A General Equilibrium Theory of Occupational Choice under Optimistic Expectations

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Abstract

We extend Lucas' (1978) by assuming that a fraction of individuals derive anticipatory utility from entrepreneurship. If these individuals are able to bias their beliefs to inflate the anticipatory benefits they endogenously become optimists. Optimism has six main effects. First, there is a misallocation of talent which lowers output. Second, optimists are more likely to become entrepreneurs than realists. Third, entrepreneurs are more optimistic than workers. Fourth, when the fraction of individuals with anticipatory utility is high, the majority of entrepreneurs are optimists. Fifth, optimism raises the wage which makes workers better off. Sixth, optimism lowers the returns to entrepreneurship.

JEL Codes: D50; H21; J24; L26.

Keywords: General Equilibrium; Entrepreneurship; Anticipatory Utility, Optimism.

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1 Introduction

Seven stylized facts stand out in the literature on entrepreneurship. First, the returns to entrepreneurship are found to be highly variable, more than wages, and more than the returns on public equity–see Borjas and Bronars (1989), Hamilton (2000), and Moskovitz and Vissing-Jorgensen (2002). Second, the average return to entrepreneurship is not significantly higher than average wages or average return to public equity.[1] Third, not only do many individuals enter entrepreneurship but many entrepreneurs persist in running businesses for long periods of time despite low risk-adjusted returns–see Åstebro et al. (2014). Fourth, the majority of entrepreneurs are optimistic about the chances that their firms will succeed–see Cooper et al. (1988), Wu and Knott (2006), Landier and Thesmar (2009), Cassar (2010, 2012), and Hyytinen et al. (2014). Fifth, optimistic individuals are more likely to become entrepreneurs–see Gentry and Hubbard (2000), Hurst and Lusardi (2004), and Cassar and Friedman (2009). Sixth, entrepreneurs are more likely to be optimists than regular wage earners–see Arabsheibani et al. (2000), Fraser and Greene (2006), Puri and Robinson (2007), and Dawson et al. (2014).[2] Seventh, realistic entrepreneurs earn more than optimistic ones–see Dawson et al. (2015).

This paper presents a fully specified general equilibrium model of occupational

[1] Measurement issues make it hard to compare the mean return to entrepreneurship to that of wage work or to public equity. Some empirical evidence shows that enterpreneurs are not deterred by the evidence of unfavorable returns to entrepreneurship. For example, Dunne et al. (1988) show that most businesses fail within a few years. Hamilton (2000) finds that after 10 years in business, median entrepreneurial earnings are 35% less than those on a paid job of the same duration. Moskovitz and Vissing-Jorgensen (2002) find that the returns from entrepreneurship are, on average, not different from the return on a diversified publicly traded portfolio (private equity puzzle) during the 1989–1998 period. In contrast, Kartashova (2014) shows that Moskowitz and Vissing-Jorgensen (2002)'s finding does not extend to the whole 1989–2010 period.

[2] For example, using data from the British Household Panel Survey for the period 1990-1996, Arabsheibani et al. (2000) find that the self-employed are 4.6 times as likely to forecast an improved financial position but experience a deterioration than to forecast a deterioration but experience an improvement. In contrast, for employees the ratio was only 2.9.

choice that is able to explain these seven stylized facts on entrepreneurship. Following Lucas (1978) we model a closed economy with a given workforce which is homogeneous with respect to productivity as an employee. Each member of the workforce is endowed with an entrepreneurial ability θ_0 which varies across individuals and is drawn from the interval $[0, 1]$. Individuals are risk neutral and maximize their expected income by choosing occupations. A firm in this economy is one entrepreneur together with the labor and capital under his control. The technology of the firm is as follows. Output is an increasing function of entrepreneurial ability, labor, and capital. Entrepreneurial ability is complementary to labor and capital. Decreasing returns to scale in labor and capital ensure that the competitive equilibrium exhibits a non-degenerate distribution of firm sizes.

We depart from Lucas (1978) by assuming that a fraction $\lambda \in (0, 1]$ of individuals in the economy derive anticipatory utility from entrepreneurship whereas the remaining fraction $1 - \lambda$ only care about the material payoff of entrepreneurship. The anticipatory utility from entrepreneurship is a nonpecuniary benefit that arises through the anticipation of future profits and of how enjoyable these will be. This nonpecuniary benefit is conceptually similar to the utility from the anticipation of the things one will do if one has the winning lottery ticket.

The assumption that a fraction of individuals in the economy derive nonpecuniary benefits from entrepreneurship is based on the literature on new business formation. Hurst and Pugsley (2011) find that a majority of small business owners within the US report nonpecuniary benefits as one of the primary reasons for starting their businesses.[3] Åstebro et al. (2014) argue convincingly that nonpecuniary benefits from entrepreneurship provide a parsimonious explanation for entry and persistence in entrepreneurship despite its low risk-adjusted returns.

Our model of anticipatory utility follows Brunnermeier and Parker (2005). Indi-

[3]The precise nature of the nonpecuniary benefits from entrepreneurship remains unclear. Besides anticipatory utility, another source of nonpecuniary benefits is an intrinsic taste for the flexibility and autonomy of being an entrepreneur (or a distaste to have rigid hours and a boss). We ignore other nonpecuniary benefits of entrepreneurship but the model could be extended to include them.

viduals who derive anticipatory utility from entrepreneurship choose their expectations of ability so as to maximize the sum of the material and anticipatory payoffs of entrepreneurship. This choice involves a trade-off between optimism, which raises the anticipatory payoff, and realism, which raises the material payoff by promoting efficient input choices. Optimal beliefs balance the anticipatory benefits of optimism with its efficiency cost. We often refer to individuals with anticipatory utility as optimists since they overestimate their entrepreneurial ability and to individuals with standard preferences as realists since they do not distort their beliefs.

The assumption that individuals may distort their beliefs is not new in the literature. Other prominent models of distorted beliefs due to anticipatory utility are Loewenstein (1987), Caplin and Leahy (2001), Koszegi (2006), Brunnermeier et al. (2007), Koszegi (2010), and Bénabou (2013). The framework of Brunnermeier and Parker (2005) implies that the optimistic bias in beliefs is a function of tastes and technology. This allows us to calibrate the model to US manufacturing data and quantify the importance of anticipatory utility driven optimism.

The timing of the model is as follows. At $t = 0$ individuals with anticipatory utility choose their beliefs of entrepreneurial ability for all future periods. At $t = 1$ individuals choose, given their ability expectations, between entrepreneurship and wage-earning. At $t = 1$ an individual with anticipatory utility becomes an entrepreneur if the sum of the material and anticipatory payoffs from entrepreneurship is greater than the wage. At $t = 1$ a realist becomes an entrepreneur if the material payoff from entrepreneurship is greater than the wage. At $t = 2$ entrepreneurs choose, given their ability expectations, how much labor and capital to hire to maximize the perceived material payoff from entrepreneurship. At $t = 2$ entrepreneurs with anticipatory utility realize the anticipatory payoff from entrepreneurship. At $t = 3$ entrepreneurs realize the material payoffs from running their firms.

Section 3 shows that individuals with anticipatory utility endogenously choose to be optimists about their entrepreneurial ability. When the weight of anticipatory utility s is not too high, being optimistic about entrepreneurial ability leads to first-order gains due to increased anticipatory utility from entrepreneurship and to second-order costs in realized profits due to distorted input choices. With a Cobb-Douglas technology with decreasing returns to scale $\eta \in (0, 1)$, the optimistic bias in beliefs is increasing with the weight of anticipatory utility s and decreasing with η. Hence, whether an individual with anticipatory utility becomes an entrepreneur or a worker depends on his entrepreneurial ability θ_0 and on his optimistic bias in beliefs which is a function of tastes and technology.[4]

Section 4 describes the competitive equilibrium assuming a Cobb-Douglas technology and a uniform distribution of talent. The competitive equilibrium is characterized by: (i) a cut-off ability level $\hat{\theta}_R$ such that realists with ability less than $\hat{\theta}_R$ become workers and those with ability greater than $\hat{\theta}_R$ become entrepreneurs, (ii) a cut-off ability level $\hat{\theta}_O$ such that optimists with ability less than $\hat{\theta}_O$ become workers and those with ability greater than $\hat{\theta}_O$ become entrepreneurs, (iii) a market clearing wage that equates labor demand to supply, and (iv) a rental cost of capital that equates capital demand to supply.

In equilibrium the marginal optimistic entrepreneur has lower ability than the marginal realistic entrepreneur, i.e., the cut-off ability level $\hat{\theta}_O$ is less than $\hat{\theta}_R$. This means that in equilibrium there is a misallocation of talent which lowers output. The ablest people do not necessarily select into entrepreneurship: the lowest ability entrepreneurs are less talented at running a firm than the highest ability workers. This is an empirically attractive implication of the model since, in reality, the income distributions of workers and entrepreneurs have overlapping supports.[5] In addition, we find that optimists are more likely to become entrepreneurs than realists and that entrepreneurs are more likely to be optimists than workers. Finally, if the

[4]In our model, like in Lucas' (1978), individuals have a choice between operating a firm or working for a wage. We focus on ability and optimistic biases in beliefs as the main determinants which explain who becomes an entrepreneur and who works as an employee. There are of course many other factors which could influence this choice. For example, risk aversion, the disutility of exerting entrepreneurial effort, and access to funds needed to create a firm.

[5]See, for example, figure 2 in Åstebro et al. (2014).

fraction of individuals with anticipatory utility is high enough, then the majority of entrepreneurs are optimists.

Section 5 provides comparative static results with respect to the fraction of individuals with anticipatory utility λ. We show that an increase in λ makes workers better off since it raises the wage. We show that an increase in λ lowers the fraction of realistic entrepreneurs as well as the fraction of realistic workers but raises the fraction of optimistic workers. We provide conditions under which an increase in λ raises the fraction of entrepreneurs. We also provide conditions under which an increase in λ raises the rental rate of capital. We close this section by discussing comparative static results with respect the weight of anticipatory utility s.

Section 6 calibrates the model to match salient features of US manufacturing data on labor's income share and the capital-output ratio. We find that anticipatory utility driven optimism can lead to a sharp decline in the mean return to entrepreneurship. This is due to three reasons. First, optimism raises input prices. Second, it distorts the input choices of optimistic entrepreneurs. Third, it lowers the fraction of realistic entrepreneurs–those with higher returns–and raises the fraction of optimistic entrepreneurs–those with lower returns. The calibration also shows that even though anticipatory utility driven optimism has a large impact on the mean return to entrepreneurship it has only a modest impact on output, the fraction of workers, and of entrepreneurs.

Our paper contributes to the literature on occupational choice using general equilibrium models. This literature uses the general equilibrium framework to study possible determinants of occupational choice. Lucas (1978) focuses on entrepreneurial ability, Kanbur (1979) on learning of entrepreneurial ability, Kihlstrom and Laffont (1979) on risk aversion, Bewley (1989) on uncertainty, Jovanovic (1994) on entrepreneurial and working abilities, Lazear (2005) on entrepreneurial and specialist abilities, Campanale (2010) and Poschke (2013) on learning of project quality with dynamics.[6]

[6] Chapter 2 of Parker (2009) discusses in detail the main extensions of Lucas' (1978) model.

The closest paper to ours is Lucas (1978). He shows, among other things, that the most talented individuals become entrepreneurs and the less talented ones become workers. This implies that the income distributions of workers and entrepreneurs have non-overlapping supports which clearly contradicts the data. We generalize Lucas (1978) by assuming that (i) a fraction of individuals in the economy derive non-pecuniary benefits from entrepreneurship through the anticipation of future profits and (ii) these individuals are able, at a cost, to bias their beliefs to increase the anticipatory benefits of entrepreneurship. These two assumptions allow us to explain the seven stylized facts on entrepreneurship. We would not be able to do so using only one of these two assumptions. On the one hand, if individuals with nonpecuniary utility from entrepreneurship are unable to bias their beliefs, then the model does not predict that the majority of entrepreneurs are optimists, that optimistic individuals are more likely to become entrepreneurs, that entrepreneurs are more likely to be optimists than regular wage earners, and that realistic entrepreneurs earn more than optimistic ones. On the other hand, if individuals are assumed to be optimists but do not derive nonpecuniary utility from entrepreneurship, then the model does not explain persistency in entrepreneurship despite the low risk-adjusted returns. In such a model optimistic entrepreneurs would eventually learn their abilities and exit entrepreneurship.

More narrowly, our paper contributes to the literature that uses the general equilibrium approach to study the impact of optimism on market outcomes (de Meza and Southey, 1996, Manove, 2000, Fraser and Greene, 2006, and Rigotti et al. 2011). There are two main difference between these papers and ours. First, in these studies optimism is exogenous whereas in our case it arises endogenously and is a function of tastes–the weight of anticipatory utility–and technology–the extent of decreasing returns to scale. Second, we make quantitative predictions about the impact of optimism on occupational choice as well as on labor, capital, and output markets.

The reminder of the paper proceeds as follows. Section 2 sets up the model. Section 3 derives the expectations of ability under a Cobb-Douglas technology. Section 4 characterizes the competitive equilibrium under a Cobb-Douglas technology and a uniform distribution of ability. Section 5 contains the comparative statics results. The model is calibrated to fit US manufacturing data in Section 6. Section 7 concludes the paper. All proofs can be found in the Appendix.

2 Set-up

The economy consists of a continuum of risk-neutral individuals of measure 1. They derive utility from consumption, and can earn income either as workers or by running their own firm. Individuals are ranked by their entrepreneurial ability, θ_0, which is distributed on $[0, 1]$ according to the cumulative distribution function $G(\theta_0)$. Each individual has one unit of labor. If an individual becomes a worker he supplies his unit of labor on the labor market and receives the competitive wage w. Thus, we assume all individuals have the same productivity (or ability) as workers. If an individual becomes an entrepreneur he can use without cost a technology defined by the continuous production function

$$y = f(l, k, \theta_0),$$

where y is output, l is labor, and k is capital. Any individual can run at most one firm. We assume that f is twice continuously differentiable with $f_l > 0$, $f_k > 0$, $f_{\theta_0} > 0$, $f_{ll} < 0$, $f_{kk} < 0$, $f_{l\theta_0} > 0$, $f_{k\theta_0} > 0$, and $f(0, k, \theta_0) = f(l, 0, \theta_0) = 0$. This production function combines as inputs one entrepreneur, who is essential to operate the firm, with a labor input of l units and a capital input of k units. The stock of capital in the economy is fixed and equal to $\bar{K}$. Entrepreneurs rent capital in the capital market at the competitive rental cost of capital r.

Production exhibits decreasing returns to scale in the variable inputs, labor and capital, so that the competitive equilibrium exhibits a non-degenerate firm size distribution.[7] The assumption that entrepreneurial ability and labor are complements

[7]This assumption implies that the size of firms is finite. This could be due for instance to limits

in production, i.e. $f_{l\theta_0} > 0$, is a critical one. This assumption implies that an optimistic entrepreneur will demand more labor than a realist with the same ability. If entrepreneurial ability and labor were substitutes in production, i.e. $f_{l\theta_0} < 0$, the opposite result would hold, namely, an optimistic entrepreneur would demand less labor than a realist with the same ability. The assumption that entrepreneurial ability and capital are complements in production, i.e. $f_{k\theta_0} > 0$, is also critical.

If an individual becomes an entrepreneur and employs l workers he receives a material payoff equal to

$$\pi = pf(l, k, \theta_0) - wl - rk.$$

From now on the price of output p is normalized to be 1. Individuals can belong to one of two types: those with anticipatory utility and those who have standard preferences. Fraction $\lambda \in (0, 1)$ of the population has anticipatory utility and fraction $1 - \lambda$ has standard preferences. The distributions of entrepreneurial abilities and types are independent.

At $t = 0$ an individual with anticipatory utility observes θ_0 and chooses his expectation of entrepreneurial ability θ so as to maximize the undiscounted sum of $f(l, k, \theta_0) - wl - rk$, his material payoff of being an entrepreneur at $t = 3$; and $s[f(l, k, \theta) - wl - rk]$, his anticipatory payoff of being an entrepreneur at $t = 2$. At $t = 1$ the individual, given his expectation of entrepreneurial ability θ, decides whether to be an entrepreneur or a worker and receive the market wage w. The individual becomes an entrepreneur if the sum of the material and anticipatory payoffs of being an entrepreneur is higher than w. At $t = 2$ an entrepreneur chooses l and k to maximize his material payoff given his expectation of entrepreneurial ability θ. At $t = 2$ the entrepreneur receives anticipatory utility from his expectation of material payoffs evaluated with belief θ: $s[f(l, k, \theta) - wl - rk]$. At $t = 3$ an entrepreneur realizes the material payoff $f(l, k, \theta_0) - wl - rk$.

According to this approach the total payoff at $t = 0$ of an individual with anti-

in entrepreneurs' span of control (Lucas 1978): as activity expands, it becomes more difficult to control, and the marginal product of the variable inputs diminishes.

cipatory utility who selects to be an entrepreneur is

$$f(l,k,\theta_0) - wl - rk + s[f(l,k,\theta) - wl - rk],$$

where the parameter $s > 0$ measures the weight the individual places on anticipatory utility relative to material payoffs. Note that the material payoff component depends on the individual's actual entrepreneurial ability θ_0, while the anticipatory utility depends on the individual's expectation of entrepreneurial ability θ.

