UNIL | Université de Lausanne
Département de finance

ÉCOLE POLYTECHNIQUE
FÉDÉRALE DE LAUSANNE

FINANCE RESEARCH SEMINAR
SUPPORTED BY UNIGESTION

"Precision of Ratings"
joint with Bilge Yilmaz

Anastasia KARTASHEVA
Bank of International Settlements

Abstract

We analyze the equilibrium precision of ratings. Our results suggest that ratings become less precise as the share of uninformed investors and the gains of trade increase. The results provide an explanation for low accuracy of ABS ratings before the financial crisis. We apply the model to evaluate the effectiveness of the recent reform proposals, including Dodd-Frank Act. We show that some policies, in particular, rating standardization and expert liability, reduce market efficiency.

Friday, November 16, 2012, 10.30-12.00
Room 126, 1st floor of the Extranef building at the University of Lausanne

Precision of Ratings*

Anastasia V. Kartasheva† Bilge Yılmaz‡

November 12, 2012

Abstract

We analyze the equilibrium precision of ratings. Our results suggest that ratings become less precise as the share of uninformed investors and the gains of trade increase. The results provide an explanation for low accuracy of ABS ratings before the financial crisis. We apply the model to evaluate the effectiveness of the recent reform proposals, including Dodd-Frank Act. We show that some policies, in particular, rating standardization and expert liability, reduce market efficiency.

JEL codes: D82, D83, G01, G18, G24, G28, L15.

Keywords: credit rating agencies, ratings accuracy, differentially informed investors, information production and selling.

1 Introduction

Credit rating agencies (CRAs) rate securities in various asset classes. The US Securities and Exchange Commission identifies five classes of ratings, (1) financial institutions, brokers and dealers; (2) insurance companies; (3) corporate issuers; (4) issuers of asset-backed securities; and (5) issuers of government, municipal or sovereign securities. These asset classes differ substantially in terms of the extent of information asymmetries between the issuers and investors. For example, the investors' assessment of the credit risk of Brazil's sovereign securities can be based on publicly available sources such as the IMF website

*We are grateful to Franklin Allen, Vincent Glode, Itay Goldstein, Richard Kihlstrom, Doron Levit, Alessandro Lizzeri, Christian Opp, Marcus Opp, Christine Parlour, Joel Shapiro and the seminar participants at ASU, Berkeley, BC Winter Finance Conference, BIS, Boston Fed, Federal Reserve Board, Hebrew University of Jerusalem, IESE Barcelona, INSEAD, Moody's Analytics, OCC, Paris Systemic Risk Conference, Philadelphia Fed, SEC, WFA, and Wharton for helpful comments and discussions. The views expressed in the paper are those of the authors and do not necessarily represent the views of the Bank for International Settlements.

†Bank for International Settlements, anastasia.kartasheva@bis.org

‡The Wharton School, University of Pennsylvania, yilmaz@wharton.upenn.edu

summarizing countries macroeconomic conditions. To the contrary, investors need access to non-public information and specialized expertise to assess the credit quality of a mid-sized industrial company located in Oklahoma, US or the credit risk of an ABS portfolio. As a result, the population of investors can be differentially informed about the assets quality. The asset classes also can differ in terms of the issuer's value of liquidity and the availability of investment opportunities in a particular asset class.

The performance of CRAs during the financial crises suggests that ratings's precision varies across asset classes. Several empirical studies report that the ratings of asset-backed securities were uninformative and inflated to highest AAA rating.[1] In August 2011, the US Justice Department started an investigation whether one of the major CRAs, Standard and Poor's (S&P), improperly rated mortgage securities in the year prior to the financial crisis. At the same time, the performance of ratings in corporate bond market, utilities and insurance sectors was stable, even during the times of the financial crisis.[2]

The purpose of the paper is to explain what determines the precision of ratings. We argue that the incentives of the CRA to produce accurate ratings depend on the market conditions measured by gains from trade, the distribution of assets in the economy and the extent of the winner's curse problem among the heterogeneously informed investors. We build a rational model that incorporates these factors and apply it to analyze the effect of the recent CRAs reforms proposals, in particular, the Dodd-Frank Act, on the equilibrium precision of ratings.

We model a market with issuers, investors and a monopolistic CRA. Issuers are privately informed about the value of the asset and aim to sell the issue at the highest price. Issuers need a rating to signal the asset quality to investors. The CRA designs the rating system that is composed of the information technology and a rating fee. This set up follows the information intermediation literature (Lizzeri 1999).

We introduce two novel features to the information intermediation literature, the is-

[1] Ashcraft, Goldsmith-Pinkham, and Vickery find that subprime and Alt-A mortgage backed securities (MBS) experienced a significant decline in rating standards, and 80-95% of deals were assigned a AAA rating. Stanton and Wallace (2011) document that ratings of commercial mortgage-backed securities allowed for lower subordination levels that inflated the ratings. The size of the AAA tranche of collateralized debt obligation (CDO) deals was larger than suggested by the CRA's rating model (Griffin and Tang, 2009), with low B+ credit quality of the collateral that supported CDO issues (Benmelech and Dlugosz, 2009).

[2] According to Standard and Poor's report on corporate default rates and rating transitions, during 2008-09 only 25 companies initially rated as investment grade were in default, and the number of investment grade defaults was at most one per year during the rest of the period of 2004-2011. The rate of speculative grade companies defaults peaked to 9.5% in 2009, which is comparable to 9.7% rate following the high-tech bubble in 2001.

suers' type dependent value of outside option and the presence of differentially informed investors. We assume that issuer's outside option is increasing in the asset quality and proportional to gains from trade in an asset class. Then the issuers with attractive investment opportunities are willing to accept a higher discount to sell the issue. As we show, gains from trade will have an important negative feedback effect on the precision of ratings.

In the setting with differentially informed investors, the CRA plays an active role in creating the market surplus. Increasing the precision of ratings limits the ability of informed investors to capture the information rent, and thus enlarges the market surplus by solving the winner's curse problem. However, we show that more precise ratings also reduce the ability of the CRA to extract the surplus. Then the CRA's information structure is obtained as a trade-off of the two countervailing incentives, and it can be inefficient.

The value of issuers' outside option and the presence of differentially informed investors is what distinguishes asset classes and varies through time. Thus the model can explain the heterogeneous performance of CRAs. Also it provides a general framework that can be applied to evaluate a variety of the recent proposals on the reforms of CRAs in the US and in Europe. We discuss the policy implications of reforms on standardization of ratings across different asset classes, regulation of rating fees, introducing expert liability for overrated securities and reducing the reliance on ratings in regulation.

We obtain five main results. The first result is that the CRA's ratings are informative but noisy. It is driven by the fact that the profit of the CRA is a product of market penetration and the fee. In the extreme case when ratings are perfectly informative about asset values, the rating fee is determined by the willingness to pay of the lowest rated issuer. The CRA can increase this issuer's willingness to pay by assigning it high ratings with a positive probability. However, in doing so the CRA is limited by the high quality issuers decision to trade. As ratings become less informative, high quality issuers prefer to hold the asset instead of selling it at a substantial discount. This result contrasts with Lizzeri (1999) where the ratings are completely uninformative. The trade off between increasing the willingness to pay of lower quality issuers and revealing enough information to induce participation of high quality issuers determines the precision of ratings.

Second, we provide several results about the optimal information structure of the CRA. We show that the information structure is asymmetric and, under certain conditions, must entail rating inflation. That is, lower quality issuers must be assigned higher ratings with a positive probability, but higher quality issuers are always assigned high ratings.

Otherwise, higher quality issuers can refuse to trade following a low rating, which reduces CRAs profits. Thus the CRA designs a rating system under which its "mistake" is always optimistic.

The third result is about the precision of ratings in the market with differentially informed investors. When all investors are uninformed, the CRA's information structure affects the distribution of surplus between the issuers and the CRA, but it does not affect the size of surplus. In the presence of the winner's curse problem, the information structure of the CRA changes the size of surplus as informed investors capture informational rent. The reason is that more informative ratings reduce the winner's curse problem of uninformed investors and increase the surplus. At the same time, more informative ratings reduce the ability of the CRA to extract issuers' surplus. We show that as the extent of winner's curse problem increases, the CRA reduces the precision of ratings. Also we show that the winner's curse problem makes rating inflation feature more likely to prevail. Furthermore, when the winner's curse problem becomes substantial, the CRA reduces the market coverage to the best quality issuers. In this case, unrated issuers do not trade and it leads to inefficiency.

The forth result is that precision of ratings depends on the market conditions. When an economy is in boom and issuers face profitable investment opportunities, they are willing to accept a higher discount to sell the asset. It gives the CRA a possibility to extract more surplus by making ratings less informative. Thus as the aggregate gains of trade in the economy increase, the precision of ratings is compromised.

Finally, we show that the precision of ratings depends on the distribution of asset values in the economy. As the high quality assets become more scarce, the precision of ratings decreases. The reason is that rating lower quality assets becomes a more important source of CRAs profits, and these issuers willingness to pay is increasing as ratings become less informative.

We apply the model to evaluate the recent reform proposals of the credit rating industry. Following disappointing performance of ratings of asset backed securities, regulators both in the US and in the EU developed an array of policies that aim to improve incentives of CRAs to produce accurate ratings. We discuss the Dodd-Frank proposals on ratings standardization, introducing expert liability and reliance on ratings in regulation, as well as several proposals regarding regulation of rating fees. Our analysis suggests that some of these policies can be detrimental to market efficiency. In particular, we show that standardization of ratings across asset classes and introducing expert liability can reduce the precision of ratings and limit trade. To the contrary, reducing reliance on ratings in regulation and regulating rating fees can increase market efficiency.

The rest of the paper is organized as follows. The next section reviews related literature. Section 3 describes the model. Section 5 derives several properties of the information structure, and Section 6 applies the results to characterize the optimal precision of ratings. Evaluation of policy proposals on the CRA reform is contained in Section 7, and the conclusion follows. All proofs are devoted to the Appendix.

2 Related literature

We build on a framework developed in Lizzeri (1999) that delivers several important results in the information intermediation literature.[3] Lizzeri analyzes a model with a continuum of seller types, risk neutral buyers and no restrictions on the disclosure rules that the information intermediary can offer to sellers willing to pay for certification. The type of the seller denotes its quality and is equal to the valuation of a good by the buyer.

The optimal disclosure policy is derived from the following trade-off. The profit of an intermediary is a product of the market coverage and the fee charged for the certification services. As buyers' certification decision is voluntary, a certification fee is determined by the willingness to pay of the lowest rated seller. If an intermediary discloses the type of this seller perfectly, the seller is willing to pay at most the difference between its type and the expected value of uncertified sellers with lower types. However, an intermediary can increase the willingness to pay of the lowest rated type by pooling it with higher types. As higher types have no means to signal their quality other than the intermediary's certification, they have to accept more noisy certification. Hence, the optimal disclosure of an intermediary is to pool all types. It implies that ex-post the intermediary discloses no information except that a seller is better than the lowest type. Also it is able to extract all the surplus by charging the fee equal to the expected value in the market.

One of our important modelling innovation is that we introduce winner's curse problem to Lizzeri's framework by assuming that investors are heterogeneously informed. Also we depart from Lizzeri's framework in that issuers of securities have type dependent outside option. Both of these features are crucial in a financial market. Incorporating them in the Lizzeri's model permits to explain the effect of changing market conditions and characteristics of an asset class on the precision of ratings. These effects cannot be explained without extending the model and are novel in the literature. Furthermore, our

[3] Admati and Pfleiderer (1986) and (1990) are the first papers that study the role of information sellers in financial markets.

model provides a unified framework that allows to analyze the consequences of an array of policy proposals on market efficiency in a single tractable model.

An important difference between Lizzeri (1999) and our model is that in our framework CRA creates surplus and helps to restore the market inefficiency. In Lizzeri's context, information intermediation results in a pure transfer of surplus from sellers to an intermediary. The key friction in our model that makes the CRA's services value-enhancing is the "lemon's problem" of Akerlof (1970). As issuers prefer to hold the asset when the market price is low, their decision to trade is endogenous and depends on the rating technology of the CRA.

Our paper is closely related to recent theoretical literature that explores the incentive problems of the credit rating agencies leading to poor performance of ratings during the financial crisis. Mathis, McAndrews and Rochet (2009) and Bolton, Freixas and Shapiro (2012) show that asset complexity in the environment with naive investors can be detrimental to CRA's incentives. In Mathis, McAndrews and Rochet (2009), reputation is sufficient to discipline CRAs only when a large fraction of their income comes from rating simple assets. Bolton, Freixas and Shapiro (2012) build a model where a CRA may overstate the seller's quality when there are more naive investors. Skreta and Veldkamp (2008) study how higher complexity of rated assets affects incentives for ratings shopping and rating inflation. Their model is based on the assumption that investors cannot correct for ratings selection bias. Then ability of sellers to obtain ratings from different CRAs and to decide which ratings to disclose leads to ratings shopping and inflation. In our framework investors can be differentially informed, but their decisions are rational, and the CRA is strategic in designing its rating technology. Thus our results are not driven by investors' ignorance or naiveté.[4]

The reliance on ratings in regulation and regulatory arbitrage are other factors that can potentially explain poor ratings performance. White (2010) documents that the role of ratings in prudential regulation of financial institutions has been increasing over the years. As a result, the CRAs' compensation has been shifting from producing credit risk analysis to issuing regulatory licences. Opp, Opp and Harris (2012) develop a model where investors value highly rated bonds due to regulatory benefits. They show that rating-based regulations lead to rating inflation. But it has an ambiguous effect on CRA's information

[4] In practice, sophisticated investors such as investment banks are large market makers in security markets who act on both the buy and the sell side of the market. It is implausible that these institutions are unaware about the security structure. However, there is abandon evidence of rating inflation in structured finance markets documented in Benmelech and Dlugosz (2009), Coval, Jurek and Stafford (2009) and Stanton and Wallace (2010).

production which depends on the distribution of firms. In contrast, our model has a richer information technology than most of the previous literature[5]. This allows us to reconsider whether rating inflation is driven only by rating-based regulation. We find that inflation can occur even in the absence of rating-based regulation, though the use of ratings in regulation makes inflation more pronounced and ratings less informative.[6]

Lack of disclosure can also reduce the quality of ratings. Sangiorgi and Spatt (2012) show that lack of disclosure about the decision to solicit ratings by issuers leads to rating biases and excessive number of ratings per issuer. Pagano and Volpin (2012) analyze the effect of information transparency on the primary and secondary market liquidity. In the presence of unsophisticated investors who are unable to process disclosed information, transparency harms primary market liquidity. The reason is that transparency of information generates the winner's curse problem between the sophisticated and unsophisticated investors. We consider a different type of winner's curse problem. Kurlat and Veldkamp (2012) analyze the pros and cons of mandating the information disclosure in a general equilibrium framework.

