive self-image leads individuals to overestimate the probability of favorable outcomes. If this is the case individuals should, on average, prefer incentive schemes featuring payments contingent on relative performance (e.g., rank-order tournaments or incentive schemes composed partly by fixed pay and partly by variable pay dependent on the magnitude of relative performance) to individualistic incentive schemes (e.g., fixed salary plans or piece rates).

In this paper the firm is a monopsonist in the market for workers' services. This assumption implies that the firm can make a take-it-or-leave it offer to the workers and get all the surplus from the employment relationship. This assumption is appropriate when there is a large pool of workers and a small number of firms. If there is a large number of firms competing for the services of a few workers, then it would be better to assume that the firm chooses tournament prizes to maximize workers' expected utility given incentive compatibility and zero-profit constraints. In this case, there is no impact of worker mistaken beliefs on the firm's profits since the zero-profit constraint implies that workers get all the surplus from the employment relationship.[24]

The paper studies tournaments with two workers and two prizes to make the analysis simpler. However, the results obtained extend to tournaments with more than two workers and more than two prizes. The model also assumes that the firm faces an homogeneous workforce in terms of productivity and self-image.[25] These assumptions simplify the firm's problem by looking only at a representative worker. If one of these assumptions is dropped the tournament becomes asymmetric but the main findings will hold.

[24] The impact of worker overestimation of skill on worker utility when firms compete for workers' services depends on worker preferences towards risk. If workers are risk neutral and have accurate perceptions of productivity, the tournament elicits the first-best effort level. However, if workers are risk neutral and overestimate their skills the tournament no longer implements the first-best effort level and workers are worse off. When workers are risk averse the impact of positive self-image on worker utility is ambiguous.

[25] Workers are likely to differ in their productivities and their perceptions of skill. If there is no correlation between skill and perceptions of skill, the results in the paper apply to the average worker in the firm.

Appendix

A.1.

Proof of Proposition 1 To show that a pure-strategy equilibrium exists I need to show that (a) worker i's strategy set is nonempty, convex, and a compact subset of $\mathbf{R}$; and (b) worker i's expected utility is continuous in a^i and a^j, and quasiconcave in a^i. Let us start by verifying (a). Worker i's effort belong to the set $[0, \infty)$ which is not compact. However, for a^i too large costs must dominate benefits, so these strategies are dominated. This follows from the assumption that costs are convex. So, in effect, worker i's effort will belong to a set $\left[0, \bar{a}^i\right]$, with $\bar{a}^i$ finite, which is a nonempty, convex and a compact subset of $\mathbf{R}$. Let us now verify (b). The assumptions that u, c and P are twice differentiable imply that worker i' s perceived expected utility function is continuous in a^i and a^j in the set $\left[0, \bar{a}^i\right]$. The assumption of strictly concave of expected utility in a^i for all $a^i \in \left[0, \bar{a}^i\right]$ implies quasiconcave of expected utility in $a^i \in \left[0, \bar{a}^i\right]$. Thus, since all the required conditions are satisfied there exists a pure-strategy Nash equilibrium.[26] The strict concavity of the expected utility function implies that the pure-strategy equilibrium is unique. The assumption that the global

[26] The assumption that P is differentiable in a^i and a^j implies that P is continuous in a^i and a^j. It is a well know result that continuity of P rules out situations where the variance of the idiosyncratic shocks is so small that there is no equilibrium in pure-strategies (but there is an equilibrium in mixed-strategies). See discussion in Nalebuff and Stiglitz (1983).

incentive compatibility condition is satisfied rules out a pure-strategy equilibrium with zero effort. Finally, the equilibrium is symmetric. Suppose, by contradiction, there exists an asymmetric pure-strategy Nash equilibrium such that $a^1 > a^2$. Then, by the workers' first-order conditions, we have

$$P_{a^1}\left(a^1, a^2, \lambda\right)\left[u\left(y_W\right) - u\left(y_L\right)\right] = c'\left(a^1\right), \tag{11}$$

and

$$P_{a^2}\left(a^2, a^1, \lambda\right)\left[u\left(y_W\right) - u\left(y_L\right)\right] = c'\left(a^2\right), \tag{12}$$

with

$$P_{a^i}\left(a^i, a^j, \lambda\right) = -e^i_{a^i}(a^i, \lambda) \int g^i\left(e^j(a^j) - e^i(a^i, \lambda) + \varepsilon^j\right) g^j(\varepsilon^j) d\varepsilon^j,$$

$j \neq i = 1, 2$. The assumption that the marginal productivity of effort is subject to diminishing returns to effort implies that

$$e^1_{a^1}(a^1, \lambda) < e^2_{a^2}(a^2, \lambda) \text{ for } a^1 > a^2. \tag{13}$$

