House Price Dynamics with Dispersed Information

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Abstract

We use a user-cost model to study how dispersed information among housing market participants affects the equilibrium house price. In the model, agents consume housing services, speculate on price changes and are disparately informed about local economic conditions. Information dispersion leads agent to form heterogeneous expectations about housing demand and prices. Optimists, who expect high house price growth, buy in anticipation of capital gains, while pessimists prefer to rent housing units to avoid capital losses. The upshot is that pessimistic expectations are not incorporated in the price of owned houses and, thus, the equilibrium price is higher relative to the benchmark case of common information. We test the predictions of the model on US cities, using the dispersion of city income as a proxy for the dispersion of information of local economic condition. The empirical evidence supports the prediction that house prices are higher and more volatile in cities where information is more dispersed.

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1 Introduction

The US housing market has experienced substantial price fluctuations in the last two decades. Figure 1 gives an example of such fluctuations for the aggregate US economy and a representative sample of US cities. As shown, housing prices not only have different trends in different cities, but display also heterogeneous short-run dynamics.\footnote{In some cities, such as Los Angeles, the housing price has moved in tandem with the overall national index, though it has moved much more. In other cities prices movements have been quite hetereogenous. In Miami, for example, the house price index has been steady for almost two decades before experiencing an exponential increase beginning in 2000; in San Antonio, it has declined since the 1980s; in Rochester, it has displayed an inverse “U shaped” history; in Memphis the same index has gone through periodic cycles.}

In the opinion of many housing-market observers (see e.g. Glaeser and Gyourko, 2007) these dynamics are difficult to explain through the lens of a user cost model in which house prices are determined by an indifference condition between owning and renting. The reason is that in such a model (Poterba, 1984, Henderson and Ioannides, 1982), the cost of owning depends on variables that either do not vary much over time (e.g., property taxes) or a constant across markets (e.g., interest rates).\footnote{While there is consensus that differences in state level property taxes cannot explain the house price behavior across markets, the debate concerning the relationship between interest rates and house prices is less conclusive. McCarthy and Peach (2004) and Himmelberg, Mayer and Sinai (2005) argue that the recent house price boom in the US was largely determined by low interest rates. In contrast, Shiller (2005, 2006) documents a non-significant relationship between house prices and interest rates over a longer period of time.}

The goal of this paper is to propose a variant of the user cost model to helps us rationalize the heterogeneous behavior of housing prices in the US. Our extension of the standard model is that information about local economic conditions is dispersed among housing market participants. Agents receive idiosyncratic noisy signals and thus form heterogeneous views about the economy and house prices growth. But since the expected resale value of a house affects the cost of owning, only optimists prefer to buy houses. As a result, the price of housing will be higher and more volatile the larger the dispersion of information about local economic conditions. To the extent house price expectations depend on local economic conditions and economic conditions vary across markets and time, our model provides a novel interpretation behind the price fluctuations observed in the US housing market.

Our analysis is based on three building blocks, motivated by several aspects of the US housing market. First, the available evidence suggests income is the main determinant of housing demand, either because richer agents may afford to spend more on homes (Poterba, 1991, Englund and Ioannides, 1997) or because higher income relaxes credit constraints (Ortalo-Magné and Rady, 2006, Almeida et al., 2006). Second, surveys of housing market participants (Case and Shiller, 1988, 2003, Piazzesi and Schneider, 2009) reveal agents’ desire to buy is strongly influenced by their expectations of reselling houses at higher prices. These surveys document also that home buyers’ disagree about the causes of house price movements and expectations are largely influenced by past and current economic conditions (see, e.g., Case, Quigley and Shiller, 2003). Third, housing supply is inelastic and adjust slowly to local demand shocks because of regulations, zoning laws or technological constraints (see, e.g., Glaeser and Gyourko (2003), Glaeser, Gyourko and Saks (2005, 2006)).

Taken together, these ingredients suggest a specific mechanism through which changes in income may generate large swings in house prices: if income not only influences housing demand but also shapes expectations of future house prices, an income shock may initiate a
self-reinforcing process that, through heterogeneous expectations and the fixed housing supply, runs from expected prices to house demand and back to house prices.

To formalize this mechanism we consider a closed-city model with a fixed supply of housing and two groups of agents with no housing endowment. Agents consume housing services by either buying or renting, and speculate on future price changes. The demand for houses fluctuates stochastically because information about local economic conditions is dispersed and agents use their own income, together with other available signals, in estimating the unknown state of the economy. As a result, idiosyncratic income shocks translate into heterogeneous expectations about aggregate housing demand, and — given the fixed supply — into heterogeneous expectations about house prices. As in a standard user-cost model, the equilibrium price is pinned down by an indifference condition between owning and renting. However, since expectations are heterogeneous the equilibrium price no longer reflects the indifference condition of the average resident. Instead, it is pinned down by the expectations of the more optimistic agents in the market. This is because pessimists, who expect future capital losses, perceive the user cost to be higher than the cost of renting. Consequently, they prefer to consume housing services through the rental market, where rental units are supplied by the optimists who, for speculative reasons, buy units in excess of their demand for housing services.

The direct implication of this model is that the price of owned houses, reflecting only the opinion of the optimists, is higher and more volatile relative to a benchmark scenario where information is not dispersed. In addition, information shocks have an asymmetric effect on prices depending on whether they convey positive or negative signals: while positive shocks move the equilibrium price upwards negative shocks do not affect the equilibrium price. This last prediction arises because agents have the option to consume housing services through the rental market. Were the rental market absent, pessimists would be forced to consume housing services by buying housing units. Hence, private signals would cancel out when the demand is aggregated across agents and the equilibrium price would depend only on the average expectation. It turns out our results survive even if agents use the equilibrium price to update their inference about the state of the economy, provided the house price is not fully revealing.

To test the predictions of the model we run regressions on a panel of approximately 350 US cities and use the dispersion of city-industry income shocks as a proxy for the dispersion of information about local income. This proxy is suggested by the logic of the model, which can be interpreted as describing the dynamics of local housing prices when the demand for housing depends on expected local economic conditions. More precisely, if city residents are employed in different industries and are imperfectly informed about the city income, shocks to industry’s income can become a source of information about current local economic conditions. In line with our model predictions, we find house prices are higher and more volatile in cities where our proxy of information dispersion is higher. We also find an asymmetric response of housing prices to information shocks: positive shocks explain significantly house price increases, while negative shocks lack statistical predictive power.

In the rest of the paper we proceed as follows. In Section 2 we relate our model to the relevant literature. In Section 3 we introduce the baseline model and discuss the determinants of the equilibrium house price. In Section 4 we study a benchmark scenario where agents hold imperfect but common information about local economic conditions. In Section 5 we derive the main model’s predictions when agents’ information is not only imperfect but also dispersed. In this section we also allow agents to update their inference through learning from
the equilibrium house price. In Section 6 we discuss our proxy of information dispersion, and present our empirical analysis and results. Our conclusions are in section 7 and all proofs in the Appendix.

2 Related Literature

Methodologically, our paper follows the user-cost approach of Poterba (1984) and Henderson and Ioannides (1982) in which a prospective buyer is indifferent between renting and owning, and the cost of owning depends, among other variables, on property taxes, the opportunity cost of capital and the expected capital gains on the housing unit. While some papers have studied the implications for house prices of changes in taxes (Poterba, 1991) and interest rates (Himmelberg et al., 2006, McCarthy and Peach, 2004), the role played by heterogeneity in the expected rate of price changes has remained so far unexplored. This is so because differences in expectations cannot arise in a standard user cost model with homogeneous information. We complement this literature by showing that information dispersion across markets, and within markets over time, help to rationalize part of the house price changes documented in Figure 1, more than changes in property taxes — which are fairly constant over time — or interest rates — which are constant across markets.

The idea of our paper that changes in income may have more than proportional effects on house prices is similar in spirit to the work of Stein (1995) and Ortalo-Magné and Rady (2006). In these papers agents buy houses by borrowing and the ability to borrow is directly tied to the value of houses. Therefore, a positive income shock that increases the housing demand and price relaxes the borrowing constraint which increases further the demand for houses. Our paper differs from Stein, and Ortalo-Magne and Rady, in three important ways. First, in our story there are no borrowing constraints, and the amplification mechanism runs only from changes in expected prices, via household income, to current prices, via changes in the speculative demand. Second, in our model agents do not need to own houses but can also use the rental market to consume housing services. Third, it is not only the level but also the dispersion of income that affects house prices.

For this reason our paper is also related to Gyourko, Mayer and Sinai (2006) and Van Nieuwerburgh and Weil (2008). Gyourko et al. argue that the interaction between an inelastic supply of houses and the skewing of the income distribution generates significant price appreciations in superstar cities (i.e., cities with unique characteristics preferred by the majority of the population) because wealthy agents are willing to pay a financial premium to live in these areas, bidding up prices in the face of a relatively inelastic supply of houses. Van Nieuwerburgh and Weil use a similar mechanism to explain the correlation between the dispersion of US house prices and the cross-sectional dispersion of US wages, though in their model agents move across cities for productive rather than preference reasons. Our paper differs from these contributions because it highlights a different channel through which income dispersion affects house prices. In our framework income shocks affect agents’s perception of local economic conditions, leading to heterogeneous expectations about current and future economic fundamentals. As a consequence, difference in expectations are more pronounced when, ceteris paribus, income is more dispersed. Moreover, our model features an asymmetric effect on prices of positive and negative income shocks, a prediction absent in Gyourko et al., and van Nieuwerburgh and Weil. In our model the speculative motive for housing units is enhanced when agents perceive better
economic conditions; conversely, the speculative motive does not arise after negative income shocks, as in this case agents consume housing services through the rental market. Another important difference is methodological. In our model prices are determined by a no-arbitrage condition between buying and renting, while in Gyourko et al., and van Nieuwerburgh and Weil prices are determined by a spatial no-arbitrage condition with owners indifferent between different locations, given local wages and amenities. The spatial equilibrium approach is, however, more suitable for studying the long-run distribution of housing prices as opposed to high frequency price variations, which is the main focus of our analysis.

Our paper is also related to a large literature in macroeconomics and finance that studies the role of imperfect information among decision makers. In fact, our story can be seen as an adaptation of the Phelps-Lucas hypothesis to the housing market, in the sense that imperfect information about the nature of disturbances to the economy makes agents react differently to changes in market conditions. Part of our work shares also many features with the literature on the pricing of financial assets in the presence of heterogeneous beliefs and short-sale constraints (e.g., Miller, 1977, Harrison and Kreps, 1979, Hong, Scheinkman, and Xiong, 2004 and Scheinkman and Xiong, 2003). In this literature, if agents have heterogeneous beliefs about asset fundamentals and face short sales constraint, the equilibrium asset price reflects the opinion of the most optimistic investors. We adapt the same idea to the housing market. In our model pessimistic agents would like to short their houses if they had the option to do so. Thus, to consume housing services pessimists move out of the market of “home for sale” and the price of owned houses ends up reflecting only the more optimistic opinion in the market, rather than the average opinion.

