A Cognitive Hierarchy Model of Behavior in Endogenous Timing Games*

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Abstract

We apply the cognitive hierarchy model of Camerer, Ho and Chong (2004)–where players have different levels of reasoning–to the action commitment game of Hamilton and Slutsky (1990)–a duopoly with endogenous timing of entry. We show that, for a reasonable average number of thinking steps, the model predicts a high percentage of Cournot outcomes including delay, and outcomes where the first mover chooses a quantity greater than Cournot but smaller than Stackelberg leader. We also show that the model can explain the most important features of the experimental evidence on the action commitment game in Huck, Müeller and Normann (2002).

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1 Introduction

The theoretical literature on endogenous timing tries to identify factors that might lead to the endogenous emergence of sequential or simultaneous play in oligopolistic markets.[1] In Hamilton and Slutsky (1990)'s action commitment game, two firms must decide a quantity to be produced in *one* of two periods before the market clears. If a firm commits to a quantity in the first period, it will have to make its decision without knowing whether the other firm has chosen to commit early or not. If a firm commits to a quantity in the second period, then it observes the first period production of the rival (or its decision to wait).

Hamilton and Slutsky show that this game has three subgame perfect Nash equilibria: both firms committing in the first period to the simultaneous-move Cournot-Nash equilibrium quantities, and each waiting and the other playing its Stackelberg leader quantity in the first period. They also show that only the Stackelberg equilibria survive elimination of weakly dominated strategies.

Observed behavior in experiments on this canonical model of endogenous timing is at odds with the theory. For example, Huck, Müeller and Normann (2002) test experimentally the predictions of the action commitment game. They find that: (i) market outcomes are very heterogeneous, (ii) Stackelberg equilibria are rare, (iii) Cournot outcomes are modal, (iv) simultaneous-move Cournot outcomes are often played in the second period (delay), and (v) there is a high percentage of outcomes where the first mover commits to a quantity higher than Cournot but lower than Stackelberg leader.

The questions that the endogenous timing literature tries to address are particularly relevant in terms of new markets, where two or more firms will enter. The experimental evidence suggests that simultaneous-move play may be a better predictor of behavior in markets for new goods than sequential play.[2] It also suggests that there may be substantial heterogeneity in behavior in these markets.

Why do we observe this gap between the theoretical predictions and the experimental evidence? One explanation might be that subjects have trouble coordinating their play in

[1] The seminal papers are Saloner (1987), Hamilton and Slutsky (1990), and Robson (1990)

[2] As we have seen the prediction of Stackelberg equilibria rests on equilibrium selection arguments. Simultaneous-move Cournot-Nash equilibria typically exist, however, they do not survive the application of equilibrium refinements.

one of the two Stackelberg equilibria: if both equilibria are exactly the same then it is far from clear which of the two firms is going to assume the leading role in the first period–see Matsumura (2001) on the instability of leader-follower relationships.

It is possible to think of explanations for some aspects of the experimental evidence. However, it is much harder to explain most of them. For example, Harsanyi and Selten's (1988) risk-payoff equilibrium selection argument can explain why Cournot outcomes are more frequently played than Stackelberg outcomes since playing the Stackelberg leader's quantity is risky by comparison with playing the Cournot-Nash quantity. However, risk-payoff considerations cannot explain delay, collusive or double Stackelberg leadership outcomes.

Another explanation might be inequity aversion. Santos-Pinto (2008) generalizes Hamilton and Slutsky's (1990) action commitment game by assuming that players are averse to inequality in payoffs. He shows that relatively high levels of inequity aversion rule out asymmetric equilibria, and inequity aversion gives rise to a continuum of simultaneous-move equilibria which include the Cournot-Nash outcome, collusive outcomes as well as double Stackelberg leadership. However, inequity aversion is not able to explain delay. Although inequity aversion can cast some light into the experimental evidence on endogenous timing games, we believe that the discussion can be further enriched with a different focus.

Recent experiments suggest that in strategic settings without clear precedents, individuals' initial responses often deviate systematically from equilibrium. Moreover, different players seem to employ different levels of reasoning in games. Nagel (1995) was one of the first to provide evidence for this using the p-beauty contest, a dominance solvable game. She found that most people do not follow the Nash equilibrium prediction of behavior; rather, their degree of strategic thinking is limited to a finite number of iterations when eliminating weakly dominated strategies. Other important references on non-equilibrium models of behavior in games are Stahl and Wilson (1995), Costa-Gomes, Crawford and Broseta (2003), and Camerer, Ho and Chong (2004). In these models, a level-k player computes his best response assuming that his rivals employ less thinking steps; the sole exception is that of the level zero players, who do not behave strategically and choose randomly across the strategy set.[3]

This paper applies Camerer, Ho and Chong's (2004) cognitive hierarchy model to Hamilton and Slutsky's (1990) action commitment game. Since this is a dynamic game and the cognitive hierarchy model is usually applied to static games we assume that: (1) a level zero player randomizes independently at each information set, and (2) players of higher levels choose best responses at information sets using backward induction and use Bayes' rule to update beliefs about their rivals' level of strategic sophistication.

We show that, for a reasonable average number of thinking steps, the cognitive hierarchy model predicts that standard Stackelberg equilibria will not be played. We also show that the model predicts that Cournot outcomes will be frequent, specially simultaneous play of Cournot quantities in the second period (delay). Finally, we show that the model predicts a high percentage of outcomes where the first mover commits to a quantity higher than Cournot but lower than Stackelberg leader.

The intuition behind these results is as follows. A level 1 ($L1$ from now on) prefers to wait since by doing that it can best respond to a level 0 ($L0$ from now on) who commits to period 1. If the $L1$ does not observe commitment by the rival in period 1, then he chooses a Cournot quantity since this is his best response to the $L0$'s expected quantity. Thus, when two $L1$s meet there will be delay. A level 2 ($L2$ from now on) faces a trade-off between committing to period 1 and waiting. If she knew the rival was an $L1$, then she would prefer to commit to period 1 with a Stackelberg leader's quantity. If she knew that the rival was an $L0$, then she would prefer to wait so that she could best respond to the rival when he commits to period 1. For reasonable average levels of thinking steps, an $L2$ commits to period 1–because there is a high probability the rival is an $L1$–and chooses a quantity higher than Cournot but lower than Stackelberg leader–because the rival might turn out to be an $L0$.

Next, we take the model to the data and see if it is able to explain the experimental evidence on the action commitment game in Huck, Müeller and Normann (2002) (HMN from now on). Since the cognitive hierarchy model typically delivers small sets of pre-

[3] Ivanov, Levin and Peck (forthcoming) apply a non-equilibrium model of behavior in games to a model of endogenous timing in investment where players decide if they want to invest in a market and, if yes, when they want to carry that action out.

dicted behavior we introduce noise into players' quantity choices (see Camerer, Palfrey and Rodgers (2007) and Östling, Wang, Chou and Camerer (2008)).

We estimate the model that best fits the data and compare its predictions to observed behavior. Since the cognitive hierarchy model is particularly suited to explain initial responses we focus on the first five rounds of play.

We find that the cognitive hierarchy model is able to generate the heterogeneity in market outcomes observed in HMN. The model predicts sequential play of Cournot quantities, first movers punished by second movers, double Stackelberg leadership, and collusive outcomes, among others. More importantly, we find that the model is able to explain two main features of the experimental data: a high percentage of Cournot outcomes in period 2 (delay) and a high percentage of outcomes where the first mover commits to a quantity higher than Cournot but lower than Stackelberg leader. Notwithstanding, the model underestimates first period commitment. This leads to underestimation of sequential play of Cournot quantities, one player plays a Stackelberg quantity and the other a Cournot quantity (both do it in period 1), double Stackelberg leadership, and collusion.

The remainder of this paper is organized as follows. Section 2 describes the empirical evidence in HMN. Section 3 applies the cognitive hierarchy model to the action commitment game. Section 4 reports the maximum likelihood estimates and discusses the results. Section 5 concludes the paper. The appendix contains the payoff matrix of the action commitment game, the classification of market outcomes, and the details of the maximum likelihood estimation.

2 Experimental Evidence

HMN use a laboratory experiment to test the action commitment model in Hamilton and Slutsky (1990). Subjects in the experiment were students of various backgrounds who were paid according to their results in the game and a participation fee to cover eventual negative profits. Subjects were told that they would act as a firm which, together with another firm, serves one market, and that in each round both were to choose the period of production and the quantity. Subjects were informed that in each round pairs of participants would be randomly matched and were not informed of who their rival was. After each round the subjects got individual feedback about what happened in their market.

