Macroeconomics

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Rational Expectations

- Traditional macroeconomics (i.e. pre-1970) essentially took expectations as given. In these models, agents do not anticipate anything - or if they do, that is exogenous..
- For instance, the traditional output-inflation model contends that inflating the economy stimulates the economy and lowers unemployment (for instance as real wages fall).
- And vice versa, if inflation is too high, cutting inflation means curtailing the economy and increasing unemployment (for instance as real wages rise)
- Thus, policymaker (the Central Bank, say) has a menu of choices between Inflation and Unemployment. Just need to pick its favourite point.
- But crucially one missing ingredient: why are agents not modifying their wage demands on the basis of what they EXPECT inflation to be?
- This is based on a very simple "reduced form" economy (i.e. on equations that are not derived from agent's fundamental behavior, but rather from ad-hoc behavioral assumptions)

A Simple Model of the Phillips Curve

- Model has a wage setting equation

$$w = p^e(1 - \alpha u)$$

with w the nominal wage, p^e the expected price level and u prevailing unemployment.

- Wage setting depends positively on expected price level (it is a real wage), and negatively on ongoing unemployment (since the unemployed create a threat and limit wage demands)
- Model has a price setting equation

$$p = (1 + \mu)w$$

where μ denotes the markup margin charged by the producing firm.

- This simply implies

$$p = (1 + \mu)p^e(1 - \alpha u)$$

which holds true at all periods t

A Simple Model of the Phillips Curve

- It is easy to use this equilibrium condition to infer (assuming constant markups)

$$\frac{p_t}{p_{t-1}} = (1+\mu)\frac{p_t^e}{p_{t-1}}(1-\alpha u_t)$$

- Let π_t denote the realized inflation rate, and π_t^e its expected counterpart (i.e. expected as of the past). We have

$$1+\pi_t = (1+\mu)\left(1+\pi_t^e\right)(1-\alpha u_t)$$

- Now expand and omit negligible products to obtain

$$\pi_t = \mu + \pi_t^e - \alpha u_t$$

  There is a negative relation between inflation and unemployment - a "menu choice" of policy. But it depends on expected inflation.

- NB: Higher expected inflation leads to higher inflation today, because it means agents expect higher prices, and thus demand higher nominal wages w.

- IF π_t^e is constant, then there is indeed a negative relation between inflation and unemployment - a Phillips curve.
- For stable inflation expectations, low unemployment will indeed translate in high current inflation, as agents demand high nominal wages.
- But a logical flaw: π_t^e is assumed to be constant, yet the model predicts inflation accelerates. Inflation expectations are formed bizarrely. Certainly not rationally.
- In reality, π_t^e cannot be held constant. It will respond endogenously to policy. Modelling this rational response is what "rational expectations" are all about.
- In this particular instance, if expectations are rational and make no systematic mistakes in anticipating inflation, then π_t and π_t^e move hand in hand, and unemployment becomes entirely disconnected from inflation.
- The "policy ineffectiveness" result. It is only to the extent that π_t^e differs from π_t that real effects - here on unemployment - are possible. Graphical Interpretation: AS responds to shift in AD.

Policy Ineffectiveness

- Is it possible to construct a model with rational expectations (solved endogenously), and yet one where policy does matter?
- Point of today's session.
- First trick will be to introduce imperfect information: agents cannot fully expect what will happen to the overall price level because they only observe an incomplete signal.
- So called Lucas "island model", due to Robert J Lucas (1978). Island because it is a model where agents live in separate, isolated islands, and only information they have is about the price of the good on *their* island.
- Thus they are unable to form accurate expectations as regards overall inflation, even though they ARE rational.

