A General Equilibrium Theory of Firm Formation under Optimal Expectations
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Abstract

We extend Lucas’ (1978) general equilibrium model of occupational choice by assuming that a fraction of individuals in the economy derive anticipatory utility from entrepreneurship and are able to bias their beliefs to inflate these anticipatory benefits. We show that these individuals endogenously choose to be optimists about their entrepreneurial ability. We show that optimism has five main effects. First, it raises the equilibrium wage and the rental cost of capital. Second, optimists crowd out realists from entrepreneurship. Third, there is a misallocation of talent: the lowest ability entrepreneurs are less talented at running a firm than the highest ability workers. Fourth, when the fraction of optimists is moderate, the majority of entrepreneurs are optimists and the majority of workers are realists. Fifth, an increase in the fraction of optimists can lower the number of entrepreneurs.

JEL Codes: D50; H21; J24; L26.
Keywords: General Equilibrium; Occupational Choice; Entrepreneurship; Optimism.

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1 Introduction

Four stylized facts stand out in the literature on entrepreneurship. First, the returns to entrepreneurship are highly variable (more than wages, more than returns on public equity), but do not offer higher compensation (same average returns as public equity; same average earnings as average wages, but lower median).

Second, entrepreneurs are overconfident about their skills and optimistic about the chances that their firms will succeed—see Cooper et al. (1988), Wu and Knott (2006), Landier and Thesmar (2009), Cassar (2010, 2012), and Hyytinen et al. (2014). Third, optimistic individuals are more likely to become entrepreneurs—see Gentry and Hubbard (2000), Hurst and Lusardi (2004), and Cassar and Friedman (2009). Fourth, self-employed people are more optimistic than regular wage earners—see Arabsheibani et al. (2000), Fraser and Greene (2006), and Puri and Robinson (2007).

This paper presents a fully specified general equilibrium model of occupational choice that can explain these stylized facts. The model also generates new and testable predictions of the impact of optimism on the labor market, capital market, and firm formation.

Following Lucas (1978) we model a closed economy with a given workforce which is homogeneous with respect to productivity as an employee. Each member of the workforce is endowed with an entrepreneurial ability which varies across individuals. Individuals are risk neutral and maximize their expected income by choosing occupations. A firm in this economy is one entrepreneur together with the labor and capital under his control. The technology of the firm is as follows. Output is an increasing function of entrepreneurial ability, labor, and capital. Entrepreneurial

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1The empirical evidence shows that entrepreneurs are not deterred by the evidence of unfavorable returns to entrepreneurship. Dunne et al. (1988) show that most businesses fail within a few years. Hamilton (2000) finds that after 10 years in business, median entrepreneurial earnings are 35% less than those on a paid job of the same duration. Moskovitz and Vissing-Jorgensen (2002) find that the returns from entrepreneurship are, on average, not different from the return on a diversified publicly traded portfolio (private equity puzzle).
ability is complementary to labor and capital in production. There are decreasing returns to scale to the use of labor and capital.

We depart from Lucas (1978) by assuming that a fraction $\lambda \in (0, 1]$ of individuals in the economy derive anticipatory utility from entrepreneurship and are able to bias their beliefs to inflate these anticipatory benefits. This assumption captures the idea that the anticipation of future profits—and of how enjoyable these will be—plays a major role in the decision to become an entrepreneur. The remaining fraction $1 - \lambda$ of individuals in the economy has standard preferences (i.e., do not derive anticipatory utility from entrepreneurship) and hence has no reason to distort beliefs. There is a continuum of both types of individuals ranked by their entrepreneurial ability $\theta_0$ which is distributed on $[0, 1]$.

Following Brunnermeier and Parker (2005), we assume that individuals with anticipatory utility choose their expectations of ability so as to maximize the sum of the anticipatory and the material payoffs of entrepreneurship. This choice involves a trade-off between optimism, which raises anticipatory utility, and realism, which raises material payoff by promoting efficient input choices. Optimal beliefs balance the anticipatory benefits of optimism with its efficiency cost. From now on, we refer to individuals who do not distort beliefs as realists and to those who do as individuals with optimal expectations.\(^2\)

The timing of the model is as follows. At $t = 0$ individuals with optimal expectations choose their beliefs of entrepreneurial ability for all future periods. At $t = 1$ individuals choose, given their ability expectations, between entrepreneurship and wage-earning. At $t = 1$ an individual with optimal expectations becomes an entrepreneur if the sum of the anticipatory and materials payoffs of entrepreneurship is greater than the wage. At $t = 1$ a realist becomes an entrepreneur if the material payoff of entrepreneurship is greater than the wage. At $t = 2$ entrepreneurs choose,

\(^2\)The assumption that individuals may distort their beliefs is not new in the economics literature. Our model of beliefs follows the optimal expectations framework of Brunnermeier and Parker (2005). Other prominent models of distorted beliefs due to anticipatory utility are Brunnermeier et al. (2007) and Bénabou (2013).
given their ability expectations, how much labor and capital to hire to maximize the material payoffs of entrepreneurship. At \( t = 2 \) entrepreneurs with optimal expectations realize the anticipatory utility from entrepreneurship. At \( t = 3 \) entrepreneurs realize the material payoffs from running their firms.\(^3\)

We start by showing that individuals with optimal expectations endogenously choose to be optimists about their entrepreneurial ability. When the weight of anticipatory utility is not too high, being optimistic about entrepreneurial ability leads to first-order gains due to increased anticipatory utility from entrepreneurship and to second-order costs in realized profits due to distorted input choices. The optimistic bias in beliefs is increasing with the weight of anticipatory utility and with the degree of decreasing return to scale.

Next we show that a competitive optimal expectations equilibrium exists. In equilibrium: (i) there exists a cut-off ability level \( \hat{\theta}_R \) such that realists with ability less than \( \hat{\theta}_R \) become workers and those with ability greater than \( \hat{\theta}_R \) become entrepreneurs, (ii) there exists a cut-off ability level \( \hat{\theta}_O \) such that optimists with ability less than \( \hat{\theta}_O \) become workers and those with ability greater than \( \hat{\theta}_O \) become entrepreneurs, (iii) labor demand equals labor supply, and (iv) capital demand equals capital supply.

We show that entrepreneurial optimism raises the market clearing wage. The intuition behind this result is as follows. The assumption that entrepreneurial ability and labor are complements in production implies that, for any given ability level, an optimistic entrepreneur demands more labor than a realistic one. This leads to an expansion of labor demand. In addition, optimists, by comparison to realists, find entrepreneurship more attractive than paid work. This leads to a contraction of labor supply. The expansion of labor demand and contraction of labor supply lead

\(^3\)In our model, like in Lucas (1978), individuals have a choice between operating a firm or working for a wage. We focus on entrepreneurial ability and expectations of ability as the main determinants which explain who becomes an entrepreneur and who works as an employee. There are of course many other factors which should influence this choice. The most important ones would include differences in the degree of risk aversion, the disutility of exerting entrepreneurial effort, the taste for being an entrepreneur, and access to funds needed to create a firm.
to an increase in the wage.

We also show that entrepreneurial optimism raises the market clearing rental cost of capital. The assumption that entrepreneurial ability and capital are complements in production implies that, for any given ability level, an optimistic entrepreneur demands more capital than a realistic one. This leads to an expansion of capital demand. The expansion of capital demand leads to an increase in the rental cost of capital since the supply of capital is exogenous.

The higher market clearing wage and rental cost of capital make entrepreneurship less attractive to realists. Hence, optimists crowd out realists from entrepreneurship. When the fraction of optimists is moderate, the crowding out effect implies that the majority of entrepreneurs are optimists and the majority of workers are realists. The crowding out effect also leads to a misallocation of talent. The ablest people do not necessarily select into entrepreneurship: the lowest ability entrepreneurs are less talented at running a firm than the highest ability workers.

Entrepreneurial optimism raises the material payoff of workers since it raises the market clearing wage. It lowers the material payoff of realistic entrepreneurs since it raises input prices. It lowers the material payoff of optimistic entrepreneurs in two ways: distorting input choices and raising input prices. Finally, it lowers welfare since it introduces a distortion in the economy.

This paper contributes to the literature that studies occupational choice using general equilibrium models (Lucas, 1978, Kanbur, 1979, Kihlstrom and Laffont, 1979, Bewley, 1989, and Lazear, 2005). We generalize Lucas (1978) by assuming that a fraction of individuals in the economy derive anticipatory utility from entrepreneurship and are able to bias their beliefs of entrepreneurial skill. The paper also contributes to the literature that studies the impact of optimism on individual decisions and market outcomes (de Meza and Southey, 1996, Manove, 2000, Fraser and Greene, 2006, and Rigotti et al. 2011). Our approach differs from previous studies since optimism arises endogenously instead of being fixed and exogenous to the model.
The reminder of the paper proceeds as follows. Section 2 sets-up the model. Section 3 derives the optimal expectations. Section 4 characterizes the competitive optimal expectations equilibrium and contains the main findings of the paper. Section 5 provides comparative statics’ results. Section 6 discusses the paper’s contribution to the literature. Section 7 concludes the paper. All proofs can be found in the Appendix.

2 Set-up

The economy consists of a continuum of risk-neutral individuals of measure 1. They derive utility from consumption, and can earn income either as workers or by running their own firm. Individuals are ranked by their entrepreneurial ability, \( \theta_0 \), which is distributed on \([0, 1]\) according to the cumulative distribution function \( G(\theta_0) \). Each individual has one unit of labor. If an individual becomes a worker he supplies his unit of labor on the labor market and receives the competitive wage \( w \). Thus, we assume all individuals have the same productivity (or ability) as workers. If an individual becomes an entrepreneur he can use without cost a technology defined by the continuous production function

\[
y = f(l, k, \theta_0),
\]

where \( y \) is output, \( l \) is labor, and \( k \) is capital. Any individual can run at most one firm. We assume that \( f \) is twice continuously differentiable with \( f_l > 0, f_k > 0, f_{\theta_0} > 0, f_{ll} < 0, f_{kk} < 0, f_{\theta_0} > 0, f_{k\theta_0} > 0 \), and \( f(0, k, \theta_0) = f(l, 0, \theta_0) = 0 \). This production function combines as inputs one manager/owner, who is essential to operate the firm, with a labor input of \( l \) units and a capital input of \( k \) units. The stock of capital in the economy is fixed and equal to \( K \). Entrepreneurs rent capital in the capital market at the competitive rental cost of capital \( r \).

Production exhibits decreasing returns to scale in the variable inputs, labor and capital, so that optimal firm size is finite.\(^4\) The assumption that entrepreneurial

\(^4\)This could be due for instance to limits in entrepreneurs’ span of control (Lucas 1978): as
ability and labor are complements in production, i.e. $f_{l\theta_0} > 0$, is a critical one. This assumption implies that an optimistic entrepreneur will demand more labor than a realist who has the same ability. This, in turn, implies that optimism leads to an expansion of labor demand. If entrepreneurial ability and labor are substitutes in production, i.e. $f_{l\theta_0} < 0$, the opposite result would hold, i.e., optimism would lead to a contraction of labor demand. Similarly, the assumption that entrepreneurial ability and capital are complements in production, i.e. $f_{k\theta_0} > 0$, is also a critical one.

If an individual becomes an entrepreneur and employs $l$ workers he receives a material payoff equal to
\[
\pi = pf(l, k; \theta_0) - wl - rk.
\]

From now on the price of output $p$ is normalized to be 1. Individuals can belong to one of two types: those who have optimal expectations of entrepreneurial ability and those who have rational expectations. Fraction $\lambda \in (0, 1)$ of the population has optimal expectations and fraction $1 - \lambda$ has realistic expectations. The distributions of entrepreneurial abilities and types are independent.

At $t = 0$ an individual with optimal expectations observes $\theta_0$ and chooses his expectation of entrepreneurial ability $\theta$ so as to maximize the undiscounted sum of $f(l, k, \theta_0) - wl - rk$, his material payoff of being an entrepreneur at $t = 3$; and $s[f(l, k, \theta) - wl - rk]$, his anticipatory payoff of being an entrepreneur at $t = 2$. At $t = 1$ the individual, given his expectation of entrepreneurial ability $\theta$, decides whether to be an entrepreneur or a worker and receive the market wage $w$. The individual becomes an entrepreneur if the sum of the material and anticipatory payoffs of being an entrepreneur is higher than $w$. At $t = 2$ an entrepreneur chooses $l$ and $k$ to maximize his material payoff given his expectation of entrepreneurial ability $\theta$. At $t = 2$ the entrepreneur receives anticipatory utility from his expectation of material payoffs evaluated with belief $\theta$: $s[f(l, k, \theta) - wl - rk]$. At $t = 3$ an entrepreneur activity expands, it becomes more difficult to control, and the marginal product of the variable inputs diminishes.
realizes the material payoff $f(l, k, \theta_0) - wl - rk$.

According to this approach the total payoff at $t = 0$ of an individual with optimal expectations who selects to be an entrepreneur is

$$f(l, k, \theta_0) - wl - rk + s[f(l, k, \theta) - wl - rk],$$

where the parameter $s > 0$ measures the weight the individual places on anticipatory utility relative to material payoffs. Note that the material payoff component depends on the individual’s actual entrepreneurial ability $\theta_0$, while the anticipatory utility depends on the individual’s expectation of entrepreneurial ability $\theta$.

An individual who becomes an entrepreneur will choose to employ $l(w, r, \theta)$ workers and $k(w, r, \theta)$ units of capital at $t = 2$ where $l(w, r, \theta)$ and $k(w, r, \theta)$ are the values of $l$ and $k$ that solve the following problem

$$\max_{l, k} [f(l, k, \theta) - wl - rk].$$

The first-order conditions to this problem are

$$f_l(l, k, \theta) = w. \tag{1}$$

and

$$f_k(l, k, \theta) = r. \tag{2}$$

It follows from (1), the assumption of decreasing returns to labor, $f_{ll} < 0$, and complementarity between entrepreneurial ability and labor, i.e., $f_{l\theta_0} > 0$, that entrepreneurs with a higher $\theta$ hire more workers: $\partial l(w, r, \theta)/\partial \theta = -f_{l\theta}/f_{ll} > 0$. Similarly, it follows from (2), the assumption of decreasing returns to capital, $f_{kk} < 0$, and complementarity between entrepreneurial ability and capital, i.e., $f_{k\theta_0} > 0$, that entrepreneurs with a higher $\theta$ hire more capital: $\partial k(w, r, \theta)/\partial \theta = -f_{k\theta}/f_{kk} > 0$. At $t = 1$ the optimal expectation of an individual with ability $\theta_0$ is the $\theta$ that solves the following problem

$$\max_{\theta \in [0,1]} \left\{ f(l(w, r, \theta), k(w, r, \theta), \theta_0) - wl(w, r, \theta) - rk(w, r, \theta) \\
+ s [f(l(w, r, \theta), k(w, r, \theta), \theta) - wl(w, r, \theta) - rk(w, r, \theta)] \right\}. \tag{8}$$
If the wage is \( w \), a realistic individual with entrepreneurial ability \( \theta_0 \) chooses to become a worker at wage \( w \) when

\[
f(l(w, r, \theta_0), k(w, r, \theta_0), \theta_0) - wl(w, r, \theta_0) - rk(w, r, \theta_0) \leq w. \tag{3}
\]

He selects to be an entrepreneur if

\[
f(l(w, r, \theta_0), k(w, r, \theta_0), \theta_0) - wl(w, r, \theta_0) - rk(w, r, \theta_0) \geq w, \tag{4}
\]

and he is indifferent if the equality holds in (3) and (4). If the wage is \( w \), an individual with optimal expectations of ability \( \theta^* \) and with entrepreneurial ability \( \theta_0 \) chooses to become a worker at wage \( w \) when

\[
\begin{align*}
f(l(w, r, \theta^*), k(w, r, \theta^*), \theta_0) - wl(w, r, \theta^*) - rk(w, r, \theta^*) \\
+s [f(l(w, r, \theta^*), k(w, r, \theta^*), \theta^*) - wl(w, r, \theta^*) - rk(w, r, \theta^*)] \leq w. \tag{5}
\end{align*}
\]

He selects to be an entrepreneur if

\[
\begin{align*}
f(l(w, r, \theta^*), k(w, r, \theta^*), \theta_0) - wl(w, r, \theta^*) - rk(w, r, \theta^*) \\
+s [f(l(w, r, \theta^*), k(w, r, \theta^*), \theta^*) - wl(w, r, \theta^*) - rk(w, r, \theta^*)] \geq w \tag{6}
\end{align*}
\]

and he is indifferent if the equality holds in (5) and (6). Since there are only three markets—output, labor, and capital—by Walras’ Law, general equilibrium is realized when the labor and capital markets clear. At the equilibrium wage, the labor demanded by individuals who choose to become entrepreneurs equals that supplied by individuals who choose to become workers. At the equilibrium rental cost of capital, the capital demanded by individuals who choose to become entrepreneurs equals the exogenous capital stock of the economy, \( K \). Formally, an equilibrium is (i) a partition \( \{[0, \hat{\theta}_R], [\hat{\theta}_R, 1]\} \) of \([0, 1]\) where for all \( \theta_0 \in [0, \hat{\theta}_R] \) (3) holds and for all \( \theta_0 \in [\hat{\theta}_R, 1] \) (4) holds, (ii) a partition \( \{[0, \hat{\theta}_O], [\hat{\theta}_O, 1]\} \) of \([0, 1]\) where for all \( \theta_0 \in [0, \hat{\theta}_O] \) (5) holds and for all \( \theta_0 \in [\hat{\theta}_O, 1] \) (6) holds, (iii) a wage \( w \) for which labor demand equals labor
supply
\[
(1 - \lambda) \int_{\hat{\theta}_R}^{1} l(w, r, \theta_0) dG(\theta_0) + \lambda \int_{\hat{\theta}_O}^{1} l(w, r, \theta^*) dG(\theta_0) = (1 - \lambda) \int_{0}^{\hat{\theta}_R} dG(\theta_0) + \lambda \int_{0}^{\hat{\theta}_O} dG(\theta_0),
\]
and (iv) a rental cost of capital \( r \) for which capital demand equals the exogenous capital supply
\[
(1 - \lambda) \int_{\hat{\theta}_R}^{1} k(w, r, \theta_0) dG(\theta_0) + \lambda \int_{\hat{\theta}_O}^{1} k(w, r, \theta^*) dG(\theta_0) = \bar{K}.
\]
In equilibrium, realists with ability below \( \hat{\theta}_R \) become workers whereas those with ability above \( \hat{\theta}_R \) become entrepreneurs. Similarly, individuals with optimal expectations and ability below \( \hat{\theta}_O \) become workers whereas those with ability above \( \hat{\theta}_O \) become entrepreneurs. We refer to a realist with ability \( \hat{\theta}_R \) as the marginal realistic entrepreneur. We refer to an individual with optimal expectations and ability \( \hat{\theta}_O \) as the marginal optimistic entrepreneur.

