Simultaneous Reporting of Credit Ratings May Discipline Rating Agencies

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Abstract

We show that requiring a credit rating agency to report many ratings at once may discipline it against rating inflation. When the rating agency reports ratings simultaneously, issuing one more good rating lowers the credibility of all the good ratings it issues, diminishing borrowers’ willingness to pay for them (and consequently the rating fee). If the number of borrowers is large, this mechanism ensures an allocation that asymptotically approaches the first best. While the mechanism may work under sequential rating, it is less effective than under simultaneous rating. This suggests an additional benefit to synchronizing the issuance of corporate bonds.

Keywords: Credit Rating Agencies, Simultaneous Rating, Synchronization of Debt Issuance.

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1 Introduction

Credit rating agencies play an important role in the functioning of financial markets and allocation of capital. However, after the recent financial crisis there has been a growing concern and evidence of rating inflation. Take for example the case of Alt-A mortgage-backed securities (MBS). About 10% of the tranches issued in the period 2005-2007 rated most safe –triple AAA– were either downgraded to junk status or lost their principal by 2009. The case of CDO bonds was not better. More than 71.3% of such bonds had the same fate despite being initially rated as Aaa.\textsuperscript{1,2} Credit rating agencies have also been involved in lawsuits because of rating inflation. In 2008 a group of investors led by the Abu Dhabi Commercial Bank initiated a lawsuit against the investment bank Morgan Stanley and the two rating agencies: Moody’s and Standard and Poor’s. The investors accused the rating agencies of collaborating with Morgan Stanley in arranging for some of its financial products to receive ratings as high as triple-A, even though much of the underlying collateral was low-quality or subprime mortgage debt.\textsuperscript{3} More recently, in February 2013, the U.S. Department of Justice filed a lawsuit against Standard & Poor’s accusing it of inflating ratings and understating risks associated with mortgage securities with the purpose of gaining more market share.\textsuperscript{4}

Inflation of credit ratings may cause huge losses to investors, as was evident in the recent crisis. The $5 billion compensation which the U.S. government seeks from Standard & Poor’s is just one signal of the magnitude of such losses. However, this is not the only cost of credit ratings inflation. The inflation of credit ratings also damages the credibility of rating agencies and thus affects the functioning of financial markets and the efficiency of capital allocation. This begs the question: how to discipline the credit rating agencies against rating inflation? The economics and finance literatures, which we review in more detail below, pays considerable attention to the reputation mechanism: if a credit rating agency gives a good rating to an issuer with a bad project, which is likely to perform poorly, then this poor performance will damage the agency’s reputation for issuing credible ratings.

\textsuperscript{1}See the “Final Report of the National Commission on the Causes of the Financial and Economic Crisis in the United States”, pages 228-229. This report is authenticated U.S. government information.

\textsuperscript{2}As a reference point, observe that the historic cumulative default rate (up to 2007) of corporate bonds rated AAA by Standard & Poor’s is only 0.6%. In the case of Moody’s the analogous figure is 0.52%. (See the ”House Report 110-835 - Municipal Bond Fairness Act” of September 2008, U.S. Government Printing Office.)

\textsuperscript{3}The case is Abu Dhabi Commercial Bank et al v. Morgan Stanley & Co et al, U.S. District Court, Southern District of New York, No. 08-07508. The parties reached a settlement agreement in April 2013. The settlement amount was almost 9.5 million dollars.

\textsuperscript{4}The case is United States of America v. McGraw-Hill Companies, Inc and Standard and Poor’s Financial Services LLC, U.S. District Court, General District of California, No. CV13-00779.
The mechanism relies on repeated interaction and on the comparison between the ratings obtained by issuers and their ex-post performance. This paper suggests a different disciplinary mechanism, which relies neither on repeated interaction nor on the comparison between issuers’ ratings and their ex-post performance. It simply requires a credit rating agency to simultaneously report the ratings of many issuers. An important feature of this simultaneity in reporting the ratings is that investors make their investment decisions only after observing the ratings given by the rating agency to many issuers. This situation is opposed to that of sequential rating where the rating agency reports issuers’ ratings (and investors decide whether to invest in each issuer) sequentially, which is what presently occurs in reality. We show that simultaneous rating engenders disincentives for a credit rating agency to inflate ratings. This is underpinned by two economic effects.

First, simultaneous rating generates a negative link between the value of a good rating and the number of such ratings reported. If a credit rating agency rates many issuers simultaneously, the higher the number of issuers to which it gives a good rating, the lower the credibility of such ratings. This negative link between the value of a good rating and the number of such ratings forces the credit rating agency to face a trade-off in the choice of the number of good ratings it reports. By reporting one more good rating it earns one more fee but it lowers the credibility of all the good ratings from that agency, which then lowers the issuers’ willingness to pay for its ratings (and consequently the rating fee). Because of this trade-off, the credit rating agency has an incentive to limit the number of good ratings it gives to issuers; it may even optimally refrain from giving a good rating to a high-quality issuer. In other words, it is optimal for the credit rating agency to self-impose a quota on the number of good ratings it issues.

Second, it is always optimal for a credit rating agency to fill that quota with issuers that have a high-quality project first before it starts to fill it with issuers that have a low-quality project. This is because given the cost of finance, high-quality projects create more value than low-quality projects, and therefore, the issuers of the former can (and are willing to) pay more for a good rating than those of the latter. Put differently, high-quality issuers can always beat low-quality issuers when competing for good ratings.

The combination of these two effects implies that in equilibrium, a rating agency will not give a good rating to all issuers and will give a good rating to a high-quality issuer before giving it to a low-quality issuer. Hence, an issuer with a good rating is more likely to be of higher quality than an
issuer without a good rating. This is why ratings will be credible, convey information about issuers' qualities to investors, and increase the efficiency in the allocation of capital, even in a static setting.

We show that under simultaneous rating a credit rating agency creates value even when the number of issuers is small. In the case of a large number of issuers, the value created by a credit rating agency asymptotically approaches the first-best total surplus. These results contrast with those obtained in the case of sequential rating, which we also analyze in the paper. Under sequential rating, the two effects mentioned above may emerge and the rating agency may create value if the number of issuers is infinite. However, the value created is bounded from above by a fraction of the first-best value.

One major reason for this difference is that while the negative link between the number of good ratings issued and their value may also emerge under sequential rating, it generates weaker disincentives against rating inflation than it does under simultaneous ratings. Under simultaneous rating, the decision of the credit rating agency to issue one additional good rating lowers the value and rating fees of all the ratings issued by the rating agency. Under sequential rating, because of its dynamic nature, that decision can lower only the agency’s future rating fees. The rating fees obtained by the rating agency in the past are not affected by its present rating decision.

This paper highlights that simultaneous reporting of ratings generates a mechanism to discipline credit rating agencies against rating inflation and to improve the credibility of the ratings they give. Other mechanisms have been studied in the literature. The most notable of them is the reputation mechanism – see, among others, Kuhner (2001); Mathis, McAndrews and Rochet (2009); Bar-Isaac and Shapiro (2012); Bolton, Freixas and Shapiro (2012); Mariano (2012); and Frenkel (2015). The reputation mechanism highlights that a rating agency will refrain from giving a good rating to a bad issuer because of the concern that its future credibility and revenues will be damaged following the (likely) default by the issuer. Unlike the reputation mechanism, the mechanism studied in this paper depends neither on repeated interaction nor on the comparison between projects’ rating with their ex-post performance. In fact, we abstract from the reputation mechanism in this paper by assuming that projects’ performance is never observed by investors. As the two mechanisms rely on different economic effects, they are complementary in disciplining rating agencies.

The literature has also highlighted that the credibility of credit ratings could be improved by

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5Interestingly, Frenkel (2015) shows that when a rating agency has two reputations—one with investors and another with issuers—reputation concerns may actually lead to the inflation of ratings by the rating agency.
addressing, in the first place, the conflicts of interest that may generate the bias in the ratings. One source of such conflicts of interest is the fact that credit rating agencies are paid by issuers – precisely those who they rate – and such payment usually occurs only if the issuer agrees with the disclosure of the rating.\textsuperscript{6} Mathis, McAndrews and Rochet (2009), for example, advocate a new business model in which the platforms where the securities are traded pay for the ratings of the securities. In the present paper, we assume that issuers pay the credit rating agency to rate them. Interestingly, this helps the mechanism highlighted in this paper because it allows issuers with high-quality projects to outbid issuers with low-quality projects when competing for good ratings, rendering good ratings credible.

Several articles have studied the impact of competition on the credibility and informativeness of ratings provided by a credit rating agency or more generally by a certifier (e.g., Lizzeri, 1999; Miao, 2009; Skreta and Veldkamp, 2009; Camanho, Deb and Liu, 2012; Bolton, Freixas and Shapiro, 2012). While Lizzeri (1999) shows that competition between certifiers can lead to full information revelation, Skreta and Veldkamp (2009), Camanho, Deb and Liu (2012) and Bolton, Freixas and Shapiro (2012) show that competition between credit rating agencies can in fact decrease the informativeness of credit ratings and the reputation of the rating agencies.\textsuperscript{7} In Skreta and Veldkamp (2009) and Bolton, Freixas and Shapiro (2012), this is because competition allows for credit rating shopping. In Camanho, Deb and Liu (2012) it is because it hinders rating agencies’ ability to sustain a high reputation.

An important difference amongst these papers is that Lizzeri (1999) assumes that the certifier can commit to a disclosure rule and the other articles assume that it cannot.\textsuperscript{8} While we abstract from competition issues in this paper, we also assume that credit rating agencies cannot commit to a disclosure rule. That is, the credit rating agency is totally free to give any rating to an issuer after having been approached to rate it and having observed its quality.

\textsuperscript{6}Griffin and Tang (2011) provide empirical evidence of ratings inflation due to conflict of interest by comparing the CDO assumptions made by the ratings department and by the surveillance department within the same rating agency. Xia and Strobl (2012) provide empirical evidence of rating inflation due to the issuer-pay model by comparing the ratings issued by Standard & Poor’s which follows the issuer-pay model to those issued by the Egan-Jones Rating Company which adopts the investor-pay model.

\textsuperscript{7}Becker and Milbourn (2011) provide evidence that increased competition due to Fitch’s entry in the credit ratings market in 1997 coincides with lower quality ratings from the incumbents. While Fitch was founded in 1913, the authors argue that only in 1997, following a merger with IBCA Limited, Fitch became an alternative global, full-service ratings agency capable of successfully competing with Moody’s and S&P.

\textsuperscript{8}Another important difference is that Lizzeri (1999) takes a mechanism design approach to the modelling of the certifier. Faure-Grimaud, Peyrache and Quesada (2009) is another example of an article that follows this line of modelling certifiers. They also use a mechanism design approach to model the certifier and assume that the certifier can commit to a disclosure rule.
Finally, some results in this paper are reminiscent of those in Damiano, Li and Suen (2008). The authors compare a rating agency that rates several clients separately (individual rating) with a rating agency that rates all clients together (centralized rating) and show that centralization of rating may enhance the credibility of the ratings. We instead compare simultaneous rating with sequential rating, which is not considered in their paper. Furthermore, they consider a costly signaling model as in Spence (1973), where producing a rating affects the agency’s payoff in itself, like obtaining a different educational degree incurs a different cost per se. Instead, we consider a model of cheap-talk, where reporting a rating affects the credit rating agency only through equilibrium interactions. This difference in modeling implies that in our paper the improvement in rating credibility under simultaneous rating is driven by the two economic effects aforementioned (i.e., the negative link between the number of good ratings issued and their value and the fact that high-quality issuers can pay more for a rating than low-quality issuers), which do not appear in their paper.