3 The Model

3.1 Information

The economy is populated by an infinite sequence of overlapping generations of agents with constant population. Each generation has unit mass and lives for two periods. In the first period, agents supply labor and make saving and housing decisions; in the second period, they consume the return on savings and housing. The wage $W^j_t$, at which labor is supplied inelastically, is equal to

$$W^j_t = \exp(\theta_t + \varepsilon^j_t),$$

where $\theta_t$ is the economy income and $\varepsilon^j_t$ an individual-specific wage shock. The individual-specific shocks, $\varepsilon^j_t$, which are the only source of income heterogeneity, are serially independent and have normal distribution with zero mean and variance $\sigma^2_\varepsilon$. We make the assumption that $\theta_t$ follows an AR(1) process,

$$\theta_t = \rho \theta_{t-1} + \eta_t, \quad \text{with } \rho \in (0, 1)$$

where $\eta_t$ is independently and normally distributed with zero mean and variance $\sigma^2_\eta$. When agents cannot observe the realization of $\theta_t$, $\varepsilon^j_t$ becomes a source of information heterogeneity. In other words, individual wage $W^j_t$ is the agent $j$’s private signal about the unobservable aggregate shock, $\theta_t$. 

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To make the analysis simple, we consider only two groups of agents, $j = 0, 1$, each with equal mass. We also make the standard assumption that idiosyncratic shocks cancel out in aggregate, or equivalently, the average private signal is an unbiased estimate of $\theta_t$:

**Assumption 1:** $\sum_j \varepsilon_j^t = 0$.

### 3.2 Preferences

Agents have logarithmic preferences over housing services, $V^j_t$, and second period consumption, $C^j_{t+1}$:

$$U^j_t = A^j_t \log V^j_t + E^j_t \log C^j_{t+1},$$

(3)

where $E^j_t$ denotes the expectation operator based on household $j$’s information set at time $t$ (to be specified later) and the parameter $A^j_t$ is a preference shock,

$$A^j_t = \exp \left( 2 \left( a_t + \nu^j_t \right) \right),$$

which consists of an aggregate taste shock, $a_t$, and an idiosyncratic noise $\nu^j_t$. We assume that $a_t$ and $\nu^j_t$ are independent and normally distributed with zero mean and variance $\sigma_a^2$ and $\sigma^2_{\nu}$. We also consider the limiting case where the variance of $\nu^j_t$ is arbitrarily large, so that knowing one’s own individual taste provides no information about the aggregate taste.

### 3.3 Budget constraint

In the first period, after the realization of the idiosyncratic income, agents decide how many housing units to buy, $H^j_t \geq 0$, at the unit price, $P_t$. They also choose the quantity of housing services to consume, $V^j_t$, and the units to rent out, $H^j_t - V^j_t$, at the rental price $Q_t$. The stock of houses owned by the old is sold to the young at the beginning of each period. At the end of the period, the residual income of the young is saved at the gross interest rate, $R$.

For type-$j$ agents, the resource constraint is thus:

$$C^j_{t+1} = R \left( W^j_t - P_t H^j_t + Q_t \left( H^j_t - V^j_t \right) \right) + P_{t+1} H^j_t,$$

(4)

with

$$H^j_t \geq 0.$$

(5)

### 3.4 Optimal house demand

Agents’ intertemporal decisions consist of choosing $H^j_t$ and $V^j_t$ to maximize (3) subject to (4) and (5). It is immediate to establish that the optimal demand for $V^j_t$ and $H^j_t$ satisfy the following first-order conditions:

$$\frac{A^j_t}{V^j_t} = E^j_t \left[ \frac{RQ_t}{C^j_{t+1}} \right],$$

(6)

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*Underlying this utility function is the assumption that the demand for second-period housing services is constant which, for simplicity, we normalize to one.*
\[ E_i^j \left[ \frac{R(U_t - Q_t)}{C_{t+1}^j} \right] \geq 0, \quad (7) \]

where
\[ U_t = P_t - \frac{P_{t+1}}{R}, \quad (8) \]
denotes the (per unit) user cost of housing, which decreases with next period house price, \( P_{t+1}/R. \)

According to equation (6) agents consume housing services until the marginal benefit (the LHS) equal the marginal cost, defined in terms of next period consumption (the RHS). The optimal demand of housing units is implicit in equation (7), which relates the cost of owning, \( U_t, \) to the cost of renting housing services, \( Q_t. \)

### 3.5 The linearized optimality conditions

To deliver explicit solutions, we linearize equations (6) and (7) around the “certainty” equilibrium; i.e., the equilibrium prevailing when both aggregate and idiosyncratic shocks are zero. Denoting with lower case letters variables in percentage deviations from the equilibrium with certainty, Appendix I shows that a linear approximation of (7) leads to

\[ E_i^j u_t \geq q_t, \quad (9) \]

where
\[ u_t = \frac{(1 + r)p_t - p_{t+1}}{r}, \quad (10) \]
and \( r \equiv R - 1 > 0. \) Further, a linear approximation of (6) leads to

\[ v_i^j = w_i^j + a_i^j - q_t, \quad (11) \]
indicating that the demand for housing services increases with income, is shifted by preferences shocks, and is negatively related to the rental price.

From now on, we adopt the convention that agents in group \( j = 1 \) are relatively more optimistic about the next-period house price; i.e., \( E_i^1 p_{t+1} \geq E_i^0 p_{t+1}. \) Using (10), equation (9) can be written as:

\[ E_i^0 u_t > q_t \quad \text{and} \quad h_i^0 = 0 \quad (12) \]

\[ E_i^1 u_t = q_t \quad \text{and} \quad h_i^1 > 0. \quad (13) \]

\( ^4 \)Our specification of the user cost is deliberately simple. We could have assumed that for each unit owned, agents incur also a cost equal to a fraction \( M_t \) of the nominal value of housing, \( P_t H_t. \) \( M_t \) can be thought of as including maintenance and depreciation costs, property taxes, interest payments on mortgages, etc. Under this alternative specification, the user cost of housing would be

\[ U_t = P_t (1 + M_t) - \frac{P_{t+1}}{R}. \]

As long as house market participants are homogeneously informed about \( M_t, \) none of the results presented below are affected, though the algebra would be more cumbersome.
Thus, pessimists choose to own no housing units, $h^0_t = 0$, as they perceive the cost of ownership to be higher than the cost of renting. Optimists, instead, who expect higher prices in the future, become indifferent between owning and renting. The upshot is that optimists consume housing services, $v^1_t$, out of the units of houses owned, $h^1_t$, and rent out the difference, $h^1_t - v^1_t$, to the pessimists:

$$h^1_t - v^1_t = v^0_t.$$  

(14)

### 3.6 The equilibrium rental and house price

Assuming a fixed housing supply, $s$, the rental price is pinned down by the market clearing condition for housing services,

$$s = \frac{v^1_t + v^0_t}{2},$$

which, together with (11), yields

$$q_t = \theta_t + a_t - s,$$

(15)

where

$$\theta_t = \frac{w^1_t + w^0_t}{2} \text{ and } a_t = \frac{a^1_t + a^2_t}{2},$$

denote the average income and the average preference shock for housing services.

The equilibrium house price is determined by the indifference condition (13), which can be rewritten as:

$$p_t = \frac{r}{1 + r} q_t + \frac{1}{1 + r} \bar{E}^1_t p_{t+1}.$$  

(16)

Using (15) to substitute out $q_t$, we obtain the following pricing equation:

$$p_t = \frac{r}{1 + r} f_t + \frac{1}{1 + r} \bar{E}^t p_{t+1} + \frac{1}{1 + r} \tilde{E}^t p_{t+1},$$

(17)

where

$$f_t = \theta_t + a_t - s,$$

(18)

summarizes average fundamental variables, and

$$\bar{E}^t p_{t+1} \equiv \frac{E^1_t p_{t+1} + E^0_t p_{t+1}}{2}, \quad \tilde{E}^t p_{t+1} \equiv \frac{E^1_t p_{t+1} - E^0_t p_{t+1}}{2},$$

denotes, respectively, the average expectation and the difference in expectations about tomorrow’s price.

In equation (17), as in a standard house pricing equation, $p_t$ depends on fundamentals, $f_t$, and the average expectation on the future house price. The extra term, $\tilde{E}^t p_{t+1}$, is non-standard and arises because agents may hold heterogenous expectations. In the next two sections, we make different assumptions about agents’ information sets in order to evaluate how $\bar{E}^t p_{t+1}$ and $\tilde{E}^t p_{t+1}$ influence the determination of the equilibrium house price.
4 Homogenous Information

We start with the benchmark case in which agents are homogeneously informed about the state of the economy, \( \theta_t \). Specifically, we assume that agents only rely on the public information. \( \theta_{t-1} \), to infer \( \theta_t \). In other words, agents share a common information set so that individual expectations coincide with the average expectation; i.e., \( E_t^j p_{t+1} = E_t p_{t+1} \). In this case, the difference in expectations is zero: \( \tilde{E}_t p_{t+1} = 0 \).

Iterating equation (17) forward and imposing a stationary condition on prices, Appendix II shows that the average expectation of tomorrow’s price can be written as

\[
E_t p_{t+1} = \phi \rho \theta_{t-1} - s, \tag{19}
\]

with

\[
\phi \equiv \frac{r \rho}{1 + r - \rho}.
\]

The expectation depends on \( \theta_{t-1} \) since \( \theta_t \) (which is not observable) follows an AR(1) process so that agents’ estimation of \( \theta_t \) depends on its past realization. The preference shock, \( a_t \), does not influence the expected price because it has, by assumption, zero mean. Inserting (19) into (17), and recalling that \( \tilde{E}_t p_{t+1} = 0 \), the equilibrium price under homogenous information, \( p^* \), can be written as

\[
p_t^* = f_t + \Lambda_t, \tag{20}
\]

where \( f_t \) is given in (18) and

\[
\Lambda_t \equiv \frac{\phi \rho \theta_{t-1} - \theta_t - a_t}{1 + r},
\]

is an expectation error. We interpret \( p_t^* \) as the “fundamental” price of owned houses, because it reflects the average opinion in the market which is an unbiased estimate of the unknown fundamental.

5 Heterogeneous Information

We now consider a setting where agents use the current realization of their income, \( w^j_t \), as well as the public signal, \( \theta_{t-1} \), to make an optimal inference about \( \theta_t \). Agent \( j \)’s information set at \( t \) is therefore

\[
\Omega^j_t = \{w^j_t, \theta_{t-1}\}, \quad j = 0, 1.
\]

It is important to notice that the equilibrium house price is not included in \( \Omega^j_t \). This assumption is made only to simplify the characterization of the channels through which information dispersion affects the equilibrium price. As we will discuss in the following section, this assumption is not essential for our results.\(^6\)

\(^5\)It is superfluous to know the entire history of aggregate shocks since \( \theta_t \) follows an AR(1) process. Similarly, knowing the past realization of agents’ private signals is irrelevant, given the iid assumption for \( \varepsilon^j_t \).