The assumption that $G^i\left(q^i | e^i\right)$ satisfies the monotone likelihood ratio property and the symmetry assumptions imply that

$$g^1\left(e^2(a^2) - e^1(a^1,\lambda) + \varepsilon^2\right) < g^2\left(e^1(a^1) - e^2(a^2,\lambda) + \varepsilon^1\right) \text{ for } a^1 > a^2. \tag{14}$$

It follows from (13) and (14) that

$$P_{a^1}\left(a^1, a^2, \lambda\right) < P_{a^2}\left(a^2, a^1, \lambda\right). \tag{15}$$

Dividing (11) by (12) and making use of (15) we obtain $c'\left(a^1\right) < c'\left(a^2\right)$, which contradicts $a^1 > a^2$. The case $a^1 < a^2$ is similar. *Q.E.D.*

Proof of Proposition 2 Let a denote an arbitrary effort level that the firm can implement when workers are risk neutral and have accurate self-images. If workers are risk neutral and have beliefs of productivity given by λ and the firm selects a prize spread equal to $\triangle y = c'(a)/P_a(a,\lambda)$, then the firm can implement effort level a. If workers have accurate self-images the symmetry of the specialized tournament model implies that $P(\gamma) = 1/2$ and the last term on the right hand side of (10) is zero. In this case implementation cost is equal to $C(a,\gamma) = \bar{U} + c(a)$. If workers have positive self-image $P(\lambda) > 1/2$ and the last term on the right hand side of (10) is negative. Thus, $C(a,\lambda) < C(a,\gamma)$, for all $\lambda \in (\gamma, \hat{\lambda})$. *Q.E.D.*

Proof of Proposition 3 Let a denote an arbitrary effort level that the firm can implement when workers have accurate self-images. If workers have a degree of positive self-image given by λ and the firm selects a utility prize spread equal to $\triangle u(\lambda) = c'(a)/P_a(a,\lambda)$, then the firm can implement effort level a. If $P_{a\lambda} = 0$ then positive self-image has no impact on the incentive compatibility constraint and there is no need to alter the prize spread. If $P_{a\lambda} > 0$ it follows that $\triangle u(\lambda) < \triangle u(\gamma)$ for all $\lambda \in (\gamma, \hat{\lambda})$. If the prize spread decreases and workers are risk averse they face less risk. Furthermore, positive self-image relaxes the workers' participation constraint. This implies that when $P_{a\lambda} \geq 0$ the firm can implement the same effort with lower prizes for all $\lambda \in (\gamma, \hat{\lambda})$. *Q.E.D.*

Proof of Proposition 4 Let a denote an arbitrary effort level that the firm can implement when workers are risk averse and have accurate self-images. If workers are risk averse and have a degree of positive self-image given by λ and the firm selects a utility prize spread equal to $\triangle u(\lambda) = c'(a)/P_a(a,\lambda)$ then the firm can implement effort level a. I will now prove that the firm can lower implementation cost. Solving the incentive compatibility constraint and the participation constraint for the utility of the losing and winning prizes we obtain

$$u_L(\lambda) = \bar{U} + c(a) - \frac{P(\lambda)}{P_a(a,\lambda)} c'(a),$$

$$u_W(\lambda) = \bar{U} + c(a) + \frac{1 - P(\lambda)}{P_a(a,\lambda)} c'(a).$$

Implementation cost is given by

$$C(a, \lambda) = \frac{1}{2}[h(u_L(\lambda)) + h(u_W(\lambda))].$$

The fact that $P(\lambda)$ is increasing with λ and the assumption that $P_{a\lambda}(a, \lambda) < 0$ imply that $\frac{P(\lambda)}{P_a(a,\lambda)}$ is increasing with λ and so $u_L(\lambda)$ is decreasing with λ. The assumption that $-\frac{P'(\lambda)P_a(a,\lambda)}{1-P(\lambda)} \leq P_{a\lambda}(a, \lambda)$ implies that $\frac{1-P(\lambda)}{P_a(a,\lambda)}$ is nonincreasing with λ. This in turn implies that $u_W(\lambda)$ is nonincreasing with λ. If $u_L(\lambda)$ is decreasing with λ and $u_W(\lambda)$ is nonincreasing with λ, then $C(a, \lambda) < C(a, \gamma)$ for all $\lambda \in (\gamma, \hat{\lambda})$. *Q.E.D.*

Proof of Proposition 5 To prove this result I will show that the assumptions made imply $a^{SB}(\gamma) < a^{SB}(\lambda) \leq a^{FB}$, for all $\gamma < \lambda < \min(\hat{\lambda}, \tilde{\lambda})$. I'll start by showing that if workers have increasing absolute risk aversion (u' is concave) and hold accurate perceptions of skill, then $a^{SB}(\gamma) < a^{FB}$.[27]

[27] I'll apply the method of proof in Nalebuff (1982). The main difference here is that I consider a monopsonistic firm–a firm that maximizes profits subject to the representative

The second-best effort level and prizes with accurate workers, the triple $(a^{SB}(\gamma), x^{SB}(\gamma), y^{SB}(\gamma))$, are the solution to

$$\max_{a,x,y} \quad \pi(a,x,y) = B(a) - y$$
$$\text{s.t.} \quad P_a(a,\gamma)\left[u(y+x) - u(y-x)\right] = c'(a)$$
$$\frac{1}{2}\left[u(y+x) + u(y-x)\right] - c(a) \geq \bar{U}.$$

At the optimal prize structure both constraints are binding. Since u is concave, a larger prize (increasing x but keeping y fixed) implies that each worker puts in more effort to increase the probability of winning, that is, $da/dx > 0$.

