Illegal Immigration, Deportation Policy, and the Optimal Timing of Return

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Abstract

Countries with strict immigration policies often resort to deportation measures to reduce their stocks of illegal immigrants. Many of their undocumented foreign workers, however, are not deported but rather choose to return home voluntarily. This paper studies the optimizing behavior of undocumented immigrants who continuously face the risk of deportation, modeled by a stochastic process, and must decide how long to remain in the host country. It is found that the presence of uncertainty with respect to the length of stay abroad unambiguously reduces the desired migration duration and may trigger a voluntary return when a permanent stay would otherwise be optimal. Voluntary return is motivated by both economic and psychological factors. Calibration of the model to match the evidence on undocumented Thai migrants in Japan suggests that the psychological impact of being abroad as an illegal alien may be equivalent to as large as a 68% cut in the consumption rate at the point of return.

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1 Introduction

Illegal immigration is considered to be undesirable by most of the receiving countries. In an effort to control this problem they have introduced various barriers to entry, internal enforcement measures to impede labor-market access for unauthorized workers, and they even resort to deportations to remove illegal immigrants from their territory. Deportation policies and levels of enforcement vary across nations. The somewhat lenient measures applied in the U.S.A. and countries of Western Europe are in sharp contrast with the very strict policies on illegal immigration in the Gulf Cooperation Council (GCC) States and East Asian economies, such as Hong Kong, Singapore, Malaysia, South Korea, Taiwan, and Japan.\(^1\) The main purpose of this paper is to show how the risk of deportation affects an undocumented migrant’s decision to return home *voluntariness*. This analysis has important implications for the effectiveness of deportations as a policy instrument aimed at reducing the stock of illegal immigrants.

In countries where enforcement is strict, undocumented migrants cannot live normal lives. They are in constant fear of being detected, apprehended and deported. In the words of one illegal immigrant in Japan, "We stay to ourselves. Outside don’t talk."\(^2\) In such circumstances undocumented immigrants typically intend to stay and work in the host country for only a limited period of time, just to accumulate enough savings for the purpose of raising their consumption after return, starting a business or improving the quality of their housing back in the source country.\(^3\) In a study of Thai overseas migration, Jones and Pardthaisong (1999) find that in 61 out of 63 surveyed villages, the proportion of overseas workers who voluntarily returned to Thailand was 95% or more. Sobieszczynk (2000) is another informative study of Thai overseas migrants, both docu-

\(^1\)In East Asia and especially in the GCC States, illegal entry, as well as an overstay of the officially allowed duration of residence, are considered to be very serious infractions entailing severe penalties (see Section 3.2 for more details).


\(^3\)See Jones and Pardthaisong (1999) and Sobieszczynk (2000).
mented and undocumented. The documented ones in her sample worked on contracts of fixed duration (typically 2 years), while unauthorized migrants were able to choose how much time to spend in a foreign country if they managed to avoid getting deported. This availability of choice with respect to the duration of stay (rather than being obliged to return when the official work contract expires) was, in fact, one of the reasons for migrating illegally rather than through official recruitment channels. The duration of stay for such migrants was on average 30 months.

The fact that an overwhelming majority of undocumented immigrants returns voluntarily from countries with strict deportation policies and low tolerance for illegal immigration raises a series of interesting questions that have not been addressed in the literature: Given that apprehension and deportation are uncertain, how is the optimal migration duration determined in this risky environment? What are the characteristics of the migrant’s optimal consumption and saving profiles abroad and after return, given the conditions prevailing in labor, commodity, and capital markets in both countries? With respect to the timing of voluntary return, how important is the role of social factors, such as homesickness or the lack of access to social capital that is available to migrants in their country of origin? The present study addresses these questions within a stochastic life-cycle model. The stochastic element comes from the fact that the event of deportation is assumed to follow a random process.

assume that once an illegal immigrant manages to get into the host country, she does not go back to the country of origin. Moreover, when apprehension is considered explicitly, as in the works of Woodland and Yoshida (2006), Djajić (2014) and Auriol and Mesnard (2012), it is modeled deterministically as a zero-one event at the point of entry, with the probability of forced repatriation dropping to zero after a migrant successfully crosses the border of the host country. This framework allows the authors to study the effectiveness of apprehensions in deterring illegal entries but it does not provide a suitable setting for examining the question of the optimal return date of an undocumented migrant who is already working in the host country and is continuously facing the risk of deportation. More recent papers by Friebel and Guriev (2006), Djajić (2013), Djajić and Mesnard (2015) and Djajić and Vinogradova (2013), allow for the possibility of deportation after a successful entry, although this is modeled deterministically and the option of voluntary return is not considered.

The present study contributes to the literature by developing a framework of temporary illegal immigration where uncertainty with respect to the duration of stay abroad is the fundamental feature of the environment, as is the case in East Asia or the Middle East.\footnote{It is important to emphasize that the focus of the paper is on the voluntary return decisions of undocumented aliens facing the prospect of apprehension in countries with strict deportation policies. It does not apply to lenient deportation regimes, such as the one in the U.S.A., where non-criminal illegal aliens were gradually being released in February 2013 due to the lack of funding needed to proceed with the deportations (Hamilton 2013). Voluntary return of undocumented Mexican migrants from the U.S.A. requires a conceptually different framework of analysis. See, for example, Cornelius (2005) and the survey of the literature on migration from Mexico to the U.S.A. by Hanson (2006). The International Handbook on the Economics of Migration, edited by Constant and Zimmermann (2013), examines international migration from a very broad perspective.} It is shown that in the presence of economic and/or psychological factors that pull the migrant back to her home country, the possibility of deportation unambiguously reduces the desired migration duration. Moreover, the prospect of deportation can trigger a voluntary return under conditions that would otherwise, in the absence of deportation risk, make a permanent undocumented stay in the host country optimal.
A direct implication of the analysis is that, for any given inflows of illegal immigrants, countries with stricter deportation policies host a smaller stock of undocumented aliens. This is not only because they are more active in physically removing them from their territory but also because by doing so, they indirectly induce illegal immigrants to return voluntarily sooner to their countries of origin. The optimal migration duration is shown to be decreasing and convex in the deportation rate. This implies that a tightening of the deportation policy is most effective in shortening the length of an unauthorized stay when the initial risk of apprehension is relatively low.

The fact that the risk of deportation unambiguously reduces the desired duration of stay in the host country is rather intuitive, although new to the literature. What makes this problem more intriguing and complex, is that the result stems from two distinct forces operating on an illegal immigrant’s behavior. First, the risk of being suddenly apprehended and sent back from a high-wage to a low-wage environment affects the migrant’s saving behavior. The saving rate increases, wealth accumulation accelerates, and the utility gain from staying longer in the host country diminishes more quickly over time to advance the date of voluntary return. Second, there are psychological factors that influence behavior of undocumented aliens residing in countries with strict deportation measures. Living without proper documentation and under a constant threat of apprehension restricts one’s freedom of movement and access to public and even private goods and services. It imposes, as well, severe limitations on a migrant’s social interactions. Descriptive studies suggest that under such conditions migrants tend to exhibit strong signs of homesickness. The desire to reunite with their family and to return to the social environment of the home country also affects the timing of return.

Accounting for the risk of deportation and the homesickness phenomenon allows us to obtain more realistic predictions for migration durations of undocumented workers than those arising from a deterministic model. The latter tends to overestimate the
optimal migration duration and, if used as a basis for policy prescriptions, may lead to implementation of overly restrictive and thus more costly policies.

How the economic and psychological factors individually and jointly influence the optimal duration of stay of undocumented aliens is a complex question. One of the objectives of this paper is to disentangle and evaluate these two elements. We use the data on unauthorized Thai migrants in Japan from the Sobieszczyk (2000) study to extract the intensity of homesickness which is consistent with the observed behavior, given the other (economic) factors that influence the timing of return. It is found that, on average, Thai migrants experience a loss of welfare due to family separation and social isolation during their last month abroad which is equivalent to a change in utility associated with a 53 to 89% cut in the consumption rate, depending on the assumed degree of concavity of the utility function. Finally, we use the calibrated model to predict the impact of a change in the deportation policy on return decisions of undocumented Thai migrants in Japan. A doubling of the deportation rate from its average in the late 1990s and early 2000s would have induced unauthorized migrants to return voluntarily to Thailand 1.25 - 2.3 months earlier.

The paper is organized as follows. A migrant’s optimization problem under uncertainty is presented and solved in Section 2, assuming that wages, prices, and rates of return on savings differ across countries. The solution is then illustrated with a numerical example and analyzed in Section 3. Section 4 extends the model to account for psychological and social factors influencing voluntary return, referred to as "homesickness", and calibrates the model for the case of undocumented Thai migrants in Japan. Section 5 concludes the paper with a summary of the main results.
2 The Framework

Consider an individual who migrates abroad as an undocumented alien (U for short) at time $t = 0$ with a possibility to return to her country of origin at some future date $\tau$. Due to the unauthorized status, U is subject to the risk of deportation. The length of the planning horizon is $T$. U’s initial asset holdings net of migration costs are given by $a_0 \geq 0$. At each instant in the host country, U earns the time-invariant wage $w^*$, which is assumed to be higher than the time-invariant source-country wage, $w$. The real rates of return on accumulated savings are given by $r$ and $r^*$ at home and abroad, respectively, with $r > r^*$.\(^5\) The price levels at home and abroad are constant at $p$ and $p^*$, with $p^* > p$. The subjective rate of time preference is a constant $\rho$ and the utility function is assumed to be of the CRRA form: $u(c) = \frac{c^{1-\theta}}{1-\theta}$, where $1/\theta$ is the elasticity of intertemporal consumption substitution (hereafter EICS).

2.1 Simple Two-Period Case

Let us first analyze the simple two-period version of the model in order to gain a clear understanding of how the optimal return date is chosen and the mechanisms at work. In this setting the migrant has to decide whether to return home after having spent one period abroad (at $t = 1$) or not to return after one period, in which case he is exposed to the risk of being deported with probability $\lambda$ and not being deported with probability $1 - \lambda$. Since the model has only 2 periods, it is not relevant whether the migrant returns home at $t = 2$ or stays abroad. Deportation entails a monetary penalty $\pi \geq 0$.\(^6\) The decision to return home at $t = 1$ or not is made by comparing the present value of lifetime

\(^5\)See Djajić (2010) for the analysis of an optimal return decision with interest rate differentials across countries in a deterministic setting. The same paper also provides supportive evidence for the assumption that $r > r^*$. See also footnote 16 in this paper.

\(^6\)Some countries, such as Singapore, South Korea, and numerous Middle Eastern economies impose monetary and other forms of penalties on apprehended illegal aliens as part of deportation proceedings (see Human Development Report 2009 and OECD 2002). This issue will be discussed in greater detail later in the paper.
welfare in the two scenarios: "return" or "stay" (with possibility of deportation). We shall refer to the former as $W^R$ and to the latter as $W^S$.