The remainder of the paper is organized as follows. In Section 2 we present the baseline model. In the model, a set of firms seeks funds from investors to implement their projects and can ask a credit rating agency to rate their creditworthiness. The credit rating agency evaluates the creditworthiness of all the firms and then investors decide whether or not to invest in each firm after having observed all these ratings. In Section 3 we present and analyze two benchmark cases. The baseline model is analyzed in Section 4. In Section 5 we analyze the case of sequential rating by considering a dynamic version of the baseline model. In this version of the model the credit rating agency rates one firm per period and investors decide whether to invest in the firm immediately after observing its credit rating (as well as the rating history of the agency). We then compare the informativeness of credit ratings and efficiency in the allocation of funds in both cases and argue that they are higher under simultaneous rating than under sequential rating. Section 6 discusses some robustness issues related to our key findings and Section 7 draws a conclusion. All the proofs are in the Appendix.

2 Model

There are $N$ penniless firms, a pool of investors, and one credit rating agency (CRA hereafter). We consider here the case of a monopolist CRA because it allows us to highlight the importance of the simultaneity in the reporting of credit ratings in the simplest possible way. In Section 6 we discuss the implications of competition between CRAs for our mechanism.
zero. Each firm (issuer) has one investment project and seeks funds from investors to implement it. A project requires an investment of one unit of funds and either succeeds and returns \( R \) or fails and returns nothing. The projects are of two types: good \((g)\) or bad \((b)\). A good project succeeds with probability \( q_g \), whereas a bad project succeeds with probability \( q_b \). We assume that a good project has a positive net present value (NPV), while a bad project destroys value, i.e.,

\[
q_b R < 1 < q_g R. \tag{1}
\]

In what follows, we denote by \( V_i \) the value created by a project of type \( i \), i.e., \( V_i \equiv q_i R - 1 \) for \( i = b, g \).

Observe that the conditions in (1) imply that investors are willing to finance good projects, but not bad projects. It is common knowledge that ex-ante a project is good with probability \( p \) and projects’ qualities are independent. We assume

\[
[p q_g + (1 - p) q_b] R < 1. \tag{2}
\]

This condition means that financing a randomly drawn project destroys value.

The quality of a firm’s project is known to the firm, but not to the investors. To overcome this informational barrier, firms can hire the CRA to rate their projects before they seek funds from investors. If hired by a firm, the CRA perfectly observes the quality of the firm’s project at no cost.\(^{10}\) After observing the quality of all the projects for which a rating was solicited, the CRA simultaneously proposes to each issuer \( i \) that requested a rating a contract specifying a rating \( r_i \in \{ \text{good, bad} \} \) and a rating fee \( f_i \). Those issuers then accept or reject the contract proposed by the CRA. Issuers who reject the contract pay nothing to the CRA and remain unrated. Issuers who accept the contract pay the rating fee to the CRA if and when their projects are financed and succeed, as before that they have no funds to make the payment. Rating fees are not observed by investors. Investors learn that an issuer hired the CRA only if a rating for that issuer is publicly disclosed.

Investors first observe all the ratings issued by the CRA (if any) and then decide which issuers to fund (if any). If investors decide to fund an issuer they demand a repayment \( C \), which the issuer is able to pay only if its project succeeds. If the project fails, the issuer defaults, and investors obtain

\(^{10}\)The assumption that the CRA observes the quality of project at no cost is made for simplicity of exposition. The assumption that the CRA evaluates a firm’s project only if hired by the firm rules out the possibility of unsolicited reporting of credit ratings.
nothing. We assume that investors have more funds than those that can be absorbed by firms and, as a result, they are satisfied with an expected net return of zero. Specifically, if investors believe that a given project is good with probability $\phi$ then they require a repayment $C$ to invest in the project such that

$$[\phi q_g + (1 - \phi) q_b]C - 1 = 0. \quad (3)$$

In other words, investors set the repayment $C$ so as to receive back in expectation precisely the unit of funds invested in the project. Hence, the lower the belief $\phi$ that the project is good, the higher the compensation they demand from the project. For sufficiently low values of $\phi$, that compensation may exceed $R$, in which case they refrain from investing in the project. Of course, investors’ beliefs about the quality of an issuer are endogenous in equilibrium and depend on the rating of the issuer and on the CRA’s equilibrium rating strategy. A good rating helps an issuer if it improves the investors’ beliefs about the quality of the issuer’s project to the point where investors demand a repayment $C < R$ to finance it.

Given the investors’ funding decisions, projects (if any) are implemented. In the case of success, the issuer first pays investors the agreed repayment, then pays the CRA the agreed fee, and keeps the remainder as profit. In the case of failure, because of limited liability, all parties obtain nothing.

The strategies of the issuers, CRA, and investors consist of the following. Based on the quality of its project, an issuer decides whether to hire the CRA and then whether to accept or reject the contract proposed by the CRA. Given the qualities of the projects evaluated, the CRA decides the rating offered to each issuer that requested a rating and the respective rating fee. Investors decide on which projects to fund (if any) and on the repayment required. We use Perfect Bayesian Equilibrium as the equilibrium concept. In this framework, the ratings of the CRA are no more than cheap talk. As a result, there exists an equilibrium in which they are totally disregarded by investors and the CRA just “babbles” when rating issuers. There might even exist equilibria in which investors believe that a good rating means something bad about the issuer’s quality and a bad rating something good. In this paper, we discard all these equilibria and focus on equilibria in which a good rating signals good quality and a bad rating bad quality. That is, investors believe that an unrated issuer or an issuer with a bad rating is of good quality with a probability no greater than the prior $p$. Given this, issuers accept to pay only for a good rating and reject any offer of a bad rating.
3 Two Benchmark Cases

Before delving into the analysis of the equilibrium outcomes, let us consider two benchmark cases, one in which no CRA operates in the market, the other in which there is only one issuer (i.e. $N = 1$).

Consider the first benchmark where there is no CRA. In this case no value is created in equilibrium. Either no project is financed, or the probability of a good project being financed and the probability of a bad project being financed are in such a proportion that the value created by the former is completely dissipated by the latter. That is, investors demand $C = R$ to finance the projects on the market. To see this, suppose for a moment that value is created, i.e. that $C < R$ in a certain equilibrium. In this equilibrium, issuers obtain $R - C > 0$ if their projects succeed. Thus ex-ante a good issuer obtains in expectation $q_g(R - C) > 0$ if it enters the market to finance its project, and a bad issuer obtains $q_b(R - C) > 0$. Therefore, both types of issuers will enter the market. As a result, the probability that a project on the marker is good is identical to the prior, and, by condition (3), the investors demand $C = 1/(pq_g + (1 - p)q_b) > R$ in order to finance the projects, contradicting the starting supposition that $C < R$.

Consider now the second benchmark where there is only one issuer and the CRA. Applying a similar argument to this benchmark, we show that the CRA creates no value. Again, suppose to the contrary that the it creates value. Then the CRA obtains a positive fee by giving a good rating to the issuer. Since the CRA obtains nothing by withholding a good rating, it will give a good rating to the issuer regardless of its quality. As a result, the CRA’s rating bears no information on the issuer’s quality, contradicting the supposition that CRA creates value.

It is a rating inflation problem that prevents the CRA from creating value in this benchmark case. If there is only one issuer and this issuer is of bad quality, the CRA obtains nothing by withholding a good rating, whereas it earns the rating fee by issuing it. It is because of this motive to earn one (more) fee that the CRA’s rating loses all its credibility in this benchmark case. Indeed, in any static setting, this motive to earn one more fee will always destroy the credibility of the CRA’s ratings as long as the value of the rating fee is not related to the number of good ratings issued by the CRA. However, as will see, a negative relationship between these two can emerge if the CRA rates many projects simultaneously. That partly contains the CRA’s motive to obtain one more fee, making the CRA’s ratings trustworthy enough to serve the purpose of screening the projects and create value.
4 Simultaneous Rating

In this section we analyze the model. As the CRA rates several issuers simultaneously and investors make their decisions only after observing all the ratings given by the CRA to issuers, the model captures the case of simultaneous rating. We are interested in characterizing the CRA’s rule when rating issuers, the informativeness of the CRA’s ratings, and the efficiency in the allocation of funds. We analyze separately two cases regarding the total number of issuers. We first consider the case of two issuers (i.e., $N = 2$). This is the case with the least simultaneity (measured here by the number of projects simultaneously rated). As such, it allows us to identify and characterize in the simplest possible setting the main effects that render ratings informative under simultaneous rating. We then consider the case of a large number of issuers and provide asymptotic results on the value created by the CRA and on the efficiency under simultaneous ratings.

4.1 Two Issuers

Consider the case with only two issuers (i.e., $N = 2$). Our first observation is that there is no equilibrium in which the CRA gives a good rating to an issuer if and only the issuer is good. To see this suppose that such an equilibrium exists. In this equilibrium, investors believe that a project is good when the CRA rates it as good. That is, $\phi = 1$ for any project that receives a good rating. This means, using (3), that investors require a repayment $C = 1/q_g < R$ to finance a project with a good rating. Then, in the event that no project is good, the CRA is better off if, instead of being honest and offering no good ratings obtaining no fee, it deviates by offering good ratings to both issuers. Specifically, the CRA can charge a fee $f = R - 1/q_g$ for each good rating and obtain an expected profit of $2q_b(R - 1/q_g) > 0$. The issuers accept this fee because unrated issuers obtain zero profit.\(^\text{11}\)

While credit ratings cannot fully eliminate the asymmetry of information between issuers and investors, we next show that the presence of the CRA can reduce it and create value. In what follows, let $C_k$ denote the repayment demanded by investors to fund an issuer with a good rating when $k$ good ratings are issued by the CRA, and $\Delta \equiv q_g - q_b$. Observe that condition (2) can be written as $p < -V_b/(\Delta R)$ and that the ex-ante probability that at least one project is good is $1 - (1 - p)^2 = p(2 - p)$. We can claim the following.

\(^{11}\)Recall that we focus on the equilibria in which a good rating signals good quality of the issuer. Thus, an issuer who has not received a good rating is regarded by investors to be of a quality no better than the prior and obtains no profit.
Proposition 1 If \( p \left( 2 - p \right) \leq -V_b/\Delta R \), then in no equilibrium the CRA creates value. If \( p \left( 2 - p \right) > -V_b/\Delta R \), however, there exists a continuum of equilibria in which the CRA creates value. Specifically, for each \( x \in [u, 1] \), with \( u = \max \{0, 1 - \frac{(1-p)^2}{p} \frac{1-u}{q_g-V_b} \} \), there exists an equilibrium in which:

(a) if both issuers are good, the CRA issues one good rating with probability \( x \) and two good ratings with probability \( (1-x) \);

(b) if only one issuer is good, the CRA issues one good rating and gives it to the good issuer;

(c) if no issuer is good, the CRA issues one good rating with a certain probability \( y \) and two good ratings with probability \( (1-y(x)) \), where \( y > 0 \), \( y \) increases with \( x \), and \( y = 1 \) at \( x = 1 \).

