Evaluating Monetary Policy Rules in Estimated Forward-Looking Models: A Comparison of US and German Monetary Policies

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Abstract

In this paper, we estimate two small macroeconomic models for the US and Germany and we compare the implied optimal monetary policy rules. We consider a model which has been used extensively in the literature (including a Phillips curve, an I-S curve, and a monetary policy rule) and which incorporates some forward-looking features. We estimate this model over the period from 1968 to 1998, using the full-information maximum-likelihood procedure, so that forward-looking expectations are fully model-consistent. On the basis of stability tests, the model is shown to have some robustness with respect to the Lucas critique. Then, we compute optimal monetary policy rules in the class of Taylor rules with interest-rate smoothing. We find that optimal policies imply a strong degree of interest-rate smoothing. Moreover, German optimal monetary policy is found to require a more persistent and slightly stronger response to inflation and output than the US optimal policy. Last, we provide evidence on the robustness of the German optimal monetary policy to parameter uncertainty.

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1 Introduction

In this paper, we aim at computing optimal monetary policy rules for the US as well as for Germany, conditional on an estimated small, macroeconomic model with some forward-looking components. The optimal monetary policy is expressed as a Taylor-type rule, with the policy-rule parameters depending on the parameters of the economy and on the parameters of the central-bank loss function. The central-bank loss function is designed in the form of a weighted average of inflation, output-gap and interest-rate variances. For various sets of weights, we compute the ex-post optimal rule, consistent with the estimated macroeconomic model. This approach has been extensively used to investigate monetary policy rules in the case of the US (see, e.g., Taylor, 1979, Fair and Howrey 1996, or the contributions in the Taylor, 1999b, volume).

Our contribution is twofold. First, we address the issue of the optimal monetary policy in Europe, with a focus on Germany. This paper is among the first to perform evaluation of the optimal monetary policy on European data using an estimated model with forward-looking components.[1] A distinctive feature of our approach is the use of German data rather than aggregate euro-area data in the empirical estimations. We thus consider Germany to be a representative country.[2] The main advantage of this choice is to avoid the bias of aggregating countries with different historical policy regimes. Focusing on Germany rather than on the euro area, we expect the policy rule to be more precisely estimated, in order to provide consistent estimates in a model with rational expectations. Since most of the research has concentrated on the US, we view the US model and optimal monetary policy rules as benchmarks for interpreting our results on German data. To ensure comparability, we consider the same specification and sample period for the two countries.

Second, we investigate the gap between estimated and optimal monetary policy rules, for the US and Germany. It is noteworthy that the macroeconomic model is estimated without assuming an optimal behavior of the central bank, so that we are not able to recover the central bank preferences from the estimated policy rule. However it is interesting to explore rationales for such the observed gap between estimated and optimal rules. We focus, for this purpose, on the robustness of optimal rules to parameter uncertainty in Germany. Indeed, the gap may be explained by an imprecise estimation or a wrong specification of the macroeconomic model. We thus consider various alterations of the key model parameters.

The estimated models we use for analysis have the following common structure. Inflation and output dynamics are described using Phillips and I-S curves, respectively. The output gap is assumed to depend on the sequence of expected future short-term real interest rates. Monetary policy is represented using a reaction function, i.e. a Taylor-type rule for the short-term nominal rate. While incorporating some forward-looking features, this model is designed to provide a good fit of the data. Indeed, many studies have highlighted that models with strong micro-foundations (and with a purely forward-looking structure) are not able to capture the dynamics of the data very well (Estrella and Fuhrer, 1999). Therefore, an increasing literature promotes the use of hybrid specifications, which combine backward as well as forward-looking behavior.

However, such hybrid models give rise to the Lucas critique, since estimated parameters are not likely to be structural ones. To assess vulnerability of our monetary-policy evaluation

[1] Taylor (1999a) and Artus, Penot, and Pollin (1999) evaluate optimal policy in estimated backward-looking models. Coenen and Wieland (2000) use a euro-area forward-looking model to analyze of the standard (non-optimal) Taylor rule.

[2] Rudebusch and Svensson (1999b) used a model estimated on US data to discuss the eurosystem monetary strategy.

exercise to the Lucas critique, we perform tests for stability of non-policy parameters. Since for both countries non-policy parameters are shown to be stable over a sample with monetary-policy shifts, the estimated macroeconomic model can be used to further compute optimal monetary policy.

The paper is organized as follows. Section 2 provides a short description of the theoretical framework. Section 3 presents the full-information maximum-likelihood (FIML) estimates of the model for the US and Germany over the 1968-98 period. The German economy is found to be somewhat less reactive than the US economy. Over the recent period, the Bundesbank's monetary policy appears to have been more responsive to changes in the output gap than the Fed's, yet more smoothed. The inflation weight in the two reaction functions are similar. We also perform stability tests, which provide some evidence on the robustness of the model to the Lucas critique. Since macroeconomic (or non-policy) parameters are found to be stable over the sample period, we then address, in Section 4, the issue of the optimal monetary policy, conditionally on the estimated model. We compute the optimal monetary frontiers for the US and Germany. The Bundesbank's optimal response is shown to be more smoothed than the Fed's one, a result which is consistent with the estimated rules. However, for both countries, the optimal monetary policy implies larger responses to inflation and output gap than the estimated policy. In order to address the issue of the gap between the estimated and the optimal policy rules, we evaluate the robustness of the computed German optimal policy rules to parameter uncertainty. Section 5 presents our main conclusions.

2 The model

The first step of our work is to design a model which specifies the dynamics of the instrument and targets of monetary authorities. We adopt a model with some forward-looking features, while providing a rather good fit of the data. Inflation dynamics is described as a standard backward-looking Phillips curve. Output-gap dynamics is an I-S curve, in which the driving term is the long-term real interest rate. Last, the short-term rate is modelled as a Taylor-rule type reaction function. We now briefly describe these specifications.

2.1 The Phillips and I-S curves

Prices are set according to an augmented Phillips curve, which can be interpreted as an aggregate supply equation. Various forms of the Phillips curve have been discussed in the policy-rule literature. We adopt the purely backward-looking specification suggested by Rudebusch and Svensson (1999a):

$$\pi_t = \sum_{k=1}^{K} \alpha_{\pi k}\pi_{t-k} + \alpha_y y_{t-1} + \alpha_0 + \varepsilon_t \quad (1)$$

where $\pi_t = 4\,(p_t - p_{t-1})$ is the annualized quarter-on-quarter inflation rate at time t, with p_t the domestic (log) price index. The output gap (y_t) is defined as $y_t = y_t^d - y_t^n$, where y_t^d is the (log) aggregate demand and y_t^n the (log) potential output. The innovation ε_t is a zero-mean iid cost-push supply shock. Parameter α_0 is a constant term. Denoting $\alpha_\pi = \sum_{k=1}^{K} \alpha_{\pi k}$, we constraint $\alpha_\pi = 1$ to avoid a long-run inflation/output gap *level* trade-off, since we are interested in the long-run inflation/output-gap *volatility* trade-off.[3]

[3] Estimating this model freely yields $\alpha_\pi = 0.941$ (the Wald statistic for the null hypothesis $\alpha_\pi = 1$ is equal to 1.702, with a p-value equal to 0.636) for US data and 0.851 (the Wald statistic is equal to 4.032, with a p-value equal to 0.133) for German data.

An alternative to this traditional Phillips curve is the 'new' Phillips curve, which can be derived from an optimization model of price setting under nominal rigidities (see, e.g., Galí, 2000, for a thorough presentation). In such an approach, inflation is found to have a purely forward-looking dynamics. An encompassing specification is the hybrid Phillips curve which has been put forward in the recent empirical literature (Fuhrer and Moore, 1995a, Galí and Gertler, 1999). Fuhrer (1997b) has highlighted the unimportance of the forward-looking component of inflation.[4] In this paper, we proceed assuming that inflation dynamics can be described by a pure backward-looking specification. Investigating further the relevance of the forward-looking component is left for further research.

Aggregate demand is described as an I-S curve. As in Fuhrer and Moore (1995a), output gap is assumed to depend on the long real rate, denoted ρ_t:

$$y_t = \beta_{y1} y_{t-1} + \beta_{y2} y_{t-2} - \beta_\rho \rho_t + \beta_0 + \eta_t \tag{2}$$

where η_t is a demand shock. Lagged terms are introduced in the equation to capture persistence in the output-gap dynamics. The interest-rate elasticity in the I-S curve is crucial for the dynamics of the full model, since monetary policy affects inflation through the impact of the interest rate on the output gap.

The long-term real interest rate is the yield to maturity on a hypothetical long-term real bond. The expectations hypothesis of the term structure implies that the expected holding-period return on a long real bond equals the short real rate, denoted r_t:

$$\rho_t - D\left(E_t \rho_{t+1} - \rho_t\right) = r_t \tag{3}$$

where r_t is defined as $r_t = i_t - E_t \pi_{t+1}$, with i_t the short nominal rate, i.e. the monetary policy instrument. D is a constant approximation to Macaulay's duration.[5] Throughout the paper, $E_t x_{t+1}$ denotes the conditional expectation on date t of variable x_{t+1}. Expectations are assumed to be model-consistent. Solving this equation recursively for ρ_t allows to express the long real rate as an exponentially weighted moving average of the expected short real rates

$$\rho_t = \frac{1}{1+D} \sum_{\tau=0}^{\infty} \left(\frac{D}{1+D}\right)^{\tau} E_t r_{t+\tau}. \tag{4}$$

Hence, the inclusion of the long real rate makes the I-S curve (2) a hybrid equation, combining backward as well as a forward-looking components. Future price developments affect current output through their impact on expected future short real rates. Note that the hybrid specification (2) can be motivated on theoretical grounds by rule-of-thumb consumers or by habit formation (Fuhrer, 2000).

2.2 The monetary policy reaction function

The design of monetary policy rules is a widely discussed issue (see Taylor, 1999b, for a recent survey). The research has developed along two lines. The first strand of the literature deals with the empirical estimation of reaction functions (Taylor, 1993, Clarida, Galí, and

[4] Galí and Gertler (1999) provided some evidence against this result, suggesting that the fraction of forward-looking price setters may be well above 50 percent. Recent work (Ruud and Whelan, 2001), however, criticized this result, showing that the GMM estimates of Galí and Gertler are biased in favor of forward-lookingness.

[5] In empirical application, we assume a constant duration. The average duration over the sample of the 10-year corporate bond is about 7 years for the US as well as for Germany. Therefore, we set $D = 28$ in the empirical estimates.

Gertler, 1998 and 2000, Judd and Rudebusch, 1998). The second strand investigates the determination of the optimal monetary policy rule (Svensson, 1997, 2000, Williams, 1999).

In the estimation part of the paper, we estimate a dynamic Taylor-type reaction function. The short nominal rate is related to its own lagged value, to expected and lagged inflation and to lagged output gap:

$$i_t = (1-\delta_i)\left[E_{t-1}\pi_t + (\delta_\pi - 1)\pi_{t-1} + \delta_y y_{t-1} + \delta_0\right] + \delta_i i_{t-1} + u_t. \qquad (5)$$

In the short run, the central bank moves its short nominal rate as a fraction of the gap between the lagged real rate $(i_{t-1} - E_{t-1}\pi_t)$ and the Taylor rule defined as $(\delta_\pi \pi_{t-1} + \delta_y y_{t-1} + \delta_0)$. We assume that the contemporaneous inflation rate and output gap are not known by the central bank. This assumption is consistent with the claim of McCallum and Nelson (1998), who promoted the use of information that is available to the central bank. Therefore, inflation and output gap are lagged to account for the delay in the observation of the data by the central bank. In the context of our simultaneous equation model, inflation expectations are model-consistent, so that $E_{t-1}\pi_t = \sum_{k=1}^{K} \alpha_\pi \pi_{t-k} + \alpha_y y_{t-1}$ is known at date $t-1$.

