# Experimental Cournot Oligopoly and Inequity Aversion

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This version: December 25, 2012

**Abstract**

This paper explores the role of inequity aversion as an explanation for observed behavior in experimental Cournot oligopolies. We show that inequity aversion can change the nature of the strategic interaction: quantities are strategic substitutes for sufficiently asymmetric output levels but strategic complements otherwise. We find that inequity aversion can explain why: (i) some experiments result in higher than Cournot-Nash production levels while others result in lower, (ii) collusion often occurs with only two players whereas with three or more players market outcomes are very close to Cournot-Nash, and (iii) players often achieve equal profits in asymmetric Cournot oligopoly.

JEL Classification Numbers: D43, D63, L13, L21.
Keywords: Inequity Aversion; Cournot Oligopoly; Experiments.

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# 1 Introduction

Although quantity-setting oligopoly is one of the "workhorse models" of industrial organization, experimentally there is much ambiguity about its outcome. A recent survey by Georgantzis (2006) indicates that many experimental Cournot oligopoly games reject the hypothesis that the outcome is in line with the Cournot-Nash equilibrium of the corresponding one-shot game. Interestingly, however, outcomes on both sides of the Cournot-Nash outcome are found: some experiments result in higher than Cournot-Nash production levels while others result in lower production levels (Holt, 1995).[1]

Why does the theory perform poorly in the experiments? One possibility is that players are averse to inequality in earnings, that is, they are concerned about their own material payoff but also about the consequences of their acts on payoff distributions.

Inequity aversion has been shown to explain a broad range of data for many different games. The clearest evidence for these type of preferences comes from bargaining and trust games. For example, in ultimatum games offers are usually much more generous than predicted by subgame perfect equilibrium, and low offers are often rejected. According to the inequity aversion explanation, these offers are consistent with an equilibrium in which players make offers knowing that other players may reject allocations that appear unfair.[2]

In this paper we study formally the role of inequity aversion on Cournot competition. We assume that a player cares about her own monetary payoff and, in addition, would like to reduce the difference between her payoff and those of her rivals. More specifically, a inequity averse player dislikes advantageous inequity: she feels compassion towards her rivals when the average material payoff of her rivals is smaller than her own material payoff. Additionally, an inequity averse player also dislikes disadvantageous inequity: she feels envy towards her rivals when the average material payoff of her rivals is greater than her own material payoff.

We find that inequity aversion can change the nature of the strategic interaction: quantities are strategic substitutes when players choose asymmetric output levels but strategic complements when they choose similar output levels. This can give rise to a continuum of equilibria. We show that the set of Nash equilibria of Cournot competition with inequity averse players changes monotonically with compassion and envy. If players' degree of envy increases, then the largest Nash equilibria of the Cournot game moves closer to the Walrasian outcome. In contrast, if players' degree of compassion increases, then the smallest Nash equilibria of the Cournot game moves closer to the collusive

[1] A rather general finding in finitely-repeated symmetric experimental Cournot oligopoly is that, while some learning occurs during the session, in many sessions total output is not significantly different from the collusive prediction, while in other sessions, total output oscilates between the collusive and the Cournot outcome. Additionally, in finitely-repeated asymmetric experimental Cournot oligopoly, subjects' strategies fail to converge towards the Nash equilibrium prediction. See Rassenti et al. (2000) and Huck et al. (2000, 2001).

[2] Camerer (2003) and Sobel (2005) provide excellent reviews of this literature.

outcome. However, as the number of players grows the impact of inequity aversion vanishes. This happens because it takes only one self-interested player to destroy the continuum of equilibria generated by inequity aversion.

We find that relatively low levels of inequity aversion generate less asymmetries in profits than those predicted when self-interested players play asymmetric Cournot oligopolies. We also show that relatively high levels of inequity aversion can explain why often players attain equal profits in asymmetric experimental Cournot oligopolies. The intuition for this result is straightforward. For relatively high levels of inequity aversion, attaining asymmetric profits imposes inequity costs that are too high in relation to the material benefits.

Our findings are more relevant in experimental Cournot oligopolies with a small number of players. In fact we show that increasing the number of players reduces the impact of inequity aversion on Cournot oligopoly. This finding is consistent with Huck et al. (2004) who find some collusion with two firms and no collusion as the number reaches four firms. Our findings are also more relevant in experimental Cournot oligopolies where individuals rather than teams play the role of firms. Hildenbrand (2012) studies a Stackelberg experiment in which the firms are either represented by individuals or teams. He finds that individuals exhibit more inequity aversion than teams.[3]

This paper is related to a recent strand of literature in economics that studies the consequences of relaxing the assumption of pure self interest. Rabin (1993) is the first using fairness considerations in game theory. Sappington and Desiraju (2007) study inequity aversion in adverse selection contexts. Biel (2008) studies how the optimal incentive contract in team production is affected when workers are averse to inequity. Santos-Pinto (2008) shows that inequity aversion is able to organize several experimental regularities of endogenous timing games. Englmaier and Wambach (2010) study optimal contracts when the agent suffers from being better off or worse off than the principal.

The paper proceeds as follows. Section 2 sets-up the model. Section 3 characterizes equilibria of Cournot oligopoly with symmetric costs and inequity averse players. Section 4 considers Cournot oligopoly with asymmetric costs. Section 5 concludes the paper. All proofs are in the Appendix.

# 2 The Model

Many experiments indicate that individuals are motivated not only by material self-interest, but also by the distribution of payoffs. We incorporate this possibility in the Cournot oligopoly game by assuming that players are averse to inequality in profits.

