The Impact of Firm Cost and Market Size Asymmetries on National Mergers in a Three-Country Model*

Luís Santos-Pinto†

This version: March 8, 2010

Abstract

This paper studies the impact of firm cost and market size asymmetries on merger decisions. I consider a model where a small and a large country compete in a third (world) market. Each of the two countries has two firms (with potentially different costs) that supply the domestic market and export to the third market. Merger decisions in the two countries are modeled as a simultaneously move game. The paper finds that firms in the large country have more incentives to merge than firms in the small country. In contrast, the government of the large country has more incentives to block a merger than the government of the small country. Thus, the model predicts that conflicts of interest between governments and firms concerning national mergers are more likely in large countries than in small ones.

JEL Codes: F13, H77, L11, L41.
Keywords: Mergers; International Trade; Merger Policy; Size Asymmetry.

*I am thankful to Pedro Barros, Fernando Branco, Eileen Fumagalli, Chrysostomos Tabakis, Costas Hadjiyiannis, the editor, and an anonymous referee for helpful comments and suggestions. I am also thankful to participants at the 4th Bank of Portugal Conference on Portuguese Economic Development in the European Area and to the seminar audience at University of Cyprus. I am grateful to Tiago Pires and João Jalles for excellent research assistance.

†University of Lausanne, Faculty of Business and Economics, Internef 535, CH-1015, Lausanne, Switzerland. Ph:+41216923658. Fax: +41216923435. E-mail address: LuisPedro.SantosPinto@unil.ch.

1 Introduction

In many European countries there is a heated debate over whether governments and competition authorities should favor or oppose the creation of national champions.[1] An argument often put forth in favor of national champions is that bigger firms will be in a better position to compete in world markets. A recent example in Germany has been the approval of the merger between the *E.ON* and *Ruhrgas* corporations by the German Ministry of Economics and Technology. A main reason for this decision was the argument that size was very important at the onset of the energy market liberalization in Europe.[2]

It's true that the emergence of a national champion might improve a country's welfare if it has strong efficiency gains and shifts profits away from competitors in export markets. However, the emergence of a national champion might also reduce a country's welfare if the efficiency gains from cost savings are smaller than the loss from reduced domestic output. A national champion might also not be able to shift profits away from competitors in export markets due to losses in market share.

This paper contributes to this debate by setting up a three-country model in which firms in two countries serve their respective domestic markets and compete in a third (world) market. This market structure captures the situation in many network industries, such as electricity, natural gas, telecommunications or railways. A typical example is the electricity market, where the German duopolists *E.ON* and *RWE* compete with other 'national champions' in several European markets. It is also relevant in markets like iron ore and cement where large national players compete in third markets, but less so in the respective home markets of their competitors. For example, global leaders in the iron ore industry like Vale (Brazil), Rio Tinto (Australia), and BHP Billiton (Australia) compete in Europe and Asia. Global leaders in the cement industry like Lafarge (France) and Holcim (Switzerland) compete in India and Africa.

I use the model to study endogenous mergers and mergers that improve national welfare. By comparing the equilibrium outcomes of these two games I am able to clarify which factors contribute to the existence of conflicts of interest between firms and governments about the desirability of national champions.

In this model mergers have efficiency gains and are modeled as a simultaneous move game. Firms compete à la Cournot, markets are segmented, and there are no producers in the third market.[3] The novelty of my approach is

[1] For example, the French government advocated a merger between the electricity and gas company *SUEZ* with the firm *GAZ DE FRANCE*.

[2] E.ON, one of Germany's largest energy companies in the electricity sector, intended to acquire a 60% majority in the gas company Ruhrgas, resulting in E.ON/Ruhrgas becoming Europe's biggest energy company. Preliminarily, permission for the merger was denied by the German Federal Cartel Office on the grounds that the merger would have a detrimentally strengthen E.ON's already dominant position. Nevertheless, the German Ministry of Economics and Technology eventually cleared the merger, overulling the decision by the cartel authority. The importance of the national champion argument in the context of this merger is stressed by Sinn (2002, pp. 10-12).

[3] This set-up captures the idea that domestic markets are less competitive than export

that it allows for both firm cost and market size asymmetries. Firms can have different costs of production and the three countries can have different market demands.[4]

The questions that this paper addresses have many links with the existing literature on merger and competition policy, specially with papers which extend the analysis to open economies.[5] This literature has taken two different directions. One line of research focuses on nationally optimal merger policies and merger profitability when trade policy instruments are available to national governments–e.g., Richardson (1999), Horn and Levinsohn (2001), and Huck and Conrad (2004). The other line of research is based on the concept of "external effects" of a merger to outsiders. An important early contribution to this topic is Farrel and Shapiro (1990). This concept was extended to open economies by Barros and Cabral (1994). This literature has derived rather general conditions under which a merger benefits, or harms, the parties not participating in the merger.

This paper takes a different approach by analyzing mergers with a three-country model like Haufler and Nielsen (2008) and Sudekum (2008). Haufler and Nielsen (2008) find that there is a range of cost reductions for which a merger is in the private interest of domestic firms, but not in the interest of the country has a whole. They also find that when the export market is larger the range for which the merger is blocked decreases. Sudekum (2008) finds that the promotion of national mergers can be in the interest of individual countries if rent extraction possibilities are strong enough when firms compete on all markets and are subject to transport costs. My model generalizes Haufler and Nielsen (2008) and Sudekum (2008) since they assume all firms have the same cost and exporting countries have the same market size. This allows me to state new results that show how firm cost and market size asymmetries influence merger incentives, merger equilibrium outcomes, and conflicts of interest.

2 Set-up

Consider three countries: a small country, s, a large country, L, and a third country, e. Initially there are 2 firms in the small country and 2 firms in the large country. There are no firms in the third country. Firms in the small and the large countries sell their product in the domestic markets and export it to the third country. Thus, there is no bilateral trade between the small country and the large country and firms compete in the third (or export) market.

The inverse demand function in the small country is $P_s = 1 - Q_s$. The

markets. See Brander and Spencer (1985).

[4] The analysis of cost asymmetries applies to cases where differences in market shares reflect cost asymmetries. The results in this paper might not extend to cases where differences in market shares come from other factors (e.g., rigid capacity constraints).

[5] The traditional analysis of mergers and acquisitions in industrial organization–Salant, Switzer, and Reynolds (1983) and Deneckere and Davidson (1985)–usually neglects the effects of country borders.

inverse demand function in the large country is $P_L = 1 - Q_L/\gamma$, with $\gamma \geq 1$.[6] The inverse demand function in the export market is $P_e = 1 - Q_e/\beta$, with $\beta \geq 1$. The parameters γ and β measure market size asymmetries.

Firms in the small and large countries are fully owned by residents and produce a homogeneous good. There are no fixed costs (this rules out gains from economies of scale in mergers).[7] Marginal costs of firms are given by $c_{v1} = 0$, $c_{v2} = \delta$, where $v = s, L$ and $\delta \in [0, 1/3]$. I assume $\delta \leq 1/3$ so that, in the absence of mergers, even the less efficient firm makes nonnegative profits in all markets. Here δ is a summary measure of cost asymmetry.

I assume that if two firms merge, the high-cost firm ceases production and only the low-cost unit produces. Therefore, a merger can be viewed as an acquisition of a high-cost firm by a low-cost firm.[8] I assume that firms play separate Cournot games in each market which implies that each market can be analyzed independently of the other markets.[9] Thus, before any merger has taken place, firm si chooses q_{si} and $q^e_{si}$to maximize

$$\pi^s_{si} + \pi^e_{si} = \left[1 - \sum_{k=1}^{2} q_{sk} - c_{si}\right] q_{si} + \left[1 - \frac{1}{\beta}\left(\sum_{k=1}^{2} q^e_{sk} + \sum_{k=1}^{2} q^e_{Lk}\right) - c_{si}\right] q^e_{si}. \quad (1)$$

Similarly, at the start, firm Li chooses q_{Li} and q^e_{Li} to maximize

$$\pi^L_{Li} + \pi^e_{Li} = \left[1 - \frac{1}{\gamma}\sum_{k=1}^{2} q_{Lk} - c_{Li}\right] q_{Li} + \left[1 - \frac{1}{\beta}\left(\sum_{k=1}^{2} q^e_{Lk} + \sum_{k=1}^{2} q^e_{sk}\right) - c_{Li}\right] q^e_{Li}. \quad (2)$$

3 Profitability of Conditional Mergers

When a merger takes place there are three effects that the merging firms need to take into consideration. First, there is an efficiency gain since the high cost firm transfers production to the low cost firm. Second, a merger leads to less competition both in the domestic market as well as in the export market. These two effects allow the merged firm to have a higher mark-up than the highest mark-up of the individual firms. Thus, the market power of the merging firms increases in both markets. However, in the export market the merger implies

[6] The direct demand functions are $Q_s = 1 - P_s$ and $Q_L = \gamma(1 - P_L)$. Thus, γ measures market size asymmetry between the large and the small country. For example, if $\gamma = 4$ the market of the large country is four times the market of the small country.

[7] Transportation costs between s and e and between L and e are assumed to be equal to zero. Transportation cost can be greater than zero without changing qualitatively the results in the paper.

[8] This approach is also used by Barros (1998) and Qiu and Zhou (2007). Perry and Porter (1985) and Farrell and Shapiro (1990) use other approaches to model the impact of a merger on an industry's cost structure.

[9] The assumption of Cournot competition is in line with much of the literature on mergers. Theoretical and empirical arguments in defence of the Cournot model are presented by Haufler and Nielsen (2008). The model proposed by Kreps and Scheinkman (1983) in which firms choose capacities in the first period and compete in prices in the second period generates Cournot outcomes.

that the market share of the merging firms is lower than the sum of the pre-merger market shares of the merging firms. This third effect, akin to the well-known Salant, Switzer, and Reynolds (1983) effect, reduces the incentive to merge.[10] Thus, a merger increases profits in the domestic market but it might reduce profits in the export market.

Lemma 1 provides conditions under which a merger in one country is profitable for a given market structure in the other country.

Lemma 1:
(i) If firms in the large country are not merged and either (a) $\delta > 7/61$ *or (b)* $\delta \leq 7/61$ *and* $\beta \leq f_{sN} = \frac{100}{9}\frac{1+8\delta-20\delta^2}{7-82\delta+183\delta^2}$, *then a merger in the small country is profitable;*
(ii) If firms in the large country are merged and either (a) $\delta > 1/15$ *or (b)* $\delta \leq 1/15$ *and* $\beta \leq f_{sM} = 2\frac{1+8\delta-20\delta^2}{1-18\delta+45\delta^2}$, *then a merger in the small country is profitable;*
(iii) If firms in the small country are not merged and either (a) $\delta > 7/61$ *or (b)* $\delta \leq 7/61$ *and* $\beta \leq \gamma f_{sN} = f_{Ln}$, *then a merger in the large country is profitable;*
(iv) If firms in the small country are merged and either (a) $\delta > 1/15$ *or (b)* $\delta \leq 1/15$ *and* $\beta \leq \gamma f_{sM} = f_{Lm}$, *then a merger in the large country is profitable.*

Figure 1 illustrates how the incentives for a merger in the small country depend on market structure in the large country, firm cost asymmetries and the size of the export market.

Insert Figure 1 here

The f_{sM} and f_{sN} curves in Figure 1 characterize, respectively, incentives for a profitable merger in the small country when firms in the large country are merged (f_{sM}) or not merged (f_{sN}). To the right (left) of each curve firms in the small country merge (do not merge).

