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“Mortgage Loan-Flow Networks and Financial Norms”

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Mortgage Loan-Flow Networks and Financial Norms*

Richard Stanton†  Johan Walden‡  Nancy Wallace§
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1 Introduction

Several recent studies highlight the importance of network linkages between intermediaries and financial institutions in explaining systemic risk in financial markets (see, for example, Allen and Gale, 2000; Allen, Babus, and Carletti, 2012; Cabrales, Gottardi, and Vega-Redondo, 2014; Glasserman and Young, 2015; Acemoglu, Ozdaglar, and Tahbaz-Salehi, 2015b; Elliott, Golub, and Jackson, 2014; Babus, 2013; Di Maggio and Tahbaz-Salehi, 2014). These studies show that financial networks may create resilience against shocks in a market via diversification and insurance, but may also generate contagion and systemic vulnerabilities by allowing shocks to propagate and amplify. The network structure is thus a pivotal determinant of the riskiness of a financial market.

These theoretical studies of networks and risk in financial markets typically focus on ex-post effects of the financial network: how the network redistributes risk between participants, and the consequences for the system’s solvency and liquidity after a shock. Ex-ante effects should also be important: the presence and structure of a financial network should affect—and be affected by—the investments and other actions of individual intermediaries and financial institutions, even before shocks are realized. Understanding the equilibrium interaction between network structure, the actions taken by market participants, and the market’s riskiness is the main theoretical focus of our study.\footnote{Babus (2013) studies contagion in a network of financial intermediaries with endogenous network formation, but does not include strategic (e.g., investment) decisions beyond link formation in her model. This is also the case for the model in Chang and Zhang (2015). A growing literature deals with trading networks in OTC markets, e.g., Gofman (2011); Babus and Kondor (2013); Zhong (2014). Systemic risk and contagion are not the focus of this literature.}

We build upon the approach in Stanton, Walden, and Wallace (2014), who empirically study the mortgage market from a network perspective and find that, despite the large total number of firms, the market is highly concentrated, with significant inter-firm linkages among the loan originators, aggregators,\footnote{Aggregators assemble the loans for sale to special purpose entities (SPEs) (see Inside Mortgage Finance, 2015).} special purpose entities (SPEs), securitization shelves,\footnote{When private-label issuers file a registration statement to register an issuance of a REMIC security, they typically use a “shelf registration.” The sponsor first files a disclosure document, known as the “core” or “base” prospectus, which outlines the parameters of the various types of REMIC securities offerings that will be conducted in the future through the sponsor’s shelf registration. The rules governing shelf issuance are part of the Secondary Mortgage Market Enhancement Act (SMMEA) (see Simplification of Registration Procedures for Primary Securities Offerings, Release No. 33-6964, Oct. 22, 1992, and SEC Staff Report: Enhancing Disclosure in the Mortgage-Backed Securities Markets, January, 2003, http://www.sec.gov/news/studies/mortgagebacked.htm#secii).} and shelf holding companies. For example, the private-label mortgage originations in 2006 were sourced from 11,103 mortgage originators of record, their loans were assembled by 2,030 aggregators, and these in turn sold the newly originated loans to SPEs that belonged to 146...
separate securitization shelves. These shelves were controlled by only 56 holding companies. Of the 1.4 million first-lien, private-label mortgages originated in 2006, sales of the loans among affiliated entities (i.e., where the lender of record, aggregator, and holding company were all subsidiaries of the same firm) accounted for 47.41% of transactions, while 52.59% of loan sales were between unaffiliated firms.

We introduce a model with multiple agents, representing financial intermediaries, which are connected in a network. Network structure in our model, in addition to determining the ex post riskiness of the financial system, also affects—and is affected by—what we call the financial norms in the network, inspired by the literature on influence and endogenous evolution of opinions and social norms in networks (see, for example, Friedkin and Johnsen, 1999; Jackson and López-Pintado, 2013; López-Pintado, 2012). Financial norms are defined as the quality and riskiness of the actions agents take, which are influenced in turn by the actions of other agents in the network.

Our model is parsimonious, in that the strategic action space of agents and the contract space are limited. Links in the network represent risk-sharing agreements, as in Allen et al. (2012). Agents may add and sever links, in line with the concept of pairwise stability in games on networks (see Jackson and Wolinsky, 1996), and also have the binary decision of whether to invest in a costly screening technology, which improves the quality of the projects they undertake.

The equilibrium concept used is subgame-perfect Nash. In an equilibrium network, each agent optimally chooses whether to accept the network structure, as well as whether to invest in the screening technology, given (correct) beliefs about all other agents’ actions and risk. Shocks are then realized and distributed among market participants according to a clearing mechanism similar to that defined in Eisenberg and Noe (2001). As in Elliott et al. (2014), we assume that there are costs associated with the insolvency of an intermediary, potentially creating contagion and propagation of shocks through the clearing mechanism, and thereby making the market systemically vulnerable. The model is simple enough to allow us to analyze the equilibrium properties of large-scale networks computationally, using numerical approximation methods.

Our model has several general implications. First, network structure is related to financial norms. Given that an agent’s actions influence and are influenced by the actions of those with whom the agent interacts, this result is natural and intuitive. Importantly, an agent’s actions affect not only his direct counterparties but also those who are indirectly connected through a sequence of links. As a consequence, there is a rich relationship between equilibrium financial norms and network structure, in turn suggesting a further relationship between the network and the financial strength of the market, beyond the mechanical relationship generated by
shock propagation.

Second, heterogeneous financial norms may coexist in the network in equilibrium. Thus, two intermediaries that are ex ante identical may be very different when their network positions are taken into account, not just in how they are affected by the other nodes in the network but also in their own actions. Empirically, this suggests that network structure is an important determinant not only of the aggregate properties of the economy but also of the actions and performance of individual intermediaries.

Third, proximity in the network is related to financial norms: nodes that are close tend to develop similar norms, just like in the literature on social norms in networks. This result suggests the possibility of decomposing the market’s financial network into “good” and “bad” parts, and addressing vulnerabilities generated by the latter.

Fourth, the behavior of a significant majority of nodes can typically be analyzed in isolation, while a small proportion of nodes affect the whole network through their actions. Such systemically pivotal nodes are especially important, suggesting a “too pivotal to fail” characterization of the systemically most important intermediaries in the market, rather than a “too big to fail” focus.

We analyze the mortgage-origination and securitization network of financial intermediaries in the U.S. empirically, using a data set containing all fixed-rate, private-label mortgages (i.e., mortgages not securitized by either Fannie Mae or Freddie Mac) originated and securitized in 2006 and 2007. We use loan flows to identify the network structure of this market and ex-post default rates to measure performance, and use the model to estimate the evolution of risk and financial norms in the network. We document a positive relationship between network position and performance, in line with the predictions of our model, which is present even after controlling for other observable characteristics like type and geographical position of the lender. We also document a positive relationship between predicted and actual out-of-sample performance of the intermediaries. Altogether, the results suggest that our proposed network decomposition is fruitful.

The rest of the paper is organized as follows. Section 2 describes the structure and properties of the U.S. residential mortgage market, related literature and available data. Section 3 introduces the model. Section 4 analyzes the properties of equilibrium, and Section 5 applies our approach to the 2006–2007 U.S. private-label mortgage market. Section 6 concludes. The appendix contains a detailed description of the mortgage data, the network game in the model, and all proofs.
2 Performance and structure of the U.S. residential mortgage market

The pre-crisis residential-mortgage origination market comprised thousands of firms and subsidiaries, including commercial banks, savings banks, investment banks, savings and loan institutions (S&Ls), mortgage companies, real estate investment trusts (REITs), mortgage brokers and credit unions. These had various roles in handling the loan flows from origination to ultimate securitization. Except for the closure of the Office of Thrift Supervision in 2011, the types of firms in the market are largely the same today.

A major driver of the subprime mortgage crisis was increased credit supply, as shown by Mian and Sufi (2009, 2011, 2014), who study heterogeneous loan performance at the zip-code level and show that performance was closely related to credit availability. Other authors (see, for example, Bernanke, 2007; Rajan, 2010; Kermani, 2014; Di Maggio and Kermani, 2014) have argued that the cumulative effect of low interest rates over the decade leading up to the financial crisis lowered user costs and increased the demand for credit to purchase housing services. Others have argued that the rapid expansions in the pre-crisis mortgage market arose due to widely held beliefs concerning continued house price growth (see, for example, Cheng, Raina, and Xiong, 2014; Shiller, 2014; Glaeser and Nathanson, 2015). Alternative explanations have focused on how mortgage securitization led to the expansion of mortgage credit to risky or marginal borrowers (see, for example, Nadauld and Sherlund, 2009; Loutskina and Strahan, 2009; Demyanyk and Van Hemert, 2011). Palmer (2015) suggests that vintage effects were important in explaining heterogeneous loan performance: mortgages originated earlier had more time for house price appreciation in the booming market, which created a cushion against default. Our focus complements this literature, since as we shall see, pre-crisis network effects were also important in explaining heterogeneous loan performance in the securitized residential-mortgage market.

Figure 1 presents the market structure for loan sales, decomposed into three levels. Loans flow from the mortgage originator of record to the aggregator of the loans (either the correspondent or the warehouse lender), then to the securitization shelf and to the holding company that owns the securitization shelf. Figure 1 portrays two possible holding-company types. The left-hand side of the schematic, shown in yellow, represents bank or thrift holding company operations, while the right-hand side of the schematic, shown in green, represents investment bank or large independent mortgage company operations in the pre-crisis period.

The gray arrows represent loan sales between entities that are subsidiaries (they may or may not be fully consolidated) of the same holding company and the blue arrows represent loan sales and the symmetric put-back liability between two unaffiliated firms and holding
companies. The graph of the gray arrows represents two trees. However, since loans in the market also flow between these entities, represented by the additional blue arrows in the figure, the market is really a network. The network structure of the market is important, because it suggests a complex structure of interaction between intermediaries, potentially affecting their behavior and providing a channel through which shocks can spread.

Note that the arrows in Figure 1 are double-headed, representing bidirectional links in the network. In 2006, the preponderance of loan sales into the pools, typically organized as Real Estate Mortgage Investment Conduits (REMICs), occurred within sixty days of the origination date of the loan due to the contractual structure of the wholesale lending mechanisms used to fund mortgage origination. These contractual funding structures assign the cash flows of the originated mortgages forward to each purchaser. However, the lender of record, or in some cases the aggregator, retains a contractual put-back option on the loan, which makes the risk structure of the loan flows bidirectional. In line with this observation, Stanton et al. (2014) find that the mortgage market is well represented by a bidirectional network, and that the performance of an individual node is closely related to the performance of the node’s neighbors (i.e., the nodes with which it shares a link) in the network. We therefore follow Stanton et al. (2014) and use a bidirectional network representation.

Table 1 shows an example of actual loan flows for a sample of seven loans originated and securitized in 2006. Each of the seven loans was aggregated by the same subsidiary of Bank of America. As shown in the table, Bank of America (BofA) aggregated loans from independent mortgage companies (Accubanc, Ameriquest, GMAC, Taylor, Bean & Whittaker Mortgage (TB&W Mtg)), from a S&L (World Savings), from a Bank of America branch, and from the subsidiary of another bank depository (Wells Fargo Bank). Bank of America then sold the mortgages to REMICs created within five different shelf-registration facilities. Three of these shelves were owned by Lehman Brothers and two by Bear Stearns. Thus, this Bank of America aggregator had a one-to-many relation both with lenders (the bottom level in the figure) and with holding companies (the top level in Figure 1), in line with the network description in Figure 1. We stress that since links in the mortgage network represent loan

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4 For visual clarity, we do not show sales between entities within the investment bank or independent mortgage holding company (the green entity on the right-hand side of Figure 1) to the bank and thrift entities, but these sales would also exist. There would also be sales between bank/thrift entities and investment bank/mortgage company entities.

5 Technically, a network is a general graph of nodes and links, with no restriction with respect to the existence of cycles or connectivity, whereas a tree is a connected graph with no cycle.

6 The two most important of these funding mechanisms were, and continue to be, 1) the master repurchase agreement, a form of repo, which received safe-harbor protections under the Bankruptcy Abuse Prevention and Consumer Protection Act of 2005 (BAPCPA) (Pub.L. 109-8, 119 Stat. 23) enacted April 20, 2005; and 2) extendable asset-backed commercial paper programs. In 2006, both of these had forty-five day “repurchase” maturities.
flows, a neighboring relationship between two nodes has no geographical interpretation.

Table 1: Example of a network loan sales for a sample of seven loans originated and securitized in 2006. Bank of America (B of A) was the aggregator for all seven loans. The geographic location of the loans varied, as did the lender of record, the shelf registration of the loan, and the holding company that owned the shelf.

![Network Diagram]

Figure 2 shows the network structure for a small subset of the market containing 21 lenders, eight aggregators and four holding companies, which are connected via loans originated and securitized in 2006. The network has 61 links, compared with 28 links in a tree representation, with 4 trees “rooted” at the holding company level. Thus, in a tree representation, more than half of the links would be unaccounted for. In Appendix A, a so-called minimum spanning tree (MST) representation of the subset is presented, which by including the links with the highest loan flow volumes provides the most complete tree representation of the market. For the full market of intermediaries, consisting in 2006 of 11,103 lenders, 2,030 aggregators, and 56 holding companies, the MST representation only
accounts for about 20% of the links. The fraction of loans accounted for in the MST is higher—about 50%—but still a significant fraction of the market is excluded when using a tree representation rather than a network representation of the market.

