Overconfidence and Timing of Entry*

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Abstract

We analyze the impact of overconfidence on the timing of entry in markets, profits, and welfare using an extension of Hamilton and Slutsky's (1990) quantity commitment game. Players have private information about costs, one player is overconfident, and the other one rational. We find that for moderate levels of overconfidence there is a unique *cost-dependent* equilibrium where the overconfident player has a higher ex-ante probability of being the Stackelberg leader. Overconfidence lowers the profit of the rational player but can increase that of the overconfident player. Consumer rents increase with overconfidence while producer rents decrease which leads to an ambiguous welfare effect.

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1 Introduction

This paper studies the impact of overconfidence–one of the most robust biases in judgment–on the timing of entry into a market. Our main research question is whether overconfident players enter markets before rational players. Additionally, we evaluate the impact of overconfidence on profits, consumer surplus and welfare.

Evidence from psychology and economics shows that most individuals hold overly favorable views of their skills. According to Myers (1996), "(...) on nearly any dimension that is both subjective and socially desirable, most people see themselves as better than average." Experimental work suggests that excess entry of new businesses that fail within several years may be due to overconfidence of entrepreneurs about their own ability in comparison with that of other entrepreneurs (Camerer and Lovallo, 1999). Interviews with new entrepreneurs revealed that their self-assessed chances of success were uncorrelated with objective predictors like education, prior experience, and initial capital, and were on average wildly off the mark (Cooper et al., 1988). Empirical evidence further suggests that firms hire and retain overconfident managers (Malmendier and Tate, 2008; Galasso and Simcoe, 2011; Hirshleifer, Low and Teoh, 2012), thus raising the question if this behavior can be explained by a relationship between overconfidence and the timing of entry.

To study the impact of overconfidence on the timing of entry into a market we use Branco's (2008) extension of Hamilton and Slutsky's (1990) quantity commitment game. In this endogenous timing model two players are privately informed about their cost (which can be either high or low), compete in quantities, and must decide whether to enter a market at date 1 or at date 2. The novelty here is that we assume that one of the players is overconfident whereas the other one is rational. The rational player has a correct belief about his cost of production. The overconfident player can be mistaken about his cost with positive probability. More precisely, we assume that if the overconfident player's cost is low, his perception is correct and he thinks that he has a low cost. However, if the overconfident player's cost is high, his perception can be mistaken and he might think that he has low cost.

Our main finding is that there exists a unique *cost-dependent* equilibrium where the overconfident player has a higher ex-ante probability of moving at date 1 than the rational player. In other words, the overconfident player is more likely to be the leader than the rational player. We also show that this equilibrium only exists if the level of overconfidence is moderate.

The intuition behind this result is as follows. In a cost-dependent equilibrium a player with a low cost perception enters the market at date 1 whereas a player with a high cost perception enters the market at date 2. Since an overconfident player has a higher ex-ante probability of having a low cost perception than a rational player, he also has a higher ex-ante probability of entering the market before the rational player. This equilibrium breaks down if overconfidence is high since a rational player with low cost would be better off by deviating and producing at date 2.[1]

Next we study the effects of overconfidence on players' profits and welfare in the cost-dependent equilibrium. We find that moderate overconfidence is good for the overconfident player as long as cost asymmetries are small. The impact of high overconfidence on the overconfident player's profits depends on a trade-off between a "leadership gain" and an "overproduction loss." If players compete in quantities, the Stackelberg leader's profits are higher than those of the follower. The overconfident player's mistaken perception gives him a Stackelberg leadership gain since high overconfidence increases the probability that he enters the market before the rational player. However, the mistaken perception yields overproduction, which lowers market price and reduces profit. If cost asymmetries are small the leadership gain dominates the overproduction loss and the bias is beneficial for the overconfident player; if cost asymmetries are large the opposite happens and the bias reduces the overconfident player's profits. The bias of the overconfident player always hurts the rational player.

Finally, we show that overconfidence has an ambiguous impact on welfare (the sum of consumer and producer surplus). This happens because overconfidence increases market output, which raises consumer surplus but reduces producer surplus. We find that when cost asymmetries are small the increase in consumer surplus is of first-order but the reduction in producer surplus is of second-order and so overconfidence increases welfare. In contrast, when cost asymmetries are moderate or large the reverse happens and overconfidence reduces welfare. These findings are consistent with the theory of the second-best. It is well known that in a world where at least one distortion is present (duopoly), introducing a new distortion (overconfidence) can increase or reduce welfare.

Our paper is related with two branches of economic literature: endogenous timing and overconfidence. The literature on endogenous timing provides conditions and criteria under which firms play either a sequential-move Stackelberg game or a simultaneous-move Cournot game in oligopolistic markets.

A seminal contribution to the endogenous timing literature is Hamilton and Slutsky's (1990) quantity commitment game.[2] In this model two players must commit to a certain quantity in one of the two periods before the market clears. If a player commits to a quantity in the first period, he acts as a leader but he does not know if the other player has chosen to commit also in the first period or not. If a player waits until the second period to do a commitment, then he observes the action of the other player in the first period. This game has three subgame perfect Nash equilibria: one Cournot equilibrium in the first period, and two Stackelberg equilibria. Only the Stackelberg equilibria survive elimination of weakly dominated strategies.

[1] For all levels of overconfidence there exist two *cost-independent* equilibria where one of the players produces at date 1 independently of his cost perception and the other player produces at date 2.

[2] Albaek (1990) contributed to the literature on endogenous timing by considering a model in which the firms commit to a timing of production (i.e., choose between sequential and simultaneous moves) before knowing their costs. If the difference between the variance of the firms' costs is sufficiently large, there will be Stackelberg leadership of the firm with the greater variance.

A two-stage game in which each player can either commit to a quantity in stage 1 or stage 2 is considered in van Damme and Hurkens (1999). It is shown that committing is more risky for the high cost firm, and thus risk dominance considerations lead to the conclusion that only the low cost firm will choose to commit. Hence, the low cost firm will emerge as the Stackelberg leader. Branco (2008) extends Hamilton and Slutsky's model by assuming that players are privately informed about their costs. He shows that there exists a cost-dependent Perfect Bayesian equilibrium where the player with a low cost produces in the first period and the player with a high cost produces in the second period.[3]

Our paper is also related to the fast growing literature on the impact of overconfidence on economic decisions and welfare. De Meza and Southey (1996) propose a model with optimistic entrepreneurs that is able to rationalize some of the stylized facts of small-scale businesses. In their model most of the facts characterizing small-scale businesses–including high failure rates, reliance on bank credit rather than equity finance, and credit rationing–can be explained by a tendency for those who are excessively optimistic to dominate new entries. Brocas and Carrillo (2004) find that there is a negative correlation between the risk free rate and the proportion of bold entrepreneurs in the economy, realist and bold agents can coexist and achieve the same payoff, and entrepreneurs with highest ability are most likely to keep optimistic prospects and make entry mistakes. Heifetz, Shannon and Spiegel (2007) show that in a large class of strategic interactions the equilibrium payoffs of overconfident players may be higher than those of rational players. This happens because overconfidence can lead the adversary to change equilibrium behavior, possibly to the benefit of overconfident agent. Our finding that the overconfident player does better than a rational player with small cost asymmetries is consistent with Heifetz, Shannon and Spiegel (2007).[4]

Our work further relates to the empirical literature on managerial overconfidence and its effects on firms' decisions. Malmendier and Tate (2005a, 2005b, 2008) show that executives' optimism affects firms' investment decisions and cash flow sensitivity. They suggest that the relation between executives beliefs and market entry timing may explain some of the firms incentives to hire executives with biased beliefs. Yet managerial overconfidence tends to destroy value through unprofitable mergers and suboptimal investment behavior. Galasso and Simcoe (2011) evaluate the relation between managerial overconfidence and corporate innovation. They find a positive correlation between overconfidence and citation-weighted patent counts on a panel of publicly traded firms. Hence, results suggest that overconfident CEOs are more likely to take their firm in a new technological direction. Hirshleifer, Low and Teoh (2012) evaluate the relation between managerial overconfidence and corporate innovation. They find that firms with overconfident CEOs have greater return variability, invest more in innovation, obtain more patents and patent citations, and achieve greater inno-

[3]See also Maggi (1996), Mailath (1993), Normann (1997), Normann (2002), and van Damme and Hurkens (2004).