The optimal reward for each worker in the first-best contract is independent of outcome, i.e., $x = 0$. The first-best effort level and reward, the pair (a^{FB}, y^{FB}), are the solution to $\max_{a,y} \pi(a,y) = B(a) - y$ subject to $u(y) - c(a) \geq \bar{U}$. The first-order conditions to this problem are given by

$$\begin{aligned} u'(y^{FB})B'(a^{FB}) &= c'(a^{FB}) \\ u(y^{FB}) - c(a^{FB}) &= \bar{U}. \end{aligned} \tag{16}$$

worker's incentive compatibility and participation constraints–whereas Nalebuff (1982) considers a firm that faces a competitive labor market–a firm that maximizes the representative worker's utility subject to the incentive compatibility and zero-profit constraints.

If a^{FB} can be implemented by a tournament in the second-best setting, there must exist (x^D, y^D) such that

$$\begin{aligned} P_a\left(a^{FB},\gamma\right)\left[u(y^D+x^D)-u(y^D-x^D)\right] &= c'\left(a^{FB}\right) \\ \frac{1}{2}\left[u(y^D+x^D)+u(y^D-x^D)\right] &= u(y^{FB}). \end{aligned} \tag{17}$$

In general, $a^{FB} \neq a^{SB}(\gamma)$. To compare the effort level at the second-best solution with that at the first-best, I consider the prize spread x^D and the income y^D, that motivate the first-best level of effort in the second-best solution and see if the prize spread is "too big" or "too small." If the sign of $d\pi/dx$ evaluated at (x^D, y^D) is negative, then the prize x^D is too big and the second-best level of effort is smaller than the first-best. If the sign of $d\pi/dx$ evaluated at (x^D, y^D) is positive, then the prize x^D is too small and the second-best level of effort is larger than the first-best.

From the firm's objective function we have that

$$\frac{d\pi}{dx} = B'(a)\frac{da}{dx} - \frac{dy}{dx}. \tag{18}$$

Differentiating the incentive compatibility and participation constraints we obtain

$$P_a\left[u'(y+x)\left(\frac{dy}{dx}+1\right)-u'(y-x)\left(\frac{dy}{dx}-1\right)\right] = c''(a)\frac{da}{dx} \quad (19)$$

$$\frac{1}{2}\left[u'(y+x)\left(\frac{dy}{dx}+1\right)+u'(y-x)\left(\frac{dy}{dx}-1\right)\right] = c'(a)\frac{da}{dx}. \quad (20)$$

Solving (19) and (20) for dy/dx we find that

$$\frac{dy}{dx}=\frac{\frac{c'(a)}{c''(a)}P_aS-\frac{1}{2}\triangle u'}{\frac{1}{2}S-\frac{c'(a)}{c''(a)}P_a\triangle u'}>0, \quad (21)$$

where $S=u'(y-x)+u'(y+x)$ and $\triangle u'=u'(y+x)-u'(y-x)$. From (20) we have

$$\frac{da}{dx}=\frac{1}{2c'(a)}\left(S\frac{dy}{dx}+\triangle u'\right)>0. \quad (22)$$

Substituting (22) into (18) we obtain

$$\begin{aligned}\frac{d\pi}{dx} &= \frac{B'(a)}{2c'(a)}\left(S\frac{dy}{dx}+\triangle u'\right)-\frac{dy}{dx}\\ &= \left(\frac{B'(a)}{2c'(a)}S-1\right)\frac{dy}{dx}+\frac{B'(a)}{2c'(a)}\triangle u'.\end{aligned}$$

