On Effects of Asymmetric Information on Non-Life Insurance Prices under Competition

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Abstract: We extend a game-theoretic model of Dutang et al. (2013) for non-life insurance pricing under competition among insurance companies and investigate the effects of asymmetric information on the equilibrium premium. We study Bayesian Nash equilibria as well as Bayesian Stackelberg equilibria and illustrate the sensitivity of equilibrium prices to various forms and magnitudes of information asymmetry through some numerical examples.

Keywords: Non-Life Insurance Pricing; Premium; Non-Cooperative Game Theory; Asymmetric Information; Nash Equilibrium; Stackelberg equilibrium; Price Sensitivity.

1 Introduction

The pricing of insurance contracts is a classical research topic. In practice, insurance companies use various approaches including general principles of premium calculation (often based on moments of the claim distribution), credibility theory and generalised linear models (GLM), see for instance Kaas et al. (2008). Game theory concepts have been suggested as a way to evaluate the strategic options available to the insurance companies with respect to the premium choices in the presence of competition. Although applications of game theory in insurance have a long history, the potential of this approach has not yet been fully exploited, particularly with respect to non-cooperative games. The classical reference is Rothschild and Stiglitz (1976), where insurance firms offer contracts with different...
premium and deductible. The model demonstrates that there might be no equilibrium point in this type of competition in the insurance market. However, when an equilibrium exists, high risk clients will choose full coverage, while low risk clients will choose partial coverage. Lemaire (1980) developed a number of insurance applications of cooperative game theory concepts and implemented them for different situations. Emms & Haberman (2005) present a model based on control theory in which they use a demand function to describe the number of underwriting policies, and the objective function of the insurer is the expected terminal wealth. Their findings include two pricing strategies; (i) set a low premium, which creates a loss in order to gain a large market share, followed by higher premium which produces profit, (ii) market withdrawal, which means that the insurer leaves the market and does not compete. This model does, however, not cover the situation of a very competitive insurance market, with only a few strong insurance companies. Emms (2012) describes an extension using game theory concepts. Other papers dealing with game theory concepts for the insurance market include e.g. Taylor (1986), Polborn (1998), Rees et al. (1999) and Powers and Shubik (2006). For pricing under competition with an exogenously given market price and underwriting cycles, see Malinovskii (2015).

In this paper, we adopt a one-period model of Dutang et al. (2013) based on a non-cooperative game among non-life insurers (see Section 2 for details) to investigate the effects on resulting premium equilibria. In particular, we study Bayesian Nash equilibria and Bayesian Stackelberg equilibria through a numerical implementation for several different scenarios.

Section 2 introduces the model assumptions and the considered solution concepts. Section 3 then presents and interprets the numerical results. Finally, Section 4 concludes.

2 One-Period Model and Solution Concepts

Let us assume a market with \( I \) insurers competing for a fixed number \( n \) of policyholders with homogeneous risks. The insurance contracts are issued for one period (one year). Each insurer \( j \) sets his premium \( x_j \) at time 0, choosing from a possible 'action' set \( A_j \). After \( x_j \) is set by all the insurers, each policyholder can decide either to renew his policy with the present insurer or switch to a competitor. It was assumed that there is no price distinction between new insurance contracts and renewed ones as the insurers offer the same product to all clients. Furthermore, the insurers do not cooperate to set their prices. In Dutang et al. (2013) it was assumed that the lapse rate of the policyholders as a function of the different prices \( x_j \) can be estimated through a multinomial logit function which motivates the demand function

\[
D_j = 1 - \beta_j \left( \frac{x_j}{m_j(x)} - 1 \right)
\]

of insurer \( j \). This demand function gives an approximation for the rate of renewed policies, where \( \beta_j > 0 \) is a price sensitivity (elasticity) parameter representing the lapse behavior of the policyholders relative to company \( j \) and \( m_j(x) = \frac{1}{I-1} \sum_{j \neq l} x_k \).
is the average premium of the competitors. For each insurer, the objective is to maximize the expected operational profit

\[ O_j(x) = D_j(x_j - \pi_j), \tag{1} \]

where \( \pi_j \) is the break-even premium calculated as a weighted average of the actuarial premium \( \overline{\pi}_{j,0} \) of insurer \( j \) (based on the individual claim experience of each insurer) and the market premium \( \overline{\pi}_0 \) (based on the collective claim experience of the entire market).