[4]See also Bénabou and Tirole (2002), and Santos-Pinto (2008).

vative success within innovative industries. Their findings suggest that overconfident CEOs are better at translating external growth opportunities into firm value. The findings of Galasso and Simcoe (2011) and Hirshleifer, Low and Teoh (2012) suggest that overconfident CEOs are more likely to invest in innovation, thus generating value for the firm. Firms may therefore want to hire optimistic managers to move earlier into a new technological direction with positive effects on firm value.

The remainder of the paper is organized as follows. Section 2 presents the model and Section 3 describes and characterizes the equilibria. In Section 4 we evaluate the effects of overconfidence on profits and welfare. Section 5 concludes the paper. All proofs are in the Appendix.

2 The Model

There are two players. One is rational, denoted by r, and the other one is overconfident, denoted by o. The two players produce a homogeneous good with price given by $p = a - q_r - q_o$, where q_r and q_o are the quantities produced by the rational and the overconfident player, respectively, with $a > 0$. To produce the good, players incur a cost. We assume that marginal cost of production is constant and that there are no fixed costs. The marginal cost of each player, C_i, $i = r, o$, might take on the values 0 (low) and c (high) with equal probability, where $a > c > 0$.[5]

Players are privately informed about their costs. Each player receives a signal X_i that is correlated with his cost, where $X_i \in \{0, c\}$. For the rational player the relation between the signal and his cost his given by $\Pr(X_r = c | C_r = c) = 1$ and $\Pr(X_r = 0 | C_r = 0) = 1$, that is, the rational player is perfectly informed about his cost. For the overconfident player the relation between the signal and his cost is given by $\Pr(X_o = c | C_o = c) = 1 - s$, $\Pr(X_o = 0 | C_o = c) = s$, and $\Pr(X_o = c | C_o = 0) = 0$, where $s \in [0, 1]$. The parameter s captures the degree of overconfidence since it represents the probability that the overconfident player receives a signal that his cost is 0 when his cost is c. If $s = 0$ there is no overconfidence and the model collapses to Branco (2008). If $s = 1$ we have the maximum level of overconfidence since the overconfident player always thinks that his cost is 0. Higher values of s imply a higher level of overconfidence.

Players must make a quantity commitment at one of two dates. Any player who does not commit to date 1, must decide his commitment quantity at date 2 after observing if the rival has committed to date 1 or not. Finally, at date 3, given the quantity commitments, the market clears. The timing of the model is:

1. Nature draws players' costs
2. Each player receives a signal about his cost.

[5] It would be straightforward to extend the model by assuming that costs can be $\underline{c}$ or $\overline{c}$ where $0 \leq \underline{c} < \overline{c}$. The assumption that the prior probability is 1/2 could also be relaxed. This would complicate the algebra but it would not change the main findings of the paper.

3. Players decide simultaneously the commitment date and quantity.

4. A player who has not committed at date 1 decides his commitment quantity at date 2.

5. The market clears at date 3.

For a player there is a clear trade-off between the timing decisions. A player who commits at date 1 gets the possible benefit of acting as a leader, producing first and influencing the other player's decision, if the latter has decided to commit at date 2. However, by committing at date 1, a player does not observe the timing of the opponent's move, risking that the opponent also commits at date 1. To the contrary, a player who decides to wait and commit at date 2, cannot influence the rival's decision, if the rival has decided to wait, but will have more information when deciding since he can observe the quantity commitment by the rival or the rival's decision to wait. The choices commitment date can be described in terms of the leader-follower dichotomy: a player who commits at date 1 acts as the leader, while a player who commits at date 2 acts as a follower. Thus, the structure of the model provides a framework for the study of the choice of moment of production in a quantity setting duopoly, with asymmetric information about costs and overconfidence as the driving forces.

3 Equilibria

In this section we analyze the equilibria of the model. The equilibrium concept used to solve this game is the Perfect Bayesian Equilibrium (PBE) which requires that strategies yield a Bayesian equilibrium in every "continuation game" given the posterior beliefs of the players, and beliefs are required to be updated in accordance with Bayes' law whenever it is applicable.

To incorporate overconfidence into this setting we follow the approach introduced by Squintani (2006) who considers events where the players self-perception may be mistaken but such that player i is playing a given game, that player j thinks that player i thinks that player j's perception is correct, that player i knows that player j believes that her perception is correct and so on. In other words, we assume that the rational player knows that if the overconfident player's cost is c the overconfident player can think that his cost is 0 with probability s. In turn, the overconfident player knows that the rational player thinks that if the overconfident player's cost is c the overconfident player can think that his cost is 0 with probability s. However, the overconfident player thinks that the rational player is mistaken about that. Hence, in this model players might agree to disagree. The rational player knows that the ex-ante probability that the overconfident player perceives a signal of high cost is

$$\begin{aligned}\Pr(X_o = c) &= \Pr(C = c)\Pr(X_o = c|C = c) + \Pr(X_o = 0)\Pr(X_o = c|C = c) \\ &= \frac{1}{2}(1-s) + \frac{1}{2}0 = \frac{1}{2}(1-s),\end{aligned}$$

and the ex-ante probability that the overconfident player perceives a signal of low cost is

$$\begin{aligned}\Pr(X_o = 0) &= \Pr(C=0)\Pr(X_o=0|C=0) + \Pr(C=c)\Pr(X_o=0|C=c)\\ &= \frac{1}{2}1 + \frac{1}{2}s = \frac{1}{2}(1+s).\end{aligned}$$

We characterize the set of equilibria of this game by describing the players' equilibrium strategies and we distinguish between two types of equilibria: cost-dependent and cost-independent. In a cost-dependent equilibria each player chooses a different period to produce according to his cost perception (high or low). In cost-independent equilibria each player chooses to produce in a certain period independently of his cost perception.

