Simultaneous Reporting of Credit Ratings May Discipline Rating Agencies

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Abstract

We show that requiring a credit rating agency to report many ratings at once may discipline it against rating inflation. When the rating agency has to report ratings simultaneously, it faces a trade-off in the choice of the number of good ratings to report. While reporting one more good rating earns the agency one more fee, it also lowers the credibility of the good ratings the agency gives, thus diminishing borrowers’ willingness to pay for a good rating (and consequently the rating fee). In the case of a large number of borrowers, this mechanism ensures an allocation that asymptotically approaches the first best. While the mechanism may also emerge under sequential rating, it is less effective than under simultaneous rating. In the functioning of this mechanism, interestingly, the fact that borrowers pay for the ratings plays a necessary role. This paper suggests an additional benefit to synchronizing the issuance of corporate bonds.

Keywords: Credit Rating Agencies, Simultaneous Rating, Synchronization of Debt Issuance.

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1 Introduction

Credit rating agencies play an important role in the functioning of financial markets and allocation of capital. However, after the recent financial crisis there has been a growing concern (and evidence) of rating inflation. Take for example the case of Alt-A mortgage-backed securities (MBS). About 10% of the tranches issued in the period 2005-2007 rated most safe – triple AAA – were either downgraded to junk status or lost their principal by 2009. The case of CDO bonds was not better. More than 71.3% of such bonds had the same fate despite being initially rated as Aaa. Credit rating agencies have also been involved in lawsuits because of rating inflation. In 2008 a group of investors led by the Abu Dhabi Commercial Bank initiated a lawsuit against the investment bank Morgan Stanley and the two rating agencies Moody’s and Standard and Poor’s. The investors accused the rating agencies of collaborating with Morgan Stanley in arranging for some of its financial products to receive ratings as high as triple-A, even though much of the underlying collateral was low-quality or subprime mortgage debt. More recently, in February 2013, the U.S. Department of Justice filled a lawsuit against Standard & Poor’s accusing the rating agency of inflating ratings and understating risks associated with mortgage securities with the purpose of gaining more market share.

Inflation of credit ratings may cause huge losses to investors, as was evident in the recent crisis. The $5 billion compensation which the U.S. government seeks from Standard & Poor’s is just one signal of the magnitude of such losses. However, this is not the only cost of credit ratings inflation. The inflation of credit ratings also damages the credibility of rating agencies and thus affects the functioning of financial markets and the efficiency of capital allocation. This begs the question: how to discipline the credit rating agencies against rating inflation? The economics and finance literatures, which we review in more detail below, pays considerable attention to the reputation mechanism: if a credit rating agency gives a good rating to an issuer with a bad project, which is likely to perform poorly, then this bad performance will damage the agency’s reputation for issuing credible ratings.

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1 See the “Final Report of the National Commission on the Causes of the Financial and Economic Crisis in the United States”, pages 228-229. This report is authenticated U.S. government information.

2 As a reference point, observe that the historic cumulative default rate (up to 2007) of corporate bonds rated AAA by Standard & Poor’s is only 0.6%. In the case of Moody’s the analogous figure is 0.52%. (See the "House Report 110-835 - Municipal Bond Fairness Act" of September 2008, U.S. Government Printing Office.)

3 The case is Abu Dhabi Commercial Bank et al v. Morgan Stanley & Co et al, U.S. District Court, Southern District of New York, No. 08-07508. The parties reached a settlement agreement in April 2013. The settlement amount was almost 9.5 million dollars.

The mechanism relies on repeated interaction and on the comparison between the ratings obtained by issuers and their ex-post performance. This paper suggests a different disciplinary mechanism, which relies neither on repeated interaction nor on the comparison between issuers’ ratings and their ex-post performance. It simply requires a credit rating agency to simultaneously report the ratings of many issuers. An important consequence of this simultaneity in the reporting of ratings is that investors make their investment decisions only after observing the ratings given by the rating agency to many issuers. This situation is opposed to that of sequential rating where the rating agency reports issuers’ ratings (and investors decide whether to invest in each issuer) sequentially, which currently observed in reality. We show that simultaneous rating engenders disincentives for a credit rating agency to inflate ratings. This is underpinned by two economic effects.

First, simultaneous rating generates a negative link between the value of a good rating and the number of such ratings reported. When a credit rating agency rates many issuers simultaneously, the higher the number of issuers to which it gives a top rating, the lower the credibility of such ratings. This negative link between the value of a good rating and the number of such ratings forces the credit rating agency to face a trade-off in the choice of the number of good ratings it reports. By reporting one more good rating it earns one more fee but it also lowers the credibility of a good rating, which then lowers issuers’ willingness to pay for the agency’s ratings (and consequently the rating fee). Because of this trade-off, the credit rating agency has an incentive to limit the number of good ratings it gives to issuers; it may even optimally refrain from giving a good rating to a high-quality issuer. In other words, it is optimal for the credit rating agency to self-impose a quota as for the number of good ratings it issues.

