Session 5: Nominal Rigidities

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Keynesian Economics

- The key distinction between neoclassical and Keynesian economics is the assumption on price adjustment.
- In neoclassical economics, prices adjust so that supply and demand equate and equilibrium obtains - without need for any government intervention (fiscal or monetary). Fluctuations in activity are just variations in that equilibrium
- In Keynesian economics, in contrast, prices do not adjust (or do so slowly) - so that it is possible to have situation of supply$>$demand (unemployment) or supply$<$demand (shortages).
- And government intervention will bring the economy back to equilibrium faster than waiting for prices to do it.

New Keynesian Economics

- Enormous literature trying to see whether empirical properties of the data can be reproduced in a (dynamic general equilibrium) model with flexible vs. rigid prices.
- Key in this literature are the "micro-foundations" for price-rigidities. Rather than assuming that prices are simply fixed (like Keynes -or IS/LM- did in the 30s and 40s), try to think of micro-founded reasons why they are.
- Difficult to take stock as controversy still ongoing. But seems consensus is that price rigidities are required - and plausible.
- So called "New Keynesian" models try to combine the rigorous micro-foundations from general equilibrium, and introducing price rigidities that are "optimal" in the model. I.e. motives for firms NOT to change their prices

Mircro-foundations of Price Rigidities

- Three ingredients
- Firms are STILL optimizing
- There is imperfect competition (either in goods or in labor markets), so that firms are price-SETTERS.
- And thus can choose optimally potentially NOT to adjust prices. Or to adjust them slowly. In perfect competition, prices are GIVEN by the market!

Consequences

- So now individual firms may choose to change their price sluggishly. First, this requires a model of individual price setting, for individual goods (as opposed to a model of the overall price level - i.e. of CPI)
- Second this has implications on the sluggishness of inflation as a whole - which may become more sticky as a result
- So we need a framework with monopolistic competition, either in goods or in labor markets.

Monopolistic Competition

- Under perfect competition, prices are equal to marginal costs, since otherwise they are competed down.To be able to set prices, firms must have a degree of monopoly power, i.e. something that prevents the consumers from bidding the price down.
- With monopoly power on the goods market, firms can charge prices at a markup over the marginal cost. (Same with monopoly power on the labor market -either on employer or employee side- resulting in wages different from the MPL)
- The conventional theory is one of "monopolistic competition", where there is a large range of different goods and services, each one being imperfect substitute for the others, and each one being produced by a firm.
- That firm therefore exerts some monopoly power over its own variety

Monopolistic Competition - Firms

- The production function for the i^{th} firm is given by

$$y_t(i) = n_t(i)$$

where $n_t(i)$ is the labor input of firm i.

- Profits are given by

$$\Pi_t(i) = P_t\,(i)\,y_t(i) - W_t(i)n_t(i)$$

with $P_t\,(i)$ the output price and $W_t(i)$ the wage rate.

Monopolistic Competition - Households

- Each household works for one type of firm i, and has utility

$$U\left[c_t, n_t(i)\right] = U\left[c_t\right] + \eta l_t\left(i\right)^{\varepsilon}$$

where c_t is TOTAL consumption (i.e. of all varieties i), $l_t\left(i\right)$ is leisure and $l_t\left(i\right) + n_t\left(i\right) = 1$. In other words, households spend their 1 unit of time working or enjoying life.

- Crucially now, total consumption is obtained by aggregating over the N different types of goods and services $c_t\left(i\right)$ using the constant elasticity of substitution function

$$c_t = \left[\sum_{i=1}^{N} c_t\left(i\right)^{\frac{\phi-1}{\phi}}\right]^{\frac{\phi}{\phi-1}}$$

Crucially, $\phi > 1$.

Price Indices

- Total household expenditures is

$$P_t c_t = \sum_{i=1}^{N} P_t(i) c_t(i)$$

So that the general price index verifies

$$P_t = \sum_{i=1}^{N} P_t(i) \frac{c_t(i)}{c_t}$$

- Finally, the budget constraint writes

$$P_t c_t = \sum_{i=1}^{N} P_t(i) c_t(i) = W_t(i) n_t(i) + \sum_{i=1}^{N} \Pi_t(i)$$

where we have assumed that each households holds a perfectly diversified portfolio of shares.

