Building the city: sunk capital, sequencing, and institutional frictions

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Abstract
This paper models a growing city and takes the model to the data on Nairobi, focusing on investment decisions and the evolution of the built environment over time and at different points in the city. We distinguish between formal and informal, or slum sector construction. The former can be built tall, but structures once built are durable and cannot be modified without complete demolition. In contrast, slum structures are malleable and do not involve sunk costs. As the city grows, areas will initially be developed informally, and then formally; formal areas are redeveloped periodically. This process can be hindered by land right issues and formalisation costs of converting slums to formal sector usage, which may vary over space. The size, shape and appearance of the city are sensitive to formalisation costs varying by location, and can result in a hotchpotch of developments. In the empirics, we analyse Nairobi for 2003/4 and 2015 using unique data, developing a novel set of facts about the evolution of the built environment. We study the evolution of building footprints and heights, churning, infill, and redevelopment of the formal sector. Volume of building space is growing at about 4.4% a year. We discuss the loss in revenues and land value from high formalisation costs, which inhibit conversion of slums near the centre to a higher and better use.

Keywords: city, urban, urban growth, slum development, urban structure, urban form, housing investment, capital durability.

JEL classification: O14, O18, R1, R3

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1. Introduction

This paper examines housing development and redevelopment in a growing city, in the context of a developing country where there are both formal and informal land markets. We look at formal sector development and redevelopment, at the (mis)allocation of land between sectors, at the transition between the two, and at the role of property rights and expectations in altering paths of urban development. We develop a model of a growing city in which buildings are durable, so that investment decisions are taken on the basis of expectations about the future growth of the city. The work builds on the standard monocentric urban model. However, with a few exceptions (e.g., Braid, 2001), the urban literature uses static models, designed to analyse slowly changing developed country urban areas. The objective of this paper is to capture key features of developing country cities. The paper takes the model to the data for Nairobi, constructing a unique data set on the built environment in 2003/4 and 2015, detailing building footprints and heights. We can then track the physical transformation of a fast growing city currently 5-6 million in population, with issues that seem common to many large developing country cities.

The model captures the following features. First, the city is growing in income, population and area. Second, the city contains ‘formal’ or modern structures. Formal buildings involve sunk capital costs, can be built tall, and are hard to modify once constructed. As the city grows there will be periodic demolition and redevelopment of formal areas, in response to rising land rents, driven by growth fundamentals. Third, the city may also contain informal structures, which we sometimes refer to as slums. Given the technology and materials used in construction, slum buildings do not involve sunk capital, are not likely to be built tall and their volume can be continuously adjusted. Finally, and most critically, there is a cost of conversion of informal to formal land use, which we call a formalisation cost which may vary across properties in the city.

We show that as the city grows land will initially be developed with informal structures which are then replaced by formal structures, which will themselves be subject to intermittent redevelopment. The share of urban population in informal structures will generally decline through time. This decline is a consequence of rising land values (and hence a greater return to achieving density by building upwards) as the city expands. Formalisation costs means that informal structures may be very persistent, and spatial heterogeneity of these costs mean that they will continue to exist alongside formal structures, having long-lasting implications for the fabric of the city.

We take the model to the data for Nairobi, for which we have a detailed data base of buildings. We know the counts and footprint of buildings throughout the urban area for 2003/4 and 2015 based on tracings of all building polygons from aerial photos. For 2015 we also know heights of these buildings based on LiDAR data. The primary measure that we
work with is the total volume of building space (height x built cover) per unit area, by
building type (formal or informal) and varying across the city and over time. We use the data
to analyse how Nairobi transforms from 2003/4 to 2015, tracing demolition, redevelopment,
and in-fill at all locations. We have also high resolution satellite data for 2003/4 and 2013,
from which roads can be extracted. We also have data on housing rents and land prices by
location for single points in time.

Nairobi conforms to predictions in our model and, in static monocentric models, that, in the
formal sector: (1) house rents and land prices decline with distance to the centre and (2)
building heights and volume per unit area decrease with distance to the centre. Beyond that,
for our developing country context, we derive a novel set of facts. We start with the cross-
section. (3) Consistent with the model, slums provide housing volume with high coverage to
area ratio and low height while, in contrast, the formal sector provides volume with high
height but fairly constant and low cover to area ratios, with much more land set aside for side
streets and green space. (4) In comparing slum vs formal sector volume, in the core part of
the city, slum and formal sectors actually provide a similar stock of built volume per unit
area, albeit with slums at lower quality of building materials and amenities. However, (5)
overall total volume of slum housing is only about 10% of total building volume, given slums
cover a small fraction of the land area of the city and do not include formal sector
commercial and industrial use. (6) In contrast to formal areas, the slum rent gradient is flat or
even modestly rises with distance from the centre. Our model suggests that this is due to
greater ‘crowding’ near the centre, as slum building volume is provided by increased cover
rather than height.

For dynamics, the city changes dramatically from 2003/4 to 2015. Dynamics are driven by
infill (a building in 2015 whose footprint overlaps with no 2004 building) and teardowns of
2004 buildings which are divided into 2 categories, demolition (no new building, yet) and
redevelopment (a new building(s) overlapping torn down 2004 building(s)). What do we see?
(7) There is rapid growth, with total built volume just within the 2003/4 city effective
boundary increasing by 50%, growth of about 4% a year, a substantial rate of capital
accumulation. (8) Between 1-6 kms from the centre, redevelopment of formal sector
buildings into higher new buildings alone accounts for large volume increases, with the total
net increase in volume from redevelopment as a fraction of initial volume peaking at 35% at
about 3 kms out. (9) Throughout the core of the city, within 1-8 kms of the city centre
(CBD), there is enormous churning. About 35% of buildings from 2003/4 were torn down
and about half of those were left vacant (‘demolitions’). Infill adds 40% to 2003/4 building
counts at distance 1.5 to 4kms, rising to 80% and beyond as distance from the centre
increases.
For slums, (10) there are no slums very near the city centre and there is vast expansion towards the city fringe. (11) The rate of increase in volume for slums is just below that of the formal sector and the slum volume increase is about 10% of total city increase. (12) While there is extensive churning and redevelopment within existing slums, there is little mid-city development of slum into formal sector buildings over the 11 years. We explore the institutional context of Nairobi, to suggest there are high formalisation costs in traditional mid-city slums and a significant amount of land is not in its highest and best use. An illustrative calculation suggests that formalisation of slum areas in the distance band 2 to 6 kms from the CBD would result in an estimated $US 15,000 per household increase in inferred land values.

There are four novel aspects to the paper. First is the modelling. While Braid (2001) has a dynamic monocentric model with durable capital, no dynamic model deals with informality, formalisation costs, and expectations. Second are the data. While there is work on the USA using demographic census data to try to analyse redevelopment over of periods of time (Rosenthal and Brueckner, 2009), no work we know of utilizes city-wide data on buildings, with demolition, redevelopment, and intensification, to detail the changes in the urban landscape. Third is a new set of facts about city development and redevelopment of the built environment. Fourth, we focus on a major developing country city, where population growth is much more rapid than in developed countries and where land market institutions are weak.

The analytical framework makes clear some of the risks faced by a growing city, and the role of policy in addressing these risks. There are major market failures that deter investment, because of lack of transparency and strong institutions governing land markets. Expectations are fundamentally important in investment decisions and may be influenced by institutions and policies affecting markets. Low expectations of the future development of the city would have a major impact in distorting investment levels below their efficient levels.

The paper is organised as follows. The basic model and core theoretical results are set out in section 2. Section 3 presents data and analysis of Nairobi. Section 4 looks specifically at Nairobi slums, conversions costs, and misallocation. Section 5 notes key missing topics and Section 6 concludes.

2. Theory

In this section we present the model of a growing city, focusing on investment decisions and consequent patterns of land use and urban density. The analysis assumes that prices of housing increase through time. In the body of the paper we take this time path as exogenous, and in the appendix we show how it can be endogenised in an open city equilibrium. Section 2.1 examines building decisions at a particular point in the city. There are two alternative
building technologies – formal and informal (or slum) – that shape the volume and form of buildings in the city. Section 2.2 examines the choices of technology at each location, and how those choices evolve over time. Through time, each place transitions from agricultural use to informal development, then formalises and goes through successive waves of formal sector demolition and reconstruction. Section 2.3 pulls in the spatial dimension, giving a complete description of both the cross-section of the city and its evolution through time. Section 2.4 adds some frictions to the model, looking at the role of expectations and focusing on how barriers to conversion from informal to formal development can lead to a ‘hotchpotch’: co-existence of different building types and sizes throughout the city.

2.1 Building technology and housing supply

There are two distinct building technologies, formal and informal. Both deliver building volume but by different means, the formal sector \( (F) \) being able to build tall, and the informal sector \( (I) \) being able to ‘crowd’, increasing cover (the total building footprint) per unit of land. The volume of building delivered on a unit of land at a particular place, \( x \), and time \( t \), is the product of height and cover, \( v_i(x,t) = h_i(x,t)c_i(x,t) \), \( i = I, F \). Generally we think of \( x \) as denoting distance from the city centre.

The informal sector is modelled as follows. First, it is unable to build tall, so has height fixed at \( h_I \); it can however increase the proportion of each unit of land that is covered with buildings, \( c_I(x,t) \). Informal sector construction materials are malleable and construction costs are a flow, occurring continuously through the life of the structure. This can be thought of as the rental on ‘lego blocks’ or ‘meccano parts’ used in construction, or as the cost of material whose life is one instant. We assume that these flow construction costs per unit volume (which, given \( h_I \), is proportional to cover), are constant \( k_I \), so construction costs per unit land are \( k_I v_I(x,t) \).

In informal areas (slums) crowding has the effect of reducing the quality of housing. We capture this by supposing that the price (and willingness to pay) for a unit of informal housing is the product of two elements; the price of informal housing of unit quality at place \( x \) at date \( t \), \( p_I(x,t) \), and a quality factor, \( q(v_I(x,t)) \), diminishing and convex in crowding (as measured by volume per unit area). With this, land rent (i.e. revenue minus construction cost)\(^1\), is

\[
r_I(x,t) = p_I(x,t)q(v_I(x,t))v_I(x,t) - k_I v_I(x,t) .
\]

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\(^1\) We reserve the word ‘rent’ for income accruing to land, and use the word ‘price’ (per unit volume) for housing services, although this is a per period flow, not a capital value.
The volume of housing supplied is chosen to maximise rent, taking \( p_j(x,t) \) as exogenous and internalising the effect of crowding on quality, \( q(v_j(x,t)) \). The first order condition is

\[
\frac{\partial r_j(x,t)}{\partial v_j(x,t)} = p_j(x,t)q(v_j(x,t)) + v_j(x,t)q'(v_j(x,t)) - k_j = 0.
\]

(2)

If informal house quality is isoelastic in cover, \( q(v_j(x,t)) = v_j(x,t)^{(1-\lambda)/\lambda} \), \( \lambda > 1 \), then optimally chosen volume and maximised rent are respectively

\[
v_j(x,t) = \left[ \frac{p_j(x,t)}{k_j \lambda} \right]^{\frac{1}{\lambda-1}}, \quad r_j(x,t) = k_j \left( \lambda - 1 \right) \left[ \frac{p_j(x,t)}{k_j \lambda} \right]^{\frac{1}{\lambda-1}}.
\]

(3)

It follows that land rent is fraction \((1 - 1/\lambda)\) of revenue earned by informal sector housing, i.e. \( r_j(x,t) = \left[1 - 1/\lambda \right] p_j(x,t) q(v_j(x,t)) v_j(x,t) \).  

(2)

The formal sector differs in a number of key respects. First, buildings are ‘putty-clay’, malleable at the date of construction but not thereafter. We assume that formal sector land cover is not a choice variable but is set exogenously at \( c_F \), and that volume is achieved by choice of height, \( h_F(x,\tau_i) \). This is chosen at date of construction, denoted \( \tau_i \), and then fixed for the life of the structure, i.e. \( v_F(x,\tau_i) = c_F h_F(x,\tau_i) \) is fixed at date \( \tau_i \) until demolition at date \( \tau_{i+1} \); subscript \( i \) is used to denote successive redevelopments of formal structures.

Construction costs are one-off and sunk, and are an increasing and convex function of building volume, \( k_F(v(x,\tau_i)) \). Demolition incurs neither costs nor benefits as materials cannot be recycled back to putty.

This sunk cost of construction differs fundamentally from the flow cost in the slum sector, and we think captures key differences in construction technology. In Nairobi, from the 2009 Census, residential formal and slum sector wall materials are distinctly different. In slums, the majority (about 55%) of housing walls are corrugated iron sheets which can be easily reconfigured; most other slum housing involves mud construction (about 20%) and other material with short duration. Both sets of materials are not sufficiently load bearing to allow much in the way of height. In contrast, over 90% of formal sector housing is made of stone or some type of brick/block. We note that some on-going studies focus on classifying slums by the use of corrugated iron for roofs. This would not work in Nairobi, where over 50% of formal sector residential buildings also have corrugated iron sheet roofs (88% in slums).

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2 Total revenue is \( p_j(x,t)q(v_j(x,t)) v_j(x,t) = \left[ p_j(x,t) / k_j \lambda \right]^{1/(\lambda-1)} / k_j \lambda \).
While volume is fixed at date of construction, \( \tau_i \), the price of a unit of formal sector building volume, \( p_F(x,t) \), will vary over the life of the building. The present value of rent (per unit land) that accrues over the life of a structure, \( t \in [\tau_i, \tau_{i+1}] \), discounted to construction date \( \tau_i \) at interest rate \( \rho \) is denoted \( R_F(x, \tau_i) \),

\[
R_F(x, \tau_i) \equiv \int_{\tau_i}^{\tau_{i+1}} p_F(x,t)v_F(x, \tau_i)e^{-\rho(t-\tau_i)}dt - k_F(v_F(x, \tau_i)). \tag{4}
\]

The first order condition for choice of volume is,

\[
\partial R_F(x, \tau_i)/\partial v_F(x, \tau_i) = \int_{\tau_i}^{\tau_{i+1}} p_F(x,t)e^{-\rho(t-\tau_i)}dt - k_F'(v_F(x, \tau_i)) = 0. \tag{5}
\]

We define the present value of the price of a unit of formal housing space over its life relative to its price at date of construction as

\[
\Phi(x, i) \equiv \int_{\tau_i}^{\tau_{i+1}} [p_F(x,t)/p_F(x, \tau_i)]e^{-\rho(t-\tau_i)}dt. \tag{6}
\]

This is akin to the ‘value-to-rent ratio’ on a newly constructed property in the terminology of the real-estate literature (noting the time horizon is cut at \( \tau_{i+1} \) in (6)).