Second, it is always optimal for a rating agency to fill the quota with issuers that have a high-quality project first and only afterwards with issuers that have low-quality projects. This is because given the cost of finance, high-quality projects create a greater value than low-quality projects, and therefore, the issuers of the former can (and are willing to) pay more for a good rating than those of the latter. Put differently, high-quality issuers can always beat low-quality issuers when competing for good ratings.

The combination of these two effects implies that in equilibrium, rating agencies will not give a good rating to all issuers and will give a good rating to a high-quality issuer before they give it to a low-quality issuer. Hence, an issuer with a good rating is more likely to be of higher quality than an
issuer without a good rating. This is why ratings will be credible, convey information about issuers’ qualities to investors, and increase the efficiency in the allocation of funds in the financial market.

We show that under simultaneous rating the presence of a credit rating agency (CRA) creates value even when the number of issuers is small. Moreover, in the case of a large number of issuers, the value created by a CRA asymptotically approaches the first-best total surplus. These results contrast with those obtained in the case of sequential rating, which we also analyze in the paper. Under sequential rating, a CRA may create value when the number of issuers is infinite. However, the surplus created is at most only a fraction of the first-best total surplus.

Two factors explain this difference. First, while the negative link between the number of good ratings issued by the CRA and their value may also emerge under sequential rating, it generates weaker incentives for the CRA than under simultaneous ratings. Under simultaneous rating, the CRA’s decision to issue one additional good rating lowers the value and rating fees of all the ratings issued by the CRA. Under sequential rating, because of its dynamic nature, that decision can lower only the CRA’s future rating fees. Second, under sequential rating, the order with which good and bad projects (issuers) appear in the market may matter. To build reputation, a CRA may have to wait a certain number of periods after issuing a good rating, until it can issue another good rating. If many good projects appear in the market consecutively, there is a congestion of good projects, many will not be rated and, consequently, abandoned due to the lack or higher cost of financing. Under simultaneous rating, the order with which good projects appear is irrelevant, and there is no loss of surplus due to congestion of good projects.

This paper highlights that simultaneous reporting of ratings generates a mechanism to discipline credit rating agencies against rating inflation and to improve the credibility of the ratings they give. Other mechanisms have been studied in the literature. The most notable of them is the reputation mechanism – see, among others, Kunher (2001); Mathis et al. (2009); Bolton et al. (2012), Mariano (2012); and Frenkel (2013). The reputation mechanism highlights that a rating agency will refrain from giving a good rating to a bad issuer because of the concern that its future credibility and revenues will be damaged later on following the (likely) default by the issuer. Unlike the reputation mechanism, the mechanism studied in this paper depends neither on repeated interaction nor on the comparison between projects’ rating with their ex-post performance. In fact, we abstract from

Interestingly, Frenkel (2013) shows that when a rating agency has two reputations—one with investors and another with issuers—reputation concerns may actually lead to the inflation of ratings by the rating agency.
the reputation mechanism in this paper by assuming that projects’ performance is never observed by investors. As the two mechanisms rely on different economic effects, they are complementary in disciplining rating agencies.

The literature has also highlighted that the credibility of credit ratings could be improved by addressing, in the first place, the conflicts of interest that may generate the bias in the ratings. One source of such conflicts of interest is the fact that credit rating agencies are paid by issuers – precisely those who they rate – and such payment usually occurs only if the issuer agrees with the disclosure of the rating.\footnote{Griffin and Tang (2011) provide empirical evidence of ratings inflation due to conflict of interest by comparing the CDO assumptions made by the ratings department and by the surveillance department within the same rating agency. Xia and Strobl (2012) provide empirical evidence of rating inflation due to the issuer-pay model by comparing the ratings issued by Standard & Poor’s which follows the issuer-pay model to those issued by the Egan-Jones Rating Company which adopts the investor-pay model.} Mathis, McAndrews and Rochet (2009), for example, advocate a new business model in which the platforms where the securities are traded pay for the ratings of the securities. In the present paper, we assume that issuers pay the credit rating agency to rate them. Interestingly, this helps the mechanism highlighted in this paper because it allows issuers with high-quality projects to beat issuers with low-quality projects when competing for good ratings, rendering good ratings credible.