There are two main theories of inequity aversion: Fehr and Schmidt's (1999) and Bolton and Ockenfels (2000). According to Fehr and Schmidt a player cares

[3] Charness and Sutter (2012) review the group vs. individual behavior experiments for variety of games and documented that groups behave more rationally and have stronger self-interested preferences.

about his own payoff and dislikes absolute payoff differences between his own payoff and the payoff of any other player.[4]

According to Bolton and Ockenfels's (2000) an inequity averse player is concerned with both his own payoff and his relative share of the total group payoff. So, a player would be equally happy if all players received the same payoff or if some were rich and some were poor as long as he received the average payoff, while according to Fehr and Schmidt (1999) he would clearly prefer that all players get the same.[5]

We follow Fehr and Schmidt's (1999) approach to model inequity aversion. Consider a Cournot oligopoly with $n$ players where the profit of player $i$ is the difference between revenue and cost, that is,

$$\pi_i(q_i, Q_{-i}) = R_i\left(q_i, Q_{-i}\right) - C_i(q_i) = P(Q)q_i - C_i(q_i),$$

where $R_i(q_i, Q_{-i})$ is revenue, $C_i(q_i)$ is the cost of production, and $P(Q)$ is the inverse market demand with $Q = \sum q_i$. We assume that $P(Q)$ is strictly positive on some bounded interval $(0, \bar{Q})$ with $P(Q) = 0$ for $Q \geq \bar{Q}$. We also assume that $P(Q)$ is twice continuously differentiable with $P'(Q) < 0$ (in the interval for which $P(Q) > 0$). Players costs of production are assumed to be twice continuously differentiable with $C_i'(q_i) \geq 0$. It is also assumed that the decreasing marginal revenue property holds, that is, $P'(Q) + P''(Q)\,q_i < 0$ (this implies that quantities are strategic substitutes). Furthermore, we assume that $P'(Q) - C_i''(q_i) \leq 0$ (this implies the profit function is strictly concave).

According to Fehr and Schmidt (1999), the payoff function of player $i$ is

$$U_i(\pi_i, \pi_{-i}) = \pi_i - \left[\frac{\alpha_i}{n-1}\sum_{j\neq i}\max\left(\pi_j - \pi_i, 0\right) + \frac{\beta_i}{n-1}\max\sum_{j\neq i}\left(\pi_i - \pi_j, 0\right)\right]. \quad (1)$$

The terms in the square bracket are the payoff effects of compassion $\beta_i$ and envy $\alpha_i$. We see that if player $i$'s profits are greater than the average profits of its rivals then player $i$ feels compassion towards its rivals. However, if player $i$'s profits are smaller than the average profits of its rivals then player $i$ feels envious of his rivals.[6] This model of inequity aversion has piecewise linear indifference curves over a player's own profits and its rivals' profits.

Player $i$'s inequity aversion towards its rivals is characterized by the pair of parameters $(\alpha_i, \beta_i)$, $i = 1, 2, \ldots, n$.[7] player $i$ exhibits strict inequity aversion when both $\alpha_i$ and $\beta_i$ are strictly greater than zero. player $i$ only cares about

[4]Neilson (2006) provides an axiomatic foundation for Fehr and Schmidt (1999).

[5]Bolton and Ockenfels's (2000) payoff function is $U_i(\pi) = v(\pi_i, \pi_i / \sum_{j=1}^{n}\pi_j)$, where $v$ is assumed to be globally non-decreasing and concave in the first argument, to be strictly concave in the second argument (relative payoff), and to satisfy $v(\pi_i, 1/n) = 0$ for all $\pi_i$. This type of inequity aversion has no impact on equilibrium outcomes in symmetric Cournot games. This result is driven by the assumption that $v(\pi_i, 1/n) = 0$ for all $\pi_i$.

[6]When there are only two players in the market, player $i$'s payoff function becomes $U_i(\pi_i, \pi_j) = \pi_i - \left[\alpha_i \max\left(\pi_j - \pi_i, 0\right) + \beta_i \max\left(\pi_i - \pi_j, 0\right)\right]$, $i \neq j = 1, 2$.

[7]Alternatively, we could have assumed that player $i$ has different feelings of compassion and envy towards each rival. To simplify the analysis, we assume that player $i$ feels the same degree of compassion and envy towards all rivals.

maximizing profits when $\alpha_i = \beta_i = 0$. In all other cases player is (weakly) averse to inequity. We assume that $\alpha_i$ and $\beta_i$, $i = 1, \ldots, n$, are common knowledge. Let $\alpha = (\alpha_1, \ldots, \alpha_n)$ and $\beta = (\beta_1, \ldots, \beta_n)$.

Fehr and Schmidt assume that the dislike of disadvantageous inequity is stronger than that of advantageous inequity, i.e. $\alpha_i > \beta_i$ and that $\beta_i$ is smaller than 1. We make no assumptions about the relation between $\alpha_i$ and $\beta_i$ but we assume, like Fehr and Schmidt, that $\beta_i$ is smaller than 1.

# 3 Equilibria with Symmetric Costs

In this section we characterize the impact of inequity aversion on Cournot oligopoly with symmetric costs. Our first result characterizes the best reply of an inequity averse player.