We work with an iso-elastic form of the cost function, \( k_F(v_F) = k_Fv_F^\gamma \), \( \gamma > 1 \), so the first order condition for choice of volume and maximised present value rent are,

\[
v_F(x, \tau_i) = \left[\frac{p_F(x, \tau_i)\Phi(x, i)}{k_F^\gamma}\right]^{1/\gamma}, \quad R_F(x, \tau_i) = k_F(\gamma - 1)\left[\frac{p_F(x, \tau_i)\Phi(x, i)}{k_F^\gamma}\right]^{\gamma/(\gamma - 1)}. \tag{7}
\]

It is useful to have a continuous flow measure of rent, given by amortizing the one-off construction cost continuously over the life of the structure. If amortization is constant proportion \( a \) of revenue, then costs are fully covered by setting \( a \) to satisfy

\[
k_F(v_F(x, \tau_i)) = \int_{\tau_i}^{\tau_{i+1}} ap_F(x,t)v_F(x, \tau_i)e^{-\rho(t-\tau_i)}dt = ap_F(x, \tau_i)v_F(x, \tau_i)\Phi(x, i). \tag{8}
\]

With \( k_F(v_F) = k_Fv_F^\gamma \) and (7), the amortization rate is \( a = 1/\gamma \). Thus, flow land rent net of amortization is fraction \((1 - 1/\gamma)\) of revenue earned by land and structure together (while in the informal sector this ratio is \((1 - 1/\lambda)\)).

### 2.2 Land development and construction phases
Continuing to focus on a particular unit of land, $x$, we now look at the choice of when to develop (or redevelop) informal or formal structures. At some date (say time 0) the present value of rent from a unit of land at $x$ that has not yet been developed is

$$R(x) = \int_{t_0}^{\tau_0} r_0 e^{-\rho t} dt + \int_{t_0}^{\tau_1} r_I (x, t) e^{-\rho t} dt + \left[ R_F (x, \tau_1) - D(x) \right] e^{-\rho \tau_1} + \sum_{i=2}^{\infty} R_F (x, \tau_i) e^{-\rho \tau_i}.$$  \(8\)

The first term is the present value of rent from undeveloped land (flow rent $r_0$ which we take to be constant), discounted at rate $\rho$ and calculated up to the date of first development, denoted $\tau_0$. The second term gives the present value of rent from informally developed land during interval $\tau_0, \tau_1$. The first formal sector development, occurring at date $\tau_1$ yields rent and incurs a one-time fixed cost $D(x)$ of converting to formality, the formalisation cost. Formal sector development requires reasonably well defined property rights, such as land titling or formal leaseholds on land granted by the government. Obstacles to obtaining these rights may be substantial, particularly in African countries where much land is held traditionally under possessory and communal rights. $D(x)$ includes the cost of obtaining formal title, which is highly variable even within a city depending on the history of the plot, as discussed later. The final term in (8) gives the discounted value of rents earned over the lives of consecutive formal sector buildings, constructed at dates $\tau_2, \tau_3...$ The terms for rent in this expression depend on prices $p_i(x, t)$ and $p_F(x, t)$ (equations (3) and (7)), which we assume to be monotonically increasing and exogenous.

Dates of development and redevelopment are chosen to maximise $R(x)$. For the first development (which we assume for the moment to be informal), the optimal $\tau_0$ simply equates flow land-rents on undeveloped and informal land, and is implicitly defined by

$$\frac{\partial R(x)}{\partial \tau_0} = e^{-\rho \tau_0} \left[ r_0 - r_I (x, \tau_0) \right] = 0.$$  \(9\)

The first formal development takes place at date $\tau_1$ satisfying

$$\frac{\partial R(x)}{\partial \tau_1} = e^{-\rho \tau_1} \left[ r_I (x, \tau_1) - p_F (x, \tau_1) v_F (x, \tau_1) + \rho [k_F (v_F (x, \tau_1)) + D(x)] \right] = 0.$$  \(10\)

The first redevelopment of formal land is at date $\tau_2$ satisfying

$$\frac{\partial R(x)}{\partial \tau_2} = e^{-\rho \tau_2} \left[ p_F (x, \tau_2) v_F (x, \tau_1) - p_F (x, \tau_2) v_F (x, \tau_2) + \rho k_F (v_F (x, \tau_2)) \right] = 0.$$  

Generalising this for all redevelopments gives (see appendix for derivation):

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3 For simplicity, we do not let this depend on time. The dependence on location is drawn out in section 2.4.
\[ p_F(x, \tau_{i+1})[v_F(x, \tau_{i+1}) - v_F(x, \tau_i)] = \rho k_F(v_F(x, \tau_{i+1})) \quad \text{for } i \geq 1. \] (11)

This condition says that demolition and reconstruction occurs at the date at which the revenue gain from the change in volume equals the interest cost of the construction expenditure incurred. Similar intuition applies to equation (10).

Assuming iso-elasticity of formal sector construction costs, \( k_F(v_F) = k_F v_F^\gamma \), and of the informal house-quality relationship, \( q(v_t(x, t)) = v_t(x, t)^{(1-\lambda)/\lambda} \), we can use the optimised values of \( v \) in equations (3) and (7) in equations (9) – (11) to generate expressions for the dates at which sites are (re-)developed. The date at which site \( x \) becomes informally developed, \( \tau_0 \), given by equation (9), is implicitly defined by

\[ p_I(x, \tau_0) = k_I \lambda \left[ \frac{r_0}{(\lambda - 1)k_I} \right]^{(1-1/\lambda)}. \] (9a)

The right hand side of this expression is constant, and can be thought of as giving a trigger value; location \( x \) becomes informally developed on the date at which the price of housing at \( x \) reaches this trigger level.

The date at which informal settlement becomes formalised, \( \tau_1 \), is given by equation (10) which using (3) and (7) becomes

\[ \left[ \frac{p_I(x, \tau_i)}{k_I \lambda} \right]^{\lambda - 1} (\lambda - 1)k_I = \left[ \frac{p_F(x, \tau_i)\Phi(x, i)}{k_F \gamma} \right]^{\gamma - 1} k_F \left( \frac{\gamma}{\Phi(x, i) - \rho} \right) - \rho D(x). \] (10a)

The dates at which successive formal redevelopments of \( x \) take place, \( \tau_i, i > 1 \), given by equation (11) can, using the iso-elastic functional forms, be expressed as

\[ \left[ \frac{p_F(x, \tau_i)\Phi(x, i)}{p_F(x, \tau_{i+1})\Phi(x, i+1)} \right]^{1/\gamma - 1} = \frac{\gamma - \rho \Phi(x, i + 1)}{\gamma}. \] (11a)

These three equations, (9a)-(11a) together with the definition of the value-to-rent ratio, \( \Phi(x, i) \), equation (6), form the basis of the analysis that follows.

### 2.3 Analysis

What do we learn from the characterisation of development stages given above? A benchmark case in which house prices are growing at constant exponential rates \( \hat{p}_I, \hat{p}_F > 0 \)
yields analytical results. The full general equilibrium model that supports constant exponential price growth is discussed in section 2.4 and detailed in the Theory Appendix; but for the present we simply assume these price paths. We look at the time series development of a particular place, $x$, and then at the urban cross-section.

**Urban dynamics**: To draw out results we look first at successive redevelopments of formal areas of the city, and then turn to the city edge and informal development.

**Proposition 1**: If formal sector constructions costs are iso-elastic in height (with elasticity $\gamma$), informal sector quality is iso-elastic in cover (with elasticity $1/\lambda - 1$), prices are growing at constant exponential rates $\rho > \hat{p}_t$, $\hat{p}_F > 0$, and agents have perfect foresight then:

(i) The value-to-rent ratio takes constant value $\Phi$, and the time interval between successive formal redevelopments is constant $\Delta\tau$,

$$
\Phi = \int_0^{\Delta\tau} e^{(\hat{h}_r - \rho)\gamma} dt = \frac{1 - e^{(\hat{h}_r - \rho)\Delta\tau}}{\rho - \hat{p}_F}, \quad \Delta\tau = \frac{(\gamma - 1)}{\hat{p}_F} \ln \left[ \frac{\gamma}{\gamma - \rho \Phi}\right].
$$

(ii) Successive rounds of formal sector building have greater volume (height) by a constant proportional factor.

$$
\frac{V_F(x, \tau_{i+1})}{V_F(x, \tau_i)} = e^{\frac{\hat{h}_r \Delta\tau}{(\gamma - 1)}} = \frac{\gamma}{\gamma - \rho \Phi}.
$$

(iii) If the rate of growth of prices is the same in all locations, $x$, then so too are $\Phi$, $\Delta\tau$, and volume growth.

The first part of this proposition comes from integrating equation (6) and using it in (11a), and noting that there is a unique solution solving the two parts of (12) with constant $\Phi$ and $\Delta\tau$. The second part follows by using this in the first order condition for volume, (7). The third comes from noting that (12) and (13) do not depend on $x$. While value ratios and time intervals do not vary with $x$, the actual dates of redevelopment do, as discussed below.

What about the earlier stages of informal development? The first transition is (we have assumed) from agriculture to informal settlement. This occurs for land at $x$ when $p_I(x, t)$, the quality un-adjusted price of informal sector housing, space reaches the trigger value on the right hand side of (9a). A period of informal settlement exists only if the return to informal settlement at this date, $\tau_0$, is greater than the return to commencing formal settlement,

$$
r_I(x, \tau_0) > p_F(x, \tau_0) v_F(x, \tau_0) - \rho \hat{k}_F(v_F(x, \tau_0)) + D(x, \tau_0) \quad \text{(see equation (10))}
$$

If not, then initial
development will be formal, with date $\tau_1$ implicitly defined by
\[ r_0 = p_F(x, \tau_0) v_F(x, \tau_0) - \rho [k_F(v_F(x, \tau_0)) + D(x, \tau_0)]. \]

The transition from informal to formal settlement is given by date $\tau_1$ that solves (10a). There is a unique transition date satisfying the second order condition if the return to formal development is rising faster than the return to informal settlement (i.e. the right hand side of (10a) increasing faster than the left). If $D = 0$, a necessary and sufficient condition for this is that $\hat{p}_F \gamma (\gamma - 1) > \hat{p}_F \lambda (\lambda - 1)$. If $D > 0$, then this condition is sufficient but not necessary. We assume the condition to be satisfied, as it will be if prices (before being deflated for crowding) increase at the same rate and $\lambda > \gamma$. The condition $\lambda > \gamma$ means that the share of land rent in revenue is higher (and the share of construction costs lower) in informal development than in formal. It also means that the elasticity of land rent with respect to price is lower in informal sector housing than formal (compare equations (3) and (7)). Essentially, there are sharper decreasing returns to increases in volume in the informal sector (where crowding reduces price) than in the formal sector (where building taller raises unit construction costs).

Figure 1 pulls these stages together and illustrates the development path. Building volume is given on the vertical axis, and on the horizontal plane axes are time $t$ and location $x$ (distance from the CBD). The figure is constructed with house prices increasing exponentially with time and falling exponentially with distance from the CBD. We discuss the cross-section – variation across $x$ at a given $t$ – in the next sub-section, and now look just at the development of a particular location through time, i.e. fix $x$ and look along a line sloping up and to the right. Initially (at low $t$) this land is rural. Building volume becomes positive at date $\tau_0$ (specific to location $x$) when informal development takes place. The volume of informal development increases steadily as increasing $p_t$ causes meccano pieces to be rearranged and building cover to increase, although that may not be visually obvious in Figure 1. Formal development takes place at $\tau_1$ and, as illustrated, leads to an increase in volume, indicated by the second step. However, the sign of the change in volume depends on parameters, and it is possible that edge slums deliver more volume than does first stage formal development. Subsequent redevelopments occur at fixed time interval $\Delta \tau$ and bring the same proportionate increase in volume, achieved by building taller. The timing and volume of each of these formal investments is based on perfect foresight about the growth of prices and the date of subsequent redevelopments.

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\[ ^4 \] Parameters used in the simulation are given in the appendix.
The urban cross-section. We have so far concentrated on a single location, point \( x \), and now place this in the context of a city where \( x \) measures distance from the CBD and house prices decrease with distance due to commuting costs (see Theory Appendix). With prices decreasing in \( x \), (9a), (10a) and (11a) can be interpreted as implicitly defining, for each date \( t \), the city edge, \( x_0(t) \), the location of current formalisation, \( x_1(t) \), and locations of successive current redevelopments, \( x_i(t), i > 1 \).\(^5\) We illustrate and derive results for the urban cross-section assuming that the spatial price gradient is exponential with distance, at rates \( \theta_f, \theta_r \), so together with exponential growth of prices through time,\(^6\)

\[
p_f(x, t) = \bar{p}_f e^{b_f t} e^{-\theta_f x}, \quad p_F(x, t) = \bar{p}_F e^{b_F t} e^{-\theta_F x}.
\]

The urban cross-section at a point in time is indicated on Figure 1 by fixing a date and moving along a line sloping upwards to the left towards the CBD, with steps in volume occurring at locations \( x_i(t), i = 0, 1, 2 \ldots \). At the city edge land is informal and, moving towards the centre, locations that have been urban for longer have been through more stages of development and offer greater building volume per unit land. The increase in volume is achieved by increasing land cover in the informal area and by greater height in formal areas closer to the centre. We will see empirical data on these relationships in the following section.