In addition, the model in Dutang et al. (2013) includes a solvency constraint imposed by the insurance regulator with respect to the choice of premium \( x_j \) as well as possible general premium bounds, i.e. \( x_j \in [\underline{x}, \overline{x}] \). For the scenarios implemented in the next section, realistic magnitudes for the solvency constraint and such bounds on the premium turn out to be automatically fulfilled by the obtained equilibrium prices, so we will not consider these constraints further in the present paper.

As insurance companies will typically only have limited information about their competitors, in this paper we extend the model of Dutang et al. (2013) to the case of asymmetric information and we apply the concept of Bayesian games to determine the resulting premiums. In a Bayesian game, players only have prior beliefs about some characteristics of other players, so player \( j \) is assumed to be of ‘type’ \( i \left( t_{ji} \right) \) with probability \( p \left( t_{ji} \right) \). In this paper, the characteristic for which the information is not symmetric is either assumed to be the elasticity parameter \( \beta \) (which actually is determined by the behavior of the current policyholders) or the internal break-even premium.

Harsanyi (1966) established a solution to such an incomplete information game, where each player ‘plays’ against all the different types of other players at the same time and maximizes the expected value of the objective function with respect to the (subjective) probabilities for the various types of the other players. A Bayesian Nash equilibrium (BNE) is hence a set of actions \( x^* = (x_1^*, \ldots, x_M^*) \) such that

\[ \forall x_j \in A_j : E(O_j(x_j^*(t_j), x_{-j}^*(t_{-j}))) \geq E(O_j(x_j(t_j), x_{-j}^*(t_{-j}))) \tag{2} \]

for all players \( j \) and all types \( t_j \), where player \( j \) knows his own type, so that the index \( \cdot_{-j} \) refers to all the possible types of the other players, and \( M \) is the number of all the considered types in the game. As shown in Dutang et al. (2013), the objective function (1) ensures the existence of a unique Nash equilibrium, so that the same holds true for the BNE \( x^* \).

While the BNE is an intuitively appealing solution concept in a competitive environment, in some markets there may be a clear market leader who takes his decision first, and the other companies will take this premium choice into account to then choose their own optimal premium. This approach was formalized by Stackelberg (1934) (originally in a duopoly setup). In an oligopoly market, the model assumes that the leader chooses a price and the followers then react with a Nash equilibrium model, given the leader’s choice. Assume w.l.o.g. that Player 1
is the leader, then the Stackelberg equilibrium is the vector $x^* = (x^*_1, \ldots, x^*_I)$ if $x^*_1$ solves the subproblem

$$\forall j, x_j \in X_j : O_1(x^*_1, x^*_{-1}(x_1)) \geq O_1(x_1, x^*_{-1}(x_1)),$$

and $x^*_{-1}(x_1)$ is the Nash Equilibrium for all other (types of) players, given the leader’s choice $x^*_1$.

### 3 Numerical Implementation and Results

In the following, we calculate Bayesian Nash premiums and Bayesian Stackelberg premiums according to the setup of Section 2 under some particular choices for model parameters.

#### 3.1 Bayesian Nash Equilibrium Premiums

In order to illustrate the effect of the information asymmetry compared to the case of full information, we start with the situation that only one insurer (Player 1) has unshared information.

##### 3.1.1 Bayesian Nash Premiums for three companies

In general, $K_i$ possible types for Player $i$ lead to $\sum_{i=1}^I K_i$ equations, each one maximizing the choice of one type according to (1). Assume now a market with three insurance companies (players), where the price sensitivity parameter $\beta_1$ of Insurer 1 is unknown to the other players, and based on their beliefs they assign probabilities $p(t_{1i})$ for Player 1 to have price sensitivity parameter $\beta_1$ (cf. 1). For simplicity we assume that these values and their probabilities coincide for Insurer 2 and 3. If they assign two possible values for $\beta_1$, then (2) consists of four equations (two types for Player 1, one type for Player 2 and Player 3) and the equilibrium premiums are the solution of the linear equation system

$$4\beta_{11}x_{11} - (1 + \beta_{11})(x_2 + x_3) - 2\beta_{11}\pi_1 = 0$$

$$4\beta_{12}x_{12} - (1 + \beta_{12})(x_2 + x_3) - 2\beta_{12}\pi_1 = 0$$

$$4\beta_{2}x_{2} - (1 + \beta_{2})[p(t_{11})(x_{11} + x_3) + p(t_{12})(x_{12} + x_3)] - 2\beta_{2}\pi_2 = 0$$

$$4\beta_{3}x_{3} - (1 + \beta_{3})[p(t_{11})(x_{11} + x_2) + p(t_{12})(x_{12} + x_2)] - 2\beta_{3}\pi_3 = 0.$$  

The break-even premiums $\pi_j = w_j\pi_{j,0} + (1 - w_j)\pi_0$ are chosen corresponding to the loss model and the respective individual claim experience. For the present approach and for the purpose of comparison, we stick to the setup of Dutang et al. (2013) and assume $\pi_{j,0} = (1.1, 1.15, 1.05)$ and $\pi_0 = 1.1$ as well as weighting parameters $w_j = (1/3, 1/3, 1/3)$, i.e. $\pi_j = (1.1, 1.117, 1.083)$.