"Return Scenario"

The migrant’s problem is to choose consumption rates in period 1, $c_1$, and in period 2, $c_2$, such that $W^R = u(c_1) + \frac{1}{\beta} u(c_2)$, where $\beta = 1/(1 + \rho)$ denotes the discount factor, is maximized. Formally,

$$\max_{c_1, c_2} u(c_1) + \frac{1}{\beta} u(c_2), \quad \text{s.t. } c_2 = [R(a_0 R^* + w^* - p^* c_1) + w]/p,$$

where $R = 1 + r$ and $R^* = 1 + r^*$. The constraint above essentially says that the real consumption rate in the second period is equal to the home-country wage, $w$, plus interest earnings on the savings brought from abroad, $R(a_0 R^* + w^* - p^* c_1)$, adjusted by the home price level, $p$. Note that the savings brought from abroad include the initial net asset holdings, $a_0$, augmented by the gross rate of return abroad, $R^*$. We could also envisage a situation where the migrant leaves her net assets in the home country in order to benefit from the higher rate of return $R$. In this case her second-period consumption is given by $c_2 = [R(a_0 R + w^* - p^* c_1) + w]/p$. In the rest of the analysis of this subsection we shall simplify the exposition by assuming that $a_0 = 0$ (zero net initial assets) and $\beta = 1$ (no discounting). Obviously, the qualitative analysis is not affected by the elimination of these constants. We shall reintroduce them in the general framework presented in the following sections.

It is straightforward to show that the optimal consumption rates are given by

$$c_1 = \left( \frac{R^* p}{p} \right)^{-\frac{1}{\beta}} \frac{(R^* w + w)/p}{\left( \frac{R^* p}{p} \right)^{1 - \frac{1}{\beta}} + 1}, \quad c_2 = \frac{(R^* w + w)/p}{\left( \frac{R^* p}{p} \right)^{1 - \frac{1}{\beta}} + 1}. \quad (1)$$

Clearly, the second-period consumption rate is larger than the one in the first-period,
indicating that the migrant saves while abroad and consumes in excess of her home wage after return. The total lifetime welfare is obtained by substituting the optimal consumption rates into the expression for $W^R$.

"Stay-Abroad Scenario"

If the migrant does not return at $t = 1$, she is subject to deportation risk. The maximization problem reads as

$$\max_{c_1^*, c_2^*, \tilde{c}_2} u(c_1) + \frac{1}{\beta} \left\{ (1 - \lambda)u(c_2^*) + \lambda u(\tilde{c}_2) \right\},$$

s.t. $c_2^* = \left[ R^*(a_0R^* + w^* - p^*c_1^*) + w^* \right]/p^*$, $\tilde{c}_2 = \left[ R(a_0R^* + w^* - p^*c_1^*) + w - \pi \right]/p,$

where $\pi$ is the deportation penalty. Let us denote the savings abroad by $s_1 = a_0R^* + w^* - p^*c_1^*$. The Lagrangian function for this problem may be written as

$$L = u(c_1^*) + \frac{1}{\beta} \left\{ (1 - \lambda)u \left( \frac{R^*s_1 + w^*}{p^*} \right) + \lambda u \left( \frac{Rs_1 + w - \pi}{p} \right) \right\} + \mu \left( a_0R^* + w^* - p^*c_1^* - s_1 \right),$$

where $\mu$ is the Lagrange multiplier and the first-order conditions are

$$c_1^* : \quad u'(c_1^*) - \mu p^* = 0,$$

$$s_1 : \quad \frac{1 - \lambda}{\beta} u'(c_2^*) \frac{R^*}{p^*} + \frac{\lambda}{\beta} u'(\tilde{c}_2) \frac{R}{p} - \mu = 0,$$

$$\mu : \quad a_0R^* + w^* - p^*c_1^* - s_1 = 0.$$

Combining these conditions to eliminate $\mu$ and setting $a_0 = 0$ and $\beta = 1$, for simplicity, yields an implicit solution for $c_1^*$:

$$\left( 1 - \lambda \right) \left[ \frac{R^*(w^* - p^*c_1^*) + w^*}{p^*} \right]^{-\theta} R^* + \lambda \left[ \frac{R(w^* - p^*c_1^*) + w - \pi}{p} \right]^{-\theta} \frac{Rp^*}{p} = c_1^{* - \theta}. \quad (2)$$

The consumption rates in the second period ($c_2^*$ and $\tilde{c}_2$) can be found by substituting
the solution for $c_1^*$ into the two constraints. And finally the expected total welfare can
be found by substituting the optimal consumption rates into the expression for $W^S$. In
the case of logarithmic utility the solution can be found explicitly. However, an implicit
solution is sufficient for our further analysis.

"The Optimal Return Decision"

The migrant will choose to return back home at $t = 1$ if and only if $W^R \geq W^S$,
otherwise she will stay in the host country. Consider a situation in which the migrant
is exactly indifferent between staying or returning, $W^R = W^S$, and let us examine the
impact of various parameters of the model on the decision to return. In other words, any
parameter which makes $W^R$ increase relative to $W^S$ will induce the migrant to return
and, vice versa, any parameter which makes $W^R$ fall relative to $W^S$ will induce the
migrant to stay abroad. Specifically, we are interested in the effects of two parameters
describing the immigration policy – the deportation rate ($\lambda$) and the penalty for overstay
($\pi$); and the two parameters representing economic pull-back factors – the foreign price
level ($p^*$) and the home real rate of return on savings ($R$). Define $D \equiv W^R - W^S$ as
the expected welfare difference and totally differentiate with respect to our parameters
of interest.

It is useful in the first step to find the effects on $c_1$, $c_2$ and $c_1^*$ by totally differentiating
Eqs. (1) and (2).

Comparative Statics for $c_1$ and $c_2$:

Since neither $c_1$ nor $c_2$ depend on $\lambda$ or $\pi$, we need to consider only the effects of $p^*$
and $R$:

$$
p^* : \quad \frac{dc_1}{dp^*} = -c_1 \frac{R}{p} \left[ \frac{1}{\bar{\theta}} \left( \frac{Rp^*}{p} \right)^\frac{1}{\bar{\theta}} - 1 \right] \left[ \left( \frac{Rp^*}{p} \right)^\frac{1}{\bar{\theta}} + \frac{Rp^*}{p} \right]^{-1} < 0, \quad \frac{dc_2}{dp^*} = c_2 \frac{R}{p} \left( \frac{1}{\bar{\theta}} - 1 \right) \left[ \left( \frac{Rp^*}{p} \right)^\frac{1}{\bar{\theta}} + \frac{Rp^*}{p} \right]^{-1} \geq 0 \Leftrightarrow \frac{1}{\bar{\theta}} \geq 1,
\"
\[
R : \quad \frac{dc_1}{dR} = - \frac{w^* \left( \frac{R_p^*}{p} \right)^{\frac{1}{g}} \left( \frac{1}{g} - 1 \right) + w \left[ \frac{1}{p^m} \left( \frac{R_p^*}{p} \right)^{\frac{1}{g}} + \frac{p^*}{p} \right]}{p \left( \left( \frac{R_p^*}{p} \right)^{\frac{1}{g}} + \frac{R_p^*}{p} \right)^2} \geq 0,
\]
\[
dc_2 \quad \frac{d}{dR} = \frac{w^* \left[ 1 + \frac{1}{g} \left( \frac{R_p^*}{p} \right)^{-\frac{1}{g}} \right] + w \left( \frac{1}{g} - 1 \right) \left( \frac{R_p^*}{p} \right)^{-\frac{1}{g}}}{p \left( \left( \frac{R_p^*}{p} \right)^{\frac{1}{g}} + \frac{R_p^*}{p} \right)^2} \geq 0
\]

Clearly, the foreign price level has a negative effect on the first-period consumption, while the effect on the second-period consumption hinges on the elasticity of intertemporal consumption substitution (EICS). If EICS is relatively high \( \left( \frac{1}{g} > 1 \right) \), then the migrant is more easily willing to reallocate her consumption from the first-period to the second. In this case a cut in \( c_1 \) following an increase in \( p^* \) is such that the generated savings actually result in an increase in \( c_2 \). If EICS is relatively low \( \left( \frac{1}{g} < 1 \right) \), the migrant prefers a relatively smoother time profile of consumption and so both \( c_1 \) and \( c_2 \) decline. A similar reasoning applies to a change in the rate of return on savings. If EICS is relatively high, \( c_1 \) declines and \( c_2 \) increases.

**Comparative Statics for \( c_1^* \):**

\[
\lambda : \quad \frac{dc_1^*}{d\lambda} = - \frac{\tilde{c}_2^{-\theta} R_p^* - c_2^{-\theta} R^*}{p \Delta_{cs}} < 0,
\]
\[
\pi : \quad \frac{dc_1^*}{d\pi} = - \frac{\theta R_p^* \tilde{c}_2^{-\theta - 1}}{p^2 \Delta_{cs}} < 0,
\]
\[
p^* : \quad \frac{dc_1^*}{dp^*} = - \frac{(1 - \lambda) R_p^* \tilde{c}_2^{-\theta - 1} R^* w^* + \lambda R_p^* \tilde{c}_2^{-\theta - 1} (\tilde{c}_2 + \theta R_p^* \tilde{c}_2)}{\Delta_{cs}} < 0,
\]
\[
R : \quad \frac{dc_1^*}{dR} = - \frac{\lambda \tilde{c}_2^{-\theta - 1} p^* [R_s (1 - \theta) + w - \pi]}{\Delta_{cs}} \geq 0,
\]

where \( \Delta_{cs} = \theta \left( (1 - \lambda) R_p^* \tilde{c}_2^{-\theta - 1} R^* + \lambda \left( \frac{R_p^*}{p} \right)^2 \tilde{c}_2^{-\theta - 1} + \tilde{c}_1^{-\theta - 1} \right) > 0 \). As expected, an increase in either \( \lambda, \pi \) or \( p^* \) decreases the first-period consumption and thus increases savings while the migrant is abroad. The effect of the home interest rate is in general
ambiguous. For $1/\theta \geq 1$, however, it is unambiguously negative. For $1/\theta < 1$ the effect might be reversed, so that an increase in the home rate of return induces an increase in the consumption rate abroad. This is akin to the income effect: Since the migrant anticipates a possible deportation and knows that the home return on assets is larger than abroad, she will try to smooth her consumption intertemporally by optimally increasing her current consumption in anticipation of a higher return on her savings in case she happens to be deported. This is true, however, only if the elasticity of intertemporal consumption substitution is low enough, i.e. less than unity ($1/\theta < 1$). If the elasticity of intertemporal consumption substitution is high enough ($1/\theta \geq 1$), then the migrant is more easily willing to accept a relatively low current consumption in exchange for a possibility to have a higher consumption in the future. Let us assume for the rest of the exposition that EICS is large enough and for the sake of clarity we shall only focus on the case $1/\theta \geq 1$.

