A General Equilibrium Theory of Occupational Choice under Optimistic Expectations

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Abstract

We present a general equilibrium model of occupational choice where individuals can be optimists about their ability. We show optimism has six main effects. First, it leads to a misallocation of talent which lowers output. Second, optimists are more likely to become entrepreneurs than realists. Third, entrepreneurs are more optimistic than workers. Fourth, optimism drives up input prices. Fifth, optimism raises the number of entrepreneurs. Sixth, optimism make workers better off and entrepreneurs worse off. We calibrate the model to match U.S. manufacturing data and show that optimism can explain the observed low returns to entrepreneurship.

JEL Codes: D50; H21; J24; L26.

Keywords: General Equilibrium; Entrepreneurship; Optimism.

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1 Introduction

This paper studies the impact of optimism on occupational choice using a general equilibrium framework. The model shows that optimism can lead to a sharp reduction in the mean returns to entrepreneurship. Optimism leads to a misallocation of talent, bids up input prices, and crowds out realists from entrepreneurship. Optimism increases the number of entrepreneurs and lowers the number of workers. We calibrate the model to match salient features of U.S. manufacturing data and show that optimism can explain quantitatively the observed low returns to entrepreneurship.

The paper is motivated by the following stylized facts about entrepreneurship and paid employment.\(^1\)

First, the mean return to entrepreneurship is not significantly higher than average wages or average return to public equity. Most businesses fail within a few years (Dunne et al. (1988)). After 10 years in business the median entrepreneurial earnings are 35 percent less than those on a paid job of the same duration (Hamilton (2000)). The returns from entrepreneurship are, on average, not different from the return on a diversified publicly traded portfolio—the private equity puzzle—during the 1989-1998 period (Moskovitz and Vissing-Jorgensen (2002)).\(^2\)

Second, the returns to entrepreneurship are found to be highly variable, more than wages, and more than the returns on public equity (Borjas and Bronars (1989), Hamilton (2000), and Moskovitz and Vissing-Jorgensen (2002)).

Third, entrepreneurs are optimistic about the future of their firms. Entrepreneurs report the odds of their business ‘succeeding’ to be significantly higher than historically observed and substantially better than the odds of success for other similar businesses (Cooper et al. (1998)). Direct comparison of entrepreneur expectations to new venture outcomes shows that a representative sample of French entrepreneurs

\(^1\)See Shane and Venkataraman (2000) and Åstebro et al. (2014) for surveys on this topic.
\(^2\)However, Kartashova (2014) shows that the private equity puzzle does not extend to the 1989–2010 period.
tend to overestimate employment expansion and sales growth (Landier and Thesmar (2009)). Nascent entrepreneurs overestimate the probability that their projects will result in operating ventures and, for those ventures that achieve operation, 62 percent overestimate future sales and 46 percent overestimate the number of employees in the first year of operation (Cassar (2010)). 48.8 percent of a sample of U.S. nascent entrepreneurs think that the likelihood of exit of their venture is zero in five years time (Hyytinen et al. (2014)). Individuals who switch into self-employment have an optimistic view of their future prior to switching into self-employment (Dawson et al. (2014)).

Fourth, optimistic individuals are more likely to become entrepreneurs. Optimism is an important determinant of self-employment after controlling for a range of family, demographic, and wealth characteristics (Puri and Robinson (2007)).

Fifth, entrepreneurs are more optimistic than non-entrepreneurs. Entrepreneurs are more optimistic than managers with equal responsibilities (Busenitz and Barney (1997)). Arabsheibani et al. (2000) compare expectations of future prosperity with actual outcomes using a sample of entrepreneurs and employees from the British Household Panel Survey (BHPS) during the years 1990-96. Arabsheibani et al. find that entrepreneurs are 4.6 times as likely to forecast an improved financial position but experience a deterioration than to forecast a deterioration but experience an improvement. In contrast, for employees the ratio was only 2.9. Self-employed Britons have higher income expectations than employees during the years 1984-99, but the difference diminishes with experience (Fraser and Greene (2006)). Entrepreneurs expect to live about 2 years longer than non-entrepreneurs after controlling for differences in smoking, race, and education-related mortality risk across groups (Puri and Robinson (2013)). Koudstaal et al. (2015) conduct a lab-in-the field experiment in the Netherlands and find that 58 percent of entrepreneurs, 54 percent of the managers, and 32 percent of the employees can be classified as ‘very optimistic’ (i.e. have a score of 18 or more in the Revised Life Orientation Test, a commonly used measure of dispositional optimism).
Sixth, optimistic entrepreneurs earn less than either realists or pessimists. Optimism is on average bad for performance (Landier and Thesmar (2009)). Entrepreneurs’ level of optimism has, on average, a negative relationship with the performance of their new ventures (Hmieleski and Baron (2009)). Dawson et al. (2015) examine how entrepreneurs’ forecasts predict entrepreneurship performance using the BHPS during the years 1991-2008. Dawson et al. find that optimists, on average, earn less than pessimists.

We present a fully specified general equilibrium model of occupational choice which is able to explain these stylized facts on entrepreneurship. Following Lucas (1978) we model a closed economy with a workforce of size $N$ and a capital stock of $K$ units of capital. Each individual is endowed with one unit of labor, with capital stock $K/N$, and with a one-dimensional ability $\theta$. Individuals are risk neutral and maximize their expected income by choosing occupations. A firm in this economy is one entrepreneur together with the labor and capital under his control. The technology of the firm is as follows. Output is an increasing function of ability, labor, and capital. Ability is complementary to labor and capital. Decreasing returns to scale in labor and capital ensure that the competitive equilibrium exhibits a non-degenerate distribution of firm sizes.

We depart from Lucas (1978) by assuming that a fraction $\lambda \in (0, 1]$ of individuals is optimist about ability whereas the remaining fraction $1 - \lambda$ is realist. Realists know their ability is $\theta$ whereas optimists think, mistakenly, that their true ability is $\gamma \theta$, with $\gamma > 1$. Hence, realists who enter entrepreneurship know the true production function of their firms, whereas optimists believe their firms are more productive than they really are.

The competitive equilibrium is characterized by: (i) a cut-off ability level $\hat{\theta}_R$ such that realists with ability less than $\hat{\theta}_R$ become workers and those with ability greater than $\hat{\theta}_R$ become entrepreneurs, (ii) a cut-off ability level $\hat{\theta}_O$ such that optimists with ability less than $\hat{\theta}_O$ become workers and those with ability greater than $\hat{\theta}_O$ become entrepreneurs, (iii) a market clearing wage that equates labor demand to supply, and
(iv) a rental cost of capital that equates capital demand to supply.

We solve the competitive equilibrium assuming a generalized Cobb-Douglas production function and a uniform distribution of ability. We show that in equilibrium there is a misallocation of talent. The ablest people do not necessarily select into entrepreneurship: the lowest ability entrepreneurs are less talented at running a firm than the highest ability workers. This is an empirically attractive implication of the model since, in reality, the income distributions of workers and entrepreneurs have overlapping supports.³ We also show that, in equilibrium, optimists are more likely to become entrepreneurs than realists. In addition, we show that an increase in the fraction of optimists raises input prices, lowers output, and raises the number of entrepreneurs.

We calibrate the model to match salient features of U.S. manufacturing data. The production function and the capital stock are calibrated following Atkeson and Kehoe (2005) and Adler (2016). The fraction of optimists and the intensity of optimism are calibrated using empirical evidence on the financial expectations of entrepreneurs and employees. The calibration shows that optimism can explain the observed low mean returns to entrepreneurship. This happens due to four reasons. First, optimism leads to the misallocation of talent we described above. Second, it raises input prices. Third, it distorts the input choices of optimistic entrepreneurs. Fourth, it raises the number of entrepreneurs. The calibration also shows that optimism has a relatively modest impact on input prices, output, and on the percentage of workers in the economy.

The reminder of the paper proceeds as follows. Section 2 reviews related literature. Section 3 sets up the model. Section 4 characterizes the competitive equilibrium. Section 5 contains comparative statics results. Section 6 calibrates the model. Section 7 discusses assumptions, extensions, and policy implications. Section 8 concludes the paper. All proofs can be found in the Appendix.

³See, for example, figure 2 in Åstebro et al. (2014).
2 Related Literature

In this section we explain how our work contributes to the literature on occupational choice and entrepreneurship. We divide this literature into studies that use the general equilibrium approach and those who use a partial equilibrium approach.

2.1 General Equilibrium Theories of Entrepreneurship

We divide the literature that uses the general equilibrium approach to understand occupational choice and entrepreneurship into static and dynamic models.

2.1.1 Static Models

The seminal paper of occupational choice and firm size distribution of an economy is Lucas (1978).\textsuperscript{4} Individuals have heterogenous one-dimensional abilities as entrepreneurs and choose between entrepreneurship and paid employment. The most talented individuals become entrepreneurs and the less talented ones become workers. The ability differentials across entrepreneurs give rise to different spans of control (firm sizes). The income distributions of workers and entrepreneurs have non-overlapping supports. This last prediction stands in contradiction to the observed low returns to entrepreneurship and the fact that the income distributions of workers and entrepreneurs overlap.

Kanbur (1979) studies the role of learning about ability on entrepreneurship. Individuals choose between entrepreneurship and paid employment. The payoff of an entrepreneur’s project depends on his ability which is unknown to new entrepreneurs. Individuals make their initial occupational choice before they learn anything at all about their ability. Only those who become entrepreneurs can make an informed future occupational choice. Hence, entering entrepreneurship has an immediate cost (risk taking) and a delayed reward (informed future occupational choice) relative to

\textsuperscript{4}Chapter 2 of Parker (2009) discusses in detail the main extensions of Lucas’ (1978) model.
the safe alternative of being employed. They find that more patient societies have a larger share of entrepreneurs.

Kilhstrom and Laffont (1979) study the role of risk aversion on entrepreneurship. All individuals have the same ability but their attitudes towards risk are heterogeneous. Individuals choose between operating a risky firm or working for a riskless wage. Kilhstrom and Laffont find that less risk averse individuals become entrepreneurs while more risk averse individuals become employees. However, the empirical evidence indicates that entrepreneurs’ risk attitudes are indistinguishable from those of wage earners (see Wu and Knott (2006), Parker (2009), Holm et al. (2013), Koudstaal et al. (2016)).

Bewley (1989) studies the role of uncertainty (or ambiguity) aversion on entrepreneurship. According to Knight (1921) a project is risky when the probability of success is known. In contrast, a project is uncertain (or ambiguous) when the probability of success is unknown. Bewley finds that entrepreneurs should be individuals with low uncertainty aversion. However, the empirical evidence indicates that entrepreneurs’ ambiguity attitudes are indistinguishable from those of wage earners (see Holm et al. (2013)).

Jovanovic (1994) studies the joint role of entrepreneurial and working abilities on entrepreneurship. Jovanovic generalizes Lucas (1978) by allowing for heterogeneous working abilities, i.e., the labor income of a worker is given by $w_y$ where $y$ represents working ability. Working ability $y$ is correlated with entrepreneurial ability $\theta$ if $y = \psi(\theta)$. Jovanovic shows that when $\psi$ is either (i) strictly decreasing or (ii) strictly increasing and not very steep at high levels of $\theta$, then the best potential entrepreneurs are drawn into entrepreneurship. In contrast, when $\psi$ is strictly increasing and very steep at high levels of $\theta$, then the best potential entrepreneurs end up as wage workers.

Lazear (2005) studies the role of entrepreneurial and specialist abilities on entrepreneurship. He posits that entrepreneurs need to be competent (though not excellent) in many skills. Individuals choose to become entrepreneurs using a variety of skills or to become specialists using only one skill. Lazear finds that individuals
with more balanced skill sets are found to be more likely to become entrepreneurs.

