Do Real Balance Effects Invalidate the Taylor Principle in Open Economies?

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Abstract
This paper derives the necessary and sufficient conditions for local equilibrium determinacy in a two-country Neo-Wicksellian model that incorporates real balance effects. It is shown that the Taylor principle remains valid in preventing indeterminacy of equilibrium provided monetary policy is characterized by a forecast-based domestic price inflation rule.

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1 Introduction

Over recent years the defining characteristic in the conduct of monetary policy has been the explicit targeting of expected future inflation rather than actual inflation by central banks (see e.g. Huang, Meng and Xue (2009)). A key issue in the design of monetary policy is that the interest rate rule adopted by a central bank should ensure a determinate equilibrium. That is, monetary policy should be designed to avoid generating real indeterminacy which can destabilize the economy through the emergence of sunspot equilibria and self-fulfilling fluctuations.\(^1\) Such fluctuations are completely unrelated to economic fundamentals and can result in large reductions in the welfare of the economy.

It has been well established in the closed economy literature that under the Taylor Principle, i.e. a policy that adjusts the nominal interest rate by proportionally more than the increase in inflation, a central bank can easily prevent the emergence of indeterminacy and thus welfare-reducing self-fulfilling fluctuations, provided the central bank is not overly aggressive.\(^2\) However recent studies have found that for inflation targeting feedback rules, the Taylor principle may not be effective for open economies in preventing indeterminacy of equilibrium.\(^3\) One crucial factor upon which this depends is the inflation index targeted by central banks. Using a small open economy framework, Linnemann and Schabert (2006) and Llosa and Tuesta (2008) both find that the Taylor principle guarantees equilibrium determinacy under plausible parameter constellations if the central bank reacts to expected domestic price inflation. This is in stark contrast to a policy rule that responds to expected consumer price inflation, where the Taylor principle may not be appropriate, since the upper bound on the inflation response coefficient is more likely to bind with a sufficient degree of trade openness. Like the vast majority of the literature, these studies analyze monetary policy in an environment where money demand plays no role for equilibrium determination.\(^4\)

However, there are strong intuitive and empirical reasons for considering the real balance

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\(^1\)By real indeterminacy we mean that there exists a continuum of equilibrium paths, starting from the same initial conditions, which converge to the steady state. Our focus of attention rests solely with the consideration of local (real) determinacy as opposed to global determinacy. For further discussion of these issues see Woodford (2003).

\(^2\)See for example, Bernanke and Woodford (1997), Clarida et al. (2000) and Woodford (2003).


\(^4\)These studies either assume a cashless economy or adopt a money-in-the-utility function model with separable preferences.
effects of money. Empirical estimates suggest that such effects, while small, are found to exist in the data. Furthermore, recent closed-economy studies suggest that the introduction of small real balance effects can have important implications for equilibrium determinacy.

This paper considers the robustness of the Taylor principle for open economies under forward-looking inflation feedback rules when real balance effects are explicitly modelled. These effects are introduced via a money-in-the-utility-function set-up where consumption and real money balances enter non-separably. Unlike Linnemann and Schabert (2006) and Llosa and Tuesta (2008), the determinacy properties are analyzed using a two-country model. We find that incorporating real balance effects can make indeterminacy more likely if consumer price inflation is the price index targeted. When expected domestic price inflation is targeted we find that real balance effects have no practical significance for the range of indeterminacy generated. Consequently in this case, for commonly employed parameterizations, policies that are consistent with equilibrium determinacy for a closed economy also preclude indeterminacy in open economies. The robustness of the Taylor principle to such specifications provides further evidence that reacting to forecast-based domestic price inflation is preferable to consumer price inflation on the grounds of local determinacy of equilibrium.

While there are a few recent papers that study the (in)determinacy of equilibrium under interest rate rules using a two-country, sticky price model, these other studies all abstract from issues of non-separability in the utility function between consumption and real balances. The closest analysis to this paper is the work by De Fiore and Liu (2005) which also allows for real balance effects. The main differences between the two are as follows. Firstly, their framework assumes a small-open economy, whereas this paper employs a two-country approach. In a two-country model the optimizing decisions of the foreign country can affect prices and allocations in the home country. This differs from a small open economy set-up, as stressed by Woodford (2003), if money is considered to provide transaction services then the benefits of this should be related to the individual’s volume of transactions. See, for example, Woodford (2003), Ireland (2004) using US data, and Andrés, López-Salido and Vallés (2006) using Euro-zone data and Kremer, Lombardo and Werner (2003) using German data. See Benhabib et al. (2001), Schabert and Stoltenberg (2005) and Kurozumi (2006). However as a source of indeterminacy, trade openness is much more important than real balance effects. For example, Batini et al. (2004) consider inflation forecast rules that can be more than one-period into the future. Bullard and Schaling (2009) study contemporaneous inflation rules for the case when trade openness is asymmetric. McKnight (2007b) looks at the determinacy implications when capital and investment spending are introduced and Leith and Wren-Lewis (2009) when consumers are assumed to be finite-lived.