Evaluating $d\pi/dx$ at (y^D, x^D) gives us

$$\left.\frac{d\pi}{dx}\right|_{(x^D,y^D)} = \left(\frac{B'(a^{FB})}{2c'(a^{FB})}S(x^D,y^D)-1\right)\frac{dy}{dx}+\frac{B'(a^{FB})}{2c'(a^{FB})}\triangle u'$$
$$= \left(\frac{S(x^D,y^D)}{2u'(y^{FB})}-1\right)\frac{dy}{dx}+\frac{1}{2u'(y^{FB})}\triangle u' \qquad (23)$$

where the second equality comes from (16). I will now show that if u' is concave, then the sign of $d\pi/dx$ at (y^D, x^D) is negative and so the second-best level of effort is less than the first-best. The concavity of u implies that $\triangle u'(x^D, y^D) < 0$. We also know that $dy/dx > 0$. Thus, the sign of $d\pi/dx$ at (y^D, x^D) is negative provided that $S(x^D, y^D) < 2u'(y^{FB})$ or

$$\frac{1}{2}[u'(y^D+x^D)+u'(y^D-x^D)] < u'(y^{FB}). \qquad (24)$$

Since y is increasing with x we know that $y^D > y^{FB}$ for $x > 0$. This together with concavity of u implies that

$$\frac{1}{2}[u'(y^D+x^D)+u'(y^D-x^D)] < \frac{1}{2}[u'(y^{FB}+x^D)+u'(y^{FB}-x^D)], \qquad (25)$$

but if u' is concave we have that

$$\frac{1}{2}[u'(y^{FB}+x^D)+u'(y^{FB}-x^D)] < u'(y^{FB}). \qquad (26)$$

Inequality (24) follows from (25) and (26).

I will now show that if $C(a, \lambda) < C(a, \gamma)$ for all a, and $C_{aa}(a, \lambda) > 0$, then $a^{SB}(\gamma) < a^{SB}(\lambda)$. We know that $a^{SB}(\gamma)$ is the solution to $\max_{a \geq 0} B(a) - C(a, \gamma)$, and that $a^{SB}(\lambda)$ is the solution to $\max_{a \geq 0} B(a) - C(a, \lambda)$. The first-order conditions to these two problems are $B'(a) = C_a(a, \gamma)$ and $B'(a) = C_a(a, \lambda)$, respectively. It is a straightforward to see that $C(a, \lambda) < C(a, \gamma)$ for all a, concavity of $B(a)$, strict convexity of $C(a, \lambda)$ in a, and the first-order condition of each effort selection problem imply that $a^{SB}(\gamma) < a^{SB}(\lambda)$. Finally, the assumption that $\lambda < \tilde{\lambda}$, where $a^{SB}(\tilde{\lambda}) = a^{FB}$ guarantees that positive self-image is not so large as to lead the firm to choose a second-best level of effort that is greater than the first-best. Thus, under the assumptions made we have that $a^{SB}(\gamma) < a^{SB}(\lambda) \leq a^{FB}$, for all $\lambda < \min(\hat{\lambda}, \tilde{\lambda})$. *Q.E.D.*

A.2. This appendix shows that if there is a weak complementarity in workers' effort choices and if workers' self-image and effort levels are complements, then the firm's profits are higher with a positive self-image workforce than with an accurate workforce.

To prove this result I need to provide conditions under which a worker's effort is increasing or decreasing with changes in positive self-image for fixed prizes. Worker i's effort choice problem, for a given realization of the common shock, is given by

$$\max_{a^i \in \mathcal{A}^i} U^i\left(y_L, a^i\right) + P^i\left(a^i, a^j, \lambda^i\right)\left[U^i\left(y_W, a^i\right) - U^i\left(y_L, a^i\right)\right]. \qquad (27)$$

Let

$$A^i\left(\lambda^i, y_L, y_W\right) \equiv \arg\max_{a^i \in \mathcal{A}^i} V^i\left(a^i, a^j, \lambda^i, y_L, y_W\right),$$

denote the set of maximizers in problem (27) as a function of λ^i, y_L, and y_W. For fixed prizes, the worker will never want to choose an infinite effort. So, the worker's effort choice set is compact. I also assume that V^i is order upper semi-continuous in a^i. This assumption together with the fact that the worker's effort choice set is compact guarantees that the set of maximizers $A^i\left(\lambda^i, y_L, y_W\right)$ is nonempty.

To make operational the view that higher self-image increases workers' effort I use the definition of increasing differences. This definition tells us that a function $h : \mathbf{R}^{\mathbf{2}}_{+} \to \mathbf{R}$ has *increasing differences* in (x, θ) if for all $x'' > x'$, the difference $h(x'', \theta) - h(x', \theta)$ is nondecreasing in θ. The property of increasing differences represents the economic notion of complementarity.[28]

[28] If h is a benefit function and $x'' > x'$, then the incremental benefit of increasing x' to x'' is $h(x'', \theta) - h(x', \theta)$. If h has increasing differences in (x, θ), then the incremental benefit from increasing x' to x'' when $\theta = \theta''$ is higher than the incremental benefit from increasing x' to x'' when $\theta = \theta'$, for any $\theta'' > \theta'$.