**Comparative Statics for $D$:**

It is useful at this stage to recall that the expected welfare maximization leads to condition (2) which implies that the term \( \left\{ u'(c_1^*) + (1 - \lambda)u'(\tilde{c}_2) \frac{dc_2^*}{dc_1^*} + \lambda u'\tilde{c}_2) \frac{d\tilde{c}_2}{dc_1^*} \right\} \) is equal to zero. The effects on the welfare difference under "return" and "stay" scenarios are summarized below:

$$
\lambda : \quad \frac{dD}{d\lambda} = - \frac{dW^S}{d\lambda} = - \left\{ u'(c_1^*) + (1 - \lambda)u'(\tilde{c}_2) \frac{dc_2^*}{dc_1^*} + \lambda u'\tilde{c}_2) \frac{d\tilde{c}_2}{dc_1^*} \right\} \frac{dc_1^*}{d\lambda} - \frac{c_2^{1-\theta} - \tilde{c}_2^{1-\theta}}{1 - \theta} = \\
= \frac{c_2^{1-\theta} - \tilde{c}_2^{1-\theta}}{1 - \theta} = u(c_2^*) - u(\tilde{c}_2) > 0,
$$

$$
\pi : \quad \frac{dD}{d\pi} = - \frac{dW^S}{d\pi} = - \left\{ u'(c_1^*) + (1 - \lambda)u'(\tilde{c}_2) \frac{dc_2^*}{dc_1^*} + \lambda u'(\tilde{c}_2) \frac{d\tilde{c}_2}{dc_1^*} \right\} \frac{dc_1^*}{d\pi} - \lambda u'\tilde{c}_2) \frac{d\tilde{c}_2}{d\pi} = \\
= \frac{\lambda u'(\tilde{c}_2)}{p \left( \frac{R^e \rho'}{p} \right)^{1 - \hat{\gamma}} + 1} > 0,
$$

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\[ p^* : \frac{dD}{dp^*} = \frac{dW^R}{dp^*} - \frac{dW^S}{dp^*} = 1 - \frac{\theta}{p^*}[u(c_1^*) - u(c_1)] > 0, \]
\[ R : \frac{dD}{dR} = \frac{dW^R}{dR} - \frac{dW^S}{dR} = u'(c_2) \frac{w^* - p^* c_1}{p} - \lambda u'(\tilde{c}_2) \frac{w^* - p^* c_1}{p} > 0. \]

We find that an increase in the deportation rate leads to a decrease in the expected welfare under the "stay" option (while \( W^R \) is not affected) and thus to an increase in \( D \). Similarly, an increase in the overstay penalty leads to a decrease in \( W^S \) (and has no effect on \( W^R \)) and thus makes it less attractive to remain abroad. An increase in the foreign price level decreases both \( W^S \) and \( W^R \), although the fall in the former is larger than in the latter. This is because under the "return" scenario the migrant faces only the home price level in the second period, while under the "stay" scenario she may happen to remain abroad for another period (with probability \( 1 - \lambda \)) and therefore she may be exposed to the higher foreign price level. Thus the overall effect on \( D \) is positive. In spite of the ambiguities in the effect of the home real interest rate on the consumption rates, the overall effect on the welfare difference is unambiguously positive. Under "return" the migrant is exposed to \( R \) in the second period with probability one but under "stay" only with probability \( \lambda < 1 \). Therefore a higher \( R \) increases the attractiveness of the "return" over the "stay" option.

### 2.2 Continuous-Time Framework

In this and the following sections time is continuous and we assume that \( U \) may be deported to her country of origin at any moment due to her unauthorized status in the host country. The event of deportation is assumed to follow the Poisson process with a constant mean arrival rate \( \lambda \). As before, deportation entails a monetary penalty \( \pi \geq 0 \). \( U \)'s optimization problem consists of maximizing the expected lifetime welfare by choosing the consumption rate abroad, \( c^*_1 \), the consumption rate after return, \( c_2 \), if return
is voluntary, or $\tilde{c}_t$, if return occurs due to deportation. The optimal date of voluntary return to the source country, $\tau$, must also be chosen in the event that deportation does not occur before $\tau$. The length of the planning horizon is $T$.

Let $a_t$ denote U’s asset position at time $t$ and $\dot{a}_t$ its time derivative. The dynamic budget constraint while abroad is then given by

$$\dot{a}_t = r^*a_t + w^* - p^*\tilde{c}_t^*, \quad a_0 \text{ given,}$$

(3)

and after return

$$\dot{a}_t = ra_t + w - p\bar{c}_t, \quad a_T = 0 \text{ (if voluntary return),}$$

(4)

$$\dot{a}_t = ra_t + w - p\tilde{c}_t, \quad a_T = 0 \text{ (if deportation),}$$

(5)

If $U$ returns voluntarily to the source country at some date $\tau$, she brings back savings equal to $a_{\tau}$. If she is deported by the immigration authorities at date $\varepsilon$, she brings back only $a_\varepsilon - \pi$ after having paid the deportation penalty.\(^7\)

In this framework, an illegal immigrant faces a variant of an optimal stopping problem which can be analyzed in the following way. At each instant $U$ must decide whether to return voluntarily to her country of origin, i.e., to stop the ongoing program and receive the termination payoff, or to continue the program for an extra instant and stop then. Continuation of the program for an extra instant may result in deportation with probability $\lambda dt$. The termination payoff in case of voluntary return is denoted by $\Omega_t$ and it is a function of the state variable - the asset position, $a_t$. If, on the contrary, $U$ decides

\(^7\)The model implicitly assumes that if by time $\varepsilon U$ has not accumulated sufficient amount of savings to pay for the penalty, she will be repaying the difference $\pi - a_\varepsilon$ from her income back home, having chosen her consumption-saving plan accordingly. In reality, deportation penalties are not prohibitively high, so that undocumented migrants typically have enough savings to cover the penalty. Alternatively, they can, in some countries, choose to serve a jail term instead of paying the fine. The penalty $\pi$ in the model can thus be also interpreted as an opportunity cost of being in jail in terms of forgone income.
to remain abroad, her expected payoff is $EW_t$, which is also a function of $a_t$. Thus at each instant $t \in (0, T]$, $U$ must compare the termination payoff $\Omega(a_t)$ with $EW(a_t)$ and if $\Omega(a_t) \geq EW(a_t)$, $U$ returns home voluntarily, otherwise she remains abroad until at least the next instant. Thus the original problem of choosing the optimal return date includes two subproblems. One is to optimally choose the consumption rate after return, $c_t$, so as to maximize $\Omega_t$ for a given return date. The other is to optimally choose $c_t^*$ and $\bar{c}_t$ to maximize $EW_t$. Once the optimal $\Omega_t$ and $EW_t$ are computed as functions of the return date, it is straightforward to find the optimal point of return, $\tau$, that sets $EW_{\tau} = \Omega_{\tau}$. A rigorous analysis is presented below in three steps: first, we obtain the maximized value of the termination payoff; second, we obtain the maximized expected continuation payoff; third, we compare the payoffs at each point in time to determine the optimal $\tau$.

**Termination Payoff**

As the first step, consider the subproblem associated with choosing the optimal paths of consumption and asset holdings in case of a voluntary return at some date $\tau \in (0, T]$:

$$\max_{c_t} \int_{\tau}^{T} u(c_t) e^{-\rho(t-\tau)} dt$$

subject to the constraint (4). The solution for the optimal $c_t$ is obtained in a straightforward manner using the standard dynamic optimization technique:\(^8\)

$$c_t = c_{\tau} e^{-\frac{\rho}{\tau}(t-\tau)}, \quad c_{\tau} = \frac{1}{p} \left[ a_\tau + w \left( 1 - \frac{e^{-r(T-\tau)}}{r} \right) \right] \frac{g}{e^{g(T-\tau)} - 1}, \quad t \in [\tau, T], \quad (6)$$

where $g \equiv \frac{r - \rho}{\rho} - r$. The optimal consumption rate follows the standard Keynes-Ramsey rule such that its growth rate is equal to the difference between the rate of interest, $r$, and the rate of time preference, $\rho$, adjusted by the elasticity of intertemporal consumption

\(^8\)The derivations of all the equations that follow are relegated to the Appendix.
substitution, $1/\theta$. An interest rate greater than the time-preference rate stimulates saving and therefore implies a positive consumption growth rate. The opposite is true when $r < p$. The starting point of the path, $c_\tau$, is increasing in the stock of assets accumulated up to time $\tau$, $a_\tau$, and in the home-country wage, $w$. It is obviously decreasing in the price level, $p$. The termination payoff is equal to

$$\Omega(a_\tau) = u(c_\tau) \frac{e^{\theta(T-\tau)} - 1}{g}.$$  \hfill (7)

**Expected Continuation Payoff**

Consider the second subproblem associated with the choice of the optimal paths of consumption and asset holdings while $U$ is abroad. Facing the risk of deportation, $U$ is confronted with a stochastic optimization problem, which can be analyzed with the aid of the Hamilton-Jacobi-Bellman equation (HJB). Denoting the value function while abroad by $\tilde{V}$ and the value function after deportation by $\tilde{\tilde{V}}$, the HJB equation may be written as

$$\rho V(a_\hat{t}) = \max_{c_\hat{t}} \left\{ u(c_\hat{t}^*) + \frac{\partial V(a_\hat{t})}{\partial a_\hat{t}} (r^* a_\hat{t} + w^* - p^* c_\hat{t}^*) + \lambda \left( \tilde{V}(a_\hat{t}) - V(a_\hat{t}) \right) \right\}.$$  \hfill (8)

The optimality conditions with respect to $c_\hat{t}^*$ and $a_\hat{t}$ yield the following differential equation describing the growth rate of consumption while abroad:

$$\frac{\dot{c}_\hat{t}}{c_\hat{t}^*} = \frac{1}{\theta} \left\{ \lambda \left[ \frac{p^*}{p} \left( \frac{\dot{c}_\hat{t}}{c_\hat{t}^*} \right)^{-\theta} - 1 \right] + r^* - \rho \right\}.$$  \hfill (9)

Note that if there is no uncertainty, i.e., $\lambda = 0$, the first term in the curly brackets in eq. (9) vanishes and the usual Keynes-Ramsey equation for the consumption growth rate applies. Furthermore, it is easy to see that the term in the square brackets is unambiguously positive, so that the presence of uncertainty results in a higher consumption
growth rate relative to the certainty case. This higher growth rate is supported by a high saving rate at the beginning of the planning horizon, implying that uncertainty triggers precautionary saving. Eq. (9), however, is not sufficient to characterize the path of $c_t^*$, since $\tilde{c}_t$ is yet to be determined. The optimal $\tilde{c}_t$ can be easily obtained by noting that if deportation occurs at some date $\varepsilon$, U’s optimal consumption in the source country must be such that it maximizes

$$
\int_{\varepsilon}^{T} u(\tilde{c}_t)e^{-\rho(t-\varepsilon)}dt
$$

subject to (5) and the initial condition for asset holdings given by $a_\varepsilon - \pi$, i.e., the amount of assets accumulated up to time $\varepsilon$ net of the deportation penalty. The solution to this deterministic control problem is given by

$$
\tilde{c}_t = \tilde{c}_e e^{-\frac{\pi e^{r(T-\varepsilon)}}{p}}, \quad \tilde{c}_e = \frac{1}{p} \left[ a_\varepsilon - \pi + w \frac{1 - e^{-r(T-\varepsilon)}}{r} \right] \frac{g}{e^{g(T-\varepsilon)} - 1}, \quad t \in [\varepsilon, T],
$$

and the maximized value of (10) is

$$
\tilde{V}(a_\varepsilon) = u(\tilde{c}_e) \frac{e^{g(T-\varepsilon)} - 1}{g},
$$

