Subjective Performance Evaluation of Employees with Biased Beliefs[1]

Matteo Foschi
European University Institute
matteo.foschi@eui.eu

Luís Santos-Pinto
HEC Lausanne
luispedro.santospinto@unil.ch

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Abstract

We analyze how optimism and overconfidence affect subjective performance evaluation (SPE) contracts. An optimistic worker overestimates the probability of observing an acceptable SPE given the realization of the principal's SPE. An overconfident worker believes his SPE signal is more informative about the outcome of the project than it actually is. The manager of the firm is better informed about performance than the worker and knows the worker's bias. We show that both optimism and overconfidence: *i)* change the optimal incentive scheme under SPE, *ii)* may lower the deadweight loss associated with SPE contracts, and *iii)* can lead to a Pareto improvement.

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1. Introduction

Most workers perform jobs in which objective performance measures are extremely difficult to obtain (Prendergast, 1999). This is because, very often, the ultimate quality of a worker's performance, output or service is not directly observable. This happens in the production of complex goods and services like movies, technological gadgets, academic research papers or in the sports industry. In these types of jobs, in order to

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provide work incentives, firms typically use subjective performance measures like, for example, subjective evaluations of supervisors, co-workers, or consumers.

The absence of objective performance measures creates a natural environment in which biases like optimism and overconfidence may influence economic behavior.[2] In this paper we ask how worker optimism and overconfidence (as well as their counterparts pessimism and underconfidence) affect the optimal design of subjective performance evaluation contracts (SPE). Is there a way for the firm's manager to take advantage of the worker's bias? Does the worker lose or gain from being optimistic/overconfident? How does the presence of biased workers affect social welfare? We provide precise answers to these questions and show that the SPE contracts offered to biased workers may differ substantially (qualitatively and quantitatively) from those offered to unbiased workers. For example, the manager may want to use a biased worker's self-evaluation to set compensation. This goes against the general recommendation in the management literature to avoid the use of self-evaluations (or "self-assessment") in setting compensation (Milkovich, Newman, and Gerhart, 2011; Bratton and Gold, 2012), a practice that is, instead, widely used nowadays.

In our model, we consider a contractual environment where a risk neutral manager (or principal) offers a one-period contract to a risk neutral worker (or agent) who is protected by limited liability. If the agent accepts the contract, he chooses an effort level — high or low — to exert towards the realization of a project that may or may not be successful. The probability that the project is a success is larger under high effort than under low effort and the cost of exerting high effort is larger than the cost of exerting low effort.

The effort choice of the agent as well as the outcome of the project are not directly observable. However, the outcome of the project generates separate, private (and hence subjective) signals for the principal and the agent. These signals represent the parties' *opinion* about the performance of the project (and can be thought of as sent by "nature"). We assume that each signal has only two possible realizations: acceptable and

[2]Felson (1981) and Dunning, Meyerowitz, and Holzberg (1989) show empirically that the more ambiguous or subjective is the definition of an ability, the more individuals overestimate their relative skills (a form of overconfidence). Van Den Steen (2004) and Santos-Pinto and Sobel (2005) provide mechanisms whereby an increase in subjectivity raises optimism and overconfidence. In general, optimism is a well documented psychological phenomenon as we argue in section 3.2.

unacceptable performance. The signals are imperfectly positively correlated and the extent to which performance evaluations are subjective depends on the degree of correlation between the signals. The principal's signal is more informative than that of the agent.

We focus on the case where the principal designs an incentive compatible contract in order to implement high effort.[3] A contract in our set-up specifies a wage cost for the principal and a compensation for the agent under each possible pairs of reported signals. The wage is the principal's dollar cost of employing the agent and the compensation is the dollar amount the agent receives. We allow the agent to impose without cost a deadweight loss upon the principal. This captures the notion of conflict in a relationship, which might happen when the parties disagree on their performance evaluations. For instance, the worker may decide to "punish" the employer by performing badly, changing manager or even sabotaging future projects.[4]

In this framework, we assume that an agent has biased beliefs about his private SPE signal. We study the following types of biased beliefs. First, we consider optimism and pessimism. An optimistic (pessimistic) agent overestimates (underestimates) the probability of observing an acceptable SPE, given the realization of the principal's SPE signal. Second, we consider overconfidence and underconfidence. An overconfident (undercondifent) agent believes his SPE signal is more (less) informative about the outcome of the project than it actually is.[5] The principal is fully informed about the agent's bias.

When the principal and the agent disagree over the distribution of the agent's signal, the principal can "speculate" on the compensation granted to the agent in each state,

[3]While we assume this purely because it is more realistic, one can always find a value for the successful project such that the principal always wants to implement high effort as opposed to low effort. Nevertheless, we solve for the optimal contract implementing low effort in Section 6 and study the effect of the agent's bias on the optimal effort implementation choice.

[4]Mas (2006, 2008) provides direct evidence of employees imposing direct costs upon employers through private actions. These costs include a decrease in future effort (Mas, 2006) or a direct reduction in the quality of the output (Mas, 2008). However, there are also examples of employers imposing direct costs upon employees. In the sports and entertainment businesses, athletes and performers (e.g., actors and musicians) are often subject to fines or are not called up for a particular game or show.

[5]Moore and Healy (2008) distinguish between three different types of overconfidence: (i) overestimation of one's absolute skill or performance, (ii) overestimation of one's relative skill or performance (overplacement or the "better-than-average effect"), and (iii) excessive confidence in the precision of one's information or forecasts (overprecision or miscalibration). We use the term overconfidence in the sense of overprecision or miscalibration.

i.e. each possible realizations of the SPEs. To do so she promises more (less) in states the agent deems more (less) probable than she does. This and other features of the optimal contracts we derive lead to four main welfare results.

First, the principal may benefit by taking advantage of the bias of the worker in order to decrease the cost of implementing high effort (Proposition 4). That is, the principal is always (at least weakly) better off when the worker is either optimistic or overconfident, compared to the case of an unbiased worker. Second, optimism and overconfidence lower the deadweight loss of subjective performance evaluation contracts (Proposition 4).[6] Third, workers' optimism as well as overconfidence can lead to a Pareto improvement by simultaneously lowering the firm's expected wage cost and raising the worker's expected compensation (Proposition 8). When the conditions for a Pareto improvement are not met, however, the agent is always (at least weakly) worse off because of his bias. Finally, in line with previous contributions (Santos-Pinto, 2008, 2010; De la Rosa, 2011; Gervais, Heaton, and Odean, 2011), we show that optimism and overconfidence have different implications for the design of optimal contracts than pessimism and underconfidence, respectively (Propositions 5 and 6).

The rest of the paper is organized as follows. Section 2 discusses the related literature and compares it to our findings. Section 3 sets up the model, shows how we introduce workers' biases, formalizes the principal's effort implementation problem, and sets out some basic features of optimal contracts in our set-up. Section 4 solves the model for an optimistic and a pessimistic agent. Section 5 solves the model for an overconfident and an underconfident agent. Section 6 presents a thorough welfare analysis of each of the contracts derived and discusses the main results on welfare and social value of workers' biases. Section 7 considers the case of an agent with a large outside option value. Section 8 discusses extensions of the model. All proofs are relegated to the appendix.

2. Related Literature

Our paper contributes to the literature on subjective performance evaluation. Within this literature the closest paper to ours is MacLeod (2003). He shows that if SPE signals

[6] Deadweight loss is a standard feature of SPE contracts (see the seminal paper MacLeod (2003) and the literature we cite below) and is often also referred to as "money burning".

are perfectly correlated, then the incentive constraints for the revelation of subjective private information are not binding and the optimal principal-agent SPE contract is the same as the optimal contract with objective performance measures. In this case there is no welfare loss due to the incentive constraints arising from SPE. This is no longer the case when SPE signals are imperfectly correlated. MacLeod (2003) also shows that the agent's ability to harm the principal is an essential input into an optimal SPE contract. Furthermore, MacLeod (2003) shows that a higher level of correlation between the parties' information reduces the expected level of conflict in an optimal SPE contract.[7]

Our paper also contributes to the growing literature on the impact of biased beliefs on the employment relationship.[8] In accordance with our results, this literature highlights further cases where workers' overconfidence may have positive welfare implications. Hvide (2002) shows that worker overconfidence about productivity outside the firm improves worker welfare. Bénabou and Tirole (2003) show that if a firm is better informed about a worker's skill than the worker, effort and overconfidence are complements, then the firm has an incentive to boost the worker's overconfidence by offering low-powered incentives that signal trust to the worker and increase motivation. Gervais and Goldstein (2007) find that a firm is better off with a team of workers who overestimate their skill when there are complementarities between workers' efforts. Further, in this literature we can find a set of papers that, like ours, study the implications of the presence of biased agents on key contractual aspects. Santos-Pinto (2008, 2010) and De la Rosa (2011) show how firms can design optimal objective performance evaluation contracts to take advantage of worker overconfidence about productivity. Fang and Moscarini (2005) and Santos-Pinto (2012) show that worker overconfidence about productivity can lead to wage compression inside and outside the firm, respectively.

[7]More recently, authors have investigated SPE contracts in several settings. Among others: Fuchs (2007) looks at repeated moral hazard problems under SPE, Chan and Zheng (2011) study dynamic contracting with SPE, Zabojnik (2014) shifts the attention on SPE as feedbacks rather than incentive tools, Fuchs (2015) studies the connection between SPE and discretional bonuses, MacLeod and Tan (2017) show respectively how malfeasance and the timing of the two parties' SPE reporting have a strong impact on optimal contracting. Finally, the literature on relational contracts (the seminal contribution is due to Levin, 2003) has originated simultaneously and parallel to SPE contracting. This literature investigates similar settings where compensation is tied to performance, but the performance evaluation of the latter is not entirely objective.

[8]We will focus on papers that use the principal-agent framework to model the worker-firm relationship.

Finally, our paper contributes to the literature on exploitative contracting. Section 6 provides conditions under which the new contracts we derive do not necessarily feature an "exploitative nature", in the sense of making the principal better off and the agent worse off (compared to the case of an unbiased agent). When these conditions aren't met, however, the principal does *exploit* the agent's biased beliefs. Notable and related contributions are Della Vigna and Malmendier (2004), Gabaix and Laibson (2006), Eliaz and Spiegler (2008), Heidhues and Koszegi (2010), and Foschi (2017).[9]

3. A Binary Model of Subjective Evaluation

In this section we set up the model, define an optimistic and a pessimistic agent, formalize the principal's problem, and describe some basic features of optimal contracts in our set-up.

3.1. **Set-up.**

A risk neutral principal (she) offers a one period contract to an agent (he). If the agent accepts, he chooses effort $\lambda \in \{\lambda^L, \lambda^H\}$, where λ is the probability that output Y is realized. We let $1 \geq \lambda^H > \lambda^L \geq 0$. The net benefit to the principal is:

$$E(\Pi) = \lambda Y - E(w)$$

where w represents the dollar costs of employing the agent and return Y is always strictly positive. We say that the result of the project is "good" ("bad") if Y is (is not) realized.

When the agent exerts effort λ and receives a compensation of c for his work, he obtains an ex post utility of $U(c, \lambda) = u(c) - V(\lambda)$, where $u(c)$ is the utility of compensation c, and $V(\lambda)$ is the cost of the effort exerted (with $V(\lambda^H) > V(\lambda^L) \geq 0$). In this paper we derive the optimal contract when the principal faces a risk neutral agent and there is limited liability. Hence, we assume $u(c) = c$ and $c \geq 0$. We also assume that the agent has access to an outside option granting him $\overline{u} \geq 0$.

[9] Della Vigna and Malmendier (2004) show how firms can design contracts to take advantage of consumers with quasi-hyperbolic preferences. Gabaix and Laibson (2006) show how firms can use base-good and add-on pricing schemes to exploit consumers who are unaware of the existence of the add-on. Eliaz and Spiegler (2008) study optimal dynamic contracting when agents are uncertain about their own preferences (naïve agents) at the time of signing the contract, and they may be more optimistic than the principal about the better state occurring. Heidhues and Koszegi (2010) study exploitative credit contracts. Foschi (2017) studies the design of optimal contracts for naïve agents introducing the assumption that naïveté may depend on the same ability agents are trying to estimate.

Following MacLeod (2003), neither the outcome of the project nor the effort exerted are observable. The outcome of the project generates separate private (and hence subjective) signals for the principal and the agent. The principal observes a measure of performance or signal T and the agent observes a measure of performance or signal S. Signals T and S have realizations $t \in \{a, u\}$ and $s \in \{a, u\}$ respectively. Realization a (u) corresponds to an "acceptable" ("unacceptable") performance.[10] In particular, we let $\gamma_t^G = \Pr\{T = t|G\}$ and $\gamma_t^B = \Pr\{T = t|B\}$ be the probability that signal T results in $t \in \{a, u\}$ when the outcome of the project is good (G) or bad (B) respectively.

Assumption 1. *Signal T is positively correlated with the outcome of the project, i.e.*

$$\gamma_a^G > \gamma_a^B \quad \textit{and} \quad \gamma_u^G < \gamma_u^B$$

The realization of signal S is described as a function of T. Let $P_{ts} = \Pr\{S = s|T = t\}$. The probability of $(T, S) = (t, s)$ occurring in states G and B is $\gamma_{ts}^G = \Pr\{T = t, S = s|G\} = P_{ts}\gamma_t^G$ and $\gamma_{ts}^B = \Pr\{T = t, S = s|B\} = P_{ts}\gamma_t^B$, respectively. The probability of $(T, S) = (t, s)$ occurring given effort levels λ^H and λ^L is:

$$\gamma_{ts}^H = \Pr\{T = t, S = s|\lambda^H\} = \lambda^H\gamma_{ts}^G + (1 - \lambda^H)\gamma_{ts}^B$$
$$\gamma_{ts}^L = \Pr\{T = t, S = s|\lambda^L\} = \lambda^L\gamma_{ts}^G + (1 - \lambda^L)\gamma_{ts}^B,$$

respectively.

Assumption 2. *For any λ^j, signals are positively correlated in the following sense:*

$$P_{aa}P_{uu} - P_{au}P_{ua} > 0$$

This assumption implies that the principal's signal is *more informative* than that of the agent in the sense defined by Blackwell (1951, 1953). Assumption 2 also has implications on the conditional distributions of signals.

Lemma 1. *Given the positive correlation of signals the following are true:*

(i) $\gamma_{ts}^j = P_{ts} \Pr[T = t|\lambda^j] \equiv P_{ts}\Gamma_t^j$ *for* $j = H, L$,

[10]The model allows for two different interpretations. The acceptable or unacceptable performance may be either the agent's or the project's overall.

(ii) $\gamma^j_{aa}\gamma^j_{uu} - \gamma^j_{au}\gamma^j_{ua} > 0$ *for* $j = H, L$,

(iii) $P_{aa} > P_{ua}$ *and* $P_{uu} > P_{au}$.

(iv) $\Delta\Gamma_a + \Delta\Gamma_u = 0$, *where* $\Delta\Gamma_t = \Gamma^H_t - \Gamma^L_t$.

3.2. **Optimistic and Pessimistic Agents.** Individuals tend to have biased expectations about the future. Most individuals tend to overestimate their chances of experiencing positive events and underestimate their chances of experiencing negative events (e.g. Weinstein, 1980; Taylor and Brown, 1988). Entrepreneurs, managers and workers have often been proven to be optimistic about their financial outcomes (Arabsheibani, De Meza, Maloney, and Pearson, 2000) and to display dispositional optimism (Koudstaal, Sloof, and Van Praag, 2015).[11] Optimism matters for economic decisions like market entry, portfolio, and career choices (e.g. Puri and Robinson, 2007). While optimistic biases are a robust and widespread psychological phenomenon, pessimistic biases are rare. Still, some individuals tend to underestimate their chances of experiencing positive events and overestimate their chances of experiencing negative ones.[12]

To model biased beliefs, we assume that the agent and the principal agree to disagree on the conditional distribution of S. In particular, we assume that, given that the principal has observed $T = t$, the agent's perceived probability of observing $S = s$ is altered by some amount b_t. As we show in Section 5, this formulation allows us not only to study optimistic (and pessimistic) agents but also overconfident (and underconfident) agents, i.e., agents who believe their SPE signals are more (less) informative about the outcome of the project than they actually are.

Assumption 3. *Regardless of the effort exerted, the agent has biased beliefs such that:*

$$\tilde{P}_{aa} = P_{aa} + b_a$$

$$\tilde{P}_{au} = P_{au} - b_a$$

$$\tilde{P}_{ua} = P_{ua} + b_u$$

$$\tilde{P}_{uu} = P_{uu} - b_u$$

[11]Dispositional optimism is defined as the global expectation that good things will be plentiful in the future and bad things will be scarce (e.g. Scheier and Carver, 1985; Scheier, Carver, and Bridges, 1994; Peterson, 2000).

[12]Carver, Scheier, and Segerstrom (2010) review the literature on optimism and show how "it is [...] possible to identify people who are pessimists in an absolute sense" and that "doing this reveals that pessimists are a minority".

where

$$b_a \in [-P_{aa}, P_{au}] \quad b_u \in [-P_{ua}, P_{uu}] \, .^{13}$$

This modelling of the bias is more general than it seems at first glance and it allows us to study several different forms of bias observed in the laboratory and in the field. We start by stating the definition of optimistic (and pessimistic) agents in this model, while we delay the definitions of overconfident (and underconfident) agents to Section 5.

Definition 1. *The agent is "*optimistic*" if he overestimates the probability that his signal is acceptable given the realization of the principal's signal, i.e., if* b_a *and* b_u *are positive.*

Definition 2. *The agent is "*pessimistic*" if he underestimates the probability that his signal is acceptable given the realization of the principal's signal, i.e., if* b_a *and* b_u *are negative.*

As described in its definition, the nature of optimism is for an agent to overestimate the probability that his performance is deemed "acceptable." In fact, denoting the biased probabilistic beliefs of the agent with $\tilde{\Pr}\{\cdot\}$, from basic probability theory we get:

$$\begin{aligned}
\tilde{\Pr}\{S = a|\lambda^j\} =& \tilde{\Pr}\{S = a|T = a\} \Pr\{T = a|\lambda^j\} + \tilde{\Pr}\{S = a|T = u\} \Pr\{T = u|\lambda^j\} \\
=& \tilde{P}_{aa}\Gamma_a^j + \tilde{P}_{ua}\Gamma_u^j \\
=& \Pr\{S = a|\lambda^j\} + b_a\Gamma_a^j + b_u\Gamma_u^j
\end{aligned}$$

which is increasing in both b_a and b_u for any $j = H, L$.[14]

A second aspect that needs attention is how the bias of the agent affects his beliefs about the correlation between the two signals, that is, the direction and extent of the agent's *perceived* correlation between signals. Given Lemma 1, for an effort level λ^j,

[13] The posed boundaries are needed for all $\tilde{P}_{ts} \in [0, 1]$ to hold.

[14] An alternative formulation for the bias would be simply to let

$$\tilde{\Pr}\{S = a|\lambda^j\} = \Pr\{S = a|\lambda^j\} + b$$

and define an optimistic agent as one where $b > 0$. In our formulation we decompose the bias as $b = b_a\Gamma_a^j + b_u\Gamma_u^j$. This makes our analysis more general and allows for the study of more complicated beliefs.

biases b_a and b_u imply the following for every t:

$$\tilde{\gamma}^j_{ta} = \tilde{P}_{ta}\Gamma^j_t = (P_{ta} + b_t)\Gamma^j_t$$

$$\tilde{\gamma}^j_{tu} = \tilde{P}_{tu}\Gamma^j_t = (P_{tu} - b_t)\Gamma^j_t$$

Lemma 2. *Given Assumption 2, when an agent has a bias that satisfies:*

$$b_u - b_a \leq P_{aa} - P_{ua}, \tag{1}$$

he believes that signals are positively correlated, i.e., $\tilde{P}_{aa}\tilde{P}_{uu} - \tilde{P}_{au}\tilde{P}_{ua} > 0$.

Assuming perceived positive correlation ensures the agent and the principal agree on the fact that the principal is the expert, i.e., they agree that the principal has the most informative signal or information structure. If (1) fails, a biased agent has beliefs that satisfy $\tilde{P}_{aa}\tilde{P}_{uu} - \tilde{P}_{au}\tilde{P}_{ua} < 0$ and believes that signals are negatively correlated. For now, we rule out this type of biased beliefs. This assumption is the most natural one to make and allows us to directly compare our results to SPE settings where the principal is assumed to be the expert and the agent is unbiased. However, we present a short summary of the results for this type of bias in Section 8 and their full derivation in Appendix C.

3.3. **The Principal's Effort Implementation Problem.** In the model, a contract is a set $\{w_{ts}, c_{ts}\}_{t,s\in\{a,u\}}$, where both the agent's compensation, c, and the principal's dollar cost of employing the agent, w, may depend on the realization of T and S. The principal is assumed to be perfectly informed about the agent's biased beliefs. The objective of the principal is to incentivize the agent to exert the level of effort that maximizes profits. Following Grossman and Hart (1983), we know that this problem can be divided into two steps. First, deriving the minimum cost $E^*(w|\lambda)$ of implementing a certain λ and then solving $\max_\lambda \lambda Y - E^*(w|\lambda)$. The rest of this paper is focused on minimizing the cost of implementing the high level of effort for different types of agents. However, in Section 6, we discuss the effect of the bias on the implementation of low effort and the optimal effort implementation choice of the principal as a function of the bias.

By the revelation principle, it is sufficient to consider only contracts where both parties have an incentive to reveal their private information in equilibrium. Hence, the principal faces the following constrained minimization problem:

$$\min_{\{w_{ts},c_{ts}\}_{t,s\in\{u,a\}}} w_{aa}\gamma^H_{aa} + w_{au}\gamma^H_{au} + w_{ua}\gamma^H_{ua} + w_{uu}\gamma^H_{uu} \tag{2}$$

$$\text{s.t.} \quad \sum_{ts} c_{ts}\tilde{\gamma}^H_{ts} - V(\lambda^H) \geq \bar{u} \tag{PC}$$

$$\sum_{ts} c_{ts}\tilde{\gamma}^H_{ts} - V(\lambda^H) \geq \sum_{ts} c_{ts}\tilde{\gamma}^L_{ts} - V(\lambda^L) \tag{IC}$$

$$w_{aa}\gamma^H_{aa} + w_{au}\gamma^H_{au} \leq w_{ua}\gamma^H_{aa} + w_{uu}\gamma^H_{au} \tag{TR^a_P}$$

$$w_{ua}\gamma^H_{ua} + w_{uu}\gamma^H_{uu} \leq w_{aa}\gamma^H_{ua} + w_{au}\gamma^H_{uu} \tag{TR^u_P}$$

$$c_{aa}\tilde{\gamma}^H_{aa} + c_{ua}\tilde{\gamma}^H_{ua} \geq c_{au}\tilde{\gamma}^H_{aa} + c_{uu}\tilde{\gamma}^H_{ua} \tag{TR^a_A}$$

$$c_{au}\tilde{\gamma}^H_{au} + c_{uu}\tilde{\gamma}^H_{uu} \geq c_{aa}\tilde{\gamma}^H_{au} + c_{ua}\tilde{\gamma}^H_{uu} \tag{TR^u_A}$$

$$w_{ts} \geq c_{ts} \geq 0 \qquad \forall t, s \in \{a, u\}. \tag{LL_{ts}}$$

The first two constraints are the classical participation and incentive compatibility constraints. They ensure that the agent is willing to accept the contract, (PC), and to exert high effort instead of low effort, (IC). Constraints (TR^t_P) are called *truthful reporting constraints for the principal*, they ensure that she is willing to report t truthfully when she observes $T = t$. Similarly, (TR^s_A) are the truthful reporting constraints for the agent, they ensure that he is willing to report s truthfully when he observes $S = s$. Notice that this implies an important distinction. The realizations of signals, t and s, are not necessarily equal to the performance evaluation reported to the other party. Hereafter we use the word "signal" to indicate the actual realization of T and S and "Performance Evaluation report" (PE report) to indicate the reported value of his/her own signal by one party to the other. The (TR) constraints ensure that the two coincide in equilibrium.[15] Last, is a set of four constraints that ensure limited liability on the side of the agent ($c_{ts} \geq 0$) and feasibility ($w_{ts} \geq c_{ts}$).[16]

[15]Technically we could define $\hat{t}$ and $\hat{s}$ as the PE report. Since the (TR) ensures that $\hat{t} = t$ and $\hat{s} = s$ we omit this notation in order not to complicate the exposition.

[16]Notice that Problem (2) solves the model under the assumption of *good-faith*, introduced by MacLeod and Tan (2017). A contract solving (2) is incentive compatible with respect to one-shot deviations. In other words, the problem above ensures that the worker neither intends to misreport his SPE realization, when he exerts high effort — the level that the principal wants to implement —, nor to

The final assumption of our model requires $\bar{u}$ to be small enough. Its only function is to improve the tractability of the problem, since when it holds the (PC) is always satisfied and can be ignored from the analysis. We relax this assumption in Section 7.[17]

Assumption 4. *Let*

$$\bar{u} \leq \frac{V(\lambda^H)\Gamma_a^L - V(\lambda^L)\Gamma_a^H}{\Delta\Gamma_a} \equiv \bar{u}_1 \tag{3}$$

Given Assumption 4, and the limited liability assumption, the (IC) implies that the (PC) is satisfied. Therefore we disregard the (PC) in the solution of the problem and check that it holds afterwards. We present this in Corollary 2.

$$\min_{\{w_{ts},c_{ts}\}_{t,s\in\{u,a\}}} w_{aa}\gamma_{aa}^H + w_{au}\gamma_{au}^H + w_{ua}\gamma_{ua}^H + w_{uu}\gamma_{uu}^H \tag{4}$$

$$\text{s.t.} \quad \sum_{ts} c_{ts}\tilde{\gamma}_{ts}^H - V(\lambda^H) \geq \sum_{ts} c_{ts}\tilde{\gamma}_{ts}^L - V(\lambda^L) \tag{IC}$$

$$w_{aa}\gamma_{aa}^H + w_{au}\gamma_{au}^H \leq w_{ua}\gamma_{aa}^H + w_{uu}\gamma_{au}^H \tag{TR_P^a}$$

$$w_{ua}\gamma_{ua}^H + w_{uu}\gamma_{uu}^H \leq w_{aa}\gamma_{ua}^H + w_{au}\gamma_{uu}^H \tag{TR_P^u}$$

$$c_{aa}\tilde{\gamma}_{aa}^H + c_{ua}\tilde{\gamma}_{ua}^H \geq c_{au}\tilde{\gamma}_{aa}^H + c_{uu}\tilde{\gamma}_{ua}^H \tag{TR_A^a}$$

$$c_{au}\tilde{\gamma}_{au}^H + c_{uu}\tilde{\gamma}_{uu}^H \geq c_{aa}\tilde{\gamma}_{au}^H + c_{ua}\tilde{\gamma}_{uu}^H \tag{TR_A^u}$$

$$w_{ts} \geq c_{ts} \geq 0 \qquad \forall t, s \in \{a, u\}. \tag{LL_{ts}}$$

Before splitting the analysis for the resulting optimal contract for each type of biased agent, we present a set of findings on problem (2) which are valid for biased as well as for unbiased agents.

3.4. **Basic Features of Optimal Contracts.** In order for the truthful reporting constraints to hold, it cannot be optimal for either party to always report the same PE regardless of that party's SPE realization. Truthful reporting imposes constraints on

exert low effort when he truthfully reports his SPE realization. However, it does not incentivise the agent to report truthfully when he exerts low effort. It is assumed that he will do so in "good-faith". While this is a weaker requirement compared to Perfect Bayesian Equilibrium (also called *guile-free* contracts of equilibrium in MacLeod and Tan, 2017) it simplifies the computations substantially while not affecting the results. We discuss this in Appendix D, where we also show that the (TR_P) constraints do incentivise the principal to always report truthfully, regardless of the effort exerted by the agent.

[17]We show how the main results of the paper, namely the way the principal uses the SPEs in contracting and the welfare results, continue to hold, while the optimal contracts may feature small qualitative modifications.

the equilibrium wages and compensation levels. For example, suppose that we had $w_{ua} \geq w_{aa}$ and $w_{uu} \geq w_{au}$, with at least one inequality holding strictly. In this case, it would be optimal for the principal to always report an unacceptable performance, regardless of her SPE realization, and this would violate the principal's truthful reporting constraints. Similarly, suppose that we had $c_{aa} \geq c_{au}$ and $c_{ua} \geq c_{uu}$, with at least one inequality holding strictly. In this case, it would be optimal for the agent to always report an acceptable perfomance, regardless of his SPE realization, and this would violate the agent's truthful reporting constraints. Since the principal wants to pay the lowest possible wage, the direction of the inequalities must be such that the wages are the lowest when the PE reports are identical, i.e., $t = s$, the most probable outcome (under truthful reporting) given that signals are positively correlated. This produces the following Lemma.

Lemma 3. *Given Assumption 2, any optimal contract implementing high effort features either (i)* $w_{ua} = w_{aa}$ *and* $w_{au} = w_{uu}$ *or (ii)* $w_{ua} > w_{aa}$ *and* $w_{au} > w_{uu}$.

Similarly, since the agent wants to obtain the highest possible compensation, if he believes signals are positively correlated, then the direction of the inequalities must be such that the compensations are the highest when $t = s$, the most probable believed outcome.

Lemma 4. *If the agent believes signals are positively correlated, i.e. (1) holds, then any optimal contract implementing high effort features either (i)* $c_{aa} = c_{au}$ *and* $c_{uu} = c_{ua}$ *or (ii)* $c_{aa} > c_{au}$ *and* $c_{uu} > c_{ua}$.

These first two Lemmas are already enough for us to state the first Proposition of the model, which confirms one of the main results of MacLeod (2003) for an agent who believes signals are positively correlated. That is, unless the optimal contract features a deadweight loss, it is impossible to implement high effort under truthful reporting.

Proposition 1. *If the principal wishes to implement high effort under truthful reporting and the agent believes signals are positively correlated, then there ought to exist at least one combination of realizations of* t *and* s *where* $w_{ts} > c_{ts}$.

To understand fully Proposition 1 notice that, intuitively, the principal always has the incentive to report that the performance of the project (and of the agent) is unacceptable, while the agent always has the incentive to report the opposite. If the principal and the agent play a constant sum game, these incentives are the only ones present and truthful reporting becomes impossible. We define the expected deadweight loss from using a subjective performance evaluation contract that implements high effort as $\sum_{ts}(w_{ts} - c_{ts})\gamma_{ts}^H$.

4. Optimism, Pessimism and Optimal Contracting

In this section, we derive the optimal contract for an optimistic and for a pessimistic agent. We also show how optimism and pessimism have different implications for the design of optimal contracts. Optimism may lead to a new type of contract designed by the principal but pessimism never does.

4.1. **Optimistic Agent.**

We start by considering the case of an optimistic agent who believes signals are positively correlated. This happens when

$$b_a > 0,\ b_u > 0,\ \text{and } b_u - b_a \leq P_{aa} - P_{ua}.$$

In order to solve problem (4), we present a set of Lemmas in Appendix A that select the binding constraints for this case and reduce the choice variables of the problem to c_{aa} and c_{au} only. All together this reduces the problem to:

$$\min_{c_{aa},c_{au}} c_{aa}[(\gamma_{aa}^H)^2\tilde{\gamma}_{uu}^H + \gamma_{aa}^H\gamma_{ua}^H\tilde{\gamma}_{uu}^H + \tilde{\gamma}_{au}^H\gamma_{uu}^H\gamma_{aa}^H - \tilde{\gamma}_{au}^H\gamma_{au}^H\gamma_{ua}^H]$$
$$+ c_{au}\left(\gamma_{aa}^H\gamma_{au}^H\tilde{\gamma}_{uu}^H + \gamma_{au}^H\gamma_{ua}^H\tilde{\gamma}_{uu}^H - \tilde{\gamma}_{au}^H\gamma_{uu}^H\gamma_{aa}^H + \tilde{\gamma}_{au}^H\gamma_{au}^H\gamma_{ua}^H\right) \quad (5)$$
$$\text{s.t.} \quad c_{aa}\left(\Delta\tilde{\gamma}_{aa} + \frac{\tilde{\gamma}_{au}^H}{\tilde{\gamma}_{uu}^H}\Delta\tilde{\gamma}_{uu}\right) + c_{au}\left(\Delta\tilde{\gamma}_{au} - \frac{\tilde{\gamma}_{au}^H}{\tilde{\gamma}_{uu}^H}\Delta\tilde{\gamma}_{uu}\right) \geq \Delta V \quad (IC)$$
$$c_{aa} \leq \left(1 + \frac{\tilde{\gamma}_{uu}^H}{\tilde{\gamma}_{au}^H}\right)c_{au} \quad (TR_P^u)$$
$$c_{aa} \geq c_{au}, \quad (TR_A^a)$$

where $\Delta V = V(\lambda^H) - V(\lambda^L)$.

The next Lemma presents a condition that selects the binding constraints of (5). Combined with Proposition 2, it presents a result original to our model. That is, as we show later, the existence of a new contract where the principal's wage cost is determined only by the agent's PE report and the worker's compensation is determined by both parties' PE reports. This stands in contrast to the baseline SPE contract in the literature where the principal's wage cost is determined by both parties' PE reports and the worker's compensation is tied only to the principal's PE report.

Lemma 5. *If an optimistic agent holds beliefs that satisfy:*

$$b_a \geq P_{au} \frac{\Gamma_u^H \Gamma_a^H \left(P_{aa} - P_{ua}\right) + \left(P_{uu} - b_u\right) \left(P_{aa} \Gamma_a^H + P_{ua} \Gamma_u^H\right) \Gamma_a^H}{\Gamma_u^H \Gamma_a^H \left(P_{aa} - P_{ua}\right) + \left(P_{uu} - b_u\right) \left(P_{aa} \Gamma_a^H + P_{ua} \Gamma_u^H\right)} \tag{6}$$

then the optimal contract implementing high effort features $c_{aa} > c_{au}$, (TR_A^a) slack and (TR_P^u) binding. If the agent believes signals are positively correlated and holds beliefs that violate (6)*, then the optimal contract implementing high effort features $c_{aa} = c_{au}$, (TR_A^a) binding and (TR_P^u) slack.*

Lemma 5 follows from a graphical analysis of the problem. Figure 1 below shows the three constraints binding in (c_{au}, c_{aa}) space and highlights the set of contracts satisfying all constraints of (5) — and therefore of (2).

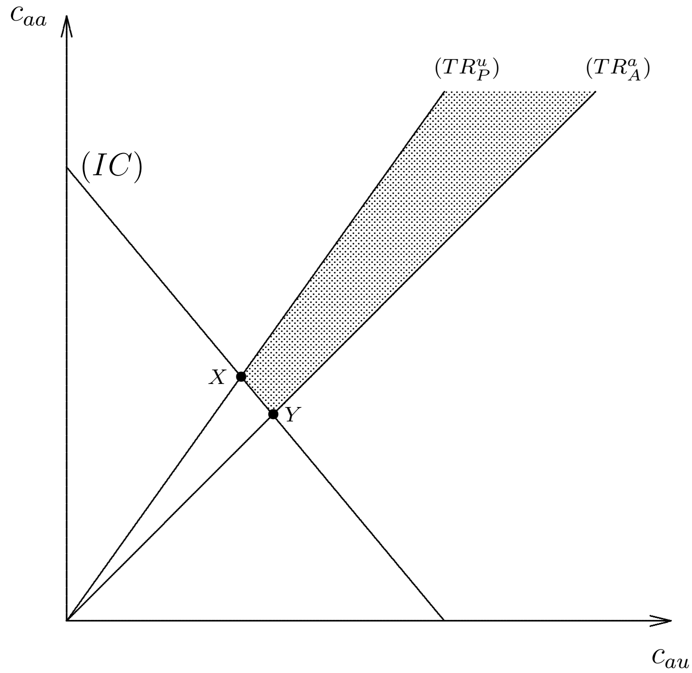

FIGURE 1. The shaded area represents the set of contracts satisfying all the constraints in the minimization problem (5) — and therefore (2).

In order to understand whether at optimum it is the (TR_P^u) or the (TR_A^a) that binds, and therefore where the optimal contract lies in Figure 1, we study the sign and magnitude of the slope of the iso-costs and the (IC). Hence, Lemma 5 shows that the optimal contract lies either at point X or Y of Figure 1, depending on how the slope of the (IC) and of the iso-costs compare.

From this analysis we can also derive the optimal contract offered to an unbiased agent, which we refer to as the *Baseline Performance Evaluation* (BPE) contract $\{w_{ts}^*, c_{ts}^*\}_{t,s=a,u}$.

Proposition 2 (BPE Contract)**.** *If the agent holds unbiased beliefs, then the optimal contract implementing high effort is given by:*

$$\begin{array}{llll} w_{aa}^* = c_{aa}^* & w_{au}^* = c_{aa}^* & w_{uu}^* = 0 & w_{ua}^* = \frac{c_{aa}^*}{P_{aa}} \\ c_{aa}^* = \frac{\Delta V}{\Delta \Gamma_a} & c_{au}^* = c_{aa}^* & c_{uu}^* = 0 & c_{ua}^* = 0. \end{array}$$

The BPE contract features:

(i) *a compensation that depends only on the principal's PE report;*

(ii) *a positive compensation when the principal reports an acceptable performance and no compensation otherwise;*

(iii) *a wage cost that depends on both parties' PE reports;*

(iv) *the highest wage cost when the principal reports an unacceptable performance and the agent reports an acceptable performance;*

(v) *a deadweight loss when the principal reports an unacceptable performance and the agent reports an acceptable performance;*

The above replicates the standard result of the literature with unbiased agents (à la MacLeod, 2003) and it provides us with a basis of comparison for the contracts derived hereafter. The key features of the BPE contract are as follows. First, the agent's compensation depends only on the principal's PE report. Second, the principal's wage cost depends on both parties' PE reports. Third, there is a deadweight loss when the principal deems unacceptable a performance deemed acceptable by the agent.

The fact that in a BPE contract the agent's compensation depends only on the principal's PE report is due to Holmström's *informativeness principle*. According to

Holmström (1979), T is a sufficient statistic for the pair (T, S) with respect to effort λ if $f(t, s; \lambda) = g(t, s)h(t, \lambda)$ for almost every (t, s).[18] This is true in our set-up since $\gamma^j_{ts} = P_{ts}[\lambda^j \gamma^t_G + (1 - \lambda^j)\gamma^t_B]$. In other words, T carries all the relevant information about effort λ, and S adds nothing to the power of inference about λ (even though S is positively correlated with T and the project). Hence, if Y, T, and S were all observable and verifiable (they were objective performance measures), then only Y and T would be used in a contract in order to incentivize the agent. In our model Y, T, and S are neither observable nor verifiable, however, truth telling by both parties, implies that the principal's PE report is a sufficient statistic for the pair of PE reports with respect to the agent's effort choice λ. Hence, the agent's PE report does not add to the power of inference about λ and, therefore, it should not be used to compensate the agent. In other words, the principal should only use her PE report to compensate the agent.

We now explain why, in a BPE contract, the principal's wage cost depends on both parties' PE reports. The principal wishes to minimize the expected wage cost and, for possible combination of PE reports, the wage cost must be greater or equal than the agent's compensation. Since the agent's compensation depends only on the principal's PE it is not surprising that the principal's wage cost also depends on the principal's PE. But, why does the principal's wage cost also depend on the agent's PE? In a BPE contract there is a deadweight loss when the principal deems unacceptable a performance deemed acceptable by the agent. This deadweight loss ensures that the principal has an incentive to reveal acceptable PEs that result in higher compensation for the agent. Otherwise, the principal would always report unacceptable PEs and the agent, knowing this, would exert low effort. Hence, the agent's PE is *valuable* as it allows truthful reporting by the principal. Truthful reporting by both parties allows the principal to implement high effort, which leaves both parties better off than if low effort is implemented.[19]

[18]Notice that Holmström (1979) assumes a moral hazard problem with a risk-averse agent. Chaigneau, Edmans, and Gottlieb (2017) extend the result to a model with limited liability and show that under risk-neutrality the principle still holds. They also show that the conditions for a signal to be of "zero value" to contracting are different from the standard informativeness principle. A signal can be of zero value even when it carries information about states where the agent obtains no compensation, which holds in our model.

[19]According to Holmström (1979), pp.83: "A signal y is said to be *valuable* if both the principal and the agent can be made strictly better off with a contract of the form $s(x, y)$ than they are with a contract of the form $s(x)$."

Now that we have seen why the deadweight loss takes place, let's discuss why it happens following $(t, s) = (u, a)$ and not in other cases. First, notice that the principal wants to punish the agent in the lowest state, that is, when the principal's PE is unacceptable, $t = u$. This is the state where a deviation to low effort is more likely to have occurred. Second, since the signals are positively correlated, a deadweight loss following $(t, s) = (u, u)$ would be suboptimal, since it would take place more often than following $(t, s) = (u, a)$. Finally, if the deadweight loss were not in place, the principal (agent) would have the dominant strategy of reporting the performance as unacceptable (acceptable). Hence, it happens after $(t, s) = (u, a)$ in order to take both parties away from their dominant strategy.

Next we present a set of results that show how, in the presence of an optimistic agent, the principal may find it optimal to offer the agent either the baseline contract or a new contract that makes different use of information and takes advantage of the agent's bias — which we call the *Agent's Performance Evaluation* (APE) contract. The APE contract is original to the present model.

Proposition 3. *If an optimistic agent believes signals are positively correlated and holds beliefs that violate* (6)*, then the optimal contract implementing high effort is given by* $\{w'_{ts}, c'_{ts}\}_{t,s=a,u}$ *where both wages and compensations equal the ones of the BPE contract, i.e.,* $c'_{ts} = c^*_{ts}$ *and* $w'_{ts} = w^*_{ts}$ $\forall t, s = a, u$*.*

Proposition 3 shows that the optimal contract for an optimistic agent is the same as that for an unbiased agent when the optimistic agent believes signals are positively correlated and his optimistic bias b_a is small, i.e., condition (6) is violated.

Now suppose the agent's optimistic bias b_a is large, i.e., condition (6) holds. In this case the APE $\{w^\dagger_{ts}, c^\dagger_{ts}\}_{t,s=a,u}$ is set at optimum. This new contract differs qualitatively from the BPE contract as we discuss below.[20]

Proposition 4 (APE Contract)**.** *If an optimistic agent holds beliefs that satisfy* (6)*, then the optimal contract implementing high effort* $\{w^\dagger_{ts}, c^\dagger_{ts}\}_{t,s=a,u}$ *is given by:*

[20]Notice that, if condition (6) holds with equality, the slopes of the (IC) and isocosts are identical and the problem has many solutions. In particular any point lying between X and Y in Figure 1 solves problem (5). At this point of indifference, we assume the principal sets up a APE contract.

$$\begin{array}{llll} w^{\dagger}_{aa} = c^{\dagger}_{aa} & w^{\dagger}_{au} = c^{\dagger}_{au} & w^{\dagger}_{uu} = c^{\dagger}_{au} & w^{\dagger}_{ua} = c^{\dagger}_{aa} \\ c^{\dagger}_{aa} = c^{\dagger}_{au}\left(1 + \frac{\tilde{\gamma}^H_{uu}}{\tilde{\gamma}^H_{au}}\right) & c^{\dagger}_{au} = \frac{\Delta V}{\Delta \Gamma_a} \frac{\tilde{P}_{au}\Gamma^H_a}{\tilde{P}_{au}\Gamma^H_a + \tilde{P}_{uu}(\tilde{P}_{aa} - \Gamma^H_a)} & c^{\dagger}_{uu} = c^{\dagger}_{au} & c^{\dagger}_{ua} = 0. \end{array}$$

The APE contract features:

(i) *a compensation that depends on both parties' PE reports;*

(ii) *the highest compensation when both parties report an acceptable performance;*

(iii) *a wage cost that depends only on the agent's PE report;*

(iv) *a high wage when the agent reports an acceptable performance and a low wage otherwise;*

(v) *a deadweight loss when the principal reports an unacceptable performance and the agent reports an acceptable performance which is smaller than the deadweight loss in a BPE contract.*

Proposition 4 shows that if the optimistic agent's bias b_a is large, then the optimal contract is very different from the BPE contract described in Proposition 2. First, the agent's compensation depends on both parties' PE reports. Second, the principal's wage cost depends only on the agent's PE report. Third, the agent's optimism reduces the deadweight loss associated with SPE contracts. Hence, the APE contract provides a surprising result: when the agent is optimistic enough, the principal uses the agent's self-evaluation as an input to set the agent's compensation. This goes against the general recommendation in the management literature against the use of self-evaluations to set compensation. Next, we provide the economic intuition behind these key differences.

In contrast to the BPE contract in Proposition 2, in the APE contract in Proposition 4 the agent's compensation depends on both parties' PE reports. On the one hand, we still have the old effect, namely, the fact that the principal's PE report is a sufficient statistic for the pair of PE reports with respect to the agent's effort choice. When the agent's optimistic bias b_a is small, the principal only wants to use her PE report to compensate the agent. However, when the agent's optimistic bias b_a is large, the principal also wants to use the agent's PE report to compensate the agent since this allows her to exploit the agent's bias. By cleverly manipulating the compensation and the wage cost according to the agent's bias, the principal is able to simultaneously raise the agent's perceived expected compensation and lower her expected wage cost. The principal does this by increasing the agent's compensation when both parties deem the performance acceptable, a state overestimated by the agent, and lowering the compensation when the principal deems acceptable a performance deemed unacceptable by the agent, a state underestimated by the agent. The optimal compensation in Proposition 4 results from this trade-off between informational efficiency and exploitation of the agent's bias.

In the BPE contract in Proposition 2 the principal's wage cost depends on both parties' PEs. Moreover, the only role played by the agent's PE report is to provide incentives for truthful revelation by the principal through the imposition of a deadweight loss when the principal reports as unacceptable a performance reported as acceptable by the agent. In other words, the threat of conflict ensures that the principal has an incentive to reveal favourable observations that result in higher compensation to the agent. Hence, the possibility of a deadweight loss makes the agent's PE report valuable in the sense of Holmström (1979). In contrast, in the APE contract in Proposition 4 the principal's wage cost depends only on the agent's PE report. In this contract the agent's PE report is still valuable in the sense of Holmström (1979). However, when the agent's optimistic bias b_a is large, the principal can decrease her expected cost of implementing high effort by increasing the correlation between the wage and the agent's PE report. She does so by increasing the wage when both parties report an acceptable performance, a state overestimated by the agent, and lowering the wage when the principal reports an acceptable performance and the agent reports an unacceptable performance, a state underestimated by the agent. The principal exploits the agent's bias maximally by making the wage depend only on the agent's PE report.

Point (v) of Proposition 4 states that the APE contract leads to a smaller deadweight loss than the BPE contract. As stated, the deadweight loss creates the threat of conflict in order to ensure that the principal truthfully reports her own signal. When the optimistic agent's bias b_a is large, however, the principal is willing to offer a contract tailored to the agent's signals and beliefs (in order to take advantage of them). The contract is tied mostly to the agent's opinion, that is, the principal is "giving in" to the agent at the contracting stage. This ensures that the agent is happy to sign a contract with a less threatening conflict, since his opinion and report is going to drive the economic interaction almost fully. Furthermore, notice that while the BPE contract sets the (TR_P^a) binding it leaves the (TR_P^u) slack. On the contrary, the APE contract features both (TR_P) binding. In other words, the smaller threat of the APE decreases the principal's truth-telling incentives to a point where both constraints become binding. If the deadweight loss following reports $(t, s) = (u, a)$ were to decrease even further, the principal would no longer be reporting his SPE truthfully.

In order to fully describe when the APE contract is chosen by the principal we present a graphical representation of the feasible (b_a, b_u) space for an optimistic agent. Figure 2 below identifies the type of contract an optimistic agent is offered for any value of his bias within the feasible space. The dotted line represents the separation between positive (to the right) and negative (to the left) perceived correlation. In the appendix we formally prove the shape of the area where the APE contract is set up.

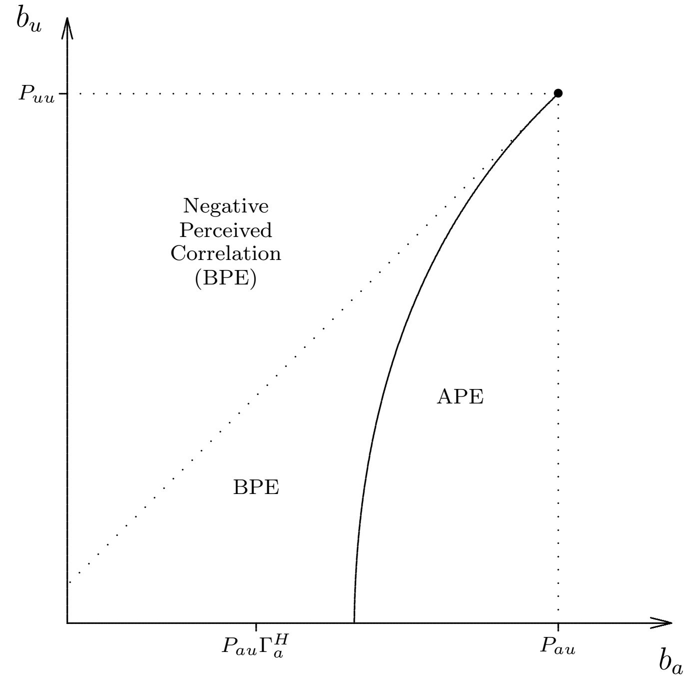

FIGURE 2. The area delimited by the solid curve on the right in the figure identifies the portion of the parameter space where a contract of the type described in Proposition 4 is set optimally. That is, when the presence of optimism generates a new contract compared to the case of an unbiased agent. The dotted line crossing the quadrant represents the condition for positive perceived correlation, (1). This specific graph was obtained for $\left(P_{aa}, P_{uu}, \Gamma_a^H\right) = (0.5, 0.6, 0.6)$. Its shape, however, generalises to all feasible parameter values.

First of all, notice that when $b_u = 0$ and $b_a = P_{au}$ the agent believes that there is no chance that he will receive a signal of an unacceptable performance conditional on the principal deeming it acceptable, i.e., he believes that conditional on $T = a$, $S = u$ is not possible. Upon observing $S = u$, he believes *fully* his signal's realization and thinks that the principal has observed exactly the same. Intuitively, this is the case where the principal can exploit most the agent's optimism. The agent, in fact, has no "suspicion" that the principal may have observed a, while in reality this may very well be the case. Hence, the agent's compensation for a performance deemed acceptable by the principal and unacceptable by the agent, becomes cheaper with an APE contract compared to a BPE contract ($c^{\dagger}_{au} < c^{*}_{au}$).

4.2. **Pessimistic Agent.** Consider now the case of a pessimistic agent who believes signals are positively correlated. That is, he holds beliefs that satisfy:

$$b_a < 0,\, b_u < 0, \text{ and } b_u - b_a \leq P_{aa} - P_{ua}.$$

Note that Lemmas 3 and 4 and Proposition 1 also hold for a pessimistic agent who believes signals are positively correlated. The next Proposition states that, when the agent believes signals are positively correlated, pessimism does not lead either to an APE contract being assigned, or to a new type of contract.

Proposition 5. *A pessimistic agent who believes signals are positively correlated is always offered the BPE contract.*

A pessimistic agent who believes signals are positively correlated does not assign enough probability to a positive (i.e. acceptable) realization of S. Hence, it is impossible for the principal to manipulate the contract in a profitable way. One could think that the principal could manipulate the standard contract increasing the agent's compensations following a realization $S = u$. However, for as much as the agent can be pessimistic, he still believes, rightfully so, that high effort leads to $S = a$ more often. This leaves no room for optimal manipulation by the principal.

The result of Proposition 5 helps us connect to the literature on optimism and pessimism. It has been observed how optimistic and pessimistic biases have asymmetric effects on contracting problems (Santos-Pinto, 2008, 2010; De la Rosa, 2011; Gervais, Heaton, and Odean, 2011). Our findings are in line with this observation. Optimism opens the way for the principal's exploitation of the agent's bias but pessimism does not. Pessimism leads the agent to assign a lower probability to state $(t, s) = (a, a)$. This latter is exactly the state the principal would like to speculate upon (as in the APE contract). Hence, pessimism hinders the ability of the principal to exploit the agent's bias by manipulating the baseline contract.

5. Overconfidence, Underconfidence and Optimal Contracting

In this section we shift our attention towards overconfidence, another well documented psychological bias. Most people, even experts, overestimate the precision of their estimates and forecasts (Oskamp, 1965; Fischhoff, Slovic, and Lichtenstein, 1977; Lichtenstein, Fischhoff, and Phillips, 1982; Wallsten, Budescu, and Zwick, 1993; Barberis and Thaler, 2003). Overconfidence is important in personal and business decisions (e.g. Russo and Schoemaker, 1992; Grubb, 2009). Overconfidence also matters for investment and financial decisions. Daniel, Hirshleifer, and Subrahmanyam (1998, 2001) show that overconfidence can lead to excess volatility and to predictability of stock returns. Scheinkman and Xiong (2003) show that it can lead to financial bubbles. While the majority of individuals are overconfident, a minority is underconfident.

Based on this discussion, we define an overconfident and an underconfident agent as follows:

Definition 3. *The agent is "*overconfident*" if he believes his PE signal is* more *informative about the outcome of the project than it actually is, that is* $\tilde{\Pr}\{S = a|G\} > \Pr\{S = a|G\}$ *and* $\tilde{\Pr}\{S = u|B\} > \Pr\{S = u|B\}$*. Equivalently, an overconfident agent's beliefs satisfy:*

$$-\frac{\gamma_a^G}{\gamma_u^G} b_a < b_u < -\frac{\gamma_a^B}{\gamma_u^B} b_a \quad \textit{and} \quad b_a > 0 \tag{7}$$

Definition 4. *The agent is "*underconfident*" if he believes his PE signal is* less *informative about the outcome of the project than it actually is, that is* $\tilde{\Pr}\{S = a|G\} < \Pr\{S = a|G\}$ *and* $\tilde{\Pr}\{S = u|B\} < \Pr\{S = u|B\}$*. Equivalently, an underconfident agent's beliefs satisfy:*

$$-\frac{\gamma_a^G}{\gamma_u^G} b_a < b_u < -\frac{\gamma_a^B}{\gamma_u^B} b_a \quad \textit{and} \quad b_a < 0 \tag{8}$$

It follows from (7) that an overconfident agent always believes signals are positively correlated. However, an underconfident agent might perceive signals to be either positively or negatively correlated. Recall that we rule out the case of negatively perceived correlation for now.