Solution - Households Side

- Now there is no capital in this model - and so everything is static. (To be precise, we also assume there is no trading in shares at any point either). Thus, the static Lagrangian writes

$$\begin{aligned}\mathcal{L} &= U\left(\left[\sum_{i=1}^{N} c_t(i)^{\frac{\phi-1}{\phi}}\right]^{\frac{\phi}{\phi-1}}\right) + \eta l_t(i)^{\varepsilon} \\ &\quad + \lambda_t\left[W_t(i)n_t(i) + \sum_{i=1}^{N}\Pi_t(i) - \sum_{i=1}^{N}P_t(i)c_t(i)\right]\end{aligned}$$

- First order conditions

$$\begin{aligned}\frac{\partial\mathcal{L}}{\partial c_t(i)} &= \left[\sum_{i=1}^{N} c_t(i)^{\frac{\phi-1}{\phi}}\right]^{\frac{\phi}{\phi-1}-1} c_t(i)^{\frac{\phi-1}{\phi}-1} U'(c_t) - \lambda_t P_t(i) = 0 \\ \frac{\partial\mathcal{L}}{\partial l_t(i)} &= \eta\varepsilon l_t(i)^{\varepsilon-1} - \lambda_t W_t(i) = 0\end{aligned}$$

Solution - Households Side

- So we have

$$
\begin{aligned}
c_t^{\frac{1}{\phi}} c_t\left(i\right)^{-\frac{1}{\phi}} U'\left(c_t\right) &= \lambda_t P_t(i) \\
\frac{c_t\left(i\right)}{c_t} &= \left[\frac{\lambda_t P_t(i)}{U'\left(c_t\right)}\right]^{-\phi}
\end{aligned}
$$

Solution - Households Side

- Note that the problem can be rewritten in function of aggregate prices and consumption only:

$$\begin{aligned}\mathcal{L} &= U(c_t) + \eta l_t(i)^{\varepsilon} \\ &\quad + \lambda_t \left[W_t(i) n_t(i) + \sum_{i=1}^{N} \Pi_t(i) - P_t c_t \right]\end{aligned}$$

- Then the first order condition with respect to aggregate consumption is

$$\frac{\partial \mathcal{L}}{\partial c_t} = U'(c_t) - \lambda_t P_t = 0$$

- In other words, we must have

$$\frac{1}{P_t} = \frac{\lambda_t}{U'(c_t)}$$

Solution - Households Side

- Substitute back in the i-specific condition to get

$$\frac{c_t(i)}{c_t} = \left[\frac{P_t(i)}{P_t}\right]^{-\phi}$$

This is a demand function for good i. RELATIVE demand for good i increases with the overall price level, but decreases with the price of that variety. And that finding also makes it possible to solve for the price index in function of i-specific prices only, i.e.

$$\begin{aligned} P_t &= \sum_{i=1}^{N} P_t(i) \left[\frac{P_t(i)}{P_t}\right]^{-\phi} \\ P_t &= \left[\sum_{i=1}^{N} P_t(i)^{1-\phi}\right]^{\frac{1}{1-\phi}} \end{aligned}$$

Solution - Households Side

- Finally, the first order condition with respect to labor implies

$$
\begin{aligned}
l_t(i) &= \left[\frac{\lambda_t W_t(i)}{\eta\varepsilon}\right]^{\frac{1}{\varepsilon-1}} \\
l_t(i) &= \left[\frac{U'(c_t)\, W_t(i)}{\eta\varepsilon P_t}\right]^{\frac{1}{\varepsilon-1}}
\end{aligned}
$$

which the labor supplied by the household. Notice that if all households have the same utility function and work equally hard (i.e. firms are indifferent as regards who they hire), then all wages are the same for all firms.