Figure 2: Between-firm networks for a subsample of private-label mortgage originations in 2006. There are 21 lenders (outer level), 8 aggregators (middle level) and 4 holding companies (inner level). Links represent loan flows.

Stanton et al. (2014) find a positive relation between loan performance and the intermediary’s network position. For example, they document a correlation of 0.23 between the average default rates of loans handled by an aggregator and those of other aggregators indirectly connected to that aggregator via a common node.\(^7\)

Why is this network effect present? One potential reason is that a shock that affects an intermediary spills over to its neighbors through contractual and business interactions. This effect, which plays out once a shock occurs and which we therefore call an ex post effect, is in line with the propagation mechanism described, e.g., in Elliott et al. (2014). Another reason is that the very presence of ex post effects influences the behavior of intermediaries ex ante, before shocks occur, in their decisions regarding whom to interact with and which standards to choose. Indeed, a common explanation for the observed heterogeneous performance of banks and other financial institutions is that they vary in their standards, through their ability to monitor and evaluate the projects they undertake (see Billett, Flannery, and

\(^7\)We verify and extend these results in Section 5.2.
Garfinkel, 1995; Berger and Humphrey, 1997, and references therein). Heterogeneous and decreasing lending standards were documented in the mortgage market in the years before the crisis (see Demyanyk and Van Hemert, 2011; Poon, 2008; MacKenzie, 2011). In what follows, we develop a model in which both ex ante and ex post network effects are important for explaining default rates in the U.S. mortgage market.\(^8\)

3 A Strategic Network Model of Intermediaries

Our network model has the fundamental properties that agents 1) act strategically when entering into contractual agreements among themselves, 2) are influenced by the actions of others to whom they are only indirectly connected, 3) make unobservable quality choices that impact outcomes, locally in the network, as well as potentially in aggregate.

We introduce the model in several steps. We first describe the risk environment and possible actions of agents that affect project payoffs, and analyze the outcome when agents act in isolation. We then study in detail the case when two agents interact, before analyzing equilibrium in the general \(N\)-agent model. A detailed description of the strategic game between agents is provided in the appendix.

3.1 Intermediaries and projects

There are \(N\) intermediaries with limited liability, each owned by a different risk-neutral agent. Each intermediary initially has full ownership of a project that generates risky cash flows at \(t = 1, \widetilde{CF}_p^n\), and may moreover incur some costs at \(t = 0\). The one-period discount rate is normalized to 0. Agent \(n\)’s objective is to maximize the expectation at \(t = 0\) of the value of the intermediary’s cash flows at \(t = 1, \widetilde{CF}_1^n\), net of any costs incurred at time 0,\(^9\)

\[
V^n = E_0 \left[\widetilde{CF}_1^n\right] - C_0^n.
\]

The risky project has scale \(s^n > 0\), with two possible returns represented by the Bernoulli-distributed random variables \(\xi^n\), so that \(R^n = R_H\) if \(\xi^n = 1\), and \(R^n = R_L\) if \(\xi^n = 0\). The

\(^8\)A third possibility is that a heterogeneous exogenous shock, which happened to be related to network position, hit all intermediaries jointly. Such a shock would have to be unrelated to all other observable characteristics of loans and intermediaries, e.g., geographical position at the zip code level, and could therefore not be identified in our data (see, for example, Manski, 1993; Bramoullé, Djebbari, and Fortin, 2009; Goldsmith-Pinkham and Imbens, 2013; Lee, Liu, and Lin, 2010; Acemoglu, García-Jimeno, and Robinson, 2015a). We cannot rule out such a scenario, but even if true, our results would be of interest in raising the question of what such a shock might represent.

\(^9\)The intermediary’s cash flow \(\widetilde{CF}_1^n\) may differ from \(\widetilde{CF}_p^n\) because of insolvency costs and because intermediaries enter into risk-sharing agreements with each other, as will be explained shortly.
probability, $p$, that $\xi^n = 0$ is exogenous, with $0 < p \ll 1$. We assume complete symmetry of risks in that the probability is the same for each of the $N$ projects.\(^\text{10}\)

Each agent has the option to invest a fixed amount, $C^n_0 = cs^n$, at $t = 0$, where $c > 0$, to increase the quality of the project. This cost is raised externally at $t = 0$. If the agent invests, then in case of the low realization, $\xi^n = 0$, the return on the project is increased by $\Delta R > c$, to $R_L + \Delta R$. This investment cost could, for example, represent an investment in a screening procedure that allows the agent to filter out the parts of the project that are most vulnerable to shocks. We represent this investment choice by the variable $q^n \in \{0, 1\}$, where $q^n = 1$ denotes that the intermediary invests in quality improvement. Intermediaries who choose $q = 1$ are said to be of high quality, whereas those who choose $q = 0$ are said to be of low quality. For the time being, we assume that $c$ is the same for all intermediaries. We will subsequently allow $c$ to vary across intermediaries, representing exogenous quality variation (as opposed to the quality differences that arise endogenously because of intermediaries’ investment decisions).\(^\text{11}\)

There is a threshold, $d > 0$, such that if the return on the investment for an intermediary falls below $d$, additional costs are immediately imposed, and no cash flows can be recovered by the agent.\(^\text{12}\) For simplicity, we call these costs “insolvency costs,” in line with Nier, Yang, Yorulmazer, and Alentorn (2008), who note that systemic events typically originate from insolvency shocks, although they are often also associated with liquidity constraints. We stress, however, that $d$ would not necessarily represent the point at which a firm becomes insolvent, but more broadly a region below which additional costs are incurred. Also, although only important from a policy viewpoint and not for the actual predictions of the model, we take the view that these additional costs are wasteful in that they impose real costs on society rather than representing transfers.

It will be convenient to define the functions

$$X(z) = \begin{cases} 
1, & z > d, \\
0, & z \leq d,
\end{cases} \quad \text{and} \quad Y(z) = X(z)z.$$

\(^\text{10}\)Note that each project may be viewed as a representative project for a portfolio of a large number of small projects with idiosyncratic risks that cancel out, and an aggregate risk component measured by $\xi^n$.

\(^\text{11}\)Several variations of the model are possible, e.g., assuming that screening costs, $c$, are part of the $t = 1$ cash flows, and fixing the scale of all intermediaries to $s \equiv 1$, all leading to qualitatively similar results. The version presented here was chosen for its tractability in combination with empirical relevance.

\(^\text{12}\)This assumption, similar to assumptions made, for example, in Elliott et al. (2014) and Acemoglu et al. (2015b), is a stylized way of modeling the additional costs related to risk for insolvency, e.g., direct costs of bankruptcy, costs of fire sales, loss of human capital, customer and supplier relationship capital, etc. It could also more generally represent other types of convex costs of capital faced by a firm with low capitalization, along the lines described in Froot, Scharfstein, and Stein (1993).
We also make the following parameter restrictions:

\[ 0 \leq R_L < d, \quad \text{and} \quad (1) \]

\[ R_L + \Delta R < R_H. \quad (2) \]

The first restriction implies that there will be insolvency costs for a low-quality intermediary after a low realization. The second restriction states that the outcome in the high realization is always higher than in the low realization, even for a high-quality intermediary.

### 3.2 Isolated intermediaries

We first focus on the setting in which intermediaries do not interact, and study the choice of whether to be of high or low quality. The cash flows generated by project \( n \in \{1, 2\} \) are then

\[ \tilde{CF}_n^P = \begin{cases} s^n R_H, & \xi = 1, \\ s^n (R_L + q \Delta R), & \xi = 0, \end{cases} \quad (3) \]

the total cash flows generated to the owner after insolvency costs are accounted for are

\[ \tilde{CF}_1^n = s^n Y \left( \frac{\tilde{CF}_P^n}{s^n} \right) = \begin{cases} s^n Y(R_H), & \xi = 1, \\ s^n Y(R_L + q \Delta R), & \xi = 0, \end{cases} \quad (4) \]

and the \( t = 0 \) value of the intermediary is

\[ V^n = V^n(q) = E_0 \left[ \tilde{CF}_1^n \right] - C_0^n = s^n \left( (1 - p) Y(R_H) + p Y(R_L + q \Delta R) - qc \right). \quad (5) \]

Given the parameter restrictions (1), we have \( V^n(0) = s^n (1 - p) R_H \). We make the technical assumption that if an intermediary is indifferent between being high and low quality, it chooses to become low quality. Therefore, \( q = 1 \) if and only if \( V^n(1) > s^n (1 - p) R_H \), immediately leading to the following result:

**Proposition 1.** An intermediary chooses to be of high quality, \( q = 1 \), if and only if

\[ R_L + \Delta R > \max \left( d, \frac{c}{p} \right). \quad (6) \]

Proposition 1 is very intuitive, implying that increases in the probability of a high outcome and costs of being insolvent, as well as decreases in the costs of information acquisition, make it less attractive for an intermediary to be of high quality. The first argument in the
maximum function on the RHS ensures that a high-quality firm avoids insolvency in the low state. If the condition is not satisfied, there is no benefit to being high quality even in the low state. The second argument ensures that the expected increase of cash flows in the low state outweighs the cost of investing in quality. The value of the intermediary when acting in isolation and following the rule (6) is then \( V_I^o = s^n V_I \), where

\[
V_I = \begin{cases} 
(1 - p)R_H, & q = 0, \\
(1 - p)R_H + p(R_L + \Delta R) - c, & q = 1.
\end{cases}
\]

Note that the intermediary’s optimal choice in (6) is decreasing in the solvency threshold, \( d \), i.e., for low thresholds it is potentially optimal to invest in quality, whereas for high thresholds it is not.

Also note that the objective functions of the agents coincide with that of society in this special case. Specifically, given that society has the social welfare function, \( V = \sum_n V^n \), under the constraint that intermediaries must act in isolation, the socially optimal outcome is realized by the intermediaries’ joint actions. We obviously do not expect this to be the case in general, when agents interact.

### 3.3 Network with two intermediaries

We explore the case with two interacting intermediaries, allowing us to gain intuition in a fairly simple setting, before formally introducing the general \( N \)-agent model. Intermediaries may enter into contracts that transfer risk. These contracts are settled according to a market-clearing system along the lines of that in Eisenberg and Noe (2001). Because of the high dimensionality of the problem when we allow agents to act strategically, we necessarily have to assume a limited contract space between intermediaries. Specifically, we assume that the contracts available are such that intermediaries swap claims on the aggregate cash flows generated by their projects in a one-to-one fashion, similar to what is assumed in Allen et al. (2012). Although obviously a simplification, this assumption qualitatively captures the bidirectional risk structure between intermediaries, discussed in the previous section.

We focus on the case where the two intermediaries have the same scale \( s^1 = s^2 = 1 \). The contract is then such that intermediary 1 agrees to deliver \( \pi \times \tilde{CF}_1^1 \) to intermediary 2 at \( t = 1 \), and in turn receives \( \pi \times \tilde{CF}_1^2 \) from intermediary 2, for some \( 0 \leq \pi \leq 1 \). Our focus is on the two cases where project risks are shared equally (\( \pi = 0.5 \)) and when intermediaries act in isolation (\( \pi = 0 \)). We use the general \( \pi \) notation, since in the general case with \( N \) agents, which we will analyze subsequently, \( \pi \) will typically take on other values.

The probability for both shocks to be low is \( p_2 \), and for either one of the shocks to be low
but not the other is \( p_1 \), so the probability for both shocks to be high is \( p_0 = 1 - 2p_1 - p_2 \). Consider a situation in which the intermediaries choose qualities \( q^1 \) and \( q^2 \), respectively, and let \( f^n(\xi^1, \xi^2) \) denote the binary variable that takes on value 0 if intermediary \( n \in \{1, 2\} \) is insolvent in state \((\xi^1, \xi^2)\), and 1 otherwise. Define

\[
\begin{align*}
  z^1(\xi^1, \xi^2) &= (1 - \pi)f^1(\xi^1, \xi^2)\tilde{CF}_p^1 + \pi f^2(\xi^1, \xi^2)\tilde{CF}_p^2, \\
  z^2(\xi^1, \xi^2) &= \pi f^1(\xi^1, \xi^2)\tilde{CF}_p^1 + (1 - \pi)f^2(\xi^1, \xi^2)\tilde{CF}_p^2.
\end{align*}
\]

(7)

(8)

Because of the assumed insolvency costs, it follows that the realized cash flows to intermediary \( n \) are

\[
\tilde{CF}_n^1(\xi^1, \xi^2) = f^n(\xi^1, \xi^2)z^n(\xi^1, \xi^2),
\]

(9)

and

\[
f^n(\xi^1, \xi^2) = X(z^n(\xi^1, \xi^2)).
\]

(10)

The time-0 value of an intermediary is then

\[
V^n(q^1, q^2) = \tilde{CF}_n^1(1, 1)p_0 + \tilde{CF}_n^1(1, 0)p_1 + \tilde{CF}_n^1(0, 1)p_1 + \tilde{CF}_n^1(0, 0)p_2 - cq^n.
\]

Equations (7)–(10) provide the adaptation of the clearing system in Eisenberg and Noe (2001) to our setting. The importance difference is that insolvency is costly in our setting, represented by the insolvency threshold, \( d > 0 \). As a consequence of such insolvency costs, there may be multiple solutions to the clearing mechanism (7–10), which lead to different net cash flows to intermediaries. This is because the insolvency of one intermediary can trigger the insolvency of another in a self-generating circular fashion (see Elliott et al., 2014).