A merger in country i makes a merger in country j profitable for a wider set of parameter values. This happens because (a) a merger in country i lowers the number of active firms in the export market from three to two if firms in country j are merged and from four to three if the firms in country j are not merged, and (b) a move from three firms to two creates a larger increase in mark-up than a move from four firms to three and both moves lead to the same loss in market share. Thus, mergers are strategic complements in the sense that the incentives for firms in country i to merge increase when firms in country j are merged.[11]

When cost asymmetries are high a merger increases profits in the domestic and export markets and so a merger is profitable no matter the size of the export

[10] The fact that a merger always reduces the firms' combined market share in the export market, despite the cost savings, is driven by the assumption that the merger merely eliminates the high cost firm, thereby reshuffling some output to the lower cost merger partner. If the merger resulted in synergies that reduced the marginal cost of the lower cost merging firm, then mergers would become more attractive.

[11] Qiu and Zhou (2007) show that mergers are strategic complements and, as a result, tend to occur in waves.

market. However, when cost asymmetries are low a merger increases profits in the domestic market but reduces profits in the export market and so a merger is only profitable when the export market is relatively small. Similarly, a merger of domestic firms is profitable for a wider range of parameter values in a large country because the losses in the export market are relatively smaller for a large country than for a small one.

A similar figure applies in the case of the large country. The only difference being that the curves that characterize incentives for a profitable merger in the large country when firms in the small country are merged (f_{Lm}) or not merged (f_{Ln}) are located to the left of f_{sM} and f_{sN}, respectively.

Proposition 1 summarizes the impact of market structure, firm cost and market size asymmetries on incentives for firms to merge.

Proposition 1: *A merger of domestic firms is profitable for a wider set of parameter values when: (i) foreign firms are merged, (ii) cost asymmetries are high, (iii) the export market is small, and (iv) the domestic market is large.*

4 Merger Game Played by Firms

I will now characterize the Nash equilibria of the merger game played by firms assuming that governments do not intervene in markets. As the starting point, I assume that no merger has taken place in either country. The decisions of firms in each country to merge or not to merge are taken simultaneously.

Table I displays the payoffs in the merger game played by firms. The strategies of the firms of the small country are m (merger) and n (no merger) and those of the firms of the large country are M (merger) and N (no merger). The upper left part of each cell displays the profits of the merged firm or the sum of profits of the two firms in s. The lower right part of each cell displays the profits of the merged firm or the sum of profits of the two firms in L. Profits in s and in L are obtained by substituting the equilibrium quantities into (1) and (2), respectively.

Table I

$s\backslash L$	M	N
m	$\frac{1}{4}+\frac{\beta}{9}$, $\frac{\gamma}{4}+\frac{\beta}{9}$	$\frac{1}{4}+\frac{\beta(1+\delta)^2}{16}$, $\frac{\gamma\left[(1+\delta)^2+(1-2\delta)^2\right]}{9}+\frac{\beta\left[(1+\delta)^2+(1-3\delta)^2\right]}{16}$
n	$\frac{(1+\delta)^2+(1-2\delta)^2}{9}+\frac{\beta\left[(1+\delta)^2+(1-3\delta)^2\right]}{16}$, $\frac{\gamma}{4}+\frac{\beta(1+\delta)^2}{16}$	$\frac{(1+\delta)^2+(1-2\delta)^2}{9}+\frac{\beta\left[(1+2\delta)^2+(1-3\delta)^2\right]}{25}$, $\frac{\gamma\left[(1+\delta)^2+(1-2\delta)^2\right]}{9}+\frac{\beta\left[(1+2\delta)^2+(1-3\delta)^2\right]}{25}$

Propositions 2 and 3, respectively, address the case where the size of the large country's market relative to the small country's market is at most 1.26 or at least 1.26. The curves f_{sM} and f_{Ln} in Figure 2 determine the equilibria of the merger game played by firms when $\gamma = 1.1$ (the set of equilibria is similar for $\gamma \leq 1.26$).

Insert Figure 2 here

Proposition 2: *Let $\gamma \leq 1.26$. Then, as figure 2 shows:*
(i) Firms in either country do not merge if cost asymmetries are low and the export market is large, that is, $\delta \leq 1/15$ and $f_{sM} \leq \beta$;
(ii) Firms in either country merge if either (a) cost asymmetries are high, that is, $\delta > 7/61$ or (b) cost asymmetries are moderate or low and the export market is small, that is, $\delta \leq 7/61$ and $\beta < f_{Ln}$;
(iii) In the remaining cases, there are two pure-strategy equilibria–firms in either country do not merge and firms in either country merge–, and one mixed-strategy equilibrium in which firms in the small country merge with probability

$$p_s = \frac{(63\beta - 100\gamma) - (738\beta + 800\gamma)\delta + (1647\beta + 2000\gamma)\delta^2}{(13 + 162\delta - 603\delta^2)\beta}, \tag{3}$$

and firms in the large country merge with probability

$$p_L = \frac{(63\beta - 100) - (738\beta + 800)\delta + (1647\beta + 2000)\delta^2}{(13 + 162\delta - 603\delta^2)\beta}; \tag{4}$$

Proposition 2 tells us that if firm cost asymmetries are high there will be mergers in both countries due to the profit gains in the domestic and in the export markets. When firm cost asymmetries are moderate mergers are not as attractive since they lead to gains in the domestic market but losses in the export market. In this case we have two possible situations. If the export market is small, then domestic profit gains are larger than the losses in the export market and firms in either country merge. If the export market is big, then we have multiple equilibria. If firm cost asymmetries are low, mergers are the least attractive since they generate small profit gains in domestic market and large losses in the export market. In this case we have three outcomes. If the export market is small, firms in either country merge. If the export market is intermediate we have multiple equilibria. If the export market is big firms in either country do not merge.

Figure 3 illustrates the equilibria of the merger game played by firms when $\gamma = 2$ (the set of equilibria is similar for $\gamma > 1.26$). The difference between Figures 2 and 3 is due to the fact that the f_{Ln} curve shifts up as γ increases but f_{sM} remains fixed.

Insert Figure 3 here

Proposition 3: *Let $\gamma > 1.26$. Then, as figure 3 shows:*
(i) Firms in either country do not merge if cost asymmetries are low and the export market is large, that is, $\delta \leq 1/15$ and $\max[f_{sM}, f_{Ln}] \leq \beta$;
(ii) Firms in the small country do not merge and firms in the large country merge if he size of the export market satisfies $f_{sM} \leq \beta \leq f_{Ln}$;
(iii) Firms in either country merge if either (a) cost asymmetries are high, that is, $\delta > 7/61$ or (b) cost asymmetries are moderate or low and the export market is small, that is, $\delta \leq 7/61$ and $\beta < \min[f_{sM}, f_{Ln}]$;
(iv) In the remaining cases, there are two pure-strategy equilibria–firms in either country do not merge and firms in either country merge–, and one mixed-strategy equilibrium in which firms in the small country merge with probability p_s, given by (3), and firms in the large country merge with probability p_L, given by (4).

Proposition 3 shows that when $\gamma > 1.26$ there exist equilibria where firms in the large country merge and firms in the small country do not merge. This happens because the bigger the market size of the large country, the greater are the domestic gains of a merger relative to the potential losses in the export market.

5 Welfare Impact of Conditional Mergers

This section analyzes the impact of a merger in one country on that country's welfare for a given market structure in the other country. Welfare is the sum of consumer surplus and profits of domestic firms in the domestic and export markets. Hence, welfare in s is equal to

$$W^s = CS^s + \sum_{i=1,2} (\pi^s_{si} + \pi^e_{si}), \tag{5}$$

and welfare in L to

$$W^L = CS^L + \sum_{i=1,2} (\pi^s_{Li} + \pi^e_{Li}), \tag{6}$$

where $CS^s = Q_s^2/2$ and $CS^L = Q_L^2/2\gamma$ represent consumer surplus in s and in L, respectively.

A merger of domestic firms reduces domestic output (even with cost savings because we move from asymmetric Cournot duopoly to monopoly with only the low cost firm). Thus domestic consumers lose, while profits rise. Hence, the impact of a domestic merger on consumer surplus plus domestic profit is ambiguous: with low cost savings, the effect is negative, because the loss in consumer surplus outweighs the increase in profits; but with sufficiently high cost savings the effect is positive since the increase in profits outweighs the loss in consumer surplus. The impact of a merger on profits in the export market is also ambiguous: with low cost savings the effect is negative, because the loss in market share outweighs the gain from restricting total output and raising price; but with high cost savings, profits in the export market can rise.

Lemma 2 provides conditions under which a merger in one country is welfare improving for a given market structure in the other country.

Lemma 2:
(i) If firms in the large country are not merged and $\beta \geq g_{sN} = \frac{50}{9}\frac{5-32\delta+44\delta^2}{-7+82\delta-183\delta^2}$, *then a merger in the small country improves that country's welfare;*
(ii) If firms in the large country are merged and $\beta \geq g_{sM} = \frac{5-32\delta+44\delta^2}{-1+18\delta-45\delta^2}$, *then a merger in the small country improves that country's welfare;*
(iii) If firms in the small country are not merged and $\beta \geq g_{Ln} = \gamma g_{sN}$, *then a merger in the large country improves that country's welfare;*
(iv) If firms in the small country are merged and $\beta \geq g_{Lm} = \gamma g_{sM}$, *then a merger in the large country improves that country's welfare.*

Figure 4 illustrates how the incentives for a welfare improving merger in the small country depend on market structure in the large country, firm cost asymmetries and the size of the export market.

Insert Figure 4 here

The g_{sM} and g_{sN} curves in Figure 4 characterize, respectively, the incentives for a welfare improving merger in the small country when firms in the large country are merged (g_{sM}) or not merged (g_{sN}). To the right (left) of each curve the merger raises (lowers) welfare.

When cost asymmetries are high a merger of domestic firms raises welfare since it leads to a small reduction in consumer surplus and a large increase in profits in the domestic market and in the export market (the increase in mark-up makes up for the loss of market share). When cost asymmetries are low a merger of domestic firms reduces welfare since it reduces profits in the export market (the increase in mark-up does not make up for the loss in market share) and the increase in profits in the domestic market is not enough to make up for the reduction in consumer surplus. Hence, a merger of national firms is more likely to increase welfare when cost asymmetries are high.

When cost asymmetries are moderate there is a trade-off between welfare losses in the domestic market (profits in the domestic market increase less than the reduction in consumer surplus) and profit gains in the export market. Hence, a merger of domestic firms is more likely to increase welfare when the export market is large. A merger of domestic firms is welfare improving for a wider set of parameter values in a small country because the losses in the domestic market are relatively smaller for a small country than for a large one.

A similar figure applies in the case of the large country. The only difference being that the curves that characterize the incentives for a welfare improving merger in the large country when firms in the small country are merged (g_{Lm}) or not merged (g_{Ln}) are located to the right of g_{sM} and g_{sN}, respectively.

Proposition 4 summarizes the impact of market structure, firm cost and market size asymmetries on welfare improving mergers of national firms.

Proposition 4: *A merger of domestic firms is welfare improving for a wider set of parameter values when: (i) foreign firms are merged, (ii) cost asymmetries are high, (iii) the export market is large, and (iv) the domestic market is small.*

6 Merger Game Played by Governments

Whenever a merger raises national welfare it must raise profit but not vice versa (because the merger reduces consumer surplus).[12] I now assume that each country's government would only permit a domestic merger that raises national welfare (and, as noted, any such merger would be profitable). Governments in s and L decide simultaneously whether to allow or reject proposed mergers.[13]

Table II displays the payoffs in the merger game played by governments. The strategies of the government of the small country are m (allow merger) and n (reject merger) and those of the government of the large country are M (allow merger) and N (reject merger). The upper left part of each cell displays the sum of consumer surplus in s and profits of the merged firm or profits of the two firms in s. The lower right part of each cell displays the sum of consumer surplus in L and profits of the merged firm or profits of the two firms in L. National welfare in s and in L is obtained by substituting the equilibrium quantities into (5) and (6), respectively.