The reaction function also includes a lag of the short rate to capture the high degree of persistence in the interest-rate series. Sack and Wieland (2000) reviewed motivations for smoothing interest rates. They mentioned measurement errors of macroeconomic variables as a possible explanation. In models with forward-looking expectations, inertial policy is also shown to be optimal. This feature, as underlined by Levin, Wieland, and Williams (1999) and Williams (1999), derives from the forward-looking dynamics of interest rates. The same impact on the long real rate can be attained through a large but short-lived change in the short rate or through a small but expected to be persistent change in short rate. With the latter solution, the short-rate variability is kept moderate (see, for instance, Williams, 1999, Woodford, 1999, and Section 4 of the present paper).

Note that, since we do not assume that the policy was optimal over the period under study, we do not attempt to reveal central-bank preferences. In Section 4, we will compute optimal policy rules on an ex-post basis, i.e. conditional on the estimated non-policy parameters, and for alternative sets of preference parameters. We will focus on optimal simple rule of the form (5).

2.3 Long-run properties of the model

The model described above implicitly assumes that the inflation rate, the short nominal rate, and the output gap are stationary variables. The structure of the model implies that the order of integration of inflation and short nominal rate is determined by monetary policy.

If monetary authorities have a sufficient response to deviations of inflation from its target, inflation and short nominal rate are stationary. As shown in Section 3.2, ensuring stationarity of the system implies $(\delta_\pi - 1)$ to be positive (see also Taylor, 1999a). Note that imposing the sum of autoregressive parameters in the inflation equation (α_π) to be equal to one does not contradict the stationarity assumption for the inflation rate. Stationarity has to be evaluated within the full model. As long as the central bank overreacts to a shock on inflation, a negative effect of the long real rate on the output gap implies a stationary inflation, even though $\alpha_\pi = 1$. For reasons detailed in the next section, we will assume that the target of inflation is constant with a possible break, so that the inflation rate is, in fact, a stationary process with a deterministic break, rather than a random walk.

Imposing stationarity, we are now able to compute steady-state values for the model variables. The steady-state value of the output gap is, by construction, equal to 0. Since

$\alpha_\pi = 1$, the constant term α_0 has to be set equal to 0. This is straightforward from the long-run solution of the Phillips curve (equation (1)), since we have: $\pi^* = \pi^* + \alpha_y y^* + \alpha_0$, where x^* denotes the steady-state value for x. The long-run solution of the I-S curve (equation (2)) implies that the steady-state value for the long real rate is constant: $\rho^* = \beta_0 / \beta_\rho$. Last, the long-run solution of the reaction function (equation (5)) for the short real rate is simply

$$(1 - \delta_i)\, i^* = (1 - \delta_i)\, (\delta_\pi \pi^* + \delta_y y^* + \delta_0)$$

so that $i^* = \delta_\pi \pi^* + \delta_0$. Therefore, since $i^* = \rho^* + \pi^*$, the steady-state value for the inflation rate is

$$\pi^* = \frac{\rho^* - \delta_0}{\delta_\pi - 1} = \frac{\beta_0 - \beta_\rho \delta_0}{\beta_\rho\, (\delta_\pi - 1)}$$

and the steady-state value for the nominal rate is

$$i^* = \frac{\delta_\pi \beta_0}{\beta_\rho\, (\delta_\pi - 1)} - \frac{\delta_0}{\delta_\pi - 1} = \frac{\delta_\pi \beta_0 - \beta_\rho \delta_0}{\beta_\rho\, (\delta_\pi - 1)}.$$

As soon as $\delta_\pi \beta_0 > \delta_0 \beta_\rho$, the positivity of the nominal rate clearly precludes the case $\delta_\pi \leq 1$.

3 Empirical results

3.1 The data

The model is estimated on US and German data over the period from 1968:1 to 1998:4. Consistently with the quarterly frequency of the model, we use the three-month interest rate as a proxy for the central bank's intervention rate. The output gap is defined by the deviation of (log) real GDP from (log) potential GDP. Potential GDP is computed using a deterministic trend with a break in trend growth rate in 1974. The GDP deflator has been chosen as price indicator. Inflation is therefore defined as the annualized quarterly change in the GDP implicit deflator.

We address the issue of the German reunification in the following way. There is some evidence that the Bundesbank has been focusing on West Germany developments in the first years of German unification. One important reason is that, in addition to the statistical break, inflation data have been distorted by several special factors such as price freeing in East Germany, or fiscal developments (Reckwerth, 1997). Therefore, we use West Germany GDP data over the whole sample period. (West and overall output growth are very similar posterior to reunification.) Regarding the GDP deflator, we use West Germany data for the period up to 1994. Posterior to 1994, West Germany price data are not available and we use overall data. All data for West Germany are drawn from the BIS database. Other data are drawn from OECD databases (BSDB and MEI). Figure 1 displays the dynamics of inflation, output gap, and short nominal rate in the US as well as in Germany.

Our model is estimated over a rather long period, in order to estimate non-policy parameters consistently. However, over such a long period, some structural breaks are likely to occur. An abundant literature has highlighted that the period 1979-82 corresponds to a major break for the Federal Reserve reaction function. The change in the Fed operating procedures during this period (from targeting the Fed funds rate to the targeting of nonborrowed reserves) induced a large increase in both the level and the volatility of interest rates. Judd and Rudebusch (1998) estimated Fed reaction functions over the samples 1979:3-1987:2 and 1987:3-1997:4, corresponding to Volcker and Greenspan Fed Chairman tenures respectively. Similarly, in estimating the Fed reaction function, Clarida, Galí, and Gertler (2000) obtained substantial differences across periods in the sensitivity of interest rate to changes in expected inflation. During the pre-Volcker period, the Fed used to raise its nominal rate by less than the rise in expected inflation. By contrast, since 1979:3, real rate has been raised in the wake of increase in expected inflation. Last, Fuhrer (1997a) broke the 1966-93 sample into three subsamples corresponding to different monetary-policy regimes. Structural breaks were assumed to occur in 1979:3 and 1982:3. In the case of Germany, Clarida, and Gertler (1997) identify four episodes in the German monetary policy, with 1973, 1979, 1983 and 1990 as breaking dates. In particular, they claimed that 1979 corresponds to a 'shift to tightening'. Clarida, Galí, and Gertler (1998) estimate a reaction function for the Bundesbank over the period March 1979 to December 1993. They picked March 1979 as a beginning date, because it is the time Germany entered the EMS.

In this paper, we introduce a break in the reaction-function parameters between 1979:2 and 1979:3 for both countries, corresponding to the assumed shift in the monetary policy regime.[6] Reaction functions on each subsample are assumed to have the same explanatory variables (as in equation (5)), but with possibly different parameter estimates. Note that this assumption allows a shift in the steady-state value of the inflation rate. In Section 3.4, we provide statistical evidence on the stability of the reaction function as well as macroeconomic equations.

3.2 Estimation procedure

We estimate the model using a FIML approach. Though less robust to specification errors, this method has several advantages over the alternative Generalized Method of Moments (GMM). In small samples, FIML is more efficient, while GMM may suffer from important biases (see, e.g., Tauchen, 1996, Fuhrer, Moore, and Schuh, 1995). Furthermore, FIML relies on model-consistent expectations. In particular in the present context, the long real rate is not observable, but is computed consistently with the expectations hypothesis of the term structure. The GMM approach relies on observed variables and does not apply in such a case. Also, the inflation forecasts used by the central bank in the determination of the short rate are model-consistent.

The method for solving the model is the procedure developed by Anderson and Moore (1985). This procedure works as follows. First, the forward-looking model is written in the following general form

$$\sum_{i=1}^{\tau_B} H_{-i}X_{t-i} + H_0X_t + \sum_{j=1}^{\tau_F} H_jE_t\left(X_{t+j}\right) = \eta_t \tag{6}$$

where X_t contains all variables in the model, τ_B and τ_F denote the maximum number of lags and leads respectively, and η_t is the vector of error terms. Then, the procedure computes the autoregressive form of this model, using a generalized saddlepath procedure, which provides

$$\sum_{i=0}^{\tau_B} S_iX_{t-i} = \eta_t. \tag{7}$$

This so-called observable structure is then used to compute the log-likelihood function.[7] See Anderson and Moore (1985) for additional details on the methodology. This procedure has

[6] We also estimated a model with a break in 1982:3. However, we then obtained a unit root in the dynamics of the German short rate over the second subperiod.

[7] Estimation is performed using GAUSS software. The log-likelihood function is maximized using the BFGS algorithm. The parameter covariance matrix is computed as the inverse of the Hessian of the log-likelihood

been applied to a wide range of applications (see, e.g., Fuhrer, Moore, and Schuh, 1995, Fuhrer and Moore, 1995a,b, Ruud and Whelan, 2001).

As proved by Blanchard and Kahn (1980), the stability condition of a model with rational expectations is that the number of eigenvalues equal to one in the observable structure (7) is equal to the number of predetermined variables. Anderson and Moore (1985) show that, under this condition, their procedure is able to compute the observable structure (or reduced form) of the model, so that the likelihood of the model can be evaluated.

To ensure that the Blanchard-Kahn condition is verified, we have to impose $(\delta_\pi - 1)$ to be positive. This translates the fact that the central bank has to be sufficiently reactive to a shock on inflation to stabilize the system. The constraint $\delta_\pi \geq 1$ is in fact binding for the first subperiod in the US model only. The result $\delta_\pi = 1$ (in fact, δ_π is set to be slightly above 1) implies that long-run inflation is almost unbounded over the first subperiod, so that the model is almost non-stationary for the first subperiod in the US. This result is in line with Clarida, Galí, and Gertler (2000), who found that the estimated monetary policy rule for the pre-Volcker period provided greater macroeconomic instability than the Volcker-Greenspan policy rule. Therefore, they obtained an estimate $\delta_\pi < 1$, which our estimation method rules out.[8]

To check the stability of the model, we computed eigenvalues associated with the observable structure (7). The largest eigenvalue is above one, whereas the second eigenvalue is always below one. Since there is only one non-predetermined variable in the system (the long real rate), the Blanchard-Kahn condition is fulfilled and the system is stationary. Note that, over the first subperiod in the US, the second eigenvalue is very close to one (namely 0.9996). As noted above, this feature reflects the constraint we impose on the policy rule, which is binding in this case. Over the second subperiod, the second eigenvalue is 0.9635. In Germany, the second largest eigenvalues are 0.9720 and 0.9772 for the first and second subperiods, respectively.

3.3 FIML estimates

We turn now to the results of the FIML estimation of US and German models. Table 1 reports parameter estimates and residuals summary statistics.

We begin with the US model (panel A). The estimated Phillips curve is fairly standard. We introduced four lags in the price inflation. The sensitivity of inflation to movements in the output gap is rather large ($\alpha_y = 0.18$) and strongly significant, as in Rudebusch and Svensson (1999a). Regarding the I-S curve, the output gap displays the usual dynamics. The first lag on output gap is larger than one, whereas the second lag is negative, such that the sum of the two parameters is lower than one ($\beta_{y1} + \beta_{y2} = 0.95$). The output-gap sensitivity to the long real rate is crucial in our model, since this is the way monetary policy affects the economy. As predicted by the theory, we obtain a negative parameter. Its magnitude ($\beta_\rho = -0.35$) is similar to the one obtained, in the empirical literature, by Fuhrer and Moore (1995b), or Rudebusch and Svensson (1999a).

Concerning the Fed reaction function, two equations are actually estimated. For the

function. We freely estimate the covariance matrix of residuals. In a previous version of the paper, we assumed the covariance matrix of residuals to be diagonal, in order to interpret the innovations as structural shocks. Parameter estimates were broadly similar, but estimated standard errors were found to be slightly larger.