3 Optimal Expectations

In this section we derive the optimal expectations. We consider a specialized version of the model with a production function given by
\[
y = f(l, k, \theta_0) = \theta_0 g(l, k) = \theta_0 l^\alpha k^\beta,
\]
where \( \alpha + \beta = \eta < 1 \). Hence, the variable inputs, labor and capital, are combined under a Cobb-Douglas production function with decreasing returns to scale and entrepreneurial skill enters into the production function as the total factor productivity (TFP).

\footnote{This is a standard assumption in models with heterogeneous skill. See, for example, Lucas (1978), Murphy et al. (1991), de Meza and Southey (1996), and Poschke (2013).}
At $t = 3$ the material payoff of an entrepreneur is

$$\pi = \theta_0 l^\alpha k^\beta - wl - rk. \tag{9}$$

We see from (9) that this production function, the assumption that individuals are risk neutral, and the assumption that entrepreneurial skill $\theta_0$ belongs to $[0, 1]$, imply that entrepreneurial skill can be interpreted as the true probability of success of the firm (the project either succeeds with probability $\theta_0$ or fails with probability $1 - \theta_0$, in which case output is zero).

At $t = 2$ an individual with expectation of ability $\theta$ who becomes an entrepreneur chooses to employ $l$ workers and $k$ units of capital where $l$ and $k$ are the solution to

$$\max_{l,k} (\theta l^\alpha k^\beta - wl - rk).$$

The first-order conditions are

$$\alpha \theta l^{\alpha-1} k^\beta = w,$$

and

$$\beta \theta l^\alpha k^{\beta-1} = r.$$

Solving for $l$ and $k$ we obtain the input demands of an entrepreneur with expectations of ability $\theta$:

$$l(w, r, \theta) = \theta^{\frac{1}{1-\alpha-\beta}} \left( \frac{\alpha}{w} \right)^{\frac{1-\beta}{\beta}} \left( \frac{\beta}{r} \right)^{\frac{\alpha}{1-\alpha-\beta}}, \tag{10}$$

and

$$k(w, r, \theta) = \theta^{\frac{1}{1-\alpha-\beta}} \left( \frac{\alpha}{w} \right)^{\frac{\alpha}{1-\alpha-\beta}} \left( \frac{\beta}{r} \right)^{\frac{\alpha}{1-\alpha-\beta}}, \tag{11}$$

respectively.

At $t = 0$ the total payoff of an entrepreneur with expectations of ability $\theta$ is:

$$\theta_0 l^\alpha k^\beta - wl - rk + s(\theta l^\alpha k^\beta - wl - rk) = (\theta_0 + s\theta) l^\alpha k^\beta - (1 + s)(wl + rk).$$

where $l$ is given by (10) and $k$ by (11). Hence, at $t = 0$, the optimal expectation of ability (or the optimal expectation of the probability of success of the firm) of an
entrepreneur with ability \( \theta_0 \) is the \( \theta \) that solves

\[
\max_{\theta \in [0,1]} \left\{ \left( \theta_0 + s\theta \right)[l(w,r,\theta)]^\alpha \left[k(w,r,\theta)]^\beta - (1+s)[wl(w,r,\theta) + rk(w,r,\theta)] \right\}. \quad (12)
\]

Our first result characterizes the solution to (12).

**Proposition 1:** If \( f(l,k,\theta_0) = \theta_0 l^\alpha k^\beta \) and the weight of anticipatory utility \( s \) is less than \( \eta/(1-\eta) \), then optimal expectations of entrepreneurial ability are given by

\[
\theta^* = \begin{cases} \frac{\eta}{\eta-s(1-\eta)} \theta_0 & \text{if } \theta_0 < \frac{\eta-s(1-\eta)}{\eta} \\ 1 & \text{if } \theta_0 \geq \frac{\eta-s(1-\eta)}{\eta} \end{cases} \quad \text{.} \quad (13)
\]

This results tells us if the weight of anticipatory utility \( s \) is less than \( \eta/(1 - \eta) \), then individuals with optimal expectations choose to be optimists about their entrepreneurial ability. We see from (13) that the expectations are optimistic in the sense that the belief of entrepreneurial ability, \( \theta^* \), is greater than the actual ability, \( \theta_0 \). The intuition behind this result is straightforward. Being optimist about entrepreneurial ability leads to first-order gains due to increased anticipatory utility from entrepreneurship and to second-order costs in realized profits due to distorted input choices.

We also see from (13) that the optimal expectations of ability \( \theta^* \) of individuals with ability \( \theta_0 \) less than \( 1 - s(1-\eta)/\eta \) are a function of entrepreneurial ability \( \theta_0 \), the weight of anticipatory utility \( s \), and the level of decreasing returns to scale \( \eta = \alpha + \beta \). Moreover, \( \theta^* \) is increasing with the weight of anticipatory utility \( s \): everything else constant, the higher \( s \) is, the more individuals are optimistic about their abilities (the gap between \( \theta^* \) and \( \theta_0 \) becomes larger). Finally, \( \theta^* \) is increasing with the level of decreasing returns to scale \( \eta \): everything else constant, the lower \( \eta \) is, the more individuals are optimistic about their abilities (the gap between \( \theta^* \) and \( \theta_0 \) becomes larger). We also see from (13) that the optimal expectations of \( \theta^* \) of individuals with ability \( \theta_0 \) higher than \( 1 - s(1-\eta)/\eta \) do not depend on \( \theta_0 \), \( s \) and \( \eta \) since they are equal to the highest possible ability level, i.e., \( \theta^* = 1 \).
4 Optimal Expectations Equilibrium

In this section we report the main findings of the paper. We describe the optimal expectations equilibrium when the weight of anticipatory utility is relatively small ($s$ is low). We show that the lowest ability entrepreneurs are less talented at running a firm than the highest ability workers. We compare the optimal expectations equilibrium to the rational expectations equilibrium. We also provide conditions under which the majority of entrepreneurs are optimists and the majority of workers are realists. Finally, we close the section by characterizing the optimal expectations equilibrium when the weight of anticipatory utility is relatively large ($s$ is high).

A realist with ability $\hat{\theta}_R$ is indifferent between being an entrepreneur and a worker when

$$\hat{\theta}_R[l(w, r, \hat{\theta}_R)]^{\alpha} [k(w, r, \hat{\theta}_R)]^{\beta} - wl(w, r, \hat{\theta}_R) - rk(w, r, \hat{\theta}_R) = w.$$

Simplifying this equation we obtain

$$\alpha^\alpha \beta^\beta (1 - \eta)^{1-\eta} \hat{\theta}_R = w^{1-\beta} r^\beta. \quad (14)$$

When the weight of anticipatory utility is relatively small (this will be made precise later on), an optimist with optimal expectation of ability $\theta^* = \eta\hat{\theta}_O/\eta - s(1 - \eta)$ and ability $\hat{\theta}_O$ is indifferent between being an entrepreneur and a worker when

$$(\hat{\theta}_O + s\theta^*)[l(w, r, \theta^*)]^{\alpha} [k(w, r, \theta^*)]^{\beta} - (1 + s) [wl(w, r, \theta^*) + rk(w, r, \theta^*)] = w.$$

Simplifying this equation we obtain

$$\alpha^\alpha \beta^\beta (1 - \eta)^{1-\eta} \left[ \frac{\eta}{\eta - s(1 - \eta)} \right]^{\eta} \hat{\theta}_O = w^{1-\beta} r^\beta. \quad (15)$$

In equilibrium, labor demand must equal labor supply. The assumption that entrepreneurial ability is uniformly distributed on $[0, 1]$ implies that (7) becomes:

$$(1 - \lambda) \int_{\theta_R}^{1} l(w, r, \theta_0) d\theta_0 + \lambda \int_{\theta_O}^{1} l[w, r, \theta^*(\theta_0)] d\theta_0 = (1 - \lambda) \hat{\theta}_R + \lambda \hat{\theta}_O.$$
After substituting for the labor demands of the two types of entrepreneurs and integrating over \(\theta_0\) we obtain

\[
\frac{1 - \eta}{2 - \eta} \left\{ 1 + \lambda \frac{s}{\eta} - (1 - \lambda) \hat{\theta}_R^{\frac{2 - \eta}{\eta}} - \lambda \left[ \frac{\eta}{\eta - s(1 - \eta)} \right] \hat{\theta}_O^{\frac{2 - \eta}{\eta}} \right\} \left( \frac{\alpha}{w} \right)^{\frac{1 - \beta}{1 - \eta}} \left( \frac{\beta}{r} \right)^{\frac{\beta}{1 - \eta}} = (1 - \lambda) \hat{\theta}_R + \lambda \hat{\theta}_O.
\]

(16)

In equilibrium, capital demand must equal the exogenous capital supply. The assumption that entrepreneurial ability is uniformly distributed on \([0, 1]\) implies that (8) becomes:

\[
(1 - \lambda) \int_{\hat{\theta}_R}^{1} k(w, r, \theta_0) d\theta_0 + \lambda \int_{\hat{\theta}_O}^{1} k[w, r, \theta^*(\theta_0)] d\theta_0 = \bar{K}.
\]

After substituting for the capital demands of the two types of entrepreneurs and integrating over \(\theta_0\) we obtain

\[
\frac{1 - \eta}{2 - \eta} \left\{ 1 + \lambda \frac{s}{\eta} - (1 - \lambda) \hat{\theta}_R^{\frac{2 - \eta}{\eta}} - \lambda \left[ \frac{\eta}{\eta - s(1 - \eta)} \right] \hat{\theta}_O^{\frac{2 - \eta}{\eta}} \right\} \left( \frac{\alpha}{w} \right)^{\frac{1 - \beta}{1 - \eta}} \left( \frac{\beta}{r} \right)^{\frac{\beta}{1 - \eta}} = \bar{K}
\]

(17)

Equations (14), (15), (16), and (17) form a system of four equations and four unknowns \((\hat{\theta}_R, \hat{\theta}_O, w, r)\) which defines the optimal expectations equilibrium when the weight of anticipatory utility is relatively small.\(^6\) Our second result describes the equilibrium.

**Proposition 2:** If \(f(l, k, \theta_0) = \theta_0 l^a k^\beta\), \(\theta_0\) is uniformly distributed on \([0, 1]\), and the weight of anticipatory utility \(s\) is less than \(\bar{s}\), then there exists a unique optimal expectations equilibrium where the marginal realistic entrepreneur has ability

\[
\hat{\theta}_R = \left[ \frac{\alpha}{2 - \beta} \frac{1 + \lambda \frac{s}{\eta}}{1 - \lambda + \lambda \phi(\eta, \beta, s)} \right]^{\frac{\frac{2 - \eta}{\eta}}{\frac{2 - \eta}{\eta}}},
\]

(18)

\(^{6}\)Note that from \(\hat{\theta}_R\) and \(\hat{\theta}_O\) we obtain the equilibrium number of workers \(L = (1 - \lambda) \hat{\theta}_R + \lambda \hat{\theta}_O\).
the marginal optimistic entrepreneur has ability
\[ \hat{\theta}_O = \psi(\eta, s) \left[ \frac{\alpha}{2 - \beta} \frac{1 + \lambda s}{1 - \lambda + \lambda \phi(\eta, \beta, s)} \right]^{\frac{1-\eta}{2-\eta}}, \]  
the wage is
\[ w^* = \frac{\alpha^\eta (1 - \eta)^{1-\eta} K^\beta}{[1 - \lambda + \lambda \psi(\eta, s)]^\beta} \left[ \frac{\alpha}{2 - \beta} \frac{1 + \lambda s}{1 - \lambda + \lambda \phi(\eta, \beta, s)} \right]^{\frac{(1-\eta)(1-\beta)}{2-\eta}}, \]  
the number of workers is
\[ L^* = [1 - \lambda + \lambda \psi(\eta, s)] \left[ \frac{\alpha}{2 - \beta} \frac{1 + \lambda s}{1 - \lambda + \lambda \phi(\eta, \beta, s)} \right]^{\frac{1-\eta}{2-\eta}}, \]  
and the rental cost of capital is
\[ r^* = \frac{\beta (1 - \eta)^{1-\eta}}{\alpha^{1-\eta} K^{1-\beta}} [1 - \lambda + \lambda \psi(\eta, s)]^{1-\beta} \left[ \frac{\alpha}{2 - \beta} \frac{1 + \lambda s}{1 - \lambda + \lambda \phi(\eta, \beta, s)} \right]^{\frac{(1-\eta)(2-\beta)}{2-\eta}}, \]  
where \( s \) is the solution to
\[ \frac{\alpha}{2 - \beta} \left( 1 + \lambda \frac{s}{\eta} \right) = [1 - \lambda + \lambda \phi(\eta, \beta, \bar{s})] \left[ \frac{\eta - s(1 - \eta)}{\eta} \right]^{2-\eta}, \]  
and
\[ \phi(\eta, \beta, s) = \left[ 1 - \frac{s(1 - \eta)(2 - \eta)}{\eta(2 - \beta)} \right] \left[ \frac{\eta}{\eta - s(1 - \eta)} \right]^{1-\eta}, \]  
and
\[ \psi(\eta, s) = \left[ \frac{\eta - s(1 - \eta)}{\eta} \right]^\eta. \]

When the weight of anticipatory utility is relatively small, i.e., \( s \) is less than \( \bar{s} \) given by (23), the equilibrium wage, number of workers, and rental cost of capital are given by (20), (21), and (22), respectively.

We now show that the existence of individuals with optimal expectations leads to a misallocation of talent. In the optimal expectations equilibrium, in contrast
to the rational expectations equilibrium, the ablest people do not necessarily select into entrepreneurship. In the rational expectations equilibrium (when $\lambda = 0$) the marginal entrepreneur has ability

$$\hat{\theta}_0 = \left(\frac{\alpha}{2 - \beta}\right)^{\frac{1-\beta}{\beta}},$$

which implies that individuals with ability $[0, \hat{\theta}_0]$ become workers and individuals with ability $[\hat{\theta}_0, 1]$ become entrepreneurs. Hence, in the rational expectations equilibrium the ablest people become entrepreneurs. In the optimal expectations equilibrium realists with ability $[0, \hat{\theta}_R]$ become workers and those with ability $[\hat{\theta}_R, 1]$ become entrepreneurs. Furthermore, optimists with ability $[0, \hat{\theta}_O]$ become workers and those with ability $[\hat{\theta}_O, 1]$ become entrepreneurs. It follows from (18) and (19) that the marginal optimistic entrepreneur has a lower ability than the marginal realistic entrepreneur:

$$\hat{\theta}_O = \psi(\eta, s)\hat{\theta}_R < \hat{\theta}_R.$$  

Hence, amongst individuals with ability $\theta_0 \in [\hat{\theta}_O, \hat{\theta}_R]$ those who are realists become workers and those who are optimists become entrepreneurs. Therefore, in the optimal expectations equilibrium, the lowest ability entrepreneurs are less talented at running a firm than the highest ability workers. This is an empirically attractive implication of the model since, in reality, the income distributions of workers and entrepreneurs have overlapping supports.8

Our next result shows that an increase in the fraction of individuals with optimal expectations $\lambda$ raises the ability of the marginal realistic entrepreneur $\hat{\theta}_R$ and the ability of the marginal optimistic entrepreneur $\hat{\theta}_O$.

**Proposition 3:** If $f(l, k, \theta_0) = \theta_0^\alpha k^\beta$, $\theta_0$ is uniformly distributed on $[0, 1]$, and

---

7The proof of Proposition 2 shows that $\hat{\theta}_R \in (0, 1)$.

8This is in contrast to models where occupational choice is only based on heterogeneity in ability, ability is one-dimensional, and where it is assumed that one occupation rewards ability more than the other. This results in income distributions for the two occupations that occupy non-overlapping intervals (see e.g. Parker (2009)).
the weight of anticipatory utility $s$ is less than $\bar{s}$, then an increase in the fraction of individuals with optimal expectations $\lambda$ raises the ability of the marginal realistic entrepreneur and the ability of the marginal optimistic entrepreneur, i.e., $\partial \hat{\theta}_R / \partial \lambda > 0$ and $\partial \hat{\theta}_O / \partial \lambda > 0$.

It follows from $\partial \hat{\theta}_R / \partial \lambda > 0$ that the ability of the marginal entrepreneur in the rational expectations equilibrium $\hat{\theta}_0$ is lower than the ability of the marginal realistic entrepreneur $\hat{\theta}_R$, i.e., $\hat{\theta}_0 < \hat{\theta}_R$. It follows from $\partial \hat{\theta}_O / \partial \lambda > 0$, (19), and (24) that if $\lambda$ is low, then the ability of the marginal optimistic entrepreneur $\hat{\theta}_O$ is lower than the ability of the marginal entrepreneur in the rational expectations equilibrium $\hat{\theta}_0$, i.e., $\hat{\theta}_O < \hat{\theta}_0$. Hence, when $\lambda$ is low we have

$$\hat{\theta}_O < \hat{\theta}_0 < \hat{\theta}_R,$$

and when $\lambda$ is high we have

$$\hat{\theta}_0 < \hat{\theta}_O < \hat{\theta}_R.$$

Thus, in the optimal expectations equilibrium there is a particular form of misallocation of talent. A realist with ability $\theta_0 \in [\hat{\theta}_0, \hat{\theta}_R]$ chooses to become a worker in the optimal expectations equilibrium but would select to be an entrepreneur in the rational expectations equilibrium. When $\lambda$ is low, an optimist with ability $\theta_0 \in [\hat{\theta}_O, \hat{\theta}_0]$ selects to be an entrepreneur in the optimal expectations equilibrium but would choose to become a worker in the rational expectations equilibrium. When $\lambda$ is high, an optimist with ability $\theta_0 \in [\hat{\theta}_0, \hat{\theta}_O]$ chooses to become a worker in the optimal expectations equilibrium but would select to be an entrepreneur in the rational expectations equilibrium.