Manove (2000), like we do, studies the role of optimism on entrepreneurship. Individuals have heterogeneous entrepreneurial abilities and choose to become entrepreneurs or workers. Entrepreneurs use internal resources (own savings and effort) and external resources (hired labor) to produce. Some individuals are optimists and others are realists. Optimists overestimate their ability whereas realists do not. Manove finds that optimistic entrepreneurs save too much, provide too much effort, and hire too many employees relative to realistic entrepreneurs. Manove also shows that optimistic entrepreneurs can survive by working and saving extra hard to compensate for the mistakes caused by their optimism.

Rigotti et al. (2011) study the role of optimism on technology choice. Individuals choose to be entrepreneurs or employees and between employing a traditional technology or a new one which has ambiguous returns. A firm is an entrepreneur-employee pair operating a particular technology. Some individuals are optimists and others are pessimists. Rigotti et al. find that optimists are more likely to become entrepreneurs and that firms employing new technologies are run by optimistic entrepreneurs and employ optimistic employees.

We generalize Lucas (1978) by assuming that a fraction of individuals in the economy are optimists. This assumption allow us to explain the motivating stylized facts on entrepreneurship using Lucas’ extremely powerful and rigorous analytical framework. Some of our qualitative predictions are in line with Manove (2000) but others are novel. Like Manove (2000), we find that optimists bid up the wage which makes workers better off and entrepreneurs worse off. Unlike Manove (2000), we are able to characterize the impact of optimism on the capital market. We find that optimists bid up the rental cost of capital which further contributes to explain the low the returns to entrepreneurship. In addition, we provide quantitative predictions about the impact of optimism on occupational choice. As far as we know, we are the first ones to show that optimism can explain quantitatively the observed low returns to entrepreneurship.
2.1.2 Dynamic Models

Vereshchagina and Hopenhayn (2009), Campanale (2010), and Poschke (2013) use dynamic life-cycle occupational choice models to try to explain the low returns to entrepreneurship puzzle.

Vereshchagina and Hopenhayn (2009) consider a life-cycle model in which individuals can switch back and forth between entrepreneurship and paid employment. Entrepreneurship is risky and paid employment provides a fixed outside option. Individuals face financing constraints and because of them they take more risk at low wealth levels than at high wealth levels. Vereshchagina and Hopenhayn show that the combination of occupational choice and financing constraints can lead entrepreneurs to display risk-taking behavior. Hence, entrepreneurs operate in a financial environment that leads them to engage in risky investment even in the absence of a return premium.

Campanale (2010) considers a life-cycle occupational and portfolio choice model with learning. The key assumption is that the quality of a business project is not precisely known upon entry and is learned over time. The model shows that entry and private equity allocation for the majority of entrepreneurs can be rationalized even with negative expected premia on individual business investment. Since individuals can switch back to paid-employment, they find it worthwhile experimenting with entrepreneurship to find out if the project is good even if initially the expected return is low. Campanale quantifies the amount of risk premia that would justify entry into entrepreneurship in this environment, and finds that it is still substantially larger than what we see in the data.

Poschke (2013) proposes a life-cycle model in which individuals differ in their efficiency as workers and in the productivity of the firms they start. Whereas efficiency as a worker is known, the productivity of entrepreneurial projects can only be found after implementing them. Poschke shows that the option to abandon bad projects attracts low-ability agents into entrepreneurship.
2.2 Partial Equilibrium Theories of Entrepreneurship

Evans and Jovanovic (1989) study the role of liquidity constraints on entrepreneurship. They consider an occupational choice model where individuals choose between wage work and self-employment. They assume that both entrepreneurial and working skills are heterogeneous but they take the skill prices as exogenous. The earnings of self-employment are a function of capital, ability, and a random disturbance. Each individual has some wealth and can borrow up to an amount that is proportional to her wealth. Evans and Jovanovic show that liquidity constraints hinder individuals from starting businesses.

De Meza and Southey (1996) study the role of optimism on entrepreneurship, investment, and on the credit market. In their model individuals choose between working in a safe occupation with a known return or undertaking a project with a risky return. Entrepreneurs must select the right mix of self-finance and debt-finance from risk neutral banks to develop their projects. All individuals have the same ability or probability of success of their projects. Banks and realistic entrepreneurs know a project’s true probability of success but optimistic entrepreneurs overestimate it. De Meza and Southey find that optimists select maximum internal finance and any form of external finance is a standard debt contract, that optimism can lead to excessive lending, and that only optimists become entrepreneurs. De Meza and Southey also find that optimistic individuals who are denied loans and become workers may end up better off ex post than those who obtain loans and become entrepreneurs.

Manove and Padilla (1999) study the role of optimism on investment and on the credit market. While de Meza and Southey (1996) assume that banks can distinguish between optimists and realists, Manove and Padilla assume that banks cannot differentiate optimists from realists. An entrepreneur receives a signal about the quality of her project. A realist always observes a fully revealing signal, but an optimist always observes a good signal. Based on the information embodied in his signal, the entrepreneur chooses an investment level on her project. The investment level must be financed by banks since the entrepreneur has no liquid assets. The entrepreneur
requests a loan of the size of his intended investment level, and each bank responds
with an offer of a loan contract with an interest rate and a collateral requirement.
Manove and Padilla find that, in the presence of optimists, perfectly competitive
banks may be insufficiently conservative in their dealings with entrepreneurs, even if
entrepreneurs themselves may practice self-restraint to signal realism. Manove and
Padilla also show that, in the presence of optimists, the use of collateral requirements
by banks may reduce the efficiency of the credit market.

Coval and Thakor (2005) study the role of optimism and pessimism on financial
intermediation. They consider a model where individuals do not have not enough
wealth to self-finance a project that can be either good or bad. Realists correctly
assess a project’s probability of success, optimists overestimate it and pessimists
underestimate it. Coval and Thakor show that realists form a financial intermediary
that raises funds from pessimists (who become investors in the intermediary) and
lends to optimists who become entrepreneurs.

Fraser and Greene (2006) consider a dynamic occupational choice model in which
entrepreneurs are uncertain about their true talent but learn from experience. It
follows that optimism in talent lowers with experience. As a consequence, the impact
of optimism on the decision to be an entrepreneur lowers with experience.

Some of our qualitative predictions are in line with de Meza and Southey (1996)
but others are novel. Like de Meza and Southey (1996), we find that optimistic
individuals are more likely to become entrepreneurs, entrepreneurs are more likely
to be optimistic than workers, and optimism lowers the returns to entrepreneurship.
In contrast to de Meza and Southey (1996), we find that optimism raises the returns
to paid employment (and the rental cost of capital) and that some entrepreneurs are
optimists while others are realists.
3 Set-up

The economy consists of a continuum of risk-neutral individuals. The population is of size $N$ and the capital stock is of $K$ units of capital. Individuals derive utility from consumption and can earn income either as workers or by running their own firm. Each individual is endowed with 1 unit of labor, with capital stock $K/N$, and with a one-dimensional ability $\theta$ drawn from the cumulative distribution function $G(\theta)$ with support on $[0, \bar{\theta}]$, with $0 < \bar{\theta} < \infty$.

If an individual with ability $\theta$ becomes a worker he supplies his unit of labor on the labor market, receives the competitive wage $w$ for his unit of labor, and receives the competitive rental rate of capital for renting his capital $K/N$. Hence, a wage worker ends up with an income

$$w + rK/N.$$ 

If an individual with ability $\theta$ becomes an entrepreneur he can use without cost a technology defined by the continuous production function

$$y = \theta f(l, k),$$

where $y$ is output, $l$ is labor, and $k$ is capital. Following Lucas (1978), $\theta$ enters into the production function as the total factor productivity (TFP). Any individual can run at most one firm. We assume that $f$ is twice continuously differentiable with $f_l > 0$, $f_k > 0$, $f_{ll} < 0$, $f_{kk} < 0$. This production function combines as inputs one entrepreneur, who is essential to operate the firm, $l$ homogeneous employees, and $k$ units homogeneous units of capital. The production function exhibits decreasing returns to scale in the variable inputs, labor and capital, so that the competitive equilibrium exhibits a non-degenerate firm size distribution. This assumption implies that the size of firms is finite. This could be due for instance to limits in entrepreneurs’ span of control: as activity expands, it becomes more difficult to control, and the marginal product of the variable inputs diminishes.

Note that since the population size is $N$, the economy wide capital stock is fixed and equal to $K$ units of homogeneous capital.
Entrepreneurs hire labor at the competitive wage rate $w$ and rent capital at the competitive rental cost of capital $r$. Hence, an entrepreneur who employs $l$ workers and rents $k$ units of capital earns a profit of

$$
\pi(\theta, w, r) = pf(l, k) - wl + r(K/N - k).
$$

From now on the price of output $p$ is normalized to be 1. Individuals can belong to one of two types: those with optimistic beliefs and realists. Fraction $\lambda \in (0, 1)$ of the population has optimistic beliefs about their ability as entrepreneurs and fraction $1 - \lambda$ has realistic beliefs. The perceived profit of an optimistic entrepreneur with perception of ability $\gamma \theta$ who employs $l$ workers and rents $k$ units of capital is

$$
\pi(\gamma \theta, w, r) = \begin{cases} 
\gamma \theta f(l, k) - wl + r(K/N - k) & \text{if } 0 \leq \theta < \frac{\bar{\theta}}{\gamma} \\
 f(l, k) - wl + r(K/N - k) & \text{if } \frac{\bar{\theta}}{\gamma} \leq \theta \leq \bar{\theta}
\end{cases},
$$

where $\gamma > 1$. The parameter $\gamma$ measures the strength or intensity of optimistic beliefs. Under this specification the perception of ability of individuals with ability above $\bar{\theta}/\gamma$ is set equal to the highest possible ability level $\theta = \bar{\theta}$. In other words, no individual thinks that he or she can be more productive than $\bar{\theta}$. This specification of optimistic beliefs is analytically convenient. The distributions of entrepreneurial abilities and types are assumed to be independent. Hence, realists and optimists are equally endowed in terms of their entrepreneurial abilities.

An individual who selects to be an entrepreneur faces the perceived profit

$$
\gamma \theta f(l, k) - wl + r(K/N - k),
$$

where $\gamma \geq 1$. An individual who becomes an entrepreneur will choose to employ $l(\gamma \theta; w, r)$ workers and $k(\gamma \theta; w, r)$ units of capital where $l(\gamma \theta; w, r)$ and $k(\gamma \theta; w, r)$ are the values of $l$ and $k$ that solve the following problem

$$
\max_{l,k} [\gamma \theta f(l, k) - wl + r(K/N - k)].
$$

The first-order conditions to this problem are

$$
f_l(\gamma \theta, l, k) = w.
$$

13
and

\[ f_k(\gamma \theta, l, k) = r. \]  \hfill (3)

It follows from (2), the assumption of decreasing returns to labor, \( f_{ll} < 0 \), and complementarity between ability and labor, i.e., \( f_{l\theta} > 0 \), that realistic entrepreneurs with a higher \( \theta \) hire more workers: \( \partial l(\gamma \theta, w, r)/\partial \theta = -\gamma f_{l\theta}/f_{ll} > 0 \). Similarly, it follows from (3), the assumption of decreasing returns to capital, \( f_{kk} < 0 \), and complementarity between ability and capital, i.e., \( f_{k\theta} > 0 \), that realistic entrepreneurs with a higher \( \theta \) hire more capital: \( \partial k(\gamma \theta, w, r)/\partial \theta = -\gamma f_{k\theta}/f_{kk} > 0 \). The same is true for optimistic entrepreneurs but only for those with \( \theta < 1/\gamma \). It also follows from (2) that an optimistic entrepreneur will demand more labor than a realist with the same ability. Similarly, it follows from (3) that an optimistic entrepreneur will demand more capital than a realist with the same ability.

