Abstract

This paper builds a quantitative general equilibrium model with commercial banks and shadow banks to study the unintended consequences of capital requirements. In particular, we investigate how the shadow banking system responds to capital regulation for traditional banks. A key feature of our model are defaultable bank liabilities that provide liquidity services to households. The quality of the liquidity services provided by bank liabilities depends on their safety in case of default. Commercial bank debt is fully insured and thus provides full liquidity. However, commercial banks do not internalize the social costs of higher leverage in the form of greater bankruptcy losses (moral hazard), and are subject to a regulatory capital requirement. In contrast, shadow bank liabilities are generally uninsured, and their liquidity is limited by their positive probability of default. Shadow banks endogenously limit their leverage as they internalize its costs. Tightening the commercial banks' capital requirement from the status quo leads to safer commercial banks and more shadow banking activity in the economy. While the safety of the financial system increases, it provides less liquidity. Calibrating the model to data from the Financial Accounts of the U.S., the optimal capital requirement is around 15%.
Financial Regulation in a Quantitative Model of the Modern Banking System*

Juliane Begenau
Harvard University

Tim Landvoigt
University of Texas at Austin

April 2016

Abstract

This paper builds a quantitative general equilibrium model with commercial banks and shadow banks to study the unintended consequences of capital requirements. In particular, we investigate how the shadow banking system responds to capital regulation for traditional banks. A key feature of our model are defaultable bank liabilities that provide liquidity services to households. The quality of the liquidity services provided by bank liabilities depends on their safety in case of default. Commercial bank debt is fully insured and thus provides full liquidity. However, commercial banks do not internalize the social costs of higher leverage in the form of greater bankruptcy losses (moral hazard), and are subject to a regulatory capital requirement. In contrast, shadow bank liabilities are generally uninsured, and their liquidity is limited by their positive probability of default. Shadow banks endogenously limit their leverage as they internalize its costs. Tightening the commercial banks’ capital requirement from the status quo leads to safer commercial banks and more shadow banking activity in the economy. While the safety of the financial system increases, it provides less liquidity. Calibrating the model to data from the Financial Accounts of the U.S., the optimal capital requirement is around 15%.

*First draft: December 2015. Email addresses: jbegenau@hbs.edu, tim.landvoigt@mccombs.utexas.edu. We would like to thank our discussant Goncalo Gino, as well as participants at the NYU Junior Macro-Finance Meeting, Stanford Junior Faculty Workshop on Financial Regulation and Banking, MFM Winter 2016 meeting, 2016 ASSA meetings, 2015 CITE conference on New Quantitative Models of Financial Markets, Federal Reserve Bank of Boston, and the University of Texas at Austin.
1 Introduction

Unintended consequences are a challenging aspect of financial regulation. If regulated financial firms are competing with unregulated financial firms that provide similar services or products, then tighter regulation can cause a shift to the unregulated sector and thus undo the original intent of the change. As an example, consider Regulation Q that was introduced after the Great Depression. Regulators thought that excessive competition for deposit funds had weakened the banking system, and imposed a cap on deposit rates to prevent such competition. As long as interest rates remained low, savers had little incentives to pull their funds out of the traditional banking system. But as soon interest rates rose, depositors looked for alternatives and the competition for their savings generated one: money market mutual funds (Adrian and Ashcraft (2012)). This and numerous other examples\(^1\) highlight possible side-effects of regulation that often affect not only traditional banks (that are typically targeted) but also their competitors – shadow banks.\(^2\)

In this paper, we study and quantify the effects of capital requirements in a general equilibrium model that features regulated (commercial) and unregulated (shadow) banks. Tightening the capital requirement on commercial banks can shift activity to shadow banks, and thereby potentially increase the fragility of the entire financial system. Calibrating the model to aggregate data from the Flow of Funds, we find that higher capital requirements indeed shift activity away from traditional banks (though not monotonically), increasing

---

\(^1\) Asset-backed commercial paper conduits are another example for entities that emerged arguably as a response to regulation, more precisely capital regulation (see Acharya, Schnabl, and Suarez (2013)).

\(^2\) We define shadow banks as financial institutions that share features of depository institutions, either by providing liquidity services such as money market mutual funds or by providing credit either directly (e.g. finance companies) or indirectly (e.g. security-broker and dealers). At the same time, they are not subject to the same regulatory supervision as traditional banks. Importantly, we adopt a consolidated view of the shadow banking sector, i.e. money market mutual funds invest in commercial paper that fund security brokers-dealers that provide credit. We view this intermediation chain as essentially being carried out by one intermediary. In our definition of shadow banks, we abstract from shadow banks as a form of commercial banks' off-balance sheet vehicles as our focus is on liquidity provision of banks.
shadow bank fragility and leverage. However, the aggregate banking system becomes safer.
Welfare is maximized at a capital requirement of roughly 15%.

We derive this result in an endowment economy with households, commercial banks, shadow banks, and a regulator. The main features of our model are risk-neutral heterogeneous banks that make productive investments, have the option to default, and differ in their ability to guarantee the safety of their liabilities. At the same time, risk-averse households have preferences for safe and liquid assets in the form of bank liabilities whose liquidity value depends on their safety. Matching the model to aggregate data from the Flow of Funds as well as the Federal Reserve, Moody’s, and the FDIC, we find an optimal capital requirement for commercial banks that is higher compared to current regulation. The optimal requirement finds the welfare maximizing balance between a reduction in liquidity services and the increase in the safety of the financial sector and consumption.

Key to this result are (i) the different economic forces that determine the leverage of each type of bank, and (ii) the relative quality of the liquidity services produced by either type of bank. In our model economy, two trees produce a stochastic endowment. Households can only access one of the trees directly. Shadow banks and commercial banks compete over shares and intermediate access to the second tree. They fund their assets by issuing equity and debt to households. When either type of bank defaults on its debt, its equity becomes worthless and a fraction of the remaining bank value is destroyed in bankruptcy. While commercial bank debt is insured by the government, shadow bank debt is not.

Hence from the perspective of investors, shadow bank debt is exposed to default risk. Investors take this into account; that is, the price at which shadow banks issue debt takes into account the default probability implied by their leverage choice. Shadow banks internalize this trade-off and limit their leverage endogenously. In contrast, commercial bank debt is

\[ ^3 \text{Figure 1 provides an overview of the model.} \]
insured and therefore always safe. However, commercial banks do not internalize that higher leverage makes costly bankruptcy more likely. Thus they fail to take into account the social costs of higher leverage in the form of greater bankruptcy losses that the government imposes on households through taxation.

Increasing the capital requirement means that for every dollar of assets, commercial banks can issue fewer debt and produce less liquidity services, which reduces welfare. At the same, it reduces the bankruptcy rate of commercial banks and the associated deadweight losses. In an economy without shadow banks, this would be the key trade-off determining the optimal capital requirement.

In the economy with shadow banks, however, we find that higher capital requirements shift intermediation activity from commercial to shadow banks (though not necessarily monotonically). The reduced liquidity production by commercial banks following tighter regulation increases the attractiveness of all types of bank liabilities as well as the value of banking in general. This causes the price of the intermediated asset to increase and the funding costs of shadow banks to decrease. Both factors effectively relax the endogenous leverage constraint of shadow banks, thereby increasing shadow bank leverage and default risk. This opposing effect might offset the gain from making commercial banks safer through a higher capital requirement.

The net effect then crucially depends on the elasticity of shadow banks liquidity supply. If shadow banks were to increase their leverage more aggressively than that of commercial banks is reduced, their increased risk taking may dominate the increased safety of commercial banks. However, in our calibrated model shadow banks increase leverage only moderately. The reason is that shadow bank liabilities are risky, and the quality of their liquidity services negatively depends on shadow bank default risk. Therefore the degree to which shadow bank liabilities can be substituted for insured deposits is limited, and shadow banks only
partially replace the quantity of debt produced by commercial banks before the raise in the
requirement. Hence we find a net positive effect despite an expansion in shadow bank activity.

The simulated dynamic model further shows that a greater capital requirement for com-
mercial banks reduces the volatility of asset prices, liquidity provision, and consumption. The
reduction in the volatility of consumption is another source of welfare gains to households
from increased capital requirements. Intuitively, lower capital requirements lead to more
frequent bankruptcies. Since bankruptcies occur in bad aggregate states, low capital require-
ments amplify the effect of aggregate shocks and cause a more volatile stochastic discount
factor of households, which in turn makes all asset prices more volatile.

The dynamic model also features procyclical shadow banking activity: positive aggregate
shocks make shadow banks safer and therefore their debt more attractive. This increases the
share of shadow banking activity in good states of the world.

Quantitatively, our results depend on households preferences for liquidity, i.e. the pa-
rameter that governs how much households dislike variations in the liquidity service to con-
sumption ratio, the weight on liquidity services in the utility, and finally the degree of sub-
stitutability between shadow and commercial bank debt. Studies on the stability of money
demand (e.g. Lucas and Nicolini (2014) ) suggests that liquidity services and consumption
are complements implying that the first parameter value should be larger than 1. We infer
the other two parameters from shadow bank value weighted market leverage (debt over mar-
ket value of assets gives 0.65) using Compustat data and the share of shadow bank activity
(30%) as estimated by Gallin (2013). The higher the degree in the curvature, particularly
combined with a low degree in substitutability, the lower the scope for shadow banks to make
up for fall in liquidity production. Such a parametrization can actually imply a rebounding of
the commercial bank share when capital requirements exceed a value of 20-25%. Intuitively,
households care a lot about the mix of money market shares and deposits and demand rela-
tively more deposits when their share falls by too much. A higher weight on liquidity services in households’ utility increases the value of liquidity services overall and therefore all banks’ incentives to overproduce liquidity services. Thus, the improvement in welfare due to higher capital requirements is larger the higher the weight on liquidity services.

We match the model to aggregate data from the Flow of Funds on financial positions of U.S. households and financial institutions, interest rates from FRED, as well as Compustat data for publicly traded shadow banks. We also use industry reports to obtain estimates for default and recovery rates of financial intermediaries. It’s intuitive to think of commercial bank debt (mostly deposits) and shadow bank debt (mostly money market mutual fund shares, commercial paper, repo, etc) as being slightly different securities. The elasticity of substitution parameter is pinned down by the relative size of commercial banks and shadow banks.

**Related Literature**

Our paper is part of a growing literature at the intersection of macroeconomics and banking that tries to understand optimal regulation of banks in a quantitative general equilibrium framework.\(^4\) Our modeling approach draws on the recent literature on the role of financial intermediaries in the macroeconomy.\(^5\) These papers study economies with assets that investors can only access through an intermediary, as in our paper. The wealth of the intermediary then emerges as an additional state variable driving asset prices and the dynamics of the economy. By introducing limited liability and deposit insurance, and by defining the role of banks as liquidity producers, we bridge the gap to a long-standing microeconomic literature on the function of banks in the economy. Several recent papers in this literature study


the interaction of different types of banks that differ in the extent of regulation and bailout guarantees.\footnote{E.g. Goodhart, Kashyap, Tsomocos, and Vardoulakis (2012), Gennaioli, Shleifer, and Vishny (2013), Plantin (2014), Harris, Opp, and Opp (2015)}

Our paper is most closely related to Moreira and Savov (2014), Huang (2015), and Gertler, Kiyotaki, and Prestipino (2016). Moreira and Savov (2014) study an intermediary asset pricing economy with two types of assets that differ in suitability as collateral for issuing safe and liquid liabilities (money). They demonstrate that the presence of shadow banks can lead to increased economic volatility as rational investors try to determine the liquidity of the debt issued by the financial sector. Huang (2015) models shadow banks as an off-balance sheet financing option for regular banks within the Brunnermeier and Sannikov (2014) framework. Financial stability is a U-shaped function of financial regulation (i.e. very tight regulation generates more off-balance activities). Gertler, Kiyotaki, and Prestipino (2016) construct a quantitative macro-finance framework with a role for both regular banks (retail) and shadow banks (wholesale). Bank runs occur endogenously and allow the model to capture important features of the recent financial crisis. Our definition of banks is closely related to Gertler, Kiyotaki, and Prestipino (2016). A key difference to other work is our focus on liquidity provision as a fundamental role of banking and moral hazard arising endogenously from deposit insurance and limited liability.

