Capital Flows and Growth in Developing Countries: the Role of Investment Risk

Kenza Benhima

CREST (Paris) and ECONOMIX (University of Paris X - Nanterre)

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Abstract

The neoclassical growth model predicts that emerging countries with higher output growth should receive larger capital inflows. Gourinchas and Jeanne (2007) document that, in fact, countries that exhibited higher growth received less capital inflows, even though they invested more in their domestic technology. This is the allocation puzzle. The contribution of this paper is twofold. First, I document another capital flows anomaly: in the data, the distortion on capital return -or more generally the capital wedge, that is all the factors introducing a difference between the private and social return to capital- accounts for a larger share of the cross-country variations in capital flows than TFP does. This fact is also at odds with the predictions of the neoclassical growth model. This the “channel puzzle”: it says that growth should be related to capital flows through TFP while in the data they are related through the capital wedge. Second, I propose an explanation for both the channel and allocation puzzles by introducing idiosyncratic investment risk in the neoclassical growth model.

Key Words: Capital flows accounting, Investment risk.

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†CREST (Macroéconomie), Bureau 1118, Bâtiment Malakoff 2 - Timbre J320 15 Bd Gabriel Péri, 92245 MALAKOFF. TEL: (+33) 06-64-25-43-36. Email: benhima@ensae.fr
1 Introduction

The neoclassical growth model (Ramsey-Cass-Koopmans) predicts that a small open economy whose marginal return on capital is above the world’s interest rate and that opens to international bond markets increases its investment level through international borrowing. More precisely, when the return on domestic capital is higher than the cost of borrowing, it is optimal to borrow from the rest of the world to finance domestic investment. Under decreasing marginal returns, this takes place until the marginal return on capital equals the world’s interest rate. The higher the initial discrepancy between both returns, the more the country invests and the more it has to borrow. Besides, since households anticipate a rise in their labor income, they borrow in order to smooth consumption. Therefore, growth and capital inflows should be positively related through both the investment and saving behaviors.

Two main elements can account for the difference between a country’s marginal return on capital and the international interest rate: distortions to the return on capital and total factor productivity (TFP) gains. If the distortions diminish, the private return to capital increases relatively to the world interest rate. Similarly, starting from equal domestic and foreign returns, an increase in TFP pushes the former above the latter. In both cases, both investment and foreign borrowing increase. Hall and Jones (1999) and Caselli (2004) show that TFP remains the main source of cross-country differences in income. According to Caselli and Feyrer (2007), it explains half of the differences in investment. Therefore, according to the textbook growth model, countries with higher productivity growth should invest more and attract more capital.

This prediction has recently been challenged by Gourinchas and Jeanne (2007). Using a calibrated neoclassical growth model in the spirit of the development accounting literature on a sample of 69 non-OECD economies between 1980 and 2000, they show that not only does the model fail to predict the correct amount of capital inflows, but the predicted flows are negatively correlated with the actual ones. They call this paradox the “allocation puzzle”. They argue that this puzzle comes from the fact that productivity growth is negatively correlated to capital inflows in the data, while they should be correlated positively according to the model.\footnote{Other empirical studies supporting the puzzle are Aizenman and Pinto (2007) and Prasad et al. (2007).}

The contribution of this paper is twofold. First, I document another capital flows anomaly: in the data, the distortions on capital return -or more generally the “capital wedge”, that is all the factors introducing a difference between the private and social return to capital\footnote{Besides actual distortions due to institutional failures, such as expropriation risk, this capital wedge can represent taxes and, above all, it can account for the relative price of capital (see Caselli and Feyrer (2007) and} - account for...
a larger share of the cross-country variations in capital flows than TFP does. This fact is at odds with the predictions of the standard growth model. This a channel puzzle: it states that growth should be related to capital flows through TFP while in the data they are related through the capital wedge. Second, I propose an explanation for both the channel and allocation puzzles by introducing idiosyncratic investment risk in the neoclassical growth model, the same as the one used by Gourinchas and Jeanne (2007). This approach enables us to nest the “riskless” approach of Gourinchas and Jeanne (2007), while considering the more general case with risk, that is the “portfolio” approach.

The channel puzzle that we document does not challenge the main findings of the growth accounting literature. In our calibration analysis, where we follow standard methods to fit TFP and the capital wedge, we do recover the main findings of this literature, that is: (i) TFP accounts for most of the differences in output growth between countries; (ii) the capital wedge and TFP explain identical fractions of the variance of investment. Despite this, when examining how these factors account for the differences in capital flows, the role of TFP shrinks to one eighth of that of the capital wedge. This is at odds with the predictions of the neoclassical growth model, which predicts that the impact of TFP should be stronger than that of the capital wedge. In this model, both factors affect capital flows equally through the investment channel, but TFP affects them more when considering the consumption smoothing channel. This is because TFP impacts labor income twice: first directly through labor productivity, second indirectly through investment; while the capital wedge affects labor income only through investment.

Why does the introduction of idiosyncratic investment risk help solve the channel and allocation puzzles? First, investment risk introduces a precautionary saving motive. These precautionary savings can generate a positive co-movement between investment and capital outflows. In particular, when domestic financial markets are sufficiently incomplete, external asset holdings are used as a buffer stock to self-insure against investment risk. When households invest, their exposure to risk increases and they need safe assets in order to self-insure against this additional risk. Therefore, the investment channel is reversed and can overcome the consumption smoothing channel, generating a positive co-movement between output growth and capital outflows, which explains the allocation puzzle. Second, the capital wedge is more likely than TFP to drive the positive correlation between growth and capital outflows. This is because it affects less future labor income than TFP does, while it affects equally investment, as argued above. Therefore, the investment effect is more likely to dominate the consumption smoothing effect. This explains

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Hsieh and Klenow (2007)).
the channel puzzle.

The absence of labor income risk, or at least the fact that labor income is less risky than capital return, is crucial to the resolution of the channel puzzle. In our model, this is due to the assumption of no aggregate risk and no idiosyncratic labor income risk. If labor and investment were equally risky, we would still have a precautionary saving motive, and we could explain part of the allocation puzzle. But since both the investment and consumption smoothing effects would be reversed, the risky model would predict, as the riskless model, that TFP should drive most capital flows. We would therefore still have a channel puzzle.

This paper is related to the recent literature on global imbalances which have highlighted the role of financial development. Dooley et al. (2005), Mendoza et al. (2007), Matsuyama (2007) and Ju and Wei (2006, 2007) explain how low financial development in the South, through production risk, credit constraints or a poor financial intermediation system can lead to “uphill flows”, that is, positive lending to the North. However, these studies do not explain the link between capital flows and growth.

Some other studies on global imbalances, however, show a concern for growth. Caballero et al. (2008) build an intergenerational model where low financial development, that is the inability of the economy to store value, increases the demand for foreign assets. As a consequence, high growth economies can still export capital if they cannot generate enough assets. As well, Sandri (2008) explains the joint growth and capital outflows in China by introducing an exogenous trend shifting workers with safe revenues to the status of entrepreneur with risky ones. These contributions study the co-movement between growth and capital flows in the time dimension. It is different from explaining the direction of capital flows in the cross-section of countries, which is the purpose of my capital flows accounting approach.

Because I assume that households’ revenues are risky, my study relates to the heterogenous agents models à la Bewley (Aiyagari, 1994; Huggett, 1997; Krusell and Smith, 1998). These contributions usually rely on numerical methods to solve the model, mainly because of the “curse of dimensionality”. These numerical methods are not easily generalizable to capital flows accounting because of the multiplicity of countries and they make interpretation more difficult.

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3See Gourinchas and Jeanne (2007) for a more detailed survey of the potential explanations of the allocation puzzle.
4Capital flow accounting has been initiated by Lucas (1990) and has been revived recently by Gourinchas and Jeanne (2006, 2007) by building on the literature on growth accounting (Hall and Jones, 1999; Caselli, 2004; Caselli and Feyrer, 2007; Hsieh and Klenow, 2007).
5Mendoza et al. (2007) and Sandri (2008) belong to this strand of literature.
However, more recently, some articles, such as Angeletos and Calvet (2006), Angeletos (2007) and Angeletos and Panousi (2007), provide solution methods to derive approximated or exact analytical solutions under some conditions. I use Angeletos and Panousi (2007)'s framework and solution method to derive closed-form predictions for capital outflows. In this framework, CRRA utility and continuous time allows to derive policy functions which are linear in wealth and therefore easy to aggregate, thus avoiding the curse of dimensionality.

Finally, this paper relates to Kraay and Ventura (2000), who explain the Feldstein-Horioka puzzle by using the portfolio approach. They argue that the absence of aggregate cross-country correlation between savings and investment is due to the fact that the co-movement between capital flows and investment is conditional on the sign of net foreign assets, which is a prediction of the portfolio approach. They call this the New Rule. My contribution regarding the New Rule is that a positive correlation between investment and capital outflows does not necessarily suppose a positive net foreign asset position. This helps explain the allocation puzzle for the sample of developing countries with mostly negative foreign asset positions.

The rest of the paper is structured as follows: Section 2 presents the model with the two approaches and their properties; Section 3 calibrates the model; Section 4 examines the riskless approach by re-assessing the allocation puzzle and establishing the channel puzzle; Section 5 shows that the portfolio approach contributes to solving these puzzles.

2 The neoclassical growth model with idiosyncratic investment risk

In this section, I build on the neoclassical growth model developed by Gourinchas and Jeanne (2007), in which capital flows are determined by their productivity path relative to the world technology frontier. The only substantial innovation is the introduction of individual investment risk.

2.1 The household’s program

Consider a small open economy with a continuum of length 1 of infinitely-lived households, or families, indexed by $i$. Time is continuous, indexed by $t \in [0, \infty)$. Each household owns a firm which produces a homogeneous good using two factors, labor and capital, according to a
Cobb-Douglas technology:
\[ Y_t^i = K_t^{i\alpha}(A_t N_t^i)^{1-\alpha}, 0 < \alpha < 1 \]

where \( K_t^i \) is the amount of capital invested in firm \( i \) at date \( t \), \( N_t^i \) the amount of labor hired in the firm and \( A_t \) the deterministic domestic level of technology.

Households can invest only in their own firm and cannot trade equity. \( K_t^i \) is therefore the household’s holdings in private capital. They face an idiosyncratic investment risk, but cannot diversify away this risk through equity or any other means. The only freely traded asset, domestically and internationally, is the riskless bond \( B_t^i \) whose return is fixed to \( R^* \).

Following Gourinchas and Jeanne (2006, 2007), I introduce a capital wedge \( \tau \) in order account for the equalization of real returns to capital across countries (Caselli and Feyrer, 2007). This wedge can be interpreted as a tax on capital income, or as the result of other distorsions that would introduce a difference between social and private returns. We assume that this tax on capital return is distributed equally among households.

Each household is composed of \( N_t \) members, and each member is endowed with 1 unit of labor which he supplies inelastically in a perfectly competitive labor market. Since, additionally, there is no aggregate risk, all aggregate variables are deterministic, including the competitive individual wage \( w_t \). There is neither unemployment risk nor any other shock that would introduce an endowment risk besides investment risk. Therefore, labor income is completely deterministic. Moreover, the tax product is by assumption redistributed equally among households, and is therefore also deterministic.

As in Barro and Sala-i Martin (1995), the household maximizes the following discounted expected welfare of the family members:
\[ U_t^i = E_t \int_t^\infty N_s \ln c_s^i e^{-\rho(s-t)} ds \] (1)

where \( \rho > 0 \) is the discount rate and \( c_s^i \) is the individual consumption of the members of household \( i \) in period \( t \). The growth rate of population is supposed to be exogenous and equal to \( n \), with \( n < \rho \):

\[ N_t = N_0 e^{nt} \]

The household maximizes his utility subject to the evolution of his effective wealth \( w_t^i = k_t^i + b_t^i + h_t \), where \( h_t \) is the per capita human wealth, that is the present discounted sum of future safe revenues, which include labor income and the tax product: \( H_t = \int_t^\infty e^{-(s-t)} R^* (N_s w_s + Z_s) ds \), with \( Z_t = \int_0^1 \tau [K_t^{i\alpha}(A_t N_t^i)]^{1-\alpha} - w_t N_t^i] dt \) the tax product. As Angeletos and Panousi (2007) show, this evolution can be written as a linear function of the assets, which makes the analysis tractable.
when $\sigma > 0$:
\[
d\omega_t^i = [r_t k_t^i + R^i (b_t^i + h_t) - c_t^i - nw_t^i] dt + \sigma k_t^i dz_t^i
\]  
(2)
where lower case letters, except $n$, the population’s growth rate, stand for per capita (i.e. per family member) values. For each variable $X_t^i$ ($X_t$), $x_t^i$ ($x_t$) stands for $X_t^i/N_t$ ($X_t/N_t$). $r_t = (1 - \tau)\alpha (1 - \alpha)^{(\alpha - 1)/\alpha} - \delta$ is the private net return on capital, with $\delta$ the depreciation rate and $\tilde{w}_t = w_t / A_t$ the competitive wage per efficient unit of labor. Thanks to constant returns to scale, the capital income can be written as a linear function of capital $k_t^i$, and thanks to competition in the labor market, the return on capital is constant across households.\(^6\) $dv_t^i$ is a standard Wiener process which is assumed to be iid across agents and time. This risk can be interpreted as a production or a depreciation shock that affects the return on capital. It is assumed that this shock is averaged out across households, that is: $\int_0^1 dv_t^i = 0$. The parameter $\sigma$ therefore measures the amount of individual investment risk. Gourinchas and Jeanne (2007)'s specification is nested when $\sigma = 0$.\(^7\) Otherwise, the investment rules follow the classical portfolio choice rules with log utility.