\(^6\)A way to think about this assumption is to consider the special case where the variance of the aggregate unobservable preference shock, \( \sigma^2_{\varepsilon_t} \), is arbitrarily large. In such a case the house price (17) becomes uninformative about \( \theta_t \) and house market participants do not learn much upon observing \( p_t \). In excluding \( p_t \) from agents’ information set, we make our analysis akin to models where agents do not condition on the equilibrium price because they do not know how to use prices correctly (e.g., they display bounded rationality, as in Hong and Stein, 1999) or because they exhibit behavioral biases (e.g., they are overconfident, as in Scheinkman and Xiong, 2003).
With signals $w_j^t$ and $\theta_{t-1}$, the ability of agent $j$ to estimate $\theta_t$ depends on the relative magnitude of $\sigma^2_\varepsilon$ and $\sigma^2_\eta$. Because of our assumption of independently and normally distributed errors, the projection theorem implies

$$E_t^j \theta_t = (1 - \lambda) \rho \theta_{t-1} + \lambda w_j^t,$$

where the weight $\lambda \equiv \frac{\sigma^2_\eta}{\sigma^2_\varepsilon + \sigma^2_\eta}$ reflects the relative precision of the two signals. With $\lambda > 0$, expectations among agents are heterogeneous and both average expectations and expectation differences become important determinants of the equilibrium house price. Moreover, since expectations depend on $w_t^j$, the optimistic (pessimistic) are those with higher (lower) realization of the idiosyncratic shock. Iterating equations (17) and (21) forward and excluding explosive price paths, Appendix III shows that difference in expectations, and the average expectation of the future price are, respectively,

$$\tilde{E}_{t+1} p_t = \phi \lambda i_t,$$

$$E_{t+1} p_t = \left( \phi \rho \theta_{t-1} - s \right) + \frac{\phi \lambda}{r} I + \phi \lambda \left( \theta_t - \rho \theta_{t-1} \right),$$

where

$$i_t \equiv \varepsilon^1_t - \varepsilon^0_t,$$

denotes the dispersion of information between the two groups of agents and

$$I \equiv \int_0^\infty x d\Gamma (x),$$

measures the average degree of information heterogeneity in the economy, with $\Gamma$ denoting the distribution of $i_t$.

Equation (22), stems from the fact that agents are disparately informed and they assign a positive weight to their private signal in estimating $\theta_t$. Differences in expectations are therefore proportional to the dispersion in private signals.

Equation (23) is the equivalent of equation (19). It differs, however, from (19) because dispersed information introduces two additional terms, each proportional to the weight that agents assign to their private signals. The first term, $\phi \lambda I / r$ arises because prices are forward looking: it is not only the current dispersion of information that influences the price of housing, but also the dispersion of future information. The second term, $\phi \lambda (\theta_t - \rho \theta_{t-1})$, capturing the average misperception in the economy, arises because agents use only part of the information contained in the public signal $\theta_{t-1}$, to make optimal inference about $\theta_t$. The slow reaction to changes in fundamentals has the effect of introducing inertia in the way average expectations are formed, which accords well with the idea that housing market expectations tend to be extrapolative (see Case and Shiller, 1988, 2003).

Plugging these expressions into (17), the equilibrium price can be written as

$$p_t = p_t^* + \lambda \Upsilon_t,$$

where, $p_t^*$, is the fundamental price given in (20), and

$$\Upsilon_t \equiv \phi \frac{\theta_t - \rho \theta_{t-1}}{1 + r} + \frac{I}{r(1 + r)} + \phi \frac{i_t}{1 + r}.$$
summarizes the role of information dispersion among agents.

With heterogeneous information (i.e., $\lambda > 0$), $p_t$ is higher than $p_t^*$ for two reasons. The first reason is the unconditional mean of $\Theta_t$ is positive, implying information dispersion leads to a higher equilibrium house price. This is quite intuitive. Optimists expect higher house prices since they estimate a higher $\theta_t$ (see equation (21)) and, thus, a higher future prices (see equation (23)). Conversely, pessimists expect lower future prices and capital losses. As discussed in Section 3, pessimists prefer to drop out of the market of homes for sale and consume housing services through the rental market. Therefore, the equilibrium price is biased upward because it reflects only the opinion of the optimists. The second reason is the price misalignment becomes more pronounced the larger the information dispersion, $i_t$. When $\varepsilon_t^1$ increases, optimistic agents demand more houses for speculative reasons, while pessimists continue to demand no housing units. These results lead to two testable implications. First, housing prices increase with information dispersion. Second, positive information shocks increase housing prices, while negative shocks have no such effects.

A third testable prediction arises in comparing (24) and (20). It is straightforward to see that relative to the benchmark case of homogeneous information, the volatility of house prices is higher the larger the average misperception in the economy, $\sigma^2_i$ and the larger the variance of information dispersion, $\sigma^2_i$:

$$V(p_t) - V(p_t^*) = \left(\frac{\lambda \phi}{1 + \tau}\right)^2 (\sigma^2_i + \sigma^2_t) > 0. \tag{26}$$

This extra source of price volatility arises because the equilibrium price with dispersed information is not only influenced by fundamental shocks but also by noise shocks.

### 5.1 Learning from the equilibrium price

In this section we relax the assumption that agents do not use the equilibrium price to infer the unknown state of the economy. This extension is desirable because house prices, like any other financial prices, contain useful information about the dispersed information in the economy.

In extending our analysis to a set-up where households learn from the equilibrium price we run, however, into a non-trivial problem. As discussed in the previous section, if households receive symmetrically dispersed information and have the option to consume housing services by either buying or renting, the housing market becomes segmented and the equilibrium price depends on the difference in expectations between optimists and pessimists, i.e., $i_t = |\varepsilon^1_t - \varepsilon^2_t|$. However, because $i_t$ is not normally distributed, $p_t$ has a non Gaussian distribution and standard linear filtering methods cannot be applied.\footnote{See Appendix IV for a derivation of the exact distribution of $i_t$.}

To circumvent this problem we introduce the assumption that $a_t$ — the aggregate preference shock — is an independent and identically distributed random variable, drawn from a distribution $\mathcal{M}$, with zero mean and variance $\sigma^2_a$. Moreover, $\mathcal{M}$ is such that $a_t + i_t \equiv \delta_t \sim \mathcal{N}(\bar{i}, \sigma^2_a)$ where $\bar{i}$ denotes the unconditional mean of $i_t$ and $\sigma^2_a$ the variance of $a_t + i_t$.

Although ad-hoc, this assumption enables us to use standard methods to characterize the filtering problem, since it ensures that the equilibrium price is Gaussian. In addition, as in a typical noisy rational expectation model à la Grossman and Stiglitz (1976) and Hellwig (1980),
Using a standard linear solution method, Appendix IV shows that the equilibrium price with learning can be written as,

\[ p_t = p^*_t + \pi_2 \Upsilon_t + \pi_3 \Phi_t, \]  

(27)

where \( \pi_2 > 0 \) and \( \pi_3 > 0 \) are the weights on the private and the endogenous public signal (the price), respectively, and

\[
\Phi_t \equiv \frac{\phi}{1 + r} \eta_t + \frac{r \phi}{(1 + r)(r + \phi \pi_2)} a_t + \frac{\phi^2 \pi_2}{(1 + r)(r + \phi \pi_2)} i_t
\]

is a term that summarizes the degree of magnification of shocks induced by the process of learning from the price. Intuitively, in the presence of unobservable shocks, households who observe a change in house prices do not understand whether this change is driven by a change in aggregate income (\( \eta_t \)), preferences (\( a_t \)), or private signals (\( i_t \)). Thus, with \( \pi_3 > 0 \), each of these shocks will have an amplified effect on equilibrium prices, since households respond to whatever is the source of movement in the house prices.

A key observation to make in comparing equation (27) with (24) is that \( i_t \) — our measure of information dispersion — continues to shift the equilibrium price away from its fundamental value, \( p^*_t \). More specifically, \( i_t \) exerts a direct effect, via \( \Upsilon_t \), for the same reasons discussed in the previous section, and an indirect one, via \( \Phi_t \), because of the magnification of shocks induced by the process of learning.

The relative importance of \( \Upsilon_t \) and \( \Phi_t \) depends, however, on \( \pi_2 \) and \( \pi_3 \). As shown in Appendix IV, \( \pi_2 \to \lambda \) and \( \pi_3 \to 0 \) as \( \sigma_0^2 \to \infty \), while \( \pi_2 < \lambda \) and \( \pi_3 > 0 \) with a finite \( \sigma_0^2 \). In words, as the noise in the preference for housing services is sufficiently larger, the equilibrium price (27) becomes non-informative, and therefore, identical to the one prevailing in absence of learning (24).\(^8\)

6 Testing the implications of the model

Our model delivers three main predictions: 1) the deviation of house prices from the fundamental value increases with the dispersion of information; 2) the volatility of house prices is higher the larger the volatility of information dispersion; and 3) positive information shocks move the house price upward, but negative shocks have no effects.

The most difficult part in testing these predictions is to obtain data on information dispersion. To deal with this problem we adopt the following strategy. We take US cities as units of observation and use the dispersion (within cities) of shocks to industry earnings as a proxy for the dispersion of information about local housing market conditions.

While debatable, this proxy is motivated by the logic of our model, which can be seen as describing the determinants of house prices in a given city, where the speculative demand

\(^8\)Note that the volatility of the equilibrium house price continues to be higher than in the benchmark scenario of imperfect but homogenous information. By comparing (27) with (20), it is immediate to see that (26) holds true.
for housing depends on local economic conditions. Specifically, if residents in each city are employed in different industries, and they are imperfectly informed about the city income, then industry-specific income shocks can become a source of confusion about the city average income, as in the signal extraction problem discussed in our theoretical framework. With this interpretation, equation (1) and (2) in the model, can be rewritten and reinterpreted as follows,

\[ w_{j,k,t} = \theta_{k,t} + \varepsilon_{j,k,t} \quad \text{and} \quad \theta_{k,t} = \rho \theta_{k,t-1} + \eta_{k,t} \]  

(28)

where \( w_{j,k,t} \) is the time \( t \) income of a resident of city \( k \) employed in industry \( j \), \( \theta_{k,t} \) the average city income at time \( t \), and \( \varepsilon_{j,k,t} \) the time-\( t \) industry-\( j \) specific shock in city \( k \). A proxy for the dispersion of information about \( \theta_{k,t} \) can then be computed using a measure of the dispersion of income shocks across the \( j \) industries.\(^9\)

### 6.1 Data description and summary statistics

We collect annual data for a sample of approximately 350 US metropolitan areas (MSA) during the period 1980 to 2000. To infer the time series properties of local income shocks we use (per employed) earnings data for 10 one-digit industries, based on the SIC classification code.\(^{10}\)

With these data, the dispersion of earnings shocks across industries is computed in two steps. First, based on equation (28), we run 10 regressions, one for each industry, in which we pool the growth rate of industry earnings for the full sample of MSAs,

\[ \Delta w_{j,k,t} = \alpha_0 + \alpha_1 \Delta \theta_{k,t} + \alpha_2 \Delta \theta_{k,t-1} + \gamma_t + \varepsilon_{j,k,t} \quad \text{for} \quad j = 1, 2, \ldots, 10. \]  

(29)

Here \( \Delta \) is the first difference operator, and \( \gamma_t \) is a time fixed effect. In this specification, the residuals \( \varepsilon_{j,k,t} \) record shocks to earnings growth in city-\( k \) industry-\( j \), controlling for city-specific income dynamics, \( \theta_{k,t} \) and \( \theta_{k,t-1} \), and nationwide effects, \( \gamma_t \).\(^{11}\) Second, we measure the dispersion of earnings shocks across \( j \) industries and within each MSA as the weighted average of the absolute value of industry-city shocks,

\[ \hat{i}_{k,t} = \sum_{j=1}^{10} \omega_{j,k,t}^j \left| \varepsilon_{j,k,t} \right| , \]  

(30)

\(^9\)There are also empirical motivations behind our proxy of information dispersion. There is widespread consensus that high frequency variations in house prices are mostly local, not national (Glaeser and Gyourko, 2006) and evidence that the bulk of short-run movements in house prices is due to changes in demand, driven by local economic conditions, as opposed to changes in preferences for local amenities. Endogenous supply-side changes may also affect movements in house prices (Glaeser, Gyourko and Saiz, 2008). However, in the short run, due to regulations and technological constraints, supply changes tend to respond slowly to shifts in demand.