Athey et al. (1998) show that the set of maximizers defined by

$$X(\theta; z) \equiv \arg\max_{x \in S} \; h(x, \theta) + z(x),$$

is nondecreasing in θ for all functions z, that is,

$$\theta > \theta' \text{ implies } X(\theta; z) \succsim_S X^*(\theta'; z),$$

if and only if the function $h : \mathbf{R}^{\mathbf{2}}_+ \rightarrow \mathbf{R}$ has increasing differences in (x, θ).[29] This equivalence is used to state my first result. Define

$$H^i\left(a^i, a^j, \lambda^i, y_L, y_W\right) \equiv P^i\left(a^i, a^j, \lambda^i\right)\left[U^i\left(y_W, a^i\right) - U^i\left(y_L, a^i\right)\right].$$

Lemma 1 *$A^i\left(\lambda^i, y_L, y_W\right)$ is nondecreasing in λ^i if and only if H^i has increasing differences in $\left(a^i, \lambda^i\right)$.*

Proof An application of Theorem 2.3 in Athey et al. (1998) *Q.E.D.*

Lemma 1 states that if a worker's self-image and effort are complements,

[29] The symbol $\succsim_S$ stands for the *strong set order.* A set $M \subseteq \mathbf{R}$ is as high as another set $N \subseteq \mathbf{R}$ (in the strong set order), written $M \succsim_S N$, if for every $x \in M$ and $y \in N$, $y \geq x$ implies both $x \in M \cap N$ and $y \in M \cap N$. A set M is higher than N, written $M \succ_S N$ if M is as high as N but N is not as high as M. A set-valued function $V : \mathbf{R} \rightarrow \mathbf{2}^{\mathbf{R}}$ is nondecreasing if for $x > y$, $V(x) \succsim_S V(y)$.

then the higher is self-image the higher is a worker's set of optimal effort choices. When self-image and effort are complements an increase in self-image leads to an increase in effort since the increase in the perceived incremental probability of winning the tournament times the utility prize spread is higher when self-image is higher.

We are interested in finding necessary and sufficient conditions on the structure of the effort choice subgame that together with the conditions found on the workers' individual effort choice problems, allow us to know how the set of pure-strategy Nash equilibria effort levels changes with positive self-image.

Milgrom and Roberts (1990) show that if the game Γ is a supermodular game then it has a pure-strategy Nash equilibrium. Furthermore, they show that Γ is a supermodular game where the payoff functions are parameterized by τ then it is possible to provide comparative static results that link a change in τ with a change in the smallest and largest of Nash equilibrium of Γ. I make use of these results to prove existence of equilibrium in the workers' effort choice subgame and to state comparative static results relating the workers' degree of positive self-image to the smallest and the largest Nash equilibrium of the effort choice subgame.

Let $\Gamma_e\left(\lambda^1, \lambda^2, y_L, y_W\right) = \left\{\{1,2\}, \left(\mathcal{A}^i, V^i, i \in \{1,2\}\right), \geq\right\}$ denote the simultaneous effort choice subgame for levels of positive self-image (λ^1, λ^2) and for prize structure (y_L, y_W). According to Milgrom and Roberts (1990), Γ_e is a supermodular subgame if (i) $\mathcal{A}^i$ is a compact interval in $\mathbf{R}$, (ii) $V^i(a^i, a^j)$ is order upper semi-continuous in a^i for fixed a^j and order continuous in a^j for fixed a^i, and $V^i(a^i, a^j)$ has a finite upper bound, (iii) V^i has increasing differences in a^i, and (iv) V^i has increasing differences in (a^i, a^j).

We see that Γ_e satisfies condition (i) since, for any finite prize structure it is never optimal for the workers to choose an infinite amount of effort. Γ_e also satisfies the first requirement of condition (ii) since we have assumed before that V^i is order upper semi-continuous in a^i, $i = 1, 2$. Condition (iii) is satisfied trivially since workers' choice variables are scalars. So, for Γ_e to be a supermodular subgame we need to assume that it also satisfies condition (iv) and the second and third requirements in condition (ii). The next result guarantees the existence of a pure-strategy Nash equilibrium in Γ_e by imposing the remaining conditions that make it a supermodular subgame.

Lemma 2 *If H^i has increasing differences in (a^i, a^j), $j \neq i$, $i = 1, 2$, V^i is order continuous in a^j for fixed a^i, $j \neq i$, $i = 1, 2$, and V^i has a finite upper bound, then $\Gamma_e(\lambda^1, \lambda^2, y_L, y_W)$ has a pure-strategy Nash equilibrium.*

Proof The assumption that H^i has increasing differences in $\left(a^i, a^j\right)$, $j \neq i$, $i = 1, 2$, implies that V^i has increasing differences in $\left(a^i, a^j\right)$, $j \neq i$, $i = 1, 2$, since the interaction between a^1 and a^2 in the workers' interim perceived payoff functions is only through H^i. The assumption that V^i is continuous in a^j and has a finite upper bound together with the fact that V^i has increasing differences in $\left(a^i, a^j\right)$, $j \neq i$, $i = 1, 2$, imply that all the conditions required for $\Gamma_e\left(\lambda^1, \lambda^2\right)$ to be a supermodular game are satisfied. But then, by Theorem 5 in Milgrom and Roberts (1990), $\Gamma_e\left(\lambda^1, \lambda^2\right)$ has a pure-strategy Nash equilibrium. *Q.E.D.*

The assumption that H^i has increasing differences in $\left(a^i, a^j\right)$, $j \neq i$, $i = 1, 2$, imposes the missing structure in the Γ_e that, together with the two other assumptions, allows us to use the order-theoretic approach to state this existence result. This assumption restricts the type of interaction between the workers choice variables by forcing a^1 and a^2 to be weak complements. That is, we restrict attention to effort choice subgames where a worker's increase in effort makes it more desirable for his opponent to increase effort too.