3 Model

There are three groups of agents: issuers, investors and a CRA. An issuer owns an asset that is worth $v \in V = \{v_1, v_2, v_3\}$ to investors, where $0 = v_1 < v_2 < v_3$. The prior distribution of v is $\lambda = (\lambda_1, \lambda_2, \lambda_3)$, where $\lambda_i = \Pr(v_i) > 0$ and $\Sigma_i \lambda_i = 1$. Issuers are privately informed about v. The reservation value of an asset to an issuer type v is δv, with $\delta < 1$. The potential gain from trade, $v - \delta v$, can be due to several motives. It can come from the difference in discount factors between the two groups when it is more costly for the issuer to hold the asset to maturity. Then the investors' discount factor is 1 and the issuers' discount factor is δ over the holding period.[7] Another related motive is that an issuer has access to a positive NPV project and yet is capital constrained. Consequently, it needs to release capital to invest by selling the current assets. Then an issuer may be

[5] We analyze a general information structure of the CRA. In this respect our paper is related to mechanism design literature on optimal information structures in auctions, Bergemann and Pesendorfer (2007).

[6] In line with our findings, there is empirical evidence suggesting that in some asset classes ratings were becoming more tight over time. Blume, Lim and MacKinlay (1998) conclude that rating standards of U.S. corporate bonds were becoming more stringent in 1978-1995. Doherty and Phillips (2002) discover a similar trend for insurance ratings. As the regulatory reliance on ratings has been rapidly increasing after mid 1970s, these findings suggest that rating-based regulation cannot explain ratings inflation.

[7] This interpretation follows DeMarzo and Duffie (1999) and Dewatripont and Tirole (1995).

willing to sell at a discount in order to take advantage of the new investment opportunity. In this case, the difference of valution, $(1-\delta)\,v$, is equivalent to the magnitude of the value creation in the new project. Third, in case of debt issuance, the gains of trade $v-\delta v$ is the difference between the project's expected cash flow and the capital costs.

Investors consist of two groups, informed and uninformed. Uninformed investors are purely competitive and represent a group large enough to buy the entire issue. Uninformed investors know that there is a possibility that there are some informed investors who observe the value of the asset v prior to subscribing to the issue. Demand of informed investors is not sufficient to absorb the entire issue. All investors demand a fixed amount of the issue as long as the expected value is higher than the price. Uninformed investors face a winner's curse problem. They are more likely to obtain a larger allotment when informed investors decide not to subscribe to an issue. The rationing rule between the two groups of investors is summarized by the probability q that the uninformed investor's demand for an underpriced security is fulfilled. Furthermore, without loss of generality we impose a normalization such that the uninformed investors' demand is fulfilled with probability one if only uninformed investors demand. The probability q measures the severity of the winner's curse problem. If all investors are uninformed and there is no winner's curse, then $q=1$. As q decreases, the extent of the winner's curse problem increases. This approach builds on Rock (1986).

The CRA has an information technology to evaluate the value of the asset, but it cannot trade the asset. The signal space is denoted by $S=\{s_1,...,s_M\}$, $M\leq+\infty$. An information structure I is given by a pair $\langle S,F(v,s)\rangle$, where $F(v,s)$ is the joint probability distribution over the set of asset values V and the set of signals S. The joint probability distribution is defined in a usual way,

$$F(v,s)=\Pr(\widetilde{v}\leq v,\widetilde{s}\leq s),$$

with $f(v_j,s_i)=\Pr(v=v_j,s=s_i)$. For F to be part of the information structure requires that the marginal distribution with respect to v to be equal to the prior distribution of v, $\sum\limits_i f(v_j,s_i)=\lambda_j$. Let $\mathcal{I}$ denote the set of information structures that satisfy this condition. For a given set of signals S, the precision of a signal s_i on type v_j is defined by

$$p_{ij}=\Pr(s_i|v_j)=\frac{f(v_j,s_i)}{\Sigma_i f(v_j,s_i)}. \tag{1}$$

The CRA can choose any information structure. The cost of every information structure to the CRA is equal to zero. CRA charges a flat fee $\phi\geq0$ to an issuer soliciting a rating[8] and commits to reveal the signal realization to investors.[9] The profile (I, ϕ) defines the rating technology of the CRA. The choice of the rating technology is common knowledge among issuers and investors.

The information structure permits a very rich set of rating systems. For example, it is perfectly informative if $M \geq 3$ and $p_{ij} = 1$ if $i = j$ and $p_{ij} = 0$ otherwise. It is uninformative if for some $s_i \in S$, $p_{ij} = 1$ for all j, and $p_{ij} = 0$ otherwise. A rating system with rating grades can be represented with an information structure where a subset of type $V_i \subset V$ is assigned the same signal s_i, $p_{ij} = 1$ for all $v_j \in V_i$ and $p_{ij} = 0$ otherwise. A noisy rating system is a system where the same type can be assigned different signals, $p_{ij} < 1$ for all i, j.

The structure of the game is common knowledge to issuers, informed and uninformed investors and the CRA. The timing of the game is as follows.

$t = 0$. The nature chooses the issuer's type according to the prior distribution λ. Issuers privately learn their types $v \in V$. The rating agency commits to the rating technology (I, ϕ). The rating technology (I, ϕ) is observed by the issuers and the investors.

$t = 1$. Issuers decide whether to pay the fee and solicit a rating from the CRA. Informed investors learn the value of the asset for each issuer v and the CRA learns a signal s for issuers who solicited a rating. Rating agency announces the ratings of rated issuers.

$t = 2$. Issuers set the price of subscription b.

$t = 3$. Investors who have observed whether the issuer is rated and the assigned rating at $t = 1$, decide whether to subscribe to an issue. The demand of informed and uninformed investors is fulfilled according the rationing rule summarized above.

The strategy for the CRA is the information structure I and a fee ϕ. A behavioral strategy for the issuer is a pair of functions $d : V \times \mathcal{I} \times \mathcal{R}_+ \rightarrow [0, 1]$ that maps the issuer's type v and the rating technology (I, ϕ) into the probability to solicit a rating d, and $b : V \times \mathcal{I} \times \mathcal{R}_+ \times S \rightarrow \mathcal{R}_+$ that maps the issuer's type v, the rating technology (I, ϕ) and the realization of the signal s into the price of subscription b. A strategy of

[8] Our main objective is to explore the market for information where price differentiation based on assigned rating is prohibited due to the potential collusion problems between issuers and the CRA. In practice, NRSRO requirements and the principles issued by the International Organization of Securities Commissions (IOSCO) prohibit ratings fees to be contingent on assigned ratings. In theory, if the CRA is allowed to charge different fees to different assigned signals, its optimal information structure is to charge each issuer the fee equal to gains of trade, and to perfectly disclose the issuer type to investors. Clearly, this outcome is uninteresting.

[9] As discussed in Lizzeri (1999), it is the ability of the CRA to fine-tune the information structure rather than the commitment assumption that drive the results. Thus we follow the literature and assume full commitment of the CRA to the information structure.

the investor is a decision to subscribe to an issue given the information available at $t = 3$. For informed investors, the subscription decision is a function $\beta_I : V \times \mathcal{R}_+ \to \{0, 1\}$ that maps the issuer's type v and the price of subscription b into the decision to subscribe (1) or not (0). For uninformed investors, the subscription decision is a function β_U : $\mathcal{I} \times \mathcal{R}_+ \times \{0, 1\} \times S \times \mathcal{R}_+ \to \{0, 1\}$ that maps the rating technology (I, ϕ), the decision of an issuer to get rated (1) or not (0), the realization of the signal s and the price of the subscription b into the decision to subscribe (1) or not (0). We analyze the set of Perfect Bayesian equilibria of the game.

The model shares the basic framework of Lizzeri (1999). It departs from Lizzeri's model in two important dimensions. The first difference is related to the value of the asset to issuers (sellers). In Lizzeri's model, the issuer's value for the asset is equal to zero for all issuer types. Assuming that the issuers' reservation value δv is proportional to their types allows to capture an important feature of the financial market that issuers with highly desired assets often have better outside opportunities. In the context of Lizzeri's model, $\delta = 0$. The issuers' outside option also introduces a "lemon's problem" to the market. If the market price is lower than the outside option, an issuer will hold the asset and the gains of trade will not be realized. Then the CRA's rating technology affects the issuers' decision to trade and, hence, the market surplus.

The second difference is related to the information available to investors (buyers). Lizzeri assumes that all investors are uninformed which implies that competitive investors do not capture any surplus. Therefore, the total surplus captured by issuers and the CRA does not depend on the information produced by the CRA. As we will show below, in the market with differentially informed investors the CRA's choice of the information structure affects the size of this surplus as informed investors capture informational rent. The amount of information available to investors is represented by the probability q that uninformed investors' demand for underpriced issue is fulfilled. As q gets smaller, the severity of the winner's curse problem increases. In the Lizzeri's model all investors are uninformed which corresponds to the case $q = 1$.

There are three technical differences between our model and Lizzeri (1999) that do not affect the qualitative results but make the model tractable. Lizzeri considers a continuum of types v on a bounded interval while we restrict attention to discrete finite types. The restriction simplifies the equilibrium analysis of the market with differentially informed investors. Also we model the information technology of the CRA as an information structure while Lizzeri considers a general set of disclosure policies. Given that no cost is imposed on the choice of the information structure or the disclosure policy, the two approaches are equivalent. Finally, we assume that issuers are setting the price while in Lizzeri's model competing buyers (investors in our setting) are bidding for the good (an issue). Again, our assumption simplifies the analysis of differentially informed investors.

For a given information structure I, consider the decision of investors to subscribe to an issue at time $t = 3$. Let $\gamma_{ij} = \Pr(v_j | I, s_i, q)$ denote the beliefs of uninformed investors that an issuer rated $s_i \in S$ under the rating system I is type v_j, conditional on an issue offer. Also denote s_0 the event that an issuer is not rated, and $\gamma_{0j} = \Pr(v_j | I, s_0)$ the corresponding beliefs of uninformed investors conditional on the issue offer. The uninformed investors' assessment of the asset value of an issuer rated s_i, $i = 1, ..., M$ or not rated s_0, $i = 0$, under rating system I is

$$U_i = \sum_j \gamma_{ij} v_j.$$

Uninformed investors decide to subscribe to an issue rated s_i if the price of subscription b_i does not exceed their assessment of the asset value,

$$b_i \leq U_i.$$

At time $t = 2$, the issuer type v_j is better off selling the issue rated s_i as long as the price is higher than the issuer's asset value,

$$b_i \geq \delta v_j.$$

It means that there are gains from trade for issuer type v_j when

$$U_i \geq \delta v_j. \tag{2}$$

Otherwise, the issuer holds the asset, for example, by setting the subscription price equal to $\overline{v}$, where $\overline{v} > v_3$.

Condition (2) defines the set of issuers $T_i \subset V$ that are willing to trade if obtain a rating s_i under the rating system I,

$$T_i = \{v_j | U_i \geq \delta v_j\}.$$

If competitive uninformed investors make zero expected profit in equilibrium[10], issuers T_i optimally set the price

$$b_i = U_i.$$

[10] There is a continuum of equilibrium outcomes in which uninformed investors make strictly positive profits. However, as we will discuss later, these equilibria are the artifact of modelling choice where issuers set the prices. In fact, such equilibria do not arise in Lizzeri's model due to the assumption that investors bid for the issue. Moreover, in our framework none of these equilibria are robust in a sense that they do not satisfy Perfect Sequential equilibrium concept (Perry and Grossman (1986)).

At this price, the uninformed investors break even. If the issue is underpriced, informed investors gain a positive rent equal to the difference between the asset value and the price, $v_j - U_i$.

At time $t = 1$, if an issuer type v_j solicits a rating, it is assigned a rating s_i with probability p_{ij}. Given the rating, at stage $t = 2$ the issuer can either charge the price U_i or hold the asset and realize the value δv_j. Thus issuer's expected value of a rating is

$$R_j = \sum_i p_{ij} \max\{U_i, \delta v_j\}.$$

If an issuer does not solicit a rating, it can either sell the issue unrated at price U_0 or hold the asset. Then denote R_0 the payoff of unrated issuer equal to $\max\{U_0, \delta v_j\}$. Given a rating technology (I, ϕ), denote $d_j \in \{0, 1\}$ the decision of type v_j to solicit a rating,

$$d_j = \begin{cases} 1 \text{ if } R_j - \phi - R_0 \geq 0, \\ 0 \text{ otherwise.} \end{cases}$$

At stage $t = 0$, the CRA chooses a rating technology (I, ϕ) that maximizes its expected profit,

$$\Pi(I, \phi) = \sum_j \lambda_j d_j \phi.$$

The main focus of the analysis is to characterize how the choice of the information structure I depends on the extend of the winner's curse problem and the value of issuing a security.

4 Example

We start with a motivating example to illustrate the trade-offs leading to the results presented in the subsequent sections. Consider a market with a set of issuers with $v_1 = 0$, $v_2 = 4$ and $v_3 = 8$, a prior distribution $\lambda = (\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$, uninformed investors only, $q = 1$, and $\delta = \frac{3}{4}$. Under these assumptions, the expected value of the asset $E[v]$ is equal to 4, and the ex-ante market surplus $(1 - \delta)E[v]$ is equal to 1. Also we restrict the signal space to three signals, $S = \{s_1, s_2, s_3\}$.

In this market, the CRA is essential to realize the gains of trade. Indeed, if there is no CRA and buyers trade under the prior distribution, participation of all sellers results in market price of an asset equal to the expected value $E[v] = 4$. However, it is not sufficiently high to induce participation of issuers type v_3 as they obtain a higher payoff by holding an asset, $4 < \delta v_3 = 6$. Similarly, if only issuers' types v_2 and v_1 trade, the market price of 2 is lower than the outside option of type v_2, $2 < \delta v_2 = 3$. As a result, in equilibrium with no CRA the gains of trade are not realized.

Next we discuss the CRA's choice of the information structure. We start with a benchmark of perfectly informative ratings. A perfectly revealing information structure can be represented as

	v_3	v_2	v_1
s_3	1	0	0
s_2	0	1	0
s_1	0	0	1

where each element p_{ij} is the precision of signal s_i on type v_j, $p_{ij} = \Pr(s_i | v_j)$. It is perfectly revealing in a sense that a signal s_i perfectly identifies an issuer type v_i. As a result, the investors' assessment of the securities rated s_3, s_2 and s_1 equals to the true asset values, $U_3 = 8$, $U_2 = 4$ and $U_1 = 0$, respectively. Also issuer's expected value of a rating is equal to its type, $R_j = v_j$. Then the issuers' willingness to pay for a rating is $R_j - \delta v_j = v_j - \delta v_j$, where

$$R_3 - \delta v_3 = 2, \ R_2 - \delta v_2 = 1, \ R_1 - \delta v_1 = 0.$$

Given the information structure, the CRA needs to select a rating fee. Charging a lower rating fee increases the issuers' demand for ratings. In fact, the issuer type v_1 demands a rating only if its cost is zero, resulting in zero profits for CRA. Clearly, the CRA can do better by charging a positive fee and excluding participation of the lowest type v_1. It is easy to see that setting a fee $\phi = 1$ induces types v_3 and v_2 to solicit a rating and yields the profits of $(\frac{1}{3} + \frac{1}{3})1 = \frac{2}{3}$. Similarly, the same profit can be achieved by setting a fee $\phi = 1$ leading only type v_3 to pay for a rating. Under $\phi = 1$ issuer type v_3 gain a positive rent, leading to expected issuer surplus of $\frac{1}{3}(2-1) = \frac{1}{3}$. Importantly, all gains of trade are realized and the market surplus is maximized, $\frac{2}{3} + \frac{1}{3} = 1$. Thus under the perfectly revealing information structure, the CRA can rate two types of issuers and the surplus is split between the CRA and the highest type issuers v_3.

