Can Optimism Solve the Entrepreneurial Earnings Puzzle?

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Abstract

This paper applies a general equilibrium occupational choice model to study the impact of optimism on the earnings of entrepreneurs and workers. We extend Lucas' (1978) by assuming that a fraction of individuals is optimistic about their ability as entrepreneurs. The model shows that optimism leads to a misallocation of talent and inputs which raises input prices and lowers output. The model is calibrated to match salient features of the U.K. economy and the British Household Panel Survey (BHPS). The calibration shows that the size of the entrepreneurial earnings puzzle in the U.K. is quite large. The Lucas' model predicts mean returns to entrepreneurship 29.4 times larger than the wage. In contrast, according to the BHPS data, the mean returns to entrepreneurship are only 4 percent greater than the average wage. The calibration also shows that optimism can account for half of the size of the puzzle. Hence, while optimism can explain a large part of the puzzle, there are additional factors behind it.

JEL Codes: D50; H21; J24; L26.

Keywords: General Equilibrium; Entrepreneurship; Optimism.

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1 Introduction

The seminal paper of occupational choice and firm size distribution of an economy is Lucas (1978). Individuals have heterogenous one-dimensional abilities as entrepreneurs and choose between entrepreneurship and paid employment. The most talented individuals become entrepreneurs and the less talented ones become workers. The ability differentials across entrepreneurs give rise to different spans of control (firm sizes). One of the main predictions of Lucas' span-of-control model is that the mean returns to entrepreneurship are greater than average wages.

This prediction stands in contrast to empirical evidence on earnings of entrepreneurs. The returns to entrepreneurship are, on average, not higher than wages– the entrepreneurial earnings puzzle (Shane and Venkataraman 2000, Åstebro et al. 2014). For example, Hamilton (2000) finds that after 10 years in business the median entrepreneurial earnings are 35 percent less than those on a paid job of the same duration. Similarly, Moskovitz and Vissing-Jorgensen (2002) find that the mean returns to entrepreneurship are not different from the return on a diversified publicly traded portfolio–the private equity puzzle.[1] The returns to entrepreneurship are also highly variable, more than wages, and more than the returns on public equity (Borjas and Bronars 1989, Hamilton 2000, Moskovitz and Vissing-Jorgensen 2002). Hence, contrary to Lucas' prediction, the mean returns to entrepreneurship are not greater than average wages.

Moreover, research on entrepreneurs' expectations shows that these are very optimistic about the future of their firms. In the U.S. manufacturing sector 61.5 percent of all firms exit within five years (Dunne et al. 1988). However, 48.8 percent of a sample of U.S. nascent entrepreneurs think that the likelihood of exit of their venture is zero in five years time (Hyytinen et al. 2014). Another sample of US entrepreneurs report the odds of their business 'succeeding' to be significantly higher than historically observed and substantially better than the odds of success for other similar

[1] This result was obtained for the 1989-1998 period. However, Kartashova (2014) shows that the private equity puzzle does not extend to the 1989-2010 period.

businesses (Cooper et al. 1998). Direct comparison of entrepreneur expectations to new venture outcomes shows that a representative sample of French entrepreneurs tend to overestimate employment expansion and sales growth (Landier and Thesmar 2009). Yet another sample of U.S. nascent entrepreneurs overestimate the probability that their projects will result in operating ventures and, for those ventures that achieve operation, 62 percent overestimate future sales and 46 percent overestimate the number of employees in the first year of operation (Cassar 2010). The evidence also shows that entrepreneurs are more optimistic than employees (Arabsheibani et al. 2000, Fraser and Greene 2006, Koudstaal et al. 2015) and expect to live about 2 years longer than non-entrepreneurs after controlling for differences in smoking, race, and education-related mortality risk across groups (Puri and Robinson 2013).

This paper uses a general equilibrium occupational choice model to study the impact of optimism on the earnings of entrepreneurs and workers. We extend Lucas' (1978) by assuming that a fraction of individuals is optimistic about their ability as entrepreneurs. An optimistic entrepreneur overestimates the total factor productivity of the firm he or she manages. The model shows that optimism leads to a misallocation of talent and inputs which raises input prices and lowers output. Hence, optimism lowers the mean returns to entrepreneurship and raises the mean returns to paid employment. We calibrate the model to match salient features of the U.K. economy and the British Household Panel Survey. The calibration shows that optimism can account for half of the size of the entrepreneurial earnings puzzle in the U.K..

Section 2 explains how our paper contributes to the literature on optimism and entrepreneurship. The idea that optimism can be one of the forces contributing to explaining the entrepreneurial earnings puzzle is not novel and dates back to Kahneman and Lovallo (1993). A large body of existing studies on entrepreneurship recognized this role of optimism and studies its implications. For example, de Meza and Southey (1996), Bernardo and Welch (2001), and Dawson et al. (2014) argue that the optimism of entrepreneurs can explain both the entry into entrepreneurship and their comparatively low earnings. Our study differs from this literature in three main ways. First, the particular formalization of the mechanism in this paper has not been done in previous studies. Second, taking advantage of the general equilibrium approach, it makes new testable predictions about the impact of optimism on entrepreneurship, markets, and welfare. Third, it provides the first quantitative assessment of what fraction of the entrepreneurial earnings puzzle can be explained by optimism.

Section 3 sets up the model. Following Lucas (1978) we consider a closed economy with a population of size N and a capital stock of K units of capital. Each individual is endowed with one unit of labor, with capital stock K/N, and with a one-dimensional ability θ drawn from the cumulative distribution function $G(\theta)$. Individuals are risk neutral and maximize their expected returns by choosing between being a paid employee or entering entrepreneurship and managing a firm. A firm in this economy is one entrepreneur together with the labor and capital under his control. The production function of the firm is characterized by decreasing returns to scale and complementarity between inputs. The ability of the entrepreneur enters into the production function as the total factor productivity. Decreasing returns to scale in labor and capital ensure that the competitive equilibrium exhibits a non-degenerate distribution of firm sizes. The model departs from Lucas (1978) by assuming that a fraction $\lambda \in (0,1)$ of individuals is optimistic about ability whereas the remaining fraction $1-\lambda$ is realistic.[2] Realists know their ability is θ whereas optimists think, mistakenly, that their true ability is $\gamma\theta$, where $\gamma \geq 1$ measures optimism intensity. The occupational choice of a realist is a standard one but the occupational choice of an optimist is affected by his biased expectations. In other words, realists

[2] Chapter 2 of Parker (2009) discusses in detail the main extensions of Lucas' (1978) model. Kanbur (1979) studies the role of learning about ability on entrepreneurship. Kilhstrom and Laffont (1979) study the role of risk aversion on entrepreneurship. Bewley (1989) considers the role of uncertainty (or ambiguity) aversion on entrepreneurship. Jovanovic (1994) analyzes the joint role of heterogeneous entrepreneurial and working abilities on entrepreneurship. Finally, Lazear (2005) studies the role of entrepreneurial and specialist abilities on entrepreneurship.

who enter entrepreneurship know the true production function of their firms whereas optimists believe their firms are more productive than they really are.

Section 4 solves the competitive equilibrium and shows that optimism has the potential to explain the entrepreneurial earnings puzzle due to three channels. First, optimism leads to a misallocation of talent: lower ability optimists crowd out higher ability realists from entrepreneurship. This misallocation of talent lowers the average ability of the pool of entrepreneurs and implies that the lowest ability entrepreneurs are less talented at running a firm than the highest ability workers. Second, optimism leads to a misallocation of inputs: optimistic entrepreneurs hire an excessive amount of labor and capital in relation to their true ability. Third, optimism raises input prices: (i) optimism raises the equilibrium wage since it expands labor demand and contracts labor supply, and (ii) optimism raises the equilibrium rental cost of capital since it expands capital demand and capital supply is exogenous. These three channels lower the mean returns to entrepreneurship and raise the mean returns to paid employment.

Section 5 calibrates the model. The technology parameters are calibrated to match salient features of the U.K. economy and the British Household Panel Survey (BHPS). Note that the model implies a direct link between entrepreneurial ability and firm size, measured by employment. More precisely, when ability follows a Pareto distribution with a shape parameter ρ, then firm size follows a Pareto distribution with shape parameter $\xi = \rho(1 - \eta)$, where η is the degree of decreasing returns to scale. We use the BHPS firm size distribution to estimate ξ. The parameters η and ρ are calibrated using steady-state values for the U.K. economy, to satisfy $\xi = \rho(1-\eta)$, and the percentage of workers in the BHPS data. The intensity of optimistic beliefs γ is calibrated using Cassar's (2010) measure of nascent entrepreneurs' optimism which compares first-year sales expectations to ex-post realizations. The fraction of optimists λ is calibrated by imposing that the equilibrium share of optimistic entrepreneurs matches the share of self-employed in the U.K. who are optimistic about their firms at start-up (taken from Fraser and Greene 2006). The calibration provides two main results. First, the size of the entrepreneurial earnings puzzle in the U.K. is quite large. The Lucas' (1978) model predicts mean returns to entrepreneurship 29.4 times greater than the wage. In contrast, according to the BHPS data, the mean returns to entrepreneurship are only 4 percent greater than the average wage. Second, the model with optimists can account for half of the size of entrepreneurial earnings puzzle in the U.K.. This indicates that while optimism can explain a large part of the puzzle, there are additional factors behind it.

Section 6 discusses the robustness of our findings to a number of extensions: if the return to entrepreneurship is stochastic rather than deterministic; if individuals have heterogeneous abilities both as workers and as entrepreneurs; if the occupational choice is extended to consider also firms run by owners without employees; if the economy is composed by a corporate and a non-corporate sector.

The reminder of the paper proceeds as follows. Section 2 reviews related literature. Section 3 sets up the model. Section 4 characterizes the competitive equilibrium. Section 5 calibrates the model. Section 6 discusses several extensions of the model. Section 7 concludes the paper. Appendix A contains the proofs of our results. Appendix B shows that optimism leaves the firm size distribution unchanged under our baseline model. Appendix C solves and calibrates a model with non-pecuniary benefits of entrepreneurship.

2 Related Literature

This section explains how our paper contributes to the literature on optimism and entrepreneurship. First, we discuss alternative explanation to the entrepreneurial earnings puzzle. Second, we review related work on the impact of optimism on entrepreneurship, markets, and welfare.

2.1 Alternative Explanations

Besides optimism, a number of competing explanations have been proposed to account for the entrepreneurial earnings puzzle: heterogeneity in risk preferences, negative correlation between entrepreneurial and working skills, non-pecuniary benefits, underreporting of entrepreneurial earnings, labor market frictions, and learning.[3]

Kilhstrom and Laffont (1979) study the role of risk aversion in a general equilibrium model of occupational choice and show that less risk averse individuals become entrepreneurs while more risk averse individuals become employees. Heterogeneity in risk preferences among individuals seems a plausible explanation for this puzzle. However, there is little empirical evidence supporting this explanation since entrepreneurs' risk attitudes are found to be indistinguishable from those of wage earners (Wu and Knott 2006, Parker 2009, Holm et al. 2013, and Koudstaal et al. 2016).[4]

Jovanovic (2019) provides another alternative explanation. If entrepreneurial skills and working skills are negatively correlated in the population of individuals, then the mean returns to entrepreneurship can be less than the wage. However, the empirical evidence supports the assumption that entrepreneurial and wage-working abilities are positively correlated (Murphy et al. 1991, Javanovic 1994, Braguinsky et al. 2011).

Non-pecuniary benefits is another potential explanation for the puzzle (Hurst and Pugsley 2011, Åstebro et al. 2014). This explanation is compelling given that

[3] Åstebro and Chen (2014) review the key findings regarding the returns to entrepreneurship and summarize them into six stylized facts: (1) entrepreneurs earn less than employees, (2) earnings for entrepreneurs show a larger variance and larger positive skew, (3) entrepreneurs have a flatter earnings-tenure profile than do wage workers, (4) there is both positive and negative selection into entrepreneurship, (5) entrepreneurs work more hours than wage workers, and (6) many entrepreneurs persist despite the possibility of earning more in wage work.

[4] According to Åstebro et al. (2014) "(...) the evidence that entrepreneurial entry can be explained by a group of people with very different general risk attitudes than the general population is quite mixed and inconclusive. Some studies suggest that those who start firms are more risk seeking, but others find no association."

many entrepreneurs persist despite the possibility of earning more in wage work. Furthermore, non-pecuniary benefits imply the existence of small, old firms, and are identified in the literature (see, for example, Jones and Pratap 2020). Appendix C solves an calibrates a general equilibrium model of occupational choice where a fraction $\mu \in (0,1)$ of individuals derive non-pecuniary benefits $B > 0$ from entrepreneurship. We find that if half of the population derives non-pecuniary benefits from entrepreneurship and these are worth 20 percent of the equilibrium wage, then the model is able to account for about one tenth of the size of the puzzle.

Another potential explanation for the puzzle is earnings underreporting by entrepreneurs (Pissarides and Weber 1989, Hurst et al. 2014, Åstebro and Chen 2014). Pissarides and Weber (1989) use British household food expenditures to infer underreporting and find that households with an entrepreneur underreport their household income by 35 percent. Åstebro and Chen (2014) use food consumption data from U.S. households to infer underreporting by entrepreneurs and find that, after correcting for underreporting, the mean financial gain to entrepreneurship is positive and large, greater than 42 percent.[5] However, the size of the entrepreneurial earnings puzzle in the U.K. reported in Section 4 casts doubt on this explanation: the size of underreporting of earnings by entrepreneurs would have to be extremely large to account for the size of the puzzle.

Labor market frictions is yet another explanation for the puzzle (Åstebro et al. 2011). According to this explanation wages are not always properly matched to ability and frictions are the highest at the tails of the wage distribution. This implies that those with the lowest and highest abilities are more likely to enter entrepreneurship. However, as Åstebro et al. (2011) point out, the model cannot explain the fact that wage workers, on average, earn no more after entering entrepreneurship.

Finally, according to the learning explanation individuals do not know their ability as entrepreneurs and enter entrepreneurship to learn it; those who are more able

[5] However, Åstebro and Chen (2014) argue that this estimate is build on questionable model assumptions and conclude that underreporting cannot adequately explain the stylized facts regarding entrepreneurial earnings.

remain as entrepreneurs, while those who are less able leave entrepreneurship for wage work (Vereshchagina and Hopenhayn 2009, Campanale 2010, Poschke 2013). In Vereshchagina and Hopenhayn (2009) entrepreneurship is risky and paid employment provides a fixed outside option. Individuals face financing constraints and because of them they take more risk at low wealth levels than at high wealth levels. The model shows that the combination of occupational choice and financing constraints can lead entrepreneurs to display risk-taking behavior. Hence, entrepreneurs operate in an environment that leads them to engage in risky investment even in the absence of a return premium. Campanale (2010) shows that entry and private equity allocation for the majority of entrepreneurs can be rationalized even with negative expected premia on individual business investment. Since individuals can switch back to paid-employment, they find it worthwhile experimenting with entrepreneurship to find out if the project is good even if initially the expected return is low. The model also quantifies the amount of risk premia that would justify entry into entrepreneurship in this environment, and finds that it is substantially larger than what is seen in the data. In Poschke (2013) individuals differ in their efficiency as workers and in the productivity of the firms they start. Whereas efficiency as a worker is known, the productivity of entrepreneurial projects can only be found after implementing them. The model shows that the option to abandon bad projects attracts low-ability individuals into entrepreneurship.

2.2 Other Related Studies

More broadly, our study contributes to the literature on the consequences of optimism for entrepreneurship, markets, and welfare. This literature can be divided into studies which use a partial equilibrium approach (de Meza and Southey 1996, Manove and Padilla 1999, Coval and Thakor 2005) and those, like ours, which use a general equilibrium approach (Manove 2007, Bachmann and Elstner 2015).

In de Meza and Southey (1996) individuals choose between working in a safe occupation or undertaking a project with a risky return. Entrepreneurs must select the right mix of self-finance and debt-finance from risk neutral banks to develop their projects. All individuals have the same ability or probability of success of their projects. Banks and realistic entrepreneurs know a project's true probability of success but optimistic entrepreneurs overestimate it. Banks can distinguish between optimists and realists. The model shows that optimists become entrepreneurs, select maximum internal finance, any form of external finance is a standard debt contract, and that optimism can lead to excessive lending.