Several articles have studied the impact of competition on the credibility and informativeness of ratings provided by a credit rating agency or more generally by a certifier (e.g., Lizzeri, 1999; Miao, 2009; Skreta and Veldkamp, 2009; Camanho, Deb and Liu, 2012; Bolton, Freixas and Shapiro, 2012). While Lizzeri (1999) shows that competition between certifiers can lead to full information revelation, Skreta and Veldkamp (2009), Camanho, Deb and Liu (2012) and Bolton, Freixas and Shapiro (2012) show that competition between credit rating agencies can in fact decrease the informativeness of credit ratings and the reputation of the rating agencies.\footnote{Becker and Milbourne (2010) provide evidence that increased competition due to Fitch’s entry in the credit ratings market in 1997 coincides with lower quality ratings from the incumbents. While Fitch was founded in 1913, the authors argue that only in 1997, following a merger with IBCA Limited, Fitch became an alternative global, full-service ratings agency capable of successfully competing with Moody’s and S&P.} In Skreta and Veldkamp (2009) and Bolton, Freixas and Shapiro (2012), this is because competition allows for credit rating shopping. In Camanho, Deb and Liu (2012) it is because it hinders rating agencies’ ability to sustain a high reputation.

An important difference amongst these papers is that Lizzeri (1999) assumes that the certifier can commit to a disclosure rule and the other articles assume that it cannot.\footnote{Another important difference is that Lizzeri (1999) takes a mechanism design approach to the modelling of the certifier. Faure-Grimaud, Peyrache and Quesada (2009) is another example of an article that follows this line of}
competition issues in this paper, we also assume that credit rating agencies cannot commit to a disclosure rule. That is, the credit rating agency is totally free to give any rating to an issuer after having been asked to rate it and having observed its quality.

Finally, some results of this paper are reminiscent of those in Damiano, Li and Suen (2008). The authors compare a rating agency that rates several clients separately (individual rating) with a rating agency that rates all clients together (centralized rating) and show that centralization of rating may enhance the credibility of the ratings. We instead compare simultaneous rating with sequential rating. Furthermore, they consider a costly signaling model as in Spence (1973), where producing a rating affects the agency’s payoff in itself, like obtaining a different educational degree incurs a different cost per see. Instead, we consider a model of cheap-talk, where reporting a rating affects the credit rating agency only through equilibrium interaction. This difference in modeling implies that in our paper the improvement in rating credibility under simultaneous rating is driven by the two economic effects aforementioned (i.e., the negative link and the fact that high-quality issuers can pay more for a rating than low-quality issuers), which do not appear in their paper.

The remainder of the paper is organized as follows. In Section 2 we present the baseline model. In the model, a set of firms seeks funds from investors to implement their projects and can ask a credit rating agency to rate their creditworthiness. The credit rating agency (simultaneously) evaluates the creditworthiness of all firms in the market and then investors decide whether or not to invest in each firm. This baseline model, which is analyzed in Section 3, captures the case of simultaneous rating, as investors make their investment decisions only after having observed the ratings of all firms. In Section 4 we analyze the case of sequential rating by considering a dynamic version of the baseline model. In this version of the model the credit rating agency rates firms’ creditworthiness sequentially (as firms try to access the credit market) and investors decide whether to invest in each firm immediately after observing its credit rating. We then compare the informativeness of credit ratings and efficiency in the allocation of funds in both cases and argue that they are higher under simultaneous rating than under sequential rating. Section 5 discusses some robustness issues related to our key findings and Section 6 draws a conclusion. All proofs are given in the Appendix.

modelling certifiers. They also use a mechanism design approach to model the certifier and assume that the certifier can commit to a disclosure rule.
2 Model

There are $N$ cashless firms, a pool of investors, and one credit rating agency (CRA hereafter). All agents are risk neutral and protected by limited liability. The risk free interest rate is normalized to zero. Each firm (issuer) has one investment project and seeks funds from investors to implement it. A project requires an investment of one unit of funds and either succeeds and returns $R$ or fails and returns nothing. There are two types of project, good ($g$) and bad ($b$). A good project succeeds with probability $q_g$, whereas a bad project succeeds with probability $q_b$. We assume that a good project has a positive net present value, while a bad project destroys value, i.e.,

$$q_b R < 1 < q_g R. \quad (1)$$

In what follows, we denote by $V_i$ the value created by a project of type $i$, i.e., $V_i = q_i R - 1$ for $i = b, g$. Observe that the conditions in (1) imply that investors are willing to finance good projects, but not bad projects. It is common knowledge that ex-ante a project is good with probability $p$ and projects' qualities are independent. We assume

$$[pq_g + (1-p)q_b]R < 1. \quad (2)$$

This condition means that (in expectation) a randomly drawn project destroys value. As we will see below, this condition implies that without any additional information on project quality, investors finance no project.