By studying the proofs of the Propositions proven so far, it is possible to see that none of the conditions derived change in the presence of either an overconfident agent or in the presence of an underconfident agent who believes signals are positively correlated. This originates the following result.

Proposition 6. *If an overconfident agent's beliefs satisfy* (6)*, then the optimal contract implementing high effort is the APE:* $\{w^{\dagger}_{ts}, c^{\dagger}_{ts}\}_{t,s=a,u}$. *If an overconfident agent's beliefs violate* (6)*, then he is assigned a BPE contract. An underconfident agent who believes signals are positively correlated is always assigned a BPE contract.*

In Figure 3, we present the parameter space for an overconfident agent. The shaded areas rule out biases that fall outside our definition of overconfidence (i.e. they violate (7)). However, our analysis holds for these bias values as well. We omit drawing a figure for an underconfident agent who believes signals are positively correlated since he is always assigned a BPE contract.

Just as in the case of optimism and pessimism, over and underconfidence have different implications for optimal contracting (as observed by Santos-Pinto, 2008, 2010; De la Rosa, 2011; Gervais, Heaton, and Odean, 2011). The direction of the agent's bias affects the incentive and ability of the principal to manipulate the contract. When the principal faces a pessimistic agent who believes signals are positively correlated, it is the agent's belief that $s = a$ is too rare that limits the principal's ability to exploit the agent's pessimistic bias. When the principal faces an underconfident agent who believes signals are positively correlated, this role is taken by the agent's belief that his signal is not informative enough with respect to the effort exerted and ultimately to the project's outcome. Since compensation in the APE contract is tied to the agent's PE report, an underconfident agent who believes signals are positively correlated would rather be assigned a BPE contract where he feels more certain that his effort will make a difference, since in a BPE contract his compensation is tied only to the principal's PE report.

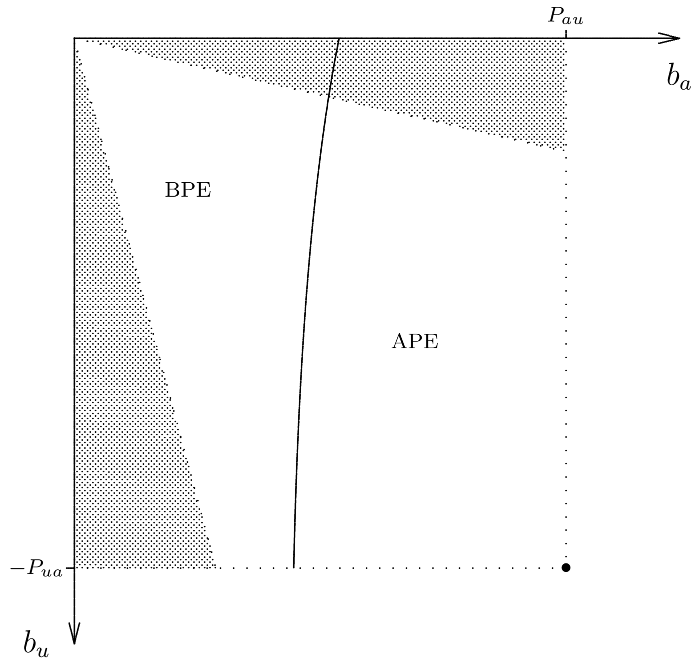

FIGURE 3. The area delimited by the solid curve on the right in the figure identifies the portion of the parameter space where an APE contract is set optimally for an overconfident agent. The shaded areas rule out biases that fall outside our definition of overconfidence. This specific graph was obtained for $\left(P_{aa}, P_{uu}, \Gamma_a^H, \gamma_a^G, \gamma_a^B\right) = (0.5, 0.4, 0.6, 0.8, 0.1)$. Its shape, however generalises to all feasible parameter values.

To conclude this section, we present a Corollary to Propositions 2 and 4 that confirms a result the discussion so far has only implied: the maximum compensation in an APE contract, $c^{\dagger}_{aa}$, is greater than the maximum compensation in a BPE contract, $c^*_{aa} = c^*_{au}$.

Corollary 1. *The maximum compensation available to either an optimistic or overconfident agent, $c^{\dagger}_{aa}$, is greater than the maximum ones available to an unbiased, pessimistic or underconfident agent, $c^*_{aa} = c^*_{au}$.*

While this result is mostly a tool to prove the welfare results in the following section, it suggests, together with Propositions 2 and 4, a potential way to test some of the implications of the model. In an SPE contract designed for an optimistic or overconfident agent, the principal rewards the agent maximally only in the "best" SPE case scenario, i.e. when both her and the agent deem the agent's work acceptable. In contrast, in an SPE contract designed for an unbiased, pessimistic or underconfident agent, the principal rewards the agent maximally simply in situations where she deems the agent's work acceptable.

The probability of $(t, s) = (a, a)$ taking place, as discussed above, is overestimated by optimistic as well as overconfident agents. This creates an opportunity for the principal to take advantage of the agent's bias.[21] As we will show next, this is not necessarily a negative feature of an APE contract as it decreases the conflict at the contracting stage. That is, the agent, because of his bias, is willing to sign a contract featuring a lower deadweight loss, one that limits his potential for retaliation, for example. When his bias goes too far, however, he signs a contract where not only the deadweight loss is decreased, but his expected compensation is also lower compared to the BPE contract.

6. Welfare and the Social Value of Biased Agents

In this section we present a welfare analysis and prove formally that some types of the agent's optimism and overconfidence may lead to a Pareto improvement. That is, compared to the BPE contract, they lead to an APE contract that can make both the principal and the agent better off. In addition, we discuss the impact of the agent's optimism and overconfidence on the principal's decision to optimally implement high or low effort.

To study the impact of the agent's bias on the agent's welfare we take the ex ante point of view of an outside observer, who knows the actual distribution of the agent's PE signal.[22]

Proposition 7. *Let $\tilde{E}(\cdot)$ denote the biased expectations of the agent. Given the BPE contract $\{w^*_{ts}, c^*_{ts}\}$ and the APE contract $\{w^\dagger_{ts}, c^\dagger_{ts}\}$, the following are true:*

(i) $E(w^*_{ts}) > E(w^\dagger_{ts})$ *whenever the APE is the contract of equilibrium.*

(ii) $E(c^*_{ts}) = \tilde{E}(c^*_{ts}) = \frac{\Delta V}{\Delta \Gamma_a}\Gamma^H_a$.

(iii) $\tilde{E}(c^\dagger_{ts}) > \tilde{E}(c^*_{ts})$ *(unless $b_a = P_{au}$ when it they are equal).*

(iv) $E(c^\dagger_{ts}) > E(c^*_{ts})$ *whenever*

$$b_a \le P_{au}\frac{P_{uu} - b_u\Gamma^H_a}{P_{uu}(1+\Gamma^H_u) - b_u}. \tag{9}$$

Proposition 7 provides a set of intuitive conclusions. First of all, point (i) obviously states that for the principal to be willing to switch to an APE contract from a BPE

[21] Hence the APE may be connected to the idea of "exploitative" contracts in the literature on agents with biased beliefs (Eliaz and Spiegler, 2006, 2008; Foschi, 2017).

[22] The agent's welfare could also be assessed as he perceives it ex ante, that is, using the agent's perception of his PE signal. However, the agent will not experience this welfare on average ex post.

one, it has to be optimal for her to do so. That is, she must be paying a lower expected wage. Point (ii) follows from the fact that in the BPE contract the agent's compensation does not depend on the agent's bias. Point (iii) shows that the agent would always be happy to be assigned the APE contract instead of a BPE contract. This is because the optimistic (or overconfident) agent overestimates the chances of obtaining $c^{\dagger}_{aa}$ which, as we show above, is the largest possible compensation available among the ones in the BPE and APE contracts. Finally, point (iv) is by far the most interesting and important. It shows that, even though the principal would like to take advantage of the agent's biased beliefs, under some conditions, the APE contract is not exploitative after all. If the agent's optimistic (or overconfident) bias satisfies (6) and (9), in fact, the contract not only allows the principal to pay a lower expected wage, but it also features, from the perspective of an outside observer, a larger expected compensation for the agent. This sets the stage for the main result of this section.

Proposition 8. *If the agent is either optimistic or overconfident and his bias satisfies* (6) *and* (9)*, the principal offers a contract that costs her a lower expected wage and, from the perspective of an outside observer, grants the agent greater expected compensation. In this region, the agent's bias is "*socially desirable*". When* (9) *fails, the agent is "*exploited*".*

Note that when the principal implements high effort with an APE contract, her expected benefit, $\lambda^H Y$, does not depend on the bias of the agent whereas her expected wage payment does. In addition, the agent's cost of exerting high effort, $V(\lambda^H)$, does not depend on his own bias but the agent's expected compensation does. Hence, if the principal implements high effort with an APE contract, then a Pareto improvement takes place when the agent's bias lowers the principal's expected wage payment and, from the perspective of an outside observer, raises the agent's expected compensation.

Proposition 8 shows that if the agent's optimistic (or overconfident) bias satisfies (6) and (9), then a Pareto improvement takes place. In contrast, when the agent's bias b_a is very large the agent is *exploited*. By exploited we mean an agent who signs a contract he would not have signed were he holding unbiased beliefs. In the proof of the Proposition we provide a formal argument to show that the region where optimism and overconfidence are socially desirable corresponds (in shape) to the one in Figure 4 below and that it always exists.[23]

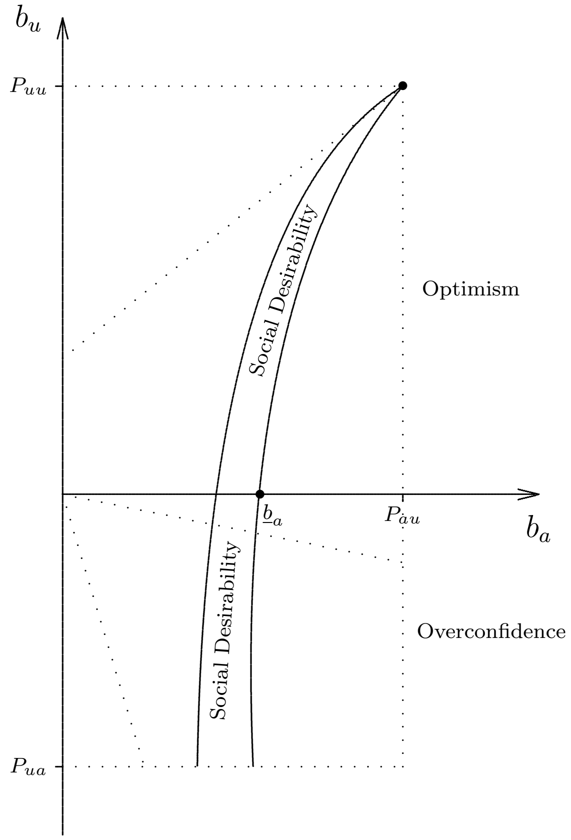

FIGURE 4. The area inside the two curves features a contract with a higher expected compensation and a lower expected wage compared to the benchmark contract assigned to an unbiased agent. Inside this area, the presence of an optimistic or overconfident agent is socially optimal. In the proof of Proposition 8 we provide a formal argument to show that this area always exists and it is shaped as is displayed here. The value of $\underline{b}_a$ is also derived in the proof. Above the dotted line in the top quadrant the agent perceives signals as negatively correlated. The dotted lines in the bottom quadrant restrict the parameter space according to our definition of overconfidence. This specific graph was obtained for $\left(P_{aa}, P_{uu}, \Gamma_a^H, \gamma_a^G, \gamma_u^B\right) = (0.7, 0.5, 0.6, 0.8, 0.1)$. Its shape, however generalises to all feasible parameter values.

Let us now discuss the effect of the agent's bias on the principal's decision to optimally implement high or low effort.

Proposition 9. *If the agent believes signals are positively correlated, then the principal's incentives to implement high effort, as opposed to low effort, are (at least weakly)*

[23]The magnitude of the area, however, is purely indicative.

higher when the agent is either optimistic or overconfident than when the agent is unbiased.

The proof of Proposition 9 shows that the principal always chooses to optimally implement low effort using a fixed wage and compensation contract, i.e., a contract $\{w^{\ell}_{ts}, c^{\ell}_{ts}\}_{t,s}$ where $w^{\ell}_{ts} = c^{\ell}_{ts} = \bar{u} + V(\lambda^L)$ for all t and s. The reason why the fixed wage and compensation contract is always optimal to implement low effort lies in the cost of truthful reporting. Under a fixed wage and compensation contract, truthful reporting is trivially true. In Lemma 13, we prove that contract $\{w^{\ell}_{ts}, c^{\ell}_{ts}\}_{t,s}$ is optimal among all contracts without deadweight loss, regardless of whether they feature a fixed or non fixed wage and/or compensation. The intuition is similar to the one behind Proposition 1. For truthful reporting to happen, none of the two available reports must lead to payoff-dominating outcomes. In the fixed wage and compensation contract (which violates the (IC) when used to implement high effort) this is trivially true since the payoff is constant. In a non fixed wage and compensation contract, this conflict of interest leads to the need for a deadweight loss, or else at least one party would have a dominant reporting strategy.

This result implies that an increase in b_a at least weakly decreases the expected cost of implementing high effort without affecting the cost of implementing low effort. This is because such a change in bias can lead to high effort being implemented via an APE contract, the cost of which is decreasing as the bias b_a of the agent increases, while it does not affect the way and cost of implementing low effort.

Finally, notice that since the expected return of the project, Y, does not affect the cost of implementation, there always exists a Y large enough for λ^H to be optimal for the principal. This would carry over to any maximal value of effort were we to let the agent pick among more than just two.[24]

7. Large Outside Option Values

In this section we generalize the model by assuming that the agent faces a large outside option value, that is, we relax Assumption 4. We solve the problem for the

[24]When Assumption 4 fails, numerical solutions show how the principal may find it optimal to implement low effort with a BPE-like contract. The latter is also independent of the bias, and therefore our discussion is unaltered.

case of an optimistic agent who believes signals are positively correlated. Our results extend to the case of an overconfident agent.[25]

We start our generalization analysis stating the promised Corollary to the Propositions of Section 4, showing that the (PC) is satisfied by all the contracts derived when $\bar{u} \leq \bar{u}_1$. The corollary also defines the limit value of $\bar{u}$ for the APE contract to satisfy the (PC), $\bar{u}_2$.[26]

Corollary 2. *When $\bar{u} \leq \bar{u}_1$, the BPE and APE contracts satisfy the (PC) constraint, which is therefore slack.*
When $\bar{u} \in (\bar{u}_1, \bar{u}_2]$, the APE satisfies the (PC) constraint while the BPE violates it. $\bar{u}_2$ is given by

$$\bar{u}_2 \equiv \frac{\Delta V}{\Delta \Gamma_a} \Gamma_a^H \frac{\tilde{P}_{au}\Gamma_a^H + \tilde{P}_{uu}\Gamma_u^H}{\tilde{P}_{au}\Gamma_a^H + \tilde{P}_{uu}(\tilde{P}_{aa} - \Gamma_a^H)} - V(\lambda^H).$$

We start by assuming that $\bar{u} \in (\bar{u}_1, \bar{u}_2]$ and then move to $\bar{u} > \bar{u}_2$. Notice that this second threshold is *not* independent of the bias, and it is therefore, agent-specific.

Our first result generalizes the BPE contract.

Proposition 10. *When $\bar{u} \in (\bar{u}_1, \bar{u}_2]$, the principal implements high effort either with the APE contract or a Generalized BPE (GBPE) contract $\{\tilde{w}^*_{ts}, \tilde{c}^*_{ts}\}_{t,s=a,u}$ given by:*

$$\begin{array}{llll} \tilde{w}^*_{aa} = \tilde{c}^*_{aa} & \tilde{w}^*_{au} = \tilde{c}^*_{aa} & \tilde{w}^*_{uu} = \tilde{c}^*_{uu} & \tilde{w}^*_{ua} = \tilde{c}^*_{aa} + \frac{\Delta V P_{au}}{\Delta \Gamma_a P_{aa}} \\ \tilde{c}^*_{aa} = \frac{\Delta V}{\Delta \Gamma_a} + \tilde{c}^*_{uu} & \tilde{c}^*_{au} = \tilde{c}^*_{aa} & \tilde{c}^*_{uu} = \bar{u} - \bar{u}_1 & \tilde{c}^*_{ua} = \tilde{c}^*_{uu}. \end{array}$$

The GBPE contract has the same properties of the BPE contract and converges to the latter as $\bar{u} \to \bar{u}_1$.

Notice that the GBPE is simply an "upward" modification of the BPE contract derived in Proposition 2. It ensures that the agent's perceived expected utility from the contract is exactly $\bar{u}$. It does so by granting the agent a minimum positive compensation (at $t = s = u$ and $t = u, s = a$) as opposed to zero in the BPE.

[25]A similar generalization is possible also for a pessimistic or underconfident agent or for one who believes signals to be negatively correlated. Numerical solutions show how optimal contracts for these cases are in line with those derived in Propositions 13 and 14 in the Appendix.
[26]In the proof of the corollary, we show how it also holds for the contracts derived for the case of perceived negative correlation.

When the principal finds the BPE contract optimal and the GBPE contract is available, it must be that the expected wage of the GBPE contract is larger. On the other hand, its competitor for optimality, the APE, hasn't changed in structure, and therefore in expected wage cost. This allows us to state the following Corollary.

Corollary 3. *When $\bar{u} \in (\bar{u}_1, \bar{u}_2]$, the APE is optimal for at least all parameter values under which it is optimal when $\bar{u} < \bar{u}_1$.*

Hence for intermediate outside option values, the APE contract becomes relatively more attractive than the GBPE contract. This is due to the fact that the APE contract relaxes the participation constraint, which holds under the GBPE one.

Proposition 8 showed that there are situations where the APE contract does in fact, from the perspective of an outside observer, raise the agent's expected compensation leading to a Pareto improvement compared to the BPE contract. Given Corollary 3, we know not only that Proposition 8 still holds in the case of the APE vs. the GBPE contract, but that it may even hold for a larger portion of the parameter space. Hence, an outside option utility such that $\bar{u} \in (\bar{u}_1, \bar{u}_2]$ reinforces (at least weakly) the welfare result on the Pareto dominance of the APE.

We now generalize the results above even further by letting the outside option be larger than $\bar{u}_2$. First of all, notice that the GBPE is feasible when $\bar{u} > \bar{u}_2$, while the APE is no longer feasible, since it violates the (PC).

Proposition 11. *Let $\bar{u} > \bar{u}_2$.*
1) If the agent's bias violates condition (37) *and satisfies condition* (39) *in Appendix A, then the principal implements high effort with a Generalized APE contract, $GAPE_1$, given by:*

$$
\begin{array}{llll}
\tilde{w}^{\dagger}_{aa} = \tilde{c}^{\dagger}_{aa} & \tilde{w}^{\dagger}_{au} = \tilde{c}^{\dagger}_{au} & \tilde{w}^{\dagger}_{uu} = \tilde{c}^{\dagger}_{au} & \tilde{w}^{\dagger}_{ua} = \tilde{c}^{\dagger}_{aa} \\
\tilde{c}^{\dagger}_{aa} = \tilde{c}^{\dagger}_{ua} + \frac{\Delta V}{\Delta \Gamma_a} \frac{\tilde{\gamma}^H_{au} + \tilde{\gamma}^H_{uu}}{\tilde{\gamma}^H_{au} + \tilde{P}_{uu}(\tilde{P}_{aa} - \Gamma^H_a)} & \tilde{c}^{\dagger}_{au} = \tilde{c}^{\dagger}_{ua} + \frac{\Delta V}{\Delta \Gamma_a} \frac{\tilde{\gamma}^H_{au}}{\tilde{\gamma}^H_{au} + \tilde{P}_{uu}(\tilde{P}_{aa} - \Gamma^H_a)} & \tilde{c}^{\dagger}_{uu} = \tilde{c}^{\dagger}_{au} & \tilde{c}^{\dagger}_{ua} = \bar{u} - \bar{u}_2.
\end{array}
$$

where the PC and (IC) bind together. Contract $GAPE_1$ has the same properties as an APE and converges to the latter as $\bar{u} \to \bar{u}_2$.

2) If the agent's bias satisfies conditions (37) *and* (38) *in Appendix A, then the principal implements high effort with a GAPE*$_2$ *contract given by:*

$$\begin{array}{llll} \tilde{w}^{\dagger'}_{aa} = \tilde{c}^{\dagger'}_{aa} & \tilde{w}^{\dagger'}_{au} = \tilde{c}^{\dagger'}_{au} & \tilde{w}^{\dagger'}_{uu} = \tilde{c}^{\dagger'}_{au} & \tilde{w}^{\dagger'}_{ua} = \tilde{c}^{\dagger'}_{aa} \\ \tilde{c}^{\dagger'}_{aa} = c^{\dagger'}_{au}\left(1 + \frac{\tilde{\gamma}^H_{uu}}{\tilde{\gamma}^{\dagger'}_{au}}\right) & \tilde{c}^{\dagger'}_{au} = \frac{\left(\bar{u}+V(\lambda^H)\right)\tilde{P}_{au}}{\left(\tilde{\gamma}^H_{au}+\tilde{\gamma}^H_{uu}\right)} & \tilde{c}^{\dagger'}_{uu} = \tilde{c}^{\dagger'}_{au} & \tilde{c}^{\dagger'}_{ua} = 0. \end{array}$$

where the PC binds but the (IC) *is slack.*

The GAPE contracts described in Proposition 11 have the same features of the APE contract derived for the case of $\bar{u} < \bar{u}_2$. The agent's compensation depends on both parties' PE reports, while the principal's wage cost depends only on the agent's PE report. Hence, our result on the use of information in SPE contracts with biased agents extends to the whole $\bar{u}$ space.

Our finding on potential Pareto dominance of the contract assigned to a biased worker also extends to the case of $\bar{u} > \bar{u}_2$. In Appendix B we derive conditions (40) and (42) for the GAPE$_1$ and GAPE$_2$, respectively, to Pareto Improve over the GBPE. We also provide examples of parameter values under which this happens.

Finally, the analysis shows how an unbiased agent obtains zero limited-liability rent (in the Laffont and Martimort, 2009, sense) when $\bar{u} \geq \bar{u}_1$, while an optimistic or overconfident agent obtains limited-liability (at least perceived) rent also for $\bar{u} \in [\bar{u}_1, \bar{u}_2)$.

8. Extensions

This section discusses the main assumptions behind our results and potential extensions.

Perceived Negative Correlation. When the agent believes signals are negatively correlated he expects his PE realization to be the opposite from that of the principal. But since the latter is informative towards the outcome of the project, it means that the agent assigns informative value to his own SPE as well. He just assigns the "wrong" one. Furthermore, the agent may think that an unacceptable realization of his own SPE, S, is a signal of a good outcome ($\tilde{\Pr}\{S = u|G\} > \tilde{\Pr}\{S = u|B\}$), while an acceptable one is a signal of a bad outcome ($\tilde{\Pr}\{S = a|B\} > \tilde{\Pr}\{S = a|G\}$). Hence, when the agent believes signals are negatively correlated, his beliefs are subject to odd interpretations.

We are not aware of substantial empirical evidence that can sustain these sort of biased beliefs. Therefore, we relegate the analysis of this case to the appendix and present a short summary here. First of all, notice that only a pessimistic, an optimistic or an underconfident agent can believe signals are negatively correlated. This is because (1) cannot fail at $b_a > 0$ and $b_u < 0$. In Appendix C we derive the following results.

(1) Proposition 1 no longer holds. The presence of a deadweight loss in the contract is no longer a necessary condition to implement high effort.
(2) An optimistic agent who believes signals are negatively correlated is always assigned a BPE contract.
(3) A pessimistic or underconfident agent who believes signals are strongly negatively correlated is assigned one of two new types of contract. These are called: the *Disagreement Performance Evaluation Deadweight Loss* (DPE-DL) contract and the *Disagreement Performance Evaluation No Deadweight Loss* (DPE-NDL) contract. These contracts feature no compensation in states where both parties observe the same PE, states that the agent deems very rare. Hence, the principal can exploit the agent's biased beliefs by compensating the agent only in states where they observe different PEs.
(4) Contract DPE-NDL features no deadweight loss and it is available only under certain parameter configurations.

These results point towards an interesting insight. While we do confirm the findings of previous contributions on how optimism and pessimism, as well as over and underconfidence, have different implications for optimal contracting, we show that optimal SPE contracts are, to some extent, symmetric when it comes to the direction and intensity of perceived correlation. Regardless of the direction, when perceived correlation is particularly strong the principal has the ability to manipulate the contract and decrease the cost of incentivising the agent to exert the amount of effort required.

As derived in Appendix C, our welfare analysis extend to the case of negative perceived correlation. There exists a parameter space where the DEP-DL contract is a Pareto Improvement over the BPE contract. Interestingly, however, the DEP-NDL contract, the one featuring no deadweight loss, never Pareto Improves over the BPE contract and is always exploitative. This is because for the deadweight loss to be completely erased, it must be that the agent is so biased that he will be exploited.

Needless to say, the DEP-NDL contract features the lowest (i.e. zero) deadweight loss, while the DEP-DL contract can be shown to always feature a lower deadweight loss than a BPE contract.

Risk Aversion. Assuming the agent is risk averse and protected by limited liability brings our model closer to MacLeod (2003).[27] We chose the approach of risk-neutrality and limited liability for two main reasons. First, we believe that this is a very natural benchmark of principal-agent models, whereas a framework with risk aversion and limited liability is not. Second, our approach improves tractability, yielding a linear programming problem (instead of a non-linear one for the case of risk aversion) and allows us to obtain closed form solutions. This, in turn, allows us to state sharp welfare results.[28]

Nevertheless, our results generalize to the case where the agent is risk-averse and protected by limited liability. In fact, a risk averse optimistic agent would value even more the attractiveness of and APE-like contract compared to a risk neutral optimistic agent. This is because the APE contract is set up either when the agent perceives signals to be more correlated than they actually are, or (for a small portion of the parameter space as in Figure 2) when the agent is almost certain that $s = a$, regardless of the principal's opinion. In the first case, he becomes almost certain of knowing the principal's SPE given his own SPE's. In the second case, he becomes almost certain of the realization $s = a$. Hence, in both situations, the bias is substantially decreasing the perceived risk of an APE contract and, as a result, increasing its expected utility for a risk averse optimistic agent.

[27]We assume the agent's utility is linear, the agent is protected by limited liability, and that effort belongs to a finite set. In contrast, MacLeod (2003) assumes the agent's utility is concave, it satisfies an Inada condition, the agent is protected by limited liability, and that effort belongs to a compact set. Note that the Inada condition makes the limited liability assumption redundant in MacLeod's analysis.

[28]Further, as discussed in Section 4, this assumption still allows us to consider results like the informativeness principle of Holmström (1979) as a benchmark, thanks to Chaigneau, Edmans, and Gottlieb (2017).

Continuum of Effort Levels. Suppose we were to let the agent choose an effort level $\lambda \in [0,1]$. Assume that $V(\lambda)$ is strictly increasing and convex in λs. When the principal wants to implement λ^* the new (IC) constraint becomes

$$\lambda^* = arg\max_{\lambda} \sum_{ts} \tilde{\gamma}_{ts}(\lambda) c_{ts} - V(\lambda^*)$$

which is equivalent to

$$\sum_{ts} \frac{\partial \tilde{\gamma}_{ts}(\lambda^*)}{\partial \lambda^*} c_{ts} = \frac{\partial V(\lambda^*)}{\partial \lambda^*}. \qquad (IC^*)$$

To see that this would not affect our results, notice that it is enough to show that the (IC^*) still satisfies the proof of Proposition 1. Suppose $w_{ts} = c_{ts}$ for all t and s, then by Lemmas 3 and 4, we have that $c_{ts} = c$ for all t and s. This implies

$$\frac{\partial V(\lambda^*)}{\partial \lambda^*} = c \sum_{ts} \frac{\partial \tilde{\gamma}_{ts}(\lambda^*)}{\partial \lambda^*} = 0,$$

which contradicts the assumption of V being increasing and convex in λ. To see why the above is true, recall that $\sum_{ts} \tilde{\gamma}_{ts}(\lambda) = 1$ for all λs. Hence, it must be that

$$\sum_{ts} \frac{\partial \tilde{\gamma}_{ts}(\lambda^*)}{\partial \lambda^*} = 0.$$

Given that Proposition 1 holds, the rest of the solution follows in a similar fashion. The use of (IC^*), however, requires the derivatives of the $\gamma_{ts}(\lambda)$ to be carried over in the algebra, complicating the calculations and derivations in an uninteresting way.

Beyond the Binary Signal Distributions. The model can be extended by allowing for signals T and S to have more than two realizations. However, this complicates the problem substantially. When PE signals have two realizations, $t \in \{a, u\}$ and $s \in \{a, u\}$, we have eight endogenous variables and four truth-telling constraints. In contrast, if PE signals have three realizations, $t \in \{a, m, u\}$ and $s \in \{a, m, u\}$, the problem has eighteen endogenous variables and twelve truth-telling constraints. Nevertheless, numerical solutions show how our main results generalize to more than two PE signal realizations. Consider the case where PE signals have three realizations, signal T is positively correlated with the outcome of the project, i.e., $\gamma_a^G/\gamma_a^B > \gamma_m^G/\gamma_m^B > \gamma_u^G/\gamma_u^B > 0$, and signals T and S are positively correlated. In this case an unbiased agent is still offered a BPE contract with a compensation that depends only on the principal's PE report, with a wage that depends on both parties' PE reports, and with money burning when $(t, s) = (u, a)$. The only novelty is that in some cases this BPE contract can feature money burning not only when $(t, s) = (u, a)$ but also when $(t, s) = (m, a)$. Hence, with more than two signal realizations the principal can spread money burning across more cases of weak PE besides the worst PE. If the agent is either optimistic or overconfident he is offered the BPE contract when his bias is low and a new type of contract when his bias is high. In the new type of contract, just like in the APE, the principal raises (lowers) compensation in states the agent deems more (less) probable than the principal. Also, like in the APE, the agent's compensation depends on both parties' PE reports. However, unlike in the APE, the wage depends on both parties' PE reports. Finally, this new type of contract can feature money burning not only when $(t, s) = (u, a)$ but also when $(t, s) = (m, a)$. The welfare results obtained with two PE signal realizations extend to the case of three PE signal realizations.

Sequential Reporting. One could argue that in some situations SPE reports do not actually happen simultaneously. In a recent paper, MacLeod and Tan (2017) solve for optimal contracts with SPE under the assumption that either the principal reports her SPE first (authority contracts) or the agent does (sales contract). They show that, under some conditions, the informational value of the SPE reported second is completely lost and deadweight loss is therefore not a necessary condition for truthful reporting any longer. It is therefore conceivable to believe that a Pareto improvement would be impossible under sequential reporting.

We consider that sequential reporting is an alternative critical assumption rather than an extension and that it therefore lies outside the scope of this paper. However, sequential reporting could also lead to interesting results in the presence of a biased agent and it is worth investigating in future research. In the sales contract setting, for example, a biased agent may find himself reporting a certain SPE with a certain expectation of what the principal will report after him which is far from the true statistic (since bias affects perceived correlation as well). We conjecture that this mechanism could lead to a similar manipulation of the contract presented in this paper by the principal. Optimism may, in fact, relax (or at least affect) the binding truthful reporting constraints of the agent in the same way it does in our setting, since the worker hasn't yet faced the principal's report. Hence, it is conceivable to think that a "manipulated" sales contract would give even more importance to the agent's report compared to a standard contract offered to an unbiased agent.

9. Concluding Remarks

This paper focuses on understanding the impact of workers' behavioral biases on subjective performance evaluation contracts. We have shown that the benchmark SPE contract assigned to an unbiased worker is also assigned to an optimistic or overconfident worker as long as the worker's bias is small. In contrast, a new SPE contract arises in equilibrium when an optimistic or an overconfident worker's bias is large. The new SPE contract shares a main driving force: the principal tries to take advantage of the bias of the agent by altering the compensation levels compared to the benchmark case. The new SPE contract always decreases the amount of conflict, i.e. deadweight loss, compared to the benchmark SPE contract. The new SPE contract always features a lower expected cost of implementing high effort but not necessarily, from the perspective of an outside observer, a lower or higher expected compensation compared to the benchmark contract (although the agent always believes he will receive a higher expected compensation). Finally, we provide conditions under which the presence of a bias in the beliefs of the agent generates a Pareto improvement.

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Appendix A. Proofs

Proof of Lemma 1

To prove (i), notice that:

$$\gamma_{ts}^{j} = \lambda^{j}\gamma_{ts}^{G} + (1-\lambda^{j})\gamma_{ts}^{B}$$
$$= \lambda^{j}P_{ts}\gamma_{t}^{G} + (1-\lambda^{j})P_{ts}\gamma_{t}^{B}$$
$$= P_{ts}\left[\lambda^{j}\gamma_{t}^{G} + (1-\lambda^{j})\gamma_{t}^{B}\right] = P_{ts}\Gamma_{t}^{j}.$$

To prove (ii), start from Assumption 2 and use (i) to obtain:

$$P_{aa}P_{uu} - P_{au}P_{ua} > 0$$
$$P_{aa}P_{uu}\Gamma_{a}^{j}\Gamma_{u}^{j} - P_{au}P_{ua}\Gamma_{a}^{j}\Gamma_{u}^{j} > 0$$
$$\gamma_{aa}^{j}\gamma_{uu}^{j} - \gamma_{au}^{j}\gamma_{ua}^{j} > 0 \tag{10}$$

by positivity of $\Gamma_{a}^{j}\Gamma_{u}^{j}$. To prove (iii), notice that $P_{aa} = 1 - P_{au}$ and $P_{ua} = 1 - P_{uu}$. Substitute for the latter in Assumption 2 to obtain:

$$(1-P_{au})P_{uu} - P_{au}(1-P_{uu}) > 0$$
$$P_{uu} - P_{au} > 0$$

Similarly, substitute for $P_{au} = 1 - P_{aa}$ and $P_{uu} = 1 - P_{ua}$ to obtain that $P_{aa} - P_{ua} > 0$.

Finally, to prove (iv), note that

$$\begin{aligned}
\Delta\Gamma_{t} &= \Gamma_{t}^{H} - \Gamma_{t}^{L} \\
&= \lambda^{H}\gamma_{t}^{G} + (1-\lambda^{H})\gamma_{t}^{B} - \left[\lambda^{L}\gamma_{t}^{G} + (1-\lambda^{L})\gamma_{t}^{B}\right] \\
&= \left(\lambda^{H} - \lambda^{L}\right)\left(\gamma_{t}^{G} - \gamma_{t}^{B}\right).
\end{aligned}$$

Therefore

$$\begin{aligned}
\Delta\Gamma_{a} + \Delta\Gamma_{u} &= \left(\lambda^{H} - \lambda^{L}\right)\left(\gamma_{a}^{G} - \gamma_{a}^{B}\right) - \left(\lambda^{H} - \lambda^{L}\right)\left(\gamma_{u}^{G} - \gamma_{u}^{B}\right) \\
&= \left(\lambda^{H} - \lambda^{L}\right)\left[(\gamma_{a}^{G} - \gamma_{u}^{G}) - (\gamma_{u}^{B} - \gamma_{a}^{B})\right] \\
&= \left(\lambda^{H} - \lambda^{L}\right)\left[(\gamma_{a}^{G} - \gamma_{u}^{G}) - (\gamma_{a}^{G} - \gamma_{u}^{G})\right] = 0.
\end{aligned}$$

Proof of Lemma 2

Simple checking yields:

$$\tilde{\gamma}^j_{aa}\tilde{\gamma}^j_{uu} - \tilde{\gamma}^j_{au}\tilde{\gamma}^j_{ua} = (\tilde{P}_{aa}\tilde{P}_{uu} - \tilde{P}_{au}\tilde{P}_{ua})\Gamma^j_a\Gamma^j_u$$

which is positive when

$$\tilde{P}_{aa}\tilde{P}_{uu} - \tilde{P}_{au}\tilde{P}_{ua} = \tilde{P}_{aa}(1-\tilde{P}_{ua}) - (1-\tilde{P}_{aa})\tilde{P}_{ua}$$
$$= \tilde{P}_{aa} - \tilde{P}_{aa}\tilde{P}_{ua} - \tilde{P}_{ua} + \tilde{P}_{aa}\tilde{P}_{ua} = P_{aa} - P_{ua} + b_a - b_u > 0.$$

Since, by Lemma 1, $P_{aa} > P_{ua}$, the latter inequality always holds for $b_a \geq b_u$. For values of $b_u > b_a$, it yields condition (1).

Proof of Lemmas 3 and 4

Rearranging the two (TR_P) constraints:

$$(w_{ua} - w_{aa}) \geq (w_{au} - w_{uu})\frac{\gamma^H_{au}}{\gamma^H_{aa}}$$

$$(w_{ua} - w_{aa}) \leq (w_{au} - w_{uu})\frac{\gamma^H_{uu}}{\gamma^H_{ua}}$$

$$\Rightarrow (w_{au} - w_{uu})\frac{\gamma^H_{au}}{\gamma^H_{aa}} \leq (w_{ua} - w_{aa}) \leq (w_{au} - w_{uu})\frac{\gamma^H_{uu}}{\gamma^H_{ua}}. \tag{11}$$

Given Assumption 2, either all the brackets in (11) are 0 (case (i)), or they have positive signs (case (ii)). This proves Lemma 3.

For Lemma 4 follow the same steps with the (TR_A) constraints to obtain:

$$(c_{uu} - c_{ua})\frac{\tilde{\gamma}^H_{ua}}{\tilde{\gamma}^H_{aa}} \leq (c_{aa} - c_{au}) \leq (c_{uu} - c_{ua})\frac{\tilde{\gamma}^H_{uu}}{\tilde{\gamma}^H_{au}}. \tag{12}$$

When the agent believes signals are positively correlated, i.e., $\tilde{\gamma}^H_{aa}\tilde{\gamma}^H_{uu} - \tilde{\gamma}^H_{au}\tilde{\gamma}^H_{ua} > 0$, we have:

$$\frac{\tilde{\gamma}^H_{ua}}{\tilde{\gamma}^H_{aa}} < \frac{\tilde{\gamma}^H_{uu}}{\tilde{\gamma}^H_{au}}.$$

Given this last inequality, either all the brackets in (12) are 0 (case (i)), or they have positive signs (case (ii)). This proves Lemma 4.

Proof of Proposition 1

Suppose not, then $w_{ts} = c_{ts}$ for all t and s. Given Lemma 3 and 4, we have:

$$c_{uu} \geq c_{ua} \geq c_{aa} \geq c_{au} \geq c_{uu},$$

where the first and third inequalities follow from Lemma 4 and the second and fourth follow from Lemma 3. Obviously, for all inequalities to hold together we need

$$c_{uu} = c_{ua} = c_{aa} = c_{au}.$$

This implies that $\tilde{E}(c_{ts}|\lambda^H) = \tilde{E}(c_{ts}|\lambda^L)$, since the agent compensation is completely independent from the realization of t and s. This, of course, violates the (IC) constraint since

$$\tilde{E}(c_{ts}|\lambda^H) - V(\lambda^H) < \tilde{E}(c_{ts}|\lambda^L) - V(\lambda^L).$$

Reducing the problem to (5)

Lemma 6 below states that an agent believing that signals are positively correlated ought to be compensated in the "most positive" case, that is, when both principal and agent report an acceptable performance. It also states that the agent obtains no compensation when the principal deems the performance unacceptable and the agent disagrees. Together with Lemma 8 below, Lemma 6 proves that a *deadweight loss* happens only when the principal deems unacceptable a performance deemed acceptable by the agent.

Lemma 6. *If the agent believes signals are positively correlated, i.e.* (1) *holds, then any optimal contract implementing high effort features $c_{aa} > c_{ua} = 0$.*

Proof. Define $\Delta\gamma_{ts} = \gamma_{ts}^H - \gamma_{ts}^L$ and $\Delta\tilde{\gamma}_{ts} = \tilde{\gamma}_{ts}^H - \tilde{\gamma}_{ts}^L$. First, we prove that $\Delta\tilde{\gamma}_{as} > 0$ and $\Delta\tilde{\gamma}_{us} < 0$ for any $s \in \{a, u\}$ (it is easy to see that the same holds for $\Delta\gamma_{as}$ and $\Delta\gamma_{us}$).

Notice that Assumption 1 is independent from Assumption 3. Therefore:

$$\begin{aligned}\Delta\tilde{\gamma}_{ts} &= \tilde{\gamma}^H_{ts} - \tilde{\gamma}^L_{ts} \\ &= \lambda^H \tilde{\gamma}^G_{ts} + (1-\lambda^H)\tilde{\gamma}^B_{ts} - \lambda^L \tilde{\gamma}^G_{ts} - (1-\lambda^L)\tilde{\gamma}^B_{ts} \\ &= \lambda^H \tilde{P}_{ts}\gamma^G_t + (1-\lambda^H)\tilde{P}_{ts}\gamma^B_t - \lambda^L \tilde{P}_{ts}\gamma^G_t - (1-\lambda^L)\tilde{P}_{ts}\gamma^B_t \\ &= (\lambda^H - \lambda^L)\tilde{P}_{ts}(\gamma^G_t - \gamma^B_t),\end{aligned}$$

which is positive at $t = a$ and negative otherwise.[29] Now we rewrite the (IC) in the following way:

$$c_{aa}\Delta\tilde{\gamma}_{aa} + c_{au}\Delta\tilde{\gamma}_{au} + c_{ua}\Delta\tilde{\gamma}_{ua} + c_{uu}\Delta\tilde{\gamma}_{uu} \geq \Delta V, \tag{13}$$

Recall that any optimal contract with truthful reporting for an agent who believes signals are positively correlated satisfies either case (i) or case (ii) of Lemma 4. Assume case (i) of Lemma 4 holds, then (13) becomes:

$$c_{aa}\underbrace{(\Delta\tilde{\gamma}_{aa} + \Delta\tilde{\gamma}_{au})}_{>0} + c_{uu}\underbrace{(\Delta\tilde{\gamma}_{ua} + \Delta\tilde{\gamma}_{uu})}_{<0} \geq \Delta V.$$

Because of the negative sign of the second bracket, and since $\Delta V > 0$ and $c_{uu} \geq 0$, the above requires $c_{aa} > 0$ to always hold. Assume now case (ii) of Lemma 4 holds, for a similar argument, we need at least one between c_{aa} and c_{au} to be positive. Since $c_{au} \geq 0$, case (ii) implies $c_{aa} > c_{au} \geq 0$. This proves the first part of Lemma 6.

To prove the second part of Lemma 6, we suppose it is false, i.e., at optimum $c_{ua} > 0$, and prove that there exists a profitable deviation from such a contract, which contradicts its optimality. First of all, from Lemma 4 we know that $c_{uu} \geq c_{ua}$ and also $c_{aa} \geq c_{au}$. The proof now depends on whether $c_{au} > 0$ or $c_{au} = 0$.

Suppose $c_{au} > 0$. Let the principal decrease both c_{uu} and c_{ua} by ϵ so that their difference remains constant (so not to affect the (TR_A) constraints). From (13) above, we see that both c_{uu} and c_{ua} enter negatively in the LHS of the (IC). Hence, decreasing them, would relax the (IC) rather than tightening it. In particular, the LHS of the (IC) constraint has increased by $-\epsilon(\Delta\tilde{\gamma}_{ua} + \Delta\tilde{\gamma}_{uu})$. Since we are in the case where $c_{au} > 0$, the principal can also decrease both c_{aa} and c_{au} by ϵ. In this way, the overall

[29]For future reference, this also proves that, as long as b_a and b_u are both positive, $\Delta\tilde{\gamma}_{ta} > \Delta\gamma_{ta}$ and $\Delta\tilde{\gamma}_{tu} < \Delta\gamma_{tu}$ for any t.

change in the LHS of the (IC) is given by

$$\begin{aligned}
&-\epsilon\left(\Delta\tilde{\gamma}_{aa}+\Delta\tilde{\gamma}_{au}+\Delta\tilde{\gamma}_{ua}+\Delta\tilde{\gamma}_{uu}\right)\\
&\quad=-\epsilon\left(\tilde{P}_{aa}\Delta\Gamma_a+\tilde{P}_{au}\Delta\Gamma_a+\tilde{P}_{ua}\Delta\Gamma_u+\tilde{P}_{uu}\Delta\Gamma_u\right)\\
&\quad=-\epsilon\left(\Delta\Gamma_a+\Delta\Gamma_u\right)=-\epsilon\left(\Delta\Gamma_a-\Delta\Gamma_a\right)=0
\end{aligned}$$

and therefore the (IC) binds again.

Finally, since both c_{ua} and c_{aa} have been decreased by ϵ, the principal can decrease also w_{ua} and w_{aa} by the same amount. This holds their difference constant and does not violate any of the relevant (LL_{ts}). Hence, it does not violate any of the (TR_P) constraints either. This new contract $\{w_{ts}, c_{ts}\}_{t,s}$ implements high effort at a lower cost. Hence, a contract where $c_{ua} > 0$ and $c_{au} > 0$ cannot be the solution to the problem.

Suppose now, instead, that the optimal contract features $c_{au} = 0$ and define $\Delta c_u = c_{uu} - c_{ua}$. Notice that this implies $c_{aa} > c_{au}$ and that we are in case (ii) of Lemma 4. We divide the proof for this case in three steps.

Step 1

When $c_{au} = 0$, the (TR_A) constraints imply

$$\Delta c_u \frac{\tilde{\gamma}^H_{ua}}{\tilde{\gamma}^H_{aa}} \leq c_{aa} \leq \Delta c_u \frac{\tilde{\gamma}^H_{uu}}{\tilde{\gamma}^H_{au}}, \tag{14}$$

where, since we are in case (ii) of Lemma 4 either only one of the two inequalities holds as equality, or none. Suppose none of the two does, or just the second one, the principal can decrease both c_{uu} and c_{ua} by ϵ keeping Δc_u constant, relaxing the (IC) constraint. In particular, the LHS of the (IC) has decreased by $\epsilon(\Delta\tilde{\gamma}_{ua}+\Delta\tilde{\gamma}_{uu})$. He can then decrease c_{aa} by $\delta \equiv \frac{\epsilon(\Delta\tilde{\gamma}_{ua}+\Delta\tilde{\gamma}_{uu})}{\Delta\tilde{\gamma}_{aa}}$ bringing the LHS of the (IC) back to its original value. Clearly, for some ϵ, this deviation can lead to the first inequality in (14) binding. Finally, to see that this is optimal for the principal, notice that according to the (LL_{ts}) constraints, she can decrease w_{ua} up to ϵ and w_{aa} up to δ. By decreasing both by $\min\{\epsilon, \delta\}$, their difference does not change. Hence, (TR_P) constraints are not affected, while the objective function decreases. This implies that, at optimum, if $c_{au} = 0$ the first inequality of (14) must bind and $\Delta c_u \frac{\tilde{\gamma}^H_{ua}}{\tilde{\gamma}^H_{aa}} = c_{aa}$.

Step 2

Given that $\Delta c_u \frac{\tilde{\gamma}^H_{ua}}{\tilde{\gamma}^H_{aa}} = c_{aa}$ when $c_{au} = 0$, we now show that the principal has at her disposal the following optimal deviation from a contract with $c_{au} = 0$. Let her decrease c_{uu} by ϵ and c_{ua} by $\epsilon_0 < \epsilon$. Then Δc_u has decreased by $(\epsilon - \epsilon_0)$. In order to keep $\Delta c_u \frac{\tilde{\gamma}^H_{ua}}{\tilde{\gamma}^H_{aa}} = c_{aa}$, the principal decreases c_{aa} by $(\epsilon - \epsilon_0)\frac{\tilde{\gamma}^H_{ua}}{\tilde{\gamma}^H_{aa}}$. It remains to check if this deviation can be made in such a way that it does not violate the (IC). The change in the (IC) is:

$$\begin{aligned}
&-(\epsilon - \epsilon_0)\frac{\tilde{\gamma}^H_{ua}}{\tilde{\gamma}^H_{aa}}\Delta\tilde{\gamma}_{aa} - \epsilon_0\Delta\tilde{\gamma}_{ua} - \epsilon\Delta\tilde{\gamma}_{uu} \\
&\quad = -(\epsilon - \epsilon_0)\frac{\tilde{P}_{ua}\Gamma^H_u}{\Gamma^H_a}\Delta\Gamma_a + \epsilon_0\tilde{P}_{ua}\Delta\Gamma_a + \epsilon\tilde{P}_{uu}\Delta\Gamma_a \\
&\quad = \Delta\Gamma_a\left[\epsilon\left(\tilde{P}_{uu} - \tilde{P}_{ua}\frac{\Gamma^H_u}{\Gamma^H_a}\right) + \epsilon_0\tilde{P}_{ua}\left(\frac{\Gamma^H_u}{\Gamma^H_a} + 1\right)\right] \\
&\quad = \frac{\Delta\Gamma_a}{\Gamma^H_a}\left[\epsilon\left(\tilde{P}_{uu}\Gamma^H_a - \tilde{P}_{ua} + \tilde{P}_{ua}\Gamma^H_a\right) + \epsilon_0\tilde{P}_{ua}\right] \\
&\quad = \frac{\Delta\Gamma_a}{\Gamma^H_a}\left[\epsilon\left(\Gamma^H_a - \tilde{P}_{ua}\right) + \epsilon_0\tilde{P}_{ua}\right],
\end{aligned}$$

which is positive when:

$$\epsilon\left(\Gamma^H_a - \tilde{P}_{ua}\right) + \epsilon_0\tilde{P}_{ua} > 0.$$

If $\Gamma^H_a > \tilde{P}_{ua}$, the above is always true. If instead $\Gamma^H_a < \tilde{P}_{ua}$ then the principal has to choose $\epsilon \in \left\{\epsilon_0, \epsilon_0\frac{\tilde{P}_{ua}}{\tilde{P}_{ua} - \Gamma^H_a}\right\}$.

Step 3

To conclude, we show that the above deviation is optimal. Given the decreases in the c_{ts}, the principal can now decrease w_{ua} up to ϵ_0 and w_{aa} up to $(\epsilon - \epsilon_0)\frac{\tilde{\gamma}^H_{ua}}{\tilde{\gamma}^H_{aa}}$. By an argument similar to the one in Step 1, she can decrease both wages by the smallest of the two limits, decreasing the objective function. This proves that a contract with $c_{ua} > 0$ and $c_{au} = 0$ cannot be optimal, since the principal can deviate optimally from it.

Hence, since a contract where $c_{ua} > 0$ and $c_{au} \geq 0$ cannot be a solution to the problem it follows that $c_{ua} = 0$. This concludes the proof of the Lemma. ∎

We now study the principal's incentives to report her PE truthfully.

Lemma 7. *If the agent believes signals are positively correlated, i.e.* (1) *holds, then constraint* (TR^a_P) *always binds in any optimal contract implementing high effort.*

Proof. Of course, in case (i) of Lemma 3 this is trivially proven. Assume now case (ii) of Lemma 3 holds and suppose (TR_P^a) is slack. Then $w_{ua} > 0$ must hold. From Lemma 6, then $c_{ua} = 0$, and the principal can simply decrease w_{ua} until (TR_P^a) binds. This would relax (TR_P^u), not affect (LL_{ua}) and decrease the objective function. ∎

We now solve for all w_{ts} as functions of the compensation c_{ts}.

Lemma 8. *If the agent believes signals are positively correlated, i.e.* (1) *holds, then any optimal contract implementing high effort features:*

(i) $w_{aa} = c_{aa}$;

(ii) $w_{uu} = c_{uu}$;

(iii) $w_{au} = \max\{c_{au}, c_{uu}\}$;

(iv) $w_{ua} = c_{aa} + (\max\{c_{au}, c_{uu}\} - c_{uu}) \frac{\gamma_{au}^H}{\gamma_{aa}^H}$.

Proof. First of all, notice that, by Lemma 7, $w_{ua} = w_{aa} + (w_{au} - w_{uu})\frac{\gamma_{au}^H}{\gamma_{aa}^H}$. Hence, the principal's objective function in (2) can be rearranged as:

$$w_{aa}\gamma_{aa}^H + w_{au}\gamma_{au}^H + \left[w_{aa} + (w_{au} - w_{uu})\frac{\gamma_{au}^H}{\gamma_{aa}^H}\right]\gamma_{ua}^H + w_{uu}\gamma_{uu}^H,$$

and further as:

$$w_{aa}\left(\gamma_{aa}^H + \gamma_{ua}^H\right) + w_{au}\left(\gamma_{au}^H + \frac{\gamma_{au}^H\gamma_{ua}^H}{\gamma_{aa}^H}\right) + w_{uu}\left(\gamma_{uu}^H - \frac{\gamma_{au}^H\gamma_{ua}^H}{\gamma_{aa}^H}\right),$$

where the last bracket is positive by Assumption 2. Furthermore, setting $w_{ua} = w_{aa} + (w_{au} - w_{uu})\frac{\gamma_{au}^H}{\gamma_{aa}^H}$ in (TR_P^u) we have

$$\left[w_{aa} + (w_{au} - w_{uu})\frac{\gamma_{au}^H}{\gamma_{aa}^H}\right]\gamma_{ua}^H + w_{uu}\gamma_{uu}^H \leq w_{aa}\gamma_{ua}^H + w_{au}\gamma_{uu}^H,$$

which is equivalent to

$$w_{uu} \leq w_{au}.$$

Hence, given the Lemmas so far, w_{aa}, w_{au}, and w_{uu} are only bound by $w_{uu} \leq w_{au}$ and the three corresponding (LL_{ts}). This implies that w_{aa}, w_{au}, and w_{uu} will be set to the lowest possible value. By Lemma 3, and in order to minimize the objective function, $w_{aa} = c_{aa}$, $w_{uu} = c_{uu}$ and $w_{au} = \max\{c_{au}, w_{uu}\}$, implying points (i), (ii) and (iii) of Lemma 8. Point (iv) follows by substitution. ∎

The next Lemma completes case (ii) of Lemma 4 by ranking c_{au} and c_{uu}. As expected, when the principal deems the performance acceptable, the agent may obtain a compensation premium even when he observes $S = u$.

Lemma 9. *If the agent believes signals are positively correlated, i.e.* (1) *holds, then any optimal contract implementing high effort features* $c_{au} \geq c_{uu}$.