This framework is similar to Kraay and Ventura (2000) and Kraay et al. (2005), who, among others, apply the portfolio choice model to an open economy. But the portfolio approach has been applied only in AK contexts, which cannot account for such phenomena as decreasing marginal returns and human wealth effects. Here, we use a transformation of the budget constraint introduced by Angeletos and Panousi (2007) in order to make it linear in wealth and apply the portfolio approach to the neoclassical growth model.

2.2 Technology

The country has an exogenous, deterministic productivity path $\{A_t\}_{t=0,\ldots,\infty}$, which is bounded by the world productivity frontier:

$$A_t \leq A_t^* = A_0^* e^{g^* t}$$

The world productivity frontier is assumed to grow at rate $g^*$. Following Gourinchas and Jeanne (2007), we define the difference between domestic productivity and the productivity

\(^6\)To understand how the evolution of wealth can be written as a function of $h_t$, consider the original budget constraint: $d(B_t^i + K_t^i) = [r_t K_t^i + R^i (B_t^i + N_t w_t + Z_t - C_t^i)] dt + \sigma K_t^i dz_t^i$. Besides, we have: $dH_t = (R^i H_t - N_t w_t - Z_t) dt$. Replacing $N_t w_t + Z_t$ in the budget constraint gives then the aggregate version of Equation (2).

\(^7\)The parameter $\tau$ is a wedge on the gross capital return, that is, before subtracting capital depreciation. This is a deviation from Gourinchas and Jeanne (2007), where the wedge is on capital returns net of depreciation. This specification is chosen only for practical reasons (the resulting amount can be expressed as a fraction of production) and does not change the results dramatically.
conditional on no technological catch-up as follows:

\[ e^{\pi_t} = \frac{A_t}{A_t e^{g t}} \]  

(3)

We assume that \( \pi = \lim_{t \to \infty} \pi_t \) is well defined. Therefore, the growth rate of domestic productivity converges to \( g^* \).

2.3 Household’s behavior

The linearity of the evolution of the budget constraint along with the homotheticity of preferences ensures that the household’s problem reduces to a homothetic problem à la Samuelson and Merton. It follows that the optimal policy rules are linear in wealth.

**Lemma 1:** Define \( \phi_t^i = k_t^i / \omega_t^i \), the fraction of effective wealth invested in private capital. For a given interest rate \( R^* \) and a given sequence of wages \( \{W_t\} \), the policy responses of the household \( i \) are given by:

\[ c_t^i = (\rho - n)\omega_t^i \]  

(4)

\[ \phi_t = \frac{r_t - R^*}{\sigma^2} \]  

(5)

Equation (4) shows the familiar result that consumption per capita equals the annualized value of wealth, taking into account population growth. It is a direct consequence of the log utility.

Equation (5) is the portfolio choice rule. It says that the risky share of the portfolio is increasing in the risk premium \( r_t - R^* \) and decreasing in the amount of risk \( \sigma \). When \( \sigma \) is large, the share of risky assets is low, while the share of safe assets is high. The share of safe assets can be viewed as a way for the household to self-insure against the potential bad shocks to the risky part of the portfolio. Even if the return on the safe assets is lower than the yield of private capital on average \( (R^* < r_t) \), they play the role of buffer-stock savings against uncertainty.

Notice that Equation (5) implies a no-arbitrage condition \( r_t = R^* \) between bonds and domestic capital when \( \sigma = 0 \). This no-arbitrage condition is an equilibrium outcome that derives from the infinite elasticity of private capital demand to the return differential between capital and bonds and from decreasing marginal returns.

Human wealth \( h_t \) and bond holdings \( b_t \) are both safe assets and are substitutes. Notice that Equation (5) implies that \( b_t^i = (1 - \phi_t)\omega_t^i - h_t \). When the household expects large labor and tax revenues in the future \( (h_t \) is large), he can borrow more \( (b_t^i \) decreases). The consumption smoothing effect on bond holdings will be channeled through this human wealth.
2.4 Steady state

The individual rules (4) and (5) are linear in wealth and can therefore be written in aggregate terms: \( c_t = (\rho - n)\omega_t \) and \( k_t = \phi_t \omega_t \), where \( \omega_t = \int_0^1 \omega'_i di \) is the aggregate value for \( \omega'_i \). By dividing each side by \( A_t \), they can also be written in terms of efficient units of labor, denoted \( \tilde{c}_t \), \( \tilde{k}_t \) and \( \tilde{\omega}_t \).

Capital per efficient units of labor at the firm level, \( K_t^i / A_t N^i_t \) is constant across firms as a result of common wages and constant returns to scale. Since the labor market clears (\( \int N^i_t di = N_t \)), it is equal to its aggregate value \( \tilde{k}_t = K_t / A_t N_t \). This defines the wage per efficient units of labor unambiguously as \( \tilde{w}_t = (1 - \alpha) \tilde{k}_t^\alpha \).

The aggregate and per efficient units of labor versions of Equations (4) and (2), along with the no-Ponzi conditions and the equilibrium values for \( \tilde{w}_t, r_t \) and \( \phi_t \), characterize the dynamics of \( \tilde{c}_t \) and \( \tilde{\omega}_t \). Once the paths of these variables are known, \( \tilde{k}_t = \phi_t \tilde{\omega}_t \), \( \tilde{h}_t = \int_0^\infty e^{-(R^* - (n + g^*))s + \pi^* - \pi^t (1 - \alpha + \tau \alpha) \tilde{k}_t^\alpha} ds \) and \( \tilde{b}_t = \tilde{\omega}_t - \tilde{k}_t - \tilde{h}_t \) can be determined. However, these equations are used here only to determine steady state.

We define the steady state by \( \dot{\tilde{c}} = 0 \) and \( \dot{\tilde{w}} = 0 \). Under the aggregate Euler condition (12) and budget constraint (13), this implies stationarity in wealth. This condition implies different constraints on the world interest rate depending on \( \sigma \).

Proposition 1:

(i) If \( \sigma > 0 \), the open economy steady state exists if and only if \( n < R^* - g^* < \rho \) and is defined by:

\[
(1 - \tau) f'(\tilde{k}^*) - \delta - R^* = \sqrt{\sigma^2 (\rho + g^* - R^*)}
\]

\[
\tilde{b}^* + \frac{(1 - \alpha + \alpha \tau) f(\tilde{k}^*)}{R^* - g^* - n} = \frac{1 - \phi^*}{\phi^*} \tilde{k}^*
\]

with \( \phi^* = \sqrt{\frac{\rho + g^* - R^*}{\sigma^2}} \).

(ii) If \( \sigma = 0 \), the open economy steady state exists if and only if \( R^* = \rho + g^* \) and is defined by (6) and:

\[
\tilde{b}^* = \frac{(1 - \alpha + \alpha \tau) f(\tilde{k}^*)}{\rho - n} = -\tilde{k}^* + \left[ \tilde{h}_0 + \tilde{k}_0 + \tilde{b}_0 \right] e^{-\pi}
\]

with \( \tilde{h}_0 = (1 - \alpha + \alpha \tau) f(\tilde{k}^*) \int_0^\infty e^{-(\rho - n) t + \pi t} dt \).
When $\sigma = 0$, stationarity implies that the growth-adjusted world interest rate $R^* - g^*$ is exactly equal to the psychological discount rate $\rho$. This condition is satisfied if we assume that the rest of the world is also characterized by the absence of risk, since $\rho + g^*$ would be the long-run autarky interest rate in an economy without risk. When $\sigma > 0$, the adjusted interest rate $R^* - g^*$ should be lower than the discount rate $\rho$ to counteract the precautionary savings motive and insure stationarity. Again, assuming that firms in the rest of the world bear some idiosyncratic risk would ensure that we indeed have $R^* < \rho + g^*$.

Equation (6) states that, in the steady state equilibrium, the risk premium (RHS) depends positively on the amount of risk $\sigma$. When capital is not risky ($\sigma = 0$), Equation (6) reduces to the no-arbitrage condition $r_t = R^*$.

Equation (7) is obtained simply by rewriting the definition of $\phi_t$ at steady state, where the steady-state share of risky capital $\phi^*$ is such that the aggregate return of the portfolio (including the risk premium), compensates for the growth-adjusted saving rate. The LHS of Equation (7) represents safe assets, that is bond holdings and human wealth. In the presence of risk, these safe assets are a constant share of the portfolio which depends only on the parameters of the model. Notice that Equation (8) is equivalent to (7) when $\sigma$ goes to zero, plus an additional term $[\tilde{h}_0 + \tilde{k}_0 + \tilde{b}_0] e^{-\pi}$. The equality between the adjusted interest rate and the discount factor $(R^* - g^* = \rho)$ makes the household completely indifferent between consumption and investment, hence the persistence of initial wealth on bond holdings. In the risky case, because the aggregate risk premium on the portfolio depends on its composition, the household is indifferent between investment and consumption only for a particular portfolio composition. The long-run portfolio composition, and hence bond holdings, are therefore determined.

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8However, the interest rate should not be too low (i.e. the world risk should not be too high). Otherwise, the long-run human wealth would not be well defined.

9To understand the link between these two expressions, consider the case with $\sigma = 0$ and $R^* < \rho + g^*$. In that case, the household is impatient: the growth rate of consumption is negative and wealth $\tilde{\omega}$ converges towards zero. We would therefore have in the long run: $\tilde{b}^* + \frac{(1-\alpha+n\tau)\tilde{f}(k^*)}{\rho-n} = -\tilde{k}^*$. This expression is exactly the same as (7) where $\sigma$ goes to zero. In the long run, the whole return from capital and human wealth is consumed by the interest payments on debt. The additional term $[\tilde{h}_0 + \tilde{k}_0 + \tilde{b}_0] e^{-\pi}$ when $R^* = \rho + g^*$ comes from the fact that consumption in per capita terms should grow exactly at rate $g^*$. Now the return from the portfolio should leave enough room for consumption, which is exactly equal to $(\rho - n) [\tilde{h}_0 + \tilde{k}_0 + \tilde{b}_0] e^{-\pi}$ in efficient units of labor in the long run. Contrary to the case where $R^* < \rho + g^*$ (with or without risk), the initial level of consumption is persistent and determined by initial wealth.
2.5 Capital flows: from the riskless to the portfolio approach

Following the method of Gourinchas and Jeanne (2007), the model is confronted with the data observed over a finite period \([0, T]\). Before deriving the level of bonds predicted by the model, some assumptions must be made. First, we abstract from unobserved future developments in productivity by assuming that all countries have the same productivity growth rate \(g^*\) after \(T\).

**Assumption 1:** \(\pi_t = \pi\) for all \(t \geq T\).

When \(\sigma = 0\), \(\bar{k}_t = \bar{k}^*\) for all \(t\). The steady state is reached immediately. However, when \(\sigma > 0\), \(\bar{k}_t\) is contingent on time, which makes it impossible to abstract from future \(\bar{k}_t\) on \(T\), except if \(\bar{k}_T\) is sufficiently close to the steady state. In particular, for \(T\) sufficiently high, \(\bar{k}_T\) is close to \(\bar{k}^*\), since \(\bar{k}\) converges to its steady state\(^{10}\). In the remainder of the analysis, it is therefore assumed that \(T\) is sufficiently large to be able to make the following approximation: \(\bar{k}_t = \bar{k}^*\) for all \(t \geq T\).