[8] In their calibrated forward-looking model, indeterminacy –rather than instability– may then arise. This is because, following a rise in expected inflation, the Fed will let short real rates decline and the output gap will rise. The expected rise in inflation will then self-fulfillingly materialize. Note that, since inflation is predetermined in our estimated Phillips curve, such an indeterminacy cannot occur.

1968-79 subperiod, we impose that the Fed reacts to an increase in inflation by an increase in short nominal rate of the same magnitude, so that the real rate remains unchanged. In the long run, a 1 percent increase in the output gap implies a 3.5 percentage point increase in the short nominal rate. For the second subperiod, i.e. the post-1979 period, we find that parameters of the reaction function have significantly changed. First, the short real rate increases strongly after a shock on inflation ($\delta_\pi = 0.43$). Second, an increase in the output gap does not affect interest rate anymore. The estimated long-run value for the real rate, ρ^*, is 3.4 percent, whereas we obtain 3.3 percent for the long-run inflation rate over the second subperiod. These values are consistent with those obtained, for instance, by Fuhrer (1997a).

Specification tests indicate that there is no evidence of serial correlation as well as heteroskedasticity in residuals of the Phillips and the I-S curves. Concerning the reaction function, we find no sign of serial correlation, yet residuals are found to be heteroskedastic, noticeably over the second subperiod. Further insight on residuals suggests that squared residuals are autocorrelated during the 1979-83 period of large interest-rate volatility. Comparison of the standard deviation of the dependent variable and the residual series shows that the model provides a very good fit in the US.

Results for Germany are reported in panel B of Table 1. Concerning the Phillips curve, three lags are sufficient to capture the inflation dynamics. The effect of output gap on inflation is noticeably weaker in Germany ($\alpha_y = 0.11$) than in the US, but it is significantly positive. The I-S curve is estimated with two lags of the output gap. German output gap has a much stronger persistence than the US one, since we obtain $\beta_{y1} + \beta_{y2} = 0.98$. The effect of the long real rate on output gap is negative ($\beta_\rho = -0.51$) and larger than the US parameter.[9] This larger sensitivity is magnified by the strong persistence of the German output gap.

The Bundesbank's reaction function displays a very stable inflation parameter over the two subperiods under study, at $\delta_\pi = 0.45$. This estimate is very close to the baseline Taylor-rule parameter. However, the Bundesbank's reaction to output gap displays very different patterns for the two subperiods. Over the 1965-79:2 period, the Bundesbank strongly reacts to shocks on the output gap, with a parameter δ_y as high as 1.2. Over the second subperiod, the response to an output-gap shock is much weaker, since the estimated of δ_y is 0.2 only. Our results for the monetary-policy reaction function are close to those found by Clarida, Galí, and Gertler (1998), although their results were obtained on monthly data with a different estimation method. To summarize, over the recent period, the reaction to inflation was similar for both central banks, whereas the Bundesbank was concerned about to output gap in contrast to the Fed.

The steady-state value of the German real rate is very close to the US one (to 3.4 percent). Conversely, the long-run inflation for Germany is found to be much lower (2 percent) than for the US over the second subperiod.

Residual check reveals that residuals of the Phillips and the I-S curves are neither serially correlated nor heteroskedastic. We only obtain some indication of serial correlation in squared residuals for the reaction function over the second subperiod. As for the US, this result is largely related to the agitated 1979-83 subperiod.

Finally, we notice that the standard deviation of residuals is slightly larger in Germany than in the US for macroeconomic equations, suggesting that the fit of the data is not as

[9] In a similar context, Taylor (1999a) obtained contrasting results. He found that the elasticity of output gap to real interest rate is five times smaller for European countries than for the US. Differences in the data may explain such a difference: Taylor estimated an I-S curve for an aggregate of German, France and Italy from 1971:1 through 1994:4. Moreover, his output-gap measure was computed using the Hodrick-Prescott filter.

good as in the US. By contrast, the estimated reaction function fits the data fairly well.

3.4 Stability tests and the Lucas critique

Since our model is not directly derived from the optimizing behavior of consumers and firms, it may be subject to the Lucas critique. Indeed, the model can be seen as a reduced form of an underlying structural model, and parameters in equations (1) and (2) may not correspond to 'deep' structural parameters.

Following Favero and Hendry (1992), Engle and Hendry (1993), and Ericsson and Irons (1995), our approach is to view the Lucas critique as a testable hypothesis. A specification is suitable for evaluating monetary policy rules if it is policy-invariant, i.e. if the non-policy parameters are stable facing a change in the policy rule.[10] To ensure that the model is immune to the Lucas critique, one has to demonstrate that the non-policy parameters are stable, once breaks in the policy rule are taken into account. To address this issue, we formally test the stability of our model over time.

We proceed in a two-stage approach. In the first step, we test for the null hypothesis of stability of all (policy and non-policy) parameters in the model (Hypothesis H1). Under the alternative, all equations contain a break, at the same date. If we cannot reject the null hypothesis, non-policy parameters are shown to be stable over the sample period. We cannot conclude that the macroeconomic model is robust with respect to the Lucas critique, however, since no monetary-policy shift is found to occur over the period. If we reject the null, then we have to identify the source of instability. This is the second step of our test procedure, which consists in two tests: (1) we test for the null hypothesis of stability of the policy-rule parameters, whereas non-policy parameters are allowed to shift (Hypothesis H2a); (2) we test for the null hypothesis of stability of the non-policy parameters, with policy-rule parameters being allowed to shift (Hypothesis H2b). If we reject H2a but not H2b, indicating that only policy-rule parameters have shifted over the sample, then the model is shown to be robust to the Lucas critique as soon as a break is introduced in the monetary policy rule. By contrast, if hypotheses H2a and H2b are rejected, policy and non-policy parameters are unstable, and the model is not immune to the Lucas critique. Last, if only hypothesis H2b is rejected, non-policy parameters are found to be unstable, but this result cannot be attributable to a change in monetary policy.

Since the model is estimated by FIML, the stability test we implement is the likelihood-ratio (LR) test with unknown break point. Considering an unknown break point provides a robustness check against an ad-hoc selection of the break point. For this purpose, we adopt the approach developed by Andrews (1993) and Andrews and Ploberger (1994). The methodology is the following. We consider a subsample in which the break is allowed to occur. Since the break may occur at any date, we choose a large subsample, covering 60 percent of the initial sample. We thus choose a subsample $[\pi_0 T, (1-\pi_0) T]$, where $\pi_0 = 0.20$ represents a fraction of the sample. For each date of this subsample (or for each π, for simplicity), we sequentially estimate our model with distinct equations for the period before and after the break. We obtain the log-likelihood under the alternative, denoted $\ln L_T^A(\pi)$. This log-likelihood is compared to the log-likelihood obtained under the null hypothesis of

[10] A model based on the optimizing behavior of agents may inaccurately reflect the true objectives and constraints facing agents or the way agents form expectations. Therefore, it may be subject to the Lucas critique as well. By contrast, a partially backward-looking (or hybrid) model may reflect the actual behavior of agents and then may not suffer from the Lucas critique (see Estrella and Fuhrer, 1999).

no break, denoted $\ln L_T^0$. Then, the 'sup' test statistic is defined as (Andrews, 1993)

$$Sup\text{-}LR_T = \sup_{\pi \in [\pi_0, (1-\pi_0)]} LR_T(\pi)$$

with $LR_T(\pi) = 2\left(\ln L_T^A(\pi) - \ln L_T^0\right)$. The asymptotic distribution of this statistic is non-standard, since the break-point parameter π_0 only appears under the alternative hypothesis. p-values of this test depend on the parameter π_0 and on the number of shifting parameters, and are reported in Andrews (1993).

Andrews and Ploberger (1994) proposed two other LR statistics, called average and exponential tests. Assuming that $\pi_0 T$ is an integer, these statistics are defined as

$$\begin{aligned} Avg\text{-}LR_T &= \frac{1}{T(1-2\pi_0)} \sum_{t=\pi_0 T}^{T-\pi_0 T} LR_T(t/T) \\ Exp\text{-}LR_T &= \ln\left(\frac{1}{T(1-2\pi_0)} \sum_{t=\pi_0 T}^{T-\pi_0 T} \exp\left(\frac{1}{2} LR_T(t/T)\right)\right). \end{aligned}$$

These statistics are shown to be asymptotically optimal in the context of structural change with unknown break date, whereas the *Sup-LR* test is not found to be an optimal test.

Table 2 reports the $Sup\text{-}LR_T$, $Avg\text{-}LR_T$, and $Exp\text{-}LR_T$ statistics for the stability tests in the US and Germany. Panels A, B, and C are devoted to the test of hypotheses H1, H2a, and H2b respectively. We begin with the first-stage test, corresponding to the null hypothesis of stability of the full model (Hypothesis H1). The null hypothesis is rejected for both countries at the usual significance level for all tests considered. In the case of the US, evidence of instability is very clear, since stability is rejected at the 1 percent level. Note that the $Sup\text{-}LR_T$ statistics is maximized, with a very large value, for the third quarter of 1980, suggesting that the optimal break date is very close to our a-priori monetary-policy shift. By contrast, in Germany, although the null hypothesis is strongly rejected by the $Avg\text{-}LR_T$ and $Exp\text{-}LR_T$ statistics, the $Sup\text{-}LR_T$ statistics is significant at the 10 percent level only. This is because the sequential pattern of the LR statistics is very flat over the first part of the sample, so that the statistic is not able to identify the break date precisely. The standard Chow test for a break in 1979:3 also rejects the null hypothesis of stability in the two countries.[11]

We turn now to the second-stage test, in order to identify the source of instability of the model parameters. Results of the test of stability of policy-rule parameters strongly support the null hypothesis H2a in both countries (panel B). The rejection is particularly strong in Germany, since the three tests statistics are significant at the 1 percent level. As for the test of global stability (H1), the Chow test for a break in 1979:3 is unambiguous in the US, since it strongly rejects the null of stability. For Germany, the p-value is 9% only. We notice that the optimal break date for the German monetary policy rule is found to occur around 1982-83, i.e. at the end of the period of high interest-rate volatility, which began in 1979, our a-priori break date. This confirms that instability in the reaction function is caused by

[11] For both countries, the LR test statistic for a break in 1979:3 (the Chow test) is slightly smaller than the *Sup-LR* test statistics for an unknown break date. This result reflects the fact that the optimal break is not found to occur in 1979:3 exactly. However, the optimal break date is estimated with a certain degree of uncertainty (Bai, 1997) and the null of stability is also rejected when the break is assumed to occur in 1979:3. Since there is a strong prior and a huge empirical evidence in favor of this break date (at least for the US), we decided to pursue the study with 1979:3 as break date.

this subperiod. Moreover, further scrutiny of estimated non-policy parameters indicates that they are essentially unaltered by the choice of 1983:1 as the break date.

Last, we perform the test of stability of non-policy parameters, whereas policy-rule parameters are allowed to shift at an unknown date (panel C). In the US, the evidence is not very strong, since only the *Avg*-LR_T is unable to reject the null hypothesis. The *Sup*-LR_T and *Exp*-LR_T statistics reject the null of stability at the 5 percent level. In Germany, by contrast, we obtain a strong evidence of stability of the non-policy parameters, since the null is not rejected whatever the test statistics. The Chow test for a break in non-policy parameters in 1979:3 provides the same pattern of evidence as tests with unknown break date. Evidence of stability is mitigated in the US, but very strong in Germany.

On the whole, there is substantial evidence of a monetary policy shift in both countries within the period. Conditional on an appropriate modelling of the shifting reaction function, there is no clear evidence of instability in non-policy parameters and thus the two macroeconomic models show some robustness to the Lucas critique.