Manove and Padilla (1999) study the role of optimism on investment and on the credit market but, unlike de Meza and Southey (1996), assume that banks cannot differentiate optimists from realists. The model shows that, in the presence of optimists, perfectly competitive banks may be insufficiently conservative in their dealings with entrepreneurs, even if entrepreneurs themselves may practice self-restraint to signal realism. In addition, the presence of optimists also implies that the use of collateral requirements by banks may reduce the efficiency of the credit market.

Coval and Thakor (2005) study the role of optimism and pessimism on financial intermediation. They consider a model where individuals do not have enough wealth to self-finance a project. Realists correctly assess a project's probability of success, optimists overestimate it and pessimists underestimate it. The model shows that realists form a financial intermediary that raises funds from pessimists (who become investors in the intermediary) and lends to optimists (who become entrepreneurs).

In Manove (2007) individuals are either realists or optimists and choose whether they become entrepreneurs or workers. Entrepreneurs start their business with an initial stock of capital, which can be used for consumption or for production. There is no external market for capital. Entrepreneurs hire labor from a perfectively competitive market, and the difference between production, net of the cost of labor, and consumption adds up to the stock of capital. The model shows that optimistic entrepreneurs may stay in business in the steady state. In addition, by bidding up wages, optimistic entrepreneurs increase the welfare of workers and decrease the welfare of realistic entrepreneurs. The effect on output is ambiguous: the overutilization of external resources (labor) reduces output, while the overutilization of internal resources (savings) increases output.

Bachmann and Elstner (2015) assess the economic significance of optimistic and pessimistic biases of firms on the lifetime utility of a representative household. In their model, as in ours, firms have to decide about their production inputs before they know their productivity level in a given period. Consequently, any expectation biases will lead to input misallocations. They calibrate the model using microdata from the German IFO Business Climate Survey and find that expectation biases create welfare losses of at most 0.2 percent of consumption.

3 Set-up

The economy consists of a continuum of risk-neutral individuals. The population is of size N and the capital stock is of K units of capital. Individuals derive utility from consumption and can earn income either as workers or by running their own firm. Each individual is endowed with 1 unit of labor, with capital stock K/N, and with a one-dimensional ability θ drawn from the cumulative distribution function $G(\theta)$ with support on $[\theta_m, \infty)$, with $0 < \theta_m < \infty$.

If an individual with ability θ becomes a worker he supplies his unit of labor on the labor market, receives the competitive wage w for his unit of labor, and receives the competitive rental rate of capital for renting his capital K/N. Hence, a wage worker ends up with an income

$$w + rK/N.$$

If an individual with ability θ becomes an entrepreneur he can use without cost a technology defined by the continuous production function

$$y = \theta f(l, k),$$

where y is output, l is labor, and k is capital. Following Lucas (1978), θ enters into the production function as the total factor productivity (TFP). Any individual can run at most one firm. We assume that f is twice continuously differentiable with $f_l > 0$, $f_k > 0$, $f_{ll} < 0$, $f_{kk} < 0$. This production function combines as inputs one entrepreneur, who is essential to operate the firm, l homogeneous employees, and k units of homogeneous capital. The production function exhibits decreasing returns to scale in the variable inputs, labor and capital, so that the competitive equilibrium exhibits a non-degenerate firm size distribution. This assumption implies that the size of firms is finite. This could be due for instance to limits in entrepreneurs' span of control: as activity expands, it becomes more difficult to control, and the marginal product of the variable inputs diminishes.

Entrepreneurs hire labor at the competitive wage rate w and rent capital at the competitive rental cost of capital r. Hence, an entrepreneur who employs l workers and rents k units of capital earns a profit of

$$\pi(\theta, w, r) = p\theta f(l, k) - wl + r(K/N - k).$$

From now on the price of output p is normalized to be 1. Individuals can belong to one of two types: optimists and realists. A fraction $\lambda \in (0, 1)$ of the population is optimistic about their ability as entrepreneurs and a fraction $1 - \lambda$ is realistic. The perceived profit of an entrepreneur who employs l workers and rents k units of capital is

$$\pi(\gamma\theta, w, r) = \gamma\theta f(l, k) - wl + r(K/N - k), \tag{1}$$

where $\gamma \geq 1$. The parameter γ measures the strength or intensity of optimism about TFP. Entrepreneurs with $\gamma = 1$ are realists and those with $\gamma > 1$ are optimists. The greater γ is, the more entrepreneurs overestimate their TFP and their future profits.[6]

[6] This specification of optimistic beliefs is analytically tractable. Furthermore, under it optimism coincides with overestimation of ability. The strongest cross-national covariate of an individual's entrepreneurial propensity is whether the person believes herself to have the sufficient skills and knowledge to start a business (Koellinger et al. 2007). The probability of becoming an entrepreneur increases with a person's confidence in his/her ability to perform entrepreneurship related tasks (Cassar and Friedman 2009). Entrepreneurs are more overconfident about their abilities than non-entrepreneurs: 59 percent of entrepreneurs, 56 percent of the managers, and 52 percent of the employees overestimate their performance on a cognitive ability test (Koudstaal et al. 2015).

The distributions of entrepreneurial ability of realists and optimists are assumed to be identical and independent.

An individual who becomes an entrepreneur will employ $l(\gamma\theta; w, r)$ workers and $k(\gamma\theta; w, r)$ units of capital where $l(\gamma\theta; w, r)$ and $k(\gamma\theta; w, r)$ are the values of l and k that solve the following problem

$$\max_{l,k}[\gamma\theta f(l, k) - wl + r(K/N - k)].$$

The first-order conditions to this problem are

$$\gamma\theta f_l(l, k) = w. \tag{2}$$

and

$$\gamma\theta f_k(l, k) = r. \tag{3}$$

It follows from (2), the assumption of decreasing returns to labor, $f_{ll} < 0$, and complementarity between ability and labor, i.e., $f_{l\theta} > 0$, that entrepreneurs with a higher θ hire more workers: $\partial l(\gamma\theta, w, r)/\partial\theta = -\gamma f_{l\theta}/f_{ll} > 0$. Similarly, it follows from (3), the assumption of decreasing returns to capital, $f_{kk} < 0$, and complementarity between ability and capital, i.e., $f_{k\theta} > 0$, that entrepreneurs with a higher θ hire more capital: $\partial k(\gamma\theta, w, r)/\partial\theta = -\gamma f_{k\theta}/f_{kk} > 0$. It also follows from (2) that an optimistic entrepreneur will demand more labor than a realist with the same ability. Similarly, it follows from (3) that an optimistic entrepreneur will demand more capital than a realist with the same ability.

A realist with ability θ chooses to become a worker at wage w and rental cost of capital r when

$$\theta f(l(\theta, w, r), k(\theta, w, r)) - wl(\theta, w, r) - rk(\theta, w, r) \leq w. \tag{4}$$

He selects to be an entrepreneur if

$$\theta f(l(\theta, w, r), k(\theta, w, r)) - wl(\theta, w, r) - rk(\theta, w, r) \geq w, \tag{5}$$

and he is indifferent if the equality holds in (4) and (5).[7] An optimist with perception of ability $\gamma\theta$ chooses to become a worker at wage w and rental cost of capital is r when

$$\gamma\theta f(l(\gamma\theta, w, r), k(\gamma\theta, w, r)) - wl(\gamma\theta, w, r) - rk(\gamma\theta, w, r) \leq w. \tag{6}$$

He selects to be an entrepreneur if

$$\gamma\theta f(l(\gamma\theta, w, r), k(\gamma\theta, w, r)) - wl(\gamma\theta, w, r) - rk(\gamma\theta, w, r) \geq w, \tag{7}$$

and he is indifferent if the equality holds in (6) and (7).

Since there are only three markets—output, labor, and capital—by Walras' Law, general equilibrium is realized when the labor and capital markets clear. At the equilibrium wage, the labor demanded by entrepreneurs equals labor supplied by workers. At the equilibrium rental cost of capital, the capital demanded by entrepreneurs equals the exogenous capital stock of the economy K. Formally, a competitive equilibrium is (i) a partition $\{[\theta_m, \hat{\theta}_R], [\hat{\theta}_R, \infty)\}$ of $[\theta_m, \infty)$ where for all $\theta \in [\theta_m, \hat{\theta}_R]$ (4) holds and for all $\theta \in [\hat{\theta}_R, \infty)$ (5) holds, (ii) a partition $\{[\theta_m, \hat{\theta}_O], [\hat{\theta}_O, \infty)\}$ of $[\theta_m, \infty)$ where for all $\theta \in [\theta_m, \hat{\theta}_O]$ (6) holds and for all $\theta \in [\hat{\theta}_O, \infty)$ (7) holds, (iii) a wage w for which labor demand equals labor supply

$$(1-\lambda)\int_{\hat{\theta}_R}^{\infty} l(\theta, w, r)dG(\theta) + \lambda\int_{\hat{\theta}_O}^{\infty} l(\gamma\theta, w, r)dG(\theta) = \left[(1-\lambda)G(\hat{\theta}_R) + \lambda G(\hat{\theta}_O)\right]. \tag{8}$$

and (iv) a rental cost of capital r for which capital demand equals the exogenous capital supply

$$N\left[(1-\lambda)\int_{\hat{\theta}_R}^{\infty} k(\theta, w, r)dG(\theta) + \lambda\int_{\hat{\theta}_O}^{\infty} k(\gamma\theta, w, r)dG(\theta)\right] = K \tag{9}$$

In equilibrium, realists with ability below $\hat{\theta}_R$ become workers whereas those with ability above $\hat{\theta}_R$ become entrepreneurs. Similarly, optimists with below $\hat{\theta}_O$ become workers whereas those with ability above $\hat{\theta}_O$ become entrepreneurs. We refer to a

[7] The term rK/N cancels out because an individual receives the rental price of his K/N unit of capital both when he decides to be a worker and an entrepreneur.

realist with ability $\hat{\theta}_R$ as the *marginal realistic entrepreneur*. We refer to an optimist with ability $\hat{\theta}_O$ as the *marginal optimistic entrepreneur*.

4 Competitive Equilibrium

In this section we determine the competitive equilibrium under a generalized Cobb-Douglas production function with decreasing returns to scale and a Pareto distribution of ability.

The production function is

$$y = \theta l^{\alpha} k^{\beta},$$

with $\theta > 0$, $\alpha > 0$, $\beta > 0$, and $\alpha + \beta \equiv \eta < 1$. Hence, the variable inputs, labor and capital, are combined under a generalized Cobb-Douglas production function with decreasing returns to scale. This is a standard assumption in general equilibrium models of occupational choice with heterogeneous ability (Evans and Jovanovic 1989, Murphy et al. 1991, de Meza and Southey 1996, Manove 2007, Poschke 2013, Bachmann and Elstner 2015).

The Pareto distribution has been shown to provide a good approximation for firm size distributions (Axtell 2001, Helpman et al. 2004, Rossi-Hansberg and Wright 2007, Luttmer 2007). Since Lucas (1978) produces a size distribution for firms that inherits the properties of the distribution of ability in the population, we solve the model assuming that ability is distributed according to a Pareto cumulative distribution:

$$G(\theta) = 1 - \left(\frac{\theta_m}{\theta}\right)^{\rho}, \tag{10}$$

with $\theta \geq \theta_m > 0$ and $\rho > 0$, where ρ is the shape parameter and θ_m is the scale parameter that marks a lower bound on ability. The density is given by $g(\theta) = \rho\theta_m^{\rho}\theta^{-\rho-1}$. Furthermore, the mean and variance are equal to $E(\theta) = \theta_m\rho/(\rho-1)$ and $V(\theta) = \theta_m^2\rho/(\rho-1)^2(\rho-2)$, respectively. Hence, the mean exists as long as $\rho > 1$ and the variance exists as long as $\rho > 2$.

The perceived profit of an entrepreneur with ability θ and perception of ability $\gamma\theta$ is

$$\pi(\gamma\theta, l, k) = \gamma\theta l^{\alpha}k^{\beta} - wl + r(K/N - k), \tag{11}$$

where $\gamma \geq 1$. Hence, an entrepreneur with perception of ability $\gamma\theta$ chooses to employ l workers and k units of capital where l and k are the solution to

$$\max_{l,k} \left[\gamma\theta l^{\alpha}k^{\beta} - wl + r(K/N - k)\right].$$

The first-order conditions are $\alpha\gamma\theta l^{\alpha-1}k^{\beta} = w$ and $\beta\gamma\theta l^{\alpha}k^{\beta-1} = r$. Solving for l and k we obtain the input demands:

$$l(\gamma\theta, w, r) = (\gamma\theta)^{\frac{1}{1-\eta}} \left(\frac{\alpha}{w}\right)^{\frac{1-\beta}{1-\eta}} \left(\frac{\beta}{r}\right)^{\frac{\beta}{1-\eta}}, \tag{12}$$

and

$$k(\gamma\theta, w, r) = (\gamma\theta)^{\frac{1}{1-\eta}} \left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\eta}} \left(\frac{\beta}{r}\right)^{\frac{1-\alpha}{1-\eta}}. \tag{13}$$

The input demands determine the size of the firm given the ability of the entrepreneur, the wage, the rental cost of capital, and the entrepreneur's perception of ability. We see from (12) and (13) that entrepreneurs' input demands are greater among those with higher ability θ. That is, more talented entrepreneurs run larger firms than less talented entrepreneurs, irrespective of whether firm size is defined in terms of labor or capital. We also see from (12) and (13) that, for a given ability level, optimists (those with $\gamma > 1$) run larger firms than realists (those with $\gamma = 1$). Substituting (12) and (13) into (11) and we obtain the perceived reduced form profit of an entrepreneur:

$$\pi(\gamma\theta, w, r) = \gamma^{\frac{1}{1-\eta}}\theta^{\frac{1}{1-\eta}}(1-\eta) \left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\eta}} \left(\frac{\beta}{r}\right)^{\frac{\beta}{1-\eta}} + r\frac{K}{N}. \tag{14}$$

We see from (14) that the assumption of decreasing returns to scale, i.e., $\eta \in (0, 1)$, implies that the perceived reduced form profit of an entrepreneur is an increasing and convex function of θ. The returns to paid employment are

$$w + r\frac{K}{N} \tag{15}$$

The ability of the marginal realistic entrepreneur, $\hat{\theta}_R$, is obtained by setting $\gamma = 1$ in (14) and equating this to (15). Hence, a realist with ability $\hat{\theta}_R$ is indifferent between being an entrepreneur and a worker when

$$\hat{\theta}_R{}^{\frac{1}{1-\eta}}(1-\eta)\left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\eta}}\left(\frac{\beta}{r}\right)^{\frac{\beta}{1-\eta}} = w. \tag{16}$$

The ability of the marginal optimistic entrepreneur, $\hat{\theta}_O$, is obtained by equating (14) to (15). Hence, an optimist with perception of ability $\gamma\hat{\theta}_O$ and ability $\hat{\theta}_O$ is indifferent between being an entrepreneur and a worker when

$$\gamma^{\frac{1}{1-\eta}}\hat{\theta}_O^{\frac{1}{1-\eta}}(1-\eta)\left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\eta}}\left(\frac{\beta}{r}\right)^{\frac{\beta}{1-\eta}} = w. \tag{17}$$

Since the perceived reduced form profit of an entrepreneur is an increasing and convex function of θ it follows from (16) and (17) that there exist a unique ability cut-off between realistic entrepreneurs and realistic workers–$\hat{\theta}_R$ is unique–and an unique ability cut-off between optimistic entrepreneurs and optimistic workers–$\hat{\theta}_O$ is unique. In addition, it follows from (16), (17), and $\gamma > 1$ that:

$$\hat{\theta}_O = \frac{\hat{\theta}_R}{\gamma} < \hat{\theta}_R, \tag{18}$$

i.e., the marginal optimistic entrepreneur has a lower ability than the marginal realistic entrepreneur. This results holds regardless of the ability distribution and implies that optimists are more likely to become entrepreneurs than realists.[8] This is in line with Puri and Robinson (2007) who find that optimism is an important determinant of self-employment after controlling for a range of family, demographic, and wealth characteristics. Inequality (18) also implies that entrepreneurs are more likely to be

[8] The probability an optimist becomes an entrepreneur is $\Pr(E|O) = \Pr(E \cap O)/\Pr(O) = \lambda(1 - G(\hat{\theta}_O))/\lambda = 1 - G(\hat{\theta}_O)$. The probability a realist becomes an entrepreneur is $\Pr(E|R) = \Pr(E \cap R)/\Pr(R) = (1-\lambda)(1-G(\hat{\theta}_R))/(1-\lambda) = 1-G(\hat{\theta}_R)$. It follows from (18) that $\Pr(E|O) > \Pr(E|R)$.

optimists than workers. In fact, the fraction of optimistic entrepreneurs is equal to

$$\frac{E_O}{E} = \frac{\lambda\left[1 - G(\hat{\theta}_O)\right]}{(1-\lambda)\left[1 - G(\hat{\theta}_R)\right] + \lambda\left[1 - G(\hat{\theta}_O)\right]},$$

and the fraction of optimistic workers to

$$\frac{L_O}{L} = \frac{\lambda G(\hat{\theta}_O)}{(1-\lambda)G(\hat{\theta}_R) + \lambda G(\hat{\theta}_O)}.$$

It follows from (18) that $E_O/E > L_O/L$. This result is valid no matter the ability distribution and is in line with the empirical evidence in Arabsheibani et al. (2000), Fraser and Greene (2006), and Koudstaal et al. (2015).[9]

Equations (8), (9), (16), and (17), form a system of four equations and four unknowns ($\hat{\theta}_R$, $\hat{\theta}_O$, w, r) which defines a unique competitive equilibrium. Solving (16) and (17) for the unique cut-offs $\hat{\theta}_R$ and $\hat{\theta}_O$ and substituting these into (8) and (9) we obtain the unique equilibrium vector of input prices (w^*, r^*). Finally, from $(\hat{\theta}_R, \hat{\theta}_O, w^*, r^*)$ we obtain the equilibrium output level Y^*. The existence and uniqueness of the equilibrium, a standard result in Lucas (1978), is not affected by the presence of optimists. Proposition 1 describes the competitive equilibrium.