Our first result characterizes the players' strategies "on the equilibrium path" of the cost-dependent equilibrium of the game.[6]

Proposition 1: *If* $\omega(s) \leq x \leq \frac{5+s}{9+3s}$, *and* $0 \leq s \leq s(x)$, *where* $x = c/a$,

$$\omega(s) = \frac{41 - 2s + 2s^2 - 2\sqrt{6}\,(2-s)\,(4+s)}{29 + 28s - 10s^2} \tag{1}$$

and

$$s(x) = \frac{3\sqrt{3}\sqrt{5 + 68x + 314x^2 + 588x^3 + 369x^4} - (10 + 28x + 106x^2)}{7 + 34x + 67x^2}, \tag{2}$$

then there is a cost-dependent equilibrium in which players will play the following strategies on the equilibrium path:

(i) If the overconfident player has the perception that his cost is equal to 0 *he produces* $q_o = \frac{3a+c+(a+c)s}{8+2s}$ *at date 1; otherwise he produces* $q_o = \frac{a-c-q_r}{2}$ *at date 2, where* q_r *is the rational player's production at date 1;*

(iii) If the rational player has cost equal to 0 *he produces* $q_r = \frac{3a+c-2sc}{8+2s}$ *at date 1; otherwise he produces* $q_r = \frac{a-c-q_o}{2}$ *at date 2, where* q_o *is the overconfident player's production at date 1.*

Proposition 1 says that if cost differences are sufficiently high [7], that is $\omega(s) \leq x \leq \frac{5+s}{9+3s}$, and overconfidence is moderate, that is $0 \leq s \leq s(x)$, then there exists a cost-dependent equilibrium where a player with a low cost perception produces at date 1 and a player with a high cost perception produces at date 2.[8] The production moments thus reveal the players' cost perceptions.[9]

[6]The players' full strategies are described in the Appendix.

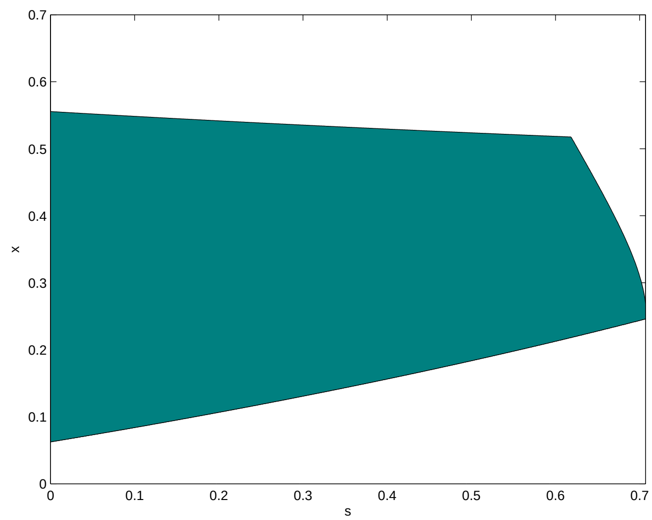
[7]For the equilibrium existence, however, the cost differences cannot be too large

[8]For the equilibrium to be well defined it must be the case that the intervals for x and s are non-empty. The interval for x is non-empty since $\max_{s\in[0,1]}\omega(s) = 0.35117 < \min_{s\in[0,1]} \frac{5+s}{9+3s} = 0.5$. The interval for s is also non-empty since $s(x) \in (0.62636, 0.68828)$. Additionally, cost differences cannot be so high that entering the market is not attractive for a rational follower with cost equal to c. This is guaranteed by $c \leq \frac{5+s}{9+3s}a$. Figure 1 plots the values of x and s for which there exists a cost-dependent equilibrium, showing that the intervals for x and s are non-empty.

[9]Proposition 1 also tells us that four outcomes are possible: a Cournot outcome will result,

More importantly, in the cost-dependent equilibrium the overconfident player has a higher ex-ante probability of being the Stackelberg leader than the rational player. The intuition behind this result is as follows. When the overconfident player's cost is low the timing decision is not affected by overconfidence since he keeps playing in the first period. However, if the overconfident player's cost is high, then he might hold a mistaken perception of cost which leads him to enter the market at date 1. In contrast, the rational player's ex-ante probability of being the Stackelberg leader is not affected by the overconfidence of the rival. Thus, for the overconfident player the ex-ante probability of being the Stackelberg leader is larger than that of the rational player.

Figure 1: Values of cost differences and overconfidence for which there exists a cost-dependent equilibrium



Note: The shaded area includes all the values of cost differences–captured by the parameter x– and overconfidence–captured by the parameter s–for which there exists a cost-dependent equilibrium.

The strategies described in Proposition 1 are an equilibrium if and only if cost differences are sufficiently high and overconfidence is moderate. Figure 1 plots the values of cost differences (x) and overconfidence (s) for which there exists a cost-dependent equilibrium. When overconfidence is moderate but cost differences are smaller than $\omega(s)$, the previous strategies will not be an equilibrium. A player with a high cost perception would gain by deviating and producing at date 1. However, both players producing at date 1 is not an equilibrium. One

if both players wait to produce at date 2; a Stackelberg outcome will emerge, if one player produces at date 1 and the other does it at date 2 (there are two of these outcomes); and a double leadership outcome appears if both players produce at date 1.

can show that if overconfidence is sufficiently low but cost differences are not sufficiently high, then the player with a low cost perception will still produce at date 1, but the player with a high cost perception will randomize between producing at date 1 or waiting to produce at date 2.[10]

When cost differences are sufficiently high but the level of overconfidence is greater than $s(x)$, the strategies described in Proposition 1 will also not be an equilibrium. The existence of a upper bound for overconfidence is quite intuitive since a high level of overconfidence implies that an overconfident player who follows his cost-dependent equilibrium strategy produces at date 1 with a very high probability. However, since the rational player knows that, he does not have incentives to play according to his cost-dependent equilibrium strategy. Particularly, if the rational player has low cost he would gain by deviating and producing at date 2.

Are there are any circumstances in which a rational player can enter the market in date 1 with a higher ex-ante probability than an overconfident player? The answer to this question is yes. The endogenous timing game has two pure strategy cost-independent equilibria: (i) the overconfident player produces at date 1 and the rational player at date 2 and (ii) the rational player produces at date 1 and the overconfident player at date 2. Proposition 2 shows that there are no other equilibria in our model.

Proposition 2: *For $\omega(s) \leq x \leq \frac{5+s}{9+3s}$, and $0 \leq s \leq s(x)$ where $\omega(s)$ and $s(x)$ are given by* (1) *and* (2)*, respectively, there does not exist a Perfect Bayesian equilibria whose strategy profiles differ from the cost-dependent equilibrium of Proposition* 1 *or from the two cost-independent equilibria.*

There might also be mixed strategy equilibria where the players mix between the actions played in the cost-dependent and independent equilibria. We do not attempt to characterize mixed strategy equilibria as we find that it is hard for a player to convince his rivals that he is committed to a randomization device.

4 Overconfidence, Profits and Welfare

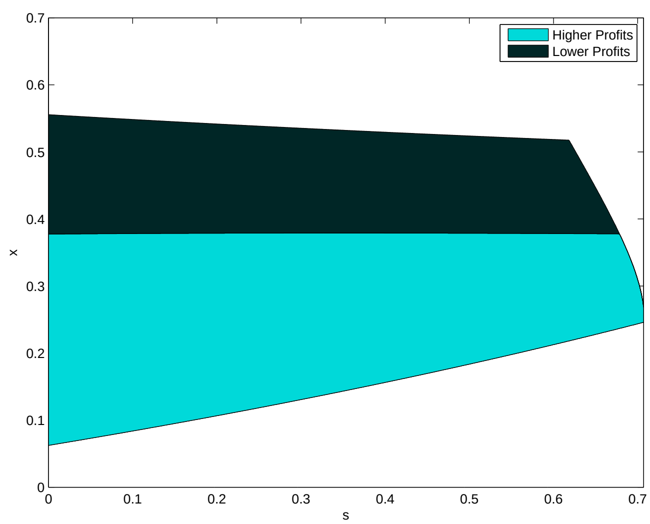
In this section we characterize effects of overconfidence on profits and welfare. Let $\Pi_o(s)$ and $\Pi_r(s)$ denote the ex-ante profits of the overconfident and the rational players, respectively, as a function of s. We take as benchmark scenario the endogenous timing game played between two rational players. Let $\Pi(0)$ denote the ex-ante profits of a player in an endogenous timing game played between two rational players. We have the following result.

