Elasticity Optimism

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February 2009
Elasticity Pessimism

- The substitutability between domestic and foreign goods is central to most calibrated models in international economics.
- Its calibrated value draws from literally decades of empirical work.
- Usually, calibration exercises infer its value from aggregate estimates of imports elasticity using:

\[ \sigma = 1 - \eta, \quad \eta = \frac{\partial M}{\partial P} \frac{P}{M} \]

with \( M \) aggregate imports in value and \( P \) a measure of relative aggregate prices (e.g. domestic vs. imported bundles).


\[ -2 \leq \eta \leq 0, \quad \text{i.e.} \ 1 \leq \sigma \leq 3 \]

⇒ Elasticity Pessimism
In micro data, the approach has been similar, estimating:

$$\sigma_i = 1 - \eta_i, \quad \eta_i = \frac{\partial m_i}{\partial p_i} \frac{p_i}{m_i}$$

with $m_i$ the value of imports in good $i$ and $p_i$ a measure of its relative price.

Identification often easier in micro studies, because exogenous movements in $p_i$ given by dedicated changes in relative price of $i$, e.g. tariff changes. Or more generally via structural estimation.


$$0 \leq \sigma_i \leq 12$$

⇒ Elasticity Heterogeneity
Macro estimates are based on assumption that

$$\sigma_i = \sigma_j$$

Can one get an estimate of the aggregate elasticity of substitution ($\sigma$) allowing for heterogeneity in the sectoral elasticities ($\sigma_i$)?

Orcutt (1950): “in aggregate trade equations, goods with relatively low price elasticities can display the largest variation in prices and therefore exert a dominant effect on the estimated aggregate price elasticity, thereby biasing the estimate downwards.”

The response of aggregate quantities to aggregate prices can be a biased estimate of the aggregate elasticity of substitution. Presumably an estimate of the parameter that accounts for the well-documented cross-sector heterogeneity is more consistent with the data. Matter for calibration purposes.

⇒ Elasticity Optimism
What We Do

- Simple framework accounting for the heterogeneity in substitutability across goods. Ask from it how to properly aggregate microeconomic estimates.
- Link between estimated import elasticity and elasticity of substitution obscured in the presence of firm dynamics (Chaney, 2008). Theory accounts for firm entry/exit.
- Estimate the disaggregated elasticities. Use the structural method proposed by Feenstra (1994) that identifies elasticities of substitution using the cross-country variation in trade flows towards the USA. Tackles endogeneity issues - at disaggregated level, and in aggregated version.
- Get sector specific elasticities vs. get one, constrained to be the same across sectors.
- Use the model to aggregate either case adequately. Compare the outcomes when elasticity is constrained to homogeneity and when it is not.
When all elasticities are forced to be equal across sectors, the estimated aggregate price elasticity of imports is around -1.9, i.e. within the ballpark of values used in the macroeconomic literature.

With heterogeneity, aggregate price elasticity of imports more than double (up to -5). The corresponding aggregate substitutability is around 7.

Robust to various alternative measures or econometric procedures.
Accommodating the well-known unambiguous fact that some goods are more substitutable than others means calibrated models should use 7 rather than 2.

This matters quantitatively and sometimes qualitatively for calibrated international macro models.

We discuss the implications of our results in various calibrated models concerned with the rebalancing of external imbalances (Obstfeld & Rogoff, 2005), the international diffusion of shocks (Kose & Yi, 2006, Corsetti, Dedola and Leduc, 2008), the extent of international risk sharing (Cole & Obstfeld, 1991), the composition of international portfolio holdings (Coeurdacier, 2005), international price differences (Atkeson & Burstein, 2008), the optimal conduct of monetary policy (Galí & Monacelli, 2005).

We finish with an illustration in a 2-sector version of the Backus, Kehoe & Kydland (1994) model.
Plan

- Theory
- Empirical estimation
- Results
- Relevance
- Conclusion
Some Theory

\[ C = \prod_{k \in K} \frac{C_k^{\alpha_k}}{\alpha_k} \]

\( k \) a good, \( \alpha_k \) preference parameter.

\[ C_k = \left[ \sum_{i \in I} (\beta_{ki} C_{ki})^{\frac{\sigma_k - 1}{\sigma_k}} + (\beta_{kd} C_{kd})^{\frac{\sigma_k - 1}{\sigma_k}} \right]^{\frac{\sigma_k}{\sigma_k - 1}} \]

\( i \) a foreign variety (an exporting country in the empirics), \( d \) the domestic variety. \( \sigma_k \) constant elasticity of substitution (different across goods, but identical across varieties). \( \beta_{ki} \) preference parameter.