The optimal paths of consumption and asset holdings pertaining to the problem in (8) can thus be fully characterized by eqs. (3), (5), (9), and (11). The expected continuation payoff at instant $\xi \in (0, T]$ is obtained as

$$
EW_\xi = (1 - \lambda d\xi) \left\{ u(c_\xi^*) + \int_{\xi}^{T} u(c_t)e^{-\rho(t-\xi)}dt \right\} + \lambda d\xi \left\{ \int_{\xi}^{T} u(\tilde{c}_t)e^{-\rho(t-\xi)}dt \right\},
$$

where $\xi$ is the next instant after $\xi$. The interpretation of (12) is the following. If

---

9Note that even if $\rho$ exceeds $r^*$ the consumption growth rate may be positive due to the deportation risk.
10In writing (12) I make use of the memorylessness property of the Poisson process. The analysis can be extended to random processes with per-unit-time probability of success being a function of the elapsed time.
the migrant decides to remain abroad for an extra instant, i.e., from $\xi$ to $\zeta$, and return home at $\zeta$, her expected welfare will consist of two terms. The first term is the present discounted value (PDV) of lifetime welfare in the event that $U$ is not deported between $\xi$ and $\zeta$, with probability $1 - \lambda d \xi$. It consists of the utility from consumption at time $\xi$ plus the present value of utility from consumption after voluntary return, from time $\zeta$ to $T$. The second term is the PDV of lifetime welfare in the event of deportation during this extra instant, which occurs with probability $\lambda d \xi$.

**The Optimal Return Date**

In the third and the final step, the difference, $D_t$, between $EW_t$ and $\Omega_t$ is computed at each instant $t \in (0, T]$, and the optimal date of return to the source country is determined as

$$\tau = \inf \{ t \in (0, T] \mid D_t = EW_t - \Omega_t = 0 \},$$

that is, the earliest date such that the termination payoff is equal to the continuation payoff (see, e.g., the value matching condition in Dixit and Pindyck, 1994).

Although the model cannot be solved in closed form, we can nonetheless examine the impact of the key parameters of interest on the optimal return date. Clearly, the parameters which increase the termination payoff relative to the continuation payoff contribute to a shorter stay abroad. Let us focus on $\lambda$, $\pi$, $p^*$, and $r$, as in the model of Section 2.1. The first two ($\lambda$ and $\pi$) affect only $EW_t$ but not $\Omega_t$. Both increase the cost relative to the benefit of staying an extra instant abroad, making it optimal to return earlier to the source country. A higher foreign price level, $p^*$, also reduces the attractiveness of staying longer abroad, as does an increase in the rate of return $r$ available on investments at home. Both make it optimal for migrants to shorten their foreign stay.

\[\text{This complication, however, produces a quite predictable outcome: If the probability (per unit of time) of being deported is an increasing (diminishing) function of the length of stay in the foreign country, the optimal migration duration is shorter (longer).}\]
Since an explicit analytical solution is unfortunately not feasible for this type of an optimal stopping problem, we shall resort to numerical methods. They will allow us to visualize the optimal time profiles of consumption and asset position, as well as the termination and continuation payoffs. We shall initially focus on two economic factors which draw migrants back to their country of origin, namely the international price and return-on-investment differentials. This analysis is useful for understanding the mechanics of the model and, in particular, the effect of uncertainty on the optimal choice of migration duration. In Section 4 the model is calibrated to match the data on illegal Thai migrants in Japan in the late 1990s. The calibration is used to evaluate the effectiveness of deportations as an immigration policy instrument in Japan.

3 Numerical Solution

There is very little systematic evidence available on the behavior of and the conditions facing unauthorized workers in the economies with strict deportation measures. In order to calibrate the model we draw on various sources of evidence on undocumented migration in East Asia. The key parameters to calibrate include the wage ratio between the host and the source country ($w^*/w$) and the deportation rate ($\lambda$). For the case of illegal Chinese migrants in South Korea, Kim (2004, p.326) writes that "...ethnic Korean Chinese reported that their average income equaled roughly one million won per month (about $800). This is six to seven times higher than the average income in Yanbian, which is about $110 per month." On the basis of information provided by Jones and Pardthaisong (1999, p. 45 and Table 6), we can calculate that $w^*/w$ of undocumented Thai migrants in Japan was in some instances as high as twenty. A survey of undocumented Filipino workers in South Korea, which compares their wages at home
and abroad, reports a wage ratio of eight.\footnote{Ignacio-esteban, Ma. Angelina. "The Dynamics of Illegal Migration: The Philippines-South Korea Case," available at http://www.filipiniana.net/publication/the-dynamics-of-illegal-migration-the-philippines-south-korea-case/13197759176833/8/1} On the basis of these and other sources of evidence\footnote{See Human Development Report (2009, p. 50), Djajić (2013), and "Illegal status in Malaysia worth the risk, migrant workers claim" available at http://mnnnews.org/?p=751}, we calibrate the international wage differential to ten by normalizing the monthly source-country wage, $w$, to unity and setting the host-country wage for undocumented workers to $w^* = 10$.

We assume that the deportation penalty, $\pi$, is initially zero and subsequently examine the implications of relaxing this assumption. The length of the planning horizon is 50 years to reflect a situation facing a young migrant in her early twenties. The benchmark value of the coefficient of relative risk aversion, $\theta$, is set at 0.9 and the sensitivity of the results to variations in $\theta$ is checked later in the paper.\footnote{Given the assumed structure of preferences, the coefficient of relative risk aversion, $\theta$, is equal to the inverse of the elasticity of intertemporal consumption substitution (EICS). Estimates of $\theta$ vary significantly, depending on the data used and the empirical strategy. See, e.g. Chetty (2006), who reports that the mean estimate in the literature is $\theta = 0.71$. He also notes that studies which combine the benefits of exogenous variation with the structural lifecycle approach, such as Blundell, Duncan, and Meghir (1998), estimate $\theta$ of around 0.93 and provide perhaps the most credible microeconomic estimates. See also Vissing-Jørgensen (2002).} The risk-free interest rate in the host country is assumed to be 3% per year and, for simplicity, is set equal to the migrant’s rate of time preference. We set the Poisson arrival rate $\lambda = 0.25$ per year, which implies that $U$ may be deported, on average, once in 4 years (by the property of the Poisson process). This corresponds, for example, to the case of Malaysia where the stock of undocumented Indonesian migrants is estimated to be 450’000 and 10’000 are deported every month (OECD 2002, p.254). These figures imply a deportation rate of 0.26 per year.\footnote{In Japan, the stock of illegal aliens in 2005 was estimated at 193’745 with 33’192 deportations, making the deportation rate roughly 0.17 per year (Vogt 2007). Kibria (2004, p.12) reports that the average duration of stay of deported Bangladeshi migrants was 2.7 years, implying a deportation rate of 0.37 per year.}

We first illustrate the implications of the model when there are only two motives for return, both of which have been emphasized in the existing literature on temporary


<table>
<thead>
<tr>
<th>planning horizon, years</th>
<th>$T$</th>
<th>50</th>
</tr>
</thead>
<tbody>
<tr>
<td>wage differential</td>
<td>$w^*/w$</td>
<td>10</td>
</tr>
<tr>
<td>risk-free interest rate</td>
<td>$r^*$</td>
<td>0.03</td>
</tr>
<tr>
<td>rate of time preference</td>
<td>$\rho$</td>
<td>0.03</td>
</tr>
<tr>
<td>Poisson deportation rate</td>
<td>$\lambda$</td>
<td>0.25</td>
</tr>
<tr>
<td>elasticity of marginal utility</td>
<td>$\theta$</td>
<td>0.9</td>
</tr>
<tr>
<td>initial assets net of migration cost</td>
<td>$a_0$</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 1: Benchmark calibration.

migration: (i) the international differential in the rates of return on accumulated assets and (ii) the differential in the price levels.\textsuperscript{15} The first simulation looks at the effect of the former and subsequently we look at the effect of the latter. We also examine the implications of a deportation penalty. Later in Section 4, we formally introduce into the model a psychological factor which can motivate a voluntary return. This will complete our analysis of the economic and social considerations influencing return decisions of undocumented foreign workers facing the prospect of deportation.

\subsection*{3.1 Rate of Return Differential}

While $r^*$ and $\rho$ are assumed to be 3\% per year, the interest rate in the source country is set at 10\% per year in the benchmark case.\textsuperscript{16} The price levels in both countries are

\textsuperscript{15}As we are considering a population of unskilled undocumented workers, the return motive based on the intention to capitalize on the skill acquisition abroad is not likely to be relevant in the present setting.

\textsuperscript{16}As mentioned in the introductory section, migrants typically seek to accumulate savings while abroad in order to improve their standard of living back home or start up a business or both. Recent empirical studies estimate rates of return for small enterprises in developing countries to be substantial, so that the assumed figure of 10\% should be interpreted as a lower bound. Udny and Anagol (2006), for example, report that Ghanaian farmers who employ new technology for pineapple cultivation enjoy average returns of 250\% per annum on median-sized plots; while for farmers employing traditional technology the average returns are 30 - 50\% annually. McKenzie and Woodruff (2006) estimate monthly returns in Mexican microenterprises to be in the range of 15\% (controlling for entrepreneurial ability) for investments below $200 and 3 - 5\% for investments above $500. The authors also note that only a small fraction (0.3\%) of firms in their sample obtained start-up financing from a formal financial institution, suggesting the importance of informal financing, which often includes remittances and savings repatriated by migrants returning from the U.S.A. It is also important to note that temporary migrants typically start their entrepreneurial activities only after return. Their physical
initially set equal to each other and normalized to unity for the purpose of isolating the effect of the interest differential. Figure 1 plots the value of \( D \), the difference between continuation and termination payoffs for the benchmark case (bold line) and for several other values of the rate of return on investment in the home country (thin lines). As has been shown in eq. (13), the optimal return date is the earliest date such that \( D = 0 \). For the chosen parameter values, the optimal \( \tau \) under the benchmark calibration is 15.82 years after arrival to the host country. For a low enough interest rate differential, e.g., \( r = 6\% \), it is in fact not optimal to return since the possibility of earning a ten times higher wage abroad yields a higher expected continuation payoff for all \( t \). By contrast, when the rate of return differential is relatively high, as shown by the thin line labeled \( r = 0.15 \), the date of return is advanced to 8.35 years. In general, an increase in the difference between \( r \) and \( r^* \) brings forward the optimal date of voluntary return as the incentive to go back home with accumulated savings is stronger.