HMN assume a linear inverse demand function $p(Q) = \max\{30 - Q, 0\}$, where $Q = q_1 + q_2$, and a cost function $C_i(q_i) = 6q_i$, with $i = 1, 2$. Table 1, taken from Huck, Müeller and Normann (2001), summarizes the quantities, profits, consumer surplus and total welfare for the Cournot and Stackelberg equilibria and for the fully collusive market outcome.[4]

Table 1: Theoretical Predictions for Equilibria and Fully Collusive Outcome

	Cournot	Stackelberg	Collusion
Individual quantities	$q_i^C = 8$	$q^L = 12; q^F = 6$	$(q_i^J = 6)_{sym}$
Total quantities	$Q^C = 16$	$Q^S = 18$	$Q^J = 12$
Profits	$\Pi_i^C = 64$	$\Pi^L = 72; \Pi^F = 36$	$(\Pi_i^J = 72)_{sym}$
Consumers' surplus	$CS^C = 128$	$CS^S = 162$	$CS^J = 72$
Total welfare	$TW^C = 256$	$TW^S = 270$	$TW^J = 216$

Subjects were handed a payoff matrix with discrete quantity values and the respective payoffs their choices would yield considering the quantities that their randomly matched rival might play and the rival's profit. The experiment was done with two payoff matrices, one large and one small. The large payoff matrix had quantities ranging from the integers 3 to 15 and the small payoff matrix had only 6, 8 and 12 as possible choices. The play lasted 30 rounds and the subjects were informed, at the end of each round, of the quantity and time of entry their rival had chosen and the respective payoffs.

We focus on the results of the game with the large payoff matrix (see appendix). In that game the quantities 6, 7, 8, and 9 are weakly dominated by the strategy "enter the market in the second period." The quantities 3, 4, 5, 13, 14, and 15 are strictly dominated. This game has three Stackelberg equilibria in undominated strategies: two asymmetric Stackelberg equilibria in pure strategies, where one player commits to quantity 12 in period 1 and the other player waits and chooses quantity 6, and a symmetric mixed equilibrium in which players commit to quantity 10 in period 1 with probability 2/5 and with probability

[4] In this earlier paper, an experiment was performed with the same design except that the timing of the decisions was previously stipulated, either for sequential and simultaneous move. The purpose was to study Stackelberg and Cournot frameworks when subjects were matched randomly or in fixed pairs.

3/5 they wait. Furthermore, there is one pure strategy equilibrium in weakly dominated strategies, namely the Cournot equilibrium in which both players commit to quantity 8 in period 1, and there is also a variety of mixed strategy equilibria in weakly dominated strategies.[5]

Table 2 displays the percentage of choices made broken down by quantity and period of play for the first five rounds and for the entire set.

Table 2: Observed Quantities per Period of Play

	First 5 rounds		Entire set	
Quantity	Period 1	Period 2	Period 1	Period 2
3	0	0.7	0.1	0.3
4	0	0.7	0.1	0.2
5	2.0	0	0.3	0.2
6	2.7	7.3	3.9	2.6
7	2.7	5.3	6.6	6.9
8	12.0	12.7	16.0	14.0
9	4.7	8.0	5.3	6.3
10	13.3	4.7	15.6	4.5
11	8.0	3.3	6.0	1.9
12	8.0	0.7	5.2	1.3
13	0.7	0	0.8	0.2
14	0.7	0	0.4	0.1
15	1.3	0.7	0.7	0.3
Total	56.0	44.0	61.0	39.0

There are three points worth stressing from inspection of Table 2. The first is the existence of three spikes in the strategy space indicating a nonrandom structure in the reasoning of players. In period 1 there is a spike in quantity 8 and another one in quantity 10. In period 2 there is a spike in quantity 8. The second is that the quantities chosen is highly concentrated in the subset $\{6, 7, 8, 9, 10, 11, 12\}$, that is, very few players choose strictly dominated quantities. The third is that more players commit to period 1 than to period 2.[6]

[5] In Hamilton and Slutsky (1990), the linearity of the demand and cost functions combined with the continuous action space guarantee that there are no equilibria where players mix a first period choice with the strategy "wait." With a discrete strategy space there exist various mixed strategy equilibria.

[6] Behavior becomes more cooperative as the number of rounds of play increases. By spliting the sample

Table 3 organizes results into market outcomes and displays the percentage of each in terms of the total. Following HMN, we count 6 and 7 as collusive quantities, 8 and 9 as Cournot quantities, and 10, 11 and 12 as Stackelberg leader's quantities.

Table 3: Observed Market Outcomes

Market outcomes (% cases)	First 5 rounds	Entire set
Cournot:		
1st period	2.7	4.5
Sequential	10.7	14.8
2nd period	6.7	4.5
Stackelberg:		
Leader 12, follower 6	4.0	0.9
Leader 11 or 10, follower 7	9.4	6.5
First mover punished or rewarded:		
Stackelberg leader punished	6.7	11.9
Stackelberg leader rewarded	0	0.2
Cournot punished	1.3	0.9
Cournot rewarded	0	0
Stackelberg and Cournot in 1st period	12.0	12.6
Double Stackelberg leadership	10.7	6.3
Collusion:		
Collusion successful	4.0	6.1
Collusion failed	5.3	10.6
Collusion exploited	4.0	4.3
Other	22.7	16.0

We will briefly go through the meaning of some of the market outcomes in Table 3. The outcome "Cournot sequential" means that the first and second movers both choose a Cournot quantity (8 or 9). The outcome "Stackelberg leader punished" means that the first mover chooses a Stackelberg leader's quantity (12, 11 or 10) and the second mover chooses a quantity greater than his best response to the first mover. The outcome "Stackelberg and Cournot in 1st period" means that one player chooses a Stackelberg leader's quantity while the other chooses a Cournot quantity. The outcome "double Stackelberg leadership"

into two parts, the first encompassing the first fifteen rounds and the second the remaining rounds, we observe that: quantities 6 and 7 were chosen less often in the first part of the sample than in the second part; quantities 9 and 11 were chosen more often in the first part and less often in the second. Throughout, quantities 8, 10 and 12 remain approximatelly constant in both subsets as well as the stricktly dominated quantities.

means that both players play a Stackelberg leader's quantity in period 1. The outcome "collusion successful" means that both players play a collusive quantity in either period. The outcome "collusion failed" means that both players move in period 1, one player chooses a collusive quantity and the other player plays either Stackelberg or Cournot. The outcome "collusion exploited" means that the first mover chooses a collusive quantity and the second a quantity greater than 7. Finally, the market outcome "others" refers to those situations that do not fit into any of the previous cases.[7]

3 A Cognitive Hierarchy in the Action Commitment Model

Camerer, Ho and Chong (2004) propose a cognitive hierarchy theory of behavior in games where different players employ different levels of reasoning. $L0$ players do not think strategically at all; they randomize equally across all strategies. Players of level $k > 0$ anticipate the decisions of lower-level players and best respond to the mixture of their decisions using normalized frequencies.[8]

Formally, players of level $k \geq 1$ know the true proportions of lower-level players $f(0)$, $f(1),\ldots, f(k-1)$. Since these proportions do not add to one, they normalize them by dividing by their sum. That is, players with $k \geq 1$ levels of reasoning have the following beliefs about players with h levels of reasoning:

$$g^{Lk}(h) = \begin{cases} f(h)/\sum_{l=0}^{k-1} f(l), & \forall h < k \\ 0, & \forall h \geq k \end{cases}.$$

Camerer, Ho and Chong (2004) discuss the properties that the appropriate distribution of levels should possess: it should be discrete because the thinking steps are integers; it should reflect the fact that, as thinking steps increase, so do the computations that the players carry out. Working memory constraints should make it likely that, the higher is k, the fewer are the players doing one further reasoning level. In other words $f(k)/f(k-1)$

[7] See Appendix A for a complete description of the quantities and periods of play that characterize each market outcome.

[8] Thus, players of level $k \geq 1$ are assumed to not realize that some players might be thinking at least as 'hard' as they are about the game. This could be due to overconfidence: players believe that their rivals have less insight regarding the game they are playing. It could also be due to the limited capacity that people have to continuously eliminate dominated strategies. However, players of level $k \geq 1$ are assumed to make an accurate guess about the relative proportions of players using fewer steps than they do.

is decreasing in k. Moreover, the authors assume that the ratio is proportional to $1/k$ and that the distribution is the Poisson $f(k|\tau) = \tau^k e^{-\tau}/k!$, with $k = 0, 1, 2, ...$ and $\tau > 0$. An advantage of this approach is that one only needs to estimate a single parameter, τ, to find out the distribution of players' types in the population.

To apply the cognitive hierarchy model to the action commitment game we follow the spirit of Camerer, Ho and Chong's (2004, p. 892) note that "Extending the model to extensive-form games is easy by assuming that 0-step thinkers randomize independently at each information set, and higher-level types choose best responses at information sets using backward induction."