Table II

$s\backslash L$	M	N
m	$\frac{1}{8}+\frac{1}{4}+\frac{\beta}{9}$, $\frac{\gamma}{8}+\frac{\gamma}{4}+\frac{\beta}{9}$	$\frac{1}{8}+\frac{1}{4}+\frac{\beta(1+\delta)^2}{16}$, $\frac{\gamma(2-\delta)^2}{18}+\frac{\gamma\left[(1+\delta)^2+(1-2\delta)^2\right]}{9}+\frac{\beta\left[(1+\delta)^2+(1-3\delta)^2\right]}{16}$
n	$\frac{(2-\delta)^2}{18}+\frac{(1+\delta)^2+(1-2\delta)^2}{9}+\frac{\beta\left[(1+\delta)^2+(1-3\delta)^2\right]}{16}$, $\frac{\gamma}{8}+\frac{\gamma}{4}+\frac{\beta(1+\delta)^2}{16}$	$\frac{(2-\delta)^2}{18}+\frac{(1+\delta)^2+(1-2\delta)^2}{9}+\frac{\beta\left[(1+2\delta)^2+(1-3\delta)^2\right]}{25}$, $\frac{\gamma(2-\delta)^2}{18}+\frac{\gamma\left[(1+\delta)^2+(1-2\delta)^2\right]}{9}+\frac{\beta\left[(1+2\delta)^2+(1-3\delta)^2\right]}{25}$

Propositions 5 and 6, respectively, address the case where the size of the large country's market relative to the small country's market is at most 2.145 or at least 2.145. The curves g_{Lm} and g_{sN} in Figure 5 determine the equilibria of the merger game played by governments when $\gamma = 2$ (the set of equilibria is similar for $\gamma \leq 2.145$).

Insert Figure 5 here

[12] The case where the government would force a merger even when the firms opposed it does not arise.

[13] I assume that the government of the third-country does not intervene in mergers in either s or L.

Proposition 5: *Let* $\gamma \leq 2.15$. *Then, as figure* 5 *shows:*
(i) Governments in either country reject a merger if either (a) cost asymmetries are low, that is, $\delta \leq 1/15$ *or (b)* $1/15 < \delta \leq 2/10$ *and the export market is small, that is,* $\beta < g_{Lm}$*;*
(ii) Governments in either country allow firms to merge if cost asymmetries are high and the export market is large, that is, $\delta > 7/61$ *and* $g_{sN} \leq \beta$*;*
(iii) In the remaining cases, there are two pure-strategy equilibria–governments in either country reject a merger and governments in either country allow a merger–, and one mixed-strategy equilibrium in which the government of the small country allows firms to merge with probability

$$q_s = \frac{(63\beta + 250\gamma) - (738\beta + 1600\gamma)\delta + (1647\beta + 2200\gamma)\delta^2}{\left(13 + 162\delta - 603\delta^2\right)\beta}, \tag{7}$$

and the government of the large country with probability

$$q_L = \frac{(63\beta + 250) - (738\beta + 1600)\delta + (1647\beta + 2200)\delta^2}{\left(13 + 162\delta - 603\delta^2\right)\beta}. \tag{8}$$

Proposition 5 tells us that if firm cost asymmetries are sufficiently low and the export market is small, then governments reject a merger of national firms since this would generate welfare losses in both the domestic and export markets. If firm cost asymmetries are sufficiently high, governments allow national firms to merge since this generates welfare gains in the export and domestic markets. Finally, when firm cost asymmetries are moderate, then a merger leads to a welfare gain in the export market but a welfare loss in the domestic market. In this case we have multiple equilibria.

Figure 6 illustrates the equilibria of the merger game played by governments when $\gamma = 3$ (the set of equilibria is similar for $\gamma > 2.15$). The difference between Figures 5 and 6 is due to the fact that the g_{Lm} curve shifts to the right as γ increases but g_{sN} remains fixed.

Insert Figure 6 here

Proposition 6: *Let* $\gamma > 2.15$. *Then, as figure* 6 *shows:*
(i) Governments in either country reject a merger if either (a) cost asymmetries are low, that is, $\delta \leq 1/15$ *or (b)* $1/15 < \delta \leq 2/10$ *and the export market is small, that is,* $\beta < \min[g_{Lm}, g_{sN}]$*;*
(ii) The small country's government allows a merger and the large country's government rejects a merger if either (a) cost asymmetries are moderate or low and the export market is large, that is, $\delta \leq 7/61$ *and* $\beta \geq g_{Lm}$ *or (b) cost asymmetries are high, that is,* $\delta \geq 7/61$, *and the size of the export market satisfies* $g_{Lm} \leq \beta \leq g_{sN}$*;*
(iii) Governments in either country allow firms to merge if cost asymmetries are high and the export market is large, that is, $\delta > 7/61$ *and* $\max[g_{Lm}, g_{sN}] \leq \beta$*;*

(iv) In the remaining cases, there are two pure-strategy equilibria–governments in either country reject a merger and governments in either country allow a merger–, and one mixed-strategy equilibrium in which the government of the small country allows firms to merge with probability q_s, given by (7), and the government of the large country with probability q_L, given by (8).

Proposition 6 shows that when $\gamma > 2.15$ there exist equilibria where the government of the small country allows national firms to merge and the government of the large country rejects a merger of national firms. This happens because the bigger the market size of the large country, the greater are the domestic losses of a merger relative to the potential gains in the export market.

7 Conflicts of Interest

This section discusses the implications of the model regarding conflicts of interest between firms and governments about merger decisions. Propositions 7, 8 and 9 in the Appendix compare the equilibria of the merger game played between firms to the equilibria of the merger game played between governments. There exists a conflict of interest when a government wants to block a merger of national firms but firms wish to merge.

Propositions 7, 8 and 9 show that if firms of a small and of a large country compete in a third country, then *the conditions under which conflicts of interest occur are less restrictive in the large country than in the small country.* This result is driven by the asymmetric equilibria of the merger game played between firms and by the asymmetric equilibria of the merger game played between governments. When these asymmetric equilibria occur, firms in the large country prefer to merge but the government of the large country opposes a merger. Thus the model predicts that, everything else constant, competition authorities should be less actively involved in the regulation of export industries in small countries than in large ones.

Figures 7, 8 and 9 display the curves that determine the equilibria of the two merger games when $\gamma = 1.1$ (the set of equilibria is similar for $\gamma \leq 1.26$), $\gamma = 2$ (the set of equilibria is similar for $1.26 < \gamma \leq 2.15$), and $\gamma = 3$ (the set of equilibria is similar for $\gamma > 2.15$), respectively. The f_{sM} and f_{Ln} curves determine the set of equilibria of the merger game played by firms and the g_{sN} and g_{Lm} curves the set of equilibria of the merger game played by governments.

Insert Figures 7, 8 and 9 here

The figures show that if firm cost asymmetries are high and the export market is large, then there are no conflicts of interest between national firms and governments: all favor a merger. The interests of national firms and governments are also aligned if firm cost asymmetries are low and the export market is large: all are against a merger. However, if firm cost asymmetries are moderate or low and the export market is relatively small, then a conflict of interest arises:

firms in either country wish to merge but governments in either country oppose the mergers.

The figures also show us that an increase in γ increases the set of parameter values where firms in either country wish to merge but governments in either country oppose mergers. Additionally, we see that, for any given γ, the set of parameter values where there are conflicts of interest in the large country is greater than or equal to the set of parameter values where there are conflicts of interest in the small country. In other words, for a given γ, the conditions under which conflicts of interest occur in the large country are less restrictive than those in the small country.[14]

8 Extensions

There are many directions in which one could extend this model. For example, one could relax the assumption that exports are only to the third country. Bilateral trade between the small and the large country has two effects. First, each exporting country has one more export market (the rival's market). Second, it increases the number of firms selling in domestic markets. The additional export market leads to a larger increase in profits in the foreign markets following a merger when cost asymmetries are high and a smaller increase when cost asymmetries are low. An increase in competition in the domestic market leads to a smaller increase in profits in the domestic market following a merger when cost asymmetries are high and a reduction in profits when cost asymmetries are low. Thus, allowing for bilateral trade makes a merger less attractive to firms for low cost asymmetries since profits in the domestic and in the export markets are lower. For high cost asymmetries the result depends on the relative sizes of the markets. Let's now turn to the impact of bilateral trade on welfare. An increase in competition in the domestic market leads to a lower reduction in consumer surplus following a merger since a move from four to three firms (when foreign firms are not merged) or from three to two (when foreign firms are merged) reduces consumer surplus by less than a move from two to one. This implies that bilateral trade makes a merger more attractive to governments when cost asymmetries are high since domestic welfare and profits in the export market are higher than without bilateral trade. When cost asymmetries are low the impact of bilateral trade on the attractiveness of a merger to governments depends on the relative size of the markets. Thus, bilateral trade between the small and the large country reduces conflicts of interest between firms and governments when cost asymmetries are high and its impact on conflicts of interest is ambiguous when cost asymmetries are low.

[14] When $1.26 < \gamma \leq 2.15$ we find parameter values where firms in the large country want to merge, firms in the small country do not want to merge, and governments oppose mergers. When $\gamma > 2.15$ there are parameter values where (i) firms in the large country want to merge, firms in the small country do not want to merge, and governments oppose mergers, and (ii) the government of the large country opposes a merger, the government of the small country favors it, and firms in either country want to merge.

Another possible extension is to assume that exports of the small and large countries must compete against domestic firms in the third country. In this case mergers would be less (more) attractive to firms if cost asymmetries are high (low) because the gains (losses) in the export market are lower. Competition from firms located in the third country market makes mergers less attractive to governments since the gains from a merger in the export market are smaller. Thus, this implies an increase in conflicts of interest when cost asymmetries are low but not when they are high.

If competition were in prices with differentiated products, mergers would be profitable for all parameter values. In this case a merger increases mark-ups both domestically and in the export market but leads to a loss of market share in the third-country, because typically the non-merging firms will raise price by less than the merging firms. However, the merger would still be profitable, because it's profitable even if the merging firms' prices were held constant (in which case the merging firms would lose even more marker share), and the fact that the non-merging firms raise price somewhat serves to further increase the profit of the merger. The effects on a country's welfare would be ambiguous due to the price increases domestically, and the conflicts of interest would still depend on the relative sizes of the markets.

9 Conclusion

This paper studies incentives for national mergers in a model where firms of two countries compete in a third country market. The main novelty of the paper is that it characterizes incentives for firms to merge and for governments to allow mergers when firms can have different costs of production and countries can have different market demands.

The model can be applied to shed light on real world incentives for mergers in markets with a high level of concentration like electricity generation and iron ore.

The three biggest firms in the European electricity generation market are EDF (90% market share in France), E.ON and RWE (27% and 25%, respectively of market share in Germany). These firms compete mostly in the U.K. and Italy. Demand for electricity in Germany, France, the U.K., and Italy is 554 TWh, 477 TWh, 390 TWh, and 322 TWh–the data is taken from Table 1 in Domanico (2009). Let France be the small country, Germany the large country with $\gamma = 554/477 = 1.16$, and the export market to have size $\beta = (390+322)/477 = 1.49$. As an approximation consider that there is a single firm in France and a duopoly in Germany. The summary index of firm cost asymmetry for Germany is the solution to $\frac{mktshare^{E.On}}{mktshare^{RWE}} = \frac{0.27}{0.25} = \frac{1+\delta}{1-2\delta}$, that is, $\delta = 0.025$.

Do E.ON and RWE want to merge? According to Lemma 1 part (iv) they do since firm cost asymmetries are small and the size of the export market is sufficiently small relative to the size of the domestic market. Should the German government approve this merger? According to Lemma 2 part (iv), the German government should reject the merger because firm cost asymmetries and the size of the export market are small.

