Session 4: Money

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Money

- So far, focused on real economy. Real quantities consumed, produced, invested. No money, no nominal influences.
- Now, introduce nominal dimension in the economy. First and foremost motivation is to be able to talk about long run inflation.
- We talked a bit about inflation when we discussed time-consistent monetary policy. But these were fluctuations of inflation, and how to trade them off against fluctuations in output.
- In reality, there is such a thing as long run inflation (2-3% now) - as well as deflations and hyperinflations.
- Money is a puzzle in macro. Its roles are well known: helps in transactions (no double coincidence of needs), stores wealth, and a credit that can be borrowed and lent.
- Surely however, other things exist that can fulfill all three roles. Why is it money that does? probably out of convenience.

Plan

- The intertemporal budget constraint with money
- Cash-in-advance: you must have cash to purchase goods and services
- Money in the utility: money is, in effect, a commodity, and holding it creates utility.
- Money as an Intermediate Good: holding money economizes on shopping time
- Transaction Cost: money economizes on the real transaction costs involved in consumption
- Applications: Hyperinflations, The Friedman Rule and Superneutrality

The Budget Constraint

- What we had previously could be written (in real terms, obviously):

$$\Delta a_{t+1} + c_t = x_t + r_t a_t$$

where a_t denotes the stock of assets at time t, c_t is consumption, x_t is (exogenous) income and r_t is the real rate of return on assets. Once again, everything is real.

- Now think of the analogy with nominal variables. In particular, assume we have two types of assets that can be used to store wealth: money M_t and bonds B_t. In particular, B_t denotes the total expenditures on one-period bonds, paying a nominal return R_t.
- The budget constraint becomes:

$$\Delta B_{t+1} + \Delta M_{t+1} + P_t c_t = P_t x_t + R_t B_t$$

where we wrote the constraint in nominal terms, with P_t the aggregate price level.

Budget Constraint

- How do we obtain the real budget constraint? simply deflate the previous expression, to obtain

$$b_{t+1}\left(1+\pi_{t+1}\right)-b_t+m_{t+1}\left(1+\pi_{t+1}\right)-m_t+c_t \quad = \quad x_t+R_t b_t$$

where $b_t=\frac{B_t}{P_t}$, $m_t=\frac{M_t}{P_t}$ and we made use of the fact that $\frac{\Delta B_{t+1}}{P_t}=\frac{B_{t+1}}{P_{t+1}}\frac{P_{t+1}}{P_t}-\frac{B_t}{P_t}=b_{t+1}\left(1+\pi_{t+1}\right)-b_t$ and $\frac{\Delta M_{t+1}}{P_t}=\frac{M_{t+1}}{P_{t+1}}\frac{P_{t+1}}{P_t}-\frac{M_t}{P_t}=m_{t+1}\left(1+\pi_{t+1}\right)-m_t$

- Rearrange

$$\left(1+\pi_{t+1}\right)\left(\Delta b_{t+1}+\Delta m_{t+1}\right)+c_t=x_t+\left(R_t-\pi_{t+1}\right)b_t-\pi_{t+1}m_t$$

Budget Constraint

- By analogy with the budget constraint without money, we can set $a_t = b_t + m_t$. The real return on bonds is $\frac{1+R_t}{1+\pi_{t+1}} - 1 \simeq R_t - \pi_{t+1}$, and the real rate on money is $-\pi_{t+1}$; holding money has zero nominal return et hence money loses it purchasing power because of inflation.
- The fall in the value of nominal money balances M_t is in effect a tax - an "inflation tax" - and it is "seignorage" income to the issuer of money, i.e. the government.

The Static Steady State

- At the steady state, all real stocks are constant, including m and b.
- So $\Delta b = \Delta m = 0$, but inflation π is not necessarily zero.
- Suppose the growth rate of nominal money $\frac{\Delta M}{M} = \mu$, then

$$\frac{\Delta m}{m} = \frac{\Delta M}{M} - \frac{\Delta P}{P} = \mu - \pi = 0$$

- So at the steady state $\mu = \pi$. But no implication on the direction of causality - money growth equals inflation; but equally true that if inflation is targetted at π then that conditions money growth.