4 Optimal policy frontiers

4.1 Methodology

We are now interested in computing the optimal monetary policy, conditional on the estimated macroeconomic model. As a reaction function, we consider the class of Taylor-rule policies with interest-rate smoothing. Although this class does not necessarily include the globally optimal rule, a large number of studies (Taylor, 1999a, Williams, 1999, among others) have shown that such simple rules provide a fairly good approximation of the optimal rule.[12] For a given set of the reaction-function parameters, we obtain a set of unconditional variances of the inflation rate, σ_π^2, the output gap, σ_y^2, and the interest rate, σ_i^2. The optimal policy frontier is then defined as the set of efficient combinations of unconditional variances attainable by the central bank. In many studies, the optimal policy frontier has been defined in the inflation/output variance plane (see, for instance, Fuhrer and Moore, 1995b, Fuhrer, 1997a, Ball, 1999). However, introducing interest rate in the central bank's loss function is a convenient way to rationalize the observed smoothing of interest rates.[13] In many models, when interest-rate variability is not taken into account, optimal rules generate implausibly large fluctuations in the interest rate (see, e.g., Artus, Penot, and Pollin, 1999). Another motivation for including interest-rate variability in the loss function is the central bank's concern for financial stability. Recent studies, as Rudebusch and Svensson (1999a) and Svensson (2000), adopted such a design and introduced the interest rate in the central bank's loss function.

Two approaches can be implemented to compute optimal policy frontiers, which incorporate interest-rate variability. The first one is to introduce interest-rate variability in the loss function. The optimal policy frontier is therefore computed by solving the following optimization program

$$\begin{cases} \min\limits_{\{\theta\}} \mu_\pi \sigma_\pi^2 + \mu_y \sigma_y^2 + \left(1 - \mu_\pi - \mu_y\right) \sigma_i^2 \\ \text{s.t.} \qquad X_t = M(\theta) X_{t-1} + v_t \end{cases} \tag{8}$$

[12] See, for instance, Svensson (1998) for the analysis of more general monetary policy rules, in an open-economy framework.

[13] An alternative approach is to restrict the policy rule to an equation for the first-difference of the interest rate, as in Fuhrer and Moore (1995b).

where $\theta = \{\delta_\pi - 1, \delta_y, \delta_i\}$ denotes the parameter vector in the monetary-policy reaction function, and the matrix M is the reduced form of the model, directly derived from the observable structure (7). Parameters μ_π and μ_y denote the weights on inflation stabilization around the inflation target, and on output-gap stabilization, respectively. They are set such that μ_π, $\mu_y \in [0,1]$ and $\mu_\pi + \mu_y \leq 1$. Therefore, μ_π and μ_y reflect the policymaker's preferences.

The other approach has been adopted by Levin, Wieland, and Williams (1999) and Williams (1999). The optimal policy frontier is then defined as the set of efficient combinations of unconditional variances of the inflation rate and the output gap, subject to the constraint that the unconditional variance of the short rate should not exceed a given value k^2. The optimization program then becomes:

$$\begin{cases} \min\limits_{\{\theta\}} \lambda\sigma_\pi^2 + (1-\lambda)\,\sigma_y^2 \\ \text{s.t.} \qquad X_t = M(\theta)\,X_{t-1} + v_t \\ \qquad\qquad \sigma_i^2 \leq k^2 \end{cases} \tag{9}$$

where $\lambda \in [0,1]$ is the weight on inflation stabilization around the inflation target.[14]

In the following, we will use both approaches in turn, because each one provides some insights. Program (9) appears to be best suited for the graphical representation of policy frontiers. Indeed, in the inflation/output variance plane, each optimal frontier obtained for a given k actually corresponds to a contour line in the third (interest-rate) variance dimension. Another advantage of the second approach is that k^2 can be set equal to the interest-rate variance under the estimated policy rule. On the other hand, the approach defined by program (8) allows the policy rules in the two countries to be compared for the same preference parameters, regardless of the respective variances of the macroeconomic shocks in the two countries.

Since we are dealing with a small, linear model, we are able to evaluate the analytic expression for the unconditional variances. This approach is likely to provide more accurate results than simulation-based methods. The procedure developed by Anderson and Moore (1985) allows to rewrite the model in a vector autoregressive form as:

$$X_{t+1} = M(\theta)\,X_t + v_{t+1}$$

where X_t is the vector of all model variables, and v_t is a vector of serially uncorrelated disturbances with mean zero and finite diagonal variance matrix Ω. Then, the unconditional covariance matrix of X_t, denoted V, is given in vector form by

$$Vec(V) = [I - M \otimes M]^{-1}\,Vec(\Omega)\,. \tag{10}$$

Unconditional variances for the inflation rate, the output gap and the interest rate are then obtained by selecting the appropriate component in $Vec(V)$. For a given interest-rate variability k, we determine the associated optimal-policy frontier as follows. For each value of λ varying from 0 to 1, we solve the optimization program in equation (9). We start with an initial guess for the policy-rule parameters θ, obtain the reduced-form solution matrices M and Ω, compute the unconditional moments V and the value of the objective function. We update the parameter vector θ iteratively, until an optimum of the objective function is attained.

[14] Williams (1999) reports that measuring interest-rate variability by the variance of the level of the interest rate or by the variance of the change in the interest rate gives similar optimal frontiers.

Note that, since we solve the forward-looking model at each step, we choose the optimal policy rule among the stabilizing rules. (All variables must have a finite unconditional variance.) In particular, this rules out the case of a central bank under-reacting to the inflation rate.

In evaluating the optimal monetary policy, we only consider policy rules of the form given by equation (5). Although it is a more general rule than the standard Taylor rule, since it incorporates interest-rate smoothing, the optimal reaction function does not depend on all state variables. Furthermore, since we use a closed-economy framework, our model is not suited to evaluate the role of the exchange rate (or of foreign interest rates) in monetary-policy rules.[15] Levin, Wieland, and Williams (1999) compare simple policy rules (based on the inflation rate, the output gap, and the interest rate as instruments) and complex policy rules (incorporating all available information on state variables). They conclude that complex rules slightly reduce inflation and output-gap variances, but that such benefits can be offset by the lower degree of transparency associated with complex rules.

4.2 Results for the US optimal monetary-policy frontier

Optimal policy rules for the Fed as well as the Bundesbank are described in Table 3. This table reports the optimal reaction-function parameters and the unconditional standard deviations computed for different values of the upper bound for σ_i, $k \in \{4.7, 5, 6\}$. The case $k = 4.7$ is close to the interest-rate unconditional standard deviation under the last subsample estimated monetary policy. Figures 2 display the associated optimal policy frontiers.

Regarding the US optimal monetary-policy frontier, the estimated values for unconditional standard deviations fall in the range of previous studies by Fuhrer (1997a), Levin, Wieland, and Williams (1999), Williams (1999) or Rudebusch and Svensson (1999a). Differences with these various papers can be explained by the sample used for the estimation and, to some extent, by the specification of the model. As in most papers, our estimated model tends to predict rather large values for unconditional variances. As it appears clearly from equation (10), unconditional variances depend on conditional variances and dominant roots in the system.[16] The larger the conditional variances and the dominant roots, the larger the unconditional variances of the variables in the system. In Section 3, we have seen that the model dynamics is quite persistent. Thus, lack of precision in estimating conditional variances is magnified in computing unconditional variances. Furthermore, in computing optimal monetary frontiers, we include the estimated conditional variance of the reaction function. We therefore consider 'monetary-policy shocks' to be a fully fledged source of variability. In practice, however, removing the monetary-policy shock variance from the computation mainly reduces the interest-rate unconditional variance, but does not affect the inflation/output variance trade-off significantly.

The unconditional inflation and output-gap variances pertaining to optimal policy rules display some usual features. First, the Fed faces a clear inflation/output variance trade-off. Moreover, if one allows for a high interest-rate volatility, one can attain low levels of inflation and output-gap volatility. The optimal frontier obtained for $k = 6$ corresponds,

[15] As pointed out by Ball (1998) and Taylor (1999a), the exchange rate can be added to the policy rule either because the central bank uses a monetary condition index, defined as a weighted average of the interest rate and the exchange rate, or because the exchange rate is added as a variable to the policy rule. Considering this type of monetary policy for the ECB, Taylor (1999a) finds no advantage for such a rule as compared to the standard Taylor rule.

[16] In a simple univariate framework, $x_t = ax_{t-1} + \varepsilon_t$, where ε_t is an error term with zero mean and σ_ε^2 variance, the unconditional variance for x is $\sigma_\varepsilon^2 / \left(1 - a^2\right)$.

in term of the optimization program (8), to a very low weight on interest-rate smoothing (with $1 - \mu_\pi - \mu_y < 0.1$). If we consider now decreasing upper bounds (or increasing weights on interest-rate smoothing), the set of attainable combinations of inflation/output variance decreases. For instance, for $k = 4.7$, the inflation standard deviation is bounded by 2.6 percent, whereas the output-gap variance is bounded by 2.1 percent. This compares to minimal inflation and output-gap standard deviations of 2.2 percent and 1.7 percent in the case $k = 6$. Furthermore, it appears that, for the case $k = 6$, the optimal frontier implies that very large output-gap variance (in fact infinite ones) are required to attain inflation standard deviations below 2 percent. Conversely, reaching an output-gap standard deviation below 1.9 percent implies very large inflation variance penalties. Above these bounds, we find rather balanced policies, with similar weights on inflation and output gap.

Figure 2a also displays the estimated actual policy, summarized by the combination of unconditional variances obtained using parameter estimates. The estimated actual policy is quite far from the computed frontier with no weight on interest-rate variability. Even if we include some concern on interest rate, the actual policy appears to be far from an optimal one.

From Table 3, several results are worth noting. First, quite intuitively, when the weight of inflation in the loss function increases, the response of interest rate to inflation increases, while the parameter on output gap decreases. Yet, as Ball (1998), we find that it is always optimal to put a positive weight on output gap, whatever the central bank's preferences. The output-gap parameter, even for large interest-rate smoothing and large inflation preference parameters, is never smaller than 0.5.

Second, even when the weight on interest-rate smoothing is assumed to be very low, we obtain a large smoothing parameter δ_i, at about 0.7 in the US and 0.8 in Germany. This is consistent with the result highlighted by Levin, Wieland, and Williams (1999), derived from the forward-lookingness of long interest rates. Interest-rate smoothing helps to stabilize the economy by generating expectations of persistent rise (or decrease) in short interest rates. This reduces the initial impact of shocks due to the forward-lookingness of aggregate demand, embodied here in long-term interest rate.

While optimal policy rules have rather large parameters, they do imply a sensible dynamics when integrated in the model. We illustrate this issue by performing a simulation experiment, with a demand (I-S) shock on each model, for the estimated as well as the optimal policy rules. We use balanced preferences, with parameters $1-\mu_\pi-\mu_y = 0.3$, $\mu_\pi = \mu_y = 0.35$ in program (8), so that the reaction-function parameters are $(\delta_\pi - 1, \delta_y, \delta_i) = (0.9; 0.9; 0.7)$. To perform the simulation, we assume that the aggregate demand is not affected contemporaneously by the price or the monetary-policy shocks, so that the shock on the I-S curve can be seen as a structural demand shock. As shown in Figure 3, the adjustment is more rapid under the optimal rule. The output gap crosses zero on the third year of simulation. Moreover, the inflation peak is twice lower and inflation does not overshoot the target.

4.3 Results for the German optimal monetary-policy frontier

The results obtained for the German model are similar to those obtained for the US, so we mainly emphasize the differences. Panel B of Table 3 reports the optimal reaction-function parameters and the unconditional standard deviations computed for different values of the upper bound for σ_i, $k \in \{4.7, 5, 6\}$. The case $k = 5$ corresponds to the interest-rate unconditional standard deviation under the last subperiod estimated monetary policy. Figure 2b displays the corresponding optimal policy frontiers.

