The Macroeconomic Effects of Housing Wealth, Housing Finance, and Limited Risk-Sharing in General Equilibrium

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Abstract

We study a two-sector general equilibrium model of housing and non-housing production where heterogeneous households face limited risk-sharing opportunities as a result of incomplete financial markets. The model generates substantial variability in national house price-rent ratios, both because they fluctuate endogenously with the state of the economy and because they rise in response to a relaxation of credit constraints and decline in housing transaction costs (financial market liberalization). We find that a financial liberalization plus an infusion of foreign capital calibrated to match the increase in foreign ownership of U.S. Treasuries from 2000-2007 generates more than half of the increase in three out of four national house price-rent ratios over this period. A financial market liberalization drives risk premia in both the housing and equity market down, shifts the composition of wealth for all age and income groups towards housing, and leads to a short-run boom in aggregate consumption but a short-run bust in investment. By contrast, an infusion of foreign capital by governmental holders increases risk-premia in both the housing and equity markets. Finally, the model implies that procyclical increases in equilibrium price-rent ratios reflect lower future housing returns, not higher future rents.

JEL: G11, G12, E44, E21
1 Introduction

Residential real estate is a large and volatile component of household wealth. In the United States, housing wealth averaged 29.6% of total household net worth over the period 1945-2007. But this average masks the significant volatility of the U.S. housing market: houses accounted for as little as 25% of household net worth at the end of 2000 but as much as 37% by the end of 2006. Moreover, volatility in housing wealth is often accompanied by large swings in house prices relative to housing fundamentals. Figure 1 shows that aggregate house price indexes relative to measures of fundamental value climbed to unusual heights by the end of 2006, but have since exhibited sharp declines.

This paper studies the macroeconomic consequences of fluctuations in housing wealth and housing finance. To what extent can episodes of national house price appreciation be attributed to a liberalization in housing finance, such as declines in collateral constraints or reductions in the costs of borrowing and conducting transactions? How do movements in house prices affect expectations about future housing fundamentals and future home price appreciation? To what extent do changes in housing wealth and housing finance affect output and investment, risk premia in housing and equity markets, measures of cross-sectional risk-sharing, life-cycle wealth-savings patterns, and the size of housing wealth effects on consumer spending? Conversely, what are the macroeconomic and financial market consequences of a subsequent increase in borrowing and/or transactions costs or of a tightening of collateralized credit constraints?

In this paper we address these questions by studying a two-sector general equilibrium model of housing and non-housing production where heterogenous households face limited risk-sharing opportunities as a result of incomplete financial markets. The multiple features of this model add to its complexity but are necessary to address the breadth of questions posed here.

A house in our model is a residential durable asset that provides utility to the household, is illiquid (expensive to trade), and can be used as collateral in debt obligations. The model economy is populated by a large number of overlapping generations of households who receive utility from both housing and nonhousing consumption and who face a stochastic life-cycle earnings profile. We introduce market incompleteness by modeling heterogeneous agents who face idiosyncratic and aggregate risks against which they cannot perfectly insure, and by imposing collateralized borrowing constraints on households.

Within the context of this model, we focus our theoretical investigation on the macroeco-
nomic consequences of three systemic changes in key features of housing finance. First, we investigate the impact of changes in housing collateral requirements. Second, we investigate the impact of changes in housing transactions costs. Third, we investigate the impact of a secular decline in interest rates driven by an infusion of foreign capital. We argue below that all three factors fluctuate over time and changed markedly during or preceding the period of rapid home price appreciation from 2000-2006. In particular, this period was marked by a widespread relaxation of collateralized borrowing constraints and declining housing transactions costs, a combination we refer to hereafter as financial market liberalization. The period was also marked by a sustained depression of long-term interest rates that coincided with an infusion of foreign governmental capital into U.S. bond markets. Some of these changes have since been partially reversed in the aftermath of the credit crisis that began in 2007. We use our framework as a laboratory for studying the impact of fluctuations in either direction of these features of housing finance.

We summarize the model’s main implications as follows.

House prices relative to measures of fundamental value are volatile. The model generates substantial variability in national house price-rent ratios, both because they fluctuate procyclically with the state of the economy, and because they rise in response to a relaxation of credit constraints and decline in housing transaction costs. In an economic expansion, a financial market liberalization adds fuel to the fire in an already heated housing market, driving up price-rent ratios more than what would occur as the result of an economic boom alone. When we add to this an infusion of foreign capital calibrated to match the increase in foreign ownership of U.S. Treasuries over the period 2000-2007, the model predicts that changes in these factors alone (without accounting for economic growth) can account for more than half of the increase in three out of four measures of national house price-rent ratios over the same period. Moreover, the rise in foreign ownership of U.S. debt generates a decline in equilibrium interest rates of greater than 50 percent, a figure that is approximately commensurate with the decline in mortgage interest rates observed in U.S. data over the period 2000-2007. These findings suggest that any subsequent tightening of credit constraints, increase in housing transactions costs, and/or decline in the willingness of foreigners to hold U.S. debt could put significant downward pressure on house prices and house price valuation ratios.

A financial market liberalization drives price-rent ratios up because it drives risk-premia down. The main driving force behind the rise in price-rent ratios after a financial market liberalization is an across-the-board decline in risk-premia in both housing
and equity assets. Risk premia fall because lower collateral requirements and housing transactions costs increase both access to credit and to the collateral required to obtain credit, thereby allowing heterogeneous households to share more of their idiosyncratic risks. This results in a decline in the cross-sectional variance of consumption growth.

It is important to note that the rise in price-rent ratios caused by a financial market liberalization must be attributed to a decline in risk premia and not to a fall in interest rates. Indeed, the very changes in housing finance that accompany a financial market liberalization drive the endogenous interest rate up, rather than down. It follows that price-rent ratios rise because the decline in risk-premia more than offsets the rise in equilibrium interest rates. These findings underscore the crucial role of foreign capital in keeping interest rates low during a financial market liberalization. Without a foreign capital infusion, any period of looser collateral requirements and lower housing transactions costs (such as that which characterized the period of rapid home price appreciation from 2000-2006) would be accompanied by an increase in equilibrium interest rates, as households endogenously respond to the improved risk-sharing opportunities afforded by financial market liberalization by reducing precautionary saving.

**Procyclical increases in equilibrium price-rent ratios reflect lower future returns, not higher future rents.** It is commonly assumed that procyclical increases in national house-price rent ratios reflect an expected increase in future housing fundamentals, such as rental growth. This reasoning, however, ignores the general equilibrium response of residential investment to economic growth. In the model, positive economic shocks stimulate both greater housing demand and greater residential investment. Under plausible parameterizations, the latter can lead to an equilibrium decline in future rental growth as the housing stock rises. Thus, high price-rent ratios in expansions must entirely reflect expectations of future house price depreciation (lower future returns), in part driven by lower risk-premia as collateral values rise with the economy. Although future rental growth is expected to be lower, price-rent ratios are still high because the decline in future housing returns more than offsets the fall in future rental growth.

**A financial market liberalization leads to a short-run boom in consumption, but a short-run bust in investment.** A financial market liberalization leads to a short-run boom in aggregate consumption, consistent with common notions of housing “wealth effects.” This result, however, occurs not for the usual partial equilibrium reason that a financial market liberalization allows credit-constrained households to borrow more against future income. On the contrary, we show that the sustained increase in consumption following
a financial market liberalization is attributable to net lenders rather than net borrowers. A
financial market liberalization is not stimulative for the economy as a whole, however, since
the short-run boom in consumption drives up interest rates and crowds out investment.

Financial market liberalization plus foreign capital leads to a shift in the
composition of wealth towards housing, increases financial wealth inequality, but
has ambiguous affects on consumption inequality. A financial market liberalization
plus an influx of foreign capital into the bond market leads households of all ages and
incomes to shift the composition of their assets towards housing. Both the magnitude and
age/income-distribution of these changes in the model are in line those observed in household-
level data from 2000 to 2007. Such changes in housing finance also have implications for
inequality. Although a financial market liberalization improves risk sharing and drives risk-
premia down, an infusion of foreign governmental capital reduces risk sharing and drives
risk premia up because it forces domestic savers out of the bond market, increasing their
exposure to systematic risk in equity markets. We show that a financial market liberalization
and foreign capital infusion have offsetting effects on consumption inequality but reinforcing
upward effects on financial wealth inequality.

The paper is organized as follows. The next subsection briefly discusses related literature.
Section 2 describes recent changes in the three key aspects of housing finance discussed
above: collateral constraints, housing transactions costs, and foreign capital in U.S. debt
markets. Section 3 presents the theoretical model. Section 4 presents our main findings,
including benchmark business cycle and financial market statistics. Here we show the model
generates a sizable equity premium and Sharpe ratio simultaneously with a plausible degree
of variability in aggregate consumption. The model also generates forecastable variation
both in long-horizon excess stock market returns and in excess returns on national house
price indexes, consistent with statistical evidence, though it produces too much cash-flow
predictability, as we discuss below. Section 5 concludes.

1.1 Related Literature

Our paper is related to a growing body of literature finance that studies the asset pricing
implications of incomplete markets models. The focus of this literature, however, is typically
on the equity market implications of such models. Moreover, the majority of this litera-
ture does not model the production side of the economy, instead studying pure exchange

Within the incomplete markets environment, our work is related to several papers that study questions related to housing and/or consumer durables more generally. These papers typically either do not model production and/or a risky asset (equity), or are analyses of partial equilibrium environments. See for example, the general equilibrium exchange-economy analyses that embed bond, stock and housing markets of Ríos-Rull and Sánchez-Marcos (2006), Lustig and Van Nieuwerburgh (2007, 2008), Piazzesi and Schneider (2008), and the partial equilibrium analyses of Peterson (2006), Ortalo-Magné and Rady (2006), and Corbae and Quintin (2009). Piazzesi and Schneider (2008) focus on the nominal aspects of borrowing and lending. They find that when household’s disagree about future inflation, those with high inflation expectations borrow more, driving up the demand for housing as collateral and, along with it, house prices. Fernández-Villaverde and Krueger (2005) study how consumption over the life-cycle is influenced by consumer durables in an incomplete markets model with production, but limit their focus to equilibria in which prices, wages and interest rates are constant over time. Kiyotaki, Michaelides, and Nikolov (2008) study a life-cycle model with housing and non-housing production, but focus their analysis on the perfect foresight equilibria of a small open economy facing an exogenous interest rate. Unlike the model investigated here, however, Kiyotaki, Michaelides, and Nikolov (2008) consider an explicit decision to rent versus own and study how welfare of owners and renters is differentially affected by changes in aggregate fundamentals.

Outside of the incomplete markets environment, a strand of the macroeconomic literature studies housing behavior in a two-sector, general equilibrium business cycle framework either with production (e.g., Davis and Heathcote (2005), Kahn (2008)) or without production (e.g., Piazzesi, Schneider, and Tuzel (2007)). The focus here is on environments with complete markets for idiosyncratic risks and a representative agent representation. These models are silent on questions involving risk-sharing, inequality, and age and income heterogeneity.

It is important to note that our paper does not address the question of why credit market conditions changed so markedly in recent decades. It is widely understood that the financial market liberalization we discuss in the next section was preceded by a number of revolu-
tionary changes in housing finance, in particular by the rise in securitization. These changes initially decreased the risk of individual home mortgages and home equity loans, making it optimal for lenders to lower collateral requirements and reduce housing transactions fees (e.g. Green and Wachter (2008); Strongin, O’Neill, Himmelberg, Hindian, and Lawson (2009)). As the researchers note, however, these initially risk-reducing changes in housing finance were accompanied by government deregulation of financial institutions that ultimately increased risk, by permitting such institutions to alter the composition of their assets towards more high-risk securities, by permitting higher leverage ratios, and by presiding over the spread of complex financial holding companies that replaced the long-standing separation between investment bank, commercial bank and insurance company. The market’s subsequent revised expectation upward of the riskiness of the underlying mortgage assets since 2007 appears, anecdotally, to have led to a reversal in collateral requirements and transactions fees. Embedding the optimal dynamic mortgage contracting problem into a general equilibrium model with limited risk-sharing remains a significant challenge for future research.

2 Changes in Housing Finance

We use the model of this paper to study the impact of changes in three features of housing finance. First, we investigate the impact of changes in housing collateral requirements, broadly defined. Collateral constraints often take the form of an explicit down payment requirement, but they also pertain to collateral required for home equity borrowing. Recent data suggests that down payment requirements declined for a range of mortgages categories in the period leading up to the broad decline in housing prices that began in 2006. Loan-to-value ratios on subprime loans rose from 79% to 86% over the period 2001-2005, while debt-income ratios rose (Demyanyk and Hemert (2008)). Other reports suggest that the increase in loan-to-value (LTV) ratios for prime mortgages was even greater, with one industry analysis finding that LTV ratios for such loans rose from 60.4% in 2002 to 75.2% in 2006.\(^2\) Moreover, there was a surge in borrowing against existing home equity between 2002 and 2006 (Mian and Sufi (2009b)).

More generally, there was a widespread relaxation of underwriting standards in the U.S. mortgage market during the period leading up to the credit crisis of 2007. The loosening of standards can be observed in the marked rise in simultaneous second-lien mortgages and in

no-documentation or low-documentation loans. Looser underwriting standards provide a back-door means of reducing collateral requirements for home purchases. By the end of 2006 households routinely bought homes with 100% financing using a piggyback second mortgage or home equity loan. (See also Mian and Sufi (2009a).) We model these changes collectively as a decline in collateralized borrowing constraints.

A second structural change in housing finance in recent years is the decline in the cost of conducting housing transactions. Specifically, there is evidence of a significant decline in fees and charges on mortgages and home equity credit. Costs associated with mortgage refinancing and home equity extraction fell sharply in the years leading up to the housing boom that ended in 2006/2007 (McCarthy and Steindel (2007)). The Federal Housing Financing Board reports monthly data on mortgage rates (based on a survey of the largest lenders). They report “contract rates”, “initial fees and charges”, and “effective rates.” The latter add to the contract rate the discounted fees and charges. Figure 2 shows that initial fees and charges on mortgages have declined from 2.70% of the loan balance in January 1985 to 0.46% in April 2008. Broken down by fixed-rate mortgages (FRMs) and adjustable rate mortgages (ARMs), both display a decline. The difference between the effective rate and the contract rate is also a measure of the initial fees and charges, but now expressed as an interest rate. This difference declined from 50 basis points to 5 basis points over the period 1985-2007.