A realist with ability \( \theta \) chooses to become a worker at wage \( w \) and rental cost of capital is \( r \) when

\[ \theta f(l(\theta, w, r), k(\theta, w, r)) - wl(\theta, w, r) - rk(\theta, w, r) \leq w. \] \hfill (4)

He selects to be an entrepreneur if

\[ \theta f(l(\theta, w, r), k(\theta, w, r)) - wl(\theta, w, r) - rk(\theta, w, r) \geq w, \] \hfill (5)

and he is indifferent if the equality holds in (4) and (5). An optimist with perception of ability \( \gamma \theta \) chooses to become a worker at wage \( w \) and rental cost of capital is \( r \) when

\[ \gamma \theta f(l(\gamma \theta, w, r), k(\gamma \theta, w, r)) - wl(\gamma \theta, w, r) - rk(\gamma \theta, w, r) \leq w. \] \hfill (6)

He selects to be an entrepreneur if

\[ \gamma \theta f(l(\gamma \theta, w, r), k(\gamma \theta, w, r)) - wl(\gamma \theta, w, r) - rk(\gamma \theta, w, r) \geq w, \] \hfill (7)

and he is indifferent if the equality holds in (6) and (7).

\footnote{The term \( rK/N \) cancels out.}
Since there are only three markets—output, labor, and capital—by Walras' Law, general equilibrium is realized when the labor and capital markets clear. At the equilibrium wage, the labor demanded by individuals who choose to become entrepreneurs equals that supplied by individuals who choose to become workers. At the equilibrium rental cost of capital, the capital demanded by individuals who choose to become entrepreneurs equals the exogenous capital stock of the economy, $K$. Formally, an equilibrium is (i) a partition $\{[0, \hat{\theta}_R], [\hat{\theta}_R, \bar{\theta}]\}$ of $[0, \bar{\theta}]$ where for all $\theta \in [0, \hat{\theta}_R]$ (4) holds and for all $\theta \in [\hat{\theta}_R, \bar{\theta}]$ (5) holds, (ii) a partition $\{[0, \hat{\theta}_O], [\hat{\theta}_O, \bar{\theta}]\}$ of $[0, \bar{\theta}]$ where for all $\theta \in [0, \hat{\theta}_O]$ (6) holds and for all $\theta \in [\hat{\theta}_O, \bar{\theta}]$ (7) holds, (iii) a wage $w$ for which labor demand equals labor supply

$$N \left[ (1 - \lambda) \int_{\hat{\theta}_R}^{\bar{\theta}} l(\theta, w, r)dG(\theta) + \lambda \int_{\hat{\theta}_O}^{\bar{\theta}} l(\gamma \theta, w, r)dG(\theta) \right]$$

$$= N \left[ (1 - \lambda) \int_{0}^{\hat{\theta}_R} dG(\theta) + \lambda \int_{0}^{\hat{\theta}_O} dG(\theta) \right],$$

and (iv) a rental cost of capital $r$ for which capital demand equals the exogenous capital supply

$$N \left[ (1 - \lambda) \int_{\hat{\theta}_R}^{\bar{\theta}} k(\theta, w, r)dG(\theta) + \lambda \int_{\hat{\theta}_O}^{\bar{\theta}} k(\gamma \theta, w, r)dG(\theta_0) \right] = K.$$  

In equilibrium, realists with ability below $\hat{\theta}_R$ become workers whereas those with ability above $\hat{\theta}_R$ become entrepreneurs. Similarly, optimists with below $\hat{\theta}_O$ become workers whereas those with ability above $\hat{\theta}_O$ become entrepreneurs. We refer to a realist with ability $\hat{\theta}_R$ as the *marginal realistic entrepreneur*. We refer to an optimist with ability $\hat{\theta}_O$ as the *marginal optimistic entrepreneur*.

### 4 Competitive Equilibrium

In this section we determine the competitive equilibrium under a generalized Cobb-Douglas production function and a uniform distribution of ability with support on
The production function given by
\[
y = \theta f(l, k) = \theta l^\alpha k^\beta, \text{ with } \alpha + \beta \equiv \eta \in (0, 1).
\]

Hence, the variable inputs, labor and capital, are combined under a generalized Cobb-Douglas production function with decreasing returns to scale.\(^7\) The profit of a realistic entrepreneur with ability \(\theta\) is
\[
\pi(\theta, w, r) = \theta l^\alpha k^\beta - wl + r(K/N - k). \tag{10}
\]

The perceived profit of an optimistic entrepreneur with perception of ability \(\gamma\theta\) is
\[
\pi(\gamma\theta, w, r) = \begin{cases} 
\gamma\theta l^\alpha k^\beta - wl + r(K/N - k) & \text{if } 0 \leq \theta < \frac{1}{\gamma} \\
l^\alpha k^\beta - wl + r(K/N - k) & \text{if } \frac{1}{\gamma} \leq \theta \leq 1
\end{cases},
\]
where \(\gamma > 1\). The assumptions: (i) individuals are risk neutral, (ii) ability \(\theta\) belongs to \([0, 1]\), and (iii) \(\theta\) is the total factor productivity, imply that \(\theta\) can be interpreted as the probability of success of the firm (the project either succeeds with probability \(\theta\) or fails with probability \(1 - \theta\), in which case output is zero).\(^8\)

An entrepreneur with perception of ability \(\gamma\theta \in [\theta, 1]\) chooses to employ \(l\) workers and \(k\) units of capital where \(l\) and \(k\) are the solution to
\[
\max_{l,k} \left[ \gamma\theta l^\alpha k^\beta - wl + r(K/N - k) \right].
\]

\(^7\)This is a standard assumption in models with heterogeneous ability. See, for example, Lucas (1978), Evans and Jovanovic (1989), Murphy et al. (1991), de Meza and Southey (1996), and Poschke (2013).

\(^8\)In other words, under this specification entrepreneurial optimism coincides with overestimation of ability. The strongest cross-national covariate of an individual’s entrepreneurial propensity is whether the person believes herself to have the sufficient skills and knowledge to start a business (Koellinger et al. (2007)). The probability of becoming an entrepreneur increases with a person’s confidence in his/her ability to perform entrepreneurship related tasks (Cassar and Friedman (2009)). Entrepreneurs are more overconfident about their abilities than non-entrepreneurs: 59 percent of entrepreneurs, 56 percent of the managers, and 52 percent of the employees overestimate their performance on a cognitive ability test (Koudstaal et al. (2015)).
The first-order conditions are
\[ \alpha \gamma \theta^{1-1} k^\beta = w, \]
and
\[ \beta \gamma \theta^{1} k^{\beta-1} = r. \]
Solving for \( l \) and \( k \) we obtain the input demands:
\[ l(\gamma \theta, w, r) = (\gamma \theta)^{1/\eta} \left( \frac{\alpha}{w} \right)^{1-\eta} \left( \frac{\beta}{r} \right)^{1/\eta}, \tag{11} \]
and
\[ k(\gamma \theta, w, r) = (\gamma \theta)^{1/\eta} \left( \frac{\alpha}{w} \right)^{1-\eta} \left( \frac{\beta}{r} \right)^{1-\eta}, \tag{12} \]
respectively. The input demands determine the size of the firm given the ability of the entrepreneur, the wage, the rental cost of capital, and the entrepreneur’s optimism. We see from (11) and (12) that entrepreneurs’ demands for labor and capital are greater among those with higher ability \( \theta \). That is, more talented entrepreneurs run larger firms than less talented entrepreneurs, irrespective of whether firm size is defined in terms of labor or capital. We also see from (11) and (12) that optimists (\( \gamma > 1 \)) run larger firms than realists (\( \gamma = 1 \)) of the same ability. Substituting (11) and (12) into (10) and setting \( \gamma = 1 \) we obtain the profit of a realistic entrepreneur:
\[ \pi(\theta, w, r) = \theta^{1-\eta} (1 - \eta) \left( \frac{\alpha}{w} \right)^{1-\eta} \left( \frac{\beta}{r} \right)^{1-\eta} \]
\[ + rK/N. \tag{13} \]
We see from (13) that the assumption of decreasing returns to scale, i.e., \( \eta \in (0, 1) \), implies that the profits of realistic entrepreneurs are an increasing and convex function of \( \theta \). The perceived profit of an optimistic entrepreneur is:
\[ \pi(\gamma \theta, w, r) = \begin{cases} 
(\gamma \theta)^{1/\eta} (1 - \eta) \left( \frac{\alpha}{w} \right)^{1-\eta} \left( \frac{\beta}{r} \right)^{1-\eta} \]
\[ + rK/N \text{ if } 0 \leq \theta < \frac{1}{\gamma} \]
\[ (1 - \eta) \left( \frac{\alpha}{w} \right)^{1-\eta} \left( \frac{\beta}{r} \right)^{1-\eta} + rK/N \text{ if } \frac{1}{\gamma} \leq \theta \leq 1. \tag{14} \]
We see from (14) that the assumption of decreasing returns to scale also implies that the perceived profits of optimistic entrepreneurs with perception of ability \( \gamma \theta \) and ability \( \theta < 1/\gamma \) are an increasing and convex function of \( \theta \) and of \( \gamma \).
A realist with ability $\hat{\theta}_R$ is indifferent between being an entrepreneur and a worker when

$$\hat{\theta}_R \frac{1}{1-\eta} \left( \frac{\alpha}{w} \right)^{\frac{\alpha}{1-\eta}} \left( \frac{\beta}{r} \right)^{\frac{\beta}{1-\eta}} = w. \quad (15)$$

An optimist with perception of ability $\gamma \hat{\theta}_O$ and ability $\hat{\theta}_O < 1/\gamma$ is indifferent between being an entrepreneur and a worker when

$$(\gamma \hat{\theta}_O) \frac{1}{1-\eta} \left( \frac{\alpha}{w} \right)^{\frac{\alpha}{1-\eta}} \left( \frac{\beta}{r} \right)^{\frac{\beta}{1-\eta}} = w. \quad (16)$$

In equilibrium, labor demand must equal labor supply. The assumption that $\theta$ is uniformly distributed on $[0, 1]$ and (1) imply that (8) becomes:

$$(1 - \lambda) \int_{\hat{\theta}_R}^{1} l(\theta, w, r)d\theta + \lambda \left[ \int_{\hat{\theta}_O}^{1/\gamma} l(\gamma \theta, w, r)d\theta + \int_{1/\gamma}^{1} l(1, w, r)d\theta \right] = (1 - \lambda) \hat{\theta}_R + \lambda \hat{\theta}_O. \quad (17)$$

In equilibrium, capital demand must equal capital supply. The assumption that $\theta$ is uniformly distributed on $[0, 1]$ and (1) imply that (9) becomes:

$$(1 - \lambda) \int_{\hat{\theta}_R}^{1} k(\theta, w, r)d\theta + \lambda \left[ \int_{\hat{\theta}_O}^{1} k(\gamma \theta, w, r)d\theta + \int_{1/\gamma}^{1} k(1, w, r)d\theta \right] = K/N \quad (18)$$

Equations (15), (16), (17), and (18) form a system of four equations and four unknowns ($\hat{\theta}_R$, $\hat{\theta}_O$, $w$, $r$) which defines a unique competitive equilibrium. Since the profits of realistic entrepreneurs are an increasing and convex function of $\theta$ it follows from (15) that there exists an unique ability cut-off between realistic entrepreneurs and realistic workers, i.e., $\hat{\theta}_R$ is unique. Similarly, since the perceived profits of optimists are an increasing and convex function of $\gamma \theta$ it follows from (16) that there exists a unique ability cut-off between optimistic entrepreneurs and optimistic workers, i.e., $\hat{\theta}_O$ is unique. Solving (15) and (16) for the unique cut-offs $\hat{\theta}_R$ and $\hat{\theta}_O$ and substituting these into (17) and (18) we obtain the unique equilibrium vector of input prices $(w^*, r^*)$. Finally, from $(\hat{\theta}_R, \hat{\theta}_O, w^*, r^*)$ we obtain the equilibrium output level $Y^*$. Hence, the existence and uniqueness of the equilibrium, a standard result in the
Lucas “span-of-control” model, is not affected by the presence of optimists. Our first result characterizes the equilibrium.