The paper is structure as follows. Section 2 lays out the main mechanism of our model in a two-period economy. Section 3 describes the dynamic model, and section 4 presents results.

2 Main mechanism in a two period model

This section demonstrates the main mechanisms of our dynamic general equilibrium model in a two-period model. We first describe the two-period model and the forces that determine
the optimal capital requirement. For a quick overview of the model see Figure 1. We discuss the key assumptions of the model in a separate section.

The basic structure of the model is as follows. Households maximize utility from consuming goods and liquidity services. The economy receives two endowments of goods: one that households directly receive as income and one that households can only access through financial intermediaries. Two types of intermediaries, C-banks and S-banks, can perform the intermediation. They issue short-term debt and equity to households to fund the intermediated asset. The short term debt of both banks provides households with liquidity services.

Both type of banks can declare bankruptcy and default on their debt. However, the debt of C-banks is riskfree to households since the government provides deposit insurance for C-banks. In return, C-banks are subject to capital regulation. S-banks, on the other hand, are not subject to regulation that limits their leverage. Their debt is risky for households since debt of defaulting S-banks only pays off a fraction of the face value. S-banks take into account the effect of their leverage choice on the expected payoff of their debt, and hence endogenously choose to limit their leverage.

2.1 Description of the model

Endowments  There are two periods. Households receive an income of $Y_0$ at date 0 and $Y_1$ at date 1. Further, the intermediated asset is in unit supply and pays off $Z$ at date 1. $Y_0$ is known at date 0 whereas $Y_1$ and $Z$ are stochastic. In particular, trees $Z$ and $Y_1$ have two realizations $G = \text{good}$ and $B = \text{bad}$. Banks are exposed to idiosyncratic valuation shocks $\rho^j_i$ that are uniformly distributed on the interval $[\underline{\rho}^j, \overline{\rho}^j]$ for bank types $j = S, C$, respectively. The shocks $\rho^j_i$ are independent of the endowment processes $Y_1$ and $Z$. 
Figure 1: Model Overview
Shadow banks  There is a unit mass of $S$-banks. $S$-bank $i$ chooses $A^S_i$ shares of the $Z$-tree at the beginning of period 0. The shares trade at a market price of $p_0$. The tree has a stochastic payoff of $Z$ in period 1. To fund their investment, $S$-banks issue short term debt $B^S_i$ that trades at the price $q^S_i$. All trades occur in 0, so all prices must be 0 in $t = 1$. At the beginning of period 1, $S-$banks face idiosyncratic valuation risks $\rho^S_i$ that are proportional to their assets, with $\rho^S_i \sim F^S$, i.i.d. across banks. After a large realization of $\rho^S_i$, $S-$bank $i$ may find it optimal to declare bankruptcy. In case of a bankruptcy, the bank’s equity is wiped out, and its assets are seized by its creditors. In case the bank does not default, it returns the debt it owes and pays out dividends to households.

The problem for shadow bank $i$ is

$$V^S_i = \max_{A^S_i, B^S_i} q(A^S_i, B^S_i)B^S_i - p_0 A^S_i + E_0 \left[ M \left( \max \left\{ 0, ZA^S_i - B^S_i - \rho^S_i A^S_i \right\} \right) \right].$$

Since $\rho^S_i$ is uniformly distributed, the probability that bank $i$ with assets $A^S_i$ and debt $B^S_i$ stays in business is given by the probability that $\rho^S_i \leq Z - \frac{B^S_i}{A^S_i}$, i.e.

$$F^S_{\rho^S} \left( Z - \frac{B^S_i}{A^S_i} \right) = \frac{Z - \frac{B^S_i}{A^S_i} - \rho^S}{\rho^S - \rho^S}.$$ (1)

The bank problem has constant returns to scale in $A^S$, allowing for direct aggregation. Since there is a unit mass of $S-$ banks, we can directly interpret $F^S$ as the mass of banks that do not fail. To further simplify the problem, we exploit the homogeneity in the scale of the bank $A^S$ by defining leverage per asset $b^S_i = B^S_i / A^S_i$.

Dropping the $i-$subscript, we rewrite the problem as

$$V^S = \max_{A^S \geq 0, b^S} A^S v^S(b^S)$$

10
where
\[ v^S(b^S) = q^S(b^S)b^S - p_0 + E_0 \left[ M \left( F^S_{\rho} \left( Z - b^S - \rho^- \right) \right) \right]. \]  \hspace{1cm} (2)

In the above expression, \( \rho^- \) is the expected value of \( \rho \) conditional on S—bank’s survival and a realization of the random variable \( Z \). We define \( \rho^+ \) as the expected value of \( \rho \) conditional on S—bank’s failure. Investors who hold debt issued by S-banks that go bankrupt recover a fraction of each bond’s value
\[ r^S = (1 - \xi\rho^-) \frac{A^S(Z - \rho^+)}{B^S} = (1 - \xi\rho^-) \frac{Z - \rho^+}{b^S}. \] \hspace{1cm} (3)

The date-0 dividend payments are equal to the initial equity required by the bank,
\[ D_0^S = A^S(p_0 - q^Sb^S). \]

and the date-1 dividends are the payout amounts of the non-bankrupt banks
\[ D_1^S = F^S A^S(Z - b^S - \rho^-). \]

**Commercial banks** The problem of C-banks is similar to the S-bank problem with one important exception. C—banks are subject to an exogenous leverage constraint and do not take into account price effects due to their default risk. For each unit of debt, commercial banks need to pay fee \( \kappa \) to fund the insurance. Defining \( b^C = B^C/A^C \), we can write the problem for a C—bank as
\[ V^C = \max_{A^C, b^C} A^C v^C(b^C) \]
subject to
\[ b^C \leq (1 - \theta)p_0, \]

where
\[ v^C(b^C) = (q^C - \kappa) b^C - p_0 + E_0 \left[ M \left( F^C_{\rho} \left( Z - b^C - \rho^- \right) \right) \right]. \] \hspace{1cm} (4)

---

7We describe the derivation in more detail in the appendix section A.1.
The term $\rho_C$ in the expression above is defined analogously to the $S-$ banks.

Households who invest in C-bank debt always receive the full face value of the bonds. The difference between the face value and the recovery value of the bonds for failing C-banks is provided by the government (deposit insurance). The insurance fund recovers a fraction of the value of the debt of bankrupt C-banks, per bond issued:

$$r^C = (1 - \xi_C) \frac{A^C(Z - \rho^C_C)}{B^C} = (1 - \xi_C) \frac{Z - \rho^C_C}{b^C}.$$  

As $S-$banks, $C-$banks payout $D_0^C$ and $D_1^C$ in period 0 and 1, respectively.

**Households** In period 0, households have power utility over consumption $C_0$. In period 1, they have power utility over consumption $C_1$ and liquidity services $H$ relative to consumption. Thus utility is

$$U(C_0, C_1, N^S, N^C) = \frac{C_0^{1-\gamma}}{1-\gamma} + \beta \left( \frac{C_1^{1-\gamma}}{1-\gamma} + \psi \left( \frac{H}{C_1} \right)^{1-\eta} \right)$$

with $\gamma > 1$ and $\eta > 1$.

Liquidity is a composite good (consisting of $N^S$ and $N^C$) that is produced in equilibrium by both types of intermediaries

$$H(N^S, N^C) = [(1 - \nu) (N^S)\alpha + \nu (N^C)\alpha]^{1/\alpha}$$

where $\alpha$ parameterizes the elasticity of substitution between liquidity services of both types of banks. Weight $\nu$ determines the quality of liquidity services provided by C-banks relative to S-banks. The weight $\nu$ depends on the fraction of defaulting S-banks $F^S$, i.e. $\nu = 1/(1+(F^S)^{\tilde{p}})$, and consequently

$$1 - \nu = \frac{(F^S)^{\tilde{p}}}{1 + (F^S)^{\tilde{p}}}.$$
Households maximize utility in equation (5) by choosing consumption $C_0$, $C_1$, and liquidity services from each bank $N^S$, $N^C$, subject to the budget constraints

$$C_0 = Y_0 - \sum_j D^j_0 - \sum_j q_j N^j,$$

$$C_1 = Y_1 + \sum_j D^j_1 + N^C + N^S (F^S + (1 - F^S) r^s) - T.$$

In period 1, households have to pay a lump-sum tax of $T$ that the government raises to make whole the depositors of failed C-banks. Households are the sole owners of all banks and therefore receive the dividend payments both at date 0 and date 1.

**Government** The government uses the deposit insurance fund to reimburse depositors of failed C-banks. The government covers the deficit by imposing a lump-sum tax on households in period 1. Therefore the amount of tax funds the government raises is

$$T = B^C[ (1 - F^C)(1 - r^C) - \kappa].$$

**Markets** Asset market clearing in period zero requires

$$1 = A^S + A^C$$

$$N^S = B^S$$

$$N^C = B^C.$$

This implies for the goods market in period 1

$$C_1 = Y_1 + Z - \sum_j A^j \left[ E^j(\rho^j) + \xi_j (1 - F^j)(Z - \rho^j_1) \right].$$

(6)

### 2.2 Discussion of assumptions

The following paragraphs discuss key assumptions of the model.
Banks’ Role as Intermediaries  In the model, banks are special because they provide liquidity (discussed below) and intermediate assets. The role of banks as intermediaries can be derived from first principles in numerous ways. For example, in models with asymmetric information between borrowers and lenders, lenders with access to a cheaper screening or monitoring technology than other lenders (regular households) become banks (see Freixas and Rochet (1998) for many other examples).

The Role of Banks as Liquidity Providers  In this model, households value bank debt because it is liquid\footnote{The idea to view banks as liquidity provider goes back to Diamond and Dybvig (1983). Other work has built upon this idea (e.g. Gorton and Pennacchi (1990))} and safe, an interpretation of bank debt in Gorton and Pennacchi (1990) and in Gorton et al. (2012). The notion of safe and liquid assets includes bank deposits, money market fund shares, commercial paper, repos, short-term interbank loans, Treasuries, agency and municipal debt, securitized debt, and high-grade financial sector corporate debt. Aside from the government, commercial banks and shadow banks are the most important providers of these securities.\footnote{Historically, money market mutual funds (a type of shadow bank) emerged precisely to satisfy demand for safe and liquid assets when Regulation Q imposed a ceiling on deposit rates.} The savings glut hypothesis articulated in Bernanke (2005) and other recent work (e.g. Caballero and Krishnamurthy (2009), Gorton, Lewellen, and Metrick (2012), and Krishnamurthy and Vissing-Jörgensen (2012)) rests on the notion that there exists a demand for safe and liquid securities. Economic agents demanding these assets are for example households that hold deposits for transaction or liquidity reasons as well as corporations, institutional investors, and high net worth individuals that carry large cash-balances and seek safe and liquid investment vehicles with higher yields than deposits, such as money market mutual funds. Commercial banks provide mostly deposits, but also issue money market fund shares, repos, and commercial paper. Some of these securities that commercial banks hold (most notably deposits) are explicitly insured through
deposit insurance. Others, such as money market fund shares and commercial papers are indirectly insured due to government guarantees.\textsuperscript{10} Shadow banks generally do not benefit from government guarantees. Nevertheless outside of recessions and banking crisis, money market mutual funds shares and collateralized short term funding sources such as repo are considered safe and liquid.