Proof. Suppose not. Then $c_{uu} > c_{au} \geq 0$. By Lemma 6, $c_{ua} = 0$. Hence $c_{uu} > c_{ua}$, implying we are in case (ii) of Lemma 4 and $c_{aa} > c_{au}$. By Lemma 8, we have $w_{uu} = w_{au} = c_{uu}$ and $w_{ua} = c_{aa} = w_{aa}$. This implies that c_{au} disappears from the objective function and from constraints. The principal can, therefore, increase c_{au} and decrease other compensation (and therefore wage payments) in such a way that the rest of the constraints are still satisfied. This operation can be repeated until $c_{au} = c_{uu}$. Hence, the contradiction. ∎

Given this, we can further decrease the amount of binding constraints by proving the following:

Lemma 10. *If the agent believes signals are positively correlated, i.e.* (1) *holds, then constraint* (TR_A^u) *always binds in any optimal contract implementing high effort. Therefore:*

$$c_{uu} = \frac{\tilde{\gamma}_{au}^H}{\tilde{\gamma}_{uu}^H}(c_{aa} - c_{au}).$$

Proof. Let $c_{uu} = 0$. Then we are in case (i) of Lemma 4 and (TR_A^u) is trivially binding. Suppose now that $c_{uu} > 0$ and (TR_A^u) is not binding. The principal can reduce c_{uu} until it binds. Given the proven Lemmas, the (TR_P) still hold, while (TR_A^a) and (IC) are relaxed by this change. To complete the proof, we need to check whether a decrease in c_{uu} would decrease the objective function as well. By Lemmas 8 and 9, we can substitute for all wages in the objective function and find that the coefficient of c_{uu} becomes $\left(\gamma_{uu}^H - \frac{\gamma_{au}^H \gamma_{ua}^H}{\gamma_{aa}^H}\right)$, which is positive by Assumption 2. Hence, decreasing c_{uu} also decreases cost and it is therefore optimal for the principal to do so. This provides the desired contradiction and proves that (TR_A^u) always binds at optimum. ∎

This concludes the set of Lemmas yielding problem (5). Notice that, when plugging in the values from Lemma 8, the objective function in (4), simplifies to (5) divided by

$\gamma^H_{aa}\tilde{\gamma}^H_{uu}$. This is however irrelevant for the minimization problem and therefore omitted.

Optimism and the (IC). We can also prove the following two Lemmas to characterize the impact of optimism on the (IC) constraint.

Lemma 11. *If the optimal contract implementing high effort features* $c_{aa} = c_{au}$ *and* $c_{uu} = c_{ua}$*, then optimism has no impact on the* (IC)*.*

Proof. see the proof for Lemma 12. ∎

Lemma 11 states that if the agent's compensation is independent of his own PE report, then his optimism over the signals' joint distribution has no effect on the (IC), and therefore on implementability of any level of effort.

Lemma 12. *If the optimal contract implementing high effort features* $c_{aa} > c_{au}$ *and* $c_{uu} > c_{ua}$*, then optimism relaxes the* (IC)*.*

Proof. The (IC)

$$\sum_{ts} c_{ts}(\tilde{\gamma}^H_{ts} - \tilde{\gamma}^L_{ts}) \geq \Delta V,$$

can be rewritten as

$$\sum_{ts} c_{ts}(\gamma^H_{ts} - \gamma^L_{ts}) + (c_{aa} - c_{au})(\Gamma^H_a - \Gamma^L_a)b_a + (c_{ua} - c_{uu})(\Gamma^H_u - \Gamma^L_u)b_u \geq \Delta V. \tag{15}$$

Note that $\Gamma^H_a > \Gamma^L_a$ and $\Gamma^H_u < \Gamma^L_u$. It follows directly from (15) that, if the optimal contract features $c_{aa} = c_{au}$ and $c_{uu} = c_{ua}$, then optimism has no impact on the (IC). This proves Lemma 11. If the optimal contract features $c_{aa} > c_{au}$ and $c_{uu} > c_{ua}$, then the second and third terms in the LHS of (15) are strictly positive and therefore optimism relaxes the (IC). This proves Lemma 12. ∎

By Lemma 4, the agent knows that given what the principal observes, he obtains a premium when he reports $T = S$. A positive b_a (b_u) increase (decreases) the agent's belief of both signals to show a (u). This means that, given effort, an optimistic agent with beliefs satisfying (1) overestimates the chances of obtains the premium $c_{aa} - c_{au}$ and underestimates the ones of obtaining $c_{uu} - c_{ua}$. Since $T = a$ is most probable when he exerts high effort, the agent requires a lower incentive to exert λ^H. That is to say, exerting high effort is part of his "strategy" to increase the chance of $(t, s) = (a, a)$.

Proof of Lemma 5

The inequality in Lemma 5 follows from the comparisons of the slope of the (IC) with the slope of the iso-costs. This produces the following condition

$$\frac{\Delta\tilde{\gamma}_{au} - \frac{\tilde{\gamma}^H_{au}}{\tilde{\gamma}^H_{uu}}\Delta\tilde{\gamma}_{uu}}{\Delta\tilde{\gamma}_{aa} + \frac{\tilde{\gamma}^H_{au}}{\tilde{\gamma}^H_{uu}}\Delta\tilde{\gamma}_{uu}} \leq \frac{\gamma^H_{aa}\gamma^H_{au}\tilde{\gamma}^H_{uu} + \gamma^H_{au}\gamma^H_{ua}\tilde{\gamma}^H_{uu} - \tilde{\gamma}^H_{au}\gamma^H_{uu}\gamma^H_{aa} + \tilde{\gamma}^H_{au}\gamma^H_{au}\gamma^H_{ua}}{(\gamma^H_{aa})^2\tilde{\gamma}^H_{uu} + \gamma^H_{aa}\gamma^H_{ua}\tilde{\gamma}^H_{uu} + \tilde{\gamma}^H_{au}\gamma^H_{uu}\gamma^H_{aa} - \tilde{\gamma}^H_{au}\gamma^H_{au}\gamma^H_{ua}}.$$

We start from simplifying the slope of the (IC)

$$LHS = \frac{\Delta\tilde{\gamma}_{au} - \frac{\tilde{\gamma}^H_{au}}{\tilde{\gamma}^H_{uu}}\Delta\tilde{\gamma}_{uu}}{\Delta\tilde{\gamma}_{aa} + \frac{\tilde{\gamma}^H_{au}}{\tilde{\gamma}^H_{uu}}\Delta\tilde{\gamma}_{uu}} = \frac{\tilde{\gamma}^H_{au} - \tilde{\gamma}^L_{au} - \tilde{\gamma}^H_{au} + \frac{\tilde{\gamma}^H_{au}}{\tilde{\gamma}^H_{uu}}\tilde{\gamma}^L_{uu}}{\tilde{\gamma}^H_{aa} - \tilde{\gamma}^L_{aa} + \tilde{\gamma}^H_{au} - \frac{\tilde{\gamma}^H_{au}}{\tilde{\gamma}^H_{uu}}\tilde{\gamma}^L_{uu}} = \frac{\frac{\tilde{\gamma}^H_{au}}{\tilde{\gamma}^H_{uu}}\tilde{\gamma}^L_{uu} - \tilde{\gamma}^L_{au}}{\tilde{\gamma}^H_{aa} - \tilde{\gamma}^L_{aa} + \tilde{\gamma}^H_{au} - \frac{\tilde{\gamma}^H_{au}}{\tilde{\gamma}^H_{uu}}\tilde{\gamma}^L_{uu}}$$

$$= \frac{\frac{\tilde{P}_{au}\tilde{P}_{uu}\Gamma^H_a\Gamma^L_u}{\tilde{P}_{uu}\Gamma^H_u} - \tilde{P}_{au}\Gamma^L_a}{\tilde{P}_{aa}\Delta\Gamma_a + \tilde{P}_{au}\Gamma^H_a\left(1 - \frac{\Gamma^L_u}{\Gamma^H_u}\right)} = \frac{\tilde{P}_{au}\left(\Gamma^H_a\Gamma^L_u - \Gamma^L_a\Gamma^H_u\right)}{\tilde{P}_{aa}\Delta\Gamma_a\Gamma^H_u + \tilde{P}_{au}\Gamma^H_a\Delta\Gamma_u}.$$

Notice that, since $\Gamma^J_a + \Gamma^J_u = 1$ for any $j = H, L$, we can substitute for $\Gamma^H_u = 1 - \Gamma^H_a$ and $\Gamma^L_u = 1 - \Gamma^L_a$. Also, from Lemma 1, $\Delta\Gamma_a = -\Delta\Gamma_u$. Hence we can further simplify the LHS:

$$\frac{\tilde{P}_{au}\left(\Gamma^H_a\Gamma^L_u - \Gamma^L_a\Gamma^H_u\right)}{\tilde{P}_{aa}\Delta\Gamma_a\Gamma^H_u + \tilde{P}_{au}\Gamma^H_a\Delta\Gamma_u} = \frac{\tilde{P}_{au}\left(\Gamma^H_a(1-\Gamma^L_a) - \Gamma^L_a(1-\Gamma^H_a)\right)}{\tilde{P}_{aa}\Delta\Gamma_a(1-\Gamma^H_a) + \tilde{P}_{au}\Gamma^H_a(-\Delta\Gamma_a)}$$

$$= \frac{\tilde{P}_{au}\Delta\Gamma_a}{\Delta\Gamma_a\left[\tilde{P}_{aa}(1-\Gamma^H_a) - \tilde{P}_{au}\Gamma^H_a\right]} = \frac{\tilde{P}_{au}}{\tilde{P}_{aa} - \Gamma^H_a} = \frac{P_{au} - b_a}{P_{aa} - \Gamma^H_a + b_a}.$$

The slope of the iso-costs, instead, is given by

$$\frac{\gamma^H_{aa}\gamma^H_{au}\tilde{\gamma}^H_{uu} + \gamma^H_{au}\gamma^H_{ua}\tilde{\gamma}^H_{uu} - \tilde{\gamma}^H_{au}\gamma^H_{uu}\gamma^H_{aa} + \tilde{\gamma}^H_{au}\gamma^H_{au}\gamma^H_{ua}}{(\gamma^H_{aa})^2\tilde{\gamma}^H_{uu} + \gamma^H_{aa}\gamma^H_{ua}\tilde{\gamma}^H_{uu} + \tilde{\gamma}^H_{au}\gamma^H_{uu}\gamma^H_{aa} - \tilde{\gamma}^H_{au}\gamma^H_{au}\gamma^H_{ua}}$$

$$= \frac{(P_{uu} - b_u)\left(P_{aa}P_{au}\Gamma^H_a + P_{au}P_{ua}\Gamma^H_u\right) - (P_{au} - b_a)\Gamma^H_a\left(P_{aa}P_{uu} - P_{au}P_{ua}\right)}{(P_{uu} - b_u)\left(P_{aa}P_{aa}\Gamma^H_a + P_{aa}P_{ua}\Gamma^H_u\right) + (P_{au} - b_a)\Gamma^H_a\left(P_{aa}P_{uu} - P_{au}P_{ua}\right)}$$

$$= \frac{(P_{uu} - b_u)P_{au}Z - (P_{au} - b_a)W}{(P_{uu} - b_u)P_{aa}Z + (P_{au} - b_a)W},$$

where $Z = \left(P_{aa}\Gamma^H_a + P_{ua}\Gamma^H_u\right)$ and $W = \Gamma^H_a\left(P_{aa}P_{uu} - P_{au}P_{ua}\right) = \Gamma^H_a(P_{aa} - P_{ua})$.

Hence the inequality in Lemma 5 is equivalent to

$$\frac{P_{au}-b_a}{P_{aa}-\Gamma_a^H+b_a} \leq \frac{(P_{uu}-b_u)P_{au}Z-(P_{au}-b_a)W}{(P_{uu}-b_u)P_{aa}Z+(P_{au}-b_a)W}$$

$$(P_{au}-b_a)\,(P_{uu}-b_u)P_{aa}Z+(P_{au}-b_a)^2\,W$$
$$\leq \left(P_{aa}-\Gamma_a^H+b_a\right)(P_{uu}-b_u)P_{au}Z-\left(P_{aa}-\Gamma_a^H+b_a\right)(P_{au}-b_a)W$$

$$(P_{au}-b_a)^2\,W+\left(P_{aa}-\Gamma_a^H+b_a\right)(P_{au}-b_a)W$$
$$\leq \left(P_{aa}-\Gamma_a^H+b_a\right)(P_{uu}-b_u)P_{au}Z-(P_{au}-b_a)\,(P_{uu}-b_u)P_{aa}Z$$

$$(P_{au}-b_a)\left(P_{au}+P_{aa}-\Gamma_a^H\right)W \leq (P_{uu}-b_u)\left[b_a\,(P_{aa}+P_{au})-P_{au}\Gamma_a^H\right]Z \tag{16}$$

$$b_u \leq P_{uu}-\frac{(P_{au}-b_a)\left(1-\Gamma_a^H\right)W}{\left(b_a-P_{au}\Gamma_a^H\right)Z}. \tag{17}$$

If the second term on the RHS of (17) is non-negative, then the inequality places no new restriction on the space (b_a, b_u). However, if the second term on the RHS of (17) is negative, then the inequality places a new restriction on the space (b_a, b_u). Since $b_a \in (0, P_{au}]$ and $\Gamma_a^H \in (0,1)$, the second term on the RHS of (17) is negative when $b_a \in (P_{au}\Gamma_a^H, P_{au})$. Further, notice that if $b_a < P_{au}\Gamma_a^H$, inequality (16) cannot hold.

To conclude the proof, solve (17) for b_u to obtain (6).

Proof of Proposition 2

The Proof is divided in two parts. First we show that when $b_a = b_u = 0$, the slope of the (IC) is never lower than the slope of the iso-costs. Then we derive the optimal contract for the unbiased agent.

Using the algebra presented in the proof of Lemma 5, consider the slope of the (IC) when the agent is unbiased:

$$\frac{P_{au}}{P_{aa}-\Gamma_a^H}$$

This implies that the (IC) is negatively sloped if and only if $\Gamma_a^H < P_{aa}$. First we assume $\Gamma_a^H < P_{aa}$ and show that the (18) always holds. Then we move to the case of $\Gamma_a^H > P_{aa}$.

Let $\Gamma_a^H < P_{aa}$. The comparison between slopes then becomes:

$$\frac{P_{au}}{P_{aa}-\Gamma_a^H} > \frac{\gamma_{aa}^H\gamma_{au}^H\gamma_{uu}^H+\gamma_{au}^H\gamma_{ua}^H\gamma_{uu}^H-\gamma_{au}^H\gamma_{uu}^H\gamma_{aa}^H+\gamma_{au}^H\gamma_{au}^H\gamma_{ua}^H}{\gamma_{aa}^H\gamma_{aa}^H\gamma_{uu}^H+\gamma_{aa}^H\gamma_{ua}^H\gamma_{uu}^H+\gamma_{au}^H\gamma_{uu}^H\gamma_{aa}^H-\gamma_{au}^H\gamma_{au}^H\gamma_{ua}^H} \tag{18}$$

We now rearrange the RHS, which is less nicely simplified.

$$RHS = \frac{\gamma^H_{aa}\gamma^H_{au}\gamma^H_{uu} + \gamma^H_{au}\gamma^H_{ua}\gamma^H_{uu} - \gamma^H_{au}\gamma^H_{uu}\gamma^H_{aa} + \gamma^H_{au}\gamma^H_{au}\gamma^H_{ua}}{\gamma^H_{aa}\gamma^H_{aa}\gamma^H_{uu} + \gamma^H_{aa}\gamma^H_{ua}\gamma^H_{uu} + \gamma^H_{au}\gamma^H_{uu}\gamma^H_{aa} - \gamma^H_{au}\gamma^H_{au}\gamma^H_{ua}}$$
$$= \frac{\gamma^H_{au}\gamma^H_{ua}\gamma^H_{uu} + \gamma^H_{au}\gamma^H_{au}\gamma^H_{ua}}{\gamma^H_{aa}\gamma^H_{aa}\gamma^H_{uu} + \gamma^H_{aa}\gamma^H_{ua}\gamma^H_{uu} + \gamma^H_{au}\gamma^H_{uu}\gamma^H_{aa} - \gamma^H_{au}\gamma^H_{au}\gamma^H_{ua}}$$

Before going ahead, notice that this proves that in the case of an unbiased agent isocosts are always negatively sloped. Carrying on we obtain

$$\frac{\gamma^H_{au}\gamma^H_{ua}\gamma^H_{uu} + \gamma^H_{au}\gamma^H_{au}\gamma^H_{ua}}{\gamma^H_{aa}\gamma^H_{aa}\gamma^H_{uu} + \gamma^H_{aa}\gamma^H_{ua}\gamma^H_{uu} + \gamma^H_{au}\gamma^H_{uu}\gamma^H_{aa} - \gamma^H_{au}\gamma^H_{au}\gamma^H_{ua}}$$
$$= \frac{P_{au}P_{ua}P_{uu}\Gamma^H_u + P_{au}P_{au}P_{ua}\Gamma^H_a}{P_{aa}P_{aa}P_{uu}\Gamma^H_a + P_{aa}P_{ua}P_{uu}\Gamma^H_u + P_{au}P_{uu}P_{aa}\Gamma^H_a - P_{au}P_{au}P_{ua}\Gamma^H_a}$$
$$= \frac{P_{au}P_{ua}(P_{uu}(1-\Gamma^H_a) + P_{au}\Gamma^H_a)}{P_{aa}P_{uu}\Gamma^H_a + P_{ua}(P_{aa}P_{uu}(1-\Gamma^H_a) - P_{au}P_{au}\Gamma^H_a)}$$
$$= \frac{P_{au}P_{ua}(P_{uu} - \Gamma^H_a(P_{uu} - P_{ua}))}{P_{aa}P_{uu}\Gamma^H_a + P_{ua}(P_{aa}P_{uu}(1-\Gamma^H_a) - P_{au}P_{au}\Gamma^H_a)}$$

This implies that comparing the slopes boils down to:

$$\frac{P_{au}}{P_{aa} - \Gamma^H_a} > \frac{P_{au}P_{ua}(P_{uu} - \Gamma^H_a(P_{uu} - P_{ua}))}{P_{aa}P_{uu}\Gamma^H_a + P_{ua}(P_{aa}P_{uu}(1-\Gamma^H_a) - P_{au}P_{au}\Gamma^H_a)}$$
$$\frac{1}{P_{aa} - \Gamma^H_a} > \frac{P_{ua}(P_{uu} - \Gamma^H_a(P_{uu} - P_{ua}))}{P_{aa}P_{uu}\Gamma^H_a + P_{ua}(P_{aa}P_{uu}(1-\Gamma^H_a) - P_{au}P_{au}\Gamma^H_a)}$$

$$P_{aa}P_{uu}\Gamma^H_a + P_{ua}(P_{aa}P_{uu}(1-\Gamma^H_a) - P_{au}P_{au}\Gamma^H_a) > (P_{aa} - \Gamma^H_a)P_{ua}(P_{uu} - \Gamma^H_a(P_{uu} - P_{ua})).$$

Recall that Lemma 1 showed $P_{aa} > P_{ua}$ and $P_{uu} > P_{au}$.

$$P_{aa}P_{uu}\Gamma^H_a + P_{ua}P_{aa}P_{uu} - P_{ua}P_{aa}P_{uu}\Gamma^H_a - P_{ua}P_{au}P_{au}\Gamma^H_a$$
$$> P_{ua}P_{aa}P_{uu} - P_{ua}P_{uu}\Gamma^H_a - P_{aa}P_{ua}\Gamma^H_a(P_{uu} - P_{ua}) + P_{ua}(\Gamma^H_A)^2(P_{uu} - P_{ua}),$$

which, by simplifying and dividing by Γ^H_a on both sides, is equivalent to:

$$P_{aa}P_{uu} - P_{ua}P_{au}P_{au} > -P_{ua}P_{uu} + P_{aa}P_{ua}P_{ua} + P_{ua}P_{uu}\Gamma^H_A - P_{ua}P_{ua}\Gamma^H_A$$
$$P_{aa}P_{uu} - P_{ua}P^2_{au} > -P_{ua}P_{uu} + P_{aa}P^2_{ua} + P_{uu}P_{ua}\Gamma^H_a - P^2_{ua}\Gamma^H_a$$
$$P_{uu}(P_{aa} + P_{ua}) - P_{ua}\Gamma^H_a(P_{uu} - P_{ua}) - P_{ua}P^2_{au} - P_{aa}P^2_{ua} > 0.$$

Now we substitute for $P_{uu} = 1 - P_{ua}$ and $P_{au} = 1 - P_{aa}$ and we get:

$$
\begin{aligned}
&(1 - P_{ua})(P_{aa} + P_{ua}) - P_{ua}\Gamma_a^H(1 - 2P_{ua}) - P_{ua}(1 - P_{aa})^2 - P_{aa}P_{ua}^2 > 0 \\
&P_{aa} + P_{ua} - P_{aa}P_{ua} - P_{ua}^2 - P_{ua}\Gamma_a^H(1 - 2P_{ua}) - P_{ua} + 2P_{aa}P_{ua} - P_{ua}P_{aa}^2 - P_{aa}P_{ua}^2 > 0 \\
&P_{aa} + P_{aa}P_{ua}(1 - P_{ua} - P_{aa}) - P_{ua}^2 + \underbrace{P_{ua}\Gamma_a^H(2P_{ua} - 1)}_{\mathbf{\Gamma}} > 0.
\end{aligned}
$$

Suppose first that $P_{ua} < \frac{1}{2}$, then $\mathbf{\Gamma} < 0$ and the LHS gets smaller the greater is Γ_a^H. Hence, to be sure the condition holds, we set $\Gamma_a^H \to P_{aa}$, the highest possible value it can get. This yields $\mathbf{\Gamma} \to 2P_{aa}P_{ua}^2 - P_{aa}P_{ua}$. Hence the condition converges to

$$
\begin{aligned}
&P_{aa} + P_{aa}P_{ua}(1 - P_{ua} - P_{aa}) - P_{ua}^2 + 2P_{aa}P_{ua}^2 - P_{aa}P_{ua} > 0 \\
&P_{aa} + P_{aa}P_{ua}^2 - P_{aa}^2P_{ua} - P_{ua}^2 > 0. \qquad (19)
\end{aligned}
$$

Notice that if this holds for all $P_{aa} > P_{ua}$ then so will the condition for the case of $P_{ua} > \frac{1}{2}$. In that case, in fact, $\mathbf{\Gamma} > 0$, which means that the LHS would increase with Γ_a^H. Hence, to check it holds we set it to 0. This would set $\mathbf{\Gamma} = 0$ and yield a condition looser than (19).

To see that (19) always holds, notice that the derivative of the LHS with respect to P_{ua} is given by:

$$
\frac{\partial LHS}{\partial P_{ua}} = 2P_{aa}P_{ua} - P_{aa}^2 - 2P_{ua} = 2P_{ua}(P_{aa} - 1) - P_{aa}^2
$$

which is negative for all $P_{aa} < 1$. Hence, the condition is monotonically decreasing in P_{ua}. We therefore check for the maximum value of P_{ua}, which in this case is $\frac{1}{2}$. At this value, condition (19) becomes simply

$$
-2P_{aa}^2 + 5P_{aa} - 1 > 0
$$

By Lemma 1, P_{aa} must be *strictly* larger than P_{ua}. The second order equation above always holds for $P_{aa} \in [\frac{1}{2}, 1]$. Hence when the (IC) is negatively sloped and $b_a = b_u = 0$, (6) always holds.

We are now left to show that the same holds when the (IC) is positively sloped. This case is omitted in the text and Figure 1. Suppose that the (IC) is positively sloped. This implies that it requires c_{aa} to be smaller than c_{au} times a positive number. First of all, notice from the (IC) that when it is positively sloped, its intercept for $c_{au} = 0$ is negative. Further, its slope is now given by

$$\frac{\Delta\tilde{\gamma}_{au} - \frac{\tilde{\gamma}^H_{au}}{\tilde{\gamma}^H_{uu}}\Delta\tilde{\gamma}_{uu}}{-\Delta\tilde{\gamma}_{aa} - \frac{\tilde{\gamma}^H_{au}}{\tilde{\gamma}^H_{uu}}\Delta\tilde{\gamma}_{uu}}$$

which is obviously larger than 1. Hence, the set of constraint compatible contracts becomes the one highlighted in Figure 5.

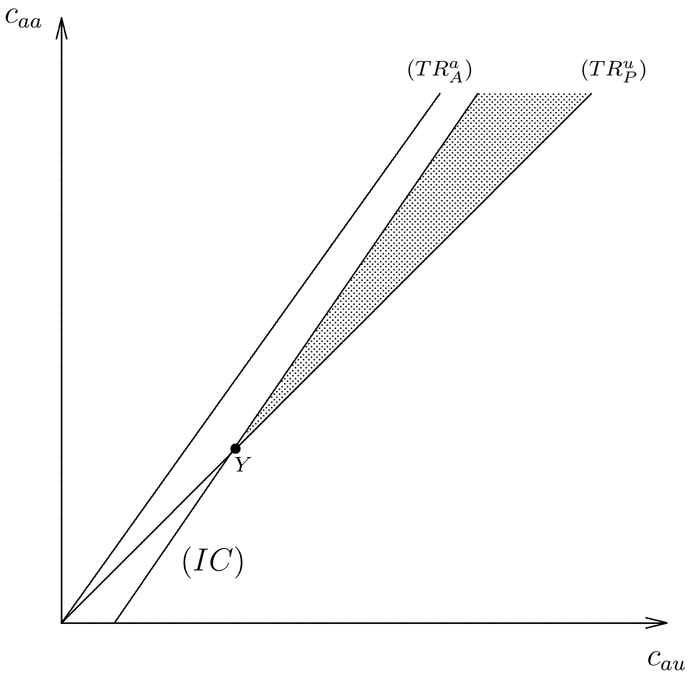

FIGURE 5. The shaded area represents the set of contracts satisfying all the constraints of the minimization problem (5) when the agent believes that signals are positively correlated and the (IC) is positively sloped.

Regardless of whether the iso-costs are positively or negatively sloped, the optimal contract lies at point Y in the graph and replicates the BPE contract.

Now that we have proven the structure of the optimal contract, we move to deriving its values. Consider problem (5) with $b_a = b_u = 0$. Given the proof so far, let $c_{aa} = c_{au}$ and the (IC) bind. We solve the (IC) to obtain

$$c_{aa}(\Delta\gamma_{aa} + \Delta\gamma_{au}) = \Delta V$$

which yields

$$c_{aa}(\Delta\Gamma_a P_{aa} + \Delta\Gamma_a P_{au}) = \Delta V$$

and

$$c_{aa}(\Delta\Gamma_a) = \Delta V \Rightarrow c_{aa} = \frac{\Delta V}{\Delta\Gamma_a} = c_{au}.$$

Wages are obtained by substituting the compensation values into the wages of Lemma 8. To obtain w_{ua} calculate

$$\begin{aligned} w_{ua} =& \frac{\Delta V}{\Delta\Gamma_a}\left(1 + \frac{\gamma^H_{au}}{\gamma^H_{aa}}\right) = \frac{\Delta V}{\Delta\Gamma_a}\left(\frac{\gamma^H_{aa} + \gamma^H_{au}}{\gamma^H_{aa}}\right) \\ =& \frac{\Delta V}{\Delta\Gamma_a}\left(\frac{P_{aa}\Gamma^H_a + P_{au}\Gamma^H_a}{P_{aa}\Gamma^H_a}\right) = \frac{\Delta V}{\Delta\Gamma_a}\left(\frac{1}{P_{aa}}\right). \end{aligned}$$

Proof of Proposition 3

To see that the contract completely resembles the baseline one, simply notice that, from Lemma 5, the binding constraints are the same of Proposition 2. Furthermore,

$$c'_{aa} = \frac{\Delta V}{\Delta\tilde{\gamma}_{aa} + \Delta\tilde{\gamma}_{au}} = \frac{\Delta V}{\Delta\Gamma_a(\tilde{P}_{aa} + \tilde{P}_{au})} = \frac{\Delta V}{\Delta\Gamma_a}.$$

Proof of Proposition 4

The proof of the proposition is divided in five parts. First, we show that when either the (IC) or the iso-costs (or both) are positively sloped, the optimal contract is the standard one. Then, we derive conditions for this case not to happen. Third, we prove that condition (6) implies all the conditions derived as well as (1) — and hence it is sufficient and necessary to our result — and we identify the shape of the area where the APE contract is set up (i.e. we provide an explanation to the shape of Figure 2). Fourth, we derive the values of wages and compensations of the APE contract. Finally, we prove how the deadweight loss of the APE contract is lower than the one of the BPE contract.

Part 1. First of all, notice from (5) that an increase of c_{aa} always increases the expected cost of implementing high effort. The effect of an increase of c_{au}, however, is not straightforward when $b_a = b_u = 0$. If it is positive, then iso-costs are negatively sloped in (c_{au}, c_{aa}) space and costs decrease towards the origin. If it is negative, then iso-costs are positively sloped and costs decrease towards the bottom right of the graph.

Suppose the latter is true. Since iso-costs are positively sloped in (c_{au}, c_{aa}) space, optimal contracts lie at point Y of Figure 1. Notice, however, that a further check is needed here. Suppose the iso-costs are positively sloped. If their slope is larger than

1, then they are steeper than the locus of points where $c_{aa} = c_{au}$. Hence, for any given $c_{aa} = c_{au} = c$, there would always exists a $c' > c$ lying on an iso-costs further to the right of Figure 1 satisfying all constraints and lowering costs. Hence, an optimal contract would feature $c_{aa} = c_{au} = c \to \infty$. In order to check that this cannot happen, we study the value of the slope of the iso-costs when the latter is positive. From the algebra in the proof of Lemma 5, we can get this value as:

$$\frac{(P_{au} - b_a)\Gamma_a^H(P_{aa} - P_{ua}) - (P_{uu} - b_u)P_{au}(P_{aa}\Gamma_a^H + P_{ua}\Gamma_u^H)}{(P_{au} - b_a)\Gamma_a^H(P_{aa} - P_{ua}) + (P_{uu} - b_u)P_{aa}(P_{aa}\Gamma_a^H + P_{ua}\Gamma_u^H)}$$

which is trivially never larger than 1. Hence in equilibrium the baseline contract is set up.

The case of a positively sloped (IC) has already been discussed in the proof of Proposition 2.

Part 2. The slope of the (IC) is negative as long as $b_a \geq \Gamma_a^H - P_{aa}$. The condition for the slope of the iso-cost to be negative, instead, can be derived as follows.

Consider the slope derived in the proof of Lemma 5 again, this time without looking at its absolute value

$$-\frac{(P_{uu} - b_u)P_{au}\left(P_{aa}\Gamma_a^H + P_{ua}\Gamma_u^H\right) - (P_{au} - b_a)\Gamma_a^H\left(P_{aa} - P_{ua}\right)}{(P_{uu} - b_u)P_{aa}\left(P_{aa}\Gamma_a^H + P_{ua}\Gamma_u^H\right) + (P_{au} - b_a)\Gamma_a^H\left(P_{aa} - P_{ua}\right)}.$$

The iso-costs are negatively sloped when the numerator of the above is positive. This happens when:

$$(P_{uu} - b_u)P_{au}\underbrace{\left(P_{aa}\Gamma_a^H + P_{ua}\Gamma_u^H\right)}_{Z} - (P_{au} - b_a)\underbrace{\Gamma_a^H\left(P_{aa} - P_{ua}\right)}_{W} > 0$$

which yields condition:

$$b_u < P_{uu} - \frac{(P_{au} - b_a)W}{P_{au}Z}. \tag{20}$$

Part 3. In this part of the proof we show how, for $b_a \in \left[P_{au}\Gamma_a^H, P_{au}\right]$, condition (6) implies the negativity of the slope of the (IC), condition (1) and condition (20). We also show how the area it delimits has a concave shape in (b_a, b_u) space and how it always lies in the interval $(P_{au}\Gamma_a, P_{au})$ on b_a. In order to study this we use version (17) of condition (6).

First of all, notice that the (IC) is negatively sloped if

$$b_a \geq \Gamma_a^H - P_{aa} = \Gamma_a^H - 1 + P_{au}$$

and that

$$\Gamma_a^H - 1 + P_{au} < P_{au}\Gamma_a^H \;\Rightarrow\; P_{au}(1-\Gamma_a^H) < 1 - \Gamma_a^H.$$

Hence, when $b_a > P_{au}\Gamma_a^H$ (which is necessary for (17) to matter) the (IC) is negatively sloped.

We now compare (17) to (1). First notice that (1) is linear and rearrange it as

$$b_u \leq P_{aa} + b_a - P_{ua}.$$

Given the possible values of b_a, the value of the RHS goes from $P_{aa} + P_{au}\Gamma_a^H - P_{au}$ to $1 - P_{au} = P_{uu}$. Its derivative in b_a is obviously 1. Similarly, we can evaluate the RHS of (6) at $b_a = P_{au}$ to see that it is simply P_{uu}. This means that the two conditions coincide at $b_a = P_{au}$. Now notice that as $b_a \to P_{au}\Gamma_a^H$ the RHS of condition (6) goes to 0 (since the second term explodes and eventually reaches P_{uu}). We therefore have that condition (1) lies above (6) at the two boundaries for the feasible interval of b_a. We are left to check that the two stay this way over the entire interval. To see this, we study the derivative of the RHS of (17) and show that it is always positive and larger than 1, i.e. larger than the derivative of the RHS of (1). This ensures that the two curves cross only once.

$$\begin{aligned}
&\frac{\partial}{\partial b_a}\left[P_{uu} - \frac{(P_{au}-b_a)\left(1-\Gamma_a^H\right)W}{\left(b_a - P_{au}\Gamma_a^H\right)Z}\right] \\
&= -\frac{-(1-\Gamma_a^H)W\left(b_a - P_{au}\Gamma_a^H\right)Z - Z\left((P_{au}-b_a)\left(1-\Gamma_a^H\right)W\right)}{\left(\left(b_a - P_{au}\Gamma_a^H\right)Z\right)^2} \\
&= \frac{P_{au}(1-\Gamma_a^H)^2WZ}{\left[\left(b_a - P_{au}\Gamma_a^H\right)Z\right]^2},
\end{aligned}$$

which is always positive. To see that it is larger than 1 we calculate:

$$\frac{P_{au}(1-\Gamma_a^H)^2WZ}{\left[\left(b_a - P_{au}\Gamma_a^H\right)Z\right]^2} > 1,$$

which yields

$$P_{au}(1-\Gamma_A^H)^2W - b_a^2Z - P_{au}^2(\Gamma_A^H)^2Z + 2P_{au}b_a\Gamma_a^HZ > 0.$$

The study of this inequality is not trivial. Consider first the derivative of the LHS with respect to b_a. It yields $P_{au}\Gamma_a^H - b_a$ which is always negative. Hence, if the condition holds at the lowest feasible value of b_a, it holds for all values of b_a. To see that this is the case, notice that as $b_a \to P_{au}\Gamma_a^H$ the LHS of the inequality above converges to:

$$P_{au}(1-\Gamma_A^H)^2 W - P_{au}^2(\Gamma_A^H)^2 Z - P_{au}^2(\Gamma_A^H)^2 Z + 2P_{au}^2(\Gamma_A^H)^2 Z = P_{au}(1-\Gamma_A^H)^2 W > 0.$$

Hence the slope of the RHS of (17) is always larger than the one of (1). This implies that the two cross only once and that (17) is always tighter than (1).

Comparing (17) with (20) is much simpler. It is enough for the RHS of (20) to be larger than (17). This comparison corresponds to comparing the second terms of the RHS of each inequality. Condition (20) is looser if

$$\frac{(P_{au}-b_a)W}{P_{au}Z} \geq \frac{(P_{au}-b_a)\left(1-\Gamma_a^H\right)W}{\left(b_a - P_{au}\Gamma_a^H\right)Z}$$

which corresponds to

$$P_{au}(1-\Gamma_a^H) \geq b_a - P_{au}\Gamma_a^H \;\Rightarrow\; P_{au} \geq b_a$$

which is always true.

To conclude this part of the proof we show that the RHS of (6) is concave in b_a. To see this, consider the first derivative above,

$$\left[\frac{P_{au}(1-\Gamma_a^H)^2 WZ}{\left[\left(b_a - P_{au}\Gamma_a^H\right)Z\right]^2}\right],$$

and notice that it is decreasing in b_a. Hence, (17) identifies a concave area.[30] To see that its lower bound is always larger than $P_{au}\Gamma_a^H$, substitute $b_u = 0$ in the condition to obtain

$$0 \leq P_{uu} - \frac{(P_{au}-b_a)\left(1-\Gamma_a^H\right)W}{\left(b_a - P_{au}\Gamma_a^H\right)Z}$$

which is equivalent to

$$b_a \geq P_{au}\frac{(1-\Gamma_a^H)W + P_{uu}\Gamma_a^H Z}{(1-\Gamma_a^H)W + P_{uu}Z}.$$

To prove our claim we then show that

$$\frac{(1-\Gamma_a^H)W + P_{uu}\Gamma_a^H Z}{(1-\Gamma_a^H)W + P_{uu}Z} > \Gamma_a^H.$$

[30] Recall that the derivative is of the entire RHS not only of the second term.

With simple algebra, it is easy to see that this condition boils down to $\Gamma_a^H \leq 1$, which is always true.

This concludes this part and proves that the area identified by the feasible values of b_a and condition (6) always features the APE contract. Its shape, furthermore, always resembles the representation in Figure 2.

Part 4. Given all the above and Lemma 5 we finally solve problem (5) by setting the (TR_P^u) binding together with the (IC). This yields the following system in two equations:

$$\begin{aligned}
&c_{aa}\left(\Delta\tilde{\gamma}_{aa} + \frac{\tilde{\gamma}_{au}^H}{\tilde{\gamma}_{uu}^H}\Delta\tilde{\gamma}_{uu}\right) + c_{au}\left(\Delta\tilde{\gamma}_{au} - \frac{\tilde{\gamma}_{au}^H}{\tilde{\gamma}_{uu}^H}\Delta\tilde{\gamma}_{uu}\right) = \Delta V \\
&c_{aa} = \left(1 + \frac{\tilde{\gamma}_{uu}^H}{\tilde{\gamma}_{au}^H}\right) c_{au}
\end{aligned}$$

from which we obtain:

$$\begin{aligned}
c_{au} &= \frac{\Delta V}{\left(1 + \frac{\tilde{\gamma}_{uu}^H}{\tilde{\gamma}_{au}^H}\right)\left(\Delta\tilde{\gamma}_{aa} + \frac{\tilde{\gamma}_{au}^H}{\tilde{\gamma}_{uu}^H}\Delta\tilde{\gamma}_{uu}\right) + \Delta\tilde{\gamma}_{au} - \frac{\tilde{\gamma}_{au}^H}{\tilde{\gamma}_{uu}^H}\Delta\tilde{\gamma}_{uu}} \\
&= \frac{\Delta V}{\Delta\tilde{\gamma}_{aa} + \Delta\tilde{\gamma}_{au} + \frac{\tilde{\gamma}_{uu}^H}{\tilde{\gamma}_{au}^H}\Delta\tilde{\gamma}_{aa} + \Delta\tilde{\gamma}_{uu}} \\
&= \frac{\Delta V}{\Delta\Gamma_a\tilde{P}_{aa} + \Delta\Gamma_a\tilde{P}_{au} + \frac{\tilde{P}_{uu}\Gamma_u^H}{\tilde{P}_{au}\Gamma_a^H}\Delta\Gamma_a\tilde{P}_{aa} - \Delta\Gamma_a\tilde{P}_{uu}} \\
&= \frac{\Delta V}{\Delta\Gamma_a}\frac{\tilde{P}_{au}\Gamma_a^H}{\tilde{P}_{au}\Gamma_a^H + \tilde{P}_{uu}(1-\Gamma_a^H)\tilde{P}_{aa} - \tilde{P}_{uu}\tilde{P}_{au}\Gamma_a^H} \\
&= c_{aa}^*\frac{\tilde{P}_{au}\Gamma_a^H}{\tilde{P}_{au}\Gamma_a^H + \tilde{P}_{uu}(\tilde{P}_{aa} - \Gamma_a^H)}.
\end{aligned}$$

To conclude the proof, we obtain $c_{aa} = \left(1 + \frac{\tilde{\gamma}_{uu}^H}{\tilde{\gamma}_{au}^H}\right) c_{au}$ from the above discussion, and $c_{au} = c_{uu}$ from Lemma 10.

Part 5. To see that the APE contract features a lower deadweight loss, notice that this is equal to $\sum_{ts}(w_{ts}^* - c_{ts}^*)\gamma_{ts}^H = (w_{ua}^* - c_{ua}^*)\gamma_{ua}^H$ in a BPE contract and to $\sum_{ts}(w_{ts}^\dagger - c_{ts}^\dagger)\gamma_{ts}^H = (w_{ua}^\dagger - c_{ua}^\dagger)\gamma_{ua}^H$ in the APE contract. Since $c_{ua}^* = c_{ua}^\dagger = 0$, the deadweight loss is smaller under the APE contract if

$$
\begin{aligned}
w^*_{ua} > w^\dagger_{ua} \quad &\iff \quad \frac{\Delta V}{\Delta\Gamma_a}\frac{1}{P_{aa}} > \frac{\Delta V}{\Delta\Gamma_a}\frac{\tilde{P}_{au}\Gamma^H_a}{\tilde{P}_{au}\Gamma^H_a + \tilde{P}_{uu}(\tilde{P}_{aa} - \Gamma^H_a)}\left(1 + \frac{\tilde{\gamma}^H_{uu}}{\tilde{\gamma}^H_{au}}\right) \\
1 > P_{aa}&\frac{\tilde{P}_{au}\Gamma^H_a}{\tilde{P}_{au}\Gamma^H_a + \tilde{P}_{uu}(\tilde{P}_{aa} - \Gamma^H_a)}\frac{\tilde{P}_{au}\Gamma^H_a + \tilde{P}_{uu}\Gamma^H_u}{\tilde{P}_{au}\Gamma^H_a} \\
\tilde{P}_{au}\Gamma^H_a + \tilde{P}_{uu}(\tilde{P}_{aa} - \Gamma^H_a) &> P_{aa}\tilde{P}_{au}\Gamma^H_a + P_{aa}\tilde{P}_{uu}\Gamma^H_u \\
(1 - P_{aa})\tilde{P}_{au}\Gamma^H_a + \tilde{P}_{uu}\tilde{P}_{aa} - \tilde{P}_{uu}\Gamma^H_a - P_{aa}\tilde{P}_{uu}\Gamma^H_u &> 0 \\
P_{au}\tilde{P}_{au}\Gamma^H_a + \tilde{P}_{uu}(P_{aa} + b_a - \Gamma^H_a - P_{aa} + P_{aa}\Gamma^H_a) &> 0 \\
P_{au}\tilde{P}_{au}\Gamma^H_a + \tilde{P}_{uu}\left[b_a - (1 - P_{aa})\Gamma^H_a\right] &> 0,
\end{aligned}
$$

which is always true since in the APE contract we have $b_a \in (P_{au}\Gamma^H_a, P_{au}]$.

Proof of Proposition 5

To see this consider the proof of Lemma 5 and notice that everything follows through in this case as well until condition (16) which can never hold for $b_a < 0$.

Proof of Proposition 6

The Propositions is easily proven by studying the case of an optimistic and a pessimistic agent. Let's start from the underconfident agent. Just as a pessimistic agent, he holds $b_a < 0$ which is incompatible with (16), and therefore he always assigned a BPE contract. The overconfident agent, instead, holds beliefs that do not violate, nor add, anything to the proof of Proposition 4 and therefore is assigned an APE contract under the same conditions of an optimistic agent.

Proof of Corollary 1

While $c^\dagger_{au} < c^*_{aa}$ we also have that $c^\dagger_{aa} > c^\dagger_{au}$. Therefore the check for $c^\dagger_{aa} > c^*_{aa}$ is given by:

$$
\left(1 + \frac{\tilde{\gamma}^H_{uu}}{\tilde{\gamma}^H_{au}}\right)\left(\frac{\tilde{P}_{au}\Gamma^H_a}{\tilde{P}_{au}\Gamma^H_a + \tilde{P}_{uu}(\tilde{P}_{aa} - \Gamma^H_a)}\right) \geq 1
$$

which is equivalent to

$$
\left(\frac{\tilde{P}_{au}\Gamma^H_a + \tilde{P}_{uu}\Gamma^H_u}{\tilde{P}_{au}\Gamma^H_a}\right)\left(\frac{\tilde{P}_{au}\Gamma^H_a}{\tilde{P}_{au}\Gamma^H_a + \tilde{P}_{uu}(\tilde{P}_{aa} - \Gamma^H_a)}\right) \geq 1
$$

and to

$$\frac{\tilde{P}_{au}\Gamma_a^H + \tilde{P}_{uu}(1-\Gamma_A^H)}{\tilde{P}_{au}\Gamma_a^H + \tilde{P}_{uu}(\tilde{P}_{aa}-\Gamma_a^H)} \geq 1,$$

which is always true since $\tilde{P}_{aa} \leq 1$.

Proof of Proposition 7

Point (i) is trivial. Condition (6) comes from the study of how to minimize cost and it selects the optimal contract precisely on the basis of the lowest possible expected wage. Since both contracts are available at the moment of minimization none of the two can minimize costs when the other is optimal.

To prove point (ii), notice that

$$\begin{aligned} E(c_{ts}^*) &= c_{aa}^*\gamma_{aa}^H + c_{au}^*\gamma_{au}^H + c_{ua}^*\gamma_{ua}^H + c_{uu}^*\gamma_{uu}^H \\ &= \frac{\Delta V}{\Delta\Gamma_a}(\gamma_{aa}^H + \gamma_{au}^H) = \frac{\Delta V}{\Delta\Gamma_a}\Gamma_a^H, \end{aligned}$$

and

$$\begin{aligned} \tilde{E}(c_{ts}^*) &= c_{aa}^*\tilde{\gamma}_{aa}^H + c_{au}^*\tilde{\gamma}_{au}^H + c_{ua}^*\tilde{\gamma}_{ua}^H + c_{uu}^*\tilde{\gamma}_{uu}^H \\ &= \frac{\Delta V}{\Delta\Gamma_a}(\tilde{\gamma}_{aa}^H + \tilde{\gamma}_{au}^H) = \frac{\Delta V}{\Delta\Gamma_a}\Gamma_a^H, \end{aligned}$$

where we used the fact that $\gamma_{ta}^H + \gamma_{tu}^H = \tilde{\gamma}_{ta}^H + \tilde{\gamma}_{tu}^H = \Gamma_t^H$ (which is easily proven from Lemma 1 and Assumption 3).

Point (iii) requires us to calculate $\tilde{E}(c_{ts}^\dagger)$.

$$\begin{aligned} \tilde{E}(c_{ts}^\dagger) &= c_{aa}^\dagger\tilde{\gamma}_{aa}^H + c_{au}^\dagger\tilde{\gamma}_{au}^H + c_{ua}^\dagger\tilde{\gamma}_{ua}^H + c_{uu}^\dagger\tilde{\gamma}_{uu}^H = c_{au}^\dagger\left[\frac{\tilde{\gamma}_{au}^H + \tilde{\gamma}_{uu}^H}{\tilde{\gamma}_{au}^H}\tilde{\gamma}_{aa}^H + \tilde{\gamma}_{au}^H + \tilde{\gamma}_{uu}^H\right] \\ &= \frac{c_{au}^\dagger}{\tilde{\gamma}_{au}^H}(\tilde{\gamma}_{aa}^H + \tilde{\gamma}_{au}^H)(\tilde{\gamma}_{au}^H + \tilde{\gamma}_{uu}^H) = \frac{c_{au}^\dagger}{\tilde{\gamma}_{au}^H}\Gamma_a^H(\tilde{\gamma}_{au}^H + \tilde{\gamma}_{uu}^H) \\ &= \frac{\Delta V}{\Delta\Gamma_a}\Gamma_a^H\frac{\tilde{P}_{au}\Gamma_a^H + \tilde{P}_{uu}\Gamma_u^H}{\tilde{P}_{au}\Gamma_a^H + \tilde{P}_{uu}(\tilde{P}_{aa}-\Gamma_a^H)} = \tilde{E}(c_{ts}^*)\frac{\tilde{P}_{au}\Gamma_a^H + \tilde{P}_{uu}\Gamma_u^H}{\tilde{P}_{au}\Gamma_a^H + \tilde{P}_{uu}(\tilde{P}_{aa}-\Gamma_a^H)}. \end{aligned}$$

Since $\Gamma_u^H = 1-\Gamma_a^H$, it is clear that the numerator is at least as large as the denominator. This proves point (iii).

Finally, for point (iv), we need to calculate $E(c^{\dagger}_{ts})$.

$$E(c^{\dagger}_{ts}) = c^{\dagger}_{aa}\gamma^H_{aa} + c^{\dagger}_{au}\gamma^H_{au} + c^{\dagger}_{ua}\gamma^H_{ua} + c^{\dagger}_{uu}\gamma^H_{uu} = c^{\dagger}_{au}\left[\frac{\tilde{\gamma}^H_{au} + \tilde{\gamma}^H_{uu}}{\tilde{\gamma}^H_{au}}\gamma^H_{aa} + \gamma^H_{au} + \gamma^H_{uu}\right]$$

$$= \frac{\Delta V}{\Delta\Gamma_a}\frac{\gamma^H_{aa}\tilde{\gamma}^H_{au} + \tilde{\gamma}^H_{uu}\gamma^H_{aa} + \gamma^H_{au}\tilde{\gamma}^H_{au} + \gamma^H_{uu}\tilde{\gamma}^H_{au}}{\tilde{P}_{au}\Gamma^H_a}\frac{\tilde{P}_{au}\Gamma^H_a}{\tilde{P}_{au}\Gamma^H_a + \tilde{P}_{uu}(\tilde{P}_{aa} - \Gamma^H_a)}$$

$$= \frac{\Delta V}{\Delta\Gamma_a}\frac{\tilde{\gamma}_{au}\Gamma^H_a + \tilde{\gamma}_{uu}P_{aa}\Gamma^H_a + \gamma^H_{uu}\tilde{P}_{au}\Gamma^H_a}{\tilde{P}_{au}\Gamma^H_a + \tilde{P}_{uu}(\tilde{P}_{aa} - \Gamma^H_a)} = E(c^*_{ts})\frac{\tilde{P}_{au}\Gamma^H_a + \tilde{P}_{uu}P_{aa}\Gamma^H_u + P_{uu}\tilde{P}_{au}\Gamma^H_u}{\tilde{P}_{au}\Gamma^H_a + \tilde{P}_{uu}(\tilde{P}_{aa} - \Gamma^H_a)}.$$

Hence, to prove our result we are left to show that

$$\frac{\tilde{P}_{au}\Gamma^H_a + \tilde{P}_{uu}P_{aa}\Gamma^H_u + P_{uu}\tilde{P}_{au}\Gamma^H_u}{\tilde{P}_{au}\Gamma^H_a + \tilde{P}_{uu}(\tilde{P}_{aa} - \Gamma^H_a)} > 1$$

which is equivalent to

$$\tilde{P}_{uu}P_{aa}\Gamma^H_u + P_{uu}\tilde{P}_{au}\Gamma^H_u \geq \tilde{P}_{uu}(\tilde{P}_{aa} - \Gamma^H_a).$$

This requires some algebra.

$$\tilde{P}_{uu}P_{aa}(1 - \Gamma^H_a) + P_{uu}(1 - \tilde{P}_{aa})(1 - \Gamma^H_a) - \tilde{P}_{uu}\tilde{P}_{aa} + \tilde{P}_{uu}\Gamma^H_a \geq 0$$

$$\tilde{P}_{uu}P_{aa} - \tilde{P}_{uu}P_{aa}\Gamma^H_a + P_{uu} - P_{uu}\tilde{P}_{aa} - P_{uu}\Gamma^H_a + P_{uu}\tilde{P}_{aa}\Gamma^H_a - \tilde{P}_{uu}\tilde{P}_{aa} + \tilde{P}_{uu}\Gamma^H_a \geq 0$$

$$\tilde{P}_{uu}P_{aa} - P_{uu}\tilde{P}_{aa} - \tilde{P}_{uu}\tilde{P}_{aa} - P_{uu}\Gamma^H_a + \tilde{P}_{uu}\Gamma^H_a - \tilde{P}_{uu}P_{aa}\Gamma^H_a + P_{uu}\tilde{P}_{aa}\Gamma^H_a + P_{uu} \geq 0$$

From here, we substitute for some of the $\tilde{P}_{ts}$ to get

$$(\tilde{P}_{uu}P_{aa} - P_{uu}\tilde{P}_{aa} - \tilde{P}_{uu}P_{aa} - \tilde{P}_{uu}b_a) + (-P_{uu}\Gamma^H_a + P_{uu}\Gamma^H_a - b_u\Gamma^H_a) +$$

$$+ (-P_{uu}P_{aa}\Gamma^H_a + b_uP_{aa}\Gamma^H_a + P_{uu}P_{aa}\Gamma^H_a + P_{uu}b_a\Gamma^H_a) + P_{uu} \geq 0$$

and finally

$$
\begin{aligned}
&- P_{uu}\tilde{P}_{aa} - \tilde{P}_{uu}b_a - b_u\Gamma_a^H + P_{uu}b_a\Gamma_a^H + b_uP_{aa}\Gamma_a^H + P_{uu} \geq 0 \\
&- P_{uu}P_{aa} - P_{uu}b_a - P_{uu}b_a + b_ub_a - b_u\Gamma_a^H + P_{uu}b_a\Gamma_a^H + b_uP_{aa}\Gamma_a^H + P_{uu} \geq 0 \\
&- P_{uu}\underbrace{(P_{aa} + b_a)}_{1-\tilde{P}_{au}} - P_{uu}b_a + b_ub_a - b_u\Gamma_a^H + P_{uu}b_a\Gamma_a^H + b_uP_{aa}\Gamma_a^H + P_{uu} \geq 0 \\
&b_u\left(b_a - \Gamma_a + P_{aa}\Gamma_a^H\right) + P_{uu}\left[1 - b_a(1-\Gamma_a^H) - (1-\tilde{P}_{au})\right] \geq 0 \\
&b_u\left(b_a - P_{au}\Gamma_a^H\right) \geq P_{uu}\left[b_a(1-\Gamma_a^H) - P_{au} + b_a\right] \\
&b_u\left(b_a - P_{au}\Gamma_a^H\right) \geq P_{uu}\left(b_a(1+\Gamma_u^H) - P_{au}\right).
\end{aligned}
$$

Notice now that the APE requires $b_a > P_{au}\Gamma_a$ as described in the proof of Proposition 4. This means that the LHS is always positive and we can therefore derive the condition presented in the Proposition.