**Proposition 1**: *The best reply of player i in a Cournot oligopoly with symmetric costs and inequity averse players is*

$$r_i(Q_{-i}) = \begin{cases} s_i(Q_{-i}), \text{ if } & 0 \leq Q_{-i} \leq (n-1)q(\beta_i) \\ \frac{Q_{-i}}{n-1}, \text{ if } & (n-1)q(\beta_i) \leq Q_{-i} \leq (n-1)q(\alpha_i) \\ t_i(Q_{-i}), \text{ if } & (n-1)q(\alpha_i) \leq Q_{-i} \end{cases}, \quad (2)$$

*where*

$$s_i(Q_{-i}) = \arg\max_{q_i} \left[ (1-\beta_i)\,\pi_i(q_i, Q_{-i}) + \frac{\beta_i}{n-1} \sum_{j \neq i} \pi_j(q_i, Q_{-i}) \right],$$

$$t_i(Q_{-i}) = \arg\max_{q_i} \left[ (1+\alpha_i)\,\pi_i(q_i, Q_{-i}) - \frac{\alpha_i}{n-1} \sum_{j \neq i} \pi_j(q_i, Q_{-i}) \right],$$

*$q(\beta_i)$ is the solution to $(1-\beta_i)\left[P(nq) - C'(q)\right] + P'(nq)q = 0$, and $q(\alpha_i)$ is the solution to $(1+\alpha_i)\left[P(nq) - C'(q)\right] + P'(nq)q = 0$.*

The best reply has three different segments. When the rivals produce low output levels the best reply has a negative slope and consists of a smaller output than the output of a self-interested player due to compassion. However, when the rivals produce intermediate output levels the best reply has a positive slope and consists in producing the average output level of the rivals. Finally, when the rivals produce high output levels the best reply has a negative slope and consists of a larger output level than the output a self-interested player due to envy.

We see that the best reply of an inequity averse player is continuous like the best reply of self-interested player. However, the best reply of an inequity averse player is non-monotonic whereas a self-interested player has a monotonic best reply. Thus, under inequity aversion quantities are strategic substitutes over low and high output levels of the rivals but strategic complements over intermediate output levels of the rivals.

**Proposition 2**: *The set of Nash equilibria of a Cournot oligopoly with symmetric costs and inequity averse players is*

$$N^{IA} = \{(q_1, \ldots, q_n) : q_i = q_j,\ \forall i \neq j,\ \text{and}\ q(\beta) \leq q_i \leq q(\alpha),\ i = 1, \ldots, n\},$$

*where* $q(\beta) = \max [q(\beta_1), \ldots, q(\beta_n)]$, *and* $q(\alpha) = \min [q(\alpha_1), \ldots, q(\alpha_n)]$.

Proposition 2 tells us that if all players are averse to inequity, then there is a continuum of equilibria in a Cournot oligopoly with symmetric costs. The smallest Nash equilibrium is determined by the preferences of the player with the highest level of compassion and the largest Nash equilibrium is determined by the preferences of the player with the lowest level of envy. Proposition 2 also tells that the market output with inequity averse players may be higher or lower than the market output with self-interested players. This depends on players' degree of envy and compassion.

**Proposition 3**: *The smallest Nash equilibrium of a Cournot oligopoly with symmetric costs and inequity averse players is a nonincreasing function of* $\beta$. *The largest Nash equilibrium is a nondecreasing function of* $\alpha$.

This result characterizes the impact of compassion and envy on the set of Nash equilibria of the Cournot oligopoly with inequity averse players. It tells us that an increase in compassion reduces the market output produced in the smallest Nash equilibria with inequity averse players. This result is quite intuitive. In fact, Fehr and Schmidt's (1999) payoff function implies that if player $i$ has a higher monetary payoff than the average payoff of his opponents and $\beta_i = 1/2$, then player $i$ is just as willing to keep one dollar to himself as to give it to his rivals. If all players have similar preferences, then they act as if they are maximizing the joint profit, $\sum \pi_i$. So, if $\beta_i = 1/2$, for all $i$, then compassion leads to the best collusive outcome.[8] In contrast, an increase in envy raises the market output produced in the largest Nash equilibria with inequity averse players.

Propositions 2 and 3 show that inequity aversion can change qualitatively the predictions of a symmetric Cournot oligopoly. In the benchmark game with self-interested players ($\alpha_i = \beta_i = 0$, for all $i$) there is a unique Nash equilibrium. Proposition 2 shows that inequity aversion can give rise to a multiplicity of symmetric equilibria. Proposition 3 shows that compassion can generate collusive outcomes in Cournot markets whereas envy can generate perfectly competitive outcomes. The existence of a multiplicity of equilibria might be the reason why in many experiments with Cournot duopolies outcomes fall on both sides

[8] Experimental evidence shows that the amount of collusion observed in repeated Cournot oligopoly depends on communication, playing with the same rival(s) and the size of the market. Daughety and Forsythe (1987a,1987b) report that face to face nonbinding groups discussions increase price in repeated Cournot games in which the quantity decision are made afterwards. Similarly, Isaac et al. (1984) report that posted-offer prices are increased when sellers are given chance to meet face to face prior to each period. Holt (1985) finds that collusion occurs only when the same subjects are matched in fixed groups for the entire experiment. With random matching the Cournot-Nash solution is a good prediction.

of the Cournot prediction and may range from perfectly collusive to relatively competitive (Holt, 1995).

Huck et al. (2004) review the literature on the role of the number of players on the outcome of Cournot oligopolies with symmetric costs. They find that (pp. 440) "(...) collusion sometimes occurs in duopolies and is very rare in markets with more than two firms. On average, total outputs in markets with more than two firms slightly exceed the Cournot prediction." They also test for number effects in oligopoly in a unified economic frame and find that (pp. 443) "collusion sometimes occurs with two firms. For three-firm oligopolies Nash equilibrium seems to be a good predictor. Markets with four or more firms are never collusive and typically settle around the Cournot outcome while some of them are very competitive with outputs close to the Walrasian outcome." Our next result shows that these findings are consistent with our model.