\(^{10}\)Specifically, we use earnings data for the following industries: 1) Farm, 2) Mining, 3) Construction, 4) Manufacturing, 5) Transportation and public utilities, 6) Wholesale trade, 7) Retail trade, 8) Finance, insurance, and real estate, 9) Services, and 10) Government and government enterprises. These data are available at http://www.bea.gov/regional/reis/. Our sample period ends in 2000 because in that year the Standard Industrial Classification (SIC) system has been replaced by the North American Industry Classification System (NAICS). This different system for classifying economic activity makes it impossible to extend our data beyond 2000. Available data based on the NICS system cover only the period 2001 to 2006. We use data based on the SIC classification codes for the period 1980-2000, to be able to exploit a longer time series variation in the data.

\(^{11}\)We have also experimented with specifications that include lags of \( \Delta w_{j,k,t} \) to control for industry-city specific dynamics. All the results reported below are robust to such changes.
where the weights $\omega_{k,t}^j$ measure the share of MSA workers employed in industry $j$, to control for the size of each industry.\textsuperscript{12}

For each MSA we take the nominal house price index for single-family houses from the Office of Federal Housing Enterprise Oversight, and per capita income data from the Bureau of Economic Analysis. Nominal variables are converted in real dollars using the national CPI index from the Bureau of Labor Statistics. We use annual observations because income data is only available annually.

Table 1 reports basic summary statistics. The data display considerable variation across MSA. Over the full period 1980-2000, our proxy of information dispersion is less than 1.5% in Minneapolis, Cleveland, Kansas City and Tampa but greater than 4% in Chicago, Dallas, Los Angeles, and New York, among other cities. Real house price changes also exhibits considerable variation. For instance, Boston, San Francisco, and San Jose all experienced growth rates in house prices over 3% per annum over the 20 year period studied, while Houston, Oklahoma City, and San Antonio experienced negative house price changes of approximately 1.5%.

### 6.2 The baseline regression

We start the analysis by examining the empirical relevance of the price equation prevailing under common information. To empirically use equation (20), we take first differences of each variable and estimate the following regression:\textsuperscript{13}

\[
\Delta p_{k,t} = \beta_0 + \beta_1 \Delta \theta_{k,t} + \beta_2 \Delta \theta_{k,t-1} + \gamma_t + \gamma_k + \epsilon_{k,t}. \tag{31}
\]

Here, $\Delta p_{k,t}$ is the log change of the real house price index in MSA $k$ in year $t$, $\Delta \theta_{k,t}$ the log change in real per capita income and $\epsilon_{k,t}$ a standard error term. In this regression, and those that follow, we include year and MSA dummies, $\gamma_t$ and $\gamma_k$, to account for unobservable aggregate and city-specific determinants of house prices.

Table 2 reports OLS estimates of this baseline regression, with standard errors clustered at the MSA level to allow for within-city autocorrelation in the errors. According to the model, $\beta_1$ and $\beta_2$ are expected positive and as shown in the first column of Table 2 these predictions are strongly supported by the data: higher current and lagged changes in income are significantly associated with higher housing prices changes.

The role of information dispersion is examined in column 2 where we report estimates of the empirical counterpart of equation (24). More specifically, we add to the baseline regression (31) our proxy of difference in expectations, $i_{k,t}$. In line with the prediction of the model, the results show a statistically significant relationship between our proxy of information dispersion and house prices. The estimated effects is also sizeable: a 1% increase in $i_{k,t}$ results in a 0.2% increase in the growth rate of house prices. To better gauge the economic effect of this result, let us consider an exogenous increase in $i_{k,t}$, from the 10th percentile value (which is

\textsuperscript{12}None of the results presented below (in terms of economic and statistical significance) change if we use squared deviations rather than absolute deviations. We prefer to use absolute deviations to be able to maintain the same unit as the change in industry earnings, so that the coefficients in the house price regressions reported below are easily interpreted.

\textsuperscript{13}We use each variable in first difference because the OFHEO house price index is not standardized to the same representative house across markets. Thus, price levels cannot be compared across cities, but they can be used to calculate growth rates.
approximately 1.2%) to the 90th percentile value (which is approximately 4%). This increase would lead to an acceleration in the growth rate of house price by 0.6% per year, which is large considering that the average annual growth rate of real house prices is 0.4% over the 1980-2000 period.

6.3 Alternative empirical specifications

The results in Table 2, although based on the price equation implied by the theoretical model, do not control for some patterns of the house price dynamics that prior works have documented to be important. For example, starting with Case and Shiller (1989), it is well known house prices exhibit momentum and mean reversion over time. To control for these effects, we add three lags of the dependent variable to our baseline regression. The results shown in column 1 of Table 3 indicate house prices indeed exhibit positive correlation at short lags and negative correlation at longer lags. However, as reported in column 2, our proxy of information dispersion continues to play a large and significant role in explaining house price changes.

Columns 3 to 4 explore the robustness of our findings to an alternative empirical specification, suggested by the work of Lamont and Stein (1999). In their study of the house price dynamics in US cities, Lamont and Stein find house prices (a) exhibit short run movements, (b) respond to contemporaneous income shocks, and (c) display a long run tendency to fundamental reversion. They thus propose to estimate the following regression,

\[ \Delta p_{k,t} = \gamma_0 + \gamma_1 \Delta p_{k,t-1} + \gamma_2 \Delta \theta_{k,t} + \gamma_3 (p/\theta)_{k,t-1} + \gamma_k + \gamma_t + \epsilon_{k,t} \]  

(32)

where \((p_k/\theta_k)_{t-1}\) is the lagged ratio of house prices to per-capita income.

As shown in column 3, these variables have all the expected sign and explain a large fraction of house price variations. To this three-variable specification we add our variable of interest, \(i_{k,t}\), in column 4. In line with the results in Table 2, we find our proxy of information dispersion continues to be related significantly to house price changes: the growth rate of house price is higher in cities where local income shocks are more dispersed.

A common objection to the correlations reported so far is that we do not control for demographic factors. Changes in the demand for housing may be driven not only by economic conditions but also by population changes — a shifter of housing demand that has been omitted in our theoretical analysis. In the attempt to control for this effect, columns 5 and 6 use population growth as an additional regressor. Population growth is expected to enter the regression with a positive sign since new potential buyers tend to move housing demand and prices up. The results show population growth has indeed a positive and significant effect on house prices. Our core findings, however, do not seem to depend on the inclusion of this additional control.

A further robustness check of our main results is in Table 4. The main prediction of our model that house prices are higher the larger the dispersion of information holds under the assumption of fixed housing supply. However, if the supply of housing is elastic, changes in demand would have a muted effect on prices. Table 4 explores this possibility using Saiz (2008) index of housing supply elasticity. The noteworthy feature of this index is that it does not depend on local market conditions but only on geographical and topographical constraints on house constructions. Using the median value of this index as a cut-off, we run the same regressions as in Table 3 for cities with high and low supply elasticity. Although our sample
of cities is substantially reduced, the results validate our prior that the speculative motive for housing demand has a more pronounced effect on prices in cities with tighter supply restrictions. The estimated coefficient for our proxy of dispersion in information is large and statistically significant in cities with low-supply-elasticity and is essentially zero and never statistically significant in cities with an elastic housing supply.

6.4 The volatility of house prices

We now turn to the second prediction of the model that the volatility of house prices increases with the variance in the dispersion of information. To examine the strength of this prediction we compute the volatility of house prices by running a pooled regression for the change in house prices, controlling for year effects, and then by taking the standard deviation of the residuals in each MSA. This gives us a measure of the volatility of house prices, within a metropolitan area, controlling for aggregate effects. Next, with one observation for MSA, we exploit the cross sectional variation of house price volatility and regress our measure of house price volatility on the standard deviation of information dispersion in each MSA.

The OLS estimates are in Table 5 and illustrated in Figure 2, which graphs the volatility of house price against the fitted values from the regression. As can be seen, MSAs with large dispersion of information have also more volatile house prices. Interestingly, this result holds even if we control for the standard deviation of aggregate MSA income, as shown in the second column of Table 5.

6.5 Positive and negative shocks to industry income growth

The final implication of our model is that house prices respond more to positive than negative information shocks. In the model, agents buy housing units for speculative reasons if they receive signals that convey information of higher future house prices, and rent if they expect house prices to depreciate. This asymmetry implies that while negative information shocks do not move house prices downward, positive shocks impart an upward shift in house prices.

It is important to check the empirical relevance of this prediction since its validity distinguishes the theoretical implications of our model from a setup where income dispersion affects house prices because agents moves across cities for productive or preference reasons, as for example in Gyourko, Mayer and Sinai (2006) or Van Nieuwerburgh and Weil (2007).

Specifically, we define positive and negative information shocks as follows:

\[
POS_{k,t} = \sum_{j=1}^{10} \omega_{k,t}^j \varepsilon_{k,t}^j \quad \text{if} \quad \varepsilon_{k,t}^j > 0
\]

\[
NEG_{k,t} = \sum_{j=1}^{10} \omega_{k,t}^j \varepsilon_{k,t}^j \quad \text{if} \quad \varepsilon_{k,t}^j < 0.
\]

Here \(\varepsilon_{k,t}^j\) are the residuals in equation (29), that is period \(t\) earning shocks in industry \(j\) in city \(k\), and \(\omega_{k,t}^j\) are weights that measure the fraction of MSA population employed in industry \(j\). \(POS\) and \(NEG\) are thus the empirical counterparts of \(\varepsilon_1^1\) and \(\varepsilon_0^0\) in the model. We enter these variables in piecewise linear form into the empirical specification (31) and (32) to allow for
differential effects between positive and negative shocks to industry earnings. The results are in Table 6, using the same specifications as in Table 3. As shown, the estimates conform with our model’s predictions that positive and negative shocks have asymmetric effects on housing prices. In fact positive shocks have a large and significant impact on house prices, while the impact of negative shocks is negligible and not statistically significant.