Figure 1 shows the resulting BNE premiums for Players 2 and 3 for the case where $\beta_1 \in \{1, 5\}$ with probability 1/2 each, compared with the case of four types $\beta_1 \in \{0.75, 1.25, 4.75, 5.25\}$ with probability 1/4 each (the constant values $\beta_2 = 3.8$ and $\beta_3 = 4.6$ for Player 2 and 3 are known to all market participants). Note that for both distributions of $\beta_1$ we have $E(\beta_1) = 3$ and $\text{Var}(\beta_1) = 4$. 

The results in Figure 1 indicate that higher uncertainty about the type of Player 1 (more possible types for Player 1 despite the first two moments of $\beta_1$ stay the same) results in higher equilibrium premiums for the other players.

Figure 1  BNE Premiums with two or four types for Player 1

![BNE Premiums with two or four types for Player 1](image)

Figure 2 shows – from the viewpoint of Player 3 – the influence on the BNE premium of the number of other players with unknown $\beta_i$ as well as the value of $\text{Var}(\beta_i)$. We depict the following five scenarios for comparison:

- a full information game where the price sensitivity parameter is $\beta_1 = 3$ (and known to all players),
- a Bayesian game where Player 1 has two possible types $\beta_1 \in \{2.75, 3.25\}$,
- a Bayesian game where Player 1 has two possible types $\beta_1 \in \{1, 5\}$,
- a Bayesian game where Player 1 and Player 2 both have two possible types $\beta_1 \in \{2.75, 3.25\}, \beta_2 \in \{3, 4.6\}$ (the expected value for the previously constant value of $\beta_2$ stays the same),
- a Bayesian game where Player 1 and Player 2 both have two possible types $\beta_1 \in \{1, 5\}$ and $\beta_2 \in \{1.8, 5.8\}$ (same expected value, higher variance)

The results show that the additional uncertainty spread (for Player 3) over several opponents changes the respective equilibrium premium only sightly while the variance of the price sensitivity has a larger (and the major) effect on the BNE premiums.

It is also of interest to see to what extent it is an advantage or disadvantage in this setup for one particular player to not communicate transparently his $\beta$-value. To that end, let us compare the BNE premium of Player 1 (having elasticity parameter $\beta_1 = 3$) in the following three situations:

- a full information game where the price sensitivity parameter $\beta_1 = 3$
• a Bayesian game where Player 1 has $\beta_1 = 3$, but the other players only know $\beta_1 \in \{1, 3, 5\}$

• a Bayesian game where Player 1 has $\beta_1 = 3$, but the other players only know $\beta_1 \in \{0.55, 3, 5.45\}$.

Figure 3 illustrates that higher uncertainty about Player 1 causes Player 2 and Player 3 to choose higher premiums. In turn, Player 1 (being of type $\beta_1 = 3$) also has to ‘rationally’ (in the sense of Nash equilibria) react by choosing a higher premium. Note that in general the interval $x_j \in [\pi_j, (1 + 1/\beta_j)m_j(x)]$ is reasonable (otherwise the objective function value is negative), and the obtained equilibrium premiums turn out to be approximately in the middle of that interval.

Table 1 depicts the actual values of the objective function for each player (which anticipates the resulting market share). One can see that it can be an advantage for Player 1 not to disclose his $\beta_1$ value to the competitors, since his objective function value increases with additional uncertainty (the increase is slightly higher for Player 1 than for the other players).