\[ C_{kj} = \left[ \int_{f} c_{kjf}^{\frac{\rho_k - 1}{\rho_k}} df \right]^{\frac{\rho_k}{\rho_k - 1}} \]

\( f \) range of firms active in sector \( k \) and country \( j \). \( \rho_k \) elasticity of substitution between producers of the same variety of good \( k \)
Optimal consumption

- At the sectoral level:

\[ C_{ki} = \beta_{ki}^{\sigma_k-1} \left( \frac{P_{ki}}{P_k} \right)^{-\sigma_k} \alpha_k \frac{P}{P_k} C, \ i \neq d \]

with

\[ P_{ki} = \left[ \int f \left( \frac{1}{P_{kif}} \right)^{\frac{1}{1-\rho_k}} df \right]^{\frac{1}{1-\rho_k}} \]

\[ P_k = \left[ \sum_{i \in I} \left( \frac{P_{ki}}{\beta_{ki}} \right)^{1-\sigma_k} + \left( \frac{P_{kd}}{\beta_{kd}} \right)^{1-\sigma_k} \right]^{\frac{1}{1-\sigma_k}} \]

\[ P = \prod_{k \in K} P_k^{\alpha_k} \]

- At the firm-level:

\[ c_{kif} = \left( \frac{p_{kif}}{P_{ki}} \right)^{-\rho_k} C_{ki} \]
Aggregate substitutability

- Define $\sigma$ as the response of aggregate quantities to changes in aggregate international relative prices, accounting or not for the heterogeneity in $\sigma_k$.
- Focus on changes in all relative prices (no cross-sector reallocation) and on uniform shocks (no reallocation across exporting economies). It is relative quantities whose responses may be heterogeneous.
- A natural candidate is a domestic shock to relative production costs ("domestic wage" shock appreciating relative prices).

$$\sigma = 1 + \frac{\partial \ln \sum_k \sum_{i \neq d} P_{ki} C_{ki} - \partial \ln P_{kd} C_{kd}}{\partial \ln w_d}$$
With

\[ n_{ki} \equiv \frac{P_{ki} C_{ki}}{\sum_{k \in K} \sum_{i \neq d} P_{ki} C_{ki}} \]

\[ n_{kd} \equiv \frac{P_{kd} C_{kd}}{\sum_{k \in K} P_{kd} C_{kd}} \]

\[ n_k = \sum_{i \neq d} n_{ki} \]
In the long-run, each domestic producer responds identically and proportionally to the shock while foreign producers do not respond to the shock.

The aggregate substitutability is given by:

\[
\sigma_{\text{NoFirm}} = \sum_{k} n_{kd} \sigma_k + \sum_{k} (n_k - n_{kd}) (\sigma_k - 1) (1 - w_k^M)
\]

where

\[
w_k^M = \frac{\sum_{i \neq d} P_{ki} C_{ki}}{P_k C_k}
\]

Second term captures response of industry-specific price indices to macroeconomic shocks. Partial vs. Total Elasticity.

Partial aggregate elasticity of substitution is a weighted average of industry-specific preference parameters.

In macroeconomic data, traded quantities are summed up to the country level before estimating substitutability. This implicitly assumes \(\sigma_k = \sigma, \forall k\) and induces a bias if \(n_{kd}\) and \(\sigma_k\) happen to be correlated.

We seek to estimate the magnitude of the difference between the unconstrained elasticity \(\sigma_{\text{NoFirm}}\) and the constrained version \((\bar{\sigma}_{\text{NoFirm}})\).
Allowing for Firm Entry

- Introduce heterogeneity à la Melitz:

\[
P_{kj} = \left[ \frac{M_{kj}}{1 - G_{kj}(\bar{\varphi}_{kj})} \int_{\bar{\varphi}_{kj}}^{\infty} p_{kjf}(\varphi)^{1-\rho_k} dG_{kj}(\varphi) \right]^{\frac{1}{1-\rho_k}}, \quad j = (i \in I)/d
\]

- \(M_{kj}\) mass of exporting firms active in sector \(k\) and country \(j\).
- \(G_{kj}(\varphi)\) CDF of firms’ productivity in sector \(k\) and country \(j\) (Pareto assumption)
- \(\bar{\varphi}_{kj}\) (endogenous) productivity cut-off level, above which firms are active on export markets.