Solution - Firm side

- Now on the firm side, each firm hires labor to maximize profits. But each firm has a monopoly onto its own good i, and so it will choose how much labor to hire keeping in mind it is facing a demand curve.
- The demand, in turn, is given by

$$c_t\left(i\right) = \left[\frac{P_t(i)}{P_t}\right]^{-\phi} c_t$$

Recognize now that, without any investment or government spending, $c_t\left(i\right) = y_t\left(i\right) = n_t(i)$, i.e all the production by each firm at time t must be fully consumed at time t.

- The problem writes

$$\Pi_t(i) = P_t(i)\left[\frac{P_t(i)}{P_t}\right]^{-\phi} c_t - W_t\left[\frac{P_t(i)}{P_t}\right]^{-\phi} c_t$$

Solution - Firm side

- These substitutions account for the dependence of quantities demanded on the price charged, so that the maximization is just on $P_t(i)$ now. We get

$$\begin{aligned}(1-\phi) P_t(i)^{-\phi} P_t^{\phi} c_t &= W_t(-\phi) P_t(i)^{-\phi-1} P_t^{\phi} c_t \\ P_t(i) &= \frac{\phi}{\phi-1} W_t\end{aligned}$$

- Notice we assume aggregate (c_t, P_t) are taken as given by the maximizing firm i.

Solution - Firm side

- This is the central result of monopolistic competition. Firms CHOOSE a price at a markup $\frac{\phi}{\phi-1} > 1$ over marginal costs, which here are just wages (since production is linear in labor). Prices vary across goods because of (potential) differences in the marginal cost. The point is, firms have some control over their prices.
- Notice that when $\phi \longrightarrow \infty$, the markup tends to unity. Makes sense since $\phi \longrightarrow \infty$ means goods become perfect substitutes, i.e. competition becomes perfect. In other words, the elasticity of substitution ϕ captures precisely the extent of competition, in that it maps directly with firms' profits margins (which end up identical across firms in equilibrium)

An Alternative Approach: Monopolistic Competition on the Production Side

- Put all the action on the production side. Suppose now households only consume a FINAL good.
- Production of that final good satisfies a CES production function:

$$y_t = \left[\sum_{i=1}^{N} y_t(i)^{\frac{\phi-1}{\phi}} \right]^{\frac{\phi}{\phi-1}}$$

with $\phi > 1$.

- Then the final output producers will maximize:

$$\begin{aligned} \Pi_t &= P_t y_t - \sum_{i=1}^{N} P_t(i) y_t(i) \\ &= P_t \left[\sum_{i=1}^{N} y_t(i)^{\frac{\phi-1}{\phi}} \right]^{\frac{\phi}{\phi-1}} - \sum_{i=1}^{N} P_t(i) y_t(i) \end{aligned}$$

An Alternative Approach: Monopolistic Competition on the Production Side

- Maximize profits

$$\frac{\partial \Pi_t}{\partial y_t(i)} = P_t \left[\sum_{i=1}^{N} y_t(i)^{\frac{\phi-1}{\phi}} \right]^{\frac{\phi}{\phi-1}-1} y_t(i)^{\frac{\phi-1}{\phi}-1} - P_t(i) = 0$$

- The demand for each input is therefore given by

$$\begin{aligned} \left[\sum_{i=1}^{N} y_t(i)^{\frac{\phi-1}{\phi}} \right]^{\frac{1}{\phi-1}} y_t(i)^{-\frac{1}{\phi}} &= \frac{P_t(i)}{P_t} \\ y_t(i) &= \left(\frac{P_t(i)}{P_t} \right)^{-\phi} y_t \end{aligned}$$

The demand for a given input falls in its relative price, but increases in the relative aggregate price (and aggregate output y_t)

An Alternative Approach: Monopolistic Competition on the Production Side

- Since profits must be zero, we have:

$$\begin{aligned} P_t y_t &= \sum_{i=1}^{N} P_t(i) y_t(i) \\ &= \sum_{i=1}^{N} P_t(i) \left(\frac{P_t(i)}{P_t} \right)^{-\phi} y_t \end{aligned}$$

- I.e.:

$$\begin{aligned} P_t &= \sum_{i=1}^{N} P_t(i)^{1-\phi} P_t^{\phi} \\ P_t &= \left(\sum_{i=1}^{N} P_t(i)^{1-\phi} \right)^{\frac{1}{1-\phi}} \end{aligned}$$

An Alternative Approach: Monopolistic Competition on the Production Side

- Now how are these intermediate goods produced?
- Suppose simply we have

$$y_t(i) = A_i n_t(i)$$

where A denotes technology.