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where the foreign sector is exogenously given. Secondly, they employ a strict cash-in-advance constraint to allow for transactions frictions. However, in this paper we do not explicitly model transactions. Instead we explicitly model the real balance effects of transactions services, by employing a standard money-in-the-utility function (MIUF) framework, where the utility function is non-separable between consumption and real money balances. Thirdly, they consider a policy rule that targets expected consumer price inflation and find that equilibrium determinacy is unlikely to be achieved if the Taylor principle is adhered to. This paper also considers a policy rule that reacts to expected domestic price inflation. This is an important policy rule to investigate since reacting to domestic price inflation has been found by Clarida et al. (2002), among others, to be the optimal monetary policy for open economies.

The remainder of the paper is organized as follows. Section 2 outlines the two-country model. The determinacy analysis is addressed in Section 3. Finally, Section 4 concludes.

2 Model

The model is a two-country extension of the Neo-Wicksellian closed-economy framework employed by Woodford (2003) and Kurozumi (2006). Below we present the linear approximation of the structural equations. These are derived from a discrete-time, MIUF model where consumption and real money balances are non-separable. Price rigidity is introduced à la Calvo (1983), with monetary policy governed by an interest rate rule. We assume that the law of one price holds and, in line with the recent literature, that the degree of trade openness is proxied by the inverse of home bias in preferences for traded goods. As is standard, we assume complete financial markets and symmetric preferences and technologies across countries.

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10 As emphasized by Leith and Wren-Lewis (2009) the fact that the difference between consumer price and domestic price inflation involves both the exchange rate and foreign prices means any results obtained under a small open economy set-up need not hold after relaxing this assumption.
11 In the standard MIUF framework, end-of-period money balances enter the utility function, whereas in a cash-in-advance economy it is the money one has before entering the goods market. As discussed by Carlstrom and Fuerst (2001) these apparently minor differences in the timing of transactions, in certain circumstances, can have serious implications in terms of equilibrium determinacy.
12 We adopt the traditional convention that end-of-period money balances enter the utility function.
13 Hence purchasing power parity is not satisfied.
The LM, IS and AS equations for the home country are given by:\textsuperscript{14}

\begin{align}
\hat{m}_t &= \eta_c \hat{C}_t - \eta_R \hat{R}_t \\
\hat{C}_t &= \hat{C}_{t+1} - \sigma \left[ \hat{R}_t - \hat{\pi}_{t+1} + \chi (\hat{m}_{t+1} - \hat{m}_t) \right] \\
\hat{\pi}_t^H &= \beta \hat{\pi}_{t+1}^H + \lambda \left[ (\sigma^{-1} + \omega a) \hat{C}_t + \omega (1 - a) \hat{C}_t^* - \chi \hat{m}_t + [1 + 2 a \theta (1 - a)] \hat{T}_t \right]
\end{align}

where \( \hat{m}, \hat{C}, \hat{R}, \hat{\pi}, \hat{\pi}^H \) are, respectively, real money balances, consumption, the (gross) nominal interest rate, consumer price inflation and domestic price inflation for the home country. \( \hat{T} \) is the terms of trade, defined as the relative price of foreign goods in terms of domestic goods (with both prices expressed in the domestic currency), and \( \hat{C}^* \) denotes foreign consumption. Analogous equations hold for the foreign country. All hatted variables denote percentage deviations from the zero-inflation symmetric steady state.

In the LM equation (1), \( \eta_c, \eta_R > 0 \) are the income elasticity and interest rate semi-elasticity of money demand. In the IS equation (2), \( \sigma > 0 \) is the intertemporal elasticity of substitution in consumption and \( \chi \) the degree of non-separability between consumption and real money balances. In the AS equation (3), \( 0 < \beta, \psi < 1 \) are the discount factor and the degree of price rigidity; \( \omega, \lambda, \theta > 0 \) are the output elasticity of real marginal cost, the real marginal cost elasticity of inflation (where \( \lambda \equiv \frac{(1 - \psi)(1 - \beta \psi)}{\psi} > 0 \)), and the intratemporal elasticity of substitution between aggregate home and foreign goods; and \( 0 < 1 - a < 0.5 \) is the degree of trade openness.