The first point to note is that the feature of the large values of unconditional variances is emphasized for the German optimal policy frontier. This result is mainly due to the large persistence in the model variables (in particular regarding the output-gap equation), rather than to large conditional error variances. If we consider first a given level of interest-rate variability, we find that inflation and output-gap unconditional standard deviations are systematically larger for Germany than for the US. Thus, to attain a given level of inflation and output-gap variability, the Bundesbank would have to accept a larger interest-rate variability than the Fed. We obtain such a feature whatever the level of interest-rate unconditional variance.

Our main result is that the German optimal policy is more persistent and slightly more aggressive in the long run than the US optimal policy. Consider the case of the same balanced preference set $1 - \mu_\pi - \mu_y = 0.3$, $\mu_\pi = \mu_y = 0.35$ (Table 4) for both countries. Then, the optimal reaction functions have the following parameter vectors $\theta = (\delta_\pi - 1, \delta_y, \delta_i) = (0.89; 0.87; 0.71)$ for the Fed and $\theta = (0.93; 0.94; 0.78)$ for the Bundesbank. In the German case, the optimal interest-rate smoothing parameter δ_i is between 0.75 and 0.80 for all policy rules. This result reflects that the autoregressive parameter should be high, in order to make full use of the 'expectation channel' mentioned above. On the other hand, the value for δ_i is bounded by 1 and unconditional variances grow dramatically for large values of δ_i.[17] The benefits of interest-rate smoothing appear to be higher in Germany than in the US. This may stem from the stronger effect of long real rate on the output-gap equation.

The estimated actual policy appears to be rather far from optimal policy frontiers, even when frontiers are obtained with a large interest-rate smoothing (for instance, $k = 4.7$).

4.4 Estimated versus optimal policy rules

Given the gap between estimated and optimal policy rules, we cannot easily recover the preferences of the central bank from our exercise. Moreover, we did not assume, at the estimation stage, that the behavior of the central bank was in any way optimal. Yet, our results provide some indications. First, they indicate a strong degree of interest-rate stabilization. Second, both central banks appear to be inflation targeters in the sense of Rudebusch and Svensson (1999a). Indeed with a high weight on inflation in the central bank's loss function, from Table 4, the reaction function parameter $(\delta_\pi - 1)$ is twice as large as δ_y. This is the case of the estimated rule of the Bundesbank. In the Fed case, no effect of the output gap was found to be empirically significant.

Still, the gap to the optimal rule may have several interpretations. First, one may argue that, given the standard errors of the estimated parameters, some optimal policy rules actually fall in the range of the estimated rules. One may also question the assumed ability to commit to a rule. Another possibility is that the estimated reaction function and the computed efficiency frontiers fail to include some relevant constraints in the central bank's loss function.[18] A last interpretation relies on model uncertainty. Our results indicate that the optimal monetary policy would require larger reaction-function parameters than the estimated one. This may indicate that central banks have underestimated the degree of persistence of shocks over the sample period.[19]

[17] We recall that the class of rules we evaluate is somewhat restrictive, since it does not include the first-difference rule studied by Fuhrer (1997a) or Williams (1999).

[18] For instance, our estimate of the reaction function may be biased because of an omitted variable, such as the exchange rate, or a constraint on the level of nominal interest rates.

[19] Another approach to parameter uncertainty relies on defining a probability distribution over the model parameters. In such a context, a gradual monetary policy is shown to be optimal over all models under consideration (Sack and Wieland, 1999, and Rudebusch, 1999).

4.5 The robustness of the German monetary policy rules

This section investigates the issue of the robustness of the monetary policy rules to parameter uncertainty, focusing on the German case. Computed optimal frontiers may be sensitive to different non-policy structural parameters, since our monetary policy evaluation is essentially model-specific. Parameters may be estimated imprecisely and/or some equations may be wrongly specified. Therefore, we measure the robustness of the computed monetary-policy frontier with respect to key model parameters. The three key parameters for the design of monetary policy in a closed economy are the following: the sensitivity of inflation to movements in the output gap (α_y), the interest-rate sensitivity of the I-S curve (β_ρ), and the persistence of the output-gap equation ($\beta_{y1} + \beta_{y2}$).

To measure the effect of parameter uncertainty in Germany, we vary each key parameter in turn by $\pm$ one standard deviation, while all other model parameters and the error covariance matrix are left unchanged. Then, we compute the corresponding optimal policy frontier. For this purpose, we use the optimization program (8), which includes the interest-rate smoothing into the objective function explicitly. We set $1 - \mu_\pi - \mu_y = 0.3$, so that the interest-rate unconditional standard deviation is close to 4.9 percent, which is the interest-rate standard deviation obtained with the estimated monetary policy.[20] Notice that we vary β_{y1} and β_{y2} parameters, so that the sum $\beta_{y1} + \beta_{y2}$ varies by $\pm$ one standard deviation (yielding 0.91 and 1.05).[21] Tables 5 to 7 and Figures 4 to 6 display the effect of a change in these parameters on the optimal policy frontier.

Increase parameter α_y (from 0.11 to 0.16) corresponds to a larger sensitivity of inflation to movements in the output gap. Therefore, the central bank is more aggressive and increases parameters δ_π and δ_y, in order to take advantage of the larger sensitivity of the economy. Thus, for given weights μ_π and μ_y, unconditional variances for the inflation rate and the output gap are lowered, by about 5 percent and 15 percent, respectively. This moves the optimal frontier inward toward to origin. The overall result is a slight increase in the interest-rate unconditional variance. We note that an increase in the α_y parameter also implies a flattening of the optimal frontier. Therefore, lowering inflation variance has a slightly smaller cost in terms of output-gap variance.

When the β_ρ parameter is increased in absolute value (from -0.51 to -0.84), we obtain a different pattern (Table 6, Figure 5). As in the case of increasing α_y, the monetary policy gains a better control over the economy. But unlike the previous case, a large interest-sensitivity of the I-S curve allows the central bank to lower the inflation-rate and the output-gap parameters in the reaction function, while at the same time decreasing by about 5 percent unconditional variances for the inflation rate and the output gap. Simultaneously, since output gap is more reactive to movement in the long real rate, the central bank is able to decrease interest-rate persistence and the unconditional variance of interest rate.

Let us now turn to the case where persistence in the output gap is lowered (from 0.96 to 0.91). A strong decrease is obtained in both inflation and output-gap parameters in the reaction function, whereas unconditional variances on the inflation rate and the output gap remain basically unchanged (Table 7, Figure 6). The central bank is thus able to obtain the same result in terms of unconditional variances with a much less aggressive monetary

[20] We prefer to use the optimization program (8) rather than program (9), because varying a structural parameter does not change the weight of interest rate in the central bank's objective function, whereas it affects the attainable interest-rate unconditional variances. Therefore, after a change in a structural parameter, the central bank adjusts reaction-function parameters to attain a new combination of unconditional variances of the inflation rate, the output gap and the short rate.

[21] We recall that values above 1 are admissible for the autoregressive sum, as soon as the whole model remains stationary.

policy, allowing the interest-rate unconditional variance to decrease significantly. Therefore, we obtain parameters in the reaction function which are significantly closer to the estimated parameters. For instance, for large values of the weight on inflation in the loss function, the output-gap parameter falls below the empirical estimate of 0.27. So the estimated reaction function seems to be consistent with a lower degree of persistence in the demand shock.

5 Conclusion

In this paper, we estimate a macroeconomic model with forward-looking features to evaluate optimal monetary policy in the US and Germany. The model has a standard I-S curve/Phillips curve/interest-rate rule structure. Our results for the US economy are rather close to previous work (in particular, Rudebusch and Svensson, 1999a, for the Phillips curve, Fuhrer and Moore, 1995b, for the I-S curve, and Clarida, Galí, and Gertler, 1998, for the reaction function). The estimated Phillips and I-S curves, though not structural, show some robustness with respect to the Lucas critique, once a break in the reaction function is incorporated in the model.

Then, we compute optimal monetary rules and frontiers for both countries. We find that, consistently with the estimated reaction function, optimal monetary policy implies a strong degree of interest-rate smoothing. This feature is related to the forward-looking dynamics of long real rates. We find that the optimal degree of interest-rate smoothing is higher for Germany than for the US. This finding rationalizes the higher degree of interest rate smoothing obtained in the estimated German reaction function.

Another main result is that optimal monetary policy implies a strong reaction to the inflation rate as well as the output gap. In particular, optimal parameters pertaining to inflation and output gap are larger than those implied by the baseline Taylor rule. This result, obtained by Ball (1998) using a calibrated model, is confirmed for Germany as well as for the US. Thus, even with a low preference for output stabilization, the output gap should always matter in the reaction function. This finding supports the view that, in spite of the positive weight of the output gap in its estimated reaction function, the Bundesbank has been targeting inflation. Nevertheless, for both countries, estimated parameters for the inflation and the output gap are lower than optimal parameters.

Several avenues can be explored to rationalize this result. First, uncertainty on the true model and on parameters of the economy may be an argument in favor of a more gradual response (Sack and Wieland, 2000). Performing sensitivity analysis demonstrates that the estimated policy rule for Germany is consistent with a lower than estimated persistence in the output gap. Second, our results may be due to the rather simple specification of the model, or to the restrictive form of the central-bank loss function. Last, our evaluation relies on the monetary authority committing to a simple rule. This assumption might not be verified over the whole sample period.

Another issue for future research is to assess whether our results for the optimal rule in Germany (i.e., a more gradual behavior in the short run, and a stronger response in the long run than for the US) extend to the euro area. Estimating an euro-area model would provide some further insights, but this requires to first define a meaningful empirical specification for the policy rule.

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Table 1: FIML estimate of the econometric models

	Panel A: The US			Panel B: Germany		
Parameter	Estimate	Std dev.	*t*-statistic	Estimate	Std dev.	*t*-statistic
Phillips curve						
$\alpha_{\pi 1}$	0.597	0.093	6.450	0.287	0.090	3.188
$\alpha_{\pi 2}$	0.079	0.105	0.751	0.394	0.083	4.772
$\alpha_{\pi 3}$	0.208	0.105	1.983	0.319	–	–
$\alpha_{\pi 4}$	0.116	–	–	–	–	–
α_y	0.175	0.046	3.843	0.106	0.053	2.009
I-S curve						
β_0 (x100)	3.395	0.701	4.844	3.366	0.659	5.109
β_{y1}	1.150	0.085	13.483	0.666	0.092	7.224
β_{y2}	-0.200	0.086	-2.333	0.314	0.088	3.571
β_ρ	-0.348	0.215	-1.618	-0.508	0.335	-1.518
Reaction function (1968:1-79:2)						
δ_0 (x100)	3.645	4.484	0.813	-1.034	2.265	-0.456
$\delta_\pi - 1$	0.001	–	–	0.446	0.397	1.123
δ_y	3.545	5.349	0.663	1.210	0.646	1.873
δ_i	0.932	0.118	7.927	0.856	0.067	12.826
(1979:3-98:4)						
δ_0 (x100)	1.925	0.784	2.454	2.504	0.993	2.523
$\delta_\pi - 1$	0.435	0.171	2.536	0.436	0.315	1.384
δ_y	0.000	–	–	0.272	0.206	1.322
δ_i	0.725	0.057	12.621	0.827	0.064	12.837
Likelihood	-495.439			-562.386		
	σ_y σ_ε	LBc(8) (*p*-value)	LM(8) (*p*-value)	σ_y σ_ε	LBc(8) (*p*-value)	LM(8) (*p*-value)
Phillips curve	2.519	3.366	13.519	2.233	7.655	7.808
	1.077	(0.909)	(0.095)	1.516	(0.468)	(0.452)
I-S curve	2.288	5.404	14.446	2.617	15.601	13.313
	0.818	(0.714)	(0.071)	1.134	(0.049)	(0.102)
Reaction function	1.990	12.441	19.738	2.894	14.400	6.511
(1968:1-79:2)	0.911	(0.133)	(0.011)	1.230	(0.072)	(0.590)
Reaction function	3.426	11.418	58.061	2.550	16.328	20.424
(1979:3-98:4)	1.076	(0.179)	(0.000)	0.673	(0.038)	(0.009)
	Real rate	Inflation (79:3-98:4)	Nominal rate	Real rate	Inflation (79:3-98:4)	Nominal rate
Steady-state values	3.382	3.335	6.717	3.361	1.965	5.326

Note: LBc(8) is the Ljung-Box statistic, corrected for heteroskedasticity, obtained by regressing residuals on 8 lags. LM(8) is the Engle statistic for heteroskedasticity, obtained by regressing squared residuals on 8 lags. These statistics are distributed as $\chi^2(8)$. Steady-state values are defined in Section 2.4.