The Cash-in-Advance model

- So far, money only entails costs (an inflation tax). So why hold it? Because it reduces transaction costs, provides store of value and a unit of account.
- Cash-in-advance model (CIA) focuses on transactions demand. Simplest model of money: all goods and services must be paid for *in full* with cash. In particular, impose

$$\begin{aligned} M_t^D &= P_t c_t \\ \text{i.e. } m_t^D &= \frac{M_t^D}{P_t} = c_t \end{aligned}$$

- Money supply assumed exogenous; equilibrium requires $M_t^D = M_t^S = M_t$

Cash in Advance

- Set up the dynamic problem:

$$\begin{aligned}\mathcal{L}_t &= \sum_{s=0}^{\infty}\{\beta^s U(c_{t+s}) + \lambda_{t+s}[x_{t+s} + (1+R_{t+s})\, b_{t+s} + m_{t+s} \\ &\quad - (1+\pi_{t+s+1})\,(b_{t+s+1} + m_{t+s+1}) - c_{t+s}] \\ &\quad + \mu_{t+s}\,[m_{t+s} - c_{t+s}]\end{aligned}$$

- First order conditions

$$\begin{aligned}\frac{\partial \mathcal{L}_t}{\partial c_{t+s}} &= \beta^s U'(c_{t+s}) - \lambda_{t+s} - \mu_{t+s} = 0 \\ \frac{\partial \mathcal{L}_t}{\partial b_{t+s}} &= \lambda_{t+s}(1+R_{t+s}) - \lambda_{t+s-1}\,(1+\pi_{t+s}) = 0 \\ \frac{\partial \mathcal{L}_t}{\partial m_{t+s}} &= \lambda_{t+s} - \lambda_{t+s-1}\,(1+\pi_{t+s}) + \mu_{t+s} = 0\end{aligned}$$

Cash in advance

- Subtracting the second and third conditions:

$$\lambda_{t+s} R_{t+s} = \mu_{t+s}$$

- Replace in the first:

$$\beta^s U'(c_{t+s}) = \lambda_{t+s} (1 + R_{t+s})$$

- Now use the implied value for λ_{t+s} in the second condition:

$$\frac{\beta U'(c_{t+s})}{U'(c_{t+s-1})} = \frac{1 + \pi_{t+s}}{1 + R_{t+s-1}}$$

- Rearrange and set $s = 1$:

$$\frac{\beta U'(c_{t+1})}{U'(c_t)} \frac{1 + R_t}{1 + \pi_{t+1}} = \frac{\beta U'(c_{t+1})}{U'(c_t)} (1 + r_{t+1}) = 1$$

The standard Euler equation

Cash in advance

- What about the dynamics of m and c? For sure, $c_t = m_t$ at all points in time. So only need to solve for c_t.
- Assume for simplicity that R_t and π_{t+1} are constant. Then the budget constraint implies

$$(1+\pi)\left(b_{t+1}+m_{t+1}\right)+c_t = x_t + (1+R)\,b_t + m_t$$

- In terms of $a = b + m$:

$$a_t = \frac{1}{1+R}\left(c_t - x_t + Rm_t\right) + \left(\frac{1+\pi}{1+R}\right)a_{t+1}$$

Cash in advance

- Which we can solve recursively to obtain

$$a_t = \frac{1}{1+R}\sum_{s=0}^{n-1}\left(\frac{1+\pi}{1+R}\right)^s (c_{t+s} - x_{t+s} + Rm_{t+s}) + \left(\frac{1+\pi}{1+R}\right)^n a_{t+n}$$

- If $r = R - \pi > 0$ (i.e. the real return on bonds is positive), then

$$\lim_{n\to\infty}\left(\frac{1+\pi}{1+R}\right)^n a_{t+n} = 0$$

and

$$a_t = \frac{1}{1+R}\sum_{s=0}^{\infty}\left(\frac{1+\pi}{1+R}\right)^s (c_{t+s} - x_{t+s} + Rm_{t+s})$$