In the model, liquidity services are generated through debt issued by shadow banks and commercial banks. Commercial bank debt represents all commercial bank liabilities precisely because the demand for safe and liquid assets goes beyond merely deposits. We apply the same idea to shadow bank debt with one notable difference: the value of shadow bank liabilities depends on the likelihood at which shadow banks default.

\textbf{Liquidity services in households preference} We capture the idea that bank liabilities provide liquidity services with our utility specification. The households in our model represent a blend of different agents with demands for different types of safe and liquid assets (deposits, money market mutual fund shares, and so forth) provided by all financial institutions. This is why our utility specification aggregates the liquidity services of both bank types.

Commercial bank debt always provides liquidity services no matter their default probability. This is different for shadow banks as the value from their liquidity service depends on their probability of default. This captures the idea that shadow bank debt is only safe as long as shadow banks are safe, that is, not too many of them go bankrupt.

The demand for liquidity services is captured with a money-in-the-utility function specification. Since Sidrauski (1967) money-in-the-utility specifications have been used to capture the benefits from money-like-securities for households in macroeconomic models. Feenstra

\textsuperscript{10}A number of empirical papers presents evidence for market expectations for government guarantees on U.S. banks (see for instance Flannery and Sorescu (1996) and Gandhi and Lustig (2013)).
(1986) proved the functional equivalence of models with money-in-the-utility and models with transaction or liquidity costs. The specific functional form is a version of Poterba and Rotemberg (1986) and Christiano, Motto, and Rostagno (2010).\textsuperscript{11} We further assumed ($\eta \geq 1$) complementarity between consumption and liquidity. This is intuitive as higher consumption levels go along with higher transaction volumes and higher savings demand.

### 2.3 Equilibrium Characterization

**Households** The stochastic discount factor $M_1$ equals

$$M = \beta E_0 \left[ \frac{C_0}{C_1} \left( 1 - \psi \left( \frac{H}{C_1} \right)^{1-\eta} \right) \right].$$

Its first term is the standard discount factor implied by log utility, while the second term reflects the adjustment to the discount factor resulting from the complementarity between liquidity services and consumption.

$$MRS^C = \nu \frac{U_1}{U_2} \left( \frac{H}{N^C} \right)^{1-\alpha} \quad \text{and} \quad MRS^S = (1 - \nu) \frac{U_1}{U_2} \left( \frac{H}{N^S} \right)^{1-\alpha}$$

denote the marginal rates of substitution between consumption and liquidity services by $C-$banks and $S-$banks respectively.

The solution to the HH’s problem is characterized by the two FOCs for deposits, which are also the HH’s intertemporal Euler equations. The FOC for C-bank deposits $N^C$ is

$$q^C = MRS^C + E_0 [M]. \quad (7)$$

The first term on the RHS of equation (7) is the marginal utility from consuming the payoff of 1 per C-bank deposit bond purchased at time 0. $MRS^C$ is the marginal utility from liquidity services.

\textsuperscript{11}In Christiano, Motto, and Rostagno (2010), the money and deposit utility parameter relates to the bank debt-consumption elasticity $\eta$ in the following way: $\sigma_q = 2 - \eta.$
The FOC for S-bank deposits $N^S$ is

$$q^S = MRS^S + E_0 \left[ M (P^S + (1 - P^S) r^S) \right]$$

(8)

The payoff of one deposit bond issued by S-banks depends on the fraction of surviving banks $P^S$. For surviving banks, the payoff is simply 1 as for C-bank deposits. For failing banks, it is given by the recovery value per bond $r^S$. The second term, the marginal utility from liquidity services, is analogous to C-banks.

**Bank Portfolio Choices** The optimal choice of asset purchases $A^j$ for each bank type $j = C, S$ imply

$$v^j(b^j) = 0,$$

(9)

for any $A^j > 0$. We can think of condition (9) as determining the relative size of the $j$-bank sector. To see this, we show in appendix A that these conditions can be rewritten as

$$p_0 - (q^C - \kappa)b^C = E_0 \left( M \frac{\sqrt{12}}{2} \sigma^C \left(F^C\right)^2 \right),$$

(10)

$$p_0 - q^S b^S = E_0 \left( M \frac{\sqrt{12}}{2} \sigma^S \left(F^S\right)^2 \right).$$

(11)

where $\sigma^j$ is the standard deviation of $\rho^j$.

The FOC for leverage per unit of assets $b^C$ is

$$(v^C)'(b^C) = 0.$$

In appendix A, we show that this condition can be expressed as

$$q^C = \lambda^C + E_0 \left( M F^C \right).$$

(12)

This implies that the Lagrange multiplier on C-banks’ leverage constraint is always positive as long as the marginal value of C-bank liquidity is positive.
The positive multiplier implies a binding constraint, i.e.

\[ b^C = (1 - \theta)p_0. \]

As for C-banks, S-bank leverage per unit of assets is determined through S-banks’ FOC for \( b^S \)

\[ (v^S)'(b^S) = 0, \]

which can be written as

\[ q^S + b^S \frac{\partial q^S(b^S)}{\partial b^S} = E_0 \left( M_1 F^S \right). \tag{13} \]

Comparing the FOC for C-bank leverage in (12) to the S-bank FOC in (13) above reveals the fundamental difference between the two types of banks. While C-banks are subject to a constant leverage constraint that in equilibrium is always binding, S-banks choose to limit their leverage because they internalize the effect of their leverage choice on the price of their deposits, \( q^S \).

Increasing leverage \( b^S \) will decrease the survival probability \( F^S \) and hence lower the value of S-bank deposits to HH, implying \( \frac{\partial q^S(b^S)}{\partial b^S} < 0 \), as can be seen in equation (8).

When choosing leverage, we assume that shadow banks take into account that their default risk is priced. In this sense, each shadow bank acts as a monopolist for its own debt. But it does not internalize how its leverage choice and default risk affects the value of liquidity services for households. This is intuitive, as shadow bank bond prices are sensitive to the specific default risk of the issuer. But they also move with changes in aggregate liquidity conditions, which are caused by actions of all shadow banks, but not by any individual bank. Thus, it is best to think of the changes in the value of aggregate liquidity services as an externality arising in general equilibrium.

The following proposition that we prove in the appendix states the resulting endogenous leverage choice of S-banks:
Proposition. Leverage per units of assets $b^S$ of S-banks is

$$b^S = \frac{E_0 (M_1 \text{MRS}^S)}{E_0 (M_1)} \sqrt{12\sigma^S} \xi^S. \quad (14)$$

For any reasonable parameter combinations, the above equilibrium choices of asset purchases and leverage for both types of banks imply that $C-$banks have a dominant position in the intermediated asset ($A^C > A^S$). The intuitive reason for this equilibrium outcome is as follows.

First, C-banks’ debt is insured and therefore C-banks do not internalize the effect of their leverage choice on the price of their debt. Since the marginal liquidity benefit is always positive, C-banks always exhaust their leverage constraint. S-banks, however, do internalize the increase in their default risk. Hence, if C-banks and S-banks are fundamentally equally risky, S-banks choose lower leverage\(^{12}\). Required initial equity for C-banks is $p_0 - q^C b^C$. Leverage $b^C$ is a constant fraction of the asset price by the collateral constraint and higher than that of S-banks if $\theta$ is sufficiently small. In equilibrium, the bond price $q^C$ adjusts in order for required initial equity to equal the expected dividend. For reasonable parameter combinations, this in turn means that the marginal benefit of C-bank debt to households must be lower than that of S-bank debt. If debt is further sufficiently substitutable, this means that C-banks must hold a greater share of the intermediated asset in equilibrium\(^{13}\).

Size of the Banking Sectors & Procyclical Shadow Banking Activity  The exact split between both types of banks depends on several parameters, particularly the elasticity of substitution between both kinds of liquidity $\alpha$.

\(^{12}\)This statement of course depends on the parameters of the model, in particular the value of $\theta$, i.e. the tightness of C-banks’ leverage constraint. For any values close to the capital requirements of commercial banks, we found this statement to be true.

\(^{13}\)Even for equal shares of asset holdings ($A^S = A^C$), C-banks will produce $N^C > N^S$ due to their higher leverage. If both types of debt are close to being complements, the higher leverage of C-banks by itself is sufficient to create the lower marginal benefit. Thus the C-bank share is increasing in the elasticity parameter $\alpha$. 

19
The relative size of both banking sectors is determined in equilibrium by the marginal benefit of opening each bank in this period and receiving dividends next period. Mathematically, this means that the holdings of the intermediated asset by both types of banks, $A_C$ and $A_S$, and thus the relative size of both banking sectors, are jointly determined by the FOCs (10) and (11).

**Figure 2:** Equilibrium Determination of $A_C$ and $A_S$ for $\alpha = 1/3$

Left-hand side (required equity at time 0, blue line) and right-hand side (expected dividend at time 1, red line) of banks’ first-order condition for asset holdings ($A_j$), while imposing market clearing $1 = A_C + A_S$ and holding fixed all other variables.

Figure (2) shows the LHS and RHS of both equations graphically for the calibrated model, depending on the current state of the economy. We first numerically compute the equilibrium values of all variables. Then we vary the share of S-banks and C-banks, $A_S$ and $A_C$, while holding all other variables fixed (and imposing market clearing $A_S + A_C = 1$). The blue lines (i.e. the equity funding costs) trace the value of the LHS of the first-order conditions (10) and (11) as we vary the shares, $p_0 - q^j b^j$, for $j = C, S$, respectively. Holding constant $p_0$
and $b^j$, the only source of variation is through the bond price $q^j$. Since total debt issued by each type is given by asset share times leverage per unit of assets, $N^j = b^j A^j$, the marginal benefit of deposits of each type changes as we vary the asset shares, as can be seen from the pricing equations for the $q^j$, (7) and (8). In particular, as we increase the share of type $j$ holding total provided liquidity constant, the marginal liquidity benefit of type $j$’s deposits will decline (for any $\alpha < 1$), and therefore type $j$’s bond price will also decline. This means that the equity required to purchase the bank’s initial asset position becomes larger for the same face amount of debt issued.

An opposing effect is that, as long as liquidity services provided by both types of debt are imperfect substitutes, the marginal benefit derived from each type’s debt is also affected by the composition of the total debt. When we increase the share of C-banks, we decrease the share of S-banks due to market clearing. Consequently the composition of liquidity services becomes more unequal and the amount of services derived from total debt issued by both banks declines, which leads to a general increase in the marginal benefit of both kinds of liquidity. Both effects, the pure effect of an increase in $A^C$ and the equilibrium effect through the implied decrease in $A^S$ can be seen in left panel of (2). Lowering $A^S$ from 0.5 to about 0.15 (= raising $A^C$ from 0.5 to 0.85) causes a decrease in the marginal benefit of C-bank debt, which in turn lowers the bond price $q^C$ and therefore raises the required initial equity of C-banks (the blue line in the graph). By lowering $A^S$ any further, the composition of debt becomes so unequal that the marginal benefit of any liquidity rises again, causing both bond prices (also $q^C$) to increase again. Hence the required equity of C-banks bends backwards, yielding the overall non-monotonic shape.