Can the CRA design an information structure that gains higher profits than the perfectly revealing information structure profits of $\frac{2}{3}$? The answer is yes. Under perfectly revealing information structure, the CRA does not extract full surplus because rated issuers types v_2 and v_3 have different willingness to pay for rating. Then the CRA is constrained to charge the fee equal to the lowest willingness to pay, in this case of issuers' type v_2. If the CRA can design an information structure that leads to the same willingness to pay by rated types, charging the fee equal to the willingness to pay will allow the CRA to extract the market surplus. Below is an example of a noisy information structure that achieves this objective.

	v_3	v_2	v_1
s_3	1	$\frac{1}{7}$	0
s_2	0	$\frac{6}{7}$	0
s_1	0	0	1

The highest issuer type v_3 is assigned a signal s_3 with probability one. The medium type v_2 is assigned a signal s_2 with probability $\frac{6}{7}$ and a signal v_3 with probability $\frac{1}{7}$. As a result, a signal s_2 reveals to investors that an issuer type is v_2, while a signal s_3 can be associated with two types of issuers, v_2 and v_3.

The noisy information structure leads to the following investors' assessment of the asset value conditional on the rating,

$$U_3 = \Pr(v_3|\, s_3) \cdot 8 + \Pr(v_2|\, s_3) \cdot 4 = \frac{\frac{1}{3} \cdot 1}{\frac{1}{3} \cdot 1 + \frac{1}{3}\frac{1}{7}} \cdot 8 + \frac{\frac{1}{3} \cdot \frac{1}{7}}{\frac{1}{3} \cdot 1 + \frac{1}{3}\frac{1}{7}} \cdot 4 = \frac{15}{2} < v_3 = 8,$$

$$U_2 = \Pr(v_3|\, s_2) \cdot 8 + \Pr(v_2|\, s_2) \cdot 4 = 0 \cdot 8 + 1 \cdot 4 = 4 = v_2.$$

As a result, the issuers' willingness to pay for the rating,

$$R_j - \delta v_j = \Pr(s_3|\, v_j)U_3 + \Pr(s_2|\, v_j)U_2,$$

is equalized between the two rated types,

$$R_3 - \delta v_3 = 1 \cdot U_3 - \frac{3}{4}8 = \frac{3}{2} \text{and } R_2 - \delta v_2 = \frac{1}{7}U_3 + \frac{6}{7}U_2 - \frac{3}{4}4 = \frac{3}{2}.$$

The noisy information structure equates issuers willingness to pay by increasing the medium type v_2 value of the rating due to a possibility of an optimistic mistake s_3 and reducing the high type v_3 value of the rating due to making the highest rating signal s_3 noisy. Then the CRA rates types v_2 and v_3, charges $\phi = \frac{3}{2}$ and extracts the market surplus, $(\frac{1}{3} + \frac{1}{3})\frac{3}{2} = 1$. An interesting feature of the noisy information structure is that it can be interpreted as rating inflation. When the CRA makes a deliberate mistake and assigns a signal s_3 to type v_2, the mistake is always optimistic in a sense that the signal s_3 is also assigned to a higher type v_3 where as s_2 is not to assigned to type v_3.[11] Another interesting feature of the noisy rating system is that the precision of ratings is asymmetric

[11] Note that assigning s_2 to type v_3 may lead type v_3 to withdraw since δv_3 may be strictly greater than U_2.

for different issuer types. While the highest type v_3 and the lowest type v_1 are assigned the corresponding signals s_3 and s_1 surely, the CRA chooses to be less precise in learning the medium issuers type v_2.

The example illustrates that the CRA can strategically choose noisy ratings, even when the precision of ratings has no costs. The analysis of a general model in the following sections builds on this economic intuition.

5 CRA's choice of noisy ratings

The example of the previous section illustrates a general idea that the CRA benefits most when the information structure leads to the equal willingness to pay among the rated issuers. Otherwise, the CRA can increase the profits by fine-tuning the information structure and changing the issuers' expected values of a rating R_i and R_j. Then a necessary condition of an optimal information structure can be written as

$$R_i - \delta v_i = R_j - \delta v_j = \phi \text{ for all } i, j.$$

In this section we develop properties of the CRA's choice of noisy ratings driven by its incentives to equalize willingness to pay among the rated issuers. In order to isolate this effect from the winner's curse problem that arises when investors are heterogeneously informed, this section focuses on the case of uninformed investors, $q = 1$. The following section extends the analysis to a general case of differentially informed investors, $q < 1$.

We first characterize an equilibrium information structure. Then we discuss its distinctive properties, and analyze whether these properties persist in any equilibrium information structure.

Consider an information structure with three signals, $S_3 = \{s_1, s_2, s_3\}$. Given S_3, the CRA chooses the probability distribution on the set of signals and types $S_3 \times V$ which can be represented in terms of $p_{ij} = \Pr(s_i | v_j)$,

	v_3	v_2	v_1
s_3	p_{33}	p_{32}	p_{31}
s_2	p_{23}	p_{22}	p_{21}
s_1	p_{13}	p_{12}	p_{11}

with $0 \leq p_{ij} \leq 1$ and $\Sigma_i p_{ij} = 1$. Then there exists an equilibrium with the following properties.

Proposition 1 *There exists an equilibrium in which CRA extracts all gains of trade. The CRA designs an information structure with three signals* S_3 *and the probability distribution*

	v_3	v_2	v_1
s_3	1	$1-p_{22}$	0
s_2	0	p_{22}	0
s_1	0	0	1

where $p_{22} = \frac{\delta(\lambda_3+\lambda_2)}{\lambda_3+\delta\lambda_2} < 1$*, and charges a positive rating fee* $\phi = (1-\delta)\frac{\lambda_3 v_3+\lambda_2 v_2}{\lambda_3+\lambda_2}$*. At* $t = 1$*, issuers types* v_3 *and* v_2 *solicit a rating, while issuers type* v_1 *do not solicit a rating. At* $t = 2$*, conditional on rating* s_3*, issuers types* v_3 *and* v_2 *set the same price* $b_3 = U_3 = \frac{\lambda_3 v_3+\lambda_2 v_2+\delta\lambda_2(v_3-v_2)}{\lambda_3+\lambda_2} > \delta v_3$*; conditional on rating* s_2*, issuers type* v_2 *set the price* $b_2 = U_2 = v_2$*. At* $t = 3$*, all rated issuers' offers are fully subscribed and traded.*

The equilibrium has several interesting features. First, as discussed earlier, the information structure induces equal willingness to pay for ratings among rated issuers,

$$\begin{aligned} R_3 - \delta v_3 &= U_3 - \delta v_3 = \phi, \\ R_2 - \delta v_2 &= p_{22}U_2 + (1-p_{22})U_3 - \delta v_2 = \phi. \end{aligned}$$

As a result, the CRA extracts the market surplus by charging the fee equal to the issuers' willingness to pay, $(\lambda_2 + \lambda_3)\phi = (1-\delta)E[v]$. The ability to extract the market surplus implies that restricting the set of signals to three signals S_3 is without loss of generality.

Second, ratings are informative but noisy. While rating s_2 reveals the issuer type v_2, rating s_3 can be assigned to two types, v_3 and v_2, and leaves investors uncertain about the type. From the perspective of issuers, while type v_3 is certain to be rated s_3, type v_2 can be rated s_2 and s_3 with probabilities p_{22} and $1-p_{22}$, respectively. Then the probability p_{22} can be interpreted as rating precision, as higher values of p_{22} make rating s_3 a more precise signal about type v_3.

Noisy ratings imply that the equilibrium security prices divert from assets' fundamental values. Issuers type v_3 are certain to sell the issue at price U_3. However, these issuers sell their security at a discount, $U_3 < v_3$. The reason is that investors rationally anticipate that an asset rated s_3 can have value v_2,

$$U_3 = \Pr(v_3|\, s_3)v_3 + \Pr(v_2|\, s_3)v_2 = \frac{\lambda_3 v_3 + (1-p_{22})\lambda_2 v_2}{\lambda_3 + (1-p_{22})\lambda_2}.$$

At the same time, issuers type v_2 rated s_3 sell the issue at a price above the fundamental value, $U_3 > v_2$. Rating s_2 perfectly reveals the issuer type v_2 and leads to price $U_2 = v_2$.

The third distinctive feature of the equilibrium information structure is that it entails rating inflation. Indeed, the intermediate type v_2 can be rated either s_2 or s_3, effectively pooling with type v_3 with probability $1 - p_{22}$. Thus the CRA intentionally makes an optimistic mistake in rating issuers v_2. The benefit of rating inflation is that it adjusts issuers' willingness to pay compared to the situation of perfectly informative ratings. Pooling of types v_2 and v_3 increases the willingness to pay of the intermediate type v_2 and decreases the willingness to pay of the highest type v_3.

Does rating inflation persist in any equilibrium of the game? The answer depends on the value of issuer's outside option. In equilibrium, an information structure needs to satisfy both ex-ante and ex-post issuers' participation constraints for the set of rated issuers. The ex-ante constraint at $t = 1$ states that the expected value of a rating is at least as high as the value of an outside option. The ex-post constraint at time $t = 2$ requires that rated issuers are willing to trade after the rating has been assigned. Under rating inflation structure of Proposition 1, trade is an optimal continuation strategy for both types of issuers. Type v_3 is certain to be rated s_3 and sell a security at price $U_3 > \delta v_3$. Type v_2 is rated s_2 or s_3 and sells at prices U_2 or U_3 which are higher than no trade alternative payoff of δv_2. Given the equilibrium security prices, the ex-post participation constraints are satisfied for any value of an outside option δ.

Intuitively, if the outside option of the issuers δ is low, the ex-post constraint can be satisfied under other information structures that permit CRA to gain the market surplus. Consider an example of an information structure with rating *deflation* where type v_3 receives a noisy signal,

	v_3	v_2	v_1
s_3	z	0	0
s_2	$1 - z$	1	0
s_1	0	0	1

with $z = \frac{\delta(\lambda_3+\lambda_2)}{\delta\lambda_3+\lambda_2}$. It exhibits rating deflation in a sense that the CRA makes a pessimistic mistake as it assigns a rating s_2 to type v_3 with a probability $1-z$. Under this information structure the equilibrium security prices are

$$\widehat{U}_3 = v_3 \text{ and } \widehat{U}_2 = \frac{(1-\delta)\lambda_3}{\lambda_3 + \lambda_2} v_3 + \frac{\delta\lambda_3 + \lambda_2}{\lambda_3 + \lambda_2} v_2.$$

Then issuers' type v_3 assigned a lower rating s_2 can trade at price $\widehat{U}_2$. This price must be sufficiently high to make trade attractive, $\widehat{U}_2 > \delta v_3$. Otherwise, holding an asset provides a higher payoff. If trade is optimal, rating deflation leads to the equal willingness to pay among the rated issuers and yields the profits to the CRA equal to the market surplus. However, as the outside option of issuers δ increases, holding an asset becomes more attractive. From the perspective of the CRA, the withdrawal from the market of issuers' type v_3 is inefficient. It will reduce issuers' willingness to pay for ratings and CRA's profits. Thus, in order to guarantee trade after the rating has been assigned, an equilibrium must entail rating inflation. The next proposition builds on the discussion and provides conditions under which rating inflation is a necessary property of an equilibrium rating system.

Proposition 2 *If the issuers have high outside option $\delta > \overline{\delta} = \frac{\lambda_2 v_2 + \lambda_3 v_3}{\lambda_2 v_3 + \lambda_3 (2v_3 - v_2)}$, the equilibrium information structure must entail rating inflation.*

The forth feature of the equilibrium information structure is the effect of the issuers' outside option δ on ratings precision. Lower value of outside option δ means that the issuer is more eager to sell the asset. In general, lower value of outside option corresponds to the situation when economy is in a boom. Then the relationship between the rating precision and the value of the outside option explains how the quality of CRA's information depends on the economic cycle.

Proposition 3 *The precision of ratings declines as the opportunity costs of holding an asset increase, $\frac{dp_{22}}{d\delta} > 0$.*

The outside option affects the informativeness of ratings due to the issuers endogenous participation decision. If ratings are uninformative, the investors' posterior assessment of issuer's type is close to the ex-ante average asset value. It implies that high value issuers type v_3 must sell an asset at a substantial discount. If the opportunity costs of holding an asset are very high, $\delta \to 0$, the issuers are willing to accept a discount. In this case ratings are uninformative, $p_{22} = 0$ and both types of issuers v_3 and v_2 are assigned the same rating. However, as the opportunity costs of holding an asset decline and the outside option δv increases, the CRA has to increase rating precision. Otherwise, high quality issuers are better off holding the asset and realizing the value δv instead of accepting to sell at a discount. In the limit case $\delta = 1$, the CRA discloses all information.

The effect of issuers' outside option on rating precision is closely related to Lizzeri (1999) striking result of uninformative ratings. In Lizzeri's model, all issuers pay a positive fee to obtain uninformative ratings in order to distinguish themselves from the worse type.[12] Proposition 3 reveals that ratings are uninformative only when the issuers have zero value of an asset, $\delta = 0$. In a general case $\delta > 0$, ratings are informative but noisy.

[12] In Lizzeri (1999), ratings are uninformative in a sense that the posterior distribution of seller types conditional on the information intermediary disclosure policy is identical to the prior distribution.

The result of Proposition 3 also shows that the ability of the CRA to gain profits must be procyclical. When the economy is in a boom, the opportunity costs of holding the assets is high due to attractive investment opportunities and δ is low. Then issuers are eager to sell the issue, yielding high profits to the CRA. When the economy is bust and δ is high, there are fewer investment opportunities, and the profits of the CRA are low.

Finally, the last feature of the equilibrium information structure of Proposition 1 regards the market coverage. The CRA rates two issuers' types v_3 and v_2, while the lowest type v_1 is not rated. Clearly, reducing the market coverage to rating either issuers type v_3 or v_2 is not profitable. By following this strategy, the CRA foregoes the surplus that can be created by issuers type v_{-j}, and achieve a profit of $(1-\delta)\lambda_j v_j$ which is always inferior to the market surplus $(1-\delta)E[v]$.