Cash in advance

- For simplicity suppose c_t and m_t are at their long run equilibrium values, i.e. $c_{t+s} = c_t$ and $m_{t+s} = m_t$. Then

$$a_t = c_t \frac{1}{1+R} \frac{1+R}{R-\pi} - \frac{1}{1+R} \sum_{s=0}^{\infty} \left(\frac{1+\pi}{1+R} \right)^s x_{t+s} + m_t \frac{R}{1+R} \frac{1+R}{R-\pi}$$

- or

$$(R-\pi)(b_t + m_t) = c_t - \frac{R-\pi}{1+R} \sum_{s=0}^{\infty} \left(\frac{1+\pi}{1+R} \right)^s x_{t+s} + Rm_t$$

Cash in advance

- Finally

$$c_t \simeq \frac{r}{1+r} \sum_{s=0}^{\infty} \frac{x_{t+s}}{(1+r)^s} + rb_t - \pi m_t$$

where we have used $r \simeq R - \pi$. If in addition we assume $x_{t+s} = x_t$, then the consumption function becomes

$$c_t \simeq x_t + rb_t - \pi m_t$$

Consumption falls with the stock of real money balances when inflation is positive. Having to pay for consumption using cash has introduced a non-neutrality into the economy. Why? because a nominal variable (inflation) affects a real variable (consumption), through the inflation tax

Solving the CIA model using the Bellman equation

- Now derive the same equilibrium using the value function. Define

$$V(b_{t-1}, m_{t-1}) = Max_{\{c_{t+s-1}, m_{t+s}, b_{t+s}\}_0^\infty} \left\{ \sum_{s=0}^{\infty} \beta^s U(c_{t+s-1}) \right\}$$

subject to

$$x_{t+s} + (1 + R_{t+s}) b_{t+s} + m_{t+s} =$$

$$(1 + \pi_{t+s+1})(b_{t+s+1} + m_{t+s+1}) + c_{t+s}$$

and

$$m_{t+s} = c_{t+s}$$

Solving the CIA model using the Bellman equation

- Rewrite an unconstrained version:

$$\begin{aligned} V(b_{t-1}) &= Max_{\{b_{t+s}\}_0^\infty} \left\{ \begin{array}{c} \sum_{s=0}^{\infty} \beta^s U(\frac{1}{1+\pi_{t+s}} x_{t+s-1} \\ + \frac{1+R_{t+s-1}}{1+\pi_{t+s}} b_{t+s-1} - b_{t+s}) \end{array} \right\} \\ &= Max_{b_t} \left\{ \begin{array}{c} U(\frac{1}{1+\pi_t} x_{t-1} + \frac{1+R_{t-1}}{1+\pi_t} b_{t-1} - b_t) \\ +\beta V(b_t) \end{array} \right\} \end{aligned}$$

where we have made use of the fact that $m_{t+s} = c_{t+s}$ in the budget constraint. In particular:

$$\frac{1}{1+\pi_{t+s+1}} x_{t+s} + \frac{1+R_{t+s}}{1+\pi_{t+s+1}} b_{t+s} - b_{t+s+1} = c_{t+s+1}$$

Solving the CIA model using the Bellman equation

- The first order condition writes:

$$U'(c_t) = \beta V'(b_t)$$

- The envelope theorem implies that

$$V'(b_{t-1}) = \frac{1+R_{t-1}}{1+\pi_t} U'(c_t)$$

- Update this once:

$$V'(b_t) = \frac{1+R_t}{1+\pi_{t+1}} U'(c_{t+1})$$

- And so back in the first order condition:

$$U'(c_t) = \beta \frac{1+R_t}{1+\pi_{t+1}} U'(c_{t+1})$$

- QED.