7 Conclusion

In this paper we have used a user-cost model to study how information dispersion about local economic conditions affect the equilibrium price of housing. In our model — in which agents consume housing services and speculate on future price changes — the equilibrium housing price is higher, the larger agents’ difference in expectations about future house prices. The intuition is that all agents face de facto a short-sale constraint in housing. Therefore, those who hold pessimistic expectations about future prices decide to rent to avoid capital losses, while those who have optimistic expectations decide to buy to speculate on future price increases. The upshot is that the equilibrium price of owner occupied houses incorporates only the expectations of the optimists and is thus higher and more volatile relative to an environment of homogenous information.

We confirm the theoretical predictions of our model in a panel data of US cities, using dispersion in industry income shocks as a proxy for dispersion in information about local economic conditions. This proxy is motivated by our model’s assumption that different realizations of individual income lead agents to form different views of the economy.

In keeping our model simple, we have abstracted from a number of issues that might play an important role in the development of a more complete model. For example, we have abstracted from the general equilibrium effects of the interest rate. Changes in \( R \), however, may affect our analysis since the return on the safe asset influences agents’ choice of renting and owning, for a given level of house price expectations. We have also prevented agents from re-trading. An extension of the model that allows for re-trading, as in Stein (1995) or Ortalo-Magné and Rady (2006), may shed new light on whether information dispersion induce a positive correlation between house prices and housing transactions. These extensions are left for future research.

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**Appendix I: Linearization**

We linearize equations (6) and (7) around the equilibrium with “certainty”, i.e., when $\varepsilon^t_i = 0$, $\eta^t_i = 0$, $a^t_i = 0$ and $\nu^t_i = 0 \forall i$. In this equilibrium $V = H$, because there is no uncertainty and no heterogeneity among agents. Denoting with $X$ any variable $X_i$ in the “certainty” equilibrium, the first order conditions (6) and (7), with interior solutions, can be written as

\[
V^j \quad = \quad V > 0 \quad \Rightarrow \quad V = \frac{C}{RQ}, \quad (33)
\]

\[
H^j \quad = \quad H > 0 \quad \Rightarrow \quad Q = U. \quad (34)
\]
Moreover, using equations (4), (8) and the fact that \( V = H \),

\[
\frac{C}{R} = W - HP \left( 1 - \frac{1}{R} \right) = W - VQ. \tag{35}
\]

Thus combining (35) and (33) one obtains

\[
V = \frac{W}{2Q}.
\]

Under the assumption of fixed housing supply, \( S \), the market clearing condition is,

\[
V = S,
\]

which implies that the following relationships must hold in a certainty equilibrium,

\[
U = Q, \quad Q = \frac{W}{2S}, \quad C = \frac{RW}{2}.
\]

Denoting with lower-case letters variables in percent deviation from the equilibrium with certainty, and recalling our definition of user cost,

\[
U_t = P_t - \frac{P_{t+1}}{R}, \tag{36}
\]

a linearization of (7) around the certainty equilibrium yields,

\[
E^j_t \left[ \frac{RP}{C} \left( 1 + p_t - c^j_{t+1} \right) - \frac{RQ}{C} \left( 1 + q_t - c^j_{t+1} \right) - \frac{P}{C} \left( 1 + p_{t+1} - c^j_{t+1} \right) \right] \geq 0.
\]

Rearranging,

\[
E^j_t \left[ \frac{RP}{C} p_t - \frac{RQ}{C} q_t - \frac{P}{C} p_{t+1} - c^j_{t+1} \left( \frac{RP}{C} - \frac{RQ}{C} - \frac{P}{C} \right) \right] \geq 0 \Rightarrow
\]

\[
E^j_t \left[ RP p_t - RQ q_t - P p_{t+1} \right] \geq 0 \Rightarrow
\]

\[
E^j_t \left[ p_t - \frac{Q}{P} q_t - \frac{1}{R} p_{t+1} \right] \geq 0,
\]

we obtain

\[
p_t \geq \frac{r}{1 + r} q_t + \frac{1}{1 + r} E^j_t p_{t+1}, \tag{37}
\]

where

\[
r = R - 1.
\]

Notice also that a linearization of (36) gives

\[
u_t = \frac{P}{U} p_t - \frac{P}{RU} p_{t+1}
\]

\[
= \left( \frac{1 + r}{r} \right) p_t - \frac{1}{r} p_{t+1}.
\]

Therefore, (37) can be rewritten as

\[
E^j_t u_t \geq q_t. \tag{38}
\]
Since, by assumption, $E_1^1 p_{t+1} > E_0^1 p_{t+1}$, it follows that $E_0^0 u_t > E_1^1 u_t$. Thus, in equilibrium, equation (38) can be written as,

$$E_1^1 u_t = q_t \quad \text{and} \quad h_1^1 > 0,$$
$$E_0^0 u_t > q_t \quad \text{and} \quad h_0^0 = 0.$$  \hfill (39, 40)

Proceeding as above, a linearization of equation (6), around the certainty equilibrium, gives

$$E_1^j R Q C_q (q_t - c_{t+1}^j) = \frac{A}{V} (2a_t^j - v_t^j)$$
$$E_0^j V (q_t - c_{t+1}^j) = \frac{1}{V} (2a_t^j - v_t^j)$$

which defines the optimal demand of housing services

$$v_t^j = 2a_t^j - q_t + E_0^j c_{t+1}^j.$$  \hfill (41)

The term $E_0^j c_{t+1}^j$ in (41) is obtained by linearizing the flow of budget constraint (4), that for the two groups of agents reads as follows,

$$C_{t+1}^1 = R (W_1^1 - P_t H_1^1 + Q_t (H_1^1 - V_1^1)) + P_{t+1} H_1^1,$$
$$C_{t+1}^0 = R (W_0^1 - Q_t V_1^0).$$  \hfill (42)

A bit of algebra establishes\(^{14}\)

$$E_1^1 c_{t+1}^1 = 2w_1^1 - v_t^1 - \left( \frac{r+1}{r} \right) p_t + \frac{1}{r} E_1^1 p_{t+1}$$
$$E_1^0 c_{t+1}^0 = 2w_0^1 - v_t^0 - q_t.$$  \hfill (43, 44)

Plugging these expressions in (41) and using equation (39) it follows that

$$v_t^1 = w_1^1 + a_1^1 - \frac{1}{2} (q_t + E_1^1 u_t)$$
$$v_t^0 = w_0^1 + a_0^1 - q_t.$$  \hfill (45)

Using the market clearing condition,

$$s = \frac{1}{2} h_t^1$$

and the fact that

$$\frac{1}{2} (h_t^1 - v_t^1) = \frac{1}{2} v_t^0$$

\(^{14}\)Linearizing (42) yields

$$E_1^1 c_{t+1}^1 = \frac{R W}{C} w_1^1 - \frac{R P H}{C} (p_t + h_t^1) + \frac{R Q H}{C} (q_t + h_t^1) - \frac{R Q V}{C} (q_t + v_t^1) + \frac{P H}{C} (E_1^1 p_{t+1} + h_t^1)$$
$$= 2w_1^1 - \frac{P}{R} (p_t + h_t^1) + (q_t + h_t^1) - (q_t + v_t^1) + \frac{P}{R U} (E_1^1 p_{t+1} + h_t^1)$$

Rearranging this equation gives (43). Proceeding in a similar way one obtains (44).
equation (45) can be written as
\[ \frac{1}{2}v_0^0 + \frac{1}{2}v_0^1 = s, \]
from which it is immediate to pin down the equilibrium rental price,
\[ q_t = \theta_t + a_t - s, \quad \text{(46)} \]
where
\[ \theta_t = \frac{w_0^1 + w_0^0}{2} \quad \text{and} \quad a_t = \frac{a_0^1 + a_0^0}{2}. \]

Finally, inserting (46) into (39) gives,
\[ p_t = \frac{r}{1 + r} (\theta_t + a_t - s) + \frac{1}{1 + r} E_{t+1}^1 p_{t+1} \]
\[ = \frac{r}{1 + r} (\theta_t + a_t - s) + \frac{1}{1 + r} E_{t+1}^1 p_{t+1} + \frac{1}{1 + r} E_{t+1}^0 p_{t+1}, \quad \text{(47)} \]
where
\[ E_{t+1}^0 = \frac{E_{t+1}^1 + E_{t+1}^0}{2} \quad \text{and} \quad E_{t+1}^1 = \frac{E_{t+1}^1 - E_{t+1}^0}{2}. \]

**Appendix II: Common Information**

When information is imperfect but homogeneous, \( E_{t+1}^i p_{t+1} = \bar{E}_{t+1} p_{t+1} \) and \( \bar{E}_{t+1} p_{t+1} = 0 \). Therefore, equation (47), shifted one period forward, gives
\[ p_{t+1} = \frac{r}{1 + r} (\theta_{t+1} + a_{t+1} - s) + \frac{1}{1 + r} \bar{E}_{t+1} p_{t+2}. \]

Taking expectations on both sides conditional on time \( t \) information, and excluding explosive price paths, a forward iteration of the expression above gives
\[ E_t p_{t+1} = \frac{r}{1 + r} \sum_{\tau=0}^{\infty} \left( \frac{1}{1 + r} \right)^\tau E_t (\theta_{t+1 + \tau} + a_{t+1 + \tau} - s) \]

Since \( \theta_t \) and \( a_t \) are unobservable at time \( t \) and
\[ \theta_t = \rho \theta_{t-1} + \eta_t, \quad \text{with} \quad \rho \in (0, 1] \]
we have
\[ E_t [\theta_{t+1} + a_{t+1} - s] = \rho^2 \theta_{t-1} - s. \]

It is therefore immediate to obtain
\[ E_t p_{t+1} = E_t f_t = \phi \rho \theta_{t-1} - s, \quad \text{(48)} \]
where \( \phi \equiv \frac{r \rho}{1 + r - \rho} \). Plugging (48) back into (47) and recalling that \( \bar{E}_t p_{t+1} = 0 \), the equilibrium price under common information can then be written as
\[ p_t^* = (\theta_t + a_t - s) + \frac{1}{1 + r} ((\phi \rho \theta_{t-1} - \theta_t) - a_t). \]
Appendix III: Heterogeneous Information

In the presence of heterogeneous expectations, \( E_t^j p_{t+1} \neq \tilde{E}_t p_{t+1} \) and \( \tilde{E}_t p_{t+1} \neq 0 \). Shifting equation (47) one period forward,

\[
p_{t+1} = \frac{r}{1+r} \left( \theta_{t+1} + a_{t+1} - s \right) + \frac{1}{1+r} E_{t+1} p_{t+2} + \frac{1}{1+r} \tilde{E}_{t+1} p_{t+2}
\]

denoting,

\[
i_t = \left| \varepsilon_t^i - \varepsilon_t^j \right| \quad \text{for} \quad i \neq j.
\]

and guessing that \( \tilde{E}_t [p_{t+1}] = \phi \lambda i_t \), we have

\[
E_t^j p_{t+1} = \frac{r}{1+r} E_t^j (\theta_{t+1} + a_{t+1} - s) + \frac{1}{1+r} E_t^j \tilde{E}_{t+1} p_{t+2} + \frac{\phi \lambda}{1+r} I,
\]

\[
\tilde{E}_t^j p_{t+1} = \frac{r}{1+r} \tilde{E}_t^j (\theta_{t+1} + a_{t+1} - s) + \frac{1}{1+r} \tilde{E}_t \tilde{E}_{t+1} p_{t+2} + \frac{\phi \lambda}{1+r} I,
\]

\[
\tilde{E}_t p_{t+1} = \frac{r}{1+r} \tilde{E}_t \theta_{t+1} + \frac{1}{1+r} \tilde{E}_t \tilde{E}_{t+1} p_{t+2},
\]

where the last equality holds because agents hold heterogeneous expectations with respect to \( \theta_{t+1} \) but not with respect to \( a_{t+1} \).