The assumptions that guarantee that condition (ii) is verified rule out the possibility that there is no equilibrium and the possibility that there exists a mixed-strategy equilibrium but not a pure-strategy equilibrium. For example, if the variability of the idiosyncratic shocks is too small (chance is not a significant factor in the outcome of the tournament) the game Γ_e does not satisfy condition (ii) and there is no pure-strategy equilibrium but there exists a mixed strategy equilibrium.[30]

Lemma 3 *If* $\Gamma_e\left(\lambda^1, \lambda^2, y_L, y_W\right)$ *is a supermodular subgame and* H^i *has increasing differences in* $\left(a^i, \lambda^i\right)$, $i = 1, 2$, *then the smallest and the largest pure-strategy Nash equilibria of* $\Gamma_e\left(\lambda^1, \lambda^2, y_L, y_W\right)$ *are nondecreasing functions of* $\left(\lambda^1, \lambda^2\right)$.

Proof An application of Theorem 6 in Milgrom and Roberts (1990). *Q.E.D.*

Lemma 3 states that if there is a weak complementarity in the workers' effort choices and if workers' self-image and effort levels are complements, then the higher is the workers' degree of positive self-image the higher will be the smallest and the largest Nash equilibria effort levels of Γ_e.[31] I use Lemma 3 to characterize the impact of worker positive self-image on the

[30] To see this consider the extreme case where the distribution of the idiosyncratic shocks is degenerate, that is, the outcome of the tournament is completely deterministic. In this case, as Nalebuff and Stiglitz (1983) point out, each worker can assure that he wins the tournament by increasing his effort slightly above that of his opponent. But then, beyond some critical effort level, it is better to shirk and be certain to receive the losing prize than incurring in a very high disutility of effort and capturing the winning prize. Although there exists no pure-strategy Nash equilibrium, Nalebuff and Stiglitz (1983) show that, in their tournament model, there exists a mixed-strategy Nash equilibrium.

[31] Note that this result does not imply that all Nash equilibria of $\Gamma_e\left(\lambda^1, \lambda^2, y_L, y_W\right)$ are nondecreasing functions of $\left(\lambda^1, \lambda^2\right)$. In fact we may have that a Nash equilibrium in the interior of the set of Nash equilibria of $\Gamma_e\left(\lambda^1, \lambda^2, y_L, y_W\right)$ is lower than the correspondent Nash equilibrium in the interior of the set of Nash equilibria of $\Gamma_e\left(\beta^1, \beta^2, y_L, y_W\right)$ with $\left(\lambda^1, \lambda^2\right)$ higher than $\left(\beta^1, \beta^2\right)$.

firm's profits when self-image and effort are complements.

Theorem 1 *If* $\Gamma_e\left(\lambda^1, \lambda^2, y_L, y_W\right)$ *is a supermodular game and* H^i *has increasing differences in* $\left(a^i, \lambda^i\right)$, $i = 1, 2$, *then the firm's profits are higher in* $T\left(\lambda^1, \lambda^2\right)$ *than in* $T\left(\gamma^1, \gamma^2\right)$, *with* $\lambda^i > \gamma^i$, $i = 1, 2$.

Proof We know from Lemma 3 that if workers' self-image and effort are complements the smallest and the largest pure-strategy Nash equilibria of $\Gamma_e\left(\lambda^1, \lambda^2, y_L, y_W\right)$ are larger than the smallest and the largest pure-strategy Nash equilibria of $\Gamma_e\left(\gamma^1, \gamma^2, y_L, y_W\right)$. Furthermore, the workers' positive self-image relaxes the workers' participation constraints. This implies that the firm can implement the same actions with lower prizes or obtain more output for the same prizes. One way or the other the firm's profits are higher in $T\left(\lambda^1, \lambda^2\right)$ than in $T\left(\gamma^1, \gamma^2\right)$. *Q.E.D.*

A.3. This appendix shows if output is exponentially or normally distributed, then there is a non-monotonic relation between effort and self-image but the firm can still be better off with a positive self-image workforce.

Proposition 6 *If workers are risk averse, the output of worker* i *has the exponential distribution with mean* a^i, *and worker* i *perceives his output to have the exponential distribution with mean* λa^i, *then the firm's profits are higher in* $T(\lambda)$ *than in* $T(1)$, *for all* $1 < \lambda$.