Proof of Proposition 8

First we study condition (9).

$$b_u \geq P_{uu}\frac{b_a(1+\Gamma_u^H) - P_{au}}{b_a - P_{au}\Gamma_a^H}$$

At $b_u = 0$, condition (9) corresponds to $b_a < P_{au}/(1+\Gamma_u^H)$. Hence, $P_{au}/(1+\Gamma_u^H)$ is the intercept of the RHS of the condition with the x-axis. Let

$$P_{au}/(1+\Gamma_u^H) \equiv \underline{b}_a.$$

To show that this condition is compatible with (6), we once again refer to (17), i.e. (6) solved for b_u, and therefore that an area where optimism is socially desirable always exists. We need to show that $\underline{b}_a$ is larger than the intercept of condition (17) (holding with equality) with the x-axis. We start from the latter, which we already calculated in Part 3 of the proof to Proposition 4.

$$b_a = P_{au}\frac{(1-\Gamma_a^H)W + P_{uu}\Gamma_a^H Z}{(1-\Gamma_a^H)W + P_{uu}Z}.$$

We then need to show that

$$P_{au}\frac{(1-\Gamma_a^H)W+P_{uu}\Gamma_a^H Z}{(1-\Gamma_a^H)W+P_{uu}Z}<\frac{P_{au}}{(1+\Gamma_u^H)}.$$

To do this, we get

$$\begin{aligned}
&(1-\Gamma_a^H)W+P_{uu}Z>(1-\Gamma_a^H)W+P_{uu}\Gamma_a^H Z+(1-\Gamma_a^H)W\Gamma_u^H+P_{uu}\Gamma_a^H Z\Gamma_u^H\\
&P_{uu}(1-\Gamma_a^H)Z-(1-\Gamma_a^H)W\Gamma_u^H-P_{uu}\Gamma_a^H\Gamma_u^H Z>0\\
&P_{uu}\Gamma_u^H Z-W(\Gamma_u^H)^2-P_{uu}\Gamma_a^H\Gamma_u^H Z>0\\
&P_{uu}\Gamma_u^H Z\underbrace{(1-\Gamma_a^H)}_{\Gamma_u^H}-W(\Gamma_u^H)^2>0\;\Rightarrow\;P_{uu}Z-W>0
\end{aligned}$$

We can now expand Z and W to get

$$\begin{aligned}
&P_{uu}Z-W>0\\
&P_{uu}P_{aa}\Gamma_a^H+P_{uu}P_{ua}\Gamma_u^H-\Gamma_a^H P_{aa}+\Gamma_a^H P_{ua}>0\\
&-P_{aa}\Gamma_a^H+P_{uu}\Gamma_u^H+\Gamma_a^H>0\\
&P_{uu}\Gamma_u^H+\Gamma_a^H(1-P_{aa})>0,
\end{aligned}$$

which is obviously always true. This proves that an area where optimism is socially desirable always exists, at least for $b_u = 0$. We now show that this area also exists for positive values of b_u. Since we know that the curve representing condition (6) intercepts the x-axis before (9), it is enough to show that the loci of points where the two conditions hold cross only once in (b_a, b_u) space and that they do so at $(b_a, b_u) = (P_{au}, P_{uu})$. To formally prove the shape of Figure 4, we are also going to show that the locus where (9) binds is concave in (b_a, b_u) space.

Take the two conditions binding and equate the two RHSs to get:

$$
\begin{aligned}
&P_{uu}\frac{b_a(1+\Gamma_u^H)-P_{au}}{b_a-P_{au}\Gamma_a^H}=P_{uu}-\frac{(P_{au}-b_a)\left(1-\Gamma_a^H\right)W}{(b_a-P_{au}\Gamma_a^H)\,Z}\\
&P_{uu}\left[(b_a(1+\Gamma_u^H)-P_{au})Z-(b_a-P_{au}\Gamma_a^H)Z\right]=-\left(P_{au}-b_a\right)\left(1-\Gamma_a^H\right)W\\
&P_{uu}(b_a-P_{au})\Gamma_u^H Z=-\left(P_{au}-b_a\right)\left(1-\Gamma_a^H\right)W\\
&(P_{uu}Z-W)(b_a-P_{au})=0\\
&(P_{uu}P_{aa}\Gamma_a^H+P_{uu}P_{ua}\Gamma_u^H-P_{aa}\Gamma_a^H+P_{ua}\Gamma_a^H)(b_a-P_{au})=0\\
&P_{ua}(P_{au}\Gamma_a^H+P_{uu}\Gamma_u^H)(b_a-P_{au})=0,
\end{aligned}
$$

which holds only if $b_a = P_{au}$.When plugged into any of the two conditions we get that the corresponding value is $b_u = P_{uu}$. Hence the two curves cross only at that point. This concludes the proof of the Proposition. To show that the RHS of (9) is concave simply calculate the first derivative and obtain:

$$
\begin{aligned}
\frac{\partial}{\partial b_a}\left[P_{uu}\frac{b_a(1+\Gamma_u^H)-P_{au}}{b_a-P_{au}\Gamma_a^H}\right]&P_{uu}\frac{b_a(1+\Gamma_u^H)-P_{au}\Gamma_a^H(1+\Gamma_u^H)-b_a(1+\Gamma_u^H)+P_{au}}{(b_a-P_{au}\Gamma_a^H)^2}\\
=&P_{uu}P_{au}\frac{1-\Gamma_a^H(1+\Gamma_u^H)}{(b_a-P_{au}\Gamma_a^H)^2}P_{uu}=P_{au}\frac{1-2\Gamma_a^H+(\Gamma_a^H)^2}{(b_a-P_{au}\Gamma_a^H)^2}=P_{uu}P_{au}\frac{(1-\Gamma_a^H)^2}{(b_a-P_{au}\Gamma_a^H)^2}>0.
\end{aligned}
$$

The second derivative is obviously negative since b_a only appears at the denominator.

Proof of Proposition 9

To prove the Proposition, we present the derivations for the optimal contract implementing λ^L for an unbiased and an optimistic agent who believes signals to be positively correlated. At the end of this proof we discuss the case of an overconfident agent. First of all, recall that SPEs are positively correlated regardless of the effort exerted (Assumption 2).

The problem the principal faces is the same as (2) with a reversed (IC) and γ_{ts}^L instead of γ_{ts}^H for all t and s.

$$\min_{\{w_{ts},c_{ts}\}_{t,s\in\{u,a\}}} \quad w_{aa}\gamma_{aa}^L + w_{au}\gamma_{au}^L + w_{ua}\gamma_{ua}^L + w_{uu}\gamma_{uu}^L \tag{21}$$

$$\text{s.t.} \quad \sum_{ts} c_{ts}\tilde{\gamma}_{ts}^L - V(\lambda^L) \geq \bar{u} \tag{PC}$$

$$\sum_{ts} c_{ts}\Delta\tilde{\gamma}_{ts} \leq \Delta V \tag{IC}$$

$$w_{aa}\gamma_{aa}^L + w_{au}\gamma_{au}^L \leq w_{ua}\gamma_{aa}^L + w_{uu}\gamma_{au}^L \tag{TR_P^a}$$

$$w_{ua}\gamma_{ua}^L + w_{uu}\gamma_{uu}^L \leq w_{aa}\gamma_{ua}^L + w_{au}\gamma_{uu}^L \tag{TR_P^u}$$

$$c_{aa}\tilde{\gamma}_{aa}^L + c_{ua}\tilde{\gamma}_{ua}^L \geq c_{au}\tilde{\gamma}_{aa}^L + c_{uu}\tilde{\gamma}_{ua}^L \tag{TR_A^a}$$

$$c_{au}\tilde{\gamma}_{au}^L + c_{uu}\tilde{\gamma}_{uu}^L \geq c_{aa}\tilde{\gamma}_{au}^L + c_{ua}\tilde{\gamma}_{uu}^L \tag{TR_A^u}$$

$$w_{ts} \geq c_{ts} \geq 0 \qquad \forall t,s \in\{a,u\}. \tag{LL_{ts}}$$

Lemma 13. *For all b_A and b_u, low effort can be implemented by the principal with a truth-telling, budget-balancing contract $w_{ts}^\ell = c_{ts}^\ell = V(\lambda^L) + \bar{u}$. Contract $\left\{w_{ts}^\ell, c_{ts}^\ell\right\}_{t,s}$ is optimal among all budget-balancing contracts. It is also guile-free.*

Proof. First, notice that Lemma 3 and 4 hold also for the case of low effort implementation, given Assumption 2 and $\frac{\tilde{\gamma}_{ua}^L}{\tilde{\gamma}_{aa}^L} > \frac{\tilde{\gamma}_{uu}^L}{\tilde{\gamma}_{au}^L}$. Hence, following the same logic behind the proof of Proposition 1, any budget balancing contract must feature $w_{ts} = c_{ts} = c$ for all t and s. Contrary to the case of high effort implementation, however, a contract like this always satisfies the (IC) since

$$c\left(\sum_{ts} \Delta\tilde{\gamma}_{ts}\right) = 0 < \Delta V.$$

All truthful reporting constraints also hold, as well as the (LL) ones. The participation constraint becomes

$$c\left(\sum_{ts} \tilde{\gamma}_{ts}^L\right) \geq V(\lambda^L) + \bar{u} \quad \Rightarrow \quad c \geq V(\lambda^L) + \bar{u}.$$

Since the objective function is now simply $\min_{ts} c$ then the restricted problem is solved by $c = V(\lambda^L) + \bar{u}$. ∎

The Lemma above yields the only solution for an unbiased agent. However, since our worker can be optimistic, we have to check whether the principal can find a way to manipulate the contract taking advantage of the worker's biased beliefs.[31] In other words, a contract that grants $\tilde{E}(c_{ts}) = \bar{u} + V(\lambda^L)$ but that in fact yields (and costs) less, i.e. $E(c_{ts}) < \bar{u} + V(\lambda^L) = \tilde{E}(c_{ts})$. Lemma 13 states that, if such a contract exists, it must feature some deadweight loss. This is because Proposition 1 does not apply here and Lemma 13 shows that $\left\{w^{\ell}_{ts}, c^{\ell}_{ts}\right\}_{t,s}$ is the *only optimal* contract featuring no deadweight loss. This implies that, if there were to exist another contract implementing optimally low effort, this ought to feature some deadweight loss. This result is key for this appendix.

A second key feature of this appendix is that we are going to assume again Assumption 4. What this does it to allow us to ignore the (IC) and solve the problem without it.[32]

We are now going to present a series of Lemmas, from 14 to 19, to prove that, under (3), $\{w^{\ell}_{ts}, c^{\ell}_{ts}\}_{t,s}$ is the only optimal contract to implement low effort.

Lemma 14. *Given* (3)*, for any value of the bias, when the principal implements low effort, the* (PC) *always binds.*

Proof. Suppose not, the principal can decrease all w_{ts} and all c_{ts} by $\epsilon > 0$. All the other constraints are unchanged and cost of implementation decreases. This does not fully prove the statement, however. It could be that $w_{ts} = c_{ts} = 0$ for some t and s, in which case, the principal cannot decrease them all. We need to prove that a deviation is possible in these cases as well.

[31] To see that an unbiased agent is always assigned $\left\{w^{\ell}_{ts}, c^{\ell}_{ts}\right\}_{t,s}$, notice that the (PC) always binds, as we argue below. Hence, an unbiased agent must be granted at least $\bar{u} + V(\lambda^L)$. Since worker and principal have the same beliefs, there is no room for the principal to manipulate the contract trying to decrease $E(w_{ts})$ below $\bar{u} + V(\lambda^L)$.

[32] Numerical simulations show that for a very large $\bar{u}$, or a very low ΔV, low effort is implemented by a BPE-like contract, the values of which are independent of b_a and b_u as in the case of high effort implementation.

Recall that the (IC) is assumed slack when (3) holds and let us rewrite the (TR) constraints as in the proofs of Lemmas 3 and 4.

$$(w_{au} - w_{uu})\frac{\gamma_{au}^L}{\gamma_{aa}^L} \leq (w_{ua} - w_{aa}) \leq (w_{au} - w_{uu})\frac{\gamma_{uu}^L}{\gamma_{ua}^L} \qquad (TR_P)$$

$$(c_{uu} - c_{ua})\frac{\tilde{\gamma}_{ua}^L}{\tilde{\gamma}_{aa}^L} \leq (c_{aa} - c_{au}) \leq (c_{uu} - c_{ua})\frac{\tilde{\gamma}_{uu}^L}{\tilde{\gamma}_{au}^L}. \qquad (TR_A)$$

By Lemma 13, we know that at least one $w_{ts} > c_{ts}$ must hold, otherwise any contract derived would be dominated by $\{w_{ts}^\ell, c_{ts}^\ell\}_{t,s}$. Also, from From (TR_A), if only one of the c_{ts} is strictly positive then at least one of the two inequalities will fail. Hence, at least two c_{ts} must be positive for the (TR_A) to hold. Further, notice that they must be c_{aa} and c_{au} or c_{uu} and c_{ua} or c_{uu} and c_{aa}. Suppose c_{aa} and c_{au} are positive, with $c_{ua} = c_{uu} = 0$, then given Lemma 3 at least $w_{ua} > 0$ must hold. The principal can decrease c_{aa}, c_{au}, w_{aa} and w_{au} by $\epsilon \in [0, w_{ua} - c_{ua})$, while also decreasing w_{ua}, to adjust for the (TR_P) to hold. This does not violate any constraints and decreases costs. The symmetric logic holds for c_{uu} and c_{ua} positive and $c_{aa} = c_{au} = 0$. For the case of c_{uu} and c_{aa} positive and $c_{ua} = c_{au} = 0$, instead, notice that, from (TR_P), it must be that w_{ua} and w_{au} are greater then zero. The principal can then decrease c_{uu} and c_{aa} in a way that (TR_A) is not violated. Further, she decreases by the same amount w_{uu} and w_{aa}, and uses a decrease in w_{ua} and w_{au} to adjust the (TR_P) to the new values of w_{uu} and w_{aa}.

Now suppose only one of the c_{ts} is 0. Notice that, from (TR_A), it can only be either the c_{au} or the c_{ua}. Suppose it is c_{au} then, by Lemma 3, $w_{au} > 0$. The principal can then decrease c_{uu}, c_{ua}, w_{uu}, w_{ua} by $\epsilon \in [0, w_{au} - c_{au})$ and use the w_{au} to adjust for the (TR_P) to hold. This does not violate any constraints and decreases costs. The symmetric logic holds for c_{ua}. This concludes the proof. ∎

Using the symmetric versions of the algebra used in all other derivations, we solve the (PC) for c_{aa} and rewrite the (TR_A) constraints.

$$c_{aa} = \frac{1}{\tilde{\gamma}_{aa}^L}\left(\bar{u} + V(\lambda^L) - c_{au}\tilde{\gamma}_{au}^L - c_{uu}\tilde{\gamma}_{uu}^L - c_{ua}\tilde{\gamma}_{ua}^L\right)$$

$$\Rightarrow \quad \bar{u} + V(\lambda^L) \geq c_{au}\Gamma_a^L + c_{uu}\Gamma_u^L \qquad (TR_A^a)$$

$$\Rightarrow \quad c_{au}\tilde{\gamma}_{au}^L + c_{uu}\tilde{\gamma}_{uu}^L - c_{ua}\left(\tilde{P}_{aa} - \tilde{P}_{ua}\right)\Gamma_u^L \geq \left(\bar{u} + V(\lambda^L)\right)\tilde{P}_{au} \qquad (TR_A^u)$$

The new problem is therefore

$$\min_{\{w_{ts},c_{ts}\}_{t,s\in\{u,a\}}} w_{aa}\gamma^L_{aa} + w_{au}\gamma^L_{au} + w_{ua}\gamma^L_{ua} + w_{uu}\gamma^L_{uu} \tag{22}$$

$$\text{s.t.} \quad (w_{au} - w_{uu})\frac{\gamma^L_{au}}{\gamma^L_{aa}} \leq (w_{ua} - w_{aa}) \tag{TR^a_P}$$

$$(w_{ua} - w_{aa}) \leq (w_{au} - w_{uu})\frac{\gamma^L_{uu}}{\gamma^L_{ua}} \tag{TR^u_P}$$

$$\bar{u} + V(\lambda^L) \geq c_{au}\Gamma^L_a + c_{uu}\Gamma^L_u \tag{TR^a_A}$$

$$c_{au}\tilde{\gamma}^L_{au} + c_{uu}\tilde{\gamma}^L_{uu} - c_{ua}\left(\tilde{P}_{aa} - \tilde{P}_{ua}\right)\Gamma^L_u \geq \left(\bar{u} + V(\lambda^L)\right)\tilde{P}_{au} \tag{TR^u_A}$$

$$w_{aa} \geq c_{aa} = \frac{1}{\tilde{\gamma}^L_{aa}}(\bar{u} + V(\lambda^L) - c_{au}\tilde{\gamma}^L_{au} - c_{uu}\tilde{\gamma}^L_{uu} - c_{ua}\tilde{\gamma}^L_{ua}) \geq 0 \tag{LL_{aa}}$$

$$w_{au} \geq c_{au} \geq 0 \tag{LL_{au}}$$

$$w_{uu} \geq c_{uu} \geq 0 \tag{LL_{uu}}$$

$$w_{ua} \geq c_{ua} \geq 0 \tag{LL_{ua}}$$

Lemma 15. *Given* (3)*, for any value of the bias, when the principal implements low effort with a contract featuring a deadweight loss, the* (TR^u_A) *always binds.*

Proof. Suppose not. The proof changes depending on which one among the (TR_P) binds at optimum.

Suppose the (TR^a_P) binds, while the (TR^u_P) is slack, then the principal can decrease c_{au} and w_{au} by ϵ. This relaxes the (TR^a_A). At the same time, it increases c_{aa} by $\epsilon\frac{\tilde{P}_{au}}{\tilde{P}_{aa}}$ and w_{aa} by a amount at most as large. This affects the (TR_P). The latter changes to the LHS by $-\epsilon\frac{P_{au}}{P_{aa}}$ and to the RHS by $-\epsilon\frac{\tilde{P}_{au}}{\tilde{P}_{aa}}$. The RHS changes less since, for an optimistic agent, $\tilde{P}_{au} < P_{au}$ and $\tilde{P}_{aa} > P_{aa}$. The (TR^u_P) is tightened by the change but, since it is assumed slack, there always exists an ϵ small enough for it to still hold. Finally, to see that this deviation is optimal, notice that the change in the objective function is given by

$$-\epsilon\gamma^L_{au} + \epsilon\frac{\gamma^L_{aa}\tilde{P}_{au}}{\tilde{P}_{aa}} = \epsilon\frac{\Gamma^L_a}{\tilde{P}_{aa}}(P_{aa}\tilde{P}_{au} - \tilde{P}_{aa}P_{au}),$$

which is negative since $\tilde{P}_{aa} > P_{aa}$ and $\tilde{P}_{au} < P_{au}$.

Now suppose it is the (TR^u_P) that binds while the (TR^a_P) is slack, then the principal can decrease c_{uu} and w_{uu} by ϵ. This relaxes the (TR^a_A). At the same time, it increases

c_{aa} by $\epsilon\frac{\tilde{\gamma}^L_{uu}}{\tilde{\gamma}^L_{aa}}$ and w_{aa} by an amount at most as large. This affects the (TR_P) constraints. First, it relaxes the (TR^u_P). It tightens the (TR^a_P) but since the latter is assumed slack, there always exist an ϵ small enough for it to still hold. Finally, to see that this deviation is optimal, notice that the change in the objective function is given by

$$-\epsilon\gamma^L_{uu} + \epsilon\frac{\gamma^L_{aa}\tilde{\gamma}^L_{uu}}{\tilde{\gamma}^L_{aa}} = \epsilon\frac{\Gamma^L_u}{\tilde{P}_{aa}}(P_{aa}\tilde{P}_{uu} - \tilde{P}_{aa}P_{uu}),$$

which is negative since $\tilde{P}_{aa} > P_{aa}$ and $\tilde{P}_{uu} < P_{uu}$.

Finally suppose that both (TR_P) bind, that is $w_{aa} = w_{ua}$ and $w_{au} = w_{uu}$. Given Lemma 13, it must be that either $w_{aa} > c_{aa}$ or $w_{ua} > c_{ua}$ (or both). When $w_{aa} > c_{aa}$, the principal can decrease c_{au}, c_{uu}, w_{au} and w_{uu} keeping w_{aa} constant. There always exists an ϵ small enough for this to be possible. This is obviously optimal and it relaxes all other constraints. When $w_{ua} > c_{ua}$, the principal can increase c_{ua} by ϵ. This will decrease c_{aa} and w_{aa} by $\epsilon\frac{\tilde{\gamma}^L_{ua}}{\tilde{\gamma}^L_{aa}}$ which, by the assumption on the (TR_P) forces the w_{ua} down by the same amount. Regardless of whether $\epsilon\frac{\tilde{\gamma}^L_{ua}}{\tilde{\gamma}^L_{aa}}$ is larger or smaller than ϵ, there always exists an ϵ small enough for $w_{ua} > c_{ua}$ to be preserved.[33] ∎

By Lemma 15 we can solve for c_{ua} from the (TR^u_A),

$$c_{ua} = \frac{c_{au}\tilde{\gamma}^L_{au} + c_{uu}\tilde{\gamma}^L_{uu} - \left(\bar{u} + V(\lambda^L)\right)\tilde{P}_{au}}{\left(\tilde{P}_{aa} - \tilde{P}_{ua}\right)\Gamma^L_u},$$

and plug it into the function for c_{aa} from the (PC) to obtain

$$c_{aa} = \frac{\left(\bar{u} + V(\lambda^L)\right)\tilde{P}_{uu} - c_{au}\tilde{\gamma}^L_{au} - c_{uu}\tilde{\gamma}^L_{uu}}{\left(\tilde{P}_{aa} - \tilde{P}_{ua}\right)\Gamma^L_a}.$$

The above poses the following additional restrictions on c_{au} and c_{uu} respectively (via the (LL_{ts}) constraints):

$$(\bar{u} + V(\lambda^L))\tilde{P}_{au} \leq c_{au}\tilde{\gamma}^L_{au} + c_{uu}\tilde{\gamma}^L_{uu},$$
$$(\bar{u} + V(\lambda^L))\tilde{P}_{uu} \geq c_{au}\tilde{\gamma}^L_{au} + c_{uu}\tilde{\gamma}^L_{uu}.$$

[33] Of course, if no (TR_P) binds, the principal can decrease c_{au}, c_{uu}, w_{au} and w_{uu} and the increasing effect on w_{aa} is not enough to offset the gain, in the same fashion as above.

Rearranging the second equation we have

$$\bar{u} + V(\lambda^L) \geq c_{au}\frac{\tilde{P}_{au}}{\tilde{P}_{uu}}\Gamma_a^L + c_{uu}\Gamma_u^L.$$

Since $\tilde{P}_{au} < \tilde{P}_{uu}$ by positive correlation, this restriction is implied by the (TR_A^a) and can be, therefore, disregarded. It also proves that c_{aa} is always at least weakly positive and that it is strictly positive as long as $c_{au} > 0$. Let us decompose each (LL_{ts}) into (LL_{ts}^1), which requires $c_{ts} > 0$, and (LL_{ts}^2), which requires $w_{ts} > c_{ts}$. The above implies that (LL_{aa}^1) is implied by the other constraints. The new problem of the principal is given by

$$\min_{\{w_{ts},c_{ts}\}_{t,s\in\{u,a\}}} w_{aa}\gamma_{aa}^L + w_{au}\gamma_{au}^L + w_{ua}\gamma_{ua}^L + w_{uu}\gamma_{uu}^L \tag{23}$$

$$\text{s.t.} \quad (w_{au} - w_{uu})\frac{P_{au}}{P_{aa}} \leq (w_{ua} - w_{aa}) \tag{TR_P^a}$$

$$(w_{ua} - w_{aa}) \leq (w_{au} - w_{uu})\frac{P_{uu}}{P_{ua}} \tag{TR_P^u}$$

$$\bar{u} + V(\lambda^L) \geq c_{au}\Gamma_a^L + c_{uu}\Gamma_u^L \tag{TR_A^a}$$

$$\bar{u} + V(\lambda^L) \leq c_{au}\Gamma_a^L + c_{uu}\frac{\tilde{P}_{uu}}{\tilde{P}_{au}}\Gamma_u^L \tag{LL_{ua}^1}$$

$$w_{au} \geq c_{au} \geq 0 \tag{LL_{au}}$$

$$w_{uu} \geq c_{uu} \geq 0 \tag{LL_{uu}}$$

$$w_{ua} \geq c_{ua} \tag{LL_{ua}^2}$$

$$w_{aa} \geq c_{aa} \tag{LL_{aa}^2}$$

Lemma 16. *Given* (3)*, for any value of the bias, if there exists a contract featuring a deadweight loss that the principal optimally sets to implement low effort, it features the* (TR_P^a) *binding.*

Proof. First of all, notice that the Lemma does not rule out the case that *suboptimal* contracts can implement low effort with the (TR_P^a) slack. Rather, it states that there exist no generally optimal way to implement low effort with a contract featuring a deadweight loss and the (TR_P^a) slack.

Suppose this is not true and let the (TR_P^a) be slack. Recall that constraint (LL_{ua}^1) ensures that $c_{ua} \geq 0$. When it binds, $c_{ua} = 0$. Suppose the (LL_{ua}^1) does bind and

$c_{ua} = 0$. This also implies that $w_{ua} > 0$. To see why, notice that, if $w_{ua} = 0$, then also $w_{aa} = 0$, by Lemma 3. Hence, c_{au} and c_{uu} have to be such that $c_{aa} = 0$ from the (PC) binding. This then implies

$$c_{ua} = \frac{\left(\bar{u} + V(\lambda^L)\right)(\tilde{P}_{uu} - \tilde{P}_{au})}{\left(\tilde{P}_{aa} - \tilde{P}_{ua}\right)\Gamma_u^L} > 0.$$

Hence, when (LL_{ua}^1) binds, it must be that $w_{ua} > 0$. It is immediate to see how the (TR_P^a) cannot be slack then, since the principal could simply decrease w_{ua} and decrease the objective function tightening the (TR_P^a).

Now suppose the (LL_{ua}^1) is slack. This implies that $c_{ua} > 0$ and, by Lemma 4, also $c_{uu} > 0$. The proof further divides depending on whether $c_{au} = 0$ or not.

Suppose $c_{au} = 0$. We are going to show that any solution either sets the (LL_{ua}^1) binding or is suboptimal to the no deadweight loss contract. First of all, it must be that $w_{au} > 0$. Otherwise $w_{uu} = 0$ would be implied by Lemma 3. Then $c_{uu} = 0$ would make $c_{ua} < 0$. Hence if $c_{au} = 0$, $w_{au} > 0$. Given this, the principal can decrease w_{au} until (TR_P^u) binds. Hence,

$$w_{au} = (w_{ua} - w_{aa})\frac{P_{ua}}{P_{uu}} + w_{uu}.$$

Further, all other w_{ts} are set equal to their respective c_{ts}. In fact, suppose this was not the case. If $w_{uu} > c_{uu}$ or $w_{ua} > c_{ua}$, the principal can simply decrease them, without violating any constraint. If $w_{aa} > c_{aa}$, instead, the principal can decrease it by ϵ while increasing w_{au} by $\epsilon\frac{P_{ua}}{P_{uu}}$. This does not violate the (TR_P^u) and it is optimal since the change in the objective function is given by

$$-\epsilon\left(\gamma_{aa}^L - \frac{P_{au}P_{ua}}{P_{uu}}\Gamma_a^L\right) = -\epsilon\frac{\Gamma_a^L(P_{aa} - P_{ua})}{P_{uu}} < 0.$$

To conclude this part of the proof, notice that given $w_{uu} = c_{uu}$, $w_{aa} = c_{aa}$, $w_{ua} = c_{ua}$, $w_{au} = (w_{ua} - w_{aa})\frac{P_{ua}}{P_{uu}} + w_{uu}$ and the fact that all c_{ts} can be written as a function of c_{uu}, the objective function depends only on c_{uu} and is subject to

$$c_{uu} \leq \frac{\bar{u} + V(\lambda^L)}{\Gamma_u^L}. \qquad (TR_A^a)$$

Depending on the sign of the coefficient of c_{uu} in the objective function, if the latter is minimized by minimizing c_{uu}, the assumption that the (LL_{ua}^1) is slack would be violated, yielding a contradiction. If it is minimized by maximizing c_{uu}, then (TR_A^a) binds and $c_{uu} = \frac{\bar{u}+V(\lambda^L)}{\Gamma_u^L}$. At this value, the rest of compensations and wages are given by

$$\begin{array}{llll} w_{aa} = 0 & w_{au} = \frac{c_{uu}}{P_{uu}} & w_{uu} = c_{uu} & w_{ua} = c_{ua} \\ c_{aa} = 0 & c_{au} = 0 & c_{uu} = \frac{\bar{u}+V(\lambda^L)}{\Gamma_u^L} & c_{ua} = c_{uu} \end{array}$$

This implies a contract with $E(w_{ts}) = w_{au}\gamma_{au}^L + \bar{u} + V(\lambda^L)$ which is clearly larger than $E(w_{ts}^\ell)$.[34] Hence, even if this contract implements λ^L, it is never optimal.

To conclude the proof, we derive a similar contradiction for the case of $c_{au} > 0$. In this case, when both (LL_{ua}^1) and (TR_P^a) are slack, the principal faces

$$\min_{\{w_{ts},c_{ts}\}_{t,s\in\{u,a\}}} \quad w_{aa}\gamma_{aa}^L + w_{au}\gamma_{au}^L + w_{ua}\gamma_{ua}^L + w_{uu}\gamma_{uu}^L \tag{24}$$

$$\text{s.t.} \quad (w_{ua} - w_{aa}) \leq (w_{au} - w_{uu})\frac{P_{uu}}{P_{ua}} \tag{TR_P^u}$$

$$\bar{u} + V(\lambda^L) \geq c_{au}\Gamma_a^L + c_{uu}\Gamma_u^L \tag{TR_A^a}$$

$$w_{au} \geq c_{au} \tag{LL_{au}^2}$$

$$w_{uu} \geq c_{uu} \tag{LL_{uu}^2}$$

$$w_{ua} \geq c_{ua} \tag{LL_{ua}^2}$$

$$w_{aa} \geq c_{aa} \tag{LL_{aa}^2}$$

where all $c_{ts} > 0$ following Lemma 4. It is immediate to see how the (LL_{uu}^2) and (LL_{ua}^2) bind in this case. If they do not, decreasing the relevant w_{ts} decreases costs and does not affect any constraint. At this point, from Lemma 13, we know that only one between (LL_{aa}^2) and (LL_{au}^2) binds. Suppose it is the latter, then $w_{aa} > c_{aa}$. In this case, the the principal can decrease c_{uu} until (LL_{ua}^1) binds, violating the assumption. Suppose instead, that $w_{aa} = c_{aa}$ while $w_{au} > c_{au}$. The (TR_P^u) becomes

$$\frac{c_{au}\tilde{\gamma}_{au}^L + c_{uu}\tilde{\gamma}_{uu}^L}{\left(\tilde{P}_{aa} - \tilde{P}_{ua}\right)\Gamma_u^L\Gamma_a^L} + c_{uu}\frac{P_{uu}}{P_{ua}} - \frac{\bar{u} + V(\lambda^L)}{\left(\tilde{P}_{aa} - \tilde{P}_{ua}\right)\Gamma_u^L\Gamma_a^L}\left(\tilde{\gamma}_{au}^L + \tilde{\gamma}_{uu}^L\right) \leq w_{au}.$$

[34] Technically, to be sure that this is indeed a potential solution, we need to check that it satisfies the (IC). It is easy to see that it does so even regardless of (3), since

$$\sum_{ts}\Delta\tilde{\gamma}_{ts}c_{ts} = \frac{(\bar{u}+V(\lambda))}{\Gamma_u^L}(\Delta\tilde{\gamma}_{uu} + \Delta\tilde{\gamma}_{ua}) = \frac{(\bar{u}+V(\lambda))}{\Gamma_u^L}\Delta\Gamma_u < 0 \leq \Delta V.$$

Since the (LL^2_{au}) is slack, the principal can decrease w_{au} until the (TR^u_P) binds. We can now calculate the new objective function where $w_{ts} = c_{ts}$ with the exception of the w_{au} (which, instead, comes from the (TR^u_P)):

$$
\begin{aligned}
& w_{aa}\gamma^L_{aa} + w_{au}\gamma^L_{au} + w_{ua}\gamma^L_{ua} + w_{uu}\gamma^L_{uu} \\
& = \left[\frac{\left(\bar{u} + V(\lambda^L)\right)\tilde{P}_{uu} - c_{au}\tilde{\gamma}^L_{au} - c_{uu}\tilde{\gamma}^L_{uu}}{\left(\tilde{P}_{aa} - \tilde{P}_{ua}\right)\Gamma^L_a} \right] \gamma^L_{aa} + w_{au}\gamma^L_{au} \\
& \qquad + \left[\frac{c_{au}\tilde{\gamma}^L_{au} + c_{uu}\tilde{\gamma}^L_{uu} - \left(\bar{u} + V(\lambda^L)\right)\tilde{P}_{au}}{\left(\tilde{P}_{aa} - \tilde{P}_{ua}\right)\Gamma^L_u} \right] \gamma^L_{ua} + c_{uu}\gamma^L_{uu} \\
& \propto w_{au}\gamma^L_{au}(\tilde{P}_{aa} - \tilde{P}_{ua}) + c_{au}\tilde{\gamma}^L_{au}(P_{ua} - P_{aa}) + c_{uu}\left(\tilde{\gamma}^L_{uu}(P_{ua} - P_{aa}) + \gamma^L_{uu}\right) \\
& \propto c_{au}\tilde{\gamma}^L_{au}\left[(P_{ua} - P_{aa}) + \frac{P_{au}}{\Gamma^L_u}\right] + c_{uu}\left[\tilde{\gamma}^L_{uu}(P_{ua} - P_{aa}) + \gamma^L_{uu} + P_{au}\tilde{P}_{uu} + \frac{\gamma^L_{au}P_{uu}(\tilde{P}_{aa} - \tilde{P}_{ua})}{P_{ua}}\right].
\end{aligned}
$$

In the reduced problem, the objective function is subject only to

$$\bar{u} + V(\lambda^L) \geq c_{au}\Gamma^L_a + c_{uu}\Gamma^L_u. \qquad (TR^a_A)$$

Since the sign of the coefficients of c_{au} and c_{uu} is not trivial, we study all possible cases and show that all of them lead to a contradiction. First, suppose the case where both c_{uu} and c_{au} increase the objective functions, then the principal wants to decrease them both. This violates the assumption that (LL^1_{ua}) is slack. Now suppose c_{uu} increases the objective function while c_{au} decreases it. Then the principal sets $c_{uu} = 0$ and c_{au} such that the (TR^a_A) binds. However, when $c_{uu} = 0$, the (TR^a_A) coincides with the (LL^1_{ua}) and therefore the latter binds, providing a contradiction again. Now suppose c_{uu} decreases the objective function while c_{au} increases it. Then $c_{au} = 0$ which violates the assumption that $c_{au} > 0$. Finally, suppose both c_{uu} and c_{au} decrease the objective function. Then the principal sets the (TR^a_A) binding and solves for c_{au} to get

$$c_{au} = \frac{\bar{u} + V(\lambda^L)}{\Gamma^L_a} - c_{uu}\frac{\Gamma^L_A}{\Gamma^L_u}. \qquad (25)$$

Substituting this into the objective function, we obtain the final form of the reduced problem:

$$\min_{c_{uu}} \; c_{uu}\left[(P_{ua}-P_{aa})\frac{(\tilde{\gamma}^L_{au}\Gamma^L_a-\tilde{\gamma}^L_{uu}\Gamma^L_u)}{\Gamma^L_u}+\gamma^L_{uu}+P_{au}\tilde{P}_{uu}+\frac{\gamma^L_{au}P_{uu}(\tilde{P}_{aa}-\tilde{P}_{ua})}{P_{ua}}-\frac{\tilde{\gamma}^L_{au}P_{au}}{\Gamma^L_u}\right]$$

If the coefficient of c_{uu} is positive, then the solution to the problem is $c_{uu}=0$ and $c_{au}=\frac{\bar{u}+V(\lambda^L)}{\Gamma^L_a}$ yielding, once again, to the (LL^1_{ua}) binding since it coincides with the (TR^a_A). If the coefficient is negative, instead, the problem is solved by the maximum possible c_{uu}. That is, the value that sets $c_{au}=0$ from (25). This violates the assumption that $c_{au}>0$. This concludes the proof for the case of a positive c_{au} and (LL^1_{ua}) slack.

This concludes the proof of the Lemma showing that, if there exists an optimal contract that implements low effort with a deadweight loss, it must be that it sets the (TR^a_P) binding. ∎

Now that we know that the (TR^a_P) binds, we are going to re-write the problem in two different ways. With the first one, we are going to prove that $c_{ua}=0$. With the second, we are going to select a value for each w_{ts} as a function of c_{ts}.

First, solve the (TR^a_P) for $w_{au}\gamma^L_{au}=w_{ua}\gamma^L_{aa}+w_{uu}\gamma^L_{au}-w_{aa}\gamma^L_{aa}$ and substitute it into the objective function. This makes the w_{aa} disappear from the objective function, which is now given by $w_{ua}(\gamma^L_{aa}+\gamma^L_{ua})+w_{uu}(\gamma^L_{au}+\gamma^L_{uu})$.

Lemma 17. *Given* (3)*, for any value of the bias, if there exists a contract featuring a deadweight loss that implements low effort, it features* $c_{ua}=0$*.*

Proof. Suppose not, and consider the objective function $w_{ua}(\gamma^L_{aa}+\gamma^L_{ua})+w_{uu}(\gamma^L_{au}+\gamma^L_{uu})$. Notice that $c_{ua}>0$ corresponds to (LL^1_{ua}) slack. Hence, the only constraint on c_{au} and c_{uu} is the (TR^a_A). Decreasing c_{au} and c_{uu} by ϵ also decreases c_{ua} and increases c_{aa}. The latter produces no effect on the objective function while the decrease in c_{au}, c_{uu} and c_{ua} allows the principal to decrease the objective function via either w_{uu} or w_{ua}. This provides a contradiction to (LL^1_{ua}) being slack. ∎

Lemma 18. *Given* (3)*, for any value of the bias, if there exists a contract, featuring a deadweight loss that implements low effort, it must feature*

$$\begin{aligned} w_{uu} &= c_{uu} \\ w_{aa} &= c_{aa} \\ w_{au} &= \max\{c_{au}, c_{uu}\} \\ w_{ua} &= (\max\{c_{au}, c_{uu}\} - c_{uu})\frac{P_{au}}{P_{aa}} + c_{aa} \end{aligned}$$

Proof. First of all, from Lemma 15, we can solve the (TR_P^a) for

$$w_{ua} = (w_{au} - w_{uu})\frac{P_{au}}{P_{aa}} + w_{aa}.$$

When plugged into the objective function, it yields

$$\begin{aligned} & w_{aa}(\gamma_{aa}^L + \gamma_{ua}^L) + w_{au}\left(\gamma_{au}^L + \frac{P_{au}}{P_{aa}}\gamma_{ua}^L\right) + w_{uu}\left(\gamma_{uu}^L - \frac{P_{au}}{P_{aa}}\gamma_{ua}^L\right) \\ & \propto w_{aa}(\gamma_{aa}^L + \gamma_{ua}^L)P_{aa} + w_{au}(\gamma_{au}^L + \gamma_{ua}^L)P_{au} + w_{uu}\Gamma_u^L(P_{aa} - P_{ua}). \end{aligned}$$

The above is subject only to the (LL_{ua}) and to $w_{au} \geq w_{uu}$, by Lemma 3. Hence, the principal can decrease the wage levels and set $w_{uu} = c_{uu}$, $w_{aa} = c_{aa}$ and $w_{au} = \max\{c_{au}, w_{uu}\} = \max\{c_{au}, c_{uu}\}$. The proof is concluded by plugging these into the function for w_{ua}. ∎

Given Lemmas 17 and 18, we have the new objective function

$$c_{aa}(\gamma_{aa}^L + \gamma_{ua}^L)P_{aa} + \max\{c_{au}, c_{uu}\}(\gamma_{au}^L + \gamma_{ua}^L) + c_{uu}\Gamma_u^L(P_{aa} - P_{ua}).$$

Before plugging in the value for c_{aa}, notice that we can solve the (LL_{ua}^1) to get

$$c_{au} = \frac{\bar{u} + V(\lambda^L)}{\Gamma_a^L} - c_{uu}\frac{\tilde{\gamma}_{uu}^L}{\tilde{\gamma}_{au}^L}.$$

We plug this into the value for c_{aa} to obtain

$$c_{aa} = \frac{\left(\bar{u} + V(\lambda^L)\right)\tilde{P}_{uu} - c_{au}\tilde{\gamma}_{au}^L - c_{uu}\tilde{\gamma}_{uu}^L}{\left(\tilde{P}_{aa} - \tilde{P}_{ua}\right)\Gamma_a^L} = \frac{\bar{u} + V(\lambda^L)}{\Gamma_a^L},$$

which is, therefore, irrelevant for the objective function of the reduced problem. This latter is given by

$$\min_{c_{au},c_{uu}} \max\{c_{au}, c_{uu}\}(\gamma^L_{au} + \gamma^L_{ua})P_{au} + c_{uu}\Gamma^L_u(P_{aa} - P_{ua}). \tag{26}$$

This allows us to state the final Lemma, that shows how there exist no optimal contract implementing low effort with deadweight loss.

Lemma 19. *When* (3) *holds, the principal implements low effort with* $\{w^\ell_{ts}, c^\ell_{ts}\}_{t,s}$.

Proof. First of all, we show that $c_{au} \geq c_{uu}$ in (26). Suppose not, and $c_{uu} > c_{au}$, then the objective function only depends (positively) on c_{uu}. The problem is then solved by $c_{uu} = 0$, which contradicts $c_{uu} > c_{au}$. Given that $c_{au} \geq c_{uu}$, the problem becomes (disregarding any constant term)

$$\min_{c_{uu}} \left[-c_{uu}\frac{\tilde{\gamma}^L_{uu}}{\tilde{\gamma}^L_{au}}(\gamma^L_{au} + \gamma^L_{ua}) + c_{uu}\Gamma^L_u(P_{aa} - P_{ua}) \right]. \tag{27}$$

Suppose the coefficient of c_{uu} in (27) is negative, then the solution would imply c_{uu} such that $c_{au} = 0$. This would violate $c_{au} \geq c_{uu}$. Hence, a solution only exists when the coefficient of c_{uu} in (27) is negative. Regardless of whether this is the case or not, notice that the solution to the problem would be

$$\begin{array}{llll} w_{aa} = c_{aa} & w_{au} = c_{aa} & w_{uu} = 0 & w_{ua} = \frac{c_{aa}}{P_{aa}} \\ c_{aa} = \frac{\bar{u}+V(\lambda^L)}{\Gamma^L_a} & c_{au} = c_{aa} & c_{uu} = 0 & c_{ua} = 0 \end{array}$$

which yields an expected wage payment of

$$E(w_{ts}) = w_{aa}\left(\Gamma^L_a + \frac{\gamma^L_{ua}}{P_{aa}}\right) = (\bar{u} + V(\lambda^L))\left(1 + \frac{\gamma^L_{ua}}{\gamma^L_{aa}}\right) > (\bar{u} + V(\lambda^L)) = E(w^\ell_{ts}).$$

Hence, even when a solution does exist, it is more expensive than the constant wage one.[35] This implies that when (3) holds, low effort is implemented with a constant wage contract. ∎

Given Lemma 19, it is immediate to see that the magnitude, or presence, of the bias does not affect the expected cost of implementing low effort. From Propositions 4 and 7, we know that the expected cost of implementing high effort, instead, is at

[35]It is possible to show that the contract with deadweight loss above satisfies the (IC) under (3).

least weakly decreasing in the bias when the latter increases the worker's perceived correlation between signals. This concludes the proof.

The case of an overconfident agent. Suppose that the agent is overconfident. The only difference with respect to an optimistic agent is that $b_u < 0$ and therefore $\tilde{P}_{uu} > P_{uu}$ and $\tilde{P}_{ua} < P_{ua}$. This contradicts the proof of Lemma 15 for the case of (TR_P^u) binding. Hence, we provide a modification of the proof for this case.

First of all, since the (TR_P^u) binds, we can re-state the objective function as

$$w_{aa}(\gamma_{aa}^L + \gamma_{ua}^L) + w_{au}(\gamma_{uu}^L + \gamma_{au}^L),$$

and set it subject to:

$$\bar{u} + V(\lambda^L) \geq c_{au}\Gamma_a^L + c_{uu}\Gamma_u^L \tag{TR_A^a}$$

$$w_{aa} \geq c_{aa} = \frac{1}{\tilde{\gamma}_{aa}^L}(\bar{u} + V(\lambda^L) - c_{au}\tilde{\gamma}_{au}^L - c_{uu}\tilde{\gamma}_{uu}^L - c_{ua}\tilde{\gamma}_{ua}^L) \geq 0 \tag{LL_{aa}}$$

$$w_{au} \geq c_{au} \geq 0 \tag{LL_{au}}$$

$$w_{uu} \geq c_{uu} \geq 0 \tag{LL_{uu}}$$

$$w_{ua} \geq c_{ua} \geq 0 \tag{LL_{ua}}$$

Given the above, it is obvious that $w_{aa} = c_{aa}$ and $w_{au} = c_{au}$. Substituting them into the objective function we get

$$\frac{(\gamma_{aa}^L + \gamma_{ua}^L)}{\tilde{\gamma}_{aa}^L}(\bar{u} + V(\lambda^L) - c_{au}\tilde{\gamma}_{au}^L - c_{uu}\tilde{\gamma}_{uu}^L - c_{ua}\tilde{\gamma}_{ua}^L) + c_{au}(\gamma_{uu}^L + \gamma_{au}^L)$$

where c_{ua} enters negatively. Compensation c_{ua} is only limited upwards by constraint (LL_{aa}). Hence, at optimum c_{ua} is such that $c_{aa} = 0$. By Lemma 4, also $c_{au} = 0$, implying we are in case (i) of the Lemma and therefore $c_{ua} = c_{uu}$. To reach the desired contradiction notice that, from the (TR_P^u) binding, we then have

$$w_{ua} = -w_{uu}\frac{\gamma_{uu}^L}{\gamma_{ua}^L}$$

which clearly is not feasible.

Proof of Corollary 2

To prove the Corollary, we need to check that

$$\min\left\{\sum_{ts} c^*_{ts}\tilde{\gamma}_{ts} - V(\lambda^H), \sum_{ts} c^\dagger_{ts}\tilde{\gamma}_{ts} - V(\lambda^H)\right\} \geq \bar{u}$$

Our welfare analysis in section 6 shows that

$$\min\left\{\tilde{E}(c^*_{ts}), \tilde{E}(c^\dagger_{ts})\right\} = \tilde{E}(c^*_{ts}).$$

Hence, it is enough to show that the BPE satisfies the (PC). From the BPE contracts, we can derive:

$$\tilde{E}(c^*_{ts}) = c_{au}(\tilde{\gamma}^H_{aa} + \tilde{\gamma}^H_{au}) = \frac{\Delta V}{\Delta\Gamma_a}(\tilde{\gamma}^H_{aa} + \tilde{\gamma}^H_{au}) = \frac{\Delta V}{\Delta\Gamma_a}\Gamma^H_a.$$

Hence, we check that

$$\frac{\Delta V}{\Delta\Gamma_a}\Gamma^H_a - V(\lambda^H) \geq \bar{u}$$

$$\left(\frac{\Gamma^H_a}{\Delta\Gamma_a} - 1\right)V(\lambda^H) - V(\lambda^L)\frac{\Gamma^H_a}{\Delta\Gamma_a} \geq \bar{u}$$

$$\frac{\Gamma^L_a}{\Delta\Gamma_a}V(\lambda^H) - V(\lambda^L)\frac{\Gamma^H_a}{\Delta\Gamma_a} \geq \bar{u}$$

which yields

$$\bar{u} \leq \frac{V(\lambda^H)\Gamma^L_a - V(\lambda^L)\Gamma^H_a}{\Delta\Gamma_a}.$$

Following from

$$\max\left\{\tilde{E}(c^*_{ts}), \tilde{E}(c^\dagger_{ts})\right\} = \tilde{E}(c^\dagger_{ts}),$$

we know that under the APE the (PC) is slack at $\bar{u} = \bar{u}_1$. From the proof of Proposition 7, we know that the perceived expected compensation of the APE equals

$$\frac{\Delta V}{\Delta\Gamma_a}\Gamma^H_a\frac{\tilde{P}_{au}\Gamma^H_a + \tilde{P}_{uu}\Gamma^H_u}{\tilde{P}_{au}\Gamma^H_a + \tilde{P}_{uu}(\tilde{P}_{aa} - \Gamma^H_a)}.$$

Hence the total utility obtained by the APE, which corresponds to the maximum value of $\bar{u}$ for which the agent accepts an APE contract, is given by

$$\frac{\Delta V}{\Delta\Gamma_a}\Gamma^H_a\frac{\tilde{P}_{au}\Gamma^H_a + \tilde{P}_{uu}\Gamma^H_u}{\tilde{P}_{au}\Gamma^H_a + \tilde{P}_{uu}(\tilde{P}_{aa} - \Gamma^H_a)} - V(\lambda^H).$$

The fact that $\bar{u}_1 < \bar{u}_2$ comes immediately from Proposition 7 as well, since

$$\bar{u}_1 = \tilde{E}(c^*_{ts}) - V(\lambda^H) < \tilde{E}(c^\dagger_{ts}) - V(\lambda^H) = \bar{u}_2.$$

Proof of Proposition 10

To prove the Proposition, we are going to show first that the GBPE is the only optimal contract when case (i) of Lemma 4 is assumed. From the analysis in the paper we know that the APE is the only optimal contract under cases (i) of Lemma 3 and (ii) of Lemma 4. We then prove that no feasible contract is possible under case (ii) of both Lemmas. This concludes the set of possible cases and leaves only the APE and the GBPE as potentially optimal contracts.

PART 1. Case (i) of Lemma 4.

Under case (i) of Lemma 4, it must be that:

$$c_{aa} = c_{au} \quad \text{and} \quad c_{uu} = c_{ua}.$$

First, we rearrange the (PC):

$$\begin{aligned}
&\sum_{ts} c_{ts}\tilde{\gamma}^H_{ts} - V(\lambda^H) \geq \bar{u} \\
&c_{aa}(\tilde{\gamma}^H_{aa} + \tilde{\gamma}^H_{au}) + c_{uu}(\tilde{\gamma}^H_{uu} + \tilde{\gamma}^H_{ua}) - V(\lambda^H) \geq \bar{u} \\
&c_{aa}\Gamma^H_a + c_{uu}\Gamma^H_u - V(\lambda^H) \geq \bar{u} \\
&c_{aa} \geq \frac{\bar{u} + V(\lambda^H)}{\Gamma^H_a} - c_{uu}\frac{\Gamma^H_u}{\Gamma^H_a}.
\end{aligned}$$

Similarly, we can rearrange the (IC) as:

$$\begin{aligned}
&\sum_{ts} c_{ts}\tilde{\gamma}^H_{ts} - V(\lambda^H) \geq \sum_{ts} c_{ts}\tilde{\gamma}^L_{ts} - V(\lambda^L) \\
&\sum_{ts} c_{ts}\Delta\tilde{\gamma}_{ts} - \Delta V \geq 0 \\
&c_{aa}(\Delta\tilde{\gamma}_{aa} + \Delta\tilde{\gamma}_{au}) + c_{uu}(\Delta\tilde{\gamma}_{ua} + \Delta\tilde{\gamma}_{uu}) - \Delta V \geq 0 \\
&c_{aa}\Delta\Gamma_a + c_{uu}\Delta\Gamma_u - \Delta V \geq 0 \\
&c_{aa} \geq \frac{\Delta V}{\Delta\Gamma_a} + c_{uu}.
\end{aligned}$$

We then draw these new versions of the (IC) and (PC) in (c_{uu}, c_{aa}) space to study which one is tighter. To do so, notice the following:

(1) the (PC) is negatively sloped;
(2) the intercept of the (PC) with the c_{aa}-axis is given by $\frac{\bar{u}+V(\lambda^H)}{\Gamma_a^H}$;
(3) contracts that lie on and *above* the locus of points satisfying the (PC) with equality satisfy the constraint;
(4) the (IC) is positively sloped with slope 1;
(5) the intercept of the (IC) with the c_{aa}-axis is given by $\frac{\Delta V}{\Delta \Gamma_a}$;
(6) contracts that lie on and *above* the locus of points satisfying the (IC) with equality satisfy the constraint;
(7) Since $\bar{u} \in (\bar{u}_1, \bar{u}_2]$ the intercept of the (IC) with the c_{aa}-axis is lower than the one of the (PC). To see this, calculate

$$\frac{\Delta V}{\Delta \Gamma_a} < \frac{\bar{u} + V(\lambda^H)}{\Gamma_a^H}$$

$$\frac{\Delta V \Gamma_a^H}{\Delta \Gamma_a} - V(\lambda^H) < \bar{u}$$

$$\bar{u} > \frac{V(\lambda^H)\Gamma_a^L - V(\lambda^L)\Gamma_a^H}{\Delta \Gamma_a},$$

which corresponds to $\bar{u} > \bar{u}_1$.

With these in mind, we can produce Figure 6 below.

In the figure, all contracts lying in the shaded area satisfy both the (PC) and the (IC). Recall that the (TR_A) constraints are trivially solved, since we are in case (i) of Lemma 4. This immediately calls for an observation.