The importance of the return-on-investment differential as a factor influencing vol-

Figure 1: Difference between continuation and termination payoffs: interest rate differential.
untary return can be better understood by comparing the evolution of the asset position at home (i.e., after return) and abroad (i.e., if the migrant suboptimally chooses not to return at the optimal $\tau$ and remains permanently in the host country continuing to face deportation risk). This is illustrated in figure 2 for the benchmark calibration. The time

![Graph](image-url)

**Figure 2:** The assets path and the optimal return date.

profile of asset holdings over the life cycle exhibits the usual hump-shaped pattern. Under the voluntary return scenario (thick solid line), however, the asset position is larger at each point in time for $t \in [15.82, 50]$, as compared to the suboptimal "stay-abroad scenario" (dashed line), due to the difference in the rates of return on savings.\(^\text{17}\) The thin solid line shows the evolution of U’s asset position if she remains permanently in the source country. Comparing the thick solid and the thin solid schedules we clearly see that migration, with the possibility of earning a higher foreign wage, entails larger asset holdings at each point in time. Even if U happens to be deported before her optimal return date, i.e., at some $t \in (0, \tau]$, she will arrive home with a larger stock of assets

\(^{17}\)With a 7% interest difference and repatriated assets of 1508.6, the interest earnings plus the home-country wage exceed the flow of income available abroad.
than the one she would have accumulated had she chosen not to migrate.

The evolution of asset holdings of a "voluntary" returnee is linked to her optimal consumption path, which is shown in figure 3 by the thick solid line. The dashed line represents the time profile of consumption in the host country, \( c_t^* \). It overlaps with the solid line for \( t \in [0, \tau] \). The dotted line shows \( c_\tau, \tau \in [0, T] \), i.e., the initial consumption rate in the home country if voluntary return occurs at \( t = \tau \) (note that the dotted line is not the time profile of \( c_t \)).

At the point of voluntary return, there is a downward jump in consumption rate from \( c_t^* \) to \( c_\tau \). Subsequently, \( c_t \) grows along the solid line at a positive rate \( \frac{r-\rho}{\sigma} \) (see eq (6)) and eventually crosses the time path of \( c_t^* \), as illustrated in the figure. Return to an environment where \( r > \rho \) from the one where \( r^* = \rho \), tilts the time profile of consumption counterclockwise. This explains both the drop in consumption at the point of return and the subsequent increase in the consumption rates above those that could be realized abroad. If, instead, the interest rate in the home country were

\[ \frac{r-\rho}{\sigma} \]

Figure 3: The optimal path of consumption.

---

18 Under the current calibration with \( \pi = 0 \), the time path of \( \hat{c}_t \) coincides with the time path of \( c_t \) and therefore the dotted line also represents \( \hat{c}_\tau, \tau \in [0, T] \).
identical to \( \rho \) (and hence to \( r^* \)), the time profile of \( c_t \) would be flat, although in this case it would not be optimal to return voluntarily under the current calibration.

### 3.2 Deportation Penalty

So far we have assumed that the deportation penalty is equal to zero. In many East Asian economies, as well as in the GCC States, both monetary and non-pecuniary penalties imposed on apprehended unauthorized migrants can be substantial. For instance, under the Immigration Act of Singapore the penalties for overstaying or illegal entry are a jail term of up to six months plus a minimum of three strokes of the cane (Singapore Immigration Act (133)). In United Arab Emirates a penalty charge of Dhs. 25 (US$ 7) - Dhs. 100 (US$ 28) per day is imposed on visitors who remain within the territory beyond the officially authorized duration of stay. If the overstay extends to a significant amount of time, then a court hearing is issued and the judge decides what penalties to impose.\(^\text{19}\) In Saudi Arabia, individuals who overstay their visit in the Kingdom are subject to a fine of 10'000 Saudi Riyals (or $2'667) and incarceration pending deportation proceedings.\(^\text{20}\) According to Malaysian Immigration Department website, "Section 15 (4) of the Immigration Act 1959/63 (Act 155) provides a fine of not less than RM10'000 or imprisonment not exceeding 5 years or both."\(^\text{21}\)

For our purpose of examining the role of penalties that accompany deportations, let us assume that they can be captured by a monetary fine. Introducing such a penalty unambiguously advances the date of voluntary return. Having to pay the fine in the case of apprehension constitutes an additional cost of remaining abroad and thus reduces the continuation payoff. With \( \pi \) set equal to six months of host-country income (\( \pi = 60 \)), the optimal \( \tau \) is reduced by 1.5 years in the benchmark calibration.

\(^\text{19}\)http://www.dubai-airport.info/visa.html
\(^\text{20}\)http://travel.state.gov/travel/cis_pa_tw/cis/cis_1012.html\#page
3.3 Price Differential

Another important factor that draws migrants back to the home country is the possibility of consuming commodities at lower prices than in the host country (Djajić 1989, Kirdar 2013). For the purpose of isolating the effect of the price differential on the decision to return, I set $r = r^* = \rho = 3\%$, normalize $p$ to unity and let $p^*$ lie in the range between 1 and 5, while keeping the values of other parameters unchanged. Figure 4 shows the optimal return date, $\tau$, as a function of the price differential, $p^*/p$, for various values of the risk aversion parameter $\theta$, with the benchmark calibration ($\theta = 0.9$) shown by the bold curve. Clearly, the larger the price differential, the shorter the desired duration of stay in the host country: Higher cost of consumption ($p^*$), for a given $w^*$, reduces the attractiveness of staying abroad. It follows that undocumented migrants from countries with a relatively low cost of living will choose to return home sooner than migrants from countries with a comparatively higher cost of living, all else equal. Moreover, for any given $\theta$ there exists a threshold level of $p^*/p$ below which it is not optimal to return. For instance, under the benchmark calibration of $\theta = 0.9$ and $\lambda = 0.25$, if the foreign price

![Figure 4: The optimal return and the price level differential.](image-url)
level is only about 30% higher than at home, there is no interior solution for the optimal voluntary return date, i.e., it is not optimal to return if \( p^* \) is sufficiently low in relation to \( p \), given the wage differential.\(^{22}\)

3.4 Role of Uncertainty

One of the key points of this paper is to show that the risk of deportation has important implications, not only for the timing, but also for the occurrence of a voluntary return. A risky environment may trigger return under conditions that would otherwise, in the absence of deportation risk, result in the optimality of a permanent stay abroad. In the section "Returning in Anticipation of Deportation" Klibia (2004, p.20) writes: "There were also accounts of returning to Bangladesh in response to anticipated trouble with the official authorities abroad...Such anticipation was often linked to the migrant’s undocumented status. But it also reflected changes in the political environment, such as a crackdown on foreign workers which could motivate undocumented workers who might otherwise have remained in the receiving country [emphasis added] to return home." Our framework is capable of rationalizing such behavior of undocumented Bangladeshi migrants. If uncertainty is completely absent, i.e., \( \lambda = 0 \), \( U \) may prefer to stay permanently abroad under certain market conditions, but would choose to return to the country of origin for a sufficiently high value of \( \lambda \). For instance, if \( p^*/p \leq 2.35 \) (with \( r = r^* = 3\% \)), it is not optimal to return when there is no deportation risk but it is optimal to do so after 26.36 years when the deportation rate is set at the benchmark value \( \lambda = 0.25 \). Alternatively, in our benchmark calibration with only interest rate differential, the optimal return occurs at \( \tau = 15.82 \) (see figure 2). However, if \( \lambda = 0 \) the solution for the optimal

\(^{22}\)Note that we are assuming constant price levels at home and abroad. In an inflationary environment the implications of our model would not be affected if nominal exchange rate adjustments were to offset price level changes so as to keep the purchasing power of foreign earnings intact in the goods market of the source country. I am assuming this to be the case.
\( \tau \) does not exist, i.e., it is optimal to remain permanently in the host country.

More generally, a higher deportation arrival rate, \( \lambda \), encourages earlier voluntary return. The reason is that a higher \( \lambda \) entails more intensive precautionary saving and hence a larger asset position at each point in time. Larger asset holdings, in turn, induce migrants to return home sooner in order to take advantage of the relatively more favorable market opportunities in the source country, be they in the capital market in the form of a higher rate of return on savings or in the commodity market in the form of lower prices. Figure 5 provides a more general illustration. It shows the optimal return date, \( \tau \), plotted on the vertical axis as a function of \( \lambda \). Clearly, a higher probability of deportation advances the date of voluntary return, although at a lower incremental rate. The convexity of the optimal return date in the deportation rate has important implications for immigration policy: A policy of increasing \( \lambda \), may quickly lose its potency in terms of creating a stronger incentive for undocumented aliens to leave the host country sooner. In addition, the figure shows that beyond a certain value of \( \lambda \) stricter deportation measures become almost totally ineffective in building on the incentives for voluntary

![Optimal return and the deportation rate](image-url)

**Figure 5:** The optimal return and the Poisson arrival rate.
return. We shall be more specific on this point in the next section when we consider the case of deportation policy in Japan.

Overall, the model predicts that when an undocumented migrant follows her optimal program over a given planning horizon, she chooses to return \textit{voluntarily} to her country of origin at some point in time if the pull-back factors are sufficiently strong and the risk of deportation sufficiently high. Otherwise, she will attempt to remain permanently abroad as an undocumented alien. Migrants from economies with a relatively low cost of living and attractive investment opportunities will choose a shorter duration of undocumented stay abroad.

4 Role of Social Factors

4.1 The Evidence

Evidence gathered on the basis of interviews with return migrants suggests that they do not derive utility only from consuming goods and services but also from \textit{where} and \textit{with whom} their consumption takes place. Many undocumented migrants explicitly mention being homesick and missing their families as a reason for not staying abroad for a longer period of time. As reported by Businessweek (2005), for example, a Sri Lankan undocumented worker in South Korea "...has little to do during his free time but to fret about his loved ones at home." The undocumented status prevents such individuals from going back and forth to visit their relatives, while the tight visa restrictions prevent family members from entering the host country. In many cases this can prevent family reunion until the migrant is deported or returns voluntarily.

To our knowledge, there is only a small number of studies that attempt to measure homesickness of immigrant workers. The few papers that deal with the issue are focused
on legal foreign-born workers or students. On the basis of a sample of 171 employees of a Dutch multinational company, Eurelings-Bontekoe et al. (2000) show that the relationship between feeling homesick and the time spent in the host country is non-linear. The prevalence of self-identified homesickness was the highest (44.2%) in the group of individuals who spent between 6 and 8 years abroad. The next highest (42.1%) was in the group who spent between 0.5 and 5 years abroad. This is followed by 21.2% among those with 9 to 18 years and, finally, 17.5% for those with 19 to 38 years abroad. These figures indicate a hump-shaped relationship with the peak of the hump at around 8 years. Although these findings are based on a sample of legally employed persons in the Netherlands, they are indicative of the fact that there is a psychological burden associated with being away from home. The intensity of homesickness for illegal immigrants is arguably stronger and increasing over time as their ability to communicate with and visit their relatives and friends in the country of origin is much more limited due to their irregular status.

More evidence on the role of homesickness in the decision to return is provided in the study by Niedomysl and Amcoff (2010). It focuses on the reasons for return migration and shows that social factors, such as the desire to be with family and friends, are just as important in explaining return migration as income and employment opportunities. This result is supported by the evidence on Bangladeshi migrants in Kibria (2004, p.16): "Besides the successful achievement of their economic objectives, the ‘voluntarily returned’ also included those who had come back for personal reasons... For Abul, who had worked in Japan for almost nine years, there were clearly multiple factors at work. But paramount was a desire to get married and to start a family, options that he did not see as available to him if he had remained in Japan...".