A third key development in the housing market of recent years is the secular decline in interest rates. Figure 3 shows that both 30-year FRMs and the 10-year Treasury bond yield have trended downward, with mortgage rates declining from around 18 percent in the early 1980s to near 6 percent by the end of 2007. At the same time, foreign ownership of U.S. Treasuries (T-bonds and T-notes) increased from $118 billion in 1984, or 13.5% of marketable Treasuries outstanding, to $2.2 trillion in 2008, or 61% of marketable Treasuries (Figure 4). Foreign holdings of U.S. agency and Government Sponsored Enterprise-backed agency securities quintupled from $261 billion to $1.3 trillion, or from 7% to 21% of total agency securities between 2000 and 2007. By pushing real interest rates lower, the rise in

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3FDIC Outlook: Breaking New Ground in U.S. Mortgage Lending, December 18, 2006. <http://www.fdic.gov/bank/analytical/regional/ro20062q/na/2006_summer04.html#10A>. No- or low-documentation loans refer to loans in which a lender sets reduced or minimal documentation standards to corroborate a borrower’s income and assets. A simultaneous second-lien loan, also referred to as a “piggyback loan,” is a lending arrangement where either a closed-end second lien or a home equity line of credit is originated at the same time as the first-lien mortgage loan, usually taking the place of a larger down payment.
foreign capital has been directly linked to the surge in mortgage originations over this period (e.g., Strongin, O’Neill, Himmelberg, Hindian, and Lawson (2009)). The role of foreign capital in driving interest rates lower has also been emphasized by economic policymakers, as in this speech by Federal Reserve Chairman Ben Bernanke in 2005:

I will argue that over the past decade a combination of diverse forces has created a significant increase in the global supply of saving—a global saving glut—which helps to explain both the increase in the U.S. current account deficit and the relatively low level of long-term real interest rates in the world today. [...] Because the dollar is the leading international reserve currency, and because some emerging-market countries use the dollar as a reference point when managing the values of their own currencies, the saving flow out of the developing world has been directed relatively more into dollar-denominated assets, such as U.S. Treasury securities.4

More recently, Bernanke tied the supply of foreign capital to the surge in U.S. house prices that peaked in 2006:

[...] The substantial increase in the net supply of saving in emerging market economies contributed to both the U.S. housing boom and the broader credit boom. [...] The pressure of these net savings flows led to lower long-term real interest rates around the world, stimulated asset prices (including house prices), and pushed current accounts toward deficit in the industrial countries—notably the United States—that received these flows.5

In the model of this paper, interest rates are determined in equilibrium by a market clearing condition for bondholders. We consider one specification of the model in which we introduce an exogenous foreign demand for domestic bonds into the market clearing condition, referred to hereafter as “foreign capital.” We think of the foreign capital in the model as primarily supplied by foreign central banks and other governmental agencies who have a specific regulatory motive for holding the safe asset, as discussed in Kohn (2002). As explained in Kohn (2002), government entities face both legal and political restrictions

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5Remarks made by Federal Reserve Chairman Ben S. Bernanke to the International Monetary Conference, Barcelona, Spain (via satellite), June 3, 2008.
on the type of assets that can be held, forcing them into safe securities. Krishnamurthy and Vissing-Jorgensen (2008) find that demand for U.S. Treasury securities by governmental holders is extremely inelastic, suggesting that when these holders receive funds to invest they buy U.S. Treasuries, regardless of their price relative to other U.S. assets. This motivates our modeling of foreign capital as both exogenous and as restricted to investments in the safe asset. In the model, we assume domestic borrowers may obtain credit at a fixed interest rate spread with the governmental rate. Because our model abstracts from default, we set this spread to zero in our calibration.

3 The Model

3.1 Firms

The production side of the economy consists of two sectors. One sector produces the non-housing consumption good, and the other sector produces the housing good. We refer to the first as the “consumption sector” and the second as the “housing sector.” Time is discrete and each period corresponds to a year. In each period, a representative firm in each sector chooses labor (which it rents) and investment in capital (which it owns) to maximize the value of the firm to its owners.

3.1.1 Consumption Sector

Denote output in the non-housing consumption good sector as

\[ Y_{C,t} = Z_{C,t}K_{C,t}^\alpha N_{C,t}^{1-\alpha}, \]

where \( Z_{C,t} \) is the stochastic productivity level at time \( t \), \( K_C \) is the capital stock in the consumption sector, \( \alpha \) is the share of capital, and \( N_C \) is the quantity of labor input in the consumption sector. Let \( I_C \) denote investment in the consumption sector. The firm’s capital stock \( K_{C,t} \) accumulates over time subject to proportional adjustment costs, \( \phi_K \left( \frac{I_{C,t}}{K_{C,t}} \right) K_{C,t} \), modeled as a deduction from the earnings of the firm. The firm maximizes the present discounted value \( V_{C,t} \) of a stream of earnings:

\[ V_{C,t} = \max_{N_{C,t},I_{C,t}} E_t \sum_{k=0}^{\infty} \frac{\beta^k}{\Lambda_t} \left( Y_{C,t+k} - w_{t+k}N_{C,t+k} - I_{C,t+k} - \phi_K \left( \frac{I_{C,t+k}}{K_{C,t+k}} \right) K_{C,t+k} \right), \]
where $\frac{\beta^{k} \lambda_{t+k}}{\lambda_t}$ is a stochastic discount factor discussed further below, and $w$ is the wage rate (equal across sectors in equilibrium). The evolution equation for the firm’s capital stock is

$$K_{C, t+1} = (1 - \delta) K_{C, t} + I_{C, t},$$

where $\delta$ is the depreciation rate of the capital stock.

The firm does not issue new shares and finances its capital stock entirely through retained earnings. The dividends to shareholders are equal to

$$D_{C, t} = Z_{C, t} K_{C, t}^\alpha N_{C, t}^{1-\alpha} - w_t N_{C, t} - I_{C, t} - \phi_K \left( \frac{I_{C, t}}{K_{C, t}} \right) K_{C, t}.$$

### 3.1.2 Housing Sector

The housing firm’s problem is directly analogous to the problem solved by the representative firm in the consumption sector. Denote output in the residential housing sector as

$$Y_{H, t} = Z_{H, t} K_{H, t}^\nu N_{H, t}^{1-\nu},$$

$Y_{H, t}$ represents construction of new housing (residential investment). $Z_{H}$ is the stochastic productivity level, $K_{H}$ is the capital stock in the housing sector, $\nu$ is the share of capital in housing output, and $N_{H}$ is the quantity of labor input in the housing sector. Let $I_{H}$ denote investment in the housing sector. $K_{H, t}$ accumulates over time subject to proportional adjustment costs, $\phi_H \left( \frac{I_{H, t}}{K_{H, t}} \right) K_{H, t}$, modeled as a deduction from the earnings of the firm.

The firm maximizes

$$V_{H, t} = \max_{N_{H, t}, I_{H, t}} \mathbb{E}_t \sum_{k=0}^\infty \mathbb{E}_t \left( p_{t+k}^{H} Y_{H, t+k} - w_{t+k} N_{H, t+k} - I_{H, t+k} - \phi_H \left( \frac{I_{H, t+k}}{K_{H, t+k}} \right) K_{H, t+k} \right),$$

where $p_{t+k}^{H}$ is the relative price of one unit of housing in units of the non-housing consumption good, and $\frac{\beta^{k} \lambda_{t+k}}{\lambda_t}$ is a stochastic discount factor discussed further below. Note that $p_{t}^{H}$ should be interpreted as the time $t$ price of a unit of housing of fixed quality and quantity.

The evolution equation for the firm’s capital stock is:

$$K_{H, t+1} = (1 - \delta) K_{H, t} + I_{H, t}.$$
where $\delta_H$ denotes the depreciation rate of the housing stock.

The firm does not issue new shares and finances its capital stock through retained earnings. The dividends to shareholders in the housing sector are denoted

$$D_{H,t} = p_t^H Z_{H,t} K_{H,t}^{\nu} N_{H,t}^{1-\nu} - w_t N_{H,t} - I_{H,t} - \phi_H \left( \frac{I_{H,t}}{K_{H,t}} \right) K_{H,t}.$$

### 3.2 Risky Asset Returns

The firms’ values $V_{H,t}$ and $V_{C,t}$ are the *cum* dividend values, measured before the dividend is paid out. Thus the *cum* dividend returns to shareholders in the housing sector and the consumption sector are defined, respectively, as

$$R_{Y_{H,t+1}} = \frac{V_{H,t+1}}{(V_{H,t} - D_{H,t})} \quad R_{Y_{C,t+1}} = \frac{V_{C,t+1}}{(V_{C,t} - D_{C,t})}.$$

We define $V^e_{j,t} = V_{j,t} - D_{j,t}$ for $j = H, C$ to be the *ex* dividend value of the firm.\(^6\)

### 3.3 Individuals

The economy is populated by $A$ overlapping generations of individuals, indexed by $a = 1, \ldots, A$, with a continuum of individuals born each period. Individuals live through two stages of life, a working stage and a retirement stage. Adult age begins at age 21, so $a$ equals this effective age minus 20. Agents live for a maximum of $A = 80$ (100 years). Workers live from age 21 ($a = 1$) to 65 ($a = 45$) and then retire. Retired workers die with an age-dependent probability calibrated from life expectancy data. The probability that an agent is alive at age $a+1$ conditional on being alive at age $a$ is denoted $\pi_{a+1|a}$.

Upon death, any remaining net worth of the individual in that period is counted as terminal “consumption,” e.g., funeral and medical expenses.\(^7\)

Individuals have an intraperiod utility function given by

$$U(C_{a,t}, H_{a,t}) = \frac{\bar{C}_{a,t}^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} \quad \bar{C}_{a,t} = \left[ \chi C_{a,t}^{\frac{\epsilon+1}{\epsilon}} + (1 - \chi) H_{a,t}^{\frac{\epsilon+1}{\epsilon}} \right]^{\frac{\epsilon}{\epsilon+1}},$$

\(^6\)Using the *ex* dividend value of the firm the return reduces to the more familiar *ex* dividend definition:

$$R^e_{j,t+1} = \frac{V^e_{j,t+1} + D_{j,t+1}}{V^e_{j,t}}.$$

\(^7\)We plan to extend our analysis to allow for bequests in future work.
where $C_{a,t}$ is non-housing consumption of an individual of age $a$, and $H_{a,t}$ is the stock of housing, $\sigma$ is the coefficient of relative risk aversion, $\chi$ is the relative weight on non-housing consumption in utility, and $\varepsilon$ is the constant elasticity of substitution between $C$ and $H$. Implicit in this specification is the assumption that the service flow from houses is proportional to the stock $H_{a,t}$.

Financial market trade is limited to a one-period riskless bond and risky capital, where the latter is a value-weighted portfolio of equity in the housing and consumption firms. The gross bond return is denoted $R_{f,t} = \frac{1}{q_{t-1}}$, where $q_{t-1}$ is the bond price. Risky capital is an asset with return equal to a portfolio weighted average of the returns in each sector

$$R_{K,t} = \frac{V_{H,t} R_{Y_{H,t}}}{V_{H,t} + V_{C,t}} + \frac{V_{C,t} R_{Y_{C,t}}}{V_{H,t} + V_{C,t}}.$$  

(3)

Individuals are born with with no initial endowment of risky capital or bonds.

Individuals are heterogeneous in their labor productivity. To denote this heterogeneity, we index individuals $i$. Before retirement households supply labor inelastically. The stochastic process for individual income for workers is

$$Y_{i,a,t} = w_t L_{i,a,t},$$

where $L_{i,a,t}$ is the individual’s labor endowment (hours times an individual-specific productivity factor), and $w_t$ is the aggregate wage per unit of productivity. Labor productivity is specified by a deterministic age-specific profile, $G_a$, and an individual shock $Z_{i,a,t}:

$$L_{i,a,t} = G_a Z_{i,a,t},$$

$$\log (Z_{i,a,t}) = \log (Z_{a-1, t-1}^{i-1}) + \epsilon_{i,a,t},$$

where $G_a$ is a deterministic function of age capturing a hump-shaped profile in life-cycle earnings and $\epsilon_{i,a,t}$ is a stochastic i.i.d. shock to individual earnings. The Appendix explains how we employ an $N$-point discrete approximation to a normal distribution for $\epsilon_{i,a,t}$ in our model solution procedure. To capture countercyclical variation in idiosyncratic risk of the type documented by Storesletten, Telmer, and Yaron (2004), we use a two-state specification for the variance of idiosyncratic earnings shocks:

$$\sigma_t^2 = \begin{cases} 
\sigma_E^2 & \text{if } Z_{C,t} \geq E(Z_{C,t}) \\
\sigma_R^2 & \text{if } Z_{C,t} < E(Z_{C,t})
\end{cases}, \quad \sigma_R^2 > \sigma_E^2,$$

This specification implies that the variance of idiosyncratic labor earnings is higher in “recessions” ($Z_{C,t} \leq E(Z_{C,t})$) than in “expansions” ($Z_{C,t} \geq E(Z_{C,t})$). The former is denoted
with an \( "R" \) subscript, the latter with an \( "E" \) subscript. Finally, labor earnings are taxed at rate \( \tau \) in order to finance social security retirement income.

At age \( a \), agents enter the period with wealth invested in bonds, \( B_{a,t}^i \), and shares \( \theta_{a,t}^i \) of risky capital. The total number of shares outstanding of the risky asset is normalized to unity. We rule out short-sales in the risky asset, \( \theta_{a,t}^i \geq 0 \).

If the individual chooses to invest in the risky capital asset, it pays a fixed, per-period participation cost, \( F_K \).