The uncovered interest parity condition, the terms of trade and the consumer price inflation differential are given by:

\begin{align}
\hat{R}_t - \hat{R}^*_t &= \Delta \hat{S}_{t+1} \\
(2a - 1) \hat{T}_t &= \sigma^{-1} \left( \hat{C}_t - \hat{C}^*_t \right) - \chi \left( \hat{m}_t - \hat{m}^*_t \right) \\
\hat{\pi}_t - \hat{\pi}^*_t &= (2a - 1) \left( \hat{\pi}_t^H - \hat{\pi}^*_t^F \right) + 2(1 - a) \Delta \hat{S}_t
\end{align}

where \( \hat{R}^*, \hat{m}^*, \hat{\pi}^*_F, \hat{\pi}^* \) are, respectively, the nominal interest rate, real money balances and domestic and consumer price inflation in the foreign country. \( \Delta \hat{S}_{t+1} \) is the depreciation of

\textsuperscript{14}See McKnight and Mihailov (2007) for a detailed derivation of these equations.
the home currency from \( t \) to \( t + 1 \). Note that an important consequence of assuming non-separability of the utility function is that real money balances now enter the IS equation (2) and the terms of trade condition (5).

Finally, to close the model, we consider a monetary policy rule that either reacts to expected domestic price inflation or expected consumer price inflation:

\[
\tilde{R}_t = \mu \tilde{\pi}^H_{t+1}; \quad \text{or} \quad \tilde{R}_t = \mu \tilde{\pi}_{t+1}
\]  

(7)

where \( \mu > 0 \) is the inflation response coefficient.

It is important to stress that in the above system there are three channels of monetary policy. Combining the AS equation of the home country (3) with its foreign equivalent and the terms of trade condition (5) generates the following:

\[
\tilde{\pi}^H_t - \tilde{\pi}^F_t = \beta (\tilde{\pi}^H_{t+1} - \tilde{\pi}^F_{t+1}) + [\kappa_C + \kappa_T \zeta_C] (\tilde{C}_t - \tilde{C}^*_t) - [\kappa_\mu + \kappa_T \zeta_\mu] (\tilde{m}_t - \tilde{m}^*_t)
\]  

(8)

where \( \kappa_T \equiv 2\lambda(1 - a)[1 + 2a \omega \theta] > 0 \), \( \kappa_C \equiv \lambda[\omega(2a - 1) + 1/\sigma] > 0 \), \( \zeta_C \equiv [\sigma(2a - 1)]^{-1} > 0 \), \( \kappa_\mu \equiv \lambda \chi > 0 \) and \( \zeta_\mu \equiv \lambda \chi/(2a - 1) > 0 \). There is the conventional aggregate demand channel, where a relative increase in the home country’s interest rate lowers home consumption and reduces the domestic price inflation differential, the sensitivity of which depends on the coefficient \( \kappa_C \). Another channel is the cost channel of monetary policy where the differential demand for money enters into (8) as a negative cost shock. Hence an increase in the relative interest rate results in a reduction in the demand for money and, given the coefficient \( \kappa_\mu \), an increase in the domestic price inflation differential. Finally there is a terms of trade channel. Here a relative increase in the interest rate leads to an improvement in the terms of trade which has two separate effects on the domestic price inflation differential. On the one hand an improvement in the terms of trade reduces the domestic price inflation differential to an extent determined by \( \kappa_T \zeta_C \), whereas on the other hand it is increased through relative changes in the demand for money depending on \( \kappa_T \zeta_\mu \).\(^{15}\)

\(^{15}\)Clearly in a closed economy both these terms of trade effects are absent since \( \kappa_T = 0 \) as \( a = 1 \).
3 Equilibrium Determinacy

This section considers the issue of local determinacy of the perfect foresight equilibrium. For analytical tractability we follow Kurozumi (2006) in imposing:

**Assumption 1** \( 0 \leq \chi < (\eta, \sigma)^{-1} \iff 0 \leq 1 - \eta, \sigma \chi = \Omega < 1 \)

According to Woodford (2003) and Kurozumi (2006), this assumption is of most empirical relevance for \( \chi \).

3.1 Closed Economy Benchmark

For comparison purposes it will be useful to derive the determinacy conditions for the closed economy implied by the model. Note that the determinacy conditions are the same in the closed economy regardless of the index of inflation targeted in the policy rule (7). This follows, since in a closed economy domestic price inflation and consumer price inflation are the same concept.

**Proposition 1** Given Assumption 1, the necessary and sufficient conditions for local equilibrium determinacy in a closed economy are:

\[ 1 < \mu < \min\{\Gamma_1^A, \Gamma_2^A\} \text{ or } \max\{1, \Gamma_3^A\} < \mu < \Gamma_2^A \]

where

\[ \Gamma_1^A = \frac{\beta \Omega}{\eta_0 \lambda \omega \chi}; \quad \Gamma_2^A = \frac{2\Omega(1 + \beta) + \lambda (\Omega + \sigma \omega)}{\lambda [\Omega + \sigma \omega (1 + 2\eta_0 \chi)]}; \quad \Gamma_3^A = \frac{\Omega(1 + \beta)}{\eta_0 \lambda \omega \sigma \chi}. \]

