

On the Consequences of State Dependent Preferences for the Pricing of Financial Assets

by

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Abstract

This paper introduces state dependent utility into the standard Mehra and Prescott (1985) economy by allowing the representative agent's coefficient of relative risk aversion to vary with the underlying economy's growth rate. Existence of equilibrium is proved and its asymptotic properties analyzed. This generalization leads to level dependent marginal rates of substitution, a property that sharply distinguishes this model from the standard construct. For very low coefficients of relative risk aversion, the equilibrium risk free and risky security returns are demonstrated to have volatilities and an associated equity premium that substantially exceed what is found in the data. This provides a contrasting perspective on the classic "equity premium puzzle."

Keywords: state dependent utility, equity premium, equity premium puzzle

JEL Classification Numbers: D91, E21, G00, G12

1. Introduction

This paper explores the implications for asset pricing of allowing the representative agent's coefficient of relative risk aversion to vary with the economy's growth rate of consumption. This variation is modeled very differently than in the habit formation literature¹ (see, e.g., Constantinides (1986) and, especially, Campbell and Cochrane (1999)), in a manner that allows us to postulate almost trivially low coefficients of relative risk aversion (CRRA).

The consequences of allowing risk aversion to be state dependent in the manner we propose are extreme. They arise from the fact that agents' demand for securities will depend not only upon the growth rate of consumption (as in the canonical case, cf. Mehra and Prescott (1985)) but upon its level as well. The end-effect is to introduce another source of variation to the pricing kernel, one that is unrelated to the volatility of the underlying fundamentals and whose importance grows disproportionately as the economy itself is growing.

The implications of this feature are such as to alter dramatically the form of the classic asset pricing puzzles: for standard parameterizations, the equity premium is easily matched or exceeded, the risk free rate is asymptotically too low, the Hansen-Jagannathan bounds are easily satisfied, and the standard deviations of equity and risk free returns are too large relative to the data. At the minimum, the articulation of this property should serve as a warning signal to those relying on state dependent preferences for solving some outstanding asset pricing puzzles. It also allows us to identify a mechanism partially underlying the results in Falato (2003), Melino and Yang (2003), and others.

¹ Myerson (1991) provides an axiomatic base for the precise form of state dependent utility considered here.

General support for the hypothesis that risk aversion is state dependent may be found in recent results from the experimental psychology and economics literatures. Isen and Patrick (1983), Isen and Geva (1987) and Nygren et al. (1996) present evidence suggesting that happy decision makers – those who have received a consumption increase – are much less willing to gamble than control groups. Isen (1996) interprets these results as suggesting that persons in a “good mood” are more reluctant to gamble because losing might undermine their good mood. Bosh-Domènech and Silvestre (1999) report the results of an experiment in which the subjects were given title to a random payout of money and were asked if they wished to insure against a 20% chance of having their personal monetary realization taken from them. Half of the subjects chose to insure, but only if their income realization fell within the high level category, a response that associates greater risk aversion with higher income levels. These results are consistent with the postulate of associating higher risk aversion with greater consumption growth and higher consumption levels. Broadly speaking, it is the perspective that risk aversion is procyclical.

Strong empirical evidence for countercyclical risk aversion is provided by Gordon and St-Amour (2002) who postulate a model with time varying risk aversion similar to the one to be considered here, and estimate the implied process on risk aversion arising from per capita consumption and financial return data. Their basic finding is that risk aversion is strongly countercyclical, rising during recessions and falling during expansions. In addition, the Gordon and St-Amour (2002) CRRA estimate moves opposite to the University of Michigan index of consumer confidence, a fact that is also consistent with countercyclical risk aversion. While we adopt the countercyclical perspective as our benchmark, our results are broadly independent of the specific pattern of risk aversion variation.

An outline of the paper is as follows: Section 2 presents the model and a number of simple analytical results. Section 3 provides a full overview and interpretation of a numerical analysis of the model, and explores the consequences of admitting a more general stochastic structure. A comparison with other models in the literature is provided in Section 4 while Section 5 concludes the paper.