The Lucas "Island" Model

- There are n islands indexed by i, each subjected to an idiosyncratic shock z_t^i. There is also an aggregate shock ε_t, which affects all islands simultaneously. Both shocks have zero mean and variances σ_z^2 and σ_ε^2, respectively. They are orthogonal.
- The key is that nobody can distinguish between z_t^i and ε_t. Agents know the distributions of both shocks, but they do not observe them separately.
- Define the price of the good being produced and sold on island i as

$$p_t^i = p_t + z_t^i$$

  where p_t is the aggregate price level. The price level on island i deviates from the aggregate by exactly the amount of the island idiosyncratic shock. But each agent only observe p_t^i, NOT its separate components.
- In turn, the aggregate price is defined as

$$p_t = \gamma + \varepsilon_t$$

  There are aggregate shocks to prices, which drive aggregate deviations from the average value γ.

- It is easy to see that
$$p_t^i = \gamma + \varepsilon_t + z_t^i$$
Agents do not observe separately p_t^i and p_t, only p_t^i, the price at home. .When I see the price of the good I produce, p_t^i, I do not know whether changes in its value come from ε_t or from z_t^i. In other words, I don't know if the price of my good changes because of a change in the overall price level everywhere on all islands (a shock in ε_t), or because of a change in the price level on my island (a shock in z_t^i)
- Why should that matter? Because producers will care about the RELATIVE price of their goods on the overall market. If my good becomes RELATIVELY more expensive (a shock in z_t^i), I will want to produce more of it. But if my good is becoming more expensive along with everything else (a shock in ε_t), I will leave production unchanged.

- Introduce output on island i, Y_t^i. Formalizing this idea, we have

$$Y_t^i = Y + a\left[p_t^i - E\{p_t \mid p_t^i\}\right]$$

where a is a constant and $E\{. \mid I_t\}$ denotes an expectation operator, where expectations about . are formed on the basis of the information set I_t. In words, $E\{p_t \mid p_t^i\}$ denotes the expectation of what the aggregate price level must be, given what is known about price on island i.

- This supply function captures the optimal supply response of producers, who increase output only when they expect the price of their goods to have increased relative to the aggregate, ongoing price level.
- Now all we need is to solve for $E\{p_t \mid p_t^i\}$. We will restrict ourselves to linear expectations, and look for the Best Linear Unbiased Estimator (BLUE) of p_t given p_t^i.

The Lucas "Island" Model

- BLUE will minimize the mean squared error $\Sigma\eta_t^2$ in

$$E\{p_t \mid p_t^i\} = \alpha + \beta p_t^i + \eta_t$$

  The expression reflects a forecast about p_t that is formed linearly on the basis of observed island prices p_t^i. We want this linear forecast to be as accurate as possible, i.e. minimize the mean squared error.

- Minimizing the mean squared error is precisely what is asked from a classic estimator, OLS. The residuals must verify two conditions:

$$\begin{aligned} E\{\eta_t\} &= E\{p_t - \alpha - \beta p_t^i\} = 0 \\ E\{\eta_t p_t^i\} &= E\{\left(p_t - \alpha - \beta p_t^i\right) p_t^i\} = 0 \end{aligned}$$

- The first condition imposes the residual be zero in expectations. The second imposes the residual be orthogonal to the regressor. Note these are UNCONDITIONAL expectations, unlike $E\{p_t \mid p_t^i\}$, which is the expectation of p_t conditional on p_t^i

- The two conditions provide a system of two equations in two unknowns, α and β.
- We have

$$E\{p_t\} = \alpha + \beta E\{p_t^i\} \quad (1)$$

$$E\{p_t p_t^i\} = \alpha E\{p_t^i\} + \beta E\{\left(p_t^i\right)^2\} \quad (2)$$

  All we have to do now is to solve for the unknowns using what we know about p_t and p_t^i.
- Firstly, $E\{p_t^i\} = E\{p_t + z_t^i\} = E\{p_t\} = \gamma$. Therefore equation (1) implies

$$\gamma = \frac{\alpha}{1-\beta}$$

The Lucas Island Model - Signal Extraction Problem

- Secondly,

$$E\{p_t p_t^i\} = E\{(\gamma+\varepsilon_t)\left(\gamma+\varepsilon_t+z_t^i\right)\} = E\{(\gamma+\varepsilon_t)^2\} = \gamma^2+\sigma_\varepsilon^2$$
$$E\{\left(p_t^i\right)^2\} = E\{\left(\gamma+\varepsilon_t+z_t^i\right)^2\} = \gamma^2+\sigma_\varepsilon^2+\sigma_z^2$$