Proposition 3: *In the cost-dependent equilibrium: (i) $\Pi_o(s) > \Pi(0)$ for all $0 < s \leq s(x)$, if $\omega(s) \leq x \leq \tau(s)$; (ii) $\Pi_o(s) < \Pi(0)$ for all $0 < s \leq s(x)$, if $\tau(s) < x \leq \frac{5+s}{9+3s}$; and (iii) $\Pi_r(s) < \Pi(0)$ for all $0 < s \leq s(x)$, where $\tau(s) = \frac{2152+1139s-32s^2+24(s+4)\sqrt{8137+2659s+96s^2}}{28648+11759s+1696s^2}$.*

[10]The implications of overconfidence are the same for both types of cost-dependent equilibria (high and low cost differences). Therefore we do not characterize the cost-dependent equilibrium with low cost differences.

This result shows that, in the cost-dependent equilibrium, moderate overconfidence can increase the profits of the overconfident player provided that cost asymmetries are small. Figure 2 illustrates this by showing the relation between the ex-ante profits of the overconfident player in the cost-dependent equilibrium and the ex-ante profits of a player in an endogenous timing game with two rational players. The intuition is as follows. By making a mistake the overconfident player has a "Stackelberg leadership gain" because it will produce at date 1 instead of date 2. However, the mistaken perception of the overconfident player will lead him to choose a quantity that is higher than the optimal one given his true cost. This leads to a loss which increases with the value of c since the difference between the optimal quantity and the quantity chosen increases with c. Hence, for low values of c the "Stackelberg leadership gain" more than compensates the loss incurred by not playing the optimal quantity. For high values of c the reverse happens.

Figure 2: Ex-ante profits of the overconfident player in the cost-dependent equilibrium versus ex-ante profits with two rational players



Note: The figure reports the relation between the ex-ante profits of the overconfident player in the cost-dependent equilibrium and the ex-ante profits of a player in an endogenous timing game with two rational players for different values of x and s.

Proposition 3 also shows that overconfidence always reduces the profits of the rational player. This happens because the mistaken perceptions of the overconfident player yield a reduction of market share for the rational player. If the rational player has low cost, then he produces at date 1. However, since the rational player knows that the overconfident player is likely to overproduce, the rational player must produce a smaller Stackelberg leader's quantity than if he faced a rational opponent. If the rational player has high cost, then he produces at date 2. In this case, no matter if the overconfident player produces at date 1 or at date 2, the rational player will have a smaller market share than if he faced a rational rival.

We now discuss the impact of overconfidence on players' profits in the cost-independent equilibria. In the cost cost-independent equilibrium where the rational player leads and the overconfident player follows, the impact of overconfidence on players' ex-ante profits is similar to that in the cost-dependent equilibrium, that is, overconfidence always hurts the rational player but can be advantageous for the overconfident player. In this equilibrium, the rational player produces at date 1 no matter if his cost is high or low and the overconfident player produces at date 2. Since the rational player knows that the overconfident player can overproduce with positive probability at date 2, he must lower his leadership output by comparison with a situation where he would be faced with a rational opponent. The mistaken beliefs of the overconfident follower allow him to increase is market share at the expense of the rational player's market share. However, if cost asymmetries are large the overconfident player will be worse off since the optimization mistake loss will be greater than the gain from the increase in market share.

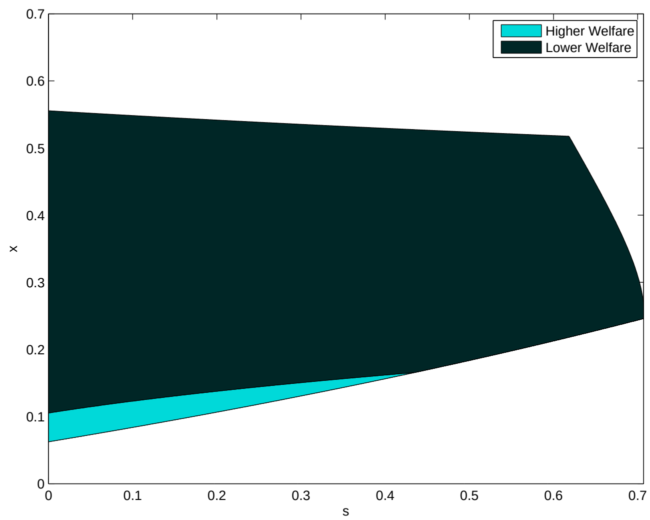
In the cost-independent equilibrium where the overconfident player is the leader and the rational player the follower, overconfidence hurts both players. In this equilibrium, the overconfident player always produces at date 1 so there is no leadership gain from holding mistaken beliefs. However, the mistaken beliefs will lead the overconfident leader to overproduce which originates a loss. The rational player is also worse off because he will have a lower market share than if he would be faced with a rational opponent.

Our last result characterizes the impact of overconfidence on welfare (the sum of consumer and producer surplus) in the cost-dependent equilibrium. Let $W(0)$ denote ex-ante welfare in the endogenous timing game played between two rational players and $W(s)$ denote the ex-ante welfare in the endogenous timing game played between a rational player and an overconfident player with confidence of $s > 0$.

Proposition 4: *In the cost-dependent equilibrium: (i)* $W(s) > W(0)$ *for all* $0 < s \leq s(x)$, *if* $\omega(s) \leq x \leq \psi(s)$*; and (ii)* $W(s) < W(0)$ *for all* $0 < s \leq s(x)$, *if* $\psi(s) < x \leq \frac{5+s}{9+3s}$, *where* $\psi(s) = \frac{1768+2027s+376s^2+24(s+4)\sqrt{209+1412s+324s^2}}{29992+10487s+808s^2}$.

Figure 3 describes the relation between the ex-ante welfare in the cost-dependent equilibrium of the endogeneous timing game with one overconfident player and the ex-ante welfare in the endogenous timing game with two rational players. The figure shows us that moderate levels of overconfidence reduce welfare in the cost-dependent equilibrium, except when cost asymmetries are small. The intuition behind this result is as follows. High overconfidence increases the output of the overconfident player but reduces that of the rational player. The net effect is an overall increase in market output since the reduction in the output of the rational player is less than the increase in that of the overconfident player. The increase in market output raises consumer surplus but reduces producer surplus. It turns out that when overconfidence is low the increase in the ex-ante profits of the overconfident player is less than the decrease in those of the rational player but when overconfidence is high the ex-ante profits of both players decrease. For sufficiently small cost asymmetries the increase in consumer surplus is of first-order and the reduction in profits is of second-order and so overconfidence increases welfare. When cost asymmetries are not sufficiently small the reverse happens and overconfidence reduces welfare.