We follow Elliott et al. (2014) and focus on the outcome with the minimal number of insolvencies.\(^{13}\) Briefly, we initially assume that (7)–(10) can be satisfied with no insolvencies. If this is not possible, because (10) implies at least one insolvency, we recalculate the cash flows given these identified insolvencies. This may trigger a new insolvency (if only one insolvency was identified in the previous step). We continue updating the set of insolvent intermediaries, calculating the cash flows after \( m \) steps in this mechanism. At the point when no more insolvencies occur, which requires at most two steps in the economy with two intermediaries, we have found the outcome that minimizes the number of insolvencies. The clearing mechanism is described in detail in Appendix C for the general case with \( N \geq 2 \) intermediaries.

We use the notation \( \mathcal{CM} \) for this clearing mechanism, which defines a mapping from

\(^{12}\)As noted in their study, since insolvencies are complements, all intermediaries as well as a social planner agree that their number should be minimized, given realized project cash flows.
realized project cash flows to actual cash flows after $\bar{m}$ steps of the mechanism,

$$\left(\tilde{CF}_{1,\bar{m}}, \tilde{CF}_{2,\bar{m}}\right)' = \mathcal{CM} \left(\tilde{CF}_{1}, \tilde{CF}_{2} \middle| \bar{m}\right).$$

By choosing $\bar{m} = 1$, we get the cash flows after only one step in the clearing mechanism, before the insolvency of one node is allowed to spread to the other. By choosing $\bar{m} \geq 2$, we get the ultimate cash flows in this example with only two intermediaries, $\left(\tilde{CF}_{1}, \tilde{CF}_{2}\right)'$, which occur after the potential propagation of an insolvency from one node to the other.

Note that the contracts offer a simple form of risk-sharing, potentially making it beneficial for agents to interact. Agents are risk neutral, but the cost of insolvency introduces a motive for avoiding low outcomes that trigger solvency costs, effectively generating risk aversion. By sharing risks, the negative effects of a low realization for the two agents can be limited. Note also that the incentive for an agent to invest in high quality is affected by the interaction with other agents.

3.3.1 Equilibrium

Intermediaries can either act in isolation ($\pi = 0$) or share risks ($\pi = 1/2$). For a risk-sharing outcome to be an equilibrium, both agents must have correct beliefs about the quality decisions made by their counterparties. We make the standard assumptions that each agent may unilaterally decide to sever a link to the other agent, and that bilaterally the two agents can decide to add a link between themselves. For an outcome with risk sharing to be an equilibrium, it follows that neither agent can be made better off by acting in isolation. For an isolated outcome to be an equilibrium, it cannot be that both agents are better off by sharing risk.

We model the mechanism via a strategic game with the sequence of events described in Figure 3, where we have formulated the game for the general $N$-agent case. At $t = -2$, given that $\pi = 1/2$, each agent may unilaterally decide to sever the link to the other agent and switch to $\pi = 0$, leading to the isolated outcome. If, on the other hand, $\pi = 0$, each agent can propose to switch to $\pi = 1/2$, in which case the other agent has the option to accept or decline at $t = -1$. Then, after the resulting network is determined, agents choose quality and outcomes are realized. Note that we implicitly assume that the actual quality decision is not contractible.\textsuperscript{14}

An equilibrium is now described by $(q^1, q^2)$ and $\pi$, such that neither agent has an incentive to sever the link (in the case $\pi = 1/2$), and it is not the case that both agents have an incentive

\textsuperscript{14}In our stylized model, the quality decision can of course be inferred from the realization of project cash flows. This issue would be avoided by assuming a small positive probability for $\Delta R = 0$ in case of a low realization with quality investments.
to form a link (in the case $\pi = 0$). Moreover, each agent’s belief about the other agent’s actions, both in the case when $\pi$ remains the same and in the case when it switches because of actions at $t = -2$ and $t = -1$, need to be correct.

For the action $q \in \{0, 1\}$, we let $-q$ denote the complementary action ($-q = 1 - q$). It follows that the three numbers, $q^1$, $q^2$ and $\pi > 0$, describe an equilibrium with risk sharing if:

$$V^1(q^1, q^2 | \pi) \geq V^1(-q^1, q^2 | \pi),$$
$$V^2(q^1, q^2 | \pi) \geq V^2(q^1, -q^2 | \pi),$$
$$V^1(q^1, q^2 | \pi) \geq V^1, 
V^2(q^1, q^2 | \pi) \geq V^2. 
$$

Condition (11) ensures that it is incentive compatible for both agents to choose the suggested investment strategies given that they share risks, while condition (12) presents participation constraints that state that risk sharing dominates acting in isolation for both agents.

Interesting dynamics arise already in this network with only two intermediaries, as seen in the following example. We choose parameter values $R_H = 1.2$, $R_L = 0.1$, $\Delta R = 0.5$, $c = 0.05$, $p_1 = 0.1$, $p_2 = 0.05$, $\pi = 0.5$, and vary the insolvency threshold, $d$. Since the setting is symmetric, it follows that $V_I^1 = V_I^2 = V_I$, and $V^1(q_1, q_2) = V^2(q_2, q_1)$, reducing the number of constraints that need to be considered.

The resulting value functions are shown in Figure 4. There are five different regions with qualitatively different equilibrium behavior. In the first region, $0 < d < 0.35$, the unique equilibrium is the one where both agents invest in quality ($q^1 = q^2 = 1$) and there is no risk-sharing, leading to values $V_I$ for both intermediaries. No intermediary ever becomes insolvent in this case (since $R_L + \Delta R > d$). The outcome where both agents invest and share

Figure 3: Sequence of events in network formation game with endogenous financial norms.
risk would lead to the same values, but cannot be an equilibrium because each agent would deviate and choose to avoid investments in this case, given that the other agent invests. For example, if intermediary 1 does not invest but intermediary 2 does, agent 1 reaches $V^1(0, 1)$ (red line with stars) which is greater than $V^1(1, 1)$ (purple line with squares) by avoiding the cost of investment but still capturing the benefits of not becoming insolvent after a low realization. Therefore, $V^1(1, 1)$ cannot be sustained in equilibrium. Now, $V^1(0, 1)$ can of course not be an equilibrium either, since under this arrangement intermediary 2 is on the $V^1(1, 0)$ (blue line with pluses) line, which is inferior to $V_I$ (black line, dotted). Similarly, $V^1(0, 0)$ (green line with crosses) is below $V_I$ so neither can it be an equilibrium for agents to share risks without quality investments. Only the isolated outcome therefore survives as an equilibrium.

In the second region, $0.35 \leq d < 0.6$, there are two equilibria, both with investments ($q^1 = q^2 = 1$) and the same value for both intermediaries, $V_I$. In addition to the isolated outcome, the outcome with risk-sharing and investments in quality by both agents is now an equilibrium. The reason is that the solvency threshold has now become so high that agent 1 has an incentive to invest in the risk-sharing outcome even when agent 2 invests, to avoid insolvency which otherwise occurs if both $\xi_1 = 0$, and $\xi_2 = 0$.

The third region is $0.6 \leq d < 0.65$, in which the unique equilibrium is for intermediaries to share risk and not invest, ($q^1 = q^2 = 0$), leading to value $V^1(0, 0)$ for both agents. Indeed,
this strategy dominates the value under isolation, $V_I$, which for $d \geq 0.6$ entails the strategy of not investing in quality since in that region insolvency occurs even when such investments are made (this is the reason for the discontinuity in $V_I$ at $d = 0.6$). Note that $V^1(0,0)$ is dominated by $V^1(1,1)$ and $V^1(0,1)$, though neither can constitute an equilibrium. The outcome $V^1(1,1)$ is not sustainable, since it is better for either agent to switch to low quality, as it is for agent 2 under $V^1(0,1)$. So the only equilibrium is the one with risk-sharing.

When $0.65 \leq d < 0.9$, i.e., in the fourth region, $V^1(0,0)$ decreases substantially compared with the third region, because for such high levels of the insolvency threshold, both intermediaries become insolvent if there is one low-shock realization, whereas two low realizations were needed in the third region. This makes $V^1(0,0)$ inferior to the isolated outcome, $V_I$ (in which both agents choose not to invest since $d$ is so high), because of a contagion effect. When risks are shared, a low realization for one intermediary not only causes that intermediary to become insolvent but also triggers the insolvency of the other intermediary. Thus, the only remaining equilibrium is now $V^1(1,1)$, i.e., for agents to share risk and for both to invest in quality ($q^1 = q^2 = 1$).

Finally, when $d \geq 0.9$, the isolated equilibrium without quality investments ($q^1 = q^2 = 0$) is the only remaining equilibrium, since any risk-sharing equilibrium will lead to contagion.

We note that equilibrium quality choice is non-monotone in $d$. For low solvency thresholds, $d < 0.6$, quality investments are optimal for both intermediaries; for solvency thresholds $0.6 \leq d < 0.65$, neither intermediary invests in quality; in the region $0.65 \leq d < 0.9$, both intermediaries invest; and finally for $d > 0.9$, investments in quality again become suboptimal for both intermediaries. This investment behavior is quite different from the isolated case in which quality choice is non-increasing in $d$, and shows how the presence of the network affects quality choices in a nontrivial way.

We also note that all equilibrium outcomes have $q^1 = q^2$. This is natural for the isolated equilibrium, but also occurs for the risk-sharing equilibria. It suggests that the “financial norm”—defined as the quality an intermediary chooses—depends on the financial norms of the intermediary with which it interacts, in line with the intuition that norms are jointly determined among interacting agents. In the two-agent case, it is straightforward to show that this property is generic:

**Proposition 2.** In any equilibrium of the network model with two agents, both agents make the same quality choice, i.e., $q^1 = q^2$.

In our terminology, intermediaries share financial norms in equilibrium. The possibility for agents to cut links is crucial for the result. As shown in the proof of Proposition 2, if

---

Note that our notion of financial norms is game-theoretic. Each intermediary maximizes its own “utility” (defined as its total expected payoff), and the utility of its counter party only enters indirectly, via
the risk-sharing network was exogenously given, it would be possible to get a risk-sharing outcome with different norms. In other words, the common financial norm in our model is a consequence of the possibility for agents to strategically influence the network structure.

3.4 General network with \( N \geq 2 \) intermediaries

We represent the network by the graph \( \mathcal{G} = (\mathcal{N}, E) \), \( \mathcal{N} = \{1, \ldots, N\} \). The relation \( E \subset \mathcal{N} \times \mathcal{N} \) describes which intermediaries are connected in the network. Specifically, the edge \( e = (n, n') \in E \), if and only if there is a connection (edge, link) between intermediary \( n \) and \( n' \). No intermediary is connected to itself, \( (n, n) \notin E \) for all \( n \), i.e., \( E \) is irreflexive. We define the transpose of the link \( (n, n')^T = (n', n) \), and assume that connections are bidirectional, i.e., \( e \in E \iff e^T \in E \). The operation \( E + e = E \cup \{e, e^T\} \), augments the link \( e \) (and its transpose) to the network, whereas \( E - e = E \setminus \{e, e^T\} \) sever the link if it exists. The number of neighbors of node \( n \) is \( Z_n(E) = |\{(n, n') \in E\}| \).

Intermediaries will in general have different scale and number of neighbors, and therefore choose to share different amounts of risk among themselves. Similar to the case with two intermediaries, we choose a simple sharing rule, represented by the sharing matrix \( \hat{\Pi} \in \mathbb{R}^{N \times N} \), where \( 0 \leq (\hat{\Pi})_{nn'} \leq s_n \) is the amount of project risk that intermediary \( n \) receives from (and in turn gives to) intermediary \( n' \), with the summing up constraint that \( \hat{\Pi} \mathbf{1} = s \), where \( s = (s^1, \ldots, s^N)' \), and \( \mathbf{1} = (1, 1, \ldots, 1)' \) is a vectors of \( N \) ones.

It will be convenient to characterize the fraction of the risk that intermediary \( n' \) provides to agent \( n \), which is represented by element \( \Pi_{nn'} \) of the the matrix \( \Pi = \hat{\Pi}\Lambda^{-1}_s \), implying that \( s = \Pi \mathbf{s} \), and also the fraction of intermediary \( n \)'s risk that it receives from \( n' \), represented by \( \Gamma_{nn'} \) of the matrix \( \Gamma = \Lambda^{-1}_s \hat{\Pi} \), which satisfies \( \mathbf{1} = \Gamma \mathbf{1} \). Here, we have used the notation that for a general vector, \( v \in \mathbb{R}^N \), we define the diagonal matrix \( \Lambda_v = \text{diag}(v) \in \mathbb{R}^{N \times N} \), with diagonal elements \( (\Lambda_v)_{ii} = v_i \).

In our previous example with two intermediaries of unit scale, \( s = (1, 1)' \) and the sharing matrices are

\[
\Pi = \hat{\Pi} = \Gamma = \begin{bmatrix}
1 - \pi & \pi \\
\pi & 1 - \pi
\end{bmatrix}.
\]

The network represents a restriction on which sharing rules are feasible. As we will discuss, this restriction can be self-imposed by intermediaries in equilibrium, who could...
choose not to interact even if they may, or it could be exogenous. Specifically, for a sharing rule to be feasible it must be that every off-diagonal element in the sharing matrix that is strictly positive is associated with a pair of agents who are linked, $\hat{\Pi}_{nn'} > 0 \Rightarrow (n, n') \in E$.