Proposition 1: *If the production function is a generalized Cobb-Douglas with decreasing returns to scale, i.e.,* $y = \theta l^\alpha k^\beta$, *with* $\alpha > 0$, $\beta > 0$, *and* $\alpha + \beta < 1$,

[9] Arabsheibani et al. (2000) compare entrepreneurs' and employees' expectations of future prosperity to actual outcomes using a sample from the BHPS during the years 1990-1996. They find that entrepreneurs are 4.6 times as likely to forecast an improved financial position but experience a deterioration than to forecast a deterioration but experience an improvement. In contrast, for employees the ratio was only 2.9. Fraser and Greene (2006) find that self-employed Britons have higher income expectations than employees during the years 1984-99, but the difference diminishes with experience. Koudstaal et al. (2015) run a lab-in-the field experiment in the Netherlands and find that 58 percent of entrepreneurs can be classified as 'very optimistic,' i.e., have a score of 18 or more in the Revised Life Orientation Test, a commonly used measure of dispositional optimism. In contrast, only 32 percent of employees can be classified as 'very optimistic.'

entrepreneurial ability is distributed according to a Pareto cumulative distribution, i.e., $G(\theta) = 1 - (\theta_m/\theta)^\rho$ *for* $\theta \geq \theta_m > 0$*, where* $\rho > 1/(1-\eta)$*, and*

$$\frac{1-\lambda+\lambda\gamma^\rho}{\gamma^\rho} \geq \frac{\rho(1-\eta)-1}{\rho(1-\beta)-1}, \tag{19}$$

then there exists a unique competitive equilibrium where the marginal realistic entrepreneur has ability

$$\hat{\theta}_R = \theta_m(1-\lambda+\lambda\gamma^\rho)^{\frac{1}{\rho}}\left[\frac{\rho(1-\beta)-1}{\rho(1-\eta)-1}\right]^{\frac{1}{\rho}}, \tag{20}$$

the marginal optimistic entrepreneur has ability

$$\hat{\theta}_O = \theta_m\frac{(1-\lambda+\lambda\gamma^\rho)^{\frac{1}{\rho}}}{\gamma}\left[\frac{\rho(1-\beta)-1}{\rho(1-\eta)-1}\right]^{\frac{1}{\rho}}, \tag{21}$$

the wage is

$$w^* = \theta_m\alpha^\alpha\beta^\beta(1-\eta)^{1-\eta}(1-\lambda+\lambda\gamma^\rho)^{\frac{1}{\rho}}\left[\frac{\rho(1-\beta)-1}{\rho(1-\eta)-1}\right]^{\frac{1}{\rho}}\left[\frac{N}{K}\frac{\beta\rho}{\rho(1-\beta)-1}\right]^{-\beta}, \tag{22}$$

and the rental cost of capital is

$$r^* = \theta_m\alpha^\alpha\beta^\beta(1-\eta)^{1-\eta}(1-\lambda+\lambda\gamma^\rho)^{\frac{1}{\rho}}\left[\frac{\rho(1-\beta)-1}{\rho(1-\eta)-1}\right]^{\frac{1}{\rho}}\left[\frac{N}{K}\frac{\beta\rho}{\rho(1-\beta)-1}\right]^{1-\beta}. \tag{23}$$

Assumption (19) implies that the marginal optimistic entrepreneur has ability greater than the lower bound for ability θ_m and thus ensures that the competitive equilibrium is well defined. Equations (20) and (21) show us that the existence of optimists leads to a misallocation of talent. In a competitive equilibrium without optimists (where $\lambda = 0$ or $\gamma = 1$) the marginal entrepreneur has ability

$$\hat{\theta}_0 = \theta_m\left[\frac{\rho(1-\beta)-1}{\rho(1-\eta)-1}\right]^{\frac{1}{\rho}},$$

which implies that individuals with ability $[\theta_m, \hat{\theta}_0]$ become workers and individuals with ability $[\hat{\theta}_0, \infty)$ become entrepreneurs. Hence, in the competitive equilibrium without optimists the ablest people become entrepreneurs. In a competitive equilibrium with optimists we have

$$\theta_m \leq \hat{\theta}_O < \hat{\theta}_0 < \hat{\theta}_R, \tag{24}$$

that is, realists with ability $[\theta_m, \hat{\theta}_R]$ and optimists with ability $[\theta_m, \hat{\theta}_O]$ become workers whereas realists with ability $[\hat{\theta}_R, \infty)$ and optimists with ability $[\hat{\theta}_O, \infty)$ become entrepreneurs. Hence, the presence of optimists replaces some above the benchmark cutoff $\hat{\theta}_0$ realistic entrepreneurs by some below the benchmark cutoff $\hat{\theta}_0$ optimistic entrepreneurs. Note that this crowding-out effect does not affect the ablest people. In addition, in a competitive equilibrium with optimists, the income distributions of workers and entrepreneurs have overlapping supports. This happens because the lowest ability entrepreneur (an optimist with ability $\hat{\theta}_O$) is less talented at running a firm than the highest ability worker (a realist with ability $\hat{\theta}_R$). This is an empirically attractive implication of the model since, in reality, the return distributions of entrepreneurs and workers have overlapping supports (Åstebro et al. 2014).[10]

Equation (22) shows that an increase in the fraction of optimists raises the equilibrium market clearing wage w^*. The intuition behind this result is as follows. Wage effects can occur through two channels: through firm's derived demand for labor and through labor-supply decisions of individuals, who must choose to be either workers or entrepreneurs. The fact that optimists overestimate their ability implies that, for given input prices, the demand for labor of an optimist is higher than the demand for labor of a realist of the same ability. This leads to an expansion of labor demand. An optimist is, for given input prices, more attracted to entrepreneurship than a realist of the same ability. This leads to a contraction of labor supply. The expansion of labor demand and contraction of labor supply raise the market clearing wage.

Equation (23) shows that an increase in the fraction of optimists raises the equilibrium rental cost of capital r^*. The fact that optimists overestimate their ability

[10] Another prediction of the Lucas' span-of-control model is that the return distributions of entrepreneurs and workers have non-overlapping supports. This is no longer true if at least one of these returns is risky.

implies that, for given input prices, the demand for capital of an optimist is higher than the demand for capital of a realist of the same ability. This leads to an expansion of capital demand. Since the supply of capital is fixed the expansion of capital demand raises the rental cost of capital.

It follows from equations (22) and (23) that a change in the fraction of optimists has no impact on the equilibrium relative input prices since

$$\frac{w^*}{r^*} = \frac{K}{N}\frac{\rho(1-\beta)-1}{\rho\beta}.$$

A change in the fraction of optimists also has no impact on the equilibrium number of workers L^* and entrepreneurs E^* ($E^* = N - L^*$) since

$$L^* = \frac{\alpha\rho}{\rho(1-\beta)-1}N. \tag{25}$$

This result is somewhat surprising. On the one hand, an increase in the fraction of optimists lowers the number of realistic entrepreneurs, but, on the other hand, it raises the number of optimistic entrepreneurs. Hence, at first sight, an increase in the fraction of optimists seems to have an ambiguous effect on the number of entrepreneurs. However, equation (25) shows that these two effects exactly off set each other, that is, there is a full crowding out effect. A change in the fraction of optimists also has no impact on the firm size distribution (the proof of this result can be found in Appendix B). Note that the marginal optimist entrepreneur's perceived ability is equal to the actual ability of the marginal realistic entrepreneur, i.e., $\gamma\hat{\theta}_O = \hat{\theta}_R$. This implies that these entrepreneurs hire the same amount of labor (and capital). Hence, the minimum size of firms run by optimists is identical to the minimum size of firms run by realists. The predictions that optimism has no impact on the equilibrium number of workers, entrepreneurs, relative input prices, and the firm size distribution are due to the assumption that ability follows a Pareto distribution.[11]

Let us now consider the impact of optimism on output, the average ability of the pool of entrepreneurs, and the mean returns to entrepreneurship. In equilibrium,

[11] If, for example, ability follows a uniform distribution, then a change in the fraction of optimists changes the number of workers, entrepreneurs, relative input prices, and the firm size distribution.

output is

$$Y^* = \theta_m \alpha^\alpha (1-\eta)^{1-\eta} N^{1-\beta} K^\beta \frac{1-\lambda+\lambda\gamma^{\rho-1}}{(1-\lambda+\lambda\gamma^\rho)^{1-\frac{1}{\rho}}} \left[\frac{\rho(1-\beta)-1}{\rho(1-\eta)-1}\right]^{\frac{1}{\rho}} \left[\frac{\rho}{\rho(1-\beta)-1}\right]^{1-\beta}. \tag{26}$$

In equilibrium, the average ability of the pool of entrepreneurs, $E(\theta|E^*)$, is the weighted average of the mean abilities of realistic and optimistic entrepreneurs, that is,

$$E(\theta|E^*) = \frac{E_R^*}{E^*} E(\theta|\theta \geq \hat{\theta}_R) + \frac{E_O^*}{E^*} E(\theta|\theta \geq \hat{\theta}_O) = \frac{E_R^*}{E^*}\frac{\rho}{\rho-1}\hat{\theta}_R + \frac{E_O^*}{E^*}\frac{\rho}{\rho-1}\hat{\theta}_O,$$

where $E^* = E_R^* + E_O^* = N - L^*$. In equilibrium, the mean returns to entrepreneurship is

$$\bar{\pi}^* = \frac{\Pi^*}{E^*} = \frac{1}{E^*}\left[Y^* - w^* L^* - r^* \left(1 - \frac{E^*}{N}\right) K\right],$$

where Π^* denotes aggregate profits in the economy.

Proposition 2: *If the production function is a generalized Cobb-Douglas with decreasing returns to scale, i.e.,* $y = \theta l^\alpha k^\beta$*, with* $\alpha > 0$*,* $\beta > 0$*, and* $\alpha + \beta < 1$*, entrepreneurial ability is distributed according to a Pareto cumulative distribution, i.e.,* $G(\theta) = 1 - (\theta_m/\theta)^\rho$ *for* $\theta \geq \theta_m > 0$*, where* $\rho > 1/(1-\eta)$*, and (19) holds, then the existence of optimists lowers: (i) output, (ii) the average ability of the pool of entrepreneurs, and (iii) the mean returns to entrepreneurship.*

The economic intuition behind Proposition 2 is straightforward. We know from Lucas (1978) that, in the absence of distortions, the competitive equilibrium maximizes output. The misguided occupational and input choices of optimists create a distortion in the economy which lowers output. The existence of optimists lowers the average ability of the pool of entrepreneurs since lower ability optimists crowd out higher ability realists from entrepreneurship.[12] Finally, the existence of optimists

[12] The presence of optimists raises the average ability of the pool of realistic entrepreneurs as a result of the higher equilibrium wage.

lowers the mean returns to entrepreneurship. This happens due to four mechanisms. First, optimism leads to a misallocation of talent. Second, optimism leads to a misallocation of inputs: optimistic entrepreneurs hire an excessive amount of labor and capital in relation to their true ability. Third, optimism raises input prices. Fourth, the misallocation of talent and inputs together with the increase in input prices lowers output.

To close this section we compare the mean returns to entrepreneurship of realists to those of optimists. In equilibrium, the mean returns to entrepreneurship of realists is

$$\bar{\pi}_R^* = \frac{\int_{\hat{\theta}_R}^{+\infty} \pi(\theta, l(\theta, w^*, r^*), k(\theta, w^*, r^*))g(\theta)d\theta}{\int_{\hat{\theta}_R}^{+\infty} g(\theta)d\theta},$$

and the mean returns to entrepreneurship of optimists is

$$\bar{\pi}_O^* = \frac{\int_{\hat{\theta}_O}^{+\infty} \pi(\theta, l(\gamma\theta, w^*, r^*), k(\gamma\theta, w^*, r^*))g(\theta)d\theta}{\int_{\hat{\theta}_O}^{+\infty} g(\theta)d\theta}.$$

Proposition 3: *If the production function is a generalized Cobb-Douglas with decreasing returns to scale, i.e., $y = \theta l^\alpha k^\beta$, with $\alpha > 0$, $\beta > 0$, and $\alpha + \beta < 1$, entrepreneurial ability is distributed according to a Pareto cumulative distribution, i.e., $G(\theta) = 1 - (\theta_m/\theta)^\rho$ for $\theta \geq \theta_m > 0$, where $\rho > 1/(1-\eta)$, and (19) holds, then the mean returns to entrepreneurship of realists are greater than the mean returns to entrepreneurship of optimists.*

This result is consistent with empirical evidence that shows that optimism is on average bad for firm performance (Landier and Thesmar 2009), and that entrepreneurs' level of optimism has, on average, a negative relationship with the performance of their new ventures (Hmieleski and Baron 2009). In addition, Dawson et al. (2015) examine how entrepreneurs' forecasts predict entrepreneurship performance using the BHPS during the years 1991-2008 and find that optimists, on average, earn less than pessimists.

5 Calibration

This section calibrates the model to illustrate quantitatively the general equilibrium effects of optimism. The technology parameters are calibrated to match salient features of the U.K. economy and the BHPS. The behavioral parameter γ is calibrated using Cassar's (2010) estimate for sales optimism of entrepreneurs. The behavioral parameter λ is calibrated using Fraser and Greene's (2006) estimate for the share of self-employed in the U.K. who are optimistic about their firms at start-up. Table I below summarizes the model parameters.