The three biggest firms in the seaborne iron ore market are Vale (Brazil) with 28% of world market share in 2004, Rio Tinto (Australia) and BHP Billiton (Australia) with 22% and 16% of world market share in 2004, respectively. Let Australia be the small country and Brazil the large country with $\gamma = 200/20 = 10$ (2004 demand for iron ore in Brazil and Australia were 200 Mmt and 20 Mmt, respectively). These firms export most of their production to China, Europe, and Japan where 2004 imports of iron ore were 444 Mmt, 148 Mmt, and 136 Mmt, respectively. Thus, take the size of the export market to be $\beta = (444 + 148 + 136)/10 = 72.8$.[15]

Do Rio Tinto and BHP Billition want to merge? The model tells us that they do since these two firms compete mainly against Vale and firm cost asymmetries between Rio Tinto and BHP Billiton are moderate ($\delta = 0.1$).[16] Should the Australian government approve such a merger? Yes, because firm cost asymmetries are moderate and the size of the export market is large.

[15] The data can be found at: http://www.ft.com/cms/s/0/3561ce38-b8e7-11de-98ee-00144feab49a.html

[16] BHP Billiton made a hostile bid for Rio Tinto in 2007 but the bid failed.

10 References

Barros, P. and L. Cabral (1994). "Merger Policy in Open Economies," *European Economic Review,* 38, 1041-1055.

Barros, P. (1998). "Endogenous Mergers and Size Asymmetry of Merger Participants," *Economic Letters,* 60, 113-119.

Brander, J., and B. Spencer (1985). "Export Subsidies and International Market Share Rivalry," *Journal of International Economics,* 18, 83-100.

Deneckere, R. and C. Davidson (1985) "Incentives to Form Coalitions with Bertrand Competition," *The Rand Journal of Economics*, 16(4), 473-486.

Domanico, F. (2009). "Liberalisation of the European Electricity Industry: Internal Market or National Champions?" World Energy Organization working paper 1227.

Farrell, J. and C. Shapiro (1990). "Horizontal Mergers: An Equilibrium Analysis," *The American Economic Review,* 80, 107-126.

Haufler, A. and S. Nielsen (2008). "Merger Policy to Promote 'Global Players'? A Simple Model," *Oxford Economic Papers*, 60(3), 517-545.

Horn, H. and J. Levinsohn (2001). "Merger Policies and Trade Liberalization," *The Economic Journal,* 111, 244-276.

Huck, S. and K. Konrad (2004), "Merger Profitability and Trade Policy," *Scandinavian Journal of Economics,* 106, 107-122.

Kreps, D. and J. Scheinkman (1983). "Capacity Precommitment and Price Competition Yield Cournot Outcomes," *Bell Journal of Economics*, 14, 326-337.

Perry, M. and R. Porter (1985). "Oligopoly and the Incentive for Horizontal Merger," *The American Economic Review*, 75(1), 219-227.

Qiu, L. and W. Zhou (2007). "Merger waves: A Model of Endogenous Mergers," *The Rand Journal of Economics*, 38(1), 214-226.

Richardson, M. (1999). "Trade and Competition Policies: Concordia Discors?," *Oxford Economic Papers,* 51, 649-664.

Salant, S., Switzer, S. and R. Reynolds (1983). "Losses from Horizontal Merger: the Effects of an Exogenous Change in Industry Structure on Cournot-Nash Equilibrium," *Quarterly Journal of Economics,* 98, 185-199.

Sinn, H-W (2002). "Fusion E.ON-Ruhrgas: Die volkswirtschaftlichen Aspekte," ifo Institut für Wirtschaftsforschung, Forschungsbericht 9.

Sudekum, J. (2008). "Cross-Border Mergers and National Champions in an Integrating Economy," *Journal of Institutional and Theoretical Economics*, 164(3), 477-508.

11 Appendix

Proof of Lemma 1: I start the proof by deriving the conditions under which a merger in s is profitable conditional on a given market structure in L. If there is no merger in any country, all fours firms choose their quantities to maximize their respective profits according to (1) and (2). In this case the equilibrium profits of s firms in the s market are $\pi_{s1} = (1+\delta)^2/9$ and $\pi_{s2} = (1-2\delta)^2/9$, the profits of L firms in the L market are $\pi_{L1} = \gamma(1+\delta)^2/9$ and $\pi_{L2} = \gamma(1-2\delta)^2/9$, and the profits of s and L firms in e are $\pi^e_{s1} = \pi^e_{L1} = \beta(1+2\delta)^2/25$ and $\pi^e_{s2} = \pi^e_{L2} = \beta(1-3\delta)^2/25$. If s firms merge and L firms are not merged, then (1) and (2) imply that the equilibrium is defined by the system $d\pi_{s1}/dq_{s1} = 0$, $d\pi^e_{s1}/dq^e_{s1} = 0$, $d\pi_{L1}/dq_{L1} = 0$, $d\pi^e_{L1}/dq^e_{L1} = 0$, $d\pi_{L2}/dq_{L2} = 0$, and $d\pi^e_{L2}/dq^e_{L2} = 0$. The resulting equilibrium profits of the merged firm are $\pi^s_m = 1/4$ in the domestic market and $\pi^e_{m,N} = \beta(1+\delta)^2/16$ in the export market. A merger of s firms is profitable when L firms are not merged if the total profits of the merged s firm are greater than the sum of the profits of the s firms before the merger, that is,

$$\frac{1}{4} + \frac{\beta(1+\delta)^2}{16} \geq \frac{(1+\delta)^2}{9} + \frac{(1-2\delta)^2}{9} + \frac{\beta(1+2\delta)^2}{25} + \frac{\beta(1-3\delta)^2}{25}.$$

Solving the inequality with respect to β we obtain $\beta \leq f_{sN}$. The inequality is satisfied if either (a) $\delta > 7/61$–the merger leads to gains in the export market–or (b) $\delta \leq 7/61$ and $\beta \leq f_{sN}$, which proves (i). If s firms are not merged but L firms are, the equilibrium profits of $s1$ in e are $\pi^e_{s1} = \beta(1+\delta)^2/16$ and the profits of $s2$ are $\pi^e_{s2} = \beta(1-3\delta)^2/16$. If s firms merge and so do L firms we have a duopoly in e. In this case the equilibrium profits of the merged s firm in e are $\pi^e_{m,M} = \beta/9$. Thus, a merger of s firms is profitable when L firms are merged if

$$\frac{1}{4} + \frac{\beta}{9} \geq (1+\delta)^2\left(\frac{1}{9} + \frac{\beta}{16}\right) + \frac{(1-2\delta)^2}{9} + \frac{\beta(1-3\delta)^2}{16}.$$

Solving the inequality with respect to β we obtain $\beta \leq f_{sM}$. The inequality is satisfied if either (a) $\delta > 1/15$–the merger leads to gains in the export market–or (b) $\delta \leq 1/15$ and $\beta \leq f_{sM}$, which proves (ii). Similarly, a merger of L firms is profitable when s firms are not merged if

$$\frac{\gamma}{4} + \frac{\beta(1+\delta)^2}{16} \geq \frac{\gamma(1+\delta)^2}{9} + \frac{\beta(1+2\delta)^2}{25} + \frac{\gamma(1-2\delta)^2}{9} + \frac{\beta(1-3\delta)^2}{25}.$$

Solving the inequality with respect to β we obtain $\beta \leq f_{Ln}$. The inequality is satisfied if either (a) $\delta > 7/61$–the merger leads to gains in the export market–or (b) $\delta \leq 7/61$ and $\beta \leq f_{Ln}$, which proves (iii). A merger of L firms is profitable when s firms are merged if

$$\frac{\gamma}{4} + \frac{\beta}{9} \geq (1+\delta)^2\left(\frac{\gamma}{9} + \frac{\beta}{16}\right) + \frac{\gamma(1-2\delta)^2}{9} + \frac{\beta(1-3\delta)^2}{16}.$$

Solving the inequality with respect to β we get $\beta \leq f_{Lm}$. The inequality is satisfied if either (a) $\delta > 1/15$–the merger leads to gains in the export market– or (b) $\delta \leq 1/15$ and $\beta \leq f_{Lm}$, which proves (iv).

Q.E.D.

Proof of Proposition 1:
(i) We know from Lemma 1 part (i) that if $\delta \in [0, 7/61)$, firms in s wish to merge when firms in L are not merged when $\beta \leq f_{sN}$. We know from Lemma 1 part (ii) that if $\delta \in [0, 7/61)$, firms in s wish to merge when firms in L are merged when $\beta \leq f_{sM}$. The conditions under which firms in s decide to merge are less restrictive when firms in L are merged since $f_{sN} < f_{sM}$ for $\delta \in [0, 7/61]$.
(ii) Suppose that firms in L are not merged. From Lemma 1 part (i) we have that if $\delta \in [7/61, 1/3]$, then firms in s wish to merge when firms in L are not merged for any β. However, if $\delta \in [0, 7/61)$, firms in s only wish to merge when firms in L are not merged as long as $\beta \leq f_{sN}$. Now, suppose that firms in L are merged. From Lemma 1 part (ii) we have that if $\delta \in [1/15, 1/3]$, then firms in s wish to merge when firms in L are merged for any β. However, if $\delta \in [0, 1/15)$, firms in s only wish to merge when firms in L are merged as long as $\beta \leq f_{sM}$.
(iii) Suppose that firms in L are not merged. From Lemma 1 part (i) we have $f_{sN}(0) = 100/63$. This means that if $\beta \in [1, 100/63]$, then firms in s wish to merge when firms in L are not merged for any δ. However, if $\beta > 100/63$, firms in s only wish to merge when firms in L are not merged as long as $\beta \leq f_{sN}$. Now, suppose that firms in L are merged. From Lemma 1 part (ii) we have $f_{sM}(0) = 2$. This means that if $\beta \in [1, 2]$, then firms in s wish to merge when firms in L are not merged for any δ. However, if $\beta > 2$, firms in s only wish to merge when firms in L are not merged as long as $\beta \leq f_{sM}$.
(iv) Lemma 1 part (i) tells us that firms in s wish to merge when firms in L are not merged when $\beta \leq f_{sN}$. Lemma 1 part (iii) tells us that firms in L wish to merge when firms in s are not merged when $\beta \leq f_{Ln} = \gamma f_{sN}$. The conditions under which firms in L decide to merge when firms in s are not merged are less restrictive than the conditions under which firms in s decide to merge when firms in L are not merged since $\gamma \geq 1$ implies $f_{sN} \leq f_{Ln}$ for all (γ, δ). Similarly, the conditions under which firms in L decide to merge when firms in s are merged are less restrictive than the conditions under which firms in s decide to merge when firms in L are merged since $\gamma \geq 1$ implies $f_{sM} \leq f_{Lm}$ for all (γ, δ). *Q.E.D.*

Proof of Proposition 2: Denote the game played by firms by F and its Nash equilibria by $NE(F)$. The assumption that $\gamma \leq 1.26$ and the definitions of f_{sN}, f_{sM}, f_{Ln}, and f_{Lm} imply $f_{sN} \leq f_{Ln} < f_{sM} \leq f_{Lm}$.
(i) If $\beta \geq f_{sM}$ and $f_{sN} < f_{sM}$, then $f_{sN} < f_{sM} \leq \beta$. Lemma 1 parts (i) and (ii) together with $f_{sN} < f_{sM} \leq \beta$ imply that m is a dominated strategy for firms in s. Thus, firms in s choose n. If $\beta \geq f_{sM}$ and $f_{sM} > f_{Ln}$, then $\beta > f_{Ln}$. Lemma 1 part (iii) together with $\beta > f_{Ln}$ imply that the best response of firms in L to n is N. So, firms in L will play N. Thus, $NE(F) = (n, N)$ for $\gamma \leq 1.26$ and $\beta \geq f_{sM}$.