We focus on a simple class of sharing rules that ensure that all weights are nonnegative and that each intermediary keeps some of its own project risk, namely

$$
(\hat{\Pi})_{nn'} = \min \left\{ \frac{s^n}{1 + Z_n(E)}, \frac{s^{n'}}{1 + Z_{n'}(E)} \right\}, \quad n \neq n', \quad (13)
$$

and

$$
(\hat{\Pi})_{nn} = s^n - \sum_{n' \neq n} \hat{\Pi}_{nn'}. \quad (14)
$$

We write $\hat{\Pi}(E)$ when stressing the underlying network from which the sharing rules is constructed.

The joint quality decision of all agents is represented by the vector $q = (q^1, \ldots, q^N)' \in \{0, 1\}^N$. In the general case, the cost of investing in quality may vary across intermediaries, represented by the vector $c = (c^1, \ldots, c^N)' \in \mathbb{R}_+^N$. The state realization is represented by the vector $\xi = (\xi^1, \ldots, \xi^N) \in \{0, 1\}^N$. We will work with a limited state space, assuming that $\xi \in \Omega \subset \{0, 1\}^N$ where $\Omega$ is a strict subset of $\{0, 1\}^N$, and mainly focus on the set $\Omega^1 = \{\xi \in \{0, 1\}^N : \xi'1 \geq N - 1\}$, with $\mathbb{P}(\xi = 1 - \delta_n) = p_1, 1 \leq n \leq N$, and associated probability space $\mathbb{P} : \Omega^1 \rightarrow [0, 1]$. Here, $\delta_n$ represents a vector of zeros, except for the $n$th element which is 1. For this set, either zero or one low realization occurs, and the probability for a low realization is the same for all intermediaries. We also define the set $\Omega^2 = \{\xi \in \{0, 1\}^N : \xi'1 \geq N - 2\}$, for which no more than two realizations may be low, with full symmetry across intermediaries, so that the probabilities of any single intermediary being hit by a low shock is the same, as is the probability for any pair of intermediaries.

Solvency is represented by the solvency vector $f \in \{0, 1\}^N$, with $f^n = 1$ if node $n$ is solvent in equilibrium and $f^n = 0$ if node $n$ is insolvent. Realized cash flows to agents are represented by the random vector $\tilde{\text{CF}}_1 = (\tilde{\text{CF}}^1_1, \ldots, \tilde{\text{CF}}^N_1)' \in \mathbb{R}_+^N$. The realized project cash-flows are represented by the vector $\tilde{\text{CF}}_P = (\tilde{\text{CF}}^1_P, \ldots, \tilde{\text{CF}}^N_P)'$, where

$$
\tilde{\text{CF}}^n_P(\xi, q) = s^n (R_H \xi^n + (R_L + q^n \Delta R)(1 - \xi^n)), \quad n = 1, \ldots, N.
$$

The general network version of the clearing mechanism described in (7–10) is $\tilde{\text{CF}}_1(\xi, q) = \mathcal{CM} [\tilde{\text{CF}}_P(\xi, q) \mid \tilde{m}],$ where we choose $\tilde{m} = \infty$ to ensure that insolvencies are allowed to fully
propagate. Given the solvency vector, \( f \), the cash flows are

\[
\tilde{CF}_1 = \Lambda_f \Pi \Lambda_f \times \tilde{CF}_P,
\]

which we decompose into two parts and write \( \tilde{CF}_1 = \Lambda_f \times \tilde{CF}_{Pre} \), where \( \tilde{CF}_{Pre} = \Pi \Lambda_f \times \tilde{CF}_P \). Here, \( CF_{Pre}^n \) represents the cash flows to node \( n \) from other nodes, allowing for these other nodes to be insolvent, but not taking into account that the cash flows paid to agent \( n \) may be zero because node \( n \) is insolvent. Therefore, the vector \( \tilde{CF}_{Pre} \) represents the “pre-insolvency” cash flows to agents. The actual cash flows paid to agents, taking into account that the cash flows to agent \( n \) is zero whenever \( f^n \) is zero, are then

\[
\tilde{CF}_1 = \Lambda_f \times \tilde{CF}_{Pre} = \Lambda_f \Pi \Lambda_f \times \tilde{CF}_P.
\]

The clearing mechanism, described in detail in Appendix C, minimizes the number of insolvencies by solving

\[
\tilde{CF}_1 = \mathcal{CM} \left[ \tilde{CF}_P \middle| \infty \right] = \max_f (\Lambda_f \Pi \Lambda_f) \times \tilde{CF}_P, \quad \text{s.t.} \quad (15)
\]

\[
f = X \left( \Lambda_s^{-1} \times \tilde{CF}_1 \right). \quad (16)
\]

Here, \( X \) operates element-wise in (16), \( X(v) = (X(v^1), X(v^2), \ldots, X(v^N))' \). We write \( f(\xi, q) \) when emphasizing that the solvency vector depends on both shock realization and nodes’ quality choices. We also write \( \tilde{CF}_{Pre} = \mathcal{CM}_{Pre} \left[ \tilde{CF}_P \middle| \infty \right] \) for the pre-insolvency cash flows. Pre-insolvency cash flows are more informative than actual cash flows, which are always zero below the insolvency threshold.

The net cash flows to the intermediaries are given by the vector

\[
w(\xi|q, E) = \tilde{CF}_1(\xi, q) - \Lambda_c \Lambda_s q, \quad (17)
\]

and \( t = 0 \) value vector of intermediaries, given quality investments, \( q \), and network \( E \) is then given by

\[
V(q|E) = \sum_{\xi \in \Omega} w(\xi|q, E) \mathbb{P}(\xi). \quad (18)
\]

### 3.4.1 Equilibrium

Let \( E^* \) denote the complete network in which all nodes are connected. We assume that there is a maximum possible network, \( \bar{E} \subset E^* \), such that only links that belong to \( \bar{E} \) may exist in the sharing network. This restriction on feasible networks could, for example, represent
environments in which it is impossible for some agents to credibly commit to deliver upon a contract written with some other agents, due to low relationship capital, limited contract enforcement across jurisdictions, etc. A network, $E$, is feasible if $E \subset \bar{E}$. If all agents who may be linked actually choose to be linked in equilibrium, i.e., if $E = \bar{E}$, we say that the equilibrium network is maximal. It may also be the case that $E$ is a strict subnetwork of $\bar{E}$, just as was the case in the economy with two intermediaries where $\bar{E} = E^*$, but $E = \emptyset$ for some parameter values because agents chose the isolated outcome in equilibrium.

To define equilibrium, we build upon the pairwise stability concept of Jackson and Wolinsky (1996). We also require equilibrium in this multistage game to be subgame perfect. The game and definition of a stable equilibrium are explained in detail in Appendix D. Here we provide a summary. The sequence of events is as in Figure 3. Consider a candidate equilibrium, represented by a network $E$ and quality choices $q$. Each agent, $n$, has the opportunity to accept the sharing rule, $\hat{\Pi}(E)$, as is, by neither severing nor proposing new links at $t = -2$. But, in line with the pairwise stability concept, any agent $n$ can also unilaterally decide to sever a link with one neighbor, $n'$, leading to the sharing network $E' = E - (n, n')$, and corresponding sharing rule $\hat{\Pi}(E')$. Also, any agent can propose an augmentation of another link $(n, n') \in \bar{E} \setminus E$, which if agent $n'$ accepts leads to the sharing network $E'' = E + (n, n')$ with sharing rule $\hat{\Pi}(E'')$. Finally, we assume that each agent can unilaterally choose the isolated outcome, $V^n_I$, by severing its links to all other agents.

The possibility to unilaterally sever all links in a sharing network—although not technically part of the standard definition of pairwise stability—is natural, in line with there being a participation constraint that no intermediary can be forced to violate. It provides a minor extension of the strategy space.

The severance, proposal, and acceptance/rejection of links occur at $t = -2$ and $t = -1$. The agents then decide whether to invest in quality or not at $t = 0$, each agent choosing $q^n \in \{0, 1\}$. A pair $(q, E)$, where $E \subset \bar{E}$, is now defined to be an equilibrium, if agents given network structure $E$ choose investment strategy $q$, if no agent given beliefs about other agent’s actions—under the current network structure as well as under all other feasible network structures in $\bar{E}$—has an incentive to either propose new links or sever links, and if every agent’s beliefs about other agents actions under network $E$ as well as under all feasible alternative network structures are correct.

4 Analysis of equilibrium

We explore whether the common financial norms documented in Section 3.3 extend to larger networks. Specifically, using simulations, we explore the relationship between network struc-
<table>
<thead>
<tr>
<th>Average</th>
<th>Number in network</th>
<th>( q(\text{neighbors}) )</th>
<th>Number of neighbors</th>
<th>cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>( q = 1 )</td>
<td>6.34</td>
<td>0.79</td>
<td>4.32</td>
<td>0.011</td>
</tr>
<tr>
<td>( q = 0 )</td>
<td>2.66</td>
<td>0.4</td>
<td>3.12</td>
<td>0.016</td>
</tr>
</tbody>
</table>

Table 2: Summary statistics of high- and low-quality intermediaries. Number of simulations: 1,000. Parameters: \( R_H = 1.1, R_L = 0.2, \Delta R = 0.3, d = 0.75, p_1 = 4/90, p_2 = 1/90, c \sim U(0, 0.025) \). Scale \( s = 1 \).

ture and quality choice for a larger class of networks. We simulate 1,000 networks, each with \( N = 9 \) nodes. We use the classical Erdős-Rényi random-graph-generation model, in which the probability that there is a link between any two nodes is i.i.d., with the probability 0.25 for a link between any two nodes, and we also randomly vary \( c \) across intermediaries, \( c^n \sim U(0, 0.025) \).\(^{16}\) For computational reasons, we focus on networks in which equilibria are maximal, \( E = \bar{E} \).

Table 2 shows summary statistics for nodes that are of high quality compared with those of low quality in equilibrium. We see that there are on average more high-quality nodes. Also, not surprisingly, the average cost of investing in quality for high-quality nodes is lower than for low-quality nodes. More interestingly, the average number of neighbors of high-quality nodes is higher, and the average quality of neighbors of high-quality nodes is higher than of low-quality nodes. All these differences are statistically significant. The last result is especially important, since it shows that the financial norms that arise in the network are indeed closely related to network position, i.e., that different clusters exist in which nodes have different norms, in line with our results in the network with two intermediaries.

Another way of measuring the presence of such clusters is to partition each network into a high-quality and a low-quality component, and study whether the number of links between these two clusters is lower than it would be if quality were randomly generated across nodes. Specifically, consider a network with a total of \( K = |\{(n, n') \in E\}| \) links, and a partition of the nodes into two clusters: \( N = N^A \cup N^B \), of size \( N^A = |N^A| \) and \( N^B = |N^B| = N - N^A \), respectively, and the number of links between the two components: \( M = |\{(n, n') \in E : n \in N^A, n' \in N^B\}| \). In the terminology of graphs, \( M \) is the size of the cut-set, and is lower the more disjoint the two clusters are. The number of links one would expect between the two clusters, if links were randomly generated, would be \( W = \frac{1}{N(N-1)} N^A N^B K \), so if the average \( M \) in the simulations is significantly lower than the average \( W \), this provides further evidence that financial norms are clustered. Indeed, the

\(^{16}\)Other possible network generation processes would also be possible, e.g., the preferential attachment model introduced in Barabasi and Albert (1999). Given the small size of our simulated networks \( (N = 9) \), the choice of network generation process is not pivotal for the results.
average $M$ in our simulations is 12.2, substantially lower than the average $W$ which is 14.2, corroborating that network position is related to financial norms.

In Appendix B, we provide an example of a network with 8 nodes for which we study equilibrium network outcomes when varying $\Delta R$, all other parameter values being held equal. The equilibria in which both high quality and low quality nodes are present always consist of two distinct clusters of nodes, again showing the presence of distinct financial norms related to network position.

An intuition for why there exists clusters with similar financial norms is that, in general, connected nodes need to coordinate to reach a high quality equilibrium, and that when such coordination fails, nodes jointly choose (i.e., coordinate on) being of low quality. Bridges between high- and low-quality clusters are provided by a few nodes for which it is optimal to be of high quality even though they are connected to some low-quality nodes, because of the diversification benefits these low-quality connections provide.

### 4.1 Characterizing equilibrium behavior

A complete characterization of the behavior of all nodes in a general network is out of reach because of the rich equilibrium behavior in many-node networks. It is, however, possible to characterize the behavior of a usually significant majority of the nodes, namely those whose quality choices do not affect the cash flows they receive from other nodes.

**Definition 1.** Node $n$ is said to be systemically pivotal in equilibrium if its quality choice influences the set of nodes that become insolvent in some state, i.e., if for some state $\xi \in \Omega$, and intermediary $m \neq n$,

$$f(\xi, q)^m \neq f(\xi, (\neg q^n, q^{-n}))^m.$$  

Here, $(\neg q^n, q^{-n})$ denotes the action vector for which agent $n$ switches to the complementary action, and all other agents choose the same actions as in equilibrium (see Appendix C).