Table I

Model Parameters

Parameter	Value	Description
Technology parameters		
α	0.4082	decreasing returns to labor
β	0.1834	decreasing returns to capital
K	13.846	capital stock
N	1	population
ρ	2.5361	shape of ability distribution
θ_m	1	scale of ability distribution
Behavioral parameters		
γ	1.4096	intensity of optimism
λ	0.5209	fraction of optimists

The BHPS is a nationally representative longitudinal survey initiated in 1991 which tracks annually a stratified random cluster sample of households, drawn from the population of British household postal addresses in Great Britain (the U.K. excluding Northern Ireland). We follow de Meza et al. (2019) and use BHPS data from 18 annual waves available between 1991 and 2008. Entrepreneurs are defined as those who self-identify as self-employed business owners.[13] de Meza et al. (2019)

[13]According to de Meza et al. (2019): "This is checked by the interviewer against their UK tax status, under which those who declare themselves to be self-employed are responsible for own

report that the mean gross monthly earnings of entrepreneurs is £1381.48 (for all firms) and £1807.71 (for firms larger than 0 employees) and that the mean gross monthly earnings of employees is £1733.92. The fact that entrepreneurs earn, on average, more or less the same as employees in the U.K. is in line with most recent studies in other countries (Åstebro et al. 2011, Åstebro et al. 2014, Åstebro and Chen 2014).

If the production function is a generalized Cobb-Douglas with decreasing returns to scale and ability follows the Pareto cumulative distribution (10), then firm size, measured by employment, follows the Pareto cumulative distribution

$$S(l) = 1 - \left(\frac{\alpha}{1-\eta}\right)^{\xi} l^{-\xi}, \text{ for } l \geq \alpha/(1-\eta),$$

where $\xi = \rho(1-\eta)$ is the Pareto shape parameter and $\alpha/(1-\eta)$ the Pareto scale parameter of the firm size distribution (i.e., the minimum firm size).[14] Previous estimates for ξ in the U.K. range from 0.995 (Fujiwara et al. 2004) to 1.2048 (Ramsden et al. 2000). To calibrate ξ we use the BHPS. Since the model implies that the minimum firm size has to be strictly greater than 0 we exclude firms with 0 employees. The mean firm size for firms larger than 0 employees in the BHPS is 29.04 employees. The method of moments estimate for ξ is the solution to $29.04 = \xi/(\xi - 1)$, that is, $\xi = 1.0357$.

According to the International Monetary Fund (2007) the capital's average income share in the U.K. is 0.31 for the period between 1991 and 2008. Hence, we set $\alpha/\beta = 0.69/0.31$. We calibrate α, β, and ρ to satisfy $1.0357 = \rho[1 - (\alpha + \beta)]$, $\alpha/\beta = 0.69/0.31$, and to match the fraction of workers employed in firms larger than 0 employees in the BHPS data $L^*/N = 0.9667$. In other words, α, β, and ρ are the

income tax declarations and payments, rather than directly through employer-made deductions. Freelancers and subcontractors who may be self-employed for tax purposes but are not business owners are excluded from the definition and the analysis, drawing on information in a questionnaire item about the nature of the self-employment. This leaves approximately 80% of the self-employed who are business owners."

[14] This result is derived in Appendix B.

solution to:

$$\begin{cases} 1.0357 = \rho(1-\alpha-\beta) \\ \alpha/\beta = 0.69/0.31 \\ 0.9667 = \frac{\alpha\rho}{\rho(1-\beta)-1} \end{cases}.$$

The solution is $\alpha = 0.4082$, $\beta = 0.1834$, and $\rho = 2.5361$. The values for α and β imply $\eta = 0.5916$. This value for the degree of decreasing returns to scale equal falls inside the range of most calibrations which goes from 0.5 up to 0.85 (Atkenson and Kehoe 2005, Hsieh and Klenow 2009, Bachmann and Elstner 2015, Garicano et al. 2016). The values for α and β imply a minimum firm size equal to $\alpha/(1-\eta) = 0.4082/0.4084 \simeq 1$. The Pareto scale parameter of the ability distribution θ_m and N can be chosen arbitrarily and are both normalized to 1. According to the International Monetary Fund (2007), the steady-state capital-output ratio K/Y in the U.K. is 2.3 for the period between 1991 and 2008. To match the steady-state capital-output ratio we use equation (26) and set the capital stock K to 13.846.

We are left with the behavioral parameters γ and λ to calibrate. Optimism intensity γ is calibrated using Cassar's (2010) measure of optimism. This measure is based on the comparison of first-year sales expectations and ex-post realizations of a sample of nascent entrepreneurs and is given by

$$\text{Sales Optimism} = \frac{\text{expected sales} - \text{realized sales}}{\text{expected sales} + \text{realized sales}}.$$

Cassar (2010) reports significant overestimation of sales, resulting in a mean sales optimism of 0.17. In our model, expected sales of optimistic entrepreneurs are equal to $\gamma\theta l(\gamma\theta, w, r)^{\alpha} k(\gamma\theta, w, r)^{\beta}$and realized sales to $\theta l(\gamma\theta, w, r)^{\alpha} k(\gamma\theta, w, r)^{\beta}$. Note that optimistic entrepreneurs hire more labor and capital (which will raise sales) but realized sales are less than the expected sales since total factor productivity, θ, is smaller than expected, $\gamma\theta$. Hence, we calibrate optimism intensity to be the solution to

$$0.17 = \frac{\gamma - 1}{\gamma + 1}. \tag{27}$$

From (27) we obtain $\gamma = 1.4096$. This value for γ implies that optimistic entrepreneurs believe they are 40.96 percent more capable than they are.[15]

The fraction of optimists λ is calibrated by assuming that the equilibrium share of optimistic entrepreneurs E_O^*/E^* is equal to 72.2 percent. The value 72.2 is from Fraser and Greene (2006). It represents the share of self-employed in the U.K. who are optimistic about their firms at start-up (for firms larger than 0 employees). Fraser and Greene (2006) obtain this value using the British Social Attitudes Survey (BSAS) for the period between 1984 and 1999.[16] Hence, to calibrate λ we impose

$$\frac{E_O^*}{E^*} = \frac{\lambda \left(\frac{\theta_m}{\hat{\theta}_O}\right)^{\rho}}{(1-\lambda)\left(\frac{\theta_m}{\hat{\theta}_R}\right)^{\rho} + \lambda \left(\frac{\theta_m}{\hat{\theta}_O}\right)^{\rho}} = \frac{\lambda\gamma^{\rho}}{1-\lambda+\lambda\gamma^{\rho}} = 0.722. \tag{28}$$

Setting $\gamma = 1.4096$ and $\rho = 2.5361$ in (28) we obtain $\lambda = 0.5209$. The competitive equilibrium is well defined since the calibration satisfies $\rho > 1/(1-\eta)$ and (19).

Table II summarizes the results of the calibration. The second column reports the competitive equilibrium of the Lucas' (1978) model ($\lambda = \gamma = 0$). The Lucas' model gives us an idea of the size of the entrepreneurial earnings puzzle in the U.K. This model generates mean returns to entrepreneurship 29.4 times greater than the wage. In contrast, according to the BHPS data, the mean returns to entrepreneurship are

[15] Note that this value is a lower bound for γ since in Cassar's (2010) survey 62 percent of entrepreneurs overestimate first-year sales but the remaining 38 percent do not. Therefore, if we were to condition the mean sales optimism in Cassar's data to be strictly positive, as it is in our model, we would get a mean sales optimism higher than 0.17 and an even higher calibrated value for γ.

[16] The BSAS survey asks self-employed individuals, in each survey year except for 1983 and 1997, whether they felt that their business prospects for the following year were better, the same or worse than the present. Fraser and Greene (2006) define optimists as respondents who believe their prospects for the coming year are better than present. It is possible that part of the optimism captured by Fraser and Green (2006) analysis might be rational, for example in the presence of a booming economy. However, the period analyzed by Fraser and Green (2006) is pretty diverse, including periods of growth and a recession at the beginning of the1990s in conjunction with Britain's entry into the European Exchange Rate Mechanism (see Hills et al. 2010).

only 4 percent greater than the average wage. Hence, the size of the entrepreneurial earnings puzzle in the U.K. is quite large.

Table II

Calibration Results

	Lucas' (1978) Model	Model with optimists	Percent change
Output	6.0200	5.8956	−2.07
Wage	2.5422	3.1507	23.95
Rental cost of capital	0.0797	0.0988	23.95
Mean returns to paid employment	3.6457	4.5190	23.95
Mean returns to entrepreneurship	74.7700	45.5680	−39.06

The third column reports the competitive equilibrium of the model with optimists. This model generates mean returns to entrepreneurship 14.5 times greater than the wage. Hence, the calibration shows that optimism can explain about one half of the size of the entrepreneurial earnings puzzle in the U.K.. This finding indicates that while optimism can explain a large part of the puzzle, there must be additional factors behind it. The fourth column reports the percent change in the variables common to both models. It shows that optimism leads to a 2.07 percent decline in output, a 23.95 percent increase in input prices and mean returns to paid employment, and a 39.06 percent decrease in the mean returns to entrepreneurship. As we have seen, the large decline in the mean returns to entrepreneurship happens due to three channels. First, the misallocation of talent due to the fact that optimists crowd out realists from entrepreneurship. This lowers the average ability of the pool of entrepreneurs by raising the fraction of optimistic entrepreneurs (who have, on average, lower ability and earn lower mean returns) and lowering the fraction of realistic entrepreneurs (who have, on average, higher ability and earn higher mean returns). Second, the misallocation of inputs due to the fact that optimistic entrepreneurs hire an excessive amount of labor and capital in relation to their true ability. Third, the increase in input prices.

6 Extensions

In this section we discuss several extensions of the model and how these would change the qualitative and quantitative general equilibrium implications of optimism.

We assume the returns from entrepreneurship are deterministic. It is possible to extend the model by including a random component ε in entrepreneurial revenues. For example, letting $y = \theta f(l, k) + \varepsilon$, where ε has mean 0 and variance $0 < \sigma^2 < \infty$. Since individuals are risk neutral all results are left unchanged as long as there is no optimism about the realization of ε. If individuals are not only optimistic about θ but also about ε, then entrepreneurship would be more attractive relative to paid employment. In this case the main qualitative effects of optimism would still hold but its quantitative effects would be larger.

We assume individuals have different abilities to run a firm and the same productivity (or ability) as workers. This implies that different entrepreneurs obtain different amounts of profit but that all workers are paid the same wage. This is a natural simplification since the empirical evidence shows that the returns to entrepreneurship are much more variable than wages (Borjas and Bronars 1989, Hamilton 2000). Still, the model could be extended by letting individuals have different abilities in both occupations. Following Jovanovic (1994), we could let the returns to paid employment be equal to $w\psi(\theta)$ where $\psi(\theta)$ is the wage-working ability of an individual with ability θ.[17] If ψ is a strictly increasing function (good entrepreneurs are also good workers) as the empirical evidence indicates (see next section), then optimists would overestimate the returns to entrepreneurship as well as the returns to paid employment. Since these two effects would partially cancel out, the main

[17] Jovanovic (1994) generalizes Lucas (1978) by allowing for heterogeneous working abilities, i.e., the labor income of a worker is given by wy where y represents working ability. Working ability y is correlated with entrepreneurial ability θ if $y = \psi(\theta)$. Jovanovic shows that when ψ is either (i) strictly decreasing or (ii) strictly increasing and not very steep at high levels of θ, then the best potential entrepreneurs are drawn into entrepreneurship. In contrast, when ψ is strictly increasing and very steep at high levels of θ, then the best potential entrepreneurs end up as wage workers.

qualitative effects of optimism would still hold but its quantitative effects would be smaller.[18]

In our model an entrepreneur hires workers and rents capital to produce output. However, the empirical evidence shows that many firms have no employed workers, i.e., the owners of these firms are self-employed without employees (Braguinsky et al. 2011, Salas-Fumas et al. 2014). The model could also be extended to incorporate this third type of occupational choice. This could be done by assuming that the returns of firms without employed workers are given by $B + \theta$, where $B > 0$ represents a non-pecuniary benefit (e.g., a preference for autonomy and control). In this case, realists with ability θ such that $w > \max\{B + \theta, \pi(\theta, w, r)\}$ would become workers, those with ability θ such that $B + \theta > \max\{w, \pi(\theta, w, r)\}$ would open a firm without employed workers, and those with ability θ such that $\pi(\theta, w, r) > \max\{B + \theta, w\}$ would become entrepreneurs. Similarly, optimists with perception of ability $\gamma\theta$ such that $w > \max\{B + \gamma\theta, \pi(\gamma\theta, w, r)\}$ would become workers, those with perception of ability $\gamma\theta$ such that $B + \gamma\theta > \max\{w, \pi(\gamma\theta, w, r)\}$ would open a firm without employed workers, and those with perception of ability $\gamma\theta$ such that $\pi(\gamma\theta, w, r) > \max\{B + \gamma\theta, w\}$ would become entrepreneurs.

In the model there is only one sector, that is, the noncorporate sector with entrepreneurs hiring all the workers in the economy. In the real world while entrepreneurs are important for hiring, the dominant role is played by the corporate sector. Following Quadrini (2000), the model could be extended by including a corporate sector with a constant returns to scale Cobb-Douglas production function $Y_c = F(L_c, K_c) = L_c^{\nu} K_c^{1-\nu}$ where Y_c is output, L_c is labor, K_c is capital, and $\nu \in (0, 1)$. The labor and capital demands of the corporate sector would be obtained from the first order conditions in the corporate sector $r = (1 - \nu)(L_c/K_c)^{\nu}$ and $w = \nu(K_c/L_c)^{1-\nu}$. In equilibrium, the sum of the labor demands from the corporate and non-corporate sectors would have to equal the endogenous labor supply. Simil-

[18] We are assuming here that ψ is strictly increasing and not very steep at high levels of θ. In this case the most talented individuals become entrepreneurs. In contrast, when ψ is strictly increasing and very steep at high level of θ, the most talented individuals become workers.

arly, the sum of the capital demands from the corporate and non-corporate sectors would have to equal the exogenous capital supply. As we have seen, part of the mechanism by which optimism lowers the mean returns to entrepreneurship are general equilibrium effects that go through labor demand and supply, capital demand, and their effects on input prices. Hence, introducing a corporate sector in the economy would not invalidate this mechanism, but might lower its quantitative importance.

We focus on differences in ability and optimism as the main determinants which explain who becomes an entrepreneur and who works as an employee. There are of course many other factors which could influence this choice. For example, entrepreneurial effort (and the disutility of exerting it), access to funds needed to create a firm, risk aversion, and learning about ability. We do not model entrepreneurial effort and therefore we rule out any positive effects of optimism on entrepreneurial effort like the ones found in Manove (2007). If ability and effort are complements, then optimistic entrepreneurs would provide more effort than realistic ones. In this case the impact of optimism on the returns to entrepreneurship and on output would be ambiguous. We assume individuals are risk neutral so we cannot discuss the role that risk aversion together with optimism might play in the decision to become an entrepreneur or a worker. In addition, our model is static so we rule out the possibility that optimists learn their true abilities over time.

7 Conclusion

This paper uses a general equilibrium occupational choice model to study the impact of optimism on the earnings of entrepreneurs and workers. It extends Lucas (1978) by assuming that a fraction of individuals is optimistic about their ability as entrepreneurs. The model shows that optimism leads to a misallocation of talent and inputs which raises input prices and lowers output.

The model is calibrated to match salient features of the U.K. economy and the BHPS. The calibration provides two main results. First, the size of the entrepreneurial earnings puzzle in the U.K. is quite large. According to the BHPS, the mean returns to entrepreneurship are only 4 percent greater than the average wage. However, the model without optimists generates mean returns to entrepreneurship 29.4 times greater than mean returns to paid employment. Second, optimism is able to account for half of the size of the puzzle. This indicates that although optimism can explain a large part of the puzzle, there must be additional factors behind it.