We assume that the housing owned by each individual depreciates at rate \( \delta_H \), the rate of depreciation of the aggregate housing stock. Households may choose to increase the quantity of housing consumed at time \( t+1 \) by making a net investment \( H_{a,t+1}^i - (1 - \delta_H) H_{a,t}^i > 0 \). But because houses are illiquid, it is expensive to change housing consumption. If the individual chooses to change its housing consumption, it pays a transaction cost \( F_{H,a,t}^i \). Denote the sum of the per period equity participation cost and housing transaction cost for individual \( i \) as

\[
F_{a,t}^i \equiv F_{H,a,t}^i + F_K.
\]

Define the individual’s gross financial wealth at time \( t \) as

\[
W_{a,t}^i \equiv \theta_{a,t}^i (V_{C,t}^e + V_{H,t}^e + D_{C,t} + D_{H,t}) + B_{a,t}^i.
\]

The budget constraint for an agent of age \( a \) who is not retired is

\[
C_{a,t}^i + B_{a+1,t+1}^i q_t + \theta_{a+1,t+1}^i (V_{C,t}^e + V_{H,t}^e) \leq W_{a,t}^i + (1 - \tau) w_t L_{a,t}^i + \left( (1 - \delta_H) H_{a,t}^i - H_{a+1,t+1}^i \right) - F_{a,t}^i,
\]

\[
W_{a+1,t+1}^i \geq - (1 - \omega) p_t^H H_{a,t}^i, \quad \forall a, t
\]

\[
\theta_{a,t}^i \geq 0, \quad \forall a, t
\]

where

\[
F_{H,a,t}^i = \begin{cases} 0 & \text{if } H_{a,t+1}^i = (1 - \delta_H) H_{a,t}^i \smallskip \\
\psi_0 + \psi_1 p_t^H H_{a,t}^i & \text{if } H_{a,t+1}^i \neq (1 - \delta_H) H_{a,t}^i
\end{cases}
\]

\[
F_K = \begin{cases} 0 & \text{if } \theta_{a+1,t+1} = 0 \\
\mathcal{F} & \text{if } \theta_{a+1,t+1} > 0
\end{cases}
\]
Equation (5) is the collateral constraint, where \( 0 \leq \varpi \leq 1 \). It says that households may borrow no more than a fraction \((1 - \varpi)\) of the value of housing, implying that they must post collateral equal to a fraction \(\varpi\) of the value of the house. This constraint can be thought of as a down-payment constraint for new home purchases, but it also encompasses collateral requirements for home equity borrowing against existing homes. The constraint gives the maximum combined LTV ratio for first and second mortgages. Notice that if the price \(p_t^h\) of the house rises and nothing else changes, the individual can finance a greater level of consumption of both housing and nonhousing goods and services.

We also prevent the individual from buying stock on margin. Thus, if the individual is a net borrower, we restrict holdings of the risky asset to be zero, \(\theta_{i+1,t+1} = 0\). This restriction is stated mathematically as follows:

\[
\text{if } W_{a,t}^i + (1 - \tau) w_t L_{a,t}^i - (C_{a,t}^i + p_t^H (H_{a+1,t+1}^i - (1 - \delta_H)H_{a,t}^i)) < 0 \quad (6)
\]

\[
\text{then } B_{a+1,t+1}^i < 0, \quad \theta_{i+1,t+1}^i = 0.
\]

Net lenders may take a positive position in the risky asset but may not short the bond to do so. This restriction is stated mathematically as

\[
\text{if } W_{a,t}^i + (1 - \tau) w_t L_{a,t}^i - (C_{a,t}^i + p_t^H (H_{a+1,t+1}^i - (1 - \delta_H)H_{a,t}^i)) \geq 0 \quad (7)
\]

\[
\text{then } B_{a+1,t+1}^i \geq 0, \quad \theta_{i+1,t+1}^i \geq 0.
\]

Let \(Z_{ar}^i\) denote the value of the stochastic component of individual labor productivity, \(Z_{a,t}^i\), during the last year of working life. Each period, retired workers receive a pension \(PE_{a,t}^i = Z_{ar}^i X_t\) where \(X_t = \tau \frac{N_W}{N_R}\) is the pension determined by a pay as you go system, and \(N_W\) and \(N_R\) are the numbers of working age and retired households.\(^8\) For agents who have reached retirement age, the budget constraint is identical to that for workers (4) except that wage income \((1 - \tau) w_t L_{a,t}^i\) is replaced by pension income \(PE_{a,t}^i\).

Let \(Z_t \equiv (Z_{C,t}, Z_{H,t})^T\) denote the aggregate shocks. The state of the economy is a pair, \((Z, \mu)\), where \(\mu\) is a measure defined over \(S = (A \times Z \times W)\), where \(A = \{1, 2, \ldots A\}\) is the set of ages, where \(Z\) is the set of all possible idiosyncratic shocks, and where \(W\) is the set of all possible beginning-of-period wealth realizations. That is, \(\mu\) is a distribution of agents across ages.

\(^8\)The decomposition of the population into workers and retirees is determined from life-expectancy tables as follows. Let \(X\) denote the total number of people born each period. (In practice this is calibrated to be a large number in order to approximate a continuum.) Then \(N_W = 45 \cdot X\) is the total number of workers. Next, from life expectancy tables, if the probability of dying at age \(a < 45\) is denoted \(p_a\) then

\[
\text{Notice that if the price } p_t^h \text{ of the house rises and nothing else changes, the individual can finance a greater level of consumption of both housing and nonhousing goods and services.}
\]

\[
N_R = \sum_{a=46}^{80} (1 - p_a) X \quad \text{is the total number of retired persons.}
\]
ages, idiosyncratic shocks, and ex-post wealth. The presence of aggregate shocks implies that $\mu$ evolves stochastically over time. We specify a law of motion, $\Gamma$, for $\mu$,

$$
\mu_{t+1} = \Gamma (\mu_t, Z_t, Z_{t+1}).
$$

### 3.4 Stochastic Discount Factor

The stochastic discount factor (SDF), $\frac{\beta \Lambda_{t+1}}{\Lambda_t}$, appears in the value maximization problem (1) and (2) undertaken by each representative firm. We assume that the representative firm in each sector discounts future profits using a weighted average of the individual shareholders’ marginal rates of substitution (IMRS) in non-housing consumption, $\frac{\beta \partial U/\partial C_{a+1,t+1}}{\partial U/\partial C_{a,t}}$, where the weights, $\theta^i_{a,t}$, correspond to the shareholder’s proportional ownership in the firm. Let $\frac{\beta \Lambda_{t+1}}{\Lambda_t}$ denote this weighted average. Recalling that the total number of shares in the risky portfolio is normalized to unity, we have

$$
\frac{\beta \Lambda_{t+1}}{\Lambda_t} \equiv \int_S \theta^i_{a+1,t+1} \frac{\beta \partial U/\partial C_{a+1,t+1}}{\partial U/\partial C_{a,t}} d\mu.
$$

\begin{equation}
\frac{\beta \partial U/\partial C_{a+1,t+1}}{\partial U/\partial C_{a,t}} = \beta \left( \left( \frac{C_{a+1,t+1}}{C_{a,t}} \right)^{-\frac{1}{\tau}} \frac{\chi + (1 - \chi) \left( \frac{H_{a+1,t+1}}{H_{a,t}} \right)^{\frac{1}{\tau}}} {\chi + (1 - \chi) \left( \frac{H_{a,t}}{C_{a,t}} \right)^{\frac{1}{\tau}}} \right).
\end{equation}

Since we weight each individual’s IMRS by its proportional ownership (and since short-sales in the risky asset are prohibited), only the marginal rates of substitution of households who have taken a positive position in the risky asset will receive non-zero weight in the SDF. Define the feasible set of stochastic discount factors as the set that includes the MRS of any individual who is long in the risky asset (shareholders) or any weighted average of these. Then in the economic environment of this paper, Carceles Poveda and Coen-Pirani (2009) show that, given the firm’s objective of value maximization, the equilibrium allocations are invariant to the choice of stochastic discount factor within the feasible set. In addition, the equilibrium allocations will be the same as the allocations obtained in an otherwise identical economy with “static” firms that rent capital from households on a period-by-period basis.\footnote{“Otherwise identical” means that the two economies are identical with respect to the specification of preference orderings, initial endowments, probability laws governing stochastic shocks, and borrowing limits.}

Furthermore, Bisin, Gottardi, and Ruta (2008) show that, as long as short-sales of the risky asset are prohibited, then in the economic environment of this paper shareholders will unanimously agree on value maximization as the firm’s objective.\footnote{What is important for this result is that the firms in our model do not make a corporate financing}
3.5 Equilibrium

An equilibrium is defined as a set of endogenously determined prices (bond prices, wages, risky asset returns) given by time-invariant functions \( q_t = q(\mu_t, Z_t) \), \( w_t = w(\mu_t, Z_t) \) and \( R_{K,t} = R_K(\mu_t, Z_t) \), respectively, a set of cohort-specific value functions and decision rules for each individual \( i \), \( \{V_a, H_{i,a+1,t+1}^i, \theta_{i,a+1,t+1}^i B_{a+1,t+1}^i\}^A_{a=1} \) and a law of motion for \( \mu_t, \mu_{t+1} = \Gamma(\mu_t, Z_t, Z_{t+1}) \) such that:

1. Households optimize:

\[
V_a(\mu_t, Z_t, Z_{i,a,t}^i, W_{a,t}^i) = \max_{H_{a+1,t+1}^i, \theta_{a+1,t+1}^i, B_{a+1,t+1}^i} \{U(C_{a,t}^i, H_{a,t}^i) + \beta \pi_{a+1} E_t[V_{a+1}(\mu_{t+1}, Z_{t+1}, Z_{a,t+1}^i, W_{a+1,t+1}^i)]\}
\tag{10}
\]

subject to (4), (5), (6), and (7) if the individual of working age, and subject to the analogous versions of (4), (5), (6), and (7) (using pension income in place of wage income), if the individual is retired.

2. Firm’s maximize value: \( V_C \) solves (1), \( V_H \) solves (2).

3. Wages \( w_t = w(\mu_t, Z_t) \) satisfy

\[
w_t = (1 - \alpha) Z_{C,t} K_{C,t}^\alpha N_{C,t}^{-\alpha}
\tag{11}
\]

\[
w_t = (1 - \nu) p_t^H Z_{H,t} K_{H,t}^\nu N_{H,t}^{-\nu}
\tag{12}
\]

4. The housing market clears:

\[
Y_{H,t} = \int_S (H_{a,t+1}^i - H_{a,t}^i (1 - \delta_t)) d\mu.
\tag{13}
\]

5. The bond market clears: \( q_t = q(\mu_t, Z_t) \) is such that

\[
\int_S B_{a,t}^i d\mu + B_t^F = 0,
\tag{14}
\]

where \( B_t^F \geq 0 \) is an exogenous supply of foreign capital discussed below.

6. The risky asset market clears:

\[
1 = \int_S \theta_{a,t}^i d\mu.
\tag{15}
\]

decision. Firms produce output, pay wages and decide on the amount of investment, which is entirely financed with retained earnings. The residual of output less total wage and investment expenses is paid out as a dividend.
7. The labor market clears:

\[ N_t \equiv N_{C,t} + N_{H,t} = \int_S L_{a,t}^i d\mu. \]  

(16)

8. The social security tax rate is set so that total taxes equal total retirement benefits:

\[ \tau N_t w_t = \int_S P E_{a,t}^i d\mu, \]  

(17)

9. The presumed law of motion for the state space \( \mu_{t+1} = \Gamma (\mu_t, Z_t, Z_{t+1}) \) is consistent with individual behavior.

Notice that (11), (12) and (16) determine the \( N_{C,t} \) and therefore determine the allocation of labor across sectors:

\[ (1 - \alpha) Z_{C,t} K_{C,t}^\alpha N_{C,t}^{-\alpha} = (1 - \nu) p_t^H Z_{H,t} K_{H,t}^\nu (N_t - N_{C,t})^{-\nu}. \]  

(18)

Also, the aggregate resource constraint for the economy must take into account the housing and risky capital market transactions/participation costs, which reduce consumption, the adjustment costs in productive capital, which reduce firm profits, and the net foreign supply of capital in the bond market, which finances domestic consumption and investment. Thus, the resource constraint implies that non-housing output minus non-housing consumption equals aggregate investment (gross of adjustment costs) less the net change in the value of foreign capital:

\[ Y_{C,t} - C_t - F_t = \left( I_{C,t} + \phi_{C} \left( \frac{I_{C,t}}{K_{C,t}} \right) K_{C,t} \right) + \left( I_{H,t} + \phi_{H} \left( \frac{I_{H,t}}{K_{H,t}} \right) K_{H,t} \right) - (B_{t+1}^F q (\mu_t, Z_t) - B_t^F), \]  

(19)

where \( C_t \) and \( F_t \) are aggregate quantities defined as:

\[ C_t \equiv \int_S C_{a,t}^i d\mu \]  

(20)

\[ F_t \equiv \int_S F_{a,t}^i d\mu. \]  

(21)

To solve the model, it is necessary to approximate the infinite dimensional object \( \mu \) with a finite dimensional object. The appendix explains the solution procedure and how we specify a finite dimensional vector to represent the law of motion for \( \mu \).

\[ \text{Note that (19) simply results from aggregating the budget constraints across all households, imposing all market clearing conditions, and using the definitions of dividends as equal to firm revenue minus costs.} \]
3.6 Model Calibration

This section discusses our calibration of the model’s primitive parameters under three alternative set of parameterizations. Model 1 is our benchmark calibration, with “normal” collateral requirements and housing transactions costs calibrated to roughly match the data prior to the housing boom of 2000-2006. Model 2 is an alternative calibration designed to match an economy that is otherwise identical to Model 1 but has undergone a financial market liberalization, where a liberalization is defined by a decline in both collateral requirements and housing transactions costs. In both Model 1 and Model 2, trade in the risk-free asset is entirely conducted between domestic residents: $B_t^F = 0$. Model 3 is calibration that is identical to that of Model 2 except that we add an exogenous foreign demand for the risk-free bond: $B_t^F > 0$.

3.6.1 Calibration of Parameters

For convenience, the model’s parameters and their calibration are summarized in the table here. We discuss these values below.
The technology shocks $Z_C$ and $Z_H$ are assumed to follow two-state Markov chains, as described in the Appendix. The parameters of this process are calibrated to roughly match the autoregressive coefficient for the Solow residual of output, and the average length of expansions relative to recessions. The Appendix also describes our calibration of the individual productivity shocks.