2. The Model

2.1 The Model

Ours is a Lucas (1978) style model modified to allow for state dependent preferences. The infinitely lived representative agent's period utility function over consumption, c_t , is of the form $u(c_t, \alpha_t) = \frac{c_t^{1-\alpha_t}}{1-\alpha_t}$ where the CRRA, α_t , is presumed to vary stochastically through time.

The representative agent's problem is

$$(2.1) \quad \begin{aligned} & \text{Max}_{\{z_{t+1}\}} E_0 \left\{ \sum_{t=0}^{\infty} \beta^t u(c_t, \alpha_t) \right\} \\ & \text{s.t.} \\ & c_t + z_{t+1}p_t \leq z_t(p_t + y_t) \\ & 0 \leq z_t \leq 1 \\ & (y_{t+1}, \alpha_{t+1}) \sim dF(y_{t+1}, \alpha_{t+1}; y_t, \alpha_t) \end{aligned}$$

taking the period t equity claims price $p_t = p(y_t, \alpha_t)$ as given, by choosing the fraction $z_{t+1} = z(y_t, \alpha_t)$ of the perfectly divisible share held he wishes to hold. The state of this economy may therefore be viewed as the ordered pair (y_t, α_t) .

The equilibrium price function $p(y_t, \alpha_t)$ thus solves:

$$(2.2) \quad u_1(y_t, \alpha_t) p(y_t, \alpha_t) = \beta \int u_1(y_{t+1}, \alpha_{t+1}) [p(y_{t+1}, \alpha_{t+1}) + y_{t+1}] dF(y_{t+1}, \alpha_{t+1}; y_t, \alpha_t)$$

where we have substituted the market clearing conditions $c_t = y_t$ and $z_t = 1$ into the representative agent's necessary and sufficient first order condition.² In a like fashion, the period t price of a risk free asset, paying one unit of the output good in every state next period and assumed to be in zero net supply, is given by

$$(2.3) \quad q(y_t, \alpha_t) = \beta E_t \left\{ \frac{u_1(y_{t+1}, \alpha_{t+1})}{u_1(y_t, \alpha_t)} \right\}.$$

The conditional period rates of return for these securities, denoted respectively by $r^e(y_{t+1}, \alpha_{t+1}; y_t, \alpha_t)$ and $r^f(y_t, \alpha_t)$, are thus:

$$r^e(y_{t+1}, \alpha_{t+1}; y_t, \alpha_t) = \frac{p(y_{t+1}, \alpha_{t+1}) + y_{t+1}}{p(y_t, \alpha_t)} - 1, \text{ and}$$

$$r^f(y_t, \alpha_t) = \frac{1}{q(y_t, \alpha_t)} - 1.$$

Under the utility specification considered here, the representative agent's equilibrium intertemporal marginal rate of substitution (IMRS) assumes the form

$$(2.4) \quad \text{IMRS}_{t,t+1} = \beta \frac{u_1(y_{t+1}, \alpha_{t+1})}{u_1(y_t, \alpha_t)} = \beta \frac{(y_t)^{\alpha_t}}{(y_{t+1})^{\alpha_{t+1}}} = \beta \frac{(y_t)^{\alpha_t - \alpha_{t+1}}}{(x_{t+1})^{\alpha_{t+1}}},$$

where x_{t+1} is defined as the growth rate of output:

$$x_{t+1} = y_{t+1} / y_t$$

Two features stand out relative to the standard formulation:

(i) the $\text{IMRS}_{t,t+1}$ depends on the output level y_t itself and not only on the (stationary on a balanced growth path) growth rate of output and

(ii) there is an added source of volatility, α_t , which is fundamentally different from the customary pure consumption uncertainty. With variation in α_t , the agent's period marginal utility varies even if there is no uncertainty in his consumption growth rate.

To distinguish this latter source of risk from the variation in utility arising from consumption uncertainty alone, it will be referred to as “mood” or “outlook” uncertainty in recognition of the observation that an individual's mood can substantially affect his assessment of his objective circumstances³. Adding this feature to an otherwise parsimonious-in-the-extreme model makes explicit, in one particularly simple way, the assertion that such mood swings can potentially influence an individual's economic behavior. Note also that these mood swings are fully anticipated by the agent and thus may be fully hedged. The results to follow are thus unrelated to any aspect of market incompleteness.