**Proof.** See Appendix A.

Note that with separability (\( \chi = 0 \)) the bounds \( \Gamma_1^A \) and \( \Gamma_3^A \) no longer apply while \( \Gamma_2^A \) reduces to \( \Gamma_2^A \chi = 0 = 1 + \frac{2(1 + \beta)}{\lambda(1 + \sigma \omega)}. \) Hence the determinacy conditions summarized in Proposition 1 collapse to:

\[ 1 < \mu < 1 + \frac{2(1 + \beta)}{\lambda(1 + \sigma \omega)}. \]  \( (9) \)

\[ ^{16}\text{Empirical estimates suggest that } 0 < \chi < 0.03. \text{ See, for example, Woodford (2003), Ireland (2004) using US data and Andrés, López-Salido and Vallés (2006) using Euro-zone data.} \]

\[ ^{17}\text{Following Aoki (1981), the dynamic system for the closed world economy is derived by aggregating the linearized equations (1)-(3) and (7) with their foreign equivalents i.e. } X^W = \frac{\xi}{\xi} + \frac{\xi^*}{\xi^*}. \]
3.2 Reacting to Domestic Price Inflation

Let us first consider the determinacy conditions for the open economy when the policy rule reacts to domestic price inflation.

**Proposition 2** Suppose that monetary policy reacts to domestic price inflation. Then given Assumption 1, the necessary and sufficient conditions for local equilibrium determinacy in an open economy are:

\[ 1 < \mu < \min \{ \Gamma_1^A, \Gamma_3^B \} \quad \text{or} \quad \max \{ 1, \Gamma_3^B \} < \mu < \Gamma_X^B \]

where

\[
\Gamma_X^B = \begin{cases} 
\Gamma_2^A & \text{if } X^B \geq 0, \\
\Gamma_2^B & \text{if } X^B < 0;
\end{cases}
\]

\[
\Gamma_1^A = \frac{\beta \Omega}{\eta_R \lambda \omega \sigma \chi}; \quad \Gamma_3^B = \frac{\Omega (1 + \beta)}{\eta_R \lambda \omega \sigma \chi (2a - 1)};
\]

\[
\Gamma_2^A = \frac{2\Omega (1 + \beta) + \lambda (\Omega + \sigma \omega)}{\lambda \left[ \Omega + \sigma \omega (1 + 2\eta_R \chi) \right]}; \quad \Gamma_2^B = \frac{2\Omega (1 + \beta) \lambda^{-1} + \Omega \left[ 1 + 4(1 - a) \omega \theta a \right] + \omega \sigma (2a - 1)^2}{\Omega \left[ 1 + 4(1 - a) \omega \theta a \right] + \omega \sigma (2a - 1)^2 + 2 \sigma \chi \omega \eta_R (2a - 1)};
\]

\[
X^B = \frac{\sigma \chi \eta R}{a} \left[ \frac{2(1 + \beta) \Omega}{\lambda} + \Omega + \sigma \omega \right] + 2 \left( \Omega \theta - \sigma \right) \left[ \sigma \omega \chi \eta_R - \frac{(1 + \beta) \Omega}{\lambda} \right].
\]

**Proof.** See Appendix B.

First consider the separability case when \( \chi = 0 \). Then the bounds \( \Gamma_1^A \) and \( \Gamma_3^B \) no longer apply and the determinacy conditions summarized in Proposition 2 collapse to:

\[
\chi = 0 \quad \Rightarrow \quad \begin{cases} 
1 < \mu < 1 + \frac{2(1 + \beta)}{(1 + \sigma \omega)} = \Gamma_{2, \chi=0}^A \text{ if } \sigma \geq \theta \\
1 < \mu < 1 + \frac{2(1 + \beta)}{(1 + \sigma \omega + 4\omega \theta a(1-a)(\theta-\sigma)}} = \Gamma_{2, \chi=0}^B \text{ if } \sigma < \theta.
\end{cases}
\]

In contrast to small open economy studies\(^{18}\) the conditions for equilibrium determinacy crucially depend on the relative size of \( \theta \) and \( \sigma \). By inspection of (9) and (10), if \( \theta \leq \sigma \) then the requirements for equilibrium determinacy are the same in both the closed and open economies. However, if \( \theta > \sigma \) then the upper bound on the inflation coefficient in the open economy is relatively lower, where the impact of the degree of trade openness on this upper

\(^{18}\)For example, Linnemann and Schabert (2006) show that determinacy of a small open economy is isomorphic to closed economy under a forward-looking domestic price inflation policy rule.
bound is given by:

\[
\frac{\partial \Gamma^B_{X=0}}{\partial a} = \frac{8(1 + \beta)\lambda\omega(\theta - \sigma)(2a - 1)}{\lambda^2 [1 + \sigma \omega + 4\omega a(1 - a)(\theta - \sigma)]^2} > 0.
\]