The quality of a firm’s project is known to the firm, but not to the investors. To overcome this informational problem, firms can hire the CRA to rate their projects before they seek funds from investors. If hired by a firm, the CRA perfectly observes the quality of its project at no cost.\footnote{We consider here the case of a monopolist CRA because it allows us to highlight the importance of the simultaneity in the reporting of credit ratings in the simplest possible way. In Section 5 we discuss the implications of competition between CRAs for our mechanism.} After observing the quality of all the projects for which a rating was solicited, the CRA simultaneously proposes to each issuer $i$ that requested a rating a contract specifying a rating $r_i \in \{\text{good, bad}\}$ and a rating fee $f_i$. Those issuers then accept or reject the contract proposed by the CRA. Issuers who

\footnote{The assumption that the CRA observes the quality of project at no cost is made for simplicity of exposition. The assumption that the CRA evaluates a firm’s project only if hired by the firm rules out the possibility of unsolicited reporting of credit ratings.}
reject the contract pay nothing to the CRA and remain unrated. Issuers who accept the contract commit to pay the rating fee to the CRA and their rating is publicly disclosed. Ratings are disclosed simultaneously. Rating fees are not observed by investors and are paid after the implementation of the project, as issuers are cashless. Investors learn that an issuer hired the CRA only if a rating for that issuer is publicly disclosed.

Investors observe the rating given to each issuer (if any) and decide which issuers to fund (if any). If investors decide to fund an issuer they demand a repayment $C$, which the issuer is able to pay only if its project succeeds. If the project fails, the issuer defaults, and investors obtain nothing. We assume that investors have more funds than those that can be absorbed by firms and, as a result, they demand an expected (net) return of zero.

Given the investors’ funding decisions, projects (if any) are implemented. In the case of successful projects, the issuer first pays investors the agreed repayment, then pays the CRA the agreed fee, and keeps the remainder as profit. In the case of unsuccessful projects all agents obtain zero.

The strategies of the issuers, CRA, and investors consist of the following. Upon observing the quality of its project, an issuer decides whether to hire the CRA and then whether to accept or reject the contract proposed by the CRA. Given the qualities of the projects it evaluates, the CRA decides the rating it offers to each issuer and the respective rating fee. Investors decide on which projects to fund (if any) and on the repayment required. We use Perfect Bayesian Equilibrium as the equilibrium concept. We focus on equilibria in which the investors’ belief that an unrated project or a project with a bad rating is of good quality is no higher than the prior $p$. Given this, issuers only accept to pay the CRA for a good rating (i.e., they reject any offers of a bad rating) and any unrated projects are not funded. Since issuers lose nothing by asking the CRA to evaluate their projects, we assume for simplicity of exposition that all issuers do so.

### 3 Simultaneous Rating

In this section we analyze the model presented in the previous section. Because the CRA rates issuers simultaneously and investors make their decisions only after observing all the ratings given by the CRA to issuers, the model captures the case of simultaneous rating. We are interested in characterizing the CRA’s rule when rating issuers, the informativeness of the CRA’s ratings, and the efficiency in the allocation of funds. We analyze separately two cases regarding the total number
of issuers. We first consider the case of two issuers (i.e., $N = 2$). This is the case with the least simultaneity (measured here by the number of projects simultaneously rated). As such, it allows us to identify and characterize in the simplest possible setting the main effects that render ratings informative under simultaneous rating. We then consider the case of a large number of issuers and provide asymptotic results on the value created by the CRA and on the efficiency under simultaneous ratings. In the next section we compare the equilibrium market outcomes obtained in both cases with those obtained in the analogous cases under sequential rating.

Before delving into the analysis of the equilibrium outcomes, let us consider the benchmark case where no CRA operates in the market. This analysis holds irrespective of the number of issuers. In the absence of a CRA, investors are unable to obtain additional information on projects’ qualities before making their investment decisions. Hence, their belief that any given project is of good quality is identical to the prior probability $p$. Even if investors require as repayment the maximum amount that a project can pay, $R$, their expected rate of return from investing in a project is

$$[pq_g + (1 - p)q_b]R - 1,$$

which by condition (2) is negative. This means that no project is financed and, consequently, no project is implemented. In particular, good projects (which may exist) are not implemented and no value is created.¹¹