An individual who becomes an entrepreneur will choose to employ $l(w,r,\theta)$ workers and $k(w,r,\theta)$ units of capital at $t = 2$ where $l(w,r,\theta)$ and $k(w,r,\theta)$ are the values of l and k that solve the following problem

$$\max_{l,k}[f(l,k,\theta) - wl - rk].$$

The first-order conditions to this problem are

$$f_l(l,k,\theta) = w. \tag{1}$$

and

$$f_k(l,k,\theta) = r. \tag{2}$$

It follows from (1), the assumption of decreasing returns to labor, $f_{ll} < 0$, and complementarity between entrepreneurial ability and labor, i.e., $f_{l\theta_0} > 0$, that entrepreneurs with a higher θ hire more workers: $\partial l(w,r,\theta)/\partial\theta = -f_{l\theta}/f_{ll} > 0$. Similarly, it follows from (2), the assumption of decreasing returns to capital, $f_{kk} < 0$, and complementarity between entrepreneurial ability and capital, i.e., $f_{k\theta_0} > 0$, that entrepreneurs with a higher θ hire more capital: $\partial k(w,r,\theta)/\partial\theta = -f_{k\theta}/f_{kk} > 0$. At $t = 1$ the expectation of ability of an individual with ability θ_0 is the θ that solves the following problem

$$\begin{aligned}\max_{\theta\in[0,1]} \{&f(l(w,r,\theta),k(w,r,\theta),\theta_0) - wl(w,r,\theta) - rk(w,r,\theta)\\ &+s\left[f(l(w,r,\theta),k(w,r,\theta),\theta) - wl(w,r,\theta) - rk(w,r,\theta)\right]\}.\end{aligned}$$

If the wage is w, a realistic individual with entrepreneurial ability θ_0 chooses to become a worker at wage w when

$$f(l(w,r,\theta_0), k(w,r,\theta_0), \theta_0) - wl(w,r,\theta_0) - rk(w,r,\theta_0) \leq w. \tag{3}$$

He selects to be an entrepreneur if

$$f(l(w,r,\theta_0), k(w,r,\theta_0), \theta_0) - wl(w,r,\theta_0) - rk(w,r,\theta_0) \geq w, \tag{4}$$

and he is indifferent if the equality holds in (3) and (4). If the wage is w, an individual with expectation of ability θ^* and entrepreneurial ability θ_0 chooses to become a worker at wage w when

$$\begin{aligned} & f(l(w,r,\theta^*), k(w,r,\theta^*), \theta_0) - wl(w,r,\theta^*) - rk(w,r,\theta^*) \\ & + s\left[f(l(w,r,\theta^*), k(w,r,\theta^*), \theta^*) - wl(w,r,\theta^*) - rk(w,r,\theta^*)\right] \leq w. \end{aligned} \tag{5}$$

He selects to be an entrepreneur if

$$\begin{aligned} & f(l(w,r,\theta^*), k(w,r,\theta^*), \theta_0) - wl(w,r,\theta^*) - rk(w,r,\theta^*), \\ & + s\left[f(l(w,r,\theta^*), k(w,r,\theta^*), \theta^*) - wl(w,r,\theta^*) - rk(w,r,\theta^*)\right] \geq w \end{aligned} \tag{6}$$

and he is indifferent if the equality holds in (5) and (6). Since there are only three markets–output, labor, and capital–by Walras' Law, general equilibrium is realized when the labor and capital markets clear. At the equilibrium wage, the labor demanded by individuals who choose to become entrepreneurs equals that supplied by individuals who choose to become workers. At the equilibrium rental cost of capital, the capital demanded by individuals who choose to become entrepreneurs equals the exogenous capital stock of the economy, $\bar{K}$. Formally, an equilibrium is (i) a partition $\{[0,\hat{\theta}_R],[\hat{\theta}_R,1]\}$ of $[0,1]$ where for all $\theta_0 \in [0,\hat{\theta}_R]$ (3) holds and for all $\theta_0 \in [\hat{\theta}_R,1]$ (4) holds, (ii) a partition $\{[0,\hat{\theta}_O],[\hat{\theta}_O,1]\}$ of $[0,1]$ where for all $\theta_0 \in [0,\hat{\theta}_O]$ (5) holds and for all $\theta_0 \in [\hat{\theta}_O,1]$ (6) holds, (iii) a wage w for which labor demand equals labor supply

$$(1-\lambda)\int_{\hat{\theta}_R}^{1} l(w,r,\theta_0)dG(\theta_0)+\lambda\int_{\hat{\theta}_O}^{1} l(w,r,\theta^*)dG(\theta_0)$$
$$=(1-\lambda)\int_{0}^{\hat{\theta}_R} dG(\theta_0)+\lambda\int_{0}^{\hat{\theta}_O} dG(\theta_0), \tag{7}$$

and (iv) a rental cost of capital r for which capital demand equals the exogenous capital supply

$$(1-\lambda)\int_{\hat{\theta}_R}^{1} k(w,r,\theta_0)dG(\theta_0)+\lambda\int_{\hat{\theta}_O}^{1} k(w,r,\theta^*)dG(\theta_0)=\bar{K}. \tag{8}$$

In equilibrium, realists with ability below $\hat{\theta}_R$ become workers whereas those with ability above $\hat{\theta}_R$ become entrepreneurs. Similarly, individuals with anticipatory utility and ability below $\hat{\theta}_O$ become workers whereas those with ability above $\hat{\theta}_O$ become entrepreneurs. We refer to a realist with ability $\hat{\theta}_R$ as the *marginal realistic entrepreneur*. We refer to an individual with anticipatory utility and ability $\hat{\theta}_O$ as the *marginal optimistic entrepreneur.*

3 Optimistic Expectations

In this section we derive the expectations of ability of individuals with anticipatory utility. We consider a specialized version of the model with a production function given by

$$y=f(l,k,\theta_0)=\theta_0 g(l,k)=\theta_0 l^{\alpha}k^{\beta},$$

where $\alpha+\beta\equiv\eta\in(0,1)$. Hence, the variable inputs, labor and capital, are combined under a Cobb-Douglas production function with decreasing returns to scale and entrepreneurial skill enters into the production function as the total factor productivity (TFP).[8]

[8]This is a standard assumption in models with heterogeneous skill. See, for example, Lucas (1978), Murphy et al. (1991), de Meza and Southey (1996), and Poschke (2013).

At $t = 3$ the material payoff of an entrepreneur is

$$\pi = \theta_0 l^\alpha k^\beta - wl - rk. \tag{9}$$

We see from (9) that this production function, the assumption that individuals are risk neutral, and the assumption that entrepreneurial skill θ_0 belongs to $[0, 1]$, imply that entrepreneurial skill can be interpreted as the true probability of success of the firm (the project either succeeds with probability θ_0 or fails with probability $1 - \theta_0$, in which case output is zero).

At $t = 2$ an individual with expectations of ability θ who becomes an entrepreneur chooses to employ l workers and k units of capital where l and k are the solution to

$$\max_{l,k}(\theta l^\alpha k^\beta - wl - rk).$$

The first-order conditions are

$$\alpha\theta l^{\alpha-1} k^\beta = w,$$

and

$$\beta\theta l^\alpha k^{\beta-1} = r.$$

Solving for l and k we obtain the input demands of an entrepreneur with expectations of ability θ:

$$l(w, r, \theta) = \theta^{\frac{1}{1-\alpha-\beta}} \left(\frac{\alpha}{w}\right)^{\frac{1-\beta}{1-\alpha-\beta}} \left(\frac{\beta}{r}\right)^{\frac{\beta}{1-\alpha-\beta}}, \tag{10}$$

and

$$k(w, r, \theta) = \theta^{\frac{1}{1-\alpha-\beta}} \left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\alpha-\beta}} \left(\frac{\beta}{r}\right)^{\frac{1-\alpha}{1-\alpha-\beta}}, \tag{11}$$

respectively.

At $t = 0$ the total payoff of an entrepreneur with expectations of ability θ is:

$$\theta_0 l^\alpha k^\beta - wl - rk + s(\theta l^\alpha k^\beta - wl - rk) = (\theta_0 + s\theta) l^\alpha k^\beta - (1 + s)(wl + rk).$$

where l is given by (10) and k by (11). Hence, at $t = 0$, the expectations of ability (or the expectation of the probability of success of the firm) of an entrepreneur with ability θ_0 is the θ that solves

$$\max_{\theta\in[0,1]} \left\{(\theta_0 + s\theta)[l(w,r,\theta)]^\alpha[k(w,r,\theta)]^\beta - (1+s)[wl(w,r,\theta) + rk(w,r,\theta)]\right\}. \quad (12)$$

Our first result characterizes the solution to (12).

Proposition 1: *If $f(l,k,\theta_0) = \theta_0 l^\alpha k^\beta$ and the weight of anticipatory utility s is less than $\eta/(1-\eta)$, then expectations of entrepreneurial ability are given by*

$$\theta^* = \begin{cases} \frac{\eta}{\eta - s(1-\eta)}\theta_0 & \text{if } \theta_0 < \frac{\eta - s(1-\eta)}{\eta} \\ 1 & \text{if } \theta_0 \geq \frac{\eta - s(1-\eta)}{\eta} \end{cases}. \quad (13)$$

This results tells us if the weight of anticipatory utility s is less than $\eta/(1-\eta)$, then individuals with anticipatory utility choose to be optimists about their entrepreneurial ability since the belief of entrepreneurial ability θ^* is greater than the actual ability θ_0. The intuition behind this result is straightforward. Being optimist about entrepreneurial ability leads to first-order gains due to increased anticipatory utility from entrepreneurship and to second-order costs in realized profits due to distorted input choices.

We see from (13) that the expectation of ability θ^* of individuals with ability θ_0 below $1 - s(1-\eta)/\eta$ is a function of ability θ_0, the weight of anticipatory utility s, and the degree of decreasing returns to scale η. We also see from (13) that the expectation of ability θ^* of individuals with ability θ_0 above $1 - s(1-\eta)/\eta$ is equal to the highest possible ability level, i.e., $\theta^* = 1$. Hence, for individuals with ability above $1 - s(1-\eta)/\eta$ the expectation of ability does not depend on θ_0, s, and η.

Let the optimistic bias in beliefs of an individual with ability θ_0 be the gap between θ^* and θ_0, i.e., $b^* = \theta^* - \theta_0$. From (13) we have

$$b^* = \begin{cases} \frac{s(1-\eta)}{\eta - s(1-\eta)}\theta_0 & \text{if } \theta_0 < \frac{\eta - s(1-\eta)}{\eta} \\ 1 - \theta_0 & \text{if } \theta_0 \geq \frac{\eta - s(1-\eta)}{\eta} \end{cases}.$$

The optimistic bias in beliefs of individuals with ability θ_0 below $1 - s(1-\eta)/\eta$ is increasing with the weight of anticipatory utility s. Everything else constant, the higher s is, the more important the anticipated payoff of entrepreneurship, $\theta l^\alpha k^\beta - wl - rk$, becomes relative to the material payoff of entrepreneurship, $\theta_0 l^\alpha k^\beta - wl - rk$, and so the greater are the benefits from holding optimistic beliefs.

The optimistic bias in beliefs of individuals with ability θ_0 below $1 - s(1-\eta)/\eta$ is decreasing with a decrease in the degree of decreasing returns to scale (an increase in η). The intuition behind this result is as follows. Everything else constant, the higher η is, the smaller is the material payoff of entrepreneurship. Similarly, everything else constant, the higher η is, the smaller is the anticipated payoff of entrepreneurship.[9] However, as η increases the decrease in the anticipated payoff of entrepreneurship is steeper than the decrease in the material payoff of entrepreneurship. Therefore, the higher η is, the smaller are the benefits from holding optimistic beliefs.

4 Competitive Equilibrium

In this section we determine the competitive equilibrium when the weight of anticipatory utility s is low. We show that the lowest ability entrepreneurs are less talented at running a firm than the highest ability workers. We also show how the competitive equilibrium can be determined when the weight of anticipatory utility s is high.

A realist with ability $\hat{\theta}_R$ is indifferent between being an entrepreneur and a worker when

$$\hat{\theta}_R[l(w,r,\hat{\theta}_R)]^\alpha[k(w,r,\hat{\theta}_R)]^\beta - wl(w,r,\hat{\theta}_R) - rk(w,r,\hat{\theta}_R) = w$$

Simplifying this equation we obtain

$$\alpha^\alpha \beta^\beta (1-\eta)^{1-\eta}\hat{\theta}_R = w^{1-\beta} r^\beta. \tag{14}$$

When the weight of anticipatory utility is low (this will be made precise further on), an optimist with an expectation of ability $\theta^* = \eta\hat{\theta}_O/[\eta - s(1-\eta)]$ and ability $\hat{\theta}_O$ is

[9] As η converges to 1 the technology converges to constant returns to scale and the material and anticipated payoffs of entrepreneurship converge to zero.

indifferent between being an entrepreneur and a worker when

$$(\hat{\theta}_O + s\theta^*)[l(w,r,\theta^*)]^\alpha[k(w,r,\theta^*)]^\beta - (1+s)\left[wl(w,r,\theta^*) + rk(w,r,\theta^*)\right] = w.$$

Simplifying this equation we obtain

$$\alpha^\alpha\beta^\beta(1-\eta)^{1-\eta}\left[\frac{\eta}{\eta - s(1-\eta)}\right]^\eta \hat{\theta}_O = w^{1-\beta}r^\beta. \tag{15}$$

In equilibrium, labor demand must equal labor supply. The assumption that entrepreneurial ability is uniformly distributed on $[0,1]$ implies that (7) becomes:

$$(1-\lambda)\int_{\hat{\theta}_R}^1 l(w,r,\theta_0)d\theta_0 + \lambda\int_{\hat{\theta}_O}^1 l[w,r,\theta^*(\theta_0)]d\theta_0 = (1-\lambda)\hat{\theta}_R + \lambda\hat{\theta}_O.$$

After substituting for the labor demands of the two types of entrepreneurs and integrating over θ_0 we obtain

$$\begin{aligned}&\frac{1-\eta}{2-\eta}\left\{1 + \lambda\frac{s}{\eta} - (1-\lambda)\hat{\theta}_R^{\frac{2-\eta}{1-\eta}} - \lambda\left[\frac{\eta}{\eta - s(1-\eta)}\right]^{\frac{1}{1-\eta}}\hat{\theta}_O^{\frac{2-\eta}{1-\eta}}\right\}\left(\frac{\alpha}{w}\right)^{\frac{1-\beta}{1-\eta}}\left(\frac{\beta}{r}\right)^{\frac{\beta}{1-\eta}} \\ = \;& (1-\lambda)\hat{\theta}_R + \lambda\hat{\theta}_O.\end{aligned} \tag{16}$$

In equilibrium, capital demand must equal the exogenous capital supply. The assumption that entrepreneurial ability is uniformly distributed on $[0,1]$ implies that (8) becomes:

$$(1-\lambda)\int_{\hat{\theta}_R}^1 k(w,r,\theta_0)d\theta_0 + \lambda\int_{\hat{\theta}_O}^1 k[w,r,\theta^*(\theta_0)]d\theta_0 = \bar{K}.$$

After substituting for the capital demands of the two types of entrepreneurs and integrating over θ_0 we obtain

$$\begin{aligned}&\frac{1-\eta}{2-\eta}\left\{1 + \lambda\frac{s}{\eta} - (1-\lambda)\hat{\theta}_R^{\frac{2-\eta}{1-\eta}} - \lambda\left[\frac{\eta}{\eta - s(1-\eta)}\right]^{\frac{1}{1-\eta}}\hat{\theta}_O^{\frac{2-\eta}{1-\eta}}\right\}\left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\eta}}\left(\frac{\beta}{r}\right)^{\frac{1-\alpha}{1-\eta}} \\ = \;& \bar{K}\end{aligned} \tag{17}$$

Equations (14), (15), (16), and (17) form a system of four equations and four unknowns ($\hat{\theta}_R$, $\hat{\theta}_O$, w, and r) which defines the competitive equilibrium when the weight of anticipatory utility s is low.[10] Our second result describes this equilibrium.

Proposition 2: *If* $f(l,k,\theta_0) = \theta_0 l^\alpha k^\beta$, θ_0 *is uniformly distributed on* $[0,1]$, *and* $s < \bar{s}$, *then there exists a unique competitive equilibrium where the marginal realistic entrepreneur has ability*

$$\hat{\theta}_R = \left[\frac{\alpha}{2-\beta}\frac{1+\lambda\frac{s}{\eta}}{1-\lambda+\lambda\phi(\eta,\beta,s)}\right]^{\frac{1-\eta}{2-\eta}}, \tag{18}$$

the marginal optimistic entrepreneur has ability

$$\hat{\theta}_O = \psi(\eta,s)\left[\frac{\alpha}{2-\beta}\frac{1+\lambda\frac{s}{\eta}}{1-\lambda+\lambda\phi(\eta,\beta,s)}\right]^{\frac{1-\eta}{2-\eta}}, \tag{19}$$

the wage is

$$w^* = \frac{\alpha^\eta(1-\eta)^{1-\eta}\bar{K}^\beta}{[1-\lambda+\lambda\psi(\eta,s)]^\beta}\left[\frac{\alpha}{2-\beta}\frac{1+\lambda\frac{s}{\eta}}{1-\lambda+\lambda\phi(\eta,\beta,s)}\right]^{\frac{(1-\eta)(1-\beta)}{2-\eta}}, \tag{20}$$

the fraction of workers is

$$L^* = [1-\lambda+\lambda\psi(\eta,s)]\left[\frac{\alpha}{2-\beta}\frac{1+\lambda\frac{s}{\eta}}{1-\lambda+\lambda\phi(\eta,\beta,s)}\right]^{\frac{1-\eta}{2-\eta}}, \tag{21}$$

and the rental cost of capital is

$$r^* = \frac{\beta(1-\eta)^{1-\eta}}{\alpha^{1-\eta}\bar{K}^{1-\beta}}[1-\lambda+\lambda\psi(\eta,s)]^{1-\beta}\left[\frac{\alpha}{2-\beta}\frac{1+\lambda\frac{s}{\eta}}{1-\lambda+\lambda\phi(\eta,\beta,s)}\right]^{\frac{(1-\eta)(2-\beta)}{2-\eta}}, \tag{22}$$

where $\bar{s}$ *is the solution to*

$$\frac{\alpha}{2-\beta}\left(1+\lambda\frac{\bar{s}}{\eta}\right) = [1-\lambda+\lambda\phi(\eta,\beta,\bar{s})]\left[\frac{\eta-\bar{s}(1-\eta)}{\eta}\right]^{2-\eta}, \tag{23}$$

and

$$\phi(\eta,\beta,s) = \left[1-\frac{s(1-\eta)(2-\eta)}{\eta(2-\beta)}\right]\left[\frac{\eta}{\eta-s(1-\eta)}\right]^{1-\eta},$$

[10] Note that from $\hat{\theta}_R$ and $\hat{\theta}_O$ we obtain the equilibrium number of workers $L = (1-\lambda)\hat{\theta}_R + \lambda\hat{\theta}_O$.

and

$$\psi(\eta, s) = \left[\frac{\eta - s(1-\eta)}{\eta}\right]^{\eta}.$$

Proposition 2 shows us that the existence of individuals with anticipatory utility leads to a misallocation of talent. In a competitive equilibrium where $\lambda = 0$ the marginal entrepreneur has ability

$$\hat{\theta}_0 = \left(\frac{\alpha}{2-\beta}\right)^{\frac{1-\eta}{2-\eta}}, \tag{24}$$

which implies that individuals with ability $[0, \hat{\theta}_0]$ become workers and individuals with ability $[\hat{\theta}_0, 1]$ become entrepreneurs. Hence, in the standard competitive equilibrium the ablest people become entrepreneurs. In contrast, in the competitive equilibrium of Proposition 2, realists with ability $[0, \hat{\theta}_R]$ become workers and those with ability $[\hat{\theta}_R, 1]$ become entrepreneurs.[11] Furthermore, optimists with ability $[0, \hat{\theta}_O]$ become workers and those with ability $[\hat{\theta}_O, 1]$ become entrepreneurs. It follows from (18) and (19) that the marginal optimistic entrepreneur has a lower ability than the marginal realistic entrepreneur:

$$\hat{\theta}_O < \hat{\theta}_R. \tag{25}$$

Hence, amongst individuals with ability $\theta_0 \in [\hat{\theta}_O, \hat{\theta}_R]$ those who are realists become workers and those who are optimists become entrepreneurs. Therefore, in the competitive equilibrium of Proposition 2, the ablest people do not necessarily become entrepreneurs. Moreover, the lowest ability entrepreneurs are less talented at running a firm than the highest ability workers. This is an empirically attractive implication of the model since, in reality, the income distributions of workers and entrepreneurs have overlapping supports.[12]

[11] The proof of Proposition 2 shows that $\hat{\theta}_R \in (0, 1)$.