**Proposition 4**: *Assume that $\alpha_i$ and $\beta_i$, for all $i$, are drawn from a uniform distribution with support on $[0,1]$. As the number of players increases the set of Nash equilibria of a Cournot oligopoly with symmetric costs and inequity averse players converges to the unique Nash equilibrium of a Cournot oligopoly with symmetric costs and self-interested players.*

This result shows that increasing the number of players reduces the impact of inequity aversion on the set of Nash equilibria of a Cournot oligopoly with symmetric costs. This happens because when there are $n$ players, the smallest Nash equilibrium of the game is determined by the preferences of the player with the lowest degree of compassion. Similarly, the largest Nash equilibrium of the game is determined by the player with the lowest degree of envy. If the levels of compassion and envy of each player are drawn from a uniform distribution with support on $[0,1]$, then an increase in the number of players makes it more likely that the lowest level of compassion as well as the lowest level of envy are both very close to zero. Thus, as the number of players increases the smallest and the largest Nash equilibria of a Cournot oligopoly with symmetric costs and inequity averse players converge to the Nash equilibrium of a Cournot oligopoly with symmetric costs and self-interested players.

# 4 Equilibria with Asymmetric Costs

In this section we analyze the impact of inequity aversion on Cournot oligopoly with asymmetric costs. To simplify the analysis let $n = 2$. Furthermore, suppose that there are no fixed costs and that player 1 has a lower marginal cost than player 2, that is, $C_1'(q) < C_2'(q)$ for all $q$.

Players will attain equal profits when $\pi_1(q_1, q_2) = \pi_2(q_1, q_2)$ or

$$P(q_1 + q_2)(q_2 - q_1) = C_2(q_2) - C_1(q_1). \tag{3}$$

Denote the solution of (3) with respect to $q_i$ as $q_i = e_i(q_j)$. The slope of the equal profit curve is

$$\frac{dq_2}{dq_1} = \frac{MR_1 - C_1' - P'q_2}{MR_2 - C_2' - P'q_1}. \tag{4}$$

We see from (4) that if $(q_1, q_2)$ is a point in the equal profit curve that satisfies $MR_1 - C_1' - P'q_2 > 0$ and $MR_2 - C_2' - P'q_1 > 0$, then the slope of the equal profit curve at that point is well-defined and positive. We assume from now on that the cost asymmetry is not too high such that the equal profit curve has a positive slope at all points $(q_1, q_2)$ with $q_2 > q_1$.

**Proposition 5**: *The best reply of player $i$ in a Cournot duopoly with asymmetric costs and inequity averse players is*

$$r_i(q_j) = \begin{cases} s_i(q_j), & \text{if} \quad 0 \leq q_j \leq e_j(q_i(\beta_i)) \\ e_i(q_j), & \text{if} \quad e_j(q_i(\beta_i)) \leq q_j \leq e_j(q_i(\alpha_i)) \\ t_i(q_j), & \text{if} \quad e_j(q_i(\alpha_i)) \leq q_j \end{cases}, \tag{5}$$

*where*

$$\begin{aligned} s_i(q_j) &= \arg\max_{q_i} \left[ (1-\beta_i)\, \pi_i(q_i, q_j) + \beta_i \pi_j(q_i, q_j) \right], \\ t_i(q_j) &= \arg\max_{q_i} \left[ (1+\alpha_i)\, \pi_i(q_i, q_j) - \alpha_i \pi_j(q_i, q_j) \right], \end{aligned}$$

*$q_i(\beta_i)$ is the solution to*

$$(1-\beta_i)\left[P'(q_i + e_j(q_i))q_i + P(q_i + e_j(q_i)) - C_i'(q_i)\right] \\ + \beta_i P'(q_i + e_j(q_i))e_j(q_i) = 0,$$

*and $q_i(\alpha_i)$ is the solution to*

$$(1+\alpha_i)\left[P'(q_i + e_j(q_i))q_i + P(q_i + e_j(q_i)) - C_i'(q_i)\right] \\ - \alpha_i P'(q_i + e_j(q_i))e_j(q_i) = 0.$$

We see from (2) and (5) that the best reply of an inequity averse player in a Cournot duopoly with asymmetric costs is qualitatively similar to the best reply of an inequity averse player in a Cournot duopoly with symmetric costs. Quantities are strategic substitutes for low and high output levels of the rival but strategic complements for intermediate output levels of the rival. The only difference is that for intermediate output levels inequity averse players wish to equalize profits. Since costs are asymmetric it is not possible to equalize profits by producing the same output level of as the rival. Thus, players will choose different output levels to equalize profits.

**Proposition 6**: *Consider a Cournot duopoly where player 1 has lower marginal cost than player 2 and players are inequity averse. If $\beta_1$ and $\alpha_2$ are sufficiently small, that is, $q_1(\beta_1) \geq e_1(q_2(\alpha_2))$, then this game has a unique Nash equilibrium $(q_1^{IA}, q_2^{IA})$, which is the solution to $q_1 = s_1(q_2)$ and $q_2 = t_2(q_1)$. In this equilibrium: (i) player 1 feels compassion of player 2, (ii) player 2 feels envy of player 1, and (iii) $q_2^S < q_2^{IA} < q_1^{IA} < q_1^S$, where $(q_1^S, q_2^S)$ is the Nash equilibrium of the game with self-interested players.*

This result says that if the low cost player has a small dislike of advantageous inequity and the high cost player has a small dislike of disadvantageous inequity, then the Cournot duopoly with asymmetric costs and inequity averse players has an asymmetric Nash equilibrium where the low cost player attains a higher profit than the high cost player. Furthermore, the inequity averse low cost player chooses a lower output level than a low cost self-interested player and the inequity averse high cost player chooses a higher output than a high cost self-interested player. The intuition behind this result is as follows.

A low cost player with a small dislike of advantageous inequity chooses a lower output than a low cost self-interested player because she knows that in equilibrium she will attain higher profits than her rival and this induces compassion towards the rival. A high cost player with a small dislike of disadvantageous inequity chooses a higher output than a high cost self-interested player because he knows that in equilibrium he will attain lower profits than his rival and this induces envy towards the rival.