Denote by \(\Delta B/Y_0 = (B_T - B_0)/Y_0\) the amount of capital flows between 0 and \(T\). Under Assumption 1 and for \(T\) large, it can be written as follows:

\[
\frac{\Delta B}{Y_0} = e^{\pi+(n+g^*)T} \frac{\bar{k}^*}{y_0} - \frac{\bar{b}_0}{y_0} \tag{9}
\]

In order to distinguish the predicted capital flows according to the riskless and portfolio approaches, denote the former \(\bar{\Delta B}/Y_0\) and the latter \(\Delta B/Y_0\). By developing equation (9), we get the following proposition:

**Proposition 2:** Under Assumption 1 and the stationarity conditions of Proposition 1, and for \(T\) sufficiently large, the ratio of cumulated capital inflows to initial input is given by:

(i) If \(\sigma = 0\):

\[
\frac{\bar{\Delta B}}{Y_0} = -\frac{\bar{k}^* - \bar{k}_0}{\bar{k}_0^\alpha} e^{(n+g^*)T} - \frac{\bar{k}^*}{\bar{k}_0^\alpha} e^{(n+g^*)T} T e^{(\alpha - \pi)T} \int_0^T e^{-\alpha n} (1 - e^{-\pi t - \pi}) dt + \left( e^{(n+g^*)T} - 1 \right) \frac{\bar{b}_0}{\bar{k}_0^\alpha} \tag{10}
\]

(ii) If \(\sigma > 0\):

\[
\frac{\Delta B}{Y_0} = \frac{1 - \phi^*}{\phi^*} \frac{\bar{k}^* - \bar{k}_0}{\bar{k}_0^\alpha} e^{(n+g^*)T} + \frac{1 - \phi^*}{\phi^*} \frac{\bar{k}^*}{\bar{k}_0^\alpha} e^{(n+g^*)T} T e^{(\alpha - \pi)T} \int_0^T e^{-\alpha n} (1 - e^{-\pi t - \pi}) dt + \left( e^{(n+g^*)T} - 1 \right) \frac{\bar{b}_0}{\bar{k}_0^\alpha} \tag{11}
\]

\(^{10}\)See Angeletos (2007) and Angeletos and Panousi (2007) for the transitional dynamics of this kind of model.
\[ + \left( \frac{1}{\phi^*} - \frac{1}{\phi_0} \right) \frac{\tilde{k}_0}{k_0^\star} e^{(n+g^*)T} \]

(11)

where \( \phi_0 = \tilde{k}_0/(\tilde{k}_0 + \tilde{b}_0 + \tilde{h}_0) \) and \( \tilde{h}_0 = (1-\alpha+\tau \alpha) \int_0^T e^{-[R^*-(n+g^*)]T+\pi_k} \tilde{L}_0^\alpha \, dt + e^{\pi+(n+g^*)T} \frac{\tilde{k}_0^\star}{R^*-(n+g^*)} \],

respectively the initial share of capital and the initial human wealth.

Equations (10) and (11) give the predicted capital outflows as a function of \( n, g^*, \rho, R^*, \tau \),

the sequence of productivity catch-up \( \{\pi_t\}_{t=1,...,T} \) and initial values \( \tilde{b}_0 \) and \( \tilde{k}_0 \). Note that \( \tilde{k}_0^\star \) is also a function of the parameters. In the risky environment, the sequence of capital per efficient

unit of labor \( \{\tilde{k}_t\}_{t=1,...,T} \) and the initial share of capital in wealth \( \phi_0 \) depend also on these parameters.

Equation (10) is the continuous-time version of Gourinchas and Jeanne’s decomposition. Consider now Equation (11), which represents the predicted flows according to the portfolio

approach, that is when \( \sigma > 0 \). This decomposition can be tracked down to Gourinchas and Jeanne’s. Notice that the first four terms are similar, except that in they are more general in the portfolio approach. Indeed, \((1 - \phi^*)/\phi^*\) is equal to \(-1\) when \( \sigma \) goes to zero. Additionally, Equation (11) features an extra term \( \left( \frac{1}{\phi^*} - \frac{1}{\phi_0} \right) \frac{\tilde{k}_0}{k_0^\star} e^{(n+g^*)T} \). 12

The sign of \( \phi^*/(1 - \phi^*) \) is critical to determine the sign of the first and second terms. We have seen that the steady-state share of capital in wealth \( \phi^* \) is strictly positive but it is not necessarily below one in the general case. Namely, the parameter \( \sigma \) must be high enough, and

more precisely satisfy the following assumption:

**Assumption 2:** \( \sigma > \rho - R^* + g^* \)

This assumption is satisfied as long as the country’s level of risk is greater than the world’s,

which is what we assume in the calibration analysis, since we are dealing with developing countries.13 It is therefore maintained in the remaining analysis for the risky case.

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11By solving the (12) and (13) system.

12Starting from Equation (11), the decomposition without risk and with \( R^* < \rho + g^* \) is obtained with \((1 - \phi^*)/\phi^* = -1 \) (i.e. \( \sigma \) goes to zero) and without the additional term.

13The constraint that \( \phi^* < 1 \) and equivalently that \( \phi^*/(1 - \phi^*) > 0 \) can be rationalized by the following general equilibrium argument. The world as a whole is in autarky, that is \( \tilde{b}_W^* = 0 \). According to (7), we have:

\[ \tilde{k}_W^* = \frac{\phi_W^*}{1 - \phi_W^*} \frac{(1 - \alpha + \sigma f_W)(\tilde{k}_W^*)^\alpha}{R^* - g^* - n_W} \]

Since \( \tilde{k}_W^* \) is positive, \( \phi_W^*/(1 - \phi_W^*) \) is necessarily positive and therefore \( \phi_W^* < 1 \). This is made possible by

the adjustment of the world interest rate \( R^* \) in order to clear the world bond market. If we assume, as in the
Consider the first term:

\[ \Delta B^c \frac{c}{Y_0} = \frac{1 - \phi^* \tilde{k}^* - \tilde{k}_0}{\phi^*} e^{(n+g^*)T} \]

This term represents the impact of convergence on capital outflows. If \( \tilde{k}_0 < \tilde{k}^* \), the country increases its capital stock. But the extent to which this translates into capital inflows depends on \( \frac{1 - \phi^*}{\phi^*} \), and therefore on the amount of risk in the country relative to the rest of the world. In the absence of risk, the difference \( \tilde{k}_0 - \tilde{k}^* \) is the amount immediately borrowed by the country to equalize its private return to capital to the world’s interest rate. In the presence of sufficient risk, agents want to self-insure against investment uncertainty by holding more bonds. Following Gourinchas and Jeanne (2007), we call this the convergence term.

The second term,

\[ \Delta B^t \frac{t}{Y_0} = \frac{1 - \phi^*(e^\pi - 1)}{\phi^*} \tilde{k}^* \frac{e^{(n+g^*)T}}{\tilde{k}_0} \]

reflects long-term productivity catch-up. Positive catch-up (\( \pi > 0 \)) implies further needs in investment but, as for the convergence term, its impact on capital outflows depends on the sign of \( \frac{1 - \phi^*}{\phi^*} \). In the riskless approach, it contribute negatively to the external position, because the country has to borrow from abroad. On the opposite, in the portfolio approach, it should trigger precautionary capital outflows.

The third term,

\[ \Delta B^s \frac{s}{Y_0} = -e^{\pi+(n+g^*)T} (1 - \alpha + \alpha \tau) \tilde{k}_0 \alpha \int_0^T e^{-(\rho-n)t} \left( 1 - \frac{\tilde{k}_0}{\tilde{k}^*} e^{\pi_t - \pi} \right) dt \]

represents the consumption smoothing behavior. Indeed, the households adjust their consumption according to their intertemporal human wealth, which depends on their discounted flow of deterministic revenue \( \tilde{w}_t + \tilde{z}_t \).\(^{14}\) This term does not depend on the amount of individual investment risk and is therefore identical in both approaches.

The fourth term,

\[ \Delta B^t \frac{t}{Y_0} = \left( e^{(n+g^*)T} - 1 \right) \frac{\tilde{b}_0}{\tilde{k}_0} \]

represents the impact of the initial external position in the presence of trend growth (\( n + g^* > 0 \)). It reflects the amount of capital outflows (or inflows) required to maintain the ratio of external position to output constant.

\(^{14}\)The path of those revenues depends on \( (1 - \alpha + \alpha \tau) \tilde{k}^* \), on the path of \( \pi_t \) and, in the presence of risk, on the path of \( \tilde{k}_0 \). This is because, in the portfolio approach, the level of capital does not immediately adjust to its steady state value; it depends on the current level of wealth and not only on the world’s interest rate.
Finally, the fifth term,  
\[
\frac{\Delta B^p}{Y_0} = \left( \frac{1}{\phi^*} - \frac{1}{\phi_0} \right) \frac{\tilde{k}_0}{k_0^*} e^{(n+g^*)T}
\]
is a portfolio adjustment term. Countries starting with a low share of capital in the portfolio relative to the long-run one should decrease their share of riskless assets, which has a negative contribution to capital outflows. This long-run adjustment in the portfolio decomposition is absent from the decomposition without risk. This is because, as we have seen, the initial wealth is persistent in the riskless approach, whereas the portfolio composition is determined in the portfolio approach.

To sum up, two main differences between the riskless and portfolio approaches are worth noting at this stage: (i) the investment channel is reversed in the portfolio approach because of precautionary savings; (ii) initial wealth is persistent in the riskless approach. The consumption or saving channel (i.e. consumption smoothing) is barely changed.

2.6 The role of productivity

Hall and Jones (1999) and Caselli (2004) show that TFP is a major source of the cross-country differences in income. Consistently, Gourinchas and Jeanne (2007) find that productivity growth is the main source of the allocation puzzle. It is therefore instructive to compare how it affects bond holdings in both approaches. It has been already noticed that \( \pi \) has opposite effects on the catch-up term \( \frac{\Delta B^p}{Y_0} \) in the two approaches. However, the consumption smoothing term \( \frac{\Delta B^s}{Y_0} \) depends in a more complicated way on \( \pi \) and the path of \( \pi_t \). In order to simplify the problem, as in Gourinchas and Jeanne, the following assumption is made:

**Assumption 3:** \( \pi_t = f(t)\pi \) where \( f(.) \) is common across countries and satisfies \( 0 \leq f(t) \leq 1 \) and \( \lim_{t \to \infty} f(t) = 1 \).

Under Assumption 3, we can rewrite \( \frac{\Delta B^s}{Y_0} \) as:

\[
\frac{\Delta B^s}{Y_0} = -e^{\pi+(n+g^*)T} \left( 1 - \alpha + \alpha \tau \right) \frac{\tilde{k}^*}{k_0^*} \int_0^T e^{-(\rho-\tau)t}(1 - \frac{\tilde{k}_0^*}{k^*} e^{\pi(f(t)-1)}) dt
\]

In the Appendix, I show that this term depends negatively on the long-run productivity catch-up \( \pi \) in the riskless approach as long as \( \pi > -100\% \), which is a weak assumption\(^{15}\). In the portfolio approach, this is also true for \( \pi \) close to zero and \( \frac{k_0^*}{k^*} \) close to one, and if \( \frac{k_0^*}{k^*} + f(t) \leq 2 \) for

\(^{15}\)In particular, it will be satisfied in the calibration analysis.
all countries.\textsuperscript{16} Faster relative productivity growth implies higher future income, leading to an increase in consumption and a decrease in savings. As a result, the external position deteriorates, including in the long run.

As a consequence, in the riskless approach, capital outflows depend unambiguously on $\pi$, since the two terms that include $\pi$ relate negatively to it, namely $\frac{\Delta B^A}{\gamma_0}$ and $\frac{\Delta B^B}{\gamma_0}$. Namely, more TFP growth implies more capital inflows, through both the investment and consumption channels.

In the portfolio approach, the effect of $\pi$ is more ambiguous. The catch-up component $\frac{\Delta B^i}{\gamma_0}$ depends now positively on $\pi$, but the consumption smoothing term $\frac{\Delta B^s}{\gamma_0}$ is still negatively related to it. Furthermore, the portfolio adjustment term $\frac{\Delta B^p}{\gamma_0}$, through initial human wealth $\tilde{h}_0$, also depends on TFP catch-up. Under assumption 3, it can be written as a function of $\pi$:

$$\tilde{h}_0 = (1 - \alpha + \tau \alpha) \left[ \int_0^T e^{-[R^*-(n+g^*)]T + \pi f(t)]} \tilde{k}^T dT + \frac{\frac{\tilde{k}}{\alpha}}{R^* - (n + g^*)} \right]$$

Since the future riskless incomes are increasing in $\pi$, the portfolio adjustment term is decreasing in $\pi$: a high initial share of riskless assets implies a large diminution of these assets in the long-run. The resulting effect on total outflows is therefore ambiguous.