Parameters pertaining to the firms’ decisions are set as follows. The adjustment costs for capital in both sectors are assumed to be the same quadratic function of the investment to capital-ratio, $\varphi \left( \frac{I}{K} - \delta \right)^2$, where the constant $\varphi$ is chosen to represent a tradeoff between the desire to match aggregate investment volatility simultaneously with the volatility of asset returns. Under our calibration of these costs, the total $\varphi \left( \frac{I}{K} - \delta \right)^2 K_t$ is less than one percent of total investment. The capital depreciation rates, $\delta$ and $\delta_H$, are set to 0.12 and 0.025,
following Tuzel (2009), which roughly corresponding to the average Bureau of Economic Analysis (BEA) depreciation rates for equipment and structures, respectively. Following Kydland and Prescott (1982) and Hansen (1985), the capital share for the non-housing sector is set to $\alpha = 0.36$. For the residential investment sector, the value of the capital share in production is taken from a BEA study of gross product originating, by industry, which delivers industry-level estimates of production shares for capital and labor.\footnote{From the November 1997 SURVEY OF CURRENT BUSINESS, “Gross Product by Industry, 1947–96,” by Sherlene K.S. Lum and Robert E. Yuskavage. http://www.bea.gov/scb/account_articles/national/1197gpo/mainext.htm} The study finds that the capital share in the construction sector ranges from 29.4% and 31.0% over the period 1992-1996. We therefore set the capital share in the housing sector to $\nu = 0.30$.

Parameters of the individual’s problem are set as follows. The subjective time discount factor is set to $\beta = 0.94$ at annual frequency, to allow the model to match the mean of a short-term Treasury rate in the data. The survival probability $\pi_{a+1|a} = 1$ for $a + 1 \leq 65$. For $a + 1 > 65$, we use the mortality tables from the U.S. Census Bureau to calibrate $\pi_{a+1|a}$ as the fraction of households over 65 born in a particular year alive at age $a + 1$. From these numbers, we compute the stationary age distribution in the model, and use it to calibrate the average earnings $G_a$ over the life-cycle observed from the Survey of Consumer Finances. Risk aversion is set to $\sigma = 8$, to help the models match the high Sharpe ratio for equity observed in the data. The static elasticity of substitution between $C$ and $H$ is set to $\varepsilon = 1$ (Cobb-Douglas utility), which we use as a benchmark. In future work, we plan to explore lower values.\footnote{Ogaki and Reinhart (1998) estimate a value of 1.167 for the elasticity of substitution between durables and nondurables in macro-level data, though without housing. Yogo (2006) estimates a value of 0.790 for the same elasticity again for durables that exclude housing. Estimates using household-level data on housing and nonhousing consumption are often lower than unity. Li, Liu, and Yao (2008), for example, estimate this elasticity to be 0.58.}

The weight, $\chi$ on $C$ in the utility function is set to 0.70, in order to match the average ratio of $I_{C,t}/I_{H,t}$ from the BEA for the non-residential and residential investment sectors, respectively. With $\varepsilon = 1$, this value for $\chi$ corresponds to a housing expenditure share of 0.30. The regime-switching conditional variance in the unit root process in idiosyncratic earnings is calibrated following Storesletten, Telmer, and Yaron (2007) to match their estimates from the Panel Study of Income Dynamics. These are $\sigma_E = 0.0768$, and $\sigma_R = 0.1296$. 

Gross Product Originating is equal to gross domestic income, whose components can be grouped into categories that approximate shares of labor and capital. Under a Cobb-Douglas production function, these equal shares of capital and labor in output.
The other parameters of the individual’s problem are less precisely pinned down from empirical observation. Precise estimates of the costs of stock market participation do not exist, and in principle they could include non-pecuniary costs as well as explicit transactions fees. Vissing-Jorgensen (2002) conducts a number of tests for the presence of a fixed, per period participation cost and again finds strong empirical support for their presence, but not for the hypothesis of variable costs. She estimates the size of these costs by estimating how large they must be in order to explain the portfolio choices of the majority of stock market participants and finds that they are small, less than 50 dollars per year in year 2000 dollars. These findings motivate our calibration of these costs so that they are no greater than 1% of per capita, average consumption, denoted $\bar{C}_i$ in the table above.

It is also difficult to obtain explicit data on average collateral requirements for mortgages and home equity loans. Our own conversations with government economists and analysts who follow the housing sector, however, indicated that prior to the housing boom that ended in 2006/2007, the combined LTV for first and second conventional mortgages (mortgages without mortgage insurance) typically was not allowed to exceed 75 to 80% of the appraised value of the home. Moreover, home equity lines of credit were not widely available until relatively recently (McCarthy and Steindel (2007)). By contrast, these same analysts suggested, during the boom years, households routinely bought homes with 100% financing using a piggyback second or home equity loan, where there were even lenders advertising loans for 125% of the home value if the borrower used the top 25% to pay off existing debt. Our Model 1, therefore, sets the maximum combined LTV (first and second mortgages) to be 75%, corresponding to $\omega = 25\%$. In Model 2, we lower this to $\omega = 1\%$.

It is similarly difficult to know how to calibrate the fixed and variable transactions costs for housing consumption, governed by the parameters $\psi_0$, and $\psi_1$. For home purchases, these costs vary considerably by region, over time, by appraised value, and by type of sale (owner versus broker). In addition, the housing transactions costs in the model are more comprehensive than the costs of buying and selling existing homes. They include costs associated with any change in housing consumption, such as home improvements and additions, but also costs associated with mortgage refinancing and home equity extraction. As discussed above, fees and costs associated with home purchases and home equity finance eroded considerably in the housing boom, and in many cases more than halved. To anchor the level of these costs, in Model 1 we set fixed and variable costs so as to match the average length of residency (in years) for home owners in our model with the average for this same variable from the Survey of Consumer Finances across the 1989-2001 waves of the survey. In the equilibrium
of our model, this amount turns out to be approximately 5% of annual per capita, aggregate consumption. In Model 2 we decrease these costs by half, setting them to approximately 2.5% of per capita aggregate consumption.

Finally, we calibrate foreign ownership of U.S. debt, $B_t^F$, by targeting a value for foreign bond holdings relative to GDP. Specifically, when we add foreign capital to the economy in Model 3, we experiment with several constant values for $B_t^F \equiv B^F$ until the model solution implies a value equal to 19% of average total output, $\overline{Y}$. Figure 5 shows that this is a conservative estimate of the fraction of foreign holdings of U.S. government securities as of the middle of 2008. Foreign holdings of long-term Treasuries alone represent 15% of GDP in 2008. Higher values are obtained if one includes foreign holdings of U.S. agency debt and/or short-term Treasuries. Depending on how many of these categories are included, the fraction of foreign holdings in 2008 ranges from 15-30%.

3.6.2 Model Returns

**Housing Return** Abstracting from transactions costs and borrowing constraints, the first-order condition for optimal housing choice is

$$\frac{\partial U}{\partial C_{a,t}} = \frac{1}{p_t^H} \beta E_t \left[ \frac{\partial U}{\partial C_{a+1,t+1}} \left( \frac{\partial U}{\partial H_{a+1,t+1}} + p_{t+1}^H (1 - \delta_H) \right) + \frac{\partial U}{\partial C_{a+1,t+1}} \right],$$

(22)

implying that each individual’s housing return is given by $\frac{\partial U}{\partial H_{a+1,t+1}} / \frac{\partial U}{\partial C_{a+1,t+1}}$ is the implicit rental price for housing services, referred to hereafter as “rent.” For the national housing return, we define national rent, $R_{t+1}$, as the equally weighted average of $\frac{\partial U}{\partial H_{a+1,t+1}} / \frac{\partial U}{\partial C_{a+1,t+1}}$ across individuals.\(^{14}\) Given this definition of national rent, we define the corresponding national housing return as

$$R_{H,t+1} = \frac{p_{t+1}^H (1 - \delta_H) + R_{t+1}}{p_t^H}. \quad (23)$$

In the model, $p_t^H$ is the price of a unit of housing stock, which holds fixed the composition of housing (quality, square footage, etc.) over time. When comparing model simulated data to historical housing return data, it is important to bear in mind that no single measure of house prices and returns in the data is directly comparable to the theoretical house price- rent ratio and housing return in the model. For this reason, we compare our model results

\(^{14}\)In practice, we specify a total of 2400 agents in the population. We experimented with this number and found that increasing it above 2400 does not affect the aggregate results. The equally weighted average is therefore computed as an average over each of the 2400 agents.
with four different measures of price-rent ratios and associated housing returns. These are (i) a measure based on aggregate housing wealth for the household sector from the Flow of Funds (FoF), (ii) a measure based on the Freddie Mac Conventional Mortgage House Price index for home purchases (Freddie Mac), (iii) a measure based on an index of Real Estate Investment Trusts (REIT), and (iv) a measure based on the Case-Shiller national house price index (CS). In each case, these measures are combined with a measure of rent, or housing services, to compute a national price-rent ratio and a housing return. The Appendix details our construction of these variables in the historical data. Although we report results for the REIT index for completeness, we note here an important disadvantage with this index: it is not a return on a portfolio of residential real estate, since it is primarily comprised of commercial real estate and has no single-family houses. Moreover, the REIT return is an investable asset return, traded in liquid markets, and is arguably better thought of as a stock market index than as an index of home prices.

**Equity Return** The risky capital return $R_{K,t}$ in the model is not comparable to a realistic equity market return because it is unlevered. To make our results comparable to a stock market return, we adjust our risky capital return to account for leverage in a simple way. Specifically, we define the equity return, $R_{E,t}$, to be

$$R_{E,t} = R_{f,t} + (1 + B/E) (R_{K,t} - R_{f,t}),$$

where $B/E$ is the fixed debt-equity ratio and where $R_{K,t}$ is the portfolio return for risky capital given in (3).\(^{15}\) Note that this calculation explicitly assumes that corporate debt in the model is completely exogenous, and must be held in fixed proportion to the value of the firm. (There is no financing decision.) For the results reported below, we set $B/E = 2/3$ to match debt-equity ratios computed in Benninga and Protopapadakis (1990).

\(^{15}\)The cost of capital $R_K$ is a portfolio weighted average of the return on debt $R_f$ and the return on equity $R_e$:

$$R_K = aR_f + (1 - a)R_e$$

$$a = \frac{B}{B + E}.$$
4 Results

4.1 Business Cycle Variables

We begin by presenting a set of benchmark results for aggregate quantities. Panel A of Table 1 presents business cycle moments from U.S. annual data over the period 1953 to 2008. Panel B of Table 1 uses simulated data to summarize the implications for these same moments in our benchmark Model 1, (with “normal” collateral constraints and housing adjustment costs, but no foreign capital). Panel C presents the same results for Model 2, where collateral constraints and housing adjustment costs are low, but where there is still no foreign capital. We report statistics for non-housing consumption, $C$, housing consumption $C_H$, and total consumption (housing and non-housing), denoted $C_T$, as well as for output and investment. In the model, housing consumption is defined $C_H \equiv R_t H_t$, price per unit of housing services times quantity of housing. Because Model 1 and Model 2 generate very similar results for these statistics, for brevity, we discuss only the results for Model 1 (Panel A).

The standard deviation of total aggregate consumption divided by the standard deviation of total output (GDP = $Y_H + Y_C$) is 0.72 in the model, which is close to the 0.70 value found in the data. Also, the level of GDP volatility in the model is close to that in the data. Thus the model produces a plausible amount of aggregate consumption volatility. Broken down by type of consumption, both the model and the data imply that housing and non-housing consumption have about the same volatility. Investment is more volatile than output, both in the model and in the data, but the model produces too little relative volatility: the ratio of the standard deviation of investment to that of output is 1.7 in the model but is 2.9 in the data. But the model does a good job of matching the relative volatility of residential investment to output: in the data the ratio of these volatilities is 4.5, while in the model it is 4.2. Finally, both in the model and the data, residential investment is less correlated with output than is consumption and total investment.

Table 2 shows the model’s implications for the cyclical properties of national house prices.

---

16 With Cobb-Douglas utility, $\varepsilon = 1$, housing and non-housing consumption are proportional. The standard deviations of housing and non-housing consumption are identical in the table because we report moments for Hodrick-Prescott (Hodrick and Prescott (1997)) detrended data.

17 Volatility of investment could be increased by adding stochastic depreciation in capital as in Storesletten, Telmer, and Yaron (2007) and Gomes and Michaelides (2008), or by adding investment-specific technology shocks. We abstract from these additional features in order to maintain a manageable level of complexity in the model.
The housing price indexes in the data are all procyclical, but not as strongly so as in the model. This may be partly attributable to the fact that the national house price indexes in the data are measured with error, whereas in the model they are not. The model implies that both the level of house prices and price-rent ratios are strongly procyclical, regardless of the calibration (Model 1, 2, or 3). Price-rent ratios are less procyclical than the level of prices because rents, in the denominator, are also procyclical. The correlation between output and national price-rent ratio ranges from 0.54 to 0.62 across the three models, whereas, in the data, these correlations are lower but vary substantially by data source and sample, ranging from 0.29 to 0.01. Finally, the model correlation between residential investment, $Y_H$, and national house price-rent ratios, $p^H/R$, is closely aligned with the data.

4.2 Life Cycle Profiles

Turning to individual-level implications, Figure 6 presents the age and income distribution of wealth, both in the model and in the historical data as given by the Survey of Consumer Finance (SCF). The figure shows total household net worth, by age, divided by average wealth across all households, for three income groups (low, medium and high earners).

In both the model and the data, total household net worth is hump-shaped over the life-cycle, and is close to zero early in life when households borrow to finance home purchases. As agents age, wealth slowly accumulates. In the data, it peaks between 60 and 70 years old (depending on the income level). In the model, the peak for all three income groups is about 65 years. After retirement, wealth is drawn down until death. Households in the model continue to hold some net worth in the final years of life to insure against the possibility of living long into old age. A similar observation holds in the data. For low and medium earners, the model gets the average amount of wealth about right, but it under-predicts the wealth of high earners.

The right-hand panels in Figure 6 plot the age distribution of housing wealth alone. Up to age 65, the model produces about the right level of housing wealth for each income group, as compared to the data. In the data, however, housing wealth peaks around age 60 for high earners and around age 67 for low and medium earners, and then declines. The model misses this hump-shape: housing wealth remains high until death. In the absence of a rental market, owning a home is the only way to generate housing consumption. For this reason, agents in the model continue to maintain a high level of housing wealth later in life even as they drawn down financial wealth.
Table 3 exhibits the age and income distribution of housing wealth relative to total net worth, both over time in the SCF data and in Models, 1, 2 and 3. Several features of the Table are notable.