- Behind the response of individual domestic producers (\(\partial p_{kdf}/\partial w_d = 1\)), price indices also respond to the shock at the extensive margin (through \(\bar{\varphi}_{kj}\) in the short run, through \(M_{kj}\) in the long run)
Consistent with our estimation strategy, we focus on long-run adjustments (≠ Chaney, 2008)

After some calculations, the aggregate elasticity of substitution becomes:

\[
\sigma_{Firm} = \sigma_{NoFirm} + \sum_k n_{kd} (\sigma_k - 1) \frac{\sigma_k}{\rho_k - \sigma_k}
\]

\[
+ \sum_k (n_k - n_{kd}) (\sigma_k - 1)(1 - w_k^M) \frac{\sigma_k}{\rho_k - \sigma_k}
\]

The second term is positive if \(\rho_k > \sigma_k > 1\). It encapsulates the relative responses of quantities produced at the extensive margin.

The third term can take either sign, but is small in magnitude.
Question: Do supply responses alter the existence of an aggregation bias?

Compare the discrepancy between the constrained and unconstrained versions of $\sigma_{Firm}$ and that obtained from $\sigma_{NoFirm}$

\[
\sigma_{Firm} - \bar{\sigma}_{Firm} = \sigma_{NoFirm} - \bar{\sigma}_{NoFirm} + \sum_k n_{kd} \left[ (\sigma_k - 1) \frac{\sigma_k}{\rho_k - \sigma_k} - (\sigma - 1) \frac{\sigma}{\rho - \sigma} \right]
+ \sum_k (n_k - n_{kd}) (1 - w_k^M) \left[ (\sigma_k - 1) \frac{\sigma_k}{\rho_k - \sigma_k} - (\sigma - 1) \frac{\sigma}{\rho - \sigma} \right]
\]

Under plausible conditions, the theory implied bias with firm entry is larger than its counterpart assuming supply responses away.

⇒ Our estimate of the bias is a lower bound.
In most of the literature, the elasticity of substitution is inferred from the price elasticity of imports ($\sigma = 1 - \eta$).

To compare our results with the literature, we also compute the price elasticity of imports:

$$\eta = \frac{\partial \ln \left[ \sum_k \sum_i P_{ki} C_{ki} \right]}{-\partial \ln w_d}$$

Without firm entry, we get:

$$\eta_{NoFirm} = 1 - \sigma_{NoFirm} + \sum_k n_{kd} (\sigma_k - 1) w_k^M - \sum_k \alpha_k (1 - w_k^M)$$

⇒ The linear relation between $\sigma$ and $\eta$ is only valid in terms of partial elasticities

⇒ The possibility that estimates based on macroeconomic data should be biased continue to prevail for the price elasticity of imports
Estimate $\sigma_k$ using Feenstra (1994)

Estimate $\sigma$ using Feenstra (1994) and imposing homogeneity.

Compare with the literature in both cases (helps validate Armington assumption)

Note both estimates are obtained with same data, same methodology, same estimator. Data embeds firm entry decisions to exact same extent for estimates of $\sigma_k$ and if $\sigma$ (Ruhl, 2005).
Use Feenstra’s (1994) methodology: identify the substitutability using the observed cross-section of traded quantities and prices across exporters to one destination.

Crucial assumption of an Armington aggregator between varieties of each good, irrespective of their origin. This is what exonerates from having data on domestic production.

Identification in the absence of any supply-shifting instruments. Augment the model with a simple supply structure with production decisions taken on the basis of the price net of transport costs, labeled in domestic currency.
Microeconomic estimates (2)

- Model at the root of the estimation:

\[
\begin{align*}
C_{kit} &= \left(\frac{P_{kit}}{P_{kt}}\right)^{1-\sigma_k} \frac{\beta_{kit}^{\sigma_k-1}P_{kt}C_{kt}}{P_{kit}} \\

P_{kit} &= \tau_{kit} \exp(\nu_{kit}) C_{kit}^{\omega_k}
\end{align*}
\]

\(\nu_{kit}\) a random technology factor, independent from the taste parameter. \(\omega_k\) the inverse supply elasticity.

- Functional form of supply can be derived from model of supply with firm entry.

- Implicitely assumes that price decisions are in LCP but assuming PCP pricing would be innocuous from an empirical standpoint.
Estimable regression (see Feenstra, 1994):

\[ Y_{kit} = \theta_{1k}X_{1kit} + \theta_{2k}X_{2kit} + u_{kit} \]

where \( Y_{kit}, X_{1kit} \) and \( X_{2kit} \) are computed using information on bilateral prices and market shares.

Identification on the cross-section of exporters in relative terms with respect to a reference country \( r \).