We now show that the optimal expectations equilibrium wage is higher than the rational expectations equilibrium wage.

**Proposition 4**: If $f(l, k, \theta_0) = \theta_0^\alpha k^\beta$, $\theta_0$ is uniformly distributed on $[0, 1]$, and the weight of anticipatory utility $s$ is less than $\bar{s}$, then the optimal expectations equilibrium wage is higher than in the rational expectations equilibrium wage, i.e., $w^* > w_0^*$.
The intuition behind Proposition 4 is as follows. The assumption that entrepreneurial ability and labor are complements in production and the fact that individuals with optimal expectations are optimists implies that, for a given wage, the demand for labor of an individual with optimal expectations is higher than the demand for labor of a realist who has the same ability. This leads to an expansion of labor demand. Since an individual with optimal expectations derives anticipatory utility from entrepreneurship and is optimist about his entrepreneurial ability he will be, for a given wage, more attracted to entrepreneurship than a realist who has the same ability. This leads to a contraction of labor supply. The expansion of labor demand and contraction of labor supply imply that the equilibrium wage in the optimal expectations equilibrium is higher than in a rational expectations equilibrium.

We now provide conditions under which the majority of entrepreneurs are optimists and the majority of workers are realists.

**Proposition 5:** If \( f(l, k, \theta_0) = \theta_0^\alpha k^\beta \), \( \theta_0 \) is uniformly distributed on \([0, 1]\), and the weight of anticipatory utility \( s \) is less than \( \bar{s} \), and

\[
\frac{1}{1 + \frac{1 - \hat{\theta}_R}{1 - \hat{\theta}_R}} < \lambda < \frac{1}{1 + \left[ \frac{s - s(1 - \eta)}{\eta} \right]^{\eta}},
\]

where \( \hat{\theta}_R \) is given by (18), then the majority of entrepreneurs are optimists and the majority of workers are realists.

We know from Proposition 1 that individuals with optimal expectations choose to be optimists. The definition of marginal optimistic entrepreneur and the assumption that \( \theta_0 \) is uniformly distributed on \([0, 1]\) imply that amongst the fraction \( \lambda \) of individuals who are optimists, a fraction \( \hat{\theta}_O \) become workers, and a fraction \( 1 - \hat{\theta}_O \) become entrepreneurs. Similarly, the definition of marginal realistic entrepreneur and the assumption that \( \theta_0 \) is uniformly distributed on \([0, 1]\) imply that amongst the fraction \( 1 - \lambda \) of individuals who are realists, a fraction \( \hat{\theta}_R \) become workers, and a fraction \( 1 - \hat{\theta}_R \) become entrepreneurs. Hence, the share of optimistic entrepreneurs
in the total number of entrepreneurs is given by
\[ E^* = \frac{\lambda(1 - \hat{\theta}_O)}{\lambda(1 - \hat{\theta}_O) + (1 - \lambda)(1 - \hat{\theta}_R)}, \]
and the share of realistic workers in the total number of workers by
\[ \gamma^*_L = \frac{(1 - \lambda)\hat{\theta}_R}{(1 - \lambda)\hat{\theta}_R + \lambda\theta_O}. \]

Proposition 5 shows us that if the fraction of optimists \( \lambda \) is between the lower and the upper bounds in (25), then the majority entrepreneurs are optimists and the majority of workers are realists, i.e., \( \gamma^*_E > 1/2 \) and \( \gamma^*_L > 1/2 \). The intuition behind Proposition 5 is as follows. When the fraction of optimists is low–\( \lambda \) is less than the lower bound in (25)–the majority of entrepreneurs and the majority of workers are realists. When the fraction of optimists is high–\( \lambda \) is greater than the upper bound in (25)–the majority of entrepreneurs and the majority of workers are optimists. Hence, \( \lambda \) can neither be too low nor too high for the majority of entrepreneurs to be optimists and for the majority of workers to be realists. Note that the higher is the weight of anticipatory utility \( s \), the easier it is to satisfy (25) since the set of \( \lambda \)s that satisfy (25) stretches. Conversely, when \( s \) approaches 0 the lower and upper bounds in (25) converge to 1/2 and the set of \( \lambda \)s that satisfy (25) shrinks.

We now define the optimal expectations equilibrium when the weight of anticipatory utility is relatively large (\( s \) is high). When the weight of anticipatory utility is relatively large, individuals with optimal expectations who choose to be entrepreneurs hold the highest possible belief of ability, i.e., \( \theta^* = 1 \). In addition, a positive mass of individuals with optimal expectations equal to \( \theta^* = 1 \) choose to be workers since their entrepreneurial ability \( \theta_0 \) is not high enough to make entrepreneurship more attractive than working as an employee.

**Proposition 6:** If \( f(l, k, \theta_0) = \theta_0 l^{\alpha} k^{\beta}, \theta_0 \) is uniformly distributed on \([0,1]\), and the weight of anticipatory utility \( s \) is greater than \( \bar{s} \) and less than \( \eta/(1 - \eta) \), then the optimal expectations equilibrium is the solution to the system of four equations and
four unknowns (\(\hat{\theta}_R, \hat{\theta}_O, w, \) and \(r\)):

\[
\alpha^{\alpha} \beta^{\beta} (1-\eta)^{1-\eta} \hat{\theta}_R = w^{1-\beta} r^{\beta},
\]

\[
\alpha^{\alpha} \beta^{\beta} \left[ \hat{\theta}_O + s - (1+s)\eta \right]^{1-\eta} = w^{1-\beta} r^{\beta};
\]

\[
\left[ (1-\lambda) \frac{1-\eta}{2-\eta} \left( 1 - \hat{\theta}_R^{2-\eta} \right) + \lambda (1 - \hat{\theta}_O) \right] \left( \frac{\alpha}{w} \right)^{\frac{1-\eta}{1-\eta}} \left( \frac{\beta}{r} \right)^{\frac{\eta}{1-\eta}} = (1-\lambda) \hat{\theta}_R + \lambda \hat{\theta}_O,
\]

\[
\left[ (1-\lambda) \frac{1-\eta}{2-\eta} \left( 1 - \hat{\theta}_R^{2-\eta} \right) + \lambda (1 - \hat{\theta}_O) \right] \left( \frac{\alpha}{w} \right)^{\frac{1-\eta}{1-\eta}} \left( \frac{\beta}{r} \right)^{\frac{\eta}{1-\eta}} = K.
\]

Proposition 6 characterizes the optimal expectations equilibrium when the weight of anticipatory utility is relatively high, i.e., \(s \in (\bar{s}, \eta/(1-\eta))\). In this case we cannot obtain closed form solutions for \(\hat{\theta}_R, \hat{\theta}_O, w, \) and \(r\). However, after having tried different parameterizations of the model we find that the results obtained in Propositions 3 to 5 are still valid when the weight of anticipatory utility is relatively large.

5 Comparative Statics

In this section we perform comparative statics on equilibrium outcomes. There are two parameters which can be used to perform this analysis: the fraction of optimists \(\lambda\) and the weight of anticipatory utility \(s\). By varying \(\lambda\) while keeping \(s\) fixed we can analyze the impact that a change in the fraction of optimists has on equilibrium outcomes. By varying \(s\) while keeping \(\lambda\) fixed we can analyze the impact that a change in the degree of optimism has on equilibrium outcomes (the higher is \(s\), the greater the degree of the optimism of individuals with optimal expectations).

We start by showing that an increase in the fraction of optimists raises the market clearing wage.

**Proposition 7:** If \(f(l, k, \theta_0) = \theta_0 \ell^\alpha k^\beta\), \(\theta_0\) is uniformly distributed on \([0, 1]\), and the weight of anticipatory utility \(s\) is less than \(\bar{s}\), then an increase in the fraction of optimists leads to an increase in the equilibrium wage, i.e., \(\partial w^*/\partial \lambda > 0\).
Our next result summarizes the impact that an increase in the fraction of optimists has on the number of realistic workers, the number of realistic entrepreneurs, and the number of optimistic workers.

**Proposition 8**: If \( f(l,k,\theta_0) = \theta_0 l^\alpha k^\beta, \theta_0 \) is uniformly distributed on \([0,1]\), and the weight of anticipatory utility \( s \) is less than \( \hat{s} \), then (i) the equilibrium number of realistic workers decreases with an increase in the fraction of optimists, i.e., \( \partial L_R^*/\partial \lambda < 0 \), (ii) the equilibrium number of optimistic workers increases with an increase in the fraction of optimists, i.e., \( \partial L_O^*/\partial \lambda > 0 \), and (iii) the equilibrium number of realistic entrepreneurs decreases with an increase in the fraction of optimists, i.e., \( \partial E_R^*/\partial \lambda < 0 \).

The number of realistic workers is \( L_R^* = (1 - \lambda)\hat{\theta}_R \) since amongst the fraction \( 1 - \lambda \) of realists, a fraction \( \hat{\theta}_R \) become workers. An increase in \( \lambda \) lowers the number of realists, \( 1 - \lambda \), and raises the ability of the marginal realistic entrepreneur, \( \hat{\theta}_R \). The first effect lowers the number of realistic workers \( L_R^* \) but the second effect raises it. Hence, at first sight, an increase in the fraction of optimists has an ambiguous effect on the number of realistic workers. However, we are able to show that the first effect always dominates. Therefore, an increase in the fraction of optimists lowers the number of realistic workers.

The number of optimistic workers is \( L_O^* = \lambda \hat{\theta}_O \) since amongst the fraction \( \lambda \) of optimists, a fraction \( \hat{\theta}_O \) become workers. An increase in \( \lambda \) raises the number of optimists and raises the ability of the marginal optimistic entrepreneur \( \hat{\theta}_O \). Both effects raise the number of optimistic workers \( L_O^* \) and therefore an increase in the fraction of optimists leads to an increase in the number of optimistic workers.

The number of realistic entrepreneurs is \( E_R^* = (1 - \lambda)(1 - \hat{\theta}_R) \) since amongst the fraction \( 1 - \lambda \) of realists, a fraction \( 1 - \hat{\theta}_R \) become entrepreneurs. An increase in \( \lambda \) lowers the number of realists, \( 1 - \hat{\theta}_R \), and raises the ability of the marginal realistic entrepreneur, \( \hat{\theta}_R \). Both effects lower the number of realistic entrepreneurs \( E_R^* \) and therefore an increase in the fraction of optimists leads to a reduction in the number of realistic entrepreneurs.
The number of optimistic entrepreneurs is $E^*_O = \lambda(1 - \hat{\theta}_O)$ since amongst the fraction $\lambda$ of optimists, a fraction $1 - \hat{\theta}_O$ become entrepreneurs. An increase in $\lambda$ raises the number of optimists and raises the ability of the marginal optimistic entrepreneur, $\hat{\theta}_O$. The first effect raises the number of optimistic entrepreneurs $E^*_O$ but the second effect lowers it. Therefore, an increase in the fraction of optimists has an ambiguous effect on the number of optimistic entrepreneurs.

The total number of workers is $L^* = L^*_R + L^*_O$. We know from Proposition 8 that, on the one hand, an increase in the fraction of optimists $\lambda$ lowers the number of realistic workers $L^*_R$, and, on the other hand, it raises the number of optimistic workers $L^*_O$. Therefore, an increase in the fraction of optimists appears to have an ambiguous effect on the total number of workers $L^*$ and on the total number of entrepreneurs $E^*$ since $E^* = 1 - L^*$. However, our next result shows how the equilibrium number of workers (and entrepreneurs) varies with the fraction of optimists.

**Proposition 9**: Assume $f(l, k, \theta_0) = \theta_0^\alpha k^\beta$, $\theta_0$ is uniformly distributed on $[0, 1]$, and the weight of anticipatory utility $s$ is less than $\bar{s}$.

(i) The equilibrium number of workers (entrepreneurs) is a concave (convex) function of the fraction of optimists, i.e., $\partial^2 L^*/\partial \lambda^2 < 0$ ($\partial^2 E^*/\partial \lambda^2 > 0$).

(ii) If

$$
1 + \frac{s}{\eta} - \phi(\eta, \beta, s) \frac{\psi(\eta, s)}{\phi(\eta, \beta, s)} > \frac{2 - \eta}{1 - \eta} [1 - \psi(\eta, s)] \left(1 + \frac{s}{\eta}\right),
$$

then the equilibrium number of workers (entrepreneurs) increases (decreases) with an increase in the fraction of optimists, i.e., $\partial L^*/\partial \lambda > 0$ ($\partial E^*/\partial \lambda < 0$).

(iii) If (26) is violated, then the equilibrium number of workers (entrepreneurs) increases (decreases) with an increase in the fraction of optimists when $\lambda \in [0, \bar{\lambda})$, i.e., $\partial L^*/\partial \lambda > 0$ ($\partial E^*/\partial \lambda < 0$), and the equilibrium number of workers (entrepreneurs) decreases (increases) with an increase in the fraction of optimists when $\lambda \in (\bar{\lambda}, 1]$, i.e., $\partial L^*/\partial \lambda < 0$ ($\partial E^*/\partial \lambda > 0$), where $\bar{\lambda}$ is the solution to

$$
1 - \bar{\lambda} + \bar{\lambda}\psi(\eta, s) = \frac{2 - \eta}{1 - \eta} \frac{1 - \psi(\eta, s)}{1 + \frac{s}{\eta} - \phi(\eta, \beta, s)} \left(1 + \frac{s}{\eta}\right).
$$

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Proposition 9 shows that an increase in the number of optimists does not necessarily lead to an increase in the number of entrepreneurs in the economy. Moreover, one of two cases might arise. First, an increase in the number of optimists raises (lowers) the total number of workers (entrepreneurs). This happens when either (a) inequality (26) is satisfied or (b) inequality (26) is violated and the fraction of optimists is small, i.e., $\lambda < \bar{\lambda}$. In this case an increase in the number of optimists raises the number of optimistic workers more than it lowers the number of realistic workers. Second, an increase in the number of optimists lowers (raises) the number of workers (entrepreneurs). This happens when inequality (26) is violated and the fraction of optimists is high, i.e., $\lambda > \bar{\lambda}$. In this case an increase in the number of optimists raises the number of optimistic workers less than it lowers the number of realistic workers.

We end this section by providing conditions under which an increase in the number of optimists raises the rental cost of capital.

**Proposition 10:** If $f(l, k; \theta_0) = \theta_0 l^\alpha k^\beta$, $\theta_0$ is uniformly distributed on $[0, 1]$, the weight of anticipatory utility $s$ is less than $\bar{s}$, and either (a) inequality (26) is satisfied or (b) inequality (26) is violated and the fraction of optimists is small enough, i.e., $\lambda < \bar{\lambda}$, then an increase in the fraction of optimists leads to an increase in the equilibrium rental cost of capital, i.e., $\partial r^*/\partial \lambda > 0$.

### 6 Contribution to the Literature

Our paper contributes to the literature that studies occupational choice using general equilibrium models. In this broad line of research, Lucas (1978), Kanbur (1979), Kihlstrom and Laffont (1979), Bewley (1989) and Lazear (2005) are five prominent papers.

Lucas (1978) proposes a general equilibrium of occupational choice where differences in entrepreneurial ability determine who becomes a worker or an entrepreneur. He considers a closed economy with a workforce of size $N$ and $K$ units of homogen-
eous capital. Individuals are risk neutral and the output of a firm is an increasing function of entrepreneurial ability, labor and capital. Lucas shows that the most talented individuals become entrepreneurs and the less talented ones become workers. He also studies a dynamic version of the model which allows him to analyze the impact that an increase in the capital stock has on the evolution of firm size distribution. We extend Lucas (1978) by allowing for a fraction of the workforce to form optimal expectations of entrepreneurial ability.\textsuperscript{9}

Kanbur (1979) considers a general equilibrium model of occupational choice where individuals learn their ability as entrepreneurs by entering entrepreneurship. Those who become entrepreneurs can therefore make an informed future occupational choice. The cost of becoming an entrepreneur is the risk exposure relative to the safe alternative of being employed. Indeed, the payoff of an entrepreneur’s project depends on his ability which is unknown to new entrepreneurs. Hence, entrepreneurship has an immediate cost (risk taking) and a postponed gain (informed future occupational choice). For this reason, more patient societies have a larger share of new entrepreneurs.

Kihlstrom and Laffont (1979) study the role of risk aversion in a general equilibrium model of occupational choice. Individuals choose between operating a risky firm or working for a riskless wage. Less risk averse individuals become entrepreneurs while more risk averse individuals become employees. Among entrepreneurs, the less risk averse are found to operate larger firms. Different risk attitudes among individuals or societies seem a plausible explanation of entry and excess entry into entrepreneurship. Yet, there is little empirical evidence supporting this explanation. Moreover, risk attitudes cannot explain individuals’ occupational choice when risks can be perfectly insured.

Jovanovic (1994) generalizes Lucas (1978) by allowing for individuals to have two dimensions of ability: managerial and working. He shows that when managerial skills are positively correlated with working skills the best potential managers could

\textsuperscript{9}Chapter 2 of Parker (2009) discusses in detail the main extensions of Lucas’ (1978) model.
end up as wage workers. Cagetti and De Nardi (2006) study the effect of borrowing constraints on wealth distribution and occupational choices in a model where individuals have managerial and working ability (assumed to be uncorrelated).

Knight (1921) distinguishes risk from uncertainty. A project is risky when the probability of outcomes are known. If the probabilities are unknown, the project is uncertain. According to Knight, entrepreneurs start uncertain projects. Building on Knight’s work, Bewley (1989) models the link between uncertainty aversion and the choice to become an entrepreneur. The main finding is that entrepreneurs are individuals with low uncertainty aversion.

Lazear (2005) considers an occupational choice model where individuals are endowed with two skills. An individual can be a specialist, in which case he receives income associated with his best skill, or he can be an entrepreneur, in which case he is limited by his weakest attribute. He shows that individuals endowed with more balanced ability sets are found to be more likely to become entrepreneurs.