Notice also that while the price of a unit volume of formal housing declines at rate \( \theta_f \) with distance from the centre by assumption in (14), the observed price of a quality adjusted unit of informal housing is constant across space (as we will see later in the data). The price is \( p_f(x, t)q(v_f(x, t)) \) and, with iso-elastic functional forms (using equation (3)), volume declines with price at rate \( \lambda/\lambda - 1 \) and quality declines with volume at rate \( (1 - \lambda)/\lambda \).\(^7\) Exact constancy is obviously a consequence of iso-elasticity, but the more general point is that crowding and quality reduction means that the price gradient for informal housing per unit volume is likely to be flatter across the city than that for formal housing, and could increase.

The evolving urban cross-section. Putting the parts together, we see how the urban cross-section evolves through time. Proposition 2 states results on how different stages of development (building types and heights) move across the city as it grows.

\(^5\) That is, instead of solving (9a)-(11a) for the date at which a particular location is developed, the equations give the location that undergoes development at a particular date.

\(^6\) For generality we allow \( \theta_f, \theta_r \) to differ, as would be the case if e.g. occupants of informal housing travelled to the CBD less frequently than occupants of formal housing (see Appendix).

\(^7\) I.e. using the house-quality relationship \( q(v_f(x, t)) = v_f(x, t)^{\lambda_1 - \lambda} \) in (3) gives constant quality adjusted price \( p_f(x, t)q(v_f(x, t)) = k_f \lambda \).
Proposition 2: If formal sector construction costs and informal sector quality are iso-elastic, prices are growing at constant exponential rates $\rho > \hat{p}_I$, $\hat{p}_F > 0$ and declining with distance at constant rates $\theta_I, \theta_F > 0$, conversion costs are the same at all locations and agents have perfect foresight then:

(i) The distance from the CBD to the edge of informal development increases through time according to $dx_I / dt = \hat{p}_I / \theta_I$.

(ii) The distance from the CBD to the edge of formal development increases through time according to $dx_F / dt = \frac{\hat{p}_F \gamma (\lambda - 1) - \hat{p}_I \lambda (\gamma - 1)}{\theta_F \gamma (\lambda - 1) - \theta_I \lambda (\gamma - 1)}$.

(iii) The distance between successive formal sector redevelopments, $\Delta x$, is constant,

$$\Delta x = \frac{(\gamma - 1)}{\theta_F} \ln \left[ \frac{\gamma}{\gamma - \rho^\Phi} \right]. \quad (15)$$

Part (i) holds because informal development first occurs when the price hits a constant trigger level, i.e. using (9a) and (14), $p_I(x_I(t), t) = k_I \lambda \left[ \frac{\rho_0}{(\lambda - 1)k_I} \right]^{(1-\lambda)} = \bar{p}_I e^{\hat{p}_I t} e^{-\theta_I x(t)}$; differentiation with respect to time gives the derivative in part (i). Part (ii) comes from differentiation of (10a) which implicitly defines the place at which first formal development occurs at date $t$, together with price equations (14). Part (iii) of the proposition follows from equations (11a), noting that $\Phi$ is a constant and that the price ratio on the left-hand side of (11a) now compares prices at different $x$ and the same $t$, $p_F(x_i(t), t) / p_F(x_{i+1}(t), t) = e^{-\theta_F \Delta x}$, where $\Delta x \equiv x_i(t) - x_{i+1}(t)$, i.e. the distance between places undergoing successive redevelopments. Notice also, comparing (15) with (12), that $\Delta x / \Delta \tau = \hat{p}_F / \theta_F$, this indicating how the price relativities that trigger redevelopment hold across space and across time.

This evolution is illustrated on Figure 1. The figure is constructed with $\hat{p}_I = \hat{p}_F$ and $\theta_I = \theta_F$ so the lines along which development and redevelopment occur are parallel, as implied by proposition 2. It follows that the width of the informal area, $x_1 - x_0$, is constant through time. Hence one can show that, even in a circular city, the share of urban land area that is informal falls with time and as the city gets larger. Generally, the area of land occupied by the informal sector becomes narrower through time if price growth is faster in the formal

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8 Whereas in proposition 1 equation (12) the price ratio was evaluated at given $x$, so $p_F(x, \tau_i) / p_F(x, \tau_{i+1}) = e^{-\hat{p}_F \Delta \tau}$. 

12
sector than informal (quality unadjusted) or price gradient of the formal sector is flatter than that of the informal sector, $\hat{p}_f \theta_f > \hat{p}_i \theta_i$.

While our analytical results are based on constant exponential price paths, we note that it is also possible to numerically compute the perfect foresight equilibrium for more general price paths, although we do not report those here.

### 2.4 Frictions and market imperfections

The analysis so far has concentrated on a benchmark case, and we now add two frictions to the model. The first is to add more heterogeneity across places by allowing formalisation costs, $D(x)$, to vary by place. Furthermore, we suppose that these costs may be due to institutional rather than real costs, creating inefficiency in the equilibrium outcome. Second, results so far have assumed perfect foresight; we relax this, and look at the implications of systematic deviations from perfect foresight.

**Formalisation costs.** Locations vary in their distance to the CBD and, potentially, in many other respects. One possibility is that the cost of formalisation, $D(x)$, varies according to place. Figure 2a illustrates the implications of there being an interval of $x$ within which $D(x)$ is particularly high. As expected, this extends the period during which the area is occupied by informal settlement.

Several other observations are noteworthy. First, a persistently informal area will see housing volume per unit area increase through time as informal buildings are reshaped and crowding increases. It is possible that it may come to have volume higher than the surrounding formal area, as illustrated in figure 2a and something we will see in the empirics; however, additional informal volume is achieved by crowding, not by height. Second, a history of informality has a persistent legacy on the area. Formal development starts later, and so therefore does subsequent redevelopment; this impacts on building volume which depends on the price (and hence date) at which redevelopment occurs. Proposition 1 still holds for the time series evolution of each place, but looking across the urban cross-section there is now more variation in building volume and height, even in areas where there is no longer an informal sector presence. This is illustrated more vividly in figure 2b, in which $D(x)$ was set randomly across space. All locations see volume increase with time, but initial and subsequent formal development takes place at different dates and builds to different heights. This means that gradients of volume, density and land rents are not monotonically decreasing from the centre in such a city. Patterns are the hotchpotch we see in the data. Of course, a cross-section slice of Figure 2 just gives volumes along a single ray from the CBD.

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9 The general expression is $\frac{dV}{dt} - \frac{dx}{dt} = \gamma(\lambda - 1)[\hat{p}_f \theta_f - \hat{p}_i \theta_i]$. 

\[ \frac{dV}{dt} = \frac{dx}{dt} \frac{\gamma(\lambda - 1)}{\theta_i \gamma(\lambda - 1) - \theta_f \lambda(\gamma - 1)} \]
The heterogeneity exists along all such rays. In the following empirical section we will look at distance bands (concentric rings), in which formal and informal structures will coexist, as will buildings of different heights.

What are the welfare implications of this hotchpotch of different land use? Under an open city model, in which time paths of residents’ utility are given, welfare change is captured entirely in rents going to land owners. Formalisation obstacles that are not real costs reduce welfare by distorting decisions, and in Section 4 we measure this by the loss of rent. Land rents are not generally observed, but housing prices are (i.e. gross revenue earned on each unit of volume). Following the structure of this model we know that – if construction costs are amortized as a constant fraction of revenues over the life of the structure – then land rents are fraction \((1-1/\gamma)\) of revenue earned in the formal sector and \((1-1/\lambda)\) of revenue earned in the informal sector.

**Expectations:** Analysis to this point has been based on optimisation with perfect foresight. What are the consequences of removing this assumption? Recall that \(\Phi(x,i)\) is the value-to-rent ratio on a newly constructed property, and equations (12) give the perfect foresight values of this and of the expected length of life of the property, \(\Delta \tau\). How do results change if construction decisions are based on a value-to-rent ratio that differs from the perfect foresight ratio?

The solid line on Figure 3 is a slice through Figure 1 at \(t = 180\), maintaining \(\hat{p}_F = \hat{p}_I\) and \(\theta_t = \theta_F\). Given the parameters used, the perfect foresight value-to-rent ratio is \(\Phi = 26\), and the interval between redevelopments is \(\Delta \tau = 70\). The dashed line is constructed on the basis that developers have less positive expectations, and build on the basis of a value-to-rent ratio of 19.5 (imposed at 75% of the perfect foresight value). The transition from rural to informal settlement is unaffected by this, but formal development is based on these less optimistic expectations. As a consequence developers build less volume and hence buildings become obsolete more rapidly, so the interval between redevelopments drops to \(\Delta \tau = 45\).

The welfare cost of this imperfection is measured by its impact on land rents. Rather than looking at flow rents, we compute the present value of these rents, integrating over the locations and dates illustrated in figures 1 and 2 (i.e. out to \(t = 250\) and to distance 60). Lower expectations reduce the present value of land rents by 13.3%. This is a substantial amount, particularly since the calculation does not take into account the fact that city population is smaller which would create further losses if there were urbanisation economies and/ or a wedge between urban and rural marginal products of labour.

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10 As a percentage of the excess of urban land rent over the rent earned by land in non-urban use, \(r_0\).
2.5 Closing the model

To this point our analysis has focused on construction of the city, given time paths for the price of housing floor-space of each type at each location. The model can be completed by specifying household behaviour and hence the demand for space. This is constructed in a way consistent with the preceding analysis, merely offering a model of price growth in terms of growth in city incomes and productivity. The technical exposition is in the Theory Appendix and based on consumers with log linear preferences and commuting costs such that income net of commuting costs declines exponentially with distance from the centre. The city is open with free migration from the outside where the outside option utility is given at any instant. The key driver of price (and population) growth is that in particular open city urban productivity relative to the outside utility level grows at a given rate.

3. Empirical work on Nairobi

The empirical work provides overall facts about the volume of built space in a city, examines key predictions of the model, and looks at a specific policy issue. For key predictions, in the cross section there are the usual items from the model and implied in the literature concerning land price, building heights, coverage and volume gradients. What is more unusual is distinguishing the role of the slum versus the formal sector. On a city wide basis we can show how house prices, heights, coverage and volume differ between the two, as well as their overall contribution to a city’s built stock.

The dynamics uses building footprint polygons from high resolution data for 2003/4 and 2015 and building heights for 2015. We derive the changes over the 11 or so years in height, cover, and volume overall and within the slum and formal sector and we note the degree of churning of individual buildings. The churning and volume changes indicate a city in rapid evolution in both the slum and formal sectors.

For slums we ask if they seem to move away from the centre and spring up on the edge and whether their role is shrinking or rising. Key to the last question is a policy issue. What is the role of formalisation costs? For Nairobi, based on “accidents” of history, we have an empirical counterpart: slum settlements where formalisation costs are high. We explore the role of these costs on building of Nairobi, in particular the lost land values (as a welfare measure), because of inability to convert slum lands to their highest and best use.

In this section we first describe the data in more detail. Then we present cross-section results, followed by dynamic ones. Section 4 focuses on slums and inefficiencies driven by artificially high formalisation costs. Section 5 examines items not directly addressed in the
model and our main presentation of results: building volume includes private uses other than housing and consumers are heterogeneous in incomes.

3.1 Data and mapping

We develop a data set for Nairobi which defines characteristics of the built environment at a very fine spatial resolution. Characteristics are defined at no more than 40 cm resolution and, based on that, then mapped for 3mx3m cells and aggregated preserving details to a grid of 150m x 150m. For the sample we focus on, the intensive margin of the 2003/4 built area of the city, we start with 6470 grid cells.

Our main data consist of building footprints based on tracings of buildings from aerial photo images for 2003/4 and 2015. We received the 2003/4 footprint data from the Nairobi City Council with digitized polygons for every building in the administrative boundary of Nairobi and the 2015 footprint data from Ramani Geosystems.\(^1\) The key methodological imagery work has been to overlay the 2003/4 polygons with those for 2015 to determine which building footprints are unchanged since 2003/4, which buildings were demolished with no replacement on the prior site, which buildings were redeveloped and finally where and to what extent infill occurred with new building on sites where no 2004 buildings existed. Overlay is complicated by variations in the way buildings were traced and specifically aligned in 2004 versus 2015, as well as by tracing error. The Data Methodology Appendix describes the issue and the algorithm used to overlay and identify types of changes.

We also have building height data for 2015 from LiDAR (0.3-1m resolution) which was used to create a Digital Elevation Model. For 2003/4 heights, we assume the height of unchanged buildings is the same as in 2015 (ignoring the possibility of adding floors to a structure). For demolished buildings in a grid square, for their 2003/4 height, we assign the average height of unchanged buildings in that sector in queen neighbouring grid squares. Both inferences are likely to overstate average 2003/4 heights and thus understate volume changes. Demolished buildings are likely shorter than unchanged ones, both from the model and the fact, as we will see later, that demolished buildings have relatively small footprints. We use high resolution SPOT satellite data for the years (circa) 2003/4 and 2013 to measure road coverage.

For Nairobi we have two classifications of slums which we utilize. For 2003/4, a land use map was prepared by the CSUD at Columbia University.\(^2\) Columbia categorized polygons as

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\(^1\) Based on documentation from the Center for Sustainable Urban Development (CSUD) at Columbia University, who use a highly detailed landuse map from the JICA, as far as we can tell, this data was created by the Japan International Cooperation Agency (JICA) and the Government of the Republic of Kenya under the Japanese Government Technical Cooperation Program, and mostly based on aerial images taken in February 2003 at a scale of 1:15,000 (Williams, et al. 2014). In January 2015, imagery at (10-20cm resolution) was recorded and digitized into building footprints by Ramani Geosystems.

\(^2\) This is based on a more detailed, copyrighted, landuse map created by the JICA and the Government of the
slums if they seemed to contain small mostly temporary buildings that are randomly distributed in high density clusters, although they also relied on classification done by the Japan International Cooperation Agency based on fieldwork at the time (See Williams, et al. 2014 for their full methodology). Second, in 2011, slums were mapped by IPE Global under the Kenya Informal Settlements program, and we digitized these maps. IPE mapping of settlements was done using satellite imagery and topographic maps with imprecisely defined criteria. The general idea is that slums are “unplanned settlements” which have some aspects of low house quality, poor infrastructure, or insecure tenure. The 2011 designation has many more slums than in 2003/4. Some 2011 areas had housing in 2003/4 not then defined as slums; in most cases these areas subsequently experienced enormous infill of small densely packed buildings. It is clear however that the effective definitions differ across years and cannot be used to distinguish slum creation and destruction per se. We rely on the 2011 mapping despite some misclassification issues we will see. But we do look within 2004 slums especially those near the centre to see what happened to those slums.