[12] This stands in contrast to models where occupational choice is only based on heterogeneity in ability and where it is assumed that one occupation rewards ability more than the other. This results in income distributions for the two occupations with non-overlapping intervals (see, e.g., Parker, 2009).

When the weight of anticipatory utility s is high, i.e., $s \in [\bar{s}, \eta/(1-\eta)]$, individuals with anticipatory utility who select to become entrepreneurs hold the highest possible belief of ability, i.e., $\theta^* = 1$. In addition, a positive mass of individuals with $\theta^* = 1$ choose to be workers since their entrepreneurial ability θ_0 is not high enough to make entrepreneurship more attractive than working as an employee. Proposition 3 characterizes the competitive equilibrium when $s \in [\bar{s}, \eta/(1-\eta)]$.

Proposition 3: *If* $f(l, k, \theta_0) = \theta_0 l^\alpha k^\beta$, θ_0 *is uniformly distributed on* $[0, 1]$, *and* $s \in [\bar{s}, \eta/(1-\eta)]$, *then the competitive equilibrium is the unique solution to the system of four equations and four unknowns (*$\hat{\theta}_R$, $\hat{\theta}_O$, w, *and* r*):*

$$\alpha^\alpha \beta^\beta (1-\eta)^{1-\eta} \hat{\theta}_R = w^{1-\beta} r^\beta,$$

$$\alpha^\alpha \beta^\beta \left[\hat{\theta}_O + s - (1+s)\eta \right]^{1-\eta} = w^{1-\beta} r^\beta,$$

$$\left[(1-\lambda) \frac{1-\eta}{2-\eta} \left(1 - \hat{\theta}_R^{\frac{2-\eta}{1-\eta}} \right) + \lambda (1 - \hat{\theta}_O) \right] \left(\frac{\alpha}{w} \right)^{\frac{1-\beta}{1-\eta}} \left(\frac{\beta}{r} \right)^{\frac{\beta}{1-\eta}} = (1-\lambda)\hat{\theta}_R + \lambda \hat{\theta}_O,$$

$$\left[(1-\lambda) \frac{1-\eta}{2-\eta} \left(1 - \hat{\theta}_R^{\frac{2-\eta}{1-\eta}} \right) + \lambda (1 - \hat{\theta}_O) \right] \left(\frac{\alpha}{w} \right)^{\frac{\alpha}{1-\eta}} \left(\frac{\beta}{r} \right)^{\frac{1-\alpha}{1-\eta}} = \bar{K}.$$

As we have seen, when the weight of anticipatory utility is low, i.e., s is less than $\bar{s}$ given by (23), the equilibrium wage, fraction of workers, and rental cost of capital are given by (20), (21), and (22), respectively. In contrast, when $s \in [\bar{s}, \eta/(1-\eta)]$ we are unable to obtain closed form solutions for w^*, L^*, and r^*. Hence, the rest of the paper focuses on the case where s is low.

We now show that the model predicts that optimists are more likely to become entrepreneurs than realists, that entrepreneurs are more likely to be optimists than workers, and, that, provided the fraction of optimists is high enough, the majority of entrepreneurs are optimists. Before doing this we need to define the probability an entrepreneur is an optimist and the probability a worker is an optimist. The fraction of entrepreneurs is equal to

$$E^* = E_O^* + E_R^* = \lambda(1 - \hat{\theta}_O) + (1-\lambda)(1 - \hat{\theta}_R).$$

Hence, the probability an entrepreneur is an optimist is given by

$$\Pr(O|E^*) = \frac{E_O^*}{E^*} = \frac{\lambda(1-\hat{\theta}_O)}{\lambda(1-\hat{\theta}_O) + (1-\lambda)(1-\hat{\theta}_R)}. \tag{26}$$

The fraction of workers is equal to

$$L^* = L_O^* + L_R^* = \lambda\hat{\theta}_O + (1-\lambda)\hat{\theta}_R.$$

Hence, the probability a worker is an optimist is given by

$$\Pr(O|L^*) = \frac{L_O^*}{L^*} = \frac{\lambda\hat{\theta}_O}{\lambda\hat{\theta}_O + (1-\lambda)\hat{\theta}_R}. \tag{27}$$

Proposition 4: *Assume* $f(l,k,\theta_0) = \theta_0 l^\alpha k^\beta$, θ_0 *is uniformly distributed on* $[0,1]$, *and* $s < \bar{s}$.
(i) The probability an optimist becomes an entrepreneur is greater than the probability a realist becomes an entrepreneur, i.e., $\Pr(E^*|O) > \Pr(E^*|R)$*;*
(ii) The probability an entrepreneur is an optimist is greater than the probability a worker is an optimist, i.e., $\Pr(O|E^*) > \Pr(O|L^*)$*;*
(iii) If

$$\lambda > \left[1 + \frac{1-\hat{\theta}_R\left[\frac{\eta - s(1-\eta)}{\eta}\right]^\eta}{1-\hat{\theta}_R}\right]^{-1}, \tag{28}$$

where $\hat{\theta}_R$ *is given by (18), then the majority of entrepreneurs are optimists, i.e.,* $\Pr(O|E^*) > 1/2$.

Proposition 4-(i) tells us that optimists are more likely to become entrepreneurs than realists. This result follows directly from (25) and is in line with the empirical evidence in Gentry and Hubbard (2000), Hurst and Lusardi (2004), and Cassar and Friedman (2009). Proposition 4-(ii) tells us that entrepreneurs are more likely to be optimists than workers. This result follows from (25), (26), and (27). This is in line with the empirical evidence in Arabsheibani et al. (2000), Fraser and Greene (2006), Puri and Robinson (2007), and Dawson et al. (2014).

According to (26) the majority of entrepreneurs can be either optimists or realists. However, the empirical evidence in Cooper et al. (1988), Wu and Knott (2006), Landier and Thesmar (2009), Cassar (2010, 2012), and Hyytinen et al. (2014) shows that entrepreneurs are overwhelmingly optimists. Proposition 4-(iii) shows if the fraction of individuals with anticipatory utility λ is higher than the lower bound in (28), then the majority of entrepreneurs are optimists.

5 Comparative Statics

In this section we perform comparative statics on equilibrium outcomes. There are two main parameters which can be used to perform this analysis: the fraction of individuals with anticipatory utility λ and the weight of anticipatory utility s. We know from Proposition 1 that all individuals with anticipatory utility endogenously become optimistic about their entrepreneurial abilities. Therefore, by changing λ while keeping everything else fixed we can analyze the impact that a change in the fraction of optimists has on equilibrium outcomes. We also know from Proposition 1 that the higher s is, the greater are optimistic biases in beliefs of individuals with ability below $1 - s(1 - \eta)/\eta$. Therefore, by changing s while keeping everything else fixed we can analyze the impact that a change in optimistic biases in beliefs have on equilibrium outcomes. We focus on comparative statics with respect to the fraction of individuals with anticipatory utility. At the end we discuss briefly the comparative statics with respect to the weight of anticipatory utility.

We start by showing that an increase in the fraction of individuals with anticipatory utility raises the ability of the marginal realistic entrepreneur $\hat{\theta}_R$ and the ability of the marginal optimistic entrepreneur $\hat{\theta}_O$.

Proposition 5: *If $f(l, k, \theta_0) = \theta_0 l^\alpha k^\beta$, θ_0 is uniformly distributed on $[0, 1]$, and $s < \bar{s}$, then an increase in the fraction of individuals with anticipatory utility raises the ability of the marginal realistic entrepreneur and the ability of the marginal optimistic entrepreneur, i.e., $\partial\hat{\theta}_R/\partial\lambda > 0$ and $\partial\hat{\theta}_O/\partial\lambda > 0$, respectively.*

It follows from $\partial\hat{\theta}_R/\partial\lambda > 0$ that the ability of the marginal realistic entrepreneur $\hat{\theta}_R$ is higher than the ability of the marginal entrepreneur in the competitive equilibrium with $\lambda = 0$, i.e., $\hat{\theta}_R > \hat{\theta}_0$. It follows from $\partial\hat{\theta}_O/\partial\lambda > 0$, (19), and (24) that if λ is low, then the ability of the marginal optimistic entrepreneur $\hat{\theta}_O$ is lower than the ability of the marginal entrepreneur in the competitive equilibrium with $\lambda = 0$, i.e., $\hat{\theta}_O < \hat{\theta}_0$. Hence, when λ is low we have

$$\hat{\theta}_O < \hat{\theta}_0 < \hat{\theta}_R,$$

and when λ is high we have

$$\hat{\theta}_0 < \hat{\theta}_O < \hat{\theta}_R.$$

Thus, the misallocation of talent depends on the fraction of individuals with anticipatory utility λ. A realist with ability $\theta_0 \in [\hat{\theta}_0, \hat{\theta}_R]$ chooses to become a worker but would select to be an entrepreneur in the competitive equilibrium where $\lambda = 0$. When λ is low, an optimist with ability $\theta_0 \in [\hat{\theta}_O, \hat{\theta}_0]$ selects to be an entrepreneur but would choose to become a worker in the competitive equilibrium where $\lambda = 0$. When λ is high, an optimist with ability $\theta_0 \in [\hat{\theta}_0, \hat{\theta}_O]$ chooses to become a worker but would select to be an entrepreneur in the competitive equilibrium where $\lambda = 0$.

We now show that an increase in the fraction of individuals with anticipatory utility raises the market clearing wage.

Proposition 6: *If $f(l, k, \theta_0) = \theta_0 l^\alpha k^\beta$, θ_0 is uniformly distributed on $[0, 1]$, and $s < \bar{s}$, then an increase in the fraction of individuals with anticipatory utility leads to an increase in the wage, i.e., $\partial w^*/\partial\lambda > 0$.*

The intuition behind Proposition 6 is as follows. The assumption that entrepreneurial ability and labor are complements and the fact that individuals with anticipatory utility become optimists implies that, for a given wage, the demand for labor of an individual with anticipatory utility is higher than the demand for labor of a realist of the same ability. This leads to an expansion of labor demand. An individual who derives anticipatory utility from entrepreneurship is optimist about his entrepreneurial ability and is, for any given wage, more attracted to entrepreneurship than a realist of the same ability. This leads to a contraction of labor supply. The expansion of labor demand and contraction of labor supply raise the market clearing wage.

Our next result summarizes the impact of an increase in the fraction of individuals with anticipatory utility on the fractions of realistic workers, optimistic workers, and realistic entrepreneurs.

Proposition 7: *If $f(l,k,\theta_0) = \theta_0 l^{\alpha} k^{\beta}$, θ_0 is uniformly distributed on $[0,1]$, $s < \bar{s}$, then an increase in the fraction of individuals with anticipatory utility leads to (i) a decrease in the fraction of realistic workers, i.e., $\partial L_R^*/\partial\lambda < 0$, (ii) an increase in the fraction of optimistic workers, i.e., $\partial L_O^*/\partial\lambda > 0$, and (iii) a decrease in the fraction of realistic entrepreneurs, i.e., $\partial E_R^*/\partial\lambda < 0$.*

Proposition 7-(i) shows that an increase in the fraction of individuals with anticipatory utility lowers the fraction of realistic workers. The intuition behind this result is the following. Amongst $1-\lambda$ realists the fraction $\hat{\theta}_R$ become workers so the fraction of realistic workers is $L_R^* = (1-\lambda)\hat{\theta}_R$. An increase in λ lowers the fraction of realists and raises the ability of the marginal realistic entrepreneur $\hat{\theta}_R$. The first effect lowers the fraction of realistic workers but the second effect raises it. Hence, at first sight, an increase in λ has an ambiguous effect on the fraction of realistic workers. However, the first effect always dominates the second and therefore an increase in λ lowers the fraction of realistic workers.

Proposition 7-(ii) shows that an increase in the fraction of individuals with anticipatory utility raises the fraction of optimistic workers. Amongst λ optimists the fraction $\hat{\theta}_O$ become workers so the fraction of optimistic workers is $L_O^* = \lambda\hat{\theta}_O$. An increase in λ raises the fraction of optimistic workers because it raises the fraction of optimists and the ability of the marginal optimistic entrepreneur $\hat{\theta}_O$.

Proposition 7-(iii) shows that an increase in the fraction of individuals with anticipatory utility lowers the fraction of realistic entrepreneurs. Amongst $1-\lambda$ realists the fraction $1-\hat{\theta}_R$ become entrepreneurs so the fraction of realistic entrepreneurs is

$E_R^* = (1-\lambda)(1-\hat{\theta}_R)$. An increase in λ lowers the fraction of realistic entrepreneurs because it lowers the fraction of realists and raises the ability of the marginal realistic entrepreneur $\hat{\theta}_R$.

Note that an increase in the fraction of individuals with anticipatory utility has an ambiguous effect on the fraction of optimistic entrepreneurs. Amongst λ optimists the fraction $1-\hat{\theta}_O$ become entrepreneurs so the fraction of optimistic entrepreneurs is $E_O^* = \lambda(1-\hat{\theta}_O)$. An increase in λ raises the fraction of optimists and the ability of the marginal optimistic entrepreneur $\hat{\theta}_O$. The first effect raises the fraction of optimistic entrepreneurs but the second effect lowers it. Without imposed additional conditions on the parameters of the model it is unclear which effect dominates.

We know from Proposition 7 that, on the one hand, an increase in the fraction of individuals with anticipatory utility lowers the fraction of realistic workers, and, on the other hand, it raises the fraction of optimistic workers. Therefore, an increase in the fraction of individuals with anticipatory utility appears to have an ambiguous effect on the fraction of workers (and entrepreneurs since $E^* = 1 - L^*$). However, our next result shows how the fraction of workers (and entrepreneurs) varies with the fraction of individuals with anticipatory utility.

Proposition 8: *Assume $f(l,k,\theta_0) = \theta_0 l^\alpha k^\beta$, θ_0 is uniformly distributed on $[0,1]$, and $s < \bar{s}$.*
(i) The fraction of workers (entrepreneurs) is a concave (convex) function of the fraction of individuals with anticipatory utility, i.e., $\partial^2 L^*/\partial\lambda^2 < 0$ ($\partial^2 E^*/\partial\lambda^2 > 0$).
(ii) If

$$\left[1+\frac{s}{\eta}-\phi(\eta,\beta,s)\right]\frac{\psi(\eta,s)}{\phi(\eta,\beta,s)} > \frac{2-\eta}{1-\eta}[1-\psi(\eta,s)]\left(1+\frac{s}{\eta}\right), \tag{29}$$

then an increase in the fraction of individuals with anticipatory utility leads to an increase (decrease) in the fraction of workers (entrepreneurs), i.e., $\partial L^*/\partial\lambda > 0$ ($\partial E^*/\partial\lambda < 0$).
(iii) If (29) is violated and $\bar{\lambda}$ is the solution to

$$\frac{1-\bar{\lambda}+\bar{\lambda}\psi(\eta,s)}{1-\bar{\lambda}+\bar{\lambda}\phi(\eta,\beta,s)} = \frac{2-\eta}{1-\eta}\frac{1-\psi(\eta,s)}{1+\frac{s}{\eta}-\phi(\eta,\beta,s)}\left(1+\bar{\lambda}\frac{s}{\eta}\right), \tag{30}$$

then an increase in the fraction of individuals with anticipatory utility leads to (a) an increase (decrease) in the fraction of workers (entrepreneurs) when $\lambda < \bar{\lambda}$, i.e., $\partial L^*/\partial\lambda > 0$ ($\partial E^*/\partial\lambda < 0$), and (b) a decrease (increase) in the fraction of workers (entrepreneurs) when $\lambda > \bar{\lambda}$, i.e., $\partial L^*/\partial\lambda < 0$ ($\partial E^*/\partial\lambda > 0$).

Proposition 8 shows that an increase in the fraction of individuals with anticipatory utility does not necessarily lead to a decrease (an increase) in the fraction of workers (entrepreneurs). Moreover, one of two cases might arise. First, an increase in λ raises (lowers) the fraction of workers (entrepreneurs). This happens when either inequality (29) is satisfied or inequality (29) is violated and λ is low, i.e., $\lambda < \bar{\lambda}$. In this case an increase in λ raises the fraction of optimistic workers more than it lowers the fraction of realistic workers. Second, an increase in λ lowers (raises) the fraction of workers (entrepreneurs). This happens when inequality (29) is violated and λ is high, i.e., $\lambda > \bar{\lambda}$. In this case an increase in λ raises the fraction of optimistic workers less than it lowers the fraction of realistic workers.