Lemma 20. *When $c_{aa} = c_{au}$ and $c_{uu} = c_{ua}$, the optimal contract features $c_{aa} > c_{uu}$.*

Proof. In Figure 6, the shaded area always lies above the 45 degree line. This is because the (IC) has slope 1 but an intercept larger than the 45 degree line (unless $\Delta V \leq 0$, which we rule out). ∎

As for the w_{ts}, of course, we can either be in case (i) or (ii) of Lemma 3. That is, either:

$$\text{(case (i))} \quad w_{ua} = w_{aa} \quad \text{and} \quad w_{au} = w_{uu}$$

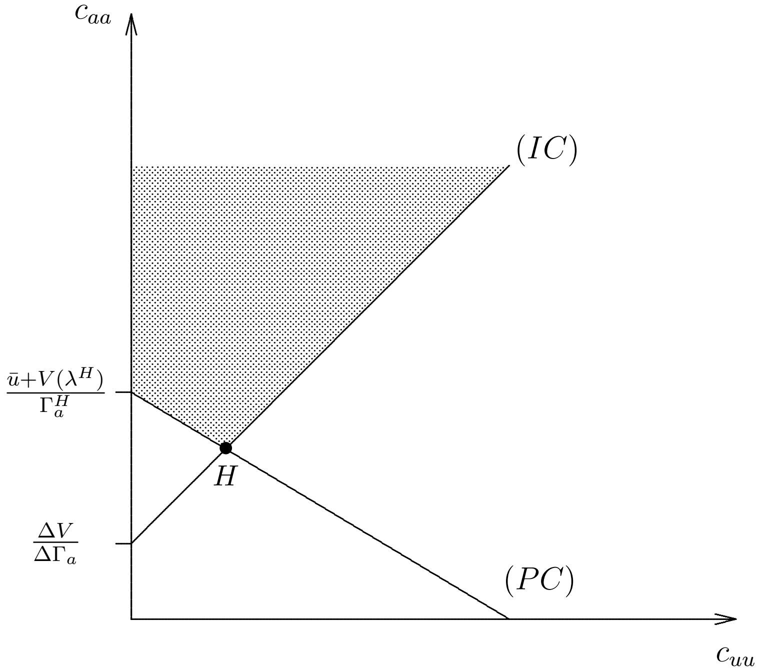

FIGURE 6. Contracts lying in the shaded area satisfy both the (PC) and the (IC) under case (i) of Lemma 4.

or

$$\text{(case (ii))} \quad w_{ua} > w_{aa} \quad \text{and} \quad w_{au} > w_{uu}.$$

Assume we are in case (i). Under $w_{ua} = w_{aa}$ and $w_{au} = w_{uu}$, the principal is left with minimizing the following cost function

$$\min_{w_{aa}, w_{uu}} w_{aa}(\gamma_{aa}^H + \gamma_{ua}^H) + w_{uu}(\gamma_{au}^H + \gamma_{uu}^H),$$

subject to the (LL_{ts}) constraints. Given the cases we are studying, these constraints imply

$$w_{aa} \geq c_{aa}, \; w_{aa} = w_{ua} \geq c_{uu}, \; w_{uu} \geq c_{uu}, \; w_{uu} = w_{au} \geq c_{aa}$$

and therefore they yield

$$w_{aa} \geq \max\{c_{aa}, c_{uu}\} = c_{aa} \; w_{uu} \geq \max\{c_{aa}, c_{uu}\} = c_{aa}.$$

All this means that, under case (i) of both Lemmas, the principal solves

$$\min_{w_{aa}, w_{uu}} w_{aa}(\gamma_{aa}^H + \gamma_{ua}^H) + w_{uu}(\gamma_{au}^H + \gamma_{uu}^H)$$

$$\text{s.t.} \quad w_{aa} \geq c_{aa} \quad \text{and} \quad w_{uu} \geq c_{aa},$$

which is trivially solved by setting $w_{aa} = w_{uu} = c_{aa}$ and setting the contract that minimizes c_{aa} among the ones that satisfy the (PC) and the (IC). That is, point H in Figure 1. To derive the final values of the optimal contract under case (i) of both Lemmas, we set the RHS of the (PC) and (IC) equal and solve for c_{uu}.

$$\frac{\bar{u} + V(\lambda^H)}{\Gamma_a^H} - c_{uu}\frac{\Gamma_u^H}{\Gamma_a^H} = \frac{\Delta V}{\Delta \Gamma_a} + c_{uu}$$
$$c_{uu} = \bar{u} + \frac{V(\lambda^L)\Gamma_a^H - V(\lambda^H)\Gamma_a^L}{\Delta \Gamma_a},$$

which consistently with our findings converges to 0 as $\bar{u} \to \bar{u}_1$.

We can then substitute the above in the (PC) to get c_{aa} and finalize the contract values. This leads to the following contract:

$$\begin{array}{llll} w_{aa} = c_{aa} & w_{au} = c_{aa} & w_{uu} = c_{aa} & w_{ua} = c_{aa} \\ c_{aa} = \frac{\Delta V}{\Delta \Gamma_a} + c_{uu} & c_{au} = c_{aa} & c_{uu} = \bar{u} + \frac{V(\lambda^L)\Gamma_a^H - V(\lambda^H)\Gamma_a^L}{\Delta \Gamma_a} & c_{ua} = c_{uu}. \end{array}$$

where the principal pays a fixed wage.

Now we move to case (ii) of Lemma 3. First of all, notice that, because of the Lemma above, we can disregard (LL_{ua}) since we have $w_{ua} > w_{aa} \geq c_{aa} > c_{uu} = c_{ua}$. On the other end, (LL_{au}) cannot be disregarded just yet, since we haven't derived the result of $\max\{w_{uu}, c_{aa}\}$. When setting wages that satisfy case (ii) of Lemma 3, the principal solves:

$$\min_{\{w_{ts}, c_{ts}\}_{t,s \in \{u,a\}}} w_{aa}\gamma_{aa}^H + w_{au}\gamma_{au}^H + w_{ua}\gamma_{ua}^H + w_{uu}\gamma_{uu}^H$$

$$w_{aa}\gamma_{aa}^H + w_{au}\gamma_{au}^H \leq w_{ua}\gamma_{aa}^H + w_{uu}\gamma_{au}^H \qquad (TR_P^a)$$

$$w_{ua}\gamma_{ua}^H + w_{uu}\gamma_{uu}^H \leq w_{aa}\gamma_{ua}^H + w_{au}\gamma_{uu}^H \qquad (TR_P^u)$$

$$w_{aa} \geq c_{aa} \qquad (LL_{aa})$$

$$w_{uu} \geq c_{uu} \qquad (LL_{uu})$$

$$w_{au} \geq c_{aa} \qquad (LL_{au})$$

Lemma 21. *Given case (i) of Lemma 4 and case (ii) of Lemma 3, at optimum the* (LL_{aa}) *binds and* $w_{aa} = c_{aa}$.

Proof. Suppose not, then the principal can decrease both w_{aa} and w_{ua} by the same amount, this has no effect on the (TR_P) constraints and it decreases the objective function. Hence, (LL_{aa}) cannot be slack at optimum. ∎

Rewriting the two (TR_P) constraints, the principal now solves:

$$\min_{\{w_{ts},c_{ts}\}_{t,s\in\{u,a\}}} c_{aa}\gamma^H_{aa} + w_{au}\gamma^H_{au} + w_{ua}\gamma^H_{ua} + w_{uu}\gamma^H_{uu}$$

$$(w_{au} - w_{uu})\frac{\gamma^H_{au}}{\gamma^H_{aa}} \leq (w_{ua} - c_{aa}) \leq (w_{au} - w_{uu})\frac{\gamma^H_{uu}}{\gamma^H_{ua}} \tag{(TR_P)}$$

$$w_{uu} \geq c_{uu} \tag{LL_{uu}}$$

$$w_{au} \geq c_{aa} \tag{LL_{au}}$$

Lemma 22. *Given case (i) of Lemma 4 and case (ii) of Lemma 3, constraints* (LL_{uu}) *and* (LL_{au}) *bind at optimum.*

Proof. Suppose not, and both constraints hold with inequality at optimum, then the principal can decrease both w_{uu} and w_{au}, keeping their difference constant so that (TR_P) is not affected, reducing the objective function. This, however, is not enough to prove the theorem. We need to show that even if only one of the two binds the other cannot be slack.

Suppose only (LL_{uu}) binds, while (LL_{au}) does not, then the principal can decrease both w_{au} and w_{ua} in such a way that (TR_P) still holds, decreasing the objective function. This is always possible, since both wages have positive signs in (TR_P).

Suppose now only (LL_{au}) binds, while (LL_{uu}) does not, then the proof is a bit trickier since decreasing both w_{uu} and w_{ua} tightens the (TR^a_P). Suppose the latter binds, the principal can decrease w_{uu} by ϵ and *increase* w_{ua} by $\epsilon_1 = \epsilon\frac{\gamma^H_{au}}{\gamma^H_{aa}}$. This has no effect on the (TR^a_P), but it affects the objective function. To see that the overall effect on costs is negative, let it be denoted by ΔC and calculate

$$\Delta C = -\epsilon\gamma^H_{uu} + \epsilon_1\gamma^H_{au} = -\epsilon\frac{(\gamma^H_{uu}\gamma^H_{aa} - \gamma^H_{au}\gamma^H_{ua})}{\gamma^H_{aa}} < 0$$

This concludes the proof. ∎

Given the above, the principal is left with

$$\min_{\{w_{ts},c_{ts}\}_{t,s\in\{u,a\}}} c_{aa}(\gamma^H_{aa}+\gamma^H_{au})+w_{ua}\gamma^H_{ua}+c_{uu}\gamma^H_{uu}$$

$$(c_{aa}-c_{uu})\frac{\gamma^H_{au}}{\gamma^H_{aa}} \leq (w_{ua}-c_{aa}) \leq (c_{aa}-c_{uu})\frac{\gamma^H_{uu}}{\gamma^H_{ua}} \qquad ((TR_P))$$

where it is easy to see that, since the first and the last element of (TR_P) are different from 0, we need $w_{ua} > c_{aa}$ for (TR_P) to hold. Now notice that the first inequality of (TR_P) corresponds to (TR^a_P).

Lemma 23. *Given case (i) of Lemma 4 and case (ii) of Lemma 3, at optimum* (TR^a_P) *binds. Hence*

$$w_{ua} = c_{aa} + (c_{aa}-c_{uu})\frac{\gamma^H_{au}}{\gamma^H_{aa}}$$

Proof. Suppose not, by decreasing w_{ua} the principal decreases the objective function and relaxes (TR^u_P). ∎

The final form of the problem that the principal solves is therefore

$$\min_{c_{aa},c_{uu}} c_{aa}\left(\gamma^H_{aa}+\gamma^H_{au}+\gamma^H_{ua}+\frac{\gamma^H_{au}\gamma^H_{ua}}{\gamma^H_{aa}}\right)+c_{uu}\left(\gamma^H_{uu}-\frac{\gamma^H_{au}\gamma^H_{ua}}{\gamma^H_{aa}}\right)$$

$$=\min_{c_{aa},c_{uu}} c_{aa}(\gamma^H_{aa}\gamma^H_{aa}+\gamma^H_{aa}\gamma^H_{au}+\gamma^H_{aa}\gamma^H_{ua}+\gamma^H_{au}\gamma^H_{ua})+c_{uu}(\gamma^H_{uu}\gamma^H_{aa}-\gamma^H_{au}\gamma^H_{ua})$$

with c_{aa} and c_{uu} that have to lie inside the shaded area of Figure 6.

Since both c_{aa} and c_{uu} enter negatively in the objective function, the cost of implementing high effort decreases towards the origin in Figure 6. This implies that the optimal contract for this case is either at point H or it corresponds to the intercept of the (PC) with the c_{aa}-axis. The next Lemma solves this dilemma and shows how the optimal contract for this case, the GBPE one, also dominates the contract derived under case (i) of both Lemmas.

Lemma 24. *Given case (i) of Lemma 4, the optimal contract is the GBPE contract.*

Proof. I start from showing that the optimal contract under case (ii) of Lemma 3 lies at H. To see this, notice that iso-costs decrease in value towards the origin in Figure 1. Hence, if they are flatter than the (PC), the contract minimising costs lies at H.

The absolute value of the slope of the iso-costs is given by:

$$\begin{aligned}
&\frac{\gamma^H_{uu}\gamma^H_{aa} - \gamma^H_{au}\gamma^H_{ua}}{\gamma^H_{aa}\gamma^H_{aa} + \gamma^H_{aa}\gamma^H_{au} + \gamma^H_{aa}\gamma^H_{ua} + \gamma^H_{au}\gamma^H_{ua}} \\
&= \frac{(P_{uu}P_{aa} - P_{au}P_{ua})\Gamma^H_a\Gamma^H_u}{P_{aa}P_{aa}(\Gamma^H_a)^2 + P_{aa}P_{au}(\Gamma^H_a)^2 + P_{aa}P_{ua}\Gamma^H_a\Gamma^H_u + P_{au}P_{ua}\Gamma^H_a\Gamma^H_u} \\
&= \frac{(P_{aa} - P_{ua})\Gamma^H_u}{P_{aa}\Gamma^H_a(P_{aa} + P_{au}) + P_{ua}\Gamma^H_u(P_{aa} + P_{au})} = \frac{(P_{aa} - P_{ua})(1 - \Gamma^H_a)}{P_{aa}\Gamma^H_a + P_{ua}(1 - \Gamma^H_a)}.
\end{aligned}$$

The absolute value of the slope of the (PC), instead, is given by $\frac{1-\Gamma^H_a}{\Gamma^H_a}$. To see that the latter is always larger than the former, calculate

$$\begin{aligned}
&\frac{(P_{aa} - P_{ua})(1 - \Gamma^H_a)}{P_{aa}\Gamma^H_a + P_{ua}(1 - \Gamma^H_a)} < \frac{1 - \Gamma^H_a}{\Gamma^H_a} \\
&\frac{P_{aa}\Gamma^H_a - P_{ua}\Gamma^H_a}{P_{aa}\Gamma^H_a + P_{ua}(1 - \Gamma^H_a)} < 1 \\
&P_{aa}\Gamma^H_a - P_{ua}\Gamma^H_a < P_{aa}\Gamma^H_a + P_{ua}(1 - \Gamma^H_a),
\end{aligned}$$

which is trivially true. Hence, the optimal contract for this case always lies at point H. Following the same calculations of case (i) of Lemma 3, we then get the values of c_{aa} and c_{uu}. To obtain the value of w_{ua}, substitute for $c_{uu} = c_{aa} - \frac{\Delta V}{\Delta \Gamma_a}$ in $w_{ua} = c_{aa} + (c_{aa} - c_{uu})\frac{\gamma^H_{au}}{\gamma^H_{aa}}$:

$$w_{ua} = c_{aa} + \left(c_{aa} - c_{aa} - \frac{\Delta V}{\Delta \Gamma_a}\right)\frac{\gamma^H_{au}}{\gamma^H_{aa}} = c_{aa} + \frac{\Delta V}{\Delta \Gamma_a}\frac{P_{au}\Gamma^H_a}{P_{aa}\Gamma^H_a} = c_{aa} + \frac{\Delta V}{\Delta \Gamma_a}\frac{P_{au}}{P_{aa}}.$$

To conclude the proof, we need to show that the cost of implementing high effort, i.e. $E(w_{ts})$, is lower under the GBPE contract compared to the one of case (i) of Lemma 3. First of all, notice that the latter is equal to c_{aa}, since the wage is constant, and that c_{aa} takes the same value in both contracts. We then check for

$$\begin{aligned}
&c_{aa}\gamma^H_{aa} + c_{aa}\gamma^H_{au} + c_{uu}\gamma^H_{uu} + \left(c_{aa} + \frac{\Delta V}{\Delta \Gamma_a}\frac{P_{au}}{P_{aa}}\right)\gamma^H_{ua} < c_{aa} \\
&c_{aa}\underbrace{\left(\gamma^H_{aa} + \gamma^H_{au} + \gamma^H_{ua} + \gamma^H_{uu}\right)}_{1} + \frac{\Delta V}{\Delta \Gamma_a}\left(\frac{P_{au}}{P_{aa}}\gamma^H_{ua} - \gamma^H_{uu}\right) < c_{aa} \\
&\frac{\Delta V}{\Delta \Gamma_a P_{aa}}\left(P_{au}P_{ua} - P_{aa}P_{uu}\right)\Gamma^H_u < 0,
\end{aligned}$$

which is true by positive correlation. ∎

PART 2. Case (ii) of both Lemmas.

We are now left to show that no feasible contract emerges from case (ii) of both Lemmas. The reason for doing so is that the GBPE is optimal for a smaller set of parameter values compared to the BPE, since it is more expensive. Call $\mathcal{P}$ the difference between this set and the one under which the BPE is optimal when $\bar{u} < \bar{u}_1$. We are looking for any potentially optimal contract that differs structurally from the APE and could be optimal in $\mathcal{P}$. If we were to find none, we would be sure that the APE is optimal in $\mathcal{P}$.[36]

Given the Lemmas, we have

$$c_{uu} > c_{ua},\, c_{aa} > c_{au} \quad \text{and} \quad w_{ua} > w_{aa},\, w_{au} > w_{uu}.$$

Hence, the problem solved by the principal is the starting one.

$$\min_{\{w_{ts},c_{ts}\}_{t,s\in\{u,a\}}} w_{aa}\gamma^H_{aa} + w_{au}\gamma^H_{au} + w_{ua}\gamma^H_{ua} + w_{uu}\gamma^H_{uu}$$

$$\text{s.t.} \quad \sum_{ts} c_{ts}\tilde{\gamma}^H_{ts} - V(\lambda^H) \geq \bar{u} \tag{PC}$$

$$\sum_{ts} c_{ts}\tilde{\gamma}^H_{ts} - V(\lambda^H) \geq \sum_{ts} c_{ts}\tilde{\gamma}^L_{ts} - V(\lambda^L) \tag{IC}$$

$$w_{aa}\gamma^H_{aa} + w_{au}\gamma^H_{au} \leq w_{ua}\gamma^H_{aa} + w_{uu}\gamma^H_{au} \tag{TR^a_P}$$

$$w_{ua}\gamma^H_{ua} + w_{uu}\gamma^H_{uu} \leq w_{aa}\gamma^H_{ua} + w_{au}\gamma^H_{uu} \tag{TR^u_P}$$

$$c_{aa}\tilde{\gamma}^H_{aa} + c_{ua}\tilde{\gamma}^H_{ua} \geq c_{au}\tilde{\gamma}^H_{aa} + c_{uu}\tilde{\gamma}^H_{ua} \tag{TR^a_A}$$

$$c_{au}\tilde{\gamma}^H_{au} + c_{uu}\tilde{\gamma}^H_{uu} \geq c_{aa}\tilde{\gamma}^H_{au} + c_{ua}\tilde{\gamma}^H_{uu} \tag{TR^u_A}$$

$$w_{ts} \geq c_{ts} \geq 0 \qquad \forall t,s \in \{a,u\}. \tag{LL_{ts}}$$

First of all, notice that at least one of the (TR_P) constraints must bind. If this were not the case, then the (LL_{ts}) would be the only constraints on the w_{ts}. Clearly these would then be set binding. This violates Proposition 1 and the resulting contract would never implement high effort under truthful reporting. We, therefore, divide the analysis in two sections depending on which one of the (TR_P) binds.

[36] This is due to the different approach taken to derive the optimal contracts under the assumption that $\bar{u} < \bar{u}_1$. Under the latter, we haven't studied the problem assuming specific cases of Lemmas 4 and 3, we have not proven yet that the APE is the only feasible contract under case (ii) of Lemma 4, but only that it is optimal w.r.t. the BPE under a certain parameter restriction.

Let the (TR_P^a) *bind.* We can then solve the (TR_P^a) for $w_{au}\gamma_{au}^H$ to get

$$w_{au}\gamma_{au}^H = w_{uu}\gamma_{au}^H + (w_{ua} - w_{aa})\gamma_{aa}^H.$$

When we plug this back into the objective function we get

$$w_{aa}\gamma_{aa}^H + w_{au}\gamma_{au}^H + w_{ua}\gamma_{ua}^H + w_{uu}\gamma_{uu}^H = w_{uu}(\gamma_{au}^H + \gamma_{uu}^H) + w_{ua}(\gamma_{ua}^H + \gamma_{aa}^H).$$

From the (PC) binding, we know that

$$c_{aa} = \frac{1}{\tilde{\gamma}_{aa}^H}\left(\bar{u} + V(\lambda^H) - c_{au}\tilde{\gamma}_{au}^H - c_{ua}\tilde{\gamma}_{ua}^H - c_{uu}\tilde{\gamma}_{uu}^H\right),$$

which plugged back into the (IC) yields

$$\bar{u} + V(\lambda^H) - \frac{\Delta V}{\Delta\Gamma_a}\Gamma_a^H \geq c_{ua}\tilde{P}_{ua} + c_{uu}\tilde{P}_{uu}.$$

The resulting maximization problem is given by

$$\min_{\{w_{ts}, c_{ts}\}_{t,s\in\{u,a\}}} w_{uu}(\gamma_{au}^H + \gamma_{uu}^H) + w_{ua}(\gamma_{ua}^H + \gamma_{aa}^H) \tag{28}$$

$$\text{s.t.} \quad \bar{u} + V(\lambda^H) - \frac{\Delta V}{\Delta\Gamma_a}\Gamma_a^H \geq c_{ua}\tilde{P}_{ua} + c_{uu}\tilde{P}_{uu} \tag{IC}$$

$$c_{aa}\tilde{\gamma}_{aa}^H + c_{ua}\tilde{\gamma}_{ua}^H \geq c_{au}\tilde{\gamma}_{aa}^H + c_{uu}\tilde{\gamma}_{ua}^H \tag{TR_A^a}$$

$$c_{au}\tilde{\gamma}_{au}^H + c_{uu}\tilde{\gamma}_{uu}^H \geq c_{aa}\tilde{\gamma}_{au}^H + c_{ua}\tilde{\gamma}_{uu}^H \tag{TR_A^u}$$

$$w_{uu} \geq c_{uu} \geq 0 \tag{LL_{uu}}$$

$$w_{ua} \geq c_{ua} \geq 0. \tag{LL_{ua}}$$

By case (ii) of Lemma 4, we know that the (TR_A) will not bind together. Suppose one of them binds, we can plug c_{aa} into it and solve for c_{au}, removing the (TR_A) constraints from the problem. At this point, in the problem above, (LL_{uu}) and (LL_{ua}) are set binding since they are the only constraints on the relevant wages. This implies that the objective function is minimised at $c_{uu} = c_{ua} = 0$. This is incompatible with case (ii) of Lemma 4. Suppose no (TR_A) constraint binds, we have the same solution, since the (TR_A) are completely disregarded, and therefore the same contradiction. Hence, there exists no optimal contract for this case when the (TR_P^a) binds.

Let the (TR_P^u) *bind.* The proof is a little more tricky. As above, solve the (TR_P^u) for $w_{uu}\gamma_{uu}^H$ to get

$$w_{uu}\gamma_{uu}^H = w_{au}\gamma_{uu}^H + (w_{aa} - w_{ua})\gamma_{ua}^H.$$

When we plug this back into the objective function we get

$$w_{aa}\gamma_{aa}^H + w_{au}\gamma_{au}^H + w_{ua}\gamma_{ua}^H + w_{uu}\gamma_{uu}^H = w_{aa}(\gamma_{aa}^H + \gamma_{ua}^H) + w_{au}(\gamma_{au}^H + \gamma_{uu}^H).$$

From the (PC), we know that

$$c_{uu} = \frac{1}{\tilde{\gamma}_{uu}^H}\left(\bar{u} + V(\lambda^H) - c_{au}\tilde{\gamma}_{au}^H - c_{ua}\tilde{\gamma}_{ua}^H - c_{aa}\tilde{\gamma}_{aa}^H\right).$$

When we plug this into the (IC) we obtain

$$\bar{u} + V(\lambda^H) + \frac{\Delta V}{\Delta \Gamma_a}\Gamma_u^H \leq c_{aa}\tilde{P}_{aa} + c_{au}\tilde{P}_{au}.$$

Differently from above, we first are going to assume that a (TR_A) binds. Suppose it is the (TR_A^u), and solve it for

$$c_{ua}\tilde{\gamma}_{uu}^H = c_{uu}\tilde{\gamma}_{uu}^H - (c_{aa} - c_{au})\tilde{\gamma}_{au}^H.$$

Substituting for c_{uu} from the (PC), we obtain

$$c_{ua} = \frac{\bar{u} + V(\lambda^H) - c_{aa}\Gamma_a^H}{\Gamma_u^H}.$$

If instead of the (TR_A^u) we set the (TR_A^a) binding, and still solve for c_{uu} and c_{ua} from the (TR_A^a) and the (PC) constraint, we get

$$c_{uu} = \frac{\bar{u} + V(\lambda^H) - c_{au}\Gamma_a^H}{\Gamma_u^H},$$

$$c_{ua} = \frac{1}{\tilde{\gamma}_{ua}^H}\left[\left(\bar{u} + V(\lambda^H)\right)\tilde{P}_{ua} + c_{au}(\tilde{P}_{aa} - \tilde{P}_{ua})\Gamma_a^H - c_{aa}\tilde{\gamma}_{aa}^H\right],$$

which leads to the symmetric problem. Hence, regardless of the (TR_A) set binding, the principal faces

$$\min_{\{w_{ts},c_{ts}\}_{t,s\in\{u,a\}}} w_{aa}(\gamma_{aa}^H+\gamma_{ua}^H)+w_{au}(\gamma_{au}^H+\gamma_{uu}^H) \tag{29}$$

$$\text{s.t.}\quad \bar{u}+V(\lambda^H)+\frac{\Delta V}{\Delta\Gamma_a}\Gamma_u^H \leq c_{aa}\tilde{P}_{aa}+c_{au}\tilde{P}_{au} \tag{IC}$$

$$w_{aa}\geq c_{aa}\geq 0 \tag{LL_{aa}}$$

$$w_{au}\geq c_{au}\geq 0 \tag{LL_{au}}$$

where it is immediate to see that the (LL_{aa}) and (LL_{au}) bind. Following this, also the (IC) must bind. We, therefore, solve it for c_{aa} to get

$$c_{aa}=\frac{\Delta V}{\Delta\Gamma_a}\frac{\Gamma_u^H}{\tilde{P}_{aa}}+\frac{\bar{u}+V(\lambda^H)}{\tilde{P}_{aa}}-c_{au}\frac{\tilde{P}_{au}}{\tilde{P}_{aa}}.$$

We can then plug this into the objective function to obtain

$$\text{(a fixed positive term)}-c_{au}\left[\left(\gamma_{aa}^H+\gamma_{ua}^H\right)\frac{\tilde{P}_{au}}{\tilde{P}_{aa}}-\left(\gamma_{au}^H+\gamma_{uu}^H\right)\right].$$

To see that the coefficient of c_{au} is always positive, notice that the bracket is always negative:

$$\begin{aligned}
&\left(\gamma_{aa}^H+\gamma_{ua^H}\right)\frac{\tilde{P}_{au}}{\tilde{P}_{aa}}-\left(\gamma_{au}^H+\gamma_{uu}^H\right)\\
&\quad=P_{au}P_{aa}\Gamma_a^H+P_{au}P_{ua}\Gamma_u^H-P_{au}P_{aa}\Gamma_a^H-P_{uu}P_{aa}\Gamma_u^H-b_a(\gamma_{aa}^H+\gamma_{ua}^H+\gamma_{au}^H+\gamma_{uu}^H)\\
&\quad=P_{au}P_{ua}\Gamma_u^H-P_{uu}P_{aa}\Gamma_u^H-b_a=(P_{ua}-P_{aa})\Gamma_u^H-b_a<0.
\end{aligned}$$

Hence, the principal sets $c_{au}=0$ to minimize the objective function. This is enough to prove our contradiction. Notice, in fact, that given c_{au} and $c_{aa}=w_{aa}$, from the (TR_P^u) we have that

$$w_{uu}=(w_{aa}-w_{ua})\frac{P_{ua}}{P_{uu}},$$

which is ≥ 0 if and only if $w_{ua}\leq w_{aa}$. This violate case (ii) of Lemma 3, and shows that there cannot be an optimal contract where one of the (TR_A) binds.

To conclude the proof, we need to show that no optimal contract exists when both the (TR_A) are slack. Problem (29) above, however, still holds when the (TR_A) are disregarded. The solution follows the same steps above and leads, therefore, to $c_{au}=0$

and to the same contradiction.
This concludes the proof of part 2 and of the Proposition.

Proof of Corollary 3

The proof follows from the text. The expected wage cost of the APE contract hasn't changed from the case of $\bar{u} < \bar{u}_1$, while the expected wage cost of the GBPE one is given by

$$E(\tilde{w}_{ts}^{*}) = \bar{u} + V(\lambda^H) + \frac{\Delta V}{\Delta\Gamma_a}\frac{\gamma_{ua}^H}{P_{aa}}$$

and it is larger than the expected wage cost of the BPE contract when $\bar{u} > \bar{u}_1$. Hence, the comparison between the APE contract and the GBPE one ends in favor of the APE contract for a larger — at least weakly — set of parameters compared to the case of the BPE vs. the APE contract.

Proof of Proposition 11

First, notice that the derivation of the optimality of the GBPE contract under case (i) of Lemma 4 does not depend on $\bar{u}$ being larger or smaller than $\bar{u}_2$. Hence, once again we focus our attention on case (ii) of Lemma 4. Second, our analysis of cases (ii) of both Lemmas in the proof of Proposition 10 also does not depend on the value of $\bar{u}$ being larger or smaller than $\bar{u}_2$. Hence we are left simply with case (i) of Lemma 3 and (ii) of Lemma 4. Any optimal contract besides the GBPE must respect these features. This proves that any GAPE contract uses information from SPE reports in the same fashion of the APE contract.

The proof of the Proposition is quite long and it is therefore split in several parts. First, focusing on case (i) of Lemma 3 we solve for the optimal w_{ts} (Lemma 25). Second, we use that to rewrite the problem and prove that at optimum, $c_{aa} \geq c_{ua}$ (Lemma 26). Third, we use a different rewriting of the problem to show that, at optimum, $c_{au} = c_{uu}$ (Lemma 27). The proof of this second result is itself divided in two parts: part 1, where we show that $c_{uu} > c_{au}$ leads to a contradiction, and part 2, where we show that also $c_{uu} < c_{au}$ leads to unfeasible contracts. Fourth, we state the final form of the problem in (35) and show graphically the potential solutions to it (Figure 9 to 11). Fifth, we select the contracts that are indeed optimal under certain parameter conditions (Lemma 28).

Finally, we derive the values for GAPE$_1$ and GAPE$_2$ together with their feasibility and optimality conditions (Lemmas 29 and 30).

Under case (i) of Lemma 3 and (ii) of Lemma 4, the principal faces the following problem (where the (TR_P) are trivially solved and we reduced the (LL_{ts}) to only two constraints)

$$\min_{\{w_{ts},c_{ts}\}_{t,s\in\{u,a\}}} w_{aa}(\gamma_{aa}^H+\gamma_{ua}^H)+w_{uu}(\gamma_{au}^H+\gamma_{uu}^H)$$

$$\text{s.t.} \quad \sum_{ts} c_{ts}\tilde{\gamma}_{ts}^H - V(\lambda^H) \geq \bar{u} \qquad (PC)$$

$$\sum_{ts} c_{ts}\Delta\tilde{\gamma}_{ts} - \Delta V \geq 0 \qquad (IC)$$

$$c_{aa}\tilde{\gamma}_{aa}^H + c_{ua}\tilde{\gamma}_{ua}^H \geq c_{au}\tilde{\gamma}_{aa}^H + c_{uu}\tilde{\gamma}_{ua}^H \qquad (TR_A^a)$$

$$c_{au}\tilde{\gamma}_{au}^H + c_{uu}\tilde{\gamma}_{uu}^H \geq c_{aa}\tilde{\gamma}_{au}^H + c_{ua}\tilde{\gamma}_{uu}^H \qquad (TR_A^u)$$

$$w_{aa} \geq \max\{c_{aa}, c_{ua}\} \qquad (LL_{aa}, LL_{ua})$$

$$w_{uu} \geq \max\{c_{au}, c_{uu}.\} \qquad (LL_{uu}, LL_{au})$$

Given that $\bar{u} > \bar{u}_2$, and that the BPE and the APE contracts are the only optimal one resulting from the (PC) being slack, we set the (PC) binding and use it to rewrite the (TR_A) constraints. First, solve for

$$c_{aa}\tilde{\gamma}_{aa}^H = \bar{u} + V(\lambda^H) - c_{au}\tilde{\gamma}_{au}^H - c_{ua}\tilde{\gamma}_{ua}^H - c_{uu}\tilde{\gamma}_{uu}^H$$

and plug this into the (TR_A^a) to obtain

$$\bar{u} + V(\lambda^H) \geq c_{au}\Gamma_a^H + c_{uu}\Gamma_u^H.$$

Similarly, solve the (PC) for

$$c_{au}\tilde{\gamma}_{au}^H = \bar{u} + V(\lambda^H) - c_{aa}\tilde{\gamma}_{aa}^H - c_{ua}\tilde{\gamma}_{ua}^H - c_{uu}\tilde{\gamma}_{uu}^H$$

and plug this into the (TR_A^u) to obtain

$$\bar{u} + V(\lambda^H) \geq c_{aa}\Gamma_a^H + c_{ua}\Gamma_u^H.$$

This allows us to rewrite the problem as

$$\begin{aligned}
\min_{\{w_{ts},c_{ts}\}_{t,s\in\{u,a\}}} \quad & w_{aa}(\gamma^H_{aa}+\gamma^H_{ua})+w_{uu}(\gamma^H_{au}+\gamma^H_{uu}) \\
\text{s.t.} \quad & c_{aa}\tilde{\gamma}^H_{aa}+c_{au}\tilde{\gamma}^H_{au}+c_{ua}\tilde{\gamma}^H_{ua}+c_{uu}\tilde{\gamma}^H_{uu}=\bar{u}+V(\lambda^H) && (PC)\\
& \sum_{ts} c_{ts}\Delta\tilde{\gamma}_{ts}-\Delta V\geq 0 && (IC)\\
& \bar{u}+V(\lambda^H)\geq c_{au}\Gamma^H_a+c_{uu}\Gamma^H_u && (TR^a_A)\\
& \bar{u}+V(\lambda^H)\geq c_{aa}\Gamma^H_a+c_{ua}\Gamma^H_u && (TR^u_A)\\
& w_{aa}\geq \max\{c_{aa},c_{ua}\} && (LL_{aa},LL_{ua})\\
& w_{uu}\geq \max\{c_{au},c_{uu}\} && (LL_{uu},LL_{au})
\end{aligned}$$

Lemma 25. *When $\bar{u}>\bar{u}_2$, given case (ii) of Lemma 4 and case (i) of Lemma 3*

$$w_{aa}=\max\{c_{aa},c_{ua}\} \quad w_{uu}=\max\{c_{au},c_{uu}\}$$

Proof. Given case (i) of Lemma 3, the (TR_P) are trivially satisfied. Both w_{aa} and w_{uu} affect the cost negatively and therefore the principal has the incentive to decrease them as much as she can. Since the (LL_{ts}) are the only constraints on wages, they are set binding. ∎

Let us rearrange the problem using the optimal w_{ts} derived.

$$\begin{aligned}
\min_{\{c_{ts}\}_{t,s\in\{u,a\}}} \quad & \max\{c_{aa},c_{ua}\}(\gamma^H_{aa}+\gamma^H_{ua})+\max\{c_{au},c_{uu}\}(\gamma^H_{au}+\gamma^H_{uu}) \\
\text{s.t.} \quad & c_{aa}\tilde{\gamma}^H_{aa}+c_{au}\tilde{\gamma}^H_{au}+c_{ua}\tilde{\gamma}^H_{ua}+c_{uu}\tilde{\gamma}^H_{uu}=\bar{u}+V(\lambda^H) && (PC)\\
& \sum_{ts} c_{ts}\Delta\tilde{\gamma}_{ts}-\Delta V\geq 0 && (IC)\\
& \bar{u}+V(\lambda^H)\geq c_{au}\Gamma^H_a+c_{uu}\Gamma^H_u && (TR^a_A)\\
& \bar{u}+V(\lambda^H)\geq c_{aa}\Gamma^H_a+c_{ua}\Gamma^H_u && (TR^u_A)
\end{aligned}$$

Lemma 26. *When $\bar{u}>\bar{u}_2$, given case (ii) of Lemma 4 and case (i) of Lemma 3, $c_{aa}\geq c_{ua}$.*

Proof. Suppose not, then $c_{ua} - c_{aa} = X > 0$. The principal can then increase c_{aa} by

$$\epsilon \equiv X \frac{\tilde{\gamma}_{ua}^H}{\tilde{\gamma}_{ua}^H + \tilde{\gamma}_{aa}^H}$$

and decrease c_{ua} by $(X - \epsilon)$. This strictly increases profits, since by assumption c_{aa} does not enter the cost function.

First of all, notice that ϵ is picked in such a way that it leaves the (PC) binding. The change to the LHS of the (PC) is in fact:

$$\epsilon \tilde{\gamma}_{aa}^H - (X - \epsilon)\tilde{\gamma}_{ua}^H = \epsilon(\tilde{\gamma}_{aa}^H + \tilde{\gamma}_{ua}^H) - X\tilde{\gamma}_{ua}^H = X\tilde{\gamma}_{ua}^H - X\tilde{\gamma}_{ua}^H = 0.$$

Now let's check that this satisfies all other constraints. Start by the (IC). Since $\Delta\tilde{\gamma}_{ua}^H < 0$ and $\Delta\tilde{\gamma}_{aa} > 0$, the (IC) is relaxed by this deviation.

Now consider the (TR_A^u). The change to its RHS has to be weakly negative. This happens when

$$\epsilon\Gamma_a^H - (X - \epsilon)\Gamma_u^H \leq 0$$
$$\epsilon - X\Gamma_u^H \leq 0 \;\Rightarrow\; \epsilon \leq X\Gamma_u^H.$$

To see that this holds, notice that

$$\frac{\tilde{\gamma}_{ua}^H}{\tilde{\gamma}_{ua}^H + \tilde{\gamma}_{aa}^H} \leq \Gamma_u^H$$
$$\tilde{P}_{ua} \leq \tilde{\gamma}_{ua}^H + \tilde{\gamma}_{aa}^H$$
$$\tilde{P}_{ua} - \tilde{P}_{ua}\Gamma_u^H - \tilde{P}_{aa}\Gamma_a^H \leq 0$$
$$\tilde{P}_{ua}\Gamma_a^H - \tilde{P}_{aa}\Gamma_a^H \leq 0,$$

which is true by positive perceived correlation. This concludes the proof since neither c_{aa} nor c_{ua} enter (TR_A^a). ∎

Before going ahead we are going to rewrite the problem in a more convenient way. From the (PC), we substitute for

$$c_{aa} = \frac{1}{\tilde{\gamma}_{aa}^H}\left(\bar{u} + V(\lambda^H) - c_{au}\tilde{\gamma}_{au}^H - c_{ua}\tilde{\gamma}_{ua}^H - c_{uu}\tilde{\gamma}_{uu}^H\right)$$

in all the constraints and the objective function. The (TR_A^a) does not change from the above. The (IC) becomes

$$c_{ts}\Delta\tilde{\gamma}_{ts} - \Delta V \geq 0$$

$$\tfrac{\Delta\Gamma_a}{\Gamma_a^H}\left(\bar{u} + V(\lambda^H) - c_{au}\tilde{\gamma}_{au}^H - c_{ua}\tilde{\gamma}_{ua}^H - c_{uu}\tilde{\gamma}_{uu}^H\right) + c_{au}\Delta\tilde{\gamma}_{au} + c_{ua}\Delta\tilde{\gamma}_{ua} + c_{uu}\Delta\tilde{\gamma}_{uu} - \Delta V \geq 0$$

$$\begin{aligned} c_{au}\left(\Delta\tilde{\gamma}_{au} - \tilde{\gamma}_{au}^H\frac{\Delta\Gamma_a}{\Gamma_a^H}\right) &+ c_{ua}\left(\Delta\tilde{\gamma}_{ua} - \tilde{\gamma}_{ua}^H\frac{\Delta\Gamma_a}{\Gamma_a^H}\right) \\ &+ c_{uu}\left(\Delta\tilde{\gamma}_{uu} - \tilde{\gamma}_{uu}^H\frac{\Delta\Gamma_a}{\Gamma_a^H}\right) + \frac{\Delta\Gamma_a}{\Gamma_a^H}\left(\bar{u} + V(\lambda^H)\right) \geq \Delta V, \end{aligned}$$

where the coefficients of each c_{ts} can be rearranged as

$$\Delta\tilde{\gamma}_{ts} - \tilde{\gamma}_{ts}^H\frac{\Delta\Gamma_a}{\Gamma_a^H} = \tilde{P}_{ts}\left(\Delta\Gamma_t - \Gamma_t^H\frac{\Delta\Gamma_a}{\Gamma_a^H}\right)$$

which is equal to 0 if $t = a$ and to

$$\tilde{P}_{us}\left(\Delta\Gamma_u - \Gamma_u^H\frac{\Delta\Gamma_a}{\Gamma_a^H}\right) = \tilde{P}_{us}\left(\Delta\Gamma_u + \Gamma_u^H\frac{\Delta\Gamma_u}{\Gamma_a^H}\right) = \tilde{P}_{us}\frac{\Delta\Gamma_u}{\Gamma_a^H}\left(\Gamma_a^H + \Gamma_u^H\right) = \tilde{P}_{us}\frac{\Delta\Gamma_u}{\Gamma_a^H}$$

otherwise. This gives us

$$\begin{aligned} &c_{ua}\tilde{P}_{ua}\frac{\Delta\Gamma_u}{\Gamma_a^H} + c_{uu}\tilde{P}_{uu}\frac{\Delta\Gamma_u}{\Gamma_a^H} + \frac{\Delta\Gamma_a}{\Gamma_a^H}\left(\bar{u} + V(\lambda^H)\right) \geq \Delta V \\ &- c_{ua}\tilde{P}_{ua}\Delta\Gamma_a - c_{uu}\tilde{P}_{uu}\Delta\Gamma_a + \Delta\Gamma_a\left(\bar{u} + V(\lambda^H)\right) \geq \Delta V\Gamma_a^H \\ &\bar{u} + V(\lambda^H) - c_{ua}\tilde{P}_{ua} - c_{uu}\tilde{P}_{uu} \geq \frac{\Delta V}{\Delta\Gamma_a}\Gamma_a^H. \end{aligned}$$

Consider the (TR_A^u) in its standard form and substitute for c_{aa} from the (PC) to obtain

$$c_{au}\tilde{\gamma}_{au}^H + c_{uu}\tilde{\gamma}_{uu}^H \geq c_{aa}\tilde{\gamma}_{au}^H + c_{ua}\tilde{\gamma}_{uu}^H$$

$$c_{au}\tilde{\gamma}_{au}^H + c_{uu}\tilde{\gamma}_{uu}^H \geq \left(\bar{u} + V(\lambda^H) - c_{au}\tilde{\gamma}_{au}^H - c_{ua}\tilde{\gamma}_{ua}^H - c_{uu}\tilde{\gamma}_{uu}^H\right)\frac{\tilde{P}_{au}}{\tilde{P}_{aa}} + c_{ua}\tilde{\gamma}_{uu}^H$$

$$c_{au}\tilde{\gamma}_{au}^H\left(1 + \frac{\tilde{P}_{au}}{\tilde{P}_{aa}}\right) + c_{uu}\tilde{\gamma}_{uu}^H\left(1 + \frac{\tilde{P}_{au}}{\tilde{P}_{aa}}\right) - c_{ua}\left(\tilde{\gamma}_{uu}^H - \tilde{\gamma}_{ua}^H\frac{\tilde{P}_{au}}{\tilde{P}_{aa}}\right) \geq \frac{\tilde{P}_{au}}{\tilde{P}_{aa}}\left(\bar{u} + V(\lambda^H)\right)$$

$$c_{au}\frac{\tilde{\gamma}_{au}^H}{\tilde{P}_{aa}} + c_{uu}\frac{\tilde{\gamma}_{uu}^H}{\tilde{P}_{aa}} - c_{ua}\frac{\Gamma_u^H}{\tilde{P}_{aa}}\left(\tilde{P}_{uu}\tilde{P}_{aa} - \tilde{P}_{ua}\tilde{P}_{au}\right) \geq \frac{\tilde{P}_{au}}{\tilde{P}_{aa}}\left(\bar{u} + V(\lambda^H)\right)$$

$$c_{au}\tilde{\gamma}_{au}^H + c_{uu}\tilde{\gamma}_{uu}^H - c_{ua}\left(\tilde{P}_{aa} - \tilde{P}_{ua}\right)\Gamma_u^H \geq \left(\bar{u} + V(\lambda^H)\right)\tilde{P}_{au}.$$

Finally, we need to rearrange the objective function.

$$
\begin{aligned}
&c_{aa}(\gamma^H_{aa}+\gamma^H_{ua})+\max\{c_{au},c_{uu}\}(\gamma^H_{au}+\gamma^H_{uu})\\
&\quad=\frac{1}{\tilde{\gamma}^H_{aa}}\left(\bar{u}+V(\lambda^H)-c_{au}\tilde{\gamma}^H_{au}-c_{ua}\tilde{\gamma}^H_{ua}-c_{uu}\tilde{\gamma}^H_{uu}\right)(\gamma^H_{aa}+\gamma^H_{ua})+\max\{c_{au},c_{uu}\}(\gamma^H_{au}+\gamma^H_{uu})\\
&\quad\propto \max\{c_{au},c_{uu}\}(\gamma^H_{au}+\gamma^H_{uu})\tilde{\gamma}^H_{aa}-(\gamma^H_{aa}+\gamma^H_{ua})\left(c_{au}\tilde{\gamma}^H_{au}+c_{ua}\tilde{\gamma}^H_{ua}+c_{uu}\tilde{\gamma}^H_{uu}\right)
\end{aligned}
$$

We can now rewrite the problem.

$$
\begin{aligned}
\min_{\{c_{ts}\}_{t,s\in\{u,a\}}}\quad & \max\{c_{au},c_{uu}\}(\gamma^H_{au}+\gamma^H_{uu})\tilde{\gamma}^H_{aa}-(\gamma^H_{aa}+\gamma^H_{ua})\left(c_{au}\tilde{\gamma}^H_{au}+c_{ua}\tilde{\gamma}^H_{ua}+c_{uu}\tilde{\gamma}^H_{uu}\right) && (30)\\
\text{s.t.}\quad & \bar{u}+V(\lambda^H)-c_{ua}\tilde{P}_{ua}-c_{uu}\tilde{P}_{uu}\geq\frac{\Delta V}{\Delta\Gamma_a}\Gamma^H_a && (IC)\\
& \bar{u}+V(\lambda^H)\geq c_{au}\Gamma^H_a+c_{uu}\Gamma^H_u && (TR^a_A)\\
& c_{au}\tilde{\gamma}^H_{au}+c_{uu}\tilde{\gamma}^H_{uu}-c_{ua}\left(\tilde{P}_{aa}-\tilde{P}_{ua}\right)\Gamma^H_u\geq\left(\bar{u}+V(\lambda^H)\right)\tilde{P}_{au} && (TR^u_A)
\end{aligned}
$$

Lemma 27. *When $\bar{u}>\bar{u}_2$, given case (ii) of Lemma 4 and case (i) of Lemma 3, $c_{au}=c_{uu}$.*

Proof. We first prove that assuming $c_{uu}>c_{au}$ leads to a contradiction. Then we prove the same for the case of $c_{au}>c_{uu}$.

PART 1: suppose $\boldsymbol{c_{uu}>c_{au}}$. We are going to show that all possible optimal contracts emerging from this case contradict this assumption. The new problem would be

$$
\begin{aligned}
\min_{\{c_{ts}\}_{t,s\in\{u,a\}}}\quad & c_{uu}\left[(\gamma^H_{au}+\gamma^H_{uu})\tilde{\gamma}^H_{aa}-(\gamma^H_{aa}+\gamma^H_{ua})\tilde{\gamma}^H_{uu}\right]-(\gamma^H_{aa}+\gamma^H_{ua})\left(c_{ua}\tilde{\gamma}^H_{ua}+c_{au}\tilde{\gamma}^H_{au}\right)\\
\text{s.t.}\quad & \bar{u}+V(\lambda^H)-c_{ua}\tilde{P}_{ua}-c_{uu}\tilde{P}_{uu}\geq\frac{\Delta V}{\Delta\Gamma_a}\Gamma^H_a && (IC)\\
& \bar{u}+V(\lambda^H)\geq c_{au}\Gamma^H_a+c_{uu}\Gamma^H_u && (TR^a_A)\\
& c_{au}\tilde{\gamma}^H_{au}+c_{uu}\tilde{\gamma}^H_{uu}-c_{ua}\left(\tilde{P}_{aa}-\tilde{P}_{ua}\right)\Gamma^H_u\geq\left(\bar{u}+V(\lambda^H)\right)\tilde{P}_{au} && (TR^u_A)
\end{aligned}
$$

where the coefficient of c_{uu} in the objective function

$$
(\gamma^H_{au}+\gamma^H_{uu})\tilde{\gamma}^H_{aa}-(\gamma^H_{aa}+\gamma^H_{ua})\tilde{\gamma}^H_{uu}
$$

is of no clear sign. We start by assuming it is positive.

It is obvious to see that the (TR_A^a) binds. Suppose it did not, then the principal could increase c_{au} enough for it to do so. As a consequence of this change, the objective function would be decreased, the (TR_A^u) relaxed and the (IC) unaffected. Hence we can solve (TR_A^a) for c_{au}

$$c_{au} = \frac{1}{\Gamma_a^H}\left(\bar{u} + V(\lambda^H) - c_{uu}\Gamma_u^H\right).$$

Plugging this into the objective function yields

$$\begin{aligned}
& c_{uu}\left[(\gamma_{au}^H + \gamma_{uu}^H)\tilde{\gamma}_{aa}^H - (\gamma_{aa}^H + \gamma_{ua}^H)\tilde{\gamma}_{uu}^H\right] - (\gamma_{aa}^H + \gamma_{ua}^H)\left(c_{ua}\tilde{\gamma}_{ua}^H + c_{au}\tilde{\gamma}_{au}^H\right) \\
&= c_{uu}\left[(\gamma_{au}^H + \gamma_{uu}^H)\tilde{\gamma}_{aa}^H - (\gamma_{aa}^H + \gamma_{ua}^H)\tilde{\gamma}_{uu}^H\right] \\
&\qquad - (\gamma_{aa}^H + \gamma_{ua}^H)\left[c_{ua}\tilde{\gamma}_{ua}^H + \frac{\tilde{\gamma}_{au}^H}{\Gamma_a^H}\left(\bar{u} + V(\lambda^H) - c_{uu}\Gamma_u^H\right)\right] \\
&\propto c_{uu}\left[(\gamma_{au}^H + \gamma_{uu}^H)\tilde{\gamma}_{aa}^H - (\gamma_{aa}^H + \gamma_{ua}^H)\tilde{\gamma}_{uu}^H\right] - (\gamma_{aa}^H + \gamma_{ua}^H)\left[c_{ua}\tilde{\gamma}_{ua}^H - \tilde{P}_{au}c_{uu}\Gamma_u^H\right] \\
&= c_{uu}\left[(\gamma_{au}^H + \gamma_{uu}^H)\tilde{\gamma}_{aa}^H - (\gamma_{aa}^H + \gamma_{ua}^H)\left(\tilde{\gamma}_{uu}^H - \tilde{P}_{au}\Gamma_u^H\right)\right] - (\gamma_{aa}^H + \gamma_{ua}^H)c_{ua}\tilde{\gamma}_{ua}^H \\
&= c_{uu}\left[(\gamma_{au}^H + \gamma_{uu}^H)\tilde{\gamma}_{aa}^H - (\gamma_{aa}^H + \gamma_{ua}^H)\Gamma_u^H\left(\tilde{P}_{uu} - \tilde{P}_{au}\right)\right] - (\gamma_{aa}^H + \gamma_{ua}^H)c_{ua}\tilde{\gamma}_{ua}^H. \qquad (31)
\end{aligned}$$

Since we assumed

$$(\gamma_{au}^H + \gamma_{uu}^H)\tilde{\gamma}_{aa}^H - (\gamma_{aa}^H + \gamma_{ua}^H)\tilde{\gamma}_{uu}^H > 0,$$

it is immediate to see how

$$(\gamma_{au}^H + \gamma_{uu}^H)\tilde{\gamma}_{aa}^H - (\gamma_{aa}^H + \gamma_{ua}^H)\Gamma_u^H\left(\tilde{P}_{uu} - \tilde{P}_{au}\right) > 0,$$

given that $\tilde{\gamma}_{uu}^H \leq \tilde{\gamma}_{uu}^H - \Gamma_u^H\tilde{P}_{au} = \Gamma_u^H\left(\tilde{P}_{uu} - \tilde{P}_{au}\right)$. Hence, the principal's objective is still to decrease c_{uu} and increase c_{ua}.

We now plug the value for c_{au} into the (TR_A^u).

$$\begin{aligned}
& \frac{\tilde{\gamma}_{au}^H}{\Gamma_a^H}\left(\bar{u} + V(\lambda^H) - c_{uu}\Gamma_u^H\right) + c_{uu}\tilde{\gamma}_{uu}^H - c_{ua}\left(\tilde{P}_{aa} - \tilde{P}_{ua}\right)\Gamma_u^H \geq \left(\bar{u} + V(\lambda^H)\right)\tilde{P}_{au} \\
& \left(\bar{u} + V(\lambda^H)\right)\left(\tilde{P}_{au} - \tilde{P}_{au}\right) + c_{uu}(\tilde{\gamma}_{uu}^H - \tilde{P}_{au}\Gamma_u^H) - c_{ua}\left(\tilde{P}_{aa} - \tilde{P}_{ua}\right)\Gamma_u^H \geq 0 \\
& c_{uu}(\tilde{P}_{uu} - \tilde{P}_{au})\Gamma_u^H - c_{ua}\left(\tilde{P}_{aa} - \tilde{P}_{ua}\right)\Gamma_u^H \geq 0 \\
& c_{uu} \geq c_{ua},
\end{aligned}$$

where we used the fact that $(\tilde{P}_{uu} - \tilde{P}_{au}) = (1 - \tilde{P}_{ua} - 1 + \tilde{P}_{aa}) = (\tilde{P}_{aa} - \tilde{P}_{ua})$.

The new (TR_A^u) and the (IC) can then be plotted in Figure 7 below. Contracts that satisfy all constraints must lie above the (TR_A^u) line and below the (IC) one.

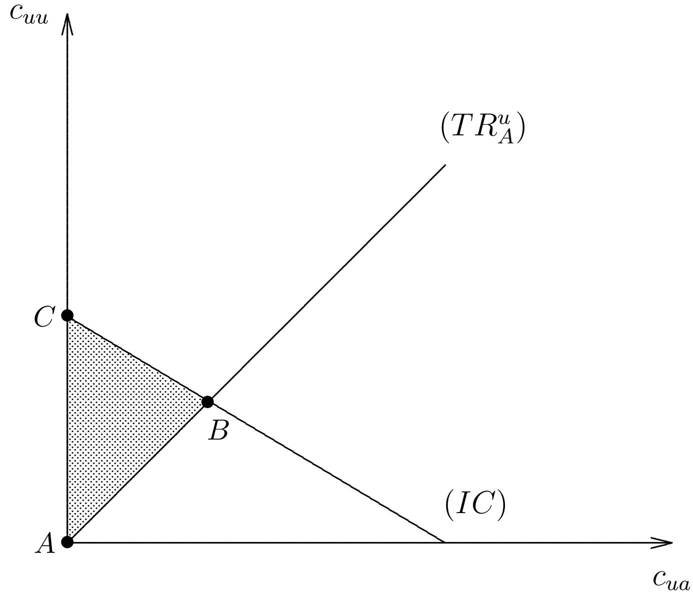

FIGURE 7. Contracts lying in the shaded area satisfy all constraints when $c_{uu} > c_{au}$.