- Intermediate goods firms maximise their profits:

$$\Pi_t(i) = P_t(i) y_t(i) - W_t n_t(i)$$

subject to the demand function $y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\phi} y_t$. W denotes the economy-wide wages.

An Alternative Approach: Monopolistic Competition on the Production Side

- Substitute to get

$$\Pi_t(i) = P_t(i)\left(\frac{P_t(i)}{P_t}\right)^{-\phi} y_t - W_t \frac{1}{A_i}\left(\frac{P_t(i)}{P_t}\right)^{-\phi} y_t$$

  which is maximized by choosing prices.

- The optimal price verifies

$$(1-\phi)P_t(i)^{-\phi}P_t^{\phi}y_t + \phi W_t \frac{1}{A_i}P_t(i)^{-\phi-1}P_t^{\phi}y_t = 0$$

  where once again aggregate variables are taken as given. Rewrite:

$$P_t(i) = \frac{\phi}{\phi-1}\frac{W_t}{A_i}$$

- The firm producing variety i of the intermediate good will charge a markup of $\frac{\phi}{\phi-1}$ over its marginal cost, which is here the wages relative to the productivity of labor.

Sources of Nominal Rigidities

- Now we have established a way to think about price-setting decisions on the part of firms.
- How does that translate in AGGREGATE price dynamics?
- Two approaches:
  - Taylor model of overlapping contracts
  - Calvo model of staggered price adjustment

Taylor "staggered" contracts

- Price is a markup over marginal costs, and the markup may be time varying.
- At any point in time, the wage rate is an average of wage contracts that were set in the past, but are still in force, and those set in the current period.
- AT THE TIME THEY WERE SET, wage contracts were profit maximizing, and reflected the prevailing MPL and the expected future price level.

Taylor "staggered" contracts

- Formally:
$$p_t = w_t + v_t$$
where lower case denote logarithms, p is the price level, w is the economy-wide wage rate and v is the markup
- The average wage w is a mean of wage contracts w_t^N and w_{t-1}^N signed at periods t and $t-1$. An N superscript means these were new at each time they were signed. We have
$$w_t = \frac{1}{2}\left(w_t^N + w_{t-1}^N\right)$$

Taylor "staggered" contracts

- Now at what levels are these wages agreed upon? Introduce a measure of the MPL, denoted with z. The real wage should equal z whenever the contract is signed. However, the contract is signed for two periods, and so the pre-agreed level should not only compensate for current price level, but also for expected changes in the price level. I.e.:

$$w_t^N - \frac{1}{2}\left(p_t + E_t p_{t+1}\right) = z_t$$

Taylor "staggered" contracts

- Put the three equations together:

$$p_t = \frac{1}{2}\left[\frac{1}{2}\left(p_t + E_t p_{t+1}\right) + z_t + \frac{1}{2}\left(p_{t-1} + E_{t-1}p_t\right) + z_{t-1}\right] + v_t$$

- The price level depends on past as well as future expected prices.
- Solve for inflation:

$$p_t - p_{t-1} = E_t p_{t+1} - p_t + E_{t-1}p_t - p_t + 2\left(z_t + z_{t-1}\right) + 4v_t$$

- Current inflation depends on its future expected level, changes in MPL (i.e. in productivity), changes in markups, and a surprise in price term. Let $\eta_t = E_{t-1}p_t - p_t$, then we have the expectations augmented Phillips curve:

$$\pi_t = E_t \pi_{t+1} + 2\left(z_t + z_{t-1}\right) + 4v_t + \eta_t$$

Calvo "staggered"price adjustment

- Aggregate price dynamics here the outcome of individual firms decisions. Still, framework very tractable - reason why probably the most popular approach.
- Firms are forward looking, and they forecast the optimal price p^*_{t+s}. They all have the same forecast, but not all are allowed to change their prices. In particular, each period, there is a probability ρ that a firm will be able to adjust its price to the optimum.
- In other words, firms have no control about when they can adjust their price - although they do it optimally when they can.