It is the associated level effect, however, that gives these mood swings potency for asset pricing. From (2.4) we see that as y_t increases the standard deviation of the $IMRS_{t, t+1}$ will similarly increase as $\alpha_t - \alpha_{t+1}$ is stochastically negative and then positive. As a result, the agent will increasingly desire to smooth his consumption without in fact being able to do so. Anticipating somewhat the results to follow this effect gives rise asymptotically to very high equilibrium risk free asset prices (and thus low risk free rates) and to very low equity prices (and, simultaneously, high equity returns). A high equity premium follows accordingly.⁴

² For the specialized functional forms specified later in the paper, we prove constructively that an equilibrium exists. See Danthine et al. (2001).

³ We borrow this terminology from Mehra and Sah (2002) who consider related issues but in a non-rational expectations framework.

⁴ The period utility function $u(c) = \frac{c^{1-\tilde{\alpha}}}{1-\tilde{\alpha}}$ is a special case of the specification $u(c) = \tilde{\theta} c^{1-\tilde{\alpha}}$, with $\tilde{\theta}$ a bounded random variable with $\tilde{\theta} < 0$ for $\tilde{\alpha} > 1$. The dramatic results we will detail are largely unrelated to variation in the implicit multiplicative factor $\tilde{\theta}$. This may be argued from two perspectives:

In order to bring out most clearly the intuition of the prior paragraph, we will, in the rest of this paper, specialize the production side of the economy as follows:

(i) $y_{t+1} = \tilde{x}_{t+1}y_t$, where the stochastic growth rate \tilde{x}_t is governed by a Markov chain probability transition matrix with $\pi_{11} = \pi_{22} = \pi$, where π_{ij} is the (time invariant) probability of passing from growth state i to growth state j . This is the Mehra and Prescott (1985) framework. With this specialization, the state vector becomes the ordered triple (y_t, x_t, α_t) . In order to illustrate the case of countercyclical risk aversion we further assume:

(ii) \tilde{x}_t and $\tilde{\alpha}_t$ are perfectly negatively correlated. This means that only state vectors of the form (y_t, x_1, α_1) and (y_t, x_2, α_2) are observed where $x_1 > x_2$ and $\alpha_2 > \alpha_1$.

2.2 Asymptotic Properties of the Risk Free Security Price

In the calibration to follow, we will need to choose reasonable values of α_1 and α_2 and the risk free security price will be seen to yield clues as to what they should be. For this reason it is useful to explore the asymptotic behavior of the risk free security price as output grows without bound through time.

Under the above restrictions, for any output level y_t , the prices of the risk free asset in the high and low growth states are, respectively

(1) If the results are solely attributable to the multiplicative factor $\tilde{\theta}$, then the specification $u(c) = -c^{1-\alpha}$ should rule them out. But it does not: the standard deviation of the IMRS $_{t,t+1}$ continues to grow without bound as y_t increases.

(2) The other extreme assumes that $u(c) = \frac{\tilde{\theta}c^{1-\alpha}}{1-\alpha}$: no variation in the CRRA, but variation in the multiplicative factor. This formulation is formally equivalent to the standard Mehra and Prescott (1985) construct where the discount factor is stochastic, a model with no unusual asset pricing features per se. The SD of the pricing kernel does not grow with output. We qualify this comment by noting that in a model with Epstein-Zinn (1991)/Weil (1989) preferences where the CRRA and the elasticity of intertemporal substitution (EIS) can be specified independently, Melino and Yang (2001) find that state dependent time preferences can have substantial effects, when employed in conjunction with a variable CRRA.

$$(2.5) \quad q(y_t, x_1) = \beta \left\{ \pi \frac{(x_1 y_t)^{-\alpha_1}}{(y_t)^{-\alpha_1}} + (1 - \pi) \frac{(x_2 y_t)^{-\alpha_2}}{(y_t)^{-\alpha_1}} \right\} = \beta \left\{ \pi (x_1)^{-\alpha_1} + (1 - \pi) x_2^{-\alpha_2} (y_t)^{\alpha_1 - \alpha_2} \right\},$$

$$(2.6) \quad q(y_t, x_2) = \beta \left\{ \pi (x_2)^{-\alpha_2} + (1 - \pi) x_1^{-\alpha_1} (y_t)^{\alpha_2 - \alpha_1} \right\}.$$

Since $\alpha_1 - \alpha_2 < 0$, $q(y_t, x_1) \mapsto \beta \pi (x_1)^{-\alpha_1}$ as $y_t \mapsto \infty$, and $r^f(y_t, x_1) \mapsto \frac{1}{\beta \pi (x_1)^{-\alpha_1}} - 1$, finite.