**Proposition 1:** If the technology is \( f(l, k, \theta) = \theta l^\alpha k^\beta \), ability \( \theta \) is uniformly distributed on \([0, 1]\), and \( \lambda - \lambda/\gamma < (1 - \alpha - \beta)/(1 - \beta) \), then there exists a unique competitive equilibrium where the marginal realistic entrepreneur has ability

\[
\hat{\theta}_R = \frac{\alpha}{2 - \beta} \frac{1 - \eta + \lambda - \frac{\lambda}{\gamma}}{(1 - \eta) \left(1 - \lambda + \frac{\lambda}{\gamma}\right)} \left[ \frac{1 - \eta}{2 - \beta} \right]^{\frac{1 - \eta}{2 - \beta}},
\]

the marginal optimistic entrepreneur has ability

\[
\hat{\theta}_O = \frac{1}{\gamma} \frac{\alpha}{2 - \beta} \frac{1 - \eta + \lambda - \frac{\lambda}{\gamma}}{(1 - \eta) \left(1 - \lambda + \frac{\lambda}{\gamma}\right)} \left[ \frac{1 - \eta}{2 - \beta} \right]^{\frac{1 - \eta}{2 - \beta}},
\]

the wage is

\[
w^* = \frac{\alpha^{\eta}(1 - \eta)^{1 - \eta}(K/N)^\beta}{(1 - \lambda + \frac{\lambda}{\gamma})^\beta} \left[ \frac{\alpha}{2 - \beta} \frac{1 - \eta + \lambda - \frac{\lambda}{\gamma}}{(1 - \eta) \left(1 - \lambda + \frac{\lambda}{\gamma}\right)} \right] \left[ \frac{1 - \eta}{2 - \beta} \right]^{\frac{(1 - \eta)(1 - \beta)}{2 - \eta}},
\]

the rental cost of capital is

\[
r^* = \frac{\beta(1 - \eta)^{1 - \eta} (1 - \lambda + \frac{\lambda}{\gamma})^{1 - \beta}}{\alpha^{1 - \eta}(K/N)^{1 - \beta}} \left[ \frac{\alpha}{2 - \beta} \frac{1 - \eta + \lambda - \frac{\lambda}{\gamma}}{(1 - \eta) \left(1 - \lambda + \frac{\lambda}{\gamma}\right)} \right] \left[ \frac{1 - \eta}{2 - \beta} \right]^{\frac{(1 - \eta)(2 - \beta)}{2 - \eta}},
\]

the fraction of workers is

\[
L^* = N \left(1 - \lambda + \frac{\lambda}{\gamma}\right) \left[ \frac{\alpha}{2 - \beta} \frac{1 - \eta + \lambda - \frac{\lambda}{\gamma}}{(1 - \eta) \left(1 - \lambda + \frac{\lambda}{\gamma}\right)} \right] \left[ \frac{1 - \eta}{2 - \beta} \right]^{\frac{1 - \eta}{2 - \beta}},
\]

and the output level is

\[
Y^* = N \left(\frac{\alpha}{w^*}\right)^{\frac{\alpha}{\lambda}} \left(\frac{\beta}{r^*}\right)^{\frac{\beta}{\lambda}} \left[ \frac{1 - \lambda + \frac{\lambda}{\gamma}}{2 - \eta} \left(1 - \eta - \frac{\alpha}{2 - \beta} \frac{1 - \eta + \lambda - \frac{\lambda}{\gamma}}{1 - \lambda + \frac{\lambda}{\gamma}}\right) + \frac{\lambda - \frac{\lambda}{\gamma}}{2}\right].
\]
Proposition 1 shows us that the existence of optimists leads to a misallocation of talent. In a competitive equilibrium without optimists (where $\lambda = 0$ or $\gamma = 1$) the marginal entrepreneur has ability

$$\hat{\theta} = \left( \frac{\alpha}{2 - \beta} \right)^{\frac{1-\gamma}{2-\gamma}},$$

(25)

which implies that individuals with ability $[0, \hat{\theta}]$ become workers and individuals with ability $[\hat{\theta}, 1]$ become entrepreneurs. Hence, in the competitive equilibrium without optimists the ablest people become entrepreneurs.

In a competitive equilibrium with optimists, realists with ability $[0, \hat{\theta}_R]$ and optimists with ability $[0, \hat{\theta}_O]$ become workers whereas realists with ability $[\hat{\theta}_R, 1]$ and optimists with ability $[\hat{\theta}_O, 1]$ become entrepreneurs. It follows from (19), (20), and $\gamma > 1$ that:

$$\hat{\theta}_O < \hat{\theta}_R.$$ (26)

Hence, in the competitive equilibrium with optimists, the ablest people do not necessarily become entrepreneurs. Moreover, the lowest ability entrepreneur (an optimist with ability $\hat{\theta}_O$) is less talented at running a firm than the highest ability worker (a realist with ability $\hat{\theta}_R$). This is an empirically attractive implication of the model since, in reality, the income distributions of workers and entrepreneurs have overlapping supports.

Proposition 1 implies that optimists are more likely to become entrepreneurs than realists. In other words, the probability an optimist becomes an entrepreneur is greater than the probability a realist becomes an entrepreneur. To see this note that the probability an optimist becomes an entrepreneur is

$$\Pr(E|O) = \frac{\Pr(E \cap O)}{\Pr(O)} = \frac{\lambda(1 - \hat{\theta}_O)}{\lambda} = 1 - \hat{\theta}_O,$$ (27)

and the probability a realist becomes an entrepreneur is

$$\Pr(E|R) = \frac{\Pr(E \cap R)}{\Pr(R)} = \frac{(1 - \lambda)(1 - \hat{\theta}_R)}{1 - \lambda} = 1 - \hat{\theta}_R.$$ (28)
It follows from (26), (27), and (28) that \( \Pr(E|O) > \Pr(E|R) \).

Proposition 1 also implies that entrepreneurs are more likely to be optimists than workers. In other words, the probability an entrepreneur is an optimist is greater than the probability a worker is an optimist. To see this note that the probability an entrepreneur is an optimist is

\[
\Pr(O|E) = \frac{\Pr(O \cap E)}{\Pr(E)} = \frac{\lambda(1 - \hat{\theta}_O)}{\lambda(1 - \hat{\theta}_O) + (1 - \lambda)(1 - \hat{\theta}_R)},
\]

(29)

and the probability a worker is an optimist is

\[
\Pr(O|L) = \frac{\Pr(O \cap L)}{\Pr(L)} = \frac{\lambda\hat{\theta}_O}{\lambda\hat{\theta}_O + (1 - \lambda)\hat{\theta}_R}.
\]

(30)

It follows from (26), (29), and (30) that \( \Pr(O|E) > \Pr(O|L) \). This result is in line with the empirical evidence in Arabsheibani et al. (2000), Fraser and Greene (2006), Puri and Robinson (2013), and Koudstaal et al. (2015).

To close this section we discuss the assumption that the competitive equilibrium is well defined as long as the overall optimism in the economy—the product of the fraction of optimists by \( (1 - 1/\gamma) \), a term that is increasing in the intensity of optimistic beliefs—is not too high.\(^9\)

5 Comparative Statics

In this section we perform comparative statics on equilibrium outcomes. There are two parameters which can be used to perform this analysis: the fraction of optimists and the intensity of optimism \( \gamma \). By changing \( \lambda \) while keeping everything else fixed we

\(^9\)When \( \lambda - \lambda/\gamma < (1 - \alpha - \beta)/(1 - \beta) \) is violated there exists a unique competitive equilibrium where optimists who select to become entrepreneurs hold the highest possible perception of ability, i.e., \( \gamma\theta = 1 \). In addition, a positive mass of individuals with \( \gamma\theta = 1 \) choose to be workers since their entrepreneurial ability \( \theta \) is not high enough to make entrepreneurship more attractive than working as an employee.
can analyze the impact that a change in the fraction of optimists has on equilibrium outcomes. By changing $\gamma$ while keeping everything else fixed we can analyze the impact that a change in optimistic biases in beliefs have on equilibrium outcomes. We focus on comparative statics with respect to $\lambda$. At the end we discuss briefly the comparative statics with respect to $\gamma$.

**Proposition 2:** If the technology is $f(l, k, \theta) = \theta l^\alpha k^\beta$, ability $\theta$ is uniformly distributed on $[0, 1]$, and $\lambda - \lambda/\gamma < (1 - \alpha - \beta)/(1 - \beta)$, then an increase in the fraction of optimists: (i) raises the market clearing wage, i.e., $\partial w^*/\partial \lambda > 0$, (ii) raises the rental cost of capital, i.e., $\partial r^*/\partial \lambda > 0$, and (iii) raises the number of entrepreneurs, i.e., $\partial E^*/\partial \lambda > 0$.

Part (i) shows that an increase in the fraction of optimists raises the market clearing wage. This is a novel prediction of the model that was absent from previous work on the impact of optimism on general equilibrium outcomes. The intuition behind this result as follows. Wage effects can occur through two channels: through firm’s derived demand for labor and through labor-supply decisions of individuals, who must choose to be either workers or entrepreneurs. The fact that optimists overestimate their ability implies that, for given input prices, the demand for labor of an optimist is higher than the demand for labor of a realist of the same ability. This leads to an expansion of labor demand. An optimist is, for given input prices, more attracted to entrepreneurship than a realist of the same ability. This leads to a contraction of labor supply. The expansion of labor demand and contraction of labor supply raise the market clearing wage.

Part (ii) shows that an increase in the fraction of optimists raises the rental cost of capital. The fact that optimists overestimate their ability implies that, for given input prices, the demand for capital of an optimist is higher than the demand for capital of a realist of the same ability. This leads to an expansion of capital demand. Since the supply of capital is fixed the expansion of capital demand raises the rental cost of capital.

Finally, part (iii) shows that an increase in the fraction of optimists raises the
number of entrepreneurs. We have that, on the one hand, an increase in the fraction of optimists lowers the number of realistic entrepreneurs, and, on the other hand, it raises the number of optimistic entrepreneurs. Hence, at first sight, an increase in $\lambda$ has an ambiguous effect on the number of entrepreneurs. However, the second effect always dominates the first and therefore an increase in $\lambda$ raises the number of entrepreneurs.

To close this section we briefly discuss the comparative statics with respect to the intensity of optimistic beliefs. An increase in $\gamma$ raises the ability of the marginal realistic entrepreneur $\hat{\theta}_R$ and lowers the ability of the marginal optimistic entrepreneur $\hat{\theta}_O$. In addition, an increase in $\gamma$ raises the wage, the rental cost of capital, and the number of entrepreneurs.

6 Calibration

This section calibrates the model to illustrate quantitatively the general equilibrium effects of optimism. The calibration parameterizes the economy to match salient features of U.S. manufacturing data and is summarized in Table I.

| Table I
<table>
<thead>
<tr>
<th>Parameters of the Model</th>
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<tbody>
<tr>
<td>Parameter</td>
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<tr>
<td>---------------------------------</td>
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<tr>
<td><strong>Standard parameters</strong></td>
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<td>$\eta$</td>
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<td><strong>Behavioral parameters</strong></td>
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<tr>
<td>$\lambda$</td>
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<tr>
<td>$\gamma$</td>
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Following Atkeson and Kehoe (2005) we set η to 0.85. Following Adler (2016), given η equal to 0.85, a value of 0.612 for α matches labor’s average income share (including managerial compensation) in U.S. manufacturing between 1998 and 2005. Again, following Atkeson and Kehoe (2005) and Adler (2016) we assume a capital-output ratio \( K/Y \) of 1.46 which together with a value for \( Y \) of 0.62032 in the model without optimists (\( \lambda = 0 \)) implies a capital stock \( K \) of 0.906.