Table 2: Stability tests

	The US				Germany			
Panel A: Test for global stability								
	p	Sup-LR	Avg-LR	Exp-LR	p	Sup-LR	Avg-LR	Exp-LR
Unknown date	12	48.89 [a]	22.09 [a]	20.20 [a]	11	27.47	19.97 [b]	11.42 [b]
Break in 1979:3	12	39.85 [a]			11	23.72 [b]		
Panel B: Test for stability of policy-rule parameters								
	p	Sup-LR	Avg-LR	Exp-LR	p	Sup-LR	Avg-LR	Exp-LR
Unknown date	4	22.05 [a]	5.74	6.88 [a]	4	22.45 [a]	11.28 [a]	8.14 [a]
Break in 1979:3	4	15.22 [a]			4	8.01		
Panel C: Test for stability of non-policy parameters								
	p	Sup-LR	Avg-LR	Exp-LR	p	Sup-LR	Avg-LR	Exp-LR
Unknown date	8	24.71 [b]	12.37	10.13 [b]	7	20.11	10.85	6.88
Break in 1979:3	8	15.40			7	10.81		

Note: Asymptotic critical values for the Sup-LR statistic are from Andrews (1993) and asymptotic critical values for the Exp-LR and the Avg-LR statistics are from Andrews and Ploberger (1994). p denotes the number of parameters allowed to shift at the break point. We assume that the break may occur over the subsample $[\pi_0 T,(1-\pi_0)T]$, with $\pi_0 = 0.20$. [a] and [b] denote that the statistic is significant at the 1 percent and 5 percent levels, respectively.

Table 3: Implied parameters for optimal monetary-policy rules using program (9)

Weights in the loss function	Optimal parameter values			Unconditional standard deviations		
$(\lambda;1-\lambda)$	$\delta_\pi - 1$	δ_y	δ_i	σ_π	σ_y	$\sigma_i\ (k)$
Panel A: The US						
k=6						
(0.00;1.00)	0.63	2.96	0.68	3.68	1.66	6.00
(0.50;0.50)	2.25	2.49	0.71	2.26	2.01	6.00
(1.00;0.00)	2.61	1.47	0.75	2.13	2.38	6.00
k=5						
(0.00;1.00)	0.63	1.55	0.71	3.21	1.93	5.00
(0.50;0.50)	1.26	1.33	0.73	2.53	2.15	5.00
(1.00;0.00)	1.38	0.86	0.73	2.44	2.39	5.00
k=4.7						
(0.00;1.00)	0.60	1.00	0.70	3.10	2.12	4.70
(0.50;0.50)	0.90	0.88	0.72	2.72	2.26	4.70
(1.00;0.00)	0.97	0.64	0.72	2.65	2.42	4.70
Model estimate	0.43	0.00	0.72	3.40	3.10	4.75
Panel B: Germany						
k=6						
(0.00;1.00)	0.67	2.43	0.76	4.66	1.98	6.00
(0.50;0.50)	2.43	2.59	0.77	2.94	2.61	6.00
(1.00;0.00)	2.55	1.08	0.81	2.59	3.59	6.00
k=5						
(0.00;1.00)	0.63	1.25	0.77	3.87	2.40	5.00
(0.50;0.50)	1.26	1.33	0.79	3.13	2.75	5.00
(1.00;0.00)	1.32	0.72	0.79	2.89	3.39	5.00
k=4.7						
(0.00;1.00)	0.60	0.84	0.77	3.62	2.70	4.70
(0.50;0.50)	0.85	0.85	0.78	3.27	2.89	4.70
(1.00;0.00)	0.91	0.59	0.78	3.11	3.29	4.70
Model estimate	0.44	0.27	0.83	3.80	4.00	4.90

Table 4: Implied parameters for optimal monetary-policy rules using program (8)

Weights in the loss function	Optimal parameter values			Unconditional standard deviations		
(μ_π ; μ_y ; 1− μ_π−μ_y)	δ_π−1	δ_y	δ_i	σ_π	σ_y	σ_i
Panel A: The US						
Inflation targeter (0.95;0.00;0.05)	2.98	1.63	0.74	2.08	2.38	6.31
Output-gap targeter (0.00;0.95;0.05)	0.63	2.85	0.69	3.64	1.67	5.91
Interest-rate targeter (0.00;0.00;1.00)	0.54	0.36	0.68	3.11	2.51	4.54
Balanced preferences (0.35;0.35;0.30)	0.89	0.87	0.71	2.72	2.26	4.71
Model estimate	0.43	0.00	0.72	3.40	3.10	4.75
Panel B: Germany						
Inflation targeter (0.95;0.00;0.05)	3.20	1.26	0.81	2.51	3.67	6.55
Output-gap targeter (0.00;0.95;0.05)	0.67	3.07	0.75	5.09	1.85	6.58
Interest-rate targeter (0.00;0.00;1.00)	0.56	0.47	0.76	3.46	3.20	4.61
Balanced preferences (0.35;0.35;0.30)	0.93	0.94	0.78	3.24	2.86	4.77
Model estimate	0.44	0.27	0.83	3.80	4.00	4.90

Table 5: Implied parameters for the German optimal monetary-policy rules for various values of α_y, using program (9)

Weights in the loss function	Optimal parameter values			Unconditional standard deviations		
$(\mu_\pi ; \mu_y ; 1-\mu_\pi-\mu_y)$	$\delta_\pi-1$	δ_y	δ_i	σ_π	σ_y	σ_i
Model estimate	0.44	0.27	0.83	4.93	4.20	4.92
Optimal rule with the estimated nonpolicy parameters (α_y=0.106)						
(0.00;0.70;0.30)	0.63	1.16	0.77	3.81	2.46	4.94
(0.35;0.35;0.30)	0.93	0.94	0.78	3.24	2.86	4.77
(0.70;0.00;0.30)	1.15	0.67	0.79	2.97	3.35	4.88
Optimal rule with α_y=0.158						
(0.00;0.70;0.30)	0.64	1.30	0.77	3.59	2.24	4.98
(0.35;0.35;0.30)	0.96	1.13	0.78	3.12	2.49	4.92
(0.70;0.00;0.30)	1.20	0.94	0.78	2.92	2.75	5.05
Optimal rule with α_y=0.054						
(0.00;0.70;0.30)	0.60	1.01	0.77	4.56	3.00	5.38
(0.35;0.35;0.30)	0.88	0.75	0.78	3.66	3.75	4.88
(0.70;0.00;0.30)	1.06	0.37	0.80	3.16	5.08	4.94

Table 6: Implied parameters for the German optimal monetary-policy rules for various values of β_ρ, using program (9)

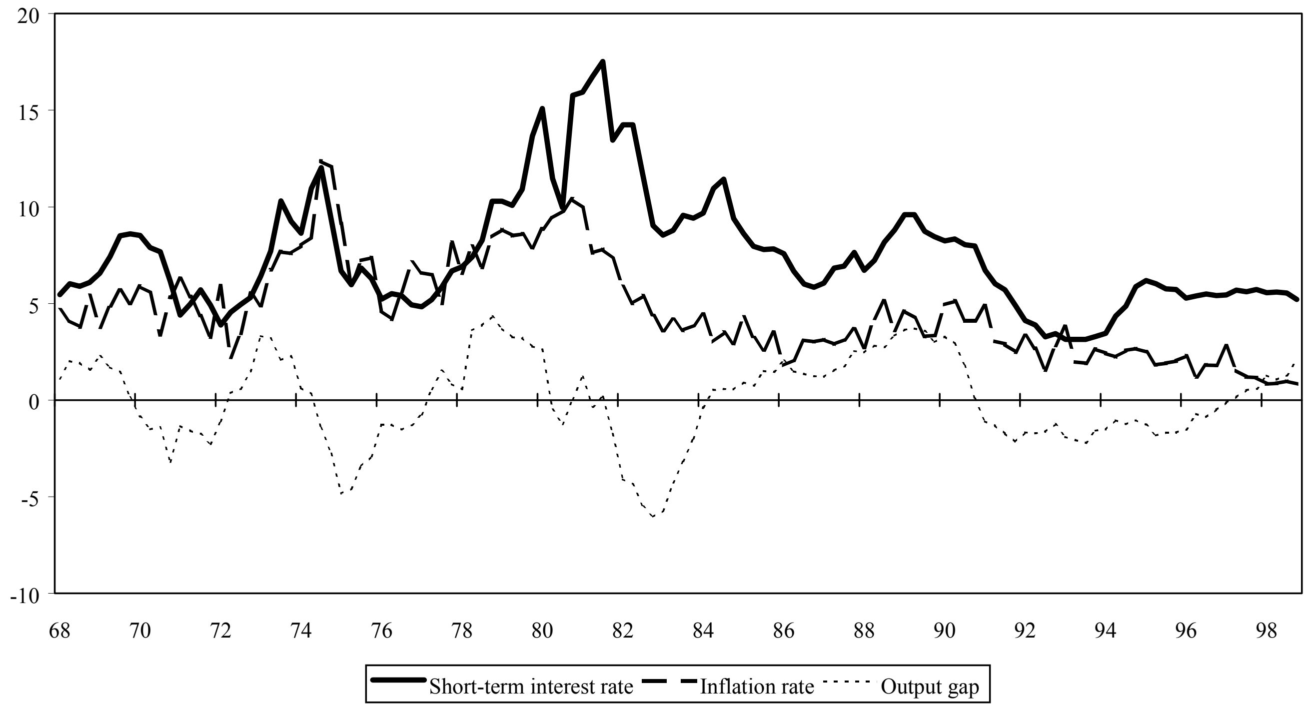
Weights in the loss function	Optimal parameter values			Unconditional standard deviations		
$(\mu_\pi ; \mu_y ; 1-\mu_\pi-\mu_y)$	$\delta_\pi - 1$	δ_y	δ_i	σ_π	σ_y	σ_i
Model estimate	0.44	0.27	0.83	4.93	4.20	4.92
Optimal rule with the estimated nonpolicy parameters (β_ρ=-0.508)						
(0.00;0.70;0.30)	0.63	1.16	0.77	3.81	2.46	4.94
(0.35;0.35;0.30)	0.93	0.94	0.78	3.24	2.86	4.77
(0.70;0.00;0.30)	1.15	0.67	0.79	2.97	3.35	4.88
Optimal rule with β_ρ=-0.84						
(0.00;0.70;0.30)	0.58	1.04	0.70	3.61	2.36	4.39
(0.35;0.35;0.30)	0.86	0.81	0.72	3.02	2.81	4.14
(0.70;0.00;0.30)	1.06	0.50	0.74	2.73	3.47	4.22
Optimal rule with β_ρ=-0.17						
(0.00;0.70;0.30)	0.73	1.49	0.86	4.53	2.75	6.80
(0.35;0.35;0.30)	1.08	1.36	0.87	3.94	3.00	6.79
(0.70;0.00;0.30)	1.35	1.20	0.87	3.67	3.24	6.97

Table 7: Implied parameters for the German optimal monetary-policy rules for various values of $\beta_{y1}+\beta_{y2}$, using program (9)