  Using these in equation (2) gives

$$\gamma^2+\sigma_\varepsilon^2 = \alpha\gamma+\beta\left(\gamma^2+\sigma_\varepsilon^2+\sigma_z^2\right)$$

- Now combining both equations, we have

$$\gamma^2+\sigma_\varepsilon^2 = \gamma(1-\beta)\gamma+\beta\left(\gamma^2+\sigma_\varepsilon^2+\sigma_z^2\right)$$

  which simplifies into

$$\beta = \frac{\sigma_\varepsilon^2}{\sigma_\varepsilon^2+\sigma_z^2}$$

- And

$$\alpha = \gamma(1-\beta) = \frac{\gamma\sigma_z^2}{\sigma_\varepsilon^2+\sigma_z^2}$$

Signal Extraction

- Classic results of a signal extraction problem. BLUE is given by

$$
\begin{aligned}
E\{p_t \mid p_t^i\} &= \frac{\gamma\sigma_z^2}{\sigma_\varepsilon^2+\sigma_z^2} + \frac{\sigma_\varepsilon^2}{\sigma_\varepsilon^2+\sigma_z^2} p_t^i \\
&= \gamma + \frac{\sigma_\varepsilon^2}{\sigma_\varepsilon^2+\sigma_z^2}\left(p_t^i - \gamma\right)
\end{aligned}
$$

I try and predict p_t, but I only observe p_t^i. Suppose I observe p_t^i above its unconditional mean γ. It is somewhat surprising, as I know prices in all island should be γ on average. So I know some shocks have happened - but I don't know which, z_t^i or ε_t. The only thing I know are the variances of both shocks, and I will try and use that information.

Signal Extraction

- In particular, if I know the aggregate shocks ε_t are much more volatile than the idiosyncratic shocks z_t^i, I will tend to ascribe most of the observed discrepancy $p_t^i - \gamma$ to an aggregate shock. In the limit, suppose idiosyncratic shocks to not exist, i.e. $\sigma_z^2 = 0$. Then BLUE implies that $E\{p_t \mid p_t^i\} = p_t^i$. Observing p_t^i is exactly identical to observing p_t, since there are no island specific shocks.
- A contrario, if I know the the idiosyncratic shocks z_t^i are much more volatile than the aggregate shocks ε_t, I will tend to ascribe most of the observed discrepancy $p_t^i - \gamma$ to an idiosyncratic shock. In the limit, suppose aggregate shocks to not exist, i.e. $\sigma_\varepsilon^2 = 0$. Then BLUE implies that $E\{p_t \mid p_t^i\} = \gamma$. Observing p_t^i carries absolutely no additional information about the aggregate price, since p_t^i is exclusively driven by island shocks.

The Lucas Supply Function

- So now back to the supply function:

$$\begin{aligned} Y_t^i &= Y + a\left[\varepsilon_t + z_t^i - \frac{\sigma_\varepsilon^2}{\sigma_\varepsilon^2 + \sigma_z^2}\left(p_t^i - \gamma\right)\right] \\ &= Y + a\frac{\sigma_z^2}{\sigma_\varepsilon^2 + \sigma_z^2}\left(p_t^i - \gamma\right) \end{aligned}$$

- Output only responds to idiosyncratic shocks: in the absence of any island specific shock, $\sigma_z^2 = 0$ and producers simply do not alter their production plans, leaving them at X. Why? Because they know any observed deviation between the price on their island and what they expect it to be (γ) comes from an aggregate shock (remember: they know the variances of the shocks). In other words, they know all prices on all islands have shifted identically. No reason to produce more.