However, the following proposition, which is an extension of the corollary of Gourinchas and Jeanne (2007), can be proven:

**Proposition 3:** Suppose that the stationarity conditions of Proposition 1 and Assumptions 1 and 3 are satisfied and consider two countries A and B, identical except for their long-run productivity catch-up $\pi$:

(i) If $\sigma = 0$ and $\pi > -100\%$: country A sends more capital outflows than country B if and only if country A catches up less than country B towards the world technology frontier:

$$\Delta B^A > \Delta B^B \iff \pi^A < \pi^B$$

(ii) If $\sigma < 0$: under Assumption 2 and if $T$ is sufficiently large, there are two cases. If $A$ and $B$ have a positive long-run external position ($\tilde{b}^* > 0$), then country A sends more capital outflows than country B if and only if country A catches up more than country B towards the world technology frontier; if $A$ and $B$ have a negative long-run external position ($\tilde{b}^* < 0$), then the opposite holds:

$$\tilde{b}^* > 0 \Rightarrow (\Delta B^A > \Delta B^B \iff \pi^A > \pi^B)$$

\textsuperscript{16}This condition is satisfied as long as the initial level of capital is not too high as compared to the long-run one and as $f(t)$ converges quickly to one.
\[ \tilde{b}^* < 0 \Rightarrow (\Delta B^A > \Delta B^B \Leftrightarrow \pi^A < \pi^B) \]

(i) follows from the above discussion. In the absence of risk, in line with Gourinchas and Jeanne (2007), countries growing faster should borrow more.

(ii) comes from the definition of capital outflows (9) and the fact that \( \tilde{b}^* \) does not depend on \( \pi \) in the portfolio approach (see Proposition 1). Therefore, for given parameters, \( \frac{\Delta B}{Y_0} \) is increasing in \( \pi \) if \( \tilde{b}^* \) is positive and decreasing otherwise. Indeed, a positive long-run external position is symptomatic of a strong precautionary motive. In that case, the investment channel is therefore sufficient to counteract the consumption channel and the adjustment in portfolio. This result is the cross-country, growth-accounting counterpart of the “New Rule” introduced by Kraay and Ventura (2000).\(^{17}\)

Regarding the role of productivity, the portfolio approach can help explain the puzzle to the extent that countries with positive long-run external positions drive the observed positive correlation between growth and capital outflows. We also observe that in the presence of risk, the relationship between growth and capital flows becomes conditional on their sign.

2.7 The role of capital wedge

In this subsection we show that not only the introduction of risk qualifies the impact of TFP growth on capital outflows, but it also changes the relationship between the capital wedge \( \tau \) and foreign investment, which could contribute to solve the puzzle. Indeed, Gourinchas and Jeanne find that their calibrated capital wedge is negatively correlated with the saving rate in the data. This is at odds with their model, since low investment should imply high savings (i.e. low indebtedness). In the portfolio approach, the opposite holds, since high investment implies high savings for precautionary motives.

**Proposition 4:** Suppose that the stationarity conditions of Proposition 1 and Assumptions 1 and 3 are satisfied and consider two countries A and B, identical except for their capital wedge \( \tau \), then:

(i) If \( \sigma = 0 \) and if \( \pi \) close to zero: country A sends more capital outflows than country B if and

\(^{17}\)Indeed, they study the impact of risk on the correlation between savings and investment, which is equivalent to studying the correlation between growth and the current account in their framework, since savings and investment are driven by productivity shocks. However, their approach focuses on the within-country correlations for industrial countries which are assumed to be at steady state. Our approach is different, since we focus on cross-country differences between countries converging towards the steady state.
only if country $A$ has a higher capital wedge than country $B$:

$$\Delta B^A > \Delta B^B \Leftrightarrow \tau^A > \tau^B$$

(ii) If $\bar{b}^* \geq -(1 - \alpha) \frac{(1-\alpha+\tau)\bar{f}(\bar{b}^*)}{R - n - \bar{y}}$: under Assumption 2 and if $T$ is sufficiently large: country $A$ sends more capital outflows than country $B$ if and only if country $A$ has a lower capital wedge than country $B$:

$$\Delta B^A > \Delta B^B \Leftrightarrow \tau^A < \tau^B$$

The proof is provided in the appendix.

To understand the result, consider the expressions for $\bar{b}^*$ (7) and (8). In the portfolio approach, decreasing the capital wedge $\tau$ has two opposing effects on the long-run external position $\bar{b}^*$: a positive precautionary savings effect through the increase in investment and a negative human wealth effect through the increase in the deterministic labor income, which is a substitute for bond holdings. An additional positive human wealth effect adds up through the insurance role of $\tau$. Indeed, $\tau$ redistribute the risky capital income equally among households, and therefore plays as an insurance device. When $\tau$ decreases, this insurance mechanism is limited, and therefore more risk is borne by the agents, which stimulates capital outflows. For the positive effects to dominate the negative human wealth effect, the external bond position must not be too negative. This condition is equivalent to assuming that the risk borne by the domestic investors is high relative to the risk borne by the rest of the world. In that case, the positive precautionary effect dominates.\(^{18}\)

Importantly, the external bond position does not need to be positive in order to generate a positive comovement between investment and capital outflows when considering the capital wedge margin, contrary to the TFP margin.

In the risky approach, the positive precautionary savings effect is replaced by a negative substitution effect: higher investment is financed by borrowing from abroad. On the other hand, the impact on long-run human wealth are the same as in the portfolio approach, but they are partly compensated by the impact on the beginning-of-period human wealth, which has a persistent effect on the long-run position in the riskless approach. If $\pi$ is close to zero, then the long-run and end-of-period human wealth compensate each other, and the negative substitution effect dominates. When $\pi$ is not close to zero, the overall effect of $\tau$ is ambiguous.

\(^{18}\)When calibrating on the data, it appears that the condition on $\bar{b}^*$ is satisfied in our sample.
Observe that, while the effect of TFP growth on capital flows is clear in the riskless approach and ambiguous in the portfolio approach, the opposite holds when considering the effect of the capital wedge. The way TFP and capital return distortions redistribute wealth between labor and capital, that is, between safe and risky income sources, is crucial. In the riskless approach, TFP has a negative effect on capital flows both through the investment and consumption smoothing channels. But it affects consumption smoothing twice: through higher labor productivity (for a given stock of capital) and through a higher capital stock, which increases future labor income. On the other hand, the capital wedge affects consumption smoothing only through a higher capital stock and with decreasing returns. This appears in the decomposition (10) where the capital stock affects the catch-up and convergence terms (i.e. the investment terms) linearly while it affects the consumption smoothing term non-linearly. On the opposite, $e^\pi$ affects both the investment and consumption terms linearly. As a consequence, TFP has a higher impact on capital flows than the capital wedge in the riskless approach. In the portfolio approach, the investment channel is reversed and can counteract the consumption smoothing effect, which is relatively smaller when originated in capital wedge changes than in TFP (see decomposition (11)). The capital wedge has therefore better chances to positively drive capital flows than has TFP.

3 Calibration

Before assessing the different approaches in explaining the allocation of capital flows, we need to either measure or calibrate the parameters on the right-hand side of (10) and (11) in order to construct the estimated capital flows.

3.1 Data and calibration method

In order to facilitate the comparison with Gourinchas and Jeanne (2007), the same sample of 69 emerging countries is used.\(^\text{19}\) However, Jordan and Angola are removed from this sample because their working-age population does not satisfy $n < \rho$. The sample is therefore reduced

\(^\text{19}\)This sample includes: Angola, Argentina, Bangladesh, Benin, Bolivia, Botswana, Brazil, Burkina Faso, Cameroon, Chile, China, Colombia, the Republic of Congo, Costa Rica, Cyprus, Côte d'Ivoire, Dominican Republic, Ecuador, Egypt, Arab Republic, El Salvador, Ethiopia, Fiji, Gabon, Ghana, Guatemala, Haiti, Honduras, Hong Kong, India, Indonesia, Iran, Israel, Jamaica, Jordan, Kenya, Republic of Korea, Madagascar, Malawi, Malaysia, Mali, Mauritius, Mexico, Morocco, Mozambique, Nepal, Niger, Nigeria, Pakistan, Panama, Papua New Guinea, Paraguay, Peru, Philippines, Rwanda, Senegal, Singapore, South Africa, Sri Lanka, Syrian Arab Republic, Taiwan, Tanzania, Thailand, Togo, Trinidad and Tobago, Tunísia, Turkey, Uganda, Uruguay and Venezuela.
to 67 countries.

The parameters which are common across countries also follow their paper: the depreciation rate $\delta$ is set to 6%, the capital share of output $\alpha$ to 0.3 and the growth rate of world productivity $g^*$ to 1.7%. Only the discount rate $\rho$ is set to a higher value of 5% (instead of 4%)\footnote{As we will see, the allocation puzzle is robust to this change in parameter.} in order to accommodate high growth rates of labor in the data.\footnote{Indeed, in the portfolio approach, the adjusted world interest rate (i.e. $R^* - g^*$) is lower than $\rho$. If $\rho$ is too small, then we could have $R^* - (g^* + n)$ negative or very close to zero. In the first case, capital flows are not well defined; in the second, their variance goes to infinity.} The time span is extended to 1980-2003 (instead of 1980-2000) in order to encompass the recent surge in capital outflows from developing countries and to shed some light on the “global imbalances” debate.

In order to determine the exogenous interest rate, we make the hypothesis that each country is too small to influence the world’s demand for bonds. We also assume that the world is composed exclusively of developed countries with zero labor growth and no distortions to the marginal capital return. The world interest rate then corresponds to the autarky steady-state interest rate with $n = 0$ and $\tau = 0$. In the portfolio approach, the amount of risk $\sigma$ in developed countries is set to 0.3, which is the amount of entrepreneurial risk commonly reported by empirical studies in the US and the Euro area (Campbell et al., 2001; Kearney and Poti, 2006). This gives $\phi^* = 13.1\%$ and $R^* = 6.55\%$. If we extend this calibration approach to the case without risk, we get $R^* = \rho + g^* = 6.7\%$, which is the long-run value of the autarky interest rate when $\sigma = 0$.

The amount of risk in emerging countries is set to $\sigma = 0.6$, in line with Koren and Tenreyro (2007), who find that the amount of both macroeconomic and idiosyncratic (sectoral) risk are both roughly twice as large in developing countries as in industrial ones.

The country-specific data are the paths for output, capital, productivity and working-age population. These data come from Version 6.2 of the Penn World Tables (Heston et al., 2006)\footnote{Gourinchas and Jeanne (2007) use Version 6.1 of Penn World Tables but we prefer Version 6.2, which is updated to take into account more recent years and allows us to extend the time span of the sample.}. Following Gourinchas and Jeanne (2007) and Caselli (2004), the capital stock is constructed with the perpetual inventory method from time series data on real investment. The level of productivity $A_t$ is calculated as $(y_t/k_t)^{1/(1-\alpha)}$ and the level of capital per efficient unit of labor $k_t$ as $(y_t/k_t)^{1/(1-\alpha)}$. The level of TFP $A_t$ and the capital per efficient unit of labor $k_t$ are filtered using the Hodrick-Prescott method in order to suppress business cycles. The parameter $n$ is measured as the annual growth rate of the working-age population. Under Assumption 1, the long-term catch-up $\pi$ can be measured as $\ln(A_T) - \ln(A_0) - T g^*$.

The capital wedge $\tau$ is calibrated in order to account exactly for the steady-state capital
per efficient units of labor. We use the fact that, assuming that $T = 20$ is a sufficiently large number, $\bar{k}^*$ is approximately equal to $\bar{k}_T$. We thus take $\bar{k}^* = \bar{k}_T$. As a result, $\tau$ will be given by: $\tau = 1 - \bar{k}^* (1 - a) R^* + \beta + \sqrt{2 (\rho + \gamma - R^*)}$. This method assigns a high capital wedge to countries with low end-of-period capital per efficient unit of labor relative to the US. The introduction of $\tau$ shuts down the Lucas paradox since this parameter is used to adjust the private marginal return to capital to the world interest rate.

Finally, actual capital flows are taken from Lane and Milesi-Ferretti (2006)'s net foreign asset data. They provide estimates for the net external position in current US dollars. Actual capital outflows during the period, as a share of initial output, are denoted $\Delta B_{Y0}$. These estimates are confronted with the predicted values given by the riskless and portfolio approaches, respectively $\Delta B_{Y0}$ and $\Delta B_{\hat{Y}}$.

### 3.2 Some key parameters

Table 1 sums up some key parameters given by the calibration method. Countries are classified by income group (World Bank classification based on 2007 GNI per capita) and by region. Finally, for robustness checks, China is excluded from the Asian group.

Consider the long-run productivity catch-up $\pi$ in column (1) of Table 1. On average, non-OECD countries have fallen behind in terms of productivity. When looking into details, only high income economies and Asia have caught up with the world productivity. Both Africa and Latin America fell behind.

The average initial share of capital in the portfolio $\phi_0$ (column (3)) is very low: around 2%. This is because human wealth accounts for an extremely large part of the household’s portfolio: not only is it an infinite discounted sum, but it is also inflated both by labor and productivity growth. Additionally, the net external position is small in absolute value, as column (2) shows. All this results in a small share of capital in the portfolio. It appears that this share is increasing with income (from 1% to 4%). This can be explained by the fact that initial capital $\hat{k}_0$ is increasing with income too.