In any of these equilibria \( C_1 < C_2 \). These are all the equilibria in which the CRA creates value if \( p \leq 0.5 \).

A good credit rating creates value if it changes investors’ belief about the quality of a project to the point where investors are willing to finance it at a repayment \( C < R \). If ex-ante the probability that at least one issuer is good is sufficiently low, then it is very likely that there are no good projects and that what the CRA rates as good is actually bad – recall that in this circumstance it will still issue a good rating if that rating is worth something. However, if the probability that at least one issuer is good is above the threshold, \( -V_b/\Delta R \), the ratings issued by the CRA can be credible enough to create value.

To understand how the CRA can create value and why the threshold is \( -V_b/\Delta R \), let us focus on one particular equilibrium amongst those identified in the proposition: the equilibrium where the CRA issues one good rating irrespective of the number of good issuers (i.e., the equilibrium where \( x = y = 1 \)). Intuitively, one can think of this equilibrium as a situation in which investors ask the CRA to recommend no more than one issuer; if it recommends both issuers, then they will regard its ratings as much less credible or even meaningless. The CRA then issues only one good rating.

More importantly, this rating will be trusted in the sense that the project rated as good will be financed at a required repayment \( C_1 < R \). The reason is that the CRA gives this good rating to a good issuer whenever there is one; it gives it to a bad issuer only in the event that neither of the issuers is good. This is because ex-ante the CRA obtains an expected profit of \( q_g(R - C_1) \) by selling
the good rating to a good issuer, while it obtains \( q_g (R - C_1) \) from a bad issuer, and the former is greater than the latter, as \( q_g > q_b \). Therefore, the issuer that receives the good rating is actually good so long as at least one of the issuers is good, and this occurs with probability \( p (2 - p) \). If this probability is greater than \(-V_b / (\Delta R)\), the project rated as good has a positive NPV, and financing it creates value.

This discussion shows that there are two effects under simultaneous rating that reduce the CRA’s incentive to inflate ratings enabling it to create value. (1) The negative link between the value of a good rating and the number of such ratings. In the equilibrium discussed above, if there is only one good rating, it is worth \( R - C_1 \) ex-post (conditional on the success of the project), whereas if there are two of them, then each is worthless or has a much smaller value (\( R - C_2 \) is negative or positive but much small than \( R - C_1 \)). (2) The CRA gives a good rating to a good issuer first before giving it to a bad issuer. This holds true in general: in competing for a good rating, a good issuer is willing to pay more than a bad issuer because the former’s project generates a higher value. Together, the first effect makes the CRA abstain from giving a good rating to all the issuers, and the second effect ensures that an issuers with a good rating is more likely to be good than one without it. Therefore, a good rating indicates a better quality than the prior; that is, it serve the function of signalling the quality of issuers.

In the equilibrium discussed above the CRA recommends only one issuer. Proposition 1 also identifies equilibria in which investors finance issuers even when the CRA recommends both issuers. In such equilibria, a good rating creates value even when both issuers receive a good rating. However, the reasons why the CRA creates value in these equilibria are the same as those in the equilibrium discussed above and result from the two key effects highlighted: the negative link between the value of a rating and the number of such ratings, which results from the fact in all equilibria \( C_1 < C_2 \); and the fact that the CRA is better of giving a good rating to a good issuer than to a bad issuer.

4.2 A Large Number of Issuers

We have seen that the existence of a CRA can improve efficiency in a simple setting with two issuers. In this section, we consider the case of a large number of issuers and obtain asymptotic results on the value created by a CRA.

In doing so, we focus on a particular type of equilibrium similar to that identified in Proposition
1 where $x = y = 1$. Specifically, we focus on equilibria where investors finance all the issuers with a good rating at a fixed repayment of $C$, so long as the total number of good ratings issued by the CRA does not exceed a threshold $k$. That is, the CRA is asked to recommend at most $k$ of the $N$ projects, or put differently, the number of good ratings is subject to a quota of $k$. The negative link, therefore, is that a good rating is worth $R - C$ ex post (conditional on the success of the issuer) if the total number of good ratings is no greater than $k$, otherwise it is worthless. Observe that in such equilibria the CRA will not miss the chance to earn one more rating fee and will always recommend $k$ issuers. We call equilibria of this type pooling equilibria, as the CRA issues the same number of good ratings irrespective of the number of good projects. There is, however, a slight abuse of language in doing so, as pooling occurs only regarding the total number of good ratings given by the CRA. The identity of the issuers that are rated as good depends on the realization of issuers’ types. As before, when rating issuers, the CRA will optimally give good ratings first to good issuers, and only if it cannot find $k$ of them to fill the quota, will it fill the gap by giving the rest of the good ratings to bad issuers. That is, the CRA will indeed recommend the best $k$ amongst the $N$ projects.

For each number of issuers $N$ sufficiently large, an equilibrium of this type always exists. Roughly speaking, by the Law of Large Numbers, the CRA will not have difficulty finding $Np$ good projects. Therefore, if $k$ is close to $Np$, investors can trust that a project recommended by the CRA is highly likely to be good. Furthermore, for the same number of issuers $N$, pooling equilibria with a different quota $k$ of good ratings may coexist. When this is the case, however, the expected total surplus (i.e. efficiency) is not the same across such equilibria. Given a quota $k$, relative to the first-best allocation in which a project is financed if and only if it is good, there are two types of loss. One occurs whenever the realized number of good projects is greater than $k$, as some of them do not receive a good rating, are not financed, and their NPV is lost. The other occurs when the realized number of good projects is smaller than $k$, as the CRA will fill the quota with bad projects, which are then financed and destroy value. With the quota increased, the first type of loss becomes smaller, the second bigger. Given the total number of issuers ($N$), the optimal value of $k$ balances these two types of loss.

Clearly, for a different total number of issuers, $N$, the optimal quota is different. The next proposition characterizes how asymptotically this optimal quota depends on $N$ and the efficiency of the optimal pooling equilibrium (i.e. the pooling equilibrium with the optimal quota) approaches
Proposition 2 Consider the case where the number of borrowers $N$ is large. The optimal quota is asymptotically $k = Np + \lambda \sqrt{Np(1 - p)}$, where $\lambda$ is implicitly defined by $\Phi(\lambda) = V_g/(V_g - V_b)$ with $\Phi(.)$ denoting the c.d.f. of the standard Normal distribution. Moreover, the ex-ante value of the pooling equilibrium with this optimal quota asymptotically approaches the value of the first-best allocation and the probability that a project with a good rating is actually good approaches one. In both cases, the approximation is in the order of $N^{-1/2}$.

At the optimal pooling equilibrium, the quota for good ratings is equal to the unconditional mean of the number of good projects, $Np$, adjusted by $\lambda$ times the standard deviation of the number of good projects. The magnitude of this adjustment depends on the value created by a good project and the value destroyed by a bad project. Specifically, as the value created by a good project $V_g$ increases relative to the value destroyed by a bad project $-V_b$, $\lambda$ increases and thus the quota is higher. The intuition for this result is simple. As $V_g$ increases relative $-V_b$, the loss from leaving good projects unfinanced becomes more important relative to the loss from financing bad projects. Therefore, it is optimal to increase the quota. In the special case in which a good project creates as much value as a bad project destroys, i.e. when $V_g = -V_b$, $\lambda = 0$ and the optimal quota precisely equals the mean of the number of good projects, $Np$.

The analysis so far highlights the potential gains generated by the presence of a CRA when rating is simultaneous. Proposition 2 is particularly important as it highlights that in the case of a large number of projects those gains can be very large. Specifically, while the first-best allocation is never attainable in equilibrium (as we have shown at the beginning of subsection 4.1), the loss of value becomes negligible relative to the first-best allocation. This contrasts sharply with the case where no CRA operates in the market, which we have considered in the first benchmark case. In that case, no value is created. (Note that the argument there applies independent of the total number of issuers.)

One may wonder whether the two effects that allow the CRA to create value under simultaneous rating – the negative link between the value of a good rating and the number of such ratings, and the effect of good issuers willing to pay more for a good rating – also work in the case of sequential rating, and, in case they do, whether they work equally well. A consideration based solely on statistics suggests they do. In particular, consider Proposition 2. It is driven by the Law of Large Numbers and we know this law applies to the dynamic settings equally well. For example, it rules the case
of throwing the same coin 1000 times equally well as it rules the case of throwing 1000 coins of the same attributes once. However, this consideration ignores that the CRA’s incentives when rating issuers may change considerably under sequential rating. As we will see, because of this change in incentives, under sequential rating the value created by the CRA can never approach the first-best value.

5 Sequential Rating

Consider now a dynamic version of the baseline model where the CRA rates issuers sequentially. Specifically, suppose there are $N$ periods and in each period a different issuer comes to seek financing its project. All the other aspects of the model remain unchanged. Hence, in any given period, that period’s issuer decides whether to obtain a credit rating from the CRA, the CRA observes the quality of the issuer and rates it if solicited to do so, then the issuer tries to obtain funding from investors and, if successful in doing so, finances its project. One fundamental difference between the baseline model and this version of the model is that in the former the investors make their decisions after observing the ratings obtained by all the $N$ issuers, whereas in here they make their decision in each given period based on the history of ratings up to that period. The period discount factor is $\beta \in (0, 1)$ and is the same for all agents.

As in the baseline model, we assume that investors do not observe the performances of the previously financed projects. This assumption allows us to abstract from the typical reputation mechanism by which investors revise their opinion on the implications of credit ratings by comparing the ratings given to issuers and the issuers’ ex-post performance. Hence, the credibility given by investors to a good credit rating in a given period depends only on the history of (good) ratings given by the CRA up that period. We follow in this section an approach similar to that in Section 4 where we investigated the case of simultaneous rating. We first analyze the case where $N = 2$ and then consider the case where $N$ is infinite. In both cases we focus on the informativeness of ratings and the value created by the CRA. Clearly, as before, if no CRA operates in the market no value is created.
5.1 Two Issuers

Suppose there are two periods and one issuer per period. We solve for the equilibrium interaction between the CRA, the issuers and the investors using backward induction.

We begin by arguing that in the second period a good rating cannot create value irrespective of what has happened in the first period. To see this, suppose to the contrary that investors finance the project with a required repayment $C < R$ if the project is rated as good. In this case, the CRA will rate the period two’s issuer as good irrespective of its quality. The CRA can charge a fee $R - C$ for the good rating (which it will receive in the event the project succeeds), while the CRA receives nothing if it issues no good rating. But in equilibrium investors anticipate this behavior on the part of the CRA. Hence, they rationally ignore the rating given by the CRA, i.e. they believe that the issuer is good with probability $p$ (the prior) even if the issuer is rated as good. However, with these beliefs, they will demand a repayment $C > R$ to finance the project – which means that they will not finance the project – contradicting the initial assumption that $C < R$. Having established that a good rating cannot create value in period two, we next argue that it cannot create value in period one either. Any decisions regarding the issuer in period one will not affect what will happen in period two. Therefore, the analysis done for the case of the period-two issuer, fully applies to the case of the period-one issuer. The observation that credit ratings can create value in neither period means that the CRA cannot create value at all. For convenience of exposition, we state this without further proof in the next proposition.