Signal Extraction

- Now we can aggregate across islands, to obtain

$$
\begin{aligned}
Y_t &= \frac{1}{N}\sum_n Y_t^i = Y + \frac{1}{N}\frac{a\sigma_z^2}{\sigma_\varepsilon^2+\sigma_z^2}\sum_n \left(\varepsilon_t + z_t^i\right) \\
&= Y + \frac{a\sigma_z^2}{\sigma_\varepsilon^2+\sigma_z^2}\left(\varepsilon_t + \frac{1}{N}\sum_n z_t^i\right) = Y + \frac{a\sigma_z^2}{\sigma_\varepsilon^2+\sigma_z^2}\varepsilon_t \\
&= Y + \frac{a\sigma_z^2}{\sigma_\varepsilon^2+\sigma_z^2}\left(p_t - E\{p_t\}\right)
\end{aligned}
$$

- Across the MACROeconomy, production only responds to changes in aggregate prices that were not expected. It does so to an extent that increases with the variance of idiosyncratic shocks.

Signal Extraction

- Reminiscent once again of the "policy ineffectiveness" argument. Changes in prices (engineered for instance via monetary policy) can have an effect on output only if they are not expected. The expectations-augmented Phillips curve told us a similar story: it is only in as much as π_t and π_t^e are disjoint that unemployment will respond.
- Now however, the agents' rational expectations are solved WITHIN the model.
- We now embed this "Lucas" supply curve in general equilibrium models of the macroeconomy - i.e. with a demand as well as a supply.

Rational Expectations in General Equilibrium

- We follow an AS-AD framework:

$$y_t = y_f + \alpha \left(p_t - Ep_t \right) \tag{AS}$$

where $\alpha = \frac{a\sigma_z^2}{\sigma_\varepsilon^2 + \sigma_z^2}$, $Y_f = Y$ and we simplified the expectations notation a bit. Lower case variables now denote logarithms (doesn't change anything, but simplifies considerably the derivations).

- As before, under exogenous constant expectations, a rise in prices will increase output (i.e. decrease unemployment). This is akin to a Phillips curve.
- Demand arises from an equation of exchange, $MV = PY$, which in logs rewrites

$$y_t = m_t - p_t + v_t \tag{AD}$$

- We want to use this framework to think about the effect of (monetary) policy on real activity in equilibrium - in particular in a RATIONAL EXPECTATIONS equilibrium.

The Conventional AS-AD model with rational expectations

- The difficulty is that we need to solve for prices and quantities, as usual, but also for expectations Ep_t.
- Use (AD) to solve for prices:

$$p_t = m_t - y_t + v_t$$

- Then (that's the key), this relation must also hold IN EXPECTATIONS. Why? Agents are rational and know the model: so they know the relation will hold in expectations. It may turn out to be invalidated (for instance, velocity suddenly collapses because of a sub-prime crisis) - but they don't know it when they form their expectations. In other words:

$$Ep_t = Em_t - Ey_t + Ev_t$$

The Conventional AS-AD model with rational expectations

- It must therefore be the case that

$$p_t - Ep_t = (m_t - Em_t) - (y_t - Ey_t) + (v_t - Ev_t)$$

  Surprises in prices must arise either from surprises in money, surprises in output, or surprises in velocity.

- Assume velocity constant (again, for simplicity). Now, use (AS) to recognize that $y_t - Ey_t = \alpha (p_t - Ep_t)$. Then

$$p_t - Ep_t = (m_t - Em_t) - \alpha (p_t - Ep_t)$$

- And finally,

$$p_t - Ep_t = \frac{1}{1+\alpha} (m_t - Em_t)$$

  In the absence of velocity shocks, suprises in prices can ONLY come from surprises in money. Why? Because surprises in output are determined endogenously, via the AS.

The Conventional AS-AD model with rational expectations

- Substitute this back into supply:

$$y_t = y_f + \frac{\alpha}{1+\alpha}\left(m_t - Em_t\right) \qquad \text{(AS)}$$

  This is a so-called reduced form, i.e. it expresses endogenous variables (output, here) in function of the model parameters (y_f, α) and policy choices (m_t). We have solved prices out - and in particular price expectations out.
- Economically, this means that money (and monetary policy) only affects economic activity in as much as it is not expected. Whatever change in money that is fully anticipated by rational agents will have zero effects on quantities produced.
- Here this happens because only surprises in money drive surprises in prices. Surprises in prices, in turn, are interpreted as a change in idiosyncratic island prices, i.e. an island specific shock. Producers respond to that by producing more (but only so long as they believe it is their price that has changed).
- Exercise: Assume $m_t = \rho m_{t-1} + \epsilon_t$. Show only ϵ_t matters for output