For the low growth state, however, $q(y_t, x_2) \mapsto \infty$ as $y_t \mapsto \infty$ because $\alpha_2 - \alpha_1 > 0$. As a result,

$r^f(y_t, x_2) \mapsto -1$. With each state having equal asymptotic likelihood, the unconditional average risk free rate Er^f satisfies,

$$(2.7) \quad Er^f \mapsto \frac{1}{2} \left[\frac{1}{\beta \pi (x_1)^{-\alpha_1}} \right] - 1, \text{ as } y_t \mapsto \infty.$$

Furthermore, it is monotonically decreasing with time.

In a like fashion the unconditional asymptotic standard deviation satisfies

$$(2.8) \quad SDr^f \mapsto \frac{1}{8} \left[\frac{(x_1)^{2\alpha_1}}{\beta^2 \pi^2} \right]; \text{ it is monotonically increasing.}$$

Notice that, in expressions (2.7) and (2.8), only the high-growth-low-risk-aversion values are represented. Otherwise, the intuition encapsulated therein is pretty much standard: agents with higher subjective discount factors β bid up security prices thereby lowering the risk free rate. In a like fashion, a higher π suggests greater risk: consumption growth rates are either highly persistently favorable or unfavorable; risk averse agents value risk free assets more highly in these circumstances similarly bidding up prices. Somewhat counterintuitively, as the agent becomes more risk averse in the high growth state (larger α_1), asymptotic risk free security prices in that same state decline, a fact that leads to a higher asymptotic standard deviation.

We also note that these results do not depend on the assumption of greater risk aversion specifically in the low growth state. If the converse were true; that is, if states one and two change roles so that $\alpha_1 > \alpha_2$, a scenario more in harmony with the experimental psychology literature, then $q(y_t, x_2) \mapsto \beta\pi(x_2)^{-\alpha_2}$ and $q(y_t, x_1) \mapsto \infty$. The asymptotic formulae for Er^f and SDr^f are altered accordingly (in particular, $Er^f \mapsto \frac{1}{2} \left[\frac{1}{\beta\pi\alpha_1(x_1)^{-\alpha_1}} \right] - 1$ as $y_t \mapsto \infty$), but their essential form remains the same. Which state displays the greater risk aversion is thus not significant for the results to follow.

Recognizing that the price of the risk free asset effectively identifies the state contingent MRS, the remarks above have the implication that the expected MRS effectively becomes asymptotically unbounded, and it is this fundamental feature that drives many of the results.

2.3 Calibration: Benchmark Formulation

To the extent possible, we initially rely on the original Mehra and Prescott (1985) calibration. In particular, Mehra and Prescott (1985) choose $\beta = .96$, $\mu = .018$, $\delta = .036$, and $\pi = .43$, where $x_1 = 1 + \mu + \delta$, $x_2 = 1 + \mu - \delta$; it remains first to specify α_1 and α_2 . As mentioned earlier, we look to the risk free security price to provide clues.

Since $x_1 > 1$, notice that (2.7) implies that the risk free rate will be made lower if α_1 is close to one.⁵ It is also reasonable to hypothesize that, at a minimum, the representative agent would only be willing to pay a smaller and smaller fraction of his income to purchase one unit of

⁵ It is possible, of course, to choose an α_1 so as to match exactly the empirically observed average real risk free rate of .8% (cf. Mehra and Prescott (1985)). For $\beta=.96$, $x_1=1.056$, $\pi=.43$, working backwards through (2.7) gives an $\alpha_1=15.96$; the agent must be dramatically risk loving in his high growth state. While this result is perhaps not objectionable qualitatively, the magnitude appears too large to be reasonable, a fact that is itself directly attributable to the assumed low persistence in the output growth rate (.43). High persistence ($\pi=.95$) yields a more conventional

the risk free security as his income grows arbitrarily large. Such a security, after all, represents a claim on only one unit of additional consumption. To summarize, it is reasonable to require

that $\frac{q(y_t, x_i)}{y_t} \mapsto 0$ as $y_t \mapsto \infty$ for any $x_i \in \{x_1, x_2\}$.