Map directly with the parameters of interest:

\[ \theta_{1k} = \frac{\omega_k}{(\sigma_k - 1)(1 + \omega_k)} \]
\[ \theta_{2k} = \frac{\omega_k \sigma_k - 2\omega_k - 1}{(\sigma_k - 1)(1 + \omega_k)} \]
The correlation between $u_{kit}$, $X_{1kit}$ and $X_{2kit}$ disappears when taking time averages (Feenstra 1994). Instrument $X_{1kit}$ and $X_{2kit}$ with $i$-specific dummy variables.

Identification therefore purely cross-sectional. Focus on long run.

Standard errors corrected for heteroscedasticity across exporters.

Include a $hs6$-specific intercept that accounts for measurement error arising from using unit values to approximate prices.

Add common correlated effects to control for aggregate shocks (Pesaran, 2006)

⇒ Estimated equation:

$$Y_{kit} = \theta_0 + \theta_{1k}\hat{X}_{1ki} + \theta_{2k}\hat{X}_{2ki} + \theta_{3k}X_{1it} + \theta_{4k}X_{2it} + u_{kit}$$
Microeconomic estimates (5)

- Using the consistent (and sector-specific) estimates of $\theta_{1k}$ and $\theta_{2k}$, it is straightforward to infer elasticities:

$$\hat{\sigma}_k = 1 + \frac{\hat{\theta}_{2k} + \Delta_k}{2\hat{\theta}_{1k}}$$  if $\hat{\theta}_{1k} > 0$ and $\hat{\theta}_{1k} + \hat{\theta}_{2k} < 1$

$$\hat{\sigma}_k = 1 + \frac{\hat{\theta}_{2k} - \Delta_k}{2\hat{\theta}_{1k}}$$  if $\hat{\theta}_{1k} < 0$ and $\hat{\theta}_{1k} + \hat{\theta}_{2k} > 1$

- For combinations of estimates that do not correspond to any theoretically consistent estimates of $\hat{\sigma}_k$, follow Broda and Weinstein (2006) and use a search algorithm that minimizes the sum of squared residuals over the intervals of admissible values. Standard errors obtained via bootstrapping using 1,000 repetitions.
Aggregation

- Estimate of aggregate substitutability allowing for heterogeneity:

\[ \sigma_{NoFirm} = \sum_{k \in K} n_{kd} \hat{\sigma}_k + \sum_{k \in K} (n_k - n_{kd})(1 - w_k^M)(\hat{\sigma}_k - 1) \]

- Constrained estimate of aggregate substitutability imposing homogeneity:

\[ \bar{\sigma}_{NoFirm} = \hat{\sigma} + (\hat{\sigma} - 1) \sum_{k \in K} (n_k - n_{kd})(1 - w_k^M) \]

with \( \hat{\sigma} \) estimated using the same method but on a pooled dataset formed by observations on all sectors and imposing coefficient equality across sectors.
Use the BACI database that describes bilateral trade at the sectoral level (6-digit level of the harmonized system, 5,000 products)

Cover the 1996-2004 period

Products grouped within industries as defined by the 3-digit level of the ISIC (revision 3) nomenclature (i.e. assume all HS6 goods to be equally substitutable within an ISIC industry)

Sampling to limit the role of extreme outliers: Exclude annual variations in prices and market shares that exceed 5 times the median value of the sector

Impose a minimum of 20 exporters for each HS6 good

Data ultimately cover 73% of the total value of US imports
In the model, $n_k$ and $n_{kd}$ depend directly on $w^M_k$ and $\alpha_k$:

$$n_k = \frac{\alpha_k w^M_k}{\sum_k \alpha_k w^M_k} \text{ and } n_{kd} = \frac{\alpha_k (1 - w^M_k)}{\sum_k \alpha_k (1 - w^M_k)}$$

- $w^M_k$ computed as the 1997 ratio of imports over domestic gross output. Source: OECD-IO Tables

- $\alpha_k$ computed as the 1997 ratio of sectoral relative to total absorption. Source: OECD-STAN

- Sensitivity analysis: Imports from BACI, Output data from STAN, I/O used to compute $n_k$, Absorption in terms of value added or gross output.
Comparison with existing studies

- Median value at the low end of the range of substitution elasticity estimates: Romalis (2007) between 6.2 and 10.9, Head & Ries between 7.9 and 11.4, Hanson (2004) between 4.9 and 7.6

- Similar to fundamental contributions to the literature on imports price elasticities, that evaluate import prices relative to their domestic counterpart: Houtakker & Magee (1969) -4.05 in manufactures, Kreinin (1967) -4.71 for manufactures. Higher estimates for manufactures, followed by semi-manufactures and crude foods and materials. The ranking is roughly prevalent in our results.