As evidenced by the analysis of this benchmark case, investors’ beliefs play a crucial role in investors’ decisions and, consequently, in the overall functioning of the financial market. Before proceeding it is useful to characterize how investors’ beliefs affect their decisions. Let $\phi$ denote the investors’ belief that a given project is good. Because investors are on the long side of the market, they require a repayment $C$ to invest in the project so that their expected return is zero, i.e., they require $C$ such that

$$[\phi q_g + (1 - \phi)q_b]C - 1 = 0.$$  

¹¹We assume here that in the absence of a CRA all issuers, irrespective of their quality, always go to the market and seek funds from investors. If we assume instead that issuers’ decision to seek funds depends on the quality of their projects, some projects may be implemented in an equilibrium where good issuers seek funds with a higher probability than bad issuers. However, even in this case, no value can be created in such equilibria. Good projects and bad projects will be financed in such proportions that the value created by the former is destroyed by the latter. To see this, observe that if value is created when a project is financed, it will be appropriated by the issuer, as investors are on the long side of the market and always break even. But then all types of issuers would decide to seek funds, rendering the decision uninformative about issuer quality.
In other words, investors set the repayment $C$ so as to receive back in expectation precisely the unit of funds invested in the project. Hence, the lower the belief $\phi$ that the project is good, the higher the compensation they demand from the project. For sufficiently low values of $\phi$, that compensation may exceed $R$, in which case they refrain from investing in the project. Of course, investors’ beliefs are endogenous and are determined in equilibrium. In the benchmark case where no CRA operates in the market, $\phi = p$. In the presence of a CRA, investors’ beliefs depend on the rating of the issuer and on the CRA’s equilibrium strategy for rating issuers. Ratings are potentially important because they may affect investors’ decisions by affecting investors’ beliefs about issuers’ qualities.

We next analyze equilibrium outcomes in the credit ratings market as well as in the allocation of funds. Note that a (good) credit rating creates value if and only if investors demand a repayment $C < R$ from an issuer a good rating. In this case, the project generates positive expected profit, and it will not be financed without a good rating.

### 3.1 Two Issuers

Consider the case with only two issuers (i.e., $N = 2$). Our first observation is that no equilibrium in which credit ratings fully reveal the quality of all issuers exists. To see this suppose that such an equilibrium exists. In this equilibrium, investors believe that a project is good when the CRA rates it as good. That is, $\phi = 1$ for any project that receives a good rating. This means, using (3), that investors require a repayment $C = 1/q_g < R$ to finance a project with a good rating. But this implies that in the event that no project is good, the CRA is better off deviating by offering a good rating to both projects. Specifically, the CRA can charge a fee $f = R - 1/q_g$ for each good rating and obtain an expected profit of $2q_b(R - 1/q_g) > 0$. Recall that the CRA obtains zero profit in case it issues no good ratings, as issuers are not willing to pay for a bad rating.\footnote{While this observation is made for the case with two issuers, it is easy to see that it holds more generally for any number $N$ issuers.}

While credit ratings cannot fully eliminate the asymmetry of information between issuers and investors, the presence of a CRA can reduce it and alleviate the market inefficiency it generates. In other words, a CRA can create value. In what follows, $C_k$ denotes the repayment demanded by investors to fund an issuer with a good rating when $k$ good ratings are issued by the CRA, and $\Delta \equiv q_g - q_b$. Observe that condition (2) can be written as $p < -V_b/\Delta R$ and that the ex-ante probability that at least one project is good is $p(2 - p)$. We can claim the following.
Proposition 1 If \( p(2 - p) \leq -V_b/\Delta R \), then in no equilibrium the CRA creates value. If \( p(2 - p) > -V_b/\Delta R \), however, there exists a continuum of equilibria in which the CRA creates value. Specifically, for each \( x \in \{u, 1\} \), with \( u = \max\{0, 1 - \frac{(1-p)^2}{p} \frac{1-q_u/q_g-V_b}{V_g} \} \), there exists an equilibrium in which:

(a) if both issuers are good, the CRA issues one good rating with probability \( x \) and two good ratings with probability \( 1 - x \);

(b) if only one issuer is good, the CRA issues one good rating and gives it to the good issuer;

(c) if no issuer is good, the CRA issues one good rating with a certain probability \( y \) and two good ratings with probability \( 1 - y \), where \( y > 0 \), \( y \) increases with \( x \), and \( y = 1 \) when \( x = 1 \).

In any of these equilibria \( C_1 < C_2 \). These are all the equilibria in which the CRA creates value.

A good credit rating creates value when it changes investors’ beliefs about the quality of a project to the point where investors are willing to finance it and require a repayment \( C < R \). When the ex-ante probability that at least one issuer is good is sufficiently low, credit ratings are never powerful enough to achieve such a change in the investors’ beliefs.