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Appendix A

Proof of Proposition 1: The first step to determine the competitive equilibrium is to find out the labor market equilibrium condition. The labor demand from realistic entrepreneurs is

$$\begin{aligned} L_R^D &= N(1-\lambda)\int_{\hat{\theta}_R}^{\infty} l(\theta,w,r)g(\theta)d\theta \\ &= N(1-\lambda)\left(\frac{\alpha}{w}\right)^{\frac{1-\beta}{1-\eta}}\left(\frac{\beta}{r}\right)^{\frac{\beta}{1-\eta}}\int_{\hat{\theta}_R}^{\infty}\theta^{\frac{1}{1-\eta}}\rho\theta_m^{\rho}\theta^{-\rho-1}d\theta \\ &= N(1-\lambda)\left(\frac{\alpha}{w}\right)^{\frac{1-\beta}{1-\eta}}\left(\frac{\beta}{r}\right)^{\frac{\beta}{1-\eta}}\rho\theta_m^{\rho}\left[\frac{\theta^{\frac{1}{1-\eta}-\rho}}{\frac{1}{1-\eta}-\rho}\right]_{\hat{\theta}_R}^{\infty} \\ &= N(1-\lambda)\left(\frac{\alpha}{w}\right)^{\frac{1-\beta}{1-\eta}}\left(\frac{\beta}{r}\right)^{\frac{\beta}{1-\eta}}\rho\theta_m^{\rho}\frac{\hat{\theta}_R^{\frac{1}{1-\eta}-\rho}}{\rho-\frac{1}{1-\eta}}. \end{aligned} \tag{29}$$

The labor demand from optimistic entrepreneurs is

$$\begin{aligned} L_O^D &= N\lambda\int_{\hat{\theta}_O}^{\infty} l(\gamma\theta,w,r)g(\theta)d\theta = N\lambda\left(\frac{\alpha}{w}\right)^{\frac{1-\beta}{1-\eta}}\left(\frac{\beta}{r}\right)^{\frac{\beta}{1-\eta}}\gamma^{\frac{1}{1-\eta}}\int_{\hat{\theta}_O}^{\infty}\theta^{\frac{1}{1-\eta}}\rho\theta_m^{\rho}\theta^{-\rho-1}d\theta \\ &= N\lambda\left(\frac{\alpha}{w}\right)^{\frac{1-\beta}{1-\eta}}\left(\frac{\beta}{r}\right)^{\frac{\beta}{1-\eta}}\gamma^{\frac{1}{1-\eta}}\rho\theta_m^{\rho}\frac{\hat{\theta}_O^{\frac{1}{1-\eta}-\rho}}{\rho-\frac{1}{1-\eta}}. \end{aligned} \tag{30}$$

From (29) and (30), labor demand is equal to

$$L^D = L_R^D + L_O^D = N\left(\frac{\alpha}{w}\right)^{\frac{1-\beta}{1-\eta}}\left(\frac{\beta}{r}\right)^{\frac{\beta}{1-\eta}}\frac{\rho\theta_m^{\rho}}{\rho-\frac{1}{1-\eta}}\left[(1-\lambda)\hat{\theta}_R^{\frac{1}{1-\eta}-\rho}+\lambda\gamma^{\frac{1}{1-\eta}}\hat{\theta}_O^{\frac{1}{1-\eta}-\rho}\right].$$

Since each worker provides a unit of labor, labor supply is

$$\begin{aligned} L^S &= N\left[(1-\lambda)L_R^S+\lambda L_O^S\right] = N\left[(1-\lambda)\int_{\theta_m}^{\hat{\theta}_R}g(\theta)d\theta+\lambda\int_{\theta_m}^{\hat{\theta}_O}g(\theta)d\theta\right] \\ &= N\left[(1-\lambda)G(\hat{\theta}_R)+\lambda G(\hat{\theta}_O)\right] = N\left[(1-\lambda)(1-\theta_m^{\rho}\hat{\theta}_R^{-\rho})+\lambda(1-\theta_m^{\rho}\hat{\theta}_O^{-\rho})\right] \\ &= N\left[1-\theta_m^{\rho}\left[(1-\lambda)\hat{\theta}_R^{-\rho}+\lambda\hat{\theta}_O^{-\rho}\right]\right]. \end{aligned}$$

In equilibrium, labor demand must equal labor supply:

$$\left(\frac{\alpha}{w}\right)^{\frac{1-\beta}{1-\eta}}\left(\frac{\beta}{r}\right)^{\frac{\beta}{1-\eta}}\frac{\rho\theta_m^\rho}{\rho-\frac{1}{1-\eta}}\left[(1-\lambda)\hat{\theta}_R^{\frac{1}{1-\eta}-\rho}+\lambda\gamma^{\frac{1}{1-\eta}}\hat{\theta}_O^{\frac{1}{1-\eta}-\rho}\right]$$
$$=1-\theta_m^\rho\left[(1-\lambda)\hat{\theta}_R^{-\rho}+\lambda\hat{\theta}_O^{-\rho}\right]. \tag{31}$$

The second step to determine the competitive equilibrium is to find out the capital market equilibrium condition. The capital demand from realistic entrepreneurs is

$$\begin{aligned}
K_R^D &= N(1-\lambda)\int_{\hat{\theta}_R}^{\infty}k(\theta,w,r)g(\theta)d\theta \\
&= N(1-\lambda)\left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\eta}}\left(\frac{\beta}{r}\right)^{\frac{1-\alpha}{1-\eta}}\int_{\hat{\theta}_R}^{\infty}\theta^{\frac{1}{1-\eta}}\rho\theta_m^\rho\theta^{-\rho-1}d\theta \\
&= N(1-\lambda)\left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\eta}}\left(\frac{\beta}{r}\right)^{\frac{1-\alpha}{1-\eta}}\rho\theta_m^\rho\frac{\hat{\theta}_R^{\frac{1}{1-\eta}-\rho}}{\rho-\frac{1}{1-\eta}}.
\end{aligned} \tag{32}$$

The capital demand from optimistic entrepreneurs is

$$\begin{aligned}
K_O^D &= N\lambda\int_{\hat{\theta}_O}^{\infty}k(\gamma\theta,w,r)g(\theta)d\theta \\
&= N\lambda\left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\eta}}\left(\frac{\beta}{r}\right)^{\frac{1-\alpha}{1-\eta}}\gamma^{\frac{1}{1-\eta}}\int_{\hat{\theta}_O}^{\infty}\theta^{\frac{1}{1-\eta}}\rho\theta_m^\rho\theta^{-\rho-1}d\theta \\
&= N\lambda\left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\eta}}\left(\frac{\beta}{r}\right)^{\frac{1-\alpha}{1-\eta}}\gamma^{\frac{1}{1-\eta}}\rho\theta_m^\rho\frac{\hat{\theta}_O^{\frac{1}{1-\eta}-\rho}}{\rho-\frac{1}{1-\eta}}.
\end{aligned} \tag{33}$$

From (32) and (33), capital demand is equal to

$$\begin{aligned}
K^D &= K_R^D+K_O^D \\
&= N\left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\eta}}\left(\frac{\beta}{r}\right)^{\frac{1-\alpha}{1-\eta}}\frac{\rho\theta_m^\rho}{\rho-\frac{1}{1-\eta}}\left[(1-\lambda)\hat{\theta}_R^{\frac{1}{1-\eta}-\rho}+\lambda\gamma^{\frac{1}{1-\eta}}\hat{\theta}_O^{\frac{1}{1-\eta}-\rho}\right].
\end{aligned}$$

In equilibrium, capital demand must equal the exogenous capital supply:

$$\left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\eta}}\left(\frac{\beta}{r}\right)^{\frac{1-\alpha}{1-\eta}}\frac{\rho\theta_m^\rho}{\rho-\frac{1}{1-\eta}}\left[(1-\lambda)\hat{\theta}_R^{\frac{1}{1-\eta}-\rho}+\lambda\gamma^{\frac{1}{1-\eta}}\hat{\theta}_O^{\frac{1}{1-\eta}-\rho}\right]=K/N. \tag{34}$$

The third step to determine the competitive equilibrium is to find out $\hat{\theta}_R$ and $\hat{\theta}_O$. A realist with ability $\hat{\theta}_R$ is indifferent between being an entrepreneur and a worker when

$$\hat{\theta}_R \left[l(\hat{\theta}_R, w, r)\right]^\alpha \left[k(\hat{\theta}_R, w, r)\right]^\beta - wl(\hat{\theta}_R, w, r) + r\left[K/N - k(\hat{\theta}_R, w, r)\right] = w + rK/N,$$

or

$$\alpha^\alpha \beta^\beta (1-\eta)^{1-\eta} \hat{\theta}_R = w^{1-\beta} r^\beta. \tag{35}$$

An optimist with perception of ability $\theta^* = \gamma\hat{\theta}_O$ and ability $\hat{\theta}_O$ is indifferent between being an entrepreneur and a worker when

$$\gamma\hat{\theta}_O \left[l(\gamma\hat{\theta}_O, w, r)\right]^\alpha \left[k(\gamma\hat{\theta}_O, w, r)\right]^\beta - wl(\gamma\hat{\theta}_O, w, r) + r\left[K/N - k(\gamma\hat{\theta}_O, w, r)\right] = w + rK/N,$$

or

$$\alpha^\alpha \beta^\beta (1-\eta)^{1-\eta} \gamma\hat{\theta}_O = w^{1-\beta} r^\beta. \tag{36}$$

It follows from (35) and (36) that

$$\alpha^\alpha \beta^\beta (1-\eta)^{1-\eta} \gamma\hat{\theta}_O = \alpha^\alpha \beta^\beta (1-\eta)^{1-\eta} \hat{\theta}_R,$$

or

$$\hat{\theta}_O = \frac{1}{\gamma}\hat{\theta}_R. \tag{37}$$

Substituting (35) and (37) into (31) we obtain

$$\alpha\rho\theta_m^\rho(1-\lambda+\lambda\gamma^\rho)\hat{\theta}_R^{\frac{1}{1-\eta}-\rho} = (1-\eta)\hat{\theta}_R^{\frac{1}{1-\eta}}\left(\rho - \frac{1}{1-\eta}\right)\left[1 - \theta_m^\rho(1-\lambda+\lambda\gamma^\rho)\hat{\theta}_R^{-\rho}\right],$$

or

$$\alpha\rho\theta_m^\rho(1-\lambda+\lambda\gamma^\rho) = (1-\eta)\left(\rho - \frac{1}{1-\eta}\right)\left[\hat{\theta}_R^\rho - \theta_m^\rho(1-\lambda+\lambda\gamma^\rho)\right],$$

or

$$\begin{aligned}
\hat{\theta}_R^\rho &= \frac{\alpha\rho\theta_m^\rho(1-\lambda+\lambda\gamma^\rho)}{(1-\eta)\left(\rho - \frac{1}{1-\eta}\right)} + \theta_m^\rho(1-\lambda+\lambda\gamma^\rho) \\
&= \theta_m^\rho(1-\lambda+\lambda\gamma^\rho)\left[\frac{\alpha\rho}{(1-\eta)\left(\rho - \frac{1}{1-\eta}\right)} + 1\right] \\
&= \theta_m^\rho(1-\lambda+\lambda\gamma^\rho)\frac{\rho(1-\beta)-1}{\rho(1-\eta)-1}.
\end{aligned}$$

Hence, the ability of the marginal realistic entrepreneur is

$$\hat{\theta}_R = \theta_m (1 - \lambda + \lambda \gamma^{\rho})^{\frac{1}{\rho}} \left[\frac{\rho(1-\beta) - 1}{\rho(1-\eta) - 1} \right]^{\frac{1}{\rho}}. \tag{38}$$

Note that (38), $\rho > 1/(1-\eta)$, and $\eta = \alpha + \beta \in (0, 1)$, imply $\hat{\theta}_R > \theta_m$. From (37) and (38) the ability of the marginal optimistic entrepreneur is

$$\hat{\theta}_O = \frac{1}{\gamma} \theta_m (1 - \lambda + \lambda \gamma^{\rho})^{\frac{1}{\rho}} \left[\frac{\rho(1-\beta) - 1}{\rho(1-\eta) - 1} \right]^{\frac{1}{\rho}}.$$

From (31) and (34) we have

$$\left(\frac{\alpha}{w} \right)^{\frac{\alpha}{1-\eta}} \left(\frac{\beta}{r} \right)^{\frac{1-\alpha}{1-\eta}} \frac{1 - \theta_m^{\rho} \left[(1-\lambda) \hat{\theta}_R^{-\rho} + \lambda \hat{\theta}_O^{-\rho} \right]}{\left(\frac{\alpha}{w} \right)^{\frac{1-\beta}{1-\eta}} \left(\frac{\beta}{r} \right)^{\frac{\beta}{1-\eta}}} = \frac{K}{N},$$

or

$$1 - \theta_m^{\rho} \left[(1-\lambda) \hat{\theta}_R^{-\rho} + \lambda \hat{\theta}_O^{-\rho} \right] = \frac{K}{N} \left(\frac{\alpha}{w} \right)^{\frac{1-\beta-\alpha}{1-\eta}} \left(\frac{\beta}{r} \right)^{\frac{\beta-1+\alpha}{1-\eta}},$$

or

$$\frac{\alpha}{w} \frac{r}{\beta} \frac{K}{N} = 1 - \theta_m^{\rho} \left[(1-\lambda) \hat{\theta}_R^{-\rho} + \lambda \hat{\theta}_O^{-\rho} \right],$$

or

$$\frac{\alpha}{w} \frac{r}{\beta} \frac{K}{N} = 1 - \left[\frac{\rho(1-\eta) - 1}{\rho(1-\beta) - 1} \right],$$

or

$$r = w \frac{N}{K} \frac{\beta \rho}{\rho(1-\beta) - 1} \tag{39}$$

Substituting (39) into (35) we obtain

$$\alpha^{\alpha} \beta^{\beta} (1-\eta)^{1-\eta} \theta_m (1 - \lambda + \lambda \gamma^{\rho})^{\frac{1}{\rho}} \left[\frac{\rho(1-\beta) - 1}{\rho(1-\eta) - 1} \right]^{\frac{1}{\rho}} = w^{1-\beta} \left[w \frac{N}{K} \frac{\beta \rho}{\rho(1-\beta) - 1} \right]^{\beta}.$$

Solving this equality with respect to w we obtain the equilibrium wage:

$$w^* = \alpha^{\alpha} \beta^{\beta} (1-\eta)^{1-\eta} \theta_m (1 - \lambda + \lambda \gamma^{\rho})^{\frac{1}{\rho}} \left[\frac{\rho(1-\beta) - 1}{\rho(1-\eta) - 1} \right]^{\frac{1}{\rho}} \left[\frac{N}{K} \frac{\beta \rho}{\rho(1-\beta) - 1} \right]^{-\beta}. \tag{40}$$

The equilibrium rental cost of capital is equal to

$$\begin{aligned} r^* &= w^* \frac{N}{K} \frac{\beta\rho}{\rho(1-\beta)-1} \\ &= \frac{\alpha^\alpha \beta^\beta (1-\eta)^{1-\eta} \theta_m (1-\lambda+\lambda\gamma^\rho)^{\frac{1}{\rho}} \left[\frac{\rho(1-\beta)-1}{\rho(1-\eta)-1}\right]^{\frac{1}{\rho}}}{\left[\frac{N}{K}\frac{\beta\rho}{\rho(1-\beta)-1}\right]^\beta} \frac{N}{K}\frac{\beta\rho}{\rho(1-\beta)-1} \\ &= \alpha^\alpha \beta^\beta (1-\eta)^{1-\eta} \theta_m (1-\lambda+\lambda\gamma^\rho)^{\frac{1}{\rho}} \left[\frac{\rho(1-\beta)-1}{\rho(1-\eta)-1}\right]^{\frac{1}{\rho}} \left[\frac{N}{K}\frac{\beta\rho}{\rho(1-\beta)-1}\right]^{1-\beta} \qquad (41) \end{aligned}$$

The equilibrium labor force is equal to

$$\begin{aligned} L^* &= N\left[1-\theta_m^\rho\left[(1-\lambda)\hat{\theta}_R^{-\rho}+\lambda\hat{\theta}_O^{-\rho}\right]\right]. \\ &= N\left[1-\theta_m^\rho(1-\lambda+\lambda\gamma^\rho)\hat{\theta}_R^{-\rho}\right] \\ &= N\left[1-\frac{\rho(1-\eta)-1}{\rho(1-\beta)-1}\right] \\ &= N\frac{\alpha\rho}{\rho(1-\beta)-1}. \end{aligned}$$

The equilibrium output level is

$$\begin{aligned} Y^* &= (1-\lambda)N\int_{\hat{\theta}_R}^\infty \theta[l(\theta,w^*,r^*)]^\alpha[k(\theta,w^*,r^*)]^\beta g(\theta)d\theta \\ &\quad +\lambda N\int_{\hat{\theta}_O}^\infty \theta[l(\gamma\theta,w^*,r^*)]^\alpha[k(\gamma\theta,w^*,r^*)]^\beta g(\theta)d\theta. \end{aligned}$$