Campanale (2010) considers a life-cycle occupational and portfolio choice model with learning. The key assumption is that the quality of a business project is not precisely known upon entry and is learned over time. The model shows that entry and private equity allocation for the majority of entrepreneurs can be rationalized even with negative expected premia on individual business investment. Since individuals can switch back to paid-employment, they find it worthwhile experimenting with entrepreneurship to find out if the project is good even if initially the expected return is low.

Markus Poschke (2013), like Campanale (2010), also studies a dynamic occupational choice model with learning. Individuals differ in their efficiency as workers and in the productivity of the firms they start. Whereas efficiency as a worker is known, the productivity of entrepreneurial projects can only be found after implementing them. He shows that the option to abandon bad projects attracts low-ability agents into entrepreneurship.

We are aware of at least four papers that explicitly study the implications of en-
entrepreneurial optimism using a general equilibrium framework: de Meza and Southey

In de Meza and Southey (1996) risk neutral individuals must choose between
becoming entrepreneurs or employees. The output of a firm is an increasing function
of entrepreneurial ability and capital. There are two types of individuals: realists
and optimists. An individual who chooses to become an entrepreneur must select
the right mix of self-finance and debt-finance from risk neutral banks to develop his
project. Banks and realistic individuals know a project’s true probability of success
but optimists overestimate it. They find that optimists select maximum self-finance
and any external finance is in the form of a standard debt contract. They also find
that, though not all optimists necessarily become entrepreneurs, all entrepreneurs
will be optimists.

Manove (2000) considers a competitive economy where individuals with different
productivities as entrepreneurs choose to become entrepreneurs or employees. Entre-
preneurs use their own capital, effort and labor provided by employees to produce.
All individuals consume the good produced. There are two types of individuals:
realists and optimists. Optimists overestimate their productivity as entrepreneurs.
Manove shows that optimism increases the savings rates and work effort of optim-
ists, which can have a positive effect on steady-state income (though the optimist’s
utility will be reduced). However, optimism may also tend to reduce income through
a negative effect on economic efficiency. The sources of the two effects on income
are distinct: the negative efficiency effect is associated with the overuse of external
resources, such as hired labor and borrowed capital, whereas the positive incentive
effect is associated with the overuse of resources internal to the entrepreneur, such
as his personal savings and effort.

Fraser and Greene (2006) consider an occupational choice model in which entre-
preneurs are uncertain about their true talent but learn from experience. It follows
that optimism in talent lowers with experience. As a consequence, the impact of
optimism on the decision to be an entrepreneur lowers with experience.
In Rigotti et al. (2011) individuals choose to be entrepreneurs or employees and between employing a traditional technology or a new one about which little is known. A firm is an entrepreneur-employee pair operating a particular technology. Individuals face ambiguity about firms’ return and are either optimistic or pessimistic. Optimists are more likely to become entrepreneurs. Moreover, firms employing new and highly ambiguous technologies are run by optimistic entrepreneurs and employ optimistic employees.

The main innovation of our paper, by comparison with these four papers, is the assumption that optimism arises endogenously. This assumption allows us link the degree of optimism observed in the economy to tastes—the weight of anticipatory utility—and to technology—entrepreneurial ability and the degree of decreasing returns to scale.

7 Conclusion

We extend Lucas’ (1978) general equilibrium model of occupational choice by assuming that fraction of the workforce has optimal expectations of entrepreneurial ability. Optimal expectations are modeled according to Brunnermeier and Parker (2005). We show that individuals with optimal expectations choose to be optimists about their entrepreneurial ability.

We show that entrepreneurial optimism has four main effects on the labor market and firm formation. First, it raises the equilibrium wage. Second, optimists crowd out realists from entrepreneurship. Third, when the fraction of optimists is moderate, the majority of entrepreneurs are optimists and the majority of workers are realists. Fourth, it leads to a misallocation of talent: the lowest ability entrepreneurs are less talented at running a firm than the highest ability workers.
8 Appendix

Proof of Proposition 1: Consider an individual with entrepreneurial ability $\theta_0$ and expectation of entrepreneurial ability $\theta$. Assume that this individual decides to be an entrepreneur at $t = 1$. At $t = 2$ this individual solves the following problem

$$\max_{l,k} (\theta l^\alpha k^\beta - wl - rk)$$

The first-order conditions are

$$\alpha \theta l^{\alpha - 1} k^\beta = w,$$

and

$$\beta \theta l^\alpha k^{\beta - 1} = r.$$

Solving for $l$ and $k$ we obtain

$$l(w, r, \theta) = \theta^{\frac{1}{1-\eta}} \left( \frac{\alpha}{w} \right)^{\frac{\eta}{1-\eta}} \left( \frac{\beta}{r} \right)^{\frac{\beta}{1-\eta}}, \quad (28)$$

and

$$k(w, r, \theta) = \theta^{\frac{1}{1-\eta}} \left( \frac{\alpha}{w} \right)^{\frac{\eta}{1-\eta}} \left( \frac{\beta}{r} \right)^{\frac{1-\eta}{1-\eta}}, \quad (29)$$

where $\eta = \alpha + \beta$. At $t = 0$ this individual solves the problem:

$$\max_{\theta \in [0,1]} \left\{ (\theta_0 + s\theta)[l(w, r, \theta)]^\alpha [k(w, r, \theta)]^\beta - (1 + s) \left[wl(w, r, \theta) + rk(w, r, \theta)\right] \right\}.$$

Substituting $l(w, r, \theta)$ by (28) and $k(w, r, \theta)$ by (29) and simplifying terms we obtain

$$\left( \frac{\alpha}{w} \right)^{\frac{\eta}{1-\eta}} \left( \frac{\beta}{r} \right)^{\frac{\beta}{1-\eta}} \max_{\theta \in [0,1]} \left[ (\theta_0 + s\theta)^{\eta} - (1 + s) \eta \theta^{\frac{1}{1-\eta}} \right].$$

The first-order condition is

$$(\theta_0 + s\theta) \frac{\eta}{1-\eta} \theta^{\frac{\eta}{1-\eta} - 1} + s\theta^{\frac{\eta}{1-\eta}} = (1 + s) \frac{\eta}{1-\eta} \theta^{\frac{1}{1-\eta} - 1}. \quad (30)$$

Multiplying both sides by $(1 - \eta)\theta$ we obtain

$$(\eta \theta_0 + s\theta)\theta^{\frac{\eta}{1-\eta}} = (1 + s)\eta \theta^{\frac{1}{1-\eta}}.$$
Dividing both sides by $\theta^{\frac{\eta}{1-\eta}}$ we have

$$\eta \theta_0 + s \theta = (1 + s) \eta \theta.$$  

Solving for $\theta$ we get

$$\theta^* = \frac{\eta}{\eta - s(1 - \eta)} \theta_0. \quad (31)$$

Note that this condition implies that all those who choose to become entrepreneurs hold optimistic expectations in the sense that $\theta^* > \theta_0$. For (31) to be an interior solution, i.e., $\theta^* \in (0, 1)$, at least two conditions must be satisfied. First, it must be that

$$\eta > s(1 - \eta),$$

or

$$s < \frac{\eta}{1 - \eta}. \quad (32)$$

This condition places an upper bound on the weight placed on anticipatory utility. We see from (32) that the upper bound only depends on $\eta$. Second, it must be that

$$\frac{\eta}{\eta - s(1 - \eta)} \theta_0 < 1,$$

or

$$\theta_0 < \frac{\eta - s(1 - \eta)}{\eta}. \quad (33)$$

This condition says that there is an interior solution only for individuals whose productivity is below an upper bound. We see from (33) that the upper bound depends on $s$ and $\eta$. Hence the optimal expectations are as follows:

$$\theta^* = \begin{cases} 
\frac{\eta}{\eta - s(1 - \eta)} \theta_0 & \text{if } \theta_0 < \frac{\eta - s(1 - \eta)}{\eta} \\
1 & \text{if } \theta_0 \geq \frac{\eta - s(1 - \eta)}{\eta}.
\end{cases}$$

To complete the proof we need to show that the second-order condition is satisfied. The first-order condition (30) is equivalent to

$$\frac{\eta}{1 - \eta} \left[ \theta_0 \theta^{\frac{\eta}{1-\eta} - 1} + s \theta^{\frac{\eta}{\eta}} + \frac{1 - \eta}{\eta} s \theta^{\frac{\eta}{\eta}} - (1 + s) \theta^{\frac{\eta}{1-\eta} - 1} \right] = 0,$$
Taking the derivative of (34) with respect to \( \theta \) we have that the second-order condition is given by

\[
\frac{2\eta - 1}{1 - \eta} \theta_0 \theta^{\frac{2n-1}{1-\eta}} + \left( \frac{s}{\eta} - 1 - s \right) \frac{\eta}{1 - \eta} \theta^{\frac{n}{1-\eta}} < 0,
\]

or

\[
(2\eta - 1)\theta_0 - \left[ \eta - s(1 - \eta) \right] \theta < 0.
\]

Since \( s < \eta/(1 - \eta) \), the second-order condition is satisfied for any \( \eta \leq 0.5 \). When \( 0.5 < \eta < 1 \) the second-order condition is satisfied as long as

\[
(2\eta - 1)\theta_0 < \left[ \eta - s(1 - \eta) \right] \theta.
\]

Replacing \( \theta \) by \( \theta^* = \eta\theta_0/[\eta - s(1 - \eta)] \) we have

\[
(2\eta - 1)\theta_0 < \eta\theta_0,
\]

or

\[
\eta < 1,
\]

which is true. \( Q.E.D. \)

**Proof of Proposition 2:** Assume \( s < \bar{s} \). Let \( \eta = \alpha + \beta \). The first step to determine the competitive optimal expectations equilibrium is to find out the labor market equilibrium condition. The labor demand from realistic entrepreneurs is

\[
L^D_R = (1 - \lambda) \int_{\theta_R}^{1} l(w, r, \theta_0) d\theta_0
\]

\[
= (1 - \lambda) \left( \frac{\alpha}{w} \right)^{\frac{1-\eta}{1-\gamma}} \left( \frac{\beta}{r} \right)^{\frac{\beta}{1-\eta}} \int_{\theta_R}^{1} \theta_0^{\frac{1}{1-\gamma}} d\theta_0
\]

\[
= (1 - \lambda) \frac{1 - \eta}{2 - \eta} \left( 1 - \bar{\theta}_R^{\frac{2n}{1-\eta}} \right) \left( \frac{\alpha}{w} \right)^{\frac{1-\eta}{1-\gamma}} \left( \frac{\beta}{r} \right)^{\frac{\beta}{1-\gamma}}.
\]

(35)
Note that for $L^D_R$ to be well defined it must be that $\hat{\theta}_R < 1$. Recall that $\hat{\theta}_O$ is the ability threshold that determines the marginal optimistic entrepreneur. If $\hat{\theta}_O < [\eta - s(1 - \eta)]/\eta$, then labor demand from optimistic entrepreneurs is the sum of the demand for labor coming from the mass of entrepreneurs with heterogeneous optimistic expectations, i.e., those with $\theta^* \in (\theta_0, 1)$, to the demand for labor coming from the mass of entrepreneurs with homogeneous optimistic expectations, i.e., those with $\theta^* = 1$:

$$L^D_O = \lambda \left\{ \int_{\theta_0}^{\eta - s(1 - \eta)}/\eta} l(w, r, \theta^*)d\theta_0 + \int_{\eta - s(1 - \eta)}/\eta} l(w, r, 1)d\theta_0 \right\}$$

$$= \lambda \left\{ \int_{\theta_0}^{\eta - s(1 - \eta)}/\eta} (\theta^*)^{1 - \eta}d\theta_0 + \int_{\eta - s(1 - \eta)}/\eta} d\theta_0 \right\} \left( \frac{\alpha}{w} \right)^{\frac{1-\beta}{1-\eta}} \left( \frac{\beta}{r} \right)^{\frac{\delta}{1-\eta}}$$

$$= \lambda \left\{ \left[ \frac{\eta}{\eta - s(1 - \eta)} \right] \int_{\theta_0}^{\eta - s(1 - \eta)}/\eta} d\theta_0 \right\} \left( \frac{\alpha}{w} \right)^{\frac{1-\beta}{1-\eta}} \left( \frac{\beta}{r} \right)^{\frac{\delta}{1-\eta}}$$

$$= \lambda \left\{ \frac{1 - \eta}{2 - \eta} \left[ \frac{\eta}{\eta - s(1 - \eta)} \right] \int_{\theta_0}^{\eta - s(1 - \eta)}/\eta} \left( \frac{\alpha}{w} \right)^{\frac{1-\beta}{1-\eta}} \left( \frac{\beta}{r} \right)^{\frac{\delta}{1-\eta}} \right\}$$

$$= \lambda \left\{ \frac{1 - \eta}{2 - \eta} \left[ \frac{\eta}{\eta - s(1 - \eta)} \right] \right\} \left( \frac{\alpha}{w} \right)^{\frac{1-\beta}{1-\eta}} \left( \frac{\beta}{r} \right)^{\frac{\delta}{1-\eta}}.$$

Note that for $L^D_O$ to be well defined it must be that $\hat{\theta}_O < [\eta - s(1 - \eta)]/\eta$. From (35)
and (36), labor demand is equal to

\[
L^D = L^D_R + L^D_O \\
= (1 - \lambda) \frac{1 - \eta}{2 - \eta} \left( 1 - \hat{\theta}^{\frac{2 - \eta}{\eta}}_R \right) \left( \frac{\alpha}{w} \right)^{\frac{1 - \beta}{1 - \eta}} \left( \frac{\beta}{r} \right)^{\frac{\beta}{1 - \eta}} \\
+ \lambda \frac{1 - \eta}{2 - \eta} \left\{ 1 + \frac{s}{\eta} - \left[ \frac{\eta}{\eta - s(1 - \eta)} \right]^{\frac{1 - \eta}{\eta}} \hat{\theta}^{\frac{2 - \eta}{\eta}}_O \right\} \left( \frac{\alpha}{w} \right)^{\frac{1 - \beta}{1 - \eta}} \left( \frac{\beta}{r} \right)^{\frac{\beta}{1 - \eta}}
\]

\[
= \frac{1 - \eta}{2 - \eta} \left\{ (1 - \lambda) \left( 1 - \hat{\theta}^{\frac{2 - \eta}{\eta}}_R \right) + \lambda + \frac{s}{\eta} - \lambda \left[ \frac{\eta}{\eta - s(1 - \eta)} \right]^{\frac{1 - \eta}{\eta}} \hat{\theta}^{\frac{2 - \eta}{\eta}}_O \right\} \left( \frac{\alpha}{w} \right)^{\frac{1 - \beta}{1 - \eta}} \left( \frac{\beta}{r} \right)^{\frac{\beta}{1 - \eta}}
\]

\[
= \frac{1 - \eta}{2 - \eta} \left\{ 1 + \frac{s}{\eta} - (1 - \lambda)\hat{\theta}^{\frac{2 - \eta}{\eta}}_R - \lambda \left[ \frac{\eta}{\eta - s(1 - \eta)} \right]^{\frac{1 - \eta}{\eta}} \hat{\theta}^{\frac{2 - \eta}{\eta}}_O \right\} \left( \frac{\alpha}{w} \right)^{\frac{1 - \beta}{1 - \eta}} \left( \frac{\beta}{r} \right)^{\frac{\beta}{1 - \eta}}. \quad (37)
\]

Since each worker provides a unit of labor, labor supply is

\[
L^S = (1 - \lambda)L^S_R + \lambda L^S_O \\
= (1 - \lambda) \int_{0}^{\hat{\theta}_R} d\theta_0 + \lambda \int_{0}^{\hat{\theta}_O} d\theta_0 \\
= (1 - \lambda)\hat{\theta}_R + \lambda \hat{\theta}_O. \quad (38)
\]

In equilibrium, labor demand must equal labor supply:

\[
\frac{1 - \eta}{2 - \eta} \left\{ 1 + \frac{s}{\eta} - (1 - \lambda)\hat{\theta}^{\frac{2 - \eta}{\eta}}_R - \lambda \left[ \frac{\eta}{\eta - s(1 - \eta)} \right]^{\frac{1 - \eta}{\eta}} \hat{\theta}^{\frac{2 - \eta}{\eta}}_O \right\} \left( \frac{\alpha}{w} \right)^{\frac{1 - \beta}{1 - \eta}} \left( \frac{\beta}{r} \right)^{\frac{\beta}{1 - \eta}}
\]

\[
= (1 - \lambda)\hat{\theta}_R + \lambda \hat{\theta}_O, \quad (39)
\]

The second step to determine the competitive optimal expectations equilibrium is to find out the capital market equilibrium condition. The capital demand from realistic entrepreneurs is

\[
K^D_R = (1 - \lambda) \int_{\hat{\theta}_R}^{1} k(w, r, \theta_0) d\theta_0 \\
= (1 - \lambda) \left( \frac{\alpha}{w} \right)^{\frac{\alpha}{1 - \eta}} \left( \frac{\beta}{r} \right)^{\frac{1 - \alpha}{1 - \eta}} \int_{\hat{\theta}_R}^{1} \theta_0^{\frac{1}{1 - \eta}} d\theta_0 \\
= (1 - \lambda) \frac{1 - \eta}{2 - \eta} \left( 1 - \hat{\theta}^{\frac{2 - \eta}{\eta}}_R \right) \left( \frac{\alpha}{w} \right)^{\frac{\alpha}{1 - \eta}} \left( \frac{\beta}{r} \right)^{\frac{1 - \alpha}{1 - \eta}}. \quad (40)
\]
Note that for \( K^D_R \) to be well defined it must be that \( \hat{\theta}_R < 1 \). Recall that \( \hat{\theta}_O \) is the ability threshold that determines the marginal optimistic entrepreneur. If \( \hat{\theta}_O < [\eta - s(1 - \eta)]/\eta \), then capital demand from optimistic entrepreneurs is the sum of the demand for capital coming from the mass of entrepreneurs with heterogeneous optimistic expectations, i.e., those with \( \theta^* \in (\theta_0, 1) \), to the demand for capital coming from the mass of entrepreneurs with homogeneous optimistic expectations, i.e., those with \( \theta^* = 1 \):

\[
K^D_O = \lambda \left\{ \int_{\hat{\theta}_O}^{\eta - s(1 - \eta)/\eta} k(w, r, \theta^*)d\theta_0 + \int_{\eta - s(1 - \eta)/\eta}^{1} k(w, r, 1)d\theta_0 \right\}
\]

\[
= \lambda \left\{ \int_{\hat{\theta}_O}^{\eta - s(1 - \eta)/\eta} (\theta^*)^{1 - \eta}\frac{1}{1 - \eta} d\theta_0 + \int_{\eta - s(1 - \eta)/\eta}^{1} d\theta_0 \right\} \left( \frac{\alpha}{w} \right)^{1 - \eta} \left( \frac{\beta}{r} \right)^{1 - \eta}
\]

\[
= \frac{1 - \eta}{2 - \eta} \left\{ 1 + \frac{s}{\eta} - \left[ \frac{\eta}{\eta - s(1 - \eta)} \right]^{1 - \eta} \theta_0^{2 - \eta} \right\} \left( \frac{\alpha}{w} \right)^{1 - \eta} \left( \frac{\beta}{r} \right)^{1 - \eta}.
\] (41)