Iterating these expressions forward and excluding explosive price paths, we obtain:

\[
E_t^j p_{t+1} = \frac{r}{1+r} E_t^j \theta_{t+1} - s + \frac{\phi \lambda}{r} I,
\]

\[
\tilde{E}_t p_{t+1} = \frac{r}{1+r} \tilde{E}_t \theta_{t+1} - s + \frac{\phi \lambda}{r} I,
\]

\[
\tilde{E}_t p_{t+1} = \frac{r}{1+r} \tilde{E}_t \theta_{t+1}.
\]

Moreover, using equation equation (21), it is easy to see that:

\[
E_t^j \theta_{t+1} = \rho E_t^j \theta_t = \rho \left[ (1-\lambda) \rho \theta_{t-1} + \lambda w_t \right],
\]

and thus,

\[
\tilde{E}_t p_{t+1} = \phi (\rho(1-\lambda)\theta_{t-1} + \lambda \theta_t) - s + \frac{\phi \lambda}{r} I,
\]

\[
\tilde{E}_t p_{t+1} = \phi \lambda \theta_t + \phi \lambda (\theta_t - \rho \theta_{t-1}) + \frac{\phi \lambda}{r} I,
\]

\[
\tilde{E}_t p_{t+1} = \phi \lambda i_t.
\]

so that \( \tilde{E}_t p_{t+1} = \phi \lambda i_t \) as claimed. Plugging \( \tilde{E}_t p_{t+1} \) and \( \tilde{E}_t p_{t+1} \) in (47), the equilibrium house prices can be written as

\[
p_t = \left( \theta_t + a_t - s \right) + \frac{1}{1+r} \left( (\phi \rho \theta_{t-1} - \theta_t) - a_t \right)
\]

\[
+ \frac{\phi \lambda}{1+r} (\theta_t - \rho \theta_{t-1}) + \frac{\phi \lambda}{r(1+r)} I + \frac{\phi \lambda}{1+r} i_t.
\]

\[
= p_t^* + \lambda Y_t
\]

23
where
\[ \gamma_t = \frac{\phi(\theta_t - \theta_{t-1})}{1 + r} + \frac{\phi I}{r(1 + r)} + \frac{\phi i_t}{1 + r}. \]

**Appendix IV: Learning from the equilibrium rental price**

In this appendix we provide a solution to the signal extraction problem when households condition on the house price to learn the unknown fundamental, \( \theta_t \). As explained in Section 6, the inference problem is involved since the equilibrium price in the presence of heterogenous information is not normally distributed. To characterize this non-standard signal extraction problem we assume that the distribution of the preference shock \( \varepsilon_t \) is such that sum of \( \varepsilon_t \) and \( \varepsilon_i \) follows a normal distribution. This assumption enables us to recover a Gaussian distribution for the equilibrium price and allows us to apply standard linear filtering techniques.

We proceed in three steps. First, we define the exact distribution for \( \varepsilon_i \). Next, we determine the form of the distribution of \( \mu_t \) that makes the equilibrium price normally distributed. Finally, using a method of undetermined coefficients we characterize the inference problem for \( \theta_t \) and the resulting equilibrium price.

**The distribution of \( i = |\varepsilon^i - \varepsilon^j| \) for \( i \neq j \)**

Consider two independent random variables, \( \varepsilon^i \) and \( \varepsilon^j \), distributed normally with zero mean and equal variance \( \sigma^2_\varepsilon \). Define,
\[ \tilde{\varepsilon} = \varepsilon^j - \varepsilon^i \sim \mathcal{N}(0, 2\sigma^2_\varepsilon). \]

The cumulative distribution function of \( i = |\tilde{\varepsilon}| \) is
\[ F_i(y) = \Pr (i = |\tilde{\varepsilon}| \leq y) = 2 \int_0^y \frac{1}{\sqrt{2\pi} \sqrt{2\sigma_\varepsilon}} \exp \left( -\frac{1}{2} \frac{z^2}{2\sigma_\varepsilon^2} \right) dz, \]
and the associated density,
\[ f_i(y) = \begin{cases} \frac{\partial F_i(y)}{\partial y} &= \frac{2}{\sqrt{2\pi} \sqrt{2\sigma_\varepsilon}} \exp \left( -\frac{1}{2} \frac{y^2}{2\sigma_\varepsilon^2} \right) \quad \text{if } y \geq 0 \\ 0 & \text{otherwise}. \end{cases} \quad (49) \]

Denote with \( \bar{\varepsilon} \), the mean of \( i \),
\[ \bar{i} = \int_0^\infty y f_i(y) dy. \]

**The distribution of the aggregate preference shock, \( a \).**

We wish to find the distribution of a random variable, \( a \), with zero mean and variance \( \sigma^2_a \), such that
\[ a + i \sim \mathcal{N}(\bar{i}, \sigma^2_a + \sigma^2_i). \]

The cumulative function of \( a + i \) is
\[ F_{a+i}(y) = \Pr (a + i \leq y) = \int_{-\infty}^\infty \left( \int_{-\infty}^{y-a} f_i(i) di \right) f_a(a) da, \]

24
where $f_a$ is the density of $a$ and $f_i$ is defined in (49). Differentiating $F_{a+i}(y)$ with respect to $y$ yields the probability density of $a+i$,

$$f_{a+i}(y) = \int_{-\infty}^{\infty} f_i(y-a) f_a(a) \, da.$$ 

Since by assumption $a+i$ follows a normal distribution, it must be

$$f_{a+i}(y) = \frac{1}{\sqrt{2\pi \sigma_a^2 + \sigma_i^2}} \exp\left( -\frac{1}{2} \frac{(y-a)^2}{\sigma_a^2 + \sigma_i^2} \right).$$

Therefore the density $f_a(a)$ is recovered by solving the following integral,

$$\int_{-\infty}^{\infty} f_i(y-a) f_a(a) \, da = \frac{1}{\sqrt{2\pi \sigma_a^2 + \sigma_i^2}} \exp\left( -\frac{1}{2} \frac{(y-a)^2}{\sigma_a^2 + \sigma_i^2} \right).$$

**Lemma 1**

The correlation coefficient between $\varepsilon^i$ and $i \equiv |\varepsilon^j - \varepsilon^i|$ is zero.

**Proof.**

$$\text{Cov}(\varepsilon^j, |\varepsilon^j - \varepsilon^i|) = \text{Cov}(\varepsilon^j, \varepsilon^j - \varepsilon^i) \Pr(\varepsilon^j > \varepsilon^i) + \text{Cov}(\varepsilon^j, -(\varepsilon^j - \varepsilon^i)) \Pr(\varepsilon^j < \varepsilon^i)$$

$$= \text{Cov}(\varepsilon^j, \varepsilon^j) \Pr(\varepsilon^j > \varepsilon^i) - \text{Cov}(\varepsilon^j, \varepsilon^j) \Pr(\varepsilon^j < \varepsilon^i)$$

$$= \text{Cov}(\varepsilon^j, \varepsilon^j) \left[ \Pr(\varepsilon^j > \varepsilon^i) - \Pr(\varepsilon^j < \varepsilon^i) \right] = 0$$

The last equation holds because $\varepsilon^j$ and $\varepsilon^i$ are independent and identically distributed normal random variable with zero mean and equal variance, so that $\Pr(\varepsilon^j > \varepsilon^i) - \Pr(\varepsilon^j < \varepsilon^i) = 0$. ■

**The method of undetermined coefficients**

Starting from equation (24), we guess that the equilibrium price is a linear function of the past observable fundamental $\theta_{t-1}$, the current unobservable fundamental $\theta_t$, preference shock $a_t$, and the difference in households private signals $i_t$; i.e.,

$$p_t = b_0 + b_\theta \theta_{t-1} + b_{\eta} \eta_t + b_a a_t + b_i i_t.$$  

(50)

where $b_0, b_\theta, b_{\eta}, b_a$ and $b_i$ are undetermined coefficients. It is convenient to rewrite equation (50) as

$$p_t = b_{\eta} \eta_t + b_a a_t + b_i i_t + X_t,$$  

(51)

where

$$X_t \equiv b_0 + b_\theta \theta_{t-1},$$

is non-stochastic. Defining

$$\hat{p}_t \equiv \frac{p_t - X_t}{b_{\eta}},$$

equation (51) can be written as

$$\hat{p}_t = \eta_t + \delta_t,$$

where

$$\delta_t = \frac{b_a}{b_{\eta}} a_t + \frac{b_i}{b_{\eta}} i_t.$$  

(52)
Under the assumption made on the distribution of $\alpha_t$, $\delta_t$ is normally distributed,

$$\delta_t \sim \mathcal{N}\left( \frac{b_{t+1}}{b_n}, \left( \frac{b_{t+1}}{b_n} \right)^2 \sigma_\alpha^2 + \left( \frac{b_{t+1}}{b_n} \right)^2 \sigma_\delta^2 \right)$$

and, as a consequence $\hat{p}_t$ is also normally distributed,

$$\hat{p}_t \sim \mathcal{N}\left( \frac{b_{t+1}}{b_n}, \sigma_\eta^2 + \frac{b_n^2 \sigma_\alpha^2 + b_n^2 \sigma_\delta^2}{b_n^2} \right). \tag{53}$$

### The inference problem

Household $j$ estimates the unknown fundamental $\theta_t$ by solving a standard filtering problem, based on the normally distributed (a) private signal, $w_i^j$, (b) exogenous public signal, $\theta_{t-1}$, and (c) endogenous public signal, $\hat{p}_t$. Recalling that

$$\begin{align*}
\theta_t &= \rho \theta_{t-1} + \eta_t, \\
w_i^j &= \theta_t + \epsilon_i^j, \\
\hat{p}_t &= \eta_t + \delta_t,
\end{align*}$$

and using (53) and Lemma 1, the log-likelihood function can be written as

$$L = -\frac{1}{2\sigma_\eta^2} \left( \rho \theta_{t-1} - E_i^j \theta_t \right)^2 - \frac{1}{2\sigma_\epsilon^2} \left( w_i^j - E_i^j \theta_t \right)^2 - \frac{1}{2\sigma_\delta^2} \left( \hat{p}_t - E_i^j \eta_t \right)^2.$$ 