Note that a node that is merely a channel through which contagion propagates will not be classified as systemically pivotal. In other words, our definition focuses on how financial norms generate systemic risk. We partition the network into the set of nodes that are systemically pivotal, $\mathcal{N}^S$, and those that are not, $\mathcal{N}^U$. Nodes that belong to $\mathcal{N}^S$ may be of especially high interest for a regulator, since their quality decisions impact the solvency of other nodes in the network, creating a systemic externality.

An equivalent characterization of $\mathcal{N}^S$ within our model is that it contains the nodes that need to take into account the effects their quality choices have on other nodes in determining their actions:
Definition 2. Node \( n \) is said to be systemically self-affected if the cash flows it receives from other nodes in some state depend on its own quality choice, i.e., if for some \( \xi \in \Omega \),

\[
\tilde{CF}_1^n(\xi, q) \neq \tilde{CF}_1^n(\xi, (-q^n, q^{-n})).
\]

Nodes that are not systemically self-affected, although their choices affect their own cash flows as well as the cash flows received by other nodes, never experience a situation where their own quality choice affects other nodes, and then feeds back to affect themselves in a second wave. The characterization of the behavior of nodes that are not systemically self-affected is therefore significantly simplified.

Proposition 3. A node is systemically pivotal if and only if it is systemically self-affected.

The intuition for why the two definitions are equivalent is that the bilateral structure of contracts in the model is such that each node must internalize the effect of a shock from one of its neighbors, if possible. Thus, if a node affects the solvency of other nodes through its choices, at least one of those other nodes must be a neighboring node, and the effect therefore feeds back to the original node. The two definitions are therefore equivalent.

The behavior of nodes in \( \mathcal{N}^U \) is straightforward to characterize, especially under the shock structure \( \Omega^1 \). We define \( \Theta^{n,z} = R_L\Gamma_{nn} + R_H\sum_{j \neq n} \Gamma_{jn}f^j(1 - \delta_n, (q^n = z, q^{-n})), z \in \{0, 1\} \), and then have

Proposition 4. In the network model with shock structure \( \Omega^1 \), any node \( n \in \mathcal{N}^U \) is of high quality in equilibrium, \( q^n = 1 \), if and only if one of the following two conditions holds:

1. \( d - \Theta^{n,1} < 0 \) \( (1a) \) and \( \frac{c^n}{p} < \Gamma_{nn}\Delta R \) \( (1b) \),
2. \( 0 \leq d - \Theta^{n,1} < \Gamma_{nn}\Delta R \) \( (2a) \) and \( \frac{c^n}{p} - \Theta^{n,1} < \Gamma_{nn}\Delta R \) \( (2b) \).

In the special case when node \( n \) is not connected to any other node, it follows that \( \Gamma_{nn} = 1, \Theta^n = R_L \), and that condition (1) is never satisfied (because of the assumption that \( R_L < d \)). Condition (2) then collapses to \( d - R_L < \Delta R \), and \( \frac{c^n}{p} - R_L < \Delta R \), which are equivalent to the condition for high quality given in Proposition 1. So Proposition 1 is a special case of Proposition 4.

Proposition 4 shows that nodes in \( \mathcal{N}^U \), behave quite similarly to isolated nodes in the quality choices they make. For low-solvency thresholds, \( d \), condition (1a) is satisfied, and

\[\footnote{Note that for nodes that are not systemically pivotal, \( n \in \mathcal{N}^U \), the insolvencies \( f^j, j \neq n \) per definition do not depend on the quality choice of node \( n \), i.e., \( \Theta^{n,j} \) does not depend on \( z \), so we can drop the \( z \) dependence for such nodes and write \( \Theta^n \).} \]
condition (1b) then determines whether or not a node is of high quality. Note that (1b) imposes a stronger condition on a connected node for being of high quality than for an isolated node, since $\Gamma_{nn} < 1$ for a connected node. Intuitively, this is because a connected node shares the benefits of quality investments with other nodes, in contrast to an isolated node.

When $d$ increases above $\Theta^n$ but is below $\Theta^n + \Gamma_{nn}\Delta R$, (1a) fails and (2a) is satisfied. If (1b) was satisfied for low $d$, then (2b) is satisfied in this region, so a node that would choose high quality for low $d$ remains high quality in this region. A node that would choose low quality for low $d$ may choose high quality in this region since (2b) is weaker than (1b). Intuitively, this is because the potential benefits of quality investment is higher in this region.

For low $d$, characterized by the region (1a), the node remains solvent regardless of whether it invests in quality or not, whereas for $d$’s characterized by (2a) the node only remains solvent after a shock if it has invested in quality, and is therefore more difficult to incentivize to invest in quality.

Finally, when $d$ increases further, such that $d \geq \Theta^n + \Gamma_{nn}\Delta R$, both (1a) and (2a) fail, and the node therefore chooses low quality. This is the region in which the node becomes insolvent after a shock regardless of whether it invests in quality or not.

Altogether, there are thus three possible quality “paths” for nodes in $N_U$ as $d$ increases: low-low-low, high-high-low, and low-high-low. The first and second paths are monotone, just as in the case for isolated nodes, whereas the third path is not. The reason that the third path is possible for connected nodes is that because these nodes share risk, they may remain solvent even after a low shock realization when $d$ is low (in contrast to the isolated case), which in turn decreases their incentives to invest in quality. This may then lead them to choose low quality for such low $d$, but high quality for intermediate $d$.

For nodes in $N^S$, the above restrictions on equilibrium behavior do not apply. We saw in the example with two intermediaries how the quality shifted between high-low-high-low as $d$ increased. The equilibrium behavior of systemically pivotal nodes is therefore not as easy to characterize. The following proposition states that systemically pivotal nodes always have at least as high incentive to be of high quality as nodes that are not systemically pivotal, and is straightforward to show.

**Proposition 5.** _In the network model with shock structure $\Omega^1$, for any node $n \in N^S$, each of conditions (1) and (2) in Proposition 4 is sufficient for $n$ to be of high quality in equilibrium, $q^n = 1$. _

The intuition behind the result is also straightforward: Systemically pivotal nodes are not only directly effected by a negative shock if they decide to be of low quality, but also
potentially by negative feedback from other nodes that they affect. The feedback effect always increases the value of being high quality compared with nodes that are not systemically pivotal.

In practice, the systemically pivotal nodes tend to make up a small fraction of the nodes in the network, i.e., there will be many non-pivotal nodes that behave in a manner that is straightforward to characterize, in line with Proposition 4, and a smaller set of pivotal nodes with richer behavior. The effects of the actions of non-pivotal nodes are always local, as shown by the following proposition.

**Proposition 6.** Only systemically pivotal nodes may through their quality choices affect the cash flows of other nodes beyond their direct neighbors.

We note that although pivotal nodes are more motivated to invest in quality than non-pivotal nodes, they still do not internalize the full effect of their quality choices, in contrast to the market with isolated nodes in Section 3.2. Because of the potential nonlocal effect of the actions of pivotal nodes, a regulator who is concerned about contagion should therefore focus on these nodes, potentially being “too pivotal to fail.”

A sufficient condition for a node not to be systemically pivotal is given by the following

**Proposition 7.** In the network model with shock structure $\Omega^1$, if for all $j$ such that $\Gamma_{jn} > 0$, i.e., for all neighbors of a node $n$ and the node itself,

$$d < R_L \Gamma_{jn} + R_H (1 - \Gamma_{jn}),$$

then node $n$ is not systemically pivotal, $n \in \mathcal{N}^U$.

### 4.2 Computation of equilibrium

An important property of our approach is that it can be applied to large scale real-world networks. Assume that the network, $E$, size vector, $s$, and the cost vector, $c$ are observable, but that the parameter values $\phi = (R_L, R_H, \Delta R, d, p)$, and the quality vector $q \in \{0, 1\}^N$ are not. The project cash-flows (3), given a state realization $\xi$, are in vector form

$$\overline{CF}_P(\xi, q) = \Lambda_s(R_H \Lambda_s \xi + \Lambda_{1-\xi}(R_L 1 + \Delta R q)).$$

We assume that pre-insolvency cash flows, including some noise, are observed after $\bar{m}$ steps of the clearing mechanism,

$$\hat{w} = \mathcal{CM}_{Pre} \left[ \overline{CF}_P | \bar{m} \right] - \Lambda c \Lambda_s q + \epsilon,$$
where $\epsilon$ is a vector of independent, identically normally distributed noise.

Given an equilibrium quality vector, $q$, we define the best response for agents (at time 0), given the actions of other agents,

$$\mathcal{F}(q) = \mathcal{F}(q|\phi) = \{\hat{q} : \hat{q}^n \in \arg\max_{x \in \{0,1\}} V^n(x, \hat{q}^n)\},$$

where we as before assume that indifferent agents choose low quality, so that $\mathcal{F}$ is a single-valued function. For $q$ to be consistent with equilibrium, it must be that $q = \mathcal{F}(q)$. We define the equilibrium set $Q \subset \{0,1\}^N$, as

$$Q(\phi) = \{q : q = \mathcal{F}(q)\}.$$

An estimate of the unobservable quality vector, $q$, and thereby of all other properties of the equilibrium network, is given by solving the problem:

$$\min_{\phi} \min_{q \in Q(\phi)} \min_{\xi \in \Omega^1} \left\| \hat{w} - \left( \mathcal{CM}_{Pre}[\hat{CF}_P(\xi, q)] \bar{m} - \Lambda_s\Lambda_c q \right) \right\|. \tag{21}$$

Here, the mean squared norm is used, and it then follows that (21) is the maximum likelihood estimator of the equilibrium, conditioned on there being a shock, i.e., conditioned on $\xi \neq 1$.

The equilibrium behavior of nodes in $\mathcal{N}^U$ is characterized by Proposition 4. For nodes in $\mathcal{N}^S$, which in our application to the U.S. mortgage market make up a small fraction (less than 20%) of the network, the characterization is not as simple. For each node in $\mathcal{N}^S$, the value of being high quality needs to be compared with the value of being low quality, leading to two full evaluations of the clearing mechanism for each such node. Importantly, however, it follows easily that the optimal quality choice of a node under the $\Omega^1$ shock structure does not depend on the quality choices made by other nodes. Thus, rather than iterating over all possible combinations of quality choices of the nodes in $\mathcal{N}^S$, each node’s behavior can be determined on its own, allowing for efficient calculation of the equilibrium set $Q(\phi)$.

Also note that although the observed network structure, size vector and cost vector are high-dimensional in a market like the U.S. residential mortgage market with thousands of intermediaries, the model is actually strongly specified in that only the five parameters in $\phi$ are free in the estimation (21).
5 Application to the Mortgage Market (2006–2007)

5.1 Data

The data for this study were assembled from Dataquick, ABSNet, and Dealrate.com. The first eight rows of the upper panel of Table 3 provide loan contract and origination summary statistics for the first-lien, fixed-rate mortgages originated and securitized in private-label mortgage-backed securities in 2006. The average loan balance was $277,425 and the average cumulative loan-to-value ratio was 79%. The average coupon on the mortgages was 7% and the average maturity was 33 years. About 33% of the loans were conventional conforming loans and the average FICO score was 670. The eighth row of Table 3 presents summary statistics for the loan costs, which averaged about 14% of the balance, with a standard deviation of 4%.

We use the default rate of loans as a measure of performance. We distinguish between the performance of loans within the same network neighborhood and the performance of loans located in the same geographic neighborhood. We use two geographic neighborhood measures: i) Loan Zip Code Average Default, the average default rate (excluding the subject loan) of loans within the same zip code as the subject loan; ii) Weighted Average Default in Contiguous Zip Codes, the weighted average, by volume and centroid distance, of default rates for loans in all zip codes contiguous to the subject loan’s zip code.18 As shown in Table 3, the average zip code default level, excluding the subject loan, was 37% for all of the zip codes in our data and the average default rate for the contiguous zip codes was 32%.

We construct the network $E$ from the loan flows, as discussed in Section 2. The performance within the network neighborhoods is measured as the average (excluding the subject loan) of the default rates of nodes that are indirectly connected to a node in the network (at distance 2). We have three measures of performance within network neighborhoods: i) Neighboring Lenders Average Default Rate, the average default rate of lenders connected to a lender via an aggregator; ii) Neighboring Aggregator Average Default Rate, the average default rate of aggregators connected to an aggregator, either via a lender or via an holding company; and iii) Neighboring Holding Company Average Default Rate, the average default rate of holding companies connected to an holding company via an aggregator.

The next section of Table 3 presents the default performance within the three network neighborhoods of the loan, again excluding the subject loan. As shown, the average lender default level for neighboring network lenders was 44%. The average default level for neighboring aggregators to the subject loan’s aggregator was 38%, and the average default level

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18The contiguous zip codes were identified using ARCGIS and ARCMap and the performance data is from ABSNet. The weights are $\text{loanbalance}_{i}/\text{distance}(\text{Zip}_{i}, \text{Zip}_{j})$ for each zip code $j$. 