We assume that an $L0$ chooses a random quantity and commits to period 1 with probability $x \in (0, 1)$.[9] Additionally, the best response function of a level-k player, with $k > 0$, to quantity q chosen by his rival is given by

$$BR^{Lk}(q) = \arg\max_{q^{Lk}} \left[P\left(q^{Lk} + q\right) - c \right] q^{Lk}.$$

The first step is to determine the optimal strategy of an $L1$. An $L1$ thinks he can only play against $L0$s. If an $L1$ commits to period 1, then his optimal quantity is the best reply to the expected output of an $L0$ since the payoff function is linear in the opponent's strategy (demand is linear). This generates a perceived expected profit of

$$\Pi_1^{L1} = \pi^{L1}\left(BR^{L1}\left(\bar{q}^{L0}\right), \bar{q}^{L0}\right),$$

where $\bar{q}^{L0}$ denotes the expected output of an $L0$ player. If, on the contrary, the $L1$ decides to wait, one of the following two situations will occur: with probability x the $L0$ will commit to period 1 and the $L1$ will be able to best respond to the quantity choice of the $L0$; with probability $1 - x$ the $L0$ will commit to period 2 and the $L1$ will choose $BR^{L1}(\bar{q}^{L0})$. Thus, at the beginning of the game, the perceived expected profit of an $L1$

[9] The assumption that an $L0$ commits to the second period with positive probability, $x < 1$, is critical to the analysis. To see this suppose that $x = 1$, that is, the $L0$ always commits to the first period. In this case the optimal choice of the $L1$ is to commit to the second period to best respond to the $L0$. Now if, for example, an $L1$ is paired against another $L1$, then both players would observe no movement in the first period from the rival. This would be inconsistent with their belief that the population is only composed of $L0$s.

who commits to period 2 is given by

$$\Pi_2^{L1} = x \sum \pi^{L1} \left(BR^{L1} \left(q^{L0} \right), q^{L0} \right) \Pr(q^{L0}) + (1-x)\pi^{L1} \left(BR^{L1} \left(\bar{q}^{L0} \right), \bar{q}^{L0} \right).$$

By definition of the best response function, we know that

$$\sum \pi^{L1} \left(BR^{L1} \left(q^{L0} \right), q^{L0} \right) \Pr(q^{L0}) > \pi^{L1} \left(BR^{L1} \left(\bar{q}^{L0} \right), \bar{q}^{L0} \right),$$

and so $\Pi_2^{L1} > \Pi_1^{L1}$. Therefore, the $L1$ is better off by waiting whatever the probability the $L0$ has of committing to period 1. The intuition behind this result is that since $L0$ players do not act strategically, the $L1$ has nothing to gain if it commits to period 1 because he cannot condition the $L0$'s response. Therefore, waiting is the optimal choice of an $L1$.

An $L2$ thinks that the population is composed exclusively of $L0$s and $L1$s. The $L2$ also knows that an $L0$ will play a random quantity and will do it in period 1 with probability x as well as that the best decision of an $L1$ is to wait. Therefore, the $L2$ is faced with a trade-off. If she knew the rival was an $L1$, then she would prefer to commit to period 1 and reap the benefits of a Stackelberg leadership position. If she knew her rival was an $L0$, then she would prefer to wait. Since the $L2$ only knows the percentage of $L0$s and $L1$s, her optimal choice will depend on τ.

If the $L2$ commits to period 1, then her optimal quantity, denote it by q_1^{L2}, is the solution to

$$\max_{q^{L2}} g^{L2}(0)\pi^{L2} \left(q^{L2}, \bar{q}^{L0} \right) + g^{L2}(1)\pi^{L2} \left(q^{L2}, BR^{L1}(q^{L2}) \right), \tag{1}$$

where $g^{L2}(0)$ is the proportion of $L0$s in the population according to $L2$'s beliefs and, likewise, $g^{L2}(1)$ is that of $L1$ players. These two proportions will be normalized by the $L2$ to sum up to one. We can thus write the $L2$'s perceived expected profit of committing to q_1^{L2} in period 1 as

$$\Pi_1^{L2} = g^{L2}(0)\, \pi^{L2} \left(q_1^{L2}, \bar{q}^{L0} \right) + [1 - g^{L2}(0)]\pi^{L2} \left(q_1^{L2}, BR^{L1} \left(q_1^{L2} \right) \right).$$

If the $L2$ waits one of the following two situations will arise. If the $L2$ observes a quantity commitment by the rival, then she believes that she is playing against an $L0$ and chooses her best response. This leads to an expected profit of $\sum \pi^{L2}\left(BR^{L2}\left(q^{L0}\right), q^{L0}\right)\Pr(q^{L0})$. If the $L2$ does not observe any quantity commitment by the rival she believes that she is either playing against an $L0$ who commits to period 2 or against an $L1$. In this case the optimal quantity choice of the $L2$, denote it by q_2^{L2}, is the solution to

$$\max_{q^{L2}} \mu^{L2}(0)\pi^{L2}\left(q^{L2}, \bar{q}^{L0}\right) + [1-\mu^{L2}(0)]\pi^{L2}(q^{L2}, BR^{L1}\left(\bar{q}^{L0}\right)), \tag{2}$$

where $\mu^{L2}(0)$ is the (posterior) belief of the $L2$ that the rival is an $L0$ given that there was no commitment in period 1. Assuming that players use Bayes' rule to update beliefs about their rivals' type we have

$$\mu^{L2}(0) = \frac{(1-x)g^{L2}(0)}{(1-x)g^{L2}(0) + g^{L2}(1)} = \frac{(1-x)g^{L2}(0)}{1-xg^{L2}(0)}.$$

Therefore, at the beginning of the game, the $L2$'s perceived expected profit of waiting is

$$\Pi_2^{L2} = xg^{L2}(0)\sum \pi^{L2}\left(BR^{L2}\left(q^{L0}\right), q^{L0}\right)\Pr(q^{L0}) + (1-x)g^{L2}(0)\pi^{L2}\left(q_2^{L2}, \bar{q}^{L0}\right) + [1-g^{L2}(0)]\pi^{L2}(q_2^{L2}, BR^{L1}\left(\bar{q}^{L0}\right)).$$

An $L2$ will commit to period 1 if $\Pi_1^{L2} > \Pi_2^{L2}$. This inequality will be satisfied for sufficiently high values of τ, that is, when, from the standpoint of an $L2$, there are relatively many $L1$s and few $L0$s in the population.

To see that an $L2$ commits to period 1 for sufficiently high values of τ, consider HMN action commitment game with a continuous quantity choice set. Assume that an $L0$ commits to either period with 50% probability and that his quantity is drawn from the continuous uniform distribution with support on $[3, 15]$. In this case we have $\bar{q}^{L0} = 9$, $g^{L2}(0) = \frac{1}{1+\tau}$, $q_1^{L2} = \frac{15+12\tau}{2+\tau}$, and the $L2$'s perceived expected profit of committing to q_1^{L2} in period 1 is

$$\Pi_1^{L2} = \frac{9}{2}\frac{16\tau^2 + 40\tau + 25}{\tau^2 + 3\tau + 2}. \tag{3}$$

When the $L2$ waits and the rival commits to period 1, the $L2$ is able to best respond and the $L2$'s expected profit is $\int_3^{15} \frac{(12-0.5q^{L0})^2}{12} dq^{L0} = 59.254$. When the $L2$ waits and the rival does not commit to period 1 the posterior belief of the $L2$ is $\mu^{L2}(0) = \frac{1}{2\tau+1}$ and she chooses $q_2^{L2} = 8 - \frac{1}{2(2\tau+1)}$. Therefore, at the beginning of the game, the $L2$'s perceived expected profit of waiting is

$$\Pi_2^{L2} = \frac{1}{2}\frac{1}{1+\tau}59.254 + \frac{1}{8}\frac{(32\tau+15)^2}{2\tau^2+3\tau+1}. \tag{4}$$

From (3) and (4) we have that $\tau > 1.05$ implies $\Pi_1^{L2} > \Pi_2^{L2}$. Hence, if $0 \leq \tau \leq 1.05$ the $L2$ chooses to wait and (i) if the rival commits to period 1 the $L2$ best responds, (ii) if the rivals waits the $L2$ chooses $q_2^{L2} = 8 - \frac{1}{2(2\tau+1)}$. If $\tau > 1.05$ the $L2$ commits to period 1 and chooses $q_1^{L2} = \frac{15+12\tau}{2+\tau}$.

The analysis also shows that if $1.05 < \tau \leq 16$, then an $L2$ will commit to period 1 and choose a quantity greater than the Cournot quantity 8 and less than the Stackelberg leader's quantity 12. This happens due to the presence of the $L0$s. In other words, an $L2$ will only choose to be a Stackelberg leader with quantity 12 for unreasonable high level of τ ($\tau > 16$). Thus, for a reasonable average level of thinking steps, the cognitive hierarchy model does not predict play of standard Stackelberg equilibria. Finally, we also see that the cognitive hierarchy model predicts delay and a spike in the Cournot quantity 8 in period 2 mostly due to the behavior of $L1$s. However, the model is not able to predict the spike in the Cournot quantity 8 in period 1.

In HMN action commitment game players are constrained to choose one of the discrete quantity levels. Applying the cognitive hierarchy model to this game gives rise to slightly different behavior than when the quantity choice set is continuous:

$L0$: Commits to either period with 50% probability and the quantity is drawn from the discrete uniform distribution with support on $\{3, 4, ..., 15\}$.

$L1$: Chooses to wait and (i) if there is commitment by the rival the $L1$ chooses a best response, (ii) if there is no commitment by the rival the $L1$ chooses the Cournot quantity 8 (the best response to quantity 9, the average quantity produced by an $L0$).