Are there equilibria in which the CRA rates all issuers? In general, the CRA cannot extract any surplus from the lowest type v_1 as it can signal its type at no cost by not soliciting a rating. It implies that participation of the lowest type must be driven by the ability to sell a rated asset at a price above the fundamental value, $R_1 - \delta v_1 \geq v_1 - \delta v_1$. Then extending the market coverage from two to three types must redistribute the surplus from higher issuer types to the lowest type. In the market with uninformed investors, $q = 1$, the size of market surplus is fixed. Thus, as long as trade remains an optimal continuation strategy of rated types, redistribution of surplus does not change CRA' profits. The following proposition explains how the CRA's choice of the market coverage affects the expected value of a rating for different types of issuers.

Proposition 4 *There exists an equilibrium in which all three types are rated. Issuers' expected value of a rating increases as the CRA's rating system excludes participation of lower types. When k highest types are rated, type v_j expected value of a rating R_j^k is*

$$R_j^3 = (1-\delta)E[v] + \delta v_j, \;\; j = 1, 2, 3,$$

$$R_j^2 = \frac{(1-\delta)E[v]}{\lambda_2 + \lambda_3} + \delta v_j, \;\; j = 2, 3.$$

When investors are uninformed, $q = 1$, the CRA is indifferent between rating all issuer types or rating the two highest types v_2 and v_3, $\sum_{j=2,3} \lambda_j (R_j^2 - \delta v_j) = \sum_{j=1,2,3} \lambda_j (R_j^3 - \delta v_j) =$ $(1-\delta)E[v]$. These strategies dominate rating only one type v_3 or v_2.

An interesting outcome in the market with uninformed investors is that the CRA is indifferent between rating two highest types v_3 and v_2, or all types. In the analysis of the market with heterogeneously informed investors of the next section, we build on these results and show how the choice of market coverage changes in the presence of winner's curse problem.

6 Noisy ratings and differentially informed investors

In the market with heterogeneously informed investors, precision of ratings affects the size of the market surplus. The reason is that it determines the amount of the information rent earned by informed investors. Uninformed investors realize that they can be offered an overpriced issue that is not demanded by informed investors. It implies that issuers must provide a winner's curse discount to uninformed investors. As a result, the winner's curse problem reduces the surplus between the issuers, the uninformed investors and the CRA. The winner's curse problem can be reduced by the CRA making ratings more informative. In fact, the CRA can eliminate the winner's curse completely by designing perfectly informative ratings. However, as the analysis of the previous section shows, perfectly informative ratings cause differences in issuers' willingness to pay, and reduces the ability of the CRA to extract the surplus. In this section we analyze the trade off between the two countervailing incentives of the CRA. We build on the results of the previous section and explore the effect of the winner's curse problem on the optimal rating system of the CRA. We show that it may lead to no trade in some market segments and thus produce inefficiencies.

The start with the effect of winner's curse problem on the market coverage.

Proposition 5 *In the market with differentially informed investors, any information structure that induces participation of the two highest types v_2 and v_3 is more profitable than the one that induces participation of all types. Consequently, there exists no equilibria in which uninformed investors break even and CRA rates all types.*

The result is in contrast to the outcome in the absence of winners curse. It implies that in the market with differentially informed investors the CRA provides partial market coverage. The economic rational for the result is that including the lowest issuer type v_1 hardens the underpricing problem without increasing the market surplus that the CRA can obtain from issuers. It reduces the price the investors are willing to pay for a rated asset, and consequently, the issuers' willingness to pay for a rating. Unlike in the case of uninformed investors $q = 1$, redistributing the surplus from higher types v_3 and v_2 to the lowest type v_1 is reducing the market surplus between the issuers, the uninformed investors and the CRA.

Winner's curse problem can induce even further reduction in the market coverage. When ratings are noisy, limiting market coverage to the highest type issuers reduces the underpricing costs. Intuitively, this strategy is preferable when the winner's curse problem is so severe that the CRA is better off to forgo the surplus created by the intermediate type issuers in order to avoid the underpricing of the highest type. Next we consider two continuation equilibria following the decision of the CRA to design a rating system that targets either both types v_3 and v_2, or only the highest type v_3. Then we characterize how the optimal choice of the market coverage depends on the winner's curse problem.

For the rest of the section, we restrict attention to information structures with three signals. As we show in Proposition 1, it is without loss of generality in the market with uninformed investors. Analyzing a rating system with three signals in the market with differentially informed investors permits to have a benchmark to evaluate the effect of asymmetric information on the market outcome. Thus we consider an information structure

	v_3	v_2	v_1
s_3	p_{33}	p_{32}	0
s_2	p_{23}	p_{22}	0
s_1	0	0	1

Under this information structure, in line with the result of Proposition 5, rating s_1 reveals perfectly type v_1 and issuers v_1 do not solicit a rating.

Suppose that the CRA designs a rating system where ratings are solicited by types v_3 and v_2. Then the CRA designs the rating system described in the following proposition.

Proposition 6 *If the CRA targets to rate types v_2 and v_3, there is a continuation equilibrium with information structure*

	v_3	v_2	v_1
s_3	1	$1-p_{22}$	0
s_2	0	p_{22}	0
s_1	0	0	1

where $p_{22} = \frac{\delta(\lambda_3 q+\lambda_2)}{\lambda_3 q+\delta\lambda_2} < 1$. The CRA charges the rating fee $\phi = (1-\delta)\frac{\lambda_3 q v_3+\lambda_2 v_2}{\lambda_3 q+\lambda_2}$ and gains profits $\Pi_{2,3} = (1-\delta)\frac{(\lambda_2+\lambda_3)(q\lambda_3 v_3+\lambda_2 v_2)}{\lambda_3 q+\lambda_2} < (1-\delta)E[v]$. At $t=1$, issuers types v_3 and v_2 solicit a rating, while issuers type v_1 do not solicit a rating. At $t=2$, conditional on rating s_3, issuers types v_3 and v_2 set the same price $b_3 = U_3 = \frac{\lambda_3 q v_3+\lambda_2 v_2+\delta\lambda_2(v_3-v_2)}{\lambda_3 q+\lambda_2} > \delta v_3$;

conditional on rating s_2, issuers type v_2 set the price $b_2 = U_2 = v_2$. At $t = 3$, all rated issuers' offers are fully subscribed and traded. Informed investors gain a positive rent equal to $\frac{\lambda_3\lambda_2(1-q)(1-\delta)(v_3-v_2)}{\lambda_3 q+\lambda_2}$.

Like in the market with uninformed investors described in Proposition 1, ratings are noisy and exhibit rating inflation. However, there are several important new features that arise due to the winner's curse problem. It introduces the winner's curse discount that issuers type v_3 need to offer to uninformed investors. A discount is proportional to the probability $\gamma_{32} = \Pr(v_2|\, s_3)$ that an issuer rated s_3 is the intermediate type v_2,

$$v_3 - U_3 = \Pr(v_2|\, s_3)(v_3 - v_2) = \frac{\lambda_2(1-\delta)}{\lambda_3 q + \lambda_2}(v_3 - v_2).$$

It is increasing as the winner's curse problem becomes more pronounced and q decreases. The discount reduces the profits of the CRA, as it reduces the willingness to pay of both rated types. However, the discount is necessary to compensate the uninformed investors for the potential of buying an issue rejected by informed investors.

Another effect of winner's curse and noisy ratings is that the CRA is unable to extract the market surplus. At the same time, the informed investors gain a positive informational rent. The decision to forgo some market surplus is optimal from the CRA's perspective. It balances the ability to equalize the willingness to pay between the rated types, and hence the need to make ratings noisy, and the size of the winner's curse discount offered to uninformed invesotrs. However, as the winner's curse problem becomes more severe, the size of the winner's curse discount increases. In the limit case of $q = 0$, the CRA's profit becomes $(1-\delta)(\lambda_3 + \lambda_2)v_2$. It means that both assets v_3 and v_2 have to be sold at price of the intermediate asset v_2, with informed investors capturing the rest of surplus. An alternative strategy of the CRA to limit the rent of informed investors is to reduce the market coverage to issuers type v_3. Next we describe the rating system under which only v_3 assets are rated, and provide the conditions when this strategy is optimal.

Suppose that the CRA designs a rating system under which ratings are solicited by one type v_3, for example, by setting $p_{ii} = 1$. Then the rating perfectly reveals the issuer's type. The issuer will set the price of subscription equal to the value of the asset, $b = v_3$. The issuers' willingness to pay for such a rating depends on the best outside option between holding the asset δv_3 and possibly selling it without a rating and thus pooling with the two lower types, v_2 and v_1. The unrated assets v_1 and v_2 are traded in the market if

$\delta v_2 < \frac{\lambda_2 q v_2}{\lambda_2 q+\lambda_1}$. It implies that the highest fee that can be charged by the CRA is

$$\phi = \begin{cases} \min\{(1-\delta)v_3, v_3 - \frac{\lambda_2 q v_2}{\lambda_2 q+\lambda_1}\}, \delta < \frac{\lambda_2 q}{\lambda_2 q+\lambda_1}, \\ (1-\delta)v_3, \delta \geq \frac{\lambda_2 q}{\lambda_2 q+\lambda_1}. \end{cases}$$

Under this rating system, types v_2 and v_1 do not solicit a rating. The next proposition summarizes the market outcome under limited market coverage of type v_3 issuers.

Proposition 7 *If the CRA targets issuers type v_3, there is a continuation equilibrium with information structure*

	v_3	v_2	v_1
s_3	1	0	0
s_2	0	1	0
s_1	0	0	1

The CRA charges the rating fee $\phi = \min\{(1-\delta)v_3, v_3 - \frac{\lambda_2 q v_2}{\lambda_2+\lambda_1}\}$ and gains profits $\Pi_3 = \lambda_3 \min\{(1-\delta)v_3, v_3 - \frac{\lambda_2 q v_2}{\lambda_2+\lambda_1}\}$. At $t=1$, issuers types v_3 solicit a rating, while issuers type v_2 and v_1 do not solicit a rating. At $t=2$, conditional on rating s_3, issuers types v_3 set the price $b_3 = U_3 = v_3$. If $\delta < \frac{\lambda_2 q}{\lambda_2 q+\lambda_1}$, issuers v_2 and v_1 set the price $b_{1,2} = \frac{\lambda_2 q v_2}{\lambda_2 q+\lambda_1}$; otherwise, these issuers set the price $b_{1,2} = v_2$. At $t=3$, rated issuers' offers are fully subscribed and traded; if $\delta < \frac{\lambda_2 q}{\lambda_2 q+\lambda_1}$, unrated types offers are fully subscribed and traded.

The CRA chooses the market coverage that provides the highest profit under a given set of market conditions.

Proposition 8 *There exists an interval $[\overline{q}, 1]$ such that for all $q \in [\overline{q}, 1]$ the optimal rating system induces two issuer types v_3 and v_2 to solicit a rating; and for all $q \in [0, \overline{q}]$ it induces only type v_3 to solicit a rating.*

As the winner's curse problem becomes more severe, the CRA decreases the market coverage. Under the market coverage with two types, the CRA's optimal information structure aims to increase the payoff of issuer type v_2 by pooling it with type v_3. Presence of informed investors leads to severe underpricing of an issue with the highest rating s_3, which ultimately reduces the fee that the CRA can charge. As the winner's curse problem becomes substantial, the CRA is better off eliminating the underpricing by restricting market coverage to the best issuer type v_3.

The rating system with two rated types v_3 and v_3 involves rating "inflation" in the sense that while the highest type receives only the highest rating s_3, type v_2 can receive two ratings, s_2 and s_3. In other words, the CRA makes an optimistic "mistake" of assigning a higher rating s_3 to type v_2. As in the case of uninformed investors, the information structure described in Proposition 6 is not unique. An interesting question though is weather the rating inflation is more prevalent in markets with substantial winner's curse problem. The next proposition shows that indeed this is the case.

Proposition 9 *Restrict attention to the set of equilibria in which only types v_2 and v_3 solicit a rating. Then for $\delta > \overline{\delta} = \frac{\lambda_2 v_2 + q\lambda_3 v_3}{\lambda_2 v_3 + q\lambda_3(2v_3 - v_2)}$ equilibrium must entail rating inflation. As winner's curse problem becomes more severe, q decreases, the set of market conditions for which rating inflation is necessary increases, $\frac{d\overline{\delta}}{dq} > 0$.*

The result is similar to Proposition 2. High values of δ imply that the participation constraint of type v_3 is more costly to satisfy. Assuring that type v_3 is always assigned a high rating guarantees that it trades, and the surplus is not lost. As the winner's curse problem increases, the valuation of the issuer rated s_2 decreases, which makes rating inflation necessary. Thus markets with high information asymmetry between among investors are most prone to exhibit rating inflation features.

When the CRA rates types v_2 and v_3, the information content of ratings depends on the market conditions. Given the information structure described in Proposition 6, the probability p_{22} that issuers type v_2 are assigned a rating s_2 can be interpreted as rating precision. Next proposition summarizes how the rating precision depends on the market conditions.

Proposition 10 *In the market with high share of uninformed investors, $q \in [\overline{q}, 1]$, the CRA reduces ratings precision*

(i) as the share of uninformed investors increases (q increases), $\frac{dp_{22}}{dq} < 0$;

(ii) as the aggregate value of liquidity increases (δ decreases), $\frac{dp_{22}}{d\delta} > 0$;

(iii) as high quality assets become more scarce ($\frac{\lambda_2}{\lambda_3}$ increases), $\frac{dp_{22}}{d(\frac{\lambda_2}{\lambda_3})} < 0$.

In the extreme case, when all investors are uninformed ($q = 1$) and the value of liquidity is very high ($\delta = 0$), ratings are uninformative, $p_{22} = 0$.

The comparative statics results suggest that under the conditions of booming economy, that is, high share of uninformed investors and high value of liquidity, ratings are less informative. It may be an explanation for the poor performance of ratings of asset backed securities. In the pre-crisis period, these assets had higher returns relative to other securities. Also the period coincided with rapid growth in several developing countries that were eager to invest in ABS assets. The other result is that the information content of ratings depends on the distribution of investment opportunities. As high quality assets become more scarce, the CRA's major revenue is driven by rating intermediate type v_2. It provides incentives to reduce the precision of ratings.

7 Policy implications

In this section, we apply our theory to evaluate the effect of recent CRA reform proposals on ratings precision and the market outcome. We discuss the proposals on standardization of rating symbols, regulation of the rating fees, expert liability and reducing the reliance on ratings in regulation.

7.1 Standardization of rating symbols

Major rating agencies use rating symbols to communicate the credit quality of issuers to investors. Usually CRAs employ a dozen of rating categories and distinguish between investment grade and non-investment grade securities. The common practice is that the same rating symbols are applied to different asset classes rated by the same CRA. At the same time, the CRAs' rating methodology documents emphasize and the empirical evidence confirms that same rated securities from different asset classes may have different credit quality.