First, the model captures an empirical stylized fact emphasized by Fernández-Villaverde and Krueger (2005), namely that young households hold most of their wealth in consumer durables (primarily housing) and hold very little in financial assets. Indeed, our calibrations imply that young households (age 35 and under), hold slightly more of their wealth as durables than do households in the data.\(^\text{18}\) Second, the model predicts that a financial market liberalization plus an influx of foreign capital leads households of all ages to shift the composition of their wealth towards housing (Model 1 to Model 3). The increase is largest for the young, where the housing wealth-total wealth ratio rises by 35% between Model 1 and Model 3, but the ratio also rises by 15% for households above age 35. The magnitudes of these changes are in line with those in individual-level data from 2001 to 2007, where the comparable increases in the housing wealth-total wealth ratio for the young and old are 37% and 21%, respectively. Finally, Table 3 shows that a financial market liberalization plus an influx of foreign capital leads households of all income levels to increase their share of housing in their wealth portfolios. Between Model 1 and 3, the housing wealth-total wealth ratio for low, medium and high earners increases by 22%, 17% and 17%, respectively. The comparable increases in the data are 15%, 22%, and 25%, respectively, for the period 2001-2007.

### 4.3 Asset Pricing

#### 4.3.1 Return Moments

Table 4 presents asset pricing implications of the model, for the calibrations represented by Models 1, 2 and 3. The statistics reported are averages over 1000 periods. We first discuss the implications of the benchmark Model 1 with normal collateral constraints and transactions costs and no foreign capital. We see that this benchmark matches the historical mean return for the risk-free rate and only slightly overstates the volatility of the risk-free rate. In addition, the model produces a sizable equity return of 5.6% per annum and an annual Sharpe ratio of 0.31, compared to 0.34 in the data.

Turning to the implications for housing assets, the average housing return in the bench-

\(^{18}\)This is likely attributable to the fact that young households in the model borrow more than young households in most waves of the SCF data, so that housing wealth exceeds net worth by an amount that is larger in the model than in the data.
mark Model 1 is 9.87% per annum. This value is close to the average annual housing return from the FoF and Freddie Mac data, equal to 9.89% and 9.11%, respectively. The average housing return from the REIT index is 6.5% per annum. The standard deviation of the housing return in the model is 5.4% per annum, again close to the values from FoF (4.9% to 5.9%, depending on sample) and Freddie Mac (4.32%), but far lower than the REIT stock index (22.5%). The housing return Sharpe ratio for Model 1 is 1.18. The FoF Sharpe ratio is between 1.2 and 1.5, the Freddie Mac Sharpe ratio is 1.4, while the REIT Sharpe ratio is 0.22. Clearly the REIT index behaves quite differently from the other housing return measures, more similar to an index of small stocks. Finally, the far right-hand column of Table 4 gives the mean price-rent ratio in Model 1 as 10.6. In the historical data, these values range from 14.7-15.2 for FoF, 13.7 for Freddie Mac, and 13 for REIT.

How are these statistics affected by a financial market liberalization? Comparing Model 2 to Model 1, we see that both the equity premium and the equity Sharpe ratio fall in an economy that has undergone a financial market liberalization. Specifically, the equity premium falls from 4% to 3.15%, while the Sharpe ratio falls from 0.31 to 0.24. A financial market liberalization lowers the risk-premium on housing assets even more. The housing risk premium is cut almost in half from Model 1 to Model 3, from 8.2% per annum to 4.37%, while the housing Sharpe ratio declines by almost 50% from 1.18 to 0.64. This decline in the riskiness of both housing and equity reflects the greater amount of risk-sharing possible after a financial market liberalization, as we discuss below.

The average price-rent ratio is about 12% higher in Model 2 than it is in the benchmark Model 1. Recalling that price-rent ratios are procyclical (Table 2), this implies that, when the economy is expanding, a financial market liberalization adds fuel to the fire in the housing market, driving up price-rent ratios more than what would occur as the result of an economic boom alone. But a financial market liberalization also leads to a sharp increase in equilibrium interest rates, which by itself decreases $p^H/R$. Indeed, the endogenous risk-free interest rate more than doubles in Model 2 to 4.14% per annum, from 1.67% in Model 1. This occurs because the relaxation of borrowing constraints and housing transactions costs drives up the demand for credit to purchase homes and for home equity extraction. Note also that there are no differences in average annual rental growth rates across Models 1, and 2 and Model 3.\(^\text{19}\) It follows that the increase in price-rent ratios following a financial market

\(^{19}\text{Because the statistics for each model are computed from averages across 1000 periods, they give the long-run annualized values of rental growth. This is the same across all three models because it is pinned down by the steady state growth of technology, which is the same in each model.}\)
liberalization is entirely attributable to the decline in risk-premia, which more than offset the rise in equilibrium interest rates.

In Model 3 we add an infusion of exogenous capital calibrated to match the increase in foreign ownership of U.S. Treasuries over the period 2000-2007. The last column of Table 4 shows that the average price-rent ratio is 24 percent higher in Model 3 than in the benchmark Model 1. This value represents more than half of the increase in three out of four measures of national house price-rent ratios over the 2000-2007 period, which can be found in Figure 1 to be 31% for the FoF and Freddie Mac price-rent ratios, 43% for the CS price-rent ratio, and 87% for the REIT price-dividend ratio. Moreover, in Model 3, the rise in foreign ownership of U.S. debt generates a decline in equilibrium interest rates of greater than 50 percent: equilibrium interest rates fall from 4.14% in Model 2 to 1.22% in Model 3, a percentage decline that is approximately commensurate with the decline in real (mortgage) interest rates over the period 2000-2007. Figure 3 shows the decline in nominal rates; subtracting off inflation to compute a real rate, we observe that the 30-year real mortgage interest rate fell from 5.5% in 2000 to 2% in 2007/early 2008, while the 5-year real Treasury bond rate fell from 3.5% to -1.0% over this period. This finding underscores the importance of foreign capital in keeping interest rates low during a financial market liberalization. Without a foreign capital infusion, the looser collateral requirements and lower housing transactions costs generate an increase in equilibrium interest rates, as households endogenously respond to the improved risk-sharing opportunities afforded by financial market liberalization.

Both the housing return risk-premium and housing Sharpe ratio are lower in Model 3 than that in Model 1. Taken together, this implies that a financial market liberalization plus foreign capital infusion leads to a decline in the riskiness of the underlying housing asset. The story is different for equity, however. The Sharpe ratio for equity is higher in Model 3 than in Model 1, as is the equity premium. Although the equity Sharpe ratio and equity risk-premium are lower in Model 2 than in Model 1, they rise substantially from Model 2 to Model 3, so much so that their values in Model 3 now exceed those in Model 1. This occurs because the exogenous supply of capital in the bond market that is included in Model 3 drives up leverage in the domestic economy, which increases the equity premium. In addition, the rise in foreign capital in the bond market means that more domestic saving must take place in the risky asset, which increases the exposure of domestic households to systematic risk in the equity market. Domestic savers are in effect “crowded out” of the bond market by foreign governmental holders who are willing to hold the safe asset at any price. In equilibrium, the equity market risk-premium and Sharpe ratio rise from Model 2 to
Model 3 as domestic savers shift the composition of their financial assets towards the risky security. Findings (not reported) verify that this generates an increase in volatility of the SDF, $\frac{\beta_{A+1}}{A}$, thereby explaining the rise in the equity Sharpe ratio.

Note that the housing risk premium and housing Sharpe ratio also rise with the infusion of foreign capital (compare Model 3 to Model 2). Unlike the case for equity, however, the rise in risk premia from Model 2 to Model 3 is not enough to fully offset the decline in risk premia from Model 1 to Model 2. This finding relates to an existing literature that attempts to estimate the impact of interest rates changes on housing price-rent ratios using partial equilibrium models of the housing market (e.g., Titman (1982)). In general equilibrium, a foreign capital infusion pushes the risk-premium up at the same time that it pushes the risk-free rate down. It follows that the net effect on the price-rent ratio is, in general, ambiguous. This result is missed in partial equilibrium investigations where the interest rate is changed holding fixed the risk-premium. In a general equilibrium setting it is far more challenging to explain a rise in house prices relative to fundamentals by appealing to a decline in interest rates.

4.3.2 Predictability in Asset Returns

Table 5 presents the model’s implications for predictability in equity and housing markets by the price-dividend ratio and price-rent ratio, respectively. Table 5 shows predictability of returns on these assets, and either dividend or rent growth, over long horizons. Table 6 shows predictability results for long horizon excess returns.

In model generated data, both the raw equity return and the excess return are forecastable over long horizons, consistent with evidence from U.S. stock market returns.\(^{20}\) High price-dividend ratios forecast low future equity returns (Table 5, right column) and low excess returns (Table 6) over horizons ranging from 1 to 30 years. Compared to the data, the model produces about the right amount of forecastability in excess equity returns (Table 6), but produces too much forecastability of dividend growth. This is not surprising since, unlike an endowment/exchange economy where dividends are set exogenously, in the model here both profits and the value of the firm respond endogenously to the persistence of aggregate shocks.\(^{21}\) Moreover, firms in the model have no dividend smoothing motive of the type

\(^{20}\)A large body of research in asset pricing finds evidence that stock returns are predictable over long horizons. See, for example, the summary evidence in Cochrane (2005), Chapter 20, and Lettau and Ludvigson (2009).

\(^{21}\)The model also produces too much predictability in raw returns (Table 5). This happens because,
suggested by Cochrane (1994).

The left panels of Tables 5 and 6 show the predictability results for housing returns. Both excess and raw housing returns are forecastable over long-horizons. In particular, high price-rent ratios forecast low future housing returns, consistent with empirical evidence in the bottom left panels of Table 5 and Table 6 (see also Campbell, Davis, Gallin, and Martin (2006)). High price-rent ratios also forecast lower future excess returns to housing assets, or risk-premia (Table 6). As price-rent ratios rise with the economy so do collateral values, which improves risk-sharing and lowers risk-premia.

It is often suggested that increases in price-rent ratios reflect an expected increase in future housing fundamentals, such as rental growth, over some subsequent horizon. For example, in a partial equilibrium setting where discount rates are held constant, higher house prices relative to fundamentals can only be generated by higher implicit rental growth rates in the future (Sinai and Souleles (2005), Campbell and Cocco (2007)). This partial equilibrium analysis, however, ignores the endogenous response of both discount rates and residential investment to economic growth. In the general equilibrium setting of this model, positive economic shocks drive discount rates down and simultaneously stimulate both greater housing demand and greater residential investment. This implies that the cost of future housing services (rent) is forecast to be lower as the housing supply expands. For this reason, high price-rent ratios forecast lower future rental growth, rather than higher. It follows that high price-rent ratios in expansions must entirely reflect lower future returns to housing, in part driven by lower risk-premia as collateral values rise with the economy. Although future rental growth is expected to be lower, price-rent ratios are still high because the decline in future housing returns more than offsets the expected fall in future rental growth. Accordingly, predictable variation in housing returns must account for more than 100 percent of the variability in price-rent ratios.

Note: although the model generates about the right amount of predictability in excess returns, it generates too much predictability in interest rates. Positive economic shocks increase consumption but not as much as income, thus saving and investment also rise. This pushes down expected rates of return to saving, implying that procyclical increases in price-dividend ratios forecast lower future interest rates, as well as lower future excess returns.

22See also the discussion in Campbell, Davis, Gallin, and Martin (2006) using the Gordon growth model as a motivation.
4.4 Macroeconomic Effects of Financial Market Liberalization

A growing body of academic work has argued that house price increases and financial liberalization are likely to stimulate a boom in consumption, and therefore have a stimulative affect on the economy as a whole (for example, Muellbauer and Murphy (1990), Mishkin (2007), and Muellbauer (2007)). Others have studied the effect of house price changes on consumption in household level data and found a positive correlation (e.g., Campbell and Cocco (2007)). In this section we study the model’s implications for so-called housing “wealth effects” on consumer spending and on other macroeconomic variables.

There are at least two ways in which the theoretical model explored here can shed new light on this question. First, causal relationships between housing wealth and consumption are difficult to assess empirically because housing wealth is not an exogenous variable to which consumption merely responds, though it is often treated as such in empirical analysis. The model environment studied here offers an advantage in this regard because it allows us to control for this endogeneity directly by studying how consumption is influenced by factors exogenous to our model, such as changes in collateralized borrowing constraints and housing transactions costs. These experiments give us some idea of the causality running from wealth to consumption and not the other way around. Second, our current theoretical understanding of housing wealth effects is based almost entirely on partial equilibrium life cycle models, as in the literature referenced above. The framework studied here allows us to ask how the conclusions of this literature change when we account for general equilibrium considerations in the relation between housing wealth and the economy. Here we focus on changes in housing wealth that arise from a financial market liberalization.

Figure 7 presents three panels that illustrate how a financial market liberalization affects macroeconomic variables. In each panel, the dynamics for transitioning between Model 1 and Model 2 are displayed. The model economy is initially started in the steady state of Model 1 and gradually transitions to the steady state of Model 2 after households learn that the parameters of the economy are now those from Model 2. In the figure, all quantities for Model 2 are expressed relative to the corresponding quantities for Model 1.

A financial market liberalization leads to a short-run boom in aggregate consumption, consistent with the implications of partial equilibrium life-cycle models. The general equilibrium framework studied here, however, does not imply that a financial market liberalization is stimulative for the economy as a whole, despite the boom in consumption. This is because a decline in collateralized borrowing constraints and housing transactions costs drives the endogenous interest rate up (Table 4), which chokes off investment. As a consequence, the
immediate impact on investment is negative and on GDP is close to zero. Moreover, in
the long-run, a financial market liberalization leads to less consumption and less aggregate
output as capital accumulation declines in the wake of lower aggregate saving rates.

The middle panel of Figure 7 shows what happens to the consumption of individual
households distinguished by age. The youngest households increase their consumption the
most, immediately upon the onset of a financial market liberalization. By contrast, the
oldest households (retirees) increase consumption very little. At first, these results appear
to differ from the findings of Campbell and Cocco (2007) who report that changes in house
prices have their smallest impact on young households, in UK household-level data. As
these authors emphasize, however, many young households in their sample rent rather than
own, in contrast to older households. In the model of this paper, all households own their
homes. When Campbell and Cocco (2007) study simulated data from a life-cycle model
that allows them to explicitly control for the selection bias attributable to the endogeneity
of homeowner status, the model predicts that house price changes have a larger effect on
the consumption of young homeowners than on the consumption of old homeowners. There
as here, the effect occurs because young households are relatively more constrained, so that
looser collateral constraints and lower housing transactions costs have the greatest influence
on their spending.