Figure 3: Ex-ante welfare in the cost-dependent equilibrium with an overconfident player versus welfare with two rational players



Note: The figure reports the relation between the welfare in the cost-dependent equilibrium with an overconfident player and the welfare in an endogenous timing game with two rational players for different values of x and s.

5 Conclusion

In this paper we characterize the impact of overconfidence on the timing of entry in a market, profits, and welfare. To do that we consider an endogenous timing model where players have private information about their cost, one of the players is rational but the other is overconfident. We find that, in a cost-dependent equilibrium, the overconfident player has a higher ex-ante probability of being the Stackelberg leader than the rational player. We show that this result is valid only if the level of overconfidence is moderate. We also show that overconfidence always hurts the rational player. However, the overconfident player can be better off by being overconfident (although he does not know it) provided that cost asymmetries are small. Finally, we show that overconfidence has an ambiguous impact on welfare since it increases consumer surplus while it reduces producer surplus. Our framework, therefore, provides a justification for the earlier entry of overconfident entrepreneurs and a motivation for firms hiring and retaining overconfident managers.

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7 Appendix

Complete statement of Proposition 1.

Proposition 1: *If* $\omega(s) \leq x \leq \frac{5+s}{9+3s}$, *and* $s \leq s(x)$, *with* $x = c/a$, $\omega(s) = \frac{41-2s+2s^2-2\sqrt{6}(2-s)(4+s)}{29+28s-10s^2}$, $s(x) = \frac{3\sqrt{3}\sqrt{5+68x+314x^2+588x^3+369x^4}-(10+28x+106x^2)}{7+34x+67x^2}$, *then here is a cost-dependent equilibrium in which the players will have the following strategies:*

Overconfident player

1. *If the overconfident player has the perception that his cost is equal to* 0:
(a) *He produces at date 1;*
(b) *He produces* $q_o = \frac{3a+c+(a+c)s}{8+2s}$;
(c) *If he had not produced at date 1 and the rational player had produced* q_r *at date 1, he would produce according to* $q_o = \frac{a-q_r}{2}$, *at date 2;*
(d) *If neither player had produced at date 1, he would produce* $q_o = \frac{2a+c}{6}$ *at date 2;*
2. *If the overconfident player has the perception that his cost is equal to* c:
(a) *He produces at date 2;*
(b) *If he were to produce at date 1, he would produce* $q_o = \frac{9a-13c+(3a-c)s}{24+6s}$;
(c) *If the rational player has produced* q_r *at date 1, he will produce according to* $q_o = \frac{a-c-q_r}{2}$, *at date 2;*
(d) *If the rational player has not produced at date 1, he will produce* $q_o = \frac{a-c}{3}$, *at date 2;*

Rational player

1. *If the rational player has cost equal to* 0:
(a) *He produces at date 1;*
(b) *He produces* $q_r = \frac{3a+c-2sc}{8+2s}$;
(c) *If he had not produced at date 1 and the overconfident player had produced* q_o *at date 1, he would produce according to* $q_r = \frac{a-q_o}{2}$, *at date 2;*
(d) *If neither player had produced at date 1, he would produce* $q_r = \frac{2a+c}{6}$ *at date 2;*
2. *If the rational player has cost equal to* c:
(a) *He produces at date 2*;
(b) *If he were to produce at date 1, he would produce* $q_r = \frac{9a-13c+(3a-9c-2cs)s}{(3+s)(8+2s)}$;
(c) *If the overconfident player has produced* q_o *at date 1, he will produce according to* $q_r = \frac{a-c-q_o}{2}$, *at date 2;*
(d) *If the overconfident player has not produced at date 1, he will produce* $q_r = \frac{a-c}{3}$ *at date 2*;

Proof of Proposition 1: Suppose player r plays according to the strategy defined. Player o has to produce according to a best response whenever possible. The proof proceeds by showing that the eight steps that describe the strategy of player o form a best response. First, we determine the optimal production levels for o in each contingency. We need to solve the game backward, so we start by looking at the problem at date 2

1. Player o has the perception that his cost is equal to 0:
(i) Player o produces at date 2, knowing that r has not produced at date 1: then o infers that r has cost equal to c and that he will produce $(a-c)/3$ at date 2; thus o must produce a quantity that solves $\max_{q_o}\left(a-q_o-\frac{a-c}{3}\right)q_o$, which leads to production of $q_o=\frac{2a+c}{6}$.
(ii) Player o produces at date 2, knowing that r has produced the quantity q_r at date 1: then o must produce the quantity that solves $\max_{q_o}\left(a-q_o-q_r\right)q_o$, which leads to production of $q_o=\frac{a-q_r}{2}$.
(iii) Player o produces at date 1: it may be that r will also produce at date 1, if he has cost equal to 0, or he will produce at date 2, if he has cost equal to c; hence, the quantity produced by o must solve $\max_{q_o}\frac{1}{2}\left(a-q_o-\frac{3a+c-2cs}{8+2s}\right)q_o+\frac{1}{2}\left(a-q_o-\frac{a-c-q_o}{2}\right)q_o$. The solution to this problem is $q_o=\frac{3a+c+(a+c)s}{8+2s}$.
2. Player o has the perception that his cost is equal to c:
(i) Player o produces at date 2, knowing that r has not produced at date 1: then o infers that r has cost equal to c and that he will produce $(a-c)/3$ at date 2; thus o must produce a quantity that solves $\max_{q_o}\left(a-q_o-\frac{a-c}{3}-c\right)q_o$, which leads to production of $q_o=\frac{a-c}{3}$.
(ii) Player o produces at date 2, knowing that r has produced the quantity q_r at date 1: then o must produce the quantity that solves $\max_{q_o}\left(a-q_o-q_r-c\right)q_o$, which leads to production of $q_o=\frac{a-c-q_r}{2}$.
(iii) Player o produces at date 1: it may be that r will also produce at date 1, if he has cost equal to 0, or he will produce at date 2, if he has cost equal to c; hence, the quantity produced by o must solve $\max_{q_o}\frac{1}{2}\left(a-q_o-\frac{3a+c-2cs}{8+2s}-c\right)q_o+\frac{1}{2}\left(a-q_o-\frac{a-c-q_o}{2}-c\right)q_o$. The solution to this problem is $q_o=\frac{9a-13c+(3a-c)s}{24+6s}$.
Now, the optimal moment of production of the overconfident player is determined by looking at the associated expected profits at dates 1 and 2:
1. Player o has the perception that his cost is equal to 0:
(i) If o produces at date 1, his perceived expected profit will be:

$$\begin{aligned}\pi_o^1 &= \frac{1}{2}\left(a-\frac{3a+c+(a+c)s}{8+2s}-\frac{3a+c-2cs}{8+2s}\right)\frac{3a+c+(a+c)s}{8+2s}\\ &+\frac{1}{2}\left(a-\frac{3a+c+(a+c)s}{8+2s}-\frac{a-c-\frac{3a+c+(a+c)s}{8+2s}}{2}\right)\frac{3a+c+(a+c)s}{8+2s}\\ &= \frac{3(3a+c)^2+3\left(4a+(a+c)(2+s)\right)(a+c)s}{(16+4s)^2}.\end{aligned}$$