The Conventional AS-AD model with rational expectations

- Note further that
$$\frac{\alpha}{1+\alpha} = \frac{a\sigma_z^2}{\sigma_\varepsilon^2 + (1+a)\sigma_z^2}$$
The response of production to surprises in money increases in the relative magnitude of σ_z^2, i.e. the importance of idiosyncratic shocks.
- Agents living in a country with a history of high AGGREGATE inflation will ascribe relatively large values to σ_ε^2, which means policy is increasingly INeffective there.
- The more policy tries to use monetary policy to boost output, the harder it becomes as the (real and perceived) volatility of σ_ε^2 increases.
- Policy Ineffectiveness here to stay apparently. This is the "neo-classical" view (Milton Friedman, Robert Lucas, Chicago School)

A Version with Wage Rigidities

- Introduce a labor market where wages are negotiated. Then we will assume wages are set for two periods (2 quarters, say). That will introduce a period of time over which prices (wages) cannot adjust to money shocks. It will make it more "easy" to engineer surprises, and thus introduce back policy effectiveness.
- Now supply is simply produced with labor:

$$Y_t = L_t^a$$

with $a < 1$. At the margin, firms pay labor its marginal product, that is

$$\frac{W_t}{P_t} = aL_t^{a-1}$$

In logarithms:

$$w_t - p_t = \ln a + (a-1)l_t$$

A Model with Wage Rigidities

- Finally (and crucially), the wage setting equation writes

$$w_t = \gamma + E_{t-2}\{p_t\}$$

Nominal wages are contracted two periods ahead, for two periods. After that, they are anchored contractually at that pre-agreed level.Rearrange all components of supply to obtain

$$\gamma + E_{t-2}\{p_t\} - p_t = \ln a + (a-1)\frac{y_t}{a}$$

that is

$$y_t = \frac{a}{1-a}(\ln a - \gamma) + \frac{a}{1-a}(p - E_{t-2}\{p_t\}) \qquad \text{(AS)}$$

This is reminiscent of a Lucas supply curve, except that now surprises in prices do matter over two periods.

A Model with Wage Rigidities

- As before model AD according to an equation of exchange:

$$y_t = m_t - p_t + v_t \tag{AD}$$

- We now want to solve for the equilibrium, and in particular to solve for $p - E_{t-2}\{p_t\}$.
- As before, it must be the case that

$$E_{t-2}\{y_t\} = E_{t-2}\{m_t\} - E_{t-2}\{p_t\} + E_{t-2}\{v_t\}$$

And therefore that

$$\begin{aligned} y_t - E_{t-2}\{y_t\} &= (m_t - E_{t-2}\{m_t\}) - (p_t - E_{t-2}\{p_t\}) \\ &\quad + (v_t - E_{t-2}\{v_t\}) \end{aligned}$$

Assume again velocity is constant for simplicity.

A Model with Wage Rigidities

- Key point is once again to recognize that (AS) implies $E_{t-2}\{y_t\} = \frac{a}{1-a}(\ln a - \gamma)$, and so

$$\frac{a}{1-a}(p - E_{t-2}\{p_t\}) = (m_t - E_{t-2}\{m_t\}) - (p_t - E_{t-2}\{p_t\})$$

- Rearrange to obtain:

$$p - E_{t-2}\{p_t\} = (1-a)(m_t - E_{t-2}\{m_t\})$$

- So reduced form output is given by

$$y_t = \frac{a}{1-a}(\ln a - \gamma) + a(m_t - E_{t-2}\{m_t\})$$

- Once again, only surprises in money matter - but now these are TWO PERIODS ahead surprises. Some systematic component of monetary policy can now affect real outcomes. Note that the effect of surprises in money increase in a, the elasticity of output with respect to labor..
- Exercise: Assume again $m_t = \rho m_{t-1} + \epsilon_t$. Show that now ρ matters for output

Now introduce a stochastic environment

- We are now augmenting the framework with shocks - to both AS and AD, and to monetary policy, as well. Consider the economy:

$$y_t = \alpha_0 + \alpha_1 (p_t - E_{t-1} p_t) + u_t \quad \text{(AS)}$$
$$y_t = \beta_0 + \beta_1 (m_t - p_t) + \beta_2 E_{t-1}\{p_{t+1} - p_t\} + v_t \quad \text{(AD)}$$
$$m_t = \mu_0 + \mu_1 m_{t-1} + \mu_2 y_{t-1} + e_t \quad \text{(MON)}$$

  where u_t, v_t and e_t are supply, demand (velocity?) and monetary shocks, respectively. Relative to previous setup, several differences are important. First, expected inflation increases quantities demanded. This could happen for instance as higher inflation means lower real interest rate FOR A GIVEN nominal interest rate.

- Second, monetary policy does now react to output, via μ_2. A purely countercyclical monetary rule would imply $\mu_1 = 0$ and $\mu_2 < 0$.

General Equilibrium

- Rearrange (AD):

$$p_t = \frac{\beta_0}{\beta_1} + m_t - \frac{1}{\beta_1} y_t + \frac{\beta_2}{\beta_1} E_{t-1}\{p_{t+1}\} - \frac{\beta_2}{\beta_1} E_{t-1}\{p_t\} + \frac{v_t}{\beta_1}$$

Then take unconditional expectations to get

$$\begin{aligned} E_{t-1}\{p_t\} &= \frac{\beta_0}{\beta_1} + E_{t-1}\{m_t\} - \frac{1}{\beta_1} E_{t-1}\{y_t\} \\ &\quad + \frac{\beta_2}{\beta_1} E_{t-1}\{p_{t+1}\} - \frac{\beta_2}{\beta_1} E_{t-1}\{p_t\} \end{aligned}$$

So

$$p_t - E_{t-1}\{p_t\} = (m_t - E_{t-1}\{m_t\}) - \frac{1}{\beta_1} (y_t - E_{t-1}\{y_t\}) + \frac{v_t}{\beta_1}$$

General Equilibrium

- As before, recognize that $E_{t-1}\{y_t\} = \alpha_0$, so that

$$p_t - E_{t-1}\{p_t\} = (m_t - E_{t-1}\{m_t\}) - \frac{1}{\beta_1}\left[\alpha_1\left(p_t - E_{t-1}p_t\right) + u_t\right] + \frac{v_t}{\beta_1}$$

  So that finally,

$$p_t - E_{t-1}\{p_t\} = \frac{\beta_1}{\alpha_1 + \beta_1}\left(m_t - E_{t-1}\{m_t\}\right) - \frac{1}{\alpha_1 + \beta_1}u_t + \frac{1}{\alpha_1 + \beta_1}v_t$$

- Substitute this back into supply to get:

$$y_t = \alpha_0 + \frac{\alpha_1\beta_1}{\alpha_1 + \beta_1}\left(m_t - E_{t-1}\{m_t\}\right) + \frac{\beta_1}{\alpha_1 + \beta_1}u_t + \frac{\alpha_1}{\alpha_1 + \beta_1}v_t$$

General Equilibrium

- Finally, we can substitute out monetary policy:

$$m_t = \mu_0 + \mu_1 m_{t-1} + \mu_2 y_{t-1} + e_t$$

and therefore

$$E_{t-1}\{m_t\} = \mu_0 + \mu_1 m_{t-1} + \mu_2 y_{t-1}$$

- So that

$$m_t - E_{t-1}\{m_t\} = e_t$$

The surprise in money one period ahead is exactly equal to the monetary shock. And finally:

$$y_t = \alpha_0 + \frac{\alpha_1 \beta_1 e_t + \alpha_1 \beta_1 v_t + \beta_1 u_t}{\alpha_1 + \beta_1}$$

- Notice once again that the policy rule (μ_1 or μ_2) is completely absent from reduced form output. All (unexpected) shocks have positive effects on output.

Issues of Dynamic Inconsistency

- To close, come back to the initial expectations-augmented Phillips curve to illustrate the importance of rational expectations for monetary policy - and in particular bring credibility issues in the picture. Consider the expectations augmented Phillips curve:

$$y_t = y + b\left(\pi_t - \pi_t^e\right)$$

  Output is above its average, long run level whenever inflation happens to be above its expected level. Once again, only surprises in inflation matter for production - yet another variation around the Lucas supply curve.

Issues of Dynamic Inconsistency

- Now suppose the Central Bank has a loss function that reflects both a concern for output AND for inflation. For instance:

$$L = \frac{1}{2}\left(y_t - y^*\right)^2 + \frac{1}{2}\theta\left(\pi_t - \pi^*\right)^2$$

where y^* denote the TARGET level of output the Central Bank would like to reach, with $y^* > y$. In other words, the Central Bank would like to push economic activity ABOVE its long run level y. Similarly π^* denotes the target inflation level - e.g. 2 or 3%, and θ captures the relative importance of inflation vs. output.

Issues of Dynamic Inconsistency

- Suppose now the Central Bank makes an announcement that it will achieve inflation at π^*. Crucially, suppose the announcement is believed, so that $\pi_t^e = \pi^*$.
- Then the Central Bank is faced with the following minimization problem:

$$Min \frac{1}{2} \left(y + b \left(\pi_t - \pi^* \right) - y^* \right)^2 + \frac{1}{2} \theta \left(\pi_t - \pi^* \right)^2$$

  Note that we have made use of the fact that, the moment the Central Bank's announcement is believed, a Phillips Curve arises at the corresponding level of π_t^e. Here at π^*.
- It will choose ACTUAL inflation π_t to solve this problem, i.e. set:

$$b \left[y + b \left(\pi_t - \pi^* \right) - y^* \right] + \theta \left(\pi_t - \pi^* \right) = 0$$

Issues of Dynamic Inconsistency

- Rearrange

$$\pi_t = \pi^* + \frac{b}{b^2 + \theta} (y^* - y)$$

- Since we have assumed the Central Bank wants to push output y^* above its long run level y, this means it will want to inflate the economy ABOVE its initial announcement of π_t^*. Why? Because the minute its initial announcement is believed, a trade-off between inflation and output arises, that corresponds to that level of π_t^e. This makes it possible to push output further up.
- So the initial announcement of π^* cannot possibly be credible. RATIONAL agents anticipate the Central Bank will renege on its initial commitment, and do not believe it when it claims it will implement $\pi_t = \pi^*$.

Issues of Dynamic Inconsistency

- The question becomes: what announcement is credible? It will be the one for which $\pi_t = \pi_t^e$: effective inflation ends up equal to what is expected. Solve the problem with that added ingredient:

$$Min \frac{1}{2} \left(y + b\left(\pi_t - \pi_t^e\right) - y^*\right)^2 + \frac{1}{2}\theta\left(\pi_t - \pi^*\right)^2$$

- The solution writes:

$$b\left[y + b\left(\pi_t - \pi_t^e\right) - y^*\right] + \theta\left(\pi_t - \pi^*\right) = 0$$

i.e

$$\pi_t = \frac{b^2}{b^2+\theta}\pi_t^e + \frac{\theta}{b^2+\theta}\pi^* + \frac{b}{b^2+\theta}\left(y^* - y\right)$$

Issues of Dynamic Inconsistency

- Which is the inflation that is credible? the one that verifies $\pi_t = \pi_t^e$. Substitute that condition to get:

$$\pi_t = \pi^* + \frac{b}{\theta}(y^* - y)$$

- This is the only announcement the Central Bank can make, that will be believed. Inflation has to be ABOVE the very target the Central Bank itself has! That so-called "inflation bias" increases with b (which captures the opportunity afforded by the Phillips curve), but decreases in θ.
- In particular, a Central Bank that has a high θ will be one with a small inflation bias. The very reason why Central Banks spend they time ensuring everyone they are above all concerned with inflation