⇒ Vindicates the Armington assumption.
Table: Estimation with common correlated effects

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<tr>
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<th>Import Elasticity</th>
<th>Substitution Elasticity</th>
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<tbody>
<tr>
<td></td>
<td>$\eta_{NoFirm}$</td>
<td>$\sigma_{NoFirm}$</td>
</tr>
<tr>
<td>Constrained total elasticity</td>
<td>-1.980$^a$</td>
<td>4.124$^a$</td>
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<tr>
<td></td>
<td>(.175)</td>
<td>(0.300)</td>
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<tr>
<td>Constrained partial elasticity</td>
<td>-2.738$^a$</td>
<td>3.738$^a$</td>
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<tr>
<td></td>
<td>(.262)</td>
<td>(0.263)</td>
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<tr>
<td>Unconstrained total elasticity</td>
<td>-4.508$^a$</td>
<td>7.226$^a$</td>
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<tr>
<td></td>
<td>(.745)</td>
<td>(0.962)</td>
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<tr>
<td>Unconstrained partial elasticity</td>
<td>-6.553$^a$</td>
<td>6.921$^a$</td>
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<tr>
<td></td>
<td>(1.100)</td>
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<td>Number of grid searches</td>
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</tr>
</tbody>
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Note: Standard errors in parentheses (obtained by bootstrapping for grid searched sectors), $^a$ denotes significance at the 1% level.
Macroeconomic estimates (3)

When it is constrained to be homogeneous across sectors, the estimated elasticity is -1.98, i.e. at the high range of values obtained in conventional estimates based on macroeconomic data (e.g. Goldstein & Kahn (1985) between -1.03 and -1.76). Consistent with the choices made in the vast majority of calibration exercises (Obstfeld & Rogoff, 2005, Backus et al., 1992)

When the elasticity is left unconstrained across sectors, the aggregate price elasticity of imports is -4.5 and the aggregate substitutability jumps to more than 7.

Magnitude of the bias robust to various alternative measures / econometric procedures

Orcutt (1950): “in aggregate trade equations, goods with relatively low price elasticities can display the largest variation in prices and therefore exert a dominant effect on the estimated aggregate price elasticity, thereby biasing the estimate downwards.”
Accommodating the well-known unambiguous fact that some goods are more substitutable than others means calibrated models should use 7 rather than 2. This matters quantitatively and qualitatively.

**Obstfeld & Rogoff (2005):** Use a calibrated model with a substitutability at 2 to argue a reversal of the US current account is compatible with a 30% depreciation of the real exchange rate. In a slightly simplified two-country version, we obtained depreciation rates of 22 or 21% for values of the parameter of 5 or 7.

**Cole & Obstfeld (1991):** The endogenous response of the terms of trade can deliver perfect insurance against country-specific shocks when the substitutability between domestic and foreign goods is unitary. Home equity bias can arise under low substitutability (Heathcote & Perri, 2008) while domestic consumers will want to hold foreign assets for high values of the parameter, when the terms-of-trade response to real shocks is muted (Coeurdacier, 2005)
Atkeson & Burstein (2008): Explain observed deviations from PPP in a model with trade costs, imperfect competition and variable markups. In their calibration, the elasticity of substitution between the (foreign and domestic) varieties equals 10; deviations from PPP virtually disappear for an alternative value set at 3.

Galí & Monacelli (2005): How exchange rates matter in the monetary policy rule. Use unitary substitutability. More generally, with non unitary elasticity policy shocks that affect the terms-of-trade also affect welfare, in a way that crucially depends on whether the calibrated parameter is above or below one.

Backus, Kehoe and Kydland (1994): we simulate a two-sector version, where the only difference between sectors is $\sigma_k$. We asked from a one-sector version what (aggregate, single) value of $\sigma$ would reproduce the J-curve implied by model with heterogeneous $\sigma_k$. Calibration as in BKK except for the $\sigma_k$s and the weights, measured in our data.
The simulation of the 1-sector model with a substitutability equal to the weighted mean of the 2-sector model’s elasticities clearly dominates the simulation using a simple average.
Enormous evidence that elasticities of substitution are heterogeneous across goods and sectors. Must be accounted for when calibrating the aggregate average value of the parameter in macroeconomics. An estimation on the basis of aggregated data does NOT pin down the aggregate parameter.

Estimates suggest the aggregate substitutability in the US is closer to 7 than to 2.

This has important quantitative and qualitative implications for calibrated macro models.