Apart from homesickness, undocumented immigrants obviously feel unwelcome in the host country. They typically have to accept a way of life which minimizes their
exposure to local authorities and even to locals who may signal the presence of an illegal alien to the police. Unauthorized migrants therefore cannot fully enjoy a range of public goods and services and even their consumption of private goods is restricted by the fear of being exposed to someone who will report them to the authorities.\(^{23}\) Moreover, as there is practically no scope for remaining permanently in a host country with strict deportation measures, undocumented workers have little incentive to invest their time in social-integration efforts abroad, focusing instead on maintaining strong ties with their relatives and friends at home (primarily over the internet or SMS). Thus, while Eurelings-Bontekoe et al. (2000) show that legal workers experience only a slight increase in homesickness over the first 8 years abroad, the above-mentioned constraints faced by undocumented workers are likely to contribute to a much higher rate of increase in homesickness over time.\(^{24}\)

### 4.2 The Model with "Homesickness"

The most natural way to capture the effect of social isolation and family separation is to generalize U’s utility function to account for the time spent abroad. In the literature on temporary migration it is commonly assumed that a given quantity of consumption good yields a smaller utility if it is consumed abroad rather than at home, i.e., there is a discount on utility in the host country (see, e.g., Hill 1987, Djajić and Milbourne 1988, Djajić 2010). In the case of undocumented migration this discount is time-dependent. Moreover, it is quite likely that the time dependence is not linear but rather convex due

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\(^{23}\)“People living and working with irregular status are often denied a whole host of basic entitlements and services and lead their lives in constant fear of arrest and deportation.” (Human Development Report 2009, p. 17.)

\(^{24}\)Homesickness is obviously a context-specific as well as an individual-specific phenomenon. Undocumented Mexican migrants in the South-West of the US, for example, face a drastically different social and policy environment in the host country when compared with undocumented migrants in East-Asian economies. This would, of course, be reflected not only in the degree to which they experience homesickness but also in the shape of their homesickness function.
to the above-mentioned constraints associated with the undocumented status. We shall therefore assume that the marginal homesickness increases as U spends more and more time in the host country. The migrant’s utility function while abroad takes the following form:

\[ u(c^*_t, t) = \frac{c^*_t^{1-\theta}}{1-\theta} - h_t, \quad h_t = e^{\alpha t} - 1, \quad \alpha > 0, \quad (14) \]

where \( h_t \) stands for "homesickness", with \( \frac{\partial u(c^*_t, t)}{\partial t} < 0 \) and \( \frac{\partial^2 u(c^*_t, t)}{\partial^2 t} < 0 \). The parameter \( \alpha \) governs the intensity of homesickness. If the migrant is located in the source country, \( \alpha \) is set to zero.\(^{25}\)

The optimization program with respect to consumption is not affected by the introduction of homesickness in the utility function, as long as time does not affect the marginal utility of consumption, which is the case here.\(^{26}\) The optimal consumption path is therefore identical to that derived in the previous subsection, although, the value functions have now two arguments, \( a_t \) and \( t \), which are the two state variables of the program. The optimality condition for the time of return is derived along the lines presented in Subsection 2.2. In our benchmark calibration of Section 3 reflecting the East-Asian context, the optimal return date is \( \tau = 9.02 \) when both economic pull-back factors are present \( (r = 10\%, \; r^* = 3\%, \; p^*/p = 3) \) and the deportation rate is \( \lambda = 0.25 \). Introducing homesickness into the model \( (\alpha = 0.01 \text{ as a starting point}) \) yields a more realistic value of \( \tau = 4.82 \) for the optimal migration duration. This is comparable to the reported 2 - 5 year durations of stay abroad of illegal aliens in Japan, South Korea, Taiwan, Malaysia, and Singapore (Jones and Pardthaisong 1999, Kibria 2004, Sobieszczynz 2000).

\(^{25}\text{Following a suggestion of an anonymous referee, we present and discuss the sensitivity of our results with respect to alternative ways of combining consumption and homesickness in the utility function in Appendix B. We also consider alternative specifications of the homesickness function, such as linear and concave. Appendix B shows that the qualitative results are not affected by the choice of the preference structure and even the quantitative results are very similar.}\)

\(^{26}\text{Appendix B.2 provides an extension where homesickness and consumption enter multiplicatively into the utility function, allowing for the marginal utility of consumption to depend negatively on the time spent abroad.}\)
4.3 The Japanese Case

The data on undocumented Thai migrants collected by Sobieszczyk (2000) allow us, in fact, to extract the value of $\alpha$ for those who worked in Japan.\textsuperscript{27} The Sobieszczyk (2000) study covers a total of 29 migrants who worked illegally in Japan between 1986 and 1997. We shall use 1995 as the benchmark year for our calibration as the data for most of the required parameters is readily available for this year (except for the deportation rate and the price ratio for which only the data in 2000 is available). For the purpose of retrieving the value of $\alpha$ on the basis of this sample, we calibrate the following parameters: $w^*/w$, $p^*/p$, $r^*$, $r$, $\lambda$, and $\tau$.

**Wage ratio.** For the 29 migrants, the average monthly wage in Japan was equal to $1,378.11 (in current dollars). Unfortunately, Sobieszczyk (2000) does not provide information on the wage earned in Thailand before or after migration. However, we know that the average monthly wage of an unskilled worker in Thailand in the mid 1990s was $626.7$ baht, which is equal to $250.68$ at the exchange rate of 25 baht per dollar in 1995-1996 (Jones and Pardthaisong 1999, footnote to Table 6). This value of the monthly wage rate is consistent with the ILO data on wages in Thailand in 1995 ($250.56$). Dividing $1,378.11$ by $250.68$ we obtain the wage ratio $w^*/w = 5.49$.

**Price ratio.** The data on the price ratio is taken from OECD Comparative price levels for 2000 (data for 1995 are not available, although one may expect that in 1995 the price differential was larger than in 2000). The value of the index for Japan was 164 and for Thailand it was 40, implying that $p^*/p = 164/40 = 4.1$.

**Interest rates.** The real interest rate in 1995 was equal to 4% in Japan and 7.3% in Thailand (World Bank Data\textsuperscript{28}). Thus we have $r^* = 0.04$ and $r = 0.073$. Since the benchmark time period used in the model simulation is one month, these values will be

\textsuperscript{27}The full sample contains information on 104 temporary migrants who worked in various destination countries including Japan, Hong-Kong, Singapore, Taiwan, Brunei, South Korea and Malaysia. However, due to the lack of data on deportation rates, we are only able to analyze the case of migration to Japan.

\textsuperscript{28}http://data.worldbank.org/indicator/FR.INR.RINR?page=3
divided by twelve in the calculations below.

**Deportation rate.** The Japanese Ministry of Justice provides information on the number of deportations and the estimated stocks of illegal immigrants as of 1999. The ratio of annual deportations of undocumented Thai nationals to the stock of illegal aliens from Thailand residing in Japan is on average 0.144 for the seven years from 1999 to 2005.\(^{29}\) I therefore set \(\lambda = 0.144\).

**Deportation penalty.** According to the Revised Immigration Control and Refugee Recognition Act (1999), the penalty for illegal stay in Japan is imprisonment of up to 3 years or a fine of up to 3 hundred thousand yen.\(^ {30}\) This monetary penalty is equal to \$2'638, applying the average exchange rate in 1999 of 113.71 yen per dollar.\(^ {31}\) Given that the average earnings of a migrant were \$1'378.11 per month, the penalty represents approximately 1.91 times the monthly wage.

**Trip duration.** The average duration of stay abroad of the 29 undocumented Thai migrants in Japan was 60.19 months or 5.015 years, which is in fact less than the hypothetical expected stay (given the deportation rates) of \(1/\lambda = 6.9\) years.

If we use these data to characterize the economic environment facing an undocumented Thai migrant and calculate the optimal duration of stay in Japan in the absence of homesickness (by setting \(\alpha = 0\)), we obtain \(\tau = 8.88\) years. This is almost 4 years longer than the observed average length of stay. The value of the homesickness parameter which is consistent with the observed average migration duration of 5.015 years is \(\alpha = 0.0117\), assuming the benchmark elasticity of marginal utility \(\theta = 0.9\). Column 2 of Table 2 reports the value of \(\alpha\) for \(\theta\) ranging from 0.75 to 1.05. These estimates are,

\(^{29}\)For each of these years, the ratio is reported to be as follows: 3886/30065 = 0.1292 in 1999, 3359/23503 = 0.1429 in 2000, 2552/19500 = 0.1308 in 2001, 2391/16925 = 0.1412 in 2002, 2272/15693 = 0.1447 in 2003, 2521/14334 = 0.1758 in 2004, and 1895/12787 = 0.1481 in 2005 (Japanese Ministry of Justice, 2005, 2011).

\(^{30}\)http://www.jca.apc.org/apfs/nyukan_rev_e.html

\(^{31}\)I use the Board of Governors of the Federal Reserve System release on monthly dollar-yen exchange rate to calculate the average. The data is available at http://research.stlouisfed.org/fred2/data/EXJPUS.txt
of course, a result of a rough calculation based on a small sample and should be only considered as suggestive. They allow us, nonetheless, to interpret the utility loss due to homesickness in more concrete terms if we relate it to the utility enjoyed from the consumption of commodities. That is, we can calculate the consumption-equivalent loss due to social isolation. Using as the point of reference the optimal consumption rate in the 30th month (middle trip duration), we find that the impact of homesickness is equivalent to a 27.64 to 55.67% reduction in consumption, depending on the degree of concavity of the utility function (column 3 of Table 2). Taking the last month of the undocumented stay as the point of reference, we find that the impact of homesickness on utility is identical to a 53.10 - 89.81% cut in the consumption rate (column 4 of Table 2). Thus homesickness appears to be a powerful pull-back factor influencing return decisions of undocumented foreign workers, as suggested by descriptive studies (see, e.g. Kibria 2004).

<table>
<thead>
<tr>
<th>1</th>
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<th>5</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>Inverse of EICS, $\theta$</td>
<td>Homesickness intensity, $\alpha$</td>
<td>Equivalent $-\Delta c_{40}^*, %$</td>
<td>Equivalent $-\Delta c_{60}^*, %$</td>
<td>Equivalent $\Delta r, %$</td>
<td>$-\Delta \tau$ due to doubling $\lambda$,%</td>
</tr>
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<td>9.63</td>
<td>2.07</td>
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<td>8.68</td>
<td>2.62</td>
<td>4.72</td>
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<tr>
<td>0.9</td>
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<td>68.32</td>
<td>8.24</td>
<td>3.01</td>
<td>5.14</td>
</tr>
<tr>
<td>0.95</td>
<td>0.010973</td>
<td>33.65</td>
<td>62.53</td>
<td>7.83</td>
<td>3.03</td>
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<tr>
<td>1.05</td>
<td>0.0096935</td>
<td>27.64</td>
<td>53.10</td>
<td>7.15</td>
<td>3.81</td>
<td>6.38</td>
</tr>
</tbody>
</table>

Table 2: Exponential homesickness function, $h_t = e^{\alpha t} - 1$.