**Proposition 7**: *Consider a Cournot duopoly where player 1 has lower marginal cost than player 2 and players are inequity averse. If* $\beta_1$ *and* $\alpha_2$ *are sufficiently large, that is,* $q_1(\beta_1) < e_1(q_2(\alpha_2))$*, then the set of Nash equilibria of this game is*

$$N^{IA} = \{(q_1, q_2) : \pi_1(q_1, q_2) = \pi_2(q_1, q_2), \text{ and } q(\beta) \leq q_1 \leq q(\alpha)\}, \quad (6)$$

*where* $q(\beta) = \max[q_1(\beta_1), e_1(q_2(\beta_2))]$, *and* $q(\alpha) = \min[q_1(\alpha_1), e_1(q_2(\alpha_2))]$.

Proposition 7 tells us that if the low cost player has a high dislike of advantageous inequity and the high cost player has a high dislike of disadvantageous inequity, then the Cournot duopoly with asymmetric costs and inequity averse players has a continuum of asymmetric Nash equilibria where players attain equal profits.

Propositions 6 and 7 are consistent with experimental evidence on Cournot oligopolies with asymmetric costs. Keser (1993) studies two stage duopoly games with asymmetric costs and demand inertia. She finds that the high cost player has higher profits and the low cost player lower profits than the self-interested subgame perfect equilibrium profits. Selten et al. (1997) study a 20-period repeated Cournot duopoly with asymmetric costs and find that players often try to achieve equal profits.

# 5 Conclusion

This paper studies the impact of inequity aversion on Cournot competition. We find that inequity aversion can change the nature of the strategic interaction: quantities are strategic substitutes when players choose asymmetric output levels but strategic complements when they choose similar output levels. We show that inequity aversion is able to organize at least three empirical regularities in experimental Cournot oligopolies.

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# 6 Appendix

**Proof of Proposition 1**: The quantity $q(\alpha_i)$ is the interception of $t_i(Q_{-i})$ with the 45 degree line. From the definition of $t_i(Q_{-i})$ we have

$$(1+\alpha_i)\left[P'(Q)q_i + P(Q) - C'(q_i)\right] - \frac{\alpha_i}{n-1}\sum_{j\neq i} P'(Q)q_j = 0.$$

In a symmetric equilibrium we have $q_1 = ... = q_n = q$. So,

$$(1+\alpha_i)\left[P'(nq)q + P(nq) - C'(q)\right] - \alpha_i P'(nq)q = 0.$$

or

$$(1+\alpha_i)\left[P(nq) - C'(q)\right] + P'(nq)q = 0.$$

Similarly, the quantity $q(\beta_i)$ is the interception of $s_i(Q_{-i})$ with the 45 degree line. From the definition of $s_i(Q_{-i})$ we have

$$(1-\beta_i)\left[P'(Q)q_i + P(Q) - C'(q_i)\right] + \frac{\beta_i}{n-1}\sum_{j\neq i} P'(Q)q_j = 0.$$

In a symmetric equilibrium we have $q_1 = ... = q_n = q$. So,

$$(1-\beta_i)\left[P'(nq)q + P(nq) - C'(q)\right] + \beta_i P'(nq)q = 0.$$

or

$$(1-\beta_i)\left[P(nq) - C'(q)\right] + P'(nq)q = 0.$$

We now show that $q(\alpha_i)$ is an increasing function of $\alpha_i$ and $q(\beta_i)$ a decreasing function of $\beta_i$ for $i = 1, \ldots, n$. Let

$$\begin{aligned} h(q,\alpha_i) &= (1+\alpha_i)\left[P(nq) - C'(q)\right] + P'(nq)q = 0, \\ g(q,\beta_i) &= (1-\beta_i)\left[P(nq) - C'(q)\right] + P'(nq)q = 0, \end{aligned}$$

which imply

$$\begin{aligned} \frac{\partial q}{\partial \alpha_i} &= -\frac{\partial h/\partial \alpha_i}{\partial h/\partial q} = -\frac{P(Q) - C'(q)}{(1+n(1+\alpha_i))\,P'(Q) + nP''(Q)q - C_i''(q)} > 0, \\ \frac{\partial q}{\partial \beta_i} &= -\frac{\partial g/\partial \beta_i}{\partial g/\partial q} = -\frac{-\left[P(Q) - C'(q)\right]}{(1+n(1-\beta_i))\,P'(Q) + nP''(Q)q - C_i''(q)} < 0, \end{aligned}$$

since $P'(Q) < 0$, $P'(Q) \leq 0$, and $C''(q_i) \geq 0$.
We will now show that $q_i = \frac{1}{n-1}\sum_{j\neq i} q_j$ is a best response for player $i$ when the rivals produce

$$q_i^N \leq \bar{q}_j \leq q(\alpha_i), \tag{7}$$

where $\bar{q}_j = \frac{1}{n-1}\sum_{j\neq i} q_j$. To do that we will show that player $i$ can not gain from deviating from $q_i = \bar{q}_j$ when (7) holds. Suppose, that (7) holds and that player $i$ produces $q_i = \bar{q}_j + \varepsilon$, with $\varepsilon > 0$. In this case player $i$'s payoff is

$$U_i = (1-\beta_i)\left[P(Q)\,q_i - C(q_i)\right] + \frac{\beta_i}{n-1}\sum_{j\neq i}\left[P(Q)\,q_j - C(q_j)\right]$$

and the change in player $i$'s payoff from producing $q_i = \bar{q}_j + \varepsilon$, $\varepsilon > 0$, instead of $\bar{q}_j$ is approximately equal to

$$\begin{aligned} dU_i &\approx \left.(1-\beta_i)\left[P'(Q)\,q_i + P(Q) - C'(q_i)\right] + \frac{\beta_i}{n-1}\sum_{j\neq i} P'(Q)\,q_j\right|_{q_i=\bar{q}_j} (\varepsilon) \\ &= \left[\left(P'(n\bar{q}_j)\,\bar{q}_j + P(n\bar{q}_j) - C'(\bar{q}_j)\right) - \beta_i\left(P(n\bar{q}_j) - C'(\bar{q}_j)\right)\right]\varepsilon. \end{aligned}$$