The explanation for this result depends on the nature of the international spillovers between the two countries. If \(\sigma > \theta\) then home and foreign bundles of goods are complements in the utility function. Thus in response to changes in the terms of trade, production of home and foreign goods will expand or contract together. However if \(\sigma < \theta\) then home and foreign goods are substitutes and terms of trade changes will lead to different production responses in the two countries. Only if \(\sigma = \theta\) are production spillover effects absent.\(^{19}\) According to the determinacy conditions given in (10), if home and foreign goods are complements then trade openness has no indeterminacy implications. However if these bundles of goods are substitutes then the range of indeterminacy increases the larger the degree of trade openness and the larger the difference between \(\theta\) and \(\sigma\).\(^{20}\)

Now consider the case when the utility function is assumed to be non-separable. Then the conditions for determinacy of the open economy are analogous to the closed economy provided \(X^B \geq 0\). In order to gain some further insight we numerically calculate the upper bounds on the inflation response coefficient (\(\mu\)) required for equilibrium determinacy. The following parameters are chosen based on Kurozumi (2006): \(\beta = 0.99; \omega = 0.47; \eta_c = 1\) and \(\eta_R = 28\). We consider alternative values of \(\chi = 0.01, 0.02, 0.03\) and set \(\psi = 0.75\) which constitutes an average price duration of one year.\(^{21}\) Finally for illustrative purposes, three alternative values for the degree of trade openness are also chosen, which are roughly consistent with the ratio of imports to GDP of the USA \((a = 0.85)\), UK \((a = 0.7)\) and Canada \((a = 0.6)\). Given these parameter values, the numerical analysis shows that \(\theta\) must be significantly greater than \(\sigma\) for \(X^B < 0\) (i.e. home and foreign goods must be substitutes) and thus for \(\Gamma^B_2 < \Gamma^A_2 (\Gamma^A_1)\). Consequently we set \(\theta = 5\) and \(\sigma = 1.5\), values which are

\(^{19}\)As discussed by Benigno and Benigno (2003) when \(\theta = \sigma\) no spillover effects on production exist as the two economies are insular.

\(^{20}\)However, similar to Linnemann and Schabert (2006) and Llosa and Tuesta (2008), the practical relevance of the upper bound on the inflation coefficient in this case is of limited value. As shown in Table 1, the numerical analysis suggests that this upper bound is of too large a magnitude to have any practical significance.

\(^{21}\)Note that these parameter constellations satisfy Assumption 1.
broadly consistent with Cashin and McDermott (2002). Table 1 summarizes the relevant upper bounds on the inflation response coefficient ($\mu$): $\Gamma_{2,\chi=0}^A$ and $\Gamma_{2}^A$ when $\chi = 0$ and $\chi > 0$, respectively, for the closed economy; $\Gamma_{2,\chi=0}^B$ and $\Gamma_{2}^B$ when $\chi = 0$ and $\chi > 0$, respectively, for the open economy. While these upper bounds are lower in the open economy relative to the closed economy for all cases of $\chi$, they are still of a sizable magnitude to be deemed unlikely to bind despite setting values of $\theta$ and $\sigma$ to ensure $X^B < 0$. Thus, while the presence of real balance effects introduces a negative cost effect on the inflation differential equation (8) of magnitude $\kappa + \kappa_T \zeta \mu$, provided the policy rules target expected domestic price inflation in both countries, there are no serious consequences for real indeterminacy. Therefore the impact of real balance effects on equilibrium determinacy in open economies does not seem to matter at a practical level.

### 3.3 Reacting to Consumer Price Inflation

To see the policy importance of the previous analysis, Proposition 3 derives the determinacy conditions for the open economy when monetary policy responds to consumer price inflation.

**Proposition 3** Suppose that monetary policy reacts to consumer price inflation. Then given Assumption 1, the necessary and sufficient conditions for local equilibrium determinacy in an open economy are:

$$1 < \mu < \min\{\Gamma_0^C, \Gamma_1^C, \Gamma_2^C\} \quad \text{or} \quad \max\{1, \Gamma_0^C\} < \mu < \min\{\Gamma_0^C, \Gamma_2^C\}$$