We are left with the behavioral parameters \( \lambda \) and \( \gamma \) to calibrate. Recall that \( \lambda \) represents the fraction of optimists and \( \gamma \) the intensity of optimistic beliefs. The ideal data to calibrate \( \lambda \) and \( \gamma \) would consists of representative samples of entrepreneurs and employees with measures of optimism that compare expectations to realized financial outcomes in the U.S. manufacturing sector. We are unaware of such data so we take the following approach. First, we review the evidence of empirical studies on optimism of entrepreneurs and employees in the U.S. and elsewhere. Second, we use the empirical evidence to obtain a lower bound for the fraction of optimists in the U.S. \( \lambda \) and a lower bound for the share of optimistic entrepreneurs in the U.S., \( E_O^*/E^* \). Third, using the lower bounds for \( \lambda \) and \( E_O^*/E^* \) together with the values for \( \alpha \) and \( \beta \) we calibrate \( \gamma \) to satisfy the equilibrium condition

\[
\frac{\lambda \left(1 - \hat{\theta}_R/\gamma \right)}{\lambda(1 - \hat{\theta}_R/\gamma) + (1 - \lambda)(1 - \hat{\theta}_R)} = \frac{E_O^*}{E^*}.
\] (31)

Finally, we verify that the calibration satisfies \( \lambda - \lambda/\gamma < (1 - \alpha - \beta)/(1 - \beta) \).

Cooper et al. (1980) find that 33 percent of a sample of 2994 U.S. nascent entrepreneurs perceive their chances of success as 10 out of 10 or “absolutely certain.” Arabsheibani et al. (2000) report results from a sample of 2909 entrepreneurs and 20056 employees from the British Household Panel Study. They find that 31 percent of individuals are optimists, 34.754 percent of entrepreneurs are optimists, and 30.46 percent of employees are optimists. Cassar (2010) reports that 62 percent of a sample of 386 U.S. nascent entrepreneurs from the Panel Study of Entrepreneurial Dynamics (PSED) who achieve operation overestimate projected first-year sales. Hyytinen et al. (2014) report that 48.8 percent of a sample of 487 U.S. nascent entrepreneurs from
the PSED think that the likelihood of exit of their venture is zero in five years time. They also report that 34.5 percent of a sample of 891 Finnish nascent entrepreneurs think that the likelihood of exit of their new venture is zero in three years’ time. Koudstaal et al. (2015) find that 58 percent of entrepreneurs, 54 percent of the managers, and 32 percent of the employees can be classified as very optimistic.

The empirical evidence shows that: (i) the fraction of optimists in the U.K. is 31 percent, (ii) the fraction of optimistic entrepreneurs in the U.S. varies from 33 to 48.8 to 62 percent, and (iii) the fraction of optimistic entrepreneurs is quite high in the U.S., U.K., Finland, and Netherlands. Based on these studies we set the lower bound for the fraction of optimists in the U.S. at $\lambda = 0.31$ and we set the lower bound for the fraction of optimistic entrepreneurs in the U.S. at $E_O^*/E^* = (33+48.8+62)/3 = 0.48$. Setting $\alpha = 0.612$, $\beta = 0.238$, $\lambda = 0.31$ and $E_O^*/E^* = 0.48$ in (31) and solving for $\gamma$ we obtain $\gamma = 1.275$. The calibration satisfies $\lambda - \lambda/\gamma < (1 - \alpha - \beta)/(1 - \beta)$ since $0.31 - 0.31/1.275 = 0.06863 < 0.19685 = (1 - 0.85)/(1 - 0.238)$.

Table II summarizes the results of the calibration. The first column in Table II lists the variables. The second and the third columns report the competitive equilibrium without and with optimists, respectively. The fourth column reports the percent change in the variables common to both models.

We now discuss the results of the calibration. We see that optimism leads to a 3.93 percent decline in output. This result is expected since we know from Lucas (1978) that, in the absence of distortions, the competitive equilibrium maximizes output. We also see that optimism leads to an overuse scarce resources in equilibrium and bids up input prices: the wage increases by 5.94 percent and the rental rate of capital by 4.90 percent. Furthermore, optimism leads to 57.05 percent decline in the mean return to entrepreneurship. The mean return to entrepreneurship without optimism ($\lambda = 0$) is equal to

$$\frac{\pi^*_0}{E^*_0} = \frac{Y^*_0 - w^*_0L^*_0 - r^*_0K}{N(1 - \theta_0)} = 0.71506,$$
whereas the mean return to entrepreneurship with optimism ($\lambda = 0.31$ and $\gamma = 1.275$) is equal to

$$\frac{\pi^*}{E^*} = \frac{Y^* - w^*L^* - r^*K}{N\lambda(1 - \hat{\theta}_O) + N(1 - \lambda)(1 - \hat{\theta}_R)} = 0.30713.$$ 

| Calibration |
|-----------------|-----------------|-----------------|
| Model $\lambda = 0$ | Model $\lambda = 0.31$ and $\gamma = 1.275$ | Percent change |
| Output ($Y^*$) | 0.62032 | 0.59595 | -3.93 |
| Wage ($w^*$) | 0.43682 | 0.46276 | 5.94 |
| Rental cost of capital ($r^*$) | 0.16297 | 0.17096 | 4.90 |
| Mean returns to entrepreneurship | 0.71506 | 0.30713 | -57.05 |
| Mean returns of realistic entrep | - | 0.61542 | - |
| Mean returns of optimistic entrep | - | -0.02685 | - |
| Fraction of workers ($L^*$) | 0.87116 | 0.86069 | -1.20 |
| Ability of mg realistic entrep ($\hat{\theta}_R$) | - | 0.92236 | - |
| Ability of mg optimistic entrep ($\hat{\theta}_O$) | - | 0.72342 | - |
| Fraction of entrep ($E^*$) | 0.12884 | 0.13931 | 8.13 |
| Fraction of entrep optimists ($E_O^*/E^*$) | - | 0.48000 | - |

The sharp decline in the mean return to entrepreneurship happens for four reasons. First, optimism leads to a misallocation of talent since lower skill optimistic individuals crowd out higher skill realistic individuals from entrepreneurship.\(^{10}\) Second, optimism raises input prices. Third, optimism distorts the input choices of optimistic entrepreneurs. Fourth, optimism raises the number of entrepreneurs.\(^{11}\) Overall,\(^{10}\) the misallocation of talent affects 8.11 percent of the population since $N\lambda(\hat{\theta}_O - \hat{\theta}_R) + N(1 - \lambda)(\hat{\theta}_R - \hat{\theta}_O) = 0.0811$.\(^{11}\) Optimism leads to a 1.20 percent decline in the fraction of workers (from 87 to 86 percent) and to a 8.11 percent increase in the fraction of entrepreneurs (from 13 to 14 percent).
the calibration shows that optimism can explain quantitatively the observed low returns to entrepreneurship. In addition, the calibration show that optimism leads to a moderate decrease in output and a moderate increase in input prices.

7 Assumptions, Extensions and Implications

In this section, we discuss the main assumptions of the model and some extensions. Then we discuss policy implications of the analysis.

7.1 Assumptions and Extensions

We assume that the returns from entrepreneurship are deterministic. It is possible to extend the model by including a random component $\varepsilon$ in entrepreneurial revenues. For example, letting $y = \theta f(l, k) + \varepsilon$, where $\varepsilon$ has mean 0 and variance $0 < \sigma^2 < \infty$. Since individuals are risk neutral all results are left unchanged as long as there is no optimism about the realization of $\varepsilon$. If individuals are not only optimistic about $\theta$ but also about $\varepsilon$, then entrepreneurship would be more attractive relative to paid employment. In this case the main qualitative effects of optimism would still hold but its quantitative effects would be larger.

We assume individuals have different abilities to run a firm and the same productivity (or ability) as workers. This implies that different entrepreneurs obtain different amounts of profit but that workers receive the same wage. This is a natural simplification since the empirical evidence shows that the returns to entrepreneurship are much more variable than wages ((Borjas and Bronars (1989), Hamilton (2000)). Still, the model could be extended by letting individuals have different abilities in both occupations. Following Jovanovic (1994), we could let the returns to paid employment be equal to $w\psi(\theta)$ where $\psi(\theta)$ is the wage-working ability of an individual with ability $\theta$. If $\psi$ is a strictly increasing function (good entrepreneurs are also good workers), then optimists would overestimate the returns to entrepreneurship as well.
as the returns to paid employment.\textsuperscript{12} Since these two effects would partially cancel out, the main qualitative effects of optimism would still hold but its quantitative effects would be smaller.\textsuperscript{13}

In our model an entrepreneur hires workers and rents capital to produce output. However, the empirical evidence shows that many firms have no employed workers, i.e., the owners of these firms are self-employed without employees (see Braguinsky et al. (2011) and Salas-Fumas et al. (2014)). The model could also be extended to incorporate this third type of occupational choice. This could be done by assuming that the returns of firms without employed workers are given by $B + \theta$, where $B > 0$ represents a non-pecuniary benefit like the utility derived from “being your own boss” (see Hurst and Pugsley (2011) and Åstebro et al. (2014)). In this case, realists with ability $\theta$ such that $w > \max\{B + \theta, \pi(\theta, w, r)\}$ would become workers, those with ability $\theta$ such that $B + \theta > \max\{w, \pi(\theta, w, r)\}$ would open a firm without employed workers, and those with ability $\theta$ such that $\pi(\theta, w, r) > \max\{B + \theta, w\}$ would become entrepreneurs. Similarly, optimists with perception of ability $\gamma \theta$ such that $w > \max\{B + \gamma \theta, \pi(\gamma \theta, w, r)\}$ would become workers, those with perception of ability $\gamma \theta$ such that $B + \gamma \theta > \max\{w, \pi(\gamma \theta, w, r)\}$ would open a firm without employed workers, and those with perception of ability $\gamma \theta$ such that $\pi(\gamma \theta, w, r) > \max\{B + \gamma \theta, w\}$ would become entrepreneurs.

We assume ability is uniformly distributed. However, empirical evidence shows that firm size might be better described by a lognormal distribution (Cabral and Mata (2003)). A right-skewed distribution of ability, like the lognormal, interacts with the optimistic beliefs of individuals. Under our simple specification for optimistic beliefs, individuals with intermediate ability levels are the ones who overestimate their

\footnote{The empirical evidence supports the assumption that entrepreneurial and wage-working abilities are positively correlated, i.e., $\psi$ is a strictly increasing function. See Murphy et al. (1991), Jovanovic (1994), and Braguinsky et al. (2011).}

\footnote{We are assuming here that $\psi$ is strictly increasing and not very steep at high levels of $\theta$. In this case the most talented individuals become entrepreneurs. In contrast, when $\psi$ is strictly increasing and very steep at high level of $\theta$, the most talented individuals become workers.}
abilities the most. If the number of these individuals is small, then the qualitative effects of optimism would still hold but its quantitative effects would be smaller.

We also assume that the intensity of optimistic beliefs is exogenous. We extended the model by endogeneizing the intensity of optimistic beliefs using the optimal expectations model of Brunnermeier and Parker (2005). We found that the qualitative and quantitative impact of optimism on labor, capital and output markets would be similar as the ones obtained under exogenous optimistic beliefs. The main novelty of this extension would be that the intensity of optimistic beliefs would become a function of tastes and technology.

We focus on differences in ability and optimism as the main determinants which explain who becomes an entrepreneur and who works as an employee. There are of course many other factors which could influence this choice. For example, entrepreneurial effort (and the disutility of exerting it), access to funds needed to create a firm, risk aversion, and learning about ability. We do not model entrepreneurial effort and therefore we rule out any positive effects of optimism on entrepreneurial effort like the ones found in Manove (2000). If ability and effort are complements, then optimistic entrepreneurs would provide more effort than realistic ones. In this case the impact of optimism on the returns to entrepreneurship and on output would be ambiguous. We assume individuals are risk neutral so we cannot discuss the role that risk aversion together with optimism might play in the decision to become an entrepreneur or a worker. In addition, our model is static so we rule out the possibility that optimists learn their true abilities over time. We believe these are interesting avenues for future research.