Proposition 3 The CRA creates no value if rating is sequential and there are two issuers.

This proposition constitutes the first step in highlighting the difference between simultaneous rating and sequential rating. With two issuers, ratings can create value under simultaneous rating, but they do not create value under sequential rating. Recall that credit ratings create value under simultaneous rating because of the negative link between the value of a good rating offered by the CRA and the number of such ratings, which gives the CRA an incentive to limit the number of good ratings to be issued. Under sequential rating, with two issuers, these incentives vanish. In the last period, the offer of a good rating by the CRA does not reduce the value of the good ratings issued in previous periods. This means that the negative link is not present in the last period. As a result, the CRA is solely driven by the motive of acquiring one (more) rating fee and rates the issuer as
good irrespective of its quality, if it can charge a positive fee for this good rating, rendering credit ratings totally uninformative about issuer quality. This, in turn, means that rating cannot generate any fee in the second period. Moreover, the fact that rating generates no fee in period two regardless of what happens in period one, means that the negative link is also not present in period one. As a result the CRA creates no value.

One may wonder whether the CRA could solve the problem of the lack of credibility of its ratings under sequential rating by being able to commit ex-ante to issue only a limited number of good ratings. The answer to this questions is no. To see this, suppose the CRA can commit ex-ante to issue only one good rating over the two periods. Observe that if the CRA does not rate the period one issuer as good, it will have the incentive to rate the second period issuer as good regardless of its quality (if it can charge a positive fee for the good rating). This implies that investors will never trust a good rating issued in period two, meaning that a good rating in period two is necessarily worthless. Hence, the CRA will always issue a good rating in period one. Then, by the same reasoning, we can conclude that this good rating must be worthless in period one also. Hence, commitment by the CRA to issue a limited number of good ratings is not sufficient to ensure the credibility of its credit ratings under sequential rating.

While we have analyzed here the case of two issuers, the analysis and results obtained in this subsection hold for any finite number of issuers.\textsuperscript{12} Therefore, the CRA cannot create value under sequential rating when the number of issuers is finite. We next consider the case in which there are infinitely many issuers (periods) and analyze the potential for value creation by the CRA. In this case, the negative link between the number of good ratings and the value of a good rating does not necessarily vanish, as it does in the case of a finite number of issuers. However, as we will see, it is weaker than it is in the case of simultaneous rating.

5.2 An Infinite Number of Issuers

Suppose now that each period a new issuer enters the market and that this occurs indefinitely. In other words, the CRA expects to be in the rating business forever. In the context of our model, this situation corresponds to the case where the number of periods is infinite. In this case, backward induction can no longer be used to obtain the equilibrium interaction between the issuers, CRA and

\textsuperscript{12}To obtain this, one can simply apply the argument for why rating creates no value in period two backwardly, first to the last period, then to the second last period, and so on.
investors. As the CRA expects to rate issuers indefinitely, the CRA’s rating decision at any given period can potentially affect the value of its ratings in all the future periods. This creates room for the negative link to operate and for the CRA to create value. In this subsection we derive an upper bound for the value created by a CRA in equilibrium. We then compare the value of this upper bound with the value created by a CRA under simultaneous ratings when the number of issuers is large.

Before presenting such an upper bound, it is instructive to briefly discuss how equilibria where the CRA creates value might emerge. The CRA can create value if its ratings are at least partially credible. This occurs when the CRA has an incentive not to rate every issuer as good and also an incentive to give good ratings to good issuers (as opposed to giving them to bad issuers). The following is an example of a situation where both of these incentives emerge. Suppose investors fully trust a rating given by the CRA if the CRA has not issued a good rating for a certain period of time, say in the last $l$ periods, and totally disregard a rating given by the CRA if otherwise. In such a setting, the CRA faces a cost when issuing a good rating: it has to wait $l$ periods until its ratings are trusted again and issuers are willing to pay for them. Because of this cost and because a good issuer can pay more for a good rating than a bad issuer, the CRA may prefer to wait until a good issuer appears in the market to issue a good rating, even if $l$ periods have elapsed since it rated an issuer as good for the last time. If the CRA indeed prefers to do so, investors will be right to fully trust a good rating issued by the CRA after an abstinence in issuing good ratings for at least $l$ periods.

One can easily envision other situations where the credibility of the ratings given by the CRA evolves in a different way over time. An important observation though is that the efficiency in the allocation of funds (and therefore the value created by the CRA) may differ from one situation to the other. The next proposition presents an upper bound for the value created by the CRA across all possible equilibria. Note that the discounted expected first-best total surplus, namely, the value created in the case where in each period the project is financed if and only if it is good, is $pv_g/(1-\beta)$. In what follows we let $V^{FB}_s = pv_g/(1-\beta)$.

Proposition 4 The value created by the CRA under sequential rating is no larger than $(1-q_b/q_g) \times V^{FB}_s$.

There are two potential sources for the loss of value relative to first-best allocation: the finance of bad projects; and the abandonment of some good projects. Interestingly, one can show that the most
efficient equilibria (those where the value created by the CRA is maximal) are of the sort described in the example given above.\textsuperscript{13} In that type of equilibria, only the second type of loss occurs. Once the CRA issues a good rating it has to wait \( l \) periods until it can issue another good rating that will be trusted by investors. However, any good issuer arriving during this waiting period will not be financed and its project will be abandoned.

Proposition 4 states that at most a fraction \( 1 - q_b / q_g \) of the first-best total surplus can be realized under sequential rating. This result, which is independent of the value of the discount factor \( \beta \), contrasts with that obtained in the case of simultaneous rating. As stated in Proposition 2, the value created by the CRA under simultaneous rating asymptotically approaches the first-best value. Two factors explain the difference between the two cases.

First, the effect of the negative link is stronger under simultaneous rating than under sequential rating, and thus it is easier to provide the CRA with the incentives to limit the number of good ratings under simultaneous rating than under sequential rating. Under simultaneous rating, the CRA’s decision to issue one additional good rating lowers the value of \textit{all} the ratings issued by the CRA. Under sequential rating, that decision can lower only the value of the ratings issued by the CRA in the \textit{future}, and cannot affect what the CRA has obtained in the past. Observe that this is the same reason why the CRA fails to create value under sequential rating when the number of issuers is finite, as argued in the discussion that follows Proposition 3.

Second, good projects are more likely to be abandoned under sequential rating than under simultaneous rating. This is because the order with which good and bad projects appear in the market matters under sequential rating. To illustrate this point, consider the event where \( k \) out of \( N \) projects are good. Suppose again the situation under sequential rating where, after issuing a good rating, the CRA has to wait \( l \) periods until it can issue another good rating. If the \( k \) good projects appear in the market consecutively, there is a congestion of good projects and many will necessarily be abandoned. Under simultaneous rating, the order with which good projects appear is irrelevant, and there is no loss of surplus due to congestion of good projects.

\textsuperscript{13}This result is obtained as part of the proof of Proposition 4.
5.3 Synchronization of Bond Issuance

The dominance of simultaneous rating over sequential rating in terms of the value created by the CRA (as indicated by a comparison of Propositions 2 and 4) suggests that in the sequential setting where issuers enter the market sequentially, there may be a gain in waiting so that a group of them are rated by the CRA simultaneously. Thereby the issuance of their bonds is synchronized. Therefore, this paper suggests an extra benefit to the idea of synchronizing bond issuance, which the practitioners have been considering mainly for the benefit of improving the liquidity of corporate bonds.\textsuperscript{14} Waiting, however, is costly. The optimal length of waiting associated with the functioning of credit rating, then, should balance the benefit of simultaneous rating and the cost of waiting.

To formalize this idea, let the length of waiting be \( N \) periods. That is, the issuers arriving in the market in periods 1, 2, \ldots, \( N-1 \) wait until period \( N \) and then, in this period, all the issuers (including the issuer that arrives in period \( N \)) are simultaneously rated by the CRA and seek funding from investors. Similarly, the issuers arriving in periods \( N+1, N+2, \ldots, 2N-1 \) wait until period \( 2N \) to be simultaneously rated, and so on. Consider one such cycle of \( N \) periods. By Proposition 2, for \( N \) large, the expected value created with all the \( N \) projects, measured at period \( N \), is approximately

\[
NpVg(1 - \sqrt{1/N}c),
\]

where \( c > 0 \) is independent of \( N \).\textsuperscript{15} Calculated at period \( t = 1 \), the value that the CRA creates with these \( N \) projects is thus \( V_N = \beta^{N-1} \cdot NpV_g(1 - \sqrt{1/N}c) \), while the expected first-best value at \( t = 1 \) with these projects is \( V^{FB} = pV_g \cdot (1 + \beta + \ldots + \beta^{N-1}) \). The efficiency relative to the first-best for these \( N \) projects is thus given by

\[
S_N = \frac{V_N}{V^{FB}} = \frac{\beta^{N-1}N}{1 + \beta + \ldots + \beta^{N-1}}(1 - \sqrt{1/N}c).
\]

Observe that \( S_N \) is also the ratio of values created with all (infinite) projects. The first term of \( S_N \) represents the cost of waiting that issuers incur. The denominator, \((1 + \beta + \ldots + \beta^{N-1})/N\), is the average discount factor in the first-best case in which the good project at period \( t \) is implemented at \( t \) immediately and thus its present value calculated at \( t = 1 \) is discounted with factor \( \beta^{t-1} \), whereas \( \beta^{N-1} \) is the discount factor in the simultaneous rating in which all projects are implemented in period

\textsuperscript{14}See the article “The Debt Penalty” in the Financial Times, 11 September 2013.

\textsuperscript{15}While we use here the asymptotic approximation, observe that in the case of the binomial distribution this involves only a small error even for low values of \( N \). For example, for a binomial with probability of success of 1/2 and \( N = 16 \), the difference between the true cdf and its the asymptotic approximation never exceeds 0.002 (see Mosteller, Rourke and Thomas, 1961, page 277).
and thus discounted with $\beta^{N-1}$. The second term, measures the efficiency relative to the first-best; as we noted, the loss is in order of $\sqrt{1/N}$.

Using this formalization, we can clearly see how the length of waiting, $N$, balances the the benefit of simultaneous rating and the cost of waiting. The first term of $S_N$ decreases with $N$, which captures the fact that the cost of waiting increases with $N$. The second term increases with $N$, which captures the economy of scale associated with simultaneous rating: the greater the number of issuers rated simultaneously, the smaller the efficiency loss relative to the first-best value. It is not difficult to show that at a unique $N$ these two sides are perfectly balanced and $S_N$ is maximized. Moreover, this optimal $N$ increases with the discount factor $\beta$ and goes to infinity as $\beta$ goes to one because a higher value of $\beta$ implies a lower waiting cost and if $\beta$ approaches to 1, the waiting cost approaches 0.