The difference of credit quality was especially stark for the asset backed securities that experienced massive downgrades during the 2007-2008 financial crisis. Combined with the fact that the majority of these securities were designed to have a high initial rating, the abrupt downgrades were followed by several policy proposals that aim to eliminate the potential investors' confusion about the credit risk of different asset classes. The European Union regulators imposed the requirement that rating symbols of structured securities must have an additional "s" qualifier to identify the asset class. The Dodd-Frank Act in the US followed a different approach. It requested the SEC to conduct a study on standardization of rating symbols that would request that symbols for different asset classes correspond to the same credit quality. Also for the rating scale of a given rating class, the policy would require that different ratings have the same rating precision.

The effect of rating symbols standardization proposals can be evaluated within the scope of our model. Suppose that the CRA is required to provide the same accuracy for both asset classes, or across different ratings within the same rating class. Effectively it means that the CRA is restricted to a given rating precision $\overline{p}_{22}$ for different asset classes but has the flexibility to set the rating fees. The policy has the following effect.

Proposition 11 *Imposing rating standardization may decrease the market coverage and reduce market liquidity.*

Rating standardization limits the CRA's ability to design the rating system. Given precision level $\overline{p}_{22}$, the CRA optimizes the profits by adjusting the fee. If the required level of precision in a particular rating class exceed the optimal precision derived in Proposition 6, the CRA may choose to increase the fee so that only the highest quality issuers solicit a rating. This strategy can induce no trade for unrated v_2 issuers and leads to inefficiency.

The other rating standardization policy is to require CRA to provide the same precision for different ratings. In terms of our model, the CRA is required to set equal precision for the two types of issuers v_2 and v_3, $p_{22} = p_{33}$. Then the CRA's adjustment to the policy can take one of the following forms. It can provide ratings of high precision, $p_{ii} = 1$ but reduce the market coverage to the highest quality issuers, leading to illiquidity for issuers type v_2. Alternatively, it can sell ratings with precision $p_{ii} < 1$ to both types. However, it means that following a low rating, high quality issuers will refuse to trade. Thus it results in lower liquidity for high quality assets. Both outcomes reduce liquidity and lead to inefficiencies.

7.2 Regulation of rating fees

Rating agencies receive compensation for rating services from the issuers of securities or the parties participating in marketing the securities. Normally fee schedules are communicated to issuers prior to the issuance of a rating. The precise fee amounts are determined by various factors including the assets class of the rated security and the principal amount of the debt issuance that is rated. According to the code of conduct of the major NRSROs, the receipt of the compensation cannot influence the process of assigning a rating.

The rating fees have high variation across different asset classes. S&P US rating fees disclosure in 2008 indicates that the price of rating corporate debt was limited at 4.25 basis points while the structured finance fees ranged up to 12 basis points. In this section we analyze the effect of imposing a cap on the fee that a CRA may charge for rating a particular class of assets.

The effectiveness of the regulation that imposes a limit on the rating fee depends on the initial equilibrium outcome. If the market coverage involves rating two types v_2 and v_3, then limiting the fee does not change the optimal information structure. Indeed, given the fee, the CRA's objective is to maximize the market coverage. Then the CRA can choose to rate two or three types. In the former case, the trade occurs under the terms described in Proposition 6. In the later case of rating all types, the underpricing becomes more severe. It results in transfer of wealth from the CRA to the informed investors. However, in either case the original information structure remains optimal, and the policy has no effect on welfare.

The policy becomes effective when the original equilibrium involves limited market coverage. According to Proposition 7, this outcome occurs in the market with a high share of uninformed investors, $q > \overline{q}$. In this case, limiting the rating fee can improve efficiency.

Proposition 12 *Consider the market in which the CRA rates only one type* v_3 *and charges a fee* ϕ*. Imposing a fee cap* $\overline{\phi} < \phi$ *induces the CRA to rate two issuer types* v_2 *and* v_3 *and increases efficiency.*

In the market with a high share of uninformed investors, the CRA reduces the market coverage in order to eliminate the underpricing problem that would force it to reveal a lot of information. It charges a high fee that discourages participation of v_2 issuers. If a regulator can limit the fee to the level that is compatible with coverage of both types v_2 and v_3, the CRA's optimal reaction to the policy is to maximize the market coverage, which leads to the information structure described in Proposition 6. The regulation is efficient because it increases market liquidity by inducing issuers v_2 to trade.

7.3 Expert liability

Traditionally CRAs has been exempt from legal liability for inaccurate ratings under the First Amendment. The courts viewed ratings as an opinion about the credit quality. There were several cases where CRAs were sued by investors when the credit quality of highly rated securities quickly deteriorated. However, the nature of the rating business makes it hard to demonstrate that the default could have been foreseen by the CRA at the time of assigning the rating.

Dodd-Frank Act has removed the First Amendment protection. Now investors can bring private rights of action against CRAs for a knowing or reckless failure to conduct a reasonable investigation of the facts or to obtain analysis from an independent source. Under Dodd- Frank Act, NRSROs are be subject to the same expert liability as auditors or security analysts. In particular, the new rules imply that the CRAs have to give their consent for their ratings report to be included in a new issue security prospectus. Facing higher legal risks, in many instances the CRAs refused to provide the consent.

What is the effect of introducing legal liability on the precision of ratings? Suppose that the CRA has to pay a fine when an issue rated s_3 realizes the value v_2. Then the following result holds.

Proposition 13 *Consider a market in which the CRA rates issuers types v_2 and v_3. Imposing a fine for overrating an issue increases ratings precision but may reduce market coverage to the highest type v_3.*

Imposing a fine makes rating inflation more costly, and thus increases the precision of ratings. However, more informative ratings reduce the ability of the CRA to extract the surplus as pooling type v_2 with the highest type v_3 has the liability cost. As a result, the CRA may choose to reduce the market coverage to the highest issuer type and provide precise rating, eliminating the legal liability risk. If this occurs, the market outcome is inefficient because the issuers type v_2 are not rated and may not trade.

7.4 Reliance on ratings in regulation

The US regulators has been using credit ratings from 1930s to control the risk taking behavior of regulated financial institutions and insurance companies. The regulatory use of ratings has expanded significantly starting the 1970s when the SEC adopted the rule which uses ratings as a basis for calculating capital requirements for broker-dealers. During the next thirty years the use of ratings in regulation has become common practice. National Association of Insurance Commissioners uses ratings to assess the risk of the insurance investment portfolio that ultimately determines the insurance company capital requirements. Department of Labor requires that the pension funds investment in asset-backed securities is restricted to securities rated A or higher. Investment grade mutual funds must sell any security rated B or below, and cannot hold more than 5% of non-investment grade securities. The Basel II proposes to use credit ratings to determine the securitization exposures for banks.

Rating based regulatory policies effectively imposed a regulatory premium on higher rated bonds and potentially decrease the liquidity on the market of lower rated bonds. The CRAs have been criticized for providing the "regulatory licence" instead of unbiased credit analysis. Following the crisis, the regulators in the US and in the EU discussed several policy options to reduce the regulatory reliance on ratings. So far, the task has been challenging as there are few alternatives that can substitute ratings in prudential regulation.

Consistent with the observations of the investment community, our model predicts that regulatory use of ratings reduces their precision. Indeed, suppose that an issuer gains a regulatory premium $\rho > 0$ if an issue obtains the highest rating s_3. Then the CRA will adjust its rating system to extract the regulatory rent.

Proposition 14 *Consider a market where issuers types v_2 and v_3 are rated. Introducing a regulatory premium ρ for securities rated s_3 reduces the precision of ratings.*

The economic intuition of this result is the following. In the basic model, the reason for rating inflation is that the CRA increases the willingness to pay of issuers v_2 by assigning them a high rating with probability $1 - p_{22}$. If the high rating value is increased by the regulatory premium, it makes rating inflation more desirable for the CRA. It is also feasible because high quality issuers value of trade is increased by regulation, and they are willing to accept less precise ratings. The regulatory premium increases the profits of the CRA. Also it permits informed investors to gain higher rent as lower rating precision aggravates the winner's curse problem.

8 Conclusion

In this paper we analyze the equilibrium precision of ratings. Our results suggest that the information content of ratings depends on the market conditions and the presence of differentially informed investors. In particular, we show that as the share of uninformed investors and the aggregate value of liquidity increase, ratings become less informative. Also we show that the ratings become less informative when the share of high quality assets in the economy decreases. The results offer an explanation for heterogenous performance of ratings in different asset classes and through the cycle. We apply the model to analyze the merits of the recent reform proposals and show that some policies, in particular, rating standardization and expert liability, reduce market efficiency.

Understanding the incentives of CRAs to produce information is important for guiding policies to improve the efficiency of the financial market. Several aspect of the problem are left for future research. The focus of the paper was to evaluate the performance of ratings under the current issuer pays model. Many commentators in academic and investment communities suggest that conversion to investor pays model may increase ratings precision. It is unclear, however, whether switching the side will lead to better ratings or shift the CRAs incentives to provide ratings that are biased in favor of investors' needs. Also our focus was on a monopoly CRA. High industry concentration has recently led to Duopoly Relief Act of 2006 that aims to encourage competition among CRAs. The effect of competition on the information content of ratings is another area that needs further analysis.

Appendix: Proofs

Proof of Proposition 1. Consider an information structure with three signals. We characterize an equilibrium in which CRA chooses $p_{11} = p_{33} = 1$ and $p_{22} = \frac{\delta(\lambda_3+\lambda_2)}{\lambda_3+\delta\lambda_2} \leq 1$ and offers a fee $\phi = (1-\delta)\frac{\lambda_3 v_3+\lambda_2 v_2}{\lambda_3+\lambda_2}$. Types v_2 and v_3 solicit rating whereas type v_1 does not solicit a rating. An issuer with a rating s_i sets the price of the issue at b_i where $b_3 = \frac{\lambda_3 v_3+\lambda_2 v_2+\delta\lambda_2(v_3-v_2)}{\lambda_3+\lambda_2}$, and $b_2 = v_2$. All investors subscribe to the issue. In terms of off–the–equilibrium–path–beliefs, investors believe that the issuer is of type v_1 if (1) an issuer solicits a rating and receives a signal s_1; or (2) the issue price is different than b_i following rating s_i.[13]

At $t = 3$, investors have the following assessment of an issue following rating s_i and offer price b_i where $i \in \{2, 3\}$:

$$U_3 = \frac{\lambda_3}{\lambda_3 + \lambda_2(1-p_{22})}v_3 + \frac{\lambda_2(1-p_{22})}{\lambda_3 + \lambda_2(1-p_{22})}v_2,$$

$$U_2 = v_2.$$

Inserting $p_{22} = \frac{\delta(\lambda_3+\lambda_2)}{\lambda_3+\delta\lambda_2}$ leads to $U_3 = \frac{\lambda_3 v_3+\lambda_2 v_2+\delta\lambda_2(v_3-v_2)}{\lambda_3+\lambda_2} = b_3$. Therefore, investors are indifferent about subscribing to an issue following rating s_i and offer price b_i where $i \in \{2, 3\}$. Any other rating and price combination is not on the equilibrium path so investors believe that the issuer is of type v_1 and therefore have assessment equal to v_1. Consequently, they always reject any positive price offers that are not on the equilibrium path.

At $t = 2$, issuer type v_3 always receives rating s_3 and if he sets the price at $b_3 = \frac{\lambda_3 v_3+\lambda_2 v_2+\delta\lambda_2(v_3-v_2)}{\lambda_3+\lambda_2}$. Given that investors always buy the issue at this price, he raises $R_3 = U_3$. Note that his outside option δv_3 is not higher. Furthermore, setting any other price leads to investors believing that he is of type v_1 and leads to nonpositive profits. Consequently, he has no incentive to deviate. On the other hand, at $t = 2$, issuer type v_2 can receive ratings s_2 and s_3. If he sets the price at $b_3 = \frac{\lambda_3 v_3+\lambda_2 v_2+\delta\lambda_2(v_3-v_2)}{\lambda_3+\lambda_2}$ following rating s_3 then his payoff will be strictly greater than his outside option, δv_2. Similarly, if he sets the price at $b_2 = v_2$ following rating s_2 then his payoff will be greater than his outside option since $v_2 \geq \delta v_2$. Thus, he has no incentive to set any other price given investors beliefs. Finally, type v_1 is indifferent between issuing and his outside option.

[13] Note that we can support this equilibrium outcome with other beliefs as well. This set of beliefs simplify arguments.

At $t = 1$, issuer type v_3 knows that if he pays for a rating he will raise $R_3 = U_3$ in period $t = 3$. Then his payoff after soliciting a rating will be

$$\begin{aligned} U_3 - \phi &= \frac{\lambda_3 v_3 + \lambda_2 v_2 + \delta\lambda_2 (v_3 - v_2)}{\lambda_3 + \lambda_2} - (1-\delta)\frac{\lambda_3 v_3 + \lambda_2 v_2}{\lambda_3 + \lambda_2} \\ &= \delta v_3. \end{aligned}$$

Therefore, he is indifferent between soliciting rating and his outside option. If he does not solicit rating, then investors will believe that he is of type v_1 and selling the issue is worse for him. Consequently, there are no incentives for him to deviate. Similarly, the issuer type v_2 knows that if he pays for a rating, the issue expects to raise

$$R_2 = p_{22}U_2 + (1 - p_{22})U_3$$

Then his payoff due to soliciting rating will be

$$\begin{aligned} R_2 - \phi &= p_{22}U_2 + (1-p_{22})U_3 - (1-\delta)\frac{\lambda_3 v_3 + \lambda_2 v_2}{\lambda_3 + \lambda_2} \\ &= \frac{\delta(\lambda_3 + \lambda_2)}{\lambda_3 + \delta\lambda_2} v_2 + (1 - \frac{\delta(\lambda_3 + \lambda_2)}{\lambda_3 + \delta\lambda_2})\frac{\lambda_3 v_3 + \lambda_2 v_2 + \delta\lambda_2 (v_3 - v_2)}{\lambda_3 + \lambda_2} - (1-\delta)\frac{\lambda_3 v_3 + \lambda_2 v_2}{\lambda_3 + \lambda_2} \\ &= \delta v_2 \end{aligned}$$

Therefore, he is indifferent between soliciting rating and his outside option. If he does not solicit rating, then investors will believe that he is of type v_1 and selling the issue is worse than outside option. Consequently, there are no incentives for him to deviate, either. Finally, type v_1 is indifferent between soliciting rating and his outside option, because he knows that he will get the s_1 rating if he solicits one.