The third panel of Figure 7 shows the differential consumption response of net savers
and net borrowers to a financial market liberalization. Immediately following the onset of
the financial market liberalization, net borrowers and net lenders raise their consumption,
by about the same percentage amount. As the transition proceeds, however, the stimula-
tive effect of the financial liberalization is entirely attributable to the higher consumption of
savers, rather than borrowers. This occurs because, unlike the partial equilibrium setting, all
households raise their consumption initially as part of an endogenous response to improved
risk-sharing opportunities, which leads to less precautionary saving. As the transition pro-
ceeds, the consumption of net lenders continues to grow for several years and remains high,
whereas the consumption of borrowers falls quickly after the initial period. Lenders benefit
from the rise in endogenous interest rates throughout the transition, while borrowers suffer
for the same reason. Twenty years out (when the new steady state of Model 2 has been
reached), there is a switch: the consumption of borrowers is about the same as it was in
Model 1, while the consumption of lenders is lower than in Model 1. This is because the
steady state aggregate capital stock is lower in Model 2 than in Model 1, which reduces the
total asset cash-flow of the wealthy more than that of borrowers.
Table 3 showed that a financial market liberalization lowers risk premia in both housing and equity assets. Let $C_T$ denote total (housing plus non-housing) consumption. Table 7 shows that the decline in risk premia coincides with a decline in the cross-sectional variance of consumption growth as risk-sharing increases. The cross-sectional standard deviation of both the individual consumption share $C_{iT,t}^{a} / C_{T,t}$, and of individual consumption growth $\ln C_{T,a,t}^{i} - \ln C_{T,a-1,t-1}^{i}$ are both lower in Model 2 than in Model 1. Moreover, the age dispersion in the consumption-GDP ratio also declines (bottom panel). Risk-sharing improves because the decline in credit constraints and housing transactions costs increases both access to credit and to the collateral required to obtain credit, thereby allowing heterogeneous households to share more of their idiosyncratic risks. As a result, consumption inequality falls.

By contrast, these same measures of consumption inequality rise from Model 2 to Model 3. The foreign presence in the bond market forces domestic savers to bear more aggregate risk, which drives up risk-premia (Table 4). In effect, foreign capital makes existing financial markets more incomplete because the foreign holders’ perfectly inelastic demand for the risk-free asset forces some domestic savers out of the bond market, reducing the availability of this asset for insurance. The resulting rise in financial market incompleteness increases consumption inequality.

This shows that the fall in consumption inequality resulting from a financial market liberalization is offset by a rise in inequality resulting from foreign demand for the risk-free asset. In the calibration here, the latter more than offsets the former so that the net change in consumption inequality is small but positive moving from Model 1 (benchmark) to Model 3 (financial liberalization plus foreign capital).

What about wealth inequality? Unlike consumption inequality, a financial market liberalization and foreign demand for the risk-free asset have reinforcing effects on financial wealth inequality. Figure 8 shows the Gini Index for inequality in total net worth, and for net worth decomposed into financial wealth and housing wealth, for Models 1, 2, and 3 (right scale), as well as the Gini indexes based on the SCF data for the years 2001, 2004 and 2007 (left scale). The Figure compares the change in the wealth Gini index from 2001 to 2007 with the change in the model Gini index between Models 1, 2 and 3.

None of the models explain the extreme degree of wealth inequality in the data.\(^{23}\) (The

\(^{23}\)It is understood (e.g., Cagetti and Nardi (2008)) that general equilibrium models with heterogeneous agents cannot explain the extreme concentration of wealth in the upper tail of the distribution unless these
level of the Gini index in the model is lower than that in the data.) But the model does a good job of capturing recent trends in wealth inequality. In the data, the Gini index for wealth rises by almost 20 percent between 2001 and 2007. In the model, the Gini for financial wealth increases by about 7 percent as a result of financial market liberalization (Model 1 to Model 2), and by another 15 percent as a result of foreign governmental demand for the safe asset (Model 2 to Model 3). In addition, both in the model and in the data, housing wealth inequality increases far less than financial wealth inequality: the Gini index for housing wealth in the SCF data is flat between 2001 and 2007, while in the model the Gini index for housing wealth falls slightly between Model 1 and Model 3. The rise in the Gini index for total wealth (financial plus housing) from Model 1 to Model 3 is comparable to that in the data from 2001 to 2007.

Why do a financial market liberalization and a foreign capital infusion have reinforcing affects on financial wealth inequality but offsetting affects on consumption inequality? A financial market liberalization relaxes the constraints of households, both by making it easier to borrow against home equity and by making it less costly to transact. This reduces consumption inequality, and to a lesser extent, housing inequality. At the time, financial wealth inequality rises because, as some (mostly young) households take advantage of the market liberalization to increase current consumption, their net worth position becomes more negative. At the same time, older households who are primarily concerned about saving for retirement are now able to earn a higher return on their savings, which drives their wealth more positive. As a consequence, a financial market liberalization increases wealth inequality even though it decreases consumption inequality.

By contrast, foreign demand for the risk-free asset increases both types of inequality because it reduces risk-sharing among domestic agents. This pushes up the risk-premium on risky assets and the average rate of return to saving, which increases disparities in both wealth and consumption between young borrowers and older savers. Because a financial market liberalization and a foreign capital infusion have reinforcing affects on financial wealth inequality but offsetting affects on consumption inequality, the model has the potential to explain why wealth inequality has risen far more than consumption inequality in recent decades (Heathcoat, Perri, and Violante (2009)).

\footnote{Heathcoat, Perri, and Violante (2009) study income and consumption inequality directly, and show that consumption inequality has risen far less than income inequality (see also Krueger and Perri (2006)). But their results for saving inequality strongly suggest that wealth inequality has risen more than consumption inequality over time.}
5 Conclusion

In this paper we have studied the macroeconomic and individual-level consequences of fluctuations in housing wealth and housing finance, in a general equilibrium model with heterogeneous agents and limited risk-sharing. We have focused much of our investigation on studying the impact of changes in housing collateral requirements, changes in housing transactions costs, and an exogenous infusion of foreign capital into U.S. bond markets.

The model implies that national house price-rent ratios may fluctuate considerably in response to changes in these factors, as well as in response to movements in the aggregate economy. Price-rent ratios fluctuate because both risk-premia and interest rates respond endogenously to changes in housing finance and to the state of the economy. We found that the general equilibrium environment is particularly important for understanding some features of these results. For example, the model implies that procyclical increases in national house price-rent ratios must reflect lower future housing returns rather than higher future rents, a finding that is difficult to understand without taking into account the endogenous response of residential investment to positive economic shocks.

A financial market liberalization drives risk premia in both the housing and equity market down and shifts the composition of wealth for all age and income groups towards housing. It also leads to a a short-run boom in aggregate consumption. A financial market liberalization is not stimulative for the economy as a whole, however, because the higher equilibrium interest rates that accompany it lead to a short-run bust in investment that offsets the consumption boom.

We also found that—in contrast to a financial market liberalization—an infusion of foreign capital by governmental holders lowers interest rates but raises consumption and wealth inequality, as well as risk-premia in both housing and equity assets. This occurs because foreign governmental holders’ highly inelastic demand for the safe asset crowds out domestic savers from the bond market, thereby exposing them to greater systematic risk in the risky asset market.

Finally, the model implies that a financial market liberalization and foreign capital infusion have reinforcing effects on financial wealth inequality (and drive it up), but have offsetting effects on consumption inequality. As a consequence, changes in these economic factors have the potential to explain why wealth inequality has risen more than consumption inequality in recent times.
Appendix

This appendix describes how we calibrate the stochastic shock processes in the model, describes the historical data we use to measure house price-rent ratios and returns, and describes our numerical solution strategy.

Calibration of Shocks

The aggregate technology shock processes $Z_C$ and $Z_H$ are calibrated following a two-state Markov chain, with two possible values for each shock, \( \{Z_C = Z_{Cl}, Z_C = Z_{Ch}\} \), \( \{Z_H = Z_{Hi}, Z_H = Z_{Hh}\} \), implying four possible combinations:

\[
\begin{align*}
Z_C &= Z_{Cl}, & Z_H &= Z_{Hi} \\
Z_C &= Z_{Ch}, & Z_H &= Z_{Hi} \\
Z_C &= Z_{Cl}, & Z_H &= Z_{Hh} \\
Z_C &= Z_{Ch}, & Z_H &= Z_{Hh}.
\end{align*}
\]

Each shock is modeled as,

\[
\begin{align*}
Z_{Cl} &= 1 - e_C, & Z_{Ch} &= 1 + e_C \\
Z_{Hi} &= 1 - e_H, & Z_{Ch} &= 1 + e_H,
\end{align*}
\]

where $e_C$ and $e_H$ are calibrated to roughly match the volatilities of $Y_C$ and $Y_H$ in the data.

We assume that $Z_C$ and $Z_H$ are independent of one another. Let $P^C$ be the transition matrix for $Z_C$ and $P^H$ be the transition matrix for $Z_H$. The full transition matrix equals

\[
P = \begin{bmatrix}
p_{ll}^H & p_{lh}^H & p_{ch}^H & p_{hh}^H \\
p_{hl}^H & p_{lh}^H & p_{ch}^H & p_{hh}^H
\end{bmatrix}.
\]

where

\[
P^H = \begin{bmatrix}
p_{ll}^H & p_{lh}^H \\
p_{hl}^H & p_{hh}^H
\end{bmatrix} = \begin{bmatrix}
p_{ll}^H & 1 - p_{ll}^H \\
1 - p_{hh}^H & p_{hh}^H
\end{bmatrix},
\]

and where we assume $P^C$, defined analogously, equals $P^H$. We calibrate values for the
matrices as

\[
P^C = \begin{bmatrix}
.60 & .40 \\
.25 & .75 \\
\end{bmatrix}
\]

\[
P^H = \begin{bmatrix}
.60 & .40 \\
.25 & .75 \\
\end{bmatrix}
\Rightarrow
\]

\[
P = \begin{bmatrix}
.36 & .24 & .24 & .16 \\
.15 & .45 & .10 & .30 \\
.15 & .10 & .45 & .30 \\
.0625 & .1875 & .1875 & .5625 \\
\end{bmatrix}
\]

With these parameter values, we roughly match the first-order autocorrelation of \(Z^C\) (which we estimate as a Solow residual), and the average length of expansions divided by the average length of recessions (equal to 2.2 in NBER data from over the period 1854-2001). For the latter, we define a recession as the event with joint probability \(p^H p^C_{ll} = 0.36\), so that a recession persists on average for \(1/(1 - .36) = 1.56\) years. If we define an expansion as the event given by the sum of joint probabilities \(p^H p^C_{hh} = 0.75\), so that an expansion will persist on average for \(1/(1 - .75) = 4\) years. Thus the average length of expansions relative to that of recessions is then \(4/(1.46) = 2.56\) years.

Idiosyncratic income shocks follow the first order Markov process \(\log(Z^i_{a,t}) = \log(\frac{Z^i_{a-1,t-1}}{\bar{Z}^i_{C,t}}) + \epsilon^i_{a,t}\), where \(\epsilon^i_{a,t}\) takes on one of two values in each aggregate state:

\[
\epsilon^i_{a,t} = \begin{cases} 
\sigma_E & \text{with Pr} = 0.5 \\
-\sigma_E & \text{with Pr} = 0.5 \\
\end{cases}
, \quad \text{if } Z^C_{t} \geq \bar{E}(Z^C_{t})
\]

\[
\epsilon^i_{a,t} = \begin{cases} 
\sigma_R & \text{with Pr} = 0.5 \\
-\sigma_R & \text{with Pr} = 0.5 \\
\end{cases}
, \quad \text{if } Z^C_{t} < \bar{E}(Z^C_{t})
\]

\[
\sigma^2_R > \sigma^2_E
\]

Housing Price and Return Data

Our first measure of house prices uses aggregate housing wealth for the household sector from the Flow of Funds (FoF) (which includes the part of private business wealth which is residential real estate wealth) and housing consumption from the National Income and Products Accounts. The price-rent ratio is the ratio of housing wealth in the fourth quarter of the year divided by housing consumption summed over the year. The return is constructed as housing wealth in the fourth quarter plus housing consumption over the year divided by
housing wealth in the fourth quarter of the preceding year. We subtract CPI inflation to
express the return in real terms and population growth in order to correct for the growth
in housing quantities that is attributable solely to population growth. (Since the return is
based on a price times quantity, it grows mechanically with the population. In the model,
population growth is zero.) The advantage of this housing return series is that it is for
residential real estate and for the entire population. The disadvantages are that it is not a
per-share return (it has the growth in the housing stock in it, which we only partially control
for by subtracting population growth), it is not an investable asset return, and it does not
control for quality changes in the housing stock.\footnote{There is also measurement error in how the Flow of Funds imputes market prices to value the housing
stock as well as in how the BEA imputes housing services consumption for owners. These errors, however, may be more likely to affect the level of the index than the change in the index.}

Our second series combines the Freddie Mac Conventional Mortgage House Price index
for home purchases (Freddie Mac) and the rental price index for shelter from the Bureau of
Labor Statistics (BLS). The price-rent ratio is the ratio of the price index in the last quarter
of the year, divided by the rent index averaged over the quarters in the year. Since the level
of the price-rent ratio is indeterminate (given by the ratio of two indexes), we normalize the
level of the series by assuming that the 1970 Freddie Mac price-rent ratio is the same as that
of the FoF price-rent ratio in 1970. The return is the price index plus the rent divided by
the price index at the end of the previous year. We subtract CPI inflation to express the
return in real terms. The FoF return has a correlation of 82% with the Freddie Mac return
over 1973-2008. Since the Freddie Mac price index is a repeat-sales price index, it controls
for quality changes in the housing stock (price changes are computed on the same house). It
also is a per-share returns (no quantities). Alternative repeat-sale price indices such as the
Freddie Mac CMHPI which includes refinancing and purchases, or the OFHEO house price
index, deliver similar results. The same is true if we use the BLS rental index for housing
instead of shelter. (The rental index for housing includes utilities while the rental price index
for shelter excludes them).

The third method uses an index of Real Estate Investment Trusts (REIT). The REIT
index is a price-dividend ratio of publicly traded stocks of companies which invest solely in
real estate, and which have to distribute 90% of their revenue in the form of dividends. The
index consists of both residential and commercial REITs, with a dominance of the latter.
It is available from 1972 to 2008. We construct annual returns by compounding monthly
returns over the year. We subtract realized CPI inflation from realized housing returns to
form monthly real housing returns. As mentioned, the disadvantage with this index is that it is not a return on a portfolio of residential real estate, since it is primarily comprised of commercial real estate and has no single-family houses. Moreover, the REIT return is an investable asset return, traded in liquid markets, and is arguably better thought of as a stock market index than as an index of home prices. The correlation between the REIT index return and the overall stock market return is 65% over 1973-2008 whereas the correlation with the stock market with the FoF and Freddie Mac housing returns is 42% and 28%.