The square brackets are negative since $q_i = \bar{q}_j > \arg\max\left[P(Q)\,q_i - C(q_i)\right]$ and $P(n\bar{q}_j) - C'(\bar{q}_j) > 0$. So, when (7) holds, player $i$ can not gain by producing more than $\bar{q}_j$. Now, suppose that (7) holds and that player $i$ produces $q_i = \bar{q}_j + \varepsilon$, with $\varepsilon < 0$. In this case player $i$'s payoff is

$$U_i = (1+\alpha_i)\left[P(Q)\,q_i - C(q_i)\right] - \frac{\alpha_i}{n-1}\sum_{j\neq i}\left[P(Q)\,q_j - C(q_j)\right],$$

and the change in player $i$'s payoff from producing $q_i = \bar{q}_j + \varepsilon$, $\varepsilon < 0$, instead of $\bar{q}_j$ is approximately equal to

$$\begin{aligned} dU_i &\approx \left.(1+\alpha_i)\left[P'(Q)\,q_i + P(Q) - C'(q_i)\right] - \frac{\alpha_i}{n-1}\sum_{j\neq i} P'(Q)\,q_j\right|_{q_i=\bar{q}_j} (\varepsilon) \\ &= \left[(1+\alpha_i)\left[P(n\bar{q}_j) - C'(\bar{q}_j)\right] + P'(n\bar{q}_j)\bar{q}_j\right]\varepsilon = \left.h(q,\alpha_i)\right|_{q=\bar{q}_j}(\varepsilon). \end{aligned}$$

Since $\varepsilon < 0$, we have that $sign\ dU_i = -sign\ \left.h(q,\alpha_i)\right|_{q=\bar{q}_j}$. If $\bar{q}_j = q(\alpha_i)$ we have that $sign\ dU_i = 0$. If $q_i^N \leq \bar{q}_j < q(\alpha_i)$, the fact $h(q,\alpha_i)$ is a decreasing function of $q$ implies that $\left.h(q,\alpha_i)\right|_{q=\bar{q}_j} > 0$, which in turn implies that $sign\ dU_i < 0$. So, when (7) holds, player $i$ can not gain by producing less than $\bar{q}_j$. From this result is follows immediately that if player $i$'s rivals produce $q(\alpha_i) < \frac{1}{n-1}\sum_{j\neq i} q_j$, then the best response of player $i$ is given by $t_i(q_{-i})$.

We will now show that $q_i = \frac{1}{n-1}\sum_{j\neq i} q_j$ is a best response for player $i$ when the rivals produce

$$q(\beta_i) \leq \bar{q}_j \leq q_i^N, \tag{8}$$

To do that we will show that player $i$ can not gain from deviating from $q_i = \bar{q}_j$ when (8) holds. Suppose, that (8) holds and that player $i$ produces $q_i = \bar{q}_j + \varepsilon$, with $\varepsilon < 0$. In this case player $i$'s payoff is given by

$$U_i = (1+\alpha_i)\left[P(Q)\,q_i - C(q_i)\right] - \frac{\alpha_i}{n-1}\sum_{j\neq i}\left[P(Q)\,q_j - C(q_j)\right],$$

and the change in player $i$'s payoff from producing $q_i = \bar{q}_j + \varepsilon$, $\varepsilon < 0$, instead of $\bar{q}_j$ is approximately equal to

$$\begin{aligned} dU_i &\approx \left.(1+\alpha_i)\left[P'(Q)\,q_i + P(Q) - C'(q_i)\right] - \frac{\alpha_i}{n-1}\sum_{j\neq i} P'(Q)\,q_j\right|_{q_i=\bar{q}_j} (\varepsilon) \\ &= \left[(1+\alpha_i)\left[P'(n\bar{q}_j)\bar{q}_j + P(n\bar{q}_j) - C'(\bar{q}_j)\right] - \alpha_i P'(n\bar{q}_j)\bar{q}_j\right]\varepsilon. \end{aligned}$$

The square brackets are positive since $q_i = \bar{q}_j < \arg\max \left[P\left(Q\right) q_i - C(q_i)\right]$ and $P'(n\bar{q}_j) < 0$. So, when (8) holds, player $i$ can not gain by producing less than $\bar{q}_j$. Now, suppose that (8) holds and that player $i$ produces $q_i = \bar{q}_j + \varepsilon$, with $\varepsilon > 0$. In this case player $i$'s payoff is given by

$$U_i = (1-\beta_i)\left[P\left(Q\right) q_i - C(q_i)\right] + \frac{\beta_i}{n-1}\sum_{j\neq i}\left[P\left(Q\right) q_j - C(q_j)\right]$$

and the change in player $i$'s payoff from producing $q_i = \bar{q}_j + \varepsilon$, $\varepsilon > 0$, instead of $\bar{q}_j$ is approximately equal to

$$\begin{aligned} dU_i &\approx (1-\beta_i)\left[P'\left(Q\right) q_i + P(Q) - C'(q_i)\right] + \frac{\beta_i}{n-1}\sum_{j\neq i} P'\left(Q\right) q_j \Bigg|_{q_i=\bar{q}_j} (\varepsilon) \\ &= \left[(1-\beta_i)\left[P(n\bar{q}_j) - C'(\bar{q}_j)\right] + P'\left(n\bar{q}_j\right)\bar{q}_j\right]\varepsilon = \left. g(q,\beta_i)\right|_{q=\bar{q}_j}(\varepsilon). \end{aligned}$$