At $t = 0$, the CRA aims to sell rating to issuers with a positive willingness to pay, v_3 and v_2. Then an optimal information structure solves

$$\max \ (\lambda_3 + \lambda_2)\phi$$

$$R_i - \delta v_i - \phi \geq 0, \ i = 2, 3. \tag{3}$$

Constraints (3) imply that

$$R_3 - \delta v_3 = R_2 - \delta v_2 = \phi,$$

which simplifies to

$$p_{22}(U_3 - U_2) = \delta(v_3 - v_2),$$

and consequently,

$$p_{22} = \frac{\delta(\lambda_3 + \lambda_2)}{\lambda_3 + \delta\lambda_2} < 1.$$

Then the CRA sets the fee

$$\phi = U_3 - \delta v_3 = (1-\delta)\frac{\lambda_3 v_3 + \lambda_2 v_2}{\lambda_3 + \lambda_2}$$

and gains the market surplus, $(\lambda_3 + \lambda_2)\phi = (1-\delta)(\lambda_3 v_3 + \lambda_2 v_2)$. Thus the information structure achieves the first best for the CRA and he has no incentive to deviate. $\blacksquare$

Proof of Proposition 2. Recall from Proposition 1 that there exists a rating system that entails inflation, that is an equilibrium outcome. Without loss of generality let $p_{22} = p < 1$ characterize this rating system with an optimal fee $\phi = (1-\delta)\frac{\lambda_3 v_3 + \lambda_2 v_2}{\lambda_3 + \lambda_2}$. One can verify the symmetric information structure, i.e., $p_{33} = p < 1$ and $p_{22} = 1$, can also achieve $R_3 - \delta v_3 = R_2 - \delta v_2 = (1-\delta)\frac{\lambda_3 v_3 + \lambda_2 v_2}{\lambda_3 + \lambda_2}$. This leads to $U_2 = \frac{\lambda_3(1-p)v_3 + \lambda_2 v_2}{\lambda_3(1-p) + \lambda_2}$ and substituting in $p = \frac{\delta(\lambda_2 + \lambda_3)}{\lambda_2 + \delta\lambda_3}$ leads to $U_2 = \frac{\lambda_3 v_3 + \lambda_2 v_2 - \delta\lambda_3(v_3 - v_2)}{\lambda_3 + \lambda_2} = \overline{U}$. Furthermore, one can show that for all $p_{33} \in (p, 1)$ there exists $p_{22} \in (p, 1)$ such that $R_3 - \delta v_3 = R_2 - \delta v_2 = (1-\delta)\frac{\lambda_3 v_3 + \lambda_2 v_2}{\lambda_3 + \lambda_2}$ is satisfied. Furthermore, from Bayesian updating, it is immediate that for all $(p_{22}, p_{33}) \in (p, 1)^2$, $U_2 < \overline{U}$. Therefore, if $\delta v_3 > \overline{U}$ then we must have rating inflation as the only rating system that can achieve $R_3 - \delta v_3 = R_2 - \delta v_2 = (1-\delta)\frac{\lambda_3 v_3 + \lambda_2 v_2}{\lambda_3 + \lambda_2}$ and also satisfy the participation constraint of type v_3, i.e., $\delta v_3 \geq U_2$. Solving for $\delta v_3 = \overline{U}$ leads to $\overline{\delta} = \frac{\lambda_3 v_3 + \lambda_2 v_2}{\lambda_2 v_3 + \lambda_3(2v_3 - v_2)}$. $\blacksquare$

Proof of Proposition 3. From Proposition 1,

$$\frac{dp_{22}}{d\delta} = \frac{(\lambda_3 + \lambda_2)\lambda_3}{(\lambda_3 + \delta\lambda_2)^2} > 0.$$

$\blacksquare$

Proof of Proposition 4. First, we show that there exists an information structure such that $R_j - \delta v_j = \phi$ for all $j \in \{1, 2, 3\}$. Suppose that $R_j - \delta v_j = \phi$ for all $j \in \{1, 2, 3\}$ does not hold. Without loss of generality, there must exist types k, j such that $R_k - \delta v_k > R_j - \delta v_j = \phi$, and yet the CRA cannot increase its profit by increasing R_j and decreasing R_k. Then we claim that we must have $k > j$. To see this, first note that R_k is a continuos function of p_{ij} for all k, i and j. Furthermore, perfectly informative signals, lead to $R_j = v_j$ so that $R_k - \delta v_k > R_j - \delta v_j$ implies $k > j$. However, if $k > j$, then we can make the signals more uninformative until $R_k - \delta v_k = R_j - \delta v_j$ since (i) R_k is a continuos function of p_{ij} for all k, i and j; and (ii) fully noisy signals result in $R_k - \delta v_k < R_j - \delta v_j$ as long as $k > j$. Therefore, there exists an information structure such that $R_j - \delta v_j = \phi$ for all $j \in \{1, 2, 3\}$. Characterization of the equilibrium in which CRA uses an information structure such that $R_j - \delta v_j = \phi$ for all $j \in \{1, 2, 3\}$ and rates all three types capturing all of the surplus is similar to the equilibrium characterization in Proposition 1, and therefore is omitted.

If all three types are rated, then

$$\sum_j \lambda_j R_j = \sum_j \lambda_j v_j = E[v].$$

In equilibrium, the CRA equates the issuers willingness to pay, i.e., $R_j - \delta v_j = \phi$ for all $j \in \{1, 2, 3\}$, implying

$$\begin{aligned}\sum_j \lambda_j R_j &= \lambda_3(R_1 + \delta v_3) + \lambda_2(R_1 + \delta v_2) + R_1 \\ &= R_1 + \delta(\lambda_3 v_3 + \lambda_2 v_2) = E[v].\end{aligned}$$

Thus,

$$\begin{aligned}\phi &= R_1 = E[v] - \delta(\lambda_3 v_3 + \lambda_2 v_2) = (1-\delta)E[v], \\ R_j &= (1-\delta)E[v] + \delta v_j, \\ \pi^3 &= (1-\delta)E[v],\end{aligned}$$

where π^3 denotes the profit of the CRA when all three types are rated.

If two types v_3 and v_2 are rated, we know from Proposition 1 that

$$R_2 - \delta v_2 = R_3 - \delta v_3 = \phi,$$

$$\lambda_3 R_3 + \lambda_2 R_2 = E[v].$$

Thus,

$$\begin{aligned}(\lambda_3 + \lambda_2)R_2 &= E[v] - \lambda_3\delta(v_3 - v_2), \\ \pi^2 &= (\lambda_3 + \lambda_2)\phi = (\lambda_3 + \lambda_2)R_2 - (\lambda_3 + \lambda_2)\delta v_2 \\ &= (1-\delta)E[v],\end{aligned}$$

where π^2 denotes the CRA's profit when types v_3 and v_2 are rated. Hence, $\pi^2 = \pi^3$ and the CRA is indifferent between rating two or three types.

Finally, we show that rating one type will lead to lower profits for the CRA. If one type v_3 is rated, then it is most profitable to rate type v_3 such that

$$R_3 - \delta v_3 = \phi,$$

$$R_3 = v_3,$$

$$\pi^1 = (1-\delta)\lambda_3 v_3 < \pi^2 = \pi^3.$$

Note that there exists a rating system that implements this outcome. Indeed, consider the rating system with $p_{33} = p_{11} = 1$, $p_{22} = \frac{\delta(\lambda_3+\lambda_2)}{\lambda_3+\delta\lambda_2}$, and $p_{ij} = 0$ for the other i, j. It is immediate to verify that this system implements R_j and permits the CRA to extract the surplus. However, this leads to lower profits for the CRA. ■

Proof of Proposition 5. We restrict attention to equilibria in which uninformed investors make zero expect profit. In particular, we impose that the issue price must be equal to the expected value of the issue given Bayesian updating on the equilibrium path. We omit characterization of the equilibria in order to avoid the repetition of arguments in Proof of Proposition 1. Instead, we show that in such equilibria the CRA makes strictly higher expected profit by rating only two highest types, i.e., v_3 and v_2. We consider a general information structure and let $\beta_{ij} = \Pr(v_j|s_i)$ stand for the posterior belief. Recall that γ_{ij} denotes the belief of an uninformed investor who is offered an issue that the issuer rated s_i is of type v_j.

We consider two cases where the CRA rates (i) all types, or (ii) two highest types v_3 and v_2.

Case (i). Suppose the CRA rates all issuers. Denote b_i^* the equilibrium price of a security rated s_i. Two cases are possible, $b_i^* \in (v_1, v_2)$ and $b_i^* \in (v_2, v_3)$.

If $b_i^* \in (v_1, v_2)$, the posterior beliefs of uninformed investors are

$$\begin{aligned}
\gamma_{i3} &= \frac{q\beta_{i3}}{q(\beta_{i3}+\beta_{i2})+\beta_{i1}}, \\
\gamma_{i2} &= \frac{q\beta_{i2}}{q(\beta_{i3}+\beta_{i2})+\beta_{i1}}, \\
\gamma_{i1} &= \frac{\beta_{i1}}{q(\beta_{i3}+\beta_{i2})+\beta_{i1}}.
\end{aligned}$$

Then

$$b_i^* = \Sigma_j \gamma_{ij} v_j = \frac{q\beta_{i3}v_3 + q\beta_{i2}v_2}{q(\beta_{i3}+\beta_{i2})+\beta_{i1}},$$

and $b_i^* \in (v_1, v_2)$ holds when

$$\frac{q\beta_{i3}v_3 + q\beta_{i2}v_2}{q(\beta_{i3}+\beta_{i2})+\beta_{i1}} < v_2,$$

which simplifies to

$$\frac{q\beta_{i3}}{\beta_{i1}} < \frac{v_2}{v_3 - v_2}.$$

The issue raises

$$\frac{q\beta_{i3}v_3 + q\beta_{i2}v_2}{q(\beta_{i3}+\beta_{i2})+\beta_{i1}}.$$

If $b_i^* \in (v_2, v_3)$, then the posterior beliefs of uninformed investors are

$$\begin{aligned}\gamma_{i3} &= \frac{q\beta_{i3}}{q\beta_{i3}+\beta_{i2}+\beta_{i1}},\\ \gamma_{i2} &= \frac{\beta_{i2}}{q\beta_{i3}+\beta_{i2}+\beta_{i1}},\\ \gamma_{i1} &= \frac{\beta_{i1}}{q\beta_{i3}+\beta_{i2}+\beta_{i1}}.\end{aligned}$$

Then

$$b_i^* = \Sigma_j \gamma_{ij} v_j = \frac{q\beta_{i3}v_3+\beta_{i2}v_2}{q\beta_{i3}+\beta_{i2}+\beta_{i1}},$$

and $b_i^* \in (v_2, v_3)$ holds when

$$\frac{q\beta_{i3}v_3+\beta_{i2}v_2}{q\beta_{i3}+\beta_{i2}+\beta_{i1}} > v_2$$

which simplifies to

$$\frac{q\beta_{i3}}{\beta_{i1}} > \frac{v_2}{v_3-v_2}.$$

In this case the issue raises

$$\frac{q\beta_{i3}v_3+\beta_{i2}v_2}{q\beta_{i3}+\beta_{i2}+\beta_{i1}}.$$

Case (ii). When only two higher types are rated, the price must satisfy $b_i^* \in (v_2, v_3)$. The posterior beliefs are

$$\begin{aligned}\gamma_{i3} &= \frac{q\beta_{i3}}{q\beta_{i3}+\beta_{i2}},\\ \gamma_{i2} &= \frac{\beta_{i2}}{q\beta_{i3}+\beta_{i2}}.\end{aligned}$$

Then

$$b_i^* = \Sigma_j \gamma_{ij} v_j = \frac{q\beta_{i3}v_3+\beta_{i2}v_2}{q\beta_{i3}+\beta_{i2}}.$$

The issue raises

$$(\lambda_3+\lambda_2)b_i^* = \frac{(\lambda_3+\lambda_2)(q\beta_{i3}v_3+\beta_{i2}v_2)}{q\beta_{i3}+\beta_{i2}}.$$

The price satisfies $b_i^* \in (v_2, v_3)$ for all parameter values.

Next, we consider any signal s_i and show that the expected revenue with two ratings is higher than that with three ratings for each signal.

If $b_i^* \in (v_2, v_3)$, then we need to show that

$$\frac{q\beta_{i3}v_3 + \beta_{i2}v_2}{q\beta_{i3} + \beta_{i2} + \beta_{i1}} < \frac{(\beta_{i3} + \beta_{i2})(q\beta_{i3}v_3 + \beta_{i2}v_2)}{q\beta_{i3} + \beta_{i2}},$$
$$\text{or } \frac{(\beta_{i3} + \beta_{i2})(q\beta_{i3} + \beta_{i2} + \beta_{i1})}{q\beta_{i3} + \beta_{i2}} > 1.$$

Denote $A = q\beta_{i3} + \beta_{i2} + \beta_{i1}$. Then the condition writes

$$\begin{aligned} \frac{(1-\beta_{i1})A}{A-\beta_{i1}} &> 1, \\ \text{or } A &< 1, \end{aligned}$$

which holds for all parameter values as long as $q < 1$.

If $b_i^* \in (v_1, v_2)$, then we need to show that

$$\frac{q\beta_{i3}v_3 + \beta_{i2}v_2}{q(\beta_{i3} + \beta_{i2}) + \beta_{i1}} < \frac{(\beta_{i3} + \beta_{i2})(q\beta_{i3}v_3 + \beta_{i2}v_2)}{q\beta_{i3} + \beta_{i2}}$$

holds for all parameter values such that $\frac{q\beta_{i3}}{\beta_{i1}} < \frac{v_2}{v_3 - v_2}$ and $v_3 > v_2$, or equivalently, $v_3 \in (v_2, \frac{q\beta_{i3}+\beta_{i1}}{q\beta_{i3}}v_2)$. When we have $v_3 = \frac{q\beta_{i3}+\beta_{i1}}{q\beta_{i3}}v_2$, then $b_i^* = v_2$. In other words, for $v_3 = \frac{q\beta_{i3}+\beta_{i1}}{q\beta_{i3}}v_2$ we have the left hand side (LHS) is equal to v_2. For $v_3 = \frac{q\beta_{i3}+\beta_{i1}}{q\beta_{i3}}v_2$ the right hand side (RHS) is

$$\frac{(\beta_{i3} + \beta_{i2})(q\beta_{i3}\frac{q\beta_{i3}+\beta_{i1}}{q\beta_{i3}}v_2 + \beta_{i2}v_2)}{q\beta_{i3} + \beta_{i2}}$$

Simplifying leads to

$$\frac{(\beta_{i3} + \beta_{i2})(q\beta_{i3} + \beta_{i2} + \beta_{i1})v_2}{q\beta_{i3} + \beta_{i2}}.$$

Let $B = q\beta_{i3} + \beta_{i2}$. Then the condition becomes

$$\frac{(1-\beta_{i1})(B + \beta_{i1})v_2}{B}.$$

Given that $\frac{(1-\beta_{i1})(B+\beta_{i1})v}{B} > 1$ simplifies to $q\beta_{i1} + \beta_{i2} + \beta_{i3} < 1$, we have RHS greater than v_2.