$L2$: If $0 \leq \tau \leq 1.30$, the $L2$ chooses to wait and (i) if there is commitment by the rival the $L2$ chooses a best response, (ii) if there is no commitment by the rival the $L2$ chooses the Cournot quantity 8. If $1.30 < \tau \leq 4$, the $L2$ commits to period 1 and chooses the Stackelberg leader's quantity 10.

Further thinking steps are easily added to the model by following the same logic as above. The behavior of higher levels consists in either committing to period 1 with quantity 10 or waiting, and either best responding to the rival's quantity if the rival commits to period 1 or producing the Cournot quantity 8 if the rival also decided to wait. In appendix 6 behavior is depicted as a function of τ up to $\tau = 4$ and from $L2$ to $L10$.

4 Estimation

This section explains how we introduce errors in players' decisions, describes the likelihood function, and reports the estimation results. Technical details of the maximum likelihood estimation can be found in Appendix B.

4.1 Maximum Likelihood Estimation

Cognitive hierarchy models typically produce a rather small set of best responses. In the action commitment game, predicted behavior alternates (for players other then the $L0$) between committing to period 1 with quantity 10 or waiting and, either best responding to observed quantities in the case of sequential movement, or playing quantity 8 if no commitment has been observed.

Following El-Gamal and Grether (1995), Costa-Gomes, Crawford and Broseta (2001), Camerer, Palfrey and Rodgers (2007) and Östling, Wang, Chou and Camerer (2008) we assume that players' best responses are stochastic. More precisely, we assume that players of level $k \geq 1$ make mistakes when choosing their quantity levels but make correct decisions regarding the commitment choice. We think this is a reasonable assumption given the large set of quantity choices (13 quantities) and the small set of timing decisions (2 periods). Allowing both types of errors would substantially increase the computational burden involved in the estimation procedure.

Additionally, we assume that the probability of playing a quantity close to the predicted quantity by mistake is higher than the probability of playing a distant quantity. To incorporate this behavior, we assume that the mistakes of a player of level $k \geq 1$ follow a power function. This function assigns a high probability to quantities adjacent to the predicted one and a small probability to distant quantities.[10] We also assume that errors

[10] The power function was tested against other alternative error specifications, such as the uniform, assigning errors only to the two and four immediate quantities or assuming no errors at all. It performed better than these alternatives.

are independent across types of players.

The estimation goes through all the pairs of decisions. By pair, we mean every possible combination of period and quantity decisions of two players playing against each other. This approach captures the interaction of players taking decisions that are conditioned by their rivals' decisions. The information is thus broken down into three possible cases: both players commit to period 1; both players commit to period 2; and one player commits to period 1 and the other to period 2.[11]

The estimation method is maximum likelihood and it is done according to a standard grid search approach. The likelihood function is given by

$$L(\tau,\varepsilon)=\prod_{t_1=1}^{2}\prod_{q_1=3}^{15}\prod_{t_2=1}^{2}\prod_{q_2=3}^{15}\left[f\left(t_1,q_1,t_2,q_2|\tau,\varepsilon\right)\right]^{n_{(t_1,q_1,t_2,q_2)}},$$

where t_i and q_i are the timing and quantity predictions for player i in a given pair, t is the index of timing predictions and q for quantity predictions, $n_{(t_1,q_1,t_2,q_2)}$ is the number of cases that each pair is observed in the data, τ is the parameter of the Poisson distribution, and ε is the error term. Since cognitive hierarchy models are better suited to explain initial responses to games our benchmark estimation is done for the first five rounds of play. However, we also estimate the model for the entire set (30 rounds of play).

4.2 Results

We now present and discuss the results of the maximum likelihood estimation. Interval by interval estimates are displayed in Appendix C. A complete list of all quantity pairs, their percentage in the data, and the estimated probability in the model is available upon request.

Parameter estimates of τ and ε for both the first five rounds of play and for the entire set, as well as the respective maximum likelihood values, are displayed in Table 4.

The model's estimate of the average number of thinking steps is 2.86 for the first five rounds of play as well as for the entire set. This estimate is in line with those found in Nagel (1995). Nevertheless, it should be pointed out that other estimates for τ are not uncommon in the literature. For example, Camerer, Ho an Chong (2004) show that the

[11] Crawford and Iriberri (2007) and Harless and Camerer (1995) use a similar estimation procedure.

average estimate for τ across a wide range of games is 1.5. In contrast, the lowest estimate for τ in the seven weeks of the LUPI game in Östling, Wang, Chou and Camerer (2008) is 2.98, the remaining six are above 5 and the highest are over 10. In Camerer, Palfrey and Rodgers (2007) there are games for which the predicted τ is also rather high.[12]

Table 4: Maximum Likelihood Estimates

	τ	ε	ML
First 5 rounds	2.86	0.64	−375.34
Entire set	2.86	0.65	−2257.49

The benchmark model's estimate for the error term is 0.64 for the first five rounds of play and 0.65 for the entire set. The estimate for the first five rounds indicates that subjects choose their predicted quantity with 36% probability, quantities immediately adjacent to the predicted one with 32% probability, and quantities that are two integers away from the predicted one with 16% probability. Thus, in the first five rounds, subjects choose their predicted quantity with 36% probability, quantities that are one or two integers away from their predicted quantity with 48% probability, and quantities that are three or more integers away from their predicted quantity with only 16% probability.

Error rate estimates of this magnitude are not uncommon in this literature–see, for example, Costa-Gomes, Crawford and Broseta (2003). The main characteristic of the action commitment game is the wide range of possible strategies. When there are many strategies available and there is noise in players' behavior, the probability of not choosing the optimal strategy should be high.

We now turn towards the quality of the adjustment. Table 5 displays upper and lower bounds to the maximum likelihood value as well as log-likelihood ratios (p-values are in parenthesis). The upper bound is attained by running the likelihood function with the empirical frequencies of the pairs of play. By definition, this procedure yields the maximum

[12] Camerer, Ho and Chong (2004) argue that subjects tend do employ a cost-benefit analysis concerning the amount of thinking they do in games. They present evidence that the higher the stakes of a given game, the higher will τ be: they show that subjects tend to think harder in games that yield \$4 than games that yield \$1. Since the LUPI game is based on data from an actual lotto game that existed in Sweden with a prize money of at least €10 000, it makes sense that the game's estimates should be relatively high (even though, of course, the probability of winning the prize is much smaller). In the large matrix experiment we used, subjects received the equivalent to \$11.44, on average, which, given the reward, places our model's estimates somewhere in the middle of this range.

Table 5: Quality of Adjustment

	First 5 rounds	Entire set
Upper bound	−336.62	−1825.90
Lower bound	−442.97	−2624.53
Log-likelihood	135.26	734.07
	(0.000)	(0.000)

value attainable for the estimation. The lower bound is the maximum likelihood value of a model where all players are $L0$s. Since we are testing for the significance of τ, we have, under the null hypothesis, that τ is not statistically different than 0. As is evident from the log-likelihood p-values, we can reject the null hypothesis.

Table 6 displays the percentage of choices, broken down by quantity and period of play, in the data and predicted by the model. We see that the cognitive hierarchy model predicts the spike in the Stackelberg quantity 10 in period 1 as well as the spike in the Cournot quantity 8 in period 2. However, the model does not generate the spike in the Cournot quantity 8 in period 1. The model also predicts very little play of strictly dominated quantities which is consistent with the data. Finally, we see that the model underestimates period 1 commitment.

Table 6: Observed and Predicted Quantities per Period of Play

	First 5 rounds				Entire set			
	Period 1		Period 2		Period 1		Period 2	
Quantity	Data	Model	Data	Model	Data	Model	Data	Model
3	0	0.3	0.7	0.8	0.1	0.3	0.3	0.8
4	0	0.4	0.7	1.3	0.1	0.4	0.2	1.4
5	2.0	0.6	0	2.5	0.3	0.6	0.2	2.5
6	2.7	1.0	7.3	4.7	3.9	1.0	2.6	4.8
7	2.7	1.8	5.3	9.2	6.6	1.9	6.9	9.3
8	12.0	3.4	12.7	20.0	16.0	3.5	14.0	19.4
9	4.7	6.6	8.0	9.2	5.3	6.7	6.3	9.3
10	13.3	14.4	4.7	4.7	15.6	14.0	4.5	4.8
11	8.0	6.6	3.3	2.5	6.0	6.7	1.9	2.5
12	8.0	3.4	0.7	1.3	5.2	3.5	1.3	1.4
13	0.7	1.8	0	0.8	0.8	1.9	0.2	0.8
14	0.7	1.0	0	0.5	0.4	1.0	0.1	0.5
15	1.3	0.6	0.7	0.4	0.7	0.6	0.3	0.4
Total	56.0	42.3	44.0	57.7	61.0	42.3	39.0	57.7

Table 7 compares the market outcomes predicted by the cognitive hierarchy model to those observed in the data for first five rounds of play and the entire set.