We now provide conditions under which an increase in the fraction of individuals with anticipatory utility raises the rental cost of capital.

Proposition 9: *If $f(l, k, \theta_0) = \theta_0 l^\alpha k^\beta$, θ_0 is uniformly distributed on $[0, 1]$, $s < \bar{s}$, and either inequality (29) is satisfied or inequality (29) is violated and the fraction of individuals with anticipatory utility is low, i.e., $\lambda < \bar{\lambda}$, then an increase in λ raises the rental cost of capital, i.e., $\partial r^*/\partial\lambda > 0$.*

To close this section we briefly discuss the comparative statics with respect to the weight of anticipatory utility. An increase in the weight of anticipatory utility s raises the ability of the marginal realistic entrepreneur $\hat{\theta}_R$ and lowers the ability of the marginal optimistic entrepreneur $\hat{\theta}_O$. Hence, an increase in s lowers the fraction of realistic entrepreneurs E_R^* and raises the fraction of optimistic entrepreneurs E_O^*. In addition, an increase in s raises the fraction of realistic workers L_R^* and lowers the fraction of optimistic workers L_O^*. Finally, an increase in the weight of anticipatory utility s raises the wage w^*, lowers the number of workers L^* and raises the number of entrepreneurs E^*.

6 Calibration

This section calibrates the model to illustrate quantitatively the general equilibrium effects of entrepreneurial optimism. The calibration parameterizes the economy to match salient features of US manufacturing data, it follows Atkeson and Kehoe (2005) and Adler (2016). The calibration is summarized in Table I.

The degree of decreasing returns to scale η is an important parameter in the model since it simultaneously characterizes the firms' technology and constrains the optimistic bias in beliefs. Following Atkeson and Kehoe (2005) and Adler (2016) we set η to 0.85.

Given η equal to 0.85, a value of 0.612 for α matches labor's average income share (including managerial compensation) in manufacturing between 1998 and 2005. Again, following Atkeson and Kehoe (2005) and Adler (2016) we assume a capital-output ratio $\bar{K}/Y$ of 1.46 which together with a value for Y of 0.620 in the benchmark Lucas' model implies a capital stock $\bar{K}$ of 0.906.

Table I
Parameters of the Model

Parameter	Value	Description
Standard parameters		
η	0.85	decreasing returns to scale
α	0.612	labor's average income share
β	0.238	capital's average income share
$\bar{K}$	0.906	capital stock
Additional parameters		
λ	0.5	fraction with anticipatory utility
s	0.683	weight of anticipatory utility

We are left with the parameters λ and s to calibrate. The parameter λ measures the fraction of individuals with anticipatory utility and can vary between 0 and 1.

Someone who experiences positive anticipatory benefits while waiting for the resolution of risk should display a preference for delayed resolution of risk. In addition, if such a person is offered several identical lottery tickets for different drawing dates with the same expected value, then he or she should have a preference for spreading the days of drawing. Kocher et al. (2014) test these two predictions using a laboratory experiment and find that, although 41.5 percent of participants prefer real lottery tickets for an immediate drawing rather than one for the subsequent day, 21.5 percent actually prefers delayed resolution, and the remaining 37 percent is indifferent. They also find that around 70.8 percent of participants want tickets on two days rather than on one, 17.9 percent want tickets on one day, and the remaining 12.3 percent is indifferent. Based on these two findings we set λ to 0.5.

The weight of anticipatory utility s is also an important parameter in the model since it constrains the optimistic bias in beliefs. For the competitive equilibrium to be well defined, s must be smaller than the upper bound $\bar{s}$ in (23). Setting $\eta = 0.85$, $\alpha = 0.612$, $\beta = 0.238$, and $\lambda = 0.5$ in (23) and solving for $\bar{s}$ we obtain $\bar{s} = 1.5$. Hence, s can vary between 0 and 1.5. We are not aware of empirical work that quantifies s. Letting s equalize the mean returns to entrepreneurship to the wage we obtain $s = 0.683$. This value for s satisfies the upper bound $\bar{s} = 1.5$ and means that individuals with anticipatory utility place more weight on the material payoff than on the anticipatory payoff of entrepreneurship.

Note that in the competitive equilibrium with $\lambda \neq 0$ and $s \neq 0$ output is equal to

$$Y^* = (1-\lambda)\int_{\hat{\theta}_R}^{1} \theta_0[l(w^*, r^*, \theta_0)]^\alpha [k(w^*, r^*, \theta_0)]^\beta d\theta_0 + \lambda \int_{\hat{\theta}_O}^{\frac{\eta - s(1-\eta)}{\eta}} \theta_0[l(w^*, r^*, \theta^*)]^\alpha [k(w^*, r^*, \theta^*)]^\beta d\theta_0 + \lambda \int_{\frac{\eta - s(1-\eta)}{\eta}}^{1} \theta_0[l(w^*, r^*, 1)]^\alpha [k(w^*, r^*, 1)]^\beta d\theta_0.$$

Setting $\lambda = 0$ in the above expression we obtain output in the benchmark Lucas' model:

$$Y_0^* = \int_{\hat{\theta}_0}^{1} \theta_0 [l(w_0^*, r_0^*, \theta_0)]^{\alpha} [k(w_0^*, r_0^*, \theta_0)]^{\beta} d\theta_0 = (1-\eta)^{1-\eta} \alpha^{\frac{\alpha(1-\eta)}{2-\eta}} (2-\beta)^{\frac{-(2-\beta)(1-\eta)}{2-\eta}} \bar{K}^{\beta}.$$

Table II summarizes the results of the calibration. The first column in Table II lists the variables. The second column reports the competitive equilibrium values with $\lambda = 0$. The third column the competitive equilibrium values with $\lambda = 0.5$ and $s = 0.683$. The fourth column reports the percent change in the values of the variables common to both models.

Table II

Calibration

	Model $\lambda = 0$ Lucas (1978)	Model $\lambda = 0.5$ and $s = 0.683$	Percent change
Output (Y^*)	0.620	0.612	-1.29
Wage (w^*)	0.436	0.458	5.05
Rental cost of capital (r^*)	0.163	0.170	4.29
Mean returns to entrepreneurship	0.722	0.458	-36.57
Mean returns of realistic entrep	-	0.630	-
Mean returns of optimistic entrep	-	0.376	-
Fraction of workers (L^*)	0.871	0.867	-0.46
Ability of mg realistic entrep ($\hat{\theta}_R$)	-	0.914	-
Ability of mg optimistic entrep ($\hat{\theta}_O$)	-	0.820	-
Fraction of entrep (E^*)	0.129	0.133	3.10
Fraction of entrep optimists	-	0.677	-
Fraction of workers optimists	-	0.473	-

We now discuss the results of the calibration. Output in the competitive equilibrium with $\lambda = 0$ is higher than in the competitive equilibrium with $\lambda = 0.5$ and $s = 0.683$, i.e., $Y_0^* = 0.620 > 0.612 = Y^*$. This corresponds to a 1.29 percent decline in output. This result is expected since we know from Lucas (1978) that, in the absence of distortions, the decentralized competitive equilibrium maximizes output.

In the competitive equilibrium with $\lambda = 0$ the mean return to entrepreneurship is $\left(Y_0^* - w_0^* L_0^* - r_o^* \bar{K}\right) / E_0^* = 0.722$. In the competitive equilibrium with $\lambda = 0.5$ and $s = 0.683$ the mean return to entrepreneurship is 0.458. This corresponds to a 36.57 percent decline. This sharp decline in the mean return to entrepreneurship happens for three reasons. First, optimism raises input prices: the wage increases by 5.02 percent and the rental rate of capital by 4.29 percent. Second, optimism distorts the input choices of optimistic entrepreneurs. Third, optimism lowers the fraction of realistic entrepreneurs–those with higher returns–and raises the fraction of optimistic entrepreneurs–those with lower returns.

In both models the fraction of workers is approximately 87 percent and the fraction of entrepreneurs 13 percent. This is not far from the fractions reported in Cagetti and de Nardi (2006), Blanchflower (2010), Hipple (2010), and Poschke (2013). Furthermore, the calibration shows a 0.46 percent decline in the fraction of workers, and a 3.10 percent increase in the fraction of entrepreneurs. The misallocation of talent affects only 4.7 percent of the population:

$$\lambda(\hat{\theta}_0 - \hat{\theta}_O) + (1 - \lambda)(\hat{\theta}_R - \hat{\theta}_0) = 0.5(0.871 - 0.820) + 0.5(0.914 - 0.871) = 0.047.$$

In the competitive equilibrium with $\lambda = 0.5$ and $s = 0.683$ we find that 67.7 percent of entrepreneurs are optimists but only 47.3 percent of workers are optimists. Hence, entrepreneurs are more optimistic than workers and the majority of entrepreneurs are optimists. This is in line with Arabsheibani et al. (2000), Fraser and Greene (2006), Puri and Robinson (2007), and Dawson et al. (2014). Finally, realistic entrepreneurs earn, on average, 0.630 whereas optimistic entrepreneurs 0.376. The fact that realistic entrepreneurs earn, on average, more than optimistic ones is in line with the empirical evidence reported in Dawson et al. (2015).

What are we to take away from these results? As we have seen, the calibration shows that anticipatory utility driven optimism can have a sharp negative impact on the returns to entrepreneurship and a positive impact on the wage. However, the calibration also shows that having a large fraction of optimistic individuals in the economy can have a relatively modest impact on output and on occupational choices.

This tells us that not all general equilibrium effects of biased beliefs are as large as one could have imagined a priori.

Are there any policy implications one can take away from this model? Given their preferences, individuals in this economy are maximizing their utility and so the economy is in a first-best situation which implies that welfare is maximized. However, if the goal of a policymaker is to raise the output of the economy, then it is possible to do so with a revenue-neutral tax-subsidy scheme. The scheme consists of a lump-sum tax to (optimistic) entrepreneurs with profits below the market clearing wage and a lump-sum subsidy to workers. The tax revenues come only from low ability optimistic entrepreneurs and induces them to stay in the labor force. The tax revenues are redistributed to workers as a lump-sum subsidy which further induces low ability optimists to stay in the labor force. The full characterization of this tax-subsidy scheme is available upon request.

7 Conclusion

We extend Lucas' (1978) by assuming that fraction of individuals in the population derives nonpecuniary anticipatory payoffs from entrepreneurship. If the anticipatory payoffs depend on beliefs about entrepreneurial skill, then these individuals will choose to be optimists about their entrepreneurial ability.

We find that anticipatory utility driven optimism has six main effects. First, there is a misallocation of talent which lowers output. Second, optimists are more likely to become entrepreneurs than realists. Third, entrepreneurs are more optimistic than workers. Fourth, when the fraction optimists is high, the majority of entrepreneurs are optimists. Fifth, optimism drives up the wage which makes workers better off. Sixth, optimism lowers the returns to entrepreneurship.

We calibrate the model to match salient features of US manufacturing data. We find that anticipatory utility driven optimism may significantly change the distribution of income by driving up the wage and lowering the returns to entrepreneurship.

The calibration also shows that even though optimism has a large impact on the returns to entrepreneurship it has only a modest impact on output, the fraction of workers, and of entrepreneurs.

8 Appendix

Proof of Proposition 1: Consider an individual with entrepreneurial ability θ_0 and expectation of entrepreneurial ability θ. Assume that this individual decides to be an entrepreneur at $t = 1$. At $t = 2$ this individual solves the following problem

$$\max_{l,k}(\theta l^{\alpha} k^{\beta} - wl - rk)$$

The first-order conditions are

$$\alpha\theta l^{\alpha-1}k^{\beta} = w,$$

and

$$\beta\theta l^{\alpha}k^{\beta-1} = r.$$

Solving for l and k we obtain

$$l(w,r,\theta) = \theta^{\frac{1}{1-\eta}} \left(\frac{\alpha}{w}\right)^{\frac{1-\beta}{1-\eta}} \left(\frac{\beta}{r}\right)^{\frac{\beta}{1-\eta}}, \tag{31}$$

and

$$k(w,r,\theta) = \theta^{\frac{1}{1-\eta}} \left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\eta}} \left(\frac{\beta}{r}\right)^{\frac{1-\alpha}{1-\eta}}, \tag{32}$$

where $\eta = \alpha + \beta$. At $t = 0$ this individual solves the problem:

$$\max_{\theta\in[0,1]} \left\{(\theta_0 + s\theta)[l(w,r,\theta)]^{\alpha}[k(w,r,\theta)]^{\beta} - (1+s)\left[wl(w,r,\theta) + rk(w,r,\theta)\right]\right\}.$$

Substituting $l(w,r,\theta)$ by (31) and $k(w,r,\theta)$ by (32) and simplifying terms we obtain

$$\left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\eta}} \left(\frac{\beta}{r}\right)^{\frac{\beta}{1-\eta}} \max_{\theta\in[0,1]} \left[(\theta_0 + s\theta)\theta^{\frac{\eta}{1-\eta}} - (1+s)\eta\theta^{\frac{1}{1-\eta}}\right].$$

The first-order condition is

$$(\theta_0 + s\theta)\frac{\eta}{1-\eta}\theta^{\frac{\eta}{1-\eta}-1} + s\theta^{\frac{\eta}{1-\eta}} = (1+s)\frac{\eta}{1-\eta}\theta^{\frac{1}{1-\eta}-1}. \tag{33}$$

Solving for θ we obtain

$$\theta^* = \frac{\eta}{\eta - s(1-\eta)}\theta_0. \tag{34}$$

This solution implies that all individuals with anticipatory utility hold optimistic beliefs since $\theta^* > \theta_0$. For (34) to be an interior solution, i.e., $\theta^* \in (0,1)$, at least two conditions must be satisfied. First, it must be that

$$\eta > s(1-\eta),$$

or

$$s < \frac{\eta}{1-\eta}. \tag{35}$$

This condition places an upper bound on the weight of anticipatory utility. Second, it must be that

$$\frac{\eta}{\eta - s(1-\eta)}\theta_0 < 1,$$

or

$$\theta_0 < \frac{\eta - s(1-\eta)}{\eta}. \tag{36}$$

This condition says that there is an interior solution only for individuals whose ability is below an upper bound. We see from (36) that the upper bound depends on s and η. Hence the expectations of ability are as follows:

$$\theta^* = \begin{cases} \frac{\eta}{\eta-s(1-\eta)}\theta_0 & \text{if } \theta_0 < \frac{\eta-s(1-\eta)}{\eta} \\ 1 & \text{if } \theta_0 \geq \frac{\eta-s(1-\eta)}{\eta} \end{cases}.$$

To complete the proof we need to show that the second-order condition is satisfied. The first-order condition (33) is equivalent to

$$\frac{\eta}{1-\eta}\left[\theta_0\theta^{\frac{\eta}{1-\eta}-1} + s\theta^{\frac{\eta}{1-\eta}} + \frac{1-\eta}{\eta}s\theta^{\frac{\eta}{1-\eta}} - (1+s)\theta^{\frac{1}{1-\eta}-1}\right] = 0,$$

or

$$\frac{\eta}{1-\eta}\left[\theta_0\theta^{\frac{2\eta-1}{1-\eta}}+\left(\frac{s}{\eta}-1-s\right)\theta^{\frac{\eta}{1-\eta}}\right]=0. \tag{37}$$

Taking the derivative of (37) with respect to θ, the second-order condition is given by

$$\frac{2\eta-1}{1-\eta}\theta_0\theta^{\frac{2\eta-1}{1-\eta}-1}+\left(\frac{s}{\eta}-1-s\right)\frac{\eta}{1-\eta}\theta^{\frac{\eta}{1-\eta}-1}<0,$$

or

$$(2\eta-1)\theta_0-\left[\eta-s(1-\eta)\right]\theta<0.$$

Since $s<\eta/(1-\eta)$, the second-order condition is satisfied for any $\eta\leq 0.5$. When $0.5<\eta<1$ the second-order condition is satisfied as long as

$$(2\eta-1)\theta_0<\left[\eta-s(1-\eta)\right]\theta.$$

Replacing θ by $\theta^*=\eta\theta_0/[\eta-s(1-\eta)]$ we have

$$(2\eta-1)\theta_0<\eta\theta_0,$$

or

$$\eta<1,$$

which is true. *Q.E.D.*

Proof of Proposition 2: Assume $s<\bar{s}$. The first step to determine the competitive equilibrium is to find out the labor market equilibrium condition. The labor demand from realistic entrepreneurs is

$$\begin{aligned}L_R^D &= (1-\lambda)\int_{\hat{\theta}_R}^{1} l(w,r,\theta_0)d\theta_0=(1-\lambda)\left(\frac{\alpha}{w}\right)^{\frac{1-\beta}{1-\eta}}\left(\frac{\beta}{r}\right)^{\frac{\beta}{1-\eta}}\int_{\hat{\theta}_R}^{1}\theta_0^{\frac{1}{1-\eta}}d\theta_0\\ &= (1-\lambda)\frac{1-\eta}{2-\eta}\left(1-\hat{\theta}_R^{\frac{2-\eta}{1-\eta}}\right)\left(\frac{\alpha}{w}\right)^{\frac{1-\beta}{1-\eta}}\left(\frac{\beta}{r}\right)^{\frac{\beta}{1-\eta}}. \end{aligned}\tag{38}$$

Note that for L_R^D to be well defined it must be that $\hat{\theta}_R<1$. Recall that $\hat{\theta}_O$ is the ability threshold that determines the marginal optimistic entrepreneur. If $\hat{\theta}_O<$