(iii) If $\beta < f_{Ln}$ and $f_{Ln} < f_{Lm}$, then $\beta < f_{Ln} < f_{Lm}$. Lemma 1 parts (iii) and (iv) together with $\beta < f_{Ln} < f_{Lm}$ imply that N is a dominated strategy for firms in L. Thus, firms in L choose M. If $\beta < f_{Ln}$ and $f_{Ln} < f_{sM}$, then $\beta < f_{sM}$. Lemma 1 part (ii) together with $\beta < f_{sM}$ imply that the best response of firms in s to M is m. So, firms in s choose m. Thus, for $\gamma \geq 1.26$ and $\beta < f_{Ln}$, we have $NE(F) = (m, M)$.
(iii) If $f_{Ln} < \beta \leq f_{sM}$ and $f_{sN} \leq f_{Ln} < f_{sM} \leq f_{Lm}$, then $f_{sN} < \beta \leq f_{sM}$ and $f_{Ln} < \beta \leq f_{Lm}$. If $f_{sN} < \beta \leq f_{sM}$, then Lemma 1 part (i) implies that the best response of firms in s to N is n and Lemma 1 part (ii) implies that the best response of firms in s to M is m. If $f_{Ln} < \beta \leq f_{Lm}$, then Lemma 1 part (iii) implies that the best response of firms in L to n is N and Lemma 1 part (iv) implies that the best response of firms in L to m is M. Thus, (n, N) and (m, M) are pure-strategy Nash equilibria of F when $\gamma \geq 1.26$ and $f_{Ln} < \beta \leq f_{sM}$. There exists also a mixed-strategy Nash equilibrium where firms in s randomize between m and n to make firms in L indifferent between M and N:

$$\begin{aligned} p_s\left(\frac{\gamma}{4}+\frac{\beta}{9}\right)+(1-p_s)\left(\frac{\gamma}{4}+\frac{\beta(1+\delta)^2}{16}\right) \\ = p_s\left(\frac{\gamma\,(1+\delta)^2}{9}+\frac{\beta\,(1+\delta)^2}{16}+\frac{\gamma\,(1-2\delta)^2}{9}+\frac{\beta(1-3\delta)^2}{16}\right) \\ +(1-p_s)\left(\frac{\gamma\,(1+\delta)^2}{9}+\frac{\beta\,(1+2\delta)^2}{25}+\frac{\gamma\,(1-2\delta)^2}{9}+\frac{\beta\,(1-3\delta)^2}{25}\right), \end{aligned}$$

where p_s is the probability that firms in s choose m. Solving this equation for p_s we obtain (3). Firms in L randomize between M and N to make firms in s indifferent between m and n:

$$\begin{aligned} p_L\left(\frac{1}{4}+\frac{\beta}{9}\right)+(1-p_L)\left(\frac{1}{4}+\frac{\beta(1+\delta)^2}{16}\right) \\ = p_L\left(\frac{(1+\delta)^2}{9}+\frac{\beta\,(1+\delta)^2}{16}+\frac{(1-2\delta)^2}{9}+\frac{\beta(1-3\delta)^2}{16}\right) \\ +(1-p_L)\left(\frac{(1+\delta)^2}{9}+\frac{\beta\,(1+2\delta)^2}{25}+\frac{(1-2\delta)^2}{9}+\frac{\beta\,(1-3\delta)^2}{25}\right), \end{aligned}$$

where p_L is the probability that firms in L choose M. Solving this equation for p_L we obtain (4). Thus, for $\gamma \geq 1.26$ and $f_{Ln} < \beta \leq f_{sM}$, we have $NE(F) = \{(n, N), (m, M), (p_s, m; p_L, M)\}$. *Q.E.D.*

Proof of Proposition 3: Denote the game played by firms by F and its Nash equilibria by $NE(F)$. The assumption that $\gamma > 1.26$ and the definitions of f_{sN}, f_{sM}, f_{Ln}, and f_{Lm} imply that (a) $f_{sN} < f_{sM} < f_{Ln} < f_{Lm}$ for $\delta \leq \frac{50\gamma-63}{750\gamma-549}$ and (b) $f_{sN} < f_{Ln} < f_{sM} < f_{Lm}$ for $\delta > \frac{50\gamma-63}{750\gamma-549}$.
(i) If $\beta \geq f_{sM}$ and $f_{sN} < f_{sM}$, then $f_{sN} < f_{sM} \leq \beta$. Lemma 1 parts (i) and (ii) together with $f_{sN} < f_{sM} \leq \beta$ imply that m is a dominated strategy for firms in

s. Thus, firms in s choose n. If $\beta \geq f_{Ln}$, then Lemma 1 part (iii) implies that the best response of firms in L to n is N. So, firms in L will play N. Thus, for $\gamma > 1.26$ and $\beta \leq \max[f_{sM}, f_{Ln}]$, we have $NE(F) = (n, N)$.
(ii) If $f_{sM} \leq \beta \leq f_{Ln}$ then (a) holds and we have $f_{sN} < f_{sM} \leq \beta \leq f_{Ln} < f_{Lm}$. Lemma 1 parts (iii) and (iv) together with $\beta \leq f_{Ln} < f_{Lm}$ imply that N is a dominated strategy for firms in L. Thus, firms in L choose M. Lemma 1 parts (i) and (ii) together with $f_{sN} < f_{sM} \leq \beta$ imply that m is a dominated strategy for firms in s. So, firms in s choose n. Thus, for $\gamma > 1.26$ and $f_{sM} \leq \beta \leq f_{Ln}$ we have $NE(F) = (n, M)$.
The proofs of (iii) and (iv) are similar to those of (ii) and (iii) of Proposition 2, respectively. *Q.E.D.*

Proof of Lemma 2: I start the proof by stating conditions under which a domestic merger improves national welfare for a given market structure in L. Consumer surplus at s is given by $CS_s = Q_s^2/2$,where Q_s is total output produced by s firms. If s firms do not merge, then $CS_s = (2-\delta)^2/18$. If s firms merge, then $CS_s = 1/8$. Thus, a merger of firms in s improves national welfare when L firms are not merged if

$$\frac{1}{8}+\frac{1}{4}+\frac{\beta(1+\delta)^2}{16} \geq \frac{(2-\delta)^2}{18}+\frac{(1+\delta)^2+(1-2\delta)^2}{9}+\frac{\beta\left[(1+2\delta)^2+(1-3\delta)^2\right]}{25}. \tag{9}$$

Solving (9) with respect to β we obtain $\beta \geq g_{sN}$ which proves (i). A merger of firms in s improves national welfare when L firms are merged if

$$\frac{1}{8}+\frac{1}{4}+\frac{\beta}{9} \geq \frac{(2-\delta)^2}{18}+\frac{(1+\delta)^2+(1-2\delta)^2}{9}+\frac{\beta\left[(1+\delta)^2+(1-3\delta)^2\right]}{16}. \tag{10}$$

Solving (10) with respect to β we obtain $\beta \geq g_{sM}$ which proves (ii). I will now state conditions under which a L merger improves L welfare conditional on a given market structure in s. Consumer surplus in L is given by $CS_L = Q_L^2/2\gamma$,where Q_L is total output produced by L firms. If L firms do not merge, then $CS_L = \gamma(2-\delta)^2/18$. If L firms merge, then $CS_L = \gamma/8$. So, a merger of firms in L improves national welfare when s firms are not merged if

$$\frac{\gamma}{8}+\frac{\gamma}{4}+\frac{\beta(1+\delta)^2}{16} \geq \frac{\gamma(2-\delta)^2}{18}+\frac{\gamma\left[(1+\delta)^2+(1-2\delta)^2\right]}{9}+\frac{\beta\left[(1+2\delta)^2+(1-3\delta)^2\right]}{25}. \tag{11}$$

Solving (11) with respect to β we have $\beta \geq g_{Ln}$ which proves (iii). A merger of firms in L improves national welfare when s firms are merged if

$$\frac{\gamma}{8}+\frac{\gamma}{4}+\frac{\beta}{9} \geq \frac{\gamma(2-\delta)^2}{18}+\frac{\gamma\left[(1+\delta)^2+(1-2\delta)^2\right]}{9}+\frac{\beta\left[(1+\delta)^2+(1-3\delta)^2\right]}{16}. \tag{12}$$

Solving (12) with respect to β we have $\beta \geq g_{Lm}$ which proves (iv). *Q.E.D.*

Proof of Proposition 4: It follows from Lemma 2 just as the proof of Proposition 1 follows from Lemma 1. *Q.E.D.*

Proof of Proposition 5: Denote the merger game played by governments by G and its set of Nash equilibria by $NE(G)$. The assumption that $\gamma \leq 2.15$ and the definitions of g_{sN}, g_{sM}, g_{Ln}, and g_{Lm} imply that $g_{sM} \leq g_{Lm} < g_{sN} \leq g_{Ln}$.
(i) If $\beta < g_{Lm}$ and $g_{Lm} < g_{sN}$, then $\beta < g_{sN}$. Lemma 2 part (i) together with $\beta < g_{sN}$ imply that the best response of the government of s to N is n. If $\beta < g_{Lm}$ and $g_{Lm} < g_{Ln}$, then $\beta < g_{Lm} < g_{Ln}$. Lemma 2 parts (iii) and (iv) together with $\beta < g_{Lm} < g_{Ln}$ imply M is a dominated strategy for the government of L. So, the government of L chooses N. Thus, for $\gamma \leq 2.15$ and $\beta < g_{Lm}$ we have $NE(G) = (n, N)$.
(ii) If $\beta \geq g_{sN}$ and $g_{sM} < g_{sN}$, then $g_{sM} < g_{sN} \leq \beta$. Lemma 2 parts (iii) and (iv) together with $g_{sM} < g_{sN} \leq \beta$ imply that n is a dominated strategy for the government of s. So, the government of s chooses m. If $\beta \geq g_{sN}$ and $g_{Lm} < g_{sN}$, then $\beta > g_{Lm}$. Lemma 2 part (iv) together with $\beta > g_{Lm}$ imply that the best response of the government of L to m is M. So, the government of L plays M. Thus, $NE(G) = (m, M)$ for $\gamma \leq 2.15$ and $\beta \geq g_{sN}$.
(iii) If $g_{Lm} \leq \beta < g_{sN}$ and $g_{sM} \leq g_{Lm} < g_{sN} \leq g_{Ln}$, then $g_{sM} \leq g_{Lm} \leq \beta < g_{sN} \leq g_{Ln}$. If $g_{sM} \leq \beta < g_{sN}$, then Lemma 2 part (i) implies that the best response of the government of s to N is n and Lemma 2 part (ii) implies that the best response of the government of s to M is m. If $g_{Lm} \leq \beta < g_{Ln}$, then Lemma 2 part (iii) implies that the best response of the government of L to n is N and Lemma 2 part (iv) implies that the best response of the government of L to m is M. Thus, (n, N) and (m, M) are pure-strategy Nash equilibria of G when $\gamma \leq 2.15$ and $g_{Lm} \leq \beta < g_{sN}$. There also exists a mixed-strategy Nash equilibrium where the government of s randomizes between m and n to make the government of L indifferent between M and N:

$$q_s\left(\frac{\gamma}{8}+\frac{\gamma}{4}+\frac{\beta}{9}\right)+(1-q_s)\left(\frac{\gamma}{8}+\frac{\gamma}{4}+\frac{\beta(1+\delta)^2}{16}\right)$$
$$=q_s\left(\frac{\gamma\,(2-\delta)^2}{18}+\frac{\gamma\left[(1+\delta)^2+(1-2\delta)^2\right]}{9}+\frac{\beta\left[(1+\delta)^2+(1-3\delta)^2\right]}{16}\right)_s$$
$$+(1-q_s)\left(\frac{\gamma\,(2-\delta)^2}{18}+\frac{\gamma\left[(1+\delta)^2+(1-2\delta)^2\right]}{9}+\frac{\beta\left[(1+2\delta)^2+(1-3\delta)^2\right]}{25}\right),$$

where q_s is the probability that the government of s chooses m. Solving this equation for q_s we obtain (7). The government of L randomizes between M and