Money in the Utility

- Model due to Sidrausky (1967). Money simply enters utility. I am happy to have cash in my pocket (though there's no reason why).
- The transaction motive (comes next) offers some justification for MIU.
- Lagrangean becomes

$$\begin{aligned}\mathcal{L}_t &= \sum_{s=0}^{\infty}\{\beta^s U(c_{t+s}, m_{t+s}) + \lambda_{t+s}[x_{t+s} + (1+R_{t+s})\, b_{t+s} \\ &\quad + m_{t+s} - (1+\pi_{t+s+1})\,(b_{t+s+1} + m_{t+s+1}) - c_{t+s}]\}\end{aligned}$$

Money in the Utility

- First order conditions

$$
\begin{aligned}
\frac{\partial \mathcal{L}_t}{\partial c_{t+s}} &= \beta^s U_c(c_{t+s}, m_{t+s}) - \lambda_{t+s} = 0 \\
\frac{\partial \mathcal{L}_t}{\partial b_{t+s}} &= \lambda_{t+s}(1 + R_{t+s}) - \lambda_{t+s-1}\,(1 + \pi_{t+s}) = 0 \\
\frac{\partial \mathcal{L}_t}{\partial m_{t+s}} &= \beta^s U_m(c_{t+s}, m_{t+s}) + \lambda_{t+s} - \lambda_{t+s-1}\,(1 + \pi_{t+s}) = 0
\end{aligned}
$$

Money in the Utility

- Subtracting the second and third conditions:

$$\lambda_{t+s} R_{t+s} = \beta^s U_m(c_{t+s}, m_{t+s})$$

- Replace in the first:

$$R_{t+s} U_c(c_{t+s}) = U_m(c_{t+s}, m_{t+s})$$

The right hand side reflects the marginal utility from holding an extra unit of real balances. The left hand side reflects the opportunity cost of doing so: namely not holding bonds, not earning interest on them, and not consuming the proceeds.

- The second equation becomes

$$\beta U_c(c_{t+s}, m_{t+s})(1 + R_{t+s}) = U_c(c_{t+s-1}, m_{t+s-1})\,(1 + \pi_{t+s})$$

- Rearrange and set $s = 1$:

$$\frac{\beta U_c(c_{t+1}, m_{t+1})}{U_c(c_t, m_t)} \frac{1 + R_{t+1}}{1 + \pi_{t+1}} = 1$$

The standard Euler equation - just slight timing difference.

Money in the Utility

- Condition on the marginal utilities of c and m has a natural interpretation. To see this, choose utility

$$U(c_t, m_t) = \frac{c_t^{1-\sigma} - 1}{1-\sigma} + \eta \left(\frac{m_t^{1-\sigma} - 1}{1-\sigma} \right)$$

- Then the condition rewrites

$$R_t c_t^{-\sigma} = \eta m_t^{-\sigma}$$

- Rewrite the demand for money:

$$m_t = c_t \left(\frac{R_t}{\eta} \right)^{-\frac{1}{\sigma}}$$

Money demand increases in consumption (a transaction motive) and decreases in the interest rate (an opportunity cost). Reminiscent of the classic IS-LM: $\frac{M}{P} = L(Y, i)$.

MIU model using Bellman equation

- As an exercise, derive the same equilibrium using the value function. Define

$$V(b_{t-1}, m_{t-1}) = Max_{\{c_{t+s-1}, m_{t+s}, b_{t+s}\}_0^\infty} \left\{ \sum_{s=0}^{\infty} \beta^s U(c_{t+s-1}, m_{t+s-1}) \right\}$$

subject to

$$x_{t+s} + (1 + R_{t+s}) b_{t+s} + m_{t+s} =$$

$$(1 + \pi_{t+s+1}) (b_{t+s+1} + m_{t+s+1}) + c_{t+s}$$

- Do it as an exercise.

The Shopping Time Model

- Model money as an intermediate good that is held to reduce shopping time: suppose total time (1) has to be allocated between work n_t, leisure l_t and shopping s_t:

$$n_t + l_t + s_t = 1$$

- In addition,

$$s_t = S(c_t, m_t)$$

  with $S_c > 0$, $S_m < 0$, $S_{cc}, S_{mm} > 0$ and $S_{cm} < 0$.
- Utility is derived from consumption and leisure (not money anymore)

$$U = U(c_t, l_t) = U[c_t, 1 - n_t - S(c_t, m_t)]$$

  which provides a direct motivation for the MIU framework.