Table 7: Observed and Predicted Market Outcomes

Market outcomes	First 5 rounds		Entire set	
	Data	Model	Data	Model
Cournot:				
1st period	2.7	1.0	4.5	1.0
Sequential	10.7	5.9	14.8	5.9
2nd period	6.7	8.5	4.5	8.2
Stackelberg:				
Leader 12, follower 6	4.0	1.4	0.9	1.4
Leader 11 or 10, follower 7	9.4	8.4	6.5	8.0
First mover punished or rewarded:				
Stackelberg leader punished	6.7	9.2	11.9	9.3
Stackelberg leader rewarded	0	3.9	0.2	3.9
Cournot punished	1.3	1.7	0.9	1.8
Cournot rewarded	0	2.8	0	2.9
Stackelberg and Cournot 1st period	12.0	4.9	12.6	5.0
Double Stackelberg leadership	10.7	6.0	6.3	5.9
Collusion:				
Collusion successful	4.0	2.7	6.1	2.7
Collusion failed	5.3	2.0	10.6	2.0
Collusion exploited	4.0	2.3	4.3	2.3
Other	22.7	39.4	16.0	39.7

The first thing we can see from inspection of Table 7 is that the cognitive hierarchy model can generate the heterogeneity in market outcomes observed in the data. More importantly, the model predicts a high percentage of Cournot outcomes (15.4%) albeit slightly less than the percentage observed in the data (by 4.7%). Thus, the model captures the most important feature of the experimental data. The model also generates simultaneous play of Cournot quantities in period 2 (delay). This is rather important since delay cannot be explained by risk-payoff equilibrium selection arguments or inequity aversion.

Additionally, the model predicts a higher percentage of Stackelberg outcomes where the leader produces 10 or 11 and the follower 7 (8.4%) and a low percentage of Stackelberg outcomes where the leader produces 12 and the follower 6 (1.4%). This is another important feature of the experimental data that the model is able to capture.

Regarding the outcomes where the first mover is either punished or rewarded by the second mover, the model predicts well the percentage of first movers (Stackelberg leaders plus Cournot) who get punished by followers but overestimates the percentage of first movers who get rewarded by second movers (by 6.7%).

Finally, the model underestimates outcomes where one player plays a Stackelberg quantity and the other a Cournot quantity in period 1 (by 6.1%), double Stackelberg leadership (by 4.7%), and collusive outcomes (by 6.3%). As a consequence, the model overestimates the percentage of cases under the category "other" (by 16.7%).

Table 7 also shows that the cognitive hierarchy model fits the data better in the first five rounds of play than in the entire set. This happens because the percentages of Cournot, Stackelberg leader punished, and collusive outcomes observed in the data increase over time whereas the percentages predicted by the model are essentially left unchanged with rounds of play. This finding is consistent with the fact that non-equilibrium models of behavior in games are particularly good at predicting initial responses.

The maximum likelihood estimates assume that an $L0$ commits to either period with 50% probability and chooses each of the quantities in HMN with uniform probability. In cognitive hierarchy models different $L0$ specifications can sometimes lead to different predictions of behavior. To address this issue we estimate the model for alternative probabilities of period 1 commitment by the $L0$.

We find that different probabilities of period 1 commitment by the $L0$ bear a small impact on the estimates for τ and ε, the maximum likelihood values, aggregate results and predicted market outcomes. This happens because: (1) the percentage of $L0$ players is quite small for the estimated τs, and (2) the maximum likelihood τs are obtained in intervals for which the players' behavior is exactly the same–all players with $k > 1$ commit to period 2 (and play quantity 8 if no commitment is observed) except the $L2$s and the $L4$s who commit to period 1 with quantity 10. Therefore, the only impact on results is the slightly different levels of τ which are insufficient to generate appreciable differences.

5 Conclusion

This paper is an additional contribution to the literature on endogenous timing games. This literature shows that observed behavior in experiments on endogenous timing is at odds with the theory. The theory predicts Stackelberg outcomes but the experiments find that these are rare and, instead, Cournot outcomes are modal.

We apply the cognitive hierarchy model of Camerer, Ho, and Chong (2004) to Hamilton and Slutsky's (1990) action commitment game. We show that, for a reasonable average level of thinking steps, the cognitive hierarchy model rules out Stackelberg equilibria, generates Cournot outcomes including delay, and outcomes where the first mover produces more than Cournot but less than Stackelberg leader.

We also show that a cognitive hierarchy model where players make mistakes when they choose their optimal quantities can explain the most important features of the experimental evidence on the action commitment game in Huck, Müeller and Normann (2002). The model generates heterogeneity in market outcomes, a high percentage of Cournot outcomes including simultaneous play of Cournot quantities in period 2 (delay), and a high percentage of outcomes where the first mover produces less than the Stackelberg leader's quantity. The main shortcoming of the model is that it underestimates commitment to period 1.

An alternative explanation for behavior in experimental endogenous timing games is the quantal response approach–see McKelvey and Palfrey (1995)–where players' beliefs are correctly formed but players do not always choose best responses. We leave this conjecture for future research.

Finally, we show that the cognitive hierarchy model explains the data in Huck, Müeller and Normann (2002) in the first five rounds of play better than in the entire set. This happens because over time players play more Cournot, Stackelberg leader punished, and collusive outcomes. It is out of the scope of this paper to study how a learning model can account for the dynamics in the data.

6 References

Camerer, C. F., T.-H. Ho, and J.-K. Chong (2004). "A cognitive hierarchy model of games," *Quarterly Journal of Economics*, 861-898.

Camerer, C. F., T. R. Palfrey, and B. W. Rodgers (2007). "Heterogeneous quantal response equilibrium and cognitive hierarchies," California Institute of Technology, Division of the Humanities and Social Sciences Working Paper N°1260.

Costa-Gomes, M., V. Crawford (2006). "Cognition and behavior in two person guessing games: an experimental study," *American Economic Review*, Vol. 96, 1737-1768.

Costa-Gomes, M., V. Crawford and B. Broseta (2003). "Experimental studies of strategic sophistication and cognition in normal-form games," *Econometrica*, Vol. 69, 1193-1235.

Crawford, V., N. Iriberri (2007). "Fatal attraction: salience, naiveté, and sophistication in "hide-and-seek" games," *American Economic Review*, Vol. 97(5), 1731-1750.

El-Gamal, M. A., D. M. Grether (1995). "Are people Bayesian: uncovering behavioral strategies," *Journal of the American Statistical Association*, Vol. 97(432), 1137-1145.

Fonseca, M. A., W. Müeller, and H.-T. Normann (2006). "Endogenous timing in duopoly: experimental evidence," *International Journal of Game Theory*, Vol. 34, 443-456.

Hamilton, J., S. Slutsky (1990). "Endogenous timing in duopoly games: Stackelberg or Cournot equilibria," *Games and Economic Behavior*, Vol. 2, 29-46.

Harless, D. W., C. F. Camerer (1995). "An error rate analysis of experimental data testing Nash refinements," *European Economic Review*, Vol. 39, 649-660.

Harsanyi, J., and R. Selten (1988). A General Theory of Equilibrium Selection in Games, Cambridge, MA: MIT Press.

Huck, S., W. Müller, and H.-T. Normann (2001). "Stackelberg beats Cournot – on collusion and efficiency in experimental markets," *Economic Journal*, Vol. 111, 749-765.

Huck, S., W. Müller, and H.-T. Normann (2002). "To commit or not to commit: endogenous timing in experimental duopoly markets," *Games and Economic Behavior*, Vol. 38, 240-264.

Ivanov, A., D. Levin, J. Peck (forthcoming). "Hindsight, foresight and insight: an experimental study of a small-market investment game with common and private values," *American Economic Review*.

Matsumura, T. (2001). "Market instability in a Stackelberg duopoly," *Journal of Economics*, Vol. 75(3), 199-210.

McKelvey, R., T. Palfrey (1995). "Quantal response equilibria in normal form games," *Games and Economic Behavior*, Vol. 10, 6-38.

Nagel, R. (1995). "Unraveling in guessing games: an experimental study," *The American Economic Review*, Vol. 85(5), 1313-1326.

Östling, R., J. T. Y. Wang, E. Chou, and C. F. Camerer (2008). "Strategic thinking and learning in the field and lab: evidence from Poisson LUPI lottery games," SSE/EFI Working Paper Series in Economics and Finance No. 671.

Robson, A. (1990). "Duopoly with endogenous strategic timing: Stackelberg regained," *International Economic Review*, Vol. 31(2), 263-274.

Saloner, G. (1987). "Cournot duopoly with two production periods," *Journal of Economic Theory*, Vol. 42, 183-187.

Santos-Pinto, L. (2008). "Making sense of the experimental evidence on endogenous timing in duopoly markets," *Journal of Economic Behavior and Organization*, Vol. 68, 657-666.

Schwarz, G. (1978). "Estimating the dimension of a model," *The Annals of Statistics*, Vol. 6 (2), 461-464.

Stahl, D. O., P. W. Wilson (1994). "On player's models of other players: theory and experimental evidence," *Games and Economic Behavior*, Vol.10, 218-254.