In the case of $q(y_t, x_1)$, such a requirement is automatically satisfied. In the case of $q(y_t, x_2)$,

$$\frac{q(y_t, x_2)}{y_t} = \beta \frac{\{\pi(x_2)^{-\alpha_2} + (1-\pi)(x_1)^{-\alpha_1} (y_t)^{\alpha_2 - \alpha_1}\}}{y_t} = \beta \left\{ \frac{\pi(x_2)^{-\alpha_2}}{y_t} + (1-\pi)(x_1)^{-\alpha_1} (y_t)^{\alpha_2 - \alpha_1 - 1} \right\}$$

Provided $\alpha_2 - \alpha_1 < 1$, $\frac{q(y_t, x_2)}{y_t} \mapsto 0$, as $y_t \mapsto \infty$ as well. Our restrictions on α_1 and α_2 are thus

that $\alpha_1 \approx 1$ and $\alpha_2 - \alpha_1 < 1$.⁶ Both restrictions are convenient, especially the latter one, because it suggests that our subsequent results do not require dramatic changes in the CRRA, a property that would challenge our sense of what is reasonable.

These calculations also remind us that the financial return statistics for this model will be sensitive to the magnitude of output, and thus to the length of the time series we use as the basis of our statistical computations. Denoting this time series length by the variable “S”, we will choose for $S = 120$, in the case of $\beta = .96$ where the length of the period is interpreted to be one year. In the case of $\beta = .99$ (period is one quarter), we choose $S = 400$. Both of these parameterizations are roughly in conformity with the maximum length of available data sets. All reported statistics are averages of the indicated quantities computed for one thousand individual time series.

$\alpha_1 = 1.4$ under the same calculation. As with most asset pricing models, the one considered here is thus likely to yield a counterfactually high Er_f .

It remains only to fix the initial output level y_0 , a choice which is in some sense “dual” to the choice of the series length with a choice of $S=100$. For all reported cases we choose $y_0=1$

because this choice yields a $E\left(\frac{p}{y}\right)$ which is very similar to its constant CRRA counterpart

$\alpha_t \equiv \alpha = \frac{\alpha_1 + \alpha_2}{2}$.⁷ While the $E\left(\frac{p}{y}\right)$ ratio has limited empirical significance in this model, it

does constitute a natural statistic around which to standardize.⁸

3. Numerical Results

Table (1) below presents a benchmark set of cases. The first and second moments of all the series as well as their basic correlation structure are reported. In reference to the Hansen-Jagannathan bound, the ratio $SD(IMRS)/E(IMRS)$ is also reported; if the model is to explain the most basic statistical characteristics of the equity premium this ratio must exceed the value

$$\frac{E r^p}{\sigma_{r^p}} = .37 \text{ for the U.S. economy.}$$

⁶ This choice is also broadly consistent with the estimate in Gordon and St-Amour (2002). Their estimates for risk aversion are centered at .25.

⁷ There is another sense in which $y_0=1$ is a natural choice of starting point. Note from (2.6) that the SD of the $IMRS_{t,t+1}$ will increase without bound both as $y_t \rightarrow \infty$ and as $y_t \rightarrow 0$. Our results thus all apply to an endlessly shrinking economy as well. These dual asymptotic results suggest $y_0=1$ since departures (in either direction) give rise to the phenomena reported here.

⁸ By “limited empirical significance” we mean that this ratio corresponds to the (value of equity)/GDP ratio only for an economy where all the income is capital income. If we modify the model to accommodate a situation in which only a fraction of the income stream is priced (output less wage income), the resulting $E\left(\frac{p}{y}\right)$ and $SD\left(\frac{p}{y}\right)$ are much lower for the same series length. Mehra (1998) demonstrates that a properly calibrated model can account neither for the range nor the variation in the $\left(\frac{p}{y}\right)$ ratio. His CRRA is constant, however.