In Figure 4 we show these two mappings of slums and we also define the area of the city we will work with. We adopt a fairly conservative definition of the boundary: that for a (150mx150m) grid cell to be in the city on the outer edge a smoothed (by 900 meter squares) building cover must be 10% or more of the area. Figure 4 shows the city in dashed outline in 2003/4 and in solid outline in 2015. For each year we mark the slums as recorded at that time and in both periods: green if in both years, yellow if only in 2011 and blue if only in 2003/4. We mark the radius in red near the CBD in which there are no slums as defined in each time period (dashed and solid) and the city centre with a yellow star. The city centre is the brightest lit pixel in night lights data in the early 1990’s.

We will focus on the intensive margin which is the 2003/4 city for key aspects of dynamics. City shape is not a simple circle. It is bounded to the south by an airport and a large national park and to the immediate north of the centre by a preserved state forest. We can see also that there is a big extensive margin to the city. Apart from spread, what we take from the figures concerns slums. As the model predicts, slums are not prevalent near the centre; and the area with no slums as defined contemporaneously expands considerably between the two years, from a 0.775 km to a 2.0 km radius around the centre by 2011. The map suggests considerable slum expansion (yellow) at the 2004 fringe of the city and beyond, as predicted in the model. However it tells us little about removal (blue), much of which may involve differential classification issues. Finally we note the large slum of Kibera directly south-west of the centre (ranging from 3-5 kms of the centre). In Section 4, we will discuss Kibera.

Republic of Kenya under the Japanese Government Technical Cooperation Program which was published and printed by the survey of KENYA 1000 in March 2005.
In Figure 5 we show a 3-D map of the city for the 2003/4 boundary, which gives the average height of all buildings in public or private use in each grid square (assigned to slum or formal sector by where the centroid of the grid square lies). Calculations are discussed below. Blank areas are those which have missing data in 2004 (the Moi airbase, the State House, and the Ministry of State for Defence) and large areas that have no cover (in particular the Kibera golf course). The city does look monocentric with high heights but variable spikiness at the centre and then diminishing. Slum areas in red are generally low. In the north-east they also reveal misclassification problems; satellite images indicate that those tall areas are not slums!

For the empirical analysis we adjust the areas of analysis in Figures 4 and 5 in two ways. Sectoral classification focuses on slums, and does so with very tight boundaries cutting off vacant land adjacent to the slum (or a river dividing a slum) and even edge slum housing. The formal sector is a residual of everything else in the city. To do a proper comparison, we first remove all grid squares entirely in permanent public use (or not traced in 2004), which serve both slum and formal sector residents. A full list is in the Appendix but includes the President’s palace, a railyard, a garbage dump, a golf course, major stadiums and parks, colleges and universities, and the like. Overall we remove 11% of land in the 2004 city boundary; but, at the centre from 0-1 km with parks and the President’s palace, it is 25%. Note neighbourhood schools are left in and appear in both slum and non-slum areas. All roads are also left in. The second issue is the tight delineation of slum areas, whereby typically vacant land (includes roads and railyards) at the slum edges and nearby to slums are classified as 100% formal and sometimes slum mapping cuts buildings at the edge of the slum in two. To offset this, we adjust the IPE boundaries by first, classifying buildings as slum if their centre lies within the original slum boundary, and then assigning each 3mx3m pixel of non-built land to slum if the nearest building is classified as slum, and formal otherwise.

The analysis also makes use of two other data sets. First is a georeferenced household level data set from the 2012 ‘Kenya: State of the Cities’ survey by the National Opinion Research Center (NORC). This is the first data set to record household rent (with detailed house and some neighbourhood characteristics) in Nairobi for a sample that is stratified between slum and formal areas (based on the 2009 Census). Prior studies look just as slums (e.g., Guylani and Talukdar, 2008) and so offer no comparison across sectors. In addition to rent data, for 2015, we have property values that have been scraped from property24.co.ke. We focus on the vacant land listings with information on asking price and plot area, for which we have information for 80% of the listings. Listings are only found in the formal sector.

3.2 Defining the built features of a city in the cross-section
To analyse the built environment and the dynamics of change, we define some key concepts and a basic decomposition of the sources of building volume in a city.

Each cell (3x3m) is classified as either informal/slum (I) or formal (F) by the adjusted IPE map. These cells are then aggregated up to 150m x 150m grid squares. As noted earlier we remove grid squares that are in public use, so what is left is just slum and formal. We have the following definitions.

\( a_i(\chi) \) is defined as the area (m\(^2\)) of grid square \( \chi \) that is occupied by type \( i \), \( i = I, F \) (as defined by the binary classification at the 3x3m level). The total area is 
\[
a_i(\chi) + a_F(\chi) = 22500.
\]

\( c_i(\chi) \) is the type \( i \) covered area, or total of building footprints in type \( i \) area of grid square \( \chi \) (in m\(^2\)).

\( \bar{h}_i(\chi) \) , is the average height (based on 3x3m cells) of the covered area \( c_i(\chi) \).

\( v_i(\chi) = \bar{h}_i(\chi) c_i(\chi) \) is the total volume of built space of type \( i \) in the grid square (in m\(^3\)).

We will relate outcomes with respect to their distance \( x \) to the city centre. We define the area at \( x \) as all grid squares \( \chi \) within a ring at \( x \). We will also show heterogeneity within \( x \) in certain dimensions.

The first key measure is the ‘cover area ratio’ (CAR) by type of use

\[
\text{CAR}_i(x) = c_i(x) / a_i(x)
\]

where \( a_i(x) = \sum_{\chi \in x} a_i(\chi) \) is the total area of type \( i \) at \( x \) (defined from slum maps) and 
\[
c_i(x) = \sum_{\chi \in x} c_i(\chi) \] is the total building type \( i \) footprint at \( x \). Average height of built space is

\[
\bar{h}_i(x) = \sum_{\chi \in x} \bar{h}_i(\chi) c_i(\chi) / c_i(x).
\]

To measure building volume, we use a new concept: the ‘built volume to area ratio’ (BVAR). This is like a floor to area ratio (FAR) except it is in cubic meters of space, related to floor space by dividing by average height 3-3.1 m. However the area is not lot size but all unbuilt land which includes side streets, vacant lots, and small (but not large) public uses.

\[
\text{BVAR}_i(x) = v_i(x) / a_i(x),
\]
where \( v_i(x) = \sum_{x \in A} v_i(x) \) is total volume supplied by type \( i \) at \( x \). We note that at each \( x \), the share of area in slums is \( \rho_i = a_i(x) / \sum_i a_i(x) \) and the share in formal is \( (1 - \rho_i) \).

For total volume \( v(x) = \sum_i v_i(x) \), we have a fundamental decomposition:

\[
v(x) = a(x) \left\{ \rho_i \cdot \frac{BVAR_i(x)}{\text{total area}} + \frac{(1 - \rho_i) \cdot BVAR_f(x)}{\text{formal}} \right\} \]

\[
= a(x) \left\{ \rho_i \cdot \frac{\bar{h}_i(x)}{\text{total area}} \cdot \frac{CAR_i(x)}{\text{slum avg. height}} \cdot \frac{(1 - \rho_i) \cdot \bar{h}_f(x) \cdot \text{CAR}_f(x)}{\text{formal} \cdot \text{slum cover/area}} \right\}
\]

(19)

Graphs will show the components of these in the cross section and dynamics, so show both overall determinants of volume and the role of height and CAR in driving BVAR by sector.

3.3 Nairobi in the 2015 cross-section

We now show how built volume at different distances is composed of buildings of different type, height, and cover, as indicated by the decomposition (19). First however, we note how land prices vary with distance from the centre. These prices correspond to the present value of future land rents, as presented in the theory section, and are measured by the land price per square meter for sales of vacant land. The price gradient is well represented by an exponential relationship (equations 7 and 14), where each km of further distance reduces price by 15.5% as shown in column 1 of Table 1 and illustrated in Figure 6. Below we will relate this and other gradients to parameters of the model.

**Building heights:** Figure 7a gives average building height (\( \bar{h}_i(x) \)) in each sector by distance from the centre. In the formal sector this declines sharply from almost 30m at the centre until levelling out at about 7-8m. These are smoothed curves for grid squares whose centroid is in a 300m moving window going out from the centre.\(^{13}\) In the slums, height is flat at under 5m throughout, as assumed in the technology modelling in Section 2. The building materials of slum housing do not permit building high. Figure 7b reports heights in just the residential sector by floors from the NORC survey. Again heights in the formal sector decline sharply as we move away from the city centre at a rate of 7.7% per km from column 2 of Table 1, while those in slums are flat or even rise modestly. Figure 7c shows the variability of height in meters within sector. Especially near the centre in the formal sector there is enormous spikiness or variability as we combine office towers, historical buildings, all-purpose buildings like parking garages and shops wedged between tall buildings. In slums, especially the older ones from 3-6 kms out there is little variability; and the variability further out may

\(^{13}\) This is STATA local mean smoothing with an Epanechnikov kernel, with default settings
reflect misclassification issues noted in the discussion of Figure 5. One comment is that we started with an impression from some Africa experts that African cities were built without height with buildings limited to 5 story walk-ups because, for example, of unreliability of power for elevators. Nairobi clearly does not fit this description. Overall, buildings from 0-1 kms of the centre average (at the 3m x 3m pixel) 10 stories (at 3.1m a storey) and in Figure 7c, 5% of these pixels are over 16 stories.

**Volume.** We now turn to equation (19) and the decomposition of the components of volume. First in Figure 8 is the share of land in slum and formal use. The share of formal sector (with its roads) is very high: 100% near the centre. Slums occupy no more than 20% of non-public land at any distance up to 10kms from the centre. Figure 9 shows that across the two sectors, there are enormous differences in how housing is produced. In the formal sector the solid lines indicate the cover to area ratio (CAR) is flat (as assumed in Section 2), typically at 25%. In contrast, in slums the cover to area ratio is very high, over 50% between 3 and 7 kms out; and, in the older slums, nearer the city centre CAR averages near 60%. Slum CAR declines sharply with distance from the centre and opportunity cost of land, at a rate of 9% per km from column 3 of Table 1. Both the high level of cover in slum areas and the decline with respect to distance are as modelled in section 2. The high CAR means that slums have little green/open space around houses and little in the way of real side streets. For the latter, we also give coverage adding in roads within each sector by dashed lines in the figure. Much more coverage by way of paved roads is added to the formal than the slum sector, where in the formal sector roads are about 15% of coverage near the centre.14

Combining Figures 7 and 9, slums produce housing with intense ground cover but little height while in the formal sector the opposite is the case. Figure 10 gives this net: built volume to area ratio (BVAR). In the formal sector up to almost 2 kms (where there are also non slums) BVAR is very high, averaging around 7 metres of vertical space per metre of ground area, or 7 cubic metres of space per metre of ground area.15 At 2km and beyond, slums and the formal sector deliver essentially the same BVAR, so height and CAR differences cancel out. In both sectors, BVAR declines with distance from the centre. This is consistent with the theory where we saw that, dependent on parameters, cover in slums and height in formal areas could deliver similar housing volumes. At 6.5 kms, the BVAR in slums does bump up, but as we saw earlier in Figure 5 and will revisit below, this may be due to misclassification of tall formal sector buildings as slums. Figure 10b shows again the high heterogeneity in BVAR in the formal sector as different grid cells have more or less roads and other non-building use, and as building heights differ between newer and more historical

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14 We know overall roads are about 22% of total area of the city centre, implying that roads in public sector use grid squares we have removed is high near the centre.

15 If one wants to compare this to the usual floor to area measure (FAR) after dividing by 3-3.1 m per floor, we note that the base is not lot size but all land not in public use including all land in transport.
or utilitarian uses (parking garage). In the core older slum areas from 3-5 kms, there is only modest variation in BVAR.

For the opposing views of whether formal sector height trumps slum coverage in providing volume of built space or not, we have learned that, in Nairobi, they do equally well on average, albeit at very different quality levels. Later, we will show that Figure 10 for the formal sector reflects an average of locked historical BVAR and redeveloped BVAR based on current demanded height for new buildings. We will see that, for 2-5 kms out, redeveloped BVAR in the formal sector modestly dominates that in slums.

The land value, formal sector height, the slum BVAR gradient relate to parameters of the model, for a given price gradient. For the house price gradient we estimate a hedonic from NORC data of monthly prices rents per sq meter of floor space, after factoring out quality differences, to get a ‘pure’ distance premium for the formal sector. These include a 22 house attributes and living conditions and 2 measures of crowding measure of crowding, as well as a control for slum and slum distance from centre to deal with unobserved conditions in slums. The motivation for slum distance measures in the hedonic is developed more in Section 4. The gradient slope for formal sector prices of -0.0512 is in column 6 of Table 11. Table 2 gives slopes of relevant height and CAR gradients sums and how they would relate to the model (assuming a regular city ignoring the hotchpotch issue). We use the resulting parameter estimates to calibrate the model.

Figure 11 pulls the whole decomposition together and shows total volume and then the share in formal throughout the city. There are two key takeaways. First slum volume is never a big part of the picture in part because it excludes formal sector commercial and industrial use. It totals about 10% of total volume in 2015 and never exceeds 20% at any distance. Second total volume rises sharply to peak at almost 13.5 million cubic meters at 3.5 kms from the centre as the amount of potentially available land in any circumference increases; but then it falls to average around 7-8 million. As we noted in Figures 4-5, Nairobi has little available land beyond 4-5 kms to the direct north and south.