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$23$This is different from the procedure used by Gourinchas and Jeanne (2007). They compute numerically a mapping from the average investment rate to the capital wedge $\tau$, for given productivity catch-up $\pi$ and population growth $n$. Their method cannot be extended easily to the portfolio approach, where the investment rate cannot be written explicitly as a simple function of the steady-state level of capital per efficient unit of labor $\bar{k}^*$ but is contingent on its whole path $\{\hat{k}_t\}_{t=1..T}$.

$24$These estimates are calculated using the cumulated current account data and are adjusted for valuation effects. In order to be consistent with the PPP-adjusted data used here, a PPP deflator is extracted from the Penn World Table and is used to calculate a PPP-adjusted measure of net external position.
Column (4) presents the average growth rate of capital $\frac{\Delta k}{1.96}$. The main observation is that the capital stock decreased on average. Consistently with Gourinchas and Jeanne (2006, 2007), emerging countries were not capital-scarce but capital-abundant. Among regions, only Asia increased its capital per efficient unit of labor. However, in absolute value, the figures are larger than the ones estimated by Gourinchas and Jeanne (2007). This is due to differing strategies in estimating the long-run stock of capital $\hat{k}^*$ (and equivalently $\tau$). Their method assigns values for $\hat{k}^*$ which are closer to $\tilde{k}_0$ because these values are estimated on the average stock of capital, while our method use the end-of-period one. The average decrease in capital stock is therefore larger with our method.

Consider now column (5). When calibrated inside the riskless framework, the average wedge $\tau$ on capital return is equal to 38%, which is larger than the value of 12% found in Gourinchas and Jeanne (2007). This is due both to the fact that our definition of capital return is in net terms while it is in gross terms in their approach\textsuperscript{25} and to the fact that we estimate lower end-of-period capital stocks, in line with the discussion in the above paragraph. Notice that the capital wedge $\tau$ is decreasing with income. This accounts for the fact that high income countries have a higher end-of-period stock of capital. Similarly, Africa, which has the smallest end-of-period capital level, has therefore the highest estimated capital wedge, while Asia’s estimated capital wedge is the smallest, since it benefits from a high end-of-period capital level. The capital wedge calibrated inside the portfolio framework is given by column (6). It is lower than the one reported in column (5) because the risk premium accounts partially for the low levels of capital. This leaves unchanged the regions’ ranking.

4 The riskless approach

4.1 The allocation puzzle

Figures 1 and 2 summarize the outcome of the riskless approach. The upper panels report the actual net capital outflows as a share of initial output $\Delta B/Y_0$: their size is $-46\%$ on average, which means that emerging countries have received net capital inflows during the period. The middle panels report the predicted capital outflows based on equation (10). These estimates are constructed under the hypothesis that the productivity catch-up follows a linear trend: $\pi_t = \pi \min\{t/T, 1\}$, as in Gourinchas and Jeanne (2007). Our results, despite the continuous time framework and the use of a different method to calibrate the capital wedge $\tau$, are in

\textsuperscript{25}This deviation from Gourinchas and Jeanne (2007) is justified by the use of a continuous-time framework.
line with Gourinchas and Jeanne (2007). According to the model, non-OECD countries should have received capital inflows on average, which is the case. Moreover, average predicted flows in non-OECD countries are of the same order of magnitude as the actual ones. However, while satisfying in terms of global trends, the model fails when considering the direction and magnitude of flows inside the sample. According to the predictions, low-income countries should have exported capital while high-income countries should have received capital inflows (middle panel of Figure 2). Actually, the opposite happened (upper panel of the same figure). Latin America and Africa should have invested abroad while Asia should have hosted capital inflows (middle panel of Figure 1). But the signs are reversed in reality (upper panel of the same figure).

On the whole, the puzzle of Gourinchas and Jeanne (2007) seems to be robust to the continuous-time approach, to the use of an alternative method to compute the capital wedge and to the extension of the time span of the sample: capital seems to flow in the wrong direction. Figure 3 sums up the puzzle by showing the scatter plot of actual versus predicted flows. The correlation seems, at best, non-significant and, at worst, negative. Actually, when looking to the overall correlation of each component with actual capital flows (see the last line of Table 2 in the Appendix), it seems that this negative correlation comes mainly from the catch-up and consumption smoothing components.36

Gourinchas and Jeanne (2007) suggest that the puzzle lies in the fact that the riskless approach does not account properly for the link between capital flows and productivity growth. Indeed, it is possible to trace back the origin of the negative correlation between observed and predicted capital flows by considering Figure 5. The positive correlation between observed capital outflows and TFP growth is driven by the same data points as the negative correlation between observed and predicted outflows. This is because predicted outflows are negatively linked to TFP growth, as Proposition 3 suggests.

4.2 The channel puzzle

We now look more closely at the transmission mechanisms between output growth, investment and capital flows. Table 4 gives the variance decomposition of observed output growth and investment between potential determinants. These determinants are the capital gap \( \frac{\Delta k}{\bar{y}} \), the initial external position \( \frac{k_0}{\bar{y}} \), productivity growth \( \sigma^\tau \) and the capital wedge \( \tau \). As column (2) shows, in line with previous studies, differences in investment between countries are equally explained by TFP growth and distortions to the return on capital (Caselli and Feyrer, 2007).

36The analysis of the different components is detailed in Gourinchas and Jeanne (2007). The Appendix provides the results and analysis for our own results, which are mainly in line with theirs.
Since TFP has an additional effect on output, the resulting differences in growth are then mainly accounted for by TFP growth (Caselli, 2004; Hall and Jones, 1999), as represented by column (1). Since the explanatory variables are fitted on the capital stocks and on measured TFP, the explanatory power of the model is high (88-89%).

Consider now column (3), which represents the variance decomposition of observed capital flows. We can assess how the capital wedge and TFP growth, which together account for the major part of investment and growth (respectively 60% and 80%) translate to capital flows. Since the explanatory variables are not fitted on observed capital flows, the explanatory power of the model is low as compared to columns (1) and (2). Still, we can ask which variable has the highest explanatory power. The bulk of the explained variance is due to the capital wedge and not TFP catch-up (11% versus 1.5%). This is also apparent in Figure 5, which plots observed capital outflows against their potential determinants. Capital outflows are more strongly correlated with the capital wedge than with TFP catch-up. The resulting correlation is also more robust to the elimination of potential outliers.\footnote{In particular, the correlation stays significant for the capital wedge when eliminating Hong-Kong and Botswana, which is not the case for productivity catch-up.}

When contrasting these results with the variance decomposition of the predicted capital outflows reported in column (4), the picture is reversed: the explanatory power of the capital wedge is negligible, while almost the whole variability is explained by TFP catch-up, which is in line with Propositions 3 and 4. This is the channel puzzle: while, according to the model, TFP should drive capital flows, the data says that it is the capital wedge that explains most of their variance. In other words, while the standard model predicts that investment and growth are channeled to capital flows through TFP, in reality, they are channeled through the capital wedge.

To conclude, the riskless approach has two shortcomings that can be summarized by Table 4: (i) the TFP and wedge channels of investment are not transmitted to capital flows with the right sign (the allocation puzzle); (ii) the relative influence of these channels is wrong (the channel puzzle).

5 The portfolio approach

We have shown that the model without risk reproduces the allocation puzzle highlighted by Gourinchas and Jeanne (2007) and features a channel puzzle. We now turn to the extension with risk, and examine whether it solves these anomalies.
5.1 The portfolio approach and the allocation puzzle

Figures 1 and 2 sum up the results for the portfolio approach. Their lower panel report the estimated predicted net outflows according to equation (7). The estimates are computed under the assumption that the productivity catch-up follows a linear path, as in the riskless approach. The path of capital per efficient unit of labor $\tilde{k}_t$ implied by the model is approximated by the following formula: $\tilde{k}_t = \tilde{k}_0 e^{\ln(k^*/k(0))(1-\lambda)}$, where $1 - \lambda$ is the convergence rate estimated from the data (that is $1 - \lambda = 0.3$).\(^{28}\)

Note first that the magnitude of predicted flows (lower panels of Figures 1 and 2) is well above the actual ones (upper panels), by three orders of magnitude. This is a shortcoming of the portfolio approach that has been already highlighted in Kraay et al. (2005). It can also be related to the home bias in portfolio (Lewis, 1999). But this shortcoming is accentuated here by the presence of potentially huge human wealth effects, due to labor and productivity growth. Indeed, when looking into the different components provided in Table 3, it appears that the main origin of the discrepancy is the portfolio term, in column (7). This term represents the adjustment in safe assets from their initial to their long-run level. Since initial human wealth, which is part of the safe assets, is very volatile, this adjustment term is also potentially huge.\(^{29}\) Another source of volatility of predicted capital flows is the strong precautionary motive that magnifies the impact of investment on the demand of safe assets. This motive affects the predictions through the catch-up and convergence components (respectively columns (3) and (4)), which are of a higher magnitude than in the riskless approach.

When abstracting from the magnitude issue, it appears that the predicted outflows in column (2) of Table 3 exhibit the right sign, which is negative, and, contrary to the riskless approach, the right ranking between country groups. Predicted capital inflows are now decreasing with income, as the actual ones, and high income countries are accurately supposed to export capital. Also, Africa and Latin America receive capital inflows while Asia exports capital, as in the data.

The portfolio approach seems therefore to be a better predictor, if not of the magnitude of flows, at least of their direction. Figure 4 sums up this idea by plotting predicted flows according to the portfolio approach against the actual ones. A positive correlation appears. Besides, this correlation is not driven by Hong Kong and Botswana. To understand the sources of this correlation is not driven by Hong Kong and Botswana. To understand the sources of this correlation

\(^{28}\) The assumed trend is a good proxy for the capital dynamics since the theory predicts that it moves smoothly from $\tilde{k}_0$ to $\tilde{k}^*$.

\(^{29}\) More precisely, this comes from the fact that the portfolio term depends on the inverse of $\phi_0$, which is small due to large initial human wealth, as pointed to when analyzing Table 1. Small variations in $\phi_0$ then imply large variations in the portfolio term.
tion, consider the correlation of each component with the overall flows (last line of Table 3). As for the riskless approach, the convergence term does not have any explanatory power. Besides the trend component, it is the catch-up and portfolio terms that explain the good performance of the portfolio approach in terms of direction of flows.\textsuperscript{30}

More precisely, the portfolio approach helps solving the allocation puzzle because it accounts better both for the link between TFP growth and capital flows on the one hand and for the link between the capital wedge and capital flows on the other.

First, the portfolio approach accounts better for the link between TFP growth and capital flows. Table 5 shows the results of the regressions of capital outflows on their determinants. The first column shows that the unconditional correlation between TFP growth and capital outflows is significantly positive. This was the source of the allocation puzzle inside the riskless approach. However, when excluding countries with positive end-of-period net foreign assets (column (2)), this correlation becomes negative (though non-significant). Figure 5 also suggests that the relationship between TFP catch-up and capital outflows is U-shaped. This suggests that the allocation puzzle was mainly driven by countries with positive long-run external positions.\textsuperscript{31} On the opposite, as stated in Proposition 3 and shown originally by Kraay and Ventura (2000), the portfolio approach can account for the fact that the relationship between capital flows and growth depends on the sign of the long-run net foreign assets. This is simply the "New Rule". A consequence of the New Rule is that the link between TFP growth and capital flows is weak, which explains part of the channel puzzle. Therefore, not only the portfolio approach can account for the link between TFP growth and capital flows, but it can also explain why the riskless approach fails to.

Second, the New Rule by itself cannot explain the positive correlation between growth and capital outflows in the one hand, and between investment and capital outflows on the other, that has been documented in the empirical literature. Indeed, the New Rule would be a convincing explanation of these capital flows anomalies if a majority of countries had positive net foreign assets. But it is far from being the case. This is where the role of the capital wedge is crucial. According to Proposition 4, the net foreign assets position does not need to be positive to generate a positive correlation between investment and capital outflows, as long as the differences in investment are driven, at least partially, by differences in the capital wedge. As column (3) of Table 5 shows, the capital wedge has a negative impact on capital outflows, which is in line

\textsuperscript{30}For a more detailed analysis of the different components, see the Appendix.

\textsuperscript{31}These countries are: Botswana, China, Hong-Kong, Iran, Mauritius, Singapore, Taiwan and Venezuela.
with the portfolio approach. Also, consistently with the results of Table 4, the impact of TFP growth becomes non-significant. But, more importantly, this negative correlation is robust to the exclusion of countries with positive long-run net foreign assets.

5.2 The portfolio approach and the channel puzzle

We consider now whether the portfolio approach accounts accurately for the channels through which investment and growth translate to capital flows, that is: does it solve the channel puzzle?

Column (5) of Table 4 provides the variance decomposition of the predicted flows according to the portfolio approach. This time, their main variation is due to differences in the capital wedge, which is in line with the data (column (3)). This is opposite to the riskless approach, which predicted that the main variations in capital flows should be originated in TFP catch-up, as shown in column (4).