N to make the government of s indifferent between m and n:

$$q_L\left(\frac{1}{8}+\frac{1}{4}+\frac{\beta}{9}\right)+(1-q_L)\left(\frac{1}{8}+\frac{1}{4}+\frac{\beta(1+\delta)^2}{16}\right)$$
$$=q_L\left(\frac{(2-\delta)^2}{18}+\frac{(1+\delta)^2+(1-2\delta)^2}{9}+\frac{\beta\left[(1+\delta)^2+(1-3\delta)^2\right]}{16}\right)$$
$$+(1-q_L)\left(\frac{(2-\delta)^2}{18}+\frac{(1+\delta)^2+(1-2\delta)^2}{9}+\frac{\beta\left[(1+2\delta)^2+(1-3\delta)^2\right]}{25}\right),$$

where q_L is the probability that the government of L chooses M. Solving this equation for q_L we obtain (8). Thus, for $\gamma \leq 2.15$ and $g_{Lm} \leq \beta < g_{sN}$, we have $NE(G) = \{(n,N), (m,M), (q_s, m; q_L, M)\}$. *Q.E.D.*

Proof of Proposition 6: Denote the merger game played by governments by G and its set of Nash equilibria by $NE(G)$. The assumption that $\gamma > 2.15$ and the definitions of g_{sN}, g_{sM}, g_{Ln}, and g_{Lm} imply that (a) $g_{sM} < g_{Lm} < g_{sN} < g_{Ln}$ for $\delta \leq \frac{63\gamma-50}{549\gamma-750}$ and (b) $g_{sM} < g_{sN} < g_{Lm} < g_{Ln}$ for $\delta > \frac{63\gamma-50}{549\gamma-750}$.
(i) If $\beta < g_{Lm}$ and $g_{Lm} < g_{Ln}$, then $\beta < g_{Lm} < g_{Ln}$. Lemma 2 parts (iii) and (iv) together with $\beta < g_{Lm} < g_{Ln}$ imply that M is a dominated strategy for the government of L. So, the government of L plays N. If $\beta < g_{sN}$ then Lemma 2 part (ii) implies that the best response of the government of s to N is n. Thus, $NE(G) = (n, N)$ for $\gamma > 2.15$ and $\beta < \min[g_{Lm}, g_{sN}]$.
(ii) If $g_{sN} \leq \beta \leq g_{Lm}$ then (b) holds and we have $g_{sM} < g_{sN} \leq \beta \leq g_{Lm} < g_{Ln}$. Lemma 2 parts (iii) and (iv) together with $\beta \leq g_{Lm} < g_{Ln}$ imply that M is a dominated strategy for the government of L. Lemma 2 parts (i) and (ii) together with $g_{sM} < g_{sN} \leq \beta$ imply that n is a dominated strategy for the government of s. Thus, $NE(G) = (m, N)$ for $\gamma > 2.15$ and $g_{sN} \leq \beta \leq g_{Lm}$.
The proofs of (iii) and (iv) are similar to those of (ii) and (iii) of Proposition 5, respectively. *Q.E.D.*

Proposition 7: *Let* $\gamma \leq 1.26$.
(i) If $\beta \geq f_{sM}$, *then* $NE(F) = NE(G) = (n, N)$;
(ii) If $\max[f_{Ln}, g_{Lm}] \leq \beta \leq f_{sM}$, *then* $NE(G) = (n, N)$ *and* $NE(F) = \{(n,N), (m,M), (p_s, m; p_L, M)\}$;
(iii) If $\beta \geq \max[f_{Ln}, g_{Lm}]$, *then* $NE(F) = \{(n,N), (m,M), (p_s, m; p_L, M)\}$ *and* $NE(G) = \{(n,N), (m,M), (q_s, m; q_L, M)\}$;
(iv) If $\beta \leq \min[f_{Ln}, g_{Lm}]$, *then* $NE(F) = (m, M) \neq (n, N) = NE(G)$;
(v) If $g_{Lm} \leq \beta \leq \min[f_{Ln}, g_{sN}]$, *then* $NE(F) = (m, M)$ *and* $NE(G) = \{(n,N), (m,M), (q_s, m; q_L, M)\}$;
(vi) If $\beta \geq g_{sN}$, *then* $NE(F) = NE(G) = (m, M)$.

Proof of Proposition 7: The proof follows from Propositions 2 and 5. *Q.E.D.*

Proposition 8: *Let* $1.26 < \gamma \leq 2.15$.
(i) If $\beta \geq \max[f_{sM}, f_{Ln}]$, *then* $NE(F) = NE(G) = (n, N)$;
(ii) If $f_{Ln} \leq \beta \leq f_{sM}$, *then* $NE(F) = \{(n, N), (m, M), (p_s, m; p_L, M)\}$ *and* $NE(G) = (n, N)$;
(iii) If $f_{sM} \leq \beta \leq f_{Ln}$, *then* $NE(F) = (n, M) \neq (n, N) = NE(G)$;
(iv) If $\beta \leq \min[f_{sM}, f_{Ln}, g_{Lm}]$, *then* $NE(F) = (m, M) \neq (n, N) = NE(G)$;
(v) If $g_{Lm} \leq \beta \leq g_{sN}$, *then* $NE(G) = \{(n, N), (m, M), (q_s, m; q_L, M)\}$ *and* $NE(F) = (m, M)$;
(vi) If $\beta \geq g_{sN}$, *then* $NE(F) = NE(G) = (m, M)$.

Proof of Proposition 8: The proof follows from Propositions 3 and 5. *Q.E.D.*

Proposition 9: *Let* $\gamma > 2.15$.
(i) If $\beta \geq \max[f_{sM}, f_{Ln}]$, *then* $NE(F) = NE(G) = (n, N)$;
(ii) If $f_{Ln} \leq \beta \leq f_{sM}$, *then* $NE(F) = \{(n, N), (m, M), (p_s, m; p_L, M)\}$ *and* $NE(G) = (n, N)$;
(iii) If $f_{sM} \leq \beta \leq f_{Ln}$, *then* $NE(F) = (n, M) \neq (n, N) = NE(G)$;
(iv) If $\beta \leq \min[f_{sM}, f_{Ln}, g_{Lm}, g_{sN}]$, *then* $NE(F) = (m, M) \neq (n, N) = NE(G)$;
(v) If $g_{Lm} \leq \beta \leq g_{sN}$, *then* $NE(G) = \{(n, N), (m, M), (q_s, m; q_L, M)\}$ *and* $NE(F) = (m, M)$;
(vi) If $g_{sN} \leq \beta \leq g_{Lm}$, *then* $NE(F) = (m, M) \neq (m, N) = NE(G)$;
(vii) If $\beta \geq \max[g_{sN}, g_{Lm}]$, *then* $NE(F) = NE(G) = (m, M)$.

Proof of Proposition 9: The proof follows from Propositions 3 and 6. *Q.E.D.*

12 Technical Appendix

Derivation of Equilibrium Profits in Lemma 1 and Table 1: If s firms are not merged they sell $q_{s1} = (1+\delta)/3$ and $q_{s2} = (1-2\delta)/3$ in the s market. In this case, profits of s firms in the s market are given by $\pi_{s1}^{s} = (1+\delta)^2/9$ and $\pi_{s2}^{s} = (1-2\delta)^2/9$. If L firms are not merged they sell $q_{L1} = \gamma(1+\delta)/3$ and $q_{L2} = \gamma(1-2\delta)/3$ in the L market. Profits of L firms in the L market are $\pi_{L1}^{L} = \gamma(1+\delta)^2/9$ and $\pi_{L2}^{L} = \gamma(1-2\delta)^2/9$. The market equilibrium in e is given by:

$$\begin{aligned}
q_{s1,N}^{e} &= \frac{\beta}{2} - \frac{1}{2}\left(q_{s2,N}^{e} + q_{L1,n}^{e} + q_{L2,n}^{e}\right) \\
q_{s2,N}^{e} &= \frac{\beta(1-\delta)}{2} - \frac{1}{2}\left(q_{s1,N}^{e} + q_{L1,n}^{e} + q_{L2,n}^{e}\right) \\
q_{L1,n}^{e} &= \frac{\beta}{2} - \frac{1}{2}\left(q_{s1,N}^{e} + q_{s2,N}^{e} + q_{L2,n}^{e}\right) \\
q_{L2,n}^{e} &= \frac{\beta(1-\delta)}{2} - \frac{1}{2}\left(q_{s1,N}^{e} + q_{s2,N}^{e} + q_{L1,n}^{e}\right)
\end{aligned}$$

Solving this system we obtain $q_{s1,N}^{e} = q_{L1,n}^{e} = \beta(1+2\delta)/5$ and $q_{s2,N}^{e} = q_{L2,n}^{e} = \beta(1-3\delta)/5$. The profits of s and L firms in e are given by $\pi_{s1,N}^{e} = \pi_{L1,n}^{e} = \beta(1+2\delta)^2/25$ and $\pi_{s2,N}^{e} = \pi_{L2,n}^{e} = \beta(1-3\delta)^2/25$.
If s firms merge the s market becomes a monopoly and the equilibrium quantity is $Q = 1/2$. The monopoly profits are $\pi_{m}^{s} = 1/4$. If s firms merge and L firms are not merged, then the equilibrium in e is given by

$$\begin{aligned}
q_{m,N}^{e} &= \frac{\beta}{2} - \frac{1}{2}\left(q_{L1,m}^{e} + q_{L2,m}^{e}\right) \\
q_{L1,m}^{e} &= \frac{\beta}{2} - \frac{1}{2}\left(q_{m,N}^{e} + q_{L2,m}^{e}\right) \\
q_{L2,m}^{e} &= \frac{\beta(1-\delta)}{2} - \frac{1}{2}\left(q_{m,N}^{e} + q_{L1,m}^{e}\right)
\end{aligned}$$

Solving this system we obtain $q_{m,N}^{e} = q_{L1,m}^{e} = \beta(1+\delta)/4$ and $q_{L2,m}^{e} = \beta(1-3\delta)/4$. The profits of the merged s firm in e are $\pi_{m,N}^{e} = \beta(1+\delta)^2/16$.
If s firms are not merged but L firms are, the equilibrium in e is given by:

$$\begin{aligned}
q_{s1,M}^{e} &= \frac{\beta}{2} - \frac{1}{2}\left(q_{s2,M}^{e} + q_{M,n}^{e}\right) \\
q_{s2,M}^{e} &= \frac{\beta(1-\delta)}{2} - \frac{1}{2}\left(q_{s1,M}^{e} + q_{M,n}^{e}\right) \\
q_{M,n}^{e} &= \frac{\beta}{2} - \frac{1}{2}\left(q_{s1,M}^{e} + q_{s2,M}^{e}\right)
\end{aligned}$$

The solution to this system is $q_{M,n}^{e} = q_{s1,M}^{e} = \beta(1+\delta)/4$ and $q_{s2,M}^{e} = \beta(1-3\delta)/4$. The profits of $s1$ in e are $\pi_{s1,M}^{e} = \beta(1+\delta)^2/16$ and the profits of $s2$ are $\pi_{s2,M}^{e} = \beta(1-3\delta)^2/16$. If s firms merge and so do L firms we have a duopoly

in e. In this case the equilibrium quantities in e are $q^e_{m,M} = q^e_{M,m} = \beta/3$ and profits of the merged firms by $\pi^e_{M,m} = \pi^e_{m,M} = \beta/9$.
Hence, the equilibrium profit of the merged firm in s when firms in L are merged is

$$\Pi^s_{m,M} = \pi^s_m + \pi^e_{m,M} = \frac{1}{4} + \frac{\beta}{9}.$$

The equilibrium profit of the merged firm in s when firms in L are not merged is

$$\Pi^s_{m,N} = \pi^s_m + \pi^e_{m,N} = \frac{1}{4} + \frac{\beta(1+\delta)^2}{16}.$$