Calvo "staggered"price adjustment

- When firms CAN adjust their prices, they will do it to minimize the present value of the cost of deviations (assumed quadratic). Whenever they can, they'll choose $\bar{p}_t$ to minimize

$$\frac{1}{2}\sum_{s=0}^{\infty}\beta^s\left(1-\rho\right)^s E_t\left(\bar{p}_t-p^*_{t+s}\right)^2$$

since $\left(1-\rho\right)^s$ is the probability that the firm will not be able to change prices for the next s periods.

Calvo "staggered"price adjustment

- Take a first order condition

$$\sum_{s=0}^{\infty} \beta^s (1-\rho)^s E_t \left(\bar{p}_t - p^*_{t+s}\right) = 0$$

- So after adjustment, the new price is:

$$\begin{aligned} \frac{\bar{p}_t}{1-\beta(1-\rho)} &= \sum_{s=0}^{\infty} \beta^s (1-\rho)^s E_t \left(p^*_{t+s}\right) \\ \bar{p}_t &= (1-\gamma) \sum_{s=0}^{\infty} \gamma^s E_t \left(p^*_{t+s}\right) \end{aligned}$$

with $\gamma = \beta(1-\rho)$.

Calvo "staggered"price adjustment

- Which can rewrite:

$$\begin{aligned}\bar{p}_t &= (1-\gamma)\,p_t^* + (1-\gamma)\sum_{s=1}^{\infty}\gamma^s E_t\left(p_{t+s}^*\right)\\ &= (1-\gamma)\,p_t^* + (1-\gamma)\,\gamma\sum_{s=0}^{\infty}\gamma^s E_t\left(p_{t+s+1}^*\right)\\ &= (1-\gamma)\,p_t^* + \gamma E_t\bar{p}_{t+1}\end{aligned}$$

Prices are still forward looking.

Calvo "staggered"price adjustment

- Recognize finally that at each point in time, the actual price level is an average of all prices, with a proportion ρ of firms adjusting at period t. So the actual price level is given by

$$\begin{aligned} p_t &= \rho \bar{p}_t + (1-\rho)\, p_{t-1} \\ &= \rho\,(1-\gamma) \sum_{s=0}^{\infty} \gamma^s E_t\left(p^*_{t+s}\right) + (1-\rho)\, p_{t-1} \end{aligned}$$

Calvo "staggered"price adjustment

- So that inflation is given by

$$p_t - p_{t-1} = \rho\left(1-\gamma\right)\sum_{s=0}^{\infty}\gamma^s E_t\left(p^*_{t+s}\right) - \rho p_{t-1}$$
$$= \rho\left(1-\gamma\right)\sum_{s=0}^{\infty}\gamma^s E_t\left(p^*_{t+s} - p_{t+s-1}\right)$$

- Or recursively:

$$\pi_t = \rho\left(1-\gamma\right)\left(p^*_t - p_{t-1}\right) + \gamma E_t \pi_{t+1}$$

- Once again a forward looking solution for equation: the actual change in inflation is related to the "desired" change, and to the expected future change in price.

Taking stock

- Price rigidities central to keynesian economics, i.e. to any model where there is a role for monetary policy.
- Rigorous treatment of nominal rigidities requires that firms can set their price, i.e. that they have market power.
- The "Dixit-Stiglitz" approach (i.e. monopolistic competition between imperfect substitutes) made it possible to model that rigorously.
- Imperfect substitutes can be either on the consumption or on the production side.
- Gives rise to markups over marginal costs.
- And to forward looking behavior of inflation - akin to what we saw in "expectations augmented Phillips curves"