$[\eta - s(1-\eta)]/\eta$, then labor demand from optimistic entrepreneurs is the sum of the demand for labor coming from the mass of entrepreneurs with heterogeneous optimistic expectations, i.e., those with $\theta^* \in (\theta_0, 1)$, to the demand for labor coming from the mass of entrepreneurs with homogeneous optimistic expectations, i.e., those with $\theta^* = 1$:

$$\begin{aligned} L_O^D &= \lambda \left\{ \int_{\hat{\theta}_O}^{\frac{\eta - s(1-\eta)}{\eta}} l(w,r,\theta^*) d\theta_0 + \int_{\frac{\eta-s(1-\eta)}{\eta}}^{1} l(w,r,1) d\theta_0 \right\} \\ &= \lambda \left\{ \int_{\hat{\theta}_O}^{\frac{\eta - s(1-\eta)}{\eta}} (\theta^*)^{\frac{1}{1-\eta}} d\theta_0 + \int_{\frac{\eta-s(1-\eta)}{\eta}}^{1} d\theta_0 \right\} \left(\frac{\alpha}{w}\right)^{\frac{1-\beta}{1-\eta}} \left(\frac{\beta}{r}\right)^{\frac{\beta}{1-\eta}} \\ &= \lambda \left\{ \left[\frac{\eta}{\eta - s(1-\eta)}\right]^{\frac{1}{1-\eta}} \int_{\hat{\theta}_O}^{\frac{\eta - s(1-\eta)}{\eta}} \theta_0^{\frac{1}{1-\eta}} d\theta_0 + s\frac{1-\eta}{\eta} \right\} \left(\frac{\alpha}{w}\right)^{\frac{1-\beta}{1-\eta}} \left(\frac{\beta}{r}\right)^{\frac{\beta}{1-\eta}} \\ &= \lambda \left\{ \frac{1-\eta}{2-\eta} \left[\frac{\eta}{\eta - s(1-\eta)}\right]^{\frac{1}{1-\eta}} \left[\theta_0^{\frac{2-\eta}{1-\eta}}\right]_{\hat{\theta}_O}^{\frac{\eta - s(1-\eta)}{\eta}} + s\frac{1-\eta}{\eta} \right\} \left(\frac{\alpha}{w}\right)^{\frac{1-\beta}{1-\eta}} \left(\frac{\beta}{r}\right)^{\frac{\beta}{1-\eta}} \\ &= \lambda \frac{1-\eta}{2-\eta} \left\{ 1 + \frac{s}{\eta} - \left[\frac{\eta}{\eta - s(1-\eta)}\right]^{\frac{1}{1-\eta}} \hat{\theta}_O^{\frac{2-\eta}{1-\eta}} \right\} \left(\frac{\alpha}{w}\right)^{\frac{1-\beta}{1-\eta}} \left(\frac{\beta}{r}\right)^{\frac{\beta}{1-\eta}}. \end{aligned} \tag{39}$$

Note that for L_O^D to be well defined it must be that $\hat{\theta}_O < [\eta - s(1-\eta)]/\eta$. From (38) and (39), labor demand is equal to

$$\begin{aligned} L^D &= L_R^D + L_O^D \\ &= (1-\lambda)\frac{1-\eta}{2-\eta}\left(1 - \hat{\theta}_R^{\frac{2-\eta}{1-\eta}}\right) \left(\frac{\alpha}{w}\right)^{\frac{1-\beta}{1-\eta}} \left(\frac{\beta}{r}\right)^{\frac{\beta}{1-\eta}} \\ &\quad + \lambda \frac{1-\eta}{2-\eta} \left\{ 1 + \frac{s}{\eta} - \left[\frac{\eta}{\eta - s(1-\eta)}\right]^{\frac{1}{1-\eta}} \hat{\theta}_O^{\frac{2-\eta}{1-\eta}} \right\} \left(\frac{\alpha}{w}\right)^{\frac{1-\beta}{1-\eta}} \left(\frac{\beta}{r}\right)^{\frac{\beta}{1-\eta}} \\ &= \frac{1-\eta}{2-\eta} \left\{ 1 + \lambda\frac{s}{\eta} - (1-\lambda)\hat{\theta}_R^{\frac{2-\eta}{1-\eta}} - \lambda \left[\frac{\eta}{\eta - s(1-\eta)}\right]^{\frac{1}{1-\eta}} \hat{\theta}_O^{\frac{2-\eta}{1-\eta}} \right\} \left(\frac{\alpha}{w}\right)^{\frac{1-\beta}{1-\eta}} \left(\frac{\beta}{r}\right)^{\frac{\beta}{1-\eta}} \end{aligned}$$

Since each worker provides a unit of labor, labor supply is

$$\begin{aligned} L^S &= (1-\lambda)L_R^S + \lambda L_O^S = (1-\lambda)\int_0^{\hat{\theta}_R} d\theta_0 + \lambda \int_0^{\hat{\theta}_O} d\theta_0 \\ &= (1-\lambda)\hat{\theta}_R + \lambda\hat{\theta}_O. \end{aligned} \tag{40}$$

In equilibrium, labor demand must equal labor supply:

$$\begin{aligned} & \frac{1-\eta}{2-\eta}\left\{1+\lambda\frac{s}{\eta}-(1-\lambda)\hat{\theta}_R^{\frac{2-\eta}{1-\eta}}-\lambda\left[\frac{\eta}{\eta-s(1-\eta)}\right]^{\frac{1}{1-\eta}}\hat{\theta}_O^{\frac{2-\eta}{1-\eta}}\right\}\left(\frac{\alpha}{w}\right)^{\frac{1-\beta}{1-\eta}}\left(\frac{\beta}{r}\right)^{\frac{\beta}{1-\eta}} \\ = \; & (1-\lambda)\hat{\theta}_R+\lambda\hat{\theta}_O, \end{aligned} \tag{41}$$

The second step to determine the competitive equilibrium is to find out the capital market equilibrium condition. The capital demand from realistic entrepreneurs is

$$\begin{aligned} K_R^D &= (1-\lambda)\int_{\hat{\theta}_R}^1 k(w,r,\theta_0)d\theta_0 = (1-\lambda)\left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\eta}}\left(\frac{\beta}{r}\right)^{\frac{1-\alpha}{1-\eta}}\int_{\hat{\theta}_R}^1 \theta_0^{\frac{1}{1-\eta}}d\theta_0 \\ &= (1-\lambda)\frac{1-\eta}{2-\eta}\left(1-\hat{\theta}_R^{\frac{2-\eta}{1-\eta}}\right)\left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\eta}}\left(\frac{\beta}{r}\right)^{\frac{1-\alpha}{1-\eta}}. \end{aligned} \tag{42}$$

Note that for K_R^D to be well defined it must be that $\hat{\theta}_R < 1$. Recall that $\hat{\theta}_O$ is the ability threshold that determines the marginal optimistic entrepreneur. If $\hat{\theta}_O < [\eta - s(1-\eta)]/\eta$, then capital demand from optimistic entrepreneurs is the sum of the demand for capital coming from the mass of entrepreneurs with heterogeneous optimistic expectations, i.e., those with $\theta^* \in (\theta_0, 1)$, to the demand for capital coming from the mass of entrepreneurs with homogeneous optimistic expectations, i.e., those with $\theta^* = 1$:

$$\begin{aligned} K_O^D &= \lambda\left\{\int_{\hat{\theta}_O}^{\frac{\eta-s(1-\eta)}{\eta}} k(w,r,\theta^*)d\theta_0 + \int_{\frac{\eta-s(1-\eta)}{\eta}}^1 k(w,r,1)d\theta_0\right\} \\ &= \lambda\left\{\int_{\hat{\theta}_O}^{\frac{\eta-s(1-\eta)}{\eta}} (\theta^*)^{\frac{1}{1-\eta}}d\theta_0 + \int_{\frac{\eta-s(1-\eta)}{\eta}}^1 d\theta_0\right\}\left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\eta}}\left(\frac{\beta}{r}\right)^{\frac{1-\alpha}{1-\eta}} \\ &= \lambda\frac{1-\eta}{2-\eta}\left\{1+\frac{s}{\eta}-\left[\frac{\eta}{\eta-s(1-\eta)}\right]^{\frac{1}{1-\eta}}\hat{\theta}_O^{\frac{2-\eta}{1-\eta}}\right\}\left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\eta}}\left(\frac{\beta}{r}\right)^{\frac{1-\alpha}{1-\eta}}. \end{aligned} \tag{43}$$

Note that for K_O^D to be well defined it must be that $\hat{\theta}_O < [\eta - s(1-\eta)]/\eta$. From

(42) and (43), capital demand is equal to

$$\begin{aligned}
K^D &= K_R^D + K_O^D \\
&= (1-\lambda)\frac{1-\eta}{2-\eta}\left(1-\hat{\theta}_R^{\frac{2-\eta}{1-\eta}}\right)\left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\eta}}\left(\frac{\beta}{r}\right)^{\frac{1-\alpha}{1-\eta}} \\
&\quad +\lambda\frac{1-\eta}{2-\eta}\left\{1+\frac{s}{\eta}-\left[\frac{\eta}{\eta-s(1-\eta)}\right]^{\frac{1}{1-\eta}}\hat{\theta}_O^{\frac{2-\eta}{1-\eta}}\right\}\left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\eta}}\left(\frac{\beta}{r}\right)^{\frac{1-\alpha}{1-\eta}} \\
&= \frac{1-\eta}{2-\eta}\left\{1+\lambda\frac{s}{\eta}-(1-\lambda)\hat{\theta}_R^{\frac{2-\eta}{1-\eta}}-\lambda\left[\frac{\eta}{\eta-s(1-\eta)}\right]^{\frac{1}{1-\eta}}\hat{\theta}_O^{\frac{2-\eta}{1-\eta}}\right\}\left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\eta}}\left(\frac{\beta}{r}\right)^{\frac{1-\alpha}{1-\eta}}
\end{aligned}$$

In equilibrium, capital demand must equal the exogenous capital supply:

$$\begin{aligned}
&\frac{1-\eta}{2-\eta}\left\{1+\lambda\frac{s}{\eta}-(1-\lambda)\hat{\theta}_R^{\frac{2-\eta}{1-\eta}}-\lambda\left[\frac{\eta}{\eta-s(1-\eta)}\right]^{\frac{1}{1-\eta}}\hat{\theta}_O^{\frac{2-\eta}{1-\eta}}\right\}\left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\eta}}\left(\frac{\beta}{r}\right)^{\frac{1-\alpha}{1-\eta}} \\
= &\ \bar{K}. \qquad (44)
\end{aligned}$$

The third step to determine the competitive equilibrium is to find out the ability level of the marginal realistic entrepreneur $\hat{\theta}_R$ and of the marginal optimistic entrepreneur $\hat{\theta}_O$. At $t=1$ a realist with ability $\hat{\theta}_R$ is indifferent between being an entrepreneur and a worker when

$$\hat{\theta}_R[l(w,r,\hat{\theta}_R)]^\alpha[k(w,r,\hat{\theta}_R)]^\beta - wl(w,r,\hat{\theta}_R) - rk(w,r,\hat{\theta}_R) = w,$$

or

$$\begin{aligned}
&\hat{\theta}_R\left[\hat{\theta}_R^{\frac{1}{1-\eta}}\left(\frac{\alpha}{w}\right)^{\frac{1-\beta}{1-\eta}}\left(\frac{\beta}{r}\right)^{\frac{\beta}{1-\eta}}\right]^\alpha\left[\hat{\theta}_R^{\frac{1}{1-\eta}}\left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\eta}}\left(\frac{\beta}{r}\right)^{\frac{1-\alpha}{1-\eta}}\right]^\beta \\
&-w\hat{\theta}_R^{\frac{1}{1-\eta}}\left(\frac{\alpha}{w}\right)^{\frac{1-\beta}{1-\eta}}\left(\frac{\beta}{r}\right)^{\frac{\beta}{1-\eta}}-r\hat{\theta}_R^{\frac{1}{1-\eta}}\left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\eta}}\left(\frac{\beta}{r}\right)^{\frac{1-\alpha}{1-\eta}} = w,
\end{aligned}$$

or

$$\hat{\theta}_R^{\frac{1}{1-\eta}}\left[\left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\eta}}\left(\frac{\beta}{r}\right)^{\frac{\beta}{1-\eta}}-w\left(\frac{\alpha}{w}\right)^{\frac{1-\beta}{1-\eta}}\left(\frac{\beta}{r}\right)^{\frac{\beta}{1-\eta}}-r\left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\eta}}\left(\frac{\beta}{r}\right)^{\frac{1-\alpha}{1-\eta}}\right] = w,$$

or

$$\hat{\theta}_R{}^{\frac{1}{1-\eta}} \left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\eta}} \left(\frac{\beta}{r}\right)^{\frac{\beta}{1-\eta}} \left[1 - w\left(\frac{\alpha}{w}\right) - r\left(\frac{\beta}{r}\right)\right] = w,$$

or

$$\hat{\theta}_R{}^{\frac{1}{1-\eta}} \left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\eta}} \left(\frac{\beta}{r}\right)^{\frac{\beta}{1-\eta}} (1-\eta) = w,$$

or

$$\hat{\theta}_R{}^{\frac{1}{1-\eta}} \alpha^{\frac{\alpha}{1-\eta}} \beta^{\frac{\beta}{1-\eta}} (1-\eta) = w^{\frac{1-\beta}{1-\eta}} r^{\frac{\beta}{1-\eta}},$$

or

$$\alpha^\alpha \beta^\beta (1-\eta)^{1-\eta} \hat{\theta}_R = w^{1-\beta} r^\beta. \tag{45}$$

At $t = 1$ an individual with expectations of ability $\theta^* = \eta\hat{\theta}_O/[\eta - s(1-\eta)]$ and ability $\hat{\theta}_O$ is indifferent between being an entrepreneur and a worker when

$$(\hat{\theta}_O + s\theta^*)[l(w,r,\theta^*)]^\alpha [k(w,r,\theta^*)]^\beta - (1+s)\left[wl(w,r,\theta^*) + rk(w,r,\theta^*)\right] = w,$$

or

$$(\hat{\theta}_O + s\theta^*) \left[(\theta^*)^{\frac{1}{1-\eta}} \left(\frac{\alpha}{w}\right)^{\frac{1-\beta}{1-\eta}} \left(\frac{\beta}{r}\right)^{\frac{\beta}{1-\eta}}\right]^\alpha \left[(\theta^*)^{\frac{1}{1-\eta}} \left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\eta}} \left(\frac{\beta}{r}\right)^{\frac{1-\alpha}{1-\eta}}\right]^\beta$$
$$-(1+s)\left[w(\theta^*)^{\frac{1}{1-\eta}} \left(\frac{\alpha}{w}\right)^{\frac{1-\beta}{1-\eta}} \left(\frac{\beta}{r}\right)^{\frac{\beta}{1-\eta}} + r(\theta^*)^{\frac{1}{1-\eta}} \left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\eta}} \left(\frac{\beta}{r}\right)^{\frac{1-\alpha}{1-\eta}}\right] = w,$$

or

$$\left[\frac{\eta - s(1-\eta)}{\eta}\theta^* + s\theta^*\right] \left[(\theta^*)^{\frac{\eta}{1-\eta}} \left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\eta}} \left(\frac{\beta}{r}\right)^{\frac{\beta}{1-\eta}}\right]$$
$$-(1+s)(\theta^*)^{\frac{1}{1-\eta}} \left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\eta}} \left(\frac{\beta}{r}\right)^{\frac{\beta}{1-\eta}} \left[w\left(\frac{\alpha}{w}\right) + r\left(\frac{\beta}{r}\right)\right] = w,$$

or

$$(\theta^*)^{\frac{1}{1-\eta}} \left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\eta}} \left(\frac{\beta}{r}\right)^{\frac{\beta}{1-\eta}} \left[\frac{\eta - s(1-\eta)}{\eta} + s - (1+s)\eta\right] = w,$$

or

$$(\theta^*)^{\frac{1}{1-\eta}} \left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\eta}} \left(\frac{\beta}{r}\right)^{\frac{\beta}{1-\eta}} \left[1 - \frac{s}{\eta} + 2s - (1+s)\eta\right] = w$$

or

$$\left[\frac{\eta}{\eta-s(1-\eta)}\right]^{\frac{1}{1-\eta}}\hat{\theta}_O^{\frac{1}{1-\eta}}\left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\eta}}\left(\frac{\beta}{r}\right)^{\frac{\beta}{1-\eta}}\left[1-\frac{s}{\eta}+2s-(1+s)\eta\right]=w$$

or

$$\hat{\theta}_O\left[\frac{\eta}{\eta-s(1-\eta)}\right]\alpha^\alpha\beta^\beta\left[1-\eta-\frac{s}{\eta}(1-\eta)^2\right]^{1-\eta}=w^{1-\beta}r^\beta,$$

or

$$\hat{\theta}_O\left[\frac{\eta}{\eta-s(1-\eta)}\right]\alpha^\alpha\beta^\beta(1-\eta)^{1-\eta}\left[\frac{\eta-s(1-\eta)}{\eta}\right]^{1-\eta}=w^{1-\beta}r^\beta,$$

or

$$\alpha^\alpha\beta^\beta(1-\eta)^{1-\eta}\left[\frac{\eta}{\eta-s(1-\eta)}\right]^\eta\hat{\theta}_O=w^{1-\beta}r^\beta. \tag{46}$$

It follows from (45) and (46) that

$$\alpha^\alpha\beta^\beta(1-\alpha-\beta)^{1-\alpha-\beta}\left[\frac{\eta}{\eta-s(1-\eta)}\right]^\eta\hat{\theta}_O=\alpha^\alpha\beta^\beta(1-\alpha-\beta)^{1-\alpha-\beta}\hat{\theta}_R,$$

or

$$\hat{\theta}_O=\left[\frac{\eta-s(1-\eta)}{\eta}\right]^\eta\hat{\theta}_R. \tag{47}$$