One may go a step further to measure the influence of homesickness in relation to other pull-back factors, such as the expected rate of return on investments in Thailand. How large would this rate have to be in the absence of the homesickness effect in order to induce undocumented Thai migrants to return after having spent 5 years in Japan? The model predicts that, if $\theta = 0.9$, the real interest rate in Thailand would need to be 15.5% (instead of the observed rate of 7.3%), implying an increase of 8.2 percentage
points. The sensitivity of this result to variations in the elasticity of marginal utility ($\theta$) can be seen in the 5th column of Table 2, which shows that, within a realistic range of values of $\theta$, the overall effect of homesickness is equivalent to that of a 7.15 to 9.63 percentage-point increase in the rate of return on investment in the sending country.

As the final step, we examine the predicted impact of a change in $\lambda$ on undocumented Thai migrants’ duration of stay in Japan. Given the estimated value of the homesickness parameter, the model implies that a doubling of the deportation rate to 0.2882 reduces the optimal $\tau$ by 2.07 - 3.81% or 1.25 to 2.29 months (depending on the value of $\theta$), as shown in the 6th column of Table 2. The last column shows the change in $\tau$ following a 3-fold increase in the deportation rate. It amounts to a decline in the duration of stay of 2.5 ($\theta = 0.75$) to 3.84 months ($\theta = 1.05$). Not surprisingly, the higher is the elasticity of intertemporal consumption substitution ($1/\theta$), the smaller is the implied change in $\tau$. Table 2 also illustrates the point established in Section 3.4, that while an increase in the deportation rate may be quite effective in advancing the date of voluntary return when the initial rate is relatively low, a further tightening of the policy may quickly run into diminishing returns (see figure 5).

In closing our discussion on the role of homesickness in a model of temporary migration we would like to emphasize that it allows us to improve our understanding of undocumented migrants’ behavior and, in particular, their incentives to return to the country of origin. Homesickness is only one of the drivers of return decision but its role is reinforced in environments characterized by sharp intolerance of illegal immigration and strict deportation policies. This is, for example, the case in East-Asian economies and the Gulf States. It is relevant to a lesser extent in analyzing the behavior of illegal immigrants working in countries that tolerate the presence of undocumented foreign labor and rarely resort to deportations of non-criminal aliens.
5 Conclusion

The present paper develops a framework for the analysis of temporary illegal immigration. Its main contribution is three-fold. Starting with the characterization of the solution for the optimal timing of return to the source country when the event of deportation follows a stochastic process, we show that a more vigorous deportation policy advances the date of voluntary return. This helps explain why undocumented immigrants choose to remain for only a relatively short period of time in countries with very strict deportation measures, such as Japan, South Korea, Singapore, Malaysia and the Gulf States, while tending to stay for longer durations or even permanently in countries with a more lenient stance, such as the U.S.A. or the EU. A higher deportation risk creates stronger incentives for voluntary return by operating through two channels: (a) it contributes to an acceleration of asset accumulation and (b) it acts as a homesickness catalyst which has a powerful influence in advancing the return date of undocumented immigrants. Thus the risk of deportation can trigger voluntary return under conditions that would otherwise make a permanent undocumented stay optimal.

An important policy implication of this analysis is that for any given flow of undocumented workers, a host country with stricter deportation policies will have a smaller stock of illegal immigrants at each point in time. This is not only because it physically removes more undocumented aliens, but also because the policy indirectly induces those who are not apprehended to voluntarily leave the host country sooner. In addition to these direct and indirect effects which contribute to a reduction in the stock of illegal aliens for a given flow, deportations obviously have a deterrent effect on the inflow.

The paper also shows that consideration of just the economic pull-back factors, such as international return-on-investment or cost-of-living differentials, which have been extensively studied in the literature on temporary migration, is not sufficient to account for the observed behavior of undocumented aliens. Although these factors do play an im-
portant role in drawing migrants back to their country of origin, they do not explain why deportable illegal immigrants return voluntarily after only 2 to 5 years of work abroad. Accounting, in addition, for the effects of "homesickness" and using the evidence on the behavior of undocumented Thai migrants in Japan, enables us to measure the influence of these psychological and social factors on the timing of voluntary return. They are found to be just as important as economic considerations. On average, the loss of welfare associated with family separation and social isolation experienced by undocumented Thai migrants during their last month in Japan is equivalent to a change in utility associated with a 53 to 68% cut in the consumption rate abroad (in the empirically relevant range of the intertemporal substitution elasticity). For illegal immigrants in countries with very strict deportation measures, the degree of social isolation and separation from other family members is indeed unique across the spectrum of migrant types. It therefore deserves particular attention in both theoretical and empirical studies.

Our calibration of the model for the case of undocumented Thai migrants in Japan also allows us to draw conclusions on the effectiveness of a change in the intensity of deportations in terms of its impact on the optimal duration of an undocumented stay. Our model predicts that while stricter deportation measures can be quite effective in building on the incentives for voluntary return at relatively low rates of deportation, it is also true that, after a certain point, a further tightening of this policy becomes less effective. In the case of Japan, we find that doubling the deportation rate from its level in the late 1990s would have reduced the optimal duration of stay of illegal aliens from Thailand by about 1.25 to 2.3 months, depending on the intertemporal substitution elasticity.
Compliance with Ethical Standards:

Funding: This research has not benefited from any funding.

Conflict of interest: The author declares that she has no conflict of interest.
References


[38] Singapore Immigration Act (Chapter 133), available at: http://statutes.agc.gov.sg/non_version/cgi-bin/cgi_retrieve.pl?actno=Reved-133&date=latest&method=part


A Optimal Return with Risk of Deportation

A.1 Derivation of optimal consumption growth rate abroad

The Hamilton-Jacobi-Bellman equation is given by

$$\rho V(a_t) = \max \left\{ \frac{u(c^*_t) + \partial V(a_t)}{\partial a_t} (r^* a_t + w^* - p^* c^*_t) + \lambda \left( \hat{V}(a_t) - V(a_t) \right) \right\}. \quad (15)$$

The first order conditions with respect to $c^*_t$ and $a_t$ yield

$$u'(c^*_t) - p^* \frac{\partial V_t}{\partial a_t} = 0, \quad (16)$$

$$\rho \frac{\partial V_t}{\partial a_t} = \frac{\partial^2 V_t}{\partial a_t^2} \dot{a}_t + r^* \frac{\partial V_t}{\partial a_t} + \lambda \left( \frac{\partial \hat{V}_t}{\partial a_t} - \frac{\partial V_t}{\partial a_t} \right). \quad (17)$$

Differentiating (16) with respect to time and using the result in (17) yields

$$\frac{u''(c^*_t)}{u'(c^*_t)} \dot{c}_t^* + r^* + \lambda \left( \frac{u'(c^*_t)p^*}{u'(c^*_t)p} - 1 \right) - \rho = 0.$$ 

After rearranging terms and using $u'(x) = (x)^{-\theta}$ ($x = \tilde{c}_t, c^*_t$), we obtain

$$\frac{\dot{c}_t^*}{c_t^*} = \frac{1}{\theta} \left\{ \lambda \left[ \frac{\tilde{c}_t}{c_t} \right]^{-\theta} \frac{p^*}{p} - 1 \right\} + r^* - \rho. \quad (18)$$

A.2 Derivation of optimal consumption growth rate after deportation

If $U$ is deported at time $\varepsilon$, her objective is to maximize

$$\int_{\varepsilon}^{T} u(\tilde{c}_t) e^{-\rho(t-\varepsilon)} dt$$
subject to
\[ \dot{a}_t = ra_t + w - p\tilde{c}_t, \quad t \in [\varepsilon, T], \quad a_\varepsilon \text{ given}, \quad a_T = 0. \]

The solution to this problem is well known. The optimal growth rate of consumption is constant at \( \frac{r-\theta}{\theta} \) and hence \( \tilde{c}_t = \tilde{c}_\varepsilon e^{\frac{r-\theta}{\theta}(t-\varepsilon)} \). Using this in the differential equation for the asset position allows to solve for \( \tilde{c}_\varepsilon \) (see eq. (11) in the text) and subsequently for the time profile of assets:

\[ a_t = a_\varepsilon e^{r(t-\varepsilon)} + w \frac{e^{r(t-\varepsilon)} - 1}{r} - p\tilde{c}_\varepsilon \frac{e^{\frac{r-\theta}{\theta}(t-\varepsilon)} - e^{r(t-\varepsilon)}}{g}, \quad t \in [\varepsilon, T]. \]

### B Alternative Functional Forms

#### B.1 Other forms of homesickness function

The results presented in Section 4.3 are undoubtedly sensitive to some extent to the chosen specification of the homesickness function. Suppose, for example, that homesickness increases with time spent abroad following a linear trend or increases at a decreasing rate or even declines over time due to better assimilation. The last case is, however, highly unlikely for illegal Thai migrants in Japan as they typically do not learn the Japanese language and restrain from being exposed to the public.

We consider here two alternative specifications of the \( h_t \) function in Eq. (14): linear \( (h_t = \alpha t) \) and concave \( (h_t = t^\alpha - 1) \), where the latter captures decaying marginal homesickness effect. In the absence of homesickness \( (\alpha = 0) \) both functions are normalized to zero. For each functional form the parameter \( \alpha \) is extracted from the data on undocumented Thai migrants in Japan and the results are summarized in Tables B1 and B2. The predicted consumption-equivalent loss is reported in columns 3 and 4 for alternative
values of $\theta$ and two consumption reference points: the 30th month (half average trip duration) and the 60th month. If we consider the 30th month as the point of reference, then the effect of homesickness is equivalent to a 33.65% (for $\theta = 0.95$) to 55.6% (for $\theta = 0.75$) cut in the migrant’s consumption rate in that month when $h_t$ is exponential, a 38.84% to 65.6% cut when $h_t$ is linear, and a 53.94% to 83.1% cut when $h_t$ is concave. Logically, the convex functional form yields a relatively smaller loss than the linear form which in turn yields a smaller loss than the concave function. If we consider the last month of the trip as the point of reference, then the effect of homesickness translates into approximately a 62% ($\theta = 0.95$) to 89% ($\theta = 0.75$) decline in the consumption rate for all the three specifications. Thus, regardless of the chosen functional form, homesickness remains a powerful element influencing return decisions of undocumented foreign workers, as suggested by descriptive studies (see, e.g., Kibria 2004).