The effect on the RHS of equation (11) is depicted by the red lines and quantitatively smaller. The discounted expected dividend at time 1 (per unit of assets) is only indirectly affected by a variation in asset shares through the households’ discount factor which depends
on consumption $C_1$. As the share is shifted towards C-banks, consumption $C_1$ decreases since S-banks have lower leverage and cause lower bankruptcy-induced consumption losses to households. This decrease in $C_1$ raises the discount factor, leading to a higher present value of the expected dividend.

The share of shadow banking activity also depends on the economic state. In a boom (solid line), shadow banks can issue debt at a higher price because they are less likely to fail next period. Moreover, because the default probability is lower, the expected dividend payment is larger. This means that shadow banking activity is pro-cyclical.

The overall take-away from the left panel of figure (2) is that the FOC of C-banks is satisfied at two points, the actual equilibrium share of $A^C = 0.80$ ($A^S = 0.20$) and an even higher share of $A^C = 0.95$. The unique equilibrium split is then determined by the FOC of S-banks, depicted in the right panel of the figure. The forces determining the shapes of required equity (blue line) and expected dividend (red line) are the same as for C-banks. With respect to required initial equity, both the isolated effect of lowering $A^S$ on the marginal benefit of S-bank debt, the equilibrium effect through debt composition work in the same direction. Thus the required initial equity of S-banks is strictly decreasing as we lower $A^S$. The unique intersection with the expected discounted dividend of S-banks is at $A^S = 0.20$.

The split between the two types in equilibrium will also depend on other parameters of the model, most importantly the relative quality of C-bank debt compared to S-bank (parameterized by $\nu$).
Figure 3: Equilibrium Determination of $A^C$ and $A^S$ for $\alpha = 0.7$

Left-hand side (required equity at time 0, blue line) and right-hand side (expected dividend at time 1, red line) of banks’ first-order condition for asset holdings ($A^j$), while imposing market clearing $1 = A^C + A^S$ and holding fixed all other variables.

Figure (3) shows the equilibrium split for the identical parameters as in figure (2), but with close to perfectly substitutable debt at $\alpha = 0.7$. We can see that for this parameter combination the marginal benefits of liquidity of both types are less responsive to changes in the relative scale, and only at $A^S$ very close to zero does the marginal benefit of S-bank debt increase sufficiently to make required equity equal to expected dividend. Thus as the elasticity of substitution approaches infinity, S-banks’ share goes to zero.

Moreover, the figure also shows that the model features procyclical shadow banking activity. During booms the aggregate shadow banking failure rate is low, leading to a higher value of shadow banking liquidity services and thus a higher value of shadow banks. Given a $A^S$, the required initial equity to open up a shadow bank is lower, leading to a higher equilibrium
share of shadow banking activity.

**Welfare-maximizing Capital Requirement** The two-period model delivers a qualitative welfare result with respect to the optimal capital requirement for C-banks, $\theta$. The general conditions for this result to obtain are

C1. C-banks are sufficiently risky such that there exists a range for low values of $\theta$ for which some C-banks default, i.e. $F^C < 1$.

C2. S-banks are at least as risky as C-banks; in other words, the standard deviation of their idiosyncratic shocks $\sigma^S$ is at least as large as that of C-banks.

C3. Households derive a strictly positive utility benefit from liquidity services ($\psi > 0$), and the liquidity services provided by C-banks are at least as good as those of S-banks.

Under these fairly general conditions there exists a trade-off in the model that leads to a unique utility maximum in $\theta$. Figure (4) illustrates the trade-off: increasing the capital requirement leads to an increase in numeraire consumption (top right graph), as fewer C-banks default due to lower leverage and hence bankruptcy losses become smaller. At the same time, decreasing C-bank leverage through tighter capital requirements lowers the amount of liquidity services provided to households (bottom left), as a greater share of the intermediated asset is shifted to S-banks. The non-monotonicity in liquidity services occurs at the point where the commercial bank share rebounds. Since total utility is a weighted sum of both components, household utility is maximized when C-banks have become completely safe ($F^C = 1$) and no further increase in consumption is possible (but liquidity decreases further with higher $\theta$).
3 Dynamic Model

This section presents the quantitative general equilibrium model. The main structure is similar to the two-period.

3.1 Model Description

Agents and Environment  Time is discrete and infinite. The agent, intermediary, and endowment ownership structure is identical to the two-period model. The two types of
intermediaries finance \( Z \)-tree investments by issuing equity and debt to households. They have limited liability, i.e., they can choose to declare bankruptcy.

**S-Banks** There is a unit mass of \( S \)-banks, indexed by \( i \). \( S \)-bank \( i \) holds \( A_{t,i}^S \) shares of the \( Z \)-tree at the beginning of period \( t \). The shares trade at a market price of \( p_t \). To fund their investment, \( S \)-banks can issue short term debt. The debt of \( S \)-bank \( i \) trades at the price \( q_{t,i}^S \).

At the beginning of the period, \( S \)-bank \( i \) has \( B_{t,i}^S \) bonds outstanding.

The payoff per share held by bank \( i \) is \( Z_t \). Each period, \( S \)-banks face idiosyncratic valuation risks \( \rho_{t,i}^S \) that are proportional to the market value of their assets. \( \rho_{t,i}^S \) is an idiosyncratic loss with \( \rho_{t,i}^S \sim F_{\rho}^S \), i.i.d. across banks and time. At the beginning of each period, \( S \)-banks can decide to declare bankruptcy. In case of a bankruptcy, banks’ equity is wiped out, and their assets are seized by their creditors. Further, the bank’s managers incur a utility penalty that is a fraction \( \delta \) of the value of the bank’s assets.

Since the idiosyncratic valuation shocks are uncorrelated over time, it is convenient to write the optimization problem of surviving banks after the bankruptcy decision and asset payoffs. All banks have the same value and face identical problems:

\[
\max_{A_{t+1}^S, B_{t+1}^S} q_{t}^S B_{t+1}^S - p_t A_{t+1}^S \\
+ E_t \left[ M_{t,t+1} \max \left\{ (1 - \rho_{t+1}^S) A_{t+1}^S (Z_{t+1} + p_{t+1}) - B_{t+1}^S, -\delta A_{t+1}^S (Z_{t+1} + p_{t+1}) \right\} \right].
\]

We can define book leverage \( b_t^S = \frac{B_t^S}{A_t^S} \) and market leverage

\[
L_t^S = \frac{B_t^S}{A_t^S (Z_t + p_t)} = \frac{b_t^S}{Z_t + p_t}.
\]

to rewrite the maximum operator that is the continuation value as

\[
\max \left\{ (1 - \rho_{t}^S) (Z_t + p_t) - b_t^S, -\delta (Z_t + p_t) \right\} \\
= 1[1 + \delta - L_t^S > \rho_{t}^S] \left\{ (1 - \rho_{t}^S) (Z_t + p_t) - b_t^S \right\} - (1 - 1[1 + \delta - L_t^S > \rho_{t}^S]) \delta (Z_t + p_t).
\]
Taking the expectation of this expression with respect to $\rho_t^S$, one obtains
\[(Z_t + p_t) \left[ F^S_{\rho} (1 + \delta - L_t^S) (1 - L_t^S - \rho_t^{S,-}) - (1 - F^S_{\rho} (1 + \delta - L_t^S)) \delta \right] \equiv (Z_t + p_t) F^S(L_t^S),\]
where $\rho_t^{S,-} = \mathbb{E}(\rho_t^S | \rho_t^S < 1 + \delta - L_t^S)$, and we defined the function
\[F^S(L_t^S) = F^S_{\rho} (1 + \delta - L_t^S) (1 - L_t^S - \rho_t^{S,-}) - (1 - F^S_{\rho} (1 + \delta - L_t^S)) \delta\]
that describes the state-contingent discount of the S-bank’s value due to the default option.

We can now define the per-asset value function
\[v^S(Z_t) = \max_{b_{t+1}^S} q_t^S b_{t+1}^S - p_t + E_t [M_{t,t+1}(Z_{t+1} + p_{t+1}) F^S(L_{t+1}^S)],\]
(16)
such that the full optimization problem of the S-bank is given by
\[\max_{A_{t+1}^S} A_{t+1}^S v^S(Z_t),\]
subject to $A_{t+1}^S \geq 0$.

**C-Banks** There is a unit mass of C-banks. C-banks are different from S-banks in two ways: (i) they issue short-term debt that is insured and risk free from the perspective of creditors, and (ii) they are subject to regulatory capital requirements. The pay an insurance fee of $\kappa$ for each bond they issue. Using the same notation as for S-banks, the problem of all surviving C-banks is identical and given by
\[\max_{A_{t+1}^C} A_{t+1}^C v^C(Z_t),\]
subject to $A_{t+1}^C \geq 0$, and where the per-asset value is given by
\[v^C(Z_t) = \max_{b_{t+1}^C} (q_t^C - \kappa) b_{t+1}^C - p_t + E_t [M_{t,t+1}(Z_{t+1} + p_{t+1}) F^C(L_{t+1}^C)],\]
(17)
such that the equity requirement
\[(1 - \theta) p_t = b_t^C.\]
Bankruptcy The idiosyncratic asset valuation shock is realized at the beginning of each period before any decisions have been made.

If a bank declares bankruptcy, its equity becomes worthless, and creditors seize all of the banks assets, which are liquidated. The recovery amount per bond issued is hence

\[ r^j(b^j_t) = (1 - \xi^j)(Z_t + p_t)(1 - \rho^{j,+})/b^j_t, \]

for \( j = S, C \), with a fraction \( \xi^j \) lost in the bankruptcy proceedings, and with \( \rho^{S,+}_t = E(\rho^S_t | \rho^S_t > 1 + \delta - L^S_t) \). After the bankruptcy proceedings are completed, a new bank is set up to replace the failed one. This bank sells its equity to new owners, and is otherwise identical to a surviving bank after asset payoffs.

If a S-bank defaults, the recovery value per bond is used to pay the claims of bondholders to the extent possible. We further consider the possibility that the government bails out the bond holders of the defaulting S-bank with a probability \( \pi_B \), known to all agents ex ante. If a C-bank declares bankruptcy, the bank is then taken over by the government that uses lump sum taxes and revenues from deposit insurance, \( \kappa B^C_{t+1} \), to pay out the bank’s creditors in full. Summing up over defaulting C-banks and S-banks that are bailed out, lump sum taxes are defined

\[ T_t = (1 - F^C_{\rho,t})(1 - r^C(b^C_t))B^C_t - \kappa B^C_{t+1} + \pi_B(1 - F^S_{\rho,t})(1 - r^S(b^S_t))B^S_t. \]

The beginning-of period dividend paid by banks to households, conditional on survival, for S-banks is

\[ D^S_t = A^S_t (Z_t + p_t) - B^S_t + A^S_{t+1} (q^S_t b^S_{t+1} - p_t), \]

and for C-banks

\[ D^C_t = A^C_t (Z_t + p_t) - B^C_t + A^C_{t+1} ((q^C_t - \kappa)b^C_{t+1} - p_t). \]
Households  Households derive utility from the consumption $C_t$ of the fruit of both trees. Households hold a portfolio of all securities that both types of intermediaries issue. In particular, they buy equity shares of both types of intermediaries, $S_j^t$, that trade at price of $p_j^t$, for $j = S, C$ respectively. They further buy the short terms bonds both types issue, $N_j^t$, trading at prices $q_j^t$, for $j = S, C$.