The Shopping Time Model

- Consider Lagrangean:

$$\begin{aligned}\mathcal{L}_t &= \sum_{s=0}^{\infty}\{\beta^s U(c_{t+s}, l_{t+s}) + \lambda_{t+s}[w_{t+s}n_{t+s} + (1+R_{t+s})\, b_{t+s} \\ &\quad + m_{t+s} - (1+\pi_{t+s+1})\,(b_{t+s+1} + m_{t+s+1}) - c_{t+s}] \\ &\quad + \mu_{t+s}\,[n_t + l_t + S\,(c_t, m_t) - 1]\}\end{aligned}$$

The Shopping Time Model

- First order conditions

$$\begin{aligned}
\frac{\partial \mathcal{L}_t}{\partial c_{t+s}} &= \beta^s U_c(c_{t+s}) - \lambda_{t+s} + \mu_{t+s} S_c(c_t, m_t) = 0 \\
\frac{\partial \mathcal{L}_t}{\partial l_{t+s}} &= \beta^s U_l(c_{t+s}) + \mu_{t+s} = 0 \\
\frac{\partial \mathcal{L}_t}{\partial n_{t+s}} &= \lambda_{t+s} w_{t+s} + \mu_{t+s} = 0 \\
\frac{\partial \mathcal{L}_t}{\partial b_{t+s}} &= \lambda_{t+s}(1 + R_{t+s}) - \lambda_{t+s-1}(1 + \pi_{t+s}) = 0 \\
\frac{\partial \mathcal{L}_t}{\partial m_{t+s}} &= \lambda_{t+s} - \lambda_{t+s-1}(1 + \pi_{t+s}) + \mu_{t+s} S_m(c_t, m_t) = 0
\end{aligned}$$

The Shopping Time Model

- Subtracting the fourth and fifth conditions:

$$\lambda_{t+s} R_{t+s} = \mu_{t+s} S_m \left(c_t, m_t \right)$$

- Now, use first and second equations and substitute, for $s = 0$:

$$\left[U_c(c_t) - U_l(c_t) S_c \left(c_t, m_t \right) \right] R_t = -U_l(c_t) S_m \left(c_t, m_t \right)$$

  The right hand side reflect the leisure saved from shopping time thanks to an additional unit of money. Now holding that additional unit means forgone interet R_t and thus some loss of consumption $U_c(c_t) R_t$.But that lost consumption also saves on shopping time, which adds to utility. The left hand side therefore captures the net loss of utility from holding an additional unit of money.

- We can get a money demand function from this condition - but it requires some specific functional forms:

$$U \left(c_t, l_t \right) = \ln c_t + \eta \ln l_t, \text{ and } s_t = \psi \frac{c_t}{m_t}$$

The Shopping Time model

- Then we have:

$$\begin{aligned}\left[\frac{1}{c_t} - \eta\psi\frac{1}{l_t}\frac{1}{m_t}\right] R_t &= \eta\psi\frac{1}{l_t}\frac{c_t}{m_t^2} \\ \frac{1}{c_t}\left[1 - \eta\frac{s_t}{l_t}\right] R_t &= \eta\frac{s_t}{m_t l_t} \\ m_t &= c_t\left[\frac{\eta\frac{s_t}{l_t}}{1 - \eta\frac{s_t}{l_t}}\right] R_t^{-1}\end{aligned}$$

Again we have a transaction demand for money, and a negative interest rate effect. In addition to previous M_d we now have a dependence on the shopping to leisure time ratio $\frac{s_t}{l_t}$. An increase in the shopping time raises the demand for money. Online shopping, for instance, will reduce the demand for money in two ways: first, use credit cards, and second, a fall in $\frac{s_t}{l_t}$.