<table>
<thead>
<tr>
<th>Var($\beta_1$)</th>
<th>Player 1 ($\beta_1 = 3$)</th>
<th>Player 2</th>
<th>Player 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.397</td>
<td>0.391</td>
<td>0.454</td>
</tr>
<tr>
<td>2.67</td>
<td>0.468</td>
<td>0.451</td>
<td>0.523</td>
</tr>
<tr>
<td>4</td>
<td>0.570</td>
<td>0.527</td>
<td>0.611</td>
</tr>
</tbody>
</table>

But what if Player 1 is of type $\beta_1 = 1$ or $\beta_1 = 5$? Table 2 compares the objective function values for this case between the full information game and the game where the other players only know $\beta_1 \in \{1, 3, 5\}$ with probability 1/3. Here Player 1 of type $\beta_1 = 1$ turns out to have an incentive to communicate his true (low) $\beta$ value to his opponents. On the other hand, if Player 1 is of type $\beta_1 = 5$, comparing the objective function values shows that he has an incentive to hide his real type. So one
interpretation may be that the disclosure of information is beneficial if the value of $\beta$ is under-estimated by competitors, but not if there is symmetric uncertainty.

Table 2  Objective function values with $\beta_1 = 1$

<table>
<thead>
<tr>
<th></th>
<th>$\text{Var}(\beta_1)=0$</th>
<th>$\text{Var}(\beta_1)=2.67$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Player 1 ($\beta_1 = 1$)</td>
<td>0.956</td>
<td>0.659</td>
</tr>
<tr>
<td>Player 1 ($\beta_1 = 5$)</td>
<td>0.331</td>
<td>0.537</td>
</tr>
</tbody>
</table>

Up to now the uncertainty in the game was about the price sensitivity parameter $\beta$ of (the customers of) competitors. Let us now investigate a situation where Player 1 considers reducing the used value of the break-even premium to, say, 80% of its real value (e.g. in order to gain market share). A natural question is whether this information should be passed on to the competitors or not. Figure 4 gives equilibrium premiums (and Table 3 the respective objective function values) for the following situations:

- using a low $\pi_1$-value and disclosing it to the other players (left column in Fig. 4): as may be expected, this results in a substantially reduced premium and objective function value.

- asymmetric information scenario, where the other players only know that Player 1 either uses the (true) higher break-even premium (probability 0.5) or the reduced one (probability 0.5). In case Player 1 actually uses the higher $\pi_1$, the BNE premium as well as the objective function value is lower than for the complete information scenario, so it is better to communicate this to the competitors. However, in case Player 1 uses the smaller $\pi_1$, he has an incentive not to tell the competitors about it (in other words, creating uncertainty can be an advantage), cf. Table 3. For the other players, even if Player 1 uses the smaller value, not knowing this with certainty is an advantage in terms of their objective function value, and they will also use higher premiums in that case.
Table 3  Objective function values

<table>
<thead>
<tr>
<th></th>
<th>$\pi_1 = 1.1$</th>
<th>$\pi_1 \in {0.88, 1.1}$</th>
<th>$\pi_1 = 0.88$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Player 1</td>
<td>0.397</td>
<td>0.307</td>
<td>0.364</td>
</tr>
<tr>
<td>Player 1 ($\pi_1 = 1.1$)</td>
<td>0.391</td>
<td>0.337</td>
<td>0.283</td>
</tr>
<tr>
<td>Player 2</td>
<td>0.454</td>
<td>0.392</td>
<td>0.330</td>
</tr>
</tbody>
</table>

Figure 4  BNE Premiums for two possible $\pi_1$

3.1.2 Bayesian Nash Premiums for seven companies

Let us now look at the effect of the number of players on the resulting premiums. For that purpose, add four insurers to the original game in such a way that the game consists now of three insurers with the same characteristics as Insurer 2 before, and three insurers with the same characteristics as Insurer 3 before. We choose $\pi_j = (1.1, 1.117, 1.083, 1.117, 1.083, 1.117, 1.083)$ and $\beta_2 = \beta_4 = \beta_6 = 3.8, \beta_3 = \beta_5 = \beta_7 = 4.6$ throughout, and calculate the resulting premiums for a

- a full information game with $\beta_1 = 3$,
- two possible types for Player 1 with $\beta_1 \in \{2.75, 3.25\}$, specifications as before
- two possible types for Player 1 with $\beta_1 \in \{1, 5\}$, specifications as before.