(ii) If o produces at date 2, his perceived expected profit will be:

$$\begin{aligned}\pi_o^2 &= \frac{1}{2}\left(a-\frac{a-\frac{3a+c-2cs}{8+2s}}{2}-\frac{3a+c-2cs}{8+2s}\right)\frac{a-\frac{3a+c-2cs}{8+2s}}{2}\\ &+\frac{1}{2}\left(a-\frac{2a+c}{6}-\frac{a-c}{3}\right)\frac{2a+c}{3}=\frac{1}{2}\left(\frac{5a-c+2s(a+c)}{16+4s}\right)^2+\frac{1}{2}\left(\frac{2a+c}{6}\right)^2\end{aligned}$$

Comparing the two possible profits of o, one obtains:

$$\frac{3(3a+c)^2+3\left(4a+(a+c)(2+s)\right)(a+c)s}{(16+4s)^2} - \frac{1}{2}\left(\frac{5a-c+2(a+c)s}{16+4s}\right)^2 - \frac{1}{2}\left(\frac{2a+c}{6}\right)^2 = \frac{(5a^2+158ac-19c^2)+2(8+s)(a^2+10ac+7c^2)s}{18(16+4s)^2},$$

which, given the restrictions on the parameters, is positive. So, when player o perceives that his cost is equal to 0, his perceived expected profit from producing at date 1 is greater than that of producing at date 2.

2. Player o has the perception that his cost is equal to c:

(i) If o produces at date 1, his expected profit will be:

$$\pi_o^1 = \frac{1}{2}\left(a - \frac{9a-13c+(3a-c)s}{6(4+s)} - \frac{3a+c-2cs}{8+2s} - c\right)\frac{9a-13c+(3a-c)s}{6(4+s)} + \frac{1}{2}\left(a - \frac{3a-15c+3as+3cs}{12(4+s)} - c\right)\frac{9a-13c+(3a-c)s}{6(4+s)}$$

Doing some algebra we find that:

$$\pi_o^1 = \frac{(81a^2-234ac+169c^2)+(54a^2-96ac+26c^2+(9a^2-6ac+c^2)s)s}{3(16+4s)^2}.$$

(ii) If o produces at date 2, his expected profit will be:

$$\pi_o^2 = \frac{1}{2}\left(a - \frac{a-c-\frac{3a+c-2cs}{8+2s}}{2} - \frac{3a+c-2cs}{8+2s} - c\right)\frac{a-c-\frac{3a+c-2cs}{8+2s}}{2} + \frac{1}{2}\left(a-\frac{a-c}{3}-\frac{a-c}{3}-c\right)\frac{a-c}{3} = \frac{1}{2}\left(\frac{5a-9c+2as}{16+4s}\right)^2 + \frac{1}{2}\left(\frac{a-c}{3}\right)^2.$$

Comparing the two expected profits of o, he will prefer to produce at date 2 if:

$$\frac{(81a^2-234ac+169c^2)+(54a^2-96ac+26c^2+(9a^2-6ac+c^2)s)s}{3(16+4s)^2} \leq \frac{1}{2}\left(\frac{5a-9c+2as}{16+4s}\right)^2 + \frac{1}{2}\left(\frac{a-c}{3}\right)^2$$

Solving this expression with respect to c, one concludes that o will prefer to produce at date 2 if:

$$c > \frac{41-2s+2s^2-2\sqrt{6}\,(s+4)\,(2-s)}{29+28s-10s^2}a = \omega(s)a$$

We now determine the optimal production levels of the rational player in each contingency.
1. Player r has cost equal to 0:
(i) Player r produces at date 2, knowing that o has not produced at date 1: then r infers that o has perceived that his cost is equal to c and that he will produce $(a-c)/3$ at date 2; thus r must produce a quantity that solves $\max_{q_r} \left(a - q_r - \frac{a-c}{3}\right) q_r$, which leads to production of $q_r = \frac{2a+c}{6}$.
(ii) Player r produces at date 2, knowing that o has produced the quantity q_o at date 1: then r must produce the quantity that solves $\max_{q_r} \left(a - q_o - q_r\right) q_r$, which leads to production of $q_r = \frac{a-q_o}{2}$.
(iii) Player r produces at date 1: it may be that o will also produce at date 1, if he perceives that he has cost equal to 0, or he will produce at date 2, if he perceives that he has cost equal to c; hence, the quantity produced by r must solve:

$$\max_{q_r} \frac{1}{2}(1+s)\left(a - q_r - \frac{3a+c+(a+c)s}{8+2s}\right) q_r \\ + \frac{1}{2}(1-s)\left(a - q_r - \frac{a-c-q_r}{2}\right) q_r.$$

The solution to this problem is $q_r = \frac{3a+c-2cs}{8+2s}$.
2. Player r has cost equal to c:
(i) Player r produces at date 2, knowing that o has not produced at date 1: then r infers that o has perceived that his cost is equal to c and that he will produce $(a-c)/3$ at date 2; thus r must produce a quantity that solves $\max_{q_r} \left(a - q_r - \frac{a-c}{3} - c\right) q_r$, which leads to production of $q_r = \frac{a-c}{3}$.
Now, the optimal moment of production of the rational player is determined by looking at the associated expected profits at dates 1 and 2:
(ii) Player r produces at date 2, knowing that o has produced the quantity q_o at date 1: then r must produce the quantity that solves $\max_{q_r} \left(a - q_o - q_r - c\right) q_r$, which leads to production of $q_r = \frac{a-c-q_o}{2}$.
(iii) Player r produces at date 1: it may be that o will also produce at date 1, if he perceives that he has cost equal to 0, or he will produce at date 2, if he perceives that he has cost equal to c; hence, the quantity produced by r must solve:

$$\max_{q_r} \frac{1}{2}(1+s)\left(a - q_r - \frac{3a+c+(a+c)s}{8+2s} - c\right) q_r \\ + \frac{1}{2}(1-s)\left(a - q_r - \frac{a-c-q_r}{2} - c\right) q_r.$$

The solution to this problem is $q_r = \frac{9a-13c+(3a-9c-2cs)s}{(3+s)(8+2s)}$.
1. Player r has cost equal to 0:

(i) If r produces at date 1, his expected profit will be:

$$\begin{aligned}
\pi_r^1 &= \frac{1+s}{2}\left(a-\frac{3a+c-2cs}{8+2s}-\frac{3a+c+(a+c)s}{8+2s}\right)\frac{3a+c-2cs}{8+2s} \\
&\quad +\frac{1-s}{2}\left(a-\frac{3a+c-2cs}{8+2s}-\frac{a-c-\frac{3a+c-2cs}{8+2s}}{2}\right)\frac{3a+c-2cs}{8+2s} \\
&= \frac{3(3a+c)^2+(9a^2-30ac-11c^2+4(2c-3a+cs)cs)s}{(16+4s)^2}.
\end{aligned}$$

(ii) If r produces at date 2, his expected profit will be:

$$\begin{aligned}
\pi_r^2 &= \frac{1+s}{2}\left(a-\frac{a-\frac{3a+c+(a+c)s}{8+2s}}{2}-\frac{3a+c+(a+c)s}{8+2s}\right)\frac{a-\frac{3a+c+(a+c)s}{8+2s}}{2} \\
&\quad +\frac{1-s}{2}\left(a-\frac{2a+c}{6}-\frac{a-c}{3}\right)\frac{2a+c}{3} \\
&= \frac{1+s}{2}\left(\frac{5a-c+s(a-c)}{16+4s}\right)^2+\frac{1-s}{2}\left(\frac{2a+c}{6}\right)^2
\end{aligned}$$