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$^{22}$Cashin and McDermott (2002) for five OECD countries estimate $\sigma$ to lie between 0.72 – 2.65 and $\theta$ to lie within 0.69 – 5.62. For the USA they estimate $\theta = 5.62$ and $\sigma = 0.72$. 
where\[ \Gamma_\Omega^C = \begin{cases} \Gamma_1^A & \text{if } \Omega \leq \frac{2a\lambda\omega\eta_R}{\beta}, \\ \Gamma_0^C & \text{if } \Omega > \frac{2a\lambda\omega\eta_R}{\beta}; \end{cases} \]

and\[ \Gamma_\beta^C = \begin{cases} \Gamma_3^A & \text{if } \Omega \leq \frac{2a\lambda\omega\eta_R}{1+\beta}, \\ \Gamma_3^C & \text{if } \Omega > \frac{2a\lambda\omega\eta_R}{1+\beta}; \end{cases} \]

\[ \Gamma_1^A = \frac{\beta\Omega}{\eta_R\lambda\omega\sigma\chi}; \quad \Gamma_2^A = \frac{2\Omega(1+\beta) + \lambda(\Omega + \sigma\omega)}{\lambda[\Omega + \sigma\omega(1+2\eta_R\chi)]}; \quad \Gamma_3^A = \frac{\Omega(1+\beta)}{\eta_R\lambda\omega\sigma\chi}; \quad X^C \equiv \Gamma_2^A - \Gamma_2^C; \]

\[ \Gamma_0^C = \frac{1}{2(1-a)}; \quad \Gamma_2^C = \frac{\lambda\sigma\omega(2a-1)^2 + 2\Omega(1+\beta) + \lambda\chi[1+4(1-a)\omega\theta a] + \Omega4(1-a)(1+\beta) + \lambda\omega\sigma(2a-1)^2[1+2\chi\eta_R]}{\Omega}; \]

\[ \Gamma_3^C = \frac{\eta_R\lambda\omega\sigma\chi(2a-1)^2 + 2(1-a)(1+\beta)\Omega}{\Omega}. \]

**Proof.** See Appendix C.

First consider the separability case when \( \chi = 0 \). Then the determinacy conditions summarized in Proposition 3 collapse to:

\[ 1 < \mu < \min\left\{ \frac{1}{2(1-a)} - \frac{2(1+\beta) + \lambda[1 + \sigma\omega + 4\omega a(1-a)(\theta - \sigma)]}{\lambda[1 + \sigma\omega + 4\omega a(1-a)(\theta - \sigma)] + 4(1+\beta)(1-a)} \right\}. \quad (11) \]

Under separability the inflation coefficient is constrained by two upper bounds. Comparison of conditions (9) and (11) suggests that unlike domestic price inflation targeting, indeterminacy can be greater in the open economy regardless of the relative size of \( \theta \) and \( \sigma \) under consumer price inflation targeting.\(^{23}\) This follows since the first upper bound on \( \mu \) given in (11) is independent of these parameters and only depends on the degree of trade openness \( (1-a) \). Intuitively, this arises because by targeting consumer price inflation, monetary policy responds directly to changes in both the exchange rate and foreign prices.

With non-separability, the inflation coefficient is now potentially constrained by three upper bounds. Table 2 summarizes the upper bound on the inflation response coefficient \( \mu \) for determinacy for selected values of \( a, \theta \) and \( \sigma \), when \( \chi = 0 \) and \( \chi > 0 \). Two observations emerge from the numerical analysis. First, the lower the degree of trade openness (the higher is \( a \)), the greater increases in \( \chi \) have on the determinacy upper bounds. However as the degree of trade openness increases, the indeterminacy impact of real balance effects is reduced until they ultimately have a negligible influence. Second, when compared to

\(^{23}\) If \( \theta = \sigma \) the second upper bound collapses to the first.
Table 2: Reacting to expected consumer price inflation: upper bounds on the inflation response coefficient \((\mu)\) for determinacy

<table>
<thead>
<tr>
<th>(\theta)</th>
<th>(\sigma)</th>
<th>(\chi = 0)</th>
<th>(\chi = 0.01)</th>
<th>(\chi = 0.02)</th>
<th>(\chi = 0.03)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Closed economy: -</td>
<td>5</td>
<td>(\mu &lt; 14.8)</td>
<td>(\mu &lt; 10.3)</td>
<td>(\mu &lt; 7.65)</td>
<td>(\mu &lt; 5.96)</td>
</tr>
<tr>
<td>Open economy: 1.5</td>
<td>5</td>
<td>(a = 0.95)</td>
<td>(\mu &lt; 6.43)</td>
<td>(\mu &lt; 5.57)</td>
<td>(\mu &lt; 4.84)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(a = 0.90)</td>
<td>(\mu &lt; 4.08)</td>
<td>(\mu &lt; 3.79)</td>
<td>(\mu &lt; 3.50)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(a = 0.85)</td>
<td>(\mu &lt; 2.98)</td>
<td>(\mu &lt; 2.85)</td>
<td>(\mu &lt; 2.73)</td>
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<tr>
<td></td>
<td></td>
<td>(a = 0.80)</td>
<td>(\mu &lt; 2.34)</td>
<td>(\mu &lt; 2.28)</td>
<td>(\mu &lt; 2.22)</td>
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<tr>
<td></td>
<td></td>
<td>(a = 0.70)</td>
<td>(\mu &lt; 1.62)</td>
<td>(\mu &lt; 1.61)</td>
<td>(\mu &lt; 1.60)</td>
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<tr>
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<td></td>
<td>(a = 0.60)</td>
<td>(\mu &lt; 1.24)</td>
<td>(\mu &lt; 1.24)</td>
<td>(\mu &lt; 1.24)</td>
</tr>
</tbody>
</table>