Proof In this case $P(Q^{ii} > q^j) = \exp\left\{-q^j/\lambda a^i\right\}$, and

$$P\left(a^i, a^j, \lambda\right) = P(Q^{ii} > Q^j) = \frac{1}{a^j}\int_0^{+\infty} \exp^{-\left(\frac{q^j}{\lambda a^i}+\frac{q^j}{a^j}\right)} dq^j = \frac{\lambda a^i}{a^j + \lambda a^i}. \quad (28)$$

The cross partial of $P\left(a^i, a^j, \lambda\right)$ with respect to a^i and λ is

$$P_{a^i\lambda}(a^i, a^j, \lambda) = \frac{a^j\left(a^j - \lambda a^i\right)}{\left(a^j + \lambda a^i\right)^3}.$$

The sign of $P_{a^i\lambda}$ is positive when $a^j/a^i > \lambda$ and negative when $a^j/a^i < \lambda$. Now, let a denote an arbitrary effort level that the firm can implement when workers are risk averse and have accurate self-images. The assumption of symmetry and (28) imply that $P(\lambda) = \lambda/(1+\lambda)$ and $P_{a^i}(a, \lambda) = \lambda/\left[a(1+\lambda)^2\right]$. Thus, the utility of the losing and winning prizes is

$$u_L(\lambda) = \bar{U} + c\left(a\right) - (1+\lambda)ac'\left(a\right), \quad (29)$$

$$u_W(\lambda) = \bar{U} + c\left(a\right) + \frac{1+\lambda}{\lambda}ac'\left(a\right). \quad (30)$$

The utility prize spread that implements effort level a is

$$\triangle u(\lambda) = \frac{(1+\lambda)^2}{\lambda}ac'(a). \quad (31)$$

If workers are accurate, then $\lambda = 1$ and $u_L(1) = \bar{U} + c(a) - 2ac'(a)$, $u_W(1) = \bar{U} + c(a) + 2ac'(a)$, and $\triangle u(1) = 4ac'(a)$. If workers have positive self-image, then (29), (30) and (31) imply that the firm is able to implement effort level a by increasing the prize spread but simultaneously reducing the winner's and the loser's prizes. This implies that $C(a, \lambda) < C(a, 1)$ for all $\lambda > 1$. *Q.E.D.*

Proposition 7 *If workers are risk averse, the output of worker i has the normal distribution $N(a^i, \sigma^2)$, and worker i perceives his output to have the normal distribution $N(\lambda a^i, \sigma^2)$, then the firm's profits are higher in $T(\lambda, a)$ than in $T(1, a)$, for all (λ, a) such that $1 < \lambda < \hat{\lambda}$ and $a < \sigma\sqrt{\frac{2}{(\lambda-1)\lambda}}$.*

Proof In this case worker i's unconditional perceived probability of winning the tournament is given by

$$\begin{aligned} P\left(a^i, a^j, \lambda\right) &= P(Q^{ii} > Q^j) = P(\lambda a^i + \varepsilon^i > a^j + \varepsilon^j) \\ &= P(\lambda a^i - a^j > \varepsilon^i - \varepsilon^j) = \Phi(\lambda a^i - a^j), \end{aligned}$$

where $\Phi()$ is the distribution function of a normal random variable with mean 0 and variance $2\sigma^2$. The cross partial of $P\left(a^i, a^j, \lambda\right)$ with respect to a^i and λ is

$$P_{a^i\lambda}(a^i, a^j, \lambda) = \frac{1}{2\sigma\sqrt{\pi}}\left(1 - \frac{\lambda a^i - a^j}{2\sigma^2}\lambda a^i\right)\exp^{-\frac{1}{4\sigma^2}(\lambda a^i - a^j)^2}. \qquad (32)$$

We see from (32) that the sign of $P_{a^i\lambda}$ is positive when $2\sigma^2 > (\lambda a^i - a^j)\lambda a^i$ and negative when $2\sigma^2 < (\lambda a^i - a^j)\lambda a^i$.[32] Now, let a denote an arbitrary effort level that the firm can implement when workers are risk averse and have accurate self-images. The assumption of symmetry, (32), and that $\lambda < \tilde{\lambda}$ imply that $P_{a^i\lambda} > 0$ at the symmetric equilibrium. It follows from Proposition 3 that the firm will reduce the mean utility prize and lower the prize spread when it wishes to implement a and the workforce has positive self-image. This implies that $C(a, \lambda) < C(a, 1)$, for all (λ, a) such that $1 < \lambda < \hat{\lambda}$ and $a < \sigma\sqrt{\frac{2}{(\lambda-1)\lambda}}$. *Q.E.D.*

References

Athey, S., P. Milgrom, and J. Roberts, "Robust Comparative Statics," mimeo, Stanford University, 1998.