Households consume the liquidity services provided by the short term debt they hold at the beginning of the period. This reflects that the liquidity services accrue at the time when the deposits from last period are redeemed. Let $N_j^t = \int_0^1 N_{j,t}^i \, di$, for $j = S, C$. Then the total liquidity services produced are

$$H(N^S_t, N^C_t),$$

and household utility in period $t$ is

$$U(C_t, H(N^S_t, N^C_t)).$$

We specify utility as

$$U(C_t, H(N^S_t, N^C_t)) = \frac{C_t^{1-\gamma} - 1}{1 - \gamma} + \psi \frac{H(N^S_t, N^C_t) / C_t}{1 - \eta},$$

with

$$H(N^S_t, N^C_t) = [\Lambda_{S,t} (N^S_t)^\alpha + (N^C_t)^\alpha]^{1/\alpha}.$$

The elasticity of substitution between the two types of bank liabilities is $1/(1 - \alpha)$.

We define the weight on the liquidity services of shadow banks $\Lambda_{S,t}$ as follows:

$$\Lambda_{S,t} = (F_{S,t})^\nu,$$

with $\nu > 0$. The liquidity productivity of shadow banks is lower than that of commercial banks. The discount depends on the fraction of surviving shadow banks, introducing endogenous time-variation in the quality of liquidity services.
Denoting household wealth at the beginning of the period by \( W_t \), the complete intertemporal problem of households is

\[
V^R(W_t, Y_t) = \max_{C_t, N_t^S, N_t^C, S_t^S, S_t^C} U(C_t, H(N_t^S, N_t^C)) + \beta E_t[V(W_{t+1}, Y_{t+1})]
\]

subject to

\[
\begin{align*}
W_t + Y_t - T_t &= C_t + \sum_{j=S,C} p_t^j S_t^j + \sum_{j=S,C} q_t^j N_t^j \\
W_{t+1} &= \sum_{j=S,C} F^j_{\rho}(L_{t+1}^j) \left( D_{t+1}^j + p_{t+1}^j - \rho^j - A_{t+1}^j (Z_{t+1} + p_{t+1}) \right) S_t^j \\
&\quad + N_t^S \left[ F^S_{\rho}(L_{t+1}^S) \left( 1 - F^S_{\rho}(L_{t+1}^S) \right) (\pi_B + (1 - \pi_B) r_{t+1}^S) \right] \\
&\quad + N_t^C.
\end{align*}
\]

The budget constraint in equation (18) shows that households spend their wealth and income on consumption and purchases of equity and debt of both types of intermediaries. The securities issued are the same for all banks, independent of the previous bankruptcy status. The equity purchases for banks that have gone through bankruptcy at the beginning of period \( t \) can be understood as initial equity offerings for these banks, while the purchases of equity of surviving banks are in a secondary market. However, since both new and surviving banks hold identical portfolios, their securities have the same price and there is no need to distinguish primary and secondary markets.\(^{14}\)

\(^{14}\)It is possible to show that the price to an equity claim of bank types \( j, p_t^j \), is equal to the value of that bank’s security portfolio, \( A_{t+1}^j p_t - q_t^j B_{t+1}^j \).
Market Clearing  Asset markets

\[
A_{t+1}^S + A_{t+1}^C = 1 \\
B_{t+1}^S = N_{t}^S \\
B_{t+1}^C = N_{t}^C \\
S_{t}^S = 1 \\
S_{t}^C = 1.
\]

Resource constraint

\[
C_t = Y_t + Z_t - \sum_{j=S,C} A_{t}^j \left[ \mu_{t}^j + \xi^j (Z_t + p_t) (1 - \rho_t^{j+}) (1 - F^j_{t+}) \right].
\]

3.2 Equilibrium Conditions

**Household**  The household’s first-order conditions for purchases of bank equity are, for \( j = S, C, \)

\[
p_t^j = E_t \left[ M_{t,t+1} F^j_{\rho} \left( L_{t+1}^j \right) \left( D_{t+1}^j + p_{t+1}^j - \rho_{t+1}^j \right) \right],
\]

where we have defined the stochastic discount factor

\[
M_{t,t+1} = \beta \frac{U_1 (C_{t+1}, H_{t+1})}{U_1 (C_{t}, H_{t})}.
\]

We further define the intratemporal marginal rate of substitution between consumption and liquidity services

\[
Q_t = \frac{U_2 (C_t, H_t)}{U_1 (C_t, H_t)}.
\]

Then the first-order conditions for purchases of bonds of either type of bank are

\[
q_t^C = Q_t \Lambda_{C,t} \left( \frac{H_t}{N_{t}^C} \right)^{1-\alpha} + E_t \left[ M_{t,t+1} \right], \quad \text{(20)}
\]

\[
q_t^S = Q_t \Lambda_{S,t} \left( \frac{H_t}{N_{t}^S} \right)^{1-\alpha} + E_t \left[ M_{t,t+1} \left[ F^S_{\rho} \left( L_{t+1}^S \right) + (1 - F^S_{\rho} \left( L_{t+1}^S \right)) \left( \pi_B + (1 - \pi_B) r_{t+1}^S \right) \right] \right]. \quad \text{(21)}
\]
The payoff of commercial bank bonds is 1, whereas the payoff of shadow bank bonds depends on their default probability and recovery value. The first terms in each expression represent the marginal benefit of liquidity services to households.

**Banks**  
$S$-banks are subject to an endogenous borrowing constraint. Each $S$-bank is a monopolist for its own debt, and hence internalizes the effect of supplying additional bonds on the bond price.

Specifically, each $S$-bank views the price of its debt as a function of its supply of bonds $q^S_t = q^S(b^S_{t+1})$ that is determined by households’ first order condition in equation 21.

It follows from differentiating equation (16) that the FOC of $S$-banks for leverage is\footnote{Appendix A.2 contains details of the derivations in this section.}

$$q(b^S_{t+1}) + b^S_{t+1} q'(b^S_{t+1}) = E_t \left[ M_{t,t+1} F^S_\rho \left( L^S_{t+1} \right) \right]. \tag{22}$$

The partial derivative $q'(b^S_{t+1})$ can be obtained directly from households’ FOC for purchases of shadow bank debt. Differentiating equation (21) yields

$$q'(b^S_{t+1}) = -E_t \left\{ (1 - \pi_B) \frac{M_{t,t+1}}{b^S_{t+1}} \left[ (1 - F^S_\rho \left( L^S_{t+1} \right)) r^S_{t+1} + f^S_\rho \left( L^S_{t+1} \right) ((1 - \xi_S) + \xi_S L^S_{t+1}) \right] \right\}. \tag{23}$$

The RHS is strictly negative, implying that the price of shadow bank debt is decreasing in shadow bank leverage $b^S_{t+1}$. The first term reflects that the recovery value per bond in case of bankruptcy decreases if the shadow bank issues more debt. The second term is the loss for lenders from a marginal increase in the probability of default.

By combining equations (22) and (23), and substituting for the bond price from households’ first-order condition (21), we get the following condition for $S$-bank leverage $L^S_{t+1}$

$$Q_t A_t \left( \frac{H_t}{N^S_t} \right)^{1-\alpha} = E_t \left\{ M_{t,t+1} \left[ (1 - \pi_B) f^S_{t+1} \left( (1 - \xi_S) + \xi_S L^S_{t+1} \right) - \pi_B \left( 1 - F^S_\rho \left( L^S_{t+1} \right) \right) \right] \right\}.$$
The debt price of commercial banks is independent of their leverage choice. Therefore the FOC of $C$-banks for leverage is

$$q_t^C - \kappa = \lambda_t^C + \mathbb{E}_t \left[ M_{t,t+1} F_{C,t+1}^C \right].$$

with $\lambda_t^C$ being the Lagrange multiplier on the leverage constraint. This FOC and the household FOC for purchases of commercial bank debt (20) jointly imply that the Lagrange multiplier is positive and hence the $C-$bank leverage constraint is binding, i.e.

$$(1 - \theta)p_t = b_t^C.$$

To be precise, the condition\textsuperscript{16} for a binding leverage requirement is

$$\lambda_t^C = MRS_{C,t} + \mathbb{E}_t \left[ M_{t,t+1} \left( 1 - F_{C,t+1}^C \right) \right] - \kappa > 0.$$

The first term is the marginal value of liquidity services derived from commercial bank deposits. As long as households are not satiated with liquidity, the marginal value is positive. The second term is the expected discounted value in case of default of carrying one unit of resources into the next period, which is positive as long as commercial banks default with some probability $F_{C,t+1}^C < 1$. The last term is the deposit insurance fee. Taken together, this means that a small enough $\kappa$ implies that the leverage requirement will always bind. This is the case for the parametrization we consider\textsuperscript{17}.

The first-order conditions for asset purchases $A_{t+1}^j$ follow from the constant returns to scale (i.e. zero-profit) nature of each type’s problem, requiring $v^S(Z_t) = 0$ and $v^C(Z_t) = 0$.

\textsuperscript{16}This condition is derived from combining the FOC of commercial banks with respect to leverage with household’s FOC with respect to debt.

\textsuperscript{17}Conversely, if a regulator could set

$$\kappa > MRS_{C,t} + \mathbb{E}_t \left[ M_{t,t+1} \left( 1 - F_{C,t+1}^C \right) \right] \forall t,$$

commercial banks may never choose leverage at the regulatory limit.
respectively, which yields

\[ p_t = q_t^S b_{t+1}^S + E_t \left[ M_{t,t+1}(Z_{t+1} + p_{t+1})F^S(L_{t+1}^S) \right], \]
\[ p_t = (q_t^C - \kappa) b_{t+1}^C + E_t \left[ M_{t,t+1}(Z_{t+1} + p_{t+1})F^C(L_{t+1}^C) \right]. \]

4 Results

We are now ready to study the properties of the model and its implication for optimal regulation.

4.1 Parametrization

We match the model to quarterly data.

The stochastic process for the Y-tree is a AR(1) in logs

\[ \log(Y_{t+1}) = (1 - \rho_Y)\log(\mu_Y) + \rho_Y\log(Y_t) + \epsilon_Y^{t+1}, \]

where \( \epsilon_Y^{t} \) is i.i.d. with mean zero and volatility \( \sigma_Y \). To capture the correlation of asset payoffs with fundamental income shocks, we model the payoff of the intermediated asset as

\[ Z_t = \phi^Y Y_t \exp(\epsilon_Z^{t}), \]

where \( \epsilon_Z^{t} \) is i.i.d. with mean zero and volatility \( \sigma_Z \), independent of \( \epsilon_Y^{t} \). This structure of the shocks implies that \( Z_t \) inherits all stochastic properties of aggregate income \( Y_t \), and is subject to an additional temporary shock that reflects risks specific to intermediated assets, such as credit risk.