We can also compare the effectiveness of a stricter deportation policy in reducing migration duration under alternative specifications of the $h_t$ function. Columns 6 and 7 of

\begin{table}[h!]
\centering
\begin{tabular}{|c|c|c|c|c|c|c|}
\hline
Inverse of EICS, $\theta$ & Homesickness intensity, $\alpha$ & Equivalent $-\Delta c_{30}$, % & Equivalent $-\Delta c_{60}$, % & Equivalent $\Delta r$, % & $-\Delta \tau$ due to doubling $\lambda$, % & $-\Delta \tau$ due to tripling $\lambda$, % \\
\hline
0.75 & 0.212093 & 83.11 & 89.93 & 9.63 & 3.40 & 5.98 \\
0.9 & 0.17209 & 59.75 & 68.45 & 8.24 & 4.35 & 7.37 \\
0.95 & 0.16119 & 53.94 & 62.65 & 7.83 & 4.67 & 7.82 \\
\hline
\end{tabular}
\caption{Concave homesickness function, $h_t = t^\alpha - 1$.}
\end{table}

\begin{table}[h!]
\centering
\begin{tabular}{|c|c|c|c|c|c|c|}
\hline
Inverse of EICS, $\theta$ & Homesickness intensity, $\alpha$ & Equivalent $-\Delta c_{30}$, % & Equivalent $-\Delta c_{60}$, % & Equivalent $\Delta r$, % & $-\Delta \tau$ due to doubling $\lambda$, % & $-\Delta \tau$ due to tripling $\lambda$, % \\
\hline
0.75 & 0.0230045 & 65.63 & 89.86 & 9.63 & 2.59 & 4.58 \\
0.9 & 0.0170148 & 43.69 & 68.37 & 8.24 & 3.40 & 5.78 \\
0.95 & 0.0155459 & 38.84 & 62.57 & 7.83 & 3.66 & 6.17 \\
\hline
\end{tabular}
\caption{Linear homesickness function, $h_t = \alpha t$.}
\end{table}
Tables B1 and B2 report the reduction in $\tau$ following a two-fold and, respectively, a three-fold increase in the deportation rate. The concave $h_t$ function captures a relatively fast-growing negative impact of homesickness at the beginning of the trip and a subsequent slow-down. The linear function captures a steady increase, while the convex function captures a slowly growing homesickness at the beginning of the trip but an explosively high growth at longer durations. The effect of an increase in the deportation rate on the optimal-return date is thus more pronounced for a concave specification (regardless of $\theta$) since the marginal impact of homesickness (the slope of $h_t$) is larger for $t < \tau$. For instance, a doubling of $\lambda$ reduces $\tau$ by 2.81 months if $h_t$ is concave and by only 1.82 months if it is convex (assuming $\theta = 0.95$).

### B.2 A more general utility function

So far we have presented a model where homesickness entered a migrant’s utility function in an additively-separable fashion. It was useful to adopt separability because in this case the optimal consumption growth rate was not affected by the time spent in the host country. As noted by an anonymous referee, one should consider alternative ways of combining consumption and homesickness in the utility function. In this appendix we propose a more general specification to allow for the marginal utility of consumption to depend on homesickness, denoted by $H_t$. This implies that the value function in the HJB equation depends not only on the asset position but also on time. Consequently the expression for the optimal consumption growth rate has extra terms. Formally, the HJB equation now reads:

$$\rho V(a_t, t) = \max \left\{ u(c^*_t, H_t) + \frac{\partial V(a_t, t)}{\partial a_t} (r^* a_t + w^* - p^* c^*_t) + \lambda \left( \tilde{V}(a_t) - V(a_t) \right) + \frac{\partial V(a_t, t)}{\partial \tau} \right\}.$$
The first-order conditions are given by:

\[
\frac{\partial u(c^*_t, t)}{\partial c^*_t} - p^* \frac{\partial V}{\partial a_t} = 0,
\]

\[
\rho \frac{\partial V}{\partial a_t} = \frac{\partial^2 V}{\partial a_t^2} \hat{a}_t + r^* \frac{\partial V}{\partial a_t} + \lambda \left( \frac{\partial V}{\partial a_t} - \frac{\partial V}{\partial a_t} \right) + \frac{\partial^2 V}{\partial t \partial a_t}.
\]

Differentiation of the first condition yields:

\[
u^*_c c^* + u^*_t = p^* [V_a \hat{a} + V_{at}],
\]

where we used short-hand notation for the partial derivatives \( u^*_c \equiv \frac{\partial u(c^*_t, t)}{\partial c^*_t} \) and similarly \( u^*_t \equiv \frac{\partial^2 u(c^*_t, t)}{\partial c^*_t \partial t} \). Combining this with the second condition yields:

\[
u^*_c c^* + u^*_t = -u^*_c \left\{ r^* - \rho + \lambda \left[ \left( \frac{\hat{u}_c}{u^*_c} \right) \frac{p^*}{p} - 1 \right] \right\}
\]

\[
\frac{\dot{c}^*}{c^*} = -\frac{u^*_c}{c^* u^*_cc} \left\{ r^* - \rho + \lambda \left[ \left( \frac{\hat{u}_c}{u^*_c} \right) \frac{p^*}{p} - 1 \right] \right\} - \frac{u^*_c}{c^* u^*_cc}
\]

We can immediately see that time spent abroad affects the growth rate of consumption through two channels. One is represented by the last negative term (recall that \( u^*_c \leq 0 \)) and the other is the indirect effect through the marginal utility of consumption in the term multiplying \( \lambda \). Both effects work to speed up consumption growth: the longer the migrant stays abroad, the higher is the growth rate of consumption.

Let us consider a specific example where consumption and homesickness enter multiplicatively in the utility function. For instance, let \( u(c^*, H_t) = c^* \frac{1-\theta}{1-\theta} H_t^{-1} \), where \( H_t = 1 + h_t \). Note that we added 1 to the function \( h \) because now it enters utility multiplicatively and therefore we need to ensure that when the homesickness parameter \( \alpha \) is set to zero, the utility of consumption is not affected by the homesickness component. Earlier we considered three types of \( h_t \) function (exponential, linear and concave), which
were normalized to zero when $\alpha = 0$. In the current setup they need to be normalized to unity and this is why we now work with the function $H_l$ and not $h_l$.

The above functional form of $u(c^t, H_l)$ has the desired properties, in the sense that it is increasing and concave in consumption (standard CRRA), decreasing in homesickness (for $\theta \in [0, 1]$) and the marginal utility of consumption is decreasing in homesickness as well. The three specifications of the $H_l$ function — exponential ($H_l = e^{ct}$), concave ($H_l = t^\alpha$), and linear ($H_l = 1 + \alpha t$) — result in three different expressions for the optimal consumption growth rate abroad:

$$g^{exp} = \frac{1}{\theta} \left\{ r^s - (\rho + \alpha) + \lambda \left[ \frac{p^s}{p} \left( \frac{\bar{c}}{c^s} \right)^{-\theta} e^{ct} - 1 \right] \right\},$$

$$g^{con} = \frac{1}{\theta} \left\{ r^s - \rho + \lambda \left[ \frac{p^s}{\bar{p}} \left( \frac{\bar{c}}{c^s} \right)^{-\theta} t^\alpha - 1 - \frac{\alpha}{t} \right] \right\},$$

$$g^{lin} = \frac{1}{\theta} \left\{ r^s - \rho + \lambda \left[ \frac{p^s}{p} \left( \frac{\bar{c}}{c^s} \right)^{-\theta} (1 + \alpha t) - 1 - \frac{\alpha}{1 + \alpha t} \right] \right\}. $$

Note that with the exponential specification the intensity of homesickness essentially alters the subjective rate of time preference from $\rho$ to $\rho + \alpha$.

Let us now turn to the numerical results. We are primarily interested in exploring robustness of our findings presented in Section 4 with respect to alternative specifications of the utility function. Given that time spent abroad increases the optimal consumption growth rate abroad, we expect that the consumption-equivalent loss due to homesickness is larger with a multiplicative utility function than with an additive function. Tables B3 to B5 below replicate Tables 2, B1 and B2 assuming that $u(c^*_t, H_l) = \frac{c^*_t^{1-\theta}}{1-\theta} H_t^{-1}$, where the homesickness function $H_t$ takes three alternative forms, specified in the caption of each table.\(^{32}\)

\(^{32}\)Due to the requirement $\partial u(c^*_t, H_l) / \partial H_t < 0$, we only consider the values of the elasticity of marginal utility of consumption ($\theta$) which do not exceed unity.
In the 5th column of Tables B3 - B5, we see that when the homesickness function is concave, tightening of the deportation policy (increase in \( \lambda \)) is more effective than when \( H \) is linear, which in turn is more effective than when \( H \) is exponential. (2) The larger is the elasticity of intertemporal consumption substitution, the smaller is the policy impact on migration duration. (3) We see in the 4th column of Tables B3 - B5, that consumption-equivalent loss due to the psychological effect, evaluated at the 60th month abroad, is quantitatively similar across the three functional forms capturing homesickness. These are essentially the same conclusions which we have drawn from our baseline model with additive utility.
Comparing the results stemming from an additive-utility model to those stemming from a multiplicative-utility model, we find no major differences. The quantitative impact of a change in $\lambda$ is very similar. For instance, when homesickness is represented by an exponential function, a doubling of $\lambda$ results in a 2.07% decline in $\tau$ with additive utility and in a 2.87% decline with a multiplicative utility, for $\theta = 0.75$; a 3.01% and a 3.40% decline, respectively, for $\theta = 0.9$; and in a 3.03% and 3.66% decline, respectively, for $\theta = 0.95$. When homesickness is represented by a linear function, the numbers are 2.59% and 2.94% for $\theta = 0.75$, 3.40% and 3.46% for $\theta = 0.9$, 3.66% and 3.69% for $\theta = 0.95$. The numbers are also very similar in the case of a concave function. Not surprisingly, with the multiplicative structure $\tau$ declines slightly more than with additive preferences.

There is one dimension, however, along which the additive and the multiplicative-utility models produce slightly different results. This occurs when we calculate the fall in the consumption rate which would replicate the homesickness effect on welfare (shown in columns 3 and 4). For instance, the additive-utility model with exponential $h$ produces a 89.8% drop in $c^*$ for $\theta = 0.75$, while in the multiplicative case we obtain a 93.76% drop. As $\theta$ increases, the percentage difference becomes less pronounced: a 68.32% drop vs. 69.20% for $\theta = 0.9$; and a 62.53% vs. 62.90% for $\theta = 0.95$. With linear $h$ we have 89.86% vs. 93.40% for $\theta = 0.75$, 68.37% vs. 69.20% for $\theta = 0.9$, and 62.57% vs. 62.90% for $\theta = 0.95$. The multiplicative specification produces larger percentage fall in consumption compared to the additive function because in the former case homesickness has a negative impact not only on the welfare level but also on the marginal utility of consumption. This is especially relevant when the elasticity of marginal utility ($\theta$) is small. This result is quite intuitive. For an identical drop in consumption the marginal utility increases by less in the multiplicative case. Thus the decline in consumption should be larger in order to counteract the negative effect of homesickness and this is
especially so when the marginal utility is less elastic (small $\theta$).

Overall, we can safely conclude that changing the preference structure from additive to multiplicative does not significantly alter our qualitative or even quantitative results. On the qualitative side, we note that with the latter preferences the optimal migration duration is shorter and the effect of a tighter deportation policy on the length of the trip is stronger. On the quantitative side, we find that the differences are fairly marginal, especially if we focus on a relatively elastic utility of consumption.