3.4 The dynamics of the built environment in Nairobi

We start by evaluating the total change in volume in the city in an 11 year period, which highlights the rapid evolution of cities like Nairobi in the developing world. There are the two margins, the intensive margin of the 2003/4 city area and the extensive margin between the 2003/4 and 2015 boundaries in Figure 4 as the city expands spatially. In the intensive margin, total built volume for non-public use increases by 53%, about a 4% annual increase in this major form of wealth. Including (unchanging) public in the base, total change is about 50%. The extensive margin accounts for 19% of total volume in all uses in 2004 and 24% by 2015, given a 96% increase in volume at that margin. Overall within the 2015 boundary, total
volume increases by 61%, about a 4.4% annual rate of increase. This compares with a similar annual population growth rate. The increase in the formal sector of 62% is modestly more than the 55% for slums, so slums only account for about 10% of the increase in total volume. The small decline in the slum share of volume reflects slum population share stagnation: from the censuses in 1999 and 2009, the slum share of population in the 2015 city area declines modestly from 29.2 to 28.8%.

Figure 12 shows volume changes by distance and the breakdown between the formal and slum sectors. Volume changes are large everywhere even near the city centre: there are 45-70% increases in total volume from 2-8 kms. These huge increases in already highly built areas are an aspect upon which we will focus. Until 9 kms out within sector percent increases in the formal sector generally dominate those in the slums, showing the increasing relative role of the formal sector in the main part of the city. However slum changes dominate at the city edge as the model predicts. In the figure, since slums have such a small weight in total area, total changes generally mimic those in the formal sector.

We next return to the decomposition analysis and develop the empirical framework for understanding what underlies volume changes. We then analyse the formal sector, followed by the slum sector.

3.4.1 Defining the dynamics of the built environment in a city

For changes, recall \( v(x) = a(x) \{ \rho_f \frac{BVAR_f(x)}{\text{built vol to area}} + (1 - \rho_f) \frac{BVAR_f(x)}{\text{formal vol to area}} \} \). We cannot distinguish and thus treat as constant the classification of cells by I or F, therefore holding constant \( \rho_f \) and \( a_i(x) \). What can change at any \( x \) (and hence \( x \)) are \( \bar{h}_i(x) \) and the \( c_i(x) \) in \( CAR_i(x) \) and hence \( BVAR_i(x) = CAR_i(x) \cdot \bar{h}_i(x) \). These then give the percent changes in \( v(x) \) and \( v_i(x) \).

Of particular interest is to decompose the changes in \( c_i(x) \bar{h}_i(x) \) into changes due to infill (a new building with a footprint that did not overlap with any building in 2003/4), demolition (a building in 2003/4 that has been demolished is now all open space) and redevelopment (a 2003/4 building which has been replaced by a new building with a different footprint). Note for redevelopment, we have both a net change in footprint (\( \Delta c_i^R(x) \)) and a new footprint (\( c_i^R(x) + \Delta c_i^R(x) \)). We obtain the three by overlaying images polygons from 2003/4 with 2015.
3.4.2. Results on dynamics in the formal sector

Between 2003/4 and 2015 there is dramatic change in the city. There is substantial infill especially farther from the centre and substantial increase in heights nearer the centre achieved through redevelopment. In the first kilometre from the centre however there is less change. Use in the centre is locked in historical buildings and roads, and sky-scrapers built over the last 35 years.

Figure 13 shows for the formal sector height of unchanged buildings, redeveloped ones, and infill. From 1.5 to 5 kms, redeveloped buildings generally average twice the height of unchanged buildings. This is building higher with redevelopment which the model predicts for the formal sector, as a city grows and land prices rise. Infill is at a lower height than either redeveloped or existing buildings at least out to 6 kms, a detail we discuss below.

Figures 14a and 14b give the percentage change in cover and in volume in total and decomposed by infill, redevelopment and demolition. To highlight changes in the core part of the city, we cut-off the graph at 8kms. The numbers at 10 kms are given at the bottom of each table. For totals since slum and formal sector areas are held constant, these also give the percent increases in total CAR and BVAR. While the components should add up to 100% of total change they don’t, because the totals for 2003/4 and 2015 include unchanged buildings. The ascribed footprints for unchanged buildings can vary between years, with buildings often outlined more tightly in 2004 (see Appendix). Demolition (without redevelopment) involves small coverage and volume changes throughout. Further from the centre where there is more available land, infill in coverage is enormous.

Next we directly compare infill and redevelopment. Beyond 1 km infill dominates in contribution to total coverage change, noting that redevelopment cover is net (2015 minus 2004 footprint size). The key observation, however, is that because of the greater heights of redeveloped buildings, the increase in building volume in the range 1-4.5 kms is dominated by redevelopment rather than infill. The net increase in volume just at 3kms out is over 35% due to redevelopment, with over another 20% from infill. We don’t have comparable numbers on volume changes for other cities; but, for a rapidly growing city like Nairobi, as we will discuss next, even near the centre there is an extraordinary pace of change.

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16 Infill (N): $\Delta c_i^{\text{N}}(x), \Delta v_i^{\text{N}}(x) = h_i^{\text{N}}(x)\Delta c_i^{\text{N}}(x), \text{where 1 is 2015.}$

Demolition (D): $-\Delta c_i^{\text{D}}(x), -\Delta v_i^{\text{D}}(x) = h_i^{\text{D}}(x)\Delta c_i^{\text{D}}(x) \text{ where 0 is 2003/4}$

Redevelopment (R): $\Delta c_i^{\text{R}}(x), \Delta v_i^{\text{R}}(x) = h_i^{\text{R}}(x)[c_i^{\text{R},0}(x) + \Delta c_i^{\text{R}}(x)] - h_i^{\text{R}}(x) c_i^{\text{R},0}(x)$
Churning of small lots

In Figure 15, we focus on an unusual feature: churning. In solid lines we show the changes in formal sector building counts as a percent of 2003/4 counts and in dashed lines the same for area covered. Even within 1.4 to 4kms about 35% of buildings are torn down and 50% of those are redeveloped. To benchmark that demolition rate of 35% we note in the USA from the American Housing Survey, for 2009 to 2013, the annual rate of demolition and removal by disaster is 1.18% a year. For 11 years this would involve 12-13% of building removal. Nairobi is 3 times that, the outcome of a pace of redevelopment beyond the current USA experience.

While there is high degree of churning in counts, the coverage area involved is small. Demolition (without replacement) is about 15-20% of 2003/4 building counts from 2-6 kms out but only involves about only 5% of cover. Infill by counts adds about 40% to 2003/4 counts from 1.5 to 4kms out, but involves just 10-18% to 2003/04 cover. After 4kms, the infill percent escalates as \( x \) increases (which is why the graph is cut at 8kms rather than 10). The 2003/4 footprint size \( (c_{i,0}^{R,0}(x)) \) involved in redevelopment from 2-6 kms bounces along at 6-10% of 2003/4 cover, while the rate of counts redeveloped is double that. Finally, however, redeveloped buildings have a distinct and very large increase \( (\Delta c_i^R(x)) \) in average footprint size. The increase in footprint size of redeveloped building averages 100% at 3 kms and rises to 200% by 6kms.

Redevelopment often involves situations where coverage can be extended and/or land assembled to increase footprint size, which is needed to build to a higher height. In contrast infill and demolition involve historically smaller hemmed-in lots, on which it is hard to build high. For example, in a sampling of 50 in-fill buildings from 0-1.5kms, 32% involve building on top of small parking lots. Land released by demolition (which could be later redeveloped) seems to go to other needed uses. In a sample of 50 demolitions from 0-1.5kms, current usage is parking areas (27%), roads (15%), gardens for others (10%), and small sandwiched spaces (19%); only 29% are more open spaces, mostly with vegetation. Further out at (1.5 – 3kms and at 5-6kms which are similar), a sample of 100 demolitions has more garden (19%) and road usage (40%) with less open space (18%) and parking (14%).

4. Slum redevelopment and lack thereof

What goes on within the slum sector? We know from Figure 4 that slums seem to have disappeared up to 2kms out. However we can’t directly capture slum conversion, so we focus on changes within areas defined as slums in 2011. In the Appendix we show the same graphs
(13-15) as we did for the formal sector. Average slum CAR percent increases really escalate near the edge (9-10 kms out), hitting 200% at 10kms, indicating the rapid development of slum areas at the city edge. Note from the notes on Figure 14a the formal sector has only 140% increase in CAR at 10kms. Heights of almost all slum buildings remain low, so that in slums coverage and volume percent changes mimic each other. The graph on churning is fairly noisy but is not dissimilar to the formal sector. However, redevelopment within slum areas is not building to a higher height, but rearranging the meccano parts or replacing rapidly depreciating mud structures. The graph on slum heights shows that heights of redeveloped buildings in slums are only slightly higher (at about 5m) then unchanged buildings (at about 4m). Further much of this change could involve some modest degree of conversion of slum areas to formal sector ones. To try to get a sense of this, Figure 16 looks one aspect of changes with the 2011 defined slums: a right tail of tall new buildings. The vast majority (65%) of buildings which are at 3m high are not shown in the figure. The rest of the figure shows the height distribution of unchanged versus redeveloped buildings within city slums overall. From the right tail, we can see that slums in 2004 did have a very few high buildings which we suspect is misclassification. However in general there is an increase everywhere beyond 4 meters high in the share of tall developed buildings even ones than are 16-20 meters high. We suspect much of this right tail is redevelopment to a higher height in the formal sector, but the amounts involved are small.

We expected much more evidence of slum redevelopment especially near the city centre. We know from various images for example that changes to formal sector use in Kibera, the huge slum at about 3.5-5kms from the centre, are limited to a few edge areas of the slum. We think the reason is that due to land market ‘institutions’ and lack of reform, formalisation costs nearer the centre are very high. This idea is based on two sources: patterns of slum ownership and the substantial literature on Nairobi slums. IPE (2012) produced a map of which slum lands are under government control versus under private ownership. As Figure 17 shows, government ownership is 100% near the centre and then declines as we move out, while private rises and there is a residual (Nairobi City Council, mixed private and government, temporary occupation licenses, and road and riparian reserves). If private is truly private, formalisation in response to market forces should be more forthcoming. However that will not be the case for the government owned slums near the centre. Why?

The literature on Nairobi slums, some focused on Kibera, suggests government owned slums are intractable problems. Research studies and government reports discuss corruption, the array of actors involved in slums, and ‘outright plunder’ (Marx, Stoker and Suri 2013 and Southall 2005). Studies suggest slum housing is almost all rented and the housing is operated by slum lords who make high profits. Guylani and Talukdar (2008) estimate payback periods on an investment in a single room of just 20.4 months. In Kibera, of 120 slum lords surveyed,
41% were government officials, 16% (often the biggest holders) were politicians, and 42% were other absentee owners (Syagga, Mitullah, and Karirah-Gitau 2002 as cited in Gulyani and Talukdar 2008). The political economy issue is that if the government were to take the land and auction it for formal use, the slumlords would have no claim to the revenue since they don’t own the land and their presence is at best quasi-legal. They would simply lose profitable businesses. Having well connected bureaucrats and political figures opposed to conversion presents a political problem.

For Kibera as an example, the problem is accentuated by Kibera’s history, and we suspect the history of many government owned slums. The 1000 acres in Kibera was awarded to Nubian soldiers in 1912. They immediately occupied a portion of the land but at independence their claims (but not tenancy) were revoked, and land reverted to the government. The large portion of Kibera not occupied by Nubians was settled on by others and had titles illegally allocated by local chiefs and bureaucrats. The moral claim of the Nubian descendants to at least the land they occupy is well recognized but the unwillingness to grant them title is yet another road block to redevelopment (Joireman and Vanderpoel, 2011).

We now turn to welfare costs, estimates of the benefits of redevelopment of slum lands as measured by increases in land values. In Table 3 we show the amount of land in slums in different rings from 2-6kms from the city centre which is what is at stake. For this land, we know the BVAR differences for redeveloped buildings as also pictured in Figure 18. The BVAR takes 2015 CAR in slums versus the formal sector at each but applies heights of corresponding redeveloped buildings to get BVAR for new slum vs formal sector developments. Differences are modest but the formal sector does now provide greater BVAR in general than slums, unlike Figure 10a. Finally we have the main difference upon which we focus: house price per square meter of floor space.

Figure 18a plots log price per square meter of floor space from the NORC data for 2012 in slums versus the formal sector by distance from the centre (or where the data start at 2.5kms from the centre). We use a regression to smooth out the gaps and variation by distance in coverage by NORC, where the only covariates are distance, a slum indicator, and their interaction as reported in Table 1, column 5. We infer that the gap in price between the formal and slum sector reflects all quality differences: quality in floor space provided in iron sheet or mud dwellings including facilities compared to permanent structures and quality of amenities offered by green space and side roads, as well as disamenities from crowding per

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17 Further documentation on the Nubian settlers in Kibera can be found online at Kenya’s Nubians, who also argue that the Nubians have a valid claim to the land in Kibera. (http://www.nubiansinkenya.com/)

18 Note CAR is not well defined for infill vs redevelopment since it is an overall area concept bringing in side streets and green spaces.
se. One might argue that socio-economic status also differs, but based on results in the Appendix, it would seem little of the price gap would be explained by such differences.

In Figure 18a, the formal sector unit volume (floor space) rent gradient declines with distance from the city centre. But the slum one does not; if anything, it rises. The key explanation comes from the model in Section 2 and Figure 9. In Figure 9, slums nearer the centre have much less green and road space, and more crowding compared to those further out. In the benchmark case in Section 2, quality adjusted price of slum housing across the city is constant and, as we saw there, for some functional forms this price could increase with distance. A second explanation has to do with possible job access. Slum residents in Kibera for example are a long way from industrial sector locations on the east side of the city and may not value much their access to the professional and tradeable business service sector in the city centre. Finally, in government slums near the centre, there may be less incentives for individual slumlords to invest in slum amenities because they do not own the land. In private slums, owners have a longer view and can potentially reap the benefits of some infrastructure investments since they own the land. By 9kms out there are no price/quality differences between slums and the formal sector.

We treat the rent differential at any distance as a reflection of the revenue gains to be made from switching one unit of floor space from slum to formal sector usage. In Table 3, for 3-4 kms out, the rent differential per sq meter of floor space is 285 KES. Since we have no cost information to compare slum vs formal sector house costs, if we assume that land’s share in revenues is 40% in a major city (see below), this implies there is 114KES in surplus land rents which could be generated per month per sq meter of floor space. That is 70% of what tenants are paying already in total per sq m floor space and is a surplus that could be split between tenants, the state and the slumlords (who have no legal claims). This is the marginal gain in flow terms for converting one unit of floor space.