It is easy to see that waiting is worthwhile if $\beta \rightarrow 1$. In this case, $1 + \beta + ... \beta^{N-1} \rightarrow N$ and hence $S_N \rightarrow 1 - \sqrt{1/N}$. This means that for $N$ large enough, $S_N$ is close to one. However, from Proposition 4, under pure sequential rating, the ratio of the equilibrium value to the first-best one is no greater than $1 - q_b / q_g$. Therefore, for any given large number $N$, performing simultaneous rating every $N$ periods (projects) dominates pure sequential rating if $\beta$ is close to one. The intuition is simple, when $\beta$ is close to one, the cost of waiting to achieve the economy of scale in rating vanishes, and only the advantages of the simultaneous rating emphasized in previous subsections remain.

6 Discussion and Extensions

The analysis above highlights that simultaneous reporting of credit ratings can provide a mechanism to contain rating inflation and enhance rating credibility. In this section, we explore the robustness of our key findings to a set of alternative modeling assumptions and explore some possible generalizations of our setting.

Competition in the market for credit ratings We have considered so far the case in which only one CRA operates in the market for credit ratings. We have seen that under simultaneous rating credit ratings can be credible in equilibrium and signal the quality of issuers to investors, and that this is because of the two following effects. First, a monopolist CRA has the incentive to limit the number of good ratings it issues, since the value of each good rating negatively depends on the total number of good ratings issued. Second, a monopolist CRA gains more by giving a good rating to
a good issuer than by giving it to a bad issuer. Here, we briefly discuss how competition between CRAs might affect these two effects.

We begin with the observation that under some extreme beliefs the equilibria obtained in the case of a monopoly continue to exist even if there are two or more CRAs available for rating. Suppose, for example, investors (potentially) trust only one particular rating agency. That is, investors’ beliefs about issuers’ quality are never affected by the ratings given by any of the other CRAs. Given these beliefs, the trusted CRA can and will act as monopolist in the market. The ratings given by the other CRAs will be ignored in equilibrium and eventually will not even be issued. Another system of beliefs that sustains the monopoly outcomes is that where investors change their beliefs about an issuer’s quality only if the issuer obtains a good rating from all the CRAs. In this case, as long as coordination between CRAs is feasible, CRAs will coordinate on the issuers to whom they give a good rating and (jointly) act as the monopolist, each obtaining the respective share of the monopoly profits of the industry. This observation holds both in the case of simultaneous rating and in the case of sequential rating.

The beliefs considered above essentially rule out the possibility of competition between CRAs for issuers. In the perhaps more plausible environments where all or some CRAs are individually trusted (at least partially) by investors, competition between CRAs is likely to emerge. If competition results in lower fees charged by CRAs to issuers, then it may weaken CRAs’ incentives to rate issuers well under simultaneous rating. The reason is the following. By reducing rating fees, competition weakens CRAs’ ability to extract surplus from issuers. In particular it weakens CRAs ability to extract surplus from good issuers whose projects generate more value, reducing CRAs’ incentive to give good ratings to the good issuers (instead of the bad issuers). However, this effect of competition on CRAs’ incentives is also present under sequential rating with infinite issuers. Hence, from this effect one cannot conclude that competition weakens the benefits of simultaneous rating relative to sequential rating. Moreover, observe that even if the CRA obtains the same rating fee from a good issuer and from a bad issuer, potential reputation concerns will still drive the CRA to give a good rating first to the good issuer. This illustrates how the mechanism considered in this paper and the reputation mechanisms considered in the literature may be complementary.

Another implication of competition is that it may reduce the benefits of simultaneous rating by decreasing the magnitude of the economy of scale. Observe that if one CRA is asked to rate one issuer
only, the rating cannot be informative in equilibrium. However, as shown in Section 4.1, credit ratings can be informative when one CRA is asked to rate two issuers. Also, as shown in Section 4.2, for a large number of issuers, the value created by a CRA asymptotically approaches the first-best total surplus. All these suggest an economy of scale associated with simultaneous rating. If competition between CRAs leads to a split of the rating market between them, the scale of each CRA is reduced and part of the benefit of the economy of scale is lost. Consider again the case of a large number \(N\) of issuers studied in Section 4.2. In the case of a monopolist CRA, the loss in expected surplus (relatives to first-best) is \(c\sqrt{N}\), for some \(c > 0\).\(^{16}\) Suppose now there are two CRAs with market shares \(\alpha\) and \(1 - \alpha\), respectively. The surplus loss in this case is given by \(c\sqrt{\alpha N} + c\sqrt{(1 - \alpha) N}\). Since \(\sqrt{N} < \sqrt{\alpha N} + \sqrt{(1 - \alpha) N}\), such loss is clearly greater in the case of two CRAs than in the case of a monopolist CRA.

**Issuers with correlated qualities** We have assumed that issuers’ quality draws are independent. In some cases this assumption may be unrealistic. Some exogenous factors may affect simultaneously the intrinsic quality of all issuers. For example, during a period of fast economic growth all projects may be more likely to succeed. Similarly, during a recession, all projects are less likely to succeed and their probability of default may increase. It is straightforward to extend the analysis in Section 4.1 to allow for intrinsic correlation between issuers’ qualities. Let \(P_n\) denote the probability that exactly \(n\) in \(\{0, 1, 2\}\) issuers are of good quality. Clearly, \(\sum_{n=0}^{2} P_n = 1\). Continue to assume that issuers are symmetric so that the probability that a given issuer is good is \(P_1/2 + P_2\), and that this probability is low enough to discourage investors from financing an issuer at random, i.e. assume \([(P_1/2 + P_2)(q_g - q_b) + q_b] R < 1\). This condition, which can be written as \(P_1/2 + P_2 < -V_b/\Delta R\), is the counterpart of (2). Observe that this formulation allows for any correlation between the qualities’ of the two issuers. Following the steps in Section 4.1, one can obtain a generalized version of Proposition 1.\(^{17}\) Specifically, if \(P_1 + P_2 \leq -V_b/\Delta R\), then the CRA cannot create value in any equilibria. However, if \(P_1 + P_2 > -V_b/\Delta R\), then the equilibria identified in Proposition 1 exist and these are the unique equilibria in which the CRA creates value under some further conditions. This result suggests that the advantages of simultaneous rating remain in more general environments where issuers’ qualities may be correlated.

\(^{16}\)See the proof of Proposition 2 for the complete mathematical expression of this loss and its derivation.

\(^{17}\)The proof of this more general version of Proposition 1 follows the exact same steps as those of Proposition 1 and are omitted.
Actually, a negative correlation may enhance the value created by the CRA under simultaneous rating. Consider the case in which there are two issuers and $P_1 = 1$, that is, one and only one of the issuers is good, but investors do not know the identity of the good issuer. Consider the equilibrium stated in Proposition 1 with $x = y = 1$, namely the equilibrium in which investors ask the CRA to recommend one and only one issuer. The CRA, in equilibrium, recommends the good issuer, which is willing to pay more for the good rating. In this equilibrium the first best allocation is implemented, that is, a project is financed if and only if it is good. Applying the same argument that we used in Subsection 5.1 for why the CRA creates no value under sequential rating if there are two (or any finite number) of issuers, we can show that the CRA creates no value either for this case of perfect negative correlation.

**Commitment by investors** Section 5.1 shows that a CRA cannot create value when issuers are rated sequential and the number of issuers is finite. Furthermore, we have argued in that section that such failure to create value persists even if the CRA could commit ex-ante to issue only a limited number of good ratings. Interestingly, a CRA can create value under sequential rating if investors (instead of the CRA) could commit ex-ante to finance only a limited number of issuers if these issuers have a good rating. In particular, some of the outcomes obtained under simultaneous rating can be achieved under sequential rating if such commitment by investors is possible.

Consider for example the outcome obtained under simultaneous rating (and identified in Proposition 1) where only one issuer receives a good rating regardless of the realized number of good issuers and that issuer is financed. This outcome can be replicated under sequential rating if investors commit ex-ante to finance one and only one issuer (at a repayment identical to that in the outcome under simultaneous rating).\(^{18}\) Given this commitment by investors, the CRA will give the good rating to the issuer in period one only if that issuer is good, and otherwise will wait until period two to give the good rating; it is willing to wait rather than give the good rating to the bad issuer in period one because the issuer in the next period is on average of higher quality and can pay the CRA more for the good rating. In equilibrium, one issuer will be financed irrespective of the realized number of good issuers. Commitment by investors is necessary for the financing to happen because otherwise, they will not finance the issuer in period two in the event that the period-one issuer does not receive

\(^{18}\)This repayment, which is equal to $1/(q_b + p(2-p)\Delta)$, is precisely the one under which investors (ex-ante) break-even also in the sequential setting.
the good rating: the investors know that in such a situation, the period two issuer will receive a good rating regardless of its quality and thus investing in it will make a loss. However, a good rating issued in period one means that the period-one issuer is good with probability one, and thus investing in it generates a net gain. In equilibrium, ex-ante the loss incurred by investors when they finance the period two issuer is offset by the gain obtained when they finance the period one issuer, and investors break even.

Observe that more commitment by investors is required to implement other equilibria obtained under simultaneous rating; for example, the equilibria identified in Proposition 1 where the number of good ratings issued by the CRA depends on the number of good issuers (i.e., equilibria where \( x < 1 \) and \( y < 1 \)). To achieve such outcomes under sequential rating, investors would have to be able to commit ex-ante to a repayment schedule that is contingent on the number of good ratings issued by the CRA over time. This means, for example, that the terms of the contract offered by investors to one particular issuer would have to depend on the number of ratings offered by the CRA to other issuers in the future. While conceptually conceivable, this type of commitment requires substantial coordination by investors and may be infeasible in practice.

7 Conclusion

There have been concerns that credit rating agencies have inflated the ratings of some financial products. This is in part because during the recent financial crisis, highly rated products performed very poorly and their ratings had to be significantly downgraded. We show in this paper that requiring a rating agency to report the ratings of many issuers at once provides a mechanism to discipline it against rating inflation and to increase the credibility of its ratings. The simultaneity in the reporting of credit ratings disciplines rating agencies because it allows for a link between the CRA’s decisions on the ratings of different borrowers: by giving more good ratings, the rating agency lowers the credibility of its ratings and the fee it can charge for a good rating. Moreover, it provides the rating agency with incentives to give good ratings to good projects, as these are the projects that can pay more for a good rating. We also show that there is an economy of scale associated with simultaneous rating. When the number of issuers simultaneously rate is sufficiently large, the surplus generated in equilibrium asymptotically approaches the first-best surplus.

This paper’s findings suggest an extra benefit to the idea of synchronizing bond issuance, which
the practitioners have been considering mainly for the benefit of improving the liquidity of corporate bonds. Specifically, the synchronization of bonds issuance may allow the mechanism highlighted in the paper to work, increasing the value created by rating agencies. There are, however, potential costs associated with the implementation of such a mechanism. One of such costs is the potential delay in the implementation of projects. Firms needing funds to implement a new project may have to wait longer for a credit rating and, consequently, for the needed funds. In this respect, the periodicity with which credit ratings can be reported seems important, as it affects the magnitude of both the benefits and the cost of the scheme. The implementation of the mechanism proposed in this paper in practice may require a more comprehensive analysis of all of its effects, including those on bond liquidity.
8 Appendix

Proof of Proposition 1. Let $\phi_k$ denote the investors’ belief that an issuer with a good rating has a good project if the CRA issues $k \in \{1, 2\}$ good ratings. Both $\phi_k$ and $C_k$ are determined in equilibrium. Let $n \in \{0, 1, 2\}$ denote the number of good issuers and $P_n$ denote the exogenous ex-ante probability that state $n$ occurs. That is, $P_0 = (1-p)^2$, $P_1 = 2p(1-p)$, and $P_2 = p^2$. In what follows, we first show that $p(2-p) > -V_0/\Delta R$ is a necessary condition for the existence of equilibria where the CRA creates value, referred to as value creating equilibria, and then list all these equilibria when that condition holds. The proof is given in the following steps.