The other extreme condition under $v_3 \in (v_2, \frac{q\beta_{i3}+\beta_{i1}}{q\beta_{i3}}v_2)$ is $v_3 = v_2$. Then LHS becomes $\frac{(q\beta_{i3}+q\beta_{i2})v_2}{q(\beta_{i3}+\beta_{i2})+\beta_{i1}}$ and RHS becomes $\frac{(\beta_{i3}+\beta_{i2})(q\beta_{i3}+\beta_{i2})v_2}{q\beta_{i3}+\beta_{i2}}$. For our result to hold for this extreme point, we must have

$$\frac{q\beta_{i3} + q\beta_{i2}}{q(\beta_{i3} + \beta_{i2}) + \beta_{i1}} < \frac{(\beta_{i3} + \beta_{i2})(q\beta_{i3} + \beta_{i2})}{q\beta_{i3} + \beta_{i2}}$$

This term simplifies to

$$\frac{q}{q(\beta_{i3}+\beta_{i2})+\beta_{i1}} < 1$$

Further iteration leads to $q < q(\beta_{i3}+\beta_{i2})+\beta_{i1}$. Subtracting $(1-q)\beta_{i1}$ from both sides we have $q-(1-q)\beta_{i1} < q(\beta_{i3}+\beta_{i2}+\beta_{i1})$. Since $\beta_{i3}+\beta_{i2}+\beta_{i1}=1$, we must have LHS less than RHS for $v_3 = v_2$.

Next we show that the difference between LHS and RHS is monotonic, and thus LHS is less than RHS for all interior points, $v_3 \in (v_2, \frac{q\beta_{i3}+\beta_{i1}}{q\beta_{i3}} v_2)$.

Define

$$F(v_3, a) = \frac{q\beta_{i3}v_3 + q\beta_{i2}v_2}{q(\beta_{i3}+\beta_{i2})+\beta_{i1}} - \frac{(\beta_{i3}+\beta_{i2})(q\beta_{i3}v_3+\beta_{i2}v_2)}{q\beta_{i3}+\beta_{i2}},$$

where a denotes all the parameters, $a = (\lambda_H, \lambda_M, q)$. Then

$$\begin{aligned} F'_{v_3}(v_3,a) &= \frac{q\beta_{i3}}{q(\beta_{i3}+\beta_{i2})+\beta_{i1}} - \frac{(\beta_{i3}+\beta_{i2})q\beta_{i3}}{q\beta_{i3}+\beta_{i2}} \\ &= \frac{q\beta_{i3}}{(q(\beta_{i3}+\beta_{i2})+\beta_{i1})(q\beta_{i3}+\beta_{i2})}(q\beta_{i3}+\beta_{i2}-(\beta_{i3}+\beta_{i2})(q(\beta_{i3}+\beta_{i2})+\beta_{i1})) \end{aligned}$$

The first term is positive, thus the sign of $F'_{v_3}(v_3,a)$ is defined by the sign of

$$\begin{aligned} & q\beta_{i3}+\beta_{i2}-(\beta_{i3}+\beta_{i2})(q(\beta_{i3}+\beta_{i2})+\beta_{i1}) \\ = \; & (1-q)(\beta_{i3} - 2(\frac{1}{2}-\beta_{i2})\beta_{i3}+\beta_{i2}^2). \end{aligned}$$

Solve the inequality with respect to β_{i3},

$$\beta_{i3}^2 - 2(\frac{1}{2}-\beta_{i2})\beta_{i3}+\beta_{i2}^2 > 0. \tag{4}$$

The roots are

$$\frac{1}{2}-\beta_{i2} \pm \sqrt{(\frac{1}{2}-\beta_{i2})^2-\beta_{i2}^2} = \frac{1}{2}-\beta_{i2} \pm \sqrt{\frac{1}{4}-\beta_{i2}}.$$

If $\beta_{i2} > \frac{1}{4}$, then (4) holds for any $a \in A_1 = \{\beta_{i3} \in (0,1), \beta_{i2} \in (\frac{1}{4}, 1), q \in (0,1)\}$.

If $\beta_{i2} \leq \frac{1}{4}$, then note that the low root is positive, $\frac{1}{2}-\beta_{i2}-\sqrt{\frac{1}{4}-\beta_{i2}} > 0$ and the high root is less than 1, $\frac{1}{2}-\beta_{i2}+\sqrt{\frac{1}{4}-\beta_{i2}} < 1$. Therefore, if $a \in A_2$, where

$$A_2 = \{\beta_{i2} \in (0, \frac{1}{4}), \beta_{i3} \in (0, \frac{1}{2}-\beta_{i2}-\sqrt{\frac{1}{4}-\beta_{i2}}) \cup (\frac{1}{2}-\beta_{i2}+\sqrt{\frac{1}{4}-\beta_{i2}}, 1), q \in (0,1)\},$$

then $F' > 0$. If $a \in A_3$, where

$$A_3 = \{\beta_{i2} \in (0, \frac{1}{4}), \beta_{i3} \in (\frac{1}{2}-\beta_{i2}-\sqrt{\frac{1}{4}-\beta_{i2}}, \frac{1}{2}-\beta_{i2}+\sqrt{\frac{1}{4}-\beta_{i2}}), q \in (0,1)\},$$

then $F' < 0$.

Hence, for $a \in A_1 \cup A_2$, $F' > 0$, and $F(\frac{q\beta_{i3}+\beta_{i1}}{q\beta_{i3}} v_2, a) < 0$ implies the result. For $a \in A_3$, $F' < 0$ and $F(v_2, a) < 0$ implies the result.

Given that the expected revenue with two ratings is higher than that with three ratings, the price the issuer sets in the two types rated case must be greater than $\frac{1}{\lambda_2+\lambda_2}$ times that of the other case. Considering the fact that this holds for any signal, the CRA can charge a higher fee that enables it to extract more expected profit when it rates two highest types rather than all types.

Consequently, we are left to show that if we restrict attention to equilibria in which uninformed investors make zero expected profit, rating all types is never an equilibrium. Suppose that such an equilibrium exists. Then consider the following deviation by the CRA. CRA increases the fee while retaining the same information structure and makes it unattractive for type v_1 to solicit a rating. By above arguments, this deviation leads to higher expected profit for the CRA as long as he sets a sufficiently high fee. Therefore, for any candidate equilibrium with all types rated, there exists a profitable deviation. ■

Proof of Proposition 6. Conditional on observing ratings s_3 and s_2, the uninformed investors hold beliefs $\gamma_{ij} = \Pr(v_j | s_i)$ with

$$\gamma_{33} = \frac{\lambda_3 q p_{33}}{\lambda_3 q p_{33} + \lambda_2(1-p_{22})},$$

$$\gamma_{22} = \frac{\lambda_2 p_{22}}{\lambda_3 q(1-p_{33}) + \lambda_2 p_{22}},$$

$$\gamma_{32} = 1 - \gamma_{33} \text{ and } \gamma_{23} = 1 - \gamma_{22},$$

$$\gamma_{31} = \gamma_{31} = \gamma_{13} = \gamma_{12} = 0, \gamma_{11} = 1$$

The resulting uninformed investors' assessment of the assets rated s_3 and s_2 are

$$U_3 = \gamma_{33} v_3 + \gamma_{32} v_2 = \frac{\lambda_3 q p_{33} v_3 + \lambda_2(1-p_{22}) v_2}{\lambda_3 q p_{33} + \lambda_2(1-p_{22})},$$

$$U_2 = \gamma_{23} v_3 + \gamma_{22} v_2 = \frac{\lambda_3 q(1-p_{33}) v_3 + \lambda_2 p_{22} v_2}{\lambda_3 q(1-p_{33}) + \lambda_2 p_{22}}.$$

■

After receiving a rating s_i, an issuer v_j can either sell the issue at price U_i or hold the asset and realize the value δv_j. Then the expected payoff of soliciting a rating for types v_3 and v_2 are

$$R_3 = p_{33} \max\{U_3, \delta v_3\} + (1-p_{33}) \max\{U_2, \delta v_3\},$$

$$R_2 = (1-p_{22}) \max\{U_3, \delta v_2\} + p_{22} \max\{U_2, \delta v_2\}.$$

Consider the information structure

$$\begin{array}{cccc} & v_3 & v_2 & v_1 \\ s_3 & p_{33} & 1-p_{22} & 0 \\ s_2 & 1-p_{33} & p_{22} & 0 \\ s_1 & 0 & 0 & 1 \end{array}$$

Under this information structure, rating s_1 perfectly reveals the types v_1, $\gamma_{13} = \gamma_{12} = 0$, $\gamma_{11} = 1$. Ratings s_2 and s_3 lead to beliefs updating $\gamma_{ij} = \Pr(v_j | \, s_i)$, with

$$\begin{aligned} \gamma_{33} &= \frac{\lambda_3 q p_{33}}{\lambda_3 q p_{33} + \lambda_2(1-p_{22})}, \\ \gamma_{32} &= \frac{\lambda_2(1-p_{22})}{\lambda_3 q p_{33} + \lambda_2(1-p_{22})}, \\ \gamma_{31} &= 0. \end{aligned}$$

$$\begin{aligned} \gamma_{23} &= \frac{\lambda_3 q(1-p_{33})}{\lambda_3 q(1-p_{33}) + \lambda_2 p_{22}}, \\ \gamma_{22} &= \frac{\lambda_2 p_{22}}{\lambda_3 q(1-p_{33}) + \lambda_2 p_{22}}, \\ \gamma_{21} &= 0. \end{aligned}$$

Under this information structure, the issuers expected payoff if they solicit a rating are equal to

$$\begin{aligned} R_3 &= p_{33} \max\{U_3, \delta v_3\} + (1-p_{33}) \max\{U_2, \delta v_3\}, \\ R_2 &= (1-p_{22}) \max\{U_3, \delta v_2\} + p_{22} \max\{U_2, \delta v_2\}, \\ R_1 &= v_1 = 0. \end{aligned}$$

The payoff of type v_1 implies that it does not solicit a rating.

Proof. The investor's assessment of the asset values under this information structure is

$$U_3 = \gamma_{33} v_3 + \gamma_{32} v_2 = \frac{\lambda_3 q p_{33} v_3 + \lambda_2(1-p_{22}) v_2}{\lambda_3 q p_{33} + \lambda_2(1-p_{22})},$$

$$U_2 = \gamma_{23} v_3 + \gamma_{22} v_2 = \frac{\lambda_3 q(1-p_{33}) v_3 + \lambda_2 p_{22} v_2}{\lambda_3 q(1-p_{33}) + \lambda_2 p_{22}}.$$

Consider the case when all rated issuers trade, $U_i \geq \delta v_j$ for all $i, j = 2, 3$. (Note that if $U_i < \delta v_j$ for all i, j, then types v_j do not solicit a rating.) Then the optimal information structure solves

$$\max \; (\lambda_3 + \lambda_2)\phi$$

$$
\begin{array}{ll}
\rho_3: & p_{33}U_3 + (1-p_{33})U_2 - \delta v_3 - \phi \geq 0,\\
\rho_2: & (1-p_{22})U_3 + p_{22}U_2 - \delta v_2 - \phi \geq 0,\\
\mu_3: & p_{33}(U_3 - \delta v_3) \geq 0,\\
\mu_2: & (1-p_{33})(U_2 - \delta v_3) \geq 0,\\
\tau_3: & 1 - p_{33} \geq 0,\\
\tau_2: & 1 - p_{22} \geq 0,\\
\kappa_3: & p_{33} \geq 0,\\
\kappa_2: & p_{22} \geq 0.
\end{array}
$$

Note that we did not include the constraints $U_i - \delta v_2 \geq 0$ because these are implied by the other two constraints.

The first order conditions of the problem are:

with respect to p_{33}

$$
\rho_3(U_3 - U_2 + p_{33}\frac{dU_3}{dp_{33}} + (1-p_{33})\frac{dU_2}{dp_{33}}) + \rho_2((1-p_{22})\frac{dU_3}{dp_{33}} + p_{22}\frac{dU_2}{dp_{33}})
$$
$$
+\mu_3(U_3 - \delta v_3 + p_{33}\frac{dU_3}{dp_{33}}) + \mu_2(-(U_2 - \delta v_3) + (1-p_{33})\frac{dU_2}{dp_{33}}) - \tau_3 + \kappa_3 = 0,
$$

with respect to p_{22}

$$
\rho_3(p_{33}\frac{dU_3}{dp_{22}} + (1-p_{33})\frac{dU_2}{dp_{22}}) + \rho_2(U_2 - U_3 + (1-p_{22})\frac{dU_3}{dp_{22}} + p_{22}\frac{dU_2}{dp_{22}})
$$
$$
+\mu_3 p_{33}\frac{dU_3}{dp_{22}} + \mu_2(1-p_{33})\frac{dU_2}{dp_{22}} - \tau_2 + \kappa_2 = 0,
$$

with respect to ϕ

$$
(\lambda_3 + \lambda_2) - \rho_3 - \rho_2 = 0.
$$

In terms of the information structure, the values $\frac{dU_i}{dp_{kk}}$, $i,k = 2,3$ write

$$
\begin{aligned}
\frac{dU_3}{dp_{33}} &= \frac{\lambda_3\lambda_2 q(1-p_{22})(v_3 - v_2)}{(\lambda_3 q p_{33} + \lambda_2(1-p_{22}))^2} > 0,\\
\frac{dU_3}{dp_{22}} &= \frac{\lambda_3\lambda_2 q p_{33}(v_3 - v_2)}{(\lambda_3 q p_{33} + \lambda_2(1-p_{22}))^2} > 0,\\
\frac{dU_2}{dp_{33}} &= -\frac{\lambda_3\lambda_2 q p_{22}(v_3 - v_2)}{(\lambda_3 q(1-p_{33}) + \lambda_2 p_{22})^2} < 0,\\
\frac{dU_2}{dp_{22}} &= -\frac{\lambda_3\lambda_2 q(1-p_{33})(v_3 - v_2)}{(\lambda_3 q(1-p_{33}) + \lambda_2 p_{22})^2} < 0.
\end{aligned}
$$

Rated issuers must have the same willingness to pay, and thus $\rho_i > 0$, $i = 2,3$. Then

$$
(p_{33} - (1-p_{22}))(U_3 - U_2) = \delta(v_3 - v_2). \tag{5}
$$

Case (a). Suppose that $\mu_i > 0$ for $i = 2, 3$. Then three cases are possible, (a1) $p_{33} = 0$ and $U_2 = \delta v_3$; (a2) $U_3 = \delta v_3$ and $p_{33} = 1$; (a3) $U_3 = U_2 = \delta v_3$. Case (a1) is not feasible due to (5) and $U_3 \geq \delta v_3$. Case (a2) is not feasible due to (5) and $p_{22} \leq 1$. Case (a3) is not feasible due to (5). Thus $\mu_i > 0$ for $i = 1, 2$ is not a solution.