Table (1)
Summary Return Statistics: Representative Cases
(Unless Otherwise Indicated, $\beta = .96$, $\pi = .43$, $\mu = .018$, $\delta = .036$, $S = 120$, $y_0=1$)
All Returns Expressed in Percent^{(i), (ii)}

	A U.S. Data	B Mehra- Prescott (1985) $\alpha_1=\alpha_2=3$	C $\alpha_1 = 1.0$ $\alpha_2 = 1.5$	D $\alpha_1 = .5$ $\alpha_2 = 1.0$	E $\alpha_1 = 4.0$ $\alpha_2 = 4.5$	F $\alpha_1 = 1.5$ $\alpha_2 = 1.0$	G $\alpha_1 = 1.0$ $\alpha_2 = 1.5$ $\delta=0$
$E r^e$	6.98	9.58	16.95	16.14	21.5	19.56	18.77
$S.D.r^e$	16.54	4.99	53.04	53.17	51.94	59.81	58.36
$E r^f$.80	9.10	8.47	7.19	15.74	6.84	7.79
$S.D.r^f$	5.67	1.61	34.21	33.90	35.78	34.61	35.75
$E r^p$	6.18	.48	8.48	8.94	5.76	12.72	10.98
$S.D.r^p$	16.67	4.70	34.92	35.26	32.75	43.21	40.32
$\text{Corr}(r_t^e, r_{t-1}^e)$	-.03	-.33	-.47	-.47	-.47	-.48	-.48
$\text{Corr}(r_t^f, r_{t-1}^f)$.87	-.15	-.15	-.15	-.14	-.15	-.15
$\text{Corr}(r_t^e, r_t^f)$	-.09	.34	.77	.77	.79	.70	.73
$SD(IMRS)/E(IMRS)$.11	.44	.46	.39	.51	.49
$E\left(\frac{p}{y}\right) / E\left(\frac{p}{y}; \alpha = \frac{\alpha_1 + \alpha_2}{2}\right)$		1	1.05	1.16	1.08	1.14	1.13

- (i) Data Sources: Mehra and Prescott (1985), Mehra (1998), and Jones (2001); the correlations are based on data for the period 1897-1998, where r^f is the real commercial paper rate and r^e is obtained from Cowles Foundation and CRISP data.
- (ii) Panel B reports results from the original Mehra and Prescott (1985) model when the CRRA is fixed at $\alpha = 3$

Lastly, the ratio

$$E\left(\frac{p}{y}; \alpha_1 \neq \alpha_2\right) / E\left(\frac{p}{y}; \alpha_1 = \frac{\alpha_1 + \alpha_2}{2}\right)$$

is reported in order to give some sense of the relative magnitude of the price-output ratio vis-à-vis its value in a corresponding pure Mehra and Prescott (1985) economy. For comparison purposes, Panel A provides the by-now-familiar summary financial statistics for the U.S.

economy while Panel B provides the corresponding Mehra and Prescott (1985) results for a representative choice of α . The classic equity premium puzzle is clearly evident, as are the other dimensions of model failure not emphasized in Mehra and Prescott (1985): the return standard deviations are much too low, the mean security returns are uniformly too high, and the correlation structure generally does not conform to the data. The low value of the $SD(IMRS)/E(IMRS)$ ratio is fully consistent with these results.

Panel C reports results under the presumption of both consumption and “outlook” uncertainty and we will take it as the benchmark case. While the coefficient of relative risk aversion in each state is very low, the premium rises to 8.48%. The standard deviation of all the series have increased enormously and now substantially exceed their corresponding values in the data. Mean security returns are also excessive although less so than for standard deviations, particularly in the case of the risk free rate and the premium. The pattern of correlations departs little from the Mehra and Prescott (1985) case and the $SD(IMRS)/E(IMRS)$ more than exceeds the level necessary to resolve the equity premium puzzle. All of this is accomplished in a context in which $E(\frac{p}{y})$ does not depart much from its corresponding value in the standard Mehra and Prescott (1985) economy. From the perspective of this particular adaptation of the Mehra and Prescott (1985) model, the “puzzle” is not that the actual premium and security return standard deviations are so high but that they are so low.