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<table>
<thead>
<tr>
<th>Variable Name</th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>Minimum</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Original Loan Balance ($)</td>
<td>277,425.00</td>
<td>224,182.00</td>
<td>50,000.00</td>
<td>989,560.00</td>
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<tr>
<td>Original Cumulative Loan to Value Ratio</td>
<td>0.79</td>
<td>0.16</td>
<td>0.14</td>
<td>1.65</td>
</tr>
<tr>
<td>Original Mortgage Contract Rate</td>
<td>0.07</td>
<td>0.03</td>
<td>0.04</td>
<td>0.13</td>
</tr>
<tr>
<td>Original Mortgage Amortization Term (years)</td>
<td>31.38</td>
<td>4.41</td>
<td>10.00</td>
<td>40.00</td>
</tr>
<tr>
<td>Conventional Conforming Loan Indicator</td>
<td>0.33</td>
<td></td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>FICO Score</td>
<td>670.00</td>
<td>67.00</td>
<td>370.00</td>
<td>850.00</td>
</tr>
<tr>
<td>No Documentation</td>
<td>0.68</td>
<td></td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Loan Origination Cost</td>
<td>0.15</td>
<td>0.04</td>
<td>0.07</td>
<td>0.28</td>
</tr>
<tr>
<td>Loan Zip Code Average Default</td>
<td>0.37</td>
<td>0.13</td>
<td>0</td>
<td>1</td>
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<tr>
<td>Weighted Average Default in Contiguous Zip Codes</td>
<td>0.32</td>
<td>0.12</td>
<td>0</td>
<td>1</td>
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<tr>
<td>Conventional Conforming Loan Indicator</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Neighboring Lenders Average Default Rate</td>
<td>0.44</td>
<td>0.01</td>
<td>0.0000</td>
<td>1.0000</td>
</tr>
<tr>
<td>Neighboring Aggregators Average Default Rate</td>
<td>0.38</td>
<td>0.01</td>
<td>0.24</td>
<td>0.53</td>
</tr>
<tr>
<td>Neighboring Holding Company Average Default Rate</td>
<td>0.38</td>
<td>0.02</td>
<td>0.03</td>
<td>0.49</td>
</tr>
<tr>
<td>Average Adjusted Gross Income</td>
<td>$53,115.00</td>
<td>$39,308.00</td>
<td>$14,635.00</td>
<td>$2,157,816.00</td>
</tr>
<tr>
<td>Average Median Sales Price</td>
<td>$392,545.00</td>
<td>$323,080.00</td>
<td>$17,666.00</td>
<td>$16,143,015.00</td>
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<tr>
<td>Bank Indicator</td>
<td>0.30</td>
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<td>Savings and Loan Institution Indicator</td>
<td>0.08</td>
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<td>Mortgage Company Indicator</td>
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<td>Credit Union Indicator</td>
<td>0.03</td>
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<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Number of Loans</td>
<td>1,152,312</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Table 3:** Summary statistics for loan-origination characteristics, default performance (60 days delinquent), the performance of loans in the loan’s zip code (excluding subject loan), and the performance over the network path of the loan (excluding subject loan).
for the neighboring holding company was 38%.

The local market economy is measured using zip code level averages from 2006 through 2013 for adjusted gross income obtained from the Internal Revenue Service and for median sales prices for homes obtained from Zillow. As shown in Table 3, the average zip code adjusted gross income over the period was $53,115 and the average median house price was $302,545.

To control for possible regulatory effects (see, for example, Agarwal, Lucca, Seru, and Trebbi, 2014), we introduce indicator variables for the type of lending institution that originated the loan. We have five lender-types: banks, savings and loan institutions, credit unions, mortgage companies, and other. The banks are primarily regulated by the Office of the Controller of the Currency, the savings and loan institutions were regulated at the time by the Office of Thrift Supervision, the credit unions by the National Credit Union Administration, and the mortgage companies were primarily regulated by the Department of Housing and Urban Development. The composition of the types of lenders in the data is 30% bank originated, 8% savings and loan originated, 59% mortgage company originated, 3% credit union originated, and the remaining mortgages in the sample were originated by finance companies and home builders.

The lower panel of Table 3 presents summary statistics for the first-lien, fixed-rate mortgages originated and securitized in private-label mortgage-backed securities in 2007. As shown, the loan balances at origination were larger in 2007, the cumulative loan-to-value ratios were larger, and the contract rates were higher than in the 2006 originations. A larger share of the fixed rate mortgages are conventional conforming and the ex post default rates are equivalent.

5.2 Network position versus performance

Table 4 presents the results of regressing loan defaults, defined as loans that are 60 or more days delinquent at any time between their origination date and December 2013, on the physical geography and macroeconomic environment of the loan collateral, the performance of the originators, aggregators, and holding companies within the loan’s network, the contractual characteristics of the loan, and the type of originator. The regression is intended to highlight the empirical relationship between network position and loan performance, controlling for a wide range of contractual, local economy, and regulatory effects that are typically found in default estimation strategies (see, for example, Mian and Sufi, 2009; Adelino, Schoar, and

19The IRS data were obtained from https://www.irs.gov/uac/SOI-Tax-Stats-Individual-Income-Tax-Statistics-ZIP-Code-Data-(SOI) and the Zillow median house sales price data were obtained from http://www.zillow.com/research/data/#median-home-value.
As shown in Table 4, the default rate of a loan is significantly affected by the average mortgage default rate within the same zip code (excluding the sample loan), the default rates of contiguous zip codes and the local macroeconomy, as well as by the average default rate for the lenders, aggregators and holding companies within the loan’s network. To see the importance of network effects relative to purely geographical effects, a 1% increase in the default rates of neighboring lenders within the network leads the subject loan’s default rate to increase by 2.28%; a 1% increase in purely geographically determined local-market default rates leads to an increase in the subject loan’s default rate of only 0.54%. Similarly, an increase of 1% in the default rates of the neighboring aggregators within the network leads the subject loan’s default rate to increase by 1.15%; a 1% increase in the average default rates of contiguous zip codes leads to an increase of only 0.37%. Even for the more network-distant holding companies, a 1% increase in default rates leads to an increase of 0.84% in the subject loan’s default rate.

All the other coefficients reported in Table 4 have the expected association with default. Interestingly, we find that zip codes with higher gross income levels and higher median sales prices are associated with elevated default levels.\textsuperscript{20} Surprisingly, mortgage companies, banks and savings and loan institutions are associated with lower loan-level default rates relative to credit unions and finance companies, despite the presence of loans from several bankrupt firms that were mortgage companies and savings and loan institutions.

Overall, we interpret these results to suggest that the mortgage loan flow network represents an important channel by which heterogeneous risk exposure and underwriting quality affected loan performance. These network effects appear to be over-and-above the well known effects of local house price and income dynamics, loan contract characteristics, and other local market conditions and are broadly consistent with the implications of our model.

5.3 Estimated financial norms

We use the number of links a node has as a proxy for its size, \( s^n \), and estimate the origination cost. For each node we compute the average of all costs of loans flowing through that node in a given year, which we use as a proxy for the cost \( c^n \). As a proxy for \( \hat{w}^n \) in (20) we use \( 1 - r^n \), where \( r^n \) is the average default rate for loans flowing through node \( n \).

We use the method described in Section 4.2 to estimate the quality vector, \( q \), and the parameters \( R_L, R_H, \Delta R, d, \) and \( p \), under the assumption that observations occur after \( \bar{m} = 4 \) steps of the clearing mechanism. We adopt the visualization approach in Stanton et al.\textsuperscript{20}

\textsuperscript{20}This result is consistent with Adelino et al. (2016).
<table>
<thead>
<tr>
<th></th>
<th>Coefficient Estimate</th>
<th>Standard Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>−1.5431***</td>
<td>0.0422</td>
</tr>
<tr>
<td>Geographic effects</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Loan Zip Code Average Default (excluding subject loan)</td>
<td>0.5437***</td>
<td>0.0065</td>
</tr>
<tr>
<td>Weighted Average Default in Contiguous Zip Codes</td>
<td>0.3745***</td>
<td>0.0059</td>
</tr>
<tr>
<td>Average House Prices in Zip Code (0000)</td>
<td>0.1054***</td>
<td>0.0033</td>
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<tr>
<td>Average IRS Median Gross Income in Zip Code (000)</td>
<td>0.0012***</td>
<td>0.0016</td>
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<tr>
<td>Network effects (excluding subject loan)</td>
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<tr>
<td>Neighboring Network Lenders Average Default Rate</td>
<td>2.2783***</td>
<td>0.0626</td>
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<td>Neighboring Network Aggregators Average Default Rate</td>
<td>1.1512***</td>
<td>0.0673</td>
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<td>Neighboring Network Holding Companies Average Default Rate</td>
<td>0.8363***</td>
<td>0.0237</td>
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<td>Contract Structure</td>
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<td>Loan Origination Cost</td>
<td>0.0135</td>
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</tr>
<tr>
<td>Original Loan Balance</td>
<td>−0.0019***</td>
<td>0.0003</td>
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<tr>
<td>Original Cumulative Loan to Value Ratio</td>
<td>0.6593***</td>
<td>0.0030</td>
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<td>Original Mortgage Contract Rate</td>
<td>0.2699***</td>
<td>0.0215</td>
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<td>Original Mortgage Amortization Term</td>
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<td>FICO Score</td>
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<td>−0.0184***</td>
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<td>Month fixed effects</td>
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<td></td>
</tr>
<tr>
<td>N</td>
<td>969,317</td>
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Table 4: Linear regression of loan-level default on the average default rate within the loan’s zip code (excluding the subject loan) and the average default rate within the loan’s network neighborhood (excluding the subject loan).
(2014), in which the high- and low-default segments of the network are shown separately. Figure 5 presents the network with a low-default part (left panel) and a high-default part (right panel). As is clear from Figure 5, the high-default segment of the network constitutes a concentrated part.

In total, there are 2,521 estimated high-quality nodes, making up 19% of the network. The aggregators and HCs are over-represented among the high quality nodes: 21 of 54 HCs are of high quality, as are 1,071 out of 2,030 aggregators, whereas only 1,429 out of 11,103 lenders are of high quality. There are 2,170 systemically pivotal nodes. A disproportionately high fraction of these, 66%, were of high quality, compared with only 9.8% of the remaining 11,019 nodes. Thus, the additional incentives that pivotal nodes had to invest in quality were important for the outcome. The systemically pivotal nodes are shown in Figure 6. The black nodes in the figure represent the 1,443 pivotal nodes of high quality, whereas the red nodes represent the remaining 727 nodes of low quality. The figure also shows the links between pivotal nodes, for those pairs of pivotal nodes that had at least 3 common loans in 2006. The links between pivotal nodes mainly include nodes within the intermediate range of default rates, suggesting that it was within this intermediate range that the choice of financial norms actually had the potential to influence the market outcome.

Figure 7 shows the estimated shock propagation. The shock initially hits a lender with a realized default rate in the top quartile (the large node close to “9 o’clock” among the lenders). In fact, it is the lender with the 2,573rd highest default rate out of 11,103, placing it in the 23rd percentile. The shock then causes contagion in the network through steps 2, 3, and from thereon, ultimately causing the vast majority (12,423) of the nodes to become insolvent. These are represented by the red nodes in lower right panel of Figure 7.

The node that was hit by the original shock in step 1 was one of the 727 systemically pivotal low-quality nodes. Had the node been of high quality, the shock would not have propagated through the network and become systemic, in support of the view that it may be fruitful for a regulator to identify and monitor nodes that are “too pivotal to fail.” The total loan flow volume handled by systemically pivotal nodes was about 6.5% of the total volume in the network, so with respect to volume, these 19% of the nodes were actually smaller than average. In contrast, the 19% of nodes with the highest loan flow volume, which may be identified under a “too big to fail” paradigm, handled 96% of the total volume.

5.4 Predicted versus actual default rates

We use the estimated network for out-of-sample predictions of loan performance development between 2006 and 2007. Specifically, we calculate the average default rate of the loans for
Figure 5: High- (left) and low-default (right) part of network, between originators and aggregators (above) and between aggregators and holding companies (below). The outer circle of nodes represents originators of record (11,103 in total), sorted clockwise in increasing order of quality of loans. The middle circle represents aggregators (2,030 in total) with the same ordering. The inner circle represents the holding companies (56 in total). The cutoff is made so that links with a higher than 80% default rate are shown in the right panel, whereas links with a lower default rate are shown in the left panel. Year: 2006.
Figure 6: The figure shows the estimated systemically pivotal nodes, as well as links between nodes that share at least 3 loans. Red nodes in the figure represent pivotal nodes of low quality, and black nodes represent those of high quality. There are 1,443 high-quality and 727 low-quality nodes in the pivotal part of the network, $N^S$. Year: 2006.

Each node that was also present in the mortgage loan-flow network in 2007. We define the change in default rates,

$$\Delta r^n = r^n_{2007} - r^n_{2006},$$

for such nodes which we use as a measure of a node’s actual (under-) performance during the period. There were 6,876 such nodes that were present in the 2007 sample, 45 HCs, 687 aggregators, and 6,144 lenders, altogether slightly more than half of the sample from 2006 of 13,189 nodes.

As mentioned, our estimation of the market in 2006 is based on $\bar{m} = 4$ steps in the clearing mechanism. The predicted out-of-sample change in performance is then

$$\Delta \hat{w} = CM_{Pre} \left[ \hat{CF}_P|5 \right] - CM_{Pre} \left[ \hat{CF}_P|4 \right],$$

i.e., the difference captures the predicted change in defaults after the shock is allowed to propagate one step further in the clearing mechanism. Assuming the time period for such a step is a year (an assumption that we will vary), we compare this predicted performance development with the actual development between 2006 and 2007. Since we assume that $\hat{w}^n = 1 - r^n$, the predicted relationship between $\Delta \hat{w}^n$ and $\Delta r^n$ is negative.