This can be simplified to

$$\begin{aligned} Y^* &= N\left(\frac{\alpha}{w^*}\right)^{\frac{\alpha}{1-\eta}}\left(\frac{\beta}{r^*}\right)^{\frac{\beta}{1-\eta}}\left[(1-\lambda)\int_{\hat{\theta}_R}^\infty \theta^{\frac{1}{1-\eta}}g(\theta)d\theta+\lambda\gamma^{\frac{\eta}{1-\eta}}\int_{\hat{\theta}_O}^\infty \theta^{\frac{1}{1-\eta}}g(\theta)d\theta\right] \\ &= N\left(\frac{\alpha}{w^*}\right)^{\frac{\alpha}{1-\eta}}\left(\frac{\beta}{r^*}\right)^{\frac{\beta}{1-\eta}}\left[(1-\lambda)\rho\theta_m^\rho\frac{\hat{\theta}_R^{\frac{1}{1-\eta}-\rho}}{\rho-\frac{1}{1-\eta}}+\lambda\gamma^{\frac{\eta}{1-\eta}}\rho\theta_m^\rho\frac{\hat{\theta}_O^{\frac{1}{1-\eta}-\rho}}{\rho-\frac{1}{1-\eta}}\right] \\ &= N\left(\frac{\alpha}{w^*}\right)^{\frac{\alpha}{1-\eta}}\left(\frac{\beta}{r^*}\right)^{\frac{\beta}{1-\eta}}\frac{\rho\theta_m^\rho}{\rho-\frac{1}{1-\eta}}(1-\lambda+\lambda\gamma^{\rho-1})\hat{\theta}_R^{\frac{1}{1-\eta}-\rho} \\ &= N\left(\frac{\alpha}{w^*}\right)^{\frac{\alpha}{1-\eta}}\left(\frac{\beta}{r^*}\right)^{\frac{\beta}{1-\eta}}\frac{\rho\theta_m^{\frac{1}{1-\eta}}}{\rho-\frac{1}{1-\eta}}\frac{1-\lambda+\lambda\gamma^{\rho-1}}{(1-\lambda+\lambda\gamma^\rho)^{1-\frac{1}{\rho(1-\eta)}}}\left[\frac{\rho(1-\beta)-1}{\rho(1-\eta)-1}\right]^{\frac{1}{\rho(1-\eta)}-1}. \end{aligned}$$

Substituting w^* and r^* by (40) and (41), respectively, and simplifying terms we obtain

$$\begin{aligned} Y^* &= N\alpha^\alpha\beta^\beta\theta_m \frac{1}{(1-\eta)^\eta}\frac{\rho}{\rho-\frac{1}{1-\eta}}(1-\lambda+\lambda\gamma^\rho)^{\frac{1-\rho}{\rho}}(1-\lambda+\lambda\gamma^{\rho-1})\times \\ & \left[\frac{\rho(1-\beta)-1}{\rho(1-\eta)-1}\right]^{\frac{1-\rho}{\rho}}\left[\frac{N}{K}\frac{\beta\rho}{\rho(1-\beta)-1}\right]^{-\beta} \\ &= N^{1-\beta}K^\beta\alpha^\alpha\theta_m(1-\eta)^{1-\eta}\frac{1-\lambda+\lambda\gamma^{\rho-1}}{(1-\lambda+\lambda\gamma^\rho)^{1-\frac{1}{\rho}}}\left[\frac{\rho(1-\beta)-1}{\rho(1-\eta)-1}\right]^{\frac{1}{\rho}}\left[\frac{\rho}{\rho(1-\beta)-1}\right]^{1-\beta} \end{aligned}$$

For the equilibrium to be well defined we must have that

$$\hat{\theta}_O \geq \theta_m,$$

or

$$\frac{1}{\gamma}\theta_m(1-\lambda+\lambda\gamma^\rho)^{\frac{1}{\rho}}\left[\frac{\rho(1-\beta)-1}{\rho(1-\eta)-1}\right]^{\frac{1}{\rho}} \geq \theta_m$$

or

$$(1-\lambda+\lambda\gamma^\rho)\left[\rho(1-\beta)-1\right] \geq \gamma^\rho\left[\rho(1-\eta)-1\right],$$

or

$$\frac{1-\lambda+\lambda\gamma^\rho}{\gamma^\rho} \geq \frac{\rho(1-\eta)-1}{\rho(1-\beta)-1}.$$

Q.E.D.

Proof of Proposition 2:

(i) We know from Proposition 1 that output in a competitive equilibrium with optimists is

$$Y^* = \theta_m\alpha^\alpha(1-\eta)^{1-\eta}N^{1-\beta}K^\beta\frac{1-\lambda+\lambda\gamma^{\rho-1}}{(1-\lambda+\lambda\gamma^\rho)^{1-\frac{1}{\rho}}}\left[\frac{\rho(1-\beta)-1}{\rho(1-\eta)-1}\right]^{\frac{1}{\rho}}\left[\frac{\rho}{\rho(1-\beta)-1}\right]^{1-\beta}.$$

Setting $\lambda = 0$ (or $\gamma = 1$) in (26) we obtain output in the competitive equilibrium without optimists:

$$Y_0^* = \theta_m\alpha^\alpha(1-\eta)^{1-\eta}N^{1-\beta}K^\beta\left[\frac{\rho(1-\beta)-1}{\rho(1-\eta)-1}\right]^{\frac{1}{\rho}}\left[\frac{\rho}{\rho(1-\beta)-1}\right]^{1-\beta}.$$

Hence, the existence of optimists (the case $\lambda > 0$ and $\gamma > 1$) lowers output provided that

$$\frac{1-\lambda+\lambda\gamma^{\rho-1}}{(1-\lambda+\lambda\gamma^{\rho})^{1-\frac{1}{\rho}}} < 1,$$

when $\rho > 1$, $\gamma > 1$, and $\lambda \in (0,1)$. Define

$$\psi(\lambda) = \frac{1-\lambda+\lambda\gamma^{\rho-1}}{(1-\lambda+\lambda\gamma^{\rho})^{1-\frac{1}{\rho}}}. \tag{42}$$

We prove this result by showing that (a) $\psi(0) = \psi(1) = 1$, (b) $\psi\prime(0) < 0$, (c) $\psi\prime(1) > 0$, and (d) there exists only one $\lambda \in (0,1)$ such that $\psi\prime(\lambda) = 0$. Results (a), (b), (c), and (d) imply that: $\psi(\lambda)$ is convex in $[0,1]$, $\psi(\lambda)$ attains a maxima of 1 at $\lambda = 0$ and at $\lambda = 1$, and $\psi(\lambda)$ attains a minimum at an $\lambda \in (0,1)$. Hence, $\psi(\lambda) < 1$ when $\rho > 1$, $\gamma > 1$, and $\lambda \in (0,1)$. Substituting $\lambda = 0$ in (42) we have $\psi(0) = 1$. Substituting $\lambda = 1$ in (42) we obtain $\psi(1) = 1$. Hence, $\psi(0) = \psi(1) = 1$. This proves (a). Next, we show that $\psi'(0) < 0$ when $\rho > 1$ and $\gamma > 1$. The first derivative of $\psi(\lambda)$ with respect to λ is:

$$\psi'(\lambda) = \frac{\gamma^{\rho-1} - \left(1-\frac{1}{\rho}\right)\gamma^{\rho} - \frac{1}{\rho} + \frac{\lambda}{\rho}(\gamma^{\rho-1}-1)(\gamma^{\rho}-1)}{(1-\lambda+\lambda\gamma^{\rho})^{2-\frac{1}{\rho}}}. \tag{43}$$

From (43) we have

$$\psi'(0) = \gamma^{\rho-1} - \left(1-\frac{1}{\rho}\right)\gamma^{\rho} - \frac{1}{\rho}.$$

Define

$$\varphi(\gamma) = \gamma^{\rho-1} - \left(1-\frac{1}{\rho}\right)\gamma^{\rho} - \frac{1}{\rho}.$$

Setting $\gamma = 1$ in $\varphi(\gamma)$ we obtain $\varphi(1) = 0$. Taking the derivative of $\varphi(\gamma)$ with respect to γ we obtain

$$\varphi'(\gamma) = (\rho-1)\gamma^{\rho-2} - \left(1-\frac{1}{\rho}\right)\rho\gamma^{\rho-1} = -(\rho-1)\gamma^{\rho-1}\left(1-\frac{1}{\gamma}\right) < 0,$$

when $\rho > 1$ and $\gamma > 1$. If $\varphi(1) = 0$ and $\varphi'(\gamma) < 0$ when $\rho > 1$ and $\gamma > 1$, then $\varphi(\gamma) < 0$ when $\rho > 1$ and $\gamma > 1$. Since $\psi'(0) = \varphi(\gamma)$ it follows that $\psi'(0) < 0$ when

$\rho > 1$ and $\gamma > 1$. This proves (b). Next, we show that $\psi'(1) > 0$ when $\rho > 1$ and $\gamma > 1$.

From (43) we have

$$\psi'(1) = \frac{\left[\gamma^{\rho-1} - \left(1 - \frac{1}{\rho}\right)\gamma^{\rho} - \frac{1}{\rho}\right] + \frac{1}{\rho}(\gamma^{\rho-1} - 1)(\gamma^{\rho} - 1)}{\gamma^{2\rho-1}} = \frac{\rho - 1 + \gamma^{\rho} - \rho\gamma}{\rho\gamma^{\rho}}.$$

Define

$$\omega(\gamma) = \rho - 1 + \gamma^{\rho} - \rho\gamma.$$

Setting $\gamma = 1$ in $\omega(\gamma)$ we obtain $\omega(1) = 0$. Taking the derivative of $\omega(\gamma)$ with respect to γ we obtain

$$\omega'(\gamma) = \rho(\gamma^{\rho-1} - 1) > 0,$$

when $\rho > 1$ and $\gamma > 1$. If $\omega(1) = 0$ and $\omega'(\gamma) > 0$ when $\rho > 1$ and $\gamma > 1$, then $\omega(\gamma) > 0$ when $\rho > 1$ and $\gamma > 1$. Since $sign\,(\psi'(1)) = sign\,(\omega(\gamma))$ it follows that $\psi'(1) > 0$ when $\rho > 1$ and $\gamma > 1$. This proves (c). Finally, we show that there exists only one $\lambda \in (0,1)$ such that $\psi\prime(\lambda) = 0$. From (43), $\psi'(\lambda) = 0$ is equivalent to

$$\gamma^{\rho-1} - \left(1 - \frac{1}{\rho}\right)\gamma^{\rho} - \frac{1}{\rho} + \frac{\lambda}{\rho}(\gamma^{\rho-1} - 1)(\gamma^{\rho} - 1) = 0.$$

Hence, the unique λ which solves $\psi'(\lambda) = 0$ is equal to

$$\tilde{\lambda} = \frac{-\rho\gamma^{\rho-1} + (\rho - 1)\,\gamma^{\rho} + 1}{(\gamma^{\rho-1} - 1)(\gamma^{\rho} - 1)}. \tag{44}$$

We see from (44) that $\tilde{\lambda} > 0$ since $-\rho\gamma^{\rho-1} + (\rho - 1)\,\gamma^{\rho} + 1 = -\rho\psi'(0)$, and, as we have shown above, $\psi'(0) < 0$. We now show that $\tilde{\lambda} < 1$. This is the case as long as

$$(\gamma^{\rho-1} - 1)(\gamma^{\rho} - 1) > -\rho\gamma^{\rho-1} + (\rho - 1)\,\gamma^{\rho} + 1,$$

or

$$\frac{1}{\gamma}\gamma^{2\rho} - \frac{1}{\gamma}\gamma^{\rho} - \gamma^{\rho} + 1 > -\rho\gamma^{\rho-1} + \rho\gamma^{\rho} - \gamma^{\rho} + 1,$$

or

$$\frac{1}{\gamma}\gamma^{\rho}(\gamma^{\rho} - 1) > \rho\gamma^{\rho}(1 - \gamma^{-1}),$$

or

$$\gamma^\rho - \rho\gamma + \rho - 1 > 0,$$

which is true since $\gamma^\rho - \rho\gamma + \rho - 1 = \omega(\gamma)$, and, as we have shown above, $\omega(\gamma) > 0$. This proves (d).

(ii) The mean ability of the pool of entrepreneurs in a competitive equilibrium without optimists is

$$E(\theta|\theta \geq \hat{\theta}_0) = \frac{\int_{\hat{\theta}_0}^{+\infty} \theta g(\theta) d\theta}{\int_{\hat{\theta}_0}^{+\infty} g(\theta) d\theta} = \frac{\rho}{\rho - 1}\hat{\theta}_0. \tag{45}$$

In a competitive equilibrium with optimists, the mean ability of realistic entrepreneurs is

$$E(\theta|\theta \geq \hat{\theta}_R) = \frac{\int_{\hat{\theta}_R}^{+\infty} \theta g(\theta) d\theta}{\int_{\hat{\theta}_R}^{+\infty} g(\theta) d\theta} = \frac{\rho}{\rho - 1}\hat{\theta}_R,$$

and the mean ability of optimistic entrepreneurs is

$$E(\theta|\theta \geq \hat{\theta}_O) = \frac{\int_{\hat{\theta}_O}^{+\infty} \theta g(\theta) d\theta}{\int_{\hat{\theta}_O}^{+\infty} g(\theta) d\theta} = \frac{\rho}{\rho - 1}\hat{\theta}_O.$$

Hence, the mean ability of the pool of entrepreneurs in a competitive equilibrium with optimists is equal to

$$\begin{aligned} E(\theta|E^*) &= \frac{E_R^*}{E^*}E(\theta|\theta \geq \hat{\theta}_R) + \frac{E_O^*}{E^*}E(\theta|\theta \geq \hat{\theta}_O) \\ &= \frac{(1-\lambda)\theta_m^\rho \hat{\theta}_R^{-\rho}}{(1-\lambda)\theta_m^\rho \hat{\theta}_R^{-\rho} + \lambda\theta_m^\rho \hat{\theta}_O^{-\rho}} \frac{\rho}{\rho - 1}\hat{\theta}_R + \frac{\lambda\theta_m^\rho \hat{\theta}_O^{-\rho}}{(1-\lambda)\theta_m^\rho \hat{\theta}_R^{-\rho} + \lambda\theta_m^\rho \hat{\theta}_O^{-\rho}} \frac{\rho}{\rho - 1}\hat{\theta}_O \\ &= \frac{1-\lambda}{1-\lambda+\lambda\gamma^\rho}\frac{\rho}{\rho-1}\hat{\theta}_R + \frac{\lambda\gamma^\rho}{1-\lambda+\lambda\gamma^\rho}\frac{\rho}{\rho-1}\hat{\theta}_O. \end{aligned} \tag{46}$$

It follows from (45) and (46) that $E(\theta|E^*) < E(\theta|\theta \geq \hat{\theta}_0)$ as long as

$$\frac{1-\lambda}{1-\lambda+\lambda\gamma^\rho}\frac{\rho}{\rho-1}\hat{\theta}_R + \frac{\lambda\gamma^\rho}{1-\lambda+\lambda\gamma^\rho}\frac{\rho}{\rho-1}\hat{\theta}_O < \frac{\rho}{\rho-1}\hat{\theta}_0$$

or

$$\frac{1-\lambda}{1-\lambda+\lambda\gamma^\rho}(1-\lambda+\lambda\gamma^\rho)^{\frac{1}{\rho}} + \frac{\lambda\gamma^{\rho-1}}{1-\lambda+\lambda\gamma^\rho}(1-\lambda+\lambda\gamma^\rho)^{\frac{1}{\rho}} < 1$$

or

$$\frac{1-\lambda+\lambda\gamma^{\rho-1}}{(1-\lambda+\lambda\gamma^{\rho})^{1-\frac{1}{\rho}}}<1,$$

which we know to hold from part (i). This proves result (ii).