By the analysis above, isocosts are positively sloped and decrease in value towards the bottom right of Figure 7. This implies that, depending on the relative slope of the isocosts and the (TR_A^u), the optimal contract for this case lies either at point A or B.[37] Both of these points, however, contradict the assumption that $c_{uu} > c_{au}$.

To see why A contradicts the assumption, simply notice that, at A, $c_{uu} = 0$ and therefore it cannot be $c_{uu} > c_{au} \geq 0$.

To see why also point B contradicts our assumption, we can solve for the optimal c_{uu} and c_{au}. At B, we have that $c_{ua} = c_{uu}$. Hence we solve for c_{uu} in the (IC) to get

$$c_{uu} = \bar{u} + V(\lambda^H) - \frac{\Delta V}{\Delta \Gamma_a} \Gamma_a^H.$$

[37]Or any linear combination fo the two when the slopes exactly coincide.

While we can prove that the above is always positive, notice that plugging this value in the formula for c_{au} yields

$$\begin{aligned} c_{au} =& \frac{1}{\Gamma_a^H}\left\{\bar{u}+V(\lambda^H)-\left[\bar{u}+V(\lambda^H)-\frac{\Delta V}{\Delta\Gamma_a}\Gamma_a^H\right]\Gamma_u^H\right\} \\ =& \frac{1}{\Gamma_a^H}\left[\left(\bar{u}+V(\lambda^H)\right)\underbrace{(1-\Gamma_u^H}_{\Gamma_a^H}+\frac{\Delta V}{\Delta\Gamma_a}\Gamma_a^H\Gamma_u^H\right] \\ =& \bar{u}+V(\lambda^H)+\frac{\Delta V}{\Delta\Gamma_a}\Gamma_u^H \;\geq\; \bar{u}+V(\lambda^H)-\frac{\Delta V}{\Delta\Gamma_a}\Gamma_a^H = c_{uu}. \end{aligned}$$

This proves that, when

$$(\gamma_{au}^H+\gamma_{uu}^H)\tilde{\gamma}_{aa}^H-(\gamma_{aa}^H+\gamma_{ua}^H)\tilde{\gamma}_{uu}^H>0,$$

c_{au} must be larger or equal than c_{uu}. We are now going to prove the same for the opposite case.

Start again from the problem

$$\begin{aligned} \min_{\{c_{ts}\}_{t,s\in\{u,a\}}} \quad & c_{uu}\left[(\gamma_{au}^H+\gamma_{uu}^H)\tilde{\gamma}_{aa}^H-(\gamma_{aa}^H+\gamma_{ua}^H)\tilde{\gamma}_{uu}^H\right]-(\gamma_{aa}^H+\gamma_{ua}^H)\left(c_{ua}\tilde{\gamma}_{ua}^H+c_{au}\tilde{\gamma}_{au}^H\right) \\ \text{s.t.} \quad & \bar{u}+V(\lambda^H)-c_{ua}\tilde{P}_{ua}-c_{uu}\tilde{P}_{uu}\geq\frac{\Delta V}{\Delta\Gamma_a}\Gamma_a^H \qquad (IC) \\ & \bar{u}+V(\lambda^H)\geq c_{au}\Gamma_a^H+c_{uu}\Gamma_u^H \qquad (TR_A^a) \\ & c_{au}\tilde{\gamma}_{au}^H+c_{uu}\tilde{\gamma}_{uu}^H-c_{ua}\left(\tilde{P}_{aa}-\tilde{P}_{ua}\right)\Gamma_u^H\geq\left(\bar{u}+V(\lambda^H)\right)\tilde{P}_{au} \qquad (TR_A^u) \end{aligned}$$

and suppose now that the coefficient of c_{uu} is negative. That is,

$$(\gamma_{au}^H+\gamma_{uu}^H)\tilde{\gamma}_{aa}^H-(\gamma_{aa}^H+\gamma_{ua}^H)\tilde{\gamma}_{uu}^H<0.$$

First of all, notice that the (TR_A^a) still binds, since the discussion above still holds. This implies that we can solve for c_{au} again and substitute it in the objective function and other constraints, exactly as we did above. Now, however, the new coefficient of c_{uu} featured in the new objective function may itself be positive or negative. Suppose it is positive, then the problem is exactly the same as above and the proof for the case of $(\gamma_{au}^H+\gamma_{uu}^H)\tilde{\gamma}_{aa}^H-(\gamma_{aa}^H+\gamma_{ua}^H)\tilde{\gamma}_{uu}^H>0$ holds.

Things change when the new coefficient is still negative. That is, when

$$(\gamma_{au}^H + \gamma_{uu}^H)\tilde{\gamma}_{aa}^H - (\gamma_{aa}^H + \gamma_{ua}^H)\Gamma_u^H \left(\tilde{P}_{uu} - \tilde{P}_{au}\right) < 0.$$

In this case, while the contracts that satisfy all constraints are still the ones in the shaded area of Figure 7, the isocosts now decrease in value towards the top right corner. Hence, depending on the relative slope of isocosts and (IC), the optimal contract lies either at point B or C of the Figure. We have already shown how point B contradicts $c_{uu} > c_{au}$. Instead of checking that the same is true for point C, we are going to show that this point is never an equilibrium and that B is the sole optimal contract for this case. To do so, we show that the isocosts are always strictly steeper than the (IC) when

$$(\gamma_{au}^H + \gamma_{uu}^H)\tilde{\gamma}_{aa}^H - (\gamma_{aa}^H + \gamma_{ua}^H)\Gamma_u^H \left(\tilde{P}_{uu} - \tilde{P}_{au}\right) < 0.$$

The slope of the (IC) is given by $\frac{\tilde{P}_{ua}}{\tilde{P}_{uu}}$. The one of the isocosts, according to the new objective function (31), is given by

$$\frac{(\gamma_{aa}^H + \gamma_{ua}^H)\tilde{\gamma}_{ua}^H}{(\gamma_{au}^H + \gamma_{uu}^H)\tilde{\gamma}_{aa}^H - (\gamma_{aa}^H + \gamma_{ua}^H)\Gamma_u^H \left(\tilde{P}_{uu} - \tilde{P}_{au}\right)}.$$

The optimal contract lies at point C iff

$$\frac{\tilde{P}_{ua}}{\tilde{P}_{uu}} \geq \frac{(\gamma_{aa}^H + \gamma_{ua}^H)\tilde{\gamma}_{ua}^H}{(\gamma_{au}^H + \gamma_{uu}^H)\tilde{\gamma}_{aa}^H - (\gamma_{aa}^H + \gamma_{ua}^H)\Gamma_u^H \left(\tilde{P}_{uu} - \tilde{P}_{au}\right)}.$$

To see that the above never holds calculate

$$\begin{aligned}
&\frac{\tilde{P}_{ua}}{\tilde{P}_{uu}} \geq \frac{(\gamma_{aa}^H + \gamma_{ua}^H)\tilde{\gamma}_{ua}^H}{(\gamma_{au}^H + \gamma_{uu}^H)\tilde{\gamma}_{aa}^H - (\gamma_{aa}^H + \gamma_{ua}^H)\Gamma_u^H \left(\tilde{P}_{uu} - \tilde{P}_{au}\right)} \\
&(\gamma_{au}^H + \gamma_{uu}^H)\tilde{\gamma}_{aa}^H \tilde{P}_{ua} - (\gamma_{aa}^H + \gamma_{ua}^H)\Gamma_u^H \left(\tilde{P}_{uu} - \tilde{P}_{au}\right)\tilde{P}_{ua} \geq (\gamma_{aa}^H + \gamma_{ua}^H)\tilde{P}_{ua}\Gamma_u^H \tilde{P}_{uu} \\
&(\gamma_{au}^H + \gamma_{uu}^H)\tilde{\gamma}_{aa}^H \tilde{P}_{ua} - (\gamma_{aa}^H + \gamma_{ua}^H)\left[\tilde{P}_{ua}\Gamma_u^H \left(\tilde{P}_{uu} - \tilde{P}_{au}\right) + \tilde{P}_{uu}\tilde{P}_{ua}\Gamma_u^H\right] \geq 0 \\
&(\gamma_{au}^H + \gamma_{uu}^H)\tilde{\gamma}_{aa}^H - (\gamma_{aa}^H + \gamma_{ua}^H)\left[\Gamma_u^H \left(\tilde{P}_{uu} - \tilde{P}_{au}\right) + \tilde{P}_{uu}\Gamma_u^H\right] \geq 0 \\
&(\gamma_{au}^H + \gamma_{uu}^H)\tilde{\gamma}_{aa}^H - (\gamma_{aa}^H + \gamma_{ua}^H)\Gamma_u^H \left(2\tilde{P}_{uu} - \tilde{P}_{au}\right) \geq 0,
\end{aligned}$$

which is never true since we are in the case of

$$(\gamma_{au}^H + \gamma_{uu}^H)\tilde{\gamma}_{aa}^H - (\gamma_{aa}^H + \gamma_{ua}^H)\Gamma_u^H \left(\tilde{P}_{uu} - \tilde{P}_{au}\right) < 0,$$

and $\Gamma_u^H\left(2\tilde{P}_{uu}-\tilde{P}_{au}\right)>\Gamma_u^H\left(\tilde{P}_{uu}-\tilde{P}_{au}\right)$.

This concludes the proof of Part 1 by showing that all potentially optimal contracts resulting from the assumption that $c_{uu}>c_{au}$ in (30) actually feature $c_{au}\geq c_{uu}$.

PART 2: suppose $c_{au}>c_{uu}$. We start by re-stating the problem

$$\min_{\{c_{ts}\}_{t,s\in\{u,a\}}} c_{au}\Gamma_a^H\left[\left(P_{aa}-P_{ua}\right)\Gamma_u^H+b_a\right]-\left(\gamma_{aa}^H+\gamma_{ua}^H\right)\left(c_{ua}\tilde{\gamma}_{ua}^H+c_{uu}\tilde{\gamma}_{uu}^H\right) \tag{32}$$

$$\text{s.t.}\quad \bar{u}+V(\lambda^H)-c_{ua}\tilde{P}_{ua}-c_{uu}\tilde{P}_{uu}\geq\frac{\Delta V}{\Delta\Gamma_a}\Gamma_a^H \tag{IC}$$

$$\bar{u}+V(\lambda^H)\geq c_{au}\Gamma_a^H+c_{uu}\Gamma_u^H \tag{TR_A^a}$$

$$c_{au}\tilde{\gamma}_{au}^H+c_{uu}\tilde{\gamma}_{uu}^H-c_{ua}\left(\tilde{P}_{aa}-\tilde{P}_{ua}\right)\Gamma_u^H\geq\left(\bar{u}+V(\lambda^H)\right)\tilde{P}_{au} \tag{TR_A^u}.$$

We then can see that the (TR_A^u) binds. Suppose not. The principal can decrease c_{au} by an ϵ such that $c_{au}>c_{uu}$ is preserved. This does not affect the (IC), it relaxes the (TR_A^a) and decreases the objective function.

We can now solve for c_{au} from the (TR_A^u) to get

$$\begin{aligned}c_{au}=&\frac{1}{\tilde{\gamma}_{au}^H}\left[\left(\bar{u}+V(\lambda^H)\right)\tilde{P}_{au}-c_{uu}\tilde{\gamma}_{uu}^H+c_{ua}\left(\tilde{P}_{aa}-\tilde{P}_{ua}\right)\Gamma_u^H\right]\\=&\frac{\left(\bar{u}+V(\lambda^H)\right)}{\Gamma_a^H}-c_{uu}\frac{\tilde{\gamma}_{uu}^H}{\tilde{\gamma}_{au}^H}+c_{ua}\frac{\left(\tilde{P}_{aa}-\tilde{P}_{ua}\right)\Gamma_u^H}{\tilde{\gamma}_{au}^H}\end{aligned}$$

and plug it in the objective function to obtain the following:

$$\left(\left(\bar{u}+V(\lambda^H)\right)-c_{uu}\frac{\tilde{\gamma}_{uu}^H}{\tilde{P}_{au}}+c_{ua}\frac{\left(\tilde{P}_{aa}-\tilde{P}_{ua}\right)\Gamma_u^H}{\tilde{P}_{au}}\right)\left[\left(P_{aa}-P_{ua}\right)\Gamma_u^H+b_a\right]+\\-\left(\gamma_{aa}^H+\gamma_{ua}^H\right)\left(c_{ua}\tilde{\gamma}_{ua}^H+c_{uu}\tilde{\gamma}_{uu}^H\right),$$

which is equivalent to minimizing

$$\left(-c_{uu}\frac{\tilde{\gamma}_{uu}^H}{\tilde{P}_{au}}+c_{ua}\frac{\left(\tilde{P}_{aa}-\tilde{P}_{ua}\right)\Gamma_u^H}{\tilde{P}_{au}}\right)\left[\left(P_{aa}-P_{ua}\right)\Gamma_u^H+b_a\right]-\left(\gamma_{aa}^H+\gamma_{ua}^H\right)\left(c_{ua}\tilde{\gamma}_{ua}^H+c_{uu}\tilde{\gamma}_{uu}^H\right).$$

Let us study the coefficients of c_{uu} and c_{ua} separately. Start from the coefficient of c_{ua} and recall that we obtained $\left(P_{aa}-P_{ua}\right)\Gamma_u^H+b_a$ from the simplification of $(\gamma_{au}^H+$

$\gamma^H_{uu})\tilde{P}_{aa} - (\gamma^H_{aa} + \gamma^H_{ua})\tilde{P}_{au}$.

$$\begin{aligned}
&\frac{\left(\tilde{P}_{aa} - \tilde{P}_{ua}\right)\Gamma^H_u}{\tilde{P}_{au}}\left[(P_{aa} - P_{ua})\,\Gamma^H_u + b_a\right] - (\gamma^H_{aa} + \gamma^H_{ua})\tilde{\gamma}^H_{ua} \\
&= \frac{\left(\tilde{P}_{aa} - \tilde{P}_{ua}\right)\Gamma^H_u}{\tilde{P}_{au}}\left[(\gamma^H_{au} + \gamma^H_{uu})\tilde{P}_{aa} - (\gamma^H_{aa} + \gamma^H_{ua})\tilde{P}_{au}\right] - (\gamma^H_{aa} + \gamma^H_{ua})\tilde{\gamma}^H_{ua} \\
&\propto \left(\tilde{P}_{aa} - \tilde{P}_{ua}\right)\left[(\gamma^H_{au} + \gamma^H_{uu})\tilde{P}_{aa} - (\gamma^H_{aa} + \gamma^H_{ua})\tilde{P}_{au}\right] - (\gamma^H_{aa} + \gamma^H_{ua})\tilde{P}_{ua}\tilde{P}_{au} \\
&= \left(\tilde{P}_{aa} - \tilde{P}_{ua}\right)(\gamma^H_{au} + \gamma^H_{uu})\tilde{P}_{aa} - (\gamma^H_{aa} + \gamma^H_{ua})\left[\left(\tilde{P}_{aa} - \tilde{P}_{ua}\right)\tilde{P}_{au} + \tilde{P}_{ua}\tilde{P}_{au}\right] \\
&= \left(\tilde{P}_{aa} - \tilde{P}_{ua}\right)(\gamma^H_{au} + \gamma^H_{uu})\tilde{P}_{aa} - (\gamma^H_{aa} + \gamma^H_{ua})\tilde{P}_{au}\tilde{P}_{aa} \\
&\propto \left(\tilde{P}_{aa} - \tilde{P}_{ua}\right)(\gamma^H_{au} + \gamma^H_{uu}) - (\gamma^H_{aa} + \gamma^H_{ua})\tilde{P}_{au} \\
&= \left(1 - \tilde{P}_{au} - \tilde{P}_{ua}\right)(\gamma^H_{au} + \gamma^H_{uu}) - (\gamma^H_{aa} + \gamma^H_{ua})\tilde{P}_{au} \\
&= \left(1 - \tilde{P}_{ua}\right)(\gamma^H_{au} + \gamma^H_{uu}) - \tilde{P}_{au} \\
&= \tilde{P}_{uu}(\gamma^H_{au} + \gamma^H_{uu}) - \tilde{P}_{au}.
\end{aligned}$$

This implies that the coefficient of c_{ua} in the new objective function has no clear sign. It is positive only if

$$b_a \geq P_{au} - (\gamma^H_{au} + \gamma^H_{uu})(P_{uu} - b_u). \tag{33}$$

The coefficient of c_{uu}, instead, is always negative and it simplifies to

$$
\begin{aligned}
&-\frac{(P_{aa}-P_{ua})\,\Gamma_u^H+b_a}{\tilde{P}_{au}}\tilde{\gamma}_{uu}^H-(\gamma_{aa}^H+\gamma_{ua}^H)\tilde{\gamma}_{uu}^H\\
&=-\frac{\tilde{\gamma}_{uu}^H}{\tilde{P}_{au}}\left[(P_{aa}-P_{ua})\,\Gamma_u^H+b_a+(\gamma_{aa}^H+\gamma_{ua}^H)\tilde{P}_{au}\right]\\
&=-\frac{\tilde{\gamma}_{uu}^H}{\tilde{P}_{au}}\left[P_{aa}\Gamma_u^H-P_{ua}\Gamma_u^H+b_a+\gamma_{aa}^H\tilde{P}_{au}+\gamma_{ua}^H\tilde{P}_{au}\right]\\
&=-\frac{\tilde{\gamma}_{uu}^H}{\tilde{P}_{au}}\left[P_{aa}\Gamma_u^H-P_{ua}\Gamma_u^H+b_a+P_{aa}\Gamma_a^H P_{au}+P_{ua}\Gamma_u^H P_{au}-b_a(P_{aa}\Gamma_a^H+P_{ua}\Gamma_u^H)\right]\\
&=-\frac{\tilde{\gamma}_{uu}^H}{\tilde{P}_{au}}\left[P_{aa}\Gamma_u^H-P_{ua}\Gamma_u^H(1-P_{au})+P_{aa}\Gamma_a^H P_{au}+b_a\underbrace{(1-\gamma_{aa}^H-\gamma_{ua}^H)}_{\gamma_{au}^H+\gamma_{uu}^H}\right]\\
&=-\frac{\tilde{\gamma}_{uu}^H}{\tilde{P}_{au}}\left[P_{aa}\Gamma_u^H-P_{ua}\Gamma_u^H P_{aa}+P_{aa}\Gamma_a^H P_{au}+b_a(\gamma_{au}^H+\gamma_{uu}^H)\right]\\
&=-\frac{\tilde{\gamma}_{uu}^H}{\tilde{P}_{au}}\left[P_{aa}\Gamma_u^H P_{uu}+P_{aa}\Gamma_a^H P_{au}+b_a(\gamma_{au}^H+\gamma_{uu}^H)\right]\\
&=-\frac{\tilde{\gamma}_{uu}^H}{\tilde{P}_{au}}(\gamma_{au}^H+\gamma_{uu}^H)\,(P_{aa}+b_a)=-\frac{\tilde{\gamma}_{uu}^H}{\tilde{P}_{au}}(\gamma_{au}^H+\gamma_{uu}^H)\tilde{P}_{aa}.
\end{aligned}
$$

Before re-writing the problem fully, let us calculate the new (TR_A^u).

$$
\begin{aligned}
&\bar{u}+V(\lambda^H)\geq\left(\bar{u}+V(\lambda^H)\right)-c_{uu}\frac{\tilde{\gamma}_{uu}^H}{\tilde{P}_{au}}+c_{ua}\frac{\left(\tilde{P}_{aa}-\tilde{P}_{ua}\right)\Gamma_u^H}{\tilde{P}_{au}}+c_{uu}\Gamma_u^H\\
&c_{uu}\left(\frac{\tilde{\gamma}_{uu}^H}{\tilde{P}_{au}}-\Gamma_u^H\right)\geq c_{ua}\frac{\left(\tilde{P}_{aa}-\tilde{P}_{ua}\right)\Gamma_u^H}{\tilde{P}_{au}}\\
&c_{uu}\Gamma_u^H\left(\tilde{P}_{uu}-\tilde{P}_{au}\right)\geq c_{ua}\left(\tilde{P}_{aa}-\tilde{P}_{ua}\right)\Gamma_u^H\\
&c_{uu}\geq c_{ua}.
\end{aligned}
$$

Hence, the new problem for the case of $c_{au}>c_{uu}$ is given by

$$
\min_{\{c_{ts}\}_{t,s\in\{u,a\}}} c_{ua}\frac{\Gamma_u^H}{\tilde{P}_{au}}\left(\tilde{P}_{uu}(\gamma_{au}^H+\gamma_{uu}^H)-\tilde{P}_{au}\right)-c_{uu}\frac{\tilde{\gamma}_{uu}^H}{\tilde{P}_{au}}(\gamma_{au}^H+\gamma_{uu}^H)\,(P_{aa}+b_a) \tag{34}
$$

$$
\text{s.t.}\quad \bar{u}+V(\lambda^H)-c_{ua}\tilde{P}_{ua}-c_{uu}\tilde{P}_{uu}\geq\frac{\Delta V}{\Delta\Gamma_a}\Gamma_a^H \tag{IC}
$$

$$
c_{uu}\geq c_{ua}. \tag{TR_A^a}
$$

We now represent the constraints and feasible contracts in Figure 8 below. In the graph, contracts above the (TR_A^a) and below the (IC) satisfy all constraints (are feasible). Recall, however, that we are in case (ii) of Lemma 4. Hence, all contracts lying on the locus of points that sets the (TR_A^a) binding, although the lie in the feasible set, if optimal, would contraddict $c_{uu} > c_{ua}$.

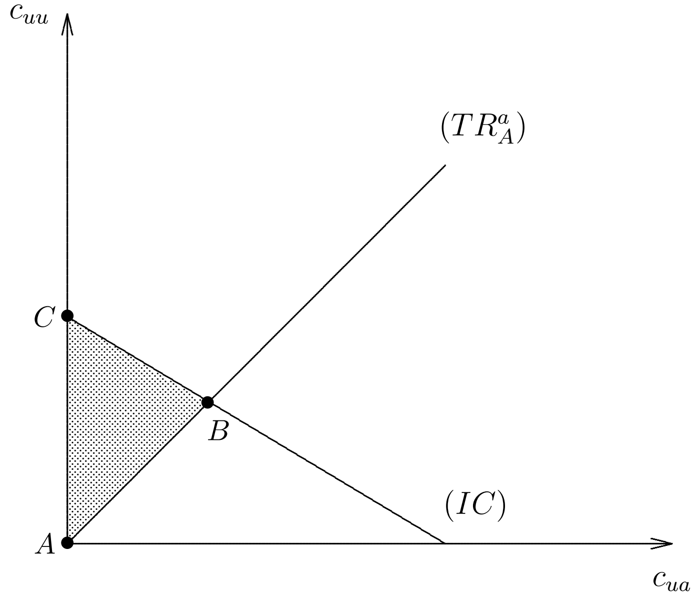

FIGURE 8. Contracts lying in the shaded area satisfy all constraints when $c_{au} > c_{uu}$. The ones lying on the (TR_A^a), however, violate case (ii) of Lemma 4.

Suppose (33) holds, then the objective function is minimized towards the top left corner of the Figure. This, inevitably, yields as an optimal contract the one lying in point C.

Suppose (33) fails, then the objective function is minimized towards the top right corner of the Figure. If the isocosts are flatter than the (IC) then point C lies on an isocosts that features a lower cost than the one where B lies. The reverse is true when the isocost is steeper. We are now going to show that at point C the assumption $c_{au} > c_{uu}$ is violated as well, leading to the desired contradiction.

At point C, $c_{ua} = 0$ is trivial. c_{uu} comes from the (IC) binding and c_{au} is derived from

$$\begin{aligned}
c_{au} =& \frac{1}{\tilde{\gamma}^H_{au}} \left[\left(\bar{u} + V(\lambda^H)\right) \tilde{P}_{au} - c_{uu}\tilde{\gamma}^H_{uu} + c_{ua}\left(\tilde{P}_{aa} - \tilde{P}_{ua}\right)\Gamma^H_u \right] \\
=& \frac{\left(\bar{u} + V(\lambda^H)\right)}{\Gamma^H_a} - \frac{\bar{u} + V(\lambda^H) - \frac{\Delta V}{\Delta \Gamma_a}\Gamma^H_a}{\tilde{P}_{uu}} \cdot \frac{\tilde{\gamma}^H_{uu}}{\tilde{\gamma}^H_{au}} \\
=& \frac{1}{\tilde{P}_{au}\Gamma^H_a} \left\{ \left(\bar{u} + V(\lambda^H)\right)\tilde{P}_{au} - \left(\bar{u} + V(\lambda^H)\right)\Gamma^H_u + \tfrac{\Delta V}{\Delta \Gamma_a}\Gamma^H_a\Gamma^H_u \right\} \\
=& \frac{1}{\tilde{P}_{au}\Gamma^H_a} \left\{ \left(\bar{u} + V(\lambda^H)\right)\left(\tilde{P}_{au} - \Gamma^H_u\right) + \tfrac{\Delta V}{\Delta \Gamma_a}\Gamma^H_a\Gamma^H_u \right\}.
\end{aligned}$$

Finally, to get c_{aa} simply plug the values in

$$c_{aa} = \frac{1}{\tilde{\gamma}^H_{aa}} \left(\bar{u} + V(\lambda^H) - c_{au}\tilde{\gamma}^H_{au} - c_{ua}\tilde{\gamma}^H_{ua} - c_{uu}\tilde{\gamma}^H_{uu}\right).$$

We now derive the contradiction. Start by the following calculations

$$\begin{aligned}
& c_{au} > c_{uu} \\
& \frac{\left(\bar{u}+V(\lambda^H)\right)\left(\tilde{P}_{au}-\Gamma^H_u\right)+\frac{\Delta V}{\Delta\Gamma_a}\Gamma^H_a\Gamma^H_u}{\tilde{P}_{au}\Gamma^H_a} > \frac{\bar{u} + V(\lambda^H) - \frac{\Delta V}{\Delta \Gamma_a}\Gamma^H_a}{\tilde{P}_{uu}} \\
& \left(\bar{u} + V(\lambda^H)\right)\left[\tilde{P}_{uu}(\tilde{P}_{au} - \Gamma^H_u) - \tilde{P}_{au}\Gamma^H_a\right] + \tfrac{\Delta V}{\Delta \Gamma_a}\Gamma^H_a\left(\tilde{P}_{uu}\Gamma^H_u + \tilde{P}_{au}\Gamma^H_a\right) > 0 \\
& \left(\bar{u} + V(\lambda^H)\right)\left[\tilde{P}_{uu}(1 - \tilde{P}_{aa} - 1 + \Gamma^H_a) - \tilde{P}_{au}\Gamma^H_a\right] + \tfrac{\Delta V}{\Delta \Gamma_a}\Gamma^H_a\left(\tilde{P}_{uu}\Gamma^H_u + \tilde{P}_{au}\Gamma^H_a\right) > 0 \\
& -\left(\bar{u} + V(\lambda^H)\right)\left[\tilde{P}_{au}\Gamma^H_a + \tilde{P}_{uu}(\tilde{P}_{aa} - \Gamma^H_a)\right] + \tfrac{\Delta V}{\Delta \Gamma_a}\Gamma^H_a\left(\tilde{P}_{uu}\Gamma^H_u + \tilde{P}_{au}\Gamma^H_a\right) > 0.
\end{aligned}$$

and notice that $\tilde{P}_{au}\Gamma^H_a + \tilde{P}_{uu}(\tilde{P}_{aa} - \Gamma^H_a)$ corresponds to the denominator of the first component of $\bar{u}_2$, which is always positive.[38] Hence, the sign of the inequality is not trivial.

Let's now substitute $\bar{u}$ for $\bar{u}_2$. We are going to show that, when this holds, the inequality above fails. Hence, since the LHS is decreasing in $\bar{u}$, for values larger than

[38]To see this calculate:

$$\begin{aligned}
&\tilde{P}_{au}\Gamma^H_a + \tilde{P}_{uu}(\tilde{P}_{aa} - \Gamma^H_a) = \tilde{P}_{au}\Gamma^H_a + \tilde{P}_{uu}(1 - \Gamma^H_a - 1 + \tilde{P}_{aa}) \\
&\quad = \tilde{P}_{au}\Gamma^H_a + \tilde{P}_{uu}\Gamma^H_u - \tilde{P}_{au}\tilde{P}_{uu} = (1 - \tilde{P}_{au})\tilde{P}_{uu}\Gamma^H_u + \tilde{P}_{au}\Gamma^H_a(1 - \tilde{P}_{uu}) \\
&\quad = \tilde{P}_{aa}\tilde{P}_{uu}\Gamma^H_u + \tilde{\gamma}^H_{au}\tilde{P}_{ua} > 0.
\end{aligned}$$

$\bar{u}_2$ the inequality fails too. To see this, calculate

$$-\left(\bar{u}+V(\lambda^H)\right)\left[\tilde{P}_{au}\Gamma_a^H+\tilde{P}_{uu}(\tilde{P}_{aa}-\Gamma_a^H)\right]+\tfrac{\Delta V}{\Delta\Gamma_a}\Gamma_a^H\left(\tilde{P}_{uu}\Gamma_u^H+\tilde{P}_{au}\Gamma_a^H\right)>0$$

$$-\left(\frac{\Delta V}{\Delta\Gamma_a}\Gamma_a^H\frac{\tilde{P}_{au}\Gamma_a^H+\tilde{P}_{uu}\Gamma_u^H}{\tilde{P}_{au}\Gamma_a^H+\tilde{P}_{uu}(\tilde{P}_{aa}-\Gamma_a^H)}-V(\lambda^H)+V(\lambda^H)\right)\left[\tilde{P}_{au}\Gamma_a^H+\tilde{P}_{uu}(\tilde{P}_{aa}-\Gamma_a^H)\right]$$
$$+\tfrac{\Delta V}{\Delta\Gamma_a}\Gamma_a^H\left(\tilde{P}_{uu}\Gamma_u^H+\tilde{P}_{au}\Gamma_a^H\right)>0$$

$$-\tfrac{\Delta V}{\Delta\Gamma_a}\Gamma_a^H\left(\tilde{P}_{uu}\Gamma_u^H+\tilde{P}_{au}\Gamma_a^H\right)+\tfrac{\Delta V}{\Delta\Gamma_a}\Gamma_a^H\left(\tilde{P}_{uu}\Gamma_u^H+\tilde{P}_{au}\Gamma_a^H\right)>0$$

which obviously fails. Hence, point C violates our assumption and is ruled out.[39] This concludes the proof of this part and of the Lemma. ∎

Given Lemma 27, we can derive the final version of the problem:

$$\min_{\{c_{ts}\}_{t,s\in\{u,a\}}} c_{uu}\left[(\gamma_{au}^H+\gamma_{uu}^H)\tilde{\gamma}_{aa}^H-(\gamma_{aa}^H+\gamma_{ua}^H)\left(\tilde{\gamma}_{au}^H+\tilde{\gamma}_{uu}^H\right)\right]-(\gamma_{aa}^H+\gamma_{ua}^H)c_{ua}\tilde{\gamma}_{ua}^H \tag{35}$$

$$\text{s.t.}\quad \bar{u}+V(\lambda^H)-c_{ua}\tilde{P}_{ua}-\frac{\Delta V}{\Delta\Gamma_a}\Gamma_a^H\geq c_{uu}\tilde{P}_{uu} \tag{IC}$$

$$\bar{u}+V(\lambda^H)\geq c_{uu} \tag{TR_A^a}$$

$$c_{uu}(\tilde{\gamma}_{au}^H+\tilde{\gamma}_{uu}^H)-c_{ua}\left(\tilde{P}_{aa}-\tilde{P}_{ua}\right)\Gamma_u^H\geq\left(\bar{u}+V(\lambda^H)\right)\tilde{P}_{au} \tag{TR_A^u}$$

where the coefficient of c_{uu} is simplified to

$$\begin{aligned}
&(\gamma_{au}^H+\gamma_{uu}^H)\tilde{\gamma}_{aa}^H-(\gamma_{aa}^H+\gamma_{ua}^H)\left(\tilde{\gamma}_{au}^H+\tilde{\gamma}_{uu}^H\right)\\
&\qquad=\tilde{\gamma}_{aa}^H-(\gamma_{aa}^H+\gamma_{ua}^H)\left(\Gamma_a^H+\tilde{\gamma}_{uu}^H\right)\\
&\qquad=\tilde{\gamma}_{aa}^H-(\gamma_{aa}^H+\gamma_{ua}^H)\left(\Gamma_a^H+\Gamma_u^H-\tilde{\gamma}_{ua}^H\right)=\tilde{\gamma}_{aa}^H-(\gamma_{aa}^H+\gamma_{ua}^H)\left(1-\tilde{\gamma}_{ua}^H\right)\\
&\qquad=\gamma_{aa}^H+b_a\Gamma_a^H-\gamma_{aa}^H-\gamma_{ua}^H+(\gamma_{aa}^H+\gamma_{ua}^H)\tilde{\gamma}_{ua}^H
\end{aligned}$$

[39]Technically when the slopes of isocosts and (IC) are identical, any point lying on segment $\bar{CB}$ is optimal. There could very well be a contract satisfying the assumption that $c_{au}>c_{uu}$ there. However, given that our parameters are continuous values, the probability of such an event happening is zero. We therefore rule this case out.

and is positive iff

$$b_a \geq \frac{\gamma^H_{ua} - (\gamma^H_{aa} + \gamma^H_{ua})\tilde{\gamma}^H_{ua}}{\Gamma^H_a}$$
$$b_a \geq \frac{(\gamma^H_{au} + \gamma^H_{uu})\gamma^H_{ua} - (\gamma^H_{aa} + \gamma^H_{ua})b_u\Gamma^H_u}{\Gamma^H_a}. \tag{36}$$

Now, consider the constraints in (c_{ua}, c_{uu}) space. Notice that the (IC) is negatively sloped with intercept

$$\frac{1}{\tilde{P}_{uu}}\left[\bar{u} + V(\lambda^H) - \frac{\Delta V}{\Delta\Gamma_a}\Gamma^H_a\right].$$

On the contrary, the (TR^u_A) is positively sloped with intercept

$$\left(\bar{u} + V(\lambda^H)\right)\frac{\tilde{P}_{au}}{\tilde{\gamma}^H_{au} + \tilde{\gamma}^H_{uu}}.$$

Hence, we need the intercept of (TR^u_A) to be smaller than the one of the (IC), or the set of feasible contracts would be empty. To see that this is always the case for $\bar{u} > \bar{u}_2$, calculate

$$\left(\bar{u} + V(\lambda^H)\right)\left(\tilde{P}_{uu}\tilde{P}_{au} - \tilde{\gamma}^H_{au} - \tilde{\gamma}^H_{uu}\right) + \frac{\Delta V}{\Delta\Gamma_a}\Gamma^H_a\left(\tilde{\gamma}^H_{au} + \tilde{\gamma}^H_{uu}\right) < 0.$$
$$\left(\bar{u} + V(\lambda^H)\right)\left(\tilde{P}_{uu}(1 - \tilde{P}_{aa}) - \tilde{P}_{au}\Gamma^H_a - \tilde{P}_{uu}\Gamma^H_u\right) + \frac{\Delta V}{\Delta\Gamma_a}\Gamma^H_a\left(\tilde{\gamma}^H_{au} + \tilde{\gamma}^H_{uu}\right) < 0.$$
$$-\left(\bar{u} + V(\lambda^H)\right)\left(\tilde{P}_{uu}\tilde{P}_{aa} + \tilde{P}_{au}\Gamma^H_a - \tilde{P}_{uu}\Gamma^H_a\right) + \frac{\Delta V}{\Delta\Gamma_a}\Gamma^H_a\left(\tilde{\gamma}^H_{au} + \tilde{\gamma}^H_{uu}\right) < 0.$$
$$\bar{u} > \frac{\Delta V}{\Delta\Gamma_a}\Gamma^H_a\frac{\left(\tilde{\gamma}^H_{au} + \tilde{\gamma}^H_{uu}\right)}{\tilde{P}_{au}\Gamma^H_a + \tilde{P}_{uu}(\tilde{P}_{aa} - \Gamma^H_a)} - V(\lambda^H) \equiv \bar{u}_2.$$

We plot in the next set of Figures the (IC), (TR^a_A) and the (TR^u_A) and study, once again, the area of feasible contracts. Notice that we have three different cases. This is because, while we know that the intercept of the (TR^u_A) is lower than the one of the (IC), and we can prove that the intercept of the (TR^u_A) is lower than the one of the (TR^a_A), it is not straightforward to prove whether the (TR^a_A) lies above or below the intercept of the (IC), or above or below the intersection between the (TR^u_A) and the (IC).

In Figure 9, we assume the (TR^a_A) is always slack, since it is looser than the (IC).

In Figure 10, we assume the (TR^a_A) crosses the (IC) and it does so above the point where (IC) and (TR^u_A) cross each other.

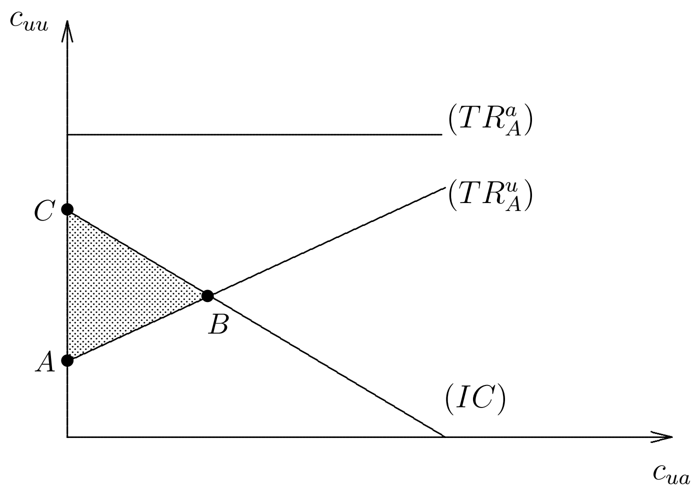

FIGURE 9. Contracts lying in the shaded area satisfy all constraints when $c_{au} = c_{uu}$. This graph represents the case where the (TR_A^a) is looser than the (IC).

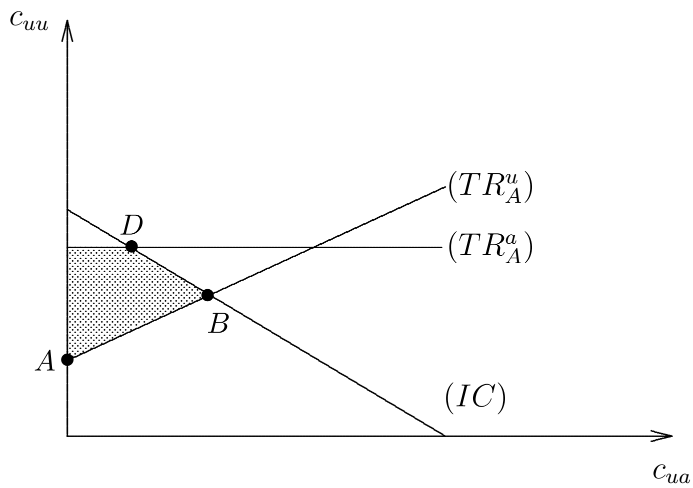

FIGURE 10. Contracts lying in the shaded area satisfy all constraints when $c_{au} = c_{uu}$. This graph represents the case where the (TR_A^a) crosses the (IC) after (in terms of the value of c_{uu}) the (TR_A^u).

In Figure 11 , we assume the (TR_A^a) crosses the (IC) and it does so below the point where (IC) and (TR_A^u) cross each other.

Point A, present in all figures, is contract GAPE$_2$ in the Proposition and it features the interesting aspect of setting the (IC) slack. Point B, in Figure 9 and 10, is the closest to the APE derived for the case of $\bar{u}_2$, since it features exactly the same binding constraints with the addition of the (PC). This corresponds to our GAPE$_1$. Point C, in Figure 9, and point D, in Figure 10, are potentially new optimal contracts where both (TR_A) are slack. Finally, notice that point E, in Figure 11, is never feasible since

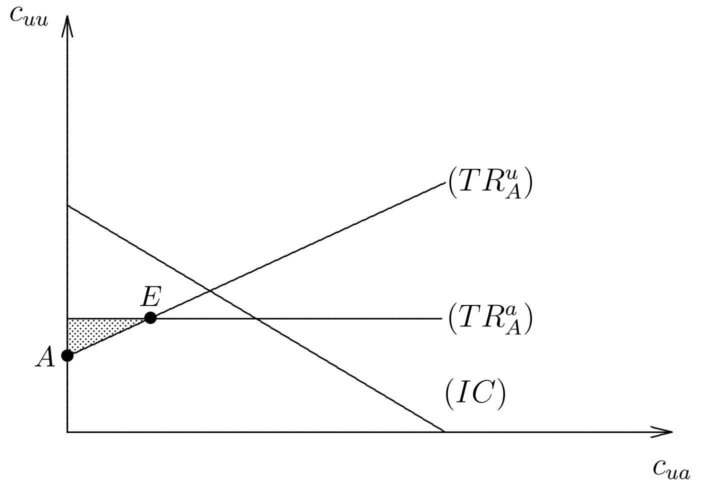

FIGURE 11. Contracts lying in the shaded area satisfy all constraints when $c_{au} = c_{uu}$. This graph represents the case where the (TR_A^a) crosses the (IC) before (in terms of the value of c_{uu}) the (TR_A^u).

both (TR_A) bind. This can only happen in case (i) of Lemma 4 and it is therefore, ruled out from the analysis.

We now show why C and D are never optimal and then present the conditions for GAPE$_1$ and GAPE$_2$ to be optimal and feasible, and derive their equilibrium values.

Lemma 28. *When $\bar{u} > \bar{u}_2$ and the GBPE is not optimal, the principal assigns the worker a Generalized APE contract that lies either at point A of Figures 9-11 or at point B of Figures 9 and 10.*

Proof. We have already shown how E can never be an equilibrium. For either C or D to be optimal, it must be that (36) fails, so that costs decrease towards the top right corner of each Figure. Second, it must be that isocosts in the reduced problem (35) are flatter than the (IC), otherwise the optimal contract would lie at point B. To prove the Lemma, we show how this second condition is never feasible.

The slope of the (IC) in (35) is given by $\frac{\tilde{P}_{ua}}{\tilde{P}_{uu}}$, while the one of isocosts — when (36) fails — is given by

$$\frac{\left(\gamma_{aa}^H + \gamma_{ua}^H\right)\tilde{P}_{ua}}{\left(\gamma_{aa}^H + \gamma_{ua}^H\right)\left(\tilde{\gamma}_{au}^H + \tilde{\gamma}_{uu}^H\right) - \left(\gamma_{au}^H + \gamma_{uu}^H\right)\tilde{\gamma}_{aa}^H}$$

Hence, for C or D to be optimal, we would need:

$$\frac{\left(\gamma_{aa}^H+\gamma_{ua}^H\right)\tilde{P}_{ua}}{\left(\gamma_{aa}^H+\gamma_{ua}^H\right)\left(\tilde{\gamma}_{au}^H+\tilde{\gamma}_{uu}^H\right)-\left(\gamma_{au}^H+\gamma_{uu}^H\right)\tilde{\gamma}_{aa}^H}<\frac{\tilde{P}_{ua}}{\tilde{P}_{uu}}$$

$$\left(\gamma_{aa}^H+\gamma_{ua}^H\right)\tilde{P}_{uu}<\left(\gamma_{aa}^H+\gamma_{ua}^H\right)\left(\tilde{\gamma}_{au}^H+\tilde{\gamma}_{uu}^H\right)-\left(\gamma_{au}^H+\gamma_{uu}^H\right)\tilde{\gamma}_{aa}^H$$

$$\left(\gamma_{aa}^H+\gamma_{ua}^H\right)\tilde{P}_{uu}\Gamma_a^H-\left(\gamma_{aa}^H+\gamma_{ua}^H\right)\tilde{\gamma}_{au}^H+\left(\gamma_{au}^H+\gamma_{uu}^H\right)\tilde{\gamma}_{aa}^H<0$$

$$\left(\gamma_{aa}^H+\gamma_{ua}^H\right)\tilde{P}_{uu}\Gamma_a^H-\left(\gamma_{aa}^H+\gamma_{ua}^H\right)\tilde{\gamma}_{au}^H+\left(\gamma_{au}^H+\gamma_{uu}^H\right)\Gamma_a^H-\left(\gamma_{au}^H+\gamma_{uu}^H\right)\tilde{\gamma}_{au}^H<0$$

$$\left(\gamma_{aa}^H+\gamma_{ua}^H\right)\left(1-\tilde{P}_{ua}\right)-\tilde{P}_{au}+\left(\gamma_{au}^H+\gamma_{uu}^H\right)<0$$

$$1-\left(\gamma_{aa}^H+\gamma_{ua}^H\right)\tilde{P}_{ua}-\tilde{P}_{au}<0$$

$$1-\left(1-\gamma_{au}^H-\gamma_{uu}^H\right)\tilde{P}_{ua}-\tilde{P}_{au}<0$$

$$\tilde{P}_{aa}-\tilde{P}_{ua}+\left(\gamma_{au}^H+\gamma_{uu}^H\right)\tilde{P}_{ua}<0,$$

which is never true by positive perceived correlation ($\tilde{P}_{aa}-\tilde{P}_{ua}>0$). Conditions for feasibility and optimality of A and B are derived in the Lemmas below. ∎

Now that we know the features of the GAPE contracts, we derive their equilibrium values and the conditions for their feasibility and optimality. In order to simplify the algebraical derivations, we start with GAPE$_2$ (point A).

Lemma 29. *When the bias of the worker is large, that is*

$$b_a\geq\frac{\left(\gamma_{au}^H+\gamma_{uu}^H\right)P_{ua}+b_u\left(\gamma_{au}^H-\gamma_{uu}^H\right)}{\Gamma_a^H}\tag{37}$$

and

$$b_a\geq P_{au}-\left(P_{uu}-b_u\right)\frac{\left[\bar{u}+V(\lambda^H)\right]\left(\gamma_{au}^H-\gamma_{ua}^H\right)+\frac{\Delta V}{\Delta\Gamma_a}\frac{\gamma_{ua}^H}{P_{aa}}\Gamma_a^H}{\bar{u}+V(\lambda^H)-\frac{\Delta V}{\Delta\Gamma_a}\frac{P_{ua}}{P_{aa}}\Gamma_a^H}.\tag{38}$$

the optimal contract set by the principal is the GAPE$_2$ of Proposition 11.

Proof. First of all, at A we have $c_{ua}=0$ and $c_{uu}=c_{au}$ equal the intercept of the (TR_A^u). That is,

$$c_{uu}=c_{au}=\frac{(\bar{u}+V(\lambda^H))\tilde{P}_{au}}{\tilde{\gamma}_{au}^H+\tilde{\gamma}_{uu}^H}.$$

Plugging these into the (PC) constraint to obtain c_{aa}, we get

$$\begin{aligned} c_{aa} &= \frac{1}{\tilde{\gamma}^H_{aa}} \left(\bar{u} + V(\lambda^H) - c_{au}\tilde{\gamma}^H_{au} - c_{ua}\tilde{\gamma}^H_{ua} - c_{uu}\tilde{\gamma}^H_{uu} \right) \\ &= \frac{\bar{u} + V(\lambda^H)}{\Gamma^H_a}. \end{aligned}$$

It is immediate to see how this contract satisfies $c_{uu} > c_{ua} = 0$. It is also easy to see that $c_{aa} > c_{au}$, since

$$\frac{1}{\Gamma^H_a} > \frac{\tilde{P}_{au}}{\tilde{\gamma}^H_{au} + \tilde{\gamma}^H_{uu}} \iff 1 > \frac{\tilde{\gamma}^H_{au}}{\tilde{\gamma}^H_{au} + \tilde{\gamma}^H_{uu}},$$

which is obvious. This ensures that the contract satisfies case (ii) of Lemma 4. Finally, to derive the wages, simply recall that we are in case (i) of Lemma 3 and that Lemma 25 above holds. Now we move to the derivations of its feasibility and optimality.

For the GAPE$_2$ contract to be optimal, quite a few conditions have to hold. First of all, point A is feasible only when the agent believes signals to be positively correlated. We know that this happens when (1) holds

$$\tilde{P}_{aa} - \tilde{P}_{ua} \geq 0 \qquad \iff \qquad b_a \geq P_{au} - (P_{uu} - b_u).$$

Second, for GAPE$_2$ to be optimal in the restricted problem of case (ii) of Lemma 4 and (i) of Lemma 3, condition (36) must hold, so that costs decrease towards the bottom right corner:

$$b_a \geq \frac{(\gamma^H_{au} + \gamma^H_{uu})\gamma^H_{ua} - (\gamma^H_{aa} + \gamma^H_{ua})b_u\Gamma^H_u}{\Gamma^H_a}$$

Third, also for GAPE$_2$ to be optimal in the restricted problem of case (ii) of Lemma 4 and (i) of Lemma 3, the isocosts of the restricted problem have to be flatter than the (TR^u_A), otherwise the optimal contract would either be at point B (Figure 9 and 10) or non-existent (Figure 11 where E yields to a contradiction). Looking at problem (35), this happens when:

$$\frac{\tilde{P}_{aa}-\tilde{P}_{ua}}{\tilde{\gamma}^H_{au}+\tilde{\gamma}^H_{uu}} \geq \frac{\left(\gamma^H_{aa}+\gamma^H_{ua}\right)\tilde{P}_{ua}}{\left(\gamma^H_{au}+\gamma^H_{uu}\right)\tilde{\gamma}^H_{aa}-\left(\gamma^H_{aa}+\gamma^H_{ua}\right)\left(\tilde{\gamma}^H_{au}+\tilde{\gamma}^H_{uu}\right)}$$
$$\left(\tilde{P}_{aa}-\tilde{P}_{ua}\right)\left(\left(\gamma^H_{au}+\gamma^H_{uu}\right)\tilde{\gamma}^H_{aa}-\left(\gamma^H_{aa}+\gamma^H_{ua}\right)\left(\tilde{\gamma}^H_{au}+\tilde{\gamma}^H_{uu}\right)\right) \geq \left(\gamma^H_{aa}+\gamma^H_{ua}\right)\tilde{P}_{ua}\left(\tilde{\gamma}^H_{au}+\tilde{\gamma}^H_{uu}\right)$$
$$\tilde{P}_{uu}\left[\Gamma^H_a\left(\gamma^H_{au}+\gamma^H_{uu}\right)-\Gamma^H_u\left(\tilde{\gamma}^H_{aa}+\tilde{\gamma}^H_{ua}\right)\right]-\tilde{\gamma}^H_{au} \geq 0$$
$$\tilde{P}_{uu}\left(P_{au}\Gamma^H_a-P_{ua}\Gamma^H_u\right)-\tilde{\gamma}^H_{au} \geq 0$$
$$b_a \geq \frac{\left(\gamma^H_{au}+\gamma^H_{uu}\right)P_{ua}+b_u\left(\gamma^H_{au}-\gamma^H_{ua}\right)}{\Gamma^H_a},$$

which generates (37). Notice that the above is true only if (36) holds, so that the denominator of the slope of the isocost is positive. However, we are now going to show that (37) is a stronger requirement compared to (36), which can therefore be disregarded. To see this, simply compare the numerators of the two and notice that

$$(\gamma^H_{au}+\gamma^H_{uu})\gamma^H_{ua}-(\gamma^H_{aa}+\gamma^H_{ua})b_u\Gamma^H_u \leq \left(\gamma^H_{au}+\gamma^H_{uu}\right)P_{ua}+b_u\left(\gamma^H_{au}-\gamma^H_{ua}\right)$$
$$\iff (\gamma^H_{au}+\gamma^H_{uu})P_{ua}(\Gamma^H_u-1)-b_u\left[(\gamma^H_{aa}+\gamma^H_{ua})\Gamma^H_u+\gamma^H_{au}-\gamma^H_{ua}\right] \leq 0.$$

The first component of the LHS is clearly negative. The second one is as well, since the bracket is always positive. To see this, notice that it is decreasing linearly in P_{ua}, so if it is positive at the maximum value of P_{ua} then it always is. By positive correlation of s an t, P_{ua} is strictly lower than P_{aa}. As $P_{ua} \to P_{aa}$, the bracket converges to

$$P_{aa}\Gamma^H_u(\Gamma^H_a+\Gamma^H_u)+P_{au}\Gamma^H_a-P_{aa}\Gamma^H_u = P_{au}\Gamma^H_a > 0.$$

Hence, we can disregard (36) in the rest of the analysis.