Transaction Costs

- Last, consider the notion that money economizes on transaction costs incurred when consuming. Modify the budget constraint accordingly:

$$b_{t+1}\left(1+\pi_{t+1}\right)-b_t+m_{t+1}\left(1+\pi_{t+1}\right)-m_t+c_t+T\left(c_t,m_t\right)=x_t$$

  where $T(0,m)=0$, T_c, T_{cc}, $T_{mm}>0$ and T_m, $T_{mc}<0$
- Consider the Lagrangean:

$$\begin{aligned}\mathcal{L}_t &= \sum_{s=0}^{\infty}\{\beta^s U(c_{t+s})+\lambda_{t+s}[x_{t+s}+\left(1+R_{t+s}\right)b_{t+s} \\ &\quad +m_{t+s}-\left(1+\pi_{t+s+1}\right)\left(b_{t+s+1}+m_{t+s+1}\right)-c_{t+s} \\ &\quad -T\left(c_{t+s},m_{t+s}\right)]\}\end{aligned}$$

Transaction Costs

- First order conditions

$$\frac{\partial \mathcal{L}_t}{\partial c_{t+s}} = \beta^s U'(c_{t+s}) - \lambda_{t+s}\left[1 + T_c\left(c_{t+s}, m_{t+s}\right)\right] = 0$$
$$\frac{\partial \mathcal{L}_t}{\partial b_{t+s}} = \lambda_{t+s}(1 + R_{t+s}) - \lambda_{t+s-1}\left(1 + \pi_{t+s}\right) = 0$$
$$\frac{\partial \mathcal{L}_t}{\partial m_{t+s}} = \lambda_{t+s}\left[1 - T_m\left(c_{t+s}, m_{t+s}\right)\right] - \lambda_{t+s-1}\left(1 + \pi_{t+s}\right) = 0$$

Transaction Costs

- The second condition implies

$$\frac{\lambda_{t+s}}{\lambda_{t+s-1}} = \frac{1+\pi_{t+s}}{1+R_{t+s}}$$

- Using the first condition (and setting $s = 1$), this implies

$$\frac{\beta U'(c_{t+1})}{U'(c_t)} \frac{1+T_c(c_t, m_t)}{1+T_c(c_{t+1}, m_{t+1})} \frac{1+R_{t+1}}{1+\pi_{t+1}} = 1$$

  An Euler equation amended for the ratio of marginal transaction costs.

- In addition

$$R_{t+1} = -T_m(c_{t+1}, m_{t+1})$$

  The forgone interest incurred by holding an additional unit of money is equal to the marginal cost saving this allows.

Money Demand and Hyperinflation - The Cagan model

- Consider the nominal money demand function we derived:

$$\begin{aligned} M_t &= P_t c_t R_t^{-\alpha} \\ &= P_t c_t \left(r_t + \pi_{t+1}\right)^{-\alpha} \end{aligned}$$

- Now we will suppose consumption and the real interest rate change little. Then money demand can be approximated as

$$M_t = \phi P_t \pi_{t+1}^{-\alpha}$$

- Taking logarithms:

$$\ln M_t - p_t = -\alpha \left(E_t p_{t+1} - p_t\right)$$

where we ignore $ln\phi$ and introduce some uncertainty - hence the expectation operator. $p_t = \ln P_t$.

Money Demand and Hyperinflation - The Cagan model

- Now rearrange:

$$p_t = \frac{1}{1+\alpha} \ln M_t + \frac{\alpha}{1+\alpha} E_t p_{t+1}$$

- Substitute recursively:

$$p_t = \frac{1}{1+\alpha} \sum_{s=0}^{\infty} \left(\frac{\alpha}{1+\alpha} \right)^s E_t \ln M_{t+s}$$

where we assumed that

$$\lim_{n \to \infty} \left(\frac{\alpha}{1+\alpha} \right)^n E_t p_{t+n} = 0$$

- Notice that the price level today responds to all future expected (discounted) realizations of monetary policy.

Hyperinflations

- Now put some structure on what monetary policy does. Assume

$$\Delta \ln M_t = \mu + \varepsilon_t$$

  Money grows at a constant rate, and there are some unexpected deviations from that rule, $E_t \varepsilon_{t+1} = 0$. Then we have

$$\ln M_{t+s} = \ln M_t + \mu s + \sum_{i=1}^{s} \varepsilon_{t+i}$$

- That is

$$E_t \ln M_{t+s} = \ln M_t + \mu s$$

- Which finally means

$$\begin{aligned} p_t &= \ln M_t + \mu \frac{1}{1+\alpha} \sum_{s=0}^{\infty} \left(\frac{\alpha}{1+\alpha} \right)^s s \\ &= \ln M_t + \mu \alpha \end{aligned}$$

Hyperinflations

- Now

$$E_t \pi_{t+1} = E_t p_{t+1} - p_t = E_t \ln M_{t+1} - \ln M_t = \mu$$

In a period of hyperinflation, consumption c_t and the real rate r_t change much less than the price level. Under these assumptions, this shows the astronomical rate of inflation requires a correspondingly high rate of monetary creation. This is what we observed in Germany 1923, in some South American countries in the 90s, and in Zimbabwe today. It happens when governments print money to pay for their expenditures.