(iii) The result follows directly from the definition of mean returns to entrepreneurship and Proposition 1. *Q.E.D.*

Proof of Proposition 3: The mean returns to entrepreneurship of realists is:

$$\begin{aligned}
\bar{\pi}_R &= E(\pi(\theta, l(\theta,w^*,r^*), k(\theta,w^*,r^*))|\theta \geq \hat{\theta}_R) \\
&= \frac{\int_{\hat{\theta}_R}^{+\infty} \pi(\theta, l(\theta,w^*,r^*), k(\theta,w^*,r^*))g(\theta)d\theta}{\int_{\hat{\theta}_R}^{+\infty} g(\theta)d\theta} \\
&= \frac{\left(\frac{\alpha}{w^*}\right)^{\frac{\alpha}{1-\eta}}\left(\frac{\beta}{r^*}\right)^{\frac{\beta}{1-\eta}}(1-\eta)\,\rho\theta_m^{\rho}\int_{\hat{\theta}_R}^{+\infty}\theta^{\frac{1}{1-\eta}}\theta^{-\rho-1}d\theta}{G(+\infty)-G(\hat{\theta}_R)} + r^*\frac{K}{N} \\
&= \frac{\left(\frac{\alpha}{w^*}\right)^{\frac{\alpha}{1-\eta}}\left(\frac{\beta}{r^*}\right)^{\frac{\beta}{1-\eta}}(1-\eta)\,\rho\theta_m^{\rho}\int_{\hat{\theta}_R}^{+\infty}\theta^{\frac{1}{1-\eta}}\theta^{-\rho-1}d\theta}{1-\left[1-\left(\frac{\theta_m}{\hat{\theta}_R}\right)^{\rho}\right]} + r^*\frac{K}{N} \\
&= \left(\frac{\alpha}{w^*}\right)^{\frac{\alpha}{1-\eta}}\left(\frac{\beta}{r^*}\right)^{\frac{\beta}{1-\eta}}(1-\eta)\,\rho\frac{\hat{\theta}_R^{\frac{1}{1-\eta}}}{\rho-\frac{1}{1-\eta}} + r^*\frac{K}{N} \\
&= \alpha^{\frac{\alpha}{1-\eta}}\beta^{\frac{\beta}{1-\eta}}(w^*)^{-\frac{\alpha}{1-\eta}}(r^*)^{-\frac{\beta}{1-\eta}}(1-\eta)\,\rho\frac{\hat{\theta}_R^{\frac{1}{1-\eta}}}{\rho-\frac{1}{1-\eta}} + r^*\frac{K}{N} \\
&= \alpha^{\frac{\alpha}{1-\eta}}\beta^{\frac{\beta}{1-\eta}}w^{-\frac{\eta}{1-\eta}}\hat{\theta}_R^{\frac{1}{1-\eta}}\left[\frac{N}{K}\frac{\beta\rho}{\rho(1-\beta)-1}\right]^{-\frac{\beta}{1-\eta}}(1-\eta)\,\frac{\rho}{\rho-\frac{1}{1-\eta}} + r^*\frac{K}{N} \\
&= \alpha^{\frac{\alpha}{1-\eta}}\beta^{\frac{\beta}{1-\eta}}\left[\theta_m\alpha^{\alpha}\beta^{\beta}(1-\eta)^{1-\eta}(1-\lambda+\lambda\gamma^{\rho})^{\frac{1}{\rho}}\left[\frac{\rho(1-\beta)-1}{\rho(1-\eta)-1}\right]^{\frac{1}{\rho}}\left[\frac{N}{K}\frac{\beta\rho}{\rho(1-\beta)-1}\right]^{-\beta}\right]^{-\frac{\eta}{1-\eta}} \\
&\times\left[\theta_m(1-\lambda+\lambda\gamma^{\rho})^{\frac{1}{\rho}}\left[\frac{\rho(1-\beta)-1}{\rho(1-\eta)-1}\right]^{\frac{1}{\rho}}\right]^{\frac{1}{1-\eta}}\left[\frac{N}{K}\frac{\beta\rho}{\rho(1-\beta)-1}\right]^{-\frac{\beta}{1-\eta}}\frac{\rho\,(1-\eta)}{\rho-\frac{1}{1-\eta}} + r^*\frac{K}{N} \\
&= \frac{\rho\theta_m\alpha^{\alpha}\beta^{\beta}(1-\eta)^{2-\eta}}{\rho(1-\eta)-1}(1-\lambda+\lambda\gamma^{\rho})^{\frac{1}{\rho}}\left[\frac{\rho(1-\beta)-1}{\rho(1-\eta)-1}\right]^{\frac{1}{\rho}}\left[\frac{N}{K}\frac{\beta\rho}{\rho(1-\beta)-1}\right]^{-\beta} + r^*\frac{K}{N}
\end{aligned} \tag{47}$$

Substituting r^* into (47) we have

$$\bar{\pi}_R = \rho\theta_m \alpha^\alpha \beta^\beta (1-\eta)^{1-\eta}(1-\lambda+\lambda\gamma^\rho)^{\frac{1}{\rho}} \left[\frac{\rho(1-\beta)-1}{\rho(1-\eta)-1}\right]^{\frac{1}{\rho}} \left[\frac{N}{K}\frac{\beta\rho}{\rho(1-\beta)-1}\right]^{-\beta} \times \left[\frac{1-\eta}{\rho(1-\eta)-1} + \frac{\beta}{\rho(1-\beta)-1}\right].$$

The mean returns to entrepreneurship of optimists is:

$$\begin{aligned}
\bar{\pi}^*_O &= E(\pi(\theta, l(\gamma\theta, w^*, r^*), k(\gamma\theta, w^*, r^*))|\theta \geq \hat{\theta}_O) \\
&= \frac{\int_{\hat{\theta}_O}^{+\infty} \pi(\theta, l(\gamma\theta, w^*, r^*), k(\gamma\theta, w^*, r^*))g(\theta)d\theta}{\int_{\hat{\theta}_O}^{+\infty} g(\theta)d\theta} \\
&= \frac{\left(\frac{\alpha}{w^*}\right)^{\frac{\alpha}{1-\eta}} \left(\frac{\beta}{r^*}\right)^{\frac{\beta}{1-\eta}} \left(\gamma^{-1}-\eta\right)\gamma^{\frac{1}{1-\eta}} \rho\theta_m^\rho \int_{\hat{\theta}_O}^{+\infty} \theta^{\frac{1}{1-\eta}}\theta^{-\rho-1}d\theta}{G(+\infty)-G(\hat{\theta}_O)} + r^*\frac{K}{N} \\
&= \frac{\left(\frac{\alpha}{w^*}\right)^{\frac{\alpha}{1-\eta}} \left(\frac{\beta}{r^*}\right)^{\frac{\beta}{1-\eta}} \left(\gamma^{-1}-\eta\right)\gamma^{\frac{1}{1-\eta}} \rho\theta_m^\rho \int_{\hat{\theta}_O}^{+\infty} \theta^{\frac{1}{1-\eta}}\theta^{-\rho-1}d\theta}{1-\left[1-\left(\frac{\theta_m}{\hat{\theta}_O}\right)^\rho\right]} + r^*\frac{K}{N} \\
&= \frac{\left(\frac{\alpha}{w^*}\right)^{\frac{\alpha}{1-\eta}} \left(\frac{\beta}{r^*}\right)^{\frac{\beta}{1-\eta}} \left(\gamma^{-1}-\eta\right)\gamma^{\frac{1}{1-\eta}} \rho\theta_m^\rho \frac{\hat{\theta}_O^{\frac{1}{1-\eta}-\rho}}{\rho-\frac{1}{1-\eta}}}{\left(\frac{\theta_m}{\hat{\theta}_O}\right)^\rho} + r^*\frac{K}{N} \\
&= \alpha^{\frac{\alpha}{1-\eta}}\beta^{\frac{\beta}{1-\eta}}(w^*)^{-\frac{\alpha}{1-\eta}}(r^*)^{-\frac{\beta}{1-\eta}}\left(\gamma^{-1}-\eta\right)\gamma^{\frac{1}{1-\eta}}\rho\frac{\hat{\theta}_O^{\frac{1}{1-\eta}}}{\rho-\frac{1}{1-\eta}} + r^*\frac{K}{N} \\
&= \alpha^{\frac{\alpha}{1-\eta}}\beta^{\frac{\beta}{1-\eta}}(w^*)^{-\frac{\eta}{1-\eta}}\hat{\theta}_O^{\frac{1}{1-\eta}}\left[\frac{N}{K}\frac{\beta\rho}{\rho(1-\beta)-1}\right]^{-\frac{\beta}{1-\eta}}\left(\gamma^{-1}-\eta\right)\gamma^{\frac{1}{1-\eta}}\frac{\rho}{\rho-\frac{1}{1-\eta}} + r^*\frac{K}{N} \\
&= \alpha^\alpha\beta^\beta(1-\eta)^{-\eta}\theta_m(1-\lambda+\lambda\gamma^\rho)^{\frac{1}{\rho}}\left[\frac{\rho(1-\beta)-1}{\rho(1-\eta)-1}\right]^{\frac{1}{\rho}}\left[\frac{N}{K}\frac{\beta\rho}{\rho(1-\beta)-1}\right]^{\frac{\beta\eta}{1-\eta}} \times \\
&\quad \gamma^{-\frac{1}{1-\eta}}\left(\frac{N}{K}\frac{\beta\rho}{\rho(1-\beta)-1}\right)^{-\frac{\beta}{1-\eta}}\frac{\rho\left(\gamma^{-1}-\eta\right)\gamma^{\frac{1}{1-\eta}}}{\rho-\frac{1}{1-\eta}} + r^*\frac{K}{N} \\
&= \frac{\rho\theta_m\alpha^\alpha\beta^\beta(1-\eta)^{1-\eta}}{\rho(1-\eta)-1}\frac{(1-\lambda+\lambda\gamma^\rho)^{\frac{1}{\rho}}}{\left(\gamma^{-1}-\eta\right)^{-1}}\left[\frac{\rho(1-\beta)-1}{\rho(1-\eta)-1}\right]^{\frac{1}{\rho}}\left[\frac{N}{K}\frac{\beta\rho}{\rho(1-\beta)-1}\right]^{-\beta} + r^*\frac{K}{N}
\end{aligned} \quad (48)$$

Substituting r^* into (48) we have

$$\begin{aligned}\bar{\pi}_O^* \quad = \quad & \frac{\rho\theta_m\alpha^\alpha\beta^\beta(1-\eta)^{1-\eta}}{\rho(1-\eta)-1}\frac{(1-\lambda+\lambda\gamma^\rho)^{\frac{1}{\rho}}}{(\gamma^{-1}-\eta)^{-1}}\left[\frac{\rho(1-\beta)-1}{\rho(1-\eta)-1}\right]^{\frac{1}{\rho}}\left[\frac{N}{K}\frac{\beta\rho}{\rho(1-\beta)-1}\right]^{-\beta} \\ & +\theta_m\alpha^\alpha\beta^\beta(1-\eta)^{1-\eta}(1-\lambda+\lambda\gamma^\rho)^{\frac{1}{\rho}}\left[\frac{\rho(1-\beta)-1}{\rho(1-\eta)-1}\right]^{\frac{1}{\rho}}\left[\frac{N}{K}\frac{\beta\rho}{\rho(1-\beta)-1}\right]^{1-\beta}\frac{K}{N},\end{aligned}$$

or

$$\begin{aligned}\bar{\pi}_O^* \quad = \quad & \rho\theta_m\alpha^\alpha\beta^\beta(1-\eta)^{1-\eta}(1-\lambda+\lambda\gamma^\rho)^{\frac{1}{\rho}}\left[\frac{\rho(1-\beta)-1}{\rho(1-\eta)-1}\right]^{\frac{1}{\rho}}\left[\frac{N}{K}\frac{\beta\rho}{\rho(1-\beta)-1}\right]^{-\beta} \\ & \times\left[\frac{\gamma^{-1}-\eta}{\rho(1-\eta)-1}+\frac{\beta}{\rho(1-\beta)-1}\right].\end{aligned}$$

Comparing (47) to (48) we see that the mean returns to entrepreneurship of realists is greater than the mean returns to entrepreneurship of optimists as long as

$$(1-\eta)^{2-\eta} > (1-\eta)^{1-\eta}\left(\gamma^{-1}-\eta\right),$$

or

$$1-\eta > \frac{1}{\gamma}-\eta,$$

which is always the case when $\gamma > 1$. *Q.E.D.*

Appendix B: Pareto Firm Size Distribution

In this appendix we show that if ability is distributed according to a Pareto distribution so is firm size. We start by doing it in the model without optimists. After that we show that the result also holds in the model with optimists.

Setting $\gamma = 1$ in the first-order conditions of an entrepreneur's problem we have

$$k = \frac{\beta}{\alpha}\frac{w}{r}l.$$

The scale of a firm is given by

$$\beta\gamma\theta l^\alpha k^{\beta-1} = r,$$

or

$$\beta\gamma\theta l^{\alpha}\left(\frac{\beta}{\alpha}\frac{w}{r}l\right)^{\beta-1}=r,$$

or

$$\theta=\frac{1}{\gamma}\left(\frac{w}{\alpha}\right)^{1-\beta}\left(\frac{r}{\beta}\right)^{\beta}l^{1-\eta}. \tag{49}$$

Let $S(l)$ denote the probability that a randomly select firm has fewer than l employees. Then under (49) $S(l)$ will be the probability that θ is less than $\left(\frac{w}{\alpha}\right)^{1-\beta}\left(\frac{r}{\beta}\right)^{\beta}l^{1-\eta}$ conditional on $\theta\geq\hat{\theta}_0^*$, or

$$\begin{aligned} S(l) &= \Pr\left[\theta\leq\left(\frac{w}{\alpha}\right)^{1-\beta}\left(\frac{r}{\beta}\right)^{\beta}l^{1-\eta}|\theta\geq\hat{\theta}_0^*\right] \\ &= \frac{G\left[\left(\frac{w}{\alpha}\right)^{1-\beta}\left(\frac{r}{\beta}\right)^{\beta}l^{1-\eta}\right]-G(\hat{\theta}_0^*)}{1-G(\hat{\theta}_0^*)} \end{aligned}$$

for $l\geq$ and 0 otherwise. If ability is distributed according to a Pareto cumulative distribution function $G(\theta)=1-\theta_m^{\rho}\theta^{-\rho}$ we have

$$\begin{aligned} S(l) &= \frac{G\left[\left(\frac{w}{\alpha}\right)^{1-\beta}\left(\frac{r}{\beta}\right)^{\beta}l^{1-\eta}\right]-G(\hat{\theta}_0^*)}{1-G(\hat{\theta}_0^*)} \\ &= \frac{1-\theta_m^{\rho}\left[\left(\frac{w}{\alpha}\right)^{1-\beta}\left(\frac{r}{\beta}\right)^{\beta}l^{1-\eta}\right]^{-\rho}-\left[1-\theta_m^{\rho}(\hat{\theta}_0^*)^{-\rho}\right]}{1-\left[1-\theta_m^{\rho}(\hat{\theta}_0^*)^{-\rho}\right]} \\ &= 1-\frac{\left[\left(\frac{w}{\alpha}\right)^{1-\beta}\left(\frac{r}{\beta}\right)^{\beta}l^{1-\eta}\right]^{-\rho}}{(\hat{\theta}_0^*)^{-\rho}} \\ &= 1-(\hat{\theta}_0^*)^{\rho}\left[\left(\frac{w}{\alpha}\right)^{1-\beta}\left(\frac{r}{\beta}\right)^{\beta}\right]^{-\rho}l^{-\rho(1-\eta)} \\ &= 1-(\hat{\theta}_0^*)^{\rho}\left(\frac{w^{1-\beta}r^{\beta}}{\alpha^{1-\beta}\beta^{\beta}}\right)^{-\rho}l^{-\rho(1-\eta)}. \end{aligned}$$