One can also do present value calculations for non-marginal changes accounting for the modest BVAR differences. We take the BVAR for redeveloped buildings in the slum vs formal sector shown in Figure 18b. We then can estimate current revenues for any area of

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19 In the Appendix we present a hedonic regression of rent per square meter on all sorts of house characteristics, a slum dummy, distance and slum x distance slum, as well as percent of the population which has some college (the key socio-economic variable after much experimentation). From that, we can take the coefficient on percent college (0.615) and multiply by 0.22, the difference in average percent between slums and formal sector. That 13% is an estimate of the portion of the price gap in Figure 18a explained by socio-economic spillover differences between sectors. At 2kms, that is a modest part of the 250% by which formal rents exceed slum ones. We use raw rents rather than predicting rents for typical slum versus formal sector houses in the hedonic for two reasons. The Rsq is modest (0.39); and related we do not think the slum and slum x distance interaction terms capture how amenities vary within slums. The latter statement requires explanation.

20 This graph also removes from consideration all slum grid squares where average height exceeds 9 meters and thus are misclassified (this is about 4% of all slum grid squares).
slums (slum BVAR x slum area x slum volume price) and what could be realized under formal sector redevelopment (formal BVAR times slum area x formal sector volume price).

Present value numbers are given in Table 1. We convert the monthly revenues to 2015 annual revenues in dollars (where house price appreciation from 2012 in Nairobi is 8% a year), apply an exchange rate of $1 = 100 KES and then obtain the present value of an indefinite stream discounting at 4%. This is done for each distance ring from 2-3 kms up to 5-6 kms.

The bottom row of Table 1 shows the present value of revenues in each ring in millions of dollars. If we sum up, we get $US about 4 billion for conversion. As noted, we have no cost side to this conversion of slum to formal sector use. If we think land rent revenues are 40% of house price revenues at this distance from the centre (Duranton and Puga, 2015), the implication is that formalisation raises land values by about $1.36 billion, a measure of welfare cost of non-conversion at today’s price conditions and heights. If land rents are a higher share of house revenues in slums than in formal areas, then the calculation depends on the actual share numbers. In the theory section we set slum land shares at 57% (λ = 2.3) and formal at 40% (γ = 1.66). The $1.36b number averages over $15000 per slum household in that distance band.

These non-marginal PV numbers have three issues, and simply give some sense that there is a huge surplus out there to play with. First they ignore the general equilibrium and spatial effects throughout the city of moving 400,000 near the centre out of slums, including the 200,000 in Kibera (overstatement). We have discounted as though we had an infinite horizon to slum life (overstatement). The differentials in rent in real terms and in volumes per unit area are maintained indefinitely, rather than potentially growing as relative and absolute demand for formal sector use increases near the centre (understatement).

Whatever the exact magnitude, there is a vast surplus in land values which could be used to buy-out vested interests of slum lords hindering formalization of lands, as well as helping with relocation. One economic solution would be to give longer term residents ownership of their units and land, allowing redevelopers to buy them out in a timely (and voluntary) fashion; but that solution would require settling with slum lords.

5. Other Considerations

The model only examines residential housing capital; and, in the data, we lump all built space together. We do not know building usage per se and in slums many buildings may have a dual residential-production purpose. We do have land use maps. Much land at the centre is classified as in commercial use. Industry is in the eastern half of the city, with older industrial areas starting to the immediate south-east of the city centre and then stretching out. Other large industrial areas are to the north-west away from the centre. All these are far from
Kibera in the western part of the city. We also can calculate the ratio of volume of built space to population in the formal and slum sectors. In the centre of the city, volume to population is very high given the intensity of commercial use. Volume to population in the formal sector then falls with distance out to 2kms; it then is similar to that in slums (which is pretty flat throughout) until 4kms, before rising again. Between 4 and 8 kms, volume to population is high perhaps because of industrial uses.

Finally we have ignored heterogeneity of the population, as being beyond the scope of the paper. We note two things however. First overall higher socio-economic families tend to live nearer the centre. Second and most relevant to our analysis is that slums are not low education-income havens. About 24% of both slum and formal sector residents are high school grads (form 4). Both areas have about equal fraction of very low and very high education families. The divide is between standard 8 where 23% of slum residents are standard but only 13% of formal; and 21% of formal sector household heads have a college degree (but not more) and only 7% of slum.

6. Conclusions

The model and data both suggest that in the formal sector house rents and land prices decline with distance to the centre; consequently building heights decrease with distance to the centre. Heights in slum areas are much lower than in the formal sector near the centre and flat throughout the city. However intensity of land cover within slums is very high; and combining height and cover differences, both sectors offer similar housing volumes per unit area. Slums account for a small fraction of total housing space overall at any distance from the centre. Between 1-6 kms from the centre from 2004 to 2015 there is major redevelopment of 2004 formal sector buildings into taller new buildings. Expansion of the informal sector is towards the city fringe. We find that there is high intensification of land use with infill of new buildings through much of the city especially on the fringes. We find that development of slum into formal sector housing mid-city over the 11 years is very slow.

Related to this slow conversion of slum to formal sector usage, in the model we explore the cost of formalisation costs. In the data for Nairobi which provides an institutional context which is similar to that in many cities in the developing world, we explore misallocation of land between slums and formal sector usage. We argue that slum ‘ownership’ by government means unresolved land right issues and corruption with vested slum interests of political figures. There is a significant welfare loss from the inability to convert slum to formal sector usage nearer the city centre. In the model we also explore the role of expectations in altering (re)development paths. Under-estimating future demand growth leads to stunted city heights and spatial size.
Theory Appendix

**Derivation of equation (11):** Derivation of (11) uses

\[
\frac{\partial R_F(x, \tau_i)}{\partial \tau_i} = -p_F(x,t)v_F(x, \tau_i) + \rho \int_{\tau_i}^{\tau_{i+1}} p_F(x,t)v_F(x, \tau_j)e^{-\rho(t-\tau_j)} \, dt
\]

\[
= -p_F(x,t)v_F(x, \tau_i) + \rho [R_F(x, \tau_i) + k_F(v_F(x, \tau_i))].
\]

And the fact that volume is optimised.

**Parameters for figures**

Parameter values in figure 1 are: \( c_F = 1, \ c_I = 0.1, \ \gamma = 2, \ \lambda = 3.33, \ \rho = 0.05, \ \hat{p}_I = \hat{p}_F = 0.015, \ \theta_F = \theta_I = 0.05, \ \tau_0 = 4, \ \bar{p}_F = 5, \ \bar{p}_I = 4 \). Simulation is done with time running to \( t = 800 \), and reported up to \( t = 250 \). Distance running to \( x = 60 \). In figure 1 formalisation cost \( D=1000 \), in figure 2a \( D = 1000 \) or \( D=5000 \) (10 \( x < 20 \)) and, for comparison, the construction cost of the first formal sector structure built is 4500).

**Section 2.5: Closing the model**

**Households:** At date \( t \) a representative urban household living at distance \( x \) from the CBD receives income net of commuting costs \( w(t)T(x) \), where \( w(t) \) is the wage at date \( t \) (the same for all households), and \( T(x) \) is the fraction remaining after commuting costs. Each household makes a discrete choice between formal and informal sector housing. For the chosen sector, the household chooses \( s_i(x,t) \) units of housing (i.e. volume), at price \( p_i(x,t) \) per unit in the formal sector, and \( p_i(x,t)q(x,t) \) in the informal sector. Utility is derived from the volume consumed, its quality and formal/informal status, and consumption of a numeraire good (equal to wage income net of commuting and housing costs). For each type of housing,

\[
u_F(x,t) = u(s_F(x,t), w(t)T_F(x) - p_F(x,t)s_F(x,t)) : F)
\]

\[
u_I(x,t) = u(s_I(x,t)q(x,t), w(t)T_I(x) - p_I(x,t)q(x,t)) : I).
\]

If preferences are Cobb-Douglas then

\[
u_F(x,t) = s_F(x,t)^{\alpha_F} \left[ w(t)T_F(x) - s_F(x,t)p_F(x,t) \right]^{1-\alpha_F}
\]

\[
u_I(x,t) = q_I(x,t)^{\alpha_I} s_I(x,t)^{\alpha_I} \left[ w(t)T_I(x) - s_I(x,t)p_I(x,t)q(x,t) \right]^{1-\alpha_I},
\]

Consumers take price and quality of housing at each place as given, and the quantity of housing space, \( s_i(x,t) \), is chosen to maximise utility. Optimal choice gives,

\[
s_F(x,t) = \alpha_F w(t)T_F(x) / p_F(x,t), \quad s_I(x,t) = \alpha_I w(t)T_I(x) / p_I(x,t)q(x,t).
\]

Maximised utility for each type of house is
\[ U_f(x,t) = A_f p_f(x,t)^{-s_f} w(t) T_f(x), \quad U_i(x,t) = A_i p_i(x,t)^{-s_i} w(t) T_i(x), \]
\[ A_i = \alpha_i^{-s_i} (1 - \alpha_i)^{1-s_i}. \]

Free choice of location and housing type means that, at any occupied location and housing type, utility equals a common city-wide utility level, \( \bar{U}(t) \). Prices of formal and informal (quality one) housing must therefore satisfy
\[ p_i(x,t) = \left( \frac{w(t) T_i(x)}{A_i \bar{U}(t)} \right)^{1/\alpha_i}, \quad p_F(x,t) = \left( \frac{w(t) T_F(x)}{A_F \bar{U}(t)} \right)^{1/\alpha_F}. \] 

(A3a)

Constant exponential growth of the price of space is achieved by assuming that urban wages relative to outside utility grow at constant rate \( g \). Similarly, constant exponential decline with respect to distance is achieved by the share of income net of commuting declining with distance at rates \( \hat{T}_i, \hat{T}_F \), so \( p_i(x,t) = \left( w e^{g t - \hat{T}_i x} / \bar{U}(t) \right)^{1/\alpha_i}, i = I, F \). This gives prices rising through time at constant rates \( \hat{p}_i = g / \alpha_i, \hat{p}_F = g / \alpha_F \), and declining with distance, \( \theta_i = -\hat{T}_i / \alpha_i, \theta_F = -\hat{T}_F / \alpha_F \).

**Labour and population:** To complete the model, we note that population at a point is \( v/s \), total volume supplied divided by consumption of floor space per household. Total city population at date \( t \) is therefore
\[ L(t) = \sum_{i=1}^{i=\text{max}(t)} \int_{\eta_i(t)}^{\xi_i(t)} v_F(x,t) / s_F(x,t) dx + \int_{\eta_i(t)}^{\xi_i(t)} v_i(x,t) / s_i(x,t) dx. \] 

(A5)

The oldest formal development has been redeveloped the most times (which, at date \( t \), we denote \( \text{max}(t) \)). Notice that this expression assumes that the city is linear (or a set of rays), not a disc; adjustment to (A5) to capture the latter is straightforward.

The final element is to close the model, either by setting \( \bar{U}(t) \) exogenously with \( L(t) \) endogenous (open city), or with \( L(t) \) exogenous and determining the equilibrium city-wide level of utility (closed city). The analysis in the body of the paper follows the open city route, with exogenous growth of urban wages relative to outside utility driving housing price growth.

**Data Methodology Appendix**

This Appendix has two components. The first deals with measures on cover/footprint and volume we use to analysis. The second gives the algorithm used to extract unchanged buildings, redeveloped buildings and infill from the overlay of 2004 and 2015 depiction of building polygons.

**Measures of cover and volume**

Our unit of analysis is 150x150m grid squares. For calculating cover within the grid square in a usage, each of these is broken into 50 3m by 3m cells and use type classified by what is at the centroid of the 3m square. There are three uses: vacant land, slum area and formal. Each 3x3 square is given the type of cover there in whichever time period. For each 150x150 square we sum across the 50 cells to get
for example total building cover in each type. If for example a 150m by 150m grid has only formal sector buildings the square meter coverage can take values of 9, 18, 27, etc. up to 450. And the same for areas that are always slums. Most 150x150 squares are either all slum or all formal sector. However there are about 12% which are mixed grid squares, for which we record the cover or volume of slum and formal separately.

For average coverage in a grid square in the formal sector, before smoothing in a year in a given distance ring, the total area of all cover in 3x3 squares is summed up for all 150x150 meter squares whose centroid falls in a narrow distance ring. That sum is then divided by the total number of 150x150 grid squares in that distance band. The same procedure follows for slums. For Volume for 2015, for each 3x3m square which is formal sector, we have the height of the building whose cover is over the centroid of that square. So volume for that 3x3 square is 9 times the height in meters of the building from LiDAR data. We then sum across the grid squares occupied with formal usage for 150x150m grid squares in each distance ring and then average by the total number of 150x150 meter grid squares in the ring. For 2004 we have no height data. To infer 2004 heights, we use what we think is an upper bound on height: the height of unchanged buildings, where we presume demolished buildings between 2004 and 2015 are likely to be of lower height than those which survive. To assign a height to a 3mx3m square in 2004 in formal sector usage, we take the average height in 2015 of all buildings that were there in 2004 for all 3x3m formal sector unchanged buildings in the own 150x150m grids square and its 8 queen neighbours. Height is the height assigned to each 3x3m square in usage in a distance ring from the centre averaged over all such cells, to effectively get a coverage weighted average of individual building heights.

How do we measure change between 2004 and 2015? For demolition, at the 3x3m level the square is defined as demolition if its centroid is covered by a 2004 building which has been replaced by open space. Demolished coverage is lost 2004 cover; demolished volume is assessed as before using the average height of unchanged buildings in the neighbourhood. Infill is new buildings which do not overlap with any 2004 buildings; a 3x3m square is infill if its centroid is covered by such a building on 2015 where there was no building in 2004. Infill cover and volume are assessed from 2015 data. Net redevelopment in coverage takes coverage in the new 2015 buildings and subtracts the coverage of old 2004 buildings. So for each 150m150m meter square we have for redeveloped buildings, we have total coverage in 2004 measured at the 3x3m level (centroid covered by the old 2004 building(s)) and we have total coverage in 2015 measured at the 3x3m squares (centroid covered by the new replacement 2015 building(s)). Net redevelopment at the 150x150square is the difference. In general, the same buildings are drawn in 2015 to have modestly more coverage than in 2004 so coverage change is likely to be an upper bound. Net volume change again assigns heights in 2004 to the 3x3m coverage based on neighbourhood averages for unchanged buildings and uses 2015 height information on the new buildings.