The tightness of (37) allows us to remove a further condition, the one of positive correlation. To show this, we calculate

$$
\begin{aligned}
&P_{au} - P_{uu} + b_u < \frac{\left(\gamma_{au}^H + \gamma_{uu}^H\right) P_{ua} + b_u \left(\gamma_{au}^H - \gamma_{uu}^H\right)}{\Gamma_a^H} \\
&b_u(-\gamma_{au}^H + \gamma_{uu}^H + \Gamma_a^H) < P_{au}P_{ua}\Gamma_a^H + P_{uu}P_{ua}\Gamma_u^H + P_{uu}\Gamma_a^H - P_{au}\Gamma_a^H \\
&b_u(\gamma_{aa}^H + \gamma_{uu}^H) < -P_{au}P_{uu}\Gamma_a^H + P_{uu}P_{ua}\Gamma_u^H + P_{uu}\Gamma_a^H \\
&b_u(\gamma_{aa}^H + \gamma_{uu}^H) < P_{uu}P_{ua}\Gamma_u^H + P_{uu}P_{aa}\Gamma_a^H \\
&b_u < P_{uu},
\end{aligned}
$$

which is always true by definition of b_u. Hence (37) is a necessary condition. Condition (38) comes, instead, from the comparison of the expected wage costs of the GBPE and GAPE$_2$ contracts. Hence, we have to calculate the two.

$$
\begin{aligned}
E(\tilde{w}_{ts}^*) &= \tilde{w}_{aa}^*\gamma_{aa}^H + \tilde{w}_{au}^*\gamma_{au}^H + \tilde{w}_{ua}^*\gamma_{ua}^H + \tilde{w}_{uu}^*\gamma_{uu}^H \\
&= \frac{\Delta V}{\Delta\Gamma_a}\frac{\gamma_{aa}^H + \gamma_{ua}^H}{P_{aa}} + \tilde{c}_{uu}^* = \bar{u} + V(\lambda^H) + \frac{\Delta V}{\Delta\Gamma_a}\frac{\gamma_{ua}^H}{P_{aa}},
\end{aligned}
$$

$$
\begin{aligned}
E(\tilde{w}_{ts}^{\dagger'}) &= \tilde{w}_{aa}^{\dagger'}\gamma_{aa}^H + \tilde{w}_{au}^{\dagger'}\gamma_{au}^H + \tilde{w}_{ua}^{\dagger'}\gamma_{ua}^H + \tilde{w}_{uu}^{\dagger'}\gamma_{uu}^H \\
&= \left[\bar{u} + V(\lambda^H)\right] \frac{\left(\tilde{\gamma}_{au}^H + \tilde{\gamma}_{uu}^H\right) - \tilde{\gamma}_{uu}^H\left(\gamma_{au}^H + \gamma_{uu}^H\right)}{\left(\tilde{\gamma}_{au}^H + \tilde{\gamma}_{uu}^H\right)\Gamma_a^H}.
\end{aligned}
$$

The GAPE$_2$ is set when

$$
\begin{aligned}
&E(\tilde{w}_{ts}^*) > E(\tilde{w}_{ts}^{\dagger'}) \\
&\bar{u} + V(\lambda^H) + \frac{\Delta V}{\Delta\Gamma_a}\frac{\gamma_{ua}^H}{P_{aa}} > \left[\bar{u} + V(\lambda^H)\right] \frac{\left(\tilde{\gamma}_{au}^H + \tilde{\gamma}_{uu}^H\right) - \tilde{\gamma}_{uu}^H\left(\gamma_{au}^H + \gamma_{uu}^H\right)}{\left(\tilde{\gamma}_{au}^H + \tilde{\gamma}_{uu}^H\right)\Gamma_a^H} \\
&\left(\bar{u} + V(\lambda^H)\right)\Gamma_u^H \left[\tilde{P}_{uu}\left(\gamma_{au}^H - \gamma_{ua}^H\right) - \tilde{\gamma}_{au}^H\right] + \frac{\Delta V}{\Delta\Gamma_a^H}\frac{\gamma_{ua}^H}{P_{aa}}\Gamma_a^H\left(\tilde{\gamma}_{au}^H + \tilde{\gamma}_{uu}^H\right) \geq 0 \\
&\tilde{\gamma}_{uu}^H\left[\left[\bar{u} + V(\lambda^H)\right]\left(\gamma_{au}^H - \gamma_{ua}^H\right) + \frac{\Delta V}{\Delta\Gamma_a}\frac{\gamma_{ua}^H}{P_{aa}}\Gamma_a^H\right] + \tilde{\gamma}_{au}^H\Gamma_u^H\left[\frac{\Delta V}{\Delta\Gamma_a}\frac{P_{ua}}{P_{aa}}\Gamma_a^H - \bar{u} - V(\lambda^H)\right] \geq 0 \\
&b_a \geq P_{au} - (P_{uu} - b_u)\frac{\left[\bar{u} + V(\lambda^H)\right]\left(\gamma_{au}^H - \gamma_{ua}^H\right) + \frac{\Delta V}{\Delta\Gamma_a}\frac{\gamma_{ua}^H}{P_{aa}}\Gamma_a^H}{\bar{u} + V(\lambda^H) - \frac{\Delta V}{\Delta\Gamma_a}\frac{P_{ua}}{P_{aa}}\Gamma_a^H}.
\end{aligned}
$$

This concludes the proof. ∎

Now we move the derivation of the GAPE$_1$.

Lemma 30. *When the worker's bias is moderate (but such that he still perceives signals as positively correlated), that is, condition* (37) *fails and*

$$b_a\left((P_{uu}-b_u+\Gamma_a^H)P_{ua}-\gamma_{aa}^H\right)\leq\gamma_{au}^H(P_{ua}+b_u)(P_{aa}-P_{ua}) \tag{39}$$

then the principal assigns contract $GAPE_1$ of Proposition 11.

Proof. Before deriving contract B, notice two things. First, there are values for b_u such that (39) holds for all b_a. Second, formally the requirement on b_a deriving from the failing of (37), should be stated as

$$b_a\in\left[P_{au}-P_{uu}+b_u,\frac{\left(\gamma_{au}^H+\gamma_{uu}^H\right)P_{ua}+b_u\left(\gamma_{au}^H-\gamma_{uu}^H\right)}{\Gamma_a^H}\right].$$

At B, we need to study the intersection between the (IC) and the (TR_A^u). From the (IC)

$$c_{uu}=\tfrac{1}{\tilde{P}_{uu}}\left(\bar{u}+V(\lambda^H)-c_{ua}\tilde{P}_{ua}-\frac{\Delta V}{\Delta\Gamma_a}\Gamma_a^H\right).$$

Recall that we need $c_{uu}>c_{ua}$. This implies that

$$c_{ua}<\bar{u}+V(\lambda^H)-\frac{\Delta V}{\Delta\Gamma_a}\Gamma_a^H.$$

We check for this later on. Plug the above in the (TR_A^u) binding to get

$$c_{ua}=\frac{\tilde{\gamma}_{au}^H+\tilde{\gamma}_{uu}^H}{\left(\tilde{P}_{aa}-\tilde{P}_{ua}\right)\tilde{P}_{uu}\Gamma_u^H}\left(\bar{u}+V(\lambda^H)-c_{ua}\tilde{P}_{ua}-\frac{\Delta V}{\Delta\Gamma_a}\Gamma_a^H\right)-\frac{\left(\bar{u}+V(\lambda^H)\right)\tilde{P}_{au}}{\left(\tilde{P}_{aa}-\tilde{P}_{ua}\right)\Gamma_u^H},$$

which is equivalent to

$$c_{ua}\left[1+\frac{\left(\tilde{\gamma}_{au}^H+\tilde{\gamma}_{uu}^H\right)\tilde{P}_{ua}}{\left(\tilde{P}_{aa}-\tilde{P}_{ua}\right)\tilde{P}_{uu}\Gamma_u^H}\right]=\frac{\left(\bar{u}+V(\lambda^H)\right)}{\left(\tilde{P}_{aa}-\tilde{P}_{ua}\right)\Gamma_u^H}\left[\frac{\tilde{\gamma}_{au}^H+\tilde{\gamma}_{uu}^H}{\tilde{P}_{uu}}-\tilde{P}_{au}\right]$$
$$-\frac{\Delta V}{\Delta\Gamma_a}\Gamma_a^H\frac{\tilde{\gamma}_{au}^H+\tilde{\gamma}_{uu}^H}{\left(\tilde{P}_{aa}-\tilde{P}_{ua}\right)\tilde{P}_{uu}\Gamma_u^H}.$$

Let's work one coefficient at a time. The one of c_{ua} is

$$\left[1+\frac{\left(\tilde{\gamma}_{au}^{H}+\tilde{\gamma}_{uu}^{H}\right)\tilde{P}_{ua}}{\left(\tilde{P}_{aa}-\tilde{P}_{ua}\right)\tilde{P}_{uu}\Gamma_{u}^{H}}\right]=\frac{\left(\tilde{P}_{aa}-\tilde{P}_{ua}\right)\tilde{P}_{uu}\Gamma_{u}^{H}+\left(\tilde{\gamma}_{au}^{H}+\tilde{\gamma}_{uu}^{H}\right)\tilde{P}_{ua}}{\left(\tilde{P}_{aa}-\tilde{P}_{ua}\right)\tilde{P}_{uu}\Gamma_{u}^{H}}$$

$$=\frac{1}{(...)}\left[\tilde{P}_{aa}\tilde{P}_{uu}\Gamma_{u}^{H}+\tilde{\gamma}_{au}^{H}\tilde{P}_{ua}\right]=\frac{1}{(...)}\left[\tilde{P}_{au}\Gamma_{a}^{H}+\tilde{P}_{uu}\Gamma_{u}^{H}-\tilde{P}_{au}\tilde{P}_{uu}\right]$$

$$=\frac{1}{(...)}\left[\tilde{P}_{au}\Gamma_{a}^{H}+\tilde{P}_{uu}(\tilde{P}_{aa}-\Gamma_{a}^{H})\right].$$

Now we work on the one of $\bar{u}+V(\lambda^{H})$ and show that it is identical.

$$\frac{1}{\left(\tilde{P}_{aa}-\tilde{P}_{ua}\right)\tilde{P}_{uu}\Gamma_{u}^{H}}\left[\tilde{\gamma}_{au}^{H}+\tilde{\gamma}_{uu}^{H}-\tilde{P}_{au}\tilde{P}_{uu}\right]$$

$$=\frac{1}{(...)}\left[\tilde{P}_{au}\Gamma_{a}^{H}+\tilde{P}_{uu}\Gamma_{u}^{H}-\tilde{P}_{au}\tilde{P}_{uu}\right]$$

$$=\frac{1}{(...)}\left[\tilde{P}_{au}\Gamma_{a}^{H}+\tilde{P}_{uu}(\tilde{P}_{aa}-\Gamma_{a}^{H})\right]$$

This implies that

$$c_{ua}=\bar{u}+V(\lambda^{H})-\frac{\Delta V}{\Delta\Gamma_{a}}\Gamma_{a}^{H}\frac{\left(\tilde{\gamma}_{au}^{H}+\tilde{\gamma}_{uu}^{H}\right)}{\tilde{P}_{au}\Gamma_{a}^{H}+\tilde{P}_{uu}(\tilde{P}_{aa}-\Gamma_{a}^{H})}=\bar{u}-\bar{u}_{2},$$

which is always positive when $\bar{u}>\bar{u}_{2}$. As promised, we now check that $c_{ua}<\bar{u}+V(\lambda^{H})-\frac{\Delta V}{\Delta\Gamma_{a}}\Gamma_{a}^{H}$.

$$c_{ua}<\bar{u}+V(\lambda^{H})-\frac{\Delta V}{\Delta\Gamma_{a}}\Gamma_{a}^{H}$$

$$\bar{u}+V(\lambda^{H})-\frac{\Delta V}{\Delta\Gamma_{a}}\Gamma_{a}^{H}\frac{\left(\tilde{\gamma}_{au}^{H}+\tilde{\gamma}_{uu}^{H}\right)}{\tilde{P}_{aa}\Gamma_{u}^{H}-\tilde{P}_{ua}(\tilde{P}_{aa}-\Gamma_{a}^{H})}<\bar{u}+V(\lambda^{H})-\frac{\Delta V}{\Delta\Gamma_{a}}\Gamma_{a}^{H}$$

$$\left(\tilde{\gamma}_{au}^{H}+\tilde{\gamma}_{uu}^{H}\right)>\tilde{P}_{aa}\Gamma_{u}^{H}-\tilde{P}_{ua}(\tilde{P}_{aa}-\Gamma_{a}^{H})$$

$$\tilde{P}_{au}\Gamma_{a}^{H}+\tilde{P}_{uu}\Gamma_{u}^{H}-\tilde{P}_{aa}+\tilde{P}_{aa}\Gamma_{a}^{H}+\tilde{P}_{ua}(\tilde{P}_{aa}-\Gamma_{a}^{H})>0$$

$$-\tilde{P}_{aa}+\Gamma_{a}^{H}+\tilde{P}_{uu}\Gamma_{u}^{H}+\tilde{P}_{ua}(\tilde{P}_{aa}-\Gamma_{a}^{H})>0$$

$$\tilde{P}_{uu}\Gamma_{u}^{H}+(\tilde{P}_{ua}-1)(\tilde{P}_{aa}-\Gamma_{a}^{H})>0$$

$$\Gamma_{u}^{H}-\tilde{P}_{aa}+\Gamma_{a}^{H})>0$$

$$\tilde{P}_{au}>0,$$

which is always true. Hence we can derive c_{uu}

$$
\begin{aligned}
c_{uu} &= \tfrac{1}{\tilde{P}_{uu}}\left(\bar{u} + V(\lambda^H) - c_{ua}\tilde{P}_{ua} - \frac{\Delta V}{\Delta\Gamma_a}\Gamma_a^H\right) \\
&= \tfrac{1}{\tilde{P}_{uu}}\left(\bar{u} + V(\lambda^H) - (\bar{u} - \bar{u}_2)\tilde{P}_{ua} - \frac{\Delta V}{\Delta\Gamma_a}\Gamma_a^H\right) \\
&= \bar{u} + V(\lambda^H) - \frac{\Delta V}{\Delta\Gamma_a}\Gamma_a^H\Gamma_u^H \frac{(\tilde{P}_{aa} - \tilde{P}_{ua})}{\tilde{P}_{au}\Gamma_a^H + \tilde{P}_{uu}(\tilde{P}_{aa} - \Gamma_a^H)} = c_{au}
\end{aligned}
$$

We now derive c_{aa} with

$$
\begin{aligned}
c_{aa} &= \frac{1}{\tilde{\gamma}_{aa}^H}\left(\bar{u} + V(\lambda^H) - c_{au}\tilde{\gamma}_{au}^H - c_{ua}\tilde{\gamma}_{ua}^H - c_{uu}\tilde{\gamma}_{uu}^H\right) \\
&= \bar{u} + V(\lambda^H) + \frac{\Delta V}{\Delta\Gamma_a}\Gamma_u^H \frac{\tilde{P}_{au}\Gamma_a^H + \tilde{P}_{uu}\Gamma_u^H}{\tilde{P}_{au}\Gamma_a^H + \tilde{P}_{uu}(\tilde{P}_{aa} - \Gamma_a^H)},
\end{aligned}
$$

where it is immediate to see that this is larger than c_{au}. This ensures that the GAPE$_1$ contract satisfies case (ii) of Lemma 4. To derive wage levels, simply recall that we are in case (i) of Lemma 3 and that Lemma 25 above holds.

For the GAPE$_1$ contract to be optimal, we need the following conditions. First of all, we need positive perceived correlation of signals ensured by (1). Second, we need isocosts in the reduced problem (35) to be steeper than (TR_A^u) when (36) holds. That is (37) failing.[40] Third, we need for the GAPE$_1$ contract to be less costly than the GBPE. This latter condition is the last calculation needed to prove the Lemma.

$$
E(\tilde{w}_{ts}^*) = \frac{\Delta V}{\Delta\Gamma_a}\frac{\gamma_{aa}^H + \gamma_{ua}^H}{P_{aa}} + \tilde{c}_{uu}^* = \bar{u} + V(\lambda^H) + \frac{\Delta V}{\Delta\Gamma_a}\frac{\gamma_{ua}^H}{P_{aa}},
$$

$$
\begin{aligned}
E(\tilde{w}_{ts}^\dagger) &= \tilde{w}_{aa}^\dagger\gamma_{aa}^H + \tilde{w}_{au}^\dagger\gamma_{au}^H + \tilde{w}_{ua}^\dagger\gamma_{ua}^H + \tilde{w}_{uu}^\dagger\gamma_{uu}^H \\
&= \bar{u} + V(\lambda^H) + \frac{\Delta V}{\Delta\Gamma_a}\frac{\left[(\tilde{\gamma}_{au}^H + \tilde{\gamma}_{uu}^H)(\gamma_{aa}^H + \gamma_{ua}^H) - (\gamma_{au}^H + \gamma_{uu}^H)(\tilde{\gamma}_{aa}^H - \tilde{P}_{ua}\Gamma_a^H)\right]\Gamma_u^H}{\tilde{\gamma}_{au}^H + \tilde{P}_{uu}(\tilde{P}_{aa} - \Gamma_a^H)}.
\end{aligned}
$$

The GAPE$_a$ is therefore set when

$$
\begin{aligned}
&E(\tilde{w}_{ts}^\dagger) < E(\tilde{w}_{ts}^*) \\
&\frac{\Delta V}{\Delta\Gamma_a}\Gamma_u^H\left[\frac{(\tilde{\gamma}_{au}^H + \tilde{\gamma}_{uu}^H)(\gamma_{aa}^H + \gamma_{ua}^H) - (\gamma_{au}^H + \gamma_{uu}^H)(\tilde{\gamma}_{aa}^H - \tilde{P}_{ua}\Gamma_a^H)}{\tilde{\gamma}_{au}^H + \tilde{P}_{uu}(\tilde{P}_{aa} - \Gamma_a^H)} - \frac{P_{ua}}{P_{uu}}\right] < 0
\end{aligned}
$$

[40]To see that no further condition is needed for optimality in the reduced problem, notice that when (36) fails, Lemma 28 ensures that the GAPE$_1$ contract is the only optimal one.

which, after some algebraical manipulation, yields (39). ∎

This Lemma concludes the proof of Proposition 11.

Appendix B. Pareto Improving GAPE Contracts

This Appendix derives conditions for the GAPE contracts to Pareto Improve over the GBPE. As in the case of $\bar{u} < \bar{u}_1$, we know that, if a GAPE contract is optimal for the firm, it is lowering the expected wage cost. Hence, we only look for *true* expected compensations. We start from GAPE_1.

First of all, notice that the true expected compensation from the GBPE is given by:

$$E(\tilde{c}^*_{ts}) = \tilde{c}^*_{aa}\gamma^H_{aa} + \tilde{c}^*_{au}\gamma^H_{au} + \tilde{c}^*_{ua}\gamma^H_{ua} + \tilde{c}^*_{uu}\gamma^H_{uu} = \bar{u} + V(\lambda^H).$$

Similarly, we can calculate the true expected compensation from the GAPE_1 contract

$$\begin{aligned} E(\tilde{c}^\dagger_{ts}) &= \tilde{c}^\dagger_{aa}\gamma^H_{aa} + \tilde{c}^\dagger_{au}\gamma^H_{au} + \tilde{c}^\dagger_{ua}\gamma^H_{ua} + \tilde{c}^\dagger_{uu}\gamma^H_{uu} \\ &= \bar{u} + V(\lambda^H) + \gamma^H_{aa}\left(\frac{\Delta V}{\Delta\Gamma_a}\Gamma^H_u \frac{(\tilde{\gamma}^H_{au} + \tilde{\gamma}^H_{uu})}{\tilde{P}_{au}\Gamma^H_a + \tilde{P}_{uu}(\tilde{P}_{aa} - \Gamma^H_a)}\right) \\ &\qquad - (\gamma^H_{au} + \gamma^H_{uu})\left(\frac{\Delta V}{\Delta\Gamma_a}\Gamma^H_a\Gamma^H_u \frac{(\tilde{P}_{aa} - \tilde{P}_{ua})}{\tilde{P}_{au}\Gamma^H_a + \tilde{P}_{uu}(\tilde{P}_{aa} - \Gamma^H_a)}\right) \\ &\qquad\qquad - \gamma^H_{ua}\left(\frac{\Delta V}{\Delta\Gamma_a}\Gamma^H_a \frac{(\tilde{\gamma}^H_{au} + \tilde{\gamma}^H_{uu})}{\tilde{P}_{au}\Gamma^H_a + \tilde{P}_{uu}(\tilde{P}_{aa} - \Gamma^H_a)}\right). \end{aligned}$$

Hence, for the GAPE_1 to be a Pareto Improvement over the GBPE, the extra terms in $E(\tilde{c}^\dagger_{ts})$ beyond $\bar{u}$ and $V(\lambda^H)$ must be positive. Given the common components, it is possible to show that this happens when

$$\gamma^H_{aa}(\tilde{\gamma}^H_{au} + \tilde{\gamma}^H_{uu}) - (\gamma^H_{au} + \gamma^H_{uu})(\tilde{P}_{aa} - \tilde{P}_{ua})\Gamma^H_a - P_{ua}\Gamma^H_a(\tilde{\gamma}^H_{au} + \tilde{\gamma}^H_{uu}) > 0,$$

which can be rearranged to yield

$$b_a < b_u \frac{P_{au}}{P_{uu}\Gamma^H_A}. \tag{40}$$

A Pareto Improving GAPE_1 can be derived, for example, setting $P_{aa} = 0.55$, $P_{uu} = 0.8$, $\Gamma^H_a = 0.65$, $b_a = 0.1$ and $b_u = 0.2$ (other parameters are irrelevant).

To prove the same for the GAPE$_2$, we calculate

$$E(\tilde{c}^{\dagger'}_{ts}) = \tilde{c}^{\dagger'}_{aa}\gamma^H_{aa} + \tilde{c}^{\dagger'}_{au}\gamma^H_{au} + \tilde{c}^{\dagger'}_{ua}\gamma^H_{ua} + \tilde{c}^{\dagger'}_{uu}\gamma^H_{uu}$$
$$= \left(\bar{u} + V(\lambda^H)\right)\left[\frac{(\tilde{\gamma}^H_{au} + \tilde{\gamma}^H_{uu})P_{aa} + \tilde{P}_{au}(\gamma^H_{au} + \gamma^H_{uu})}{\tilde{\gamma}^H_{au} + \tilde{\gamma}^H_{uu}}\right]$$

Hence, for the GAPE$_2$ to be a Pareto Improvement over the GBPE, we solve

$$\left(\bar{u} + V(\lambda^H)\right)\left[\frac{(\tilde{\gamma}^H_{au} + \tilde{\gamma}^H_{uu})P_{aa} + \tilde{P}_{au}(\gamma^H_{au} + \gamma^H_{uu})}{\tilde{\gamma}^H_{au} + \tilde{\gamma}^H_{uu}}\right] > \bar{u} + V(\lambda^H) \tag{41}$$

which can be rearranged to yield

$$b_a < b_u \frac{P_{au}}{P_{uu}}. \tag{42}$$

A Pareto Improving GAPE$_2$ can be derived setting, for example, $P_{aa} = 0.3$, $P_{uu} = 0.9$, $\Gamma^H_a = 0.9$, $\Gamma^H_u = 0.4$, $V(\lambda^L) = 0.5$, $V(\lambda^H) = 6$, $\bar{u} = 3$, $b_a = 0.5$ and $b_u = 0.66$.

Appendix C. Perceived Negative Correlation Derivations

In this Appendix, we fully derive results for the case of an agent who perceives a negative correlation between SPEs. That is, an agent holding biased belief that violate condition (1). As already mentioned, no overconfident agent perceives signals to be negatively correlated. Hence, we are left only with optimistic, pessimistic and underconfident agents. We present the analysis in this very order. We conclude with a welfare analysis of the contracts derived.

C.1. **Optimism and Perceived Negatively Correlation.** Consider the case of an optimistic agent who believes that signals are negatively correlated. This happens when

$$b_a > 0,\ b_u > 0, \text{ and } b_u - b_a > P_{aa} - P_{ua}.$$

In this case, the compensation scheme of the contract has to subdue to different properties in order to ensure truthful reporting.

Lemma 31. *If the agent believes signals are negatively correlated, i.e.,* (1) *fails to hold, then any optimal contract implementing high effort features either (i)* $c_{aa} = c_{au}$ *and* $c_{uu} = c_{ua}$ *or (ii)* $c_{aa} < c_{au}$ *and* $c_{uu} < c_{ua}$.

Proof. When the agent believes signals are negatively correlated, i.e., $\tilde{\gamma}_{aa}^H \tilde{\gamma}_{uu}^H - \tilde{\gamma}_{au}^H \tilde{\gamma}_{ua}^H < 0$, we have:

$$\frac{\tilde{\gamma}_{ua}^H}{\tilde{\gamma}_{aa}^H} > \frac{\tilde{\gamma}_{uu}^H}{\tilde{\gamma}_{au}^H}.$$

That is, the (TR_A) becomes

$$(c_{ua} - c_{uu})\frac{\tilde{\gamma}_{uu}^H}{\tilde{\gamma}_{au}^H} \leq (c_{au} - c_{aa}) \leq (c_{ua} - c_{uu})\frac{\tilde{\gamma}_{ua}^H}{\tilde{\gamma}_{aa}^H}.$$

Where either all brackets are 0 or $c_{aa} < c_{au}$ and $c_{uu} < c_{ua}$. ∎

Lemma 4 shows that, if the agent believes signals are positively correlated, then the principal might opt for designing an optimal contract where, taking as given her PE report, the compensation is higher in the agreement cases than in the disagreement cases, i.e. a contract with $c_{aa} > c_{au}$ and $c_{uu} > c_{ua}$. Lemma 31 shows that the opposite happens when the agent believes signals are negatively correlated. In this case the principal might opt for designing a contract where, taking as given her PE report, the compensation is higher in the disagreement cases than in the agreement cases, i.e., a contract with $c_{aa} < c_{au}$ and $c_{uu} < c_{ua}$. This follows the exact opposite intuition of Lemma 4.

In order to solve problem (4), when the agent believes signals are negatively correlated, we present a set of Lemmas that select the binding constraints for this case and reduce the problem to the following one.

$$\min_{\{w_{ts}, c_{ts}\}_{t,s\in\{u,a\}}} \quad c_{aa}\gamma_{aa}^H + w_{au}\gamma_{au}^H + w_{ua}\gamma_{ua}^H \tag{43}$$

$$\text{s.t.} \quad c_{aa}\tilde{P}_{aa} + c_{au}\left(\tilde{P}_{au} - \Gamma_a^H\right) > \frac{\Delta V}{\Delta \Gamma_a}\Gamma_u^H \tag{IC}$$

$$w_{au}\frac{P_{au}}{P_{aa}} \leq (w_{ua} - c_{aa}) \tag{(TR_P^a)}$$

$$(w_{ua} - c_{aa}) \leq w_{au}\frac{P_{uu}}{P_{ua}} \tag{(TR_P^u)}$$

$$c_{au} \geq c_{aa} \tag{(TR_A^u)}$$

$$w_{ua} \geq c_{ua} \geq 0 \tag{LL_{ua}}$$

$$w_{au} \geq c_{au} \geq 0. \tag{LL_{au}}$$

Reducing the problem to (43)**.** Lemma 32 below studies the effect of optimism on the (IC) in this case. Intuitively, given Lemma 31, the agent now overestimates the chances of obtaining premium $c_{ua} - c_{uu}$, and underestimates the ones of obtaining $c_{au} - c_{aa}$. Hence, his incentive to exert λ^L is higher, since $T = u$ is more probable under low effort, and the (IC) tightens.

Lemma 32. *If the optimal contract implementing high effort features $c_{aa} < c_{au}$ and $c_{uu} < c_{ua}$, then optimism tightens the (IC).*

Proof. If the optimal contract features $c_{aa} < c_{au}$ and $c_{uu} < c_{ua}$, then the second and third terms in the LHS of (15) are strictly negative and therefore optimism tightens the (IC). ∎

Lemma 33. *If the agent believes signals are negatively correlated, i.e.* (1) *fails, then the optimal contract implementing high effort features $c_{au} > c_{uu} = 0$.*

Proof. Recall that any optimal contract with truthful reporting for an agent who believes signals are negatively correlated satisfies either case (i) or case (ii) of Lemma 31. Assume case (i) of Lemma 31 holds, then the (IC) becomes:

$$c_{au} \underbrace{(\Delta\tilde{\gamma}_{aa} + \Delta\tilde{\gamma}_{au})}_{>0} + c_{uu} \underbrace{(\Delta\tilde{\gamma}_{ua} + \Delta\tilde{\gamma}_{uu})}_{<0} \geq \Delta V.$$

Because of the negative sign of the second bracket, and since $\Delta V > 0$ and $c_{uu} \geq 0$, the above requires $c_{au} > 0$ to always hold. Assume now case (ii) of Lemma 31 holds, for a similar argument, we need at least one between c_{aa} and c_{au} to be positive. If $c_{au} > 0$, the Lemma is trivially proven. If $c_{aa} \geq 0$, case (ii) implies $c_{au} > c_{aa} \geq 0$. This proves the first part of Lemma 33.

To prove the second part of Lemma 33 we suppose it is false, i.e., at optimum the contract features $c_{uu} > 0$, and we prove that there exists a profitable deviation from it, which contradicts its optimality. From Lemma 31, we know that $c_{ua} \geq c_{uu}$ and also $c_{au} \geq c_{aa}$. The proof now depends on whether $c_{aa} > 0$ or $c_{aa} = 0$.

Let $c_{aa} > 0$. Let the principal decrease both c_{uu} and c_{ua} by ϵ so that their difference remains constant (so not to affect the (TR_A) constraints). From the rearrangement of the constraint above, we see that both c_{uu} and c_{ua} enter negatively in the LHS of the (IC). Hence, decreasing them, would relax the (IC) rather than tightening it. In particular, the LHS of the (IC) constraint has increased by $-\epsilon(\Delta\tilde{\gamma}_{ua} + \Delta\tilde{\gamma}_{uu})$. Since we are in the case where $c_{aa} > 0$, the principal can also decrease both c_{aa} and c_{au} by ϵ. In this way, the overall change in the LHS of the (IC) is given by:

$$\begin{aligned}
&-\epsilon \left(\Delta\tilde{\gamma}_{aa} + \Delta\tilde{\gamma}_{au} + \Delta\tilde{\gamma}_{ua} + \Delta\tilde{\gamma}_{uu}\right) \\
&\quad = -\epsilon \left(\tilde{P}_{aa}\Delta\Gamma_a + \tilde{P}_{au}\Delta\Gamma_a + \tilde{P}_{ua}\Delta\Gamma_u + \tilde{P}_{uu}\Delta\Gamma_u\right) \\
&\quad = -\epsilon \left(\Delta\Gamma_a + \Delta\Gamma_u\right) = -\epsilon \left(\Delta\Gamma_a - \Delta\Gamma_a\right) = 0,
\end{aligned}$$

and therefore the (IC) binds again.

Finally, since both c_{ua} and c_{aa} have been decreased by ϵ, then the principal can decrease also w_{ua} and w_{aa} by the same amount. This does not violate the relevant (LL_{ts}) and holds their difference constant. Hence, it does not violate any of the (TR_P) constraints. This new contract $\{w_{ts}, c_{ts}\}_{t,s}$ implements high effort at a lower cost. Hence, a contract where $c_{uu} > 0$ and $c_{aa} > 0$ cannot be the solution to the problem.

Let now, instead, the optimal contract feature $c_{aa} = 0$ and define $\Delta c_u = c_{ua} - c_{uu}$. We divide the proof for this case in three steps.

Step 1. When $c_{aa} = 0$, the (TR_A) imply:

$$\Delta c_u \frac{\tilde{\gamma}^H_{uu}}{\tilde{\gamma}^H_{au}} \leq c_{au} \leq \Delta c_u \frac{\tilde{\gamma}^H_{ua}}{\tilde{\gamma}^H_{aa}}, \tag{44}$$

where, since we are in case (ii) of Lemma 31, either only one of the two inequalities holds as equality, or none. Suppose none of the two does, or only the second one, the principal can decrease both c_{ua} and c_{uu} by ϵ keeping Δc_u constant, relaxing the (IC) constraint. In particular, the LHS of the (IC) has decreased by $\epsilon(\Delta\tilde{\gamma}_{ua} + \Delta\tilde{\gamma}_{uu}) < 0$. He can then decrease c_{au} by $\delta \equiv \frac{\epsilon(\Delta\tilde{\gamma}_{ua}+\Delta\tilde{\gamma}_{uu})}{\Delta\tilde{\gamma}_{au}}$ bringing the LHS of the (IC) back to its original value. Clearly for some ϵ, this deviation can be done until the first inequality in (44) binds. Finally, to see that this is optimal for the principal, notice that according to the (LL) constraints, she can now decrease w_{uu} up to ϵ and w_{au} up to δ. By decreasing both by $\min\{\epsilon, \delta\}$, their difference does not change. Hence, (TR_P) constraints are not affected while the objective function decreases. This implies that at optimum if $c_{aa} = 0$, the first inequality of (44) binds.

Step 2. Given that $\Delta c_u \frac{\tilde{\gamma}^H_{uu}}{\tilde{\gamma}^H_{au}} = c_{au}$ must hold at optimum if $c_{aa} = 0$, we now show that the principal has at her disposal the following optimal deviation from a contract with $c_{aa} = 0$ and $c_{uu} > 0$. Let her decrease c_{ua} by ϵ and c_{uu} by $\epsilon_0 < \epsilon$. Then Δc_u has decreased by $(\epsilon - \epsilon_0)$. In order to keep $\Delta c_u \frac{\tilde{\gamma}^H_{uu}}{\tilde{\gamma}^H_{au}} = c_{au}$, the principal decreases c_{au} by $(\epsilon - \epsilon_0)\frac{\tilde{\gamma}^H_{uu}}{\tilde{\gamma}^H_{au}}$. It remains to check if this deviation can be made in such a way that it does not violate the (IC). The change in the (IC) is:

$$\begin{aligned}
& -(\epsilon - \epsilon_0)\frac{\tilde{\gamma}^H_{uu}}{\tilde{\gamma}^H_{au}}\Delta\tilde{\gamma}_{au} - \epsilon\Delta\tilde{\gamma}_{ua} - \epsilon_0\Delta\tilde{\gamma}_{uu} \\
= \; & -(\epsilon - \epsilon_0)\frac{\tilde{\gamma}^H_{uu}}{\Gamma^H_a}\Delta\Gamma_a + \epsilon\tilde{P}_{ua}\Delta\Gamma_a + \epsilon_0\tilde{P}_{uu}\Delta\Gamma_a \\
= \; & \Delta\Gamma_a\left[\epsilon\left(\tilde{P}_{ua} - \frac{\tilde{\gamma}^H_{uu}}{\Gamma^H_a}\right) + \epsilon_0\left(\frac{\tilde{\gamma}^H_{uu}}{\Gamma^H_a} + \tilde{P}_{uu}\right)\right] \\
= \; & \frac{\Delta\Gamma_a}{\Gamma^H_a}\left[\epsilon\left(\tilde{P}_{ua}\Gamma^H_a - \tilde{P}_{uu}\Gamma^H_u\right) + \epsilon_0\left(\tilde{P}_{uu}\Gamma^H_u + \tilde{P}_{uu}\Gamma^H_a\right)\right] \\
= \; & \frac{\Delta\Gamma_a}{\Gamma^H_a}\left[\epsilon(\Gamma^H_a - \tilde{P}_{uu}) + \epsilon_0\tilde{P}_{uu}\right],
\end{aligned}$$

which is positive when:

$$\epsilon\left(\Gamma^H_a - \tilde{P}_{uu}\right) + \epsilon_0\tilde{P}_{uu} > 0.$$

If $\Gamma^H_a > \tilde{P}_{uu}$, the above is always true. If, instead, $\Gamma^H_a < \tilde{P}_{uu}$ then the principal has to choose $\epsilon \in \left\{\epsilon_0, \epsilon_0\frac{\tilde{P}_{uu}}{\tilde{P}_{uu} - \Gamma^H_a}\right\}$.

Step 3. To conclude, given the decreases in the c_{ts}, the principal can now decrease w_{uu} up to ϵ_0 and w_{au} up to $(\epsilon - \epsilon_0)\frac{\tilde{\gamma}^H_{uu}}{\tilde{\gamma}^H_{au}}$. By an argument similar to the one of Step 1, she can decrease both by the smallest of the two limits, decreasing the objective function. This provides the desired contradiction.

Finally, since a contract where $c_{uu} > 0$ and $c_{aa} \geq 0$ cannot be a solution to the problem it follows that $c_{uu} = 0$. This concludes the proof of the Lemma. ∎

Lemma 34. *If the agent believes signals are negatively correlated, i.e.* (1) *fails, then constraint* (TR^a_A) *always binds in any optimal contract implementing high effort. Therefore:*

$$c_{ua} = \frac{\tilde{\gamma}^H_{aa}}{\tilde{\gamma}^H_{ua}}(c_{au} - c_{aa}).$$

Proof. Suppose not. Given the Lemmas proven till now and that the (TR^a_A) is slack, the problem that the principal faces is given by

$$\min_{\{w_{ts},c_{ts}\}_{t,s\in\{u,a\}}} w_{aa}\gamma^H_{aa}+w_{au}\gamma^H_{au}+w_{ua}\gamma^H_{ua}+w_{uu}\gamma^H_{uu} \tag{45}$$

$$\text{s.t.}\quad c_{aa}\Delta\tilde{\gamma}_{aa}+c_{au}\Delta\tilde{\gamma}_{au}+c_{ua}\Delta\tilde{\gamma}_{ua}-\Delta V\geq 0 \tag{IC}$$

$$w_{aa}\gamma^H_{aa}+w_{au}\gamma^H_{au}\leq w_{ua}\gamma^H_{aa}+w_{uu}\gamma^H_{au} \tag{TR^a_P}$$

$$w_{ua}\gamma^H_{ua}+w_{uu}\gamma^H_{uu}\leq w_{aa}\gamma^H_{ua}+w_{au}\gamma^H_{uu} \tag{TR^u_P}$$

$$c_{aa}\tilde{\gamma}^H_{aa}+c_{ua}\tilde{\gamma}^H_{ua}\geq c_{au}\tilde{\gamma}^H_{aa} \tag{TR^a_A}$$

$$c_{au}\tilde{\gamma}^H_{au}\geq c_{aa}\tilde{\gamma}^H_{au}+c_{ua}\tilde{\gamma}^H_{uu} \tag{TR^u_A}$$

$$w_{ts}\geq c_{ts}\geq 0\qquad \forall t,s\in\{a,u\}. \tag{LL_{ts}}$$

and we can rewrite the (TR_P) and (TR_A) constraints as

$$(w_{au}-w_{uu})\frac{\gamma^H_{au}}{\gamma^H_{aa}}\leq(w_{ua}-w_{aa})\leq(w_{au}-w_{uu})\frac{\gamma^H_{uu}}{\gamma^H_{ua}} \tag{(TR_P)}$$

$$c_{ua}\frac{\tilde{\gamma}^H_{uu}}{\tilde{\gamma}^H_{au}}\leq(c_{au}-c_{aa})\leq c_{ua}\frac{\tilde{\gamma}^H_{ua}}{\tilde{\gamma}^H_{aa}}. \tag{(TR_A)}$$

The principal can then decrease c_{ua} by ϵ, such that the (TR^a_A) still holds, and c_{au} and c_{aa} by $\epsilon\tilde{P}_{ua}$. Since the difference $c_{au}-c_{aa}$ is constant, the (TR_A) still hold. The (IC) is invariant, since its LHS has changed by

$$-\epsilon\tilde{P}_{ua}\underbrace{(\Delta\tilde{\gamma}_{aa}+\Delta\tilde{\gamma}_{au})}_{\Delta\Gamma_a}-\epsilon\Delta\tilde{\gamma}_{ua}=\Delta\Gamma_a(\epsilon\tilde{P}_{ua}-\epsilon\tilde{P}_{ua})=0.$$

We are now left to show that this is optimal for the principal. Notice that both c_{ua} and c_{aa} have decreased. Hence, the principal can decrease both w_{ua} and w_{aa} by $\epsilon\tilde{P}_{ua}$. This does not violate (TR_P) and decreases the objective function, providing the desired contradiction. ∎

Given this, we can rewrite the (IC) as

$$c_{aa}\left(\Delta\tilde{\gamma}_{aa}-\frac{\tilde{\gamma}^{H}_{aa}}{\tilde{\gamma}^{H}_{ua}}\Delta\tilde{\gamma}_{ua}\right)+c_{au}\left[\Delta\tilde{\gamma}_{au}+\frac{\tilde{\gamma}^{H}_{aa}}{\tilde{\gamma}^{H}_{ua}}\Delta\tilde{\gamma}_{ua}\right]-\Delta V>0$$

$$c_{aa}\left(\tilde{P}_{aa}\Gamma^{H}_{u}+\tilde{P}_{aa}\Gamma^{H}_{a}\right)+c_{au}\left(\tilde{P}_{au}\Gamma^{H}_{u}-\tilde{P}_{aa}\Gamma^{H}_{a}\right)>\frac{\Delta V}{\Delta\Gamma_{a}}\Gamma^{H}_{u}$$

$$c_{aa}\tilde{P}_{aa}(\Gamma^{H}_{a}+\Gamma^{H}_{u})+c_{au}\left[\tilde{P}_{au}(1-\Gamma^{H}_{a})-\tilde{P}_{aa}\Gamma^{H}_{a}\right]>\frac{\Delta V}{\Delta\Gamma_{a}}\Gamma^{H}_{u}$$

$$c_{aa}\tilde{P}_{aa}+c_{au}\left(\tilde{P}_{au}-\Gamma^{H}_{a}\right)>\frac{\Delta V}{\Delta\Gamma_{a}}\Gamma^{H}_{u}$$

Lemma 35. *If the agent believes signals are negatively correlated, i.e.* (1) *fails, then* (LL_{aa}) *and* (LL_{uu}) *bind in any optimal contract implementing high effort, i.e.,* $w_{aa}=c_{aa}$ *and* $w_{uu}=0$.

Proof. Consider the (TR_P)

$$\underbrace{(w_{au}-w_{uu})\frac{P_{au}}{P_{aa}}}_{LHS}\le\underbrace{(w_{ua}-w_{aa})}_{\text{middle term}}\le\underbrace{(w_{au}-w_{uu})\frac{P_{uu}}{P_{ua}}}_{RHS}$$

Start from (LL_{aa}). Suppose it does not bind. Then the principal can increase w_{au} by ϵ and decrease w_{aa} by $\epsilon_1\equiv\epsilon\frac{P_{uu}}{P_{ua}}$. The values in (TR_P) change. The RHS increases by ϵ_1. The middle term also increases by ϵ_1. The LHS increases by $\epsilon\frac{P_{au}}{P_{aa}}$. To see that the LHS stays lower than the middle term, notice that

$$\epsilon\frac{P_{au}}{P_{aa}}\le\epsilon\frac{P_{uu}}{P_{ua}}\quad\text{since}\quad P_{au}P_{ua}<P_{aa}P_{uu},$$

by Assumption 2. This creates an overall effect on the objective function given by

$$\epsilon\gamma^{H}_{au}-\epsilon\frac{P_{uu}}{P_{ua}}\gamma^{H}_{aa}=\epsilon\frac{1}{P_{ua}}\left(\gamma^{H}_{au}P_{ua}-\gamma^{H}_{aa}P_{uu}\right)=\epsilon\frac{1}{P_{ua}}(P_{au}P_{ua}-P_{aa}P_{uu})\Gamma^{H}_{a}<0.$$

Hence, this deviation contradicts the optimality of $w_{aa}>c_{aa}$.

For the (LL_{uu}) we follow the same logic. Suppose it does not bind. The principal can decrease w_{uu} by ϵ and increase w_{ua} by $\epsilon_1\equiv\epsilon\frac{P_{au}}{P_{aa}}$. The values in (TR_P) change. The RHS increases by $\epsilon\frac{P_{au}}{P_{aa}}$. The middle term increases by ϵ_1. The LHS increases also by ϵ_1. To see that the RHS stays larger than the middle term, notice that

$$\epsilon\frac{P_{au}}{P_{aa}}\le\epsilon\frac{P_{uu}}{P_{ua}}$$

as above. This creates an overall effect on the objective function given by

$$-\epsilon\gamma^H_{uu} + \epsilon\frac{P_{au}}{P_{aa}}\gamma^H_{ua} = \epsilon\frac{1}{P_{aa}}\left(\gamma^H_{ua}P_{au} - \gamma^H_{uu}P_{aa}\right) = \epsilon\frac{1}{P_{aa}}(P_{au}P_{ua} - P_{aa}P_{uu})\Gamma^H_u < 0.$$

Hence this deviation contradicts the optimality of $w_{uu} > c_{uu}$. ∎

This concludes the set of Lemmas that yield us problem (43).

The study of the solution of (43) is longer and more complicated than the solutions to (5). First of all, notice that this time we have $c_{uu} = 0$ (Lemma 33). When the agent believes signals to be negatively correlated the agreement payoffs (i.e. the ones following $T = S$) are now surprising for an agent. In particular if the agent observes $S = u$, he believes that the principal has observed $T = a$, and is not easily convinced that $T = u$ instead.

The first implication of the above, is that the proof of Proposition 1 does no longer hold. That is, the bias of an agent who believes signals to be negatively correlated is such that the existence of a deadweight loss is no longer a necessary condition for the implementation of high effort. Further, as we show later, there exists a portion of the parameter space where the optimal contract does not, in fact, feature any deadweight loss. This is, however, not the case for an optimistic agent. While there may exist equilibrium contracts different from the standard one for an agent with beliefs violating (1), they are never optimal when the agent is optimistic.

Proposition 12. *If the agent is optimistic and perceives signals as negatively correlated, then he is assigned the BPE contract.*

Proof. At optimum, it is of course true that either (TR^a_P) or (TR^u_P) bind, or both. Since, however, $P_{aa}P_{uu} - P_{au}P_{ua} > 0$ and $w_{uu} = 0$, the only way to have both binding would be for $w_{ua} = w_{aa}$ and $w_{au} = 0$. From Lemma 33, we know that $c_{au} > 0$. Hence, at least one for the two constraint has to be slack. The proof is quite long and therefore we divide it in several parts. First, we assume only the (TR^a_P) binds, which is split further into two cases on the basis of the sign of the slope of the (IC). Second, we assume no (TR_P) binds. Finally, we assume only the (TR^u_P) binds.

Constraint (TR_P^a) **binding.** Suppose the optimal contract sets the (TR_P^a) binding. We then have

$$w_{ua} = w_{au}\frac{P_{au}}{P_{aa}} + c_{aa},$$

which results in the following objective function

$$c_{aa}\left(\gamma_{aa}^H + \gamma_{ua}^H\right) + w_{au}\left(\gamma_{au}^H + \frac{P_{au}}{P_{aa}}\gamma_{ua}^H\right).$$

Since w_{au} has a clear positive effect on it and the only constraint left on w_{ua} is (LL_{au}), we have that $w_{au} = c_{au}$. We can further simplify the objective function

$$\begin{aligned} & c_{aa}\left(\gamma_{aa}^H + \gamma_{ua}^H\right) + w_{au}\left(\gamma_{au}^H + \frac{P_{au}}{P_{aa}}\gamma_{ua}^H\right) \\ &= c_{aa}\left(\gamma_{aa}^H + \gamma_{ua}^H\right) + c_{au}\frac{P_{au}}{P_{aa}}\left(P_{aa}\Gamma_a^H + P_{ua}\Gamma_u^H\right) \\ &= \left(\gamma_{aa}^H + \gamma_{ua}^H\right)\left[c_{aa} + c_{au}\frac{P_{au}}{P_{aa}}\right], \end{aligned}$$

which is equivalent to minimizing:

$$c_{aa} + c_{au}\frac{P_{au}}{P_{aa}}.$$

This implies that iso-costs have slope $-P_{au}/P_{aa} < 0$.

On the other hand, the (IC) is not necessarily negatively sloped. Its slope is given by

$$-\frac{\tilde{P}_{au} - \Gamma_a^H}{\tilde{P}_{aa}},$$

which is negative only if $b_a < P_{au} - \Gamma_a^H$.

The reduced problem for this case is given by

$$\min_{\{w_{ts},c_{ts}\}_{t,s\in\{u,a\}}} c_{aa} + c_{au}\frac{P_{au}}{P_{aa}} \tag{46}$$

$$\text{s.t.} \quad c_{aa}\tilde{P}_{aa} + c_{au}\left(\tilde{P}_{au} - \Gamma_a^H\right) > \frac{\Delta V}{\Delta\Gamma_a}\Gamma_u^H \tag{IC}$$

$$c_{au} \geq c_{aa} \tag{(TR_A^u)}$$

Positively Sloped (IC). Suppose $b_a > P_{au} - \Gamma_a^H$, the slope of the (IC) is positive and smaller than 1. To see this, notice that

$$\Gamma_a^H - \tilde{P}_{au} < \tilde{P}_{aa} \Rightarrow \Gamma_a^H - 1 < 0,$$

which is always true. Hence, the binding constraints can be represented in (c_{au}, c_{aa}) space as in Figure 12.

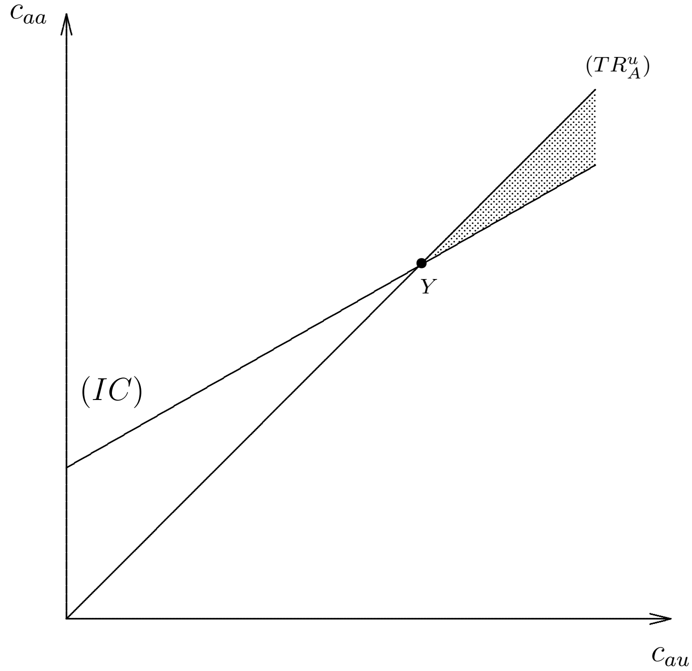

FIGURE 12. The shaded area represents the set of contracts satisfying all the constraints of the minimization problem (46), when the (IC) is positively sloped and the agent believes signals are negatively correlated.

In the Figure, costs decrease towards the origin of the graph. The shaded area represents the set of contracts satisfying all constraints and the optimal contract is therefore at point Y. At Y, $c_{au} = c_{aa} > 0 = c_{ua}$. We derive the full contract below in Lemma 36 and show that it is equivalent to the BPE contract.

Negatively Sloped (IC). Now suppose that $b_a < P_{au} - \Gamma_a^H$.[41] The problem can be represented as in Figure 13 below.

Once again, costs decrease towards the origin, but whether the minimum point lies at Y or X depends on the comparison between the slope of the (IC) and the one of the iso-costs, as in the case of an overconfident agent. In particular, the minimum lies

[41]Notice that this restriction may always fail for an optimistic agent if $P_{au} \leq \Gamma_a^H$.

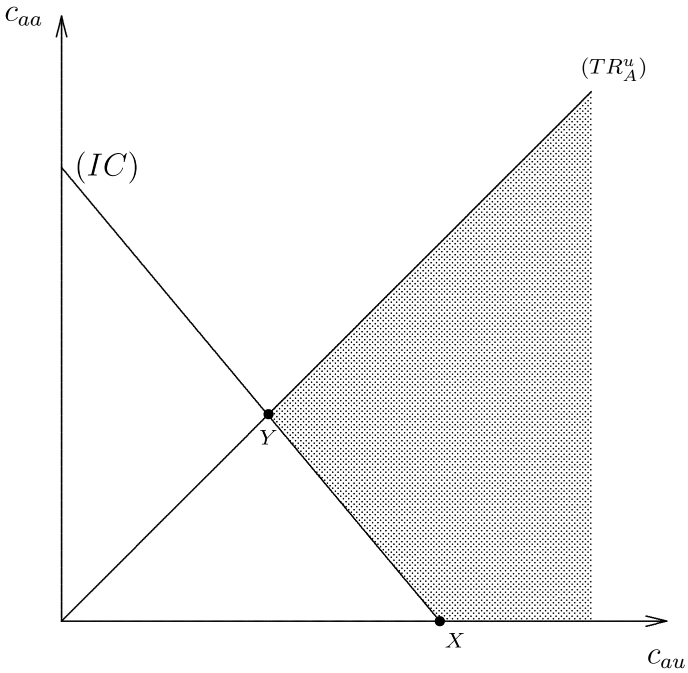

FIGURE 13. The shaded area represents the set of contracts satisfying all the constraints of the minimization problem (46), when the (IC) is negatively sloped and the agent believes signals are negatively correlated.

at X if iso-costs are flatter than the (IC). This happens when

$$\begin{aligned}
&\frac{P_{au}}{P_{aa}} \leq \frac{\tilde{P}_{au} - \Gamma_a^H}{\tilde{P}_{aa}} \\
&P_{au}\tilde{P}_{aa} \leq \tilde{P}_{au}P_{aa} - \Gamma_a^H P_{aa} \\
&b_a P_{au} + b_a P_{aa} \leq P_{au}P_{aa} - \Gamma_a^H P_{aa} - P_{au}P_{aa} \\
&b_a(P_{au} + P_{aa}) \leq -\Gamma_a^H P_{aa} \\
&b_a \leq -P_{aa}\Gamma_a^H. \qquad (47)
\end{aligned}$$

Notice that $-P_{aa}\Gamma_a^H < P_{au} - \Gamma_A^H$. Hence, (47) implies the negative slope of the (IC).

Lemma 36. *If the agent believes signals are negatively correlated, i.e.* (1) *fails, and* (TR_P^a) *binds, then the optimal contract implementing high effort is given by:*

$$\begin{aligned}
&w_{aa} = c_{au} \quad w_{au} = c_{au} \quad w_{uu} = 0 \quad w_{ua} = \tfrac{c_{au}}{P_{aa}} \\
&c_{aa} = c_{au} \quad c_{au} = \tfrac{\Delta V}{\Delta \Gamma_a} \quad c_{uu} = 0 \quad c_{ua} = 0.
\end{aligned}$$

which fully replicates the BPE contract.

Proof. Substitute $c_{aa} = c_{au}$ into the (IC) and notice that

$$c_{au}(\tilde{P}_{aa} + \tilde{P}_{au} - \Gamma_a^H) = \frac{\Delta V}{\Delta \Gamma_a}\Gamma_u$$

implies

$$c_{au} = \frac{\Delta V}{\Delta \Gamma_a}\frac{\Gamma_u}{(1-\Gamma_a^H)} = \frac{\Delta V}{\Delta \Gamma_a}.$$

For w_{ua}, notice that

$$w_{ua} = c_{au}\frac{P_{au}}{P_{aa}} + c_{aa} = c_{au}\left(\frac{P_{au}}{P_{aa}} + 1\right) = c_{au}\left(\frac{P_{au}+P_{aa}}{P_{aa}}\right) = \frac{c_{au}}{P_{aa}}$$

∎

Since for an optimistic type b_a is never smaller or equal to $-P_{aa}\Gamma_a^H$, the BPE contract is the only possible contract for an optimistic agent who believes signals are negatively correlated, if the (TR_P^a) is binding.