Substituting (45) and (47) into (41) we obtain

$$\begin{aligned}&1+\lambda\frac{s}{\eta}-(1-\lambda)\hat{\theta}_R^{\frac{2-\eta}{1-\eta}}-\lambda\left[\frac{\eta}{\eta-s(1-\eta)}\right]^{1-\eta}\hat{\theta}_R^{\frac{2-\eta}{1-\eta}}\\=\;&\frac{2-\eta}{1-\eta}\frac{w^{\frac{1-\beta}{1-\eta}}r^{\frac{\beta}{1-\eta}}}{\alpha^{\frac{1-\beta}{1-\eta}}\beta^{\frac{\beta}{1-\eta}}}\left\{(1-\lambda)\hat{\theta}_R+\lambda\left[\frac{\eta-s(1-\eta)}{\eta}\right]^\eta\hat{\theta}_R\right\},\end{aligned}$$

or

$$\begin{aligned}&1+\lambda\frac{s}{\eta}-(1-\lambda)\hat{\theta}_R^{\frac{2-\eta}{1-\eta}}-\lambda\left[\frac{\eta}{\eta-s(1-\eta)}\right]^{1-\eta}\hat{\theta}_R^{\frac{2-\eta}{1-\eta}}\\=\;&\frac{2-\eta}{1-\eta}\frac{\alpha^{\frac{\alpha}{1-\eta}}\beta^{\frac{\beta}{1-\eta}}(1-\eta)\hat{\theta}_R^{\frac{1}{1-\eta}}}{\alpha^{\frac{1-\beta}{1-\eta}}\beta^{\frac{\beta}{1-\eta}}}\left\{(1-\lambda)\hat{\theta}_R+\lambda\left[\frac{\eta-s(1-\eta)}{\eta}\right]^\eta\hat{\theta}_R\right\},\end{aligned}$$

or

$$\begin{aligned}&1+\lambda\frac{s}{\eta}-(1-\lambda)\hat{\theta}_R^{\frac{2-\eta}{1-\eta}}-\lambda\left[\frac{\eta}{\eta-s(1-\eta)}\right]^{1-\eta}\hat{\theta}_R^{\frac{2-\eta}{1-\eta}}\\=\;&\frac{2-\eta}{1-\eta}\frac{1-\eta}{\alpha}\left\{(1-\lambda)+\lambda\left[\frac{\eta-s(1-\eta)}{\eta}\right]^\eta\right\}\hat{\theta}_R^{\frac{2-\eta}{1-\eta}},\end{aligned}$$

or

$$1+\lambda\frac{s}{\eta}=\left\{\frac{(2-\beta)(1-\lambda)}{\alpha}+\lambda\left[\frac{\eta}{\eta-s(1-\eta)}\right]^{1-\eta}+\lambda\frac{2-\eta}{\alpha}\left[\frac{\eta-s(1-\eta)}{\eta}\right]^{\eta}\right\}\hat{\theta}_R^{\frac{2-\eta}{1-\eta}}$$

or

$$\hat{\theta}_R^{\frac{2-\eta}{1-\eta}}=\frac{1+\lambda\frac{s}{\eta}}{\frac{(2-\beta)(1-\lambda)}{\alpha}+\lambda\left[\frac{\eta}{\eta-s(1-\eta)}\right]^{1-\eta}+\lambda\frac{2-\eta}{\alpha}\left[\frac{\eta-s(1-\eta)}{\eta}\right]^{\eta}}.$$

Hence, the ability of the marginal realistic entrepreneur is

$$\begin{aligned}\hat{\theta}_R &= \left\{\frac{1+\lambda\frac{s}{\eta}}{\frac{(2-\beta)(1-\lambda)}{\alpha}+\lambda\left[\frac{\eta}{\eta-s(1-\eta)}\right]^{1-\eta}+\lambda\frac{2-\eta}{\alpha}\left[\frac{\eta-s(1-\eta)}{\eta}\right]^{\eta}}\right\}^{\frac{1-\eta}{2-\eta}} \\ &= \left\{\frac{\alpha}{2-\beta}\frac{1+\lambda\frac{s}{\eta}}{1-\lambda+\lambda\left[1-\frac{s(1-\eta)(2-\eta)}{\eta(2-\beta)}\right]\left[\frac{\eta}{\eta-s(1-\eta)}\right]^{1-\eta}}\right\}^{\frac{1-\eta}{2-\eta}} \\ &= \left[\frac{\alpha}{2-\beta}\frac{1+\lambda\frac{s}{\eta}}{1-\lambda+\lambda\phi(\eta,\beta,s)}\right]^{\frac{1-\eta}{2-\eta}}, \end{aligned} \tag{48}$$

where

$$\begin{aligned}\phi(\eta,\beta,s) &= \left[1-\frac{s(1-\eta)(2-\eta)}{\eta(2-\beta)}\right]\left[\frac{\eta}{\eta-s(1-\eta)}\right]^{1-\eta} \\ &= \frac{\eta-s(1-\eta)\frac{2-\eta}{2-\beta}}{\eta-s(1-\eta)}\left[\frac{\eta-s(1-\eta)}{\eta}\right]^{\eta}. \end{aligned} \tag{49}$$

From (47) and (48) the ability of the marginal optimistic entrepreneur is

$$\hat{\theta}_O=\left[\frac{\eta-s(1-\eta)}{\eta}\right]^{\eta}\left[\frac{\alpha}{2-\beta}\frac{1+\lambda\frac{s}{\eta}}{1-\lambda+\lambda\phi(\eta,\beta,s)}\right]^{\frac{1-\eta}{2-\eta}}.$$

From (41) and (44) we have

$$\left[(1-\lambda)\hat{\theta}_R+\lambda\hat{\theta}_O\right]\left(\frac{w}{\alpha}\right)^{\frac{1-\beta}{1-\eta}}\left(\frac{r}{\beta}\right)^{\frac{\beta}{1-\eta}}=\bar{K}\left(\frac{w}{\alpha}\right)^{\frac{\alpha}{1-\eta}}\left(\frac{r}{\beta}\right)^{\frac{1-\alpha}{1-\eta}},$$

or

$$\alpha r\bar{K} = \beta w\left[(1-\lambda)\hat{\theta}_R + \lambda\hat{\theta}_O\right],$$

or

$$r = \frac{\beta w}{\alpha\bar{K}}\left[1-\lambda+\lambda\psi(\alpha,\beta,s)\right]\hat{\theta}_R, \tag{50}$$

where

$$\psi(\eta,s) = \left[\frac{\eta - s(1-\eta)}{\eta}\right]^{\eta}. \tag{51}$$

Note that (49) and (51) together with $2-\eta \leq 2-\beta$ imply

$$\phi(\eta,\beta,s) \geq \psi(\eta,s). \tag{52}$$

Substituting (50) into (45) we obtain

$$\alpha^{\alpha}\beta^{\beta}(1-\eta)^{1-\eta}\hat{\theta}_R = w^{1-\beta}\left(\frac{\beta}{\alpha}\right)^{\beta} w^{\beta}\left[1-\lambda+\lambda\psi(\eta,s)\right]^{\beta}\hat{\theta}_R^{\beta}\bar{K}^{-\beta}.$$

Solving this equality with respect to w we obtain the equilibrium wage:

$$\begin{aligned} w^* &= \frac{\alpha^{\eta}(1-\eta)^{1-\eta}\bar{K}^{\beta}}{\left[1-\lambda+\lambda\psi(\eta,s)\right]^{\beta}}\hat{\theta}_R^{1-\beta} \\ &= \frac{\alpha^{\eta}(1-\eta)^{1-\eta}\bar{K}^{\beta}}{\left[1-\lambda+\lambda\psi(\eta,s)\right]^{\beta}}\left[\frac{\alpha}{2-\beta}\frac{1+\lambda\frac{s}{\eta}}{1-\lambda+\lambda\phi(\eta,\beta,s)}\right]^{\frac{(1-\eta)(1-\beta)}{2-\eta}}. \end{aligned}$$

The equilibrium rental cost of capital is equal to

$$\begin{aligned} r^* &= \frac{\beta w^*}{\alpha\bar{K}}\left[1-\lambda+\lambda\psi(\eta,s)\right]\hat{\theta}_R \\ &= \frac{\beta\alpha^{\eta}(1-\eta)^{1-\eta}\bar{K}^{\beta}\hat{\theta}_R^{1-\beta}}{\alpha\bar{K}\left[1-\lambda+\lambda\psi(\eta,s)\right]^{\beta}}\left[1-\lambda+\lambda\psi(\eta,s)\right]\hat{\theta}_R \\ &= \frac{\beta(1-\eta)^{1-\eta}}{\alpha^{1-\eta}\bar{K}^{1-\beta}}\left[1-\lambda+\lambda\psi(\eta,s)\right]^{1-\beta}\left[\frac{\alpha}{2-\beta}\frac{1+\lambda\frac{s}{\eta}}{1-\lambda+\lambda\phi(\eta,\beta,s)}\right]^{\frac{(1-\eta)(2-\beta)}{2-\eta}}. \end{aligned}$$

The equilibrium labor force is equal to

$$\begin{aligned} L^* &= (1-\lambda)\hat{\theta}_R + \lambda\hat{\theta}_O = \left\{1-\lambda+\lambda\left[\frac{\eta-s(1-\eta)}{\eta}\right]^{\eta}\right\}\hat{\theta}_R \\ &= \left[1-\lambda+\lambda\psi(\eta,s)\right]\left[\frac{\alpha}{2-\beta}\frac{1+\lambda\frac{s}{\eta}}{1-\lambda+\lambda\phi(\eta,\beta,s)}\right]^{\frac{1-\eta}{2-\eta}}. \end{aligned}$$

For the equilibrium to be well defined we need to make sure that $\hat{\theta}_O$ is less than $\frac{\eta - s(1-\eta)}{\eta}$, i.e.,

$$\left[\frac{\eta - s(1-\eta)}{\eta}\right]^{\eta}\left[\frac{\alpha}{2-\beta}\frac{1+\lambda\frac{s}{\eta}}{1-\lambda+\lambda\phi(\eta,\beta,s)}\right]^{\frac{1-\eta}{2-\eta}} < \frac{\eta - s(1-\eta)}{\eta}$$

or

$$\left[\frac{\alpha}{2-\beta}\frac{1+\lambda\frac{s}{\eta}}{1-\lambda+\lambda\phi(\eta,\beta,s)}\right]^{\frac{1-\eta}{2-\eta}} < \left[\frac{\eta - s(1-\eta)}{\eta}\right]^{1-\eta}$$

or

$$\frac{\alpha}{2-\beta}\frac{1+\lambda\frac{s}{\eta}}{1-\lambda+\lambda\phi(\eta,\beta,s)} < \left[\frac{\eta - s(1-\eta)}{\eta}\right]^{2-\eta}$$

or

$$\frac{\alpha}{2-\beta}\left(1+\lambda\frac{s}{\eta}\right) < (1-\lambda)\left[\frac{\eta - s(1-\eta)}{\eta}\right]^{2-\eta} + \lambda\left[1-\frac{s(1-\eta)(2-\eta)}{\eta(2-\beta)}\right]\left[\frac{\eta - s(1-\eta)}{\eta}\right]. \tag{53}$$

The LHS of (53) is increasing in s whereas the RHS of (53) is decreasing in s. When s is equal to 0 the LSH of (53) is equal to $\alpha/(2-\beta) < 1$ and the RHS of (53) is equal to 1. When s is equal to $\eta/(1-\eta)$ the LSH of (53) is equal to $\alpha(1-\eta+\lambda)/(2-\beta)(1-\eta)$ and the RHS of (53) is equal to 0. Hence, there exists a unique $s \in (0, \eta/(1-\eta))$ such that the LHS and RHS of (53) are the same, which is given by

$$\frac{\alpha}{2-\beta}\left(1+\lambda\frac{\bar{s}}{\eta}\right) = (1-\lambda)\left[\frac{\eta - \bar{s}(1-\eta)}{\eta}\right]^{2-\eta} + \lambda\left[1-\frac{\bar{s}(1-\eta)(2-\eta)}{\eta(2-\beta)}\right]\left[\frac{\eta - \bar{s}(1-\eta)}{\eta}\right].$$

or

$$\frac{\alpha}{2-\beta}\left(1+\lambda\frac{\bar{s}}{\eta}\right) = [1-\lambda+\lambda\phi(\eta,\beta,\bar{s})]\left[\frac{\eta - \bar{s}(1-\eta)}{\eta}\right]^{2-\eta},$$

which is (23). Hence, inequality (53) is satisfied as long as $s < \bar{s}$. For the equilibrium to be well defined we also need to make sure that $\hat{\theta}_R$ is less than 1. From (47) we have

$$\begin{aligned}\hat{\theta}_R &= \left[\frac{\eta}{\eta - s(1-\eta)}\right]^{\eta}\hat{\theta}_O \\ &< \left[\frac{\eta}{\eta - s(1-\eta)}\right]^{\eta}\frac{\eta - s(1-\eta)}{\eta} = \left[\frac{\eta - s(1-\eta)}{\eta}\right]^{1-\eta} < 1.\end{aligned}$$

where the first inequality follows $s \leq \bar{s}$. *Q.E.D.*

Proof of Proposition 3: Assume $s \in (\bar{s}, \eta/(1-\eta))$. The first step to determine the competitive equilibrium is to find out the labor market equilibrium condition. The labor demand from realistic entrepreneurs is given by (38). If $s > \bar{s}$, then the labor demand from optimistic entrepreneurs is

$$L_O^D = \lambda \int_{\hat{\theta}_O}^{1} l(w,r,1) d\theta_0 = \lambda(1-\hat{\theta}_O)\left(\frac{\alpha}{w}\right)^{\frac{1-\beta}{1-\eta}}\left(\frac{\beta}{r}\right)^{\frac{\beta}{1-\eta}}. \tag{54}$$

From (38) and (54), labor demand is equal to

$$\begin{aligned} L^D &= (1-\lambda)\frac{1-\eta}{2-\eta}\left(1-\hat{\theta}_R^{\frac{2-\eta}{1-\eta}}\right)\left(\frac{\alpha}{w}\right)^{\frac{1-\beta}{1-\eta}}\left(\frac{\beta}{r}\right)^{\frac{\beta}{1-\eta}} + \lambda(1-\hat{\theta}_O)\left(\frac{\alpha}{w}\right)^{\frac{1-\beta}{1-\eta}}\left(\frac{\beta}{r}\right)^{\frac{\beta}{1-\eta}} \\ &= \left[(1-\lambda)\frac{1-\eta}{2-\eta}\left(1-\hat{\theta}_R^{\frac{2-\eta}{1-\eta}}\right) + \lambda(1-\hat{\theta}_O)\right]\left(\frac{\alpha}{w}\right)^{\frac{1-\beta}{1-\eta}}\left(\frac{\beta}{r}\right)^{\frac{\beta}{1-\eta}}. \end{aligned}$$

Labor supply is given by

$$L^S = (1-\lambda)\int_0^{\hat{\theta}_R} d\theta_0 + \lambda \int_0^{\hat{\theta}_O} d\theta_0 = (1-\lambda)\hat{\theta}_R + \lambda\hat{\theta}_O.$$

In equilibrium, labor demand must equal labor supply

$$\left[(1-\lambda)\frac{1-\eta}{2-\eta}\left(1-\hat{\theta}_R^{\frac{2-\eta}{1-\eta}}\right) + \lambda(1-\hat{\theta}_O)\right]\left(\frac{\alpha}{w}\right)^{\frac{1-\beta}{1-\eta}}\left(\frac{\beta}{r}\right)^{\frac{\beta}{1-\eta}} = (1-\lambda)\hat{\theta}_R + \lambda\hat{\theta}_O. \tag{55}$$

The second step to determine the competitive equilibrium is to find out the capital market equilibrium condition. The capital demand from realistic entrepreneurs is given by (42). If $s > \bar{s}$, then the capital demand from optimistic entrepreneurs is

$$K_O^D = \lambda \int_{\hat{\theta}_O}^{1} k(w,r,1) d\theta_0 = \lambda(1-\hat{\theta}_O)\left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\eta}}\left(\frac{\beta}{r}\right)^{\frac{1-\alpha}{1-\eta}}. \tag{56}$$

From (42) and (56), capital demand is equal to

$$\begin{aligned} K^D &= (1-\lambda)\frac{1-\eta}{2-\eta}\left(1-\hat{\theta}_R^{\frac{2-\eta}{1-\eta}}\right)\left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\eta}}\left(\frac{\beta}{r}\right)^{\frac{1-\alpha}{1-\eta}} + \lambda(1-\hat{\theta}_O)\left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\eta}}\left(\frac{\beta}{r}\right)^{\frac{1-\alpha}{1-\eta}} \\ &= \left[(1-\lambda)\frac{1-\eta}{2-\eta}\left(1-\hat{\theta}_R^{\frac{2-\eta}{1-\eta}}\right) + \lambda(1-\hat{\theta}_O)\right]\left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\eta}}\left(\frac{\beta}{r}\right)^{\frac{1-\alpha}{1-\eta}}. \end{aligned}$$

In equilibrium, capital demand must equal the exogenous capital supply

$$\left[(1-\lambda)\frac{1-\eta}{2-\eta}\left(1-\hat{\theta}_R^{\frac{2-\eta}{1-\eta}}\right)+\lambda(1-\hat{\theta}_O)\right]\left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\eta}}\left(\frac{\beta}{r}\right)^{\frac{1-\alpha}{1-\eta}}=\bar{K}. \tag{57}$$

The third step to determine the competitive equilibrium is to find out $\hat{\theta}_R$ and $\hat{\theta}_O$. A realist with entrepreneurial ability $\hat{\theta}_R$ is indifferent between being an entrepreneur and a worker at $t=1$ when (45) holds:

$$\alpha^\alpha\beta^\beta(1-\eta)^{1-\eta}\hat{\theta}_R=w^{1-\beta}r^\beta. \tag{58}$$

An optimist with entrepreneurial ability $\hat{\theta}_O$ and expectation of entrepreneurial ability $\theta^*=1$ is indifferent between being an entrepreneur and a worker at $t=1$ when

$$(\hat{\theta}_O+s\theta^*)[l(w,r,\theta^*)]^\alpha[k(w,r,\theta^*)]^\beta-(1+s)\left[wl(w,r,\theta^*)+rk(w,r,\theta^*)\right]=w,$$

or

$$(\hat{\theta}_O+s)\left[\left(\frac{\alpha}{w}\right)^{\frac{1-\beta}{1-\eta}}\left(\frac{\beta}{r}\right)^{\frac{\beta}{1-\eta}}\right]^\alpha\left[\left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\eta}}\left(\frac{\beta}{r}\right)^{\frac{1-\alpha}{1-\eta}}\right]^\beta$$
$$-(1+s)\left[w\left(\frac{\alpha}{w}\right)^{\frac{1-\beta}{1-\eta}}\left(\frac{\beta}{r}\right)^{\frac{\beta}{1-\eta}}+r\left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\eta}}\left(\frac{\beta}{r}\right)^{\frac{1-\alpha}{1-\eta}}\right]=w,$$

or

$$(\hat{\theta}_O+s)\left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\eta}}\left(\frac{\beta}{r}\right)^{\frac{\beta}{1-\eta}}-(1+s)\left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\eta}}\left(\frac{\beta}{r}\right)^{\frac{\beta}{1-\eta}}\left[w\left(\frac{\alpha}{w}\right)+r\left(\frac{\beta}{r}\right)\right]=w,$$

or

$$\left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\eta}}\left(\frac{\beta}{r}\right)^{\frac{\beta}{1-\eta}}\left[\hat{\theta}_O+s-(1+s)\eta\right]=w,$$

or

$$\alpha^{\frac{\alpha}{1-\eta}}\beta^{\frac{\beta}{1-\eta}}\left[\hat{\theta}_O+s-(1+s)\eta\right]=w^{\frac{1-\beta}{1-\eta}}r^{\frac{\beta}{1-\eta}},$$

or

$$\alpha^\alpha\beta^\beta\left[\hat{\theta}_O+s-(1+s)\eta\right]^{1-\eta}=w^{1-\beta}r^\beta. \tag{59}$$

Equations (55), (57), (58), and (59) define the competitive equilibrium when $s \in (\bar{s}, \eta/(1-\eta))$. *Q.E.D.*

Proof of Proposition 4:

(i) The probability an optimist becomes an entrepreneur is equal to

$$\Pr(E^*|O) = \frac{E_O^*}{\lambda} = \frac{\lambda(1-\hat{\theta}_O)}{\lambda} = 1 - \hat{\theta}_O,$$

and the probability a realist becomes an entrepreneur to

$$\Pr(E^*|R) = \frac{E_R^*}{1-\lambda} = \frac{(1-\lambda)(1-\hat{\theta}_R)}{1-\lambda} = 1 - \hat{\theta}_R.$$

Hence, optimists are more likely to become entrepreneurs than realists as long as

$$\Pr(E^*|O) > \Pr(E^*|R),$$

or

$$1 - \hat{\theta}_O > 1 - \hat{\theta}_R,$$

or

$$\hat{\theta}_O < \hat{\theta}_R,$$

which is true by (25).