In their small-open economy model once real balance effects are ignored (by dropping the cash in advance assumption and assuming a cashless economy) they find that there is a much larger range of parameters that sustain equilibrium determinacy. In our case this is only true for low values of trade openness. This suggests that indeterminacy is a more serious problem in multi-country environments.

4 Conclusion

Recent studies have considered the relative desirability of alternative inflation indexes in the design of forward-looking feedback rules. We extend the Neo-Wicksellian framework to include a role for money demand in a two-country open economy model. We show that in the presence of real balance effects and trade openness the Taylor principle still ensures equilibrium determinacy provided the policy rule responds to expected domestic price inflation. This is in stark contrast to feedback rules that react to expected consumer price inflation where the Taylor principle is invalidated. Our results thus provide further
evidence that, in order to prevent indeterminacy, central banks should target domestic price inflation rather than consumer price inflation in the conduct of monetary policy.

**Acknowledgments**

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A Proof of Proposition 1

Equations (1)-(3) and (7), along with their foreign equivalents, can be reduced to the following two-dimensional world aggregate system in \( \mathbf{m}_t^W \mathbf{C}_t^W \), where the coefficient matrix is:

\[
\mathbf{A} = \begin{bmatrix}
\frac{\sigma(\omega+\sigma-1)+\beta_R \eta}{\mu_R \sigma X} & \frac{\eta - \left(\lambda(\omega+\sigma-1)+\beta_R \eta\right)}{\eta R + \sigma(\mu-1)\eta L}
\end{bmatrix}
\]

Its determinant and trace are: \( \det \mathbf{A} = \frac{\Omega}{\beta R - \nu R \lambda \omega \sigma} \) and \( \text{tr} \mathbf{A} = 1 + \frac{(\mu-1)\lambda(\omega+\sigma-1) - \Omega}{\eta R \mu \sigma \lambda \omega \beta R} \). Since \( \mathbf{m}_t^W \) and \( \mathbf{C}_t^W \) are non-predetermined variables, determinacy requires that the two eigenvalues of \( \mathbf{A} \) are outside the unit circle. According to the Schur-Cohn criteria this requires that (i) \( |\det \mathbf{A}| > 1 \) and (ii) \( |\text{tr} \mathbf{A}| < 1 - \text{det} \mathbf{A} \). First note that \( \det \mathbf{A} > 1 \) provided \( \mu < \Gamma_1^A \). In this case condition (ii) implies that \( 1 < \mu < \Gamma_2^A \). Next note that \( \det \mathbf{A} < -1 \) provided \( \mu > \Gamma_3^A \). Then condition (ii) implies \( 1 < \mu < \Gamma_4^A \). This completes the proof. \( \square \)

B Proof of Proposition 2

We employ the Aoki (1981) decomposition approach in order to characterize the determinacy properties of the model. Thus we solve both for cross-country differences \( X^R \equiv \hat{X} - \hat{X}^* \), and world aggregate variables \( X^W \equiv \hat{X} + \hat{X}^* \).

Equations (1)-(3) and (7), along with their foreign equivalents, and equations (4)-(6) can be reduced to the following two-dimensional cross-country difference system in \( \mathbf{m}_t^R \mathbf{C}_t^R \), where the coefficient matrix is:

\[
\mathbf{B} = \begin{bmatrix}
\frac{\sigma(\omega+\sigma-1) + (2a-1)(\mu-1)\lambda_1 - \lambda_1}{\mu_R \sigma X + (2a-1)(\mu-1)\lambda_2 - \lambda_1} & \frac{\eta - \left(\lambda(\omega+\sigma-1) + (2a-1)(\mu-1)\lambda_1\right)}{\eta R + \sigma(\mu-1)\eta L}
\end{bmatrix}
\]