No (TR_P) constraint binding (no deadweight loss contract). Suppose, now, all (TR_P) are slack. Then clearly all (LL_{ts}) constraint bind, since the principal wants to decrease the expected wage paid as much as she can, and they are the only constraints preventing her to set the $w_{ts} = 0$. We have

$$w_{ts} = c_{ts}, \quad c_{uu} = 0, \quad c_{ua} = \frac{\tilde{\gamma}_{aa}^H}{\tilde{\gamma}_{ua}^H}(c_{au} - c_{aa}).$$

Then the principal solves

$$\min_{\{w_{ts},c_{ts}\}_{t,s\in\{u,a\}}} \; c_{aa}\gamma_{aa}^H + c_{au}\gamma_{au}^H + \frac{\tilde{\gamma}_{aa}^H}{\tilde{\gamma}_{ua}^H}(c_{au} - c_{aa})\gamma_{ua}^H \tag{48}$$

$$\text{s.t.} \quad c_{aa}\tilde{P}_{aa} + c_{au}\left(\tilde{P}_{au} - \Gamma_a^H\right) > \frac{\Delta V}{\Delta \Gamma_a}\Gamma_u^H \tag{IC}$$

$$c_{au} \geq c_{aa}. \tag{(TR_A^u)}$$

The sign of the slope of the iso-costs is not as trivial as above.

The objective function can be rearranged to obtain

$$\frac{1}{\tilde{\gamma}_{ua}^H}c_{aa}(\gamma_{aa}^H\tilde{\gamma}_{ua}^H - \tilde{\gamma}_{aa}^H\gamma_{ua}^H) + c_{au}(\gamma_{au}^H\tilde{\gamma}_{ua}^H + \tilde{\gamma}_{aa}^H\gamma_{ua}^H)$$

$$\propto c_{aa}(P_{aa}\tilde{P}_{ua} - \tilde{P}_{aa}P_{ua}) + c_{au}(P_{au}\tilde{P}_{ua} + \tilde{P}_{aa}P_{ua}).$$

Hence the slope of the iso-costs is negative if:

$$
\begin{aligned}
&P_{aa}\tilde{P}_{ua} - \tilde{P}_{aa}P_{ua} \\
&= b_u P_{aa} - b_a P_{ua} > 0. \qquad (49)
\end{aligned}
$$

Now notice two things

(1) If the (IC) is positively sloped, the optimal point would be at $c_{au} = c_{aa}$, regardless of whether costs decrease towards the origin (negatively sloped iso-costs) or towards the top-left corner (positively sloped) in (c_{au}, c_{aa}) space. This, however, yields an unfeasible contract, since $c_{au} = c_{aa} \Rightarrow c_{ua} = 0 = w_{ua}$, which violates the (TR_P^a) constraint as

$$c_{au}\frac{P_{au}}{P_{aa}} > -c_{au}.$$

(2) If the (IC) is negatively sloped, the constraint of the problem are the same as the ones represented already in Figure 13. When the iso-costs are positively sloped, or when they are negatively sloped but steeper than the (IC), the optimal point would be at $c_{au} = c_{aa} (\Rightarrow c_{ua} = 0 = w_{ua})$ again.

These two observations imply that the only possible feasible contract for this case is one where the iso-costs and the (IC) are negatively sloped and the former are flatter than the latter.[42] This happens when

$$
\begin{aligned}
&\frac{P_{au}\tilde{P}_{ua} + \tilde{P}_{aa}P_{ua}}{P_{aa}\tilde{P}_{ua} - \tilde{P}_{aa}P_{ua}} \leq \frac{P_{au} - b_a - \Gamma_a^H}{P_{aa} + b_a} \\
&\frac{(P_{au} + P_{aa})P_{ua} + b_u P_{au} + b_a P_{ua}}{P_{aa}b_u - b_a P_{ua}} \leq \frac{P_{au} - b_a - \Gamma_a^H}{P_{aa} + b_a} \\
&(P_{ua} + b_u P_{au} + b_a P_{ua})(P_{aa} + b_a) \leq (P_{aa}b_u - b_a P_{ua})(P_{au} - b_a - \Gamma_a^H) \\
&P_{aa}P_{ua} + b_u P_{aa}P_{au} + b_a P_{ua}P_{aa} + b_a P_{ua} + b_a b_u P_{au} + b_a^2 P_{ua} \\
&\qquad\qquad \leq b_u P_{aa}P_{au} - b_a b_u P_{aa} - b_u \Gamma_a^H P_{aa} - b_a P_{ua}P_{au} + b_a^2 P_{ua} + b_a \Gamma_a^H P_{ua} \\
&P_{aa}P_{ua} + b_a P_{ua} + b_a P_{ua}\Gamma_u^H + b_a b_u + b_u \Gamma_a^H P_{aa} \geq 0 \\
&b_a(P_{ua}(1 + \Gamma_u^H) + b_u) \leq -P_{aa}\left(P_{ua} + b_u \Gamma_a^H\right),
\end{aligned}
$$

which generates

$$
b_a \leq -P_{aa}\frac{\left(P_{ua} + b_u \Gamma_a^H\right)}{(P_{ua}(1 + \Gamma_u^H) + b_u)} \tag{50}
$$

that always fails for $b_a < 0$.[43]

Constraint (TR_P^u) binding. Suppose finally that (TR_P^u) binds. We have

$$
w_{ua} = w_{au}\frac{P_{uu}}{P_{ua}} + c_{aa}.
$$

In this case, the objective function is given by

$$
\begin{aligned}
&c_{aa}(\gamma_{aa}^H + \gamma_{ua}^H) + w_{au}\left(\gamma_{au}^H + \frac{P_{uu}}{P_{ua}}P_{ua}\Gamma_u^H\right) \\
&= c_{aa}(\gamma_{aa}^H + \gamma_{ua}^H) + w_{au}\left(\gamma_{au}^H + \gamma_{uu}^H\right).
\end{aligned}
$$

[42] Notice that, since we assumed that the (TR_P) are slack, they cannot be considered as restrictions to the problem. On the contrary, when we assumed the (TR_P^a) binding in the previous case, we made no assumption about the (LL_{ts}) and, therefore, they were considered as potentially binding.

[43] Notice that $P_{ua}(1 + \Gamma_u^H) + b_u > 0$ is always true since

$$
-P_{ua}(1 + \Gamma_u^H) < -P_{ua} < b_u.
$$

Since w_{au} has a clear positive effect on it, and the only constraint left on w_{au} is (LL_{au}), we have that $w_{au} = c_{au}$. The reduced problem for this case is given by

$$\min_{\{w_{ts},c_{ts}\}_{t,s\in\{u,a\}}} c_{aa}(\gamma^H_{aa} + \gamma^H_{ua}) + c_{au}\left(\gamma^H_{au} + \gamma^H_{uu}\right) \tag{51}$$

$$\text{s.t.} \quad c_{aa}\tilde{P}_{aa} + c_{au}\left(\tilde{P}_{au} - \Gamma^H_a\right) > \frac{\Delta V}{\Delta \Gamma_a}\Gamma^H_u \tag{IC}$$

$$c_{au} \geq c_{aa}. \tag{(TR^u_A)}$$

We can immediately see that iso-costs are always negatively sloped.

Lemma 37. *If the agent is optimistic and believes signals are negatively correlated, then there exists no optimal contract implementing high effort where the* (TR^u_P) *binds.*

Proof. Suppose not, and the (TR^u_P) binds. Suppose $b_a > P_{au} - \Gamma^H_a$, than the IC is positively sloped and Figure 12 represents the constraints of the problem. The optimal contract would feature $c_{ua} = 0$ and the contract resemble the one of Lemma 36 with the only difference that

$$w_{ua} = c_{au}\frac{P_{uu}}{P_{ua}} + c_{au} = \frac{c_{au}}{P_{ua}}.$$

However, since $P_{ua} < P_{aa}$ (from Assumption 1) this contract is clearly dominated by the BPE contract in Lemma 36.

If instead the (IC) is negatively sloped, we are, once again, in Figure 13, where a new contract may arise if iso-costs are flatter than the (IC) (if instead they are steeper we have the BPE contract again, for the reasons just explained). As standard by now, we are going to show that this case can never happen if the agent is optimistic. Iso-costs are flatter than the (IC) if

$$(P_{ua} + b_u)(\gamma^H_{au} + \gamma^H_{uu}) < (\tilde{P}_{au} - \Gamma^H_a)(\gamma^H_{ua} + \gamma^H_{aa}).$$

Notice that the condition becomes looser the smaller is b_u. Since the agent believes signals to be negatively correlated, b_u must be at least equal to $P_{aa} + b_a - P_{ua} = \tilde{P}_{aa} - P_{ua}$.

Hence, we check the above assuming the floor value of b_u.

$$(P_{ua} + \tilde{P}_{aa} - P_{ua})(\gamma^H_{au} + \gamma^H_{uu}) < (\tilde{P}_{au} - \Gamma^H_a)(\gamma^H_{ua} + \gamma^H_{aa})$$
$$\tilde{P}_{aa}(\gamma^H_{au} + \gamma^H_{uu} + \gamma^H_{ua} + \gamma^H_{aa}) < (1 - \Gamma^H_a)(\gamma^H_{ua} + \gamma^H_{aa})$$
$$\tilde{P}_{aa} < \Gamma^H_u(\gamma^H_{ua} + \gamma^H_{aa})$$
$$P_{aa}(1 - \Gamma^H_a\Gamma^H_u) + b_a - P_{ua}\left(\Gamma^H_u\right)^2 < 0.$$

where we know that $P_{aa} > P_{ua}$ by Assumption 2 and we can calculate

$$(1 - \Gamma^H_a\Gamma^H_u) = 1 - \Gamma^H_A + \left(\Gamma^H_a\right)^2 > 1 - \Gamma^H_a > \left(1 - \Gamma^H_a\right)^2 = \left(\Gamma^H_u\right)^2.$$

Hence the LHS is always positive for $b_a > 0$ and the condition can never been satisfied for an optimistic agent who believes signals are negatively correlated. This concludes the proof of the Lemma. ∎

This concludes the proof of the Proposition. ∎

C.2. **Pessimism and Perceived Negatively Correlation.** Consider now a pessimistic agent who believes signals are negatively correlated and holds beliefs such that

$$b_a < 0,\ b_u < 0,\ \text{and}\ b_u - b_a > P_{aa} - P_{ua}.$$

Following up on the discussion started in Section C.1, we show how the optimal contract in this case can take two new forms: the *Disagreement Performance Evaluation Deadweight Loss* (DPE-DL) contract and the *Disagreement Performance Evaluation No Deadweight Loss* (DPE-NDL) contract. We are going to present each new contract in a Proposition.

As already anticipated in section C.1, when the agent believes that signals are negatively correlated, the existence of a deadweight loss is not a necessary condition for the implementation of high effort any longer. It can, however, still be optimal as the next result shows.

Proposition 13 (DPE-DL Contract)**.** *If the agent is pessimistic or underconfident, believes signals are negatively correlated, and has beliefs that satisfy*

$$b_u P_{au} \Gamma_u^H - b_a P_{aa} \Gamma_a^H \geq P_{aa}^2 \Gamma_a^H - P_{au} P_{ua} \Gamma_u^H \tag{52}$$

and

$$b_a < -P_{aa} \Gamma_A^H, \tag{53}$$

then the optimal contract implementing high effort $\{\hat{w}_{ts}, \hat{c}_{ts}\}_{t,s=a,u}$ *is given by:*

$$\begin{array}{llll} \hat{w}_{aa} = 0 & \hat{w}_{au} = \hat{c}_{au} & \hat{w}_{uu} = 0 & \hat{w}_{ua} = \frac{P_{au}}{P_{aa}} \hat{c}_{au} \\ \hat{c}_{aa} = 0 & \hat{c}_{au} = \frac{\Delta V}{\Delta \Gamma_a} \frac{\Gamma_u^H}{\tilde{P}_{au} - \Gamma_a^H} & \hat{c}_{uu} = 0 & \hat{c}_{ua} = \frac{\tilde{\gamma}_{aa}^H}{\tilde{\gamma}_{ua}^H} \hat{c}_{au}. \end{array}$$

The DPE-DL contract features:

(i) *a wage and compensation that depend on both parties' PE reports;*

(ii) *a deadweight loss when the principal reports an unacceptable performance and the agent reports an acceptable performance (unless* (52) *holds with equality);*

(iii) *a wage and a compensation only when the parties report misaligned performance evaluations.*

Proof. See the proof of Proposition 14. ∎

This result shows that if the pessimistic agent believes that signals are negatively correlated and has a "large" bias, then the optimal contract is very different from the BPE contract described in Proposition 2. First, the principal's wage cost and the agent's compensation depend on both parties' performance evaluations. Second, wage and compensation are positive only when the parties *disagree* on their performance evaluations. Next, we provide the economic intuition behind the DPE-DL contract.

When the agent believes that signals are negatively correlated, two very similar and connected effects take place: (i) he believes $(t, s) = (a, u)$ and $(t, s) = (u, a)$ more probable than $(t, s) = (a, a)$ and $(t, s) = (u, u)$ (at least jointly); (ii) his believed most probable events are the symmetric opposite of the ones believed by the principal. Hence, it is straightforward to see why a pessimistic agent who believes that signals are negatively correlated never accepts a APE contract. First, he rarely expects to obtain $c_{aa}^{\dagger}$. Second, he is not willing any longer to accept a contract that features $c_{ua} = 0$.

Similarly to the APE contract, the principal can take advantage of this in order to decrease the expected wage paid. She can "speculate" on the misalignment of beliefs by increasing (decreasing) the compensation of the agent in states he wrongly deems more (less) probable than she does. Because of (ii), the principal is therefore happy to offer the agent a positive c_{ua} and a larger c_{au} (compared to the BPE case) in exchange for a lower c_{aa} and/or c_{uu}. The fact that they no longer disagree only on the extent of the correlation, but also on the direction of it, opens up to a stronger manipulation of the standard contract, compared to the switch from BPE to APE. This is behind the result that $c_{aa} = c_{uu} = 0$.

Obviously, the above has to be feasible, i.e. the agent has to be biased enough to accept such a manipulation compared to the BPE contract, and optimal, i.e. the DPE contract has to implement high effort at a lower cost. These are precisely the meanings of condition (52) and (53). Condition (52) requires the agent to be biased enough to accept a DPE contract while (53) requires the agent to be biased enough for the principal to find it optimal to offer a DPE contract. To better understand the meaning of, and intuitions behind the conditions, let us represent them in (b_a, b_u) space. Figure 14 represents the two conditions for a pessimistic agent when $\left(P_{aa}, P_{uu}, \Gamma_a^H\right) = (0.7, 0.5, 0.6)$.[44]

When (53) holds, the bias of the agent is such that the principal has the incentive to speculate as described above. In other words, the agent perceives a negative enough correlation for the principal to be able to offer a new contract with a lower c_{aa} and higher c_{au} which is accepted by the agent.[45]

Before going ahead, notice that a key aspect of this contract lies in the full implementation of both PE reports. The disagreement about the correlation between the signals allows the principal to design contracts that take advantage of both information sources. This is confirmed in the next type of contract as well. From the informativeness principle of Holmström (1979) (and its extension to models of risk neutrality and

[44]Similarly to what we discussed in section 4.1, if condition (53) holds with equality, the slopes of the (IC) and isocosts are identical and the problem has many solutions. In particular any point lying between X and Y in Figure 13, presented in the proof of Proposition 12, solves the principal's problem. At this point of indifference, we assume the principal sets up a DPE-DL contract.

[45]To see why this is reflected in the Figure, consider the area where the DPE-DL is set. Notice that it is at its largest when b_u is large and b_a is low. This is precisely when the disagreement on correlation is at its maximum, since $\tilde{P}_{aa}$ is close to zero and $\tilde{P}_{uu}$ is at its minimum.

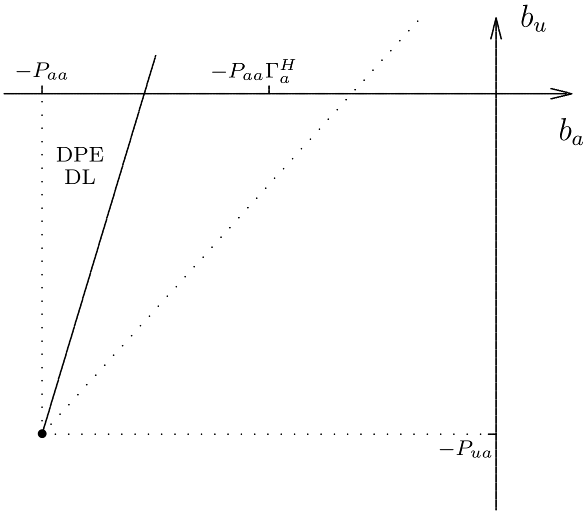

FIGURE 14. In the Figure, we represent conditions (52) and (53). Together they define the area where a pessimistic agent is assigned a DPE-DL contract. The dotted line crossing the quadrant represents (1). In the Figure, we assume $\left(P_{aa}, P_{uu}, \Gamma_a^H\right) = (0.7, 0.5, 0.6)$.

limited liability due to Chaigneau, Edmans, and Gottlieb, 2017) we know that, since T is a sufficient statistic for the pair T, S with respect to effort, the fact that the contract fully implements both sources does not add any informational value to the transaction. If anything it may even decrease the correlation between the actual performance of the project and the resulting payment of w and c.

As we mentioned already, the classical result of Proposition 1 does not necessarily hold in the presence of an agent who (wrongly) believes signals to be negatively correlated. This is originated by the disagreement on the direction of the correlation.[46] To see this, suppose the agent observes $S = a$ and that he believes signals to be negatively correlated. Clearly the agent would be very happy to hear the principal reporting $T = a$, but what happens if the principal reports $T = u$? On the one hand, the agent is upset because the principal deems his performance unacceptable, and therefore would like to punish her in general. On the other hand, however, the agent expects $T = u$ because she believes signals to be negatively correlated! So he is less prone to punish the principal because he is more convinced that $T = u$ is indeed the true realization. As

[46]It would, in fact, still hold if the true correlation were negative and the agent and principal agreed on it.

already explained above, the principal takes advantage of this by setting $c_{aa} = c_{uu} = 0$. When the agent reports $S = a$, he knows that if the principal reports $T = a$, he will get no compensation at all. This makes the agent (i) willing to report $S = a$ only when it is indeed true, (ii) less prone to punish the principal compared to the positive correlation case. Under some particular levels of bias, this effect is so strong that the presence of a deadweight loss case in the contract is no longer a necessary condition for its implementation. This is highlighted in the following proposition.

Proposition 14 (DPE-NDL Contract)**.** *If the agent is pessimistic or underconfident, believes signals are negatively correlated, has beliefs that violate* (52) *but satisfy*

$$b_u P_{uu} \Gamma_u^H - b_a P_{ua} \Gamma_a^H > P_{aa} P_{ua} \Gamma_a^H - P_{uu} P_{ua} \Gamma_u^H \tag{54}$$

and

$$b_a \leq -P_{aa} \left(\frac{P_{ua} + b_u \Gamma_a^H}{P_{ua}(1 + \Gamma_u^H) + b_u} \right), \tag{55}$$

then the optimal contract implementing high effort $\{\hat{w}'_{ts}, \hat{c}'_{ts}\}_{t,s=a,u}$ *is given by:*

$$\begin{array}{llll} \hat{w}'_{aa} = 0 & \hat{w}'_{au} = \hat{c}'_{au} & \hat{w}'_{uu} = 0 & \hat{w}'_{ua} = \hat{c}'_{ua} \\ \hat{c}'_{aa} = 0 & \hat{c}'_{au} = \frac{\Delta V}{\Delta \Gamma_a} \frac{\Gamma_u^H}{\tilde{P}_{au} - \Gamma_a^H} & \hat{c}'_{uu} = 0 & \hat{c}'_{ua} = \frac{\tilde{\gamma}_{aa}^H}{\tilde{\gamma}_{ua}^H} \hat{c}'_{au}. \end{array}$$

The DPE-NDL contract features:

(i) *a wage and compensation that depend on both parties' PE reports;*

(ii) *no deadweight loss;*

(iii) *a wage and a compensation only when the parties report misaligned performance evaluations.*

Proof. To prove the Propositions, we build on the findings of the proof of Proposition 12. First, we show that when (52) and (53) hold, the DPE-DL is optimal. Second, we show that when (52) fails but (54) and (55) hold, the DPE-NDL is feasible and optimal. Finally, we show how Lemma 37 holds in this case too.

Start from the case where (TR_P^a) binds and notice that (53) (which is the equivalent of (47)) may now hold since the agent's bias features $b_a < 0$.

From Figure 13, we see that, when (53) holds, $c_{aa} = 0$, c_{ua} follows from Lemma 34 and c_{au} comes from

$$c_{au}(\tilde{P}_{au} - \Gamma_a^H) = \frac{\Delta V}{\Delta \Gamma_a}\Gamma_u \Rightarrow c_{au} = \frac{\Delta V}{\Delta \Gamma_a}\frac{\Gamma_u}{\tilde{P}_{au} - \Gamma_a^H}.$$

Notice that given the value of w_{ua}, it is not straightforward whether (LL_{ua}) holds or not. Hence, we have

$$\begin{aligned}
&w_{ua} \geq c_{ua} \\
&\frac{P_{au}}{P_{aa}}c_{au} \geq \frac{\tilde{\gamma}_{aa}^H}{\tilde{\gamma}_{ua}^H}c_{au} \\
&P_{au}\tilde{P}_{ua}\Gamma_u^H \geq P_{aa}\tilde{P}_{aa}\Gamma_a^H \\
&P_{au}P_{ua}\Gamma_u^H - P_{aa}^2\Gamma_a^H + b_u P_{au}\Gamma_u^H - b_a P_{aa}\Gamma_a^H \geq 0,
\end{aligned}$$

which generates (52).

Now suppose no (TR_P) constraint holds and notice that (50) (which is equivalent to (55)) may now hold. When it does, we have the same contract of DPE-DL with the difference that now w_{ua} derives from the (LL_{ua}) instead of the (TR_P^a) and therefore

$$w_{ua} = c_{ua} = \frac{\tilde{\gamma}_{aa}^H}{\tilde{\gamma}_{ua}^H}c_{au}.$$

Further checks have to be carried out to be sure that the contract satisfies the (TR_P) constraints. We start from the (TR_P^a) and see that it holds as long as

$$\begin{aligned}
&w_{ua} > \frac{P_{au}}{P_{aa}}w_{au} \Rightarrow c_{ua} > \frac{P_{au}}{P_{aa}}c_{au} \\
&\frac{\tilde{\gamma}_{aa}^H}{\tilde{\gamma}_{ua}^H}c_{au} > \frac{P_{au}}{P_{aa}}c_{au} \Rightarrow \frac{\tilde{\gamma}_{aa}^H}{\tilde{\gamma}_{ua}^H}\frac{P_{aa}}{P_{au}}c_{au} > c_{au} \\
&\frac{\tilde{\gamma}_{aa}^H}{\tilde{\gamma}_{ua}^H}\frac{P_{aa}}{P_{au}} \geq 1 \Rightarrow P_{aa}\tilde{\gamma}_{aa}^H - P_{au}\tilde{\gamma}_{ua}^H \geq 0 \\
&P_{aa}\tilde{P}_{aa}\Gamma_a^H - P_{au}\tilde{P}_{ua}\Gamma_u^H > 0,
\end{aligned}$$

which yields the opposite of (52).

Now we check for (TR_P^u) to hold:

$$\begin{aligned}
& w_{ua} \leq \frac{P_{uu}}{P_{ua}} w_{au} \Rightarrow c_{ua} < \frac{P_{uu}}{P_{ua}} c_{au} \\
& \frac{\tilde{\gamma}_{aa}^H}{\tilde{\gamma}_{ua}^H} c_{au} < \frac{P_{uu}}{P_{ua}} c_{au} \Rightarrow \frac{\tilde{\gamma}_{aa}^H}{\tilde{\gamma}_{ua}^H} \frac{P_{uu}}{P_{ua}} c_{au} < c_{au} \\
& \frac{\tilde{\gamma}_{aa}^H}{\tilde{\gamma}_{ua}^H} \frac{P_{uu}}{P_{ua}} \leq 1 \Rightarrow P_{uu}\tilde{\gamma}_{ua}^H - P_{ua}\tilde{\gamma}_{aa}^H > 0 \\
& P_{uu}P_{ua}\Gamma_u^H - P_{ua}P_{aa}\Gamma_a^H + b_u P_{uu}\Gamma_u^H - b_a P_{ua}\Gamma_a^H > 0 \\
& b_u P_{uu}\Gamma_u^H - b_a P_{ua}\Gamma_a^H > P_{aa}P_{ua}\Gamma_a^H - P_{uu}P_{ua}\Gamma_u^H,
\end{aligned}$$

which yields (54).

Before proving that no optimal contract exists where (TR_P^u) binds, notice that the above contracts are feasible in completely distinct areas (since (52) separates them) and that if all the conditions derived hold, they also "dominate" the BPE, i.e. they are optimal.

To conclude the proof, we provide a different proof to Lemma 37. From the original proof, notice that it is possible now for the iso-costs to be flatter than the (IC). However, we now show that (i) the resulting contract with $c_{aa} = 0$ and (TR_P^u) binding is feasible only if (54) holds, (ii) it is always dominated by the contract without a deadweight loss derived above when the latter is feasible, (iii) it is always dominated by the contract with (TR_P^a) binding derived above when the latter is feasible. Hence, this new contract, even if optimal given the assumption of (TR_P^u) binding, is never generally optimal and can be ignored.

Let's start from (i). Notice that the contract lying at point X of Figure 13 for this case is given by

$$\begin{array}{llll}
w_{aa} = 0 & w_{au} = c_{au} & w_{uu} = 0 & w_{ua} = \frac{P_{uu}}{P_{ua}} c_{au} \\
c_{aa} = 0 & c_{au} = \frac{\Delta V}{\Delta \Gamma_a} \frac{\Gamma_u^H}{\tilde{P}_{au} - \Gamma_a^H} & c_{uu} = 0 & c_{ua} = \frac{\tilde{\gamma}_{aa}^H}{\tilde{\gamma}_{ua}^H} c_{au}
\end{array}$$

The calculations follow the same identical derivations of the case of (TR_P^a) binding, save for w_{ua} that follows from $w_{ua} = c_{au}\frac{P_{uu}}{P_{ua}} + c_{aa}$, given the (TR_P^u).

Given this, constraint (LL_{ua}) holds if

$$\frac{P_{uu}}{P_{ua}} c_{au} \geq \frac{\tilde{\gamma}_{aa}^H}{\tilde{\gamma}_{ua}^H} c_{au},$$

which generates (54) again.

For (ii), we compare the average wage payment in both contracts. Without the need of any algebra, we notice that the no deadweight loss contract features $w_{ts} = c_{ts}$ for all t and s and the compensations and wages offered by the two contracts are identical, save for w_{ua}. Hence, the only way for the contract with (TR_P^u) binding to grant a lower expected wage payment than the no deadweight loss contract is for it to feature $w_{ts} < c_{ts}$ for some ts, which is unfeasible.

Finally, for (iii), notice that the two contracts, again, feature identical c_{ts} and w_{ts}, save for w_{ua}. The contract with (TR_P^a) binding grants a lower average wage payment if

$$\frac{P_{au}}{P_{aa}}\left(\frac{\Delta V}{\Delta\Gamma_a}\frac{\Gamma_u^H}{\tilde{P}_{au}-\Gamma_a^H}\right) \leq \frac{P_{uu}}{P_{ua}}\left(\frac{\Delta V}{\Delta\Gamma_a}\frac{\Gamma_u^H}{\tilde{P}_{au}-\Gamma_a^H}\right),$$

which boils down to simply

$$P_{au}P_{ua} - P_{uu}P_{aa} \leq 0,$$

which is always true.

This proves both propositions and holds for a underconfident agent as well. ∎

Proposition 14 shows that there exist DPE contracts without deadweight loss. Identifying the set of parameter values under which a DPE-NDL contract is feasible and optimal is no easy task. In fact, none of the conditions behind it implies any of the others for all parameter values. In Figure 15 below, we plot the area where a DPE-NDL contract is feasible and optimal for a pessimistic agent when $\left(P_{aa}, P_{uu}, \Gamma_a^H\right) = (0.7, 0.5, 0.6)$.

While condition (54) and (55) play the exact same role for a DPE-NDL contract as (52) and (53) do for a DPE-DL contract, the requirement of (52) to fail needs attention. In the proof of Proposition 14, we show how condition (52) determines whether the (LL_{ua}) or the (TR_P^a) is more stringent in problem (43). When (52) holds, the (TR_P^a) is more stringent and the contract must feature a deadweight loss. When it fails, the contract features no deadweight loss. In other words, condition (52) failing together with condition (54) holding, identify an area where an agent who disagrees with the principal on the direction of the correlation of signals has a bias such that he is willing to sign a contract without deadweight loss. It is possible to show that the area

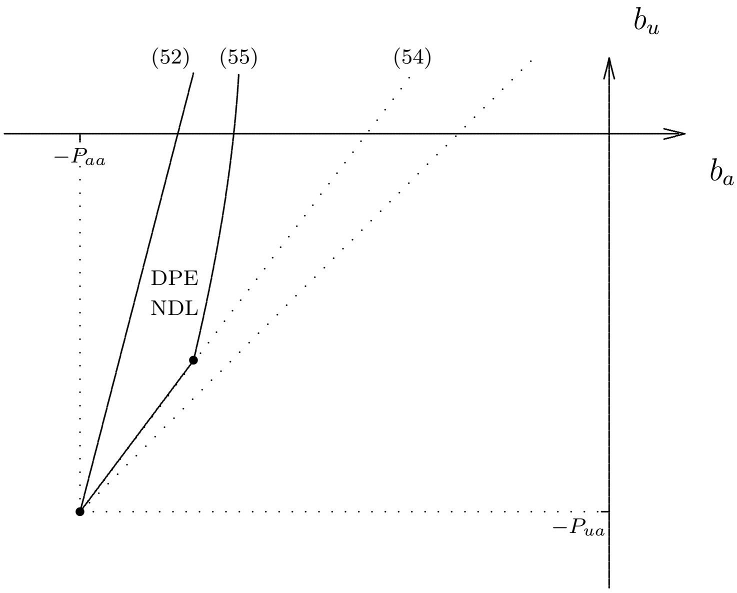

FIGURE 15. In the Figure we represent conditions (52), (54), and (55). Together they define the area where a pessimistic agent is assigned an DPE-NDL contract. The bullet indicates the point where (55) becomes tighter than (54). The dotted line crossing the quadrant represents (1). In the Figure, we assume $\left(P_{aa}, P_{uu}, \Gamma_a^H\right) = (0.7, 0.5, 0.6)$.

where a DPE-NDL contract is set may not exist, under some parameter conditions.[47] This implies that the presence of a (particularly) biased agent may not be enough for the principal to be able to set up a contract without a deadweight loss.

The proofs show that in any portion of the feasible (b_a, b_u) space for a pessimistic agent who believes signals are negatively correlated where none of the DPE contracts is assigned, the BPE contract is assigned instead.

C.3. **Underconfidence and Perceived Negatively Correlation.** By studying the Proofs of the above subsection, it is easy to see how none of the conditions derived for the pessimistic agent change when we discuss the case of a underconfident agent who perceives signals as negatively correlated. Hence, we present directly the graphical analysis for this case in Figure 16, where we also plot the restrictions following from our definition of underconfidence.

[47]For example, for the same parameters of Figure 15 but $\Gamma_a^H = 0.25$, DPE-DL is the only feasible and optimal contract other than the BPE assigned to a pessimistic agent with beliefs that violate (1).

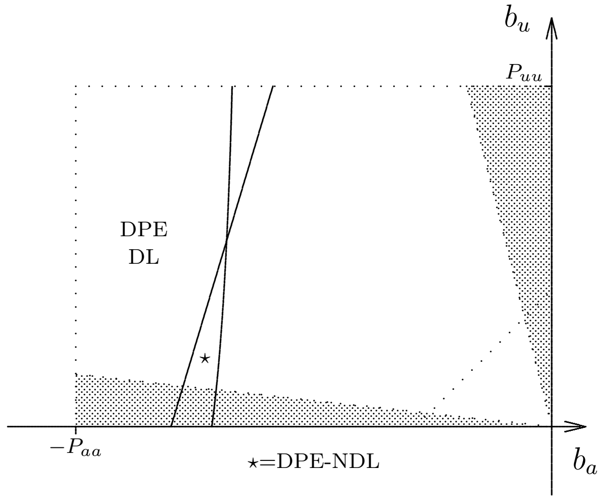

FIGURE 16. When the bias is in the area to the left of the two curves, the principal assigns a DPE-DL contract to an underconfident agent. In the area marked by a star ($\star$), the principal offers a DPE-NDL contract instead. The shaded areas rule out biases that fall outside our definition of underconfidence. The Figure assumes $\left(P_{aa}, P_{uu}, \Gamma_a^H\right) = (0.7, 0.5, 0.6)$.

C.4. **Welfare Analysis for the case of Perceived Negative Correlation.** We now carry on a comparison between the DPE contracts and the BPE, along the same lines of the one in Section 6.

Proposition 15. *Let $\tilde{E}(\cdot)$ denote the biased expectations of the agent. Given the BPE contract $\{w^*_{ts}, c^*_{ts}\}$, the DPE-DL $\{\hat{w}_{ts}, \hat{c}_{ts}\}$ and the DPE-NDL $\{\hat{w}'_{ts}, \hat{c}'_{ts}\}$ contracts, the following are true:*

(i) $E(\hat{w}'_{ts}) \leq E(\hat{w}_{ts}) < E(w^*_{ts})$ *whenever the DPE contracts are optimal.*
(ii) $\tilde{E}(\hat{c}_{ts}) = \tilde{E}(\hat{c}'_{ts}) > \tilde{E}(c^*_{ts})$ *always.*
(iii) $E(\hat{c}_{ts}) > E(c^*_{ts})$ *whenever* (55) *fails.*

Proof. Point (i) follows from the fact that the DPE contracts feature the same wage but for the $(t,s) = (u,a)$ case and the optimality of the DPE contracts (as in Proposition 7).

Point (ii)'s equality is straightforward. To see why the inequality is true we calculate

$$\begin{aligned}\tilde{E}(\hat{c}_{ts}) =& \hat{c}_{aa}\tilde{\gamma}^H_{aa} + \hat{c}_{au}\tilde{\gamma}^H_{au} + \hat{c}_{ua}\tilde{\gamma}^H_{ua} + \hat{c}_{uu}\tilde{\gamma}^H_{uu}\hat{c}_{au}\left(\tilde{\gamma}^H_{au} + \frac{\tilde{\gamma}^H_{aa}}{\tilde{\gamma}^H_{ua}}\tilde{\gamma}^H_{ua}\right)\\ =& \hat{c}_{au}\left(\tilde{\gamma}^H_{au} + \tilde{\gamma}^H_{aa}\right) = \hat{c}_{au}\Gamma^H_a = \frac{\Delta V}{\Delta\Gamma^H_a}\frac{\Gamma^H_a\Gamma^H_u}{\tilde{P}_{au} - \Gamma^H_a}.\end{aligned}$$

Hence, to prove point (ii), we simply need

$$\frac{\Gamma_u^H}{\tilde{P}_{au} - \Gamma_a^H} > 1 \quad \Rightarrow \quad \underbrace{\Gamma_a^H + \Gamma_u^H}_{1} - \tilde{P}_{au} > 0,$$

which always holds.

Finally, to prove point (iii) we calculate

$$\begin{aligned} E(\hat{c}_{ts}) =& \hat{c}_{aa}\gamma_{aa}^H + \hat{c}_{au}\gamma_{au}^H + \hat{c}_{ua}\gamma_{ua}^H + \hat{c}_{uu}\gamma_{uu}^H = \hat{c}_{au}\left(\gamma_{au}^H + \frac{\tilde{\gamma}_{aa}^H}{\tilde{\gamma}_{ua}^H}\gamma_{ua}^H\right) \\ =& \frac{\Delta V}{\Delta \Gamma_a^H}\frac{\gamma_{au}^H\tilde{\gamma}_{ua}^H + \tilde{\gamma}_{aa}^H\gamma_{ua}^H}{\tilde{P}_{au}\tilde{P}_{ua} - \Gamma_a^H\tilde{P}_{ua}}. \end{aligned}$$

Hence, we need

$$\begin{aligned} &\frac{\gamma_{au}^H\tilde{\gamma}_{ua}^H + \tilde{\gamma}_{aa}^H\gamma_{ua}^H}{\tilde{P}_{au}\tilde{P}_{ua} - \Gamma_a^H\tilde{P}_{ua}} > \Gamma_a^H \\ &\gamma_{au}^H\tilde{\gamma}_{ua}^H + \tilde{\gamma}_{aa}^H\gamma_{ua}^H > \Gamma_a^H(\tilde{P}_{au}\tilde{P}_{ua} - \Gamma_a^H\tilde{P}_{ua}) \\ &P_{au}\tilde{P}_{ua}\Gamma_u^H + \tilde{P}_{aa}P_{ua}\Gamma_u^H - \tilde{P}_{au}\tilde{P}_{ua} + \Gamma_a^H\tilde{P}_{ua} > 0, \end{aligned}$$

which, using our usual tools, further simplifies to

$$\begin{aligned} &\underbrace{P_{au}P_{ua}\Gamma_u^H + P_{aa}P_{ua}\Gamma_u^H}_{P_{ua}\Gamma_u^H} - P_{au}P_{ua} + \Gamma_a^H P_{ua} \\ &\qquad + b_u P_{au}\Gamma_u^H + b_a P_{ua}\Gamma_u^H + b_a P_{ua} - b_u P_{au} + b_a b_u + b_u\Gamma_a^H > 0 \\ &P_{ua}(\Gamma_u^H - P_{au} + \Gamma_a^H) + b_u(P_{au}\Gamma_u^H - P_{au} + \Gamma_a^H) + b_a(P_{ua}\Gamma_u^H + P_{ua} + b_u) > 0 \\ &P_{ua}(1 - P_{au}) + b_u(P_{au}\underbrace{(\Gamma_u^H - 1)}_{\Gamma_a^H} + \Gamma_a^H) + b_a\left(P_{ua}(1 + \Gamma_u^H) + b_u\right) > 0 \\ &P_{ua}P_{aa} + b_u\Gamma_a^H(1 - P_{au}) + b_a\left(P_{ua}(1 + \Gamma_u^H) + b_u\right) > 0 \\ &b_a\left(P_{ua}(1 + \Gamma_u^H) + b_u\right) > -P_{ua}P_{aa} - b_u\Gamma_a^H P_{aa} \end{aligned}$$

which generates the opposite of (55). ∎

Point (i) states, once again, that whenever DPE contracts are assigned, they must be optimal. There is a difference, however, compared to the case of the APE. It is trivial to observe that $E(\hat{w}'_{ts}) \leq E(\hat{w}_{ts})$ whenever (52) strictly holds, since the DPE contracts feature the same payments but for $\hat{w}'_{ua} \leq \hat{w}_{ua}$. As a matter of fact, the principal would always like to set up the DPE-NDL contract rather than the DPE-DL one. The former however, may not be feasible under some parameter conditions. Point (ii) is due to the wrong believed direction of correlation between signals by the agent. We know that the DPE contracts feature the largest possible compensations among the contracts set up for the pessimistic or underconfident agent, while featuring zero compensation in the agreement states. The bias of the agent is, however, enough for him to believe that his expected compensation is higher under a DPE contract than under the baseline one. This, once again, connects to the idea of exploitation, where the principal takes advantage of the bias of the agent. Point (iii) follows the same intuition behind point (iv) of Proposition 7, but provides even more interesting insights summarized in the following Proposition.

Proposition 16. *A DPE-NDL contract is never socially desirable. A DPE-DL contract is socially desirable whenever it is optimal, feasible and* (55) *fails.*

Proof. The first statement is trivial since the DPE-NDL contract is optimal only if (55) and it would be socially desirable only when (55) fails. Hence, the DPE-NDL contract never Pareto improves over the BPE contract when it is assigned.

The second statement follows from point (iii) of Proposition 15. ∎

Proposition 16 states a very controversial result on the DPE-NDL contract. On the one hand, the DPE-NDL contract features no deadweight loss. On the other hand, with a DPE-NDL contract the principal takes so much advantage of the agent's biased beliefs that the agent never gains from switching from a BPE to a DPE-NDL contract. The "exploitation motive" is so strong that the agent is, in fact, always exploited by a DPE-NDL contract.

On the positive side, social desirability may take place when the DPE-DL contract is assigned. When the principal is not capable of eliminating the deadweight loss, her taking advantage of the agent's bias may put the latter in a better position compared to the baseline contract. To see that this is possible, consider Figure 17 below where we assume $\left(P_{aa}, P_{uu}, \Gamma_a^H\right) = (0.7, 0.5, 0.6)$. The shaded area may be larger for different parameter configurations. Figure 18 represents a case where a Pareto improvement is possible also for a pessimistic agent.

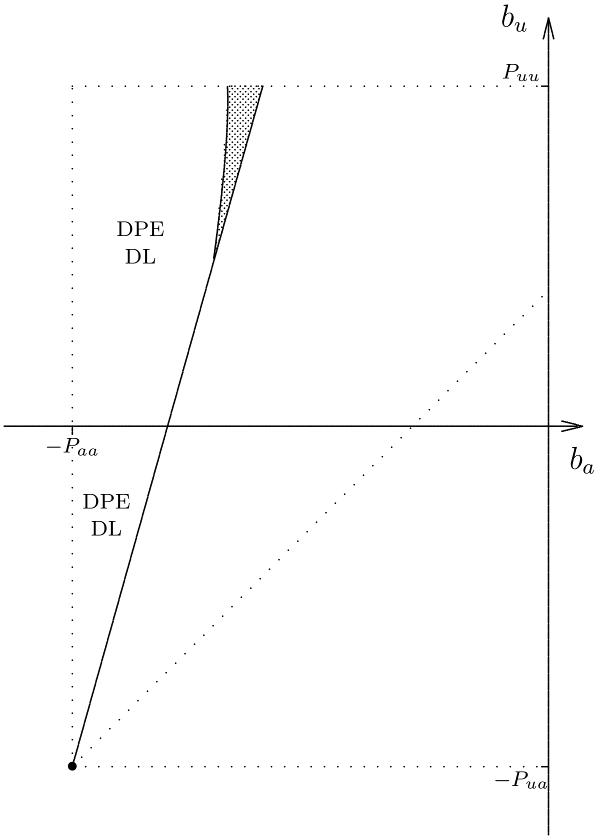

FIGURE 17. The shaded area between the two curves features a DPE-DL contract with a higher expected compensation and a lower expected wage compared to the benchmark contract. Inside this area, the presence of a underconfident agent is socially optimal. The Figure assumes $\left(P_{aa}, P_{uu}, \Gamma_a^H\right) = (0.7, 0.5, 0.6)$.

We conclude with a result on the magnitude of deadweight losses.

Proposition 17. *The BPE contract always features the highest deadweight loss also compared to the DPE contracts. That is*

$$\sum_{ts} (\hat{w}_{ts} - \hat{c}_{ts})\gamma_{ts}^H < \sum_{ts} (w_{ts}^* - c_{ts}^*)\gamma_{ts}^H.$$

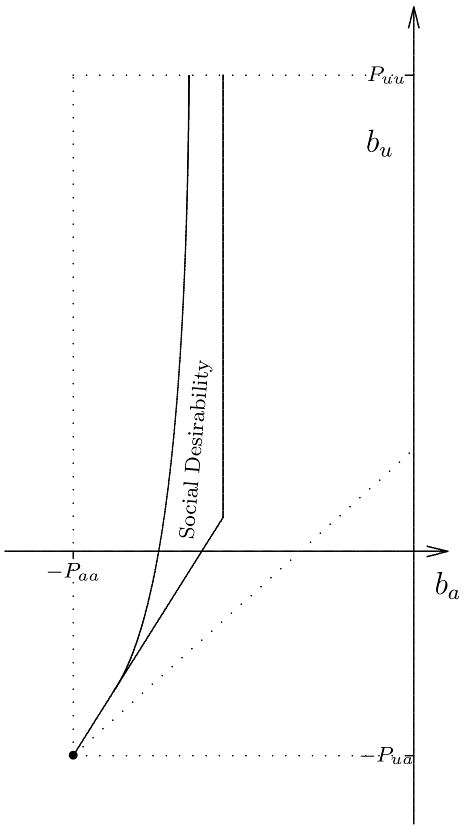

FIGURE 18. The area between the curve and the straight lines on the left features a DPE-DL contract with a higher expected compensation and a lower expected wage compared to the benchmark contract. Inside this area, the presence of a pessimistic or underconfident agent is socially optimal. The Figure assumes $\left(P_{aa}, P_{uu}, \Gamma_a^H\right) = (0.55, 0.7, 0.5)$.

Proof. The deadweight loss under the DPE-DL contract is given by

$$\sum_{ts} \left(\hat{w}_{ts} - \hat{c}_{ts}\right) \gamma_{ts}^H = (\hat{w}_{ua} - \hat{c}_{ua})\gamma_{ua}^H = \left(\frac{P_{au}}{P_{aa}} - \frac{\tilde{\gamma}_{aa}^H}{\tilde{\gamma}_{ua}^H}\right) \hat{c}_{au}$$

$$= \left(\frac{P_{au}}{P_{aa}} - \frac{\tilde{\gamma}_{aa}^H}{\tilde{\gamma}_{ua}^H}\right) \frac{\Delta V}{\Delta \Gamma_A} \frac{\Gamma_u^H}{\tilde{P}_{au} - \Gamma_A^H} = \left(\frac{P_{au}\tilde{\gamma}_{ua}^H - P_{aa}\tilde{\gamma}_{aa}^H}{P_{aa}\tilde{P}_{ua}\Gamma_u^H}\right) \frac{\Delta V}{\Delta \Gamma_A} \frac{\Gamma_u^H}{\tilde{P}_{au} - \Gamma_A^H}$$

$$= \frac{\Delta V}{\Delta \Gamma_A} \frac{P_{au}\tilde{\gamma}_{ua}^H - P_{aa}\tilde{\gamma}_{aa}^H}{P_{aa}\tilde{P}_{ua}\left(\tilde{P}_{au} - \Gamma_A^H\right)}.$$

To see that the deadweight loss in a DPE-DL contract is always lower than that in a BPE contract when the DPE-DL one is optimal, we calculate

$$\frac{\Delta V}{\Delta \Gamma_a}\frac{1}{P_{aa}} > \frac{\Delta V}{\Delta \Gamma_A}\frac{P_{au}\tilde{\gamma}^H_{ua} - P_{aa}\tilde{\gamma}^H_{aa}}{P_{aa}\tilde{P}_{ua}\left(\tilde{P}_{au} - \Gamma^H_A\right)}$$

$$1 > \frac{P_{au}\tilde{\gamma}^H_{ua} - P_{aa}\tilde{\gamma}^H_{aa}}{\tilde{P}_{ua}\left(\tilde{P}_{au} - \Gamma^H_A\right)}$$

$$P_{au}\tilde{\gamma}^H_{ua} - P_{aa}\tilde{\gamma}^H_{aa} - \tilde{P}_{ua}\tilde{P}_{au} + \tilde{P}_{ua}\Gamma^H_A < 0$$

$$P_{au}\tilde{P}_{ua}\Gamma^H_u - P_{aa}\tilde{P}_{aa}\Gamma^H_a - \tilde{P}_{ua}\tilde{P}_{au} + \tilde{P}_{ua}\Gamma^H_A < 0$$

$$P_{au}P_{ua}\underbrace{(\Gamma^H_u - 1)}_{-\Gamma^H_a} - P_{aa}P_{aa}\Gamma^H_a + P_{ua}\Gamma^H_A + b_u(P_{au}\Gamma^H_u - P_{au} + \Gamma^H_a) + b_a(P_{ua} + b_u - P_{aa}\Gamma^H_a) < 0$$

$$P_{ua}\Gamma^H_a\underbrace{(1 - P_{au})}_{P_{aa}} - P_{aa}P_{aa}\Gamma^H_a + b_u(P_{au}(\Gamma^H_u - 1) + \Gamma^H_a) + b_a(\tilde{P}_{ua} - P_{aa}\Gamma^H_a) < 0$$

$$P_{aa}\Gamma^H_a(P_{ua} - P_{aa}) + b_u\Gamma^H_a(1 - P_{au}) + b_a(\tilde{P}_{ua} - P_{aa}\Gamma^H_a) < 0$$

$$P_{aa}\Gamma^H_a(P_{ua} - P_{aa} + b_u) + b_a(\tilde{P}_{ua} - P_{aa}\Gamma^H_a) < 0$$

$$P_{aa}\Gamma^H_a(\tilde{P}_{ua} - P_{aa}) + b_a(\tilde{P}_{ua} - P_{aa}\Gamma^H_a) < 0.$$

Recall that for the DPE-DL contract to be optimal $b_a \in \left[-P_{aa}, -P_{aa}\Gamma^H_a\right]$. Since the above inequality is linear in b_a, but its effect on the LHS is not straightforward, we can check that it holds at the extremes of the interval. At $b_a = -P_{aa}$, we have

$$-P_{aa}\tilde{P}_{ua} + P^2_{aa}\Gamma^H_a + P_{aa}\tilde{P}_{ua}\Gamma^H_a - P^2_{aa}\Gamma^H_a = -P_{aa}\tilde{P}_{ua} + P_{aa}\tilde{P}_{ua}\Gamma^H_a = P_{aa}\tilde{P}_{ua}(\Gamma^H_a - 1) < 0.$$

At $b_a = -P_{aa}\Gamma^H_a$, we have

$$-P_{aa}\tilde{P}_{ua}\Gamma^H_a + P^2_{aa}(\Gamma^H_a)^2 + P_{aa}\tilde{P}_{ua}\Gamma^H_a - P^2_{aa}\Gamma^H_a = P^2_{aa}(\Gamma^H_a)^2 - P^2_{aa}\Gamma^H_a = P^2_{aa}\Gamma^H_a(\Gamma^H_a - 1) < 0.$$

This proves that the DPE-DL contract always features a smaller deadweight loss than the BPE contract. ∎

Appendix D. Guile-Free vs. Good-Faith Contracts

In order to solve for the guile-free Perfect Bayesian Equilibrium contract, rename the (TR^i_P) and (TR^i_A) constraints in (2) as $(TR^i_{P,H})$ and $(TR^i_{A,H})$ for $i = a, u$. We can add to (2) the following constraints

$$w_{aa}\gamma_{aa}^{L} + w_{au}\gamma_{au}^{L} \leq w_{ua}\gamma_{aa}^{L} + w_{uu}\gamma_{au}^{L} \tag{$TR^{a}_{P,L}$}$$

$$w_{ua}\gamma_{ua}^{L} + w_{uu}\gamma_{uu}^{L} \leq w_{aa}\gamma_{ua}^{L} + w_{au}\gamma_{uu}^{L} \tag{$TR^{u}_{P,L}$}$$

$$c_{aa}\tilde{\gamma}_{aa}^{L} + c_{ua}\tilde{\gamma}_{ua}^{L} \geq c_{au}\tilde{\gamma}_{aa}^{L} + c_{uu}\tilde{\gamma}_{ua}^{L} \tag{$TR^{a}_{A,L}$}$$

$$c_{au}\tilde{\gamma}_{au}^{L} + c_{uu}\tilde{\gamma}_{uu}^{L} \geq c_{aa}\tilde{\gamma}_{au}^{L} + c_{ua}\tilde{\gamma}_{uu}^{L}. \tag{$TR^{u}_{A,L}$}$$

These ensure that both the principal and the agent are incentivize to truthfully report also when the agent exerts low effort. First of all, notice that $(TR^{i}_{P,L})$ and $(TR^{i}_{P,H})$ coincide for all $i = a, u$. To see this, notice that they both imply

$$(w_{au} - w_{uu})\frac{P_{au}}{P_{aa}} \leq (w_{ua} - w_{aa}) \leq (w_{au} - w_{uu})\frac{P_{uu}}{P_{ua}}.$$

On the other hand, this is not true for the (TR_{A}) constraints. Among the four, we can show that $(TR^{a}_{A,L})$ implies $(TR^{a}_{A,H})$ while $(TR^{u}_{A,H})$ implies $(TR^{u}_{A,L})$. Recall that, by Assumption 1 and the definition of λ, we have $\Gamma_{a}^{H} > \Gamma_{a}^{L}$ and $\Gamma_{u}^{H} < \Gamma_{u}^{L}$.

Constraint $(TR^{a}_{A,j})$ implies

$$(c_{uu} - c_{ua})\frac{\tilde{\gamma}_{ua}^{j}}{\tilde{\gamma}_{aa}^{j}} \leq (c_{aa} - c_{au}) \quad j = H, L$$

where we have

$$\frac{\tilde{\gamma}_{ua}^{H}}{\tilde{\gamma}_{aa}^{H}} < \frac{\tilde{\gamma}_{ua}^{L}}{\tilde{\gamma}_{aa}^{L}} \quad \text{since} \quad \frac{\Gamma_{u}^{H}}{\Gamma_{a}^{H}} < \frac{\Gamma_{u}^{L}}{\Gamma_{a}^{L}}$$

proving that the $(TR^{a}_{A,L})$ is tighter. Similarly, constraint $(TR^{u}_{A,j})$ implies

$$(c_{aa} - c_{au}) \leq (c_{uu} - c_{ua})\frac{\tilde{\gamma}_{uu}^{j}}{\tilde{\gamma}_{au}^{j}} \quad j = H, L$$

where we have

$$\frac{\tilde{\gamma}_{uu}^{H}}{\tilde{\gamma}_{au}^{H}} < \frac{\tilde{\gamma}_{uu}^{L}}{\tilde{\gamma}_{au}^{L}} \quad \text{since} \quad \frac{\Gamma_{u}^{H}}{\Gamma_{a}^{H}} < \frac{\Gamma_{u}^{L}}{\Gamma_{a}^{L}}$$

proving that the $(TR^{u}_{A,H})$ is tighter. Hence, in a problem with both the $(TR^{u}_{A,H})$ and the $(TR^{u}_{A,L})$, the latter would be slack. To obtain a guile-free PBE, we can therefore solve problem (2) substituting $(TR^{a}_{A,L})$ for $(TR^{a}_{A,H})$. To see that this would not change the key tools we use to solve the model, notice that i) Proposition 1 does not depend on which of the (TR_A^a) constraint we use and ii) Lemma 4 still holds. Hence, solving, for example, the problem for the case of an optimistic worker using the $(TR_{A,L}^a)$ leads to an equivalent of condition (6) and the APE contracts that depend on the $\tilde{\gamma}_{ts}^L$. While this does not affect our results qualitatively, it makes the study of the problem, and the parameter space where the APE contract is set up, needlessly complicated. For example, as discussed in the paper, a good-faith APE contract depends on many conditions, all of which are implied by (6). The equivalent condition for a guile-free APE contract, instead, can be shown to be necessary but not sufficient. That is, an area where the equivalent of the APE contract is set up exists, but it is harder to identify and graphically describe it.