Baker, G., M. Jensen, and K. Murphy, "Compensation and Incentives: Practice vs. Theory," *Journal of Finance* XLIII (1988), 593- 616.

Bénabou, R., and J. Tirole, "Self-Confidence and Personal Motivation," *The Quarterly Journal of Economics* 117 (2002), 871-915.

———, and ———, "Intrinsic and Extrinsic Motivation," *The Review*

[32] The interpretation of these two conditions is as follows. If player i thinks that player j's effort is higher than λa^i or player i thinks that player j's effort is smaller than λa^i and the variance of output is high, then effort and self-image are complements. However, if player i thinks that player j's effort is smaller than λa^i and the variance of output is low, then effort and self-image are substitutes.

of Economic Studies 70 (2003), 489-520.

Brozynski, T., L. Menkhoff, and U. Schmidt, "The Impact of Experience on Risk Taking, Overconfidence, and Herding of Fund Managers: Complementary Survey Evidence," *European Economic Review* 50 (2006), 1753-1766.

Camerer, C., and D. Lovallo, "Overconfidence and Excess Entry: An Experimental Approach," *American Economic Review* 89 (1999), 306-318.

Debreu, D., "A Social Equilibrium Existence Theorem," *Proceedings of the National Academy of Sciences* 38 (1952), 886-893.

Deaves, R., E. Luders, and R. Luo, "An Experimental Test of the Impact of Overconfidence and Gender on Trading Activity," AFA Philadelphia Meetings, EFMA 2004 Basel Meetings Paper, (2003).

Fan, K., "Fixed Point and Minimax Theorems in Locally Convex Topological Linear Spaces," *Proceedings of the National Academy of Sciences* 38 (1952), 121-126.

Gervais, S., and I. Goldstein, "The Positive Effect of Biased Self-Perceptions in Firms," *Review of Finance* 11 (2007), 453-496.

Glicksberg, L., "A Further Generalization of the Kakutani Fixed Point Theorem with Applications to Economics," *Proceedings of the National Academy of Sciences* 38 (1952), 170-174.

Grossman, J., and O. Hart, "An Analysis of the Principal Agent Problem," *Econometrica* 51 (1983), 7-45.

Hvide, H. K., "Pragmatic Beliefs and Overconfidence," *Journal of Economic Behavior and Organization* 48 (2002), 15–28.

Koh, W., "A Note on Modelling Tournaments," *Journal of Economics* 55 (1992), 297-308.

Kyle, A. S., and F.A. Wang, "Speculation Duopoly with Agreement to Disagree: Can Overconfidence Survive the Market Test?" *Journal of Finance* 52 (1997), 2073-2090.

Lazear, E., and S. Rosen, "Rank-Order Tournaments as Optimum Labor Contracts," *Journal of Political Economy* 89 (1981), 841-864.

Lipsey, R., and K. Lancaster, "A General Theory of Second Best," *Review of Economic Studies* 24 (1956), 11-32.

Malmendier, U., and G. Tate, "Who Makes Acquisitions? CEO Overconfidence and Market's Reaction," *Journal of Financial Economics* 89 (2008), 20-43.

Milgrom, P., and J. Roberts, "Rationalizability, Learning, and Equilibrium in Games with Strategic Complementarities," *Econometrica* 58 (1990), 1255-1277.

Mirrlees, J., "The Theory of Moral Hazard and Unobservable Behavior. Part I." mimeo, Nuffield College, Oxford, 1975.

Mookherjee, D., "Optimal Incentive Schemes with Many Agents," *Review of Economic Studies* LI (1984), 433-446.

Myers, D., *Social Psychology,* (New York: McGraw-Hill, 1996).

Nalebuff, B., "Prizes and Incentives," mimeo, D. Phil. Thesis, Nuffield College, Oxford, 1982.

———, and J. Stiglitz, "Prizes and Incentives: Towards a General Theory of Compensation and Competition," *The Bell Journal of Economics* 14 (1983), 21-43.

Oberlechner, T., and C. Osler, "Overconfidence in Currency Markets," Working Paper, Brandeis University, 2004.

Park, Y., and L. Santos-Pinto, "Forecasts of Relative Performance in Tournaments: Evidence from the Field," Working Paper, Universidade Nova de Lisboa, 2005.

Santos-Pinto, L., "Positive Self-Image and Incentives in Organizations," *The Economic Journal* 118, (2008), 1315-1332.

Simon, M., and S. Houghton, "The Relationship between Overconfidence and the Introduction of Risky Products: Evidence from a Field Study," *Academy of Management Journal* 46 (2003), 139-149.

Squintani, F., "Equilibrium and Mistaken Self-Perception," *Economic Theory* 27 (2006), 615-641.

Waldman, M., "Systematic Errors and the Theory of Natural Selection," *The American Economic Review* 84 (1994), 482-497.