However, for higher values of this probability, the ratings issued by the CRA can be credible enough to create value. To see why, let us focus on one particular equilibrium amongst those identified in the proposition: the equilibrium where the CRA issues one good rating irrespective of the number of good issuers (i.e., the equilibrium where \( x = y = 1 \)). Intuitively, one can think of this equilibrium as one where investors ask the CRA to recommend no more than one issuer and the CRA always recommends one. The CRA recommends one issuer because it can charge no rating fee if it recommends no issuer and because it can charge a much higher fee for a good rating when it issues one good rating then when it issues two good ratings. This is because investors find a good rating less credible when the CRA issues two good ratings than when it issues one good rating and request a higher repayment (i.e., \( C_1 < C_2 \)). Note that the CRA charges \( R - C_k \) for a good rating when it issues \( k \) good ratings.

The reason why the CRA creates value in this equilibrium is the following. While the CRA always issues one good rating, in the event that only one issuer is good, the good rating goes to the good issuer. Hence, upon observing that one good rating has been issued, investors learn nothing about the number of good issuers. However, they know that in the event that only one issuer is good, the good issuer is the one receiving the good rating. When the ex-ante probability that at least one
issuer is good is sufficiently high, this is enough to change investors’ beliefs about the quality of an issuer with a good rating to the point where investors are willing to finance its project. Indeed, the investors’ belief that an issuer with a good rating is indeed good is precisely \( p(2 - p) \), the *ex-ante* probability that at least one issuer is good. Observe that given the CRA’s rating strategy, the only situation where an issuer with a good rating is not good occurs precisely when no issuer is good. Given these beliefs and using the relationship in (3), it is easy to obtain that \( C_1 < R \).

To close the discussion on this equilibrium, observe that in the event that only one issuer is good, the CRA gives the good rating to the good issuer (rather than to the bad issuer) because it is optimal for the CRA to do so. The CRA can charge issuers a fee \( f = (R - C_1) \) for the good rating. Thus, the expected profit of the CRA from selling the good rating to the good issuer is \( q_g(R - C_1) \), whereas the CRA’s expected profit from selling the good rating to the bad issuer is \( q_b(R - C_1) \). Clearly, the former is greater than the latter, as \( q_g > q_b \). In other words, good issuers can beat bad issuers when they “compete” for good ratings. It is easy to obtain the value created by the CRA in this equilibrium. In equilibrium, one issuer is always financed. When no issuer has a good project, one bad project is financed, and when at least one issuer has a good project, one good project is financed. The expected total surplus is therefore \( (1 - p)^2 V_b + p(2 - p)V_g \).

In the equilibrium discussed above the CRA recommends only one issuer. Proposition 1 also identifies equilibria in which investors finance issuers even when the CRA recommends both issuers. In such equilibria, a good rating provides valuable information on the quality of an issuer even when all issuers obtain a good rating. However, the reasons why the CRA creates value in this equilibria and in the equilibrium discussed above are the same and result from two key effects that emerge under simultaneous rating. First, the negative link between the value of a good rating and the number of good ratings given by the CRA. Observe that in all equilibria, \( C_1 < C_2 \). This is precisely what refrains the CRA from always giving a good rating to all issuers. Second, the fact that the CRA is better off giving a good rating to a good issuer than to a bad issuer. This means that conditional on the number of good ratings that the CRA decides to issue, it will first give the good ratings to good issuers and only afterwards to bad issuers. It is the combination of these two effects that render credit ratings informative in all these equilibria.
3.2 A Large Number of Issuers

We have seen that the existence of a CRA can improve efficiency in a simple setting with two issuers. In this section, we consider the case of a large number of issuers and obtain asymptotic results on the value created by a CRA.

In doing so, we focus on a particular type of equilibrium similar to that identified in Proposition 1 where \( x = y = 1 \). Specifically, we focus on equilibria where investors finance all issuers with a good rating and demand each issuer the same repayment \( C \), as long as the total number of good ratings issued by the CRA does not exceed a fixed number \( k \). In other words, the CRA is asked to recommend at most \( k \) of the \( N \) projects. The CRA, therefore, optimally always recommends \( k \) issuers and asks each to pay \( R - C \). We call equilibria of this type pooling equilibria, as the CRA issues the same number of good ratings irrespective of the number of good projects. Observe that there is a slight abuse of language in doing so, as pooling occurs only regarding the total number of good ratings given by the CRA. The identity of the issuers that are rated as good depends on the realization of issuers’ types. As before, when rating issuers, the CRA will optimally give good ratings first to the good issuers and then, if some of the \( k \) good ratings remain to be given, it will fill the gap by giving those good ratings to bad issuers. That is, the CRA will indeed recommend the best \( k \) amongst the \( N \) projects.