Since $\varepsilon > 0$, we have that $sign\ dU_i = sign\ \left. g(q,\beta_i)\right|_{q=\bar{q}_j}$. If $\bar{q}_j = q(\beta_i)$ we have that $sign\ dU_i = 0$. If $q(\beta_i) < \bar{q}_j \leq q_i^N$, the fact $g(q,\beta_i)$ is a decreasing function of $q$ implies that $\left. g(q,\beta_i)\right|_{q=\bar{q}_j} < 0$, which in turn implies that $sign\ dU_i < 0$. So, when (8) holds, player $i$ can not gain by producing more than $\bar{q}_j$. From this result is follows immediately that if player $i$'s rivals produce $0 \leq \frac{1}{n-1}\sum_{j\neq i} q_j < q(\beta_i)$, then the best response of player $i$ is given by $s_i(q_{-i})$. *Q.E.D.*

**Proof of Proposition 2**: The proof proceeds in two steps. First we show that the set of equilibria is non-empty. Second, we show that if all players are strictly averse to inequality, then there is a continuum of equilibria and we characterize the largest and the smallest one.
We now show that $q_i = q_i^N$ is the best reply to $q_{-i}^N = \left(q_1^N, \ldots, q_{i-1}^N, q_{i+1}^N, \ldots q_n^N\right)$ in the Cournot oligopoly with symmetric costs and inequity averse players. The welfare of player 1 under outcome $q^N$ is $\pi_1(q^N) = \left[P\left(nq_i^N\right) - C_i(q_i^N)\right] q_i^N$, where $q_i^N = \arg_{q_1}\max\left[P\left(q_i + \sum_{j\neq i} q_j^N\right) - C_i(q_i)\right] q_i$.
If player $i$ produces $q_i^N + \varepsilon$, with $\varepsilon > 0$, and all other players produce $q_{-i}^N$, then the change in player $i$'s profit is approximately equal to

$$\begin{aligned} d\pi_i &\approx \varepsilon\ \partial\pi_i/\partial q_i\big|_{q_i=q_i^N} + \frac{1}{2}\varepsilon^2\ \partial^2\pi_i/\partial q_i^2\big|_{q_i=q_i^N} \\ &= \frac{1}{2}\varepsilon^2\left[2P'(Q^N) + P''(Q^N)q_i^N - C''(q_i^N)\right]. \end{aligned} \quad (9)$$

The assumption that $P' < 0$, $P'' \leq 0$, and $C'' \geq 0$ imply that $d\pi_i < 0$. The change in the profit of one of player $i$'s rivals, say $j$, is approximately equal to

$$\begin{aligned} d\pi_j &\approx \varepsilon\ \partial\pi_j/\partial q_i\big|_{q_i=q_i^N} + \frac{1}{2}\varepsilon^2\ \partial^2\pi_j/\partial q_i^2\big|_{q_i=q_i^N} \\ &= \varepsilon P'(Q^N)q_j^N + \frac{1}{2}\varepsilon^2 P''(Q^N)q_j^N. \end{aligned}$$

Note that the change in the average profit of player $i$'s rivals is the same as the change in the profit of a single rival since

$$\begin{aligned}\frac{1}{n-1}\sum_{j\neq i} d\pi_j &\approx \frac{1}{n-1}\varepsilon P'(Q^N)\sum_{j\neq i} q_j^N + \frac{1}{2}\varepsilon^2 P''(Q^N)\sum_{j\neq i} q_j^N \\ &= \varepsilon P'(Q^N) q_j^N + \frac{1}{2}\varepsilon^2 P''(Q^N) q_j^N. \qquad (10)\end{aligned}$$

The assumption that $P' < 0$ and $P'' \leq 0$ imply that $\frac{1}{n-1}\sum_{j\neq i} d\pi_j < 0$. We see from (9) and (10) that if player $i$ produces $q_i^N + \varepsilon$, with $\varepsilon > 0$, and all other players produce $q_{-i}^N$, then there is a first order decrease in profits of player $i$ and a second order decrease in the average profit of player $i$'s rivals. Thus, if player $i$ produces $q_i^N + \varepsilon$, with $\varepsilon > 0$, it suffers a loss in profits and also a loss from an increase in inequity aversion given that the average profit of the rivals becomes smaller than player $i$'s profit. If that is the case, then player $i$ can not gain by producing $q_i^N + \varepsilon$, with $\varepsilon > 0$, instead of producing $q_i^N$.
If player $i$ produces $q_i^N + \varepsilon$, with $\varepsilon < 0$, and all other players produce $q_{-i}^N$, then the change in player $i$'s profit is given by (9) and we have that $d\pi_i < 0$. The change in the average profit of player $i$'s rivals is given by (10) and we have that $\frac{1}{n-1}\sum_{j\neq i} d\pi_j > 0$ since $\varepsilon < 0$ and the first term is of first order while the second term is of second order. Thus, if player $i$ produces $q_i^N + \varepsilon$, with $\varepsilon < 0$, it suffers a loss in profits and also a loss from an increase in inequity aversion given that the average profit of the rivals becomes greater than player $i$'s profit. If that is the case, then player $i$ can not gain by producing $q_i^N + \varepsilon$, with $\varepsilon < 0$, instead of producing $q_i^N$. This proves that $q_i = q_i^N$ is the best reply to $q_{-i}^N = \left(q_1^N, \ldots, q_{i-1}^N, q_{i+1}^N, \ldots q_n^N\right)$ and so $q^N$ is a Nash equilibrium of a Cournot oligopoly with inequity averse players.
We now know that the set $N^{IA}$ is non-empty. We still need to show that if all players are strictly averse to inequity, then $q(\beta) < q(\alpha)$, that is, $N^{IA}$ is an interval. We know that $q(\alpha_i)$ is an increasing function of $\alpha_i$ and that $q(\beta_i)$ is a decreasing function of $\beta_i$ for $i = 1, \ldots, n$. Note that if at least one player does not feel inequity aversion then $q(\beta) = q(\alpha)$, and $N^{IA}$ is a singleton. To see this suppose that player $i$ is not inequity averse, that is, $\alpha_i = \beta_i = 0$. If that is the case, then $h(q, \alpha_i) = 0$ and $g(q, \beta_i) = 0$ imply that $q(0) = q^N$. If $q(\alpha_i)$ is an increasing function of $\alpha_i$ and $q(0) = q^N$, then $q(\alpha) = q^N$. Similarly, if $q(\beta_i)$ is a decreasing function of $\beta_i$ and $q(0) = q^N$, then $q(\beta) = q^N$. So, if at least one player feels aversion to inequity we have that $q(\beta) = q(\alpha) = q^N = N^{IA}$. We will now show that if all players are strictly averse to inequity, then $q(\beta) < q(\alpha)$, that is, $N^{IA}$ is an interval. If all players are strictly averse to inequity, $q(\alpha_i)$ is an increasing function of $\alpha_i$ and $q(0) = q^N$, then $q(\alpha) > q^N = q(0)$. Also, if all players are strictly inequity averse, $q(\alpha_i)$ is an decreasing function of $\beta_i$ and $q(0) = q^N$, then $q(\beta) < q^N = q(0)$. This shows that $q(\beta) < q(\alpha)$ when all players are strictly inequity averse, that is the set $N^{IA}$ is an interval. All outcomes in the set $N^{IA}$ are equilibria of the symmetric Cournot game with inequity aversion since for any profile of quantities, $q_{-i}$, the quantity $q_i$ belongs to the best response of player $i$, $i = 1, \ldots n$. *Q.E.D.*