Step 1: Preliminary considerations. Recall that we focus on the equilibria in which an unrated issuer is of lower-than-prior quality, and thus, as shown in the first benchmark case presented in Section 3, obtains no profit. This has two implications. (1) In any value creating equilibrium, all issuers approach the CRA for a rating. This is because unrated issuers obtain no profit, while even a bad issuer has a positive chance of receiving a good rating and being financed (note that in the event no issuer is good, the CRA will still issue a good rating, otherwise its profit is zero). (2) When offering a good rating to an issuer the CRA chooses the rating fee $f$ so as leave the issuer with surplus $\epsilon > 0$ (but very small and ignored hereafter) and appropriate the remainder of surplus generated by its project (i.e. of $R-C$). Thus, conditional on issuing a total of $k$ good ratings, the CRA’s expected profit from giving a good rating to an issuer of quality $q_i$ is $q_i \times \max\{R - C_k, 0\}$.

Three observations follow.

(i): In any value creating equilibrium, the CRA issues at least one good rating, namely, $k \in \{1, 2\}$, because it obtains nothing if it issues no good rating.

(ii): In any value creating equilibrium, the CRA obtains all the surplus and therefore its profit is positive. It follows that $C_k < R$ must hold for at least one $k \in \{1, 2\}$.

(iii): As $q_g > q_b$, the CRA obtains more by giving a good rating to a good issuer than it obtains by giving a good rating to a bad issuer. Hence, if $k = 1$, the CRA gives the good rating to a good issuer unless there is none, that is, $n = 0$.

Step 2: In any value creating equilibrium, $C_1 < C_2$. Suppose to the contrary that there exists a value creating equilibrium in which $C_1 \geq C_2$. From observation (ii) in Step 1, it follows that $C_2 < R$. Given any profile of issuers’ qualities, $(q_1, q_2)$, the CRA strictly prefers issuing two good ratings (one to each issuer) to issuing only one good rating: doing the former it obtains $(q_1 + q_2) \times (R-$
the latter $\max\{q_1, q_2\} \times \max\{R-C_1, 0\}$, and $(q_1+q_2) \times (R-C_2) > \max\{q_1, q_2\} \times (R-C_2) \geq \max\{q_1, q_2\} \times \max\{R-C_1, 0\}$. As the CRA rates both issuers as good irrespective of their qualities, $\phi_2 = p$, necessarily. It follows from (3) and (2) that $C_2 = 1/(pq_g + (1-p)q_b) > R$, which contradicts the previous assertion that $C_2 < R$.

The facts that $C_1 < C_2$ and that $C_k < R$ for at least one $k \in \{1, 2\}$ imply that in any value creating equilibrium, either (a) $C_1 < R < C_2$; or (b) $C_1 < C_2 \leq R$. These cases are analyzed separately in the next two steps.

**Step 3: Value creating equilibria in which $C_1 < R < C_2$ exist if and only if $p(2-p) > -V_b/(\Delta R)$.** In any value creating equilibrium of this type, if the CRA issues one good rating, it obtains $\max\{q_1, q_2\} \times (R-C_1) > 0$; while if it issues two good ratings, as $R < C_2$, neither of the issuers is financed, and the CRA obtains nothing. Therefore, the CRA always issues only one good rating. By observation (iii) in Step 1, this good rating is issued to a good issuer whenever there is one, that is, in states $n \in \{1, 2\}$. Thus, the project of the issuer with the good rating is good in the event $n \in \{1, 2\}$ and bad in the event $n = 0$. Ex-ante, therefore, it is good with probability $P_1 + P_2$, i.e. $\phi_1 = P_1 + P_2$. Its NPV, $(P_1 + P_2) \times V_g + (1-P_1 - P_2) \times V_b$, is positive – thus the rating creates value and $C_1 < R$ if and only if $P_1 + P_2 = p(2-p) > -V_b/(\Delta R)$. Now we move on to case (b).

**Step 4: No value creating equilibrium in which $C_1 < C_2 \leq R$ exists if $p(2-p) \leq -V_b/(\Delta R)$.**

With $C_1 < C_2 \leq R$ and by observation (iii) in Step 1, the CRA’s expected profit from issuing $k \in \{1, 2\}$ good ratings in state $n \in \{0, 1, 2\}$, denoted by $\pi^n_k$, is: $\pi^n_0 = q_b \times (R-C_1)$; $\pi^n_1 = \pi^n_2 = q_g \times (R-C_1)$; $\pi^n_2 = 2q_b \times (R-C_2)$; $\pi^n_2 = (q_b + q_g) \times (R-C_2)$; and $\pi^n_2 = 2q_g \times (R-C_2)$. Note that

\[
\pi^n_1 \geq \pi^n_2 \iff R - C_1 \geq 2(R - C_2)
\]
\[
\pi^n_1 \geq \pi^n_1 \iff R - C_1 \geq (q_g + q_b)/(R-C_2)
\]
\[
\pi^n_1 \geq \pi^n_2 \iff R - C_1 \geq 2(R - C_2)
\]

From direct inspection of these conditions and the fact that $(q_g + q_b)/(R-C_2) < 2(R - C_2)$, one obtains that there are three relevant subcases regarding the values of $R - C_1$ and $R - C_2$, which are analyzed separately in next three substeps.

**Step 4.1:** Suppose $R - C_1 < ((q_g + q_b)/(R-C_2) < 2(R - C_2)$. If a value creating equilibrium
exists for this case, \( C_2 < R \) as \( 2(R - C_2) > R - C_1 > 0 \). In this case \( \pi_2^n > \pi_1^n \) for all \( n \in \{0, 1, 2\} \).

Hence, in an equilibrium with such values of \( C_1 \) and \( C_2 \), if it exists, the CRA issues two good ratings regardless of the number of good issuers. Bayesian updating by investors implies that \( \phi_2 = p \). It follows from (3) and (2) that \( C_2 = 1/(pq_g + (1-p)q_b) > R \), which contradicts the previous assertion that \( C_2 < R \). Hence, there are no value-creating equilibrium in this case.

**Step 4.2:** Suppose that \( ((q_g + q_b)/q_g)(R - C_2) \leq R - C_1 < 2(R - C_2) \). Again \( C_2 < R \) if a value-creating equilibrium exists for this case. In this case, \( \pi_2^n > \pi_1^n \) for all \( n \in \{0, 2\} \) and \( \pi_1 < \pi_2 \).

Hence, if a value-creating equilibrium exists for this case, the CRA issues two good ratings both when \( n = 0 \) and when \( n = 2 \), and in the event \( n = 1 \), the CRA either issues one good rating, or randomizes between issuing one and two good ratings. Let \( \mu \) denote the probability that the CRA issues one good rating when \( n = 1 \). Bayesian updating by investors implies that \( \phi_1 = 1 \) and

\[
\phi_2 = \frac{P_1(1 - \mu)/2 + P_2}{P_0 + P_1(1 - \mu) + P_2}.
\]

There are two possible cases depending on the relative values of \( P_0 \) and \( P_2 \). If \( P_0 \geq P_2 \Leftrightarrow (1 - p)^2 \geq p^2 \Leftrightarrow p \leq 0.5 \), then \( \phi_2 \) decreases with \( \mu \), which implies that \( \phi_2 \leq P_1/2 + P_2 = p \). It follows from (3) and (2) that \( C_2 > 1/(pq_g + (1-p)q_b) > R \), which contradicts the assertion that \( C_2 < R \). If instead \( P_0 < P_2 \), the \( \phi_2 \) increases with \( \mu \) and \( \phi_2 \leq P_2/(P_0 + P_2) \). From (3) it follows that

\[
C_2 \geq 1/[(P_0/P_2)q_g + (1 - P_2/P_0)q_b],
\]

which, again contradicts \( C_2 < R \) if \( p(2-p) \leq \frac{V_b}{R} \Leftrightarrow P_1 + P_2 \leq \frac{V_b}{R} \). This is because \( 1/[(P_0/P_2)q_g + (1 - P_2/P_0)q_b] \geq R \) which is equivalent to \( P_2 + \frac{V_b}{R} \cdot P_1 \leq \frac{V_b}{R} \), which is implied by \( P_1 + P_2 \leq \frac{V_b}{R} \) because \( \frac{V_b}{R} = \frac{V_b}{V_b - V_b} < 1 \).

**Step 4.3:** Suppose that \( ((q_g + q_b)/q_g)(R - C_2) < 2(R - C_2) \leq R - C_1 \). If a value-creating equilibrium exists for this case, \( C_1 < R \). In this case, \( \pi_1^n > \pi_2^n \) for all \( n \in \{0, 2\} \) and \( \pi_1 > \pi_2 \). Hence, if a value-creating equilibrium exists, the CRA issues one good rating when \( n = 1 \), and either issues one good rating or randomizes between issuing one and two good ratings when \( n = 0 \) or \( n = 2 \).

Let \( \mu_0 \) and \( \mu_2 \) denote the probability that the CRA issues one good rating when \( n = 0 \) and \( n = 2 \),
respectively. Bayesian updating by investors implies

\[
\phi_1 = \frac{P_1 + \mu_2 P_2}{\mu_0 P_0 + P_1 + \mu_2 P_2} \quad \text{and} \quad \phi_2 = \frac{(1 - \mu_2)P_2}{(1 - \mu_0)P_0 + (1 - \mu_2)P_2}.
\] (4)

If \(2(R - C_2) < R - C_1\), then \(\pi^1_n > \pi^2_n\) for all \(n \in \{0, 2\}\) and in equilibrium \(\mu_0 = \mu_2 = 1\). This implies \(\phi_1 = P_1 + P_2\). It follows from (3) that \(C_1 = 1/((P_1 + P_2)q_g + (1 - P_1 + P_2)q_b)\). If \(P_1 + P_2 \leq -V_b/(\Delta R)\), then \(C_1 \geq R\), whereas we saw that \(C_1 < R\) if a value-creating equilibrium exists. Thus, such equilibria do not exist.

If instead \(R - C_1 = 2(R - C_2)\), then \(C_1 < C_2 < R\). Solving the simultaneous equations in (4) with respect to \(\mu_0\) and \(\mu_2\), we obtain

\[
\mu_0 = \frac{(P_1 + P_2 - \phi_2)(1 - \phi_1)}{(1 - P_2 - P_1)(\phi_1 - \phi_2)} \quad \text{and} \quad \mu_2 = \frac{(P_1 \phi_2 + P_2 \phi_1 - \phi_1 \phi_2)}{(\phi_1 - \phi_2) P_2}.
\] (5)

Since by (3) \(C_2 = 1/(q_L + \phi_2 \Delta)\). Thus, if \(C_2 < R\), then \(\phi_2 > (-V_L)/\Delta R\). From (5) and \(\mu_0 \geq 0\) it follows that \(P_1 + P_2 \geq \phi_2\), which implies that \(P_1 + P_2 \geq \phi_2 > (-V_L)/\Delta R\), a contradiction to the condition of the proposition.