Appendix A: Large payoff matrix

Table 8: Large payoff matrix

	3	4	5	6	7	8	9	10	11	12	13	14	15
3	54 54	51 68	48 80	45 90	42 98	39 104	36 108	33 109	30 110	27 108	24 104	21 98	18 90
4	68 51	64 64	60 75	56 84	52 91	48 96	44 99	40 100	36 99	32 96	28 91	24 84	19 75
5	80 48	75 60	70 70	65 78	60 84	55 88	50 89	45 90	40 88	35 84	29 78	25 70	20 60
6	90 45	84 56	78 65	72 72	66 77	60 80	54 81	48 80	41 77	36 72	30 65	24 56	18 45
7	98 42	91 52	84 60	77 66	70 70	63 72	55 71	49 70	42 66	35 60	28 52	21 42	14 30
8	104 39	96 48	88 55	80 60	72 63	64 64	56 63	48 60	40 55	32 48	24 39	16 28	8 15
9	108 36	99 44	89 50	81 54	71 55	63 56	54 54	45 50	36 44	27 36	18 26	9 14	0 0
10	109 33	100 40	90 45	80 48	70 49	60 48	50 45	40 40	30 33	20 24	10 13	0 0	-10 -15
11	110 30	99 36	88 40	77 41	66 42	55 40	44 36	33 30	22 22	11 12	0 0	-11 -14	-22 -30
12	108 27	96 32	84 35	72 36	60 35	48 32	36 27	24 20	12 11	0 0	-12 -13	-24 -28	-36 -45
13	104 24	91 28	78 29	65 30	52 28	39 24	26 18	13 10	0 0	-13 -12	-26 -26	-39 -42	-52 -60
14	98 21	84 24	70 25	56 24	42 21	28 16	14 9	0 0	-14 -11	-28 -24	-42 -39	-56 -56	-70 -75
15	90 18	75 19	60 20	45 18	30 14	15 8	0 0	-15 -10	-30 -22	-45 -36	-60 -52	-75 -70	-90 -90

Appendix B: Classification of market outcomes

Table 9 provides the classification of market outcomes. Specifically, the table is composed of four different matrices. The upper left refers to cases where both players commit to period 1, the lower right refers to cases where both players commit to period 2, and the remaining two tables refer to sequential play. The notation employed is as follows:

Cournot outcomes:

- C_1 - Cournot 1st period
- C_{12} - Sequential play of Cournot quantities
- C_2 - Cournot 2nd period

Stackelberg outcomes:

- S_{12}- Stackelberg leader 12, follower 6
- S_{11} - Stackelberg leader 11, follower 7
- S_{10} - Stackelberg leader 10, follower 7

First mover punished or rewarded:

- SP - Stackelberg leader punished
- SR - Stackelberg leader rewarded
- CP - Cournot punished
- CR - Cournot rewarded

SC - Stackelberg and Cournot in 1st period

DL - Double Stackelberg leadership

Collusive outcomes:

- CS - Collusion successful
- CF - Collusion failed
- CE - Collusion exploited

O - Other market outcomes

Table 9: Market Outcomes Classification

		t=1							t=2						
		6	7	8	9	10	11	12	6	7	8	9	10	11	12
t=1	6	CS	CS	CF	CF	CF	CF	CF	CS	CS	CE	CE	CE	CE	CE
	7	CS	CS	CF	CF	CF	CF	CF	CS	CS	CE	CE	CE	CE	CE
	8	CF	CF	C_1	C_1	SC	SC	SC	CR	CR	C_{12}	C_{12}	CP	CP	CP
	9	CF	CF	C_1	C_1	SC	SC	SC	CR	CR	C_{12}	C_{12}	CP	CP	CP
	10	CF	CF	SC	SC	DL	DL	DL	SR	S_{10}	SP	SP	SP	SP	SP
	11	CF	CF	SC	SC	DL	DL	DL	SR	S_{11}	SP	SP	SP	SP	SP
	12	CF	CF	SC	SC	DL	DL	DL	S_{12}	SP	SP	SP	SP	SP	SP
t=2	6	CS	CS	CR	CR	SR	SR	S_{12}	CS	CS	O	O	O	O	O
	7	CS	CS	CR	CR	S_{10}	S_{11}	SP	CS	CS	O	O	O	O	O
	8	CE	CE	C_{12}	C_{12}	SP	SP	SP	O	O	C_2	C_2	O	O	O
	9	CE	CE	C_{12}	C_{12}	SP	SP	SP	O	O	C_2	C_2	O	O	O
	10	CE	CE	CP	CP	SP	SP	SP	O	O	O	O	O	O	O
	11	CE	CE	CP	CP	SP	SP	SP	O	O	O	O	O	O	O
	12	CE	CE	CP	CP	SP	SP	SP	O	O	O	O	O	O	O

Appendix C: Maximum Likelihood Estimation

In this appendix we describe in detail the procedures behind the maximum likelihood estimation. Let $f^{Lk}(t)$ denote the probability that a player of level k moves in period t, where $t \in \{1, 2\}$. In our model $f^{L0}(1) = 1/2$. We assume that players of level $k > 0$ do not make mistakes when choosing their period of entry but that they might make mistakes when choosing their quantity level. Let $f^{Lk}(q|q^{Lk})$ denote the probability that a player of level k plays quantity q where $q \in \{3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15\}$ given that his predicted quantity is q^{Lk}. We use a power function to model the probability of making mistakes in quantity choices. Let i be the distance of a given quantity level from the predicted one, let N_l be the distance between the predicted quantity and the lower bound of the interval of available quantities, i.e., quantity 3, and let N_u be the distance between the predicted quantity and the upper bound of the interval of available quantities, i.e., quantity 15. We have that

$$\sum_{i=1}^{N_l} x_{q^{Lk}}^{-2^{i-1}} + \sum_{i=1}^{N_u} x_{q^{Lk}}^{-2^{i-1}} = 1.$$

where $x_{q^{Lk}}$ will be different for different predicted quantities, depending on their position within the interval of quantities. For example, if an $L3$'s predicted behavior is to commit to period 1 with quantity 8, then $f^{L3}(1) = 1$ and

$$f^{L3}(q|8) = \left(\frac{\varepsilon}{16x_8}, \frac{\varepsilon}{8x_8}, \frac{\varepsilon}{4x_8}, \frac{\varepsilon}{2x_8}, \frac{\varepsilon}{x_8}, 1-\varepsilon, \frac{\varepsilon}{x_8}, \frac{\varepsilon}{2x_8}, \frac{\varepsilon}{4x_8}, \frac{\varepsilon}{8x_8}, \frac{\varepsilon}{16x_8}, \frac{\varepsilon}{32x_8}, \frac{\varepsilon}{64x_8} \right),$$

where $x_8 = 3.921876$.

The first row in Table 10 displays the probability of playing each quantity level of a Stackelberg leader with a predicted quantity of 10. History $\{\}$ means that we are at the beginning of the game, nothing as occurred yet. Only leaders play with this history. With probability $1 - \varepsilon$ the leader plays quantity 10 and makes no mistake; with probability $\frac{\varepsilon}{x_{10}}$ he plays either quantity 9 or 11; with probability $\frac{\varepsilon}{2x_{10}}$ he plays either quantity 8 or 12, and so on. The second row in the table displays the probability of playing each quantity for a player who decides to wait, has a theoretical predicted quantity of 8, and period 1 has elapsed and none of the players has moved–the history is $\{0\}$. Therefore, this player plays 8 with probability $1-\varepsilon$, 7 or 9 with probability $\frac{\varepsilon}{x_8}$ each, 6 or 10 with probability $\frac{\varepsilon}{2x_8}$