Comparing the predictions of Propositions 3 and 4 with the data helps explaining the better performance of the portfolio approach. According to the theory, the latter does not predict a clear relationship between TFP growth and capital flows, while the link between the capital wedge and capital flows is stronger. On the other hand, when turning to the data, it seems also that the correlation of capital flows with TFP growth is not as robust as with the capital wedge. This is apparent in Figure 5, which shows the unconditional correlations.

To conclude, the portfolio approach outperforms the riskless one along two dimensions that can be summarized by Table 4: (i) the TFP and wedge channels of investment are transmitted to capital flows with the right sign; (ii) the relative influence of these channels is right.

6 Conclusion

This paper develops an extension of the traditional neoclassical growth model to risky investment that contributes to match the actual capital flows and to solve the puzzle highlighted by Gourinchas and Jeanne (2007), as well as a “channel” puzzle that we establish. The advantage of this approach is that it does not constitute a great departure from the textbook model and therefore allows the adoption of a development accounting approach similar to Gourinchas and Jeanne (2007). The portfolio approach appears then more promising than the riskless one in explaining the allocation of capital flows among developing countries. This shows that international financial markets have to be considered not only as a financing source, but also as a way to provide insurance in the presence of domestic investment risk.
However, while the portfolio approach explains better the direction of flows than the riskless one, it fails to account for their magnitude, which is overestimated by several orders of magnitude. Still, this problem of magnitude in the portfolio choice model is commonly come across in the literature. It can be related to the home bias in portfolio (see Lewis (1999) for a survey), and especially to the findings of Kraay et al. (2005) on the magnitude of North-South bond position. This shortcoming is a natural consequence of our approach which makes some strong assumptions in order to derive analytical results and allow qualitative analysis. As a counterpoint, the quantitative performance of the model is poor. One of these strong assumptions is the fact that countries can pledge their entire human wealth, which is not realistic in the presence of sovereign risk and labor income risk. Relaxing this assumption would certainly dampen the magnitude issue.

Another direction for research consists in checking whether the portfolio approach can also account for the composition of flows. Extending the model to the possibility to trade equity could lead to predictions in terms of equity holdings. According to portfolio choice models, the more productive assets should constitute a higher share both in the domestic and foreign portfolio, which would explain why direct foreign investment is still positively correlated with productivity growth, as shown in Gourinchas and Jeanne (2007).

Last but not least, this study opens some new empirical questions. First, it would be interesting to reassess the empirical evidence on the allocation puzzle of Aizenman and Pinto (2007) and Prasad et al. (2007) in the light of our findings. Second, the channel puzzle deserves some more investigation. In particular, in our study, the capital wedge is calibrated in order to account for capital stocks differences. It is a residual, and this is a limit of our analysis. We could use the literature on growth accounting (Caselli and Feyrer (2007) for example) to provide determinants of this capital wedge and assess empirically their role in explaining capital flows, as opposed to TFP growth. The latter could also be measured alternatively than as the Solow residual, by using the composition of capital as suggested by Caselli and Wilson (2004).

References


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7 Appendix

Proof of Lemma 1

Maximizing \( U_i \) is equivalent to maximizing \( V_t^i = U_i / N_t = E_t \int_r^\infty e^{-(r-n)(s-t)} \ln(c_i^t)ds \).

Indexes are now dropped for simplicity. Define \( \phi \) such that \( \phi = k/\omega \). The constraint of the maximization problem is therefore: \( d\omega = [(r-R^*)\phi + R^*)\omega - c - n\omega]dt + \sigma \phi dz \).

The Bellman equation for this problem is:

\[
(r-n)V_t = \max_{c,\phi} \left\{ \ln(c_t) + \frac{\partial V_t}{\partial t} \right\}
\]

Then, applying Ito’s Lemma, we obtain:

\[
(r-n)V(\omega, t) = \max_{c,\phi} \left\{ \ln(c) + \frac{\partial V(\omega, t)}{\partial \omega} + \frac{\partial V(\omega, t)}{\partial \omega}[(r-R^*)\phi + R^*)\omega - c - n\omega] + \frac{\partial^2 V(\omega, t)}{\partial \omega^2} \frac{1}{2}\phi^2 \sigma^2 \right\}
\]

The first-order conditions of this problem are:

\[
\frac{1}{c} - \frac{\partial V(\omega, t)}{\partial \omega} = 0
\]

\[
\frac{\partial V(\omega, t)}{\partial \omega} (r-R^*) + \frac{\partial^2 V(\omega, t)}{\partial \omega^2} \phi \sigma^2 = 0
\]

An educated guess for the general form of the value function is:

\[
V(\omega, t) = \frac{\ln(\omega)}{\chi} + \psi
\]

where \( \chi \) and \( \psi \) have to be determined.

Substituting the derivatives of the value function into the first order conditions yields the solutions:

\[
c = \chi\omega \quad \text{and} \quad \phi = \frac{r-R^*}{\sigma^2}
\]

Plugging these expressions into the Bellman equation yields \( \chi = \rho - n \) and \( \psi = \ln(\rho - n) + (r-R^*)^2/2\sigma^2 + R^* - \rho \). This gives Equations (4) and (5).

Proof of Proposition 1

In order to prove Proposition 1, we first establish the following Lemma:

**Lemma 2:** Let \( \tilde{x}_t = X_t/(A_tN_t) \) denote the value of \( X_t \) in efficient labor terms at the aggregate level. For a given interest rate \( R^* \), the aggregate dynamics of the economy is characterized by:

\[
\tilde{c}_t = (\rho-n)\tilde{\omega}_t
\]

\[
\frac{\dot{\tilde{\omega}}_t}{\tilde{\omega}_t} = r_t\phi_t + R^*(1-\phi_t) - (\rho + g^* + \hat{n})
\]

where \( r_t = r(\phi_t\tilde{\omega}_t) = (1-\tau)\alpha(\phi_t\tilde{\omega}_t)^{\alpha-1} - \delta \) and \( \phi_t \) defined by equation (5).

Equation (12) is the counterpart of Equation (4) in terms of efficient units of labor.

Equation (13) is obtained from the aggregation of the individual budget constraints (2) written in terms of efficient units of labor and where the wage clears the labor market.
Equations (12) and (13), along with the no-Ponzi conditions and the definitions of \( r_t \) and \( \phi_t \), characterize the dynamics of \( \ddot{c}_t \) and \( \ddot{\omega}_t \). Once the paths of these variables are known, \( \ddot{h}_t = \phi_t \ddot{\omega}_t \), \( h_t = \int_0^\infty e^{-(R^*-(n+g^*))s} \pi_t \, ds \) and \( b_t = \ddot{\omega}_t - \ddot{h}_t - \dot{h}_t \) can be determined. However, these equations are used here only to determine steady state.

We now characterize stationarity.

Since, according to Equation (12), consumption is proportional to wealth, the stationarity of consumption \( \ddot{c}/\ddot{c} = 0 \) implies the stationarity of wealth \( \ddot{w}/\ddot{w} = 0 \). Additionally, the stationarity of catch-up \( \ddot{\pi} = 0 \), along with the aggregate budget constraint, implies that the long-run share of capital \( \phi^* \) satisfies:

\[
  r^* \phi^* + R^*(1 - \phi^*) - (\rho + g^*) = 0
\]

which is equivalent to:

\[
  \frac{(r^* - R^*)^2}{\sigma^2} = \rho + g^* - R^*
\]

(14)

Therefore, for the steady state to exist, we must have \( R^* \leq \rho + n \).

Since the share of capital in wealth \( \phi \) is necessarily non-negative, then \( r - R^* \) is also non-negative. Equation (14) therefore yields the expression of the return differential:

\[
  r^* - R^* = \sqrt{\sigma^2(\rho + g^* - R^*)}
\]

This equation is equivalent to Equation (6).

**Proof of (i):**

Equation (7) derives from the definition of \( \phi \) at steady state:

\[
  \phi^* = \ddot{h}^*/(\ddot{h}^* + \ddot{b}^* + \ddot{h}^*)
\]

(15)

To establish Equation (7), we must then derive the value of the steady-state share of capital \( \phi^* \) and the steady-state human wealth \( h^* \).

First, when \( \sigma > 0 \), we must have necessarily \( R^* < \rho + n \). This can be shown by noticing that Equation (14) can be rewritten as follows:

\[
  \sigma^2 \phi^2 = \rho + g^* - R^*
\]

Suppose that \( R^* = \rho + g^* \), this would imply, when \( \sigma > 0 \), that \( \phi^* = 0 \). If wealth is stationary, this means that the stock of capital converges towards zero. Given the Cobb-Douglas specification, this implies that the return to capital would tend to infinity, which contradicts the fact that the return differential is constant, as suggested by Equation (6). Since, as shown above, \( R^* \leq \rho + g^* \) for the steady state to exist, then we must have \( R^* < \rho + g^* \). As a consequence, Equation (14) implies that:

\[
  \phi^* = \sqrt{\frac{\rho + g^* - R^*}{\sigma^2}}
\]

Second, to complete Equation (7), we have to determine \( \ddot{h} \) at steady state. \( \ddot{h}_t = \int_0^\infty e^{-(R^*-(n+g^*))s} \pi_t \, ds = \int_0^\infty e^{-(R^*-(n+g^*))s} \pi_t (1 - \alpha + \tau \alpha) f(\ddot{h}_t) \, ds \). Equation (6) gives \( \ddot{k}^* \), the steady-state value of \( k \). We have also \( \pi_t = \pi \) in the long run. Therefore,

\[
  \ddot{h}^* = \frac{(1 - \alpha + \tau \alpha) f(\ddot{k}^*)}{R^* - (n + g^*)}
\]

Replacing \( \ddot{h}^* \) in Equation (15) with the steady-state \( \phi^* \) yields Equation (7).
Proof of (ii):

When $\sigma = 0$, the no-arbitrage condition $r_t = R^*$ is necessarily satisfied for all $t$. Otherwise, according to the expression of $\phi_t$ (5), the stock of capital would be infinite. Therefore, the stationarity of wealth implies:

$$R^* = \rho + g^*$$

Using (12) and (13) in per capita terms, along with the fact that $r_t = R^*$ and $R^* = \rho + g^*$, we obtain the following Euler condition:

$$\frac{\dot{c}_t}{c_t} = g^*$$

(16)

Therefore, $c_t = c_0 e^{\rho t}$, and $\tilde{c}_t = \tilde{c}_0 e^{\rho t} A_0 / A_t = \tilde{c}_0 e^{-n t}$. As a consequence, we obtain at steady state: $\tilde{c}^* = \tilde{c}_0 e^{-n}$. We know also that $\tilde{k}_t = \tilde{k}^*$ always because of the no-arbitrage condition. The stationarity of wealth therefore implies that $\tilde{b}^*$ is also constant in the long run and satisfies:

$$\tilde{\omega}^* = \tilde{k}^* + \tilde{b}^* + \tilde{h}^*$$

Since $\tilde{c}^* = (\rho - n)\tilde{\omega}^*$ and, as we have shown above, $\tilde{h}^* = (1 - \alpha + \tau \alpha) f(\tilde{k}^*)/(R^* - (n + g^*))$, this equation can be rewritten as follows:

$$\tilde{b}^* = -\tilde{k}^* - \frac{(1 - \alpha + \tau \alpha) f(\tilde{k}^*)}{\rho - n} + \tilde{c}_0 e^{-n}$$

Equation (8) is recovered by replacing $\tilde{c}_0$ using $\tilde{c}_0 = (\rho - n)\tilde{\omega}_0$.

Proof of Proposition 2

Proof of (i):

Replacing the expression for $\tilde{b}^*$ (8) in Equation (9) and substituting for $\tilde{b}_0$, we obtain:

$$\frac{\Delta B}{Y_0} = \frac{\tilde{k}^* - \tilde{k}_0}{k_0^\alpha} e^{(n + g^*)T} - (e^\pi - 1) \frac{\tilde{k}^*}{k_0^\alpha} e^{(n + g^*)T}$$

$$- e^{\pi + (n + g^*)T} \left( 1 - \alpha + \tau \alpha \right) f(\tilde{k}^*) \left( \frac{1 - \int_0^\infty e^{-(\rho - n)t + \pi - \pi t} dt}{\rho - n} \right) + (e^{(n + g^*)T} - 1) \frac{\tilde{b}_0}{k_0^\alpha}$$

Since $1/(\rho - n) = \int_0^\infty e^{-(\rho - n)t} dt$, this expression leads to Equation (10).