**Overlaying Buildings**

We match buildings across time by overlaying 2015 and 2004 building polygon data in order to track the persistency, demolition, construction and reconstruction of buildings over time. Since buildings are not identified across time our links rely on a shape matching algorithm. For each building, the algorithm determines whether it was there in the other period, or not, by comparing it with the buildings that overlap in the other time period.

This task is not straightforward, since the same building can be recorded in different ways depending on the aerial imagery used, whether building height was available, and the idiosyncrasies of the human digitizer.

**Data and definitions**
For 2004 we use a building dataset received from the Nairobi City Council with digitized polygons for every building, roughly 340,000 in the administrative boundary of Nairobi. For 2015 we use a similar dataset that was created by Ramani Geosystems using imagery (10-20cm resolution) and LiDAR (0.3-1m resolution). We have 2015 data for a wider extent, and consequently many more buildings, about 1.14 million. The LiDAR data in 2015 were used to measure heights of objects. With use of the aerial imagery and heights in 2015, a 3D model was created by hand, and rooftops extracted from this model.

Here we define the nomenclature that we use. First, a **trace** is the collection of polygon vertices that make up its outline. A **shape** is the area enclosed by the trace, and it can be thought of as a representation of the rooftop of a building. A **cavity** is an empty hole completely enclosed in a shape. A **candidate pair** is the set of any two shapes in different time periods which spatially intersect. A **link** is the relationship between a set of candidates in one period to a set of candidates in the opposite time period.

**Pre-processing**

Before running our shape matching algorithm we clean up the data sets. First we take care of no data areas. There are some areas that were not delineated in 2004, including the Moi Air Base, and the Nairobi State House. We drop all buildings in these areas for both 2004 and 2015. We drop roughly 1,500 buildings from the 2015 data, and 100 buildings from the 2004 data. Next we deal with overlapping shapes. While the 2004 data has no overlapping shapes, in the 2015 data there are some shapes that overlap. This is most often the same building traced multiple times. We identify all such overlapping polygons and discard the smaller version, until no overlaps remain. We drop about 1,400 buildings from the 2015 data this way. We also decide to drop small shapes, in part because the 2015 data has many very small shapes, while the 2004 data does not. In order to avoid complications of censoring in the 2004 data, we simply drop all shapes that have an area of less than 1m². We drop 2 small buildings in 2004, and 462 small buildings in 2015.

Another issue is that buildings are often defined as contiguous shapes in 2004, but broken up in 2015. For the majority of buildings we cannot aggregate the broken up pieces in 2015 since it is hard to identify such cases in general. To match these cases across time we rely on our one to many, and many to many matching algorithms defined below. However, in the specific case where a building is completely enclosed in another the task is much easier. First, we find all cavities present in each period, then we take all building shapes that overlap with the cavities in the same time period. After identifying all shapes that intersect a cavity, we redefine both shapes, the original shape containing the cavity and the shape intersecting it, as a single new shape.

**Shape Matching Algorithm**

After the pre-processing of each cross-section is complete, we run our shape matching algorithm to establish links between buildings across time periods. For any given building we consider 5 possible scenarios; that it has a link to no building, that it has a link to one building (one to one match), that it has a link to multiple buildings (one to many), that it is part of a group of buildings that match to one building (many to one), or that it is a part of a group of buildings that matches to a group of buildings (many to many). We follow and approach similar to Yeom et al (2015) however, due to the inherent difficulty of inconsistent tracings we contribute to their method by introducing the one to many and many to many approaches. We assign each link a measure of fit that we call the overlay ratio. We then choose optimal links based on the overlay ratio. Finally, we categorize links as matched or not using a strict cut-off on the overlay ratio of 0.5. Other cut-offs such as 0.4, 0.6 and 0.7 produced more errors in categorization.

**Candidates**
For all buildings A in the first time period, and B in the second time period we identify the set of candidates:

$$CP = \{(A, B); \text{Area}(A \cap B) \neq 0\}$$

For each candidate pair we find the ratio of the intersection area over the area of each shape, so if shapes A and B intersect, we find

$$r_{AB} = \frac{\text{Area}(A \cap B)}{\text{Area}(A)} \quad \text{and} \quad r_{BA} = \frac{\text{Area}(A \cap B)}{\text{Area}(B)}$$

We link all shapes which do not belong to a candidate pair to the empty set.

**One to One Matching**

First we consider candidate pairs to be links on their own. For each pair, we calculate the overlay ratio as the intersection area over union area, so if A and B are candidate pair, we find:

$$R_{AB} = \frac{\text{Area}(A \cap B)}{\text{Area}(A \cup B)} = \frac{\text{Area}(A \cap B)}{\text{Area}(A) + \text{Area}(B) - \text{Area}(A \cap B)}$$

**One to Many Matching**

For each time period separately, we identify all candidate pair links for which their intersection to area ratio is above threshold \( \theta \). For shape A we define a group \( G = \{B; r_{BA} \geq \theta\} \). Now we calculate the overlay ratio of one to many links as the intersection area over union area ratio:

$$R_{AG} = \frac{\text{Area}(A \cap \bigcup B \in G B)}{\text{Area}(A \cup \bigcup B \in G B)} = \frac{\sum_{B \in G} \text{Area}(A \cap B)}{\sum_{B \in G} \text{Area}(A \cup B)}$$

**Many to Many Matching**

Here we have two cases, one when the shapes are fairly similar, which we capture in previous sections (one to one, or many to one). The other is inconsistent shapes that form the same structure. To capture these we consider both time periods at the once, we clean the candidate pair list, keeping links for which either ratio is above a threshold \( \theta_1 \):

$$LC = \{(A, B); \, r_{AB} \geq \theta_1 \text{ or } r_{BA} \geq \theta_1\}$$

Then we condition to only keep shape for which the total ratio intersection is above threshold \( \theta_2 \), so shape A will be included if \( \sum_{B \in \{X(A, X) \in LC\}} r_{AB} \geq \theta_2 \). Now we are left with a new candidate list, which we convert to sets \( LC = \{(A), \{B\}\} \) and start merging them:

if \( G_i \cap G_j \neq \emptyset \text{ or } H_i \cap H_j \neq \emptyset \): \( LC = \{(G_i \cup G_j, H_i \cup H_j)\} \cup LC /\{(G_i, H_i), (G_j, H_j)\}, i \neq j \)

We keep doing this until we can no longer merge any two rows. At this point we calculate the overlay ratio of many to many links as the intersection area over union section ratio:

$$R_{GH} = \frac{\text{Area}(\bigcup A \in G \cap \bigcup B \in H B)}{\text{Area}(\bigcup A \in G \cup \bigcup B \in H B)}$$

**ICP Translation**

We encounter a problem when the two shapes or groups of shapes are similar but do not overlap well, this usually stems from the angle at which the images were taken, and is especially prevalent with tall buildings. To address this issue, we translate one trace towards the other, and then recalculate the overlay ratio. As in Besl and McKay (1992), we use the iterative closest point (ICP) method to estimate this translation. To perform the ICP we ignore any cavity points as we found they often cause less suitable translation. We found that for similar shapes this will optimize the intersection area.

**Optimal Linking**

In the end, we rank all links by their overlay ratio. We iteratively keep the link with the highest overlay ratio, or discard it if at least one of the buildings in the link has already been confirmed in a
separate link. From the list of optimal links, we define a link to be a match if its overlay ratio, or the overlay ratio after ICP translation is above 0.5. We then define all matched candidates as unchanged, and the remaining candidates as redeveloped. All buildings that were not considered as candidates are defined as infill, if from 2015, and demolished, if from 2004.

**Accuracy Assessment**

In order to assess the performance of the polygon matching algorithm we manually classified links between 2004 and 2015 for a random sample of buildings. We sampled 48 150x150m gridcells, stratifying over slum, non-slum within 3km, non-slum within 6km, and non-slum further than 6km to the CBD. The sample consists of over 2,250 buildings in 2004 and 3,500 buildings in 2015.

**Results**

We first break down matches by their mapping type. There are five types of manual link: redeveloped/infill/demolished (0), one to one match (1), one to many match (2), many to one match (3), and many to many match (4). For the algorithm we further split (0) into infill/demolished (-1) and redeveloped (0). Appendix table 1 shows the correspondence between the two mappings by building (a) and roof area (b). We can see that most errors come from the one to one matches, however, the many to many matches have the worst performance. Overall the diagonal values are quite high, which means not only are we matching buildings well, but also the algorithm is recognising the clumping of buildings as a human does (bear in mind that, for example, the one to one matches which we ‘misclassify’ as many to many will still be classified as match in the final data). Finally we have perfect correspondence for demolition and in 2015 nearly perfect for infill.

Next we compare buildings that were matched by the algorithm and those matched manually. For now we use a cut-off of the overlay ratio of 0.5, later we explore the effect of different cut-offs on performance. As seen in appendix table 1 infill and demolition are classified with almost perfect correspondence. For this reason we ignore buildings with these mappings and focus on accuracy of redevelopment and unchanged. In appendix table 2 we condense mappings 1, 2, 3, and 4 into category 1, while redevelopment, or category 0, remains the same.

We define precision $P$ (negative predictive value $NPV$) as the fraction of buildings classified as unchanged (redeveloped) by the algorithm that are correct, recall $R$ (true negative rate $TNR$) as the fraction of buildings classified as unchanged (redeveloped) by hand that the algorithm gets correct, and the F1 score $(F)$ as the weighted average of the two.

\[
P = \frac{\text{True Positive}}{\text{Positive Predictions}}, \quad NPV = \frac{\text{True Negative}}{\text{Negative Predictions}}, \quad R = \frac{\text{True Positive}}{\text{Positive Condition}}
\]

\[
TNR = \frac{\text{True Negative}}{\text{Negative Condition}}, \quad F = \frac{2 \cdot P \cdot R}{P + R}
\]

The confusion matrix in table 2 is done across all sampled buildings in 2004 and weights observations by buildings (1) and roof area (2). The F1 score is high in both cases, but in part this is due to relative success classifying unchanged buildings: precision for buildings that were classified as unchanged (redeveloped) by the algorithm is 76% of buildings and 72% of roof area, while recall of true redeveloped buildings is 83% of buildings and 74% of roof area.

In our first attempt we arbitrarily picked 50% as a cut off of the overlay ratio. Here we take a closer look at this choice. Using our manually classified links we can maximize the F1 score with respect to the cut off. In appendix figure 1 we plot the F1 score weighted by roof area against cut-offs of the
overlay ratio for the 2004 data. We find that the highest F1 score comes just below 50% suggesting our first estimate was not far off.

In figure 1 we plot lines for each method of calculating the overlay ratio: without ICP, with ICP, and the maximum of the two. Around 50% we can see that the maximum performs best, but with only a very slight improvement over the ICP alone, which is in turn marginally better than without the ICP.

Appendix Table 1 – Mapping Correspondence 2004

<table>
<thead>
<tr>
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<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Algo=-1</td>
<td>280</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Algo=0</td>
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<td>25</td>
<td>29</td>
<td>18</td>
<td>65</td>
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<td>712</td>
<td>21</td>
<td>6</td>
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<td>10</td>
<td>266</td>
<td>0</td>
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<td>1</td>
<td>0</td>
<td>137</td>
<td>63</td>
</tr>
<tr>
<td>Algo=4</td>
<td>20</td>
<td>25</td>
<td>20</td>
<td>1</td>
<td>135</td>
</tr>
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</table>

Mapping definitions: -1 demolition or infill; 0 redevelopment; 1 one to one match; 2 one to many match; 3 many to one match; 4 many to many match

Appendix Table 2 – Matching all areas 2004

<table>
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</thead>
<tbody>
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<td>Algo=-1</td>
<td>12708</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
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<tr>
<td>Algo=0</td>
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<td>3575</td>
<td>3575</td>
<td>910</td>
<td>5317</td>
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<tr>
<td>Algo=1</td>
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<td>2328</td>
<td>112762</td>
<td>1053</td>
<td>5528</td>
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<tr>
<td>Algo=2</td>
<td>2780</td>
<td>89472</td>
<td>4180</td>
<td>0</td>
<td>4795</td>
</tr>
<tr>
<td>Algo=3</td>
<td>943</td>
<td>279</td>
<td>279</td>
<td>0</td>
<td>4464</td>
</tr>
<tr>
<td>Algo=4</td>
<td>1043</td>
<td>1775</td>
<td>1775</td>
<td>2819</td>
<td>14262</td>
</tr>
</tbody>
</table>

Mapping definitions: -1 demolition or infill; 0 redevelopment; 1 one to one match; 2 one to many match; 3 many to one match; 4 many to many match

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>Recall</td>
<td>0.83</td>
<td>0.91</td>
<td>0.76</td>
<td>0.94</td>
<td>0.93</td>
</tr>
<tr>
<td>Precision</td>
<td>0.74</td>
<td>0.96</td>
<td>0.72</td>
<td>0.96</td>
<td>F=0.96</td>
</tr>
</tbody>
</table>

37
Appendix: List of public uses

Recreational
a) Impala club, Kenya Harlequins, and Rugby Union of East Africa (0.14kmsq)
b) Golf Course (0.9kmsq)
c) Arboretum (0.25kmsq)
d) Central park, Uhuru park, railway club, railway golf course (0.5kmsq)
e) Nyayo stadium (0.1kmsq)
f) City park, Simba Union, Premier Club (1.1kmsq)
g) Barclays, Stima, KCB, Ruara, Utali clubs, and FOX drive in cinema (0.3kmsq)

Undeveloped
a) Makdara Railway Yard (1kmsq)
b) John Michuki Memorial Park (0.1kmsq)

Special use -- Includes poorly traced areas
a) State House
b) Ministry of State for Defence
c) Forces Memorial Hospital and Administration Police Camp
d) Langata Army Barracks
e) Armed Forces
f) Moi Airbase
g) Kahawa Garrison