Note that for \( K^D_O \) to be well defined it must be that \( \hat{\theta}_O < [\eta - s(1 - \eta)]/\eta \). From (40) and (41), capital demand is equal to

\[
K^D = K^D_R + K^D_O
\]

\[
= (1 - \lambda) \frac{1 - \eta}{2 - \eta} \left( 1 - \hat{\theta}_R^{2 - \eta} \right) \left( \frac{\alpha}{w} \right)^{1 - \eta} \left( \frac{\beta}{r} \right)^{1 - \eta}
\]

\[
+ \lambda \frac{1 - \eta}{2 - \eta} \left\{ 1 + \frac{s}{\eta} - \left[ \frac{\eta}{\eta - s(1 - \eta)} \right]^{1 - \eta} \theta_0^{2 - \eta} \right\} \left( \frac{\alpha}{w} \right)^{1 - \eta} \left( \frac{\beta}{r} \right)^{1 - \eta}
\]

\[
= \frac{1 - \eta}{2 - \eta} \left\{ 1 + \frac{s}{\eta} - (1 - \lambda) \hat{\theta}_R^{2 - \eta} - \lambda \left[ \frac{s}{\eta - s(1 - \eta)} \right]^{1 - \eta} \theta_0^{2 - \eta} \right\} \left( \frac{\alpha}{w} \right)^{1 - \eta} \left( \frac{\beta}{r} \right)^{1 - \eta}.
\] (42)

In equilibrium, capital demand must equal the exogenous capital supply:

\[
\frac{1 - \eta}{2 - \eta} \left\{ 1 + \frac{s}{\eta} - (1 - \lambda) \hat{\theta}_R^{2 - \eta} - \lambda \left[ \frac{s}{\eta - s(1 - \eta)} \right]^{1 - \eta} \theta_0^{2 - \eta} \right\} \left( \frac{\alpha}{w} \right)^{1 - \eta} \left( \frac{\beta}{r} \right)^{1 - \eta} = \bar{K}.
\] (43)

The third step to determine the competitive optimal expectations equilibrium is to find out the ability level of the marginal realistic entrepreneur \( \hat{\theta}_R \) and of the marginal
optimistic entrepreneur \( \hat{\theta}_O \). At \( t = 1 \) a realist with ability \( \hat{\theta}_R \) is indifferent between being an entrepreneur and a worker when

\[
\hat{\theta}_R[l(w, r, \hat{\theta}_R)]^\alpha[k(w, r, \hat{\theta}_R)]^\beta - wl(w, r, \hat{\theta}_R) - rk(w, r, \hat{\theta}_R) = w,
\]

or

\[
\hat{\theta}_R \left[ \hat{\theta}_R^{\frac{1}{1-\eta}} \left( \frac{\alpha}{w} \right)^{\frac{1-\beta}{1-\eta}} \left( \frac{\beta}{r} \right)^{\frac{\beta}{1-\eta}} \right]^{\alpha} \left[ \hat{\theta}_R^{\frac{1}{1-\eta}} \left( \frac{\alpha}{w} \right)^{\frac{1-\beta}{1-\eta}} \left( \frac{\beta}{r} \right)^{\frac{1-\alpha}{1-\eta}} \right]^{\beta} - w\hat{\theta}_R^{\frac{1}{1-\eta}} \left( \frac{\alpha}{w} \right)^{\frac{1-\beta}{1-\eta}} \left( \frac{\beta}{r} \right)^{\frac{\beta}{1-\eta}} - r\hat{\theta}_R^{\frac{1}{1-\eta}} \left( \frac{\alpha}{w} \right)^{\frac{1-\alpha}{1-\eta}} \left( \frac{\beta}{r} \right)^{\frac{1-\alpha}{1-\eta}} = w,
\]

or

\[
\hat{\theta}_R^{\frac{1}{1-\eta}} \left[ \left( \frac{\alpha}{w} \right)^{\frac{1-\beta}{1-\eta}} \left( \frac{\beta}{r} \right)^{\frac{1-\eta}{1-\eta}} - w \left( \frac{\alpha}{w} \right)^{\frac{1-\beta}{1-\eta}} \left( \frac{\beta}{r} \right)^{\frac{1-\beta}{1-\eta}} - r \left( \frac{\alpha}{w} \right)^{\frac{1-\alpha}{1-\eta}} \left( \frac{\beta}{r} \right)^{\frac{1-\alpha}{1-\eta}} \right] = w,
\]

or

\[
\hat{\theta}_R^{\frac{1}{1-\eta}} \left( \frac{\alpha}{w} \right)^{\frac{1-\beta}{1-\eta}} \left( \frac{\beta}{r} \right)^{\frac{1-\beta}{1-\eta}} \left[ 1 - w \left( \frac{\alpha}{w} \right)^{\frac{1-\beta}{1-\eta}} - r \left( \frac{\beta}{r} \right)^{\frac{1-\alpha}{1-\eta}} \right] = w,
\]

or

\[
\hat{\theta}_R^{\frac{1}{1-\eta}} \left( \frac{\alpha}{w} \right)^{\frac{1-\beta}{1-\eta}} \left( \frac{\beta}{r} \right)^{\frac{1-\beta}{1-\eta}} (1 - \eta) = w,
\]

or

\[
\hat{\theta}_R^{\frac{1}{1-\eta}} \alpha^{\frac{\beta}{1-\eta}} \beta^{\frac{\beta}{1-\eta}} (1 - \eta) = w^{\frac{1-\beta}{1-\eta}} r^{\frac{\beta}{1-\eta}},
\]

or

\[
\alpha^{\beta}(1 - \eta)^{1-\eta} \hat{\theta}_R = w^{1-\beta} r^\beta.
\]

At \( t = 1 \) an individual with optimal expectations of ability \( \theta^* = \eta \hat{\theta}_O / [\eta - s(1 - \eta)] \) and ability \( \hat{\theta}_O \) is indifferent between being an entrepreneur and a worker when

\[
(\hat{\theta}_O + s\theta^*)[l(w, r, \theta^*)]^\alpha[k(w, r, \theta^*)]^\beta - (1 + s)[wl(w, r, \theta^*) + rk(w, r, \theta^*)] = w,
\]

or

\[
(\hat{\theta}_O + s\theta^*) \left[ (\theta^*)^{\frac{1}{1-\eta}} \left( \frac{\alpha}{w} \right)^{\frac{1-\beta}{1-\eta}} \left( \frac{\beta}{r} \right)^{\frac{1-\beta}{1-\eta}} \right]^{\alpha} \left[ (\theta^*)^{\frac{1}{1-\eta}} \left( \frac{\alpha}{w} \right)^{\frac{1-\alpha}{1-\eta}} \left( \frac{\beta}{r} \right)^{\frac{1-\alpha}{1-\eta}} \right]^{\beta} - (1 + s) \left[ w(\theta^*)^{\frac{1}{1-\eta}} \left( \frac{\alpha}{w} \right)^{\frac{1-\beta}{1-\eta}} \left( \frac{\beta}{r} \right)^{\frac{1-\eta}{1-\eta}} + r(\theta^*)^{\frac{1}{1-\eta}} \left( \frac{\alpha}{w} \right)^{\frac{1-\alpha}{1-\eta}} \left( \frac{\beta}{r} \right)^{\frac{1-\alpha}{1-\eta}} \right] = w,
\]

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or

\[
\left[ \frac{\eta - s(1 - \eta)}{\eta} \theta^* + s \theta^* \right] \left[ (\theta^*)^{\frac{1}{1-\eta}} \left( \frac{\alpha}{w} \right)^{\frac{\alpha}{1-\eta}} \left( \frac{\beta}{r} \right)^{\frac{\beta}{1-\eta}} \right] \\
-(1 + s)(\theta^*)^{\frac{1}{1-\eta}} \left( \frac{\alpha}{w} \right)^{\frac{\alpha}{1-\eta}} \left( \frac{\beta}{r} \right)^{\frac{\beta}{1-\eta}} \left[ w \left( \frac{\alpha}{w} \right) + r \left( \frac{\beta}{r} \right) \right] = w,
\]

or

\[
(\theta^*)^{\frac{1}{1-\eta}} \left( \frac{\alpha}{w} \right)^{\frac{\alpha}{1-\eta}} \left( \frac{\beta}{r} \right)^{\frac{\beta}{1-\eta}} \left[ \eta - s(1 - \eta) + s - (1 + s)\eta \right] = w,
\]

or

\[
(\theta^*)^{\frac{1}{1-\eta}} \left( \frac{\alpha}{w} \right)^{\frac{\alpha}{1-\eta}} \left( \frac{\beta}{r} \right)^{\frac{\beta}{1-\eta}} \left[ 1 - \frac{s}{\eta} + 2s - (1 + s)\eta \right] = w
\]

or

\[
\left[ \frac{\eta}{\eta - s(1 - \eta)} \right]^{\frac{1}{1-\eta}} \hat{\theta}_O = \frac{1}{\beta} \left( \frac{\alpha}{w} \right)^{\frac{\alpha}{1-\eta}} \left( \frac{\beta}{r} \right)^{\frac{\beta}{1-\eta}} \left[ 1 - \frac{s}{\eta} + 2s - (1 + s)\eta \right] = w
\]

or

\[
\hat{\theta}_O \left[ \frac{\eta}{\eta - s(1 - \eta)} \right] \alpha^\beta \left[ 1 - \eta - \frac{s}{\eta}(1 - \eta)^2 \right]^{1-\eta} = w^{1-\beta} r^\beta,
\]

or

\[
\hat{\theta}_O \left[ \frac{\eta}{\eta - s(1 - \eta)} \right] \alpha^\beta (1 - \eta)^{1-\eta} \left[ \eta - s(1 - \eta) \right]^{1-\eta} = w^{1-\beta} r^\beta,
\]

or

\[
\alpha^\beta (1 - \eta)^{1-\eta} \left[ \frac{\eta}{\eta - s(1 - \eta)} \right]^{\eta} \hat{\theta}_O = w^{1-\beta} r^\beta. \tag{45}
\]

It follows from (44) and (45) that

\[
\alpha^\beta (1 - \alpha - \beta)^{1-\alpha-\beta} \left[ \frac{\eta}{\eta - s(1 - \eta)} \right]^{\eta} \hat{\theta}_O = \alpha^\beta (1 - \alpha - \beta)^{1-\alpha-\beta} \hat{\theta}_R,
\]

or

\[
\hat{\theta}_O = \left[ \eta - s(1 - \eta) \right]^{\eta} \hat{\theta}_R. \tag{46}
\]

Substituting (44) and (46) into (39) we obtain

\[
1 + \lambda \frac{s}{\eta} - (1 - \lambda) \hat{\theta}_R^{\frac{1}{1-\eta}} \left[ \eta - s(1 - \eta) \right]^{1-\eta} \hat{\theta}_R^{\frac{1}{1-\eta}} = 2 - \frac{\eta}{1 - \eta} \frac{w^{\frac{1-\beta}{1-\eta}} r^{\frac{\alpha}{1-\eta}}}{\alpha^\beta} \left\{ (1 - \lambda) \hat{\theta}_R + \lambda \left[ \eta - s(1 - \eta) \right]^{\eta} \hat{\theta}_R \right\},
\]

35
\[
1 + \frac{\lambda}{\eta} - (1 - \lambda) \hat{\theta}_{R}^{2 - \eta} - \lambda \left[ \frac{\eta}{\eta - s(1 - \eta)} \right]^{1 - \eta} = \frac{2 - \eta}{1 - \eta} \left\{ (1 - \lambda) + \lambda \left[ \frac{\eta - s(1 - \eta)}{\eta} \right]^{\eta} - \hat{\theta}_{R}^{2 - \eta} \right\},
\]

or

\[
1 + \frac{\lambda}{\eta} + (1 - \lambda) \hat{\theta}_{R}^{2 - \eta} - \lambda \left[ \frac{\eta}{\eta - s(1 - \eta)} \right]^{1 - \eta} = \frac{2 - \eta}{1 - \eta} \left\{ (1 - \lambda) + \lambda \left[ \frac{\eta - s(1 - \eta)}{\eta} \right]^{\eta} - \hat{\theta}_{R}^{2 - \eta} \right\},
\]

or

\[
1 + \frac{\lambda}{\eta} = \left\{ 1 - \lambda + \lambda \left[ \frac{\eta}{\eta - s(1 - \eta)} \right]^{1 - \eta} + \frac{2 - \eta}{\alpha} (1 - \lambda) + \frac{2 - \eta}{\alpha} \lambda \left[ \frac{\eta - s(1 - \eta)}{\eta} \right]^{\eta} \right\} \hat{\theta}_{R}^{2 - \eta},
\]

or

\[
1 + \frac{\lambda}{\eta} = \left\{ \frac{(2 - \beta) (1 - \lambda)}{\alpha} + \lambda \left[ \frac{\eta}{\eta - s(1 - \eta)} \right]^{1 - \eta} + \frac{2 - \eta}{\alpha} \left[ \frac{\eta - s(1 - \eta)}{\eta} \right]^{\eta} \right\} \hat{\theta}_{R}^{2 - \eta},
\]

or

\[
\hat{\theta}_{R}^{2 - \eta} = \left\{ \frac{(2 - \beta) (1 - \lambda)}{\alpha} + \lambda \left[ \frac{\eta}{\eta - s(1 - \eta)} \right]^{1 - \eta} + \frac{2 - \eta}{\alpha} \left[ \frac{\eta - s(1 - \eta)}{\eta} \right]^{\eta} \right\} 1 + \frac{\lambda}{\eta},
\]
Hence, the ability of the marginal realistic entrepreneur is

\[
\hat{\theta}_R = \left\{ \frac{1 + \lambda_s^\alpha}{\frac{(2-\beta)(1-\lambda)}{\alpha} + \lambda \left[ \frac{\eta}{\eta - s(1-\eta)} \right]^{1-\eta} + \lambda^\beta \left[ \frac{\eta - s(1-\eta)}{\eta} \right]^{\eta}} \right\}^{\frac{1-\eta}{2-\eta}}
\]

\[
= \left\{ \frac{1 + \lambda_s^\alpha}{\frac{\alpha}{2-\beta} \left[ 1 - \lambda + \lambda \frac{\alpha}{2-\beta} \left[ \frac{\eta}{\eta - s(1-\eta)} \right]^{1-\eta} + \lambda^\beta \left[ \frac{\eta - s(1-\eta)}{\eta} \right]^{\eta}} \right\}^{\frac{1-\eta}{2-\eta}}
\]

\[
= \left\{ \frac{1 + \lambda_s^\alpha}{\frac{\alpha}{2-\beta} \left[ 1 - \lambda + \lambda \frac{\alpha}{2-\beta} \left[ \frac{\eta}{\eta - s(1-\eta)} \right]^{1-\eta} + \lambda^\beta \left[ \frac{\eta - s(1-\eta)}{\eta} \right]^{\eta}} \right\}^{\frac{1-\eta}{2-\eta}}
\]

\[
= \left\{ \frac{1 + \lambda_s^\alpha}{\frac{\alpha}{2-\beta} \left[ 1 - \lambda + \lambda \phi(\eta, \beta, s) \right] \left[ \frac{\eta}{\eta - s(1-\eta)} \right]^{1-\eta}} \right\}^{\frac{1-\eta}{2-\eta}}
\]

where

\[
\phi(\eta, \beta, s) = \left[ 1 - \frac{s(1-\eta)(2-\eta)}{\eta(2-\beta)} \right] \left[ \frac{\eta}{\eta - s(1-\eta)} \right]^{1-\eta}
\]

\[
= \left[ 1 - \frac{s(1-\eta)(2-\eta)}{\eta(2-\beta)} \right] \left[ \frac{\eta}{\eta - s(1-\eta)} \right]^{1-\eta}
\]

\[
= \frac{\eta(2-\beta) - s(1-\eta)(2-\eta)}{[\eta - s(1-\eta)](2-\beta)} \left[ \frac{\eta - s(1-\eta)}{\eta} \right]^{\eta}
\]

\[
= \frac{\eta(2-\beta) - s(1-\eta)(2-\beta)(2-\eta)/(2-\beta)}{[\eta - s(1-\eta)](2-\beta)} \left[ \frac{\eta - s(1-\eta)}{\eta} \right]^{\eta}
\]

\[
= \frac{\eta - s(1-\eta)}{\eta - s(1-\eta)} \left[ \frac{\eta - s(1-\eta)}{\eta} \right]^{\eta}
\]

From (46) and (47) the ability of the marginal optimistic entrepreneur is

\[
\hat{\theta}_O = \left[ \frac{\eta - s(1-\eta)}{\eta} \right]^{\eta} \left[ \frac{\alpha}{2-\beta} \left[ 1 + \lambda_s^\alpha \phi(\eta, \beta, s) \right] \right]^{\frac{1-\eta}{2-\eta}}
\]