7.2 Policy Implications

Are there any policy implications one can take away from this model? Given their mistaken beliefs, individuals in this economy are not maximizing their utility and so the economy is in a second-best situation. If the goal of a policymaker is to move the economy back to the first-best, then it is possible to do so with a revenue-
neutral tax-subsidy scheme. The scheme consists of a lump-sum tax to (optimistic) entrepreneurs with profits below the market clearing wage and a lump-sum subsidy to workers. The tax revenues come only from low ability optimistic entrepreneurs and induces them to stay in the labor force. The tax revenues are redistributed to workers as a lump-sum subsidy which further induces low ability optimists to stay in the labor force. The full characterization of this tax-subsidy scheme is available upon request.

8 Conclusion

We present a fully specified general equilibrium model of occupational choice where a fraction of individuals are optimists about ability. We find that optimism has six main effects. First, there is a misallocation of talent which lowers output. Second, optimists are more likely to become entrepreneurs than realists. Third, entrepreneurs are more optimistic than workers. Fourth, optimism drives up input prices. Fifth, optimism raises the number of entrepreneurs. Sixth, optimism makes workers better off and entrepreneurs worse off.

We calibrate the model to match salient features of U.S. manufacturing data. We find that optimism may significantly change the distribution of income by driving up the wage and lowering the return to entrepreneurship. The calibration also shows that even though optimism has a large impact on the return to entrepreneurship it has only a moderate impact on output and input prices.

9 Appendix

Proof of Proposition 1: Let $\lambda - \lambda/\gamma < (1 - \alpha - \beta)/(1 - \beta)$. The first step to determine the competitive equilibrium is to find out the labor market equilibrium
condition. The labor demand from realistic entrepreneurs is

\[ L^D_R = N(1 - \lambda) \int_{\hat{\theta}_R}^{1} l(\theta, w, r)d\theta \]

\[ = N(1 - \lambda) \left( \frac{\alpha}{w} \right)^{\frac{1-\eta}{1-\eta}} \left( \frac{\beta}{r} \right)^{\frac{\beta}{1-\eta}} \int_{\hat{\theta}_R}^{1} \theta^{\frac{1}{1-\eta}} d\theta \]

\[ = N(1 - \lambda) \frac{1-\eta}{2-\eta} \left( 1 - \frac{2-\eta}{r} \right) \left( \frac{\alpha}{w} \right)^{\frac{1-\eta}{1-\eta}} \left( \frac{\beta}{r} \right)^{\frac{\beta}{1-\eta}}. \quad (32) \]

Note that for \( L^D_R \) to be well defined it must be that \( \hat{\theta}_R < 1 \). Recall that \( \hat{\theta}_O \) is the ability threshold that determines the marginal optimistic entrepreneur. If \( \hat{\theta}_O < 1/\gamma \), then labor demand from optimistic entrepreneurs is the sum of the demand for labor coming from the mass of entrepreneurs with heterogeneous optimistic expectations, i.e., those with \( \theta \in (\hat{\theta}_O, 1/\gamma) \), to the demand for labor coming from the mass of entrepreneurs with homogeneous optimistic expectations, i.e., those with \( \theta \in (1/\gamma, 1) \):

\[ L^O_D = N\lambda \left[ \int_{\hat{\theta}_O}^{1} \frac{1}{\gamma} l(\gamma \theta, w, r)d\theta + \int_{\frac{1}{\gamma}}^{1} l(1, w, r)d\theta \right] \]

\[ = N\lambda \left[ \int_{\hat{\theta}_O}^{1} \frac{1}{\gamma} \theta^{\frac{1}{1-\eta}} d\theta + \int_{\frac{1}{\gamma}}^{1} d\theta \right] \left( \frac{\alpha}{w} \right)^{\frac{1-\eta}{1-\eta}} \left( \frac{\beta}{r} \right)^{\frac{\beta}{1-\eta}} \]

\[ = N\lambda \left( \gamma^{\frac{1}{1-\eta}} \int_{\hat{\theta}_O}^{1} \theta^{\frac{1}{1-\eta}} d\theta + 1 - \frac{1}{\gamma} \right) \left( \frac{\alpha}{w} \right)^{\frac{1-\eta}{1-\eta}} \left( \frac{\beta}{r} \right)^{\frac{\beta}{1-\eta}} \]

\[ = N\lambda \left( \gamma^{\frac{1}{1-\eta}} \frac{1-\eta}{2-\eta} \left[ \theta^{\frac{2-\eta}{1-\eta}} \hat{\theta}_O^{\frac{1}{1-\eta}} \right] + 1 - \frac{1}{\gamma} \right) \left( \frac{\alpha}{w} \right)^{\frac{1-\eta}{1-\eta}} \left( \frac{\beta}{r} \right)^{\frac{\beta}{1-\eta}} \]

\[ = N\lambda \left[ \gamma^{\frac{1}{1-\eta}} \frac{1-\eta}{2-\eta} \left( \gamma^{\frac{2-\eta}{1-\eta}} - \hat{\theta}_O^{\frac{2-\eta}{1-\eta}} \right) + 1 - \frac{1}{\gamma} \right] \left( \frac{\alpha}{w} \right)^{\frac{1-\eta}{1-\eta}} \left( \frac{\beta}{r} \right)^{\frac{\beta}{1-\eta}} \]

\[ = N\lambda \frac{1-\eta}{2-\eta} \left( \frac{2 - \eta - 1/\gamma}{1 - \eta} - \gamma^{\frac{2-\eta}{1-\eta}} \hat{\theta}_O^{\frac{2-\eta}{1-\eta}} \right) \left( \frac{\alpha}{w} \right)^{\frac{1-\eta}{1-\eta}} \left( \frac{\beta}{r} \right)^{\frac{\beta}{1-\eta}}. \quad (33) \]
Note that for $L_D^O$ to be well defined it must be that $\hat{\theta}_O < 1/\gamma$. From (32) and (33), labor demand is equal to

\[
L^D = L_R^D + L_O^D
\]

\[
= N(1 - \lambda) \frac{1 - \eta}{2 - \eta} \left( 1 - \hat{\theta}_R^{2-\eta} \right) \left( \frac{\alpha}{w} \right)^{\frac{1-\gamma}{1-\eta}} \left( \frac{\beta}{r} \right)^{\frac{\gamma}{1-\eta}}
+ N\lambda \frac{1 - \eta}{2 - \eta} \left( \frac{2 - \eta - 1/\gamma}{1 - \eta} - \gamma \frac{1}{1-\eta} \hat{\theta}_O^{2-\eta} \right) \left( \frac{\alpha}{w} \right)^{\frac{1-\eta}{1-\eta}} \left( \frac{\beta}{r} \right)^{\frac{\gamma}{1-\eta}}
= N \frac{1 - \eta}{2 - \eta} \left[ (1 - \lambda) \left( 1 - \hat{\theta}_R^{2-\eta} \right) + \lambda \left( \frac{2 - \eta - 1/\gamma}{1 - \eta} - \gamma \frac{1}{1-\eta} \hat{\theta}_O^{2-\eta} \right) \right] \left( \frac{\alpha}{w} \right)^{\frac{1-\eta}{1-\eta}} \left( \frac{\beta}{r} \right)^{\frac{\gamma}{1-\eta}}
\]

Since each worker provides a unit of labor, labor supply is

\[
L^S = N \left[ (1 - \lambda)L_R^S + \lambda L_O^S \right]
= N \left[ (1 - \lambda) \int_0^{\hat{\theta}_R} d\theta_0 + \lambda \int_0^{\hat{\theta}_O} d\theta_0 \right]
= N \left[ (1 - \lambda) \hat{\theta}_R + \lambda \hat{\theta}_O \right].
\] (34)

In equilibrium, labor demand must equal labor supply:

\[
\frac{1 - \eta}{2 - \eta} \left[ (1 - \lambda) \left( 1 - \hat{\theta}_R^{2-\eta} \right) + \lambda \left( \frac{2 - \eta - 1/\gamma}{1 - \eta} - \gamma \frac{1}{1-\eta} \hat{\theta}_O^{2-\eta} \right) \right] \left( \frac{\alpha}{w} \right)^{\frac{1-\eta}{1-\eta}} \left( \frac{\beta}{r} \right)^{\frac{\gamma}{1-\eta}}
= (1 - \lambda) \hat{\theta}_R + \lambda \hat{\theta}_O.
\] (35)

The second step to determine the competitive equilibrium is to find out the capital market equilibrium condition. The capital demand from realistic entrepreneurs is

\[
K_R^D = N(1 - \lambda) \int_{\hat{\theta}_R}^{1} k(\theta, w, r)d\theta
= N(1 - \lambda) \left( \frac{\alpha}{w} \right)^{\frac{\alpha}{\gamma}} \left( \frac{\beta}{r} \right)^{\frac{1-\eta}{1-\eta}} \int_{\hat{\theta}_R}^{1} \theta^{\frac{\gamma}{1-\eta}} d\theta
= N(1 - \lambda) \frac{1 - \eta}{2 - \eta} \left( 1 - \hat{\theta}_R^{2-\eta} \right) \left( \frac{\alpha}{w} \right)^{\frac{\alpha}{\gamma}} \left( \frac{\beta}{r} \right)^{\frac{1-\eta}{1-\eta}}.
\] (36)
Note that for $K^D_R$ to be well defined it must be that $\hat{\theta}_R < 1$. Recall that $\hat{\theta}_O$ is the ability threshold that determines the marginal optimistic entrepreneur. If $\hat{\theta}_O < 1/\gamma$, then capital demand from optimistic entrepreneurs is the sum of the demand for capital coming from the mass of entrepreneurs with heterogeneous optimistic expectations, i.e., those with $\theta \in (\hat{\theta}_O, 1/\gamma)$, to the demand for capital coming from the mass of entrepreneurs with homogeneous optimistic expectations, i.e., those with $\theta \in (1/\gamma, 1)$:

$$K^D_O = N\lambda \left[ \int_{\hat{\theta}_O}^{1/\gamma} k(\gamma\theta, w, r)d\theta + \int_{1/\gamma}^{1} k(1, w, r)d\theta \right]$$

$$= N\lambda \left[ \int_{\hat{\theta}_O}^{1/\gamma} (\gamma\theta)^{\frac{1}{1-\eta}} d\theta + \int_{1/\gamma}^{1} d\theta \right] \left( \frac{\alpha}{w} \right)^{\frac{\alpha}{\eta}} \left( \frac{\beta}{r} \right)^{\frac{1-\alpha}{1-\eta}}$$

$$= N\lambda \frac{1-\eta}{2-\eta} \left( \frac{2-\eta - 1/\gamma}{1-\eta} - \gamma^{\frac{1}{1-\eta}} (\hat{\theta}_O^{\frac{2-\eta}{1-\eta}}) \right) \left( \frac{\alpha}{w} \right)^{\frac{\alpha}{\eta}} \left( \frac{\beta}{r} \right)^{\frac{1-\alpha}{1-\eta}}. \quad (37)$$

Note that for $K^D_O$ to be well defined it must be that $\hat{\theta}_O < 1/\gamma$. From (36) and (37), capital demand is equal to

$$K^D = K^D_R + K^D_O$$

$$= N(1 - \lambda) \frac{1-\eta}{2-\eta} \left[ 1 - \hat{\theta}_R^{\frac{2-\eta}{1-\eta}} \right] \left( \frac{\alpha}{w} \right)^{\frac{\alpha}{\eta}} \left( \frac{\beta}{r} \right)^{\frac{1-\alpha}{1-\eta}}$$

$$+ N\lambda \frac{1-\eta}{2-\eta} \left( \frac{2-\eta - 1/\gamma}{1-\eta} - \gamma^{\frac{1}{1-\eta}} (\hat{\theta}_O^{\frac{2-\eta}{1-\eta}}) \right) \left( \frac{\alpha}{w} \right)^{\frac{\alpha}{\eta}} \left( \frac{\beta}{r} \right)^{\frac{1-\alpha}{1-\eta}}$$

$$= N\frac{1-\eta}{2-\eta} \left[ (1 - \lambda) \left( 1 - \hat{\theta}_R^{\frac{2-\eta}{1-\eta}} \right) + \lambda \left( \frac{2-\eta - 1/\gamma}{1-\eta} - \gamma^{\frac{1}{1-\eta}} (\hat{\theta}_O^{\frac{2-\eta}{1-\eta}}) \right) \right] \left( \frac{\alpha}{w} \right)^{\frac{\alpha}{\eta}} \left( \frac{\beta}{r} \right)^{\frac{1-\alpha}{1-\eta}}$$