By exhausting all the possible cases regarding the values of \(R - C_1\) and \(R - C_2\), we have shown that no value creating equilibrium in which \(C_1 < C_2 \leq R\) exists if \(p(2 - p) \leq -V_b/(\Delta R)\). Moreover, Steps 3 and 4 combined imply that no value creating equilibrium exists if \(p(2 - p) \leq -V_b/(\Delta R)\). We next analyze existence of value creating equilibria when \(p(2 - p) > -V_b/(\Delta R)\). In Step 3 we already established the existence of value creating equilibria in which \(C_1 < R < C_2\). Thus, we next focus on equilibria in which \(C_1 < C_2 \leq R\).

**Step 5: Existence and characterization of value-creating equilibria in which \(C_1 < C_2 \leq R\) if \(p(2 - p) > -V_b/(\Delta R)\).** Such equilibria exist only in the subcase considered in step 4.3 if \(p \leq 0.5\), as the argument that shows the non-existence of this type of equilibria in steps 4.1-4.2 do not require assumption \(p(2 - p) \leq -V_b/\Delta R\) and depends only on condition (2) if \(p \leq 0.5\). It is clear that there exists an equilibrium in which the CRA issues one good rating for all \(n \in \{0, 1, 2\}\), i.e. in which \(\mu_0 = \mu_2 = 1\). In this case, as shown in step 4.3, \(\phi_1 = P_1 + P_2\) and \(C_1 < R\) if \(p(2 - p) > -V_b/\Delta R\). The event where the CRA issues two good ratings is off-the-equilibrium path. Hence, we can choose \(\phi_2\), which pins down \(C_2\) through (3), so that \(R - C_1 \geq 2(R - C_2)\), which makes it optimal for the CRA to issue one good rating for all \(n \in \{0, 1, 2\}\).
Let us now consider equilibria where $\mu_n < 1$ for some $n \in \{0, 2\}$. In this type of equilibrium, $\pi^n_1 = \pi^n_2$, which is equivalent to

$$R - C_1 = 2(R - C_2).$$

(6)

Hence, such an equilibrium exists if and only if there exists $\mu_0$ and $\mu_2$ such that: (i) $\phi_1$ and $\phi_2$ satisfy (4); and (ii) $C_1$ and $C_2$ satisfy (6), and

$$C_i = \frac{1}{\phi_i q_b + (1 - \phi_i) q_g}$$

and $C_i \in [1/q_g, R]$ for both $i = 1, 2$. We next show that for each $\mu_2 \geq \max\{0, 1 - \frac{(1-p)^2 (1-q_b/q_g-V^*_b)}{V^*_g}\}$ such an equilibrium exists. By (4),

$$\mu_0 = \frac{1 - \phi_1}{\phi_1} \cdot \frac{P_1 + P_2 \mu_2}{P_0} \quad \text{and} \quad 1 - \mu_0 = \frac{1 - \phi_2}{\phi_2} \cdot \frac{P_2 (1 - \mu_2)}{P_0}.$$  

(8)

It follows that

$$\frac{1 - \phi_1}{\phi_1} \cdot \frac{P_1 + P_2 \mu_2}{P_0} + \frac{1 - \phi_2}{\phi_2} \cdot \frac{P_2 (1 - \mu_2)}{P_0} = 1.$$  

(9)

From (7) it follows that for all $i = 1, 2$,

$$\frac{1 - \phi_i}{\phi_i} = \frac{q_g C_i - 1}{1 - q_b C_i}.$$  

(10)

Substituting it into (9) and rearranging, we obtain

$$\frac{q_g C_1 - 1}{1 - q_b C_1} \cdot (P_1 + P_2 \mu_2) + \frac{q_g C_2 - 1}{1 - q_b C_2} \cdot P_2 (1 - \mu_2) = P_0.$$  

(11)

Observe that for any $(C_1, C_2) \in [1/q_g, R]^2$, by (10) $(1 - \phi_i)/\phi_i \geq 0$ for $i = 1, 2$, which by (8) implies that $\mu_0 \geq 0$ and $1 - \mu_0 \geq 0$. This means that given $\mu_2 \in [0, 1)$, if the simultaneous equations (6) and (11) have a solution for $(C_1, C_2)$ in $[1/q_b, R]^2$, then there exists $\mu_0 \in [0, 1]$ such that $\mu_0$ and $\mu_2$ constitute an equilibrium. Using (6) to eliminate $C_2$ in (11), we obtain

$$g(C_1) = \frac{q_g C_1 - 1}{1 - q_b C_1} \cdot (P_1 + P_2 \mu_2) + \frac{q_g (C_1 + R)/2 - 1}{1 - q_b (C_1 + R)/2} \cdot P_2 (1 - \mu_2) = P_0.$$  

(12)
Observe that $g$ is continuous and $g' > 0$ for all $C_1 \in (1/q_b, R]$. Moreover,

$$g(R) = \frac{V_g}{V_b} \cdot (P_1 + P_2 \mu_2) + \frac{V_g}{V_b} \cdot P_2 (1 - \mu_2) = \frac{V_g}{V_b} \cdot (P_1 + P_2).$$

Thus, $g(R) > P_0 \iff P_1 + P_2 > (-V_b)/V_g \cdot P_0 \iff P_1 + P_2 > (-V_b)/V_g \cdot [1 - (P_1 + P_2)] \iff P_1 + P_2 > (-V_b)/(V_g - V_b)$, or equivalently $p (2 - p) > -V_b/\Delta R$. It follows that equation (12) has a solution in $C_1 \in [1/q_g, R]$ (and that solution is unique) if and only if $g(1/q_g) \leq P_0$. Since

$$g(1/q_g) = \frac{V_g}{1 - q_b/q_g - V_b} \cdot P_2 (1 - \mu_2),$$

this condition is satisfied if and only if $\mu_2 \geq \max\{0, 1 - \frac{P_b}{1 - q_b/q_g - V_b} \} = \max\{0, 1 - \frac{(1-p)^2}{p^2} \frac{1 - q_b/q_g - V_b}{V_g} \}.$

Since when (6) is satisfied, $C_2 \in [1/q_g, R]$ whenever $C_1 \in [1/q_g, R]$, we obtain the existence of the equilibrium. Next, observe that by (8), $\mu_0 = 1$ when $\mu_2 = 1$. To obtain the result stated in the proposition, simply let $x = \mu_2$ and $y = \mu_0$, and also note that in the value creating equilibria where $C_1 < R < C_2$, described in Step 3 above, the CRA’s strategy is to issue one good rating regardless of the issuers’ qualities and is thus the same as the equilibrium described in Step 5 with $x = y = 1$.

**Proof of Proposition 2.** Given the number of issuers $N$, if the realized number of good issuers is $n \in \{0, 1, \ldots, N\}$, the ex post value created (or total surplus generated) in a pooling equilibrium with quota $k$ is $kV_g$ if $n \geq k$ and $nV_g + (k - n)V_b$ if $n < k$. Thus, the expected total surplus (before $n$ is realized) is given by

$$V(k, N) = \Pr(n \geq k) \times kV_g + \sum_{n=0}^{k-1} \Pr(n < k) \times nV_g + (k - n)V_b$$

$$= \Pr(n \geq k) \times kV_g + \Pr(n < k) \times kV_b + (V_g - V_b) \times \sum_{n=0}^{k-1} P_n n$$

$$= kV_g - \Pr(n < k) \times (V_g - V_b) + (V_g - V_b) \times \sum_{n=0}^{k-1} P_n n$$

$$= kV_g - (V_g - V_b) [\Pr(n < k) k - \sum_{n=0}^{k-1} P_n n],$$

where $P_n$ denotes the ex-ante probability that $n$ issuers are good. The optimal $k$ maximizes $V(k, N)$. Finding the optimal $k$ is not a trivial problem. Therefore, we next obtain an asymptotic approximation to $V(k, N)$.
Observe that
\[
\frac{V(k, N)}{N} = \frac{k}{N} V_g - (V_g - V_b) \frac{\Pr(n < k)}{N} k - \sum_{n=0}^{k-1} P_n n.
\] (13)

In what follows, it is convenient to do variable transformations to \(k\) and \(n\) with \(k = Np + \lambda N \sqrt{Np(1-p)}\) and \(n = Np + t \sqrt{Np(1-p)}\), where \(\lambda, t \in R\). Hence, we can write
\[
\frac{k}{N} = p + \lambda \sqrt{\frac{p(1-p)}{N}}.
\] (14)

By the Central Limit Theorem, asymptotically, \(t \sim N(0,1)\) with density \(\phi(t) = (\sqrt{2\pi})^{-1} e^{-\frac{t^2}{2}}\) and c.d.f. \(\Phi(t) = \int_{-\infty}^{t} \phi(s)ds\). By the Barry-Esseen Theorem, for \(N\) large,
\[
\Pr(n < k) = \Phi(\lambda N) + O\left(\frac{1}{\sqrt{N}}\right).
\] (15)

Observe also that
\[
\sum_{n=0}^{k-1} P_n \frac{n}{N} = \Pr(n < k) E\left[\frac{n}{N} \mid n < k\right] = \Pr(n < k) E\left[p + t \sqrt{\frac{p(1-p)}{N}} \mid n < k\right] = \Pr(n < k) E[p + t \sqrt{\frac{p(1-p)}{N}} \mid n < k] = \Pr(n < k) p + \sqrt{\frac{p(1-p)}{N}} \int_{-\infty}^{\lambda N} t \phi(t) dt + o\left(\frac{1}{\sqrt{N}}\right).
\]

Using the fact that \(\int_{-\infty}^{\lambda N} \phi(t) dt = \int_{-\infty}^{\lambda N} \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} dt = -\phi(\lambda)\), we can write
\[
\sum_{n=0}^{k-1} P_n \frac{n}{N} = \Pr(n < k) p - \sqrt{\frac{p(1-p)}{N}} \phi(\lambda) + o\left(\frac{1}{\sqrt{N}}\right).
\] (16)
Using (14), (15) and (16) into (13), we obtain

\[
\frac{V(k, N)}{N} = V_g \times (p + \lambda N \sqrt{\frac{p(1 - p)}{N}}) - (V_g - V_b) \times \{[\Pr(n < k) \times (p + \lambda N \sqrt{\frac{p(1 - p)}{N}})] - [\Pr(n < k) \times p - \sqrt{\frac{p(1 - p)}{N}} \phi(\lambda N) + o\left(\frac{1}{\sqrt{N}}\right)]\}
\]

\[
= \frac{pV_g + \sqrt{\frac{p(1 - p)}{N}} \lambda N V_g}{N} - \frac{\sqrt{\frac{p(1 - p)}{N}} (V_g - V_b) \left[\Phi(\lambda N) \lambda N + \phi(\lambda N)\right]}{N} + o\left(\frac{1}{\sqrt{N}}\right)
\]

\[
= \frac{pV_g + \sqrt{\frac{p(1 - p)}{N}} \{\lambda N V_g - (V_g - V_b) \left[\Phi(\lambda N) \lambda N + \phi(\lambda N)\right]\}}{N} + o\left(\frac{1}{\sqrt{N}}\right).
\]