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From (39) and (43) we have
\[
\left[ (1 - \lambda) \hat{\theta}_R + \lambda \hat{\theta}_O \right] \left( \frac{w}{\alpha} \right)^{\frac{1-\beta}{1-\eta}} \left( \frac{r}{\beta} \right)^{\frac{\beta}{1-\eta}} = K \left( \frac{w}{\alpha} \right)^{\frac{\alpha}{1-\eta}} \left( \frac{r}{\beta} \right)^{\frac{1-\alpha}{1-\eta}},
\]
or
\[
\alpha r \bar{K} = \beta w \left[ (1 - \lambda) \hat{\theta}_R + \lambda \hat{\theta}_O \right],
\]
or
\[
r = \frac{\beta w}{\alpha \bar{K}} [1 - \lambda + \lambda \psi(\alpha, \beta, s)] \hat{\theta}_R,
\]
where
\[
\psi(\eta, s) = \left[ \frac{\eta - s(1 - \eta)}{\eta} \right]^\eta.
\]
Note that (48) and (50) together with $2 - \eta \leq 2 - \beta$ imply
\[
\phi(\eta, \beta, s) \geq \psi(\eta, s).
\]
Substituting (49) into (44) we obtain
\[
\alpha^\beta (1 - \eta)^{1-\eta} \hat{\theta}_R = w^{1-\beta} \left( \frac{\beta}{\alpha} \right)^{\beta} w^\beta [1 - \lambda + \lambda \psi(\eta, s)]^\beta \hat{\theta}_R \bar{K}^{-\beta}.
\]
Solving this equality with respect to $w$ we obtain the equilibrium wage:
\[
w^* = \alpha^\eta (1 - \eta)^{1-\eta} \bar{K}^\beta [1 - \lambda + \lambda \psi(\eta, s)]^{-\beta} \hat{\theta}_R^{1-\beta}
= \alpha^\eta (1 - \eta)^{1-\eta} \bar{K}^\beta \left[ \frac{\alpha}{1 - \lambda + \lambda \psi(\eta, s)} \right]^\beta \left[ \frac{1 + \lambda \phi(\eta, \beta, s)}{2 - \beta 1 - \lambda + \lambda \phi(\eta, \beta, s)} \right]^{(1-\eta)(1-\beta) \frac{2-\eta}{2-\eta}}.
\]
The equilibrium rental cost of capital is equal to
\[
r^* = \frac{\beta w^*}{\alpha \bar{K}} [1 - \lambda + \lambda \psi(\eta, s)] \hat{\theta}_R
= \frac{\beta \alpha^\eta (1 - \eta)^{1-\eta} \bar{K}^\beta [1 - \lambda + \lambda \psi(\eta, s)]^{-\beta} \hat{\theta}_R^{1-\beta}}{\alpha \bar{K}} [1 - \lambda + \lambda \psi(\eta, s)]^{1-\beta} \hat{\theta}_R\bar{K}^{2-\beta}
= \frac{\beta (1 - \eta)^{1-\eta} \alpha^{1-\eta} \bar{K}^{1-\beta} [1 - \lambda + \lambda \psi(\eta, s)]^{1-\beta} \left[ \frac{\alpha}{2 - \beta 1 - \lambda + \lambda \phi(\eta, \beta, s)} \right]^{(1-\eta)(2-\beta) \frac{2-\eta}{2-\eta}}}{\alpha^{1-\eta} \bar{K}^{1-\beta}}.
The equilibrium labor force is equal to

\[ L^* = (1 - \lambda)\hat{\theta}_R + \lambda\hat{\theta}_O = \left\{ 1 - \lambda + \lambda \left[ \frac{\eta - s(1 - \eta)}{\eta} \right]^\eta \right\}_{\frac{\eta - s(1 - \eta)}{\eta}} \hat{\theta}_R \]

\[ = [1 - \lambda + \lambda\psi(\eta, s)] \left[ \frac{\alpha}{2 - \beta} \frac{1 + \lambda \frac{s}{\eta}}{1 - \lambda + \lambda\phi(\eta, \beta, s)} \right]^{\frac{1 - \eta}{\eta}}. \]

For the equilibrium to be well defined we need to make sure that \( \hat{\theta}_O \) is less than \( \frac{\eta - s(1 - \eta)}{\eta} \), i.e.,

\[ \left[ \frac{\eta - s(1 - \eta)}{\eta} \right]^\eta \left[ \frac{\alpha}{2 - \beta} \frac{1 + \lambda \frac{s}{\eta}}{1 - \lambda + \lambda\phi(\eta, \beta, s)} \right]^{\frac{1 - \eta}{\eta}} < \frac{\eta - s(1 - \eta)}{\eta} \]

or

\[ \left[ \frac{\alpha}{2 - \beta} \frac{1 + \lambda \frac{s}{\eta}}{1 - \lambda + \lambda\phi(\eta, \beta, s)} \right]^{\frac{1 - \eta}{\eta}} < \left[ \frac{\eta - s(1 - \eta)}{\eta} \right]^{1 - \eta} \]

or

\[ \frac{\alpha}{2 - \beta} \left( 1 + \lambda \frac{s}{\eta} \right) < (1 - \lambda) \left[ \frac{\eta - s(1 - \eta)}{\eta} \right]^{2 - \eta} + \lambda \left[ 1 - \frac{s(1 - \eta)(2 - \eta)}{\eta(2 - \beta)} \right] \left[ \frac{\eta - s(1 - \eta)}{\eta} \right]. \]

The LHS of (52) is increasing in \( s \) whereas the RHS of (52) is decreasing in \( s \). When \( s \) is equal to 0 the LSH of (52) is equal to \( \alpha / (2 - \beta) < 1 \) and the RHS of (52) is equal to 1. When \( s \) is equal to \( \eta / (1 - \eta) \) the LSH of (52) is equal to \( \alpha(1 - \eta + \lambda) / (2 - \beta)(1 - \eta) \) and the RHS of (52) is equal to 0. Hence, there exists a unique \( s \in (0, \eta / (1 - \eta)) \) such that the LHS and RHS of (52) are the same, which is given by

\[ \frac{\alpha}{2 - \beta} \left( 1 + \lambda \frac{s}{\eta} \right) = (1 - \lambda) \left[ \frac{\eta - s(1 - \eta)}{\eta} \right]^{2 - \eta} + \lambda \left[ 1 - \frac{s(1 - \eta)(2 - \eta)}{\eta(2 - \beta)} \right] \left[ \frac{\eta - s(1 - \eta)}{\eta} \right]. \]

or

\[ \frac{\alpha}{2 - \beta} \left( 1 + \lambda \frac{s}{\eta} \right) = \left[ 1 - \lambda + \lambda\phi(\eta, \beta, s) \right] \left[ \frac{\eta - s(1 - \eta)}{\eta} \right]^{2 - \eta}, \]

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which is (23). Hence, inequality (52) is satisfied as long as \( s < \bar{s} \). For the equilibrium to be well defined we also need to make sure that \( \hat{\theta}_R \) is less than 1. From (46) we have

\[
\hat{\theta}_R = \left[ \frac{\eta}{\eta - s(1 - \eta)} \right]^{\eta} \hat{\theta}_O < \left[ \frac{\eta}{\eta - s(1 - \eta)} \right]^{\eta} \frac{\eta - s(1 - \eta)}{\eta} = \left[ \frac{\eta - s(1 - \eta)}{\eta} \right]^{1 - \eta} < 1.
\]

where the first inequality follows \( s \leq \bar{s} \). Q.E.D.

**Proof of Proposition 3:** Assume \( s < \bar{s} \). We wish to show that

\[
\frac{\partial \hat{\theta}_R}{\partial \lambda} > 0.
\]

From the definition of \( \hat{\theta}_R \) we have

\[
\frac{\partial \hat{\theta}_R}{\partial \lambda} = \frac{1 - \eta}{2 - \eta} \left( \frac{\alpha}{2 - \beta} \right)^{1 - \eta} \left( \frac{1 + \frac{\lambda s}{\eta}}{1 - \lambda + \lambda \phi} \right)^{-\eta} \frac{1}{2 - \eta} \partial \left( \frac{1 + \frac{\lambda s}{\eta}}{1 - \lambda + \lambda \phi} \right)\\
= \frac{1 - \eta}{2 - \eta} \left( \frac{\alpha}{2 - \beta} \right)^{1 - \eta} \left( \frac{1 + \frac{\lambda s}{\eta}}{1 - \lambda + \lambda \phi} \right)^{-\eta} \frac{1 + \frac{s}{\eta} - \phi}{(1 - \lambda + \lambda \phi)^2}\\
= \frac{1 - \eta}{2 - \eta} \hat{\theta}_R \frac{1 + \frac{s}{\eta} - \phi}{(1 + \frac{\lambda s}{\eta})(1 - \lambda + \lambda \phi)}.
\]

(53)

Hence, \( \frac{\partial \hat{\theta}_R}{\partial \lambda} > 0 \) as long as

\[
1 + \frac{s}{\eta} > \phi = \left[ 1 - \frac{s(1 - \eta)(2 - \eta)}{\eta(2 - \beta)} \right] \left[ \frac{\eta}{\eta - s(1 - \eta)} \right]^{1 - \eta}.
\]

(54)
Note that \( \phi > 0 \) and \( \phi(\eta, \beta, 0) = 1 \). The derivative of \( \phi \) with respect to \( s \) is equal to

\[
\frac{\partial \phi}{\partial s} = -(1 - \eta)(2 - \eta) \left[ \frac{\eta}{\eta - s(1 - \eta)} \right]^{1-\eta}
\]

\[
+ \left[ 1 - \frac{s(1 - \eta)(2 - \eta)}{\eta(2 - \beta)} \right] \left[ \frac{\eta}{\eta - s(1 - \eta)} \right]^{1-\eta} \frac{(1 - \eta)^2}{\eta - s(1 - \eta)}
\]

\[
= \frac{(1 - \eta)[(2 - \beta)(1 - \eta) - (2 - \eta) + s(1 - \eta)(2 - \eta)] \left[ \frac{\eta}{\eta - s(1 - \eta)} \right]^{1-\eta}}{[\eta - s(1 - \eta)](2 - \beta)}
\]

\[
= \frac{(1 - \eta)[-\eta - \beta(1 - \eta) + s(1 - \eta)(2 - \eta)] \left[ \frac{\eta}{\eta - s(1 - \eta)} \right]^{1-\eta}}{[\eta - s(1 - \eta)](2 - \beta)}
\]

\[\text{(55)}\]

It follows from (55) that: (i) \( \phi \) decreases with \( s \) when \( s \in [0, \bar{s}] \), (ii) \( \phi \) increases with \( s \) when \( s \in (\bar{s}, \eta/(1 - \eta)) \), and (iii) \( \phi \) attains a minimum at \( \bar{s} = \frac{\eta + \beta(1 - \eta)}{(1 - \eta)(2 - \eta)} < \frac{\eta}{1 - \eta} \), which is given by

\[
\phi(\bar{s}) = \left[ 1 - \frac{\eta + \beta(1 - \eta)}{(1 - \eta)(2 - \eta)} \right] \left[ \frac{\eta}{\eta - \frac{\eta + \beta(1 - \eta)}{(1 - \eta)(2 - \eta)}(1 - \eta)} \right]^{1-\eta}
\]

\[
= \left[ 1 - \frac{\eta + \beta(1 - \eta)}{\eta(2 - \beta)} \right] \left[ \frac{\eta}{\eta - \frac{\eta + \beta(1 - \eta)}{2 - \eta}} \right]^{1-\eta}
\]

\[
= \frac{\eta(2 - \beta) - \eta - \beta(1 - \eta)}{\eta(2 - \beta)} \left[ \frac{(2 - \eta)\eta}{(2 - \eta)\eta - \eta - \beta(1 - \eta)} \right]^{1-\eta}
\]

\[
= \frac{\eta - \beta}{\eta(2 - \beta)} \left[ \frac{(2 - \eta)\eta}{(1 - \eta)(\eta - \beta)} \right]^{1-\eta} = \frac{\alpha}{\eta(2 - \beta)} \left[ \frac{(2 - \eta)\eta}{(1 - \eta)\alpha} \right]^{1-\eta}.
\]

This implies that

\[
\max_{s \in [0, \bar{s}]} \phi(s) = \max\{1, \phi(\bar{s})\}.
\]

We know from the definition of \( \bar{s} \) that

\[
\phi(\bar{s}) = \frac{\frac{\alpha}{2 - \beta} \left[ 1 + \frac{\lambda \bar{s}}{\eta} \right]}{\frac{\lambda}{\eta - s(1 - \eta)} \frac{2 - \eta}{2 - \eta}} - \frac{1 - \lambda}{\lambda}.
\]

If we can show that \( 1 + \frac{\bar{s}}{\eta} > \phi(\bar{s}) \) we are done:

\[
1 + \frac{\bar{s}}{\eta} > \frac{\frac{\alpha}{2 - \beta} \left[ 1 + \frac{\lambda \bar{s}}{\eta} \right]}{\frac{\lambda}{\eta - s(1 - \eta)} \frac{2 - \eta}{2 - \eta}} - \frac{1 - \lambda}{\lambda},
\]

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or
\[
1 + \frac{\lambda \bar{s}}{\eta} > \frac{\alpha}{2 - \beta} \left( 1 + \frac{\lambda \bar{s}}{\eta} \right) \left( \frac{\alpha}{2 - \beta} \right)^{\frac{1}{2 - \beta}},
\]

or
\[
\left( \frac{\eta - \bar{s}(1 - \eta)}{\eta} \right)^{2 - \eta} > \frac{\alpha}{2 - \beta},
\]

or
\[
\eta - \bar{s}(1 - \eta) > \eta \left( \frac{\alpha}{2 - \beta} \right)^{\frac{1}{2 - \eta}},
\]

or
\[
\bar{s} < \frac{\eta}{1 - \eta} \left[ 1 - \left( \frac{\alpha}{2 - \beta} \right)^{\frac{1}{2 - \eta}} \right].
\]

From the definition of \( \bar{s} \) this inequality is satisfied if
\[
\frac{\alpha}{2 - \beta} \left[ 1 + \frac{\lambda}{1 - \eta} - \frac{\lambda}{1 - \eta} \left( \frac{\alpha}{2 - \beta} \right)^{\frac{1}{2 - \eta}} \right] > (1 - \lambda) \left( \frac{\alpha}{2 - \beta} \right) + \lambda \left\{ 1 - \left[ 1 - \left( \frac{\alpha}{2 - \beta} \right)^{\frac{1}{2 - \eta}} \right] \frac{2 - \eta}{2 - \beta} \right\} \left( \frac{\alpha}{2 - \beta} \right)^{\frac{1}{2 - \eta}},
\]

or
\[
\left[ 1 + \frac{1}{1 - \eta} - \frac{1}{1 - \eta} \left( \frac{\alpha}{2 - \beta} \right)^{\frac{1}{2 - \eta}} \right] \left( \frac{\alpha}{2 - \beta} \right)^{\frac{1}{2 - \eta}} > 1 - \left[ 1 - \left( \frac{\alpha}{2 - \beta} \right)^{\frac{1}{2 - \eta}} \right] \frac{2 - \eta}{2 - \beta},
\]

or
\[
\left[ 1 + \frac{1}{1 - \eta} - \frac{1}{1 - \eta} \left( \frac{\alpha}{2 - \beta} \right)^{\frac{1}{2 - \eta}} \right] \left( \frac{\alpha}{2 - \beta} \right)^{\frac{1}{2 - \eta}} > 1 - \frac{2 - \eta}{2 - \beta} + \frac{2 - \eta}{2 - \beta} \left( \frac{\alpha}{2 - \beta} \right)^{\frac{1}{2 - \eta}},
\]

or
\[
\left[ 1 + \frac{1}{1 - \eta} - \frac{1}{1 - \eta} \left( \frac{\alpha}{2 - \beta} \right)^{\frac{1}{2 - \eta}} \right] \left( \frac{\alpha}{2 - \beta} \right)^{\frac{1}{2 - \eta}} > \frac{\alpha}{2 - \beta} + \frac{2 - \eta}{2 - \beta} \left( \frac{\alpha}{2 - \beta} \right)^{\frac{1}{2 - \eta}},
\]

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or
\[ 1 + \frac{1}{1 - \eta} - \frac{1}{1 - \eta} \left( \frac{\alpha}{2 - \beta} \right)^{\frac{1}{x-\eta}} > \left[ \frac{\alpha}{2 - \beta} + \frac{2 - \eta}{2 - \beta} \left( \frac{\alpha}{2 - \beta} \right)^{\frac{1}{x-\eta}} \right] \left( \frac{\alpha}{2 - \beta} \right)^{-\frac{1}{x-\eta}}, \]
or
\[ 1 + \frac{1}{1 - \eta} - \frac{1}{1 - \eta} \left( \frac{\alpha}{2 - \beta} \right)^{\frac{1}{x-\eta}} > \left( \frac{\alpha}{2 - \beta} \right)^{\frac{1}{x-\eta}} + \frac{2 - \eta}{2 - \beta} \left( \frac{\alpha}{2 - \beta} \right)^{\frac{\eta}{x-\eta}}, \]
or
\[ 2 - \eta \left[ 1 - \left( \frac{\alpha}{2 - \beta} \right)^{\frac{1}{x-\eta}} \right] > \frac{2 - \eta}{2 - \beta} \left( \frac{\alpha}{2 - \beta} \right)^{\frac{\eta}{x-\eta}}, \]
or
\[ 2 - \eta \left[ 1 - \left( \frac{\alpha}{2 - \beta} \right)^{\frac{1}{x-\eta}} \right] > \frac{2 - \alpha - \beta}{2 - \beta} \left( \frac{\alpha}{2 - \beta} \right)^{\frac{\eta}{x-\eta}}, \]
or
\[ 2 - \eta \left[ 1 - \left( \frac{\alpha}{2 - \beta} \right)^{\frac{1}{x-\eta}} \right] > \left( \frac{\alpha}{2 - \beta} \right)^{\frac{\eta}{x-\eta}} - \left( \frac{\alpha}{2 - \beta} \right)^{\frac{2}{x-\eta}}, \]
or
\[ 2 - \eta \left[ 1 - \left( \frac{\alpha}{2 - \beta} \right)^{\frac{1}{x-\eta}} \right] - \left( \frac{\alpha}{2 - \beta} \right)^{\frac{\eta}{x-\eta}} + \left( \frac{\alpha}{2 - \beta} \right)^{\frac{2}{x-\eta}} > 0, \]
or
\[ \left[ 1 - \left( \frac{\alpha}{2 - \beta} \right)^{\frac{1}{x-\eta}} \right] + \frac{1}{1 - \eta} \left[ 1 - \left( \frac{\alpha}{2 - \beta} \right)^{\frac{1}{x-\eta}} \right] - \left( \frac{\alpha}{2 - \beta} \right)^{\frac{\eta}{x-\eta}} + \left( \frac{\alpha}{2 - \beta} \right)^{\frac{2}{x-\eta}} > 0, \]
or
\[ \left[ 1 - \left( \frac{\alpha}{2 - \beta} \right)^{\frac{\eta}{x-\eta}} \right] + \left\{ \frac{1}{1 - \eta} \left[ 1 - \left( \frac{\alpha}{2 - \beta} \right)^{\frac{1}{x-\eta}} \right] - \left( \frac{\alpha}{2 - \beta} \right)^{\frac{1}{x-\eta}} \right\} + \left( \frac{\alpha}{2 - \beta} \right)^{\frac{2}{x-\eta}} > 0. \]

This inequality holds because the tree terms inside brackets in the LHS are strictly positive. Hence, we have shown that $\frac{\partial \hat{\theta}_R}{\partial \lambda} > 0$. Let us now show that

\[ \frac{\partial \hat{\theta}_O}{\partial \lambda} > 0. \]

We know from Proposition 2 that

\[ \hat{\theta}_O = \left[ \frac{\eta - s(1 - \eta)}{\eta} \right]^\eta \hat{\theta}_R. \]
Hence
\[
\frac{\partial \hat{\theta}_O}{\partial \lambda} = \left[ \frac{\eta - s(1-\eta)}{\eta} \right]^{\eta} \frac{\partial \hat{\theta}_R}{\partial \lambda} > 0,
\]
where the inequality follows from the fact that $\partial \hat{\theta}_R/\partial \lambda > 0$.