Weights in the loss function	Optimal parameter values			Unconditional standard deviations		
(μ_π ; μ_y ; $1-\mu_\pi-\mu_y$)	$\delta_\pi-1$	δ_y	δ_i	σ_π	σ_y	σ_i
Model estimate	0.44	0.27	0.83	4.93	4.20	4.92
Optimal rule with the estimated nonpolicy parameters ($\beta_{y1}+\beta_{y2}$=0.98)						
(0.00;0.70;0.30)	0.63	1.16	0.77	3.81	2.46	4.94
(0.35;0.35;0.30)	0.93	0.94	0.78	3.24	2.86	4.77
(0.70;0.00;0.30)	1.15	0.67	0.79	2.97	3.35	4.88
Optimal rule with $\beta_{y1}+\beta_{y2}$=1.05						
(0.00;0.70;0.30)	0.71	1.53	0.77	3.93	2.50	5.39
(0.35;0.35;0.30)	1.05	1.35	0.78	3.34	2.86	5.26
(0.70;0.00;0.30)	1.32	1.12	0.79	3.07	3.27	5.41
Optimal rule with $\beta_{y1}+\beta_{y2}$=0.91						
(0.00;0.70;0.30)	0.51	0.71	0.75	3.72	2.40	4.60
(0.35;0.35;0.30)	0.72	0.46	0.76	3.15	2.83	4.38
(0.70;0.00;0.30)	0.84	0.14	0.76	2.89	3.37	4.47

Fig. 1: Inflation rate, output gap and short-term interest rate

The US



Germany

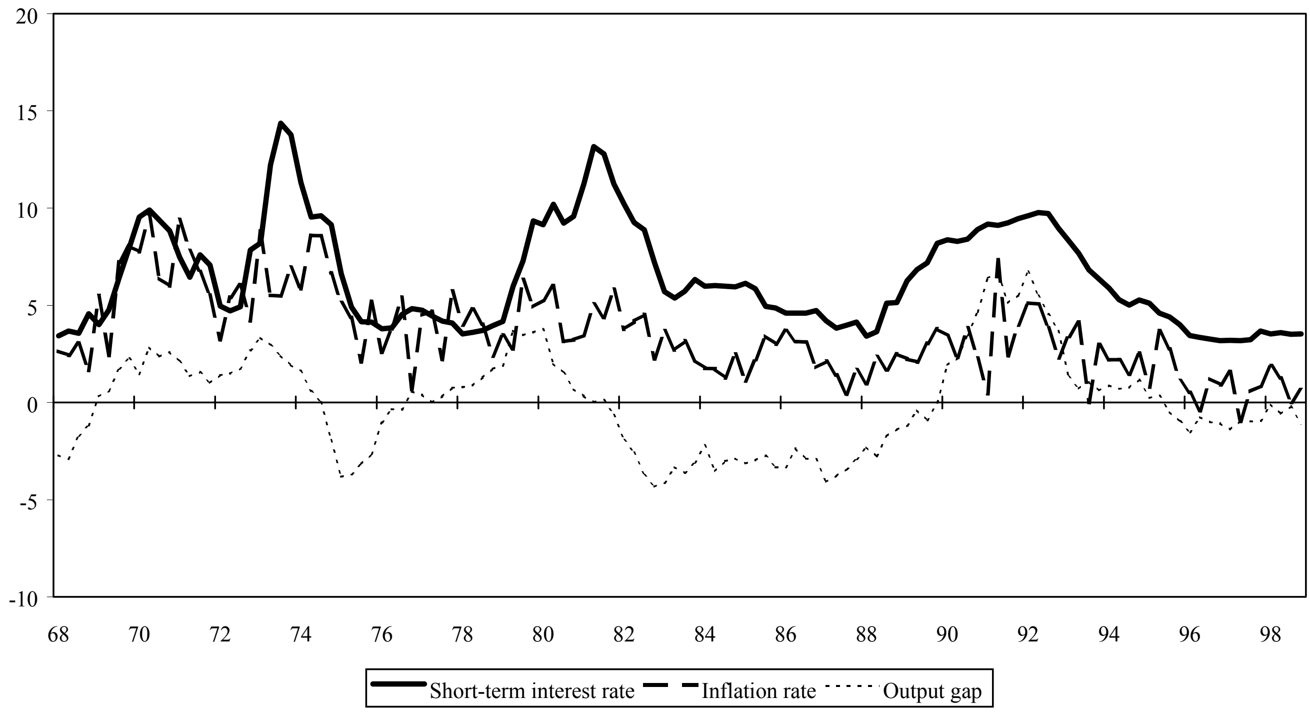


Fig. 2a: The US optimal policy frontier

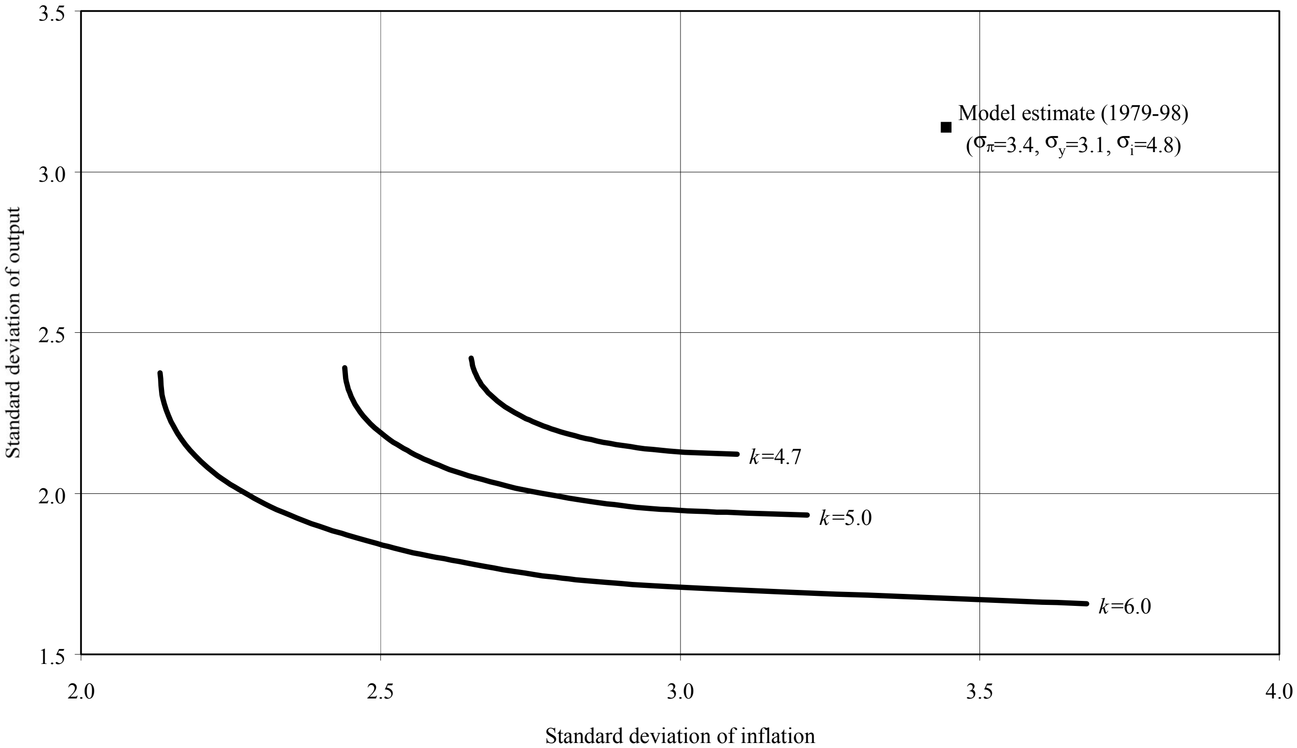


Fig. 2b: The German optimal policy frontier

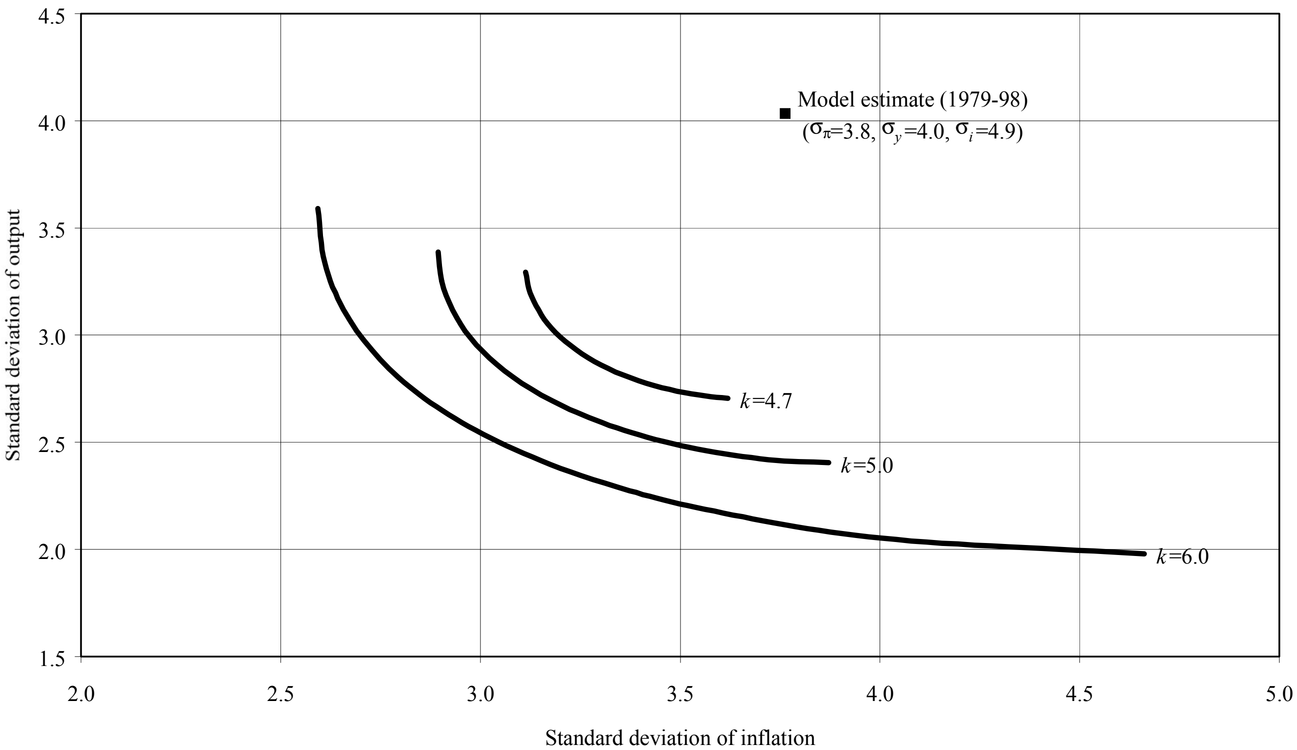
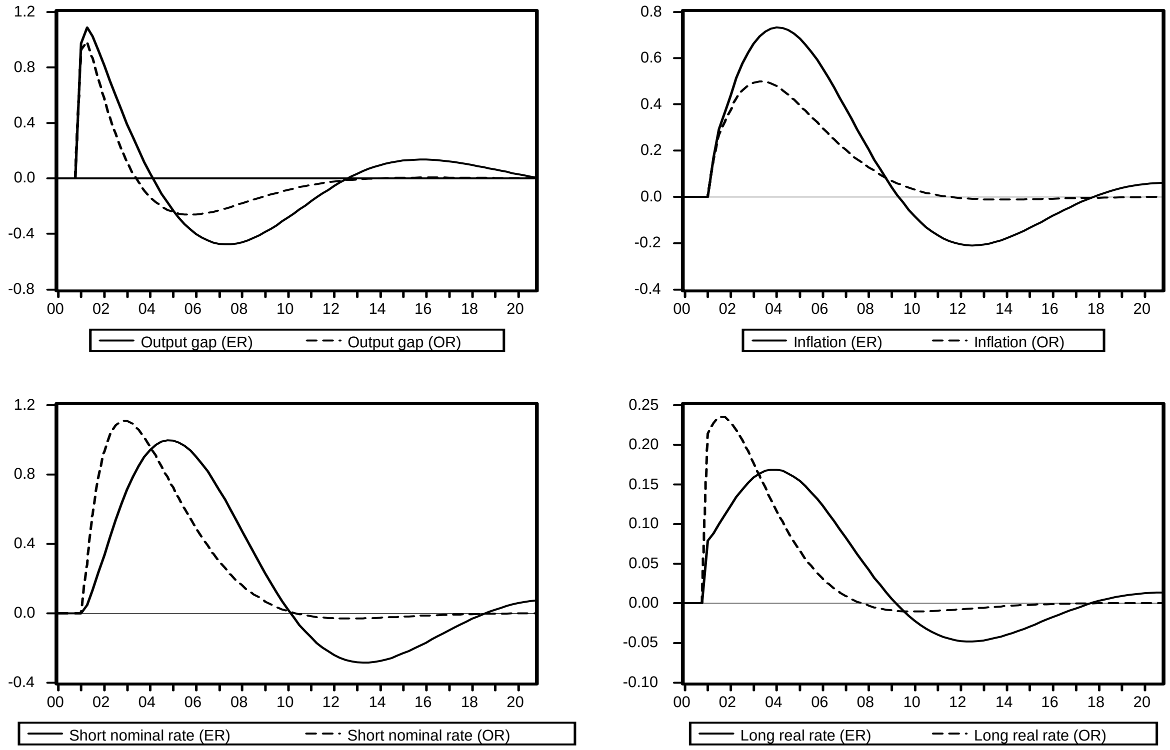