Panels D and E provide comparative statistics when the agent is made, respectively, less and more risk averse (relative to Panel C) all the while maintaining the same CRRA risk differential of one half. Comparing Panels C and D we observe that as the agent becomes less risk averse, average returns to both equity and debt decline, the more so for the risk free security. As a result, the premium rises. We attribute this latter observation to the fact that as α_1 and α_2

decline, all the while maintaining $\alpha_2 - \alpha_1 = .5$, they become more disparate on a relative basis (as measured, for example, by the ratio of $\frac{\alpha_2}{\alpha_1}$; for Panel C this ratio is .50 while for Panel D it is 2.0). In effect, although the average level of risk aversion declines, with the familiar consequences, relative outlook variation increases. The latter effects dominate with the result that the riskiness of the agent's overall environment increases. On a relative basis the risky asset becomes less valuable and the premium rises. The relative outlook affect diminishes, however, if α_1 and α_2 are increased (compare Panels C and E). In the case of $\frac{\alpha_2}{\alpha_1} = \frac{4.5}{4} = 1.25$. As a result the premium declines to 5.76%, although mean returns rise and standard deviations are little affected. But this cannot be the full story.

We have postulated that agents display less risk aversion when they are confronted by the experience of greater consumption growth. As noted in the introduction, however, there is experimental evidence to the contrary. For this reason we consider a case under the orthogonal assumption that agents display more risk aversion in the high growth state, $\alpha_2 < \alpha_1$, an assertion that suggests a strong desire "to preserve one's gains when they are high." The results of this exercise are summarized in Panel F of Table 1. Comparing Panels C and F we observe that the mean and S.D. of equity returns both increase while the opposite is true for the risk free security. As a result the Er^P rises in Panel F to 12.72 %: if anything the results are strengthened when low growth is associated with low risk aversion. Notice that the correlation structure is largely unaffected.

Our latter results strongly suggest that "outlook" variation, rather than

consumption variation, is the dominant effect. To confirm this assertion we examined the return characteristics of an economy in which there is no consumption variation at all, i.e., we fix $\alpha_2 > \alpha_1$, and $\delta = 0$. The results of this exercise are presented in Table 1, Panel G. Relative to Panel C, the premium is even larger due to a decline in $E r_f$ (standard deviations are largely unchanged, however). This suggests that within the class of Lucas (1978) asset pricing models, uncertainty in the period utility function of the type considered here will matter overwhelmingly more importantly for the characteristics of asset prices than will any postulated variation in the consumption (dividend) process.⁹

5. Related Literature

We have emphasized the non-stationarity of the pricing kernel evident in a particularly simple representation of countercyclical risk aversion. To restore stationarity in a related model Gordon and St-Amour (2002) introduce an ad hoc normalization parameter that they estimate from data. Once normalized, their pricing kernel still requires strongly countercyclical risk aversion to replicate asset return data. In a construct consistent with the procyclical risk aversion argued in the psychology literature, Falato (2003) preserves stationarity by postulating preferences defined over consumption and the consumption/wealth ratio. He achieves a remarkably accurate representation of the basic financial stylized facts. Melino and Yang (2003), in a model where the CRRA and the elasticity of the IMRS can be specified independently, achieve a similarly excellent match in a model parameterized for modest variation in the IMRS and strongly countercyclical risk aversion. In all of these excellent papers, the mechanism considered here, properly “restrained,” is present at some deep level. The contrasting evidence of risk aversion cyclicalities cited in Falato (2003) and Gordon and St-Amour

⁹ More precisely, while the dividend growth autocorrelations determines the autocorrelation of risk free and risky

(2003), however, provokes the question of how procyclical risk aversion at the individual level can translate into countercyclical risk aversion economy-wide.

6. Concluding Comments

State dependent preferences are increasingly popular. In finance the accent is applied to the plausible hypothesis that risk aversion is not constant. In this paper we have shown that the most natural representation of this hypothesis has profound implications for asset pricing. It implies in particular that the IMRS is level dependent and that this dependence introduces another source of volatility potentially more powerful than the volatility of the fundamentals being priced. This in turn leads to a reversal of the paradoxes first identified in the seminal work of Mehra and Prescott (1985). Asymptotically, the risk free rate is too low, the equity premium too high, the standard deviations of all security returns are too high and the Hansen-Jagannathan bounds are easily satisfied.

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