For each number of issuers \( N \) sufficiently large, an equilibrium of this type always exists. Roughly speaking, by the Law of Large Numbers, the CRA will not have difficulty finding \( N p \) good projects. Therefore, if \( k \) is close to \( N p \), investors can trust that a project recommended by the CRA is highly likely to be good. Furthermore, for the same number of issuers \( N \), pooling equilibria with a different “quota” \( k \) of good ratings may coexist. When this is the case, however, the expected total surplus (efficiency) is not the same across such equilibria. There is a clear trade-off for efficiency associated with different values of \( k \). If \( k \) is too low, then too few good projects, which create value, are financed. If instead \( k \) is too large, then too many bad projects, which destroy value, are financed. In the optimal pooling equilibrium—i.e., the one in which the expected total surplus is highest—, the value of \( k \) optimally balances these two effects.

Clearly, for different numbers of total issuers \( N \), the optimal pooling equilibrium may consist of a different total number \( k \) of good ratings. The next proposition characterizes the asymptotically optimal pooling equilibrium as well as the expected total surplus evaluated at that equilibrium when
$N$ is large.

Proposition 2 Consider the case where the number of borrowers $N$ is large. In the asymptotically optimal pooling equilibrium, the number of issuers who receive a good rating (and whose projects are implemented) is $k = Np + \lambda \sqrt{Np(1-p)}$, where $\lambda$ is implicitly defined by $\Phi(\lambda) = V_g/(V_g - V_b)$ with $\Phi(.)$ denoting the c.d.f. of the normal distribution. Furthermore, the expected total surplus evaluated at this equilibrium asymptotically approaches the first-best expected total surplus, and the probability that a project with a good rating is good asymptotically approaches one. In both cases, the asymptotic approximation is in the order of $N^{-1/2}$.

At the optimal pooling equilibrium, the number of projects that receive a good rating is equal to the unconditional expected number of good projects $Np$ adjusted by $\lambda$ times the standard deviation of the distribution of the number of good projects. The magnitude of this adjustment depends on the value created by a good project and the value destroyed by a bad project. Specifically, as the value created by a good project $V_g$ increases relative to the value destroyed by a bad project $-V_b$, $\lambda$ increases, meaning that at the optimal pooling equilibrium more projects receive a good rating and are implemented. The intuition for this result is simple. As $V_g$ increases relative to $-V_b$, the loss from not implementing a good project increases relative to the loss of implementing a bad project and it becomes optimal to increase the number of implemented projects. In the limit case in which a good project creates as much value as a bad project destroys, i.e. when $V_g = -V_b$, the optimal equilibrium prescribes that the number of projects that receive a good rating and are implemented is precisely the expected number of good projects $Np$.

The analysis so far highlights the potential gains generated by the presence of a CRA when rating is simultaneous. Proposition 2 is particularly important as it highlights that in the case of a large number of projects those gains can be very large. Specifically, while the first-best outcome is not attainable in equilibrium, the surplus loss becomes negligible relative to the first-best total surplus. This contrasts sharply with the case where no CRA operates in the market. Recall that in the absence of a CRA no surplus is created and that this is independent of the total number of issuers.

One may wonder if the effects that allow the CRA to create value under simultaneous rating are also present in the case of sequential rating and, if so, whether they work equally well. One may also wonder whether the Law of Large Numbers, which is behind Proposition 2, applies in the case
of sequential rating. We analyze sequential rating in next section. As we will see, under sequential rating the value created by the CRA never approaches the first-best expected total surplus.

4 Sequential Rating

Consider now a dynamic version of the baseline model where the CRA rates issuers (and these issuers seek funds from investors) sequentially. Specifically, suppose there are $N$ periods and in each period a different issuer seeks to finance its project. All the other aspects of the model remain unchanged. Hence, in a given period, that period’s issuer decides whether to obtain a credit rating from the CRA, the CRA observes the quality of the issuer and rates it if solicited to do so, then the issuer tries to obtain funding from investors and, if successful in doing so, finances its project. The fundamental difference between the baseline model and this version of the model is that in the former the investors make their decisions after observing the ratings obtained by all the $N$ issuers, whereas in the latter they make their decision in each given period based on the history of ratings up to that period. The period discount factor is $\beta \in (0, 1)$ and is the same for all agents.

As in the baseline model, we assume that investors do not observe the performance of the financed projects. This assumption allows us to abstract from the typical reputation mechanism by which investors revise their opinion on the value of credit ratings by comparing the ratings given to issuers and the issuers’ ex-post performance. Hence, the credibility given by investors to a good credit rating in a given period depends only on the history of (good) ratings given by the CRA up that period. We follow in this section an approach similar to that in Section 3 where we investigated the case of simultaneous rating. We first analyze the case where $N = 2$ and then consider the case where $N$ is infinite. In both cases we focus on the informativeness of ratings and the value created by the CRA. Clearly, as before, if no CRA operates in the market no value is created.