The fourth series is the ratio of the Case-Shiller national house price index to the Bureau of Labor Statistics’s price index of shelter (CS). The Case-Shiller price index is also a repeat-sales price index, which receives a lot of attention in the literature. It is available from 1987 on a quarterly basis.

**Numerical Solution Procedure**

The solution strategy is related to Krusell and Smith (1998) and Storesletten, Telmer, and Yaron (2007). The strategy consists of solving a individual’s problem in partial equilibrium given some set of beliefs about the aggregate behavior of the economy. The economy is then simulated for many households and this simulation is used to create a new set of beliefs about aggregate behavior for the households. The individual’s expectations are rational once this process converges and the aggregate economy behaves as the individual believes it should.

The individual’s problem is solved using dynamic programming. The vector of aggregate state variables is \( \mu_t = (Z_t, K_{C,t}, K_{H,t}, H_t, p_t^H, q_t) \); this selection will be justified below. The vector of individual state variables is given by \( \mu_{t,i} = (Z_{t,i}, W_t, H_t) \). In order to solve the dynamic programming problem, the individual must know \( \mu_{t+1} \) as a function of \( \mu_t \) and realized aggregate shocks \( Z_{t+1} \). For this it is sufficient for the individual to have beliefs about four quantities: (1) \( K_{C,t+1} \), (2) \( p_{t+1}^H \), (3) \( q_{t+1} \), and (4) \( E_t[\frac{\beta^{\Lambda_{t+1}}}{\Lambda_t}(Q_{C,t+1} - Q_{H,t+1})] \). (2) and (3) give us \( p_{t+1}^H \) and \( q_{t+1} \); \( H_{t+1} = (1 - \delta_H)H_t + Y_{H,t} \); using (1) we can solve for \( I_{C,t} \); using (4) and \( I_{C,t} \) we can solve for \( I_{H,t} \); using \( I_{C,t} \) and \( I_{H,t} \) we can solve for \( K_{H,t+1} \); doing this one step forward we can solve for \( I_{C,t+1} \) and \( I_{H,t+1} \). Combining \( I \) with the Firm’s Euler Equation gives us \( Q \) and \( D \) which allows us to compute risky returns.

The above problem can be summarized as:

\[
V_{t,i} = \max_{H_{t+1},q_{t+1},B_{t+1}} U(C_t, H_t) + \beta \pi_i E_t[V_{t+1,i+1}] \quad s.t. \\
C_t + B_{t+1}q_t + \theta_{t+1}(V_{C,t} + V_{H,t}) + p_t^H H_{t+1} + X_t = W_t + Y_t + p_t^h (1 - \delta_H)H_t
\]
\[ W_t = \theta(V_{C,t} + V_{H,t} + D_{C,t} + D_{H,t}) + B_t \]

\[ W_t + Y_t + p_t^H (1 - \delta_H) H_t - C_t - X_t \geq \omega p_t^H H_{t+1} \]

\[ \mu_{t+1} = \Gamma(\mu_t, Z_{t+1}) \]

where \( \Gamma \) is described by the previous paragraph. This dynamic programming problem is quite complex numerically because of a large number of state variables but is otherwise straightforward.

Next we simulate the economy for a large number of households using the policy functions from the dynamic programming problem. One complication is that markets must clear each period; this is why \( p_t^H \) and \( q_t \) are convenient state variables. The individual’s policy functions are a response to a menu of prices, thus simulation involves choosing \( p_{t+1}^H \) and \( q_{t+1} \) which clear markets next period. The continuum of households born each period is approximated by a number large enough to insure that the mean and volatility of aggregate variables is not affected by idiosyncratic shocks. We check this by simulating the model for successively large numbers of households in each age cohort and checking whether the mean and volatility of aggregate variables changes.

Using data from the simulation we calculate (1)-(4) as linear functions of \( \mu_t \). In particular, for every \( Z_t \) and \( Z_{t+1} \) combination we regress (1)-(4) on \( K_{C,t}, K_{C,t+1}, K_{H,t}, H_t, p_t^H, \) and \( q_t \). This is used to calculate a new \( \Gamma \) which is used to solve the individual’s problem. We repeat this procedure until all relevant aggregate variables have converged.

Finally, we must check whether the aggregate state variable vector is sufficient for this problem. Zhang (2005) report high \( R^2 \) in the \( \Gamma \) regressions as evidence of the state being sufficient. Krusell and Smith (1998) suggest adding extra variables to the state space to show that the solution does not change.
References


The figure compares four measures of the price-rent ratio. The first measure ("Flow of Funds") is the ratio of residential real estate wealth of the household sector from the Flow of Funds to aggregate housing services consumption from NIPA. The second measure ("Freddie") is the ratio of the Freddie Mac Conventional Mortgage Home Price Index for purchases to the Bureau of Labor Statistics’s price index of shelter (which measures rent of renters and imputed rent of owners). The third series ("Case-Shiller") is the ratio of the Case-Shiller national house price index to the Bureau of Labor Statistics’s price index of shelter. The fourth series is the price-dividend ratio on the All Reits index published by the FTSE NAREIT US Real Estate Index Series. All indices are normalized to a value of 100 in 2000.Q4. The data are quarterly from 1970.Q1 until 2008.Q4. The REITs series starts in 1972.Q4 and the Case-Shiller series in 1987.Q1.
Figure 2: Fixed-rate Mortgage Rate and Ten-Year Constant Maturity Treasury Rate

The solid line plots the 30-year Fixed-Rate Mortgage rate (FRM); the dashed line plots the ten-year Constant Maturity Treasury Yield (CMT). The FRM data are from Freddie Mac’s Primary Mortgage Market Survey. They are average contract rates on conventional conforming 30-year fixed-rate mortgages. The CMT yield data are from the St.-Louis Federal reserve Bank (FRED). The data are monthly from April 1971.4 until February 2009.
Figure 3: Initial Fees and Charges

The solid line plots the initial fees and charges on all mortgages. They are expressed as a percentage of the value of the loan, and averaged across all mortgage contracts. The data are from the Federal Housing Financing Board’s Monthly Interest Rate Survey. The data are monthly from January 1973 until January 2009.
Figure 4: Foreign Holdings of US Treasuries

Figure 5: Foreign Holdings of U.S. Treasuries and U.S. Agency Debt Relative to U.S. GDP

The figure plots foreign holdings of U.S. Treasury securities (T-bills, T-notes, and T-bonds) and the sum of U.S. treasuries and U.S. Agency debt (e.g., debt issued by Freddie Mac and Fannie Mae), relative to GDP. The first two series report only long-term debt holdings, while the other two series add in short-term debt holdings. Since no short-term debt holdings are available before 2002, we assume that total holdings grow at the same rate as long-term holdings before 2002. Data are from the Treasury International Capital System of the U.S. Department of the Treasury. The foreign holdings data are available for December 1974, 1978, 1984, 1989, 1994, 1997, March 2000, and annual for June 2002 through June 2008. Nominal GDP is from the National Income and Product Accounts, Table 1.1.5, line 1.
The figure plots net financial wealth ("Wealth") by age in the left columns and housing wealth ("Housing") by age in the right columns. The top panels are for the Data, the middle panels for Model 1, and the bottom panels for Model 2. We use all 9 waves of the Survey of Consumer Finance (1983-2007, every 3 years). We construct housing wealth as the sum of primary housing and other property. We construct net financial wealth as the sum of all other assets (bank accounts, bonds, IRA, stocks, mutual funds, other financial wealth, private business wealth, and cars) minus all liabilities (credit card debt, home loans, mortgage on primary home, mortgage on other properties, and other debt). We express wealth on a per capita basis by taking into account the household size, using the Oxford equivalence scale for income. For each age between 22 and 81, we construct average net financial wealth and housing wealth using the SCF weights. To make information in the different waves comparable to each other and to the model, we divide housing wealth and net financial wealth in a given wave by average net worth (the sum of housing wealth and net financial wealth) across all respondents for that wave. We do the same in the model. The Low Earner label refers to those in the bottom 25% of the income distribution, where income is wage plus private business income. The Medium Earner group refers to the 25-75 percentile of the income distribution, and the High Earner is the top 25%. The model computations are obtained from a 1,000 year simulation. The "Model 1" is the model with normal moving costs and collateral constraints, "Model 2" reports on the model with low moving costs and loose constraints. In particular, fixed and variable costs go from 5% to 2.5% and the down-payment goes from 50% to 10%.
Figure 7: The Macroeconomic Affects of Financial Market Liberalization

The figure plots transitional dynamics between Model 1, the model with tight borrowing constraints and high transaction costs and Model 2 with looser borrowing constraints and lower transaction costs. The lines trace the first 50 years of the transition from the dynamic steady state of Model 1 to the dynamic steady state of Model 2. All quantities are expressed relative to the corresponding quantities from Model 1. In particular, we start in the (dynamic) steady state of Model 1 and evaluate the policy functions at values for the state variables that are typical for Model 1 (obtained by averaging over a 1,000-period simulation of Model 1). Households learn at time 1 that the parameters of the economy are now those from Model 2. They make decisions based on the policy functions of Model 2. These decisions gradually change the values of the state variables and move the economy towards the steady state of Model 2. The plots are averages over 40 simulations. The first panel reports aggregate consumption, GDP, and investment. The second panel reports consumption by age group. The last panel reports consumption for net borrowers and net lenders.
Figure 8: Wealth Inequality in Model and Data

The figure plots the Gini coefficient of total wealth (left panel), financial wealth (middle panel), and housing wealth (right panel). In each panel, the Gini in the data is measured against the left axis, while the Gini in the model is measured against the right axis. The data are shown for the years 2001, 2004, and 2007, indicated by the solid line with dots. For the model, we report the steady state Gini values in Models 1, 2 (star), and 3 (square). The right axes are chosen so that the Model 1 Gini coincides with the value in Model 1. The data are from three waves of the Survey of Consumer Finance. We construct housing wealth as the sum of primary housing and other property. We construct financial wealth as the sum of all other assets (bank accounts, bonds, IRA, stocks, mutual funds, other financial wealth, private business wealth, and cars) minus all liabilities (credit card debt, home loans, mortgage on primary home, mortgage on other properties, and other debt). We express wealth on a per capital basis by taking into account the household size, using the Oxford equivalence scale for income. We use the SCF weights to calculate the Gini coefficients. The “Model 1” is our the model with normal moving costs and collateral constraints, “Model 2” reports on the model with low moving costs and loose constraints. In particular, fixed and variable costs go from 5% to 2.5% and the down-payment goes from 25% to 1%. Finally, “Model 3” is the same as Model 2 except with a positive demand for bonds from foreigners, equal to 19% of GDP.
Table 1: Real Business Cycle Moments

Panel A denotes business cycle statistics in annual post-war U.S. data (1953-2008). The data combine information from NIPA Tables 1.1.5, 3.9.5, and 2.3.5. Output \( (Y = Y_C + Y_H) \) is gross domestic product minus net exports minus government expenditures. Total consumption \( (C_T) \) is total private sector consumption (housing and non-housing). Housing consumption \( (C_H = R\times H) \) is consumption of housing services. Non-housing consumption \( (C) \) is total private sector consumption minus housing services. Housing investment \( (Y_h) \) is residential investment. Non-housing investment \( (I) \) is the sum of private sector non-residential structures, equipment and software, and changes in inventory. Total investment is denoted \( I_T \) (residential and non-housing). For each series in the data, we first deflate by the disposable personal income deflator, We then construct the trend with a Hodrick-Prescott (1980) filter with parameter \( \lambda = 100 \). Finally, we construct detrended data as the log difference between the raw data and the HP trend, multiplied by 100. The standard deviation (first column), correlation with GDP (second column), and the first-order autocorrelation are all based on these detrended series. The autocorrelation AC is a one-year correlation in data and model. The share of GDP (fourth column) is based on the raw data. Panel B denotes the same statistics for the Model 1 with normal moving costs and collateral constraints. Panel C reports on Model 2 with low moving costs and loose constraints. In particular, fixed and variable costs go from 5% to 2.5% and the down-payment goes from 25% to 1%.