**Q.E.D.**

**Proof of Proposition 4**: Setting $\lambda = 0$ in (20) we obtain the rational expectations equilibrium wage:

\[
w_0^* = \alpha^n (1 - \eta)^{1-\eta} K^\beta \left( \frac{\alpha}{2 - \beta} \right)^{(1-\eta)(1-\beta)}.
\]

From (20) the optimal expectations equilibrium wage is higher than the rational expectations equilibrium wage as long as

\[
\frac{\alpha^n(1 - \eta)^{1-\eta} K^\beta}{[1 - \lambda + \lambda \psi(\eta, s)]^\beta} \left[ \frac{1 + \lambda^2}{2 - \beta} 1 - \lambda + \lambda \phi(\eta, \beta, s) \right]^{(1-\eta)(1-\beta)} \frac{\alpha}{2 - \beta} > \alpha^n (1 - \eta)^{1-\eta} K^\beta \left( \frac{\alpha}{2 - \beta} \right)^{(1-\eta)(1-\beta)},
\]

or

\[
\left[ \frac{1 + \lambda^2}{1 - \lambda + \lambda \phi(\eta, \beta, s)} \right]^{(1-\eta)(1-\beta)} > [1 - \lambda + \lambda \psi(\eta, s)]^\beta .
\]

A sufficient condition for this inequality to be satisfied is:

\[
\left[ \frac{1 + \lambda^2}{1 - \lambda + \lambda \phi(\eta, \beta, s)} \right]^{(1-\eta)(1-\beta)} > 1 > [1 - \lambda + \lambda \psi(\eta, s)]^\beta .
\]

(56)

The first inequality in (56) is satisfied since

\[
\frac{1 + \lambda^2}{1 - \lambda + \lambda \phi(\eta, \beta, s)} > 1 \iff 1 + \frac{s}{\eta} > \phi(\eta, \beta, s),
\]

which is true by Proposition 3. The second inequality in (56) is satisfied since $\psi(\eta, s) < 1$ implies

\[
1 - \lambda + \lambda \psi(\eta, s) < 1,
\]

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Therefore, we have \( w^* > w_0^* \). \( \text{Q.E.D.} \)

**Proof of Proposition 5**: When \( s < \bar{s} \) the equilibrium share of realists amongst workers is equal to

\[
\gamma_L^* = \frac{(1 - \lambda) \hat{\theta}_R}{(1 - \lambda) \hat{\theta}_R + \lambda \hat{\theta}_O}.
\]

Since

\[
\hat{\theta}_O = \left[ \frac{\eta - s(1 - \eta)}{\eta} \right]^{\eta} \hat{\theta}_R,
\]

we have

\[
\gamma_L^* = \frac{(1 - \lambda) \hat{\theta}_R}{(1 - \lambda) \hat{\theta}_R + \lambda \left[ \frac{\eta - s(1 - \eta)}{\eta} \right]^{\eta} \hat{\theta}_R} = \frac{1 - \lambda}{1 - \lambda + \lambda \left[ \frac{\eta - s(1 - \eta)}{\eta} \right]^{\eta}}.
\]

Therefore, the majority of workers are realists when

\[
\frac{1 - \lambda}{1 - \lambda + \lambda \left[ \frac{\eta - s(1 - \eta)}{\eta} \right]^{\eta}} > \frac{1}{2},
\]

or

\[
1 - \lambda > \lambda \left[ \frac{\eta - s(1 - \eta)}{\eta} \right]^{\eta},
\]

or

\[
\lambda < \frac{1}{1 + \left[ \frac{\eta - s(1 - \eta)}{\eta} \right]^{\eta}}.
\]

When \( s < \bar{s} \) the equilibrium share optimists amongst entrepreneurs is equal to

\[
\gamma_E^* = \frac{\lambda(1 - \hat{\theta}_O)}{\lambda(1 - \hat{\theta}_O) + (1 - \lambda)(1 - \hat{\theta}_R)}.
\]

Therefore, the majority of entrepreneurs are optimists when

\[
\frac{\lambda(1 - \hat{\theta}_O)}{\lambda(1 - \hat{\theta}_O) + (1 - \lambda)(1 - \hat{\theta}_R)} > \frac{1}{2},
\]

or

\[
2\lambda(1 - \hat{\theta}_O) > \lambda(1 - \hat{\theta}_O) + (1 - \lambda)(1 - \hat{\theta}_R),
\]

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or

$$\lambda(1 - \hat{\theta}_O) > (1 - \lambda)(1 - \hat{\theta}_R),$$

or

$$\lambda(1 - \hat{\theta}_O) + \lambda(1 - \hat{\theta}_R) > 1 - \hat{\theta}_R,$$

or

$$\lambda > \frac{1 - \hat{\theta}_R}{2 - \hat{\theta}_R - \hat{\theta}_O} = \frac{1 - \hat{\theta}_R}{1 - \hat{\theta}_R + 1 - \hat{\theta}_R \left[ \frac{\alpha - \delta(1 - \alpha)}{\alpha} \right]^\alpha} = \frac{1}{1 + \frac{1 - \hat{\theta}_R \left[ \frac{\alpha - \delta(1 - \alpha)}{\alpha} \right]^\alpha}{1 - \hat{\theta}_R}}.$$  

Q.E.D.

**Proof of Proposition 6**: Assume $s \in (\bar{s}, \eta/(1 - \eta))$. The first step to determine the optimal expectations equilibrium is to find out the labor market equilibrium condition. The labor demand from entrepreneurs with rational expectations is given by (35). If $s > \bar{s}$, then labor demand from entrepreneurs with optimal expectations is

$$L^D_O = \lambda \int_{\hat{\theta}_O}^{1} l(w, r, 1)d\theta_0 = \lambda(1 - \hat{\theta}_O) \left( \frac{\alpha}{w} \right)^{\frac{1 - \delta}{1 - \eta}} \left( \frac{\beta}{r} \right)^{\frac{\delta}{1 - \eta}}. \quad (57)$$

From (35) and (57), labor demand is equal to

$$L^D = (1 - \lambda) \frac{1 - \eta}{2 - \eta} \left( 1 - \hat{\theta}_R \right)^{\frac{2 - \eta}{1 - \eta}} \left( \frac{\alpha}{w} \right)^{\frac{1 - \delta}{1 - \eta}} \left( \frac{\beta}{r} \right)^{\frac{\delta}{1 - \eta}} + \lambda(1 - \hat{\theta}_O) \left( \frac{\alpha}{w} \right)^{\frac{1 - \delta}{1 - \eta}} \left( \frac{\beta}{r} \right)^{\frac{\delta}{1 - \eta}}$$

$$= \left[ (1 - \lambda) \frac{1 - \eta}{2 - \eta} \left( 1 - \hat{\theta}_R \right)^{\frac{2 - \eta}{1 - \eta}} + \lambda(1 - \hat{\theta}_O) \right] \left( \frac{\alpha}{w} \right)^{\frac{1 - \delta}{1 - \eta}} \left( \frac{\beta}{r} \right)^{\frac{\delta}{1 - \eta}}.$$

Labor supply is given by

$$L^S = (1 - \lambda) \int_{0}^{\hat{\theta}_O} d\theta_0 + \lambda \int_{0}^{\hat{\theta}_O} d\theta_0 = (1 - \lambda)\hat{\theta}_R + \lambda\hat{\theta}_O.$$

In equilibrium, labor demand must equal labor supply

$$\left[ (1 - \lambda) \frac{1 - \eta}{2 - \eta} \left( 1 - \hat{\theta}_R \right)^{\frac{2 - \eta}{1 - \eta}} + \lambda(1 - \hat{\theta}_O) \right] \left( \frac{\alpha}{w} \right)^{\frac{1 - \delta}{1 - \eta}} \left( \frac{\beta}{r} \right)^{\frac{\delta}{1 - \eta}} = (1 - \lambda)\hat{\theta}_R + \lambda\hat{\theta}_O. \quad (58)$$

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The second step to determine the optimal expectations equilibrium is to find out the capital market equilibrium condition. The capital demand from entrepreneurs with rational expectations is given by (40). If \( s > \bar{s} \), then capital demand from entrepreneurs with optimal expectations is

\[
K^D_O = \lambda \int_{\theta_O}^{1} k(w, r, 1)d\theta_0 = \lambda(1 - \hat{\theta}_O) \left( \frac{\alpha}{w} \right)^{\frac{\alpha}{1-\eta}} \left( \frac{\beta}{r} \right)^{\frac{1-\alpha}{1-\eta}}. \tag{59}
\]

From (40) and (59), capital demand is equal to

\[
K^D = (1 - \lambda) \left( \frac{1-\eta}{2-\eta} \right) \left( 1 - \hat{\theta}_R^{2-\eta} \right) \left( \frac{\alpha}{w} \right)^{\frac{\alpha}{1-\eta}} \left( \frac{\beta}{r} \right)^{\frac{1-\alpha}{1-\eta}} + \lambda(1 - \hat{\theta}_O) \left( \frac{\alpha}{w} \right)^{\frac{\alpha}{1-\eta}} \left( \frac{\beta}{r} \right)^{\frac{1-\alpha}{1-\eta}}.
\]

In equilibrium, capital demand must equal the exogenous capital supply

\[
\left[ (1 - \lambda) \left( \frac{1-\eta}{2-\eta} \right) \left( 1 - \hat{\theta}_R^{2-\eta} \right) + \lambda(1 - \hat{\theta}_O) \right] \left( \frac{\alpha}{w} \right)^{\frac{\alpha}{1-\eta}} \left( \frac{\beta}{r} \right)^{\frac{1-\alpha}{1-\eta}} = K. \tag{60}
\]

The third step to determine the competitive optimal expectations equilibrium is to find out \( \hat{\theta}_R \) and \( \hat{\theta}_O \). An individual with entrepreneurial ability \( \hat{\theta}_R \) and rational expectation of ability is indifferent between being an entrepreneur and a worker at \( t = 1 \) when (44) holds:

\[
\alpha^\alpha \beta^\beta (1 - \eta)^{1-\eta} \hat{\theta}_R = w^{1-\beta} r^\beta. \tag{61}
\]

An individual with entrepreneurial ability \( \hat{\theta}_O \) and optimal expectation of entrepreneurial ability \( \theta^* = 1 \) is indifferent between being an entrepreneur and a worker at \( t = 1 \) when

\[
(\hat{\theta}_O + s\theta^*)[l(w, r, \theta^*)]^\alpha[k(w, r, \theta^*)]^\beta - (1 + s) [wl(w, r, \theta^*) + rk(w, r, \theta^*)] = w,
\]

or

\[
(\hat{\theta}_O + s) \left[ \left( \frac{\alpha}{w} \right)^{\frac{1-\beta}{1-\eta}} \left( \frac{\beta}{r} \right)^{\frac{\beta}{1-\eta}} \right]^\alpha \left[ \left( \frac{\alpha}{w} \right)^{\frac{\alpha}{1-\eta}} \left( \frac{\beta}{r} \right)^{\frac{1-\alpha}{1-\eta}} \right]^\beta
\]

\[-(1 + s) \left[ w \left( \frac{\alpha}{w} \right)^{\frac{1-\beta}{1-\eta}} \left( \frac{\beta}{r} \right)^{\frac{\beta}{1-\eta}} + r \left( \frac{\alpha}{w} \right)^{\frac{\alpha}{1-\eta}} \left( \frac{\beta}{r} \right)^{\frac{1-\alpha}{1-\eta}} \right] = w,
\]

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or
\[
(\hat{\theta}_O + s) \left( \frac{\alpha}{w} \right)^{\frac{\alpha}{\alpha - \eta}} \left( \frac{\beta}{r} \right)^{\frac{\beta}{\alpha - \eta}} - (1 + s) \left( \frac{\alpha}{w} \right)^{\frac{\alpha}{\alpha - \eta}} \left( \frac{\beta}{r} \right)^{\frac{\beta}{\alpha - \eta}} \left[ w \left( \frac{\alpha}{w} \right) + r \left( \frac{\beta}{r} \right) \right] = w,
\]
or
\[
\left( \frac{\alpha}{w} \right)^{\frac{\alpha}{\alpha - \eta}} \left( \frac{\beta}{r} \right)^{\frac{\beta}{\alpha - \eta}} \left[ \hat{\theta}_O + s - (1 + s)\eta \right] = w,
\]
or
\[
\alpha^{\frac{\alpha}{\alpha - \eta}} \beta^{\frac{\beta}{\alpha - \eta}} \left[ \hat{\theta}_O + s - (1 + s)\eta \right] = w^{\frac{1 - \beta}{\alpha - \eta}} r^{\frac{\beta}{\alpha - \eta}},
\]
or
\[
\alpha^{\frac{\alpha}{\alpha - \eta}} \beta^{\frac{\beta}{\alpha - \eta}} \left[ \hat{\theta}_O + s - (1 + s)\eta \right]^{1 - \eta} = w^{1 - \beta} r^{\beta}.
\]
Equations (58), (60), (61), and (62) define the optimal expectations equilibrium when \( s \in (\bar{s}, \eta/(1 - \eta)) \).

**Proof of Proposition 7:** Let \( s < \bar{s} \). The equilibrium wage is equal to
\[
w^* = \frac{\alpha^\eta (1 - \eta)^{1 - \eta} \bar{K}^\beta}{(1 - \lambda + \lambda\psi)^\beta} \left( \frac{\alpha}{2 - \beta} \frac{1 + \lambda s}{1 - \lambda + \lambda\phi} \right)^{\frac{(1 - \eta)(1 - \beta)}{2 - \eta}}
\]
\[
= \frac{\alpha^\eta (1 - \eta)^{1 - \eta} \bar{K}^\beta}{(1 - \lambda + \lambda\psi)^\beta} \bar{R}^{1 - \beta}.
\]
The impact of a change in \( \lambda \) on \( w^* \) is given by
\[
\frac{\partial w^*}{\partial \lambda} = \beta \alpha^\eta (1 - \eta)^{1 - \eta} \bar{K}^\beta (1 - \lambda + \lambda\psi)^{-\beta-1} (1 - \psi) \hat{\theta}_R^{1 - \beta}
\]
\[
+ (1 - \beta) \frac{\alpha^\eta (1 - \eta)^{1 - \eta} \bar{K}^\beta}{(1 - \lambda + \lambda\psi)^\beta} \hat{\theta}_R \frac{\partial \hat{\theta}_R}{\partial \lambda}
\]
\[
= \frac{\alpha^\eta (1 - \eta)^{1 - \eta} \bar{K}^\beta}{(1 - \lambda + \lambda\psi)^\beta} \left[ \beta \frac{1 - \psi}{1 - \lambda + \lambda\psi} \hat{\theta}_R + (1 - \beta) \frac{\partial \hat{\theta}_R}{\partial \lambda} \right] > 0.
\]
Hence, an increase in \( \lambda \) raises the equilibrium wage.

**Proof of Proposition 8:** Let \( s < \bar{s} \).
(i) The equilibrium fraction of realistic workers is given by

\[ L_R^* = (1 - \lambda)\hat{\theta}_R. \]

Hence,

\[ \frac{\partial L_R^*}{\partial \lambda} = -\hat{\theta}_R + (1 - \lambda)\frac{\partial \hat{\theta}_R}{\partial \lambda} \]

\[ = -\hat{\theta}_R + (1 - \lambda)\frac{1 - \eta \hat{\theta}_R}{2 - \eta} \frac{1 + \frac{s}{\eta} - \phi}{(1 + \frac{\lambda s}{\eta})(1 - \lambda + \lambda \phi)} \]

\[ = \left[ -1 + (1 - \lambda)\frac{1 - \eta}{2 - \eta} \frac{1 + \frac{s}{\eta} - \phi}{(1 + \frac{\lambda s}{\eta})(1 - \lambda + \lambda \phi)} \right] \hat{\theta}_R, \]

where the second equality follows from (53). Therefore \( \frac{\partial L_R^*}{\partial \lambda} < 0 \) when

\[ (1 - \lambda)\frac{1 - \eta}{2 - \eta} \frac{1 + \frac{s}{\eta} - \phi}{(1 + \frac{\lambda s}{\eta})(1 - \lambda + \lambda \phi)} < 1, \]

or

\[ (1 - \eta)(1 - \lambda) \left( 1 + \frac{s}{\eta} - \phi \right) < (2 - \eta)(1 - \lambda + \lambda \phi) \left( 1 + \frac{\lambda s}{\eta} \right). \]

Simplifying and rearranging terms this inequality is equivalent to

\[ 0 < -\lambda(1 - \phi) + s \left( 2\lambda - \lambda^2 \right) \left( \frac{1}{\eta} - 1 \right) + s \left( \lambda - \lambda^2 \right) \frac{1}{\eta} \]

\[ + s \lambda^2 \left( \frac{2}{\eta} - 1 \right) \phi + \left[ 1 - s \left( \frac{1}{\eta} - 1 \right) \right] + \phi(1 - \eta). \]

Since \( s < \eta/(\eta - 1) \) all terms in the RHS of the inequality are non-negative when \( \phi \geq 1 \). Hence the inequality is satisfied when \( \phi \in [1, 1 + s/\eta] \). We now show that the inequality is also satisfied when \( \phi \in (0, 1) \). When \( \phi \in (0, 1) \) the RHS is a concave function of \( \lambda \) (the second derivative of the RHS with respect to \( \lambda \) is equal to \( -2(1 - \phi)(2 - \eta)s^2/\eta \)). Hence, the RHS attains a minimum either at \( \lambda = 0 \) or at \( \lambda = 1 \). When \( \lambda = 0 \) the inequality becomes

\[ 0 < \left[ 1 - s \left( \frac{1}{\eta} - 1 \right) \right] + \phi(1 - \eta), \]

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which is true. When \( \lambda = 1 \) the inequality becomes

\[
0 < \phi + s \left( \frac{2}{\eta} - 1 \right) \phi + \phi(1 - \eta),
\]

which is true. Therefore, when \( \phi \in (0, 1) \) the inequality is satisfied. Hence, we have shown that \( \partial L^*_R / \partial \lambda < 0 \).