Public utility
a) Dandora dump (0.5kmsq)
b) Sewage works (0.25kmsq)

Public use
a) Communications Commission of Kenya (0.1kmsq)
b) Langata Womens prison (0.2kmsq)
c) Nairobi and Kenyatta hospitals, Milimani Police Station, Civil Service club
d) Mbagathi hospital, Kenya Medical Research Institute, Monalisa funeral home
e) National museums of Kenya
f) Kenya convention centre and railway museum
g) Industrial area prison
h) Mathari mental hospital, Mathare police station, traffic police, Kenya police, Ruaraka complex, and National youth service
i) Jamahuri show ground

Educational (not primary and secondary schools)
a) University of Nairobi and other colleges
b) Kenya Institute of Highways & Built Technology
c) Railway Training Institute
d) Kenya Veterinary Vaccines Production Institute
e) Moi Forces Academy
f) NYS engineering, Kenya Institute of Monetary Studies, KCA university, KPLC training, Utali college

Appendix: Hedonic regression based on NORC data for 2012

<table>
<thead>
<tr>
<th></th>
<th>(1) Ln Rent per m-sq</th>
<th>(2) Ln Rent per m-sq</th>
<th>(3) Ln Rent per m-sq</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distance to Centre</td>
<td>-0.0884***</td>
<td>-0.0512***</td>
<td>-0.0415**</td>
</tr>
<tr>
<td></td>
<td>-0.0176</td>
<td>-0.0173</td>
<td>-0.0167</td>
</tr>
<tr>
<td>Slum=1 X Distance to Centre</td>
<td>0.136***</td>
<td>0.0659**</td>
<td>0.0627**</td>
</tr>
<tr>
<td></td>
<td>-0.0318</td>
<td>-0.0274</td>
<td>-0.0258</td>
</tr>
<tr>
<td>Slum=1</td>
<td>-1.228***</td>
<td>-0.463***</td>
<td>-0.380**</td>
</tr>
<tr>
<td></td>
<td>-0.179</td>
<td>-0.174</td>
<td>-0.169</td>
</tr>
<tr>
<td>Tenancy Agreement=No Written Agreement</td>
<td>-0.19</td>
<td>-0.137</td>
<td></td>
</tr>
<tr>
<td></td>
<td>-0.119</td>
<td>-0.118</td>
<td></td>
</tr>
<tr>
<td>Piped Water in Compound=no</td>
<td>-0.204**</td>
<td>-0.193**</td>
<td></td>
</tr>
<tr>
<td></td>
<td>-0.0872</td>
<td>-0.0863</td>
<td></td>
</tr>
<tr>
<td># Bathrooms=One</td>
<td>-0.163*</td>
<td>-0.192**</td>
<td></td>
</tr>
<tr>
<td></td>
<td>-0.0905</td>
<td>-0.0876</td>
<td></td>
</tr>
<tr>
<td># Bathrooms=Two+</td>
<td>0.0371</td>
<td>0.0233</td>
<td></td>
</tr>
<tr>
<td></td>
<td>-0.103</td>
<td>-0.102</td>
<td></td>
</tr>
<tr>
<td>Type of Structure=Shared House</td>
<td>-0.629*</td>
<td>-0.67</td>
<td></td>
</tr>
<tr>
<td></td>
<td>-0.348</td>
<td>-0.428</td>
<td></td>
</tr>
<tr>
<td>Type of Structure=Single-storey with shared facilities</td>
<td>0.215**</td>
<td>0.215**</td>
<td></td>
</tr>
<tr>
<td></td>
<td>-0.0915</td>
<td>-0.0927</td>
<td></td>
</tr>
<tr>
<td>Category</td>
<td>Coefficient 1</td>
<td>Coefficient 2</td>
<td>Coefficient 3</td>
</tr>
<tr>
<td>----------------------------------------------</td>
<td>---------------</td>
<td>---------------</td>
<td>---------------</td>
</tr>
<tr>
<td>Type of Structure=Room in house</td>
<td>-0.470***</td>
<td>-0.447***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>-0.15</td>
<td>-0.15</td>
<td></td>
</tr>
<tr>
<td>Type of Structure=Shack</td>
<td>-0.339</td>
<td>-0.377</td>
<td></td>
</tr>
<tr>
<td></td>
<td>-0.217</td>
<td>-0.247</td>
<td></td>
</tr>
<tr>
<td>Type of Structure=Multi-storey private bath</td>
<td>0.167</td>
<td>0.134</td>
<td>-0.169</td>
</tr>
<tr>
<td>Type of Structure=Multi-storey shared bath</td>
<td>0.368***</td>
<td>0.382***</td>
<td>-0.0987</td>
</tr>
<tr>
<td>Type of Walls=Brick/Block</td>
<td>0.370***</td>
<td>0.359***</td>
<td>-0.127</td>
</tr>
<tr>
<td>Type of Walls=Mud/Wood</td>
<td>-0.0608</td>
<td>-0.0342</td>
<td>-0.182</td>
</tr>
<tr>
<td>Type of Walls=Mud/Cement</td>
<td>-0.238</td>
<td>-0.184</td>
<td>-0.259</td>
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<tr>
<td>Type of Walls=Wood only</td>
<td>0.390*</td>
<td>0.422*</td>
<td>-0.221</td>
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<tr>
<td>Type of Walls=Corrugated iron sheet</td>
<td>0.171</td>
<td>0.202</td>
<td>-0.14</td>
</tr>
<tr>
<td>Type of Walls=Tin</td>
<td>0.316*</td>
<td>0.337**</td>
<td>-0.17</td>
</tr>
<tr>
<td>Type of Floor=Tiles</td>
<td>0.993***</td>
<td>0.836***</td>
<td>-0.198</td>
</tr>
<tr>
<td></td>
<td>-0.192</td>
<td>-0.1942</td>
<td></td>
</tr>
<tr>
<td>Times Flooded Last Rainy Season=Once</td>
<td>-0.378**</td>
<td>-0.354**</td>
<td>-0.153</td>
</tr>
<tr>
<td>Times Flooded Last Rainy Season=2-3 times</td>
<td>-0.351**</td>
<td>-0.348**</td>
<td>-0.176</td>
</tr>
<tr>
<td>Times Flooded Last Rainy Season=More than 3 times</td>
<td>-0.343***</td>
<td>-0.317***</td>
<td>-0.112</td>
</tr>
<tr>
<td>Ln # Floors</td>
<td>0.115*</td>
<td>0.124*</td>
<td>-0.0685</td>
</tr>
<tr>
<td>EA Builing Cover 2015 (CAR)</td>
<td>-0.095</td>
<td>-0.0137</td>
<td>-0.281</td>
</tr>
<tr>
<td>EA frc household heads with some post-secondary</td>
<td>0.547***</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>6.147***</td>
<td>5.695***</td>
<td>5.367***</td>
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<tr>
<td>Observations</td>
<td>983</td>
<td>902</td>
<td>902</td>
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<tr>
<td>R-squared</td>
<td>0.132</td>
<td>0.37</td>
<td>0.382</td>
</tr>
</tbody>
</table>

Standard errors in parentheses

= ** p<0.05
*** p<0.01"
Slum Dynamics. Slum figures corresponding to Figures 12, 13 and 16

References


Figure 1: Urban development with perfect foresight

Figure 2a: Heterogeneous formalisation costs
Figure 2b: Random variation in formalisation costs

Figure 3: Expectations: volume profile of city at $t = 180$. 

$\text{Building volume}$ 

$x$: distance from CBD 

time 

Perfect foresight: $\Phi = 26$, $\Delta \tau = 70$ 

Low expectations: $\Phi = 20$, $\Delta \tau = 45$ 

Formal (1) 

Informal 

Distance from CBD
Figure 4  City shape

Figure 5. 3-D average height of buildings by grid square in the formal and slum sectors
Figure 6. Land prices per square meter land

This is a regression relationship for price per square meter as a function of distance from the centre, controlling for whether the address was imprecise (usually a lot in an inferior area) and for lot size. Overall $R^2$ is 0.59 in Table 1.

Figure 7

a. Mean height in meters

b. Residential height in floors from survey data

*Estimated from a Poisson regression of individual height floors on distance by type of use.
c. Variability in height

These are mean and percentiles based on pixel (3mx3m) heights.

**Figure 8 Share of land in slums versus formal sector**

The lines add up to 1, but we show both for ease of reading.

**Figure 9. Cover to area ratio, without (building cover, CAR) and with roads**

Dashed lines add road cover to building cover in the numerator. Note road cover in the formal sector far exceeds that in slums. Roads include any paved roads, which can accommodate at least two cars passing.
Figure 10. Built volume per unit area (BVAR)

a. Smoothed means

b. Heterogeneity

Figure 11. Total volume by distance and sector

The dashed lines add up to 1, but we show both for ease of reading.
Figure 12. Growth in total volume and by sector

![Graph showing growth in total volume and by sector](image)

Figure 13. Building higher in the formal sector

![Graph showing mean formal height 2015](image)
Figure 14. Changes in cover (and CAR) and volume (and BVAR)

a. Cover

At 10kms, the total CAR increase is about 140% with infill and net redevelopment approximating 120 and 20% each.

b. Volume

At 10kms the total volume increase is about 125%, with infill and net redevelopment approximating 120 and 20% each.
Figure 15 Churning: Changes in counts and cover due to infill, redevelopment and demolition as a fraction of 2003/4 counts and cover (in the formal sector)

In the formal sector, this takes counts of 2015 infill, 2004 demolished and 2004 buildings that are redeveloped in a radius and divides by original counts in that radius. The cover ratios are calculated on the same basis. E.g., for redeveloped, it is the 2004 footprint of 2004 buildings redeveloped divided by 2004 building cover.

Figure 16 Height changes in 2004 Slums (Not showing 65+% at 3 meters)
Figure 17. Slum ownership

![Share of Slum (2011) Roof Cover by Tenure](image)

Figure 18. The formal-slum sector differentials

a. Rent gap per sq meter of floor space (2012 monthly rent)

![Rent Gradients](image)

* From Table 1. Only covariates are slum, distance to centre and slum*distance to the centre.
b. Built volume to area ratio for redeveloped buildings

![Redeveloped BVAR, (X) 2015](image)

### Table 1. Gradients

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
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<tbody>
<tr>
<td>Ln (R(x): asking</td>
<td>Ln Formal Height</td>
<td>In Slum CAR distance from 2.0 km</td>
<td>In Formal Redeveloped Height</td>
<td>In rent p sq m. per month floor space</td>
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<td>price per unit of</td>
<td></td>
<td></td>
<td></td>
<td>2012 kenyan shilling from 2.5 km</td>
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<td>land for sale)</td>
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<td>Distance to centre</td>
<td>-0.155***</td>
<td>-0.0769***</td>
<td>-0.0902***</td>
<td>-0.0924***</td>
<td>-0.0884***</td>
<td>-0.0512***</td>
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<tr>
<td></td>
<td>(0.0157)</td>
<td>(0.0000337)</td>
<td>(0.0000741)</td>
<td>(0.000126)</td>
<td>(0.0176)</td>
<td>(0.0173)</td>
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<td>Distance to Centre*</td>
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<td>0.0659**</td>
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<tr>
<td>Slum = 1</td>
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<td></td>
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<td>(0.0318)</td>
<td>(0.0274)</td>
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<tr>
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<td>-1.228***</td>
<td>-0.463***</td>
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<tr>
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<td></td>
<td></td>
<td>(0.179)</td>
<td>(0.174)</td>
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<td>Controls (Appendix)</td>
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<tr>
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<td></td>
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<tr>
<td>Controls: lot size,</td>
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<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td>Constant</td>
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<td>5.695***</td>
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<td></td>
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<td>-0.000328</td>
<td>(0.000747)</td>
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<td>26854614</td>
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<td>902</td>
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<td>R-squared</td>
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<td>0.162</td>
<td>0.102</td>
<td>0.118</td>
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<td>0.37</td>
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Table 1. Rough estimates of parameters of model used in simulations

<table>
<thead>
<tr>
<th>Gradient parameters</th>
<th>Estimates in Table 1</th>
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<tbody>
<tr>
<td>Slope of house price gradient (eq 14) ( \theta_f = \theta_r )</td>
<td>-0.051</td>
</tr>
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<td>Slope of formal sector heights (eq 7) ( \theta_f / (1 - \gamma) )</td>
<td>-0.077</td>
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<tr>
<td>Slope of slum CAR (eq 3) ( \theta_s \lambda / (\lambda - 1) )</td>
<td>-0.090</td>
</tr>
<tr>
<td>Slope of land value gradient (eq 7) ( \theta_s \gamma / (1 - \gamma) )</td>
<td>-0.155</td>
</tr>
</tbody>
</table>

Using rows 1, 2 and 3 \( \gamma = 1.66; \ \lambda = 2.31 \). That \( \gamma \) implies a land value gradient slope of -0.128 while the estimated one is -0.155.

Table 3. Gains from conversion

<table>
<thead>
<tr>
<th></th>
<th>2-3km</th>
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<th>4-5km</th>
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<tbody>
<tr>
<td>Formal Volume Intensity 2015</td>
<td>2.73</td>
<td>3.07</td>
<td>2.42</td>
<td>1.93</td>
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<td>Slum Volume Intensity 2015</td>
<td>2.98</td>
<td>2.56</td>
<td>2.31</td>
<td>2.3</td>
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<td>Slum Land Area 2015</td>
<td>2,634,30</td>
<td>11,293,11</td>
<td>22,634,28</td>
<td>19,460,34</td>
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<tr>
<td>Raw rents:</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Formal rent, per month per sq m floor, 2012, KES</td>
<td>467</td>
<td>428</td>
<td>392</td>
<td>358</td>
</tr>
<tr>
<td>Slum rent, per month per sq m floor, 2012, KES</td>
<td>137</td>
<td>143</td>
<td>150</td>
<td>158</td>
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<tr>
<td>PV of revenue gains in USA $millions, 2015</td>
<td>278</td>
<td>1,300</td>
<td>1,659</td>
<td>782</td>
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