In equilibrium, capital demand must equal the exogenous capital supply:

$$\frac{1-\eta}{2-\eta} \left[ (1 - \lambda) \left( 1 - \hat{\theta}_R^{\frac{2-\eta}{1-\eta}} \right) + \lambda \left( \frac{2-\eta - 1/\gamma}{1-\eta} - \gamma^{\frac{1}{1-\eta}} (\hat{\theta}_O^{\frac{2-\eta}{1-\eta}}) \right) \right] \left( \frac{\alpha}{w} \right)^{\frac{\alpha}{\eta}} \left( \frac{\beta}{r} \right)^{\frac{1-\alpha}{1-\eta}} = K/N. \quad (38)$$

The third step to determine the competitive equilibrium is to find out the ability level of the marginal realistic entrepreneur $\hat{\theta}_R$ and of the marginal optimistic entrepreneur
\( \hat{\theta}_O \). A realist with ability \( \hat{\theta}_R \) is indifferent between being an entrepreneur and a worker when

\[
\hat{\theta}_R \left[ l(\hat{\theta}_R, w, r) \right]^\alpha \left[ k(\hat{\theta}_R, w, r) \right]^\beta - w l(\hat{\theta}_R, w, r) + r \left[ K/N - k(\hat{\theta}_R, w, r) \right] = w + r K/N,
\]

or

\[
\hat{\theta}_R \left[ \hat{\theta}_R^\frac{1}{1-\eta} \left( \frac{\alpha}{w} \right)^{\frac{1-\beta}{1-\eta}} \left( \frac{\beta}{r} \right)^{\frac{\beta}{1-\eta}} \right]^\alpha \left[ \hat{\theta}_R^\frac{1}{1-\eta} \left( \frac{\alpha}{w} \right)^{\frac{\alpha}{1-\eta}} \left( \frac{\beta}{r} \right)^{\frac{\beta}{1-\eta}} \right]^\beta - w \hat{\theta}_R^\frac{1}{1-\eta} \left( \frac{\alpha}{w} \right)^{\frac{1-\beta}{1-\eta}} \left( \frac{\beta}{r} \right)^{\frac{\beta}{1-\eta}} - r \hat{\theta}_R^\frac{1}{1-\eta} \left( \frac{\alpha}{w} \right)^{\frac{\alpha}{1-\eta}} \left( \frac{\beta}{r} \right)^{\frac{\beta}{1-\eta}} = w,
\]

or

\[
\hat{\theta}_R^\frac{1}{1-\eta} \left( \frac{\alpha}{w} \right)^{\frac{\alpha}{1-\eta}} \left( \frac{\beta}{r} \right)^{\frac{\beta}{1-\eta}} - w \left( \frac{\alpha}{w} \right)^{\frac{1-\beta}{1-\eta}} \left( \frac{\beta}{r} \right)^{\frac{\beta}{1-\eta}} - r \left( \frac{\alpha}{w} \right)^{\frac{\alpha}{1-\eta}} \left( \frac{\beta}{r} \right)^{\frac{\beta}{1-\eta}} = w,
\]

or

\[
\hat{\theta}_R^\frac{1}{1-\eta} \left( \frac{\alpha}{w} \right)^{\frac{\alpha}{1-\eta}} \left( \frac{\beta}{r} \right)^{\frac{\beta}{1-\eta}} \left[ 1 - w \left( \frac{\alpha}{w} \right) - r \left( \frac{\beta}{r} \right) \right] = w,
\]

or

\[
\hat{\theta}_R^\frac{1}{1-\eta} \left( \frac{\alpha}{w} \right)^{\frac{\alpha}{1-\eta}} \left( \frac{\beta}{r} \right)^{\frac{\beta}{1-\eta}} (1 - \eta) = w,
\]

or

\[
\hat{\theta}_R^\frac{1}{1-\eta} \alpha^\frac{\alpha}{1-\eta} \beta^\frac{\beta}{1-\eta} (1 - \eta) = w^{\frac{1-\beta}{1-\eta} r^{\frac{\beta}{1-\eta}}},
\]

or

\[
\alpha^\alpha \beta^\beta (1 - \eta)^{1-\eta} \hat{\theta}_R = w^{1-\beta} r^{\beta}.
\]

An optimist with perception of ability \( \theta^* = \gamma \hat{\theta}_O \) and ability \( \hat{\theta}_O \) is indifferent between being an entrepreneur and a worker when

\[
\gamma \hat{\theta}_O \left[ l(\gamma \hat{\theta}_O, w, r) \right]^\alpha \left[ k(\gamma \hat{\theta}_O, w, r) \right]^\beta - w l(\gamma \hat{\theta}_O, w, r) + r \left[ K/N - k(\gamma \hat{\theta}_O, w, r) \right] = w + r K/N,
\]

or

\[
\gamma \hat{\theta}_O \left[ (\gamma \hat{\theta}_O)^\frac{1}{1-\eta} \left( \frac{\alpha}{w} \right)^{\frac{1-\beta}{1-\eta}} \left( \frac{\beta}{r} \right)^{\frac{\beta}{1-\eta}} \right]^\alpha \left[ (\gamma \hat{\theta}_O)^\frac{1}{1-\eta} \left( \frac{\alpha}{w} \right)^{\frac{\alpha}{1-\eta}} \left( \frac{\beta}{r} \right)^{\frac{\beta}{1-\eta}} \right]^\beta = w,
\]

An optimist with perception of ability \( \theta^* = \gamma \hat{\theta}_O \) and ability \( \hat{\theta}_O \) is indifferent between being an entrepreneur and a worker when

\[
\gamma \hat{\theta}_O \left[ l(\gamma \hat{\theta}_O, w, r) \right]^\alpha \left[ k(\gamma \hat{\theta}_O, w, r) \right]^\beta - w l(\gamma \hat{\theta}_O, w, r) + r \left[ K/N - k(\gamma \hat{\theta}_O, w, r) \right] = w + r K/N,
\]

or

\[
\gamma \hat{\theta}_O \left[ (\gamma \hat{\theta}_O)^\frac{1}{1-\eta} \left( \frac{\alpha}{w} \right)^{\frac{1-\beta}{1-\eta}} \left( \frac{\beta}{r} \right)^{\frac{\beta}{1-\eta}} \right]^\alpha \left[ (\gamma \hat{\theta}_O)^\frac{1}{1-\eta} \left( \frac{\alpha}{w} \right)^{\frac{\alpha}{1-\eta}} \left( \frac{\beta}{r} \right)^{\frac{\beta}{1-\eta}} \right]^\beta = w,
\]
or

\[(\gamma \hat{\theta}_O)^{\frac{1}{1-\eta}} \left( \frac{\alpha}{w} \right)^{\frac{\alpha}{1-\eta}} \left( \frac{\beta}{r} \right)^{\frac{\beta}{1-\eta}} = w, \]

or

\[\alpha^\alpha \beta^\beta (1 - \eta)^{1-\eta} \gamma \hat{\theta}_O = w^{1-\beta} r^\beta. \quad (40)\]

It follows from (39) and (40) that

\[\alpha^\alpha \beta^\beta (1 - \alpha - \beta)^{1-\alpha-\beta} \gamma \hat{\theta}_O = \alpha^\alpha \beta^\beta (1 - \alpha - \beta)^{1-\alpha-\beta} \hat{\theta}_R, \]

or

\[\hat{\theta}_O = \frac{1}{\gamma} \hat{\theta}_R. \quad (41)\]

Substituting (39) and (41) into (35) we obtain

\[
(1 - \lambda) \left( 1 - \hat{\theta}_R^{\frac{2-\eta}{1-\eta}} \right) + \lambda \left[ \frac{2 - \eta}{1 - \eta} \left( 1 - \frac{1}{\gamma} \right) + \frac{1}{\gamma} - \gamma \frac{1}{1-\eta} \gamma^{-\frac{2-\eta}{1-\eta}} \hat{\theta}_R^{\frac{2-\eta}{1-\eta}} \right] = \frac{2 - \eta}{\alpha} \hat{\theta}_R^{\frac{1}{1-\eta}} \left[ (1 - \lambda) \hat{\theta}_R + \lambda \frac{1}{\gamma} \hat{\theta}_R \right],
\]

or

\[
\left( 1 - \lambda + \frac{\lambda}{\gamma} \right) \left( 1 - \hat{\theta}_R^{\frac{2-\eta}{1-\eta}} \right) + \lambda \frac{2 - \eta}{1 - \eta} \left( 1 - \frac{1}{\gamma} \right) = \frac{2 - \eta}{\alpha} \left( 1 - \lambda + \frac{\lambda}{\gamma} \right) \hat{\theta}_R^{\frac{2-\eta}{1-\eta}},
\]

or

\[
\lambda \frac{2 - \eta}{1 - \eta} \left( 1 - \frac{1}{\gamma} \right) = \left( 1 - \lambda + \frac{\lambda}{\gamma} \right) \left( \frac{2 - \eta}{\alpha} \hat{\theta}_R^{\frac{2-\eta}{1-\eta}} - 1 + \hat{\theta}_R^{\frac{2-\eta}{1-\eta}} \right),
\]

or

\[
\lambda \frac{2 - \eta}{1 - \eta} \left( 1 - \frac{1}{\gamma} \right) + \left( 1 - \lambda + \frac{\lambda}{\gamma} \right) = \left( 1 - \lambda + \frac{\lambda}{\gamma} \right) \frac{2 - \beta}{\alpha} \hat{\theta}_R^{\frac{2-\eta}{1-\eta}}
\]

or

\[
\hat{\theta}_R^{\frac{2-\eta}{1-\eta}} = \frac{\lambda \frac{2 - \eta}{1 - \eta} \left( 1 - \frac{1}{\gamma} \right) + \left( 1 - \lambda + \frac{\lambda}{\gamma} \right)}{\left( 1 - \lambda + \frac{\lambda}{\gamma} \right)^{\frac{2-\beta}{\alpha}}} = \frac{\alpha}{2 - \beta} \left( 1 + \frac{2 - \eta}{1 - \eta} \frac{\lambda - \frac{1}{\gamma}}{1 - \lambda + \frac{1}{\gamma}} \right)
\]

35
Hence, the ability of the marginal realistic entrepreneur is

\[
\hat{\theta}_R = \left[ \frac{\alpha}{2 - \beta} \frac{1 - \eta + \lambda - \frac{\lambda}{\gamma}}{(1 - \eta) \left(1 - \lambda + \frac{\lambda}{\gamma}\right)} \right]^{\frac{1 - \eta}{3 - \eta}}.
\] (42)

From (41) and (42) the ability of the marginal optimistic entrepreneur is

\[
\hat{\theta}_O = \frac{1}{\gamma} \left[ \frac{\alpha}{2 - \beta} \frac{1 - \eta + \lambda - \frac{\lambda}{\gamma}}{(1 - \eta) \left(1 - \lambda + \frac{\lambda}{\gamma}\right)} \right]^{\frac{1 - \eta}{3 - \eta}}.
\]