Proof of (ii):

Similarly, replacing the expression for $\tilde{b}^*$ (7) in Equation (9), we obtain:

$$\frac{\Delta B}{Y_0} = \frac{1 - \phi^* \tilde{k}^* - \tilde{k}_0}{k_0^\alpha} e^{(n + g^*)T} + \frac{1 - \phi^*}{\phi^*} (e^\pi - 1) \frac{\tilde{k}^*}{k_0^\alpha} e^{(n + g^*)T} + \frac{1 - \phi^*}{\phi^*} e^{(n + g^*)T}$$

$$- e^{\pi + (n + g^*)T} \left( 1 - \alpha + \tau \alpha \right) f(\tilde{k}^*) \left( \frac{1 - \int_0^\infty e^{-(\rho - n)t + \pi - \pi t} dt}{\rho - n} \right) \frac{\tilde{b}_0}{k_0^\alpha}$$

Further transformation yields:

$$\frac{\Delta B}{Y_0} = \frac{1 - \phi^* \tilde{k}^* - \tilde{k}_0}{k_0^\alpha} e^{(n + g^*)T} + \frac{1 - \phi^*}{\phi^*} (e^\pi - 1) \frac{\tilde{k}^*}{k_0^\alpha} e^{(n + g^*)T} + \frac{1}{\phi^*} \frac{\tilde{k}_0}{k_0^\alpha} e^{(n + g^*)T} - \frac{\tilde{b}_0}{k_0^\alpha} e^{(n + g^*)T}$$
\[-e^{\pi+(n+g^*)T} \frac{1 - \alpha + \alpha \tau}{k_0^0} f(\tilde{k}^*) \left( \frac{1}{R^* - (n + g^*)} - \int_0^\infty e^{-(R^* - (n + g^*))t + \pi_t - \pi} dt \frac{f(\tilde{k}_t)}{f(k^*)} \right) \frac{\tilde{b}_0}{k_0^0} e^{(n+g^*)T} \]

\[+ \left( e^{(n+g^*)T} - 1 \right) \frac{\tilde{b}_0}{k_0^0} \frac{\tilde{b}_0}{k_0^0} e^{(n+g^*)T} \]

Since \(1/(R^* - (n + g^*)) = \int_0^\infty e^{-(R^* - (n + g^*))t} dt\) and \(\frac{(\tilde{k}_0 + \tilde{b}_0 + \tilde{b}_0)}{k_0^0} e^{(n+g^*)T} \right) = \frac{\tilde{k}_0}{k_0^0} e^{(n+g^*)T}\), this expression leads to Equation (10).

**Productivity and consumption smoothing**

**Riskless case:**

If \(\sigma = 0\), under Assumption 3, \(\frac{\Delta B^s}{Y_0}\) can be written as follows:

\[\frac{\Delta B^s}{Y_0} = -e^{(n+g^*)T} \frac{1 - \alpha + \alpha \tau}{k_0^0} \frac{\tilde{k}^s}{\tilde{k}^s} \int_0^T e^{-(\rho-n)t} \left( e^{\pi - \pi f(t)} \right) dt \]

A sufficient condition for \(\frac{\Delta B^s}{Y_0}\) to be decreasing in \(\pi\) is that \(e^\pi - \pi f(t)\) is increasing in \(\pi\). We have:

\[\frac{\partial \left[ e^\pi - \pi f(t) \right]}{\partial \pi} = e^\pi \left[ 1 - f(t) e^{\pi f(t)-1} \right] \]

Consider the term between brackets:

\[\frac{\partial \left[ 1 - f(t) e^{\pi f(t)-1} \right]}{\partial f(t)} = -e^{\pi(1-f(t))} [1 + \pi f(t)] \]

A sufficient condition for this derivative to be negative is \(\pi \geq -1\). If it is the case, then for \(0 \leq f(t) \leq 1\):

\[1 - f(t) e^{\pi f(t)-1} \geq 0 \]

which implies that \(\frac{\partial \left[ e^\pi - \pi f(t) \right]}{\partial \pi} \geq 0\). As a consequence, if \(\pi \geq -1\), then \(\frac{\Delta B^s}{Y_0}\) is decreasing in \(\pi\).

**Risky case:**

When \(\sigma > 0\), under Assumption 3, \(\frac{\Delta B^s}{Y_0}\) can be written as follows:

\[\frac{\Delta B^s}{Y_0} = -e^{(n+g^*)T + \pi} \frac{1 - \alpha + \alpha \tau}{k_0^0} \frac{\tilde{k}^s}{\tilde{k}^s} \int_0^T e^{-(\rho-n)t} \left( 1 - \frac{\tilde{k}_t}{\tilde{k}^s} e^{\pi f(t)-1} \right) dt \]

A sufficient condition for \(\frac{\Delta B^s}{Y_0}\) to be decreasing in \(\pi\) is that \(e^\pi \left( 1 - \frac{\tilde{k}_t}{\tilde{k}^s} e^{\pi f(t)-1} \right)\) is increasing in \(\pi\) for all \(t > 0\). For \(\pi\) close to zero and \(\tilde{k}_t\) close to \(\tilde{k}^s\), we have:

\[e^\pi \left( 1 - \frac{\tilde{k}_t}{\tilde{k}^s} e^{\pi f(t)-1} \right) \approx \frac{\tilde{k}_t}{\tilde{k}^s} - \pi \left( 1 - f(t) - \frac{\tilde{k}_t}{\tilde{k}^s} e^{\pi f(t)-1} \right) \]

The sign of the derivative of \(e^\pi \left( 1 - \frac{\tilde{k}_t}{\tilde{k}^s} e^{\pi f(t)-1} \right)\) with respect to \(\pi\) is therefore negative if and only if \(1 - f(t) - \frac{\tilde{k}_t}{\tilde{k}^s} e^{\pi f(t)-1}\) is positive. Hence the sufficient condition.
Proof of Proposition 4

Notice that, with or without risk, the predicted capital flows must satisfy (9). According to this equation the derivative of $\frac{\Delta \bar{p}}{\bar{y}_0}$ with respect to $\tau$ depends only on the derivative of $\bar{b}^*$.

Proof of (i):

Without risk and following the stationarity conditions specified in Proposition 1, $\bar{b}^*$ must satisfy:

$$\bar{b}^* = \bar{k}^* + e^{-\pi} \left( \bar{b}_0 + \bar{k}_0 \right) - (1 - \alpha + \tau \alpha) f(\bar{k}^*) \int_0^T e^{-(\rho - n)t} \left( 1 - e^{\pi(f(t) - 1)} \right) dt$$

This equation is obtained by rearranging (8) under Assumption 3.

This expression is differentiated with respect to $\tau$:

$$\frac{\partial \bar{b}^*}{\partial \tau} = -\frac{\partial \bar{k}^*}{\partial \tau} \left[ 1 + (1 - \alpha + \tau \alpha) f'(\bar{k}^*) \int_0^T e^{-(\rho - n)t} \left( 1 - e^{\pi(f(t) - 1)} \right) dt \right]$$

$$-\alpha f(\bar{k}^*) \int_0^T e^{-(\rho - n)t} \left( 1 - e^{\pi(f(t) - 1)} \right) dt$$

If $\pi$ converge towards zero, then we can approximate: $\frac{\partial \bar{b}^*}{\partial \tau} = -\frac{\partial \bar{k}^*}{\partial \tau} > 0$.

Proof of (ii):

Differentiating (7) with respect to $\tau$, we obtain:

$$\frac{\partial \bar{b}^*}{\partial \tau} = \frac{\partial \bar{k}^*}{\partial \tau} \left[ \frac{1 - \phi^*}{\phi^*} - \frac{(1 - \alpha + \tau \alpha) f'(\bar{k}^*)}{R^* - n - g^*} \right] - \frac{\alpha f(\bar{k}^*)}{R^* - n - g^*}$$

$\frac{\partial \bar{k}^*}{\partial \tau}$ is negative. Therefore, if $\frac{1 - \phi^*}{\phi^*} - \frac{(1 - \alpha + \tau \alpha) f'(\bar{k}^*)}{R^* - n - g^*}$ is positive, then $\frac{\partial \bar{b}^*}{\partial \tau}$ is negative.

$$\frac{1 - \phi^*}{\phi^*} - \frac{(1 - \alpha + \tau \alpha) f'(\bar{k}^*)}{R^* - n - g^*} \geq 0 \Leftrightarrow \frac{1 - \phi^*}{\phi^*} - \alpha \frac{(1 - \alpha + \tau \alpha) f(\bar{k}^*)/\bar{k}^*}{R^* - n - g^*} \geq 0$$

$$\Leftrightarrow \frac{1 - \phi^*}{\phi^*} - \frac{(1 - \alpha + \tau \alpha) f'(\bar{k}^*)/\bar{k}^*}{R^* - n - g^*} \geq -(1 - \alpha) \frac{(1 - \alpha + \tau \alpha) f(\bar{k}^*)/\bar{k}^*}{R^* - n - g^*}$$

Therefore, if $\bar{b}^*$ satisfies the last condition, then $\frac{\partial \bar{b}^*}{\partial \tau}$ is negative.

Components analysis

The riskless approach

After looking into components, it appears that the catch-up component, in column (3) of Table 2, has a negative contribution. This average result is mainly driven by Asia, which had a strong positive long-term productivity catch-up: it should have borrowed from the rest of the world in order to finance the extra investment. Other non-OECD countries have fallen behind world productivity on average, namely Africa and Latin America. This relative fall in productivity should have led households to disinvest and enabled them to lend to the rest of the world.

The convergence term in column (4) contributes positively to the total predicted outflows. This can be explained by the fact that, as shown above, countries have started on average above
their long-term level of capital, and thus have disinvested on average. As a consequence, they should have lent to the rest of the world. This is the case in Latin America and Africa which had too much capital and should have used their extra capital stock to invest abroad, whereas Asia should have received capital from abroad to finance its growth in capital stock.\footnote{The convergence term have a higher impact on capital flows than it does in Gourinchas and Jeanne (2007). This is due to the fact we estimate larger capital gaps (see above discussion).}

The consumption smoothing component, in column (5), is negative on average despite the negative average productivity catch-up. This is because Asian countries, which have benefited from a positive productivity catch-up, contribute highly to the sample mean. When considering regions, it still appears that Latin America and Africa, which expected a fall in their revenue because of a negative catch-up, should have saved in order to smooth consumption. The contribution of the consumption smoothing term is therefore positive for these regions. On the opposite, Asian countries, which expected a relative rise in their productivity and therefore a relative rise in their revenue, should have dissaved in order to smooth consumption. Their consumption smoothing term is thus negative.

These three components (convergence, catch-up and consumption smoothing) are at odds with the data. They all imply capital inflows to Asia and capital outflows from Latin America and Africa, while actually Asia received less capital than the two other regions.

Only the last one, the trend component in column (6), is consistent with the data. Indeed, as observed capital inflows, it is decreasing with income. Also, according to this component, Asia should receive less capital than Latin America and Africa. However, its quantitative importance is not sufficient to counteract the other components.

The portfolio approach

To understand the source of the patterns described in section 5.1, consider the different components of predicted flows in Table 3. The catch-up and convergence terms (respectively columns (3) and (4)) contribute substantially to solving the puzzle. In particular, regions are now correctly ranked. While, in the riskless approach, Asia was supposed to receive more inflows than the other non-OECD countries because of high catch-up and positive convergence, the estimates here suggest that it should export more capital, which matches the data better, as well in terms of the direction of flows as in terms of hierarchy between regions. To understand this predictions’ reversal for the convergence and catch-up terms, note that both high productivity catch-up and positive convergence imply investment in domestic technology. In the riskless approach, more investment is financed through more borrowing from abroad while in the portfolio approach, more investment implies more safe assets to compensate for more risk-taking. Notice also that the portfolio approach restores the correct hierarchy between low and high income countries when considering the catch-up and convergence terms.

The consumption smoothing term in column (5) is of the same magnitude and same sign as in the riskless approach. Indeed, the two definitions are close. The only difference is that in the absence of risk, the stock of capital adjusts immediately to its long-run level while the adjustment is smoother in the presence of risk. As a consequence, the consumption smoothing term predicts more capital inflows in the portfolio approach because it predicts a slower decrease in the capital stock and therefore higher future incomes. But the hierarchy between country groups do not change. This term then does not contribute to solving the puzzle. On the opposite, the trend component in column (6), which is the same as in riskless approach, goes in the right direction.

Coming back to the portfolio term in column (7) of Table 3, we can notice that it is negative on average. This is because developing countries started on average with a low share of capital in their portfolio as compared to the long-run one. They should then substitute safe assets for capital, which implies capital inflows (i.e. a diminution in bond holdings). We seen when an
alyzing Table 1 that the initial share of capital is increasing in income. As a consequence, the portfolio term is increasing with income and as a consequence it reproduces the right income-group ranking and the right direction of flows. The implied pattern for regions is less accurate. In particular, Asia is supposed to receive large capital inflows because its initial capital share is low. This is originated in its high TFP catch-up.