with \( \lambda_1 \equiv \frac{\lambda(2a-1) + \sigma - 1}{\sigma(2a-1)} + \lambda \left[1 + \sigma(2a-1) - 1\right] \) and \( \lambda \left[1 + \sigma(2a-1) - 1\right] \) and \( \lambda \left[1 + \sigma(2a-1) - 1\right] \), det \( \mathbf{B} = 1 + \frac{(1)\lambda(2a-1) - \lambda_1}{\beta_R - \nu R \lambda \omega \sigma(2a-1) - \beta R} \) and \( \text{tr} \mathbf{B} = 1 + \frac{(1)\lambda(2a-1) - \lambda_1}{\beta_R - \nu R \lambda \omega \sigma(2a-1) - \beta R} \). Determinancy again requires that the two eigenvalues are outside the unit circle. Using the Schur-Cohn conditions, det \( \mathbf{B} > 1 \) provided \( \mu < \Gamma_1^B \) and then condition (ii) implies that \( 1 < \mu < \Gamma_2^B \). By inspection det \( \mathbf{B} < -1 \) provided \( \mu > \Gamma_3^B \). Then condition (ii) implies \( 1 < \mu < \Gamma_4^B \). Therefore \( 1 < \mu < \min\{\Gamma_1^B, \Gamma_2^B\} \) and max\{1, \Gamma_3^B\} < \mu < \Gamma_4^B \) are the necessary and sufficient conditions for the difference system. Comparing these bounds on \( \mu \) with the conditions obtained for the aggregate system (i.e. closed economy), it is straightforward to verify that \( \Gamma_1^A < \Gamma_1^B, \Gamma_3^A < \Gamma_3^B \) and if \( X^B > 0 \), then \( \Gamma_2^A < \Gamma_2^B \). This completes the proof. \( \square \)

C Proof of Proposition 3

If monetary policy responds to expected consumer-price inflation, then equations (1)-(3) and (7), along with their foreign equivalents, and equations (4)-(6) can be reduced to the
following two-dimensional cross-country difference system in $[\hat{m}_t^R \tilde{C}_t^R]^T$, where the coefficient
matrix is:

$$C \equiv \begin{bmatrix}
\sigma(2a-1)[\mu \eta \chi + (\mu - 1)]A_1 - (2(1-a)\mu) & \eta_2 \sigma(2a-1)[\mu \eta \chi + (\mu - 1)]A_2 - (2(1-a)\mu) \\
\sigma(2a-1)[\mu \eta \chi + (\mu - 1)]A_3 - (2(1-a)\mu) & \sigma(2a-1)[\mu \eta \chi + (\mu - 1)]A_4 - (2(1-a)\mu)
\end{bmatrix},$$

with $A_3 \equiv \frac{\lambda_2(1-a)[1+2\omega \eta \chi]}{(2a-1)} + \lambda \left[\omega(2a - 1) + \sigma^{-1}\right] + \frac{\beta \eta_2}{\mu \eta \chi \omega(2a-1)^2} + \lambda \left[\omega(2a - 1) + \sigma^{-1}\right] + \frac{\beta \eta_2}{\mu \eta \chi \omega(2a-1)^2}, \quad A_4 \equiv \frac{\lambda_2(1-a)[1+2\omega \eta \chi]}{(2a-1)} + \frac{\beta \eta_2}{\mu \eta \chi \omega(2a-1)^2} - \beta(1-2(1-a)\mu)$, $\text{tr} C = 1 + \frac{\beta \eta_2}{\mu \eta \chi \omega(2a-1)^2} + \beta \eta_2$, and $\text{det} C = \frac{\beta \eta_2}{\mu \eta \chi \omega(2a-1)^2}$. Determinacy again requires that the two eigenvalues are outside the unit circle. Using the Schur-Cohn conditions, if $\mu > \frac{1}{\beta(1-\sigma)} \equiv \Gamma_0^C$, then condition (ii) requires $\mu < 1$ which contradicts the determinacy requirements of the aggregate system. Furthermore $\text{det} C < -1$ can never be supported with $\mu > \Gamma_0^C$ and thus this case can be ruled out. Now suppose that $\mu < \Gamma_0^C$. Then $\text{det} C > 1$ provided $\mu < \frac{1}{\beta(1-\sigma)} \equiv \Gamma_0^C$. But by inspection $\text{det} C < -1$ provided $\mu > \Gamma_0^C$. Then condition (ii) implies $1 < \mu < \Gamma_0^C$. Therefore $1 < \mu < \min\{\Gamma_0^C, \Gamma_0^C, \Gamma_0^C\}$ and $\max\{1, \Gamma_0^C\} < \mu < \min\{\Gamma_0^C, \Gamma_0^C\}$ are the necessary and sufficient conditions for the difference system. Comparing these bounds on $\mu$ with the conditions obtained for the aggregate system, it is straightforward to verify that $\Gamma_1^A < \Gamma_0^C$ if $\Omega < \frac{2\alpha \eta \chi \omega(2a-1)^2 + \beta \eta_2 (1-a)}{\beta}$, $\Gamma_1^A < \Gamma_0^C$ if $\Omega(1 + \beta) < 2\alpha \eta \chi \omega(2a-1)^2 + \beta \eta_2 (1-a)$, and if $X^C < 0$, then $\Gamma_0^C < \Gamma_0^C$. This completes the proof. □

References