Thus, the optimal filtering solves the following first order condition,

$$-\frac{1}{\sigma_\eta^2} \left( -E_i^j \eta_t \right) + \frac{1}{\sigma_\epsilon^2} \left( w_i^j - \rho \theta_{t-1} - E_i^j \eta_t \right) + \frac{1}{\sigma_\delta^2} \left( \hat{p}_t - E_i^j \eta_t \right) = 0,$$

or,

$$E_i^j \eta_t = \frac{\sigma_\eta^2 \delta^2 \left( w_i^j - \rho \theta_{t-1} \right) + \sigma_\delta^2 \sigma_\epsilon^2 \hat{p}_t}{\sigma_\eta^2 \delta^2 + \sigma_\delta^2 \sigma_\epsilon^2 + \sigma_\eta^2 \sigma_\epsilon^2}.$$ 

The best linear estimate of $\theta_t$ is therefore,

$$E_i^j \theta_t = (\pi_1 + \pi_3) \rho \theta_{t-1} + \pi_2 w_i^j + \pi_3 \hat{p}_t, \tag{54}$$

where

$$\begin{align*}
\pi_1 &= \frac{\sigma_\eta^2 \sigma_\delta^2}{\sigma_\eta^2 \sigma_\delta^2 + \sigma_\delta^2 \sigma_\epsilon^2 + \sigma_\eta^2 \sigma_\epsilon^2}, \\
\pi_2 &= \frac{\sigma_\delta^2 \sigma_\epsilon^2}{\sigma_\eta^2 \sigma_\delta^2 + \sigma_\delta^2 \sigma_\epsilon^2 + \sigma_\eta^2 \sigma_\epsilon^2}, \\
\pi_3 &= \frac{\sigma_\eta^2 \sigma_\epsilon^2}{\sigma_\eta^2 \sigma_\delta^2 + \sigma_\delta^2 \sigma_\epsilon^2 + \sigma_\eta^2 \sigma_\epsilon^2}.
\end{align*} \tag{55, 56, 57}$$

Notice that if $\sigma_\delta^2 \to \infty$ (for example, because $\sigma_\alpha^2 \to \infty$, i.e., the preference shock has a very large variance) then

$$\begin{align*}
\pi_1 &\to \frac{\sigma_\epsilon^2}{\sigma_\eta^2 + \sigma_\epsilon^2} = 1 - \lambda, \\
\pi_2 &\to \frac{\sigma_\eta^2}{\sigma_\epsilon^2 + \sigma_\eta^2} = \lambda \quad \text{and} \quad \pi_3 \to 0.
\end{align*}$$

In other words, households have nothing to learn from the equilibrium price and the weights used for inferring the unobservable aggregate fundamental are the same as in Section 5.
The equilibrium price

To solve for the equilibrium price we follow the same steps as in Appendix III. By guessing that $\tilde{E}_{tp_{t+1}} = \phi \pi_2 i_t$, we have

\[
\begin{align*}
E_{tp_{t+1}}^j &= \frac{r}{1 + r - \rho} E_{tp_{t+1}}^j - s + \frac{\phi \pi_2 I}{r}, \\
\tilde{E}_{tp_{t+1}} &= \frac{r}{1 + r - \rho} \tilde{E}_{tp_{t+1}} - s + \frac{\phi \pi_2 I}{r}, \\
\tilde{E}_{tp_{t+1}} &= \frac{r}{1 + r - \rho} \tilde{E}_{tp_{t+1}}.
\end{align*}
\]

Moreover, using (54) the last two equations can be written as:

\[
\begin{align*}
\tilde{E}_{tp_{t+1}} &= \phi (\rho \theta_{t-1} + \pi_2 \eta_t + \pi_3 \tilde{p}_t) - s + \frac{\phi \pi_2 I}{r}, \\
\tilde{E}_{tp_{t+1}} &= \phi \pi_2 i_t.
\end{align*}
\]

The second line confirms the claim that $\tilde{E}_{tp_{t+1}} = \phi \pi_2 i_t$. Inserting now $\tilde{E}_{tp_{t+1}}$ in (17), the equilibrium price becomes

\[
\begin{align*}
p_t &= \frac{r}{1 + r} (\rho \theta_{t-1} + \eta_t + a_t - s) + \frac{1}{1 + r} \left( \phi \rho \theta_{t-1} + \phi \pi_2 \eta_t + \phi \pi_3 \tilde{p}_t - s + \frac{\phi \pi_2 I}{r} \right) + \frac{\phi \pi_2 i_t}{1 + r}
\end{align*}
\]

from which it follows,

\[
p_t = \frac{\phi}{1 + r} \left( \frac{\pi_2 I}{r} - \frac{\pi_2 b_0}{\eta_t} \right) - s + \frac{r + \phi \pi_2 I}{1 + r} \rho \theta_{t-1} + \frac{r + \phi \pi_2 I}{1 + r} \eta_t + \frac{r + \phi \pi_2 I}{1 + r} a_t + \frac{\phi \pi_2 I}{1 + r}
\]

The undetermined coefficients can therefore be written as

\[
\begin{align*}
b_0 &= \frac{\phi \pi_2}{r(1 + r)} I - s \\
b_0 &= \frac{r + \phi \pi_2}{1 + r} \\
b_0 &= \frac{r + \phi (\pi_2 + \pi_3)}{1 + r} \\
b_0 &= \frac{r + \phi \pi_2}{1 + r} \left( 1 + \frac{\phi \pi_3}{r + \phi \pi_2} \right) \\
b_0 &= \frac{\phi \pi_2}{1 + r} \left( 1 + \frac{\phi \pi_3}{r + \phi \pi_2} \right)
\end{align*}
\]

and the equilibrium price as,

\[
\begin{align*}
p_t &= \frac{\phi \pi_2}{r (1 + r)} I - s + \frac{r + \phi \pi_2}{1 + r} \rho \theta_{t-1} + \frac{r + \phi (\pi_2 + \pi_3)}{1 + r} \eta_t \\
&\quad + \frac{r}{1 + r} \left( 1 + \frac{\phi \pi_3}{r + \phi \pi_2} \right) a_t + \frac{\phi \pi_2}{1 + r} \left( 1 + \frac{\phi \pi_3}{r + \phi \pi_2} \right) i_t.
\end{align*}
\]

or, after some manipulation:

\[
p_t = p_t^* + \pi_2 \Upsilon_t + \pi_3 \Phi_t.
\]

As in section 4 and 5, $p_t^*$ denotes the fundamental price, and $\Upsilon_t$ measures the degree of dispersion in beliefs. The new term,

\[
\Phi_t = \frac{\phi}{1 + r} (\theta_t - \rho \theta_{t-1}) + \frac{r \phi}{(1 + r)(r + \phi \pi_2)} a_t + \frac{\phi^2 \pi_2}{(1 + r)(r + \phi \pi_2)} i_t
\]

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captures, instead, the degree of magnification of shocks induced by the process of learning from price.

Finally, since

\[
\sigma_\delta^2 = \left( \frac{b_a}{b_{ni}} \right)^2 \sigma_n^2 + \left( \frac{b_i}{b_{ni}} \right)^2 \sigma_i^2,
\]

(58)

\(\pi_1, \pi_2\) and \(\pi_3\) are functions of \(\sigma_\delta^2\), which, in turn, depends on \(b_{ni}, b_a\) and \(b_i\). To pin down these undetermined coefficients, it is thus necessary to use equations (56), (57) and (58). This leads to

\[
b_{ni} = \frac{r}{1 + r} + \frac{\phi}{1 + r} \left( \frac{\sigma_n^2 \left( \frac{b_i^2 \sigma_a^2 + b_n^2 \sigma_i^2}{b_{ni}} \right) + \sigma_n^2 \sigma_i^2}{(\sigma_a^2 + \sigma_n^2) \left( \frac{b_i^2 \sigma_n^2 + b_n^2 \sigma_i^2}{b_{ni}} \right) + \sigma_n^2 \sigma_i^2} \right),
\]

\[
b_{bi} = \frac{\phi \pi_2}{r} = \frac{\phi}{r} \left( \frac{\sigma_n^2 \left( \frac{b_i^2 \sigma_a^2 + b_n^2 \sigma_i^2}{b_{ni}} \right)}{(\sigma_a^2 + \sigma_n^2) \left( \frac{b_i^2 \sigma_n^2 + b_n^2 \sigma_i^2}{b_{ni}} \right) + \sigma_n^2 \sigma_i^2} \right),
\]

and

\[
b_{ni} = b_n + b_i,
\]

which define a system of three equations in the three unknowns, \(b_{ni}, b_n\) and \(b_i\). Unfortunately, this system of equations does not admit closed-form solutions. However, numerical values can easily be computed.
Figure 1: Real House Price Index in the US

Source: OFHEO and BLS
Volatility = 0.0127 (0.001) + 1.3034 (0.217) × S.D. dispersion of beliefs

Figure 3: House price volatility and volatility of dispersion in beliefs in 321 MSAs

\[ \text{Volatility} = 0.0127 (0.001) + 1.3034 (0.217) \times \text{S.D. dispersion of beliefs} \quad R^2 = 0.12 \]
<table>
<thead>
<tr>
<th>Description</th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>Minimum</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Delta P_t )</td>
<td>.004021</td>
<td>.044600</td>
<td>-.297416</td>
<td>.233374</td>
</tr>
<tr>
<td>( \Delta \theta_t )</td>
<td>.014956</td>
<td>.025590</td>
<td>-.168891</td>
<td>.179146</td>
</tr>
<tr>
<td>( \Delta Pop_t )</td>
<td>.011968</td>
<td>.015082</td>
<td>-.110857</td>
<td>.133832</td>
</tr>
<tr>
<td>( Disp_t )</td>
<td>.025007</td>
<td>.012729</td>
<td>.003807</td>
<td>.196372</td>
</tr>
<tr>
<td>( Pos_t )</td>
<td>.012200</td>
<td>.010211</td>
<td>.000029</td>
<td>.192546</td>
</tr>
<tr>
<td>( Neg_t )</td>
<td>-.011717</td>
<td>.008969</td>
<td>-.091725</td>
<td>-.000014</td>
</tr>
</tbody>
</table>