The companies that dropped out from our sample between 2006 and 2007 mainly did so
Figure 7: The figure shows the estimated shock, originally hitting a lender in step 1, and subsequently propagating through the network, ultimately causing 12,423 nodes to become insolvent. Estimation based on 2006 mortgage network.
because they defaulted.\footnote{Of the eleven holding companies that no longer appear in the 2007 sample, 82.82% of those firms went bankrupt, were closed, or were acquired. Of the twenty two aggregators that no longer appear in the 2007 sample, 72.72% went bankrupt, were closed, or were acquired. Among the top 50 lenders that no longer appear in the 2007 sample, 25% went bankrupt, were closed or were acquired. Among all of these firms, bankruptcy was the primary cause of exit from the sample.} They were thus the companies that performed the worst during the time period, which leads to our first prediction:

**Prediction 1.** *Nodes that remained in the sample in 2007 had higher $\Delta \hat{w}^n$ than nodes that dropped out of the sample.*

Our second prediction relates the actual and predicted performance among the nodes that remained in the sample in 2007, in line with the discussion above.

**Prediction 2.** *There is a negative relationship between $\Delta r^n$ and $\Delta \hat{w}^n$ for nodes that remained in the sample.*

The first prediction is straightforward to verify. We compare the mean $\Delta \hat{w}$ for the nodes that remained in the sample and those that dropped out, using a two-sample $t$-test. The mean is $-0.0036$ for the remaining nodes, and $-0.025$ for those that dropped out: a difference that is strongly statistically significant with a $t$-statistic of 23.3.

The results for the second prediction are shown in Table 5. Panel A, above, shows ordinary least squares regressions of actual performance on predicted performance, quality choice, whether a node was systemically pivotal, and also on a node’s size measured as the logarithm of the number of loans it handled. Columns 1–4 shows the results for the full sample, whereas columns 5–8 focuses lenders and columns 9–12 on aggregators.\footnote{Since only 46 HCs remained in 2007, the sample was too small for meaningful regressions at this level.} In line with our prediction, the coefficients are strongly negatively significant for the predicted performance coefficient, both in the univariate and multivariate regression. The coefficients are also significant for the aggregator subsample, whereas the coefficients have the right sign but are not significant for the lender subsample.

Interestingly, the coefficient on quality is positive (except for in the multivariate regression for the lender subset), suggesting that higher quality nodes had worse performance. Our model is of course silent about the cross sectional equilibrium relationship between quality and performance: All else equal, a node’s performance after a shock is better if it is of high quality, but in the cross section nodes that are more exposed to shocks will tend to be those that invest in quality, offsetting the positive relationship. The positive relationship we find in the data highlights the challenges, e.g., for a regulator, of identifying intermediaries of poor quality from performance data alone. Similarly, the coefficient on systemic pivotalness is in general positive but not statistically significant, as seen from the table.
For robustness, we carry out several variations of the tests. Panel B of Table 5 shows that the results are similar when using univariate and multivariate rank correlation regressions, which are more robust to nonlinearities. The choices of using $\bar{m} = 4$ steps for the clearing mechanism in the estimation, and 5 steps for the predicted 2007 performance, are also somewhat arbitrary. The results are very similar when we vary the steps between 3 and 6, and also under multivariate regressions on an arbitrary subset of the variables in the table (not reported).

The ex post effects generated by shock propagation are of course important for our results. We verify that the ex ante effects, i.e., the endogenous financial norms, are also important. By requiring that $\Delta R = 0$, we “turn off” the quality decision component of the model, since there are only costs and no benefits to an intermediary of investing in quality in this case. We find no relationship between actual and predicted performance when $\Delta R$ is set to zero, regardless of the number of steps in the clearing mechanism, $\bar{m} \in \{3, 4, 5, 6\}$, so the financial norms are indeed also important for our results.

6 Conclusions

In our network model of intermediaries, heterogeneous financial norms and performance, as well as systemic vulnerabilities arise as equilibrium outcomes. The optimal behavior of each intermediary, in terms of its attitude toward risk, the quality of the projects it undertakes, and the intermediaries it chooses to interact with, are influenced by the behavior of its prospective counterparties. These network effects, together with the intrinsic differences between intermediaries, jointly determine financial strength, quality, and systemic vulnerability, at the aggregate level of the market, as well as for individual intermediaries.

We apply the model to the mortgage-origination and securitization network of financial intermediaries, using a large data set of more than a million mortgages originated and securitized through the private-label market in 2006–2007. As predicted, the network position of an intermediary was strongly related to the default rates of its loans, above and beyond geographical and other observable factors. Moreover, out-of-sample predictive power of loan performance was shown.

Altogether, our results show the importance of network effects in the U.S. mortgage market, and suggest the possibility of detecting systemically vulnerable parts of the mortgage market’s network before shocks have had time to propagate and amplify.
### A. OLS

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<td><strong>Size, log($s$)</strong></td>
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### B. Rank correlation

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<td><strong>Size, log($s$)</strong></td>
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Table 5: **Predicted and actual performance.** The table displays realized changes of default rates of nodes between 2006–2007, regressed on predicted changes, estimated quality vector, whether nodes are systemically pivotal, and log-size measured as number of loans. Panel A shows OLS regressions, whereas Panel B shows rank correlation regressions. Predicted and actual performance are negatively related in all regressions, and the results are strongly significant in the full sample. Significance at 5% (*), 1% (**), and 0.1% level (**). Estimation based on actual mortgage network in 2006.
### A Minimum Spanning Tree

The minimum spanning trees representation of the sub-sample of the 2006 loan flow mortgage network discussed in Section 2 is shown in Figure 8.

![Minimum Spanning Tree Diagram]

Figure 8: Trees representation of a sub-sample of private-label mortgage originations in 2006.

### B Example with eight intermediaries

As mentioned in Section 4, we study a specific example with $N = 8$ intermediaries, all of which have scale equal to unity, $s = 1$, and with the shock structure $\Omega^2$. The maximal network, $\bar{E}$, which is also an equilibrium network is shown in panel A of Figure 9. As shown in panel A, for low $\Delta R$ the equilibrium outcome is for all intermediaries to be of low quality. This is simply because investing in quality does not increase the payoff in low states sufficiently to avoid insolvency for any intermediary for low $\Delta R$.

For $\Delta R = 0.12$, shown in Panel B, by investing in quality intermediaries can now avoid insolvency. This is what intermediaries 6–8 do in equilibrium. However, intermediaries 1–5 cannot sustain an equilibrium in which they also invest in quality, since it is too tempting for them to free-ride on the investments by others. This is because they have so many neighbors that their individual investment decisions are not pivotal in avoiding insolvency.

For $\Delta R = 0.15$, shown in Panel C, it becomes an equilibrium strategy also for nodes 1 and 5 to invest in quality. This is because the nodes can now avoid insolvency for several
Figure 9: Equilibrium outcomes for different $\Delta R$ in network with 8 nodes. Low quality nodes are marked in black, whereas high quality nodes are yellow (gray). In panel A (upper left corner), $\Delta R = 0.05$, and all nodes are of low quality. In panel B (upper right corner), $\Delta R = 0.12$, and nodes 6–8 are of high quality. In panel C (lower left corner), $\Delta R = 0.15$, and nodes 1, 5–8 are of high quality. In panel D, $\Delta R = 0.2$ and all nodes are of high quality. Parameters: $c = 0.0025$, $R_H = 1.2$, $R_L = 0.1$, $d = 0.68$, $p_1 = 0.0625$, $p_2 = x$, $\bar{E} = E$. Shock structure $\Omega^2$, and scale $s = 1$. 
cases in which two shocks hit the network, in which case they are pivotal. Nodes 2–4 still cannot sustain quality investments in equilibrium, though.

Finally, when $\Delta R = 0.2$, nodes 2–4 can also be made to invest in quality in equilibrium, since they are now also pivotal in avoiding insolvency after two shocks in a sufficient number of states. Thus, as shown in Panel B and C, heterogeneous financial norms may coexist in equilibrium, and intermediaries with the same quality tend to be linked (or at least to be close in the network).

C  Clearing Algorithm for general network model

As explained in the body of the paper, the initial assumption in the clearing mechanism is that all nodes remain solvent. If this is a feasible outcome, then it is the outcome of the clearing mechanism. If, however, some nodes fall below the insolvency threshold, then the new cash flows are calculated, given the insolvencies in the first step and new insolvencies are checked for. The algorithm terminates when no new insolvencies occurs, which takes at most $N$ steps.

Algorithm 1 (Clearing mechanism).

1. Set iteration $m = 0$ and the initial solvency vector $f_1 = 1$.
2. Repeat:
   - Set $m = m + 1$.
   - Calculate $z_m = \Lambda f_m \Pi \Lambda f_m \tilde{CF}_p$.
   - Calculate $f_{m+1}^n = X \left( \frac{\tilde{z}_M}{\tilde{s}^{\tilde{w}}} \right), n = 1, \ldots, N$.
3. Until $f_{m+1} = f_m$.
4. Calculate the $t = 1$ cash flow as $\tilde{CF}_1 = z_m$.

The iteration over $M$ can be viewed as showing the gradual propagation of insolvencies, where $f_M - f_{M-1}$ shows the insolvencies triggered in step $M$ by the insolvencies that occurred in step $M - 1$.

We can write the algorithm using returns, defining $R = \Lambda_s^{-1} \tilde{CF}_p$, and $\Gamma = \Lambda_s^{-1} \tilde{\Pi}$. The algorithm then becomes even simpler:

Algorithm 2 (Clearing mechanism on return form).

1. Set iteration $m = 0$ and the initial solvency vector $f_1 = 1$.
2. Repeat:
   - Set $m = m + 1$. 

• Calculate \( z_m = \Lambda f_m \Gamma \Lambda f_m R \).
• Calculate \( f_{m+1} = X(z_m) \).

3. Until \( f_{m+1} = f_m \).
4. Calculate the \( t = 1 \) cash flow as \( \widetilde{CF}_1^n = \Lambda_s z_m \).

It is straightforward to show that the two algorithms are equivalent, since \( \hat{\Pi} = \Lambda_s \Gamma = \Pi \Lambda_s \).

The \( \bar{m} \) step version of the clearing mechanism is obtained by replacing condition in step 3 of Algorithm 2 by the terminal condition \("Until \ m = \bar{m},\"\) leading to \( \widetilde{CF}_{1,\bar{m}} = \mathcal{CM} \left[ \widetilde{CF}_P|\bar{m} \right] \).

The pre-insolvency cash flows are defined as \( \mathcal{CM}_{Pre} \left[ \widetilde{CF}_P|\bar{m} \right] = (\Pi \Lambda f_{\bar{m}}) \times \widetilde{CF}_P \), which via (16) leads to the relation \( \widetilde{CF}_{1,\bar{m}} = f_{\bar{m}}^n \mathcal{CM}_{Pre} \left[ \widetilde{CF}_P|\bar{m} \right] \).

### D Network formation game

The sequence of events is as in Figure 3. We use subgame perfect, pairwise-stable Nash as the equilibrium concept. We make one extension of the pairwise stability concept in the definition of agents’ action space. Specifically, agents are allowed to unilaterally decide to become completely isolated by severing links to all agents they are connected to. In contrast, with the standard definition of pairwise stability agents are only allowed to sever exactly one link, or propose the addition of one link. The assumption that agents can choose to become isolated can thus be viewed as a network participation constraint.

The sequence of events is as follows: At \( t = -2 \) (the proposal/severance stage of the game) there is a given initial network, \( E \subset \bar{E} \). Recall that \( \bar{E} \) here is the maximal network in the economy, which arises if all possible links are present. Each agent, \( n = 1, \ldots, N \), simultaneously chooses from the following mutually exclusive set of actions:

1. Sever links to all other agents and become completely isolated.
2. Sever exactly one existing link to another agent, \((n, n') \in E\).
3. Propose the formation of a new link to (exactly) one other agent, \((n, n') \in \bar{E} \setminus E\).
4. Do nothing.

In contrast to actions 1. and 2., which are unilateral, agent \( n' \) needs to agree for the links to actually be added to the network under action 3.

The set of networks that can potentially arise from this process is denoted by \( \mathcal{E} \). We note that \( E' \subset \bar{E} \) for all \( E' \in \mathcal{E} \). The set of actual proposals for addition of links, generated by actions 3., is denoted by \( L^A \). The set of links that are actually severed, generated by actions 1. and 2., is \( L^S \). The total set of potential link modifications is \( \mathcal{L} = \{(L^A, L^S)\} \).
At $t = -1$ (the acceptance/decline stage) $L^A$ and $L^S$ are revealed to all agents, who then simultaneously choose whether to accept or decline proposed links. Formally, for each proposed link, $\ell = (n, n') \in L^A$, agent $n'$ chooses an action $a_\ell \in \{D, A\}$ (representing the actions of Declining or Accepting the proposed link). The total set of actions is then $A = \{a_\ell : \ell \in L^A\} \in \{D, A\}^{L^A}$, which for each $n'$ can be decomposed into $A^{n'} \cup A^{-n'}$, where $A^{n'} = \{(n, n') \in L^A\}$ represents the actions taken by agent $n'$, and $A^{-n'} = A \setminus A^{n'}$ the actions taken by all other agents. Altogether, $E$, $L_A$, $L_D$, and $A$ then determines the resulting network, $E'$, after the first two stages of the game, at $t = 0$.