4.1 Two Issuers

Suppose there are two periods and one issuer per period. In period one, the respective issuer is rated as good (or not) by the CRA and then investors decide whether to lend the required funds to the issuer. The same occurs in period two with that period’s issuer. We can solve for the equilibrium interaction between the CRA, the issuers and the investors by using backward induction.

We begin by arguing that in the second period a good rating cannot create value irrespective of
what has happened in the first period. To see this, suppose to the contrary that investors require a repayment $C < R$ to invest in the project if the project is rated as good. In this case, the CRA will rate the period two’s issuer as good irrespective of its quality. The CRA can charge a fee $R - C$ for the good rating, and receives nothing if it issues no good rating. But in equilibrium investors anticipate this behavior on the part of the CRA. Hence, they rationally ignore the rating given by the CRA, i.e. they believe that the issuer is good with probability $p$ (the prior) even if the issuer is rated as good. However, with these beliefs, they require a repayment $C > R$ and will not finance the project, contradicting the initial assumption that $C < R$. Having established that a good rating cannot create value in period two, we next argue that it cannot create value in period one either. Any decisions regarding the issuer in period one will not affect decisions in period two. Therefore, the analysis done for the case of the period two issuer, fully applies to the case of the period one issuer. The observation that credit ratings can neither create value in period one nor create value in period two means that the CRA cannot create value. For convenience of exposition, we state this without further proof in the next proposition.

**Proposition 3** The CRA creates no value if rating is sequential and there are two issuers.

This proposition constitutes the first step in highlighting the difference between simultaneous rating and sequential rating. With two issuers, ratings can create value under simultaneous rating, but they do not create value under sequential rating. Recall that credit ratings create value under simultaneous rating because of the negative link between the value of a good rating offered by the CRA and the number of such ratings, which gives the CRA an incentive to limit the number of good ratings. Under sequential rating, with two issuers, these incentives vanish. In the last period, the offering of a good rating by the CRA will not reduce the value of good ratings issued in previous periods. This means that the negative link is not present in the last period. As a result, the CRA has an incentive to rate an issuer (independently of its type) as good whenever it can charge a positive fee for this good rating, rendering credit ratings totally uninformative about issuer quality. This, in turn, means that rating cannot generate any fee in the second period. Moreover, the fact that rating generates no fee in period two regardless of what happens in period one, means that the negative link is also not present in period one. As a result the CRA creates no value.

One may wonder whether the CRA could solve the problem of the lack of credibility of its ratings under sequential rating by being able to commit *ex-ante* to issue only a limited number of good
ratings. The answer to this questions is no. Suppose the CRA can commit \textit{ex-ante} to issue only one good rating over the two periods. Observe that if the CRA does not rate the period one issuer as good, it will have the incentive to rate the second period issuer as good regardless of its quality (if it can charge a positive fee for the good rating). This implies that investors will never trust a good rating if issued in period two, meaning that a good rating in period two is necessarily worthless. Hence, the CRA can only charge a positive fee for a good rating in period one. However, if this is the case, by the same reasoning, we can conclude that a good rating must also be worthless in period one. Hence, commitment by the CRA to issue a limited number of good ratings is not sufficient to ensure the credibility of its credit ratings.

While we have analyzed here the case of two issuers, the analysis and results obtained in this subsection hold for any finite number of issuers. Indeed, a CRA cannot create value under sequential rating when the number of issuers is finite. The arguments used above to claim that ratings in last period cannot be informative and that this has an unravelling effect on all credit ratings remain. We next consider the case in which there are infinitely many issuers (periods) and analyze the potential for value creation by a CRA whose provision of rating services are not expected to cease at a given point in time. In this case, the negative link between the number of good ratings and the value of a good rating does not necessarily vanish, as it does in the case of a finite number of issuers. However, as we will see it is weaker than in the case of simultaneous rating.

4.2 An Infinite Number of Issuers

Suppose now that each period a new issuer enters the market and that this occurs indefinitely. In other words, the CRA expects to be in the rating business forever. In the context of our model, this situation corresponds to the case where the number of issuers (periods) is infinite. In this case, backward induction can no longer be used to obtain the equilibrium interaction between the issuers, CRA and investors. Because the CRA expects to rate issuers indefinitely, the CRA’s rating decision in any given period can potentially affect the value of its ratings in the future. Hence, unlike in the case of a finite number of issuers, it is not necessarily the case that credit ratings are