Therefore, roughly speaking, all the components except the consumption smoothing one reproduce the right pattern of flows (in terms of groups ranking): the convergence, catch-up, trend and portfolio components.
Table 1: Long-term capital per efficient unit of labor, capital wedge and potential determinants of capital flows

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5) riskless approach</th>
<th>(6) portfolio approach</th>
</tr>
</thead>
<tbody>
<tr>
<td>Non-OECD‡</td>
<td>-9%</td>
<td>-30%</td>
<td>2,1%</td>
<td>-1,1%</td>
<td>38%</td>
<td>28%</td>
</tr>
<tr>
<td>Low income‡</td>
<td>-14%</td>
<td>-32%</td>
<td>1%</td>
<td>-0,8%</td>
<td>66%</td>
<td>60%</td>
</tr>
<tr>
<td>Lower middle income‡</td>
<td>-17%</td>
<td>-35%</td>
<td>2.3%</td>
<td>-1,1%</td>
<td>31%</td>
<td>19%</td>
</tr>
<tr>
<td>Upper middle income‡</td>
<td>-11%</td>
<td>-34%</td>
<td>2.7%</td>
<td>-1,5%</td>
<td>28%</td>
<td>16%</td>
</tr>
<tr>
<td>High income‡ (non OECD †)</td>
<td>38%</td>
<td>2%</td>
<td>3,9%</td>
<td>-1,6%</td>
<td>-7%</td>
<td>-25%</td>
</tr>
<tr>
<td>Sub-Saharan Africa</td>
<td>-15%</td>
<td>-39%</td>
<td>1.6%</td>
<td>-1,9%</td>
<td>55%</td>
<td>47%</td>
</tr>
<tr>
<td>Latin America</td>
<td>-36%</td>
<td>-33%</td>
<td>2.9%</td>
<td>-1,3%</td>
<td>32%</td>
<td>20%</td>
</tr>
<tr>
<td>Asia</td>
<td>37%</td>
<td>-11%</td>
<td>1.9%</td>
<td>0.4%</td>
<td>19%</td>
<td>5%</td>
</tr>
<tr>
<td>Asia (Excluding China)</td>
<td>27%</td>
<td>-12%</td>
<td>1.9%</td>
<td>0.6%</td>
<td>19%</td>
<td>5%</td>
</tr>
</tbody>
</table>

The figures are unweighted country averages.

‡: Includes also Korea, Mexico and Turkey.

†: World Bank classification based on 2007 GNI per capita.
Table 2: Predicted and actual capital flows between 1980 and 2000 - Riskless approach

<table>
<thead>
<tr>
<th>Capital flows</th>
<th>(share of initial output)</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
<th>(7)</th>
<th>Obs.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Non-OECD†</td>
<td></td>
<td>-0.46</td>
<td>-0.48</td>
<td>-0.20</td>
<td>0.57</td>
<td>-0.34</td>
<td>-0.52</td>
<td>67</td>
<td></td>
</tr>
<tr>
<td>Low income‡</td>
<td></td>
<td>-1.01</td>
<td>1.24</td>
<td>0.17</td>
<td>0.49</td>
<td>1.17</td>
<td>-0.59</td>
<td>23</td>
<td></td>
</tr>
<tr>
<td>Lower middle income‡</td>
<td></td>
<td>-0.75</td>
<td>-0.52</td>
<td>-0.06</td>
<td>0.61</td>
<td>-0.50</td>
<td>-0.57</td>
<td>23</td>
<td></td>
</tr>
<tr>
<td>Upper middle income‡</td>
<td></td>
<td>0.01</td>
<td>0.94</td>
<td>0.26</td>
<td>0.71</td>
<td>0.55</td>
<td>-0.58</td>
<td>14</td>
<td></td>
</tr>
<tr>
<td>High income‡ (non OECD†)</td>
<td></td>
<td>1.38</td>
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<td>-2.79</td>
<td>0.47</td>
<td>-6.52</td>
<td>-0.03</td>
<td>7</td>
<td></td>
</tr>
<tr>
<td>Africa</td>
<td></td>
<td>-0.67</td>
<td>1.72</td>
<td>0.31</td>
<td>1.11</td>
<td>0.99</td>
<td>-0.70</td>
<td>28</td>
<td></td>
</tr>
<tr>
<td>Latin America</td>
<td></td>
<td>-0.56</td>
<td>6.89</td>
<td>1.27</td>
<td>0.59</td>
<td>5.60</td>
<td>-0.56</td>
<td>22</td>
<td></td>
</tr>
<tr>
<td>Asia</td>
<td></td>
<td>0.02</td>
<td>-13.65</td>
<td>-2.94</td>
<td>-0.33</td>
<td>-10.20</td>
<td>-0.18</td>
<td>17</td>
<td></td>
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<tr>
<td>Asia (Excluding China)</td>
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<td>0.01</td>
<td>-7.63</td>
<td>-1.83</td>
<td>-0.40</td>
<td>-5.20</td>
<td>-0.20</td>
<td>16</td>
<td></td>
</tr>
<tr>
<td>Correlation with $\frac{\Delta F}{Y_0}$</td>
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<td>1</td>
<td>-0.20</td>
<td>-0.26</td>
<td>-0.02</td>
<td>-0.19</td>
<td>0.25</td>
<td>66</td>
<td></td>
</tr>
</tbody>
</table>

(Excluding China)

$\Delta F/Y_0$ is the observed ratio of net capital outflows to initial output, predicted capital flows $\Delta F$ and its components $\Delta F^m/Y_0$, $\Delta F^r/Y_0$ and $\Delta F^b/Y_0$ are given by (10).

The figures are unweighted country averages.

†: Includes also Korea, Mexico and Turkey.

‡: World Bank classification based on 2007 GNI per capita.

Table 3: Predicted and actual capital flows between 1980 and 2000 - Portfolio approach

<table>
<thead>
<tr>
<th>Capital flows</th>
<th>(share of initial output)</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
<th>(7)</th>
<th>(8)</th>
<th>Obs.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Non-OECD†</td>
<td></td>
<td>-0.46</td>
<td>-28.49</td>
<td>2.91</td>
<td>-8.33</td>
<td>-1.34</td>
<td>-0.52</td>
<td>-21.20</td>
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<td></td>
</tr>
<tr>
<td>Low income‡</td>
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<td>-0.59</td>
<td>-51.37</td>
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<td>-1.54</td>
<td>-0.57</td>
<td>-15.75</td>
<td>23</td>
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<tr>
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<td>0.01</td>
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<td>-0.58</td>
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<td></td>
</tr>
<tr>
<td>High income‡ (non OECD†)</td>
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<td>-6.76</td>
<td>-5.89</td>
<td>-0.03</td>
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<tr>
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<td>5.08</td>
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<tr>
<td>Asia</td>
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<td>11.99</td>
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<td>-8.76</td>
<td>-0.18</td>
<td>-26.39</td>
<td>17</td>
<td></td>
</tr>
<tr>
<td>Asia (Excluding China)</td>
<td></td>
<td>0.01</td>
<td>7.34</td>
<td>26.53</td>
<td>5.76</td>
<td>-3.75</td>
<td>-0.20</td>
<td>-21.01</td>
<td>16</td>
<td></td>
</tr>
<tr>
<td>Correlation with $\frac{\Delta F}{Y_0}$</td>
<td></td>
<td>1</td>
<td>0.33</td>
<td>0.26</td>
<td>0.02</td>
<td>-0.11</td>
<td>0.25</td>
<td>0.20</td>
<td>66</td>
<td></td>
</tr>
</tbody>
</table>

(Excluding China)

$\Delta F/Y_0$ is the observed ratio of net capital outflows to initial output, predicted capital flows $\Delta F$ is given by (9) where $\delta^*$ satisfies Equation (7). Its components $\Delta F^m/Y_0$, $\Delta F^r/Y_0$ and $\Delta F^b/Y_0$ are given by (11). $\Delta F^p/Y_0$ is inferred from $\Delta F/Y_0$ minus the other components.

The figures are unweighted country averages.

†: Includes also Korea, Mexico and Turkey.

‡: World Bank classification based on 2007 GNI per capita.
Table 4: Variance decomposition of capital outflows and investment - The channel puzzle (Excluding China)

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>ΔY/Y₀</td>
<td>ΔK/Y₀</td>
<td>ΔB/Y₀</td>
<td>ΔM/Y₀</td>
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<td>Model</td>
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<td>88%</td>
<td>21%</td>
<td>94%</td>
<td>74%</td>
</tr>
<tr>
<td>Δbₜ /b₀</td>
<td>10%</td>
<td>26%</td>
<td>0%</td>
<td>7%</td>
<td>1%</td>
</tr>
<tr>
<td>e^n</td>
<td>80% (+)</td>
<td>29% (+)</td>
<td>1,5% (+)</td>
<td>90% (-)</td>
<td>0,001% (+)</td>
</tr>
<tr>
<td>τ</td>
<td>1% (-)</td>
<td>30% (-)</td>
<td>11% (-)</td>
<td>0,04% (+)</td>
<td>69% (-)</td>
</tr>
<tr>
<td>Residual</td>
<td>11%</td>
<td>12%</td>
<td>79%</td>
<td>6%</td>
<td>26%</td>
</tr>
<tr>
<td>Total</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
</tr>
<tr>
<td>Observations</td>
<td>66</td>
<td>66</td>
<td>66</td>
<td>66</td>
<td>66</td>
</tr>
</tbody>
</table>

Table 5: Determinants of observed capital outflows

| Dependent variable:   | (1)       | (2)       | (3)       | (4)       |
| ΔB/Y₀                 | All (Excluding China) | All (Excluding China) | All (Excluding China) | All (Excluding China) |
|                       | bₜ < 0¹   | bₜ < 0¹   | bₜ < 0¹   | bₜ < 0¹   |
| e^n                   | 0.782**   | -0.255    | 0.483     | -0.282    |
|                       | (2.02)    | (1.14)    | (1.26)    | (1.26)    |
| Δk /k₀                | 0.022     | 0.045     |           |           |
|                       | (0.09)    | (0.32)    |           |           |
| k₀ /k₀                | 1.037     | 0.007     |           |           |
|                       | (1.33)    | (0.02)    |           |           |
| τ                     | -1.393**  | -0.535*   |           |           |
|                       | (2.66)    | (1.75)    |           |           |
| Constant              | -1.228*** | -0.582*** | -0.080    | -0.320    |
|                       | (-4.31)   | (-3.02)   | (0.13)    | (1.12)    |
| Observations          | 66        | 59        | 66        | 59        |
| R-squared             | 0.058     | 0.022     | 0.215     | 0.088     |

Robust t statistics in parentheses
* significant at 10%; ** significant at 5%; *** significant at 1%

¹: bₜ refers to the observed end-of-period net foreign assets in efficient labor terms.
Figure 1: Observed and predicted capital flows by region

Source: Penn World Tables 6.2 (Heston et al., 2006), Lane and Milesi-Ferretti (2006), author’s calculations. Observed capital flows are the observed ratio of net capital outflows to initial output, predicted capital flows in the riskless and portfolio approaches are respectively $\frac{3F_{M}}{Y_{C}}$ as defined by Equation (10) and $\frac{3F_{P}}{Y_{C}}$ as given by Equation (9) where $b^{*}$ satisfies Equation (7). The figures are unweighted country averages. Non-OECD countries include also Korea, Mexico and Turkey.
Figure 2: Observed and predicted capital flows by income group

Source: Penn World Tables 6.2 (Heston et al., 2006), Lane and Milesi-Ferretti (2006), author’s calculations.
Observed capital flows are the observed ratio of net capital outflows to initial output, predicted capital flows in the riskless and portfolio approaches are respectively $\Delta \frac{B}{Y}$ as defined by Equation (10) and $\Delta \frac{b^*}{b}$ as given by Equation (9) where $b^*$ satisfies Equation (7).

The figures are unweighted country averages.
Non-OECD countries include also Korea, Mexico and Turkey.
1: World Bank classification based on 2007 GNI per capita.
Figure 3. Actual capital outflows (as a share of initial GDP) against their predicted value, according to the riskless approach, 1980-2000

Source: Penn World Tables 6.2 (Heston et al., 2006), Lane and Milesi-Ferretti (2006), author’s calculations.

Figure 4. Actual capital outflows (as a share of initial GDP) against their predicted value, according to the portfolio approach, 1980-2000

Source: Penn World Tables 6.2 (Heston et al., 2006), Lane and Milesi-Ferretti (2006), author’s calculations.
Figure 5: Actual capital outflows between 1980 and 2000 (as a share of initial GDP), against their determinants: capital gap \((k* - \bar{k}_0)/\bar{k}_0\), initial external position to GDP ratio \((\bar{b}_0)/\bar{y}_0\), long-run productivity catch-up \((\pi)\) and capital wedge \(\tau\).

Source: Penn World Tables 6.2 (Heston et al., 2006), Lane and Milesi-Ferretti (2006), author’s calculations.

Note: The capital wedge \(\tau\) used here is the one calibrated inside the portfolio approach. The negative correlation is robust to the use of the one calibrated inside the riskless approach.