We quantify the model with data from the Flow of Funds\textsuperscript{18}, Compustat, and NIPA. We use quarterly data from 1999 (after the passage of the Gramm-Leach-Bliley Act that revoked

\textsuperscript{18}The Flow of Funds tables are organized according to institutions and instruments. We focus on the balance sheet information on institutions. This is important, as we want to take into account all bank and shadow-bank positions when we quantify the model.
parts of the Glass-Steagall Act) until the second quarter of 2015. We choose depository institutions as data counterparts for $C$—banks and shadow bank institutions as data counterparts for $S$—banks. Shadow banks are defined on their asset side as security broker and dealer, finance companies, asset-backed security issuers and so on and on their liability side as money market mutual funds.

For the parameters of the shock process we use Flow of Funds tables S.1 and S.6 to obtain a time series for financial sector value added and GDP which we deflate and express in GDP per capita using NIPA data. Normalizing the mean of real per capita GDP growth process to $\mu_Y = 0.5$ we can derive $\sigma_Y$ and $\rho_Y$ from the observed volatility and autocorrelation. Similarly, using the observed volatility in the real per capital financial sector value added time series we find a value for $\sigma_Z$.

**Table 1: Parametrization**

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Function</th>
<th>Value</th>
<th>Target</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>discount rate</td>
<td>0.99</td>
<td>qrt. Tbill</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>HH risk aversion parameter</td>
<td>2</td>
<td>macro literature</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>elasticity NS vs NC</td>
<td>0.65</td>
<td>$S$—bank share / Gallin FOF 1/3</td>
</tr>
<tr>
<td>$\psi$</td>
<td>utility weight on safe assets</td>
<td>0.6</td>
<td>safe-asset share</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>deposit insurance fee</td>
<td>0.0006168</td>
<td>deposit ins. rates (25 bp p.a)</td>
</tr>
<tr>
<td>$\theta$</td>
<td>$C$— bank capital req.</td>
<td>0.10</td>
<td>average Tier 1 cap ratio</td>
</tr>
<tr>
<td>$\pi_B$</td>
<td>S-banks bailout prob.</td>
<td>0.75</td>
<td>spread on $C$— &amp; $S$—bank debt</td>
</tr>
<tr>
<td>$\xi_C$</td>
<td>bankruptcy loss</td>
<td>0.37</td>
<td>recovery rate (37%)</td>
</tr>
<tr>
<td>$\xi_S$</td>
<td>bankruptcy loss</td>
<td>0.37</td>
<td>recovery rate (37%)</td>
</tr>
<tr>
<td>$\eta$</td>
<td>compl. bw cons &amp; safe assets</td>
<td>2</td>
<td>FoF corr(liquid assets, consumption)</td>
</tr>
<tr>
<td>$\delta_C$</td>
<td>default penalty</td>
<td>0.49</td>
<td>C-bank profitability dispersion</td>
</tr>
<tr>
<td>$\delta_S$</td>
<td>default penalty</td>
<td>0.25</td>
<td>value weighted leverage of shadow banks</td>
</tr>
<tr>
<td>$\sigma_C$</td>
<td>vol of $\rho$ shock</td>
<td>0.225</td>
<td>FDIC default rate 0.04 p.a. %</td>
</tr>
<tr>
<td>$\sigma_S$</td>
<td>vol of $\rho$ shock</td>
<td>0.225</td>
<td>Moody’s financial inst. default rate</td>
</tr>
<tr>
<td>$\mu_Y$</td>
<td>mean of $Y$</td>
<td>0.5</td>
<td>normalization</td>
</tr>
<tr>
<td>$\rho_Y$</td>
<td>persistence of $Y$ growth</td>
<td>0.75</td>
<td>normalization</td>
</tr>
<tr>
<td>$\sigma_Y$</td>
<td>vol of $Y$ shock</td>
<td>0.007</td>
<td>agg. GDP growth vol</td>
</tr>
<tr>
<td>$\phi_Y$</td>
<td>relative scale of $Z$</td>
<td>0.08</td>
<td>8% fin. sec. value added relative to GDP</td>
</tr>
<tr>
<td>$\sigma_Z$</td>
<td>vol of $Z$ shock</td>
<td>0.056</td>
<td>FOF fin. sec. value added vol</td>
</tr>
</tbody>
</table>
The key parameters are those that determine households’ utility for liquidity services. These are the elasticity of substitution of HH \( (\alpha = 1 - 1/\text{elasticity}) \) between the two liquidity services provided by the two types of banking sector liabilities, the weight on liquidity services \( \psi \), and the curvature parameter \( \eta \).

To fix ideas, let’s think of commercial bank debt as deposits, and shadow bank debt as money market mutual fund shares. Deposits and money market mutual fund shares are similar in terms of risk and liquidity and hence produce liquidity services but they are not necessarily perfectly substitutable. For example, deposits can be used as a medium of exchange while money market fund shares can mostly not. In the model the degree of substitutability between these two securities is governed by \( \alpha \), the elasticity of substitution parameter. The degree of substitutability pins down the size of commercial banks relative to shadow banks. We use the estimate of the share of shadow banking intermediation developed by Gallin (2013) as a target for \( \alpha \). More precisely, Gallin (2013) used data from the Flow of Funds to carefully trace back how much shadow banking sector funding the real economy received. Since many shadow banks fund each other and not necessarily real activity the actual share of shadow activity is much lower (around 33%) than what one would expect given the total asset size of the financial sector.

The utility weight on liquidity services \( \psi \) affects how valuable liquidity services are relative to consumption. We use the safe-asset share estimate by Gorton, Lewellen, and Metrick (2012) as a target for \( \psi \).

The last key parameter is \( \eta \) that determines the curvature of the liquidity preferences relative to consumption. As such, it determines whether consumption and liquidity services are complements or substitutes. We choose a fairly conservative value of 2 that implies that consumption and liquidity services are complements and still allows for fairly high variability in their ratio.
We choose $\delta_S$ to match the market value leverage of shadow banks in Compustat. That is, we compute the value weighted market leverage of publicly traded shadow banks\(^{19}\) as total debt over the market value of assets weighted by the relative market value of each institution and average across time and banks. The result is a market value weighted leverage ratio of 0.65.

We set $\kappa$, the deposit insurance fee to 0.0006168. This is in the range of quarterly FDIC assessment rates. The parameter $\xi^S$ (loss in bankruptcy) is set to match an average recovery value of 37% percent from Moody’s report.\(^{20}\) We pick the parameters $\xi$ to match a recovery rate of 37%. The values for $\sigma^C$ and $\sigma^S$ are chosen in order to match the default probabilities of FDIC and financial institutions in the data. We set $\sigma^C$ such that the quarterly probability of default equals 1.6%. We set $\sigma^S$ such that we match an annual default probability of 2.4%.

4.2 Benchmark Economy

We solve the nonlinear model dynamic model for the benchmark level of $\theta$. We then simulate it for 2,000 periods and compute moments of the simulated series. Table (2) reports means, standard deviations, business cycle correlations (correlations with $Y$), correlations with lagged $Y$, correlations with $Z$ and lagged $Z$ as well as autocorrelations.

The first set of variables are the exogenous states. The second set comprise the intermediary asset price, the interest rates on intermediary debt and the excess returns on intermediary equity. The third set of variables affect households’ liquidity demand, i.e. shadow banks’ liquidity quality factor and the marginal rate of substitution between liquidity and consumption. The fourth set describe intermediary debt, balance sheet, and leverage. The fifth set

\(^{19}\)We define shadow banks as all institutions with SIC code 6111, 6141, 6153, 6159, 6162, 6163, 6172, 6211, and 6798.

\(^{20}\)We use Moody’s 1984-2004. Exhibit 9 in the report presents the recovery rates of defaulted bond for financial institutions. We use the mean for financial institutions over all bonds and preferred stocks.
Table 2: This table presents moments of key variables of the simulated benchmark economy ($\theta = 10\%$).

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>D</th>
<th>Corr Y</th>
<th>Corr Y(t-1)</th>
<th>Corr Z</th>
<th>Corr Z(t-1)</th>
<th>AC</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Y$</td>
<td>0.500</td>
<td>0.004</td>
<td>1.000</td>
<td>0.742</td>
<td>0.060</td>
<td>0.059</td>
<td>0.742</td>
</tr>
<tr>
<td>$Z$</td>
<td>0.040</td>
<td>0.003</td>
<td>0.060</td>
<td>0.033</td>
<td>1.000</td>
<td>-0.006</td>
<td>-0.006</td>
</tr>
<tr>
<td>$A$ Asset price</td>
<td>5.335</td>
<td>0.085</td>
<td>0.638</td>
<td>0.443</td>
<td>0.803</td>
<td>-0.001</td>
<td>0.263</td>
</tr>
<tr>
<td>Deposit rate $S$</td>
<td>0.006</td>
<td>0.014</td>
<td>-0.337</td>
<td>-0.213</td>
<td>-0.957</td>
<td>0.021</td>
<td>0.040</td>
</tr>
<tr>
<td>Deposit rate $C$</td>
<td>0.006</td>
<td>0.013</td>
<td>-0.337</td>
<td>-0.213</td>
<td>-0.957</td>
<td>0.021</td>
<td>0.039</td>
</tr>
<tr>
<td>EER Equity $S$</td>
<td>0.011</td>
<td>0.013</td>
<td>-0.280</td>
<td>-0.149</td>
<td>-0.968</td>
<td>0.082</td>
<td>-0.046</td>
</tr>
<tr>
<td>EER Equity $C$</td>
<td>0.011</td>
<td>0.014</td>
<td>-0.290</td>
<td>-0.156</td>
<td>-0.966</td>
<td>0.080</td>
<td>-0.040</td>
</tr>
<tr>
<td>$\Lambda^S$</td>
<td>0.994</td>
<td>0.001</td>
<td>0.379</td>
<td>0.058</td>
<td>0.884</td>
<td>-0.041</td>
<td>-0.019</td>
</tr>
<tr>
<td>$MRS_S$</td>
<td>0.004</td>
<td>0.000</td>
<td>0.492</td>
<td>0.319</td>
<td>0.892</td>
<td>-0.053</td>
<td>0.093</td>
</tr>
<tr>
<td>$MRS_C$</td>
<td>0.003</td>
<td>0.000</td>
<td>0.017</td>
<td>0.022</td>
<td>-0.970</td>
<td>-0.048</td>
<td>0.068</td>
</tr>
<tr>
<td>Debt $S$</td>
<td>1.396</td>
<td>0.061</td>
<td>-0.178</td>
<td>-0.219</td>
<td>0.003</td>
<td>-0.983</td>
<td>-0.008</td>
</tr>
<tr>
<td>Debt $C$</td>
<td>2.963</td>
<td>0.130</td>
<td>0.352</td>
<td>0.456</td>
<td>0.005</td>
<td>0.911</td>
<td>0.110</td>
</tr>
<tr>
<td>$S$ Share</td>
<td>0.383</td>
<td>0.017</td>
<td>-0.269</td>
<td>-0.343</td>
<td>-0.001</td>
<td>-0.954</td>
<td>0.043</td>
</tr>
<tr>
<td>$S$ Assets</td>
<td>2.046</td>
<td>0.095</td>
<td>-0.067</td>
<td>-0.230</td>
<td>0.244</td>
<td>-0.937</td>
<td>-0.243</td>
</tr>
<tr>
<td>$C$ Assets</td>
<td>3.286</td>
<td>0.117</td>
<td>0.543</td>
<td>0.510</td>
<td>0.415</td>
<td>0.806</td>
<td>0.513</td>
</tr>
<tr>
<td>$S$ Leverage</td>
<td>0.683</td>
<td>0.010</td>
<td>-0.393</td>
<td>-0.066</td>
<td>-0.880</td>
<td>0.044</td>
<td>-0.018</td>
</tr>
<tr>
<td>$C$ Leverage</td>
<td>0.900</td>
<td>0.017</td>
<td>-0.125</td>
<td>0.161</td>
<td>-0.650</td>
<td>0.661</td>
<td>-0.464</td>
</tr>
<tr>
<td>Liquidity</td>
<td>6.162</td>
<td>0.075</td>
<td>0.727</td>
<td>0.506</td>
<td>0.723</td>
<td>0.006</td>
<td>0.353</td>
</tr>
<tr>
<td>Consumption</td>
<td>0.537</td>
<td>0.005</td>
<td>0.780</td>
<td>0.540</td>
<td>0.666</td>
<td>-0.020</td>
<td>0.388</td>
</tr>
<tr>
<td>$DWLS$</td>
<td>0.002</td>
<td>0.000</td>
<td>-0.420</td>
<td>-0.123</td>
<td>-0.848</td>
<td>-0.262</td>
<td>0.260</td>
</tr>
<tr>
<td>$DWLC$</td>
<td>0.002</td>
<td>0.000</td>
<td>-0.048</td>
<td>0.219</td>
<td>-0.581</td>
<td>0.713</td>
<td>-0.439</td>
</tr>
<tr>
<td>$F^S$</td>
<td>0.994</td>
<td>0.001</td>
<td>0.379</td>
<td>0.058</td>
<td>0.884</td>
<td>-0.041</td>
<td>-0.019</td>
</tr>
<tr>
<td>$F^C$</td>
<td>0.996</td>
<td>0.001</td>
<td>0.115</td>
<td>-0.166</td>
<td>0.653</td>
<td>-0.648</td>
<td>-0.454</td>
</tr>
</tbody>
</table>
are the utility components (liquidity services and consumption). The last set of variables are banks’ deadweight loss and survival probabilities.