(ii) Entrepreneurs are more likely to be optimists than workers if

$$\Pr(O|E^*) > \Pr(O|L^*),$$

or

$$\frac{\lambda(1-\hat{\theta}_O)}{\lambda(1-\hat{\theta}_O) + (1-\lambda)(1-\hat{\theta}_R)} > \frac{\lambda\hat{\theta}_O}{\lambda\hat{\theta}_O + (1-\lambda)\hat{\theta}_R},$$

or

$$\lambda\hat{\theta}_O(1-\hat{\theta}_O) + (1-\lambda)(1-\hat{\theta}_O)\hat{\theta}_R > \lambda(1-\hat{\theta}_O)\hat{\theta}_O + (1-\lambda)(1-\hat{\theta}_R)\hat{\theta}_O,$$

or

$$(1-\hat{\theta}_O)\hat{\theta}_R > (1-\hat{\theta}_R)\hat{\theta}_O,$$

or

$$\hat{\theta}_R > \hat{\theta}_O,$$

which is true by (25).

(iii) The majority of entrepreneurs are optimists if

$$\gamma_E^* = \frac{\lambda(1-\hat{\theta}_O)}{\lambda(1-\hat{\theta}_O)+(1-\lambda)(1-\hat{\theta}_R)} > \frac{1}{2},$$

or

$$2\lambda(1-\hat{\theta}_O) > \lambda(1-\hat{\theta}_O)+(1-\lambda)(1-\hat{\theta}_R),$$

or

$$\lambda(1-\hat{\theta}_O)+\lambda(1-\hat{\theta}_R) > 1-\hat{\theta}_R,$$

or

$$\lambda > \frac{1-\hat{\theta}_R}{2-\hat{\theta}_R-\hat{\theta}_O} = \frac{1-\hat{\theta}_R}{1-\hat{\theta}_R+1-\hat{\theta}_R\left[\frac{\eta-s(1-\eta)}{\eta}\right]^{\eta}} = \left[1+\frac{1-\hat{\theta}_R\left[\frac{\eta-s(1-\eta)}{\eta}\right]^{\eta}}{1-\hat{\theta}_R}\right]^{-1}.$$

Q.E.D.

Proof of Proposition 5: Assume $s < \bar{s}$. We wish to show that

$$\frac{\partial\hat{\theta}_R}{\partial\lambda} > 0.$$

From the definition of $\hat{\theta}_R$ we have

$$\begin{aligned}
\frac{\partial\hat{\theta}_R}{\partial\lambda} &= \frac{1-\eta}{2-\eta}\left(\frac{\alpha}{2-\beta}\right)^{\frac{1-\eta}{2-\eta}}\left(\frac{1+\frac{\lambda s}{\eta}}{1-\lambda+\lambda\phi}\right)^{-\frac{1}{2-\eta}}\frac{\partial}{\partial\lambda}\left(\frac{1+\frac{\lambda s}{\eta}}{1-\lambda+\lambda\phi}\right) \\
&= \frac{1-\eta}{2-\eta}\left(\frac{\alpha}{2-\beta}\right)^{\frac{1-\eta}{2-\eta}}\left(\frac{1+\frac{\lambda s}{\eta}}{1-\lambda+\lambda\phi}\right)^{-\frac{1}{2-\eta}}\frac{1+\frac{s}{\eta}-\phi}{(1-\lambda+\lambda\phi)^2} \\
&= \frac{1-\eta}{2-\eta}\hat{\theta}_R\frac{1+\frac{s}{\eta}-\phi}{(1+\frac{\lambda s}{\eta})(1-\lambda+\lambda\phi)}. \qquad (60)
\end{aligned}$$

Hence, $\partial\hat{\theta}_R/\partial\lambda > 0$ as long as

$$1 + \frac{s}{\eta} > \phi = \left[1 - \frac{s(1-\eta)(2-\eta)}{\eta(2-\beta)}\right]\left[\frac{\eta}{\eta - s(1-\eta)}\right]^{1-\eta}. \tag{61}$$

Note that $\phi > 0$ and $\phi(\eta, \beta, 0) = 1$. The derivative of ϕ with respect to s is equal to

$$\begin{aligned}
\frac{\partial\phi}{\partial s} &= -\frac{(1-\eta)(2-\eta)}{\eta(2-\beta)}\left[\frac{\eta}{\eta - s(1-\eta)}\right]^{1-\eta} \\
&\quad + \left[1 - \frac{s(1-\eta)(2-\eta)}{\eta(2-\beta)}\right]\left[\frac{\eta}{\eta - s(1-\eta)}\right]^{1-\eta}\frac{(1-\eta)^2}{\eta - s(1-\eta)} \\
&= \frac{(1-\eta)\left[(2-\beta)(1-\eta) - (2-\eta) + s(1-\eta)(2-\eta)\right]}{[\eta - s(1-\eta)](2-\beta)}\left[\frac{\eta}{\eta - s(1-\eta)}\right]^{1-\eta} \\
&= \frac{(1-\eta)\left[-\eta - \beta(1-\eta) + s(1-\eta)(2-\eta)\right]}{[\eta - s(1-\eta)](2-\beta)}\left[\frac{\eta}{\eta - s(1-\eta)}\right]^{1-\eta}. \qquad (62)
\end{aligned}$$

It follows from (62) that: (i) ϕ decreases with s when $s \in [0, \breve{s})$, (ii) ϕ increases with s when $s \in (\breve{s}, \eta/(1-\eta))$, and (iii) ϕ attains a minimum at $\breve{s} = \frac{\eta+\beta(1-\eta)}{(1-\eta)(2-\eta)} < \frac{\eta}{1-\eta}$ which is given by

$$\begin{aligned}
\phi(\breve{s}) &= \left[1 - \frac{\frac{\eta+\beta(1-\eta)}{(1-\eta)(2-\eta)}(1-\eta)(2-\eta)}{\eta(2-\beta)}\right]\left[\frac{\eta}{\eta - \frac{\eta+\beta(1-\eta)}{(1-\eta)(2-\eta)}(1-\eta)}\right]^{1-\eta} \\
&= \left[1 - \frac{\eta+\beta(1-\eta)}{\eta(2-\beta)}\right]\left[\frac{\eta}{\eta - \frac{\eta+\beta(1-\eta)}{2-\eta}}\right]^{1-\eta} \\
&= \frac{\eta(2-\beta) - \eta - \beta(1-\eta)}{\eta(2-\beta)}\left[\frac{(2-\eta)\eta}{(2-\eta)\eta - \eta - \beta(1-\eta)}\right]^{1-\eta} \\
&= \frac{\eta - \beta}{\eta(2-\beta)}\left[\frac{(2-\eta)\eta}{(1-\eta)(\eta-\beta)}\right]^{1-\eta} = \frac{\alpha}{\eta(2-\beta)}\left[\frac{(2-\eta)\eta}{(1-\eta)\alpha}\right]^{1-\eta}.
\end{aligned}$$

This implies that

$$\max_{s\in[0,\bar{s}]} \phi(s) = \max\{1, \phi(\bar{s})\}.$$

We know from the definition of $\bar{s}$ that

$$\phi(\bar{s}) = \frac{\frac{\alpha}{2-\beta}\left(1 + \frac{\lambda\bar{s}}{\eta}\right)}{\lambda\left[\frac{\eta - \bar{s}(1-\eta)}{\eta}\right]^{2-\eta}} - \frac{1-\lambda}{\lambda}.$$

If we can show that $1 + \frac{\bar{s}}{\eta} > \phi(\bar{s})$ we are done:

$$1 + \frac{\bar{s}}{\eta} > \frac{\frac{\alpha}{2-\beta}\left(1 + \frac{\lambda\bar{s}}{\eta}\right)}{\lambda\left[\frac{\eta - \bar{s}(1-\eta)}{\eta}\right]^{2-\eta}} - \frac{1-\lambda}{\lambda},$$

or

$$1 + \frac{\lambda\bar{s}}{\eta} > \frac{\frac{\alpha}{2-\beta}\left(1 + \frac{\lambda\bar{s}}{\eta}\right)}{\left[\frac{\eta - \bar{s}(1-\eta)}{\eta}\right]^{2-\eta}},$$

or

$$\left[\frac{\eta - \bar{s}(1-\eta)}{\eta}\right]^{2-\eta} > \frac{\alpha}{2-\beta},$$

or

$$\eta - \bar{s}(1-\eta) > \eta\left(\frac{\alpha}{2-\beta}\right)^{\frac{1}{2-\eta}},$$

or

$$\bar{s} < \frac{\eta}{1-\eta}\left[1 - \left(\frac{\alpha}{2-\beta}\right)^{\frac{1}{2-\eta}}\right].$$

From the definition of $\bar{s}$ this inequality is satisfied if

$$\frac{\alpha}{2-\beta}\left[1 + \frac{\lambda}{1-\eta} - \frac{\lambda}{1-\eta}\left(\frac{\alpha}{2-\beta}\right)^{\frac{1}{2-\eta}}\right] >$$
$$(1-\lambda)\left(\frac{\alpha}{2-\beta}\right) + \lambda\left\{1 - \left[1 - \left(\frac{\alpha}{2-\beta}\right)^{\frac{1}{2-\eta}}\right]\frac{2-\eta}{2-\beta}\right\}\left(\frac{\alpha}{2-\beta}\right)^{\frac{1}{2-\eta}},$$

or

$$\left[1 + \frac{1}{1-\eta} - \frac{1}{1-\eta}\left(\frac{\alpha}{2-\beta}\right)^{\frac{1}{2-\eta}}\right]\left(\frac{\alpha}{2-\beta}\right)^{\frac{1-\eta}{2-\eta}} >$$
$$1 - \left[1 - \left(\frac{\alpha}{2-\beta}\right)^{\frac{1}{2-\eta}}\right]\frac{2-\eta}{2-\beta},$$

or

$$\left[1 + \frac{1}{1-\eta} - \frac{1}{1-\eta}\left(\frac{\alpha}{2-\beta}\right)^{\frac{1}{2-\eta}}\right]\left(\frac{\alpha}{2-\beta}\right)^{\frac{1-\eta}{2-\eta}} >$$
$$1 - \frac{2-\eta}{2-\beta} + \frac{2-\eta}{2-\beta}\left(\frac{\alpha}{2-\beta}\right)^{\frac{1}{2-\eta}},$$

or

$$\left[1+\frac{1}{1-\eta}-\frac{1}{1-\eta}\left(\frac{\alpha}{2-\beta}\right)^{\frac{1}{2-\eta}}\right]\left(\frac{\alpha}{2-\beta}\right)^{\frac{1-\eta}{2-\eta}}>\frac{\alpha}{2-\beta}+\frac{2-\eta}{2-\beta}\left(\frac{\alpha}{2-\beta}\right)^{\frac{1}{2-\eta}},$$

or

$$1+\frac{1}{1-\eta}-\frac{1}{1-\eta}\left(\frac{\alpha}{2-\beta}\right)^{\frac{1}{2-\eta}}>\left[\frac{\alpha}{2-\beta}+\frac{2-\eta}{2-\beta}\left(\frac{\alpha}{2-\beta}\right)^{\frac{1}{2-\eta}}\right]\left(\frac{\alpha}{2-\beta}\right)^{-\frac{1-\eta}{2-\eta}},$$

or

$$1+\frac{1}{1-\eta}-\frac{1}{1-\eta}\left(\frac{\alpha}{2-\beta}\right)^{\frac{1}{2-\eta}}>\left(\frac{\alpha}{2-\beta}\right)^{\frac{1}{2-\eta}}+\frac{2-\eta}{2-\beta}\left(\frac{\alpha}{2-\beta}\right)^{\frac{\eta}{2-\eta}},$$

or

$$\frac{2-\eta}{1-\eta}\left[1-\left(\frac{\alpha}{2-\beta}\right)^{\frac{1}{2-\eta}}\right]>\frac{2-\eta}{2-\beta}\left(\frac{\alpha}{2-\beta}\right)^{\frac{\eta}{2-\eta}},$$

or

$$\frac{2-\eta}{1-\eta}\left[1-\left(\frac{\alpha}{2-\beta}\right)^{\frac{1}{2-\eta}}\right]>\frac{2-\alpha-\beta}{2-\beta}\left(\frac{\alpha}{2-\beta}\right)^{\frac{\eta}{2-\eta}},$$

or

$$\frac{2-\eta}{1-\eta}\left[1-\left(\frac{\alpha}{2-\beta}\right)^{\frac{1}{2-\eta}}\right]>\left(\frac{\alpha}{2-\beta}\right)^{\frac{\eta}{2-\eta}}-\left(\frac{\alpha}{2-\beta}\right)^{\frac{2}{2-\eta}},$$

or

$$\frac{2-\eta}{1-\eta}\left[1-\left(\frac{\alpha}{2-\beta}\right)^{\frac{1}{2-\eta}}\right]-\left(\frac{\alpha}{2-\beta}\right)^{\frac{\eta}{2-\eta}}+\left(\frac{\alpha}{2-\beta}\right)^{\frac{2}{2-\eta}}>0,$$

or

$$\left[1-\left(\frac{\alpha}{2-\beta}\right)^{\frac{1}{2-\eta}}\right]+\frac{1}{1-\eta}\left[1-\left(\frac{\alpha}{2-\beta}\right)^{\frac{1}{2-\eta}}\right]-\left(\frac{\alpha}{2-\beta}\right)^{\frac{\eta}{2-\eta}}+\left(\frac{\alpha}{2-\beta}\right)^{\frac{2}{2-\eta}}>0,$$

or

$$\left[1-\left(\frac{\alpha}{2-\beta}\right)^{\frac{\eta}{2-\eta}}\right]+\left\{\frac{1}{1-\eta}\left[1-\left(\frac{\alpha}{2-\beta}\right)^{\frac{1}{2-\eta}}\right]-\left(\frac{\alpha}{2-\beta}\right)^{\frac{1}{2-\eta}}\right\}+\left(\frac{\alpha}{2-\beta}\right)^{\frac{2}{2-\eta}}>0.$$

This inequality holds because the three terms inside brackets in the LHS are strictly positive. Hence, we have shown $\partial\hat{\theta}_R/\partial\lambda > 0$. Let us now show that

$$\frac{\partial\hat{\theta}_O}{\partial\lambda}>0.$$

We know from Proposition 2 that

$$\hat{\theta}_O = \left[\frac{\eta - s(1-\eta)}{\eta}\right]^{\eta} \hat{\theta}_R.$$

Hence

$$\frac{\partial \hat{\theta}_O}{\partial \lambda} = \left[\frac{\eta - s(1-\eta)}{\eta}\right]^{\eta} \frac{\partial \hat{\theta}_R}{\partial \lambda} > 0,$$

where the inequality follows from the fact that $\partial \hat{\theta}_R / \partial \lambda > 0$. *Q.E.D.*

Proof of Proposition 6: Let $s < \bar{s}$. The wage is equal to

$$\begin{aligned} w^* &= \frac{\alpha^{\eta}(1-\eta)^{1-\eta}\bar{K}^{\beta}}{(1-\lambda+\lambda\psi)^{\beta}} \left(\frac{\alpha}{2-\beta}\frac{1+\lambda\frac{s}{\eta}}{1-\lambda+\lambda\phi}\right)^{\frac{(1-\eta)(1-\beta)}{2-\eta}} \\ &= \frac{\alpha^{\eta}(1-\eta)^{1-\eta}\bar{K}^{\beta}}{(1-\lambda+\lambda\psi)^{\beta}} \hat{\theta}_R^{1-\beta}. \end{aligned}$$

The impact of a change in λ on w^* is given by

$$\begin{aligned} \frac{\partial w^*}{\partial \lambda} &= \beta\alpha^{\eta}(1-\eta)^{1-\eta}\bar{K}^{\beta}(1-\lambda+\lambda\psi)^{-\beta-1}(1-\psi)\hat{\theta}_R^{1-\beta} \\ &+ (1-\beta)\frac{\alpha^{\eta}(1-\eta)^{1-\eta}\bar{K}^{\beta}}{(1-\lambda+\lambda\psi)^{\beta}}\hat{\theta}_R^{-\beta}\frac{\partial \hat{\theta}_R}{\partial \lambda} \\ &= \frac{\alpha^{\eta}(1-\eta)^{1-\eta}\bar{K}^{\beta}}{(1-\lambda+\lambda\psi)^{\beta}\hat{\theta}_R^{\beta}} \left[\beta\frac{1-\psi}{1-\lambda+\lambda\psi}\hat{\theta}_R + (1-\beta)\frac{\partial \hat{\theta}_R}{\partial \lambda}\right] > 0. \end{aligned}$$

Hence, an increase in λ raises the wage. *Q.E.D.*

Proof of Proposition 7: Let $s < \bar{s}$.