Case (b). Suppose that $\mu_3 = 0$ and $\mu_2 > 0$. Thus the optimal information structure is determined by the conditions

$$(1 - p_{33})(U_2 - \delta v_3) = 0,$$
$$(p_{33} - (1 - p_{22}))(U_3 - U_2) = \delta(v_3 - v_2).$$

We obtain

$$U_3 - U_2 = \frac{\lambda_3 \lambda_2 q(p_{33}p_{22} - (1 - p_{33})(1 - p_{22}))}{(\lambda_3 q p_{33} + \lambda_2(1 - p_{22}))(\lambda_3 q(1 - p_{33}) + \lambda_2 p_{22})}(v_3 - v_2),$$

and thus

$$\frac{\lambda_3 \lambda_2 q(p_{33} - (1 - p_{22}))^2}{(\lambda_3 q p_{33} + \lambda_2(1 - p_{22}))(\lambda_3 q(1 - p_{33}) + \lambda_2 p_{22})} = \delta.$$

Two alternatives are possible for $(1 - p_{33})(U_2 - \delta v_3) = 0$. Consider first the case $p_{33} = 1$. Then

$$\frac{\lambda_3 q p_{22}}{\lambda_3 q + \lambda_2(1 - p_{22})} = \delta,$$
$$\lambda_3 q p_{22} - \delta(\lambda_3 q + \lambda_2(1 - p_{22})) = 0$$
$$p_{22}(\lambda_3 q + \delta\lambda_2) - \delta(\lambda_3 q + \lambda_2) = 0$$
$$p_{22} = \frac{\delta(\lambda_3 q + \lambda_2)}{\lambda_3 q + \delta\lambda_2}.$$

Then the values $\frac{dU_i}{dp_{kk}}$ write

$$\frac{dU_3}{dp_{33}} = \frac{\lambda_3 \lambda_2 q(1 - p_{22})(v_3 - v_2)}{(\lambda_3 q + \lambda_2(1 - p_{22}))^2},$$
$$\frac{dU_3}{dp_{22}} = \frac{\lambda_3 \lambda_2 q(v_3 - v_2)}{(\lambda_3 q + \lambda_2(1 - p_{22}))^2},$$
$$\frac{dU_2}{dp_{33}} = -\frac{\lambda_3 q(v_3 - v_2)}{\lambda_2 p_{22}} < 0,$$
$$\frac{dU_2}{dp_{22}} = 0.$$

The values U_i write

$$U_3 = \frac{\lambda_3 q v_3 + \lambda_2(1-p_{22})v_2}{\lambda_3 q + \lambda_2(1-p_{22})},$$
$$U_2 = v_2.$$

We need to verify that this solution satisfies ex-ante and ex-post participation constraints. Constraints with respect to $\rho_i > 0$ for $i = 2, 3$ imply

$$\begin{aligned}&U_3 - \delta v_3 - \phi = 0,\\ &(1-p_{22})U_3 + p_{22}U_2 - \delta v_2 - \phi = 0,\end{aligned}$$

Thus

$$\begin{aligned}\phi &= U_3 - \delta v_3\\ &= \frac{\lambda_3 q v_3 + \lambda_2(1-p_{22})v_2}{\lambda_3 q + \lambda_2(1-p_{22})} - \delta v_3,\end{aligned}$$

and then

$$p_{22}(U_3 - U_2) - \delta(v_3 - v_2) = 0,$$

which holds. The ex-post constraints are

$$\begin{aligned} U_3 - \delta v_3 &> 0,\\ \lambda_3 q v_3 + \lambda_2 v_2 \frac{\lambda_3 q(1-\delta)}{\lambda_3 q + \delta\lambda_2} - \delta v_3(\lambda_3 q + \lambda_2(1 - \frac{\delta(\lambda_3 q + \lambda_2)}{\lambda_3 q + \delta\lambda_2})) &> 0,\\ \lambda_3 q v_3 + \lambda_2 v_2 &> 0.\end{aligned}$$

The CRA charges the fee

$$\phi = U_3 - \delta v_3 = (1-\delta)\frac{\lambda_3 q v_3 + \lambda_2 v_2}{\lambda_3 q + \lambda_2}.$$

it gains profit

$$(\lambda_2 + \lambda_3)\phi = (1-\delta)(\lambda_2 + \lambda_3)\frac{\lambda_3 q v_3 + \lambda_2 v_2}{\lambda_3 q + \lambda_2}.$$

The surplus of informed investors is equal to the difference between the ex-ante market surplus and the CRA's profits,

$$\lambda_2 v_2 + \lambda_3 v_3 - (\lambda_2 + \lambda_3)\phi = \frac{\lambda_3\lambda_2(1-q)(1-\delta)(v_3 - v_2)}{\lambda_3 q + \lambda_2}.$$

Type v_3 obtains the rating s_2 with zero probability, thus the ex-post constraint is satisfied,

$$(1-p_{33})(U_2 - \delta v_3) = 0.$$

The ex-post constraint for the type v_2 following rating s_3 is implied by the constraint for type v_3. The ex-post constraint for type v_2 following rating s_2 is $(1-\delta)v_2 > 0$.

It is immediate to verify that any rating system that permits no trade following the rating reduces the CRA's profits. ■

Proof of Proposition 8. In the optimal information structure with types v_2 and v_3 rated we have $p_{22} = \frac{\delta(\lambda_3 q+\lambda_2)}{\lambda_3 q+\delta\lambda_2}$, $\gamma_{33} = \frac{\lambda_3 q+\delta\lambda_2}{\lambda_3 q+\lambda_2}$ and $\gamma_{32} = \frac{(1-\delta)\lambda_2}{\lambda_3 q+\lambda_2}$.Then the expected revenue is

$$\lambda_3(1+\frac{\lambda_3 q(1-\delta)}{\lambda_3 q+\delta\lambda_2})\left(\frac{\lambda_3 q+\delta\lambda_2}{\lambda_3 q+\lambda_2}v_3+\frac{(1-\delta)\lambda_2}{\lambda_3 q+\lambda_2}v_2\right)+\lambda_2\frac{\delta(\lambda_3 q+\lambda_2)}{\lambda_3 q+\delta\lambda_2}v_2$$

The profits of CRA π^2 will be

$$\lambda_3(1+\frac{\lambda_3 q(1-\delta)}{\lambda_3 q+\delta\lambda_2})\left(\frac{\lambda_3 q+\delta\lambda_2}{\lambda_3 q+\lambda_2}v_3+\frac{(1-\delta)\lambda_2}{\lambda_3 q+\lambda_2}v_2\right)+\lambda_2\frac{\delta(\lambda_3 q+\lambda_2)}{\lambda_3 q+\delta\lambda_2}v_2-\delta(\lambda_3 v_3+\lambda_2 v_2)$$

Recall that the profits for rating only type 3 is $\pi^1 = \lambda_3\min\{(1-\delta)v_3, v_3-\frac{\lambda_2 v_2}{\lambda_1+\lambda_2}\}$. We need to solve for q such that $\pi^2=\pi^1$. Therefore, we have to solve for q: $\lambda_3(1+\frac{\lambda_3 q(1-\delta)}{\lambda_3 q+\delta\lambda_2})\left(\frac{\lambda_3 q+\delta\lambda_2}{\lambda_3 q+\lambda_2}v_3+\frac{(1-\delta)\lambda_2}{\lambda_3 q+\lambda_2}v_2\right)+\lambda_2\frac{\delta(\lambda_3 q+\lambda_2)}{\lambda_3 q+\delta\lambda_2}v_2-\delta(\lambda_3 v_3+\lambda_2 v_2) = \lambda_3\min\{(1-\delta)v_3, v_3-\frac{\lambda_2 v_2}{\lambda_1+\lambda_2}\}$.

As the winner's curse problem increases, we obtain

$$\begin{aligned}\lim_{q\to 0}\pi^2 &= \lambda_3\left(\delta v_3+(1-\delta)v_2\right)+\lambda_2 v_2-\delta(\lambda_3 v_3+\lambda_2 v_2)\\ &= \lambda_3\delta v_3+\lambda_3(1-\delta)v_2+\lambda_2 v_2-\delta\lambda_3 v_3-\delta\lambda_2 v_2.\end{aligned}$$

Simplifying leads to $\lambda_3(1-\delta)v_2+\lambda_2 v_2(1-\delta) = v_2(1-\delta)(\lambda_3+\lambda_2)$. ■

Proof of Proposition 9. Using the identical argument in Proposition 2, one gets $\overline{\delta} = \frac{\lambda_2 v_2+q\lambda_3 v_3}{\lambda_2 v_3+q\lambda_3(2v_3-v_2)}$.

Taking the derivative of $\overline{\delta}$ with respect to q, we obtain

$$\frac{\lambda_3 v_3(\lambda_2 v_3+q\lambda_3(2v_3-v_2))-\lambda_3(2v_3-v_2)(\lambda_2 v_2+q\lambda_3 v_3)}{\left(\lambda_2 v_3+q\lambda_3(2v_3-v_2)\right)^2}$$

Note that the denominator is always positive. The numerator simplifies to $\lambda_2\lambda_3\left(v_2-v_3\right)^2$ which is also positive. Therefore, as the winner's curse increases, $\overline{\delta}$ decreases leading to a larger set of δ that requires inflation. ■

Proof of Proposition 10. We obtain the following comparative statics results.

As the share of uninformed investors increases, the ratings become less informative.

$$\frac{dp_{22}}{dq} = -\frac{\lambda_3\lambda_2\delta(1-\delta)}{(\lambda_3 q+\delta\lambda_2)^2} < 0.$$

As the aggregate value of liquidity increases, that is δ decreases, the ratings become less informative.

$$\frac{dp_{22}}{d\delta} = \frac{\delta\lambda_2(\lambda_3 q + \lambda_2)}{(\lambda_3 q + \delta\lambda_2)^2} > 0.$$

As high quality assets become more scares, $\frac{\lambda_2}{\lambda_3}$ increases, ratings become less informative. Define $s = \frac{\lambda_2}{\lambda_3}$. Then $p_{22} = \frac{\delta(q+s)}{q+\delta s}$ and

$$\frac{dp_{22}}{ds} = -\frac{\delta(1-\delta)q}{(q+\delta s)^2} < 0.$$

If all investors are uninformed, $q = 1$, then

$$p_{22} = \frac{\delta(\lambda_3 + \lambda_2)}{\lambda_3 + \delta\lambda_2} < 1.$$

If the value of liquidity is very high, $\delta = 0$ (Lizzeri's case), then $p = 0$ and ratings are uninformative. ■

Proof of Proposition 11. The results follow from Propositions 7 and 6. ■

Proof of Proposition 12. Consider a market with a high share of informed investors, $q > \overline{q}$ and the rating system described in Proposition 7. Also assume that $\delta > \frac{q\lambda_2}{\lambda_2+\lambda_1}$, and thus unrated types v_2 and v_1 prefer to hold the asset. Now suppose that a regulator imposes a fee cap $\overline{\phi} = \frac{\lambda_3 q(1-\delta)}{\lambda_3 q+\delta\lambda_2}(q\lambda_3 v_3 + \lambda_2 v_2)$ which is less than the fee $(1-\delta)v_3$ charged to type v_3. Then the CRA profit is maximized under the information structure that induces rating two types v_2 and v_3 described in Proposition 6. The regulation is efficient as it allows issue type v_2 to trade. However, the regulation reduces the profits of the CRA. Informed investors gain positive rent. ■

Proof of Proposition 13. The results follow from Propositions 7 and 6. ■

Proof of Proposition 14. The result follows from Proposition 6. ■

References

[1] Admati, Anat R. and Paul Pfleiderer, 1986. A Monopolistic Market for Information. Journal of Economic Theory 39, 400-438.

[2] Admati, Anat R. and Paul Pfleiderer, 1990. Direct and Indirect Sale of Information. Econometrica 58, 901-928.

[3] Akerlof, George A., 1970. The Market for 'Lemons': Quality Uncertainty and the Market Mechanism. Quarterly Journal of Economics 84 (3): 488–500.

[4] Ashcraft, A. B., P. Goldsmith-Pinkham, and J. I. Vickery, 2010. MBS Ratings and the Mortgage Credit Boom. SSRN eLibrary.

[5] Benmelech, E., and J. Dlugosz, 2008. The Alchemy of CDO Credit Ratings. Journal of Monetary Economics 56: 617-634.

[6] Bergemann, Dirk and Martin Pesendorfer, 2007. Information structures in optimal auctions. Journal of Economic Theory 137: 580-609.

[7] Blume, Marshall E., Felix Lim, and A. Craig Mackinlay, 1998. The Declining Credit Quality of U.S. Corporate Debt: Myth or Reality? Journal of Finance 53(4), 1389-1413.

[8] Bolton, Patrick, Xavier Freixas, and Joel Shapiro, 2012. The Credit Ratings Game. Journal of Finance 76: 86-112.

[9] Coval, J.D., Jurek, J.W., Stafford, E., 2009. The Economics of Structured FInance. Journal of Economic Perspectives 23, 3-25.

[10] DeMarzo, Peter and Darrel Duffie, 1999. A Liquidity-Based Model of Security Design. Econometrica 67(1), 65-99.

[11] Dewatripont, Mathias and Jean Tirole, 1995.: The Prudential Regulation of Banks. Cambridge, MA: MIT Press.

[12] Doherty, Neil and Richard D. Phillips, 2002. Keeping up with the Joneses: Changing Rate Standards and the Buildup of Capital by U.S. Property-Liability Insurers. Journal of Financial Services Research 21, 55-78.

[13] Goel, Anand and Anjan Thakor, 2010. Credit Ratings and Litigation Risk. Unpublished working paper. Washington University in St. Louis.

[14] Griffin, J. M., and D. Y. Tang, 2009. Did Subjectivity Play a Role in CDO Credit Ratings? Working paper, SSRN.

[15] Grossman, Sanford J. and Perry, Motty, 1986. Sequential bargaining under asymmetric information. Journal of Economic Theory 39(1), 120-154.

[16] Kurlat, Pablo and Laura Veldkamp, 2012. Should We Regulate Financial Information? Unpublished working paper, SSRN.

[17] Lizzeri, Alessandro, 1999. Information Revelation and Certification Intermediaries. RAND Journal of Economics 30(2), 214-231.

[18] Mathis, Jerome, James McAndrews, and Jean-Charles Rochet, 2009. Rating the Raters: Are Reputational Concerns Powerful Enough to Discipline Rating Agencies? Journal of Monetary Economics 56(5), 657-674.

[19] Opp, Christian, Marcus Opp, and Milton Harris, 2011. Rating Agencies in the Face of Regulation. Journal of Financial Economics, forthcoming.

[20] Pagano, Marco and Paolo Volpin, 2012. Securitization, Disclosure and Liquidity. Review of Financial Studies, forthcoming.

[21] Rock, Kevin, 1986. Why New Issues are Underpriced. Journal of Financial Economics 15: 187-212.

[22] Sangriorgi, Francesco and Chester Spatt, 2012. Opacity, Credit Rating Shopping and Bias. Unpublished working paper. Stockholm School of Economics and Carnegie Mellon.

[23] Skreta, Vasiliki, and Laura Veldkamp, 2009. Rating Shopping and Asset Complexity: A Theory of Ratings Inflation. Journal of Monetary Economics 56(5), 678-695.

[24] Stanton, R., and N. Wallace, 2011. CMBS Subordination, Ratings Inflation, and the Crisis of 2007-2009, Working Paper 16206, National Bureau of Economic Research.

[25] White, Lawrence J., 2010. Markets: The credit rating agencies. Journal of Economic Perspectives 24(2), 211-226.