Using the equilibrium condition

$$w^{1-\beta}r^{\beta} = \alpha^{\alpha}\beta^{\beta}(1-\eta)^{1-\eta}\hat{\theta}_0^*,$$

we have

$$\begin{aligned} S(l) &= 1-(\hat{\theta}_0^*)^{\rho}\left[\frac{(1-\eta)^{1-\eta}\hat{\theta}_0^*}{\alpha^{1-\eta}}\right]^{-\rho} l^{-\rho(1-\eta)} \\ &= 1-\left(\frac{\alpha}{1-\eta}\right)^{\rho(1-\eta)} l^{-\rho(1-\eta)}. \end{aligned}$$

We now show that optimism does not have an impact on the firm size distribution. Note that if $\theta_r < x$:

$$\begin{aligned} \Pr(X<x) &= \Pr(X_o<x)\,\lambda + \Pr(X_r<x)\,(1-\lambda) \\ &= \left[1-\left(\frac{\hat{\theta}_O}{x}\right)^{\alpha}\right]\lambda + \left[1-\left(\frac{\hat{\theta}_R}{x}\right)^{\alpha}\right](1-\lambda) \\ &= -\lambda\left(\frac{\hat{\theta}_O}{x}\right)^{\alpha} + 1 - (1-\lambda)\left(\frac{\hat{\theta}_R}{x}\right)^{\alpha} \\ &= 1-\lambda\left(\frac{\hat{\theta}_O}{x}\right)^{\alpha} - (1-\lambda)\left(\frac{\hat{\theta}_R}{x}\right)^{\alpha} \\ &= 1-\left[\lambda\left(\hat{\theta}_O\right)^{\alpha} - (1-\lambda)\left(\hat{\theta}_R\right)^{\alpha}\right]\left(\frac{1}{x}\right)^{\alpha} \\ &= 1-\left[\frac{\left(\lambda\hat{\theta}_O^{\alpha} - (1-\lambda)\,\hat{\theta}_R^{\alpha}\right)^{\frac{1}{\alpha}}}{x}\right]^{\alpha}. \end{aligned}$$

If $\hat{\theta}_O < x < \hat{\theta}_R$:

$$\Pr(X<x) = \Pr(X_o<x) = 1-\left(\frac{\hat{\theta}_O}{x}\right)^{\alpha}$$

Note that the marginal optimist entrepreneur perceived ability is $\gamma\hat{\theta}_O = \hat{\theta}_R$. Therefore, the firm size associated to this entrepreneur is the same as that associated to the marginal realist entrepreneur. The firm size distribution can be derived as follows:

$$l > \left[\gamma\hat{\theta}_O \left(\frac{w}{\alpha}\right)^{\beta-1} \left(\frac{r}{\beta}\right)^{-\beta}\right]^{\frac{1}{1-\eta}}$$

or, using the equilibrium condition, $w^{1-\beta}r^{\beta} = \alpha^{\alpha}\beta^{\beta}(1-\eta)^{1-\eta}\gamma\hat{\theta}_O$,

$$l > \left[\gamma\hat{\theta}_O \left(\frac{1}{\alpha}\right)^{\beta-1} \left(\frac{1}{\beta}\right)^{-\beta} \alpha^{-\alpha}\beta^{-\beta}(1-\eta)^{\eta-1}\gamma^{-1}\hat{\theta}_O^{-1}\right]^{\frac{1}{1-\eta}}$$

or

$$l > \left[\left(\frac{\alpha}{1-\eta}\right)^{1-\eta}\right]^{\frac{1}{1-\eta}}$$

or

$$l > \frac{\alpha}{1-\eta}.$$

Then

$$\begin{aligned}
\Pr(S<l) &= \Pr\left(\theta \le \frac{1}{\gamma}\left(\frac{w}{\alpha}\right)^{1-\beta}\left(\frac{r}{\beta}\right)^{\beta} l^{1-\eta} \middle| \theta \ge \hat{\theta}_O\right)\lambda \\
&\quad + \Pr\left(\theta \le \left(\frac{w}{\alpha}\right)^{1-\beta}\left(\frac{r}{\beta}\right)^{\beta} l^{1-\eta} \middle| \theta \ge \hat{\theta}_R\right)(1-\lambda) \\
&= \frac{1-\theta_m^{\rho}\left(\frac{1}{\gamma}\left(\frac{w}{\alpha}\right)^{1-\beta}\left(\frac{r}{\beta}\right)^{\beta} l^{1-\eta}\right)^{-\rho} - \left(1-\theta_m^{\rho}\hat{\theta}_O^{-\rho}\right)}{1-\left(1-\theta_m^{\rho}\hat{\theta}_O^{-\rho}\right)}\lambda \\
&\quad + \frac{1-\theta_m^{\rho}\left(\left(\frac{w}{\alpha}\right)^{1-\beta}\left(\frac{r}{\beta}\right)^{\beta} l^{1-\eta}\right)^{-\rho} - \left(1-\theta_m^{\rho}\hat{\theta}_R^{-\rho}\right)}{1-\left(1-\theta_m^{\rho}\hat{\theta}_R^{-\rho}\right)}(1-\lambda) \\
&= \frac{1-\theta_m^{\rho}\left(\frac{1}{\gamma}\left(\frac{w}{\alpha}\right)^{1-\beta}\left(\frac{r}{\beta}\right)^{\beta} l^{1-\eta}\right)^{-\rho} - \left(1-\theta_m^{\rho}\hat{\theta}_O^{-\rho}\right)}{1-\left(1-\theta_m^{\rho}\hat{\theta}_O^{-\rho}\right)} \\
&\quad + \frac{1-\theta_m^{\rho}\left(\left(\frac{w}{\alpha}\right)^{1-\beta}\left(\frac{r}{\beta}\right)^{\beta} l^{1-\eta}\right)^{-\rho} - \left(1-\theta_m^{\rho}\gamma^{-\rho}\hat{\theta}_O^{-\rho}\right)}{1-\left(1-\theta_m^{\rho}\gamma^{-\rho}\hat{\theta}_O^{-\rho}\right)}(1-\lambda)
\end{aligned}$$

or λ

$$\begin{aligned}
\Pr\left(S<l\right) &= 1-\left[\lambda\frac{\left(\frac{1}{\gamma}\left(\frac{w}{\alpha}\right)^{1-\beta}\left(\frac{r}{\beta}\right)^{\beta}l^{1-\eta}\right)^{-\rho}}{\hat{\theta}_O^{-\rho}}\right]-\left[\left(1-\lambda\right)\frac{\left(\left(\frac{w}{\alpha}\right)^{1-\beta}\left(\frac{r}{\beta}\right)^{\beta}l^{1-\eta}\right)^{-\rho}}{\gamma^{-\rho}\hat{\theta}_O^{-\rho}}\right] \\
&= 1-\left\{\left[\lambda\frac{\left(\frac{1}{\gamma}\right)^{-\rho}}{\hat{\theta}_O^{-\rho}}\right]+\left[\left(1-\lambda\right)\frac{1}{\gamma^{-\rho}\hat{\theta}_O^{-\rho}}\right]\right\}\left(\left(\frac{w}{\alpha}\right)^{1-\beta}\left(\frac{r}{\beta}\right)^{\beta}\right)^{-\rho}l^{-\rho(1-\eta)} \\
&= 1-\hat{\theta}_O^{\rho}\left(\frac{\left(\frac{w}{\alpha}\right)^{1-\beta}\left(\frac{r}{\beta}\right)^{\beta}}{\gamma}\right)^{-\rho}l^{-\rho(1-\eta)} \\
&= 1-\hat{\theta}_O^{\rho}\left(\frac{\left(\frac{1}{\alpha}\right)^{1-\beta}\left(\frac{1}{\beta}\right)^{\beta}\alpha^{\alpha}\beta^{\beta}\left(1-\eta\right)^{1-\eta}\gamma\hat{\theta}_O}{\gamma}\right)^{-\rho}l^{-\rho(1-\eta)} \\
&= 1-\left(\left(\frac{1}{\alpha}\right)^{1-\beta}\left(\frac{1}{\beta}\right)^{\beta}\alpha^{\alpha}\beta^{\beta}\left(1-\eta\right)^{1-\eta}\right)^{-\rho}l^{-\rho(1-\eta)} \\
&= 1-\left(\frac{\alpha}{1-\eta}\right)^{\rho(1-\eta)}l^{-\rho(1-\eta)}.
\end{aligned}$$

Therefore the firm size distribution is the same that would apply in the absence of optimism. The intuition is that the marginal optimist behaves, in terms of labor demand, as the marginal realist, under the assumption that the ability distribution is the same for optimists and realists, the labor market is the same, etc.

Appendix C: Non-Pecuniary Benefits

This appendix derives and calibrates the competitive equilibrium with non-pecuniary benefits. Assume fraction $\mu \in (0,1)$ of the population derives a non-pecuniary benefit from entrepreneurship and fraction $1-\mu$ does not. The profit of an entrepreneur who employs l workers and rents k units of capital is

$$\pi(\theta, w, r, B) = \theta l^{\alpha} k^{\beta} - wl + r(K/N - k) + B, \tag{50}$$

where $B \geq 0$. The parameter B measures the intensity of non-pecuniary benefits. Entrepreneurs for whom $B = 0$ are called R-entrepreneurs and those with $B > 0$ are called B-entrepreneurs. An entrepreneur solves

$$\max_{l,k} \left[\theta l^{\alpha} k^{\beta} - wl + r(K/N - k) + B\right].$$

The first-order conditions are $\alpha\theta l^{\alpha-1}k^{\beta} = w$, and $\beta\theta l^{\alpha}k^{\beta-1} = r$. Solving for l and k we obtain the input demands:

$$l(\theta, w, r) = \theta^{\frac{1}{1-\eta}} \left(\frac{\alpha}{w}\right)^{\frac{1-\beta}{1-\eta}} \left(\frac{\beta}{r}\right)^{\frac{\beta}{1-\eta}}, \tag{51}$$

and

$$k(\theta, w, r) = \theta^{\frac{1}{1-\eta}} \left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\eta}} \left(\frac{\beta}{r}\right)^{\frac{1-\alpha}{1-\eta}}. \tag{52}$$

The input demands determine the size of the firm given the ability of the entrepreneur, the wage, the rental cost of capital. Substituting (51) and (52) into (50) we obtain the reduced form profit of an entrepreneur:

$$\pi(\theta, w, r, B) = \theta^{\frac{1}{1-\eta}}(1-\eta) \left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\eta}} \left(\frac{\beta}{r}\right)^{\frac{\beta}{1-\eta}} + r\frac{K}{N} + B. \tag{53}$$

The returns to paid employment are given by

$$w + r\frac{K}{N}. \tag{54}$$

The ability of the marginal R-entrepreneur, $\hat{\theta}_R$, is obtained by setting $B = 0$ in (53) and equating this to (54). Hence, an individual with ability $\hat{\theta}_R$ is indifferent between being an entrepreneur and a worker when

$$\hat{\theta}_R^{\frac{1}{1-\eta}}(1-\eta) \left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\eta}} \left(\frac{\beta}{r}\right)^{\frac{\beta}{1-\eta}} = w. \tag{55}$$

The ability of the marginal B-entrepreneur, $\hat{\theta}_B$, is obtained by equating (53) to (54). Hence, an individual with ability $\hat{\theta}_B$ is indifferent between being an entrepreneur and a worker when

$$\hat{\theta}_B^{\frac{1}{1-\eta}}(1-\eta) \left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\eta}} \left(\frac{\beta}{r}\right)^{\frac{\beta}{1-\eta}} = w - B. \tag{56}$$

Since the reduced form profit of an entrepreneur is an increasing and convex function of θ it follows from (55) and (56) that there exist a unique ability cut-off between R-entrepreneurs and R-workers–$\hat{\theta}_R$ is unique–and an unique ability cut-off between B-entrepreneurs and B-workers–$\hat{\theta}_B$ is unique. Moreover, from (55) and (56) we have

$$\hat{\theta}_B = \hat{\theta}_R \left(\frac{w-B}{w}\right)^{1-\eta} < \hat{\theta}_R.$$

The labor market equilibrium condition is

$$\left(\frac{\alpha}{w}\right)^{\frac{1-\beta}{1-\eta}} \left(\frac{\beta}{r}\right)^{\frac{\beta}{1-\eta}} \frac{\rho\theta_m^\rho}{\rho - \frac{1}{1-\eta}} \left[(1-\mu)\hat{\theta}_R^{\frac{1}{1-\eta}-\rho} + \mu\hat{\theta}_B^{\frac{1}{1-\eta}-\rho}\right] = 1 - \theta_m^\rho \left[(1-\mu)\hat{\theta}_R^{-\rho} + \mu\hat{\theta}_B^{-\rho}\right], \tag{57}$$

and the capital market equilibrium condition is

$$\left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\eta}} \left(\frac{\beta}{r}\right)^{\frac{1-\alpha}{1-\eta}} \frac{\rho\theta_m^\rho}{\rho - \frac{1}{1-\eta}} \left[(1-\mu)\hat{\theta}_R^{\frac{1}{1-\eta}-\rho} + \mu\hat{\theta}_B^{\frac{1}{1-\eta}-\rho}\right] = K/N. \tag{58}$$

Equations (55), (56), (57), and (58) form a system of four equations and four unknowns ($\hat{\theta}_R$, $\hat{\theta}_B$, w, r) which defines a unique competitive equilibrium. Solving (55) and (56) for the unique cut-offs $\hat{\theta}_R$ and $\hat{\theta}_B$ and substituting these into (57) and (58) we obtain the unique equilibrium vector of input prices (w^*, r^*). Finally, from $(\hat{\theta}_R, \hat{\theta}_B, w^*, r^*)$ we obtain the equilibrium labor force

$$L^* = N\left[1 - \theta_m^\rho \left[(1-\mu)\hat{\theta}_R^{-\rho} + \mu\hat{\theta}_B^{-\rho}\right]\right],$$

and output level

$$\begin{aligned} Y^* &= (1-\mu)N \int_{\hat{\theta}_R}^{\infty} \theta[l(\theta, w^*, r^*)]^\alpha [k(\theta, w^*, r^*)]^\beta g(\theta) d\theta \\ &\quad + \mu N \int_{\hat{\theta}_B}^{\infty} \theta[l(\theta, w^*, r^*)]^\alpha [k(\theta, w^*, r^*)]^\beta g(\theta) d\theta. \end{aligned}$$

For the equilibrium to be well defined we must have $\hat{\theta}_B \geq \theta_m$. Note that this condition together with (56) imposes an upper bound on B given by

$$B \leq w\left[1 - \left(\frac{\theta_m}{\hat{\theta}_R}\right)^{1-\eta}\right]. \tag{59}$$

It follows from (59) that B has to be smaller than the equilibrium wage. We calibrate the model using the technology parameters in Table I. The behavioral parameters μ and B are calibrated as follows. According to Hurst and Pugsley (2011, pp.75): "Over 50 percent of these new business owners cite non-pecuniary benefits—for example, "wanting flexibility over schedule" or "to be one's own boss"—as a primary reason for starting the business." Hence, we calibrate the fraction of individuals with non-pecuniary benefits at 50 percent, i.e., $\mu = 0.5$.

It is very hard to infer the nature and magnitude of non-pecuniary benefits directly from empirical data. Bartling et al. (2014) estimate experimentally the non-pecuniary benefit of decision rights (i.e., the non-pecuniary benefit of autonomy and control over decision making) to 16.7 percent of the payoff associated to a decision. Owens et al. (2014) find that the average participant in an experiment is willing to sacrifice 8 percent to 15 percent of expected asset earnings to retain control. Following these studies we calibrate B such that non-pecuniary benefits are 20 percent of the equilibrium wage. This corresponds to $B = 0.5$.

Table III reports the results of the calibration with non-pecuniary benefits.

Table III

Calibration Results: Non-Pecuniary Benefits

Non-pecuniary benefits	0	0.5
Output	6.0200	6.0190
Wage	2.5422	2.5521
Rental cost of capital	0.0797	0.0797
Mean returns to paid employment	3.6457	3.6559
Mean returns to entrepreneurship	74.7700	66.9920

The third column in Table III shows that the model with non-pecuniary benefits generates mean returns to entrepreneurship 26.2 times greater than the wage. Hence, the calibration shows that non-pecuniary benefits can explain about one tenth of the size of the entrepreneurial earnings puzzle in the U.K..