The Friedman Rule

- In principle, what was just discussed holds in the long term, since c_t and r_t are stabilised around their steady state values. Thus, in the long term, inflation is "always and everywhere a monetary phenomenon"
- So choosing μ will pin down long run inflation. Which μ should governments choose? Friedman proposed an answer.
- He argued the marginal private return to holding money is $-\pi$, while the marginal social cost of producing money is virtually zero. Friedman proposed to eliminate the private cost of holding money by setting $-\pi = \theta$, the rate of private time preferences, which in the long run will equal the real interest rate r.
- So Friedman basically suggested setting the nominal interest rate $R = r + \pi$ to zero. Then the opportunity cost of holding money balances is effectively zero.

Superneutrality

- The classical dichotomy in macroeconomics is that nominal shocks have no long-run effects on real variables. When these nominal shocks are money shocks, this is known as the superneutrality of money. Proportional changes in nominal money balances, and hence inflation, have no effect on the real variables of the economy, such as consumption, investment, production and capital.
- From the models we saw, it seems the presence of money in the household budget constraint imposes a real cost on the economy, and thus money is not super-neutral. But this was based only on the decisions of households, and thus on a partial equilibrium view of the economy. Now turn to a general equilibrium version.
- In particular, we now need to account for the fact that inflation tax (which imposes a real cost on households) actually generates seignorage revenues for the government. And the government must also satisfy a budget constraint.

Superneutrality

- So we will consider an economy where all the money is provided by the government, and seignorage revenues are returned to households in the form of transfers, which households are aware of. The household budget constraint writes

$$(1+\pi_{t+1})(b_{t+1}+m_{t+1})+c_t+T_t=x_t+(1+R_t)b_t+m_t$$

where $T_t<0$ would imply a transfer to households. The government also has a budget constraint, which writes

$$T_t=m_t-(1+\pi_{t+1})m_{t+1}$$

- So we have the consolidated constraint

$$(1+\pi_{t+1})b_{t+1}+c_t=x_t+(1+R_t)b_t$$

in which real money balances no longer appear!

Superneutrality

- The Lagrangean will write:

$$\mathcal{L}_t = \sum_{s=0}^{\infty} \{ \beta^s U(c_{t+s}) + \lambda_{t+s} [x_{t+s} + (1+R_{t+s})\, b_{t+s} - (1+\pi_{t+s+1})\, b_{t+s+1} - c_{t+s}] \}$$

- with first order conditions

$$\frac{\partial \mathcal{L}_t}{\partial c_{t+s}} = \beta^s U_c(c_{t+s}) - \lambda_{t+s} = 0$$

$$\frac{\partial \mathcal{L}_t}{\partial b_{t+s}} = \lambda_{t+s}(1+R_{t+s}) - \lambda_{t+s-1}(1+\pi_{t+s}) = 0$$

- and the classic Euler condition:

$$\frac{\beta U_c(c_{t+1})}{U_c(c_t)} \frac{1+R_{t+1}}{1+\pi_{t+1}} = 1$$

Superneutrality

- Money does not matter for consumption any more. Does it matter in the long run? Consolidated constraint implies

$$c = x + (R - \pi)\, b$$

- What is the long run real rate of interest? The Euler condition requires that

$$\beta \frac{1+R}{1+\pi} = 1$$

i.e:

$$R - \pi = \frac{1-\beta}{\beta}(1+\pi) \simeq \frac{1-\beta}{\beta}$$

- So long run consumption is given by

$$c = x + \frac{1-\beta}{\beta} b$$

and money matters not at all. The economy is super-neutral.