(ii) The equilibrium fraction of optimistic workers is given by

\[
L^*_O = \lambda \hat{\theta}_O,
\]

which is an increasing function of \( \lambda \) since \( \hat{\theta}_O \) increases with \( \lambda \). Therefore, \( \partial L^*_O / \partial \lambda > 0 \).

(iii) The equilibrium fraction of realistic entrepreneurs is given by

\[
E^*_R = (1 - \lambda)(1 - \hat{\theta}_R),
\]

which is a decreasing function of \( \lambda \) since \( \hat{\theta}_R \) increases with \( \lambda \). Therefore, \( \partial E^*_R / \partial \lambda < 0 \).

\textbf{Q.E.D.}

\textbf{Proof of Proposition 9:} Let \( s < \bar{s} \).

(i) We wish to show that the equilibrium fraction of workers \( L^* \) is a concave function of the fraction of optimists \( \lambda \). The equilibrium fraction of workers is equal to

\[
L^* = (1 - \lambda)\hat{\theta}_R + \lambda \hat{\theta}_O = (1 - \lambda)\hat{\theta}_R + \lambda \psi \hat{\theta}_R = (1 - \lambda + \lambda \psi)\hat{\theta}_R.
\]

The impact of a change in \( \lambda \) on \( L^* \) is given by

\[
\frac{\partial L^*}{\partial \lambda} = -(1 - \psi)\hat{\theta}_R + (1 - \lambda + \lambda \psi) \frac{\partial \hat{\theta}_R}{\partial \lambda}.
\]

From (63) it follows that

\[
\frac{\partial^2 L^*}{\partial \lambda^2} = -2(1 - \psi) \frac{\partial \hat{\theta}_R}{\partial \lambda} + (1 - \lambda + \lambda \psi) \frac{\partial^2 \hat{\theta}_R}{\partial \lambda^2}.
\]

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We know from Proposition 3 that
\[
\frac{\partial \hat{\theta}_R}{\partial \lambda} = \frac{\partial}{\partial \lambda} \left[ \left( \frac{\alpha}{2 - \beta} \frac{1 + \lambda \frac{s}{\eta}}{1 - \lambda + \lambda \phi} \right)^{\frac{1-\eta}{2-\eta}} \right] \\
= \frac{1 - \eta}{2 - \eta} \left( \frac{\alpha}{2 - \beta} \frac{1 + \lambda \frac{s}{\eta}}{1 - \lambda + \lambda \phi} \right)^{\frac{1-\eta}{2-\eta} - 1} \frac{\alpha}{2 - \beta} \frac{\partial}{\partial \lambda} \left( \frac{1 + \lambda \frac{s}{\eta}}{1 - \lambda + \lambda \phi} \right) \\
= \frac{1 - \eta}{2 - \eta} \left( \frac{\alpha}{2 - \beta} \right)^{\frac{1-\eta}{2-\eta}} \left( \frac{1 + \lambda \frac{s}{\eta}}{1 - \lambda + \lambda \phi} \right)^{-\frac{1}{2-\eta}} \frac{1 + \frac{s}{\eta} - \phi}{(1 - \lambda + \lambda \phi)^2} \\
= \frac{1 - \eta}{2 - \eta} \left( \frac{\alpha}{2 - \beta} \right)^{\frac{1-\eta}{2-\eta}} \left( \frac{1 + \lambda \frac{s}{\eta}}{1 - \lambda + \lambda \phi} \right)^{-\frac{1}{2-\eta}} \frac{1 + \frac{s}{\eta} - \phi}{(1 - \lambda + \lambda \phi)^2}.
\] (65)

From (65) we obtain
\[
\frac{\partial^2 \hat{\theta}_R}{\partial \lambda^2} = -\frac{z}{2 - \eta} \left( \frac{1 + \lambda \frac{s}{\eta}}{1 - \lambda + \lambda \phi} \right)^{-\frac{1}{2-\eta} - 1} (1 + \frac{s}{\eta} - \phi)^2 \\
+ 2z \left( \frac{1 + \lambda \frac{s}{\eta}}{1 - \lambda + \lambda \phi} \right)^{-\frac{1}{2-\eta}} \left( 1 + \frac{s}{\eta} - \phi \right) (1 - \phi) \\
= -\frac{z}{2 - \eta} \left( \frac{1 + \lambda \frac{s}{\eta}}{1 - \lambda + \lambda \phi} \right)^{-\frac{1}{2-\eta}} 1 - \lambda + \lambda \phi \frac{(1 + \frac{s}{\eta} - \phi)^2}{1 + \lambda \frac{s}{\eta}} \frac{1 + \frac{s}{\eta} - \phi}{(1 - \lambda + \lambda \phi)^3} \\
+ 2z (1 - \phi) \left( \frac{1 + \lambda \frac{s}{\eta}}{1 - \lambda + \lambda \phi} \right)^{-\frac{1}{2-\eta}} \frac{1 + \frac{s}{\eta} - \phi}{(1 - \lambda + \lambda \phi)^3} \\
= z \left( \frac{1 + \lambda \frac{s}{\eta}}{1 - \lambda + \lambda \phi} \right)^{-\frac{1}{2-\eta}} 1 + \frac{s}{\eta} - \phi \frac{1 + \frac{s}{\eta} - \phi}{(1 - \lambda + \lambda \phi)^3} \left[ -\frac{1}{2 - \eta} \frac{1 + \frac{s}{\eta} - \phi}{1 + \lambda \frac{s}{\eta}} + 2(1 - \phi) \right] \\
= \frac{z}{1 - \lambda + \lambda \phi} \left[ -\frac{1}{2 - \eta} \frac{1 + \frac{s}{\eta} - \phi}{1 + \lambda \frac{s}{\eta}} + 2(1 - \phi) \right] \frac{\partial \hat{\theta}_R}{\partial \lambda},
\] (66)

where
\[
z = \frac{1 - \eta}{2 - \eta} \left( \frac{\alpha}{2 - \beta} \right)^{\frac{1-\eta}{2-\eta}}.
\]
Substituting (65) and (66) into (64) we obtain

$$\frac{\partial^2 L^*}{\partial \lambda^2} = z \left\{ -2(1 - \psi) + \frac{1 - \lambda + \lambda \psi}{1 - \lambda + \lambda \phi} \left[ -\frac{1}{2 - \eta} \frac{1 + \frac{s}{\eta} - \phi}{1 + \frac{s}{\eta}} + 2(1 - \phi) \right] \right\} \frac{\partial \hat{\theta}_R}{\partial \lambda}. $$

Since $\frac{\partial \hat{\theta}_R}{\partial \lambda} > 0$ it follows that $\partial^2 L^*/\partial \lambda^2 < 0$ as long as

$$-2(1 - \psi) + \frac{1 - \lambda + \lambda \psi}{1 - \lambda + \lambda \phi} \left[ -\frac{1}{2 - \eta} \frac{1 + \frac{s}{\eta} - \phi}{1 + \frac{s}{\eta}} + 2(1 - \phi) \right] < 0.$$ 

When $\phi \in [1, 1 + s/\eta]$, the second term on the LHS is non-positive and $\partial^2 L^*/\partial \lambda^2 < 0$. When $\phi \in (0, 1)$ a sufficient condition for $\partial^2 L^*/\partial \lambda^2 < 0$ is

$$\frac{1 - \psi}{1 - \phi} \geq \frac{1 - \lambda + \lambda \psi}{1 - \lambda + \lambda \phi}. \quad (67)$$

This inequality is satisfied since $\psi \leq \phi < 1$—see (51)—implies that the LHS of (67) is greater than or equal to 1 and the RHS of (67) is less than or equal to 1. Hence, $L^*$ is a concave function of $\lambda$.

(ii) and (iii) We start by showing that

$$\frac{\partial L^*}{\partial \lambda} \bigg|_{\lambda=0} > 0,$$

which implies that $L^*$ is not a decreasing function of $\lambda$. From (63) we have

$$\frac{\partial L^*}{\partial \lambda} \bigg|_{\lambda=0} = -(1 - \psi) \hat{\theta}_R \bigg|_{\lambda=0} + (1 - \lambda + \lambda \psi) \big|_{\lambda=0} \frac{\partial \hat{\theta}_R}{\partial \lambda} \bigg|_{\lambda=0}$$

$$= - (1 - \psi) \hat{\theta}_0 + \frac{1 - \eta}{2 - \eta} \hat{\theta}_0 \left( 1 + \frac{s}{\eta} - \phi \right)$$

$$= \left[ \frac{1 - \eta}{2 - \eta} \left( 1 + \frac{s}{\eta} - \phi \right) - (1 - \psi) \right] \hat{\theta}_0.$$ 

This derivative is positive as long as

$$(1 - \eta) \left( 1 + \frac{s}{\eta} - \phi \right) > (2 - \eta)(1 - \psi),$$

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or
\[ \phi \eta - \phi - \eta - s + \frac{s}{\eta} + 1 > \psi \eta - \eta - 2\psi + 2, \]
or
\[ \phi \eta - \phi - s + \frac{s}{\eta} > \psi \eta - 2\psi + 1, \]
or
\[ \phi \eta - \phi - s + \frac{s}{\eta} - \psi \eta + \psi + \psi - 1 > 0, \]
or
\[ -s + \frac{s}{\eta} - \frac{s(1-\eta)^2}{\eta} \frac{\alpha}{\eta - s(1-\eta)} 2 - \beta \psi + \psi - 1 > 0, \]
or
\[ \frac{s(1-\eta)}{\eta} > \frac{s(1-\eta)^2}{\eta - s(1-\eta)} \frac{\alpha}{2 - \beta} \psi + (1 - \psi), \]
or
\[ \psi > \frac{s(1-\eta)^2}{\eta - s(1-\eta)} \frac{\alpha}{2 - \beta} \psi + \frac{\eta - s(1-\eta)}{\eta}, \]
or
\[ 1 > \frac{s(1-\eta)^2}{\eta - s(1-\eta)} \frac{\alpha}{2 - \beta} + \left[ \frac{\eta - s(1-\eta)}{\eta} \right]^{1-\eta}. \]

We know from Proposition 3 that
\[ \frac{\alpha}{2 - \beta} < \left[ \frac{\eta - \bar{s}(1-\eta)}{\eta} \right]^{2-\eta} < \left[ \frac{\eta - s(1-\eta)}{\eta} \right]^{2-\eta}, \]
where the second inequality follows from \( s < \bar{s} \). Therefore the inequality is satisfied as long as
\[ 1 > \frac{s(1-\eta)^2}{\eta} \left[ \frac{\eta - s(1-\eta)}{\eta} \right]^{1-\eta} + \left[ \frac{\eta - s(1-\eta)}{\eta} \right]^{1-\eta}, \]
or
\[ 1 > \left[ \frac{s(1-\eta)^2}{\eta} + 1 \right] \left[ \frac{\eta - s(1-\eta)}{\eta} \right]^{1-\eta}. \]

Note that if \( s = 0 \) the RHS of the inequality is equal to 1. We now show that the RHS is decreasing with \( s \) which implies that the inequality is satisfied for any \( s \in (0, \bar{s}) \).
The derivative of the RHS with respect to \( s \) is:
\[
\frac{(1 - \eta)^2}{\eta} \left[ \frac{\eta - s(1 - \eta)}{\eta} \right]^{1-\eta} - \frac{(1 - \eta)^2}{\eta} \left[ \frac{s(1 - \eta)^2}{\eta} + 1 \right] \left[ \frac{\eta - s(1 - \eta)}{\eta} \right]^{-\eta} = \frac{(1 - \eta)^2}{\eta} \left[ \frac{\eta - s(1 - \eta)}{\eta} \right]^{-\eta} \left[ \frac{\eta - s(1 - \eta)}{\eta} - \frac{s(1 - \eta)^2}{\eta} - 1 \right] = -\frac{(1 - \eta)^2}{\eta} \left[ \frac{\eta - s(1 - \eta)}{\eta} \right]^{-\eta} \left[ \frac{s(1 - \eta)}{\eta} + \frac{s(1 - \eta)^2}{\eta} \right] < 0.
\]

Therefore, \( L^* \) is not a decreasing function of \( \lambda \). Thus, we are left with two cases: (1) \( L^* \) is an increasing and concave function of \( \lambda \)(and \( L^*(\lambda) \) attains a maximum at \( \lambda = 1 \)), and (2) \( L^* \) is a concave function of \( \lambda \) which attains a maximum at \( \lambda \in (0, 1) \).

Case (1) happens when
\[
\frac{\partial L^*}{\partial \lambda} \bigg|_{\lambda=1} = -(1 - \psi) \frac{\partial \theta_R}{\partial \lambda} \bigg|_{\lambda=1} + (1 - \lambda + \lambda \psi) \frac{\partial^2 \theta_R}{\partial \lambda^2} \bigg|_{\lambda=1} = -(1 - \psi) \left( \frac{\alpha}{2 - \beta} \frac{1 + \frac{s}{\eta}}{\phi} \right) + \psi \left( \frac{1 - \eta}{2 - \eta} \left( \frac{\alpha}{2 - \beta} \right) \right) \left( \frac{1 + \frac{s}{\eta}}{\phi} \right) \frac{1 + \frac{s}{\eta}}{\phi^2} = -(1 - \psi) + \left( \frac{\psi}{\phi} \frac{1 - \eta}{2 - \eta} \right) \left( \frac{\alpha}{2 - \beta} \frac{1 + \frac{s}{\eta}}{\phi} \right) \frac{1 + \frac{s}{\eta}}{\phi^2} > 0.
\]

This condition is satisfied when
\[
(1 - \eta) \left( 1 + \frac{s}{\eta} - \phi \right) \psi > (2 - \eta)(1 - \psi) \left( 1 + \frac{s}{\eta} \right) \phi. \tag{68}
\]

Case (2) happens when (68) is violated. From (63) and the fact that \( \partial^2 L^*/\partial \lambda^2 < 0 \) it follows that in case (2) \( L^* \) attains a maximum at the \( \lambda \) which solves
\[
(1 - \lambda + \lambda \psi) \frac{\partial^2 \theta_R}{\partial \lambda^2} = (1 - \psi) \hat{\theta}_R. \tag{69}
\]

Note that
\[
\frac{\partial \theta_R}{\partial \lambda} = \frac{1 - \eta}{2 - \eta} \hat{\theta}_R \frac{1}{1 + \lambda \frac{s}{\eta}} \frac{1 + \frac{s}{\eta} - \phi}{1 - \lambda + \lambda \phi}. \tag{70}
\]
Substituting (70) into (69) we obtain

\[
(1 - \lambda + \lambda \psi) \frac{1 - \eta}{2 - \eta} \hat{\theta}_R \frac{1}{1 + \lambda \frac{s}{\eta}} \frac{1 + \frac{s}{\eta} - \phi}{1 - \lambda + \lambda \phi} = (1 - \psi) \hat{\theta}_R,
\]

or

\[
\frac{1 - \eta}{2 - \eta} \frac{1 + \frac{s}{\eta} - \phi}{1 + \lambda \frac{s}{\eta}} \frac{1 - \lambda + \lambda \phi}{1 - \lambda + \lambda \phi} = 1 - \psi,
\]

or

\[
\frac{1 - \lambda + \lambda \psi}{1 - \lambda + \lambda \phi} = \frac{2 - \eta}{1 - \eta} \frac{1 - \psi}{1 + \frac{s}{\eta} - \phi} \left(1 + \lambda \frac{s}{\eta}\right).
\]

Hence, \( L^* \) attains a maximum at the \( \lambda \in (0, 1) \) which solves (71): \( \bar{\lambda} \). \( Q.E.D. \)

**Proof of Proposition 10:** From (49) the equilibrium rental cost of capital is equal to

\[
r^* = \frac{\beta w^*}{\alpha K} [1 - \lambda + \lambda \psi(\eta, s)] \hat{\theta}_R = \frac{\beta}{\alpha K} w^* L^*.
\]

The impact of a change in \( \lambda \) on \( r^* \) is given by

\[
\frac{\partial r^*}{\partial \lambda} = \frac{\beta}{\alpha K} \left[ \frac{\partial w^*}{\partial \lambda} L^* + w^* \frac{\partial L^*}{\partial \lambda} \right].
\]

We know from Proposition 7 that \( \partial w^*/\partial \lambda \) > 0. Hence, \( \partial L^*/\partial \lambda \) > 0 is a sufficient condition for \( \partial r^*/\partial \lambda \) > 0. Therefore, it follows from Proposition 9 that \( \partial r^*/\partial \lambda \) > 0 when either (1) inequality (68) is satisfied or (2) inequality (68) is violated and \( \lambda \in (\bar{\lambda}, 1] \).

**References**


Manove, M. 2000. Entrepreneurs, Optimism and The Competitive Edge, Boston University and CEMFI.