Table 10: Probability of Playing a Particular Quantity

H	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	Q12	Q13	Q14	Q15
{}	$\frac{\varepsilon}{64x_{10}}$	$\frac{\varepsilon}{32x_{10}}$	$\frac{\varepsilon}{16x_{10}}$	$\frac{\varepsilon}{8x_{10}}$	$\frac{\varepsilon}{4x_{10}}$	$\frac{\varepsilon}{2x_{10}}$	$\frac{\varepsilon}{x_{10}}$	$1-\varepsilon$	$\frac{\varepsilon}{x_{10}}$	$\frac{\varepsilon}{2x_{10}}$	$\frac{\varepsilon}{4x_{10}}$	$\frac{\varepsilon}{8x_{10}}$	$\frac{\varepsilon}{16x_{10}}$
{0}	$\frac{\varepsilon}{16x_8}$	$\frac{\varepsilon}{8x_8}$	$\frac{\varepsilon}{4x_8}$	$\frac{\varepsilon}{2x_8}$	$\frac{\varepsilon}{x_8}$	$1-\varepsilon$	$\frac{\varepsilon}{x_8}$	$\frac{\varepsilon}{2x_8}$	$\frac{\varepsilon}{4x_8}$	$\frac{\varepsilon}{8x_8}$	$\frac{\varepsilon}{16x_8}$	$\frac{\varepsilon}{32x_8}$	$\frac{\varepsilon}{64x_8}$
{1,3}	$\frac{\varepsilon}{128x_{11}}$	$\frac{\varepsilon}{64x_{11}}$	$\frac{\varepsilon}{32x_{11}}$	$\frac{\varepsilon}{16x_{11}}$	$\frac{\varepsilon}{8x_{q11}}$	$\frac{\varepsilon}{4x_{11}}$	$\frac{\varepsilon}{2x_{11}}$	$\frac{\varepsilon}{x_{11}}$	$1-\varepsilon$	$\frac{\varepsilon}{x_{11}}$	$\frac{\varepsilon}{2x_{11}}$	$\frac{\varepsilon}{4x_{11}}$	$\frac{\varepsilon}{8x_{11}}$
{1,4}	$\frac{\varepsilon}{64x_{10}}$	$\frac{\varepsilon}{32x_{10}}$	$\frac{\varepsilon}{16x_{10}}$	$\frac{\varepsilon}{8x_{10}}$	$\frac{\varepsilon}{4x_{q10}}$	$\frac{\varepsilon}{2x_{10}}$	$\frac{\varepsilon}{x_{10}}$	$1-\varepsilon$	$\frac{\varepsilon}{x_{10}}$	$\frac{\varepsilon}{2x_{10}}$	$\frac{\varepsilon}{4x_{10}}$	$\frac{\varepsilon}{8x_{10}}$	$\frac{\varepsilon}{16x_{10}}$
{1,5}	$\frac{\varepsilon}{64x_{10}}$	$\frac{\varepsilon}{32x_{10}}$	$\frac{\varepsilon}{16x_{10}}$	$\frac{\varepsilon}{8x_{10}}$	$\frac{\varepsilon}{4x_{q10}}$	$\frac{\varepsilon}{2x_{10}}$	$\frac{\varepsilon}{x_{10}}$	$1-\varepsilon$	$\frac{\varepsilon}{x_{10}}$	$\frac{\varepsilon}{2x_{10}}$	$\frac{\varepsilon}{4x_{10}}$	$\frac{\varepsilon}{8x_{10}}$	$\frac{\varepsilon}{16x_{10}}$
{1,6}	$\frac{\varepsilon}{32x_9}$	$\frac{\varepsilon}{16x_9}$	$\frac{\varepsilon}{8x_9}$	$\frac{\varepsilon}{4x_9}$	$\frac{\varepsilon}{2x_{q9}}$	$\frac{\varepsilon}{x_9}$	$1-\varepsilon$	$\frac{\varepsilon}{x_9}$	$\frac{\varepsilon}{2x_9}$	$\frac{\varepsilon}{4x_9}$	$\frac{\varepsilon}{8x_9}$	$\frac{\varepsilon}{16x_9}$	$\frac{\varepsilon}{32x_9}$
{1,7}	$\frac{\varepsilon}{16x_8}$	$\frac{\varepsilon}{8x_8}$	$\frac{\varepsilon}{4x_8}$	$\frac{\varepsilon}{2x_8}$	$\frac{\varepsilon}{x_{q8}}$	$1-\varepsilon$	$\frac{\varepsilon}{x_8}$	$\frac{\varepsilon}{2x_8}$	$\frac{\varepsilon}{4x_8}$	$\frac{\varepsilon}{8x_8}$	$\frac{\varepsilon}{16x_8}$	$\frac{\varepsilon}{32x_8}$	$\frac{\varepsilon}{64x_8}$
{1,8}	$\frac{\varepsilon}{16x_8}$	$\frac{\varepsilon}{8x_8}$	$\frac{\varepsilon}{4x_8}$	$\frac{\varepsilon}{2x_8}$	$\frac{\varepsilon}{x_{q8}}$	$1-\varepsilon$	$\frac{\varepsilon}{x_8}$	$\frac{\varepsilon}{2x_8}$	$\frac{\varepsilon}{4x_8}$	$\frac{\varepsilon}{8x_8}$	$\frac{\varepsilon}{16x_8}$	$\frac{\varepsilon}{32x_8}$	$\frac{\varepsilon}{64x_8}$
{1,9}	$\frac{\varepsilon}{16x_8}$	$\frac{\varepsilon}{8x_8}$	$\frac{\varepsilon}{4x_8}$	$\frac{\varepsilon}{2x_8}$	$\frac{\varepsilon}{x_{q8}}$	$1-\varepsilon$	$\frac{\varepsilon}{x_8}$	$\frac{\varepsilon}{2x_8}$	$\frac{\varepsilon}{4x_8}$	$\frac{\varepsilon}{8x_8}$	$\frac{\varepsilon}{16x_8}$	$\frac{\varepsilon}{32x_8}$	$\frac{\varepsilon}{64x_8}$
{1,10}	$\frac{\varepsilon}{8x_7}$	$\frac{\varepsilon}{4x_7}$	$\frac{\varepsilon}{2x_7}$	$\frac{\varepsilon}{x_7}$	$1-\varepsilon$	$\frac{\varepsilon}{x_7}$	$\frac{\varepsilon}{2x_7}$	$\frac{\varepsilon}{4x_7}$	$\frac{\varepsilon}{8x_7}$	$\frac{\varepsilon}{16x_7}$	$\frac{\varepsilon}{32x_7}$	$\frac{\varepsilon}{64x_7}$	$\frac{\varepsilon}{128x_7}$
{1,11}	$\frac{\varepsilon}{8x_7}$	$\frac{\varepsilon}{4x_7}$	$\frac{\varepsilon}{2x_7}$	$\frac{\varepsilon}{x_7}$	$1-\varepsilon$	$\frac{\varepsilon}{x_7}$	$\frac{\varepsilon}{2x_7}$	$\frac{\varepsilon}{4x_7}$	$\frac{\varepsilon}{8x_7}$	$\frac{\varepsilon}{16x_7}$	$\frac{\varepsilon}{32x_7}$	$\frac{\varepsilon}{64x_7}$	$\frac{\varepsilon}{128x_7}$
{1,12}	$\frac{\varepsilon}{4x_6}$	$\frac{\varepsilon}{2x_6}$	$\frac{\varepsilon}{x_6}$	$1-\varepsilon$	$\frac{\varepsilon}{x_{q6}}$	$\frac{\varepsilon}{2x_6}$	$\frac{\varepsilon}{4x_6}$	$\frac{\varepsilon}{8x_6}$	$\frac{\varepsilon}{16x_6}$	$\frac{\varepsilon}{32x_6}$	$\frac{\varepsilon}{64x_6}$	$\frac{\varepsilon}{128x_6}$	$\frac{\varepsilon}{256x_6}$
{1,13}	$\frac{\varepsilon}{4x_6}$	$\frac{\varepsilon}{2x_6}$	$\frac{\varepsilon}{x_6}$	$1-\varepsilon$	$\frac{\varepsilon}{x_{q6}}$	$\frac{\varepsilon}{2x_6}$	$\frac{\varepsilon}{4x_6}$	$\frac{\varepsilon}{8x_6}$	$\frac{\varepsilon}{16x_6}$	$\frac{\varepsilon}{32x_6}$	$\frac{\varepsilon}{64x_6}$	$\frac{\varepsilon}{128x_6}$	$\frac{\varepsilon}{256x_6}$
{1,14}	$\frac{\varepsilon}{2x_5}$	$\frac{\varepsilon}{x_5}$	$1-\varepsilon$	$\frac{\varepsilon}{x_5}$	$\frac{\varepsilon}{2x_{q5}}$	$\frac{\varepsilon}{4x_5}$	$\frac{\varepsilon}{8x_5}$	$\frac{\varepsilon}{16x_5}$	$\frac{\varepsilon}{32x_5}$	$\frac{\varepsilon}{64x_5}$	$\frac{\varepsilon}{128x_5}$	$\frac{\varepsilon}{256x_5}$	$\frac{\varepsilon}{512x_5}$
{1,15}	$\frac{\varepsilon}{2x_5}$	$\frac{\varepsilon}{x_5}$	$1-\varepsilon$	$\frac{\varepsilon}{x_5}$	$\frac{\varepsilon}{2x_{q5}}$	$\frac{\varepsilon}{4x_5}$	$\frac{\varepsilon}{8x_5}$	$\frac{\varepsilon}{16x_5}$	$\frac{\varepsilon}{32x_5}$	$\frac{\varepsilon}{64x_5}$	$\frac{\varepsilon}{128x_5}$	$\frac{\varepsilon}{256x_5}$	$\frac{\varepsilon}{512x_5}$

each, and so on. The remaining rows in the table display the probability of playing each quantity for a player who decides to wait and who has observed the rival committing to period 1. For example, if the history is $\{1, 10\}$ then the first mover has played 10 and the follower will play his best response of 7 with probability $1-\varepsilon$, 6 or 8 with probability $\frac{\varepsilon}{x_7}$ each, 5 or 9 with probability $\frac{\varepsilon}{2x_7}$ each, and so on.