Hence, asymptotically, the optimal \(k\) for each \(N\) can be obtained by solving:

\[
\max_{\lambda N} \lambda N V_g - (V_g - V_b) \left[\Phi(\lambda N) \lambda N + \phi(\lambda N)\right] = 0,
\]

Observe that \(\Phi'(\lambda N) = \phi(\lambda N)\) and \(\phi'(\lambda N) = -\lambda N \phi(\lambda N)\). The first-order condition associated with this maximization problem is

\[
V_g - (V_g - V_b) \Phi(\lambda N) = 0,
\]

from which it follows that the optimal \(\lambda N\) does not depend on \(N\) and is implicitly defined by the condition

\[
\Phi(\lambda^*) = \frac{V_g}{V_g - V_b}.
\]

Hence, asymptotically, the optimal \(k\) is \(k^* = Np + \lambda^* \sqrt{Np(1 - p)}\) and the expected total surplus evaluated at the optimal pooling equilibrium is

\[
V(k^*) = N(pV_g - \sqrt{\frac{p(1 - p)}{N}} (V_g - V_b) \phi(\lambda^*)).
\]

Since the first-best expected total surplus is \(V^{FB}(N) = NpV_g\), we obtain that

\[
\frac{V^{FB}(N) - V(k^*, N)}{V^{FB}(N)} = \sqrt{\frac{1 - p}{Np}} (V_g - V_b) \phi(\lambda^*),
\]

which means that \(V(K^*, N)\) asymptotically approaches \(V^{FB}(N)\) in the order of \(N^{-1/2}\). Finally, observe that under the optimal \(k\), the probability that a project with a good rating is indeed good
when $N$ is sufficiently large is given by

$$
\phi^N_{k^*} = \Pr(n \geq k^*) \times 1 + \sum_{n=0}^{k^*-1} P_n \frac{n}{k^*}
$$

$$
= 1 - \Phi(\lambda^*) + \frac{N}{k^*} \times \sum_{n=0}^{k^*-1} P_n \frac{n}{N} + O\left(\frac{1}{\sqrt{N}}\right)
$$

$$
= 1 - \Phi(\lambda^*) + \frac{1}{p + \lambda^* \sqrt{\frac{p(1-p)}{N}}} \times [p\Phi(\lambda^*) - \sqrt{\frac{p(1-p)}{N}} \phi(\lambda^*)] + O\left(\frac{1}{\sqrt{N}}\right)
$$

$$
= 1 - \Phi(\lambda^*) + \frac{1}{p} \times (1 - \lambda^* \sqrt{\frac{1-p}{Np}}) \times [p\Phi(\lambda^*) - \sqrt{\frac{p(1-p)}{N}} \phi(\lambda^*)] + O\left(\frac{1}{\sqrt{N}}\right)
$$

$$
= 1 - \Phi(\lambda^*) + \frac{1}{p} \times (1 - \lambda^* \sqrt{\frac{1-p}{Np}}) \times [\Phi(\lambda^*) - \sqrt{\frac{(1-p)}{Np}} \phi(\lambda^*)] + O\left(\frac{1}{\sqrt{N}}\right)
$$

$$
= 1 - \Phi(\lambda^*) + \Phi(\lambda^*) + O\left(\frac{1}{\sqrt{N}}\right)
$$

$$
= 1 + O\left(\frac{1}{\sqrt{N}}\right).
$$

It converges to one in the order of $N^{-1/2}$. Finally, the fact that this probability converges to one ensures that all projects with a good rating are actually good with a probability close 1 and are thus financed in equilibrium, as was initially supposed. ■

**Proof of Proposition 4.** Let $E$ denote the set of Perfect Bayesian Equilibria of the game, $H_t$ the set of histories until the beginning of period $t$, $H = \bigcup_{t=1}^{\infty} H_t$ the set of all histories of the game, and $v_{e,h}$ the continuation value to the CRA in equilibrium $e$ after history $h$. We can define the set $\mathcal{V} = \{v = v_{e,h}| e \in E$ and $h \in H\}$. This is the set of all the continuation values to the CRA in any Perfect Bayesian equilibrium. $\mathcal{V}$ is non-empty, as there is always an equilibrium in which investors never trust a good rating given by the CRA and the CRA abstains from issuing any good rating. The continuation value to the CRA in this equilibrium is always zero. Moreover, $\mathcal{V}$ is bounded from above, as $v_{e,h} \leq pv_b/(1-\beta) = V_{FB}^s$ for all $e \in E$ and $h \in H$, which follows from the fact that investors always break-even. Let $\bar{\tau} = \sup \mathcal{V}$, which exists and is finite, since $\mathcal{V}$ is non-empty and bounded from above. If $\bar{\tau} = 0$, then the proposition trivially holds. Hence, the remainder of the proof supposes the case where $\tau > 0$ and consists of showing that in this case $\tau \leq (1 - q_b/q_y)V_{FB}^s$.

By definition of $\bar{\tau}$, it follows that for any given $\varepsilon > 0$, there exist an equilibrium $e$ and a history $h$ such that $v_{e,h} > \tau - \varepsilon$. Consider the continuation game at this equilibrium and history. Let $\gamma_b$ (resp. $\gamma_y$) be the probability that in the first period of the continuation game the CRA rates the
issuer as good in the event the issuer is good (resp. bad). Let also \( \gamma := p\gamma_g + (1 - p)\gamma_b \), which is the \textit{ex ante} probability that the CRA rates the issuer as good in that period. Similarly, let \( v_0 \) denote the continuation value to the CRA at the beginning of the second period if the CRA abstains from giving a good rating in the first period, and \( v_1 \) denote that value if the CRA issues a good rating in the first period. Given the CRA’s rating strategy, With Bayesian updating investors believe that if rated good, the issuer in the first period of the continuation game is actually good with probability \( \phi = p\gamma_g / (p\gamma_g + (1 - p)\gamma_b) \). This implies, by (3), that the repayment required by investors to finance the issuer is

\[
C = \frac{p\gamma_g + (1 - p)\gamma_b}{p\gamma_g q_g + (1 - p)\gamma_b q_b}.
\]

Hence, we can write

\[
v_{e,h} = p \cdot \{\gamma_g[q_g(R - C) + \beta v_1] + (1 - \gamma_g) [0 + \beta v_0]\} + (1 - p) \cdot \{\gamma_b[q_b(R - C) + \beta v_1] + (1 - \gamma_b) [0 + \beta v_0]\}
\]

\[
= [p\gamma_g q_g + (1 - p)\gamma_b q_b] \cdot (R - C) + \gamma \cdot \beta v_1 + (1 - \gamma) \cdot \beta v_0
\]

\[
= [p\gamma_g q_g + (1 - p)\gamma_b q_b] \cdot R - [p\gamma_g + (1 - p)\gamma_b] + \gamma \cdot \beta v_1 + (1 - \gamma) \cdot \beta v_0
\]

\[
= p\gamma_g V_g + (1 - p)\gamma_b V_b + \gamma \cdot \beta v_1 + (1 - \gamma) \cdot \beta v_0.
\]

(17)

By definition,

\[
v_0 \leq \bar{v}.
\]

(18)

For \( \varepsilon \) sufficiently small (i.e., for equilibria and history where the CRA’s continuation value is sufficiently close to \( \bar{v} \)), the CRA must create value in the first period of the continuation game (otherwise the continuation value to the CRA at the beginning of the next period would exceed \( \bar{v} \), which is impossible by definition of \( \bar{v} \)). Hence, the CRA must not be strictly better off giving a good rating to a bad issuer than abstaining from doing it, or

\[
0 + \beta v_0 \geq q_b \cdot (R - C) + \beta v_1 \iff 
\]

\[
\beta v_0 - q_b \cdot (R - C) \geq \beta v_1.
\]

(19)
Using inequalities (18) and (19) and equation (17), we obtain

\[ v_{e,h} = p \gamma_g V_g + (1 - p) \gamma_b V_b + \gamma \cdot \beta v_1 + (1 - \gamma) \cdot \beta v_0 \]

\[ \leq p \gamma_g V_g + (1 - p) \gamma_b V_b - \gamma_0 (R - C) + \beta v_0 \]

\[ \equiv f(\gamma_g, \gamma_b) + \beta v_0 \]

\[ \leq f(\gamma_g, \gamma_b) + \beta \bar{v}. \]

From this inequality and the fact that \( \bar{v} - \varepsilon < v_{e,h} \), it follows that \( \bar{v} - \varepsilon/(1 - \beta) < f(\gamma_g, \gamma_b)/(1 - \beta) \), which implies that \( \bar{v} < \max_{0 \leq \gamma_g, \gamma_b \leq 1} f(\gamma_g, \gamma_b)/(1 - \beta) + \varepsilon/(1 - \beta) \), and since this inequality holds for any \( \varepsilon > 0 \), we have:

\[ \bar{v} \leq \max_{0 \leq \gamma_g, \gamma_b \leq 1} f(\gamma_g, \gamma_b)/(1 - \beta). \]

(20)

We next calculate \( \max_{0 \leq \gamma_g, \gamma_b \leq 1} f(\gamma_g, \gamma_b) \). Recall that, as defined above,

\[ f(\gamma_g, \gamma_b) = p \gamma_g V_g + (1 - p) \gamma_b V_b - \gamma_0 (R - C), \]

and observe that we can write

\[ C = \frac{p + (1 - p)t}{pq_g + (1 - p)q_b t} \]

where \( t \equiv \gamma_b/\gamma_g \). Thus,

\[ f(\gamma_g, \gamma_b) = g(\gamma_g, t) \equiv \gamma_g \cdot \{ p v_g + (1 - p)v_b t - [p + (1 - p)t] \cdot q_b [R - \frac{p + (1 - p)t}{pq_g + (1 - p)q_b t}] \}. \]

(21)

Using the fact that \( V_b - \gamma_0 (R - C) = q_b R - 1 - q_b (R - C) = q_b C - 1 \), we obtain that

\[ \frac{\partial g}{\partial t} = -\gamma_g (1 - p) \left( \frac{p(q_g - q_b)}{pq_g + (1 - p)q_b t} \right)^2 < 0. \]

Therefore, to maximize \( g(\gamma_g, t) \), we set \( t = 0 \). It follows that to maximize \( f(\gamma_g, \gamma_b) \), we set \( \gamma_b = 0 \). Using (21), we obtain that \( f(\gamma_g, 0) = \gamma_g \cdot \{ p V_g - p \cdot q_b [R - 1/q_g] \} = \gamma_g \cdot p V_g (1 - q_b/q_g) \). Therefore, to maximize \( f \), we set \( \gamma_g = 1 \) and \( \max_{0 \leq \gamma_g, \gamma_b \leq 1} f(\gamma_g, \gamma_b) = p V_g (1 - q_b/q_g) \). It follows by (20) that \( \bar{v} \leq (1 - q_b/q_g) \cdot pv_g/(1 - \beta) \). This completes the proof. ■
References