The equilibrium profit in s when firms in s are not merged and firms in L are merged is

$$\begin{aligned}\Pi^s_{n,M} &= \pi^s_{s1} + \pi^s_{s2} + \pi^e_{s1,M} + \pi^e_{s2,M} \\ &= \frac{(1+\delta)^2}{9} + \frac{(1-2\delta)^2}{9} + \frac{\beta\,(1+\delta)^2}{16} + \frac{\beta(1-3\delta)^2}{16}.\end{aligned}$$

The equilibrium profit in s when firms in s are not merged and firms in L are not merged is

$$\begin{aligned}\Pi^s_{n,N} &= \pi^s_{s1} + \pi^s_{s2} + \pi^e_{s1,N} + \pi^e_{s2,N} \\ &= \frac{(1+\delta)^2}{9} + \frac{(1-2\delta)^2}{9} + \frac{\beta\,(1+2\delta)^2}{25} + \frac{\beta\,(1-3\delta)^2}{25}.\end{aligned}$$

The equilibrium profit of the merged firm in L when firms in s are merged is

$$\Pi^L_{M,m} = \pi^L_M + \pi^e_{M,m} = \frac{\gamma}{4} + \frac{\beta}{9}.$$

The equilibrium profit of the merged firm in L when firms in s are not merged is

$$\Pi^L_{M,n} = \pi^L_M + \pi^e_{M,n} = \frac{\gamma}{4} + \frac{\beta(1+\delta)^2}{16}.$$

The equilibrium profit in L when firms in L are not merged and firms in s are merged is

$$\begin{aligned}\Pi^L_{N,m} &= \pi^L_{L1} + \pi^L_{L2} + \pi^e_{L1,m} + \pi^e_{L2,m} \\ &= \frac{\gamma\,(1+\delta)^2}{9} + \frac{\gamma\,(1-2\delta)^2}{9} + \frac{\beta\,(1+\delta)^2}{16} + \frac{\beta(1-3\delta)^2}{16}.\end{aligned}$$

The equilibrium profit in L when firms in L are not merged and firms in s are not merged is

$$\begin{aligned}\Pi^L_{N,n} &= \pi^L_{L1} + \pi^L_{L2} + \pi^e_{L1,n} + \pi^e_{L2,n} \\ &= \frac{\gamma\,(1+\delta)^2}{9} + \frac{\gamma\,(1-2\delta)^2}{9} + \frac{\beta\,(1+2\delta)^2}{25} + \frac{\beta\,(1-3\delta)^2}{25}.\end{aligned}$$

Q.E.D.

Derivation of Equilibrium Welfare Levels in Lemma 2 and Table 2: Welfare in s is the sum of profits of s firms and consumer surplus at s conditional on a given market structure in L. Consumer surplus at s is given by $CS^s = (1-p_s)Q_s/2 = Q_s^2/2$,where Q_s is total output produced by s firms. If s firms do not merge, then $Q_s = (2-\delta)/3$ and $CS_n^s = (2-\delta)^2/18$. If s firms merge, then $Q_s = 1/2$ and $CS_m^s = 1/8$. Welfare in s when s firms are not merged and L firms are not merged is given by

$$\begin{aligned} W_{n,N}^s &= CS_n^s + \pi_{s1}^s + \pi_{s2}^s + \pi_{s1,N}^e + \pi_{s2,N}^e \\ &= \frac{(2-\delta)^2}{18} + \frac{(1+\delta)^2}{9} + \frac{(1-2\delta)^2}{9} + \frac{\beta(1+2\delta)^2}{25} + \frac{\beta(1-3\delta)^2}{25}. \end{aligned}$$

Welfare in s when s firms are merged and L firms are not merged is given by

$$\begin{aligned} W_{m,N}^s &= CS_m^s + \pi_m^s + \pi_{m,N}^e \\ &= \frac{1}{8} + \frac{1}{4} + \frac{\beta(1+\delta)^2}{16}. \end{aligned}$$

Welfare in s when s firms are not merged and L firms are merged is given by

$$\begin{aligned} W_{n,M}^s &= CS_n^s + \pi_{s1}^s + \pi_{s2}^s + \pi_{s1}^e + \pi_{s2}^e \\ &= \frac{(2-\delta)^2}{18} + \frac{(1+\delta)^2}{9} + \frac{\beta(1+\delta)^2}{16} + \frac{(1-2\delta)^2}{9} + \frac{\beta(1-3\delta)^2}{16}. \end{aligned}$$

Welfare in s when s firms are merged and L firms are merged is given by

$$\begin{aligned} W_{m,M}^s &= CS_{s1+s2}^s + \pi_{s1+s2}^s + \pi_{s1+s2}^e \\ &= \frac{1}{8} + \frac{1}{4} + \frac{\beta}{9}. \end{aligned}$$

Welfare in L is the sum of profits of L firms and consumer surplus at L conditional on a given market structure in s. Consumer surplus in L is given by $CS^L = (1-p_L)Q_L/2 = Q_L^2/2\gamma$,where Q_L is total output produced by L firms. If L firms do not merge, then $Q_L = \gamma(2-\delta)/3$ and $CS_N^L = \gamma(2-\delta)^2/18$. If L firms merge, then $Q_L = \gamma/2$ and $CS_M^L = \gamma/8$. Welfare in L when L firms are not merged and s firms are not merged is given by

$$\begin{aligned} W_{N,n}^L &= CS_N^L + \pi_{L1}^L + \pi_{L2}^L + \pi_{L1,n}^e + \pi_{L2,n}^e \\ &= \frac{\gamma(2-\delta)^2}{18} + \frac{\gamma(1+\delta)^2}{9} + \frac{\gamma(1-2\delta)^2}{9} + \frac{\beta(1+2\delta)^2}{25} + \frac{\beta(1-3\delta)^2}{25}. \end{aligned}$$

Welfare in L when L firms are merged and s firms are not merged is given by

$$\begin{aligned} W_{M,n}^L &= CS_M^L + \pi_M^L + \pi_{M,n}^e \\ &= \frac{\gamma}{8} + \frac{\gamma}{4} + \frac{\beta(1+\delta)^2}{16}. \end{aligned}$$

Welfare in L when L firms are not merged and s firms are merged is given by

$$\begin{aligned} W_{N,m}^{L} &= CS_{N}^{L}+\pi_{L1}^{L}+\pi_{L2}^{L}+\pi_{L1,m}^{e}+\pi_{L2,m}^{e} \\ &= \frac{\gamma\left(2-\delta\right)^{2}}{18}+\frac{\gamma\left(1+\delta\right)^{2}}{9}+\frac{\gamma\left(1-2\delta\right)^{2}}{9}+\frac{\beta\left(1+\delta\right)^{2}}{16}+\frac{\beta(1-3\delta)^{2}}{16}. \end{aligned}$$

Welfare in L when L firms are merged and s firms are merged is given by

$$\begin{aligned} W_{M,m}^{L} &= CS_{M}^{L}+\pi_{M}^{L}+\pi_{M,m}^{e} \\ &= \frac{\gamma}{8}+\frac{\gamma}{4}+\frac{\beta}{9}. \end{aligned}$$

Q.E.D.

Figure 1: The incentive for a profitable merger in the small country when firms in the large country are merged (f_{sM}) or not merged (f_{sN}).

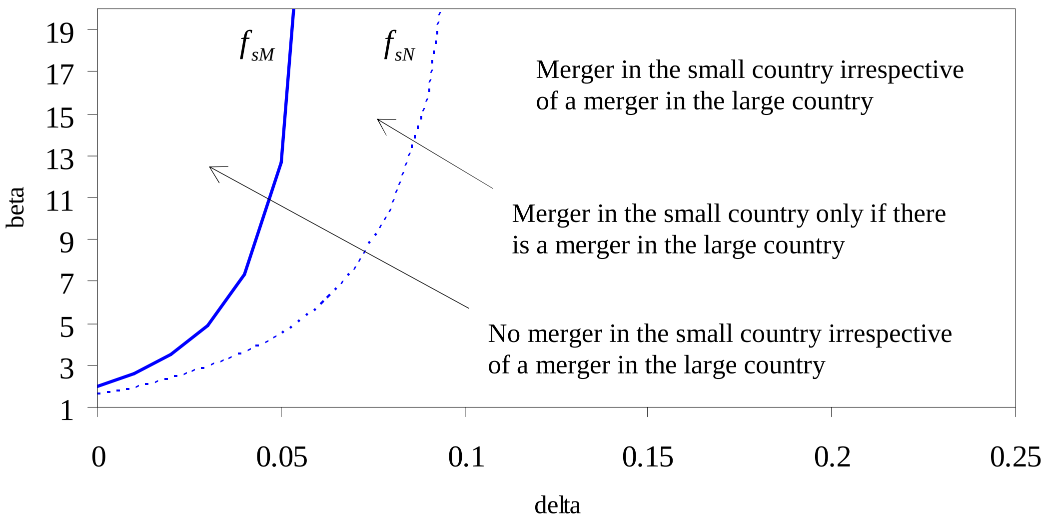

Figure 2: Equilibria of the merger game played by firms when the large country's market is 1.1 times greater than the small country's market.

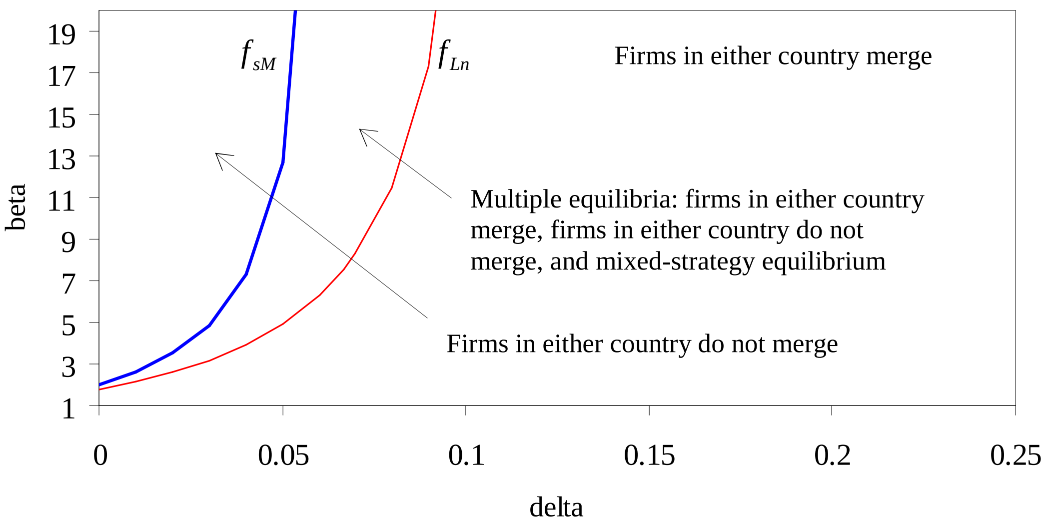

Figure 3: Equilibria of the merger game played by firms when the large country's market is 2 times greater than the small country's market.