The price for the intermediated asset moves together with $Y$ and $Z$. Good times make intermediation more valuable. Co-movements with $Z$ are not surprising, while co-movements with $Y$ are induced by the complementarity between consumption and liquidity on the demand side and the loosening of the leverage constraint (regulatory and endogenous respectively) on the supply side. The deposit rates for commercial and shadow banks are countercyclical since the marginal benefit of liquidity is higher in booms. The return on equity for both types of banks is of similar magnitude – the expected excess return for both types is 1.1 percentage points. Both bank’s equity returns contain countercyclical risk premia.

The liquidity quality factor $\Lambda^S$ is decisively procyclical as shadow banks’ survive with higher likelihood in good times. This leads to strong co-movements of $MRS_S$ with GDP. From a liquidity demand side perspective, an acyclical $MRS_C$ implies that households are effective in smoothing both the marginal utility of consumption and C-banks’ liquidity services. On the supply side, commercial banks are more effective in ramping up debt when their leverage constraint is relaxed by high asset prices during booms.

The model captures pro-cyclical book-leverage of commercial banks and counter-cyclical market value leverage for both banks. The balance sheet of commercial banks moves strongly with $Y$, a reflection of a relaxed leverage constraint in good states.\(^{21}\) Because the endogenous borrowing constraint disciplines shadow banks with regard to their ability to lever up, shadow banks cannot expand their balance sheets as easily despite an increased marginal utility of shadow bank liquidity services. As a consequence, shadow banks own a smaller share of the intermediated assets in good times relative to commercial banks. Since good times lift the

\(^{21}\)We designed the leverage constraint to reflect a Basel II type of capital requirement that has been criticized for its procyclical effects.
value of all banks’ assets the market value of shadow banks’ asset holdings remains stable over the cycle.

Overall, the financial sector is more stable in good economic times as default risk is lower. On the flipside, deadweight losses due to intermediary bankruptcy are higher in bad states of the economy, in particular those bad states that affect the financial sector most (low Z payoffs).

4.3 Welfare

As for the two-period model, there is a unique maximum in aggregate welfare. The welfare maximum for the quantitative model occurs in the interval around 15 percent. The main forces leading to this result are the same as in the simple model. Increasing $\theta$ makes the debt of both types of banks safer, thus reducing bankruptcy losses and increasing aggregate consumption. At the same time, a higher level of $\theta$ restricts the amount of liabilities and thus liquidity commercial banks can produce for each unit of assets.

**Changing the capital requirement**  The second panel of (3) shows that an increase in the capital requirement shifts a greater fraction of the intermediated asset to shadow banks. Restricting the liquidity production by commercial banks increases the marginal benefit of liquidity to households. Shadow banks take advantage of the lower funding costs by increasing their demand for intermediated assets.

Since shadow banks have a higher marginal valuation of the asset than commercial banks, the price of the Z-asset rises (by roughly 1%) when the capital requirement is increased from 10% to 20%. This is the opposite effect one would get in a model with only commercial banks. In such a model, raising the capital requirement would lower the collateral value of the intermediated asset and reduce the price. Shadow banks do not increase their leverage in
Table 3: Model Moments for Different Capital Requirements

<table>
<thead>
<tr>
<th></th>
<th>$\theta = 0.1$</th>
<th>$\theta = 0.15$</th>
<th>$\theta = 0.2$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>M</td>
<td>SD</td>
<td>M</td>
</tr>
<tr>
<td>$Z$ Asset price</td>
<td>5.332</td>
<td>0.087</td>
<td>5.346</td>
</tr>
<tr>
<td>Deposit rate $S$</td>
<td>0.007</td>
<td>0.014</td>
<td>0.007</td>
</tr>
<tr>
<td>Deposit rate $C$</td>
<td>0.007</td>
<td>0.014</td>
<td>0.007</td>
</tr>
<tr>
<td>EER Equity $S$</td>
<td>0.012</td>
<td>0.014</td>
<td>0.011</td>
</tr>
<tr>
<td>EER Equity $C$</td>
<td>0.011</td>
<td>0.014</td>
<td>0.012</td>
</tr>
<tr>
<td>$\Lambda^S$</td>
<td>0.994</td>
<td>0.001</td>
<td>0.994</td>
</tr>
<tr>
<td>$S^\text{Debt}$</td>
<td>1.398</td>
<td>0.062</td>
<td>1.596</td>
</tr>
<tr>
<td>$C^\text{Debt}$</td>
<td>2.959</td>
<td>0.132</td>
<td>2.562</td>
</tr>
<tr>
<td>$S^\text{Share}$</td>
<td>0.384</td>
<td>0.018</td>
<td>0.436</td>
</tr>
<tr>
<td>$S^\text{Leverage}$</td>
<td>0.684</td>
<td>0.010</td>
<td>0.684</td>
</tr>
<tr>
<td>$C^\text{Leverage}$</td>
<td>0.900</td>
<td>0.018</td>
<td>0.850</td>
</tr>
<tr>
<td>$S^\text{Assets}$</td>
<td>2.046</td>
<td>0.095</td>
<td>2.332</td>
</tr>
<tr>
<td>$C^\text{Assets}$</td>
<td>3.286</td>
<td>0.117</td>
<td>3.014</td>
</tr>
<tr>
<td>Liquidity</td>
<td>6.159</td>
<td>0.077</td>
<td>5.959</td>
</tr>
<tr>
<td>Consumption</td>
<td>0.537</td>
<td>0.005</td>
<td>0.537</td>
</tr>
<tr>
<td>$\text{DWL}^S$</td>
<td>0.002</td>
<td>0.000</td>
<td>0.002</td>
</tr>
<tr>
<td>$\text{DWL}^C$</td>
<td>0.002</td>
<td>0.001</td>
<td>0.001</td>
</tr>
<tr>
<td>$F^S$</td>
<td>0.994</td>
<td>0.001</td>
<td>0.994</td>
</tr>
<tr>
<td>$F^C$</td>
<td>0.996</td>
<td>0.001</td>
<td>0.998</td>
</tr>
<tr>
<td>Welfare</td>
<td>-191.672</td>
<td>0.03%</td>
<td>-0.02%</td>
</tr>
</tbody>
</table>

The table unconditional means and standard deviations of the main outcome variables from a 1,000 period simulation of three different models with different capital requirements.

$^a$: Welfare is the percentage change of mean and volatility of the household value function relative to the benchmark model with $\theta = 0.1$. 

41
response to the scaling back of commercial banks. Rather, they keep their leverage roughly constant. Hence overall liquidity production falls monotonically as long as the maximum leverage allowed by regulation for $C$-banks leverage is higher than $S$-banks’ optimal leverage.

Both types of banks become safer as in the two-period model when $\theta$ is raised. In the dynamic model increasing the requirement yields the additional benefit of slightly lower consumption volatility. For the risk-averse household, this leads to a further utility gain, albeit a small one. It is more than offset by an increase in the volatility of liquidity provision. Therefore, increasing the capital requirement leads to both fewer and more volatile liquidity provision. The result is that total welfare already becomes decreasing in the capital requirement before all bankruptcy losses are eliminated; in other words, our calibrated model says that it is not optimal to make commercial banks completely safe.

5 Conclusion

This paper proposes a novel model to study the consequences of higher bank capital requirements for the economy. The optimal level of capital regulation trades-off a reduction in liquidity services against an increase in the safety of the banking system and consumption. Increasing the capital requirement for regulated banks leads to more intermediation activity by the shadow banking system and thus a higher valuation for the intermediated asset because households’ substitute shadow bank liquidity for commercial bank liquidity. This effect is non-monotonic as shadow banks’ endogenous borrowing constraint restricts their ability to provide liquidity services. Moreover, a higher capital requirement makes both bank types safer: they affect commercial banks directly through a mandated reduction in leverage while they make shadow banks indirectly safer through their effect on the intermediated asset valuation.
References


Elenev, V., T. Landvoigt, and S. Van Nieuwerburgh (2015): “Phasing out the GSEs,”


A Appendix

A.1 Two-period model

Preliminaries  Since \( \rho^j_i \) is uniformly distributed, the probability that bank \( i \) with assets \( A^j_i \) and debt \( B^j_i \) stays in business is given by the probability that \( \rho^j_i \leq Z - \frac{B^j_i}{A^j_i} \), i.e.

\[
F^j_\rho \left( Z - \frac{B^j_i}{A^j_i} \right) = \frac{Z - B^j_i - \rho^j_i}{\rho^j_i - \rho^j}.
\]  

(24)

Furthermore, \( \rho^-_j \) is the expected value of \( \rho^j_i \) conditional on \( j \)-bank’s survival and a realization of the random variable \( Z \):

\[
\rho^-_j = E[\rho^j_i | \rho^j < Z - b^j] = \frac{1}{2} \left( Z - b^j + \rho^j \right).
\]  

(25)

The expected value of \( \rho \) conditional on \( j \)-bank’s failure is

\[
\rho^+_j = E[\rho^j_i | \rho^j > Z - b^j] = \frac{1}{2} \left( Z - b^j + \rho^j \right).
\]  

(26)

First-order conditions with respect to \( A^j \)  The first-order condition for holdings of the intermediated asset for both types of banks is

\[
v^j(b^j) = 0,
\]

which implies

\[
q^j b^j - p_0 = E_0 \left[ M \left( F^j_\rho (Z - b^j - \rho^-) \right) \right]
\]

using the expressions for \( v^j(b^j) \) defined in equations (4) and (2) for \( j = C, S \), respectively. We can now substitute the expressions for \( F^j_\rho \) given in equation (24) and for \( \rho^-_j \) given in equation (25) into the condition above, which yields

\[
q^j b^j - p_0 = E_0 \left[ \frac{M}{2(\rho^j - \rho^j)} (Z - b^j - \rho^j)^2 \right].
\]
Using once more the definition of $F^j_\rho$ this becomes

$$q^j b^j - p_0 = E_0 \left[ \frac{M(\overline{\rho}^j - \rho^j)}{2} \left( F^j_\rho \right)^2 \right].$$

Recognizing that the standard deviation of a uniform random variable with support $[\rho^j, \overline{\rho}^j]$ is given by

$$\sigma^j = \frac{\overline{\rho}^j - \rho^j}{\sqrt{12}}$$

and plugging in gives the result in equations (10) and (11), respectively.