<table>
<thead>
<tr>
<th>Panel A: Data (1953-2008)</th>
<th>st.dev.</th>
<th>corr. w. GDP</th>
<th>AC</th>
<th>share of gdp</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Y )</td>
<td>2.78</td>
<td>1.00</td>
<td>0.46</td>
<td>1.00</td>
</tr>
<tr>
<td>( C_T )</td>
<td>1.78</td>
<td>0.91</td>
<td>0.62</td>
<td>0.80</td>
</tr>
<tr>
<td>( C )</td>
<td>1.89</td>
<td>0.91</td>
<td>0.60</td>
<td>0.68</td>
</tr>
<tr>
<td>( C_H )</td>
<td>1.64</td>
<td>0.62</td>
<td>0.74</td>
<td>0.12</td>
</tr>
<tr>
<td>( I_T )</td>
<td>8.01</td>
<td>0.93</td>
<td>0.36</td>
<td>0.20</td>
</tr>
<tr>
<td>( I )</td>
<td>8.66</td>
<td>0.80</td>
<td>0.37</td>
<td>0.14</td>
</tr>
<tr>
<td>( Y_h )</td>
<td>12.77</td>
<td>0.71</td>
<td>0.49</td>
<td>0.06</td>
</tr>
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</table>

<table>
<thead>
<tr>
<th>Panel B: Model 1</th>
<th>st.dev.</th>
<th>corr. w. GDP</th>
<th>AC</th>
<th>share of gdp</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Y )</td>
<td>2.91</td>
<td>1.00</td>
<td>0.30</td>
<td>1.00</td>
</tr>
<tr>
<td>( C_T )</td>
<td>2.09</td>
<td>0.97</td>
<td>0.29</td>
<td>0.68</td>
</tr>
<tr>
<td>( C )</td>
<td>2.09</td>
<td>0.97</td>
<td>0.29</td>
<td>0.48</td>
</tr>
<tr>
<td>( C_H )</td>
<td>2.09</td>
<td>0.97</td>
<td>0.29</td>
<td>0.21</td>
</tr>
<tr>
<td>( I_T )</td>
<td>4.91</td>
<td>0.98</td>
<td>0.30</td>
<td>0.32</td>
</tr>
<tr>
<td>( I )</td>
<td>4.73</td>
<td>0.91</td>
<td>0.30</td>
<td>0.25</td>
</tr>
<tr>
<td>( Y_h )</td>
<td>12.20</td>
<td>0.54</td>
<td>0.22</td>
<td>0.07</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel C: Model 2</th>
<th>st.dev.</th>
<th>corr. w. GDP</th>
<th>AC</th>
<th>share of gdp</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Y )</td>
<td>2.93</td>
<td>1.00</td>
<td>0.30</td>
<td>1.00</td>
</tr>
<tr>
<td>( C_T )</td>
<td>2.12</td>
<td>0.98</td>
<td>0.31</td>
<td>0.71</td>
</tr>
<tr>
<td>( C )</td>
<td>2.12</td>
<td>0.98</td>
<td>0.31</td>
<td>0.50</td>
</tr>
<tr>
<td>( C_H )</td>
<td>2.12</td>
<td>0.98</td>
<td>0.31</td>
<td>0.21</td>
</tr>
<tr>
<td>( I_T )</td>
<td>5.05</td>
<td>0.97</td>
<td>0.29</td>
<td>0.29</td>
</tr>
<tr>
<td>( I )</td>
<td>4.76</td>
<td>0.92</td>
<td>0.29</td>
<td>0.21</td>
</tr>
<tr>
<td>( Y_h )</td>
<td>9.95</td>
<td>0.63</td>
<td>0.34</td>
<td>0.08</td>
</tr>
</tbody>
</table>
Table 2: Correlations House Prices and Real Activity

The table reports the correlations between house prices $p^H$ and house price-rent ratios $p^H/R$ with GDP $Y$ and with residential investment $Y_H$. The “Model 1” is our the model with normal moving costs and collateral constraints, “Model 2” reports on the model with low moving costs and loose constraints. In particular, fixed and variable costs go from 5% to 2.5% and the down-payment goes from 25% to 1%. Finally, “Model 3” is the same as Model 2 except with a positive demand for bonds from foreigners, equal to 19% of GDP. In the data, the housing price and price-rent ratio are measured three different ways. In the first row (Data 1), the housing price is the aggregate value of residential real estate wealth in the fourth quarter of the year (Flow of Funds). The price-rent ratio divides this housing wealth by the consumption of housing services summed over the four quarters of the year (NIPA). In Data 2, the housing price is the repeat-sale Freddie Mac Conventional Mortgage House Price index for purchases only (Freddie Mac). The price-rent ratio divides this price by the rental price index for shelter (BLS). It assumes a price rent ratio in 1970, equal to the one in Data 1. In Data 3, the housing price and price-rent ratio are the price index and price-dividend ratio on the REITS index (NAREIT). In Data 4, the housing price is the repeat-sale Case-Shiller National House Price index. The price-rent ratio divides this price by the rental price index for shelter (BLS). It assumes a price rent ratio in 1987, equal to the one in Data 1. For series 1, 2, and 4, the price and price-rent ratio values in a given year are the fourth quarter values; for series 3, they are the December values. The annual price index, GDP, and residential investment are first deflated by the disposable personal income price deflator and then expressed as log deviations from their Hodrick-Prescott trend.

<table>
<thead>
<tr>
<th>Correlations</th>
<th>$(Y, p^H)$</th>
<th>$(Y_H, p^H)$</th>
<th>$(Y, p^H/R)$</th>
<th>$(Y_H, p^H/R)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data 1 (1953-2008)</td>
<td>0.23</td>
<td>0.43</td>
<td>0.23</td>
<td>0.31</td>
</tr>
<tr>
<td>Data 1 (1973-2008)</td>
<td>0.33</td>
<td>0.50</td>
<td>0.27</td>
<td>0.39</td>
</tr>
<tr>
<td>Data 2 (1973-2008)</td>
<td>0.33</td>
<td>0.52</td>
<td>0.29</td>
<td>0.46</td>
</tr>
<tr>
<td>Data 3 (1973-2008)</td>
<td>0.23</td>
<td>0.40</td>
<td>0.01</td>
<td>0.23</td>
</tr>
<tr>
<td>Data 4 (1987-2008)</td>
<td>0.36</td>
<td>0.75</td>
<td>0.10</td>
<td>0.62</td>
</tr>
<tr>
<td>Model 1</td>
<td>0.97</td>
<td>0.43</td>
<td>0.62</td>
<td>0.30</td>
</tr>
<tr>
<td>Model 2</td>
<td>0.93</td>
<td>0.45</td>
<td>0.61</td>
<td>0.24</td>
</tr>
<tr>
<td>Model 3</td>
<td>0.93</td>
<td>0.38</td>
<td>0.54</td>
<td>0.26</td>
</tr>
</tbody>
</table>
Table 3: Housing Wealth Relative to Total Wealth

The first column reports average housing wealth of the young (head of household is aged 35 or less) divided by average total wealth (i.e., net worth) of the young. The second column reports average housing wealth of the old divided by average net worth of the old. The third column reports average housing wealth of the young plus average housing wealth of the old divided by average net worth of the young plus average net worth of the old. The fourth (fifth) [sixth] column reports average housing wealth of the low (medium) [high] earners divided by average net worth of the low (medium) [high] earners. Low (medium) [high] earners are those in the bottom 25% (middle 50%) [top 25%] of the income distribution, relative to the cross-sectional income distribution at each age. The data are from the Survey of Consumer Finance for 1983-2007. The last two rows report the model. In the model, housing wealth is $P_H * H$ and total wealth is $W + P_H * H$. The “Model 1” is our the model with normal moving costs and collateral constraints, “Model 2” reports on the model with low moving costs and loose constraints. In particular, fixed and variable costs go from 5% to 2.5% and the down-payment goes from 50% to 10%. Finally, “Model 3” is the same as Model 2 except with a positive demand for bonds from foreigners, equal to 15% of GDP.

<table>
<thead>
<tr>
<th>Year</th>
<th>young</th>
<th>old</th>
<th>all</th>
<th>low earn</th>
<th>medium earn</th>
<th>high earn</th>
</tr>
</thead>
<tbody>
<tr>
<td>1983</td>
<td>0.72</td>
<td>0.50</td>
<td>0.52</td>
<td>0.52</td>
<td>0.67</td>
<td>0.47</td>
</tr>
<tr>
<td>1986</td>
<td>0.88</td>
<td>0.53</td>
<td>0.55</td>
<td>0.93</td>
<td>0.79</td>
<td>0.47</td>
</tr>
<tr>
<td>1989</td>
<td>0.73</td>
<td>0.54</td>
<td>0.55</td>
<td>0.57</td>
<td>0.68</td>
<td>0.54</td>
</tr>
<tr>
<td>1992</td>
<td>0.88</td>
<td>0.54</td>
<td>0.56</td>
<td>0.58</td>
<td>0.70</td>
<td>0.51</td>
</tr>
<tr>
<td>1995</td>
<td>0.84</td>
<td>0.47</td>
<td>0.49</td>
<td>0.48</td>
<td>0.65</td>
<td>0.45</td>
</tr>
<tr>
<td>1998</td>
<td>0.67</td>
<td>0.44</td>
<td>0.46</td>
<td>0.43</td>
<td>0.63</td>
<td>0.40</td>
</tr>
<tr>
<td>2001</td>
<td>0.67</td>
<td>0.43</td>
<td>0.44</td>
<td>0.44</td>
<td>0.58</td>
<td>0.40</td>
</tr>
<tr>
<td>2004</td>
<td>1.14</td>
<td>0.53</td>
<td>0.55</td>
<td>0.49</td>
<td>0.70</td>
<td>0.51</td>
</tr>
<tr>
<td>2007</td>
<td>0.92</td>
<td>0.52</td>
<td>0.54</td>
<td>0.51</td>
<td>0.71</td>
<td>0.50</td>
</tr>
<tr>
<td>Model 1</td>
<td>1.55</td>
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<td>0.49</td>
<td>0.41</td>
<td>0.47</td>
<td>0.54</td>
</tr>
<tr>
<td>Model 2</td>
<td>1.69</td>
<td>0.52</td>
<td>0.56</td>
<td>0.46</td>
<td>0.54</td>
<td>0.62</td>
</tr>
<tr>
<td>Model 3</td>
<td>2.09</td>
<td>0.53</td>
<td>0.58</td>
<td>0.50</td>
<td>0.55</td>
<td>0.63</td>
</tr>
</tbody>
</table>
### Table 4: Return Moments

The table reports the mean and standard deviation of the return on physical capital, on a levered claim to physical capital, and on housing, as well as their Sharpe ratios. The Sharpe ratios are defined as the average excess return, i.e., in excess of the riskfree rate, divided by the standard deviation of the excess return. It also reports the mean and standard deviation of the riskfree rate. The last column is the price-rent ratio. The leverage ratio (debt divided by equity) we use in the model is 2/3: \[ R_E = R_f + (1 + B/E)(R_K - R_f). \] The “Model 1” is our the model with normal moving costs and collateral constraints, “Model 2” reports on the model with low moving costs and loose constraints. In particular, fixed and variable costs go from 5% to 2.5% and the down-payment goes from 25% to 1%. Finally, “Model 3” is the same as Model 2 except with a positive demand for bonds from foreigners, equal to 19% of GDP. In the data, the housing return and price-rent ratio are measured three different ways. In the first row (Data 1), the housing return is the aggregate value of residential real estate wealth in the fourth quarter of the year (Flow of Funds) plus the consumption of housing services summed over the four quarters of the year (NIPA) divided by the value of residential real estate in the fourth quarter of the preceding year. We subtract CPI inflation to express the return in real terms and population growth in order to correct for the growth in housing quantities due to population growth. In Data 2, the housing return uses the repeat-sale Freddie Mac Conventional Mortgage House Price index for purchases only (Freddie Mac) and the rental price index for shelter (BLS). It assumes a price rent ratio in 1970, equal to the one in Data 1. In Data 3, the housing return is for a REIT index, which is available from 1972 to 2008 (source: NAREIT). We subtract realized CPI inflation from realized housing returns to form monthly real housing returns. We construct annual real housing returns by compounding monthly real housing returns over the year. The levered physical capital return in the data is measured as the CRSP value-weighted stock return. We subtract realized annual CPI inflation from realized annual stock returns between 1953 and 2008 to form real annual stock returns. The risk-free rate is measured as the yield on a one-year government bond at the start of the year minus the realized inflation rate over the course of the year. The data are from the Fama-Bliss data set and available from 1953 until 2008.

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Table 5: Predictability

This table reports the the coefficients, t-stats, and $R^2$ of real return and real dividend growth predictability regressions. The return regression specification is: \[ r_t = \alpha + \kappa_r p_d^t + \varepsilon_{t+k} \] where $k$ is the horizon in years, $r_t$ is the log housing return (left panel) or log stock return (right panel), and $p_d^t$ is the log price-rent ratio (left panel) or price-dividend ratio on equity (right panel). The dividend growth predictability specification is similar: \[ \Delta d_t = \alpha + \kappa_d p_d^t + \varepsilon_{t+k} \] where $\Delta d_t$ is the log rental growth rate (left panel) or log dividend growth rate on equity (right panel). In the model, we use the return on physical capital for the real return on dividend growth predictability specification is similar: \[ \text{pd}_{t} \]

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This table is based on the quarterly REITs data for 1973-2008 or annual Flow of Funds data for 1953-2008. We subtract CPI inflation to obtain the real returns and real dividend or rental growth rates.
This table reports the coefficients, t-stats, and $R^2$ of excess return predictability regressions. The return regression specification is:

$$\frac{1}{k} \sum_{j=1}^{k} r_{i+j}^t = \alpha + \kappa^{r,e} p_{dt}^i + \varepsilon_{t+k},$$

where $k$ is the horizon in years, $r_{i+j}^t$ is the log real housing return in excess of a real short-term bond yield (left panel) or the log real stock return in excess of a real short-term bond yield (right panel), and $p_{dt}^i$ is the log price-rent ratio (left panel) or price-dividend ratio on equity (right panel). In the model, we use the return on physical capital for the real return on equity and the return on the one-year bond as the real bond yield. The model objects are obtained from a 1150-year simulation, where the first 150 periods are discarded as burn-in. In the data we use the CRSP value-weighted stock return minus CPI inflation, either quarterly data for 1973-2008 or annual data for 1953-2008. The housing return in the data is based on the quarterly REITS data for 1973-2008 or annual Flow of Funds data for 1953-2008. We subtract CPI inflation to obtain the real return. The real bond yield is the Fama 3-month bond yield in excess of CPI inflation for the quarterly results and the 1-year Fama-Bliss yield in excess of CPI inflation for the annual results.

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<th>$R^2$</th>
<th>Horizon</th>
<th>$\kappa^{r,e}$</th>
<th>t-stat</th>
<th>$R^2$</th>
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Table 7: Risk Sharing

This table reports the cross-sectional standard deviation of the consumption share $C_{T,a,t}^i/C_{T,T}^i$, as well as the cross-sectional standard deviation of individual-level consumption growth. The last panel reports the ratio of per capita consumption to per capita GDP. We simulate the model for $N = 2400$ households and for $T = 1150$ periods (the first 150 years are burn-in and discarded). We calculate cross-sectional means and standard deviations of individual consumption share or consumption growth within each age group for each period, and then average over periods. The first column pools households of all ages, the next four columns look at various age groups. The “Model 1” is our the model with normal moving costs and collateral constraints, “Model 2” reports on the model with low moving costs and loose constraints. In particular, fixed and variable costs go from 5% to 2.5% and the down-payment goes from 25% to 1%. Finally, “Model 3” is the model with foreign holdings of bonds to the extent of 19% of GDP.

<table>
<thead>
<tr>
<th>Cross-sectional St. Dev. Consumption Share</th>
<th>all</th>
<th>≤ 35</th>
<th>36-50</th>
<th>51-65</th>
<th>&gt;65</th>
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</thead>
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<td>Model 1</td>
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<td>44.79</td>
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<td>44.13</td>
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<th>≤ 35</th>
<th>36-50</th>
<th>51-65</th>
<th>&gt;65</th>
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<td>6.00</td>
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<th>36-50</th>
<th>51-65</th>
<th>&gt;65</th>
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<tbody>
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<td>0.35</td>
<td>0.60</td>
<td>0.78</td>
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<td>0.60</td>
<td>0.79</td>
<td>0.75</td>
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