**Proof of Proposition 3**: The quantity produced by each player in the largest Nash equilibria of $N^{IA}$ is given by $q(\alpha) = \min[q(\alpha_1), \ldots, q(\alpha_n)]$. The largest Nash equilibria of $N^{IA}$ is nondecreasing in $\alpha$ since $\min[q(\alpha_1), \ldots, q(\alpha_n)]$ is nondecreasing in $\alpha$. Similarly, the quantity produced by each player in the smallest Nash equilibria of $N^{IA}$ is given by $q(\beta) = \max[q(\beta_1), \ldots, q(\beta_n)]$. The smallest Nash equilibria of $N^{IA}$ is nonincreasing in $\beta$ since $\max[q(\beta_1), \ldots, q(\beta_n)]$ is nonincreasing in $\beta$. *Q.E.D.*

**Proof of Proposition 4**: When all players are strictly averse to inequity we have $q(\beta) < q^N < q(\alpha)$. Since $\alpha_i$ is drawn from a uniform distribution with support on $[0, 1]$, the larger is $n$ the most likely it becomes that $\min(\alpha_1, \ldots \alpha_n)$ is closer to zero, that is, $N(\alpha)$ is closer to $q^N$. Similarly, since $\beta_i$ is drawn from a uniform distribution with support on $[0, 1]$, the larger is $n$ the most likely it becomes that $\min(\beta_1, \ldots, \beta_n)$ is closer to zero, that is, that $N(\beta)$ is closer to $q^N$. *Q.E.D.*

**Proof of Proposition 5**: The quantity $q_i(\alpha_i)$ is the interception of $t_i(q_j)$ with the equal profit curve. From the definition of $t_i(q_j)$ we have

$$(1 + \alpha_i)[P'(Q)q_i + P(Q) - C_i'(q_i)] - \alpha_i P'(Q)q_j = 0.$$

In the equal profit curve we have $q_j = e_j(q_i)$. So,

$$(1 + \alpha_i)[P'(q_i + e_j(q_i))q_i + P(q_i + e_j(q_i)) - C_i'(q_i)] - \alpha_i P'(q_i + e_j(q_i))e_j(q_i) = 0.$$

Similarly, the quantity $q_i(\beta_i)$ is the interception of $s_i(q_j)$ with the equal profit curve. From the definition of $s_i(q_j)$ we have

$$(1 - \beta_i)[P'(Q)q_i + P(Q) - C'(q_i)] + \beta_i P'(Q)q_j = 0.$$

In the equal profit curve we have $q_j = e_j(q_i)$. So,

$$(1 - \beta_i)[P'(q_i + e_j(q_i))q_i + P(q_i + e_j(q_i)) - C_i'(q_i)] + \beta_i P'(q_i + e_j(q_i))e_j(q_i) = 0.$$

The rest of the proof is similar to that of Proposition 1. *Q.E.D.*

**Proof of Proposition 6**: It follows from $e_1(q_2(\alpha_2)) \leq q_1(\beta_1)$ that the best replies of players 1 and 2 only intersect when $q_1 \geq e_1(q_2(\alpha_2))$ and $q_2 \leq e_2(q_1(\beta_1))$. This together with (5) implies that the Nash equilibrium is the solution to $q_1 = s_1(q_2)$ and $q_2 = t_2(q_1)$. Hence, player 1 fells compassion of player 2 and player 2 feels envy of player 1. It follows from the definitions of $s_1(q_2)$ and $t_2(q_1)$ that $q_2^S < q_2^{IA} < q_1^{IA} < q_1^S$. *Q.E.D.*

**Proof of Proposition 7**: It follows from $q_1(\beta_1) < e_1(q_2(\alpha_2))$, that the best replies of players 1 and 2 intersect when $e_2(q_1(\beta_1)) \leq q_2 \leq e_2(q_1(\alpha_1))$ and $e_1(q_2(\beta_2)) \leq q_1 \leq e_1(q_2(\alpha_2))$. This together with (5) implies that the set of Nash equilibria is given by (6). *Q.E.D.*