Statistics shown are for annual observations pooled across MSAs. The house price data comes from the Office of Federal Housing Enterprise Oversight. Income and Population data are from the Regional Economic Accounts of the Bureau of Economic Analysis. Variables \( Disp_t \), \( Pos_t \), and \( Neg_t \) are computed as described in Section 7.1 and 7.5 using (per employed) earnings data for one-digit industries from the Regional Economic Accounts of the Bureau of Economic Analysis. The deflator for nominal variables is the aggregate CPI index from the Bureau of Labor Statistics.
<table>
<thead>
<tr>
<th>Independent Variable</th>
<th>Dependent variable: $\Delta P_t$ --- Sample: 1980-2000</th>
</tr>
</thead>
</table>
| $\Delta \theta_t$    | 0.426***  
                     | (0.041)  
| $\Delta \theta_{t-1}$| 0.639***  
                     | (0.044)  
| $\text{Disp}_t$      | 0.199** 
                     | (0.092)  
| Year effects          | Yes  
| MSA effect            | Yes  

Obs  
N. Units  
R2 within  
R2 overall

5393  
350  
0.240  
0.242

3285  
306  
0.251  
0.268

Fixed effects estimates of the log change of the real house prices, $\Delta P_t$. $\Delta \theta_{t-j}$ is the $j$-period lagged value of the log change in real per capita personal income. $\text{Disp}_t$ is the proxy of dispersion of beliefs, computed as explained in Section 7.1. Standard errors (in parenthesis below coefficients) are clustered at the MSA level. *** denotes significance at 1% level, ** denotes significance at 5% level.
Table 3
Dispersion of beliefs on house price growth

<table>
<thead>
<tr>
<th>Independent Variable</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
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</thead>
<tbody>
<tr>
<td>(\Delta P_{t-1})</td>
<td>0.511***</td>
<td>0.494***</td>
<td>0.540***</td>
<td>0.565***</td>
<td>0.373***</td>
<td>0.442***</td>
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<tr>
<td></td>
<td>(0.014)</td>
<td>(0.017)</td>
<td>(0.024)</td>
<td>(0.023)</td>
<td>(0.025)</td>
<td>(0.030)</td>
</tr>
<tr>
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<tr>
<td></td>
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<td>(0.037)</td>
<td></td>
<td></td>
<td>(0.031)</td>
<td></td>
</tr>
<tr>
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<td>(0.016)</td>
<td></td>
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<td>0.375***</td>
<td>0.347***</td>
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<tr>
<td></td>
<td>(0.028)</td>
<td>(0.038)</td>
<td>(0.029)</td>
<td>(0.038)</td>
<td>(0.035)</td>
<td>(0.037)</td>
</tr>
<tr>
<td>(\Delta \theta_{t-1})</td>
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<td>0.366***</td>
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</tr>
<tr>
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<td>(0.047)</td>
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<td>(0.037)</td>
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<tr>
<td>((P/\theta)_{t-1})</td>
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<td>-0.165***</td>
<td>-0.141***</td>
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</tr>
<tr>
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<td></td>
<td></td>
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<td>(0.008)</td>
<td>(0.007)</td>
<td></td>
</tr>
<tr>
<td>(\Delta \text{Pop}_t)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1.353***</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1.197***</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(0.123)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(0.103)</td>
</tr>
<tr>
<td>(\text{Disp}_t)</td>
<td>0.140**</td>
<td>0.209***</td>
<td>0.133**</td>
<td>0.154***</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.068)</td>
<td>(0.060)</td>
<td>(0.062)</td>
<td>(0.057)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Year effects</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>MSA effect</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Obs</td>
<td>4466</td>
<td>2616</td>
<td>5166</td>
<td>3127</td>
<td>2616</td>
<td>3127</td>
</tr>
<tr>
<td>N. Units</td>
<td>350</td>
<td>289</td>
<td>350</td>
<td>302</td>
<td>289</td>
<td>302</td>
</tr>
<tr>
<td>R2 within</td>
<td>0.553</td>
<td>0.565</td>
<td>0.521</td>
<td>0.541</td>
<td>0.618</td>
<td>0.586</td>
</tr>
<tr>
<td>R2 overall</td>
<td>0.583</td>
<td>0.603</td>
<td>0.386</td>
<td>0.419</td>
<td>0.533</td>
<td>0.452</td>
</tr>
</tbody>
</table>

Fixed effects estimates of the log change of the real house prices, \(\Delta P\). \(\Delta P_{t-j}\) is the \(j\)-period lagged value of the log change in house price. \(\Delta \theta_{t-j}\) is the \(j\)-period lagged value of the log change in real per capita personal income. \((P/\theta)_{t-j}\) is the one period lagged log-ratio of the house price to per-capita income. \(\Delta \text{Pop}_t\) is the log difference of MSA population. \(\text{Disp}_t\) is the proxy of dispersion of beliefs computed as explained in Section 7.1. Standard errors (in parenthesis below coefficients) are clustered at the MSA level. *** denotes significance at 1% level, ** denotes significance at 5% level.
Table 4
Dispersion of beliefs on house price growth in MSAs with low and high supply elasticity.

<table>
<thead>
<tr>
<th>Independent Variable</th>
<th>Low Supply Elasticity</th>
<th>High Supply Elasticity</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
</tr>
<tr>
<td>(\Delta P_{t-1})</td>
<td>0.427***</td>
<td>0.507***</td>
</tr>
<tr>
<td></td>
<td>(0.038)</td>
<td>(0.050)</td>
</tr>
<tr>
<td>(\Delta P_{t-2})</td>
<td>0.334***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.037)</td>
<td></td>
</tr>
<tr>
<td>(\Delta P_{t-3})</td>
<td>-0.098***</td>
<td>-0.163***</td>
</tr>
<tr>
<td></td>
<td>(0.034)</td>
<td>(0.053)</td>
</tr>
<tr>
<td>(\Delta \theta_t)</td>
<td>0.693***</td>
<td>0.720***</td>
</tr>
<tr>
<td></td>
<td>(0.145)</td>
<td>(0.168)</td>
</tr>
<tr>
<td>(\Delta \theta_{t-1})</td>
<td>0.089</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.167)</td>
<td></td>
</tr>
<tr>
<td>((P/\theta)_{t-1})</td>
<td></td>
<td>-0.152***</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.014)</td>
</tr>
<tr>
<td>(\Delta \text{Pop}_t)</td>
<td>1.403***</td>
<td>1.131***</td>
</tr>
<tr>
<td></td>
<td>(0.268)</td>
<td>(0.145)</td>
</tr>
<tr>
<td>\text{Disp}_t</td>
<td>0.416*</td>
<td>0.499**</td>
</tr>
<tr>
<td></td>
<td>(0.206)</td>
<td>(0.223)</td>
</tr>
</tbody>
</table>

Year effects Yes Yes Yes Yes
MSA effect Yes Yes Yes Yes

Obs 368 436 377 446
N. Units 31 31 36 38
R2 within 0.727 0.679 0.704 0.665
R2 overall 0.645 0.597 0.401 0.433

Fixed effects estimates of the log change of the real house prices, \(\Delta P\); \(\Delta P_{t-j}\) is the \(j\)-period lagged value of the log change in house price. \(\Delta \theta_{t-j}\) is the \(j\)-period lagged value of the log change in real per capita personal income. \((P/\theta)_{t-1}\) is the one period lagged log-ratio of the house price to per-capita income. \(\Delta \text{Pop}_t\) is the log difference of MSA population. \text{Disp}_t is the proxy of dispersion of beliefs, computed as discussed in Section 7.1. Low supply elasticity indicates MSA with a Saiz (2008) index of supply elasticity below the median value. Standard errors (in parenthesis below coefficients) are clustered at the MSA level. *** denotes significance at 1% level, ** denotes significance at 5% level.
### Table 5
House price volatility and dispersion of beliefs

<table>
<thead>
<tr>
<th>Independent Variable</th>
<th>Dependent variable: $\sigma(\Delta P)$ --- Sample: 1980-2000</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma(\text{Disp})$</td>
<td>1.3034*** (0.217) 1.1229*** (0.227)</td>
</tr>
<tr>
<td>$\sigma(\theta)$</td>
<td>0.1002 (0.149)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Obs 321</th>
<th>321</th>
</tr>
</thead>
<tbody>
<tr>
<td>R2 overall</td>
<td>0.12</td>
<td>0.12</td>
</tr>
</tbody>
</table>

Cross-sectional OLS estimates of the volatility of change in house prices $\sigma(\Delta P)$ against the volatility of dispersion in beliefs $\sigma(\text{Disp})$ for each MSA. $\sigma(\theta)$ denotes the standard deviation of aggregate MSA income. Robust standard errors are in parenthesis. *** denotes significance at 1% level.
<table>
<thead>
<tr>
<th>Independent variables</th>
<th>Dependent variable: $\Delta P_t$ --- Sample: 1980-2000</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
</tr>
<tr>
<td>$\Delta P_{t-1}$</td>
<td>0.505***</td>
</tr>
<tr>
<td></td>
<td>(0.015)</td>
</tr>
<tr>
<td>$\Delta P_{t-2}$</td>
<td>0.232***</td>
</tr>
<tr>
<td></td>
<td>(0.035)</td>
</tr>
<tr>
<td>$\Delta P_{t-3}$</td>
<td>-0.105***</td>
</tr>
<tr>
<td></td>
<td>(0.013)</td>
</tr>
<tr>
<td>$\Delta \theta_t$</td>
<td>0.297***</td>
</tr>
<tr>
<td></td>
<td>(0.028)</td>
</tr>
<tr>
<td>$\Delta \theta_{t-1}$</td>
<td>0.342***</td>
</tr>
<tr>
<td></td>
<td>(0.034)</td>
</tr>
<tr>
<td>$(P/\theta)_{t-1}$</td>
<td>0.161***</td>
</tr>
<tr>
<td></td>
<td>(0.006)</td>
</tr>
<tr>
<td>$\Delta P_{\text{Pop}}$</td>
<td>1.194***</td>
</tr>
<tr>
<td></td>
<td>(0.100)</td>
</tr>
<tr>
<td>$\text{Pos}_t$</td>
<td>0.229***</td>
</tr>
<tr>
<td></td>
<td>(0.071)</td>
</tr>
<tr>
<td>$\text{Neg}_t$</td>
<td>0.058</td>
</tr>
<tr>
<td></td>
<td>(0.061)</td>
</tr>
</tbody>
</table>

Year effects: Yes, Yes, Yes, Yes
MSA effect: Yes, Yes, Yes, Yes

Obs: 4391, 4391, 5076, 5076
N. Units: 350, 350, 350, 350
R2 within: 0.55, 0.60, 0.53, 0.56
R2 overall: 0.58, 0.53, 0.38, 0.41

Fixed effects estimates of the log change of the real house prices, $\Delta P$. $\Delta P_{t-j}$ is the $j$-period lagged value of the log change in house price. $\Delta \theta_{t-j}$ is the $j$-period lagged value of the log change in real per capita personal income. $(P/\theta)_{t-1}$ is the one period lagged log-ratio of the house price to per-capita income. $\Delta P_{\text{Pop}}$ is the log difference of MSA population. Pos, and Neg, refer to positive and negative "news" shocks, computed as explained in Section 7.5. Standard errors (in parenthesis below coefficients) are clustered at the MSA level. *** denotes significance at 1% level, ** denotes significance at 5% level.