The resulting BNE premiums for Insurer 1 (of type $\beta_1 = 3$) and for Insurer 2 are given in Figures 5 and 6, respectively. The results suggest that more players in the game reduce the effect of the uncertainty stemming from the unknown type of Player 1 (as, intuitively, the marginal influence of Player 1 becomes weaker, see the smaller rate of change as a function of higher variance for the 7-player case compared to the 3-player case). At the same time, the difference between equilibrium premiums of the 7-player and the 3-player case becomes more prominent the higher the uncertainty about the type of Player 1, i.e. the number of players plays a more important role in the presence of larger uncertainty.
Effects of Asymmetric Information on Non-Life Insurance Prices

Figure 5  BNE Premiums for Player 1

Figure 6  BNE Premiums for Player 2

3.2 Stackelberg Equilibrium Premiums for three companies

Let us now assume that there are three players, and Player 1 is the leader with his price sensitivity parameter $\beta$ being the lowest (which suggests that his policyholders are less sensitive to a possible price increase). We assume that the elasticity parameter $\beta_2$ of Player 2 is not known to the other players, they instead use probabilities $p(t_{21})$ for Player 2 to use $\beta_2$. If they assume two possible values, then the expected value of the objective function of the leader, Player 1 is:

$$E(O_1) = \left[p(t_{21})(1 - \beta_1(\frac{2x_1}{x_{21} + x_3} - 1)) + p(t_{22})(1 - \beta_1(\frac{2x_1}{x_{22} + x_3} - 1))\right](x_1 - \pi_1). \quad (4)$$
Once Player 1 sets his premium $x_1^*$ by maximizing (4), the other players calculate their corresponding Bayesian Nash premiums and the optimal solution is obtained by finding the solutions to the equations system

$$
4\beta_{21}x_{21} - (1 + \beta_{21})(x_1 + x_3) - 2\beta_{21}\pi_2 = 0
$$

$$
4\beta_{22}x_{22} - (1 + \beta_{22})(x_1 + x_3) - 2\beta_{22}\pi_2 = 0
$$

$$
4\beta_3x_3 - (1 + \beta_3)[p(t_{21})(x_1 + x_{21}) + p(t_{22})(x_1 + x_{22})] - 2\beta_3\pi_3 = 0.
$$

First we calculate the Stackelberg equilibria for a full information game with $\pi_j = (1.1, 1.117, 1.083)$ and $\beta_j = (3, 3.8, 4.6)$. Table 4 presents the equilibrium premiums of the different insurers for a Stackelberg set up versus a Nash set up:

<table>
<thead>
<tr>
<th>$j$</th>
<th>$x_j^*$ (Stackelberg)</th>
<th>$x_j^*$ (Nash)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Player 1</td>
<td>1.792</td>
<td>1.544</td>
</tr>
<tr>
<td>Player 2 ($\beta_2 = 3.8$)</td>
<td>1.624</td>
<td>1.511</td>
</tr>
<tr>
<td>Player 3</td>
<td>1.581</td>
<td>1.471</td>
</tr>
</tbody>
</table>

Figure 7 shows the Stackelberg premiums of Insurer 1, the leader, as a function of $\text{Var}(\beta_2)$, for constant $E(\beta_2) = 3.8$. From Table 4 and Figure 7, one sees that in general the Stackelberg equilibrium premiums are much higher than the BNE premiums and Insurer 1 will react to additional uncertainty about Insurer 2 with higher premiums.

To verify the incentive of Insurer 2 to communicate the information about his $\beta_2$-value to his competitors, in Figure 8 we compare the value of the objective function for the two types of Insurer 2 with $\beta_{21} = 2.8$ and $\beta_{22} = 4.8$. One sees that if Insurer 2 is of Type 1, he has an incentive to communicate his information to the other insurers (as in a full information game he has a higher objective function value), whereas if Insurer 2 is of Type 2, he has no incentive to communicate his type and will prefer to keep the information asymmetric.

4 Conclusion

In this paper we deal with the effects of asymmetric information about risk profiles on the pricing mechanism of a competitive insurance market. We use Bayesian Nash and Stackelberg equilibrium concepts to quantify these effects in an extension of the game-theoretic model of Dutang et al. (2013). We numerically illustrate some scenarios in which uncertainty can be beneficial, and others where market participants have incentives to be transparent to their competitors. For the cases considered in this paper, Stackelberg premiums resulting from the presence of a market leader are higher than Nash premiums. In general, from a market equilibrium perspective in the context of non-cooperative game theory, asymmetric information typically results in higher premiums. In a market with more
participants (competitors), we find the equilibrium premiums to be less sensitive to uncertainty than in a market with fewer players. Clearly, the findings in this paper rely on the particular choice of model, objective function and equilibrium concept (the choice in the present paper being partly motivated by simplicity and particularly to ensure existence and uniqueness of an equilibrium premium). However, a number of important market features have indeed been taken into account here, and the obtained results express within this framework some intuitive causal relations in a quantitative way. It is left for future research to study similar questions for other such choices, as well as for dynamic versions of the game.

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References