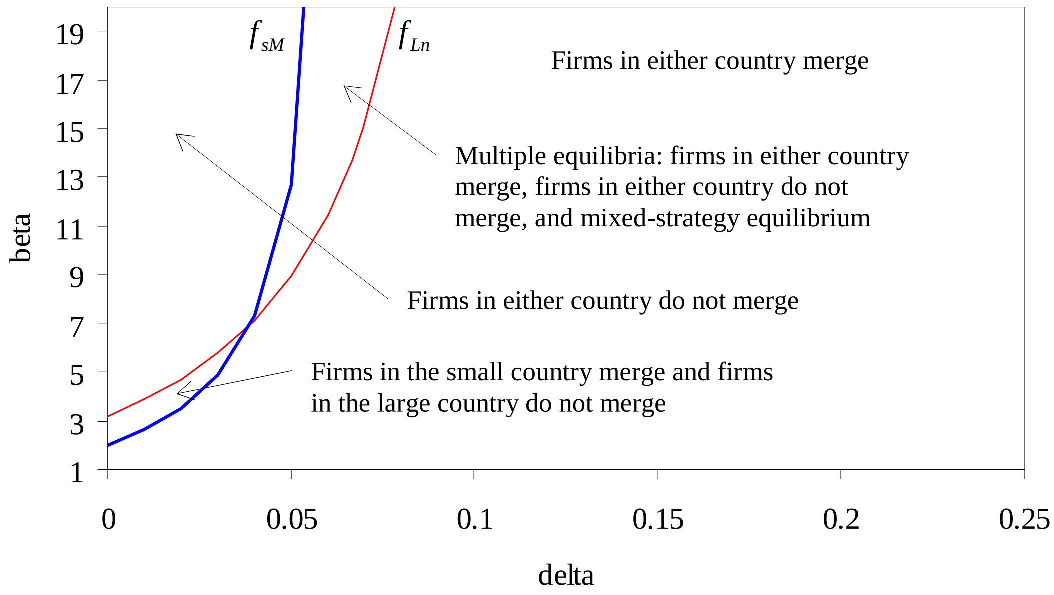

Figure 4: The incentive for a welfare improving merger in the small country when firms in the large country are merged (g_{sM}) or not merged (g_{sN}).

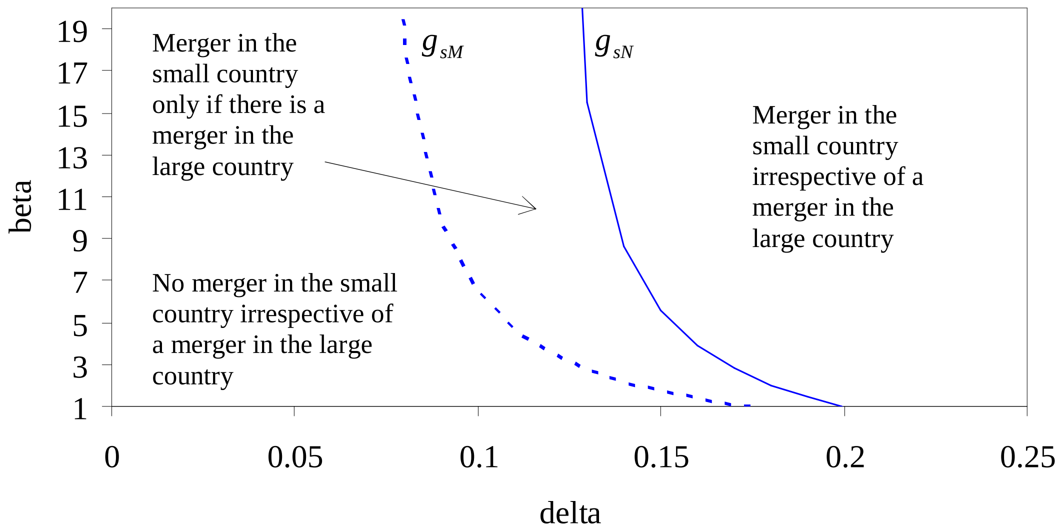

Figure 5: Equilibria of the merger game played by governments when the large country's market is 2 times greater than the small country's market.

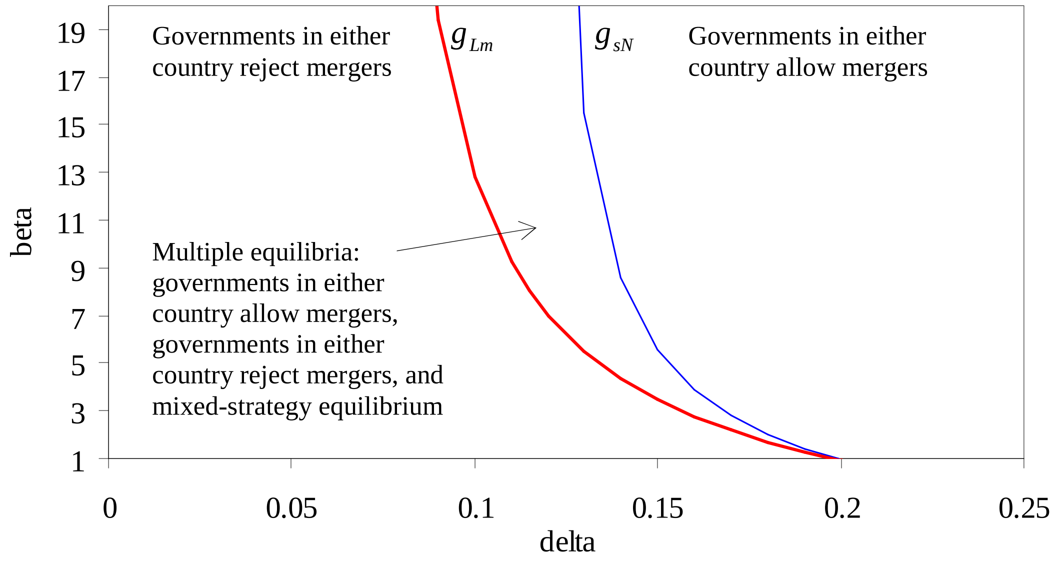

Figure 6: Equilibria of the merger game played by governments when the large country's market is 3 times greater than the small country's market.

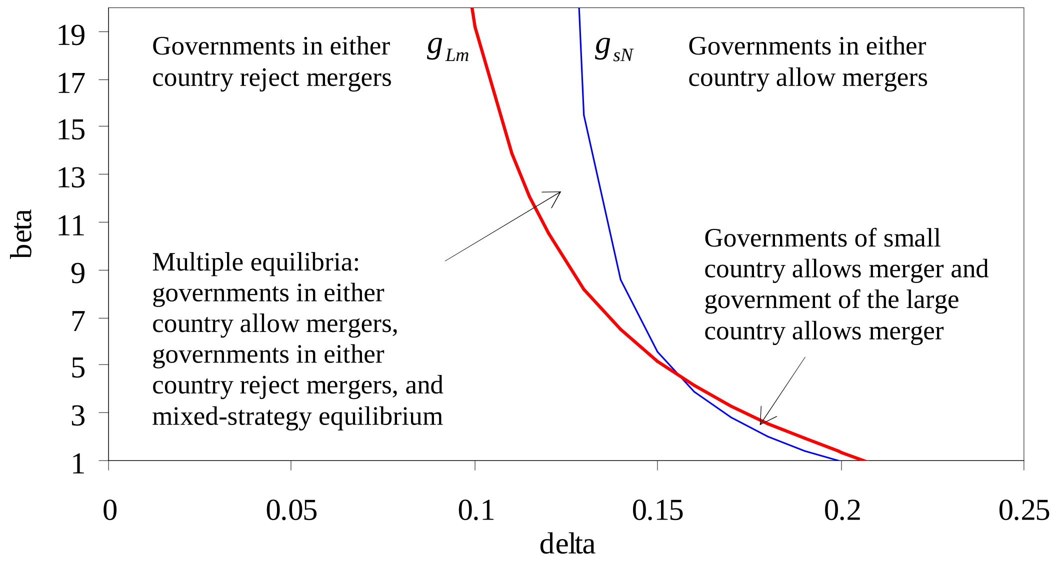

Figure 7: Equilibria in the merger game played by firms and of the merger game played by governments when the large country’s market is 1.1 times greater than the small country’s market.

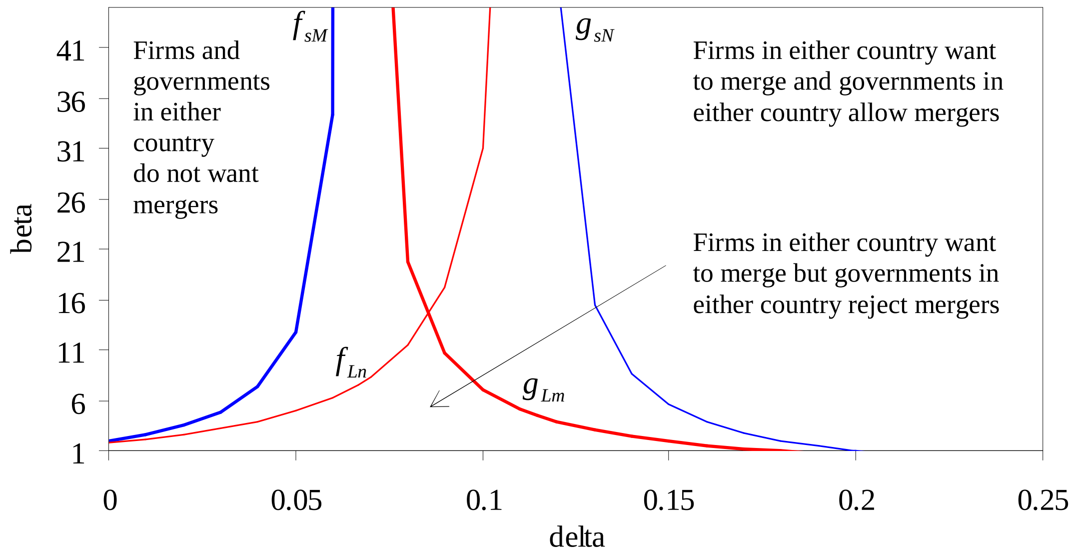

Figure 8: Equilibria in the merger game played by firms and of the merger game played by governments when the large country’s market is 2 times greater than the small country’s market.

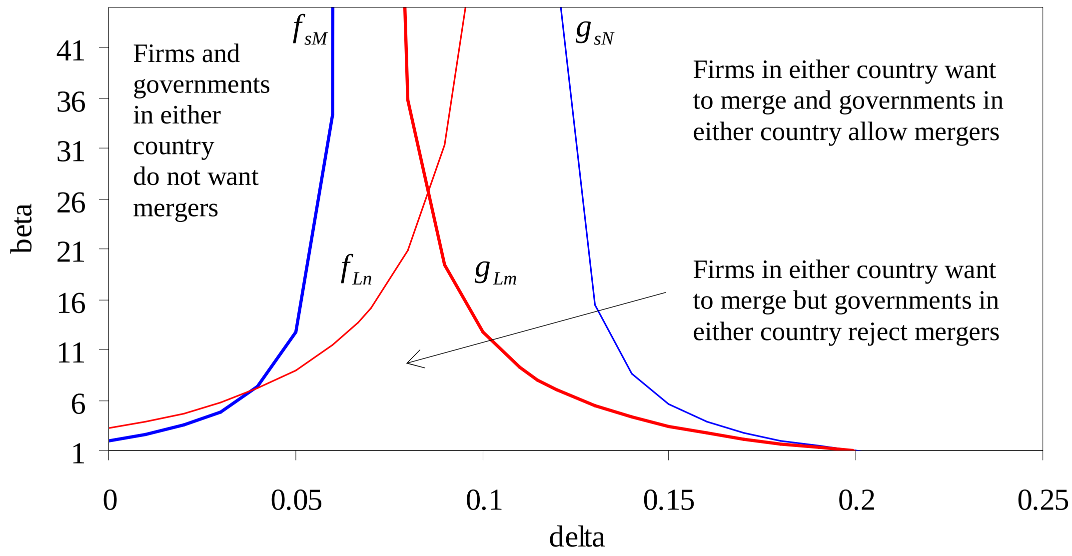

Figure 9: Equilibria in the merger game played by firms and of the merger game played by governments when the large country's market is 3 times greater than the small country's market.