Comparing the two expected profits of r, he will prefer to produce at date 1 if:

$$\begin{aligned}
&\frac{3(3a+c)^2+(9a^2-30ac-11c^2+4(2c-3a+cs)cs)s}{(16+4s)^2} \\
&\qquad -\frac{1+s}{2}\left(\frac{5a-c+s(a-c)}{16+4s}\right)^2-\frac{1-s}{2}\left(\frac{2a+c}{6}\right)^2 \geq 0
\end{aligned}$$

or

$$\begin{aligned}
&\frac{1}{18(16+4s)^2}\left\{(5a^2+158ac-19c^2)-(25a^2+214ac+193c^2)s\right. \\
&\qquad \left.+(13a^2+22ac+145c^2)s^2+(7a^2+34ac+67c^2)s^3\right\} \geq 0
\end{aligned}$$

Solving this expression with respect to s, one concludes that r will prefer to produce at date 1 if:

$$s \leq \frac{3\sqrt{3}\sqrt{5a^4+369c^4+588ac^3+68a^3c+314a^2c^2}-(10a^2+28ac+106c^2)}{7a^2+34ac+67c^2}.$$

2. Player r has cost equal to c:
(i) If r produces at date 1, his expected profit will be:

$$\begin{aligned}
\pi_r^1 &= \frac{1+s}{2}\left(\frac{6a-14c+5as-9cs+as^2-cs^2}{14s+2s^2+24}\right)\frac{9a-13c+(3a-9c-2cs)s}{(3+s)(8+2s)} \\
&+\frac{1-s}{2}\left(\frac{a-c}{2}+\frac{13c-9a-3as+9cs+2cs^2}{28s+4s^2+48}\right)\frac{9a-13c+(3a-9c-2cs)s}{(3+s)(8+2s)}
\end{aligned}$$

After doing some algebra this expression simplifies to

$$\pi_r^1 = \frac{81a^2 - 234ac + 169c^2 + \left(54a^2 - 240ac + 234c^2\right)s}{16\left(48 + 40s + 11s^2 + s^3\right)} + \frac{\left(9a^2 - 90ac + 133c^2\right)s^2 - 12acs^3 + 36c^2s^3 + 4c^2s^4}{16\left(48 + 40s + 11s^2 + s^3\right)}.$$

(ii) If r produces at date 2, his expected profit will be:

$$\begin{aligned}
\pi_r^2 &= \frac{1+s}{2}\left(\frac{a-c}{2} - \frac{(3a+c+as+cs)}{4s+16}\right)\frac{a-c-\frac{3a+c+(a+c)s}{8+2s}}{2} \\
&\quad + \frac{1-s}{2}\left(a - \frac{a-c}{3} - \frac{a-c}{3} - c\right)\frac{a-c}{3} \\
&= \frac{1+s}{2}\left(\frac{5a-9c+(a-3c)s}{16+4s}\right)^2 + \frac{1-s}{2}\left(\frac{a-c}{3}\right)^2.
\end{aligned}$$

Comparing the two expected profits of r, he will prefer to produce at date 2 if:

$$\frac{81a^2 - 234ac + 169c^2 + \left(54a^2 - 240ac + 234c^2\right)s}{16\left(48 + 40s + 11s^2 + s^3\right)} + \frac{\left(9a^2 - 90ac + 133c^2\right)s^2 - 12acs^3 + 36c^2s^3 + 4c^2s^4}{16\left(48 + 40s + 11s^2 + s^3\right)} \leq \frac{1+s}{2}\left(\frac{5a-9c+(a-3c)s}{16+4s}\right)^2 + \frac{1-s}{2}\left(\frac{a-c}{3}\right)^2$$

Solving this expression with respect to c, one concludes that r will prefer to produce at date 2 if:

$$c > \frac{123 + 143s + 67s^2 + 11s^3 - 6\sqrt{2}\sqrt{(1+s)(3+s)}(2+s)(4+s)}{87 + 53s - 5s^2 - 7s^3}a = \lambda(s)a.$$

Since $\omega(s) \geq \lambda(s)$ for $s \in [0,1]$, we have that $c > \omega(s)a$ implies $c > \lambda(s)a$. *Q.E.D.*

Proof of Proposition 2: Using the same approach as Branco (2008) we enumerate each type of strategy profile that could be considered and explain why there cannot exist equilibria with such profiles.
Both players produce at date 1, regardless of their cost perceptions:
This cannot be an equilibrium because if both players choose to produce at date 1 regardless of their perceptions they don't have information about the other player and they cannot guarantee a leadership gain. Thus, a player would gain by waiting to see the production of the other player and picking his best response quantity at date 2.
Both players produce at date 2, regardless of their cost perceptions:

This cannot be an equilibrium because if both players wait regardless of their cost perceptions, they have no information gain by waiting. If they deviate by committing to a quantity at date 1 they have a first-mover advantage gain.
A player with a high cost perception produces at date 1 and a player with a low cost perception produces at date 2:
Suppose that there is a cost-dependent equilibrium in which the player with a high cost perception produces at date 1 whereas the player with a low cost perception produces at date 2. In this case, the strategy of the overconfident player in the hypothetical equilibrium would be:
1. If $X_o = c$, then produce a quantity equal to $q_o = \frac{1}{2s-8}\left(4c - 3a + as - 2cs\right)$ at date 1.
2. If $X_o = 0$, then do not produce at date 1. Produce at date 2 according to $q_o = \frac{a}{2} - \frac{1}{2}q_r$ if r has produced q_r at date 1, otherwise produce at date 2 $q_o = \frac{a}{3}$ if neither player has produced at date 1.
The strategy of the rational player in the hypothetical equilibrium would be:
1. If $X_r = c$, then produce a quantity equal to $q_r = \frac{1}{2s-8}\left(4c - 3a + 2cs\right)$ at date 1.
2. If $X_r = 0$, then do not produce at date 1. Produce at date 2 according to $q_r = \frac{a}{2} - \frac{1}{2}q_o$ if o has produced q_o at date 1, otherwise produce at date 2 $q_r = \frac{a}{3}$ if neither player has produced at date 1.
In this hypothetical cost-dependent equilibrium, the overconfident player with a low cost perception has expected profits equal to $\frac{3}{16(s-4)^2}\left(3a - 4c - as + 2cs\right)^2$.
However, if the overconfident player deviates and produces at date 1 a quantity equal to $\frac{1}{24-6s}\left(9a + 4c - 3as + 2cs\right)$, he will obtain expected profits equal to

$$\frac{\left(9a + 4c - 3as + 2cs\right)^2}{48\left(s-4\right)^2} > \frac{3\left(3a - 4c - as + 2cs\right)^2}{16\left(s-4\right)^2}.$$

Therefore, the strategy profiles cannot be a cost-dependent equilibrium. *Q.E.D.*