The probability of commitment and quantity choices (t_1, q_1, t_2, q_2) occurring is:

$$f\left(t_1, q_1, t_2, q_2|\tau, \varepsilon\right) = \sum_{k_1=0}^{K} \sum_{k_2=0}^{K} f^{Lk_1}(t_1) f^{Lk_1}(q_1|q^{LK_1}) f\left(k_1|\tau\right) f^{Lk_2}(t_2) f^{Lk_2}(q_2|q^{LK_2}) f\left(k_2|\tau\right).$$

Therefore, the likelihood function is

$$L(\tau, \varepsilon) = \prod_{t_1=1}^{2} \prod_{q_1=3}^{15} \prod_{t_2=1}^{2} \prod_{q_2=3}^{15} \left[f\left(t_1, q_1, t_2, q_2|\tau, \varepsilon\right)\right]^{n_{(t_1,q_1,t_2,q_2)}},$$

where $n_{(t_1,q_1,t_2,q_2)}$ is the number of cases (t_1, q_1, t_2, q_2) is observed in the data. The first step in the estimation procedure is the determination of intervals where predicted behavior is constant. These intervals were computed up to $\tau = 4$. There are two reasons for this: (i) higher levels of τ are unlikely according to the cognitive hierarchy literature, and (ii) with $\tau = 4$ ten thinking steps are required which is reasonable to address this problem.

Table 11 presents the upper and lower bounds of each interval, as well as the behavior of each type of player. The inclusion of higher thinking steps is done progressively as τ increases: fewer types are needed for smaller τ. I_1 stands for period 1 commitment, I_2 for period 2 commitment, and Q_{10} for quantity 10. If the predicted behavior is commitment to period 2 and there is no commitment by a rival to period 1, then the predicted quantity is 8. However, if the predicted behavior is commitment to period 2 and there is commitment by a rival to period 1, then the predicted quantity of period 2 is the best response to the rival's period 1 commitment.

Table 11: Intervals where Predicted Behavior is Constant

Interval for τ	$L2$	$L3$	$L4$	$L5$	$L6$	$L7$	$L8$	$L9$	$L10$
[0.0001,0.8351]	I_2	I_2	I_2	I_2	–	–	–	–	–
[0.8352,0.8444]	I_2	I_2	I_2	I_1Q_{10}	–	–	–	–	–
[0.8445,0.8580]	I_2	I_2	I_1Q_{10}	I_2	–	–	–	–	–
[0.8581,0.8980]	I_2	I_2	I_1Q_{10}	I_1Q_{10}	–	–	–	–	–
[0.8981,0.9878]	I_2	I_1Q_{10}	I_2	I_2	–	–	–	–	–
[0.9879,1.0176]	I_2	I_1Q_{10}	I_2	I_1Q_{10}	–	–	–	–	–
[1.0177,1.0737]	I_2	I_1Q_{10}	I_1Q_{10}	I_2	–	–	–	–	–
[1.0738,1.0905]	I_2	I_1Q_{10}	I_1Q_{10}	I_1Q_{10}	I_2	–	–	–	–
[1.0906,1.3012]	I_2	I_1Q_{10}	I_1Q_{10}	I_1Q_{10}	I_1Q_{10}	–	–	–	–
[1.3013,1.8257]	I_1Q_{10}	I_2	I_2	I_2	I_2	I_2	–	–	–
[1.8258,1.8550]	I_1Q_{10}	I_2	I_2	I_2	I_2	I_1Q_{10}	–	–	–
[1.8551,1.9015]	I_1Q_{10}	I_2	I_2	I_2	I_1Q_{10}	I_2	–	–	–
[1.9016,1.9804]	I_1Q_{10}	I_2	I_2	I_2	I_1Q_{10}	I_1Q_{10}	–	–	–
[1.9805,2.1944]	I_1Q_{10}	I_2	I_2	I_1Q_{10}	I_2	I_2	–	–	–
[2.1945,2.2481]	I_1Q_{10}	I_2	I_2	I_1Q_{10}	I_2	I_1Q_{10}	I_2	–	–
[2.2482,2.3459]	I_1Q_{10}	I_2	I_2	I_1Q_{10}	I_2	I_1Q_{10}	I_1Q_{10}	–	–
[2.3460,2.8563]	I_1Q_{10}	I_2	I_2	I_1Q_{10}	I_1Q_{10}	I_2	I_2	I_2	–
[2.8564,3.1886]	I_1Q_{10}	I_2	I_1Q_{10}	I_2	I_2	I_2	I_2	I_2	I_2
[3.1887,3.2126]	I_1Q_{10}	I_2	I_1Q_{10}	I_2	I_2	I_2	I_2	I_2	I_1Q_{10}
[3.2127,3.2478]	I_1Q_{10}	I_2	I_1Q_{10}	I_2	I_2	I_2	I_2	I_1Q_{10}	I_2
[3.2479,3.2899]	I_1Q_{10}	I_2	I_1Q_{10}	I_2	I_2	I_2	I_2	I_1Q_{10}	I_1Q_{10}
[3.2900,3.3826]	I_1Q_{10}	I_2	I_1Q_{10}	I_2	I_2	I_2	I_1Q_{10}	I_2	I_2
[3.3827,3.4263]	I_1Q_{10}	I_2	I_1Q_{10}	I_2	I_2	I_2	I_1Q_{10}	I_2	I_1Q_{10}
[3.4264,3.4963]	I_1Q_{10}	I_2	I_1Q_{10}	I_2	I_2	I_2	I_1Q_{10}	I_1Q_{10}	I_2
[3.4964,3.5604]	I_1Q_{10}	I_2	I_1Q_{10}	I_2	I_2	I_2	I_1Q_{10}	I_1Q_{10}	I_1Q_{10}
[3.5605,3.8640]	I_1Q_{10}	I_2	I_1Q_{10}	I_2	I_2	I_1Q_{10}	I_2	I_2	I_2
[3.8641,4.0000]	I_1Q_{10}	I_2	I_1Q_{10}	I_2	I_2	I_1Q_{10}	I_2	I_2	I_1Q_{10}

Table 12: Maximum Likelihood Estimates per Interval

Interval for τ	First 5 rounds			Entire set		
	τ	ε	ML	τ	ε	ML
[0.0001,0.8351]	0.2064	0.06	-413.69	0.1564	0.04	-2505.48
[0.8352,0.8444]	0.8352	0.41	-437.30	0.8352	0.45	-2689.38
[0.8445,0.8580]	0.8445	0.40	-432.21	0.8445	0.42	-2655.10
[0.8581,0.8980]	0.8581	0.40	-431.55	0.8581	0.42	-2650.71
[0.8981,0.9878]	0.9878	0.45	-414.70	0.9878	0.46	-2536.01
[0.9879,1.0176]	1.0176	0.46	-413.34	1.0176	0.47	-2526.94
[1.0177,1.0737]	1.0737	0.49	-407.96	1.0737	0.50	-2490.39
[1.0738,1.0905]	1.0905	0.50	-406.33	1.0905	0.51	-2479.31
[1.0906,1.3012]	1.3012	0.54	-399.59	1.3012	0.55	-2434.58
[1.3013,1.8257]	1.5742	0.59	-383.06	1.5291	0.60	-2317.04
[1.8258,1.8550]	1.8258	0.61	-383.42	1.8258	0.62	-2321.09
[1.8551,1.9015]	1.8551	0.61	-382.67	1.8551	0.62	-2315.49
[1.9016,1.9804]	1.9016	0.61	-382.46	1.9016	0.62	-2314.02
[1.9805,2.1944]	1.9805	0.62	-380.14	1.9805	0.63	-2296.43
[2.1945,2.2481]	2.1945	0.62	-379.71	2.1945	0.64	-2293.26
[2.2482,2.3459]	2.2569	0.63	-379.54	2.2482	0.64	-2291.88
[2.3460,2.8563]	2.8562	0.64	-377.62	2.8563	0.65	-2275.88
[2.8564,3.1886]	2.8564	0.64	-375.34	2.8564	0.65	-2257.49
[3.1887,3.2126]	3.1887	0.65	-376.80	3.1887	0.66	-2268.42
[3.2127,3.2478]	3.2127	0.65	-376.68	3.2127	0.66	-2267.43
[3.2479,3.2899]	3.2479	0.65	-376.73	3.2479	0.66	-2267.74
[3.2900,3.3826]	3.2900	0.65	-376.37	3.2900	0.66	-2264.78
[3.3827,3.4263]	3.3827	0.65	-376.63	3.3827	0.66	-2266.59
[3.4264,3.4963]	3.4264	0.65	-376.49	3.4264	0.66	-2265.36
[3.4964,3.5604]	3.4964	0.65	-376.55	3.4964	0.66	-2265.71
[3.5605,3.8640]	3.5605	0.65	-375.72	3.5605	0.66	-2258.75
[3.8641,4.0000]	3.8641	0.65	-376.49	3.8641	0.66	-2264.30

Table 12 displays the maximum likelihood estimates for τ for each of the intervals. The estimation was carried out for each of the intervals and the maximum was extracted from the set of 27 maxima, one per each interval. The procedure used was a standard grid search: the maximum likelihood expression was calculated for all possible values of τ within each interval and for all possible values that ε can assume. The software used was GAUSS.