Fig. 3: Simulation of a temporary I-S shock under estimated rule (ER) and optimal rule (OR)

The US



Germany

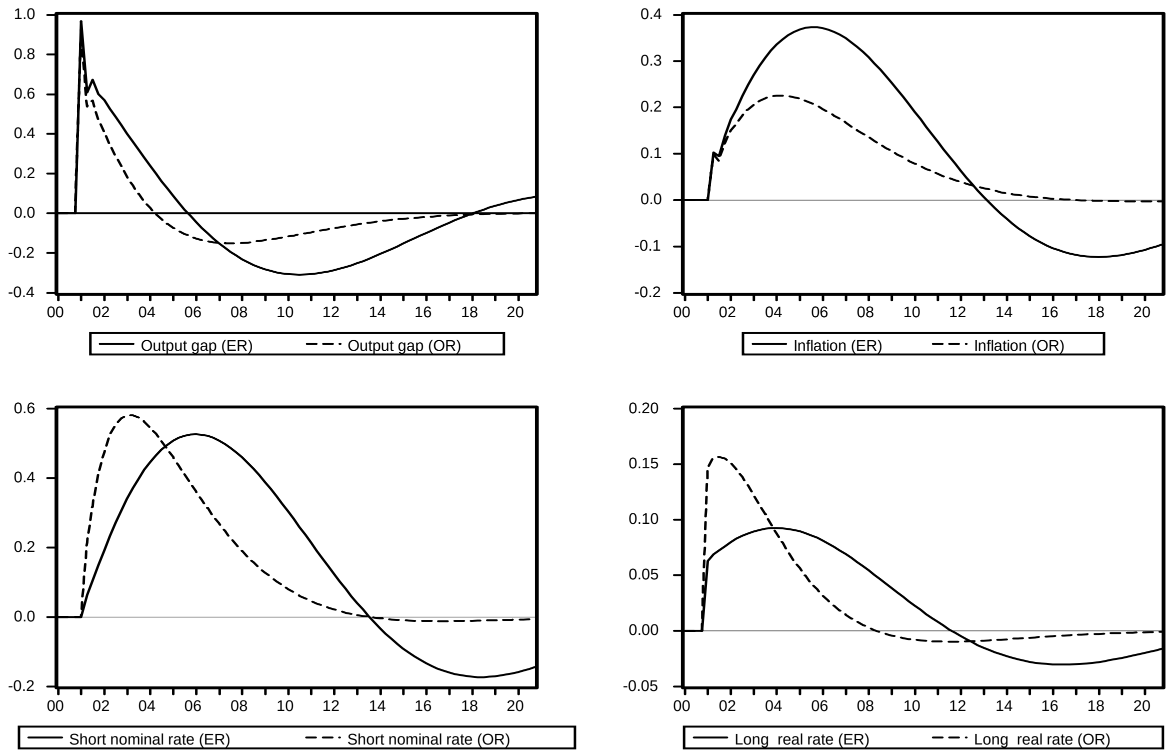


Fig. 4: Shifts in German optimal policy frontier (with μ_i=0.3) - Change in the sensitivity of inflation to movements in the output gap

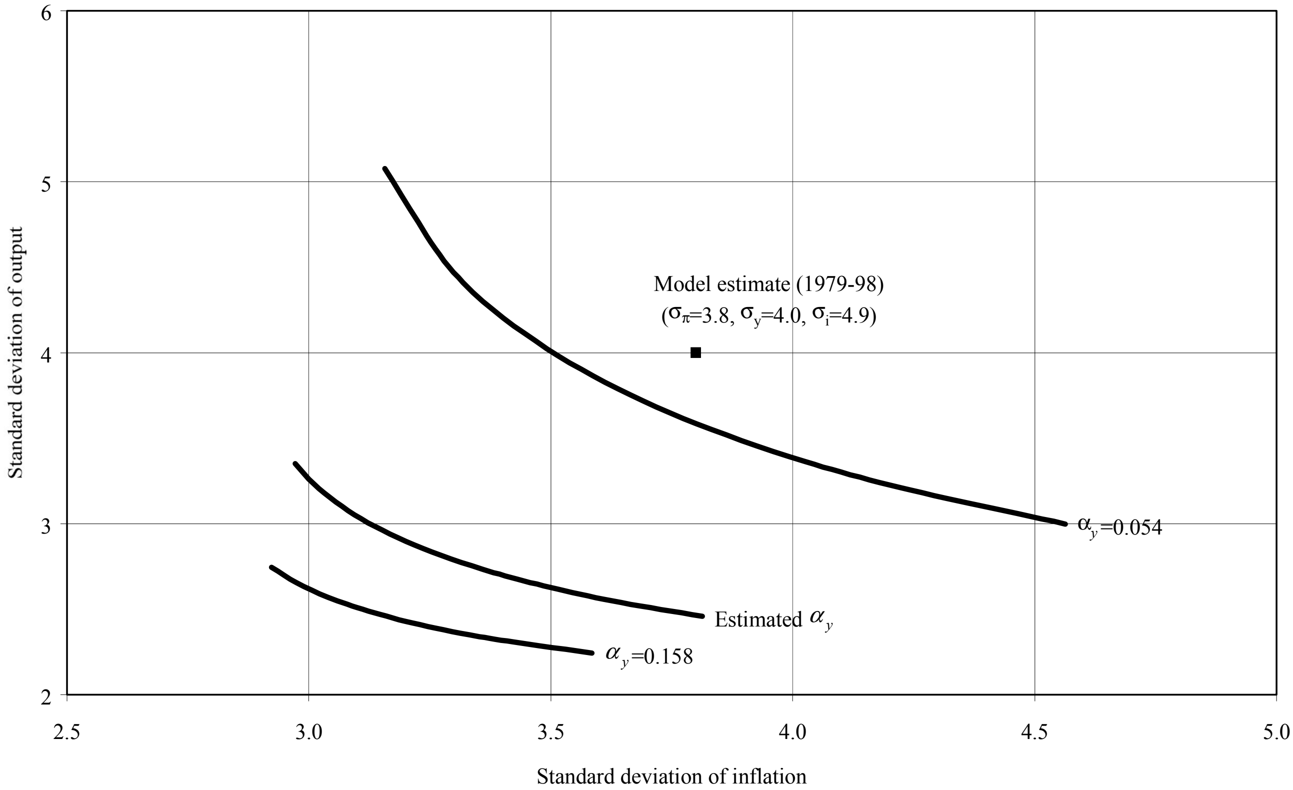


Fig. 5: Shifts in German optimal policy frontier (with μ_i=0.3) - Change in the interest-sensitivity of the I-S curve

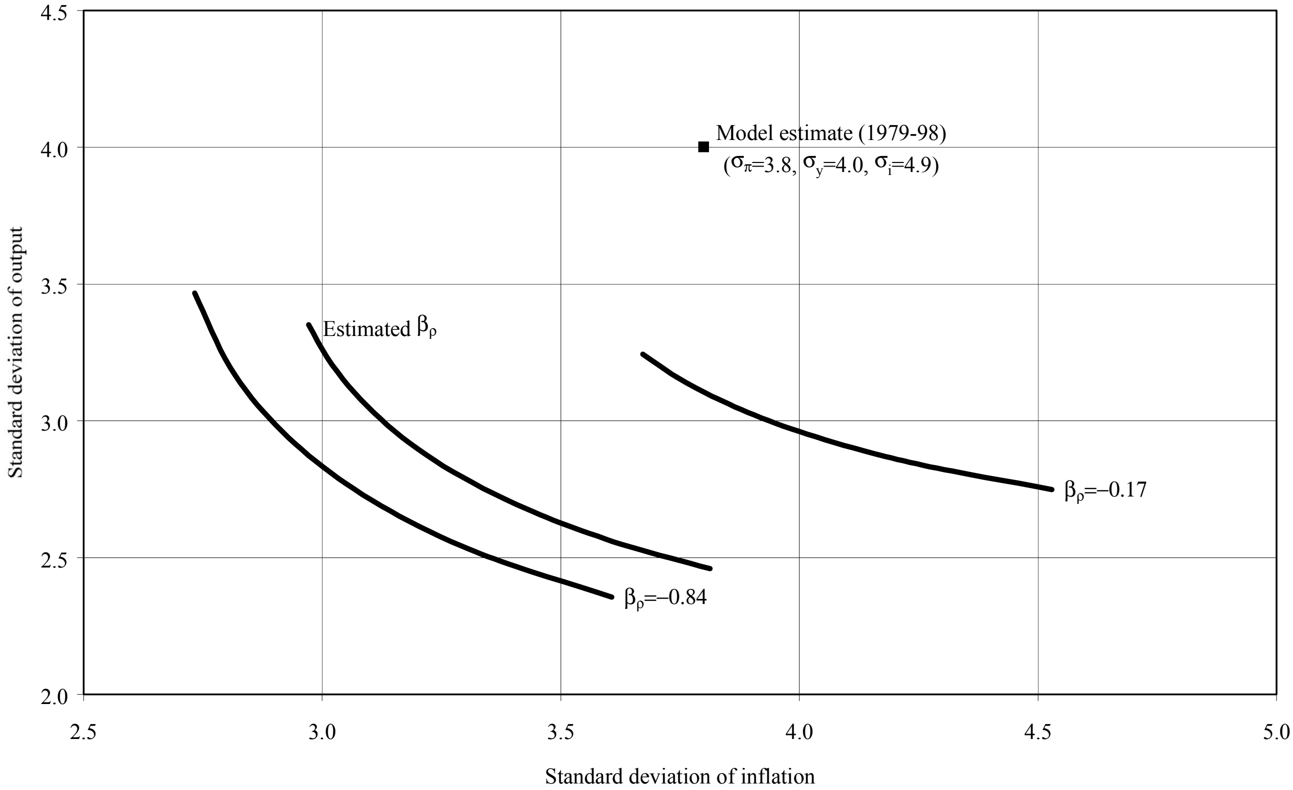


Fig. 6: Shifts in German optimal policy frontier (with μ_i=0.3) - Change in the persistence of the output-gap equation