Proof of Proposition 3: The ex-ante profits of player o are equal to

$$\Pi_o(s) = \frac{1+s}{2}\pi_o^1 - \frac{sc}{2}q_o^1 + \frac{1-s}{2}\pi_o^2.$$

Making use of the expressions obtained for π_o^1, q_o^1, and π_o^2 in Proposition 2 we have

$$\begin{aligned}\Pi_o(s) = {}& \frac{1+s}{2}\frac{3(3a+c)^2 + 3\left(4a + (a+c)(2+s)\right)(a+c)s}{(16+4s)^2} \\ & - \frac{sc}{2}\frac{3a + c + (a+c)s}{8+2s} + \frac{1-s}{2}\left(\frac{1}{2}\left(\frac{5a - 9c + 2as}{16+4s}\right)^2 + \frac{1}{2}\left(\frac{a-c}{3}\right)^2\right),\end{aligned}$$

which can be simplified to

$$\begin{aligned}\Pi_o(s) = {}& \frac{a^2}{36(16+4s)^2}\left[967 - 998x + 1039x^2 + (637 - 230x - 1271x^2)s\right. \\ & \left. +2(61 + 40x - 335x^2)s^2 + 2(1 - 2x - 53x^2)s^3\right]. \qquad (3)\end{aligned}$$

The ex-ante profits of player r, $\Pi_r(s) = \frac{1}{2}\pi_r^1 + \frac{1}{2}\pi_r^2$, are given by

$$\begin{aligned}\Pi_r(s) \quad &= \quad \frac{1}{2}\frac{3(3a+c)^2 + (9a^2 - 30ac - 11c^2 + 4(2c - 3a + cs)cs)s}{(16+4s)^2} \\ &\quad +\frac{1}{2}\left(\frac{1+s}{2}\left(\frac{5a - 9c + (a-3c)\,s}{16+4s}\right)^2 + \frac{1-s}{2}\left(\frac{a-c}{3}\right)^2\right),\end{aligned}$$

which can be simplified to

$$\begin{aligned}\Pi_r(s) \quad &= \quad \frac{a^2}{36(16+4s)^2}\left[967 - 998x + 1039x^2 + (349 - 1526x + 889x^2)s\right. \\ &\quad \left. -(13 + 478x - 599x^2)s^2 - (7 + 22x - 137x^2)s^3\right]. \qquad (4)\end{aligned}$$

The ex-ante profits of a player in an endogenous timing game with two rational players are:

$$\Pi(0) = \Pi_o(0) = \Pi_r(0) = \frac{a^2}{36}\frac{967 - 998x + 1039x^2}{16^2}. \qquad (5)$$

The difference between (3) and (5) is given by:

$$\begin{aligned}\Pi_o(s) - \Pi(0) \quad &= \quad \frac{sa^2}{36 \times 16^2(16+4s)^2}\left[-(458368 + 188144s + 27136s^2)x^2\right. \\ &\quad \left. +(68864 + 36448s - 1024s^2)x + (39296 + 15760s + 512s^2)\right]. \qquad (6)\end{aligned}$$

From (6) we have that $\Pi_o(s) > \Pi(0)$ as long as

$$x < \frac{2152 + 1139s - 32s^2 + 24(s+4)\sqrt{8137 + 2659s + 96s^2}}{28648 + 11759s + 1696s^2} = \tau(s).$$

The difference between (5) and (4) is:

$$\begin{aligned}\Pi(0) - \Pi_r(s) = &\frac{sa^2}{36 \times 16^2(16+4s)^2}\left[-(94592 + 136720s + 35072s^2)x^2\right. \\ &\left. +(262912 + 106400s + 5632s^2)x + (34432 + 18800s + 1792s^2)\right]. \qquad (7)\end{aligned}$$

From (7) we see have that $\Pi(0) > \Pi_r(s)$ for all $0 < s < s(x)$ since the restrictions on the parameters imply that $x > x^2$ and $262912 + 106400s + 5632s^2 > 94592 + 136720s + 35072s^2$. *Q.E.D.*

Proof of Proposition 4: From (6) and (7), the change in aggregate profits, $\triangle\Pi$, is equal to

$$\begin{aligned}\triangle\Pi &= \Pi_o(s) - \Pi(0) - (\Pi(0) - \Pi_r(s)) \\ &= \frac{sa^2}{36\,(2048 + 1024s + 128s^2)}\left[152 - 95s - 40s^2\right. \\ &\quad \left. -(6064 + 2186s + 208s^2)x - (11368 + 1607s - 248s^2)x^2\right].\end{aligned}$$

The ex-ante consumer surplus in the model with the overconfident player is equal to

$$
\begin{aligned}
CS(s) &= \frac{1+s}{4}CS(l,l) + \frac{1+s}{4}CS(l,f) + \frac{1-s}{4}CS(f,l) + \frac{1-s}{4}CS(f,f) \\
&= \frac{1+s}{4}\frac{1}{2}\left(\frac{6a+2c+as-cs}{2s+8}\right)^2 + \frac{1+s}{4}\frac{1}{2}\left(\frac{11a-7c+3as-cs}{4s+16}\right)^2 \\
&\quad + \frac{1-s}{4}\frac{1}{2}\left(\frac{11a-7c+2as-4cs}{4s+16}\right)^2 + \frac{1-s}{4}\frac{1}{2}\left(2\frac{a-c}{3}\right)^2.
\end{aligned}
$$

After some algebra the above expression simplifies to

$$
\begin{aligned}
CS(s) = \frac{a^2}{96\left(192+96s+12s^2\right)}&\left[4498+2206s+335s^2+17s^3\right. \\
&\left.-(3956+20s-806s^2-146s^3)x+(2050+118s-781s^2-163s^3)x^2\right].
\end{aligned}
$$

The ex-ante consumer surplus in the model with two rational players is equal to

$$
\begin{aligned}
CS(0) &= \frac{1}{4}CS(l,l) + \frac{1}{2}CS(l,f) + \frac{1}{4}CS(f,f) \\
&= \frac{1}{4}\frac{1}{2}\left(\frac{3a+c}{4}\right)^2 + 2\frac{1}{4}\frac{1}{2}\left(\frac{3a+c}{8}+\frac{5a-9c}{16}\right)^2 + \frac{1}{4}\frac{1}{2}\left(2\frac{a-c}{3}\right)^2 \\
&= \frac{a^2}{96^2}\left(2249-1978x+1025x^2\right).
\end{aligned}
$$

The change in consumer surplus, $\triangle CS = CS(s) - CS(0)$, is equal to

$$
\begin{aligned}
\triangle CS &= \frac{sa^2}{96\left(1536+768s+96s^2\right)}\left[-344+431s+136s^2\right. \\
&\quad \left.+(15664+8426s+1168s^2)x-(7256+7273s+1304s^2)x^2\right]
\end{aligned}
$$

The change in welfare, $\triangle W = \triangle\Pi + \triangle CS$, is given by

$$
\begin{aligned}
\triangle W &= \frac{sa^2}{36\times 96}\left[\frac{-(1079712+377532s+29088s^2)x^2}{\left(1536+768s+96s^2\right)\left(2048+1024s+128s^2\right)}\right. \\
&\quad \left.+\frac{(127296+145944s+27072s^2)x+(-1440+8676s+2016s^2)}{\left(1536+768s+96s^2\right)\left(2048+1024s+128s^2\right)}\right]
\end{aligned}
$$

From the expression above we have $\triangle W > 0$ as long as

$$
x < \frac{1768+2027s+376s^2+24(s+4)\sqrt{209+1412s+324s^2}}{29992+10487s+808s^2} = \psi(s).
$$

Q.E.D.