At $t = 0$ (the quality choice stage) each agent, $n = 1, \ldots, N$, simultaneously chooses the quality $q^n \in \{0, 1\}$. The joint quality actions of all agents are summarized in the action vector $q \in \{0, 1\}^N$.

At $t = 1$, shocks, $\xi$, are realized, leading to realized net cash flows $w^n(\xi|q, E')$ as defined by equations (15–16). The value of intermediary $n$ at $t = 0$ is thus

$$V^n(q|E') = \sum_{\xi \in \Omega} w^n(\xi|E', q)P(\xi), \quad n = 1, \ldots, N. \quad (22)$$

**Equilibrium**

A stable equilibrium to the network formation game is an initial network and quality strategies, together with a set of beliefs about agent actions for other feasible network structures, such that no agent has an incentive to add or sever links, given that no other agents do so, and agents’ have consistent beliefs about each others’ behavior on and off the equilibrium path.

Specifically, the action-network pair $(q, E)$, together with $t = -1$ acceptance strategies: $A : \mathcal{L} \rightarrow \{D, A\}^{L^A}$, and $t = 0$ quality strategies $Q : \mathcal{E} \rightarrow \{0, 1\}^N$ constitute an equilibrium, if

1. At $t = 0$, strategies are consistent in that for each $E' \in \mathcal{E}$, and $q = Q(E')$,

$$q^n \in \arg \max_{x \in \{0, 1\}} V^n((x, q^{-n})|E),$$

for all $n = 1, \ldots, N$, i.e., it is optimal for each agent, $n$, to choose strategy $q^n$, given that the other agents choose $q^{-n}$.

2. At $t = -1$, strategies are consistent in that for each $(L_A, L_D) \in \mathcal{L}$, $A^n(L_A, L_D)$ is the optimal action for each agent, $n$, given that the other agents choose $A^{-n}(L_A, L_D)$.

Optimal here, means that the action maximizes $V^n$ at $t = 0$.

3. At $t = -2$, the strategy $L = \emptyset$ is consistent, i.e., for each agent $n$, given that no other
agent severs links or proposes additional links, it is optimal for agent \( n \) not to do so either, given the value such actions would lead to at \( t = -1 \).

A stable equilibrium is said to be maximal if \( E = \overline{E} \). We note that since we focus on pure strategies, the existence of stable equilibrium is not guaranteed, which has not been an issue in the examples we have studied. Neither is uniqueness of stable equilibrium guaranteed.

It follows that the following conditions are necessary and sufficient for there to exist acceptance and quality strategies such that \((q, E)\) is an equilibrium:

1. For all \( n \), \( V^n(q|E) \geq V^n((-q^n, q^{-n})|E) \).
2. For all \( n \), \( V^n(q|E) \geq V^i_n \).
3. For all \( (n, n') \in E, \exists q' \in \{L, H\}^N \) such that
   - \( V^n(q|E) \geq V^n(q'|E - (n, n')) \),
   - For all \( n'' \), \( V^n''(q'|E - (n, n')) \geq V^n''((-q''n''), (q')^{-n''})|E - (n, n') \).
4. For all \( (n, n') \in \overline{E} \setminus E, \exists q' \in \{L, H\}^N \) such that
   - \( V^n(q|E) \geq V^n(q'|E + (n, n')) \) or \( V^n(q'|E) \geq V^n(q'|E + (n, n')) \),
   - For all \( n'' \), \( V^n''(q'|E + (n, n')) \geq V^n''((-q''n''), (q')^{-n''})|E + (n, n') \).

The first condition ensures that each agent makes the optimal quality choice at \( t = 0 \), given that no change to the network is made. The second condition ensures that it is not optimal for any agent to sever all links and become isolated. The third condition ensures that there are consistent beliefs about future actions, such that no agent has an incentive to sever a link at \( t = -2 \). The fourth condition ensures that there are consistent beliefs about future actions, so that no two agents can be made jointly better off by adding a link.

### E Proofs

**Proof of Proposition 1:** The value if the agent invests in quality is

\[
V_A = s^n ((1 - p)R_H + pY(R_L + \Delta R) - c),
\]

versus

\[
V_B = s^n (1 - p)R_H
\]

if not investing. It follows that \( V_A > V_B \) if and only if

\[
pY(R_L + \Delta R) - c > 0,
\]

for which \( R_L + \Delta R > d \) and \( p(R_L + \Delta R) > c \) in turn are necessary and sufficient conditions. \( QED. \)
Proof of Proposition 2:
It follows immediately from Proposition 1 that in any isolated equilibrium, agents will choose the same quality \( q^1 = q^2 \). We therefore focus on the risk sharing equilibrium. W.l.o.g., assume that intermediary 1 is of high quality and intermediary 2 of low quality in equilibrium. The value of intermediary 1 is then

\[
V_A = (1 - 2p - p^2)R_H + pY ((1 - \pi)(R_L + \Delta R) + \pi R_H) + pY ((1 - \pi)R_H + \pi R_L) + p_2Y ((1 - \pi)(R_L + \Delta R) + \pi R_L) - c. \tag{23}
\]

For it to be an equilibrium strategy for agent 1 to be of high quality, it must be the case that this value weakly dominates that of being isolated and choosing high quality, which is

\[
V_B = (1 - 2p - p^2)R_H + pY (R_L + \Delta R) + pY (R_H) + p_2Y (R_L + \Delta R) - c, \tag{24}
\]

and also that it strictly dominates the value of risk sharing and choosing low quality, which is

\[
V_C = (1 - 2p - p^2)R_H + pY ((1 - \pi)R_L + \pi R_H) + pY ((1 - \pi)R_H + \pi R_L) + p_2Y (R_L). \tag{25}
\]

i.e., \( V_A \geq V_B \) and \( V_A > V_C \). In words, these conditions ensure that the diversification benefits of sharing risk for the high quality agent outweighs his cost of giving away the payoffs of a high quality project for the payoffs of a low quality project, and also the cost of investing into being of high quality. It is easy to show that the first inequality is satisfied if and only if

\[
R_L + \Delta R \leq d < \min((1 - \pi)R_H + \pi R_L, (1 - \pi)(R_L + \Delta R) + \pi R_H), \tag{26}
\]

and that the second inequality requires that \( p(1 - \pi)\Delta R > c \) in this case.

The value for the low quality agent if switching to high quality while still sharing risk is

\[
V_D = (1 - 2p - p^2)R_H + pY (\pi(R_L + \Delta R) + (1 - \pi)R_H) + pY (\pi R_H + (1 - \pi)(R_L + \Delta R)) + p_2Y (R_L + \Delta R) - c, \tag{27}
\]

which, given the restriction (26), is equal to

\[
V_D = (1 - 2p - p^2)R_H + p(\pi(R_L + \Delta R) + (1 - \pi)R_H) + p(\pi R_H + (1 - \pi)(R_L + \Delta R)) - c, \tag{28}
\]

versus

\[
V_E = (1 - 2p - p^2)R_H + p(\pi(R_L + \Delta R) + (1 - \pi)R_H) + pY (\pi R_H + (1 - \pi)R_L), \tag{29}
\]

if remaining of low quality in the risk sharing agreement. For the case when \( \pi R_H + (1 - \pi)R_L > d \), it follows immediately that \( p(1 - \pi)\Delta R > c \) is necessary and sufficient for agent 2 to switch to high quality, i.e., for \( V_D > V_E \).

For the case when \( \pi R_H + (1 - \pi)R_L \leq d \), it follows immediately that \( p(1 - \pi)\Delta R > c \) is sufficient for agent 2 to switch.

Thus, \( q^1 = 1 \), \( q^2 = 0 \) is ruled out as a risk sharing equilibrium, since regardless of parameter values it is optimal for at least one of agent 1 and 2 to deviate. Note that if we relax the constraint \( V_A > V_B \), corresponding to not allowing agent 1 to cut the link to agent 2, it is easy to construct examples where \( q^1 = 1 \) and \( q^2 = 0 \). The proposition therefore does not in general hold if the network is exogenously given. \( \text{QED} \)

Proof of Proposition 3: As described by the clearing mechanism in Appendix B, the total cash flows to nodes are in
equilibrium given by $\widetilde{CF}_1 = \Lambda f \Pi \Lambda f \widetilde{CF}_P$. We decompose these cash flows into those received from other nodes

$$\widetilde{CF}_{1,\text{Rec}}^n = \sum_{j \neq n} \Pi_{nj} f^j \widetilde{CF}_P^j,$$

and those generated by the node’s own project, $\widetilde{CF}_{Ou}^n = \Pi_{nn}((R_L + q^n \Delta R)(1 - \xi^n) + R_H \xi^n)$, leading to

$$\widetilde{CF}_1^n = f^n(\widetilde{CF}_{Ou}^n + \widetilde{CF}_{1,\text{Rec}}^n).$$

The cash flows $\widetilde{CF}_{1,\text{Rec}}^n$ and insolvencies, $j$, $j \neq n$, in general depend on $q^n$, and the proposition states that $\widetilde{CF}_{1,\text{Rec}}^n$ depends on $q^n$ if and only if $f^j$ depends on $q^n$ for some $j \neq n$.

From (30) it is clear that a necessary condition for being systemically self affected is to be systemically pivotal, since neither $\widetilde{CF}_P$, nor $\Pi$ depends on $q^n$, and the only way for $\widetilde{CF}_{1,\text{Rec}}^n$ to depend on $q^n$ is therefore if some $f^j$ depends on $q^n$. Also note that $\widetilde{CF}_{1,\text{Rec}}^n$ is decreasing in $f^j$, so if at least one $f^j$ for which $\Pi_{nj} > 0$ depends on $q^n$, then node $n$ must be systemically self affected.

Sufficiency of systemic pivotality to imply systemic self affectedness still does not follow immediately though, since an inspection of (30) suggests that if the only $f^j$’s that depend on $q^n$ are those for which $\Pi_{nj} = 0$, then node $n$ is systemically pivotal but not systemically self affected. However, due to the propagation mechanism of insolvencies, such a situation cannot arise, as follows from the following lemma:

**Lemma 1.** If node $n$’s quality choice does not affect the insolvency of any of its neighbors, then it will not affect the cash flows or insolvencies of any node to which it is not directly connected.

**Proof:** For any given state $\xi$, a neighbor $k$ to neighbor $n$ that becomes insolvent regardless of $q^n$ will provide cash flows of 0 to all of its neighbors regardless of $q^n$, and therefore $q^n$ will not affect cash flows to or insolvency of any such neighbor of $k$, and through lack of propagation any other node. Similarly, a neighbor $k$ that does not become insolvent regardless of $q^n$ will provide $f^k\xi k(1 - \xi^k)(R_L + \Delta R^k) + \xi^k R_H$ to any of its neighbors regardless of $q^n$, and again through lack of propagation lead to the same cash flows to and insolvencies of all other nodes.

Thus, if $f^j$ varies with $q^n$ for any node, there must be a direct neighbor of $n$, $j'$, for which $f^{j'}$ varies with $q^n$, which in turn implies that node $n$ is systemically self affected. $\text{QED}$

**Proof of Proposition 4:** If node $n$ chooses high quality, its value is

$$s^n (pY(\Gamma_{nn}(R_L + \Delta R) + R_H \Theta^n) + (1 - p)V) - c^n,$$

whereas if it chooses low quality, its value

$$s^n (pY(\Gamma_{nn} R_L + R_H \Theta^n) + (1 - p)V).$$

Here $V$ is the expected value of cash flows conditioned on $\xi^n = 1$, which is the same regardless of $q^n$, since node $n$’s project cash flows are the same ($R_H$) regardless of $q^n$. So, node $n$ will choose to be of high quality, if and only if

$$pY(\Gamma_{nn}(R_L + \Delta R) + R_H \Theta^n) - c^n > pY(\Gamma_{nn} R_L + R_H \Theta^n),$$

and it is easy to verify that the requirement that either condition (1) or (2) in the proposition holds is necessary and sufficient for (31). $\text{QED}$
Proof of Proposition 5: If node $n$ chooses high quality, its value is

$$s^n \left( pY (\Gamma_{nn} (R_L + \Delta R) + R_H \Theta^{n,1}) + (1 - p)V \right) - c^n,$$

whereas if it chooses low quality, its value

$$s^n \left( pY (\Gamma_{nn} R_L + R_H \Theta^{n,1}) + (1 - p)V \right).$$

Here, $\Theta^{n,1} > \Theta^{n,0}$ since $n \in N^2$, and an identical argument as that in the proof of Proposition 4 shows that conditions (1) or (2) are sufficient for the value of being high quality to outweigh that of being of low quality. QED

Proof of Proposition 6: The proposition is immediate, since any node that is not systemically pivotal never affects whether any of its neighbors becomes insolvent, and therefore never affects the cash flows to any of its neighbors’ neighbors. QED

Proof of Proposition 7: It is easy to check that condition (19) ensures that none of node $n$’s neighbors, including itself, become insolvent if they all have project cash flow realizations of $R_H$, whereas they receive $\Gamma_{jn} R_L$ from node $n$. Therefore no node will become insolvent regardless of $n$’s quality choice. QED
References