(i) The fraction of realistic workers is $L_R^* = (1-\lambda)\hat{\theta}_R$. Hence,

$$\begin{aligned} \frac{\partial L_R^*}{\partial \lambda} &= -\hat{\theta}_R + (1-\lambda)\frac{\partial \hat{\theta}_R}{\partial \lambda} \\ &= -\hat{\theta}_R + (1-\lambda)\frac{1-\eta}{2-\eta}\hat{\theta}_R \frac{1+\frac{s}{\eta}-\phi}{(1+\frac{\lambda s}{\eta})(1-\lambda+\lambda\phi)} \\ &= \left[-1 + (1-\lambda)\frac{1-\eta}{2-\eta}\frac{1+\frac{s}{\eta}-\phi}{(1+\frac{\lambda s}{\eta})(1-\lambda+\lambda\phi)}\right]\hat{\theta}_R, \end{aligned}$$

where the second equality follows from (60). Therefore $\partial L_R^*/\partial\lambda < 0$ when

$$(1-\lambda)\frac{1-\eta}{2-\eta}\frac{1+\frac{s}{\eta}-\phi}{\left(1+\frac{\lambda s}{\eta}\right)(1-\lambda+\lambda\phi)} < 1,$$

or

$$(1-\eta)(1-\lambda)\left(1+\frac{s}{\eta}-\phi\right) < (2-\eta)(1-\lambda+\lambda\phi)\left(1+\frac{\lambda s}{\eta}\right).$$

Simplifying and rearranging terms this inequality is equivalent to

$$\begin{aligned} 0 \;&<\; -\lambda(1-\phi)+s\left(2\lambda-\lambda^2\right)\left(\frac{1}{\eta}-1\right)+s\left(\lambda-\lambda^2\right)\frac{1}{\eta} \\ &\quad +s\lambda^2\left(\frac{2}{\eta}-1\right)\phi+\left[1-s\left(\frac{1}{\eta}-1\right)\right]+\phi(1-\eta). \end{aligned}$$

Since $s < \eta/(\eta-1)$ all terms in the RHS of the inequality are non-negative when $\phi \geq 1$. Hence the inequality is satisfied when $\phi \in [1, 1+s/\eta]$. We now show that the inequality is also satisfied when $\phi \in (0,1)$. When $\phi \in (0,1)$ the RHS is a concave function of λ (the second derivative of the RHS with respect to λ is equal to $-2(1-\phi)(2-\eta)\frac{s}{\eta}$). Hence, the RHS attains a minimum either at $\lambda = 0$ or at $\lambda = 1$. When $\lambda = 0$ the inequality becomes

$$0 < \left[1-s\left(\frac{1}{\eta}-1\right)\right]+\phi(1-\eta),$$

which is true. When $\lambda = 1$ the inequality becomes

$$0 < \phi+s\left(\frac{2}{\eta}-1\right)\phi+\phi(1-\eta),$$

which is true. Therefore, when $\phi \in (0,1)$ the inequality is satisfied. Hence, we have shown that $\partial L_R^*/\partial\lambda < 0$.

(ii) The fraction of optimistic workers is $L_O^* = \lambda\hat{\theta}_O$, which is an increasing function of λ since both terms increase with λ. Therefore, $\partial L_O^*/\partial\lambda > 0$.

(iii) The fraction of realistic entrepreneurs is $E_R^* = (1-\lambda)(1-\hat{\theta}_R)$, which is a decreasing function of λ since both terms inside brackets decrease with λ. Therefore, $\partial E_R^*/\partial\lambda < 0$. *Q.E.D.*

Proof of Proposition 8: Let $s < \bar{s}$.

(i) We wish to show that the equilibrium fraction of workers L^* is a concave function of λ. The equilibrium fraction of workers is equal to

$$L^* = (1-\lambda)\hat{\theta}_R + \lambda\hat{\theta}_O = (1-\lambda+\lambda\psi)\hat{\theta}_R.$$

The impact on L^* of a change in λ is given by

$$\frac{\partial L^*}{\partial \lambda} = -(1-\psi)\hat{\theta}_R + (1-\lambda+\lambda\psi)\frac{\partial \hat{\theta}_R}{\partial \lambda}. \tag{63}$$

From (63) it follows that

$$\frac{\partial^2 L^*}{\partial \lambda^2} = -2(1-\psi)\frac{\partial \hat{\theta}_R}{\partial \lambda} + (1-\lambda+\lambda\psi)\frac{\partial^2 \hat{\theta}_R}{\partial \lambda^2}. \tag{64}$$

We know from Proposition 5 that

$$\begin{aligned}
\frac{\partial \hat{\theta}_R}{\partial \lambda} &= \frac{\partial}{\partial \lambda}\left[\left(\frac{\alpha}{2-\beta}\frac{1+\lambda\frac{s}{\eta}}{1-\lambda+\lambda\phi}\right)^{\frac{1-\eta}{2-\eta}}\right] \\
&= \frac{1-\eta}{2-\eta}\left(\frac{\alpha}{2-\beta}\frac{1+\lambda\frac{s}{\eta}}{1-\lambda+\lambda\phi}\right)^{\frac{1-\eta}{2-\eta}-1}\frac{\alpha}{2-\beta}\frac{\partial}{\partial\lambda}\left(\frac{1+\lambda\frac{s}{\eta}}{1-\lambda+\lambda\phi}\right) \\
&= \frac{1-\eta}{2-\eta}\left(\frac{\alpha}{2-\beta}\right)^{\frac{1-\eta}{2-\eta}}\left(\frac{1+\frac{\lambda s}{\eta}}{1-\lambda+\lambda\phi}\right)^{-\frac{1}{2-\eta}}\frac{1+\frac{s}{\eta}-\phi}{(1-\lambda+\lambda\phi)^2}.
\end{aligned} \tag{65}$$

From (65) we obtain

$$\begin{aligned}
\frac{\partial^2 \hat{\theta}_R}{\partial \lambda^2} &= -\frac{z}{2-\eta}\left(\frac{1+\frac{\lambda s}{\eta}}{1-\lambda+\lambda\phi}\right)^{-\frac{1}{2-\eta}-1}\frac{(1+\frac{s}{\eta}-\phi)^2}{(1-\lambda+\lambda\phi)^4} \\
&\quad +2z\left(\frac{1+\frac{\lambda s}{\eta}}{1-\lambda+\lambda\phi}\right)^{-\frac{1}{2-\eta}}\frac{\left(1+\frac{s}{\eta}-\phi\right)(1-\phi)}{(1-\lambda+\lambda\phi)^3} \\
&= z\left(\frac{1+\frac{\lambda s}{\eta}}{1-\lambda+\lambda\phi}\right)^{-\frac{1}{2-\eta}}\frac{1+\frac{s}{\eta}-\phi}{(1-\lambda+\lambda\phi)^3}\left[-\frac{1}{2-\eta}\frac{1+\frac{s}{\eta}-\phi}{1+\frac{\lambda s}{\eta}}+2(1-\phi)\right] \\
&= \frac{z}{1-\lambda+\lambda\phi}\left[-\frac{1}{2-\eta}\frac{1+\frac{s}{\eta}-\phi}{1+\frac{\lambda s}{\eta}}+2(1-\phi)\right]\frac{\partial \hat{\theta}_R}{\partial \lambda},
\end{aligned} \tag{66}$$

where

$$z = \frac{1-\eta}{2-\eta}\left(\frac{\alpha}{2-\beta}\right)^{\frac{1-\eta}{2-\eta}}.$$

Substituting (65) and (66) into (64) we obtain

$$\frac{\partial^2 L^*}{\partial \lambda^2} = z\left\{-2(1-\psi) + \frac{1-\lambda+\lambda\psi}{1-\lambda+\lambda\phi}\left[-\frac{1}{2-\eta}\frac{1+\frac{s}{\eta}-\phi}{1+\frac{\lambda s}{\eta}} + 2(1-\phi)\right]\right\}\frac{\partial \hat{\theta}_R}{\partial \lambda}.$$

Since $\partial\hat{\theta}_R/\partial\lambda > 0$ it follows that $\partial^2 L^*/\partial\lambda^2 < 0$ as long as

$$-2(1-\psi) + \frac{1-\lambda+\lambda\psi}{1-\lambda+\lambda\phi}\left[-\frac{1}{2-\eta}\frac{1+\frac{s}{\eta}-\phi}{1+\frac{\lambda s}{\eta}} + 2(1-\phi)\right] < 0.$$

When $\phi \in [1, 1+s/\eta]$, the second term on the LHS is non-positive and $\partial^2 L^*/\partial\lambda^2 < 0$. When $\phi \in (0,1)$ a sufficient condition for $\partial^2 L^*/\partial\lambda^2 < 0$ is

$$\frac{1-\psi}{1-\phi} \geq \frac{1-\lambda+\lambda\psi}{1-\lambda+\lambda\phi}. \tag{67}$$

This inequality is satisfied since $\psi \leq \phi < 1$–see (52)–implies that the LHS of (67) is greater than or equal to 1 and the RHS of (67) is less than or equal to 1. Hence, L^* is a concave function of λ.

(ii) and (iii) We start by showing that

$$\left.\frac{\partial L^*}{\partial \lambda}\right|_{\lambda=0} > 0,$$

which implies that L^* is not a decreasing function of λ. From (63) we have

$$\begin{aligned}\left.\frac{\partial L^*}{\partial \lambda}\right|_{\lambda=0} &= -(1-\psi)\left.\hat{\theta}_R\right|_{\lambda=0} + \left.(1-\lambda+\lambda\psi)\right|_{\lambda=0}\left.\frac{\partial\hat{\theta}_R}{\partial\lambda}\right|_{\lambda=0} \\ &= -(1-\psi)\hat{\theta}_0 + \frac{1-\eta}{2-\eta}\hat{\theta}_0\left(1+\frac{s}{\eta}-\phi\right) \\ &= \left[\frac{1-\eta}{2-\eta}\left(1+\frac{s}{\eta}-\phi\right) - (1-\psi)\right]\hat{\theta}_0.\end{aligned}$$

This derivative is positive as long as

$$(1-\eta)\left(1+\frac{s}{\eta}-\phi\right) > (2-\eta)(1-\psi),$$

or

$$\phi\eta - \phi - \eta - s + \frac{s}{\eta} + 1 > \psi\eta - \eta - 2\psi + 2,$$

or

$$\phi\eta - \phi - s + \frac{s}{\eta} > \psi\eta - 2\psi + 1,$$

or

$$\phi\eta - \phi - s + \frac{s}{\eta} - \psi\eta + \psi + \psi - 1 > 0,$$

or

$$-s + \frac{s}{\eta} - \frac{s(1-\eta)^2}{\eta - s(1-\eta)}\frac{\alpha}{2-\beta}\psi + \psi - 1 > 0,$$

or

$$\frac{s(1-\eta)}{\eta} > \frac{s(1-\eta)^2}{\eta - s(1-\eta)}\frac{\alpha}{2-\beta}\psi + (1-\psi),$$

or

$$\psi > \frac{s(1-\eta)^2}{\eta - s(1-\eta)}\frac{\alpha}{2-\beta}\psi + \frac{\eta - s(1-\eta)}{\eta},$$

or

$$1 > \frac{s(1-\eta)^2}{\eta - s(1-\eta)}\frac{\alpha}{2-\beta} + \left[\frac{\eta - s(1-\eta)}{\eta}\right]^{1-\eta},$$

We know from Proposition 5 that

$$\frac{\alpha}{2-\beta} < \left[\frac{\eta - \bar{s}(1-\eta)}{\eta}\right]^{2-\eta} < \left[\frac{\eta - s(1-\eta)}{\eta}\right]^{2-\eta},$$

where the second inequality follows from $s < \bar{s}$. Therefore the inequality is satisfied as long as

$$1 > \frac{s(1-\eta)^2}{\eta}\left[\frac{\eta - s(1-\eta)}{\eta}\right]^{1-\eta} + \left[\frac{\eta - s(1-\eta)}{\eta}\right]^{1-\eta},$$

or

$$1 > \left[\frac{s(1-\eta)^2}{\eta} + 1\right]\left[\frac{\eta - s(1-\eta)}{\eta}\right]^{1-\eta}$$

Note that if $s = 0$ the RHS of the inequality is equal to 1. We now show that the RHS is decreasing with s which implies that the inequality is satisfied for any $s \in (0, \bar{s})$.

The derivative of the RHS with respect to s is:

$$\begin{aligned}
& \frac{(1-\eta)^2}{\eta}\left[\frac{\eta-s(1-\eta)}{\eta}\right]^{1-\eta}-\frac{(1-\eta)^2}{\eta}\left[\frac{s(1-\eta)^2}{\eta}+1\right]\left[\frac{\eta-s(1-\eta)}{\eta}\right]^{-\eta} \\
= & \frac{(1-\eta)^2}{\eta}\left[\frac{\eta-s(1-\eta)}{\eta}\right]^{-\eta}\left[\frac{\eta-s(1-\eta)}{\eta}-\frac{s(1-\eta)^2}{\eta}-1\right] \\
= & -\frac{(1-\eta)^2}{\eta}\left[\frac{\eta-s(1-\eta)}{\eta}\right]^{-\eta}\left[\frac{s(1-\eta)}{\eta}+\frac{s(1-\eta)^2}{\eta}\right]<0.
\end{aligned}$$

Therefore, L^* is not a decreasing function of λ. Thus, we are left with two cases: (1) L^* is an increasing and concave function of λ (and $L^*(\lambda)$ attains a maximum at $\lambda=1$), and (2) L^* is a concave function of λ which attains a maximum at $\bar{\lambda}\in(0,1)$. Case (1) happens when

$$\begin{aligned}
\left.\frac{\partial L^*}{\partial\lambda}\right|_{\lambda=1} &= -(1-\psi)\left.\hat{\theta}_R\right|_{\lambda=1}+(1-\lambda+\lambda\psi)|_{\lambda=1}\left.\frac{\partial\hat{\theta}_R}{\partial\lambda}\right|_{\lambda=1} \\
&= \left[-(1-\psi)+\frac{\psi}{\phi}\frac{1-\eta}{2-\eta}\frac{1+\frac{s}{\eta}-\phi}{1+\frac{s}{\eta}}\right]\left(\frac{\alpha}{2-\beta}\frac{1+\frac{s}{\eta}}{\phi}\right)^{\frac{1-\eta}{2-\eta}}>0.
\end{aligned}$$

This condition is satisfied when

$$(1-\eta)\left(1+\frac{s}{\eta}-\phi\right)\psi>(2-\eta)(1-\psi)\left(1+\frac{s}{\eta}\right)\phi. \tag{68}$$

Case (2) happens when (68) is violated. From (63) and the fact that $\partial^2L^*/\partial\lambda^2<0$ it follows that in case (2) L^* attains a maximum at the λ which solves

$$(1-\lambda+\lambda\psi)\frac{\partial\hat{\theta}_R}{\partial\lambda}=(1-\psi)\hat{\theta}_R. \tag{69}$$

Note that

$$\frac{\partial\hat{\theta}_R}{\partial\lambda}=\frac{1-\eta}{2-\eta}\hat{\theta}_R\frac{1}{1+\lambda\frac{s}{\eta}}\frac{1+\frac{s}{\eta}-\phi}{1-\lambda+\lambda\phi}. \tag{70}$$

Substituting (70) into (69) we obtain

$$(1-\lambda+\lambda\psi)\frac{1-\eta}{2-\eta}\hat{\theta}_R\frac{1}{1+\lambda\frac{s}{\eta}}\frac{1+\frac{s}{\eta}-\phi}{1-\lambda+\lambda\phi}=(1-\psi)\hat{\theta}_R,$$

or

$$\frac{1-\eta}{2-\eta}\frac{1+\frac{s}{\eta}-\phi}{1+\lambda\frac{s}{\eta}}\frac{1-\lambda+\lambda\psi}{1-\lambda+\lambda\phi}=1-\psi,$$

or

$$\frac{1-\lambda+\lambda\psi}{1-\lambda+\lambda\phi}=\frac{2-\eta}{1-\eta}\frac{1-\psi}{1+\frac{s}{\eta}-\phi}\left(1+\lambda\frac{s}{\eta}\right). \tag{71}$$

Hence, L^* attains a maximum at the $\lambda \in (0,1)$ which solves (71): $\bar{\lambda}$. *Q.E.D.*

Proof of Proposition 9: From (50) the equilibrium rental cost of capital is equal to

$$r^*=\frac{\beta w^*}{\alpha\bar{K}}\left[1-\lambda+\lambda\psi(\eta,s)\right]\hat{\theta}_R=\frac{\beta}{\alpha\bar{K}}w^*L^*.$$

The impact of a change in λ on r^* is given by

$$\frac{\partial r^*}{\partial\lambda}=\frac{\beta}{\alpha\bar{K}}\left[\frac{\partial w^*}{\partial\lambda}L^*+w^*\frac{\partial L^*}{\partial\lambda}\right].$$

We know from Proposition 6 that $\partial w^*/\partial\lambda > 0$. Hence, $\partial L^*/\partial\lambda > 0$ is a sufficient condition for $\partial r^*/\partial\lambda > 0$. Therefore, it follows from Proposition 8 that $\partial r^*/\partial\lambda > 0$ when either (1) inequality (68) is satisfied or (2) inequality (68) is violated and $\lambda \in (\bar{\lambda}, 1]$.

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