**First-order condition for $b^C$** Starting with the definition of $v^C(b^C)$ in equation (4), and substituting the expressions for $F^C_\rho$ and $\rho^-_C$ given in equations (24) and (25), we get

$$v^C(b^C) = (q^C - \kappa)b^C - p_0 - E_0 \left[ \frac{M}{2(\overline{\rho}^C - \rho^-_C)} \left( Z - b^C - \rho^-_C \right)^2 \right].$$

Taking the derivative with respect to $b^C$ (including the leverage constraint $b^C \leq (1 - \theta)p_0$) then yields

$$q^C - \kappa = \lambda^C + E_0 \left[ M \left( \frac{Z - b^C - \rho^-_C}{\overline{\rho}^j - \rho^j} \right) \right],$$

which gives the result in equation (12).

**First-order condition for $b^S$** To derive the FOC of the S-bank, it is helpful to first rewrite several expressions. As for the C-bank, we can write the continuation value as

$$F^S(Z - b^S - \rho^-_S) = \frac{1}{2(\overline{\rho}^S - \rho^-_S)}(Z - b^S - \rho^-_S)^2.$$

Furthermore, noting that $Z - \rho^-_S = \frac{1}{2}(Z + b^S - \overline{\rho}^S)$, we get for the recovery value on the bonds of bankrupt S-banks

$$(1 - F^S) r^S = \frac{1 - \xi_S}{2(\overline{\rho}^S - \rho^-_S)} \frac{(b^S)^2 - (Z - \overline{\rho}^S)^2}{b^S}.$$
Using these expressions, we can rewrite HH’s FOC for $N^S$ (see equation 8) as

$$q^S = E_0 \left\{ M_1 \frac{1}{(p^S - \rho^S)} \left[ Z - b^S - \rho^S + 0.5 (1 - \xi_S) \frac{(b^S)^2 - (Z - \rho^S)^2}{b^S} + MRS^S \right] \right\}.$$ 

Taking the derivative of this equation with respect to $b^S$

$$\frac{\partial q^S(b^S)}{\partial b^S} = -E_0 \left\{ M_1 \left( \frac{\xi_S}{p^S - \rho^S} + (1 - F^S) \frac{r^S}{b^S} \right) \right\}. \quad (27)$$

We can now derive the S-bank’s FOC with respect to its leverage ratio $b^S$

$$q^S + b^S \frac{\partial q^S(b^S)}{\partial b^S} = E_0 \left[ M_1 \frac{Z - b^S - \rho^S}{p^S - \rho^S} \right].$$

Plugging in from equation (27), and noting that the last term on the right-hand side equal the survival probability $F^S$

$$q^S - E_0 M_1 \left[ \frac{b^S \xi_S}{p^S - \rho^S} + (1 - F^S) r^S \right] = E_0 M_1 F^S,$$

which can be rearranged to yield

$$q^S - E_0 M_1 [F^S + (1 - F^S) r^S] = E_0 M_1 \frac{1}{p^S - \rho^S} \xi_S b^S.$$

Using the HH’s FOC for shadow bank debt (8), we can substitute for the difference on the LHS

$$E_0 \left[ M_1 (1 - \nu) \frac{U_1}{U_2} \left( \frac{H}{N^S} \right)^{1 - a} \right] = E_0 \left[ M_1 \frac{1}{p^S - \rho^S} \xi_S b^S \right],$$

and solve for $b^S$, which yields the expression in equation (14).

**Equations and Solution Method** The equilibrium of the economy needs to be computed numerically. The equilibrium can be reduced to a system of four nonlinear equations in four unknowns. We numerically find a unique solution for every parameter combination we have tried. The four variables are $(p_0, A^S, C_1, b^S)$. Using the variables, we can first compute the C-bank leverage ratio as

$$b^C = (1 - \theta)p_0$$
Using the market clearing conditions $N^S = b^S A^S$, $N^C = A^C b^C (1 - (1 - F^C)(1 - r^C) + \kappa)$ and $A^C = 1 - A^S$, we can get the two bond prices $(q^S, q^C)$ from the HH FOCs for deposits. Then the four equations are

\[
\begin{align*}
    p_0 &= (q^C - \kappa)b^C + E_0 \left( M_1 \frac{1}{4\sigma_2\sqrt{3}}(Z - b^C - \rho^C)^2 \right) \\
    p_0 &= q^S b^S + E_0 \left( M_1 \frac{1}{4\sigma_2\sqrt{3}}(Z - b^S - \rho^S)^2 \right) \\
    C_1 &= Y_1 + \sum_j D^j_t + N^C + N^S(F^S + (1 - F^S)r^s) \\
    b^S &= \frac{E_0 (M_1 MRS^S)}{E_0 (M_1)} \sqrt{12\sigma^S} / \xi^S.
\end{align*}
\]

### A.2 Dynamic Model

#### A.2.1 First-order conditions with respect to $b^S_t$

Differentiating equation 16 with respect to $b^S_t$ gives

\[
q(b^S_{t+1}) + b^S_{t+1} q'(b^S_{t+1}) = E_t \left[ M_{t+1} (Z_{t+1} + p_{t+1}) \frac{\partial F^S(L^S_{t+1})}{\partial b^S_{t+1}} \right].
\]

First, rewrite

\[
F^S(L^S_{t+1}) = F^S_\rho(1 + \delta^S - L^S_{t+1})(1 - L^S_{t+1}) - (1 - F^S_\rho(1 + \delta^S - L^S_{t+1})) \delta^S - \int_{-\infty}^{1+\delta^S-L^S_{t+1}} \rho f^S_\rho(\rho) d\rho.
\]

Now compute

\[
\frac{\partial F^S(L^S_{t+1})}{\partial L^S_{t+1}} = -f^S_\rho(1 + \delta^S - L^S_{t+1})(1 - L^S_{t+1}) - F^S_\rho(1 + \delta^S - L^S_{t+1})
\]

\[
- f^S_\rho(1 + \delta^S - L^S_{t+1}) \delta^S + f^S_\rho(1 + \delta^S - L^S_{t+1})(1 + \delta^S - L^S_{t+1})
\]

\[
= - F^S_\rho(1 + \delta^S - L^S_{t+1}),
\]

which combined with the fact that

\[
\frac{\partial L^S_{t+1}}{\partial b^S_t} = \frac{1}{Z_t + p_t},
\]

yields the result in equation (22).
A.2.2 Derivation of \((q^S)'(b^S_t)\)

Recall the definition of the recovery value for S-banks as

\[
r^S_t(b^S_t) = (1 - \xi^S_t)(Z_t + p_t)(1 - \rho^{S,+}_t/b^S_t),
\]

with the conditional expectation

\[
\rho^{S,+}_t = \mathbb{E}_{\rho,S}[\rho | \rho > 1 + \delta - L^S_t].
\]

We can therefore rewrite the recovery value times the probability of default as

\[
(1 - F^S_t)r^S_t = \frac{(1 - \xi^S_t)(1 - F^S_t)(Z_t + p_t)}{b^S_t} - \frac{(1 - \xi^S_t)(Z_t + p_t)}{b^S_t} \int_{1+\delta-L^S_t}^{\infty} \rho \, dF_{\rho,S}(\rho).
\]

Differentiating this expression with respect to \(b^S_t\) gives

\[
\frac{\partial}{\partial b^S_t} (1 - F^S_t)r^S_t = -\frac{(1 - \xi^S_t)(1 - F^S_t)(Z_t + p_t)}{(b^S_t)^2} + \frac{f^S_t}{b^S_t} (1 - \xi^S_t) + \frac{(1 - \xi^S_t)(1 - F^S_t)(Z_t + p_t)\rho^{S,+}_t}{(b^S_t)^2}
\]

\[
- \frac{f^S_t}{b^S_t} (1 - \xi^S_t)(1 + \delta - L^S_t)
\]

\[
= -\frac{(1 - F^S_t)r^S_t}{b^S_t} - \frac{f^S_t}{b^S_t} (1 - \xi^S_t)(\delta - L^S_t),
\]

where we used the definition of \(L^S_t = b^S_t/(Z_t + p_t)\). The complete derivative of the household’s bond pricing equation for \(q^S(21)\) with respect to \(b^S_{t+1}\) is therefore

\[
\frac{\partial q^S}{\partial b^S_{t+1}} = -\mathbb{E}_t \left\{ (1 - \pi_B) M_{t,t+1} \left[ \frac{f^S_{t+1}}{b^S_{t+1}} \frac{1 + (1 - F^S_{t+1})r^S_{t+1}}{Z_{t+1} + p_{t+1}} + \frac{f^S_{t+1}}{b^S_{t+1}} (1 - \xi^S_t)(\delta - L^S_{t+1}) \right] \right\}
\]

\[
= -\mathbb{E}_t \left\{ (1 - \pi_B) M_{t,t+1} \left[ \frac{f^S_{t+1}}{b^S_{t+1}} \frac{L^S_{t+1}}{b^S_{t+1}} + \frac{f^S_{t+1}}{b^S_{t+1}} (1 - \xi^S_t)(\delta - L^S_{t+1}) \right] \right\}
\]

\[
= -\mathbb{E}_t \left\{ (1 - \pi_B) M_{t,t+1} \left[ \frac{(1 - F^S_{t+1})r^S_{t+1}}{b^S_{t+1}} + \frac{f^S_{t+1}}{b^S_{t+1}} ((1 - \xi^S_t)\delta + \xi^S_S L^S_{t+1}) \right] \right\}.
\]

A.2.3 Condition for S-bank Leverage

Combining the household condition with the S-bank’s condition gives

\[
MRS_{S,t} + \mathbb{E}_t \left[ M_{t,t+1} \left( F^S_\rho (L^S_{t+1}) + (1 - F^S_\rho (L^S_{t+1})) \left( \pi_B + (1 - \pi_B) r^S_{t+1} \right) \right) \right] = \mathbb{E}_t \left\{ (1 - \pi_B) M_{t,t+1} \left[ (1 - F^S_{t+1})r^S_{t+1} + f^S_{t+1} ((1 - \xi^S_t)\delta + \xi^S_S L^S_{t+1}) \right] \right\} + \mathbb{E}_t \left[ M_{t,t+1} \left( F^S_\rho (L^S_{t+1}) \right) \right].
\]

52
Collection terms gives

$$MRS_{S,t} = E_t \{ M_{t,t+1} \left[ (1 - \pi_B) f_{t+1} \left( (1 - \xi_S) \delta + \xi_S L^S_{t+1} \right) - \pi_B \left( 1 - F^S_{\rho} \left( L^S_{t+1} \right) \right) \right] \} ,$$

which is the condition in the main text.