From (35) and (38) we have

\[
\left[ (1 - \lambda)\hat{\theta}_R + \lambda\hat{\theta}_O \right] \left( \frac{w}{\alpha} \right)^{\frac{1 - \eta}{3 - \eta}} \left( \frac{r}{\beta} \right)^{\frac{1 - \eta}{3 - \eta}} = (K/N) \left( \frac{w}{\alpha} \right)^{\frac{1 - \eta}{3 - \eta}} \left( \frac{r}{\beta} \right)^{\frac{1 - \eta}{3 - \eta}},
\]

or

\[
\alpha r K/N = \beta w \left[ (1 - \lambda)\hat{\theta}_R + \lambda\hat{\theta}_O \right],
\]

or

\[
r = \frac{\beta w}{\alpha K/N} \left(1 - \lambda + \frac{\lambda}{\gamma}\right) \hat{\theta}_R,
\] (43)

Substituting (43) into (39) we obtain

\[
\alpha^3 \beta^3 (1 - \eta)^{1 - \eta}\hat{\theta}_R = w^{1 - \beta} \left( \frac{\beta}{\alpha} \right)^{\beta} w^{\beta} \left(1 - \lambda + \frac{\lambda}{\gamma}\right)^{\beta} \hat{\theta}_R (K/N)^{-\beta}.
\]

Solving this equality with respect to \( w \) we obtain the equilibrium wage:

\[
w^* = \frac{\alpha^3 (1 - \eta)^{1 - \eta} (K/N)^{\beta} \hat{\theta}_R^{1 - \beta}}{\left(1 - \lambda + \frac{\lambda}{\gamma}\right)^{\beta}},
\]

\[
= \frac{\alpha^3 (1 - \eta)^{1 - \eta} (K/N)^{\beta}}{\left(1 - \lambda + \frac{\lambda}{\gamma}\right)^{\beta}} \left[ \frac{\alpha}{2 - \beta} \frac{1 - \eta + \lambda - \frac{\lambda}{\gamma}}{(1 - \eta) \left(1 - \lambda + \frac{\lambda}{\gamma}\right)} \right]^{\frac{(1 - \eta)(1 - \beta)}{3 - \eta}}.
\]
The equilibrium rental cost of capital is equal to

\[ r^* = \frac{\beta w^*}{\alpha K/N} \left( 1 - \lambda + \frac{\lambda}{\gamma} \right) \hat{\theta}_R \]

\[ = \frac{\beta \alpha^\eta (1 - \eta)^{1-\eta}(K/N)^{\beta^2 - \beta}}{\alpha(K/N) \left( 1 - \lambda + \frac{\lambda}{\gamma} \right)^{\beta}} \left( 1 - \lambda + \frac{\lambda}{\gamma} \right) \hat{\theta}_R \]

\[ = \frac{\beta (1 - \eta)^{1-\eta}}{\alpha^{1-\eta} (K/N)^{1-\beta}} \left( 1 - \lambda + \frac{\lambda}{\gamma} \right)^{1-\beta} \left[ \frac{\alpha}{2 - \beta} \frac{1 - \eta + \lambda - \frac{\lambda}{\gamma}}{(1 - \eta)(1 - \lambda + \frac{\lambda}{\gamma})} \right]^{\frac{(1 - \eta)(2 - \beta)}{2 - \eta}} \]

\[ = \frac{\beta \left( \frac{\alpha}{2 - \beta} \right)^{\frac{(1 - \eta)(2 - \beta)}{2 - \eta}} (1 - \lambda + \frac{\lambda}{\gamma})^{\frac{\alpha}{2 - \beta}} \left( 1 - \eta + \lambda - \frac{\lambda}{\gamma} \right)^{\frac{(1 - \eta)(2 - \beta)}{2 - \eta}}}{\alpha^{1-\eta} (K/N)^{1-\beta}(1 - \eta)^{\frac{1-\eta}{2 - \eta} \alpha}} \]

The equilibrium labor force is equal to

\[ L^* = N \left[ (1 - \lambda) \hat{\theta}_R + \lambda \hat{\theta}_O \right] = N \left( 1 - \lambda + \frac{\lambda}{\gamma} \right) \hat{\theta}_R \]

\[ = N \left( 1 - \lambda + \frac{\lambda}{\gamma} \right) \left[ \frac{\alpha}{2 - \beta} \frac{1 - \eta + \lambda - \frac{\lambda}{\gamma}}{(1 - \eta)(1 - \lambda + \frac{\lambda}{\gamma})} \right]^{\frac{1-\eta}{2 - \eta}} \]

The equilibrium output level is

\[ Y^* = (1 - \lambda)N \int_{\hat{\theta}_R}^{1} \theta [l(\theta, w^*, r^*)]^\alpha [k(\theta, w^*, r^*)]^\beta d\theta \]

\[ + \lambda N \int_{\hat{\theta}_O}^{1} \theta [l(\gamma \theta, w^*, r^*)]^\alpha [k(\gamma \theta, w^*, r^*)]^\beta d\theta \]

\[ + \lambda N \int_{\hat{\theta}_O}^{1} \theta [l(1, w^*, r^*)]^\alpha [k(1, w^*, r^*)]^\beta d\theta. \]
This can be simplified to

\[
Y^* = R \left( \frac{\alpha}{\theta^*} \right)^{\frac{2}{1-\eta}} \left( \frac{\beta}{\theta^*} \right)^{\frac{\beta}{1-\eta}} \left[ (1 - \lambda) \int_{\theta_R}^{1} \theta^{1-\eta} d\theta + \lambda \gamma^{\frac{1}{1-\eta}} \int_{\theta_O}^{1} \theta^{1-\eta} d\theta + \lambda \int_{\frac{1}{\gamma}}^{1} \theta d\theta \right]
\]

\[
= R \left( \frac{\alpha}{\theta^*} \right)^{\frac{2}{1-\eta}} \left( \frac{\beta}{\theta^*} \right)^{\frac{\beta}{1-\eta}} \left\{ \frac{1 - \eta}{2 - \eta} \left[ (1 - \lambda) \left[ \theta^{\frac{2-\eta}{1-\eta}} \right]_{\theta_R}^{1} + \lambda \gamma^{\frac{1}{1-\eta}} \left[ \theta^{\frac{2-\eta}{1-\eta}} \right]_{\theta_O}^{1/\gamma} \right] + \frac{\lambda}{2} \left[ \theta^2 \right]_{1/\gamma} \right\}
\]

\[
= R \left( \frac{\alpha}{\theta^*} \right)^{\frac{2}{1-\eta}} \left( \frac{\beta}{\theta^*} \right)^{\frac{\beta}{1-\eta}} \left[ \frac{1 - \eta}{2 - \eta} \left( 1 - \lambda + \frac{\lambda}{\gamma^2} \right) \left( 1 - \frac{\lambda + \frac{\lambda}{\gamma}}{2} \right) \right]
\]

For the equilibrium to be well defined we need to make sure that \( \hat{\theta}_R \) is less than 1, i.e.,

\[
\left[ \frac{\alpha}{2 - \beta} \left( 1 + \frac{2 - \eta}{1 - \eta} \frac{\lambda - \frac{\lambda}{\gamma}}{1 - \lambda + \frac{\lambda}{\gamma}} \right) \right]^{\frac{2}{1-\eta}} < 1
\]

or

\[
\frac{2 - \eta}{1 - \eta} \frac{\lambda - \frac{\lambda}{\gamma}}{1 - \lambda + \frac{\lambda}{\gamma}} < \frac{2 - \beta}{\alpha} - 1
\]

or

\[
\frac{\lambda - \frac{\lambda}{\gamma}}{1 - \lambda + \frac{\lambda}{\gamma}} < \frac{1 - \eta}{\alpha}
\]

or

\[
\alpha \left( \lambda - \frac{\lambda}{\gamma} \right) < (1 - \alpha - \beta) \left( 1 - \lambda + \frac{\lambda}{\gamma} \right)
\]

or

\[
\alpha \left( \lambda - \frac{\lambda}{\gamma} \right) + \alpha \left( 1 - \lambda + \frac{\lambda}{\gamma} \right) < (1 - \beta) \left( 1 - \lambda + \frac{\lambda}{\gamma} \right)
\]

or

\[
\lambda - \frac{\lambda}{\gamma} < \frac{1 - \alpha - \beta}{1 - \beta},
\]

which is true by assumption. \( Q.E.D. \)
Proof of Proposition 2: Let \( \lambda - \lambda/\gamma < (1 - \alpha - \beta)/(1 - \beta) \). It follows directly from (19) and (20) that
\[
\frac{\partial \hat{\theta}_R}{\partial \lambda} > 0 \text{ and } \frac{\partial \hat{\theta}_O}{\partial \lambda} > 0.
\]
(i) It follows directly from (21) that
\[
\frac{\partial w^*}{\partial \lambda} = 0.
\]
(ii) The rental cost of capital is given by (22). To show that \( \partial r^*/\partial \lambda > 0 \) we need to show:
\[
\frac{\partial}{\partial \lambda} \left[ \left( 1 - \lambda + \frac{\lambda}{\gamma} \right) \frac{\alpha}{2 - \eta} \left( 1 - \eta + \lambda - \frac{\lambda}{\gamma} \right)^{\frac{(1-\eta)(2-\beta)}{2-\eta}} \right] > 0.
\]
We have that
\[
\frac{\alpha}{2 - \eta} \left( 1 - \lambda + \frac{\lambda}{\gamma} \right)^{\frac{\alpha}{2 - \eta} - 1} \left( 1 - \eta + \lambda - \frac{\lambda}{\gamma} \right)^{\frac{(1-\eta)(2-\beta)}{2-\eta}} \left( 1 - \eta + \lambda - \frac{\lambda}{\gamma} \right)^{-1} \left( 1 - \frac{1}{\gamma} \right) > 0
\]
or
\[
(1 - \eta)(2 - \beta) \left( 1 - \lambda + \frac{\lambda}{\gamma} \right) > \alpha \left( 1 - \eta + \lambda - \frac{\lambda}{\gamma} \right).
\]
Since \( \lambda - \lambda/\gamma < (1 - \alpha - \beta)/(1 - \beta) \) is equivalent to \( \alpha < (1 - \beta) (1 - \lambda + \lambda/\gamma) \) the above inequality is satisfied if
\[
(1 - \eta)(2 - \beta) \left( 1 - \lambda + \frac{\lambda}{\gamma} \right) > (1 - \beta) \left( 1 - \lambda + \frac{\lambda}{\gamma} \right) \left( 1 - \eta + \lambda - \frac{\lambda}{\gamma} \right),
\]
or
\[
(1 - \eta)(2 - \beta) > (1 - \beta) \left( 1 - \eta + \lambda - \frac{\lambda}{\gamma} \right),
\]
or
\[
(1 - \alpha - \beta) > (1 - \beta) \left( \lambda - \frac{\lambda}{\gamma} \right),
\]
or
\[
\lambda - \frac{\lambda}{\gamma} < \frac{1 - \beta}{1 - \alpha - \beta};
\]
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which is true.

(iii) The number of workers is given by (23). To show that $\partial L^*/\partial \lambda < 0$ we need to show:

$$\frac{\partial}{\partial \lambda} \left[ \left(1 - \lambda + \frac{\lambda}{\gamma} \right)^{\frac{1}{2-\eta}} \left(1 - \eta + \lambda - \frac{\lambda}{\gamma} \right)^{\frac{1-\eta}{2-\eta}} \right] < 0.$$ 

We have that

$$\frac{1}{2-\eta} \left(1 - \lambda + \frac{\lambda}{\gamma} \right)^{\frac{1}{2-\eta} - 1} \left( -1 + \frac{1}{\gamma} \right) \left(1 - \eta + \lambda - \frac{\lambda}{\gamma} \right)^{\frac{1-\eta}{2-\eta} - 1} \left(1 - \frac{1}{\gamma} \right) < 0$$

or

$$(1 - \eta) \left(1 - \lambda + \frac{\lambda}{\gamma} \right) < \left(1 - \eta + \lambda - \frac{\lambda}{\gamma} \right),$$

which is true. \(Q.E.D.\)

10 References


Manove, M. 2000. Entrepreneurs, Optimism and the Competitive Edge, Boston University and CEMFI.

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