Non-Falsified Expectations and General Equilibrium Asset Pricing : The Power of the Peso*

by

Jean-Pierre Danthine
University of Lausanne and CEPR

and

John B. Donaldson
Columbia University

First version: February 1996; this version: February 1999
Forthcoming in *The Economic Journal*

* We gratefully acknowledge the programming assistance of Dr. Hua-Chu and thank G. Huffman as well as seminar participants at Columbia, Padova, Venice, Paris, Tilburg, the Texas Monetary Conference and the 1997 CEPR Summer Macro Workshop in Athens for helpful comments.

Abstract

We discuss the extent to which the expectation of a rare event which happens not to materialise over the sample period, but which is not rationally excludable from the set of possibilities - the peso problem -, can affect the behaviour of rational agents and the characteristics of market equilibrium. To that end we describe quantitatively the macroeconomic and financial properties of a standard equilibrium business cycle model modified to allow for a very small probability of a depression state. We produce a reasonable model specification for which both business cycle characteristics and mean financial returns are in accord with United States observations. The 6.2% premium is obtained in an economy where agents are only moderately risk averse and where there are no frictions.

Keywords: Rational expectations, asset pricing, equity premium, peso problem, business cycle

JEL codes: D84, E32, G12

This paper examines the possibility that the large equity premium observed in the United States may result from the expectations of a disaster event, or set of events, which happen not to have materialised in the sample period of observations. Such a possibility, which falls under the rubric of a peso phenomenon, is supported by recent empirical work of Goetzman and Jorion (1997). Using return data for a wide range of countries, these authors conclude that the high historical premium in the United States is unique, and they conjecture that it may be attributable to the fact that disastrous events affecting other financial markets (e.g. WWII for Japan, Germany, and other European countries) have largely bypassed the American economy.

Ours is a theoretical examination of asset returns in a dynamic general equilibrium model with a production sector. We study how the small probability of a depression state modifies the macrodynamic and financial characteristics of a standard equilibrium business cycle model. We find that the effects are most dramatically manifest in data samples in which the feared "disaster state" is not actually present. Such a peso phenomenon is seen to have an especially powerful influence on asset returns and to be capable of generating a large equity premium in an economy with plausible macroeconomic characteristics.

The underlying idea is related to, but different from, Rietz's (1988) proposed solution to the equity premium puzzle. Rietz's (1988) results are obtained for an exchange economy model in which the disaster events are actually observed. We show that even stronger results can be obtained for an economy in which the actual disaster state is not observed, yet in which agent's believe it may occur. This is accomplished in the more demanding context of a production economy model for which we show that the *actual* occurrence of catastrophic events would not plausibly resolve the puzzle.

Relying as heavily as we do on deviations between realised and subjectively perceived probability distributions raises difficult methodological questions which the rational expectations perspective had the goal of resolving. We argue, however, that the empirical implementation (whether via estimation or calibration) of rational expectations models using short data samples (usually post-war data) runs the risk of over-disciplining the typical inquiry. Indeed, the usual procedures assume, by and large, that all states of nature relevant to agent's decision making are represented in the sample of observations and that other, significantly different, possibilities may be excluded from consideration: events that have not been observed are assumed never to occur. In the language of probability these procedures often assume that the values of the state variables observed in the historical period encompass all the values in the economy's stationary probability distribution.

Yet, it is not unreasonable to think, for example, that the experience of the Great Depression continues to have a significant influence on the behaviour of those who experienced it directly or indirectly, even though it has not recurred in sixty-five years. Similarly, the fear of 1929, although not borne out, or of a much-talked-about-but-never-experienced systemic financial meltdown, may have been significant in the crash of October 1987 (as well as in October 1997!). In other words, these events may have loomed larger in the investors' belief sets than their "objective" probability as assessed on the basis of recent history: the fact that they have not materialised in the postwar period does not prove that the *possibility* of their occurrence has not affected behaviour.

These considerations suggest to us the importance of checking the robustness of certain accepted results and modelling regularities to the feature of allowing agents to entertain, in their decision problem, the expectation of certain events which have not (or not yet) appeared in the reference historical sample. This is a perspective first adopted in studies of the foreign exchange markets, hence the "peso problem" label. If we are to generalise this viewpoint, however, we must do so in a highly disciplined way. Most specifically, we must avoid a return to the pre-rational expectations situation in which puzzling phenomena could be explained away on the basis of unverifiable assumptions on "what is in the mind " of decision makers. We maintain discipline in our inquiry by abiding by three requirements: first, we allow for expectations of significant but not implausible events; second, these events must be sufficiently unlikely (i.e. have small enough probability of occurrence) so that the fact that they have not been observed in a period corresponding to the usual sample length does not make it irrational to attach a positive probability to such events;[1] finally, and most importantly, we check the full general equilibrium implications of the adopted expectational hypotheses against a large set of observations and not exclusively against the particularly puzzling ones which have motivated their adoption. While this paper focuses on financial returns and the equity premium puzzle, we thus verify that the macroeconomic implications of our expectational hypotheses are not falsified by the data.[2]

It is already well understood that peso effects can have substantial implications for security return estimates obtained from 'small samples'. See Evans (1997) for an excellent and thorough survey of the relevant literature. Focusing on the equity premium in a technically related context, Cechetti, Lam and Mark (1990, 1993), Kandel and Stambaugh (1990) and Abel (1994) study the effects of imposing a Hamilton (1989) style Markov switching regime on the conditional and unconditional return estimates. Their study of the mean returns conditional on a particular regime parallels our peso perspective. Under the Markov regime switching approach, however, the economy alternates, with probabilities calibrated to registered frequencies, between different regimes corresponding to regularly observed historical episodes. By contrast, the peso approach centers on the effects of the possibility of switching to a small probability, catastrophic regime in a context when the ex-post frequency is not in accord with the ex-ante probability. On balance the regime switching structure has small implications only for conditional risk premia (see Cechetti et al. (1990) and Kandel and Stambaugh (1990), for example) because the pricing kernel is left mostly unaffected by uncertainty in the future regime. Although for somewhat different reasons, the effects are similarly modest in Cechetti et al. (1993). Our results for the case where the catastrophic event

---

[1] For all the parameter specifications reported below, the probability of not observing the catastrophic event in a period of 40 years (taken to represent the typical macroeconomic sample) is always higher than 20% ; it is as high as 55% for our benchmark case.

actually materialises are in accord with these contributions. Abel (1994) finds that Markov switching in the endowment process actually exacerbates the inability of the model to explain unconditional equity return premia.

More recently, Bekaert et al. (1995) attempt to explain a series of anomalies in the United States term structure of interest rates by relying on the possibility that high-interest rate regimes may have occurred less frequently in the realised sample of United States data than was rationally anticipated. By pooling British and German observations with the United States data, they construct a sample of observations which they argue is a plausible representation of the relevant probability distribution for United States decision-makers. This, in effect, assumes that the latter regard the interest rate and inflation experience of foreign countries as being drawn from the same underlying probability distribution as their own and thus equally representative of future possible realisations of the United States economy itself.

Hansen et. al (1999) show that aversion to specification error in the evolution of an economy's state variables can also lead to large consequences for security prices, in a manner that goes a long way in explaining financial regularities. This is accomplished by constructing a model in which agents' aversion to specification error is equivalent (same equilibrium time series of aggregate consumption etc.) to assuming they possess "risk sensitive" preferences in the manner of Epstein and Zinn (1989). While very different in motivation (the latter errors arise in the context of a Markov perfect game with a 'malevolent' opponent rather than as a pure expectations peso effect), this work also explores the asset pricing consequences of expectational errors."

An outline of the paper is as follows: Section 2 lays out the basic model construct whose properties are studied in Sections 3 and 4. Section 5 presents a version of our model with plausible macroeconomic properties and an equity premium in line with the United States observations.

---

[2] As advocated by Evans (1997) in his conclusion.

Section 6 comments on some related literature. We conclude in Section 8 after addressing welfare issues in Section 7.

1. Introducing a depression state into an equilibrium business cycle model.

Our benchmark is the well-known business cycle model of Hansen (1985) with technology shocks and an indivisible labour supply. The dynamic equilibrium allocations in this model solve the following central planning formulation:

$$\max_{\{x_{t+1},n_t\}} E\sum_{t=0}^{\infty} \beta^t u(c_t, 1-n_t)$$

$$\text{s.t.}\quad \begin{aligned} & c_t + x_{t+1} \leq f(k_t, n_t)z_t \\ & k_{t+1} = k_t(1-\delta) + x_{t+1} \\ & n_t \leq 1, c_t \geq 0, x_t \geq 0, n_t \geq 0, \text{ and} \\ & k_0 \text{ given} \end{aligned} \qquad \text{(P)}$$

In the above problem, c_t denotes the period t consumption of the representative agent and n_t his period t supply of labour ($1-n_t$ is thus his leisure in period t). Similarly, k_t and z_t represent, respectively, the agent's period t capital stock and investment, while $u(.,.)$ is his period utility function and $f(.,.)$ the period production technology which is subject to the technology shock sequence $\{z_t\}$. Lastly the parameter β is the subjective discount factor and δ the period depreciation rate. Under Hansen's formulation, the period utility function and production technology assume the forms:

$$u(c_t, 1-n_t) = \ln c_t + An_t \qquad (1)$$

and $$f(k_t, n_t) = Lk_t^{\alpha} n_t^{1-\alpha} \qquad (2)$$

with $\alpha = 0.36, L = 1.25$ and $A = -2.85$

The only change we wish to undertake at this stage concerns the form and the distribution of the technology shocks. As in most of the literature to date, Hansen (1985) assumes that the technology shock follows a highly persistent first order autoregressive process of the form:

$$z_{t+1} = \rho z_t + \varepsilon_{t+1}$$

with ρ = 0.95 and the $\{\varepsilon_t\}$ i.i.d. lognormally with mean 1-ρ and SD = 0.00712. The latter quantity is inferred from the variability of estimated Solow residuals over the postwar period. In view of the significant measurement problems (see Prescott, 1986) it is an appropriately conservative estimate of the indicated standard deviation. Under this specification, z_t is also lognormally distributed with

$$E(z_t) = 1; SD(z_t) = \left(\frac{1}{1-\rho^2} \operatorname{var} \varepsilon_t\right)^{1/2} = 3.2\, SD(\varepsilon) = 0.023$$

Note that the 95% confidence interval for z_t is $[0.954,\ 1.046]$ and that the probability of a shock being significantly worse than this range is essentially negligible; for example,

$$\text{Prob}[z \le 0.9] = 2.10^{-6}; \text{Prob}[z \le 0.8] = 6.10^{-23}; \text{Prob}[z \le 0.6] = 2.10^{-111}.$$

The first step in implementing the change in the shock process we want to effect is to approximate the continuous process above with a finite state discrete Markov chain. There are two relevant considerations in constructing such an approximation. First, we know from Tauchen (1986) that we can approximate the continuous process noted above to any degree of desired precision (match any number of moments) by choosing a sufficiently large number of states. Second, we know from earlier work (see, e.g., Danthine and Donaldson (1989)), that it is possible to replicate almost exactly the macroeconomic and financial results of Hansen (1985) with a coarse state partition of two carefully chosen states. This possibility reflects the fact that the performance of the propagation mechanism inherent in these models seems relatively unaffected by the precise structure of the shock process.

For economy of computation and in order that our message be more transparent we will thus rely on the latter comment and postulate, initially, a two state symmetric Markov process with $z_1 = 0.979$ and $z_2 = 1.021$ and the following probability transition matrix:[3]

$$\begin{pmatrix} 0.97 & 0.03 \\ 0.03 & 0.97 \end{pmatrix} = M'$$

As confirmed below, the latter choice allows the model to replicate nearly exactly the results presented in Hansen (1985).

We next modify the shock process to admit the possibility of a depression state occurring with small probability. In doing so, we do not mean to suggest that we view the Great Depression as having been caused by a technology shock. Our aim is to study asset pricing in a simple and consistent equilibrium model where the possibility of a major shock is entertained. It is well beyond our purpose to propose a theory of the occurrence of rare catastrophes. We believe the financial implications of our set-up are likely to be illustrative of those resulting from other catastrophic falls in consumption but admit such a statement must be substantiated in other model contexts.

Our benchmark case postulates z_1 and z_2 as per above but with $z_3 = 0.6$ and transition matrix

$$\begin{pmatrix} 0.97 & 0.03-\boldsymbol{h} & \boldsymbol{h} \\ 0.03-\boldsymbol{h} & 0.97 & \boldsymbol{h} \\ 0.5(1-\boldsymbol{f}) & 0.5(1-\boldsymbol{f}) & \boldsymbol{f} \end{pmatrix} = M$$

where $\boldsymbol{h}$ captures the likelihood of entering the third "depression" state and $\boldsymbol{f}$ determines the expected time remaining in it. An increase in either of these parameters, ceteris paribus, will increase the stationary probability of the depression state. With $\boldsymbol{h} = 0.0004$ and $\boldsymbol{f} = 0.10$, for

[3] Once again, the characteristics of the transition probability matrix such that the discrete process most closely approximates the continuous process are derived from Tauchen (1986).

example, the unconditional probability of the depression state is 0.00044 or 44 times in 100'000 periods (quarters under our parameterisation).

The focus of this paper is the study of the business cycle and financial characteristics of a model economy in which economic agents rationally anticipate the possibility of a disaster state, yet in which no such state is observed in the actual realisation of events. We formalise this idea by assuming that the process M is the true, objectively and subjectively anticipated, Markov process for this economy, but that, given its low probability, the third "depression" state happens not to materialise in the sample period of 160 observations that we take as our benchmark. In practice we compute the decision rules of the model and use these to generate time series in a standard fashion. Our statistics are obtained as averages of 200 runs each of 160 consecutive data points in which the disaster has not materialised. This latter choice is in accord with usual practice in the business cycle literature with the series length corresponding roughly to the postwar period (measured in quarters). Standard deviations of the estimates are also reported. [4]

More specifically, time series of per capita capital stock, consumption, investment, and hours were thus generated using the decision rules *x(k,z)* and *n(k,z)* obtained from the solution to problem (P)-(M). This problem was solved numerically using discrete state space dynamic programming methods based on a standard grid search with the norm of the capital stock partition equal to 0.0025. [5] See Christiano (1989) or Danthine and Donaldson (1989) for a

---

[4] Another interpretation is that agents irrationally base their decisions on a subjective perception that the Markov process for this economy is M while in fact the true shock generator is M'. While in substance the two interpretations are materially different, in practice the results are the same. In other words, we have verified that generating macro and financial statistics using the M' matrix while agents' decision rules have been derived on the basis of the M matrix yields essentially the same results as using the M matrix to represent both the expected and the actual process but purging from the observations any realisations of the depression state provided we are careful to also remove all observations posterior to a realization of the depression state until the economy has returned to its "normal" steady state values.

[5] Note that, by its very nature which implies that the "action" is far away from the steady state, the problem raised cannot be solved efficiently via computation methods based on approximations around the steady state.

detailed summary of these procedures. For all simulation runs, $\boldsymbol{b}$ = 0.99, $\boldsymbol{d}$ = 0.025 and $\boldsymbol{a}$ = 0.36.

In all cases, the time series respected the relationships:

$$c(k_t, z_t) = f(k_t, n(k_t, z_t))z_t - x(k_t, z_t) \tag{3}$$

and
$$k_{t+1} = k_t(1-\boldsymbol{d}) + x(k_t, z_t) \tag{4}$$

Financial statistics were completed using the now-standard perspective of Lucas (1978) and Mehra and Prescott (1985). In particular, the dividend stream $d(k_t, z_t)$ of the equity security was constructed as per :

$$d(k_t, z_t) = f(k_t, n(k_t, z_t))z_t - w(k_t, z_t)n(k_t, z_t) - x(k_t, z_t) \tag{5}$$

where $w(k_t, z_t)$, the period wage rate, satisfied

$$w(k_t, z_t) = f_2(k_t, n(k_t, z_t))z_t \ , \tag{6}$$

and $w(k_t, z_t)n(k_t, z_t)$ defines the representative firm's period t wage bill. Using the dividend series so constructed, the (conditional) price $q^e(k_t, z_t)$ of the equity security was recursively computed according to the equation

$$q^e(k_t, z_t) = \boldsymbol{b}\int \frac{u_1(c(k_{t+1}, z_{t+1}), 1-n(k_{t+1}, z_{t+1}))}{u_1(c(k_t, z_t), 1-n(k_t, z_t))}[q^e(k_{t+1}, z_{t+1}) + d(k_{t+1}, z_{t+1})]dF_M(z_{t+1}; z_t) \tag{7}$$

We note for later use that a recursive solution to (7) leads to the following price representation for the equity security:

$$q_t^e = E_t\left(\sum_{j=1}^{\infty} \boldsymbol{b}^j \frac{u_1(c_{t+j}, 1-n_{t+j})}{u_1(c_t, 1-n_t)} d_{t+j}\right) \tag{8}$$

At all times the above expectation was computed using the conditional shock distribution $dF_M(z_{t+1}; z_t)$ obtained from the full matrix M; i.e., at all times investors priced the equity security taking into account the possibility of a disaster state. Using these prices, the time series of equity returns was then computed in the obvious way:

$$r^e_{t,t+1} = \frac{q^e(k_{t+1}, z_{t+1}) + d(k_{t+1}, z_{t+1})}{q^e(k_t, z_t)} - 1 \quad (9)$$

In a like fashion, the price of a one period risk free debt security , q^b , is given by

$$q^b(k_t, z_t) = \boldsymbol{b} \int \frac{u_1(c(k_{t+1}, z_{t+1}), 1 - n(k_{t+1}, z_{t+1}))}{u_1(c(k_t, z_t), 1 - n(k_t, z_t))} dF_M(z_{t+1}; z_t) \quad (10)$$

with the conditional risk free rate defined correspondingly as

$$r^b(k_t, z_t) = \frac{1}{q^b(k_t, z_t)} - 1 \quad (11)$$

If such a security is in positive net supply (as we may later choose to assume), the definition of the dividend stream must be accordingly modified:

$$d(k_t, z_t) = f(k_t, n(k_t, z_t))z_t - w(k_t, z_t)n(k_t, z_t) - x(k_t, z_t) - Br^b(k_t, z_t) \quad (12)$$

where *B* denotes the (positive) net supply of bonds issued by the representative firm. Note that this abstraction requires a notion of decentralisation which admits multi-period firms. See Altug and Labadie (1994) or Danthine and Donaldson (1995) for possibilities in this regard.

2. The Benchmark case: a Macroeconomic Assessment

In this section we verify that our main expectational assumption is not falsified when the model results are confronted with macro observations. For our benchmark case (z_3 =0.60 , $\boldsymbol{h}$ = 0.008 and $\boldsymbol{f}$ = 0.20), the disaster state is expected to occur approximately once every 100 quarters or, on average, roughly once every 25 years. For these parameter values, the likelihood of not observing z_3 in a sample period of 40 years is 28% (see Appendix 1). If and when a disaster state occurs, output declines to about 50% of its mean value for one quarter and investment almost comes to a halt.[6] Consumption falls by about 30% for one quarter. Labour supply also decreases by about 20%. All variables return to normal almost immediately because the effect of the shock on the capital stock is small (k declines by less than 1%). Investment and employment overshoot to some extent.

For reference, we present in the second panel of Table (1) the case where the shocks z_1 and z_2 are chosen so that for $\boldsymbol{h}$ = 0 (no possibility of entering the disaster state), the model replicates, almost exactly, Hansen's (1985) indivisible labour economy results. Note that the model's results are generally consistent with the statistical summary of the United States economy although consumption is too smooth.

In the third panel of Table (1) we report the average statistics for a complete set of 500'000 observations derived from the (P)-(M) model. For this case, there is no short sample problem and the disaster actually occurs with the stationary frequency computed above. We see that the effect of the presence of the disaster state ex-post is to make the economy substantially more variable, especially investment. To some degree, this perception is affected by the extent to which the periods of depression are allowed to pull down the estimated trend. To correct for this effect, we report (columns (d), Table (1)) the standard deviations of the stationary probability distributions without detrending. A comparison of the corresponding numbers shows that the detrending procedure exaggerates the effect on the standard deviation of investment but that, as an approximation, the introduction of the disaster state can be said to increase the “stationary” or “unconditional” variability of the macro-aggregates by 50% (undetrended data).

Let us now turn to the case where the disaster state is anticipated but not realised. Panel (4) of Table (1) presents the results for a complete set of 200 x 160 observations. The disaster state is absent from each of these 200 runs. Comparing columns (a) and (b) of Panels (2) and (4) we see that the addition of the pure possibility of the disaster occurrence has little effect at business cycle frequencies. Neither the detrended standard deviations nor the detrended correlations appear to be materially affected. Something more is revealed if we examine the undetrended statistics (columns (c) and (d) of Panels (2) and (4)). Comparing

[6] The direct effect of technology shock is amplified by the induced decline in labor input.

mean values (column (c)), we observe a consistent increase across all macro-aggregates when a disaster state is anticipated but never realised. The percentage increase ranges from a high of 3.6% for investment and capital stock, to a low of 0.67% in the case of employment. As a result of these latter shifts, output and consumption are, on average, higher as well.

How may these findings be interpreted? Relative to consumer-investors in the standard Hansen (1985) model (no disaster state anticipated or experienced), agents in the benchmark economy perceive an increase in risk, even if the disaster state is never realised. They can partially self-insure against this risk by saving more (and, as we'll see later, by attempting to hold more risk free assets) and accumulating capital. While the consequences of this precautionary-like behaviour are not large, this is not surprising given the small stationary probability of being in the disaster state.

We may thus conclude, at this level of observation and within the confines of the adopted parameter specifications, that it is not possible to reject the hypothesis that a small probability disaster state is part of economic agents' expectations sets. In other words, whether the perceived probability of a disaster is positive or null, our ability (or inability) to explain macroeconomic observations is unchanged since the data summary is largely unaffected.

In Appendix 2, we confirm these results for alternative parameter values. For a broad range of values of ϕ , η, and z_3, consistent with the depression state remaining a small probability event, the macroeconomic properties of our model remain very close to those of the standard Hansen (1985) business cycle model. Accordingly we feel justified in exploring its financial implications for these parameter specifications.

3. The Benchmark Case: Financial Implications

Comforted that our specific expectational assumption is not falsified by macro data, we now turn to examining the financial properties of the benchmark case. As noted in the introduction, it has been understood for some time (e.g., Evans (1997)) that peso effects can have a substantial influence on financial data, and it is of interest to examine the extent to which this is manifest in our general equilibrium context. Table (2) presents the results of the model for the base case of Table (1).

Panel (2) illustrates the force of the classical equity premium puzzle of Mehra and Prescott (1985) translated into our production setting: the risk free rate is much too high and the equity return too low relative to the stylised facts of the United States economy (Panel (1)). As a result the premium is trivially small. At this stage, one may worry that the Hansen model is nothing but a strawman for the study of the equity premium puzzle. Indeed, the status of current research efforts, as reviewed by Kocherlakota (1996) for example, suggests avoiding a perfect market, frictionless equilibrium model in the quest for a solution to the puzzle. Yet, we believe that, in light of our successes to come, the weakness of the platform we build on strengthens rather than it diminishes our message vis-à-vis the importance of peso considerations for asset pricing.

If a possible disaster state (z_3=0.6) is appended to the model (our Benchmark), and if the disaster is experienced with the same relative frequency as anticipated (Panel 3(i)), the premium does increase dramatically (in fact more than 100 times), but still remains much below its observed level, despite the fact that this economy is now more than three times as variable than the United States economy (see Table (1)). While both risky and risk free returns display appropriately greater volatility than in the base Hansen (1985) model, the volatility of the risk free rate now exceeds that of the equity return, which is counterfactual. If the coefficient of relative risk aversion is increased to γ=3 (Panel 3(ii)), the upper bound on its generally accepted range of plausible values, the premium, while rising further, still falls dramatically short of its empirically observed level.[7] Increasing risk aversion does further increase volatility,

[7] In this case the preferences of the representative agent are hypothesized to be of the form:

$$u(c_t, 1-n_t) = \frac{c_t^{1-\gamma}}{1-\gamma} + An_t .$$

but the relative rankings of the equity and risk free securities remains inverted, indeed, increasingly so. These results suggest that, in the context of a production economy, the introduction of a disaster state alone does not significantly improve the ability of this class of models to replicate the generally accepted financial stylised facts.

If we examine the analogous statistics from a sample in which the anticipated disaster state has not materialised (Panel 3(iii)), however, the results are very different. The risk free rate, in particular, is seen to decline very considerably, while the equity return assumes more or less its value in the original Hansen (1985) model. These effects together give rise to a 1.62% premium. While still short of the United States historical value, the latter number fairly well approximates what is observed for non-United States developed countries. In addition, the volatilities, while too low, are now in the correct relative relationship.

The first step in the interpretation of this series of results is the realisation that the risk of a catastrophic quarter or string of quarters is borne exclusively by equity holders and not at all by bond holders. The occasional experience of a depression state in itself generates a larger premium made of a decrease in the return on the (more desirable) risk-free asset and an increase in the expected return on the (less desirable) risky asset. Indeed, this is the thrust of the Rietz (1988) solution of the equity puzzle to be discussed at length below. In addition, the presence of a catastrophic state destroys the normality of the pricing kernel. Backus, Foresi and Telmer (1996) show that an increase in the skewness and kurtosis of pricing kernels both tend to decrease the risk free rate. A disaster state does both. In our case, as already noted, the effect is significant but small: the equity premium remains below 1%.

But more is at work specifically attributable to the peso problem. As just observed, the pure possibility of a catastrophic state makes the premium larger. And this is the case despite the fact that, going into the catastrophe, the return on equity is very low (negative) while going out of the catastrophe the return on bonds is very high. The former observation is easy to understand, the latter results from the fact that the depression state is a bad time for saving: the expected consumption in the succeeding periods will be much higher, and thus the expected marginal utility of consumption much lower, than in the disaster state itself. Low expected marginal utility of consumption tomorrow and a high marginal utility of consumption today translates into low asset prices in general (see eqns. (8) and (10)). This effect is relatively much more significant for the risk free security. Because of the positive probability of exiting the depression state, i.e. of a much better shock (z_1 or z_2) in the near future, and the zero probability of being hit by a worse shock in the interim, the equity investment nearly dominates the risk free asset. The pressure on the price of the latter is thus more pronounced than on the price of the former.

When we study the statistical properties of time series in which the disaster state is absent, we precisely exclude those states where the risk-free-security-price is the lowest and the risk-free return is the highest. The lower tail of the price distribution (representing the disaster states)--equivalently, the higher tail of the return distribution -- is thus eliminated. This accounts for the substantially lower mean risk free return in the sample. For the equity security, these same events are also eliminated but they represent a much smaller proportion of the high return states than in the case of the risk free security. Accordingly, the mean returns are lower for both securities (relative to the situation in which the disaster is experienced) but the effect is much greater for the risk free security. This same logic also accounts for the substantially lower standard deviation of returns to both securities.

This intuition is formalised in the following proposition:

<u>Proposition</u>: Under appropriate assumptions, the stationary price distribution for the risk free security in the case where the disaster state is not observed stochastically dominates (1st order stochastic dominance) the corresponding distribution over the full set of shock realisations.

Proof: see Appendix 3.

4. A Plausible Macro Model without Risk Premium Puzzle

Drawing the lessons of the last two sections we can envision the possibility of simultaneously replicating the main stylised facts of the business cycle and matching the first moments of financial returns. We first try to understand if the preceding results can be strengthened by generalising the set of admissible parameters. Two possibilities immediately suggest themselves. The first is to admit a higher degree of relative risk aversion on the part of the representative agent. In the standard Hansen (1985) formulation, this change has the consequence of making the risk free security more desirable (thereby lowering its return), and the risky security less desirable (raising its return), with the joint consequence that the premium increases. In our context, the fear of the disaster may even strengthen the effect for highly risk averse agents. A drawback to this device, however, is that there is a concomitant effect in the behaviour of the economy's macroaggregates: substantial increases in risk aversion tend to force equilibrium output and consumption to be excessively smooth relative to the data.

A second parameter with which it is natural to experiment is B, the level of outstanding risk free firm debt (see eqn. (11)): positive debt issuance represents, via the attendant interest charges, a fixed claim against the firm's dividend stream. This has the consequence of making the dividend stream proportionately more variable. As a result, the price of the equity security would be expected to fall and the return rise, everything else equal. Since the agent also owns the firm's debt, he receives the interest payments and his income path (dividends, interest, and wages) is thus unaffected by a change in the firm's capital structure. As a result, his consumption and investment streams are similarly unaffected by such a modification, with the consequence that no attendant change is to be expected whether in the return characteristics of the risk free asset or in the economy's macrodynamics.

Table (3) provides results for a representative set of risk aversion — debt level parameter combinations. Panel (1) repeats the benchmark results for ease of comparison.

Panel (2) illustrates a case in which no debt is present but where the coefficient of relative risk aversion rises to $\gamma=3$. An increase in risk aversion increases the demand for both securities, but proportionately for the risk free one much more so. As a result, its expected return declines very significantly and the premium rises to 2.22%. Since greater risk aversion gives rise to an optimal consumption path that is much more smooth, however, there is less intertemporal variation in the equilibrium pricing kernel with the consequence that the volatilities of all securities, as well as of the consumption growth rate, decline.

Panel (3) captures a pure increase in the economy's debt level to B=2. In this case, the value of the firm, which equals the value of its capital stock is 11.53; a debt level of B=2 thus corresponds to a debt/equity ratio of 0.21. In the case of $\gamma=3$ (Panel (4), to be considered shortly), the average capital stock is 13.10; the debt/equity ratio for the same B=2 is 0.18.

In the case of $\gamma=1$, B=2, the mean equity return increases to 4.56% from 4.09% (B=0). With no change in the risk free return, the average premium rises to 2.10%. With both modifications (Panel (4)) the premium increases to 2.85%. If B is further increased to B=4, a premium of 3.88% is achieved (Panel (5)).[8]

Equipped with this knowledge, we present in Table (4) what can be viewed as our main model specification. The choice of parameters differs from the cases presented earlier principally in two respects. First, $z_3=0.5$; that is, the anticipated (but never experienced) decline in output is somewhat larger. Second, $\phi=0$; whenever the economy experiences a disaster it is of short duration. As a result, the stationary probability of the disaster is now less than 1% (0.00792)! This is thus a scenario of more severe but very short lived and low stationary probability disaster states. The coefficient of relative risk aversion is set at $\gamma=3$. In

[8] It also makes sense to consider variations in η and ϕ. We find, however, that unless these parameters are chosen so that the jointly determined stationary probability is relatively high (approaching 7%), the effects of increasing η or ϕ, singly or jointly, is small. For the same range of parameters as are present in Table (2) Part 2, for example, the premium varies from a low of 1.62 to a high of 1.70 (the $\phi=0.9$ case). This shows in

order to offset the reduced aggregate variation attendant to this latter choice, the shock dispersion is increased somewhat to z_1=1.028 and z_2=0.972. The level of debt is increased slightly to B = 2.5.

We see that this case replicates the first moments of United States security returns almost exactly. The premium is 6.2%, as contrasted with 6.18% in United States data. Although the mean equity return is a bit too low (and similarly for the mean risk free rate), this shortfall could easily be remedied by postulating a somewhat higher Debt/Equity ratio. Return volatilities remain much too low, however, for the reasons noted earlier. These results are obtained in a model whose business cycle properties are in line with those of the celebrated Hansen (1985) model. Consumption is a bit smoother here and the same is true for employment, but the macro literature offers several alternatives for improvements in these directions and this issue should not occupy us much here. Note, however, that the fact that we obtain our favourable financial results in a model with very little consumption volatility suggests that our peso mechanism is substantially different from those considered heretofore in the literature. The other three series display approximately the correct volatilities and the correlation structure is also very satisfactory.

We find these results striking: here is a calibrated macroeconomic model with perfectly respectable business cycle properties. Agents are moderately risk averse and there are no frictions. Yet, there is nothing puzzling in mean asset returns: the equity premium of this economy corresponds almost exactly to the 6.2% observed in the United States economy. This exercise forcefully illustrates the power of peso considerations for asset returns. More generally, it suggests the sensitivity of traditional results to small and plausible perturbations in expectations assumptions. In that light, not only is the United States equity premium somewhat

---

particular that the premium is not significantly affected, one way or another, by the expected duration of the disaster state. In the rest of this section, we do consider other changes in ϕ and z_3.

less of a puzzle, but it is also less surprising to register large variations in equity premia across time periods and geographical locations. Indeed it is clear that the full range of international premia could be replicated with small changes in our parameter specification.

5. Related literature

Results similar in spirit to those presented in the previous section are presented in Cecchetti et. al. (1997). Those authors consider a Lucas (1978) style exchange economy, in which the representative consumer-investor believes in a transition matrix that is false. They find that financial returns computed when the economy's dividend growth process follows the true transition matrix yet where agent's security demands are derived under the assumption of the false matrix allows the resolution of the equity premium puzzle at the level of first moments. While our model adds the further discipline and complication of a production setting, we are effectively endowing our agent with distorted beliefs relative to the actual time series of shocks, and in that sense our results and theirs are in perfect congruence. In view of our later discussion of second moments, it is noteworthy that Cecchetti et. al. (1997) goes on to show that a satisfactory matching of the second moments of security returns requires that the representative consumer believes not only in a false matrix but also one which is time varying.

At this stage it is also worthwhile to review Rietz's (1988) claimed solution to the equity premium puzzle. Rietz (1988) examines the implication of introducing a disaster state for the pricing of financial assets in the context of a pure exchange economy closely similar to the original setting of Mehra and Prescott (1985). In that setting, consumption and output coincide and the source of uncertainty lies is the stochastic process governing the growth rate of output. Rietz (1988) adds a disaster-growth-rate state. The probability transition matrix governing the consumption growth rate he employs is, in fact, identical to our matrix M with $\boldsymbol{f} = 0$ (no persistence in the disaster state). He is able to obtain risk premia in the range of 5-7% with combinations of the time discount factor $\boldsymbol{b}$ in the range of [0.96, 0.99] in conjunction with coefficients of relative risk aversion in the range [5, 9] when, in the disaster state, consumption falls to one half its average value (see Rietz (1988), Table (2)[9]). These results are generated for η parameter values in the range of η ε [0.0001, 0.004].

Our production economy is more complex than the economy of Rietz (1988). With z_3 = 0.5, output in our model also declines by slightly more than one-half relative to its average, but consumption does not. In a production economy, agents have the power to smooth consumption by altering their investment plans. Such smoothing ability reduces the variance of the representative agent's MRS and the pricing effects are expected to be less dramatic than in Rietz (1988). But how much less dramatic? To gain insight on this question, we examined the financial statistics for our model in the case where z_1 = 1.021, z_2 = 0.979, z_3 = 0.5, η = 0.004, γ = 9 and ϕ = 0, a situation which directly corresponds to Rietz's (1988, Table 2). All statistics were computed from the full time series in which the disaster state was present with the appropriate frequency. In this case, the economy displayed a level of variability substantially in excess of what is typically observed. In particular, the standard deviation of output achieved a level of 2.51% percent; investment displayed a variability of 34.73% percent. Nevertheless, the observed premium was only in the neighbourhood of 0.38%. If the relative magnitudes of z_1 and z_2 were reduced so that the model's variation better reflected the true business cycle stylised facts, the premium would have been even lower. These results confirm an earlier conclusion: the introduction of a disaster state into a production economy, in itself, is not an effective way to explain the financial stylised facts if the standard properties of the business cycle are to be preserved.

6. Welfare

---

9 Preferences in Mehra and Prescott (1985) and Rietz (1988) are described by a utility function of the form (no labor-leisure choice): $u(c_t) = \frac{c_t^{1-\gamma}}{1-\gamma}$

We have repeatedly insisted on the necessity maintaining discipline in an inquiry such as ours. To that end, we have considered only plausibly catastrophic events, occurring with very small attached probability, so that the macroeconomic impact of the induced behavioural changes is minimal. In this section we take an extra step by asking the question of the welfare consequences of anticipating a disaster state which never materialise. Indeed, one can conjecture that such anticipation will induce agents to deviate from their optimal investment-consumption-labour supply decisions and that, as a result, their welfare will be lower. It is of interest to measure the magnitude of this loss; i.e. how costly are the expectations "mistakes" that we have allowed so far, and, consequently, how large are the incentives to adjust these expectations?

This question was formalised in two alternative ways. The first approach runs as follows: initially, the average period utility of consumption under the matrix M' was computed for a particular set of parameter values. This corresponded to the case in which agents neither anticipate, nor experience the disaster state; it is an exact replication of Hansen's (1985) indivisible labour economy. An identical calculation (for the same parameters) was undertaken for the restricted series (z_3 not present in the sample), where the decision rules were obtained from the solution of (P)-(M) (disaster anticipation). The consumption transfer that an agent operating under the « (P)-(M)-restricted sample/no disaster » would have to pay (or receive) in order to achieve the same mean period utility as if he had solved (P)-(M') was then computed. That is, we solved for the quantity ξ which satisfied:

$$E\{U(c_t + \boldsymbol{x}, 1 - n_t); M; \text{restricted sample}\} = E\{U(c_t, 1 - n_t); M'\}$$

For the base Hansen (1985) case ((P)-(M')), $E\{U(c_t, 1 - n_t)\}$ = -1.053052. The results of this comparison for a sample of cases are presented in Table (5). The cases reported are representative in that for all cases, ξ < 0, and |ξ/Ec| assumed a value in the range of 1%. We see that, once in the stationary state – that is in terms of the new stationary probabilities resulting from his altered optimal behaviour -, the representative agent is slightly better off if he believes and prepares for the possibility of a disaster state (but none is observed). This, of course, means that the entire cost of his pessimistic expectations is borne in the transition to the stationary state, that is in the first periods of his behavioural adjustment, during which time he consumes less, saves more and accumulates additional capital. The latter translates into higher steady state consumption, at least if the disaster state does not materialise. While these adjustments appear appropriately small, it is fair to observe that, since, on the whole, agents must be worse off (otherwise the optimal policy under (M') would not be optimal), it must be that the sacrifices made in the transition are larger than the present value of the steady state gains, which, with our discount rate of 0.99 per quarter, can be estimated at 25% of the yearly consumption flow.

A question remains: once the effective transition costs are weighted against the higher steady state utility, what is the overall net cost of expecting a disaster that does not materialise? To answer this question we compute the average utility levels that the representative agent would attain, starting from the same capital stock level, in either of two scenarios for the economic future: no disaster state economy, i.e., problem P-M and disaster state expected P-M' but none actually present in the data. To this end, we first compute the optimal decision rules corresponding to each problem P-M and P-M'. Using the latter, we generate one thousand independent individual time series of equilibrium consumption and hours, each of ten thousand consecutive elements, all beginning from the corresponding problem steady state capital stock level $k_{M'}^{ss}$. Let us these be denoted by $\left\{(c_{t,j}^{M'}, n_{t,j}^{M'}) : j = 1,..,1000; t = 1,..,10000\right\}$. For these series the average discounted utility was constructed according to the formula

$$AU_{M'} = \tfrac{1}{1000}\left\{\sum_{j=1}^{1000}\left(\sum_{t=1}^{10000}\boldsymbol{b}^{t}u(c_{t,j}^{M'}, n_{t,j}^{M'})\right)\right\}$$

Lastly, the equivalent constant consumption level, $\bar{c}_{M'}$, i.e., the constant consumption level providing identical utility was obtained as the solution to

$$\sum_{t=1}^{10000} \boldsymbol{b}^t u(\bar{c}_{M'}, \hat{n}_{M'}) = AU_{M'} ,$$

where $\hat{n}_{M'}$ is the average level of employment across the thousand time series.

In an identical procedural fashion, the analogous quantities were computed using the decision rules for problem P-M, yet for the case in which the disaster state did not appear in any of the one thousand series (in practice, the actual shocks were generated according to M'). The equivalent constant consumption series corresponding to the P-M economy with no observed disaster, $c^*_{M;nd}$, was thus computed as the solution to

$$\sum_{t=1}^{10000} \boldsymbol{b}^t u(c^*_{M;nd}, \hat{n}_{M'}) = AU_{M;nd} = \tfrac{1}{1000}\left\{\sum_{j=1}^{1000}\left(\sum_{t=1}^{10000} \boldsymbol{b}^t u(c^{M;nd}_{t,j}, n^{M;nd}_{t,j})\right)\right\}$$

where the notation is as indicated with *nd* meaning 'no disaster observed'.

Table 5 contains the results of this exercise for a representative set of parameters. In all cases, the difference in the values of $\bar{c}_{M'}$ and $c^*_{M;nd}$ is in the order of 0.002% - essentially inconsequential. This suggests that the overall cost of expecting and preparing for a small probability event that does not materialise is very small. Notice that for this comparison we have standardised the employment level at $\hat{n}_{M'}$. Were it standardised at $\hat{n}_{M;nd}$, a similar result is obtained. In all cases the starting capital stock is $k^{ss}_{M'}$.

7. Conclusions

In the context of a dynamic general equilibrium model, we have explored the consequences of letting agents believe in the possibility of a disaster state. We have shown that allowing for such a possibility has substantial implications for the properties of financial returns, most dramatically so when the disaster state is <u>not</u> observed in the sample data.

Macroaggregates, by contrast, are more robust to this modification. We believe the lesson of this inquiry to be useful because most recent macroeconomic and financial modelling exercises implicitly or explicitly assume both that economic agents have an assured knowledge of the relevant stationary probability distributions and that the latter are satisfactorily characterised by the properties of relatively short historical data samples.

The idea proposed in this paper represents, in some sense, a departure from the full rational expectations perspective that has been so fruitful for macroeconomic theorising over the past twenty-five-years. Its appeal lies in its reasonableness and parsimony: we require only that agents fear, and prepare for, events which may not ultimately occur. It is, nevertheless, a dangerous path to follow precisely because our results show that our understanding of reality can be very sensitive to expectational assumptions which we cannot test directly. We have attempted to manage this danger by restricting ourselves to the expectation of relatively plausible small probability events for which the out-of-sample time series on which we based our statistical computations have a high probability of actually being observed, i.e., that, short of a much longer data sample than what is customary, agents' expectations, while not fully verified, are not falsified either. Moreover, we have argued that out-of-sample considerations, if they are to be integral to a successful theory, must contribute to an explanation of the broadest possible range of stylised facts, both macroeconomic and financial.

We have found that the possibility of a disaster state manifests itself most spectacularly by a decrease in the average equilibrium risk free rate which translates into a substantial increase in the equity premium and we have produced a reasonable model specification for which both business cycle characteristics and mean financial returns accord well with United States observations.

We do not want to argue that this is the solution to the equity premium puzzle. By the nature of our exercise, we will never be able to prove that catastrophic expectations such as those we model were indeed part of the market participants' information sets. This leaves the door open to competing or complementary explanations for the high observed premium. Moreover, we recognise that observations on the second moments of financial returns falsify our model as it stands. While it can be argued that there are well known avenues for increasing return volatilities,[10] it would not be consistent with our call for checking the broad implications of a modelling hypothesis such as ours to claim victory before second moments are satisfactorily replicated as well.

Still we find it striking that the 6.2% equity premium is obtained in an economy where agents are only moderately risk averse and where there are no frictions. In our view, this exercise forcefully illustrates the power of peso considerations for asset returns. More generally, we conclude that traditional results are highly sensitive to small and plausible perturbations in expectations assumptions. In that light, not only is the United States equity premium less of a puzzle, but we also find it less surprising to observe large variations in equity premia across time periods and geographical locations.

References

Abel, A. (1994). "Exact Solutions for Expected Rates of Return under Markov Regime Switching: Implications for the Equity Premium." *Journal of Money Credit and Banking*, vol. 26, pp. 345-361.

Altug, S. and Labadie, P. (1994). *Dynamic Choice and Asset Markets*. New York: Academic Press.

Backus, D., Foresi, S. and Telmer, C. (1996). "Affine Models of Currency Pricing." *NBER* Working paper 5623.

Bekaert, G., Hodrick, R. and Marshall, D. (1995). "Peso Problem Explanations for Term Structure Anomalies." mimeo, Northwestern University.

Cecchetti, S.J., Lam, P. and Mark, N.C. (1990). "Mean Revision in Equilibrium Asset Prices." *American Economic Review*, vol. 80, pp. 398-418.

[10] We pursue one of them in Danthine – Donaldson (1995)

____________ (1993). "The Equity Premium and the Risk Free Rate: Matching the Moments." *Journal of Monetary Economics*, vol. 31, pp. 21-46.

____________ (1997). "Asset Pricing with Distorted Beliefs: Are Equity Returns Too Good to be True." working paper, Ohio State Depart. of Economics, (January).

Christiano, L.J. (1989). "Solving the Stochastic Growth Model by Linear Quadratic Approximation and by Value-Function Iteration." *Journal of Business and Economic Statistics*, vol. 13, pp. 449-470.

Danthine, J.P. and Donaldson, J.B. (1995). "Labour Contracts, Operating Leverage, and Asset Pricing." working paper, Columbia University.

Danthine, J.P., Donaldson, J.B., and Mehra, R. (1989). "On Some Computational Aspects of Equilibrium Business Cycle Theory." *Journal of Economic Dynamics and Control*, vol. 13, pp. 449-470.

Donaldson, J. B. and Mehra, R. (1983). "Stochastic Growth with Correlated Production Shocks." *Journal of Economic Theory*, vol. 11, pp. 191-200.

Epstein L. G. and S. E. Zin (1989). "Substitution, Risk Aversion, and the Temporal Behavior of Consumption and Asset Returns: A Theoretical Framework," *Econometrica* 57,4, 937-969.

Evans, Martin, D. D. (1996). "Peso Problems: Their Theoretical and Empirical Implications." In *Handbook of Statistics 14: Statistical Methods in Finance*, ed. by G.S. Maddala and C.R Rao, North Holland: Amsterdam.

Goetzmann, W. and Jorion, P. (1997). "A Century of Global Stock Markets." *NBER Working Paper* #5901, (January).

Hansen, G. (1985). "Indivisible Labour and the Business Cycle." *Journal of Monetary Economics*, vol. 16, pp. 309-327.

Hansen, L., Sargent, T., and Tallarini T. (1999). "Robust Permanent Income and Pricing," *Carnegie-Mellon University Working Paper.*

Hamilton, J. D. (1989). "A New Approach to the Economic Analysis of Non Stationary Time Series and the Business Cycle." *Econometrica*, vol. 57, pp. 357-384.

Hodrick, R. J. and Prescott E. J., "Postwar United States Business Cycles: An Empirical Investigation." Discussion paper 451, Carnegie Mellon University

Kandel, S. and Stambaugh, R. (1990). "Expectations and the Volatility of Consumption and Asset Return." *Review of Financial Studies*, vol. 3, pp.207-232.

Kemeny, J.G. and Laurie Snell J. (1976). "Finite Markov Chains." New York Springer-Verlag.

Kocherlakota N. R. (1996). "The Equity Premium: It's Still a Puzzle, *Journal of Economic Literature*." Vol. 34, pp. 42-71.

Lucas, R. E. Jr. (1978). "Asset Prices in an Exchange Economy," *Econometrica*, vol. 66, pp. 1429-1445.

Mehra, R. and Prescott, E.C. (1985). "The Equity Premium: A Puzzle." *Journal of Monetary Economics*, vol. 15, pp. 145-161.

Rietz, T. (1988). "The Equity Premium: A Solution." *Journal of Monetary Economics*, vol. 22, pp. 117-131.

Shaked, M. and Shanthikumar, G.J. (1994). *Stochastic orders and Their Applications*. New York: Academic Press.

Tauchen, J.E. (1986). "Finite State Markov-Chain Approximations to Univariate and Vector Autoregressions." *Economics Letters*, vol. 20, pp. 177-181.

Appendix 1

A. Stationary probabilities

The stationary probability distribution (P_1 P_2, P_3), for respectively, states 1, 2, and 3 satisfies:

$$(P_1, P_2, P_3) \bullet M = (P_1, P_2, P_3)$$

Solving for the shock transition matrix given by

$$M = \begin{bmatrix} 0.97 & 0.03-\boldsymbol{h} & \boldsymbol{h} \\ 0.03-\boldsymbol{h} & 0.97 & \boldsymbol{h} \\ 0.5(1-\boldsymbol{f}) & 0.5(1-\boldsymbol{f}) & \boldsymbol{f} \end{bmatrix}$$

the implied system of equation yields $P_1 = P_2$; $P_3 = \frac{2\eta P_1}{1-\phi}$

For ϕ=0.2 and η=0.008, one thus obtain: P_1=0.495; P_2=0.495; P_3=0.0099, or approximately 99/10,000. For η=0.0008, P_3=0.00099 or 99/100,000 while P_3=0.000495 for η=0.0004 and ϕ=0.2. For the other values of ϕ and η used in the text, the following P_3 probabilities are obtained:

η= \ ϕ=	0	.1	.2
0.0004	0.000396	0.00044	0.000495
0.004	0.00396	0.0044	0.00495
0.008	0.00792	0.0088	0.0099
0.01	0.0099	0.011	0.0124
0.016	0.01584	0.0176	0.0198

B. Probability of 160 consecutive trials without the appearance of z_3

Let us define

$${}_3P^{n}_{j,k} = \text{Prob}(z_i \neq z_3,\ i=1,2,..,n-1,\ z_n = z_k \mid z_1 = z_j).$$

The probability that z_3 is not observed in 160 consecutive trials can then be expressed as

$\frac{1}{2}\left\{{}_3P^{160}_{1,1} + {}_3P^{160}_{1,2} + {}_3P^{160}_{2,1} + {}_3P^{160}_{2,2}\right\}$, assuming an equal probability of starting either from state 1 or state 2.

These probabilities are computed as follows. Let us define the submatrix $\hat{M}$ by

$$\hat{M} = \begin{bmatrix} 0.97 & 0.03-\boldsymbol{h} \\ 0.03-\boldsymbol{h} & 0.97 \end{bmatrix},$$

and let $\hat{M}^{160}$ correspond to the above matrix raised to the 160th power with entries $\hat{M}_{ij}^{160}$.
Then

$$ {}_3P_{1,1}^{160} = \hat{M}_{11}^{160}; {}_3P_{1,2}^{160} = \hat{M}_{12}^{160}; {}_3P_{2,1}^{160} = \hat{M}_{21}^{160}; {}_3P_{2,2}^{160} = \hat{M}_{22}^{160} $$

See Kemeny and Snell (1976).

In the case of the parameters underlying Table 1 (Panel 4)

$$ \hat{M} = \begin{bmatrix} 0.97 & 0.022 \\ 0.022 & 0.97 \end{bmatrix}, and\ \hat{M}^{160} = \begin{bmatrix} 0.1384 & 0.1382 \\ 0.1382 & 0.1384 \end{bmatrix}. $$

The desired probability is thus $\left(\frac{1}{2}\right)$ {0.1384 + 0.1382 + 0.1384 + .01382} = 0.277.

Appendix 2

This appendix presents and comments upon the macroeconomic implications of altering the parameter specification adopted for the benchmark case of Section 3. Table (6), Part 1 provides descriptive statistics for a wide range of η values. In every case the disaster state is anticipated but not observed in the time series from which the statistics were computed (same procedure as in Table (1), Panel 4). Focusing first on the detrended business cycle statistics (columns (a) and (b), we see that our original conclusion is sustained: the variations in the model's macro aggregates seem almost entirely unaffected by increases in η. Undetrended variation appears to be affected even less. Similar statements apply when we compare these results to the case of η=0 (Panel 2 of Table (1)). Notice that the precautionary effect is also evident: relative to the case of η=0.0004, investment and employment are higher when η=0.015. These latter effects, once again, are not large, but then neither is the stationary probability of disaster, even when η=0.015.

Part 2 of Table (6) undertakes the analogous comparison across various values of ϕ. In the right most Panel (3) (ϕ=0.9), the stationary probability of disaster is more substantial than in any of the prior cases and it is of particular interest to see what effects this has. At business cycle frequencies (column (a) and (b)) we see once again that the correlation structure is unaltered across all the cases. With the exception of consumption, all detrended macro series become progressively less variable as ϕ increase, with the effect becoming quite pronounced as ϕ=0.9 is achieved. The proportionate decline in variability in greatest for investment, followed by employment. The relative variation characteristic of the business cycle is largely maintained, however.

For all undetrended series (consumption included) variation declines as ϕ increases, with the effect being, again, especially significant when ϕ is large (high stationary probability of the disaster). There is also a simultaneous uniform increase in (columns (c)) the mean values of all the series. This latter data (Table 6, Part 2) provides a more dramatic illustration of the pseudo precautionary effect noted earlier.

On balance, these cases reinforce our earlier conclusions: the presence of an unrealised but anticipated disaster state does not materially alter the ability of this class of models to replicate the stylised facts of the business cycle for plausible parameterisations of the likelihood of disaster (ϕ=0.9 is probably not plausible). If we accept this family of models as a reasonable approximation to underlying processes generating real world data, we are forced to conclude that it is not possible to exclude the possibility of peso phenomena underlying actual reported statistics. Our results also suggest that the decline in savings experienced by the United States economy during the recent prolonged expansion may in part be interpreted as reflecting a reduced probability of disaster in the mind of the typical consumer – investor.

The remaining parameter which may have some effect is the parameter z_3 which measures the severity of the disaster (rather than the frequency as in the prior discussion). Here the effects are totally consistent with our earlier conclusions. Consider the results in Table (7) which considers a representative selection of z_3 values. The message is unambiguous: the severity of the anticipated disaster has little effect on the equilibrium time series either detrended or undetrended. This further reinforces our conclusion: for this class of models, peso considerations are not inconsistent with the observed equilibrium time series of macroaggregates.

Appendix 3

Proposition 3.1: Consider the model described by (P)-(M) and suppose the following additional assumptions are satisfied:

(i) $\phi=0$

(ii) Let $F_m(.,..)$ denote the conditional equilibrium price distribution for the risk free security under the shock transition matrix M, and let q_i^b, q_j^b. Then $F_M(q; q_j^b) \leq F_M(q; q_i^b)$, where we interpret this to mean that for any q, $\Pr(q_{t+1}^b \leq q; q_t^b = q_j^b) \leq \Pr(q_{t+1}^b \leq q; q_t = q_i^b)$.

Then there exists a disaster state z_3, sufficiently low in value, for which the stationary risk free asset price distribution under (P)-(M') first order stochastically dominates the distribution under (P)-(M).

Proof: Under the functional forms specified by (P)-(M), all the assumptions of Donaldson and Mehra (1983), section IV are satisfied. Thus the solution to (P)-(M) is a triple of continuous decision rules c(k,z), x(k,z) and n(k,z) which describe the economy's equilibrium consumption, investment and labour supply functions for each pair of state variables (k,z). These decision rules describe the evolution of the economy under (P)-(M′) as well. By the repeated application of this investment function x(k,z), in the context of the equation of motion on capital stock,

$$K_{t+1} = (1-\Omega)\, k_t + x(k_t, z_t)$$

there also exists a stationary probability distribution for the joint stochastic process on capital stock, shock pairs to which the economy converges.

We consider a discrete approximation to this continuous stationary distribution in order to correspond more directly to the results of the simulation.

By equation (10), to each capital stock-shock pair (k_i, z_j) in the stationary distribution there corresponds a unique risk free asset price $q^b(k_i, z_j)$. Let us denote by M_B, the probability transition matrix describing the evolution of the risk free asset price under (P)-(M) where we order the prices according to

$$q_1^b > q_2^b > \dots > q_n^b$$

Let $Q = \{q_1^b, q_2^b, \dots, q_n^b\}$. Select arbitrary $q_i^b, q_j^b \in Q$, where $q_i^b = q^b(k_i, z_i)$ and $q_j^b = q^b(k_j, z_j)$. Then

$$\Pi_{ij}^B = \Pr(q_{t+1}^b = q_j^b, q_t^b = q_i^b)$$

$$= \begin{cases} \Pi_{ij} & \text{if} \quad k_j = (1-\Omega)k_i + x(k_i, z_i) \\ 0 & \text{otherwise} \end{cases}$$

where $\Pi_{ij} = \Pr(z_{t+1} = z_j; z_t = z_i)$, $z_j, z_i \in \{z_1, z_2, z_3\}$.

<u>Step 1</u>: We first wish to show that there exists a z_3^* such that if $z_3 \le z_3^*$, $q^b(k_i, z_i) \le q^b(k_j, z_j)$ for all (k_i, z_i) where $z_i = z_3$ and all (k_j, z_j) where $z_j \ne z_3$.

Proof: Since $c(k,z) \le f(k,n(k,z))z$, we know there exists a $z_3^* (z_3^* = 0$ is admissible; by continuity a $z_3^* > 0$ may be chosen) such that if $z_3 \le z_3^*$,

$$\frac{c(k_j, z_3)}{c(k'(k_j, z_3), z_i)} < \frac{c(k_l, z_k)}{c(k'(k_l, z_k), z_n)}$$

for any k_j, k_l in the stationary distribution where $k'(k, z) = (1 - \Omega) k + x(k, z)$ and $z_k, z_i, z_n \in \{z_1, z_2\}$. It follows from the above inequality, since $\phi=0$, that $\beta \int \frac{c(k_j, z_3)}{c(k'(k_j, z_3), z_i)} dF_z(z_i; z_3)$ $< \beta \int \frac{c(k_l, z_k)}{c(k_l, z_k), z_n)} dF_2(z_n; z_k)$, for any k_j, k_l in the stationary distribution. Given that $U(c, 1-n) = \ell nc + A\, \ell n\, n$, we may conclude that $q^b(k_j, z_3) \le q^b(k_l, z_l)$ for any k_j, k_l in the stationary distribution provided $z_l \ne z_3$ as desired.

<u>Step 2</u>: Let F_M be as in the statement of the theorem and let F_M be the analogous conditional distribution under (P) – (M′). Then for any $q_i^b \in Q$,

$$F_{M'}(q; q_i^b) \le F_M(q; q_i^b).$$

This inequality follows from the following observation. Let $q_i^b = q^b(k_i, z_i)$. Under $F_M(q^b; q_i^b)$ there will be positive probability mass η for $q^b = q^b((1-\Omega)k_i + x(k_i,z_i),z_3)$; under $F_{M'}(q; q_i^b)$ this entry will be zero and the entry corresponding to z_1 or z_2 correspondingly greater (in probability). Since $q^b(k,z_3) < q^b(k,z_i)$, $z_i \in \{z_1,z_2\}$ for any k, $F_{M'}(q; q_i^b)$ has lower probability on the least possible (conditional) value of $q_{t+1}=q^b$ and higher probability elsewhere. This is sufficient for the result.

Step 3: conclude the proof. By assumption we know that $F_M(q; q_j^b) \le F_M(q; q_i^b)$ for all $q_j^b \le q_i^b$. Furthermore, by step 2

$$F_{M'}(q; q_j^b) \le F_M(q; q_j^b)$$

Thus, for all $q_j^b \le q_i^b$,

$$F_{M'}(q; q_j^b) \le F_M(q; q_i^b)$$

By theorem 4.B.16 in Shanked and Shanthikumar (1994), we can conclude that the stationary risk free asset price distribution under (P)-(M′) 1st order stochastically dominates the distribution under (P)-(M).

Table 1
Aggregate Statistics: Benchmark Model

	(1) U.S. Economy[i]		(2) Hansen′s Model[ii]				(3) Disaster State Anticipated and Present in the Sample[iii]				(4) Disaster State Anticipated but not Present in the Sample[iv]			
Series	(a)	(b)	(a)	(b)	(c)	(d)	(a)	(b)	(c)	(d)	(a)	(b)	(c)	(d)
Output	1.76	1.00	1.71	1.00	1.101	0.040	5.74	1.00	1.12	0.065	1.74	1.00	1.12	0.042
											(0.45)	(0)	(0.020)	(0.008)
Consumption	1.29	0.85	0.52	0.86	0.824	0.019	3.08	0.94	0.829	0.031	.51	0.85	0.832	0.019
											(0.13)	(0.175)	(0.013)	(0.005)
Investment	8.60	0.92	5.50	0.99	0.277	0.026	51.80	0.98	0.287	0.037	5.56	0.99	0.287	0.028
											(1.45)	(0.022)	(0.0075)	(0.0053)
Capital Stock	0.63	0.04	0.47	0.07	11.11	0.310	0.54	-.11	11.47	0.360	0.47	0.06	11.50	0.353
											(0.13)	(0.089)	(0.254)	(0.094)
Employment	1.66	0.76	1.27	0.98	0.300	0.006	2.74	0.98	0.302	0.010	1.32	0.98	0.302	0.007
											(0.34)	(0.024)	(0.0011)	(0.001)

(a) – standard deviation in percent, H-P filtered data
(b) – correlation with output, H-P filtered data
(c) – mean values, raw data, undetrended
(d) – standard deviation, raw data undetrended

(i) Source: Hansen (1985), Table (1); quarterly data detrended using the H-P filter (Hodrick-Prescott (1980))
(ii) Hansen′s model corresponds to the case where z_1=1.021, z_2=0.979, z_3=0, η=0; statistics computed from artificial data.
(iii) This full sample case assumes z_1=1.021, z_2=0.979, z_3=0.6, η=.008, ϕ=0.20. Statistics are computed from 500,001 data points in which z_3 was observed with frequency consistent with M.
(iv) Statistics computed for the same parameter set and decision rules as in Panel (3) but for 200 runs of 160 consecutive data points in which z_3 was not present; standard deviations of estimates in parentheses.

Table 2
Select Financial Statistics

	(1) U.S. Economy		(2) Hansen Model		(3) Benchmark Model z_1=1.021, z_2=0.979, z_3=0.6, η=0.008, ϕ=0.2, γ=0, except in (ii)					
					(i) Disaster State Anticipated and Present in Sample		(ii) Disaster State Anticipated and Present in Sample, γ=3		(iii) Disaster State Anticipated But not Present in Sample	
Series	(a)	(b)	(a)	(b)	(a)	(b)	(a)	(b)	(a)	(b)
r^e	6.98	16.54	4.11	0.53	4.22	3.20	4.29	4.14	4.09 (0.0944)	0.44 (0.0795)
r^b	0.80	5.67	4.11	0.35	3.87	6.48	3.73	8.89	2.46 (0.1215)	0.37 (0.0646)
$r^p = r^e - r^b$	6.18	16.76	0.0006	0.30	0.35	6.48	0.56	8.58	1.62 (0.044)	0.15 (0.031)
Dividend growth	1.98	13.11	0	8.81	0	10.52	0	10.52	0 (0.53)	9.63 (2.70)
Consumption growth		1.87	0	0.76	0	2.33	0	1.09	0 (0.1048)	0.75 (0.16)

(a) mean values
(b) standard deviation (raw data).
Standard deviations of estimates in parentheses

Table 3
Select Financial Statistics
Various γ, B Combinations
For all Cases, z_1=1.021, z_2=0.979, z_3=0.6, η=0.008, ϕ=0.20
z_3 Anticipated But Not Present in the Sample

	(1) γ=1, B=0 (D/E=0)		(2) γ=3, B=0 (D/E=0)		(3) γ=1, B=2 (D/E=0.21)		(4) γ=3, B=2 (D/E=0.18)		(5) γ=3, B=4 (D/E=0.36)	
Series	(a)	(b)	(a)	(b)	(a)	(b)	(a)	(b)	(a)	(b)
$\underline{r}^e$	4.09	0.44	4.02	.42	4.56	0.45	4.65	0.42	5.68	0.43
	(0.0944)	(0.0795)	(0.1285)	(0.0696)	(0.098)	(0.0811)	(0.1332)	(0.0697)	(0.1494)	(0.0720)
$\underline{r}^b$	2.46	0.37	1.80	0.33	2.46	0.37	1.80	0.33	1.80	0.33
	(0.1215)	(0.0646)	(0.1743)	(0.0584)	(0.1215)	(0.0646)	(0.1743)	(0.0584)	(0.1743)	(0.0584)
$\underline{r}^p = \underline{r}^e - \underline{r}^b$	1.62	0.15	2.22	0.21	2.10	0.16	2.85	0.20	3.88	0.20
	(0.044)	(0.031)	(0.0575)	(0.0380)	(0.0418)	(0.0349)	(0.0536)	(0.0388)	(0.0407)	(0.0427)
Dividend Growth	0	9.63	0	7.76	0	12.28	0	9.10	0	10.74
	(0.53)	(2.70)	(0.4228)	(2.136)	(0.7088)	(3.5593)	(0.4978)	(2.53)	(0.600)	(3.037)
Consumption Growth	0	0.75	0	0.34	0	0.75	0	0.34	0	0.34
	(0.1048)	(0.16)	(0.0404)	(0.0744)	(0.1048)	(0.1563)	(0.0404)	(0.0744)	(0.0404)	(0.0744)

(a) mean values
(b) standard deviation (raw data).
Standard deviations of estimates in parentheses

Table 4
Matching Macroaggregate and Financial Moments
Disaster Anticipated But Not Realized
z_1=1.028, z_2=0.972, z_3=0.5, ϕ=0, γ=3, η=0.008, B=2.5 (D/E=0.31)

Series	(a)	(b)	Series	(c)	(d)
Output	1.70 (0.45)	1.00 (0)	r^e	6.53 (0.24)	0.48 (0.09)
Consumption	.33 (0.08)	0.89 (0.209)	r^b	0.33 (0.24)	0.44 (0.08)
Investment	5.78 (1.54)	0.99 (0.141)	$r^p = r^e - r^b$	6.20 (0.025)	0.23 (0.087)
Capital Stock	0.50 (0.14)	0.000 (0.096)			
Employment	0.90 (0.25)	0.89 (0.136)			

(a) Standard deviations, in percent, H-P filtered data (b) Correlations with output, H-P filtered data
(c) Mean values in percent (d) Standard deviation in percent
Standard deviations of estimates in parentheses

Table 5
Welfare Comparisons
Disaster State Expected or Not

	M′: η=0.008 φ=0; z_3=0.5	M′: η=0.016 φ=0.2; z_3=0.6	M′: η=0.016 φ=0.2; z_3=0.5
$\bar{c}_{M'}$	.8254234	.8254234	0.8254234
$c^{*}_{M,nd}$	.8254173	.8254104	0.8254114
$EU(c_t, 1-n_t)$	-1.044579	-1.044083	-1.04387
ξ	-0.007019	-0.007436	-0.007616
ξ/Ec_t	-0.84%	-0.89%	-0.91%

Table 6[(i)]
Comparative Dynamics: z_3 Anticipated but Not Realized
z_1=1.021, z_2=0.979, z_3=0.6
Part 1
Changes in η
ϕ=0.2

	(1) η=0.0004				(2) η=0.008				(3) η=0.015			
Series	(a)	(b)	(c)	(d)	(a)	(b)	(c)	(d)	(a)	(b)	(c)	(d)
Output	1.71	1.00	1.105	0.041	1.74	1.00	1.12	0.042	1.69	1.00	1.12	0.041
	(0.44)	(0)	(0.020)	(0.008)	(0.45)	(0)	(0.020)	(0.008)	(0.44)	(0)	(0.019)	(0.008)
Consumption	0.52	0.85	0.825	0.019	0.51	0.85	0.832	0.019	0.52	0.85	0.833	0.019
	(0.13)	(0.173)	(0.013)	(0.005)	(0.13)	(0.175)	(0.013)	(0.005)	(0.13)	(0.194)	(0.013)	(0.005)
Investment	5.54	0.99	0.279	0.027	5.56	0.99	0.287	0.028	5.33	0.98	0.288	0.027
	(1.42)	(0.024)	(0.0071)	(0.0050)	(1.45)	(0.022)	(0.0075)	(0.0053)	(1.39)	(0.141)	(0.007)	(0.0051)
Capital Stock	0.47	0.07	11.170	0.332	0.47	0.06	11.50	0.353	0.45	0.07	11.54	0.334
	(0.13)	(0.068)	(0.240)	(0.088)	(0.13)	(0.089)	(0.254)	(0.094)	(0.13)	(0.093)	(0.235)	(0.088)
Employment	1.29	0.98	0.301	0.0065	1.32	0.98	0.302	0.007	1.26	0.97	0.302	0.0064
	(0.33)	(0.025)	(0.0010)	(0.001)	(0.34)	(0.024)	(0.0011)	(0.001)	(0.33)	(0.140)	(0.001)	(0.001)
Stationary Probability	0				0.0099						0.0184	

[(i)](a) (b) (c) (d) – same interpretations as in Table 1; standard deviations of estimates in parentheses

Part 2[i]

Changes in ϕ

η=0.008

	(1) ϕ=0				(2) ϕ=0.4				(3) ϕ=0.9			
Series	(a)	(b)	(c)	(d)	(a)	(b)	(c)	(d)	(a)	(b)	(c)	(d)
Output	1.74	1.00	1.119	0.042	1.71	1.00	1.12	0.041	1.64	1.00	1.19	0.040
	(0.45)	(0)	(0.020)	(0.008)	(0.45)	(0)	(0.019)	(0.008)	(0.42)	(0)	(0.018)	(0.007)
Consumption	0.51	0.85	0.832	0.019	0.52	0.85	0.833	0.019	0.55	0.87	0.864	0.019
	(0.13)	(0.194)	(0.013)	(0.005)	(0.13)	(0.184)	(0.013)	(0.005)	(0.14)	(0.189)	(0.012)	(0.004)
Investment	5.59	0.98	0.287	0.028	5.41	0.98	0.288	0.027	4.76	0.98	0.327	0.026
	(1.45)	(0.141)	(0.008)	(0.005)	(1.43)	(0.142)	(0.007)	(0.005)	(1.23)	(0.142)	(0.006)	(0.005)
Capital Stock	0.48	0.07	11.490	0.355	0.46	0.07	11.54	0.340	0.40	0.07	13.087	0.308
	(0.13)	(0.094)	(0.255)	(0.094)	(0.13)	(0.092)	(0.236)	(0.089)	(0.11)	(0.093)	(0.212)	(0.079)
Employment	1.32	0.97	0.302	0.007	1.28	0.97	0.302	0.0065	1.18	0.97	0.309	0.006
	(0.35)	(0.141)	(0.001)	(0.001)	(0.34)	(0.141)	(0.001)	(0.001)	(0.30)	(0.141)	(0.001)	(0.001)
Stationary Probability	0				0.013				0.0741			

[i] (a) (b) (c) (d) – same interpretations as in Table 1; standard deviations of estimates in parentheses

Table 7[i]

Changes in z_3: Disaster Anticipated But Not Realized

$\phi=0.2$, $\eta=0.008$, $z_1=1.201$, $z_2=0.979$

	(1) $z_3=0.6$				(2) $z_3=0.5$				(3) $z_3=0.4$			
Series	(a)	(b)	(c)	(d)	(a)	(b)	(c)	(d)	(a)	(b)	(c)	(d)
Output	1.74	1.00	1.12	0.042	1.74	1.00	1.12	0.042	1.73	1.00	1.12	0.042
	(0.45)	(0)	(0.020)	(0.008)	(0.45)	(0)	(0.020)	(0.008)	(0.45)	(0)	0(.020)	(0.008)
Consumption	0.51	0.85	0.832	0.019	0.51	0.84	0.832	0.019	0.51	0.83	0.832	0.019
	(0.13)	(0.175)	(0.013)	(0.005)	(0.13)	(0.197)	(0.013)	(0.005)	(0.13)	(0.197)	(0.013)	(0.005)
Investment	5.56	0.99	0.287	0.028	5.61	0.98	0.287	0.028	5.56	0.98	0.287	0.028
	(1.45)	(0.022)	(0.0075)	(0.0053)	(1.46)	(0.141)	(0.0075)	(0.0053)	(1.47)	(0.141)	(0.0075)	(0.0053)
Capital Stock	0.47	0.06	11.50	0.353	0.48	0.07	11.50	0.354	0.48	0.07	11.50	0.360
	(0.13)	(0.089)	(0.254)	(0.094)	(0.14)	(0.093)	(0.255)	(0.094)	(0.14)	(0.093)	(0.256)	(0.095)
Employment	1.32	0.98	0.302	0.007	1.33	0.97	0.302	0.007	1.32	0.98	0.302	0.007
	(0.34)	(0.024)	(0.0011)	(0.001)	(0.35)	(0.140)	(0.0011)	(0.0013)	(0.35)	(0.140)	(0.0011)	(0.0013)

[i] (a), (b), (c), (d) as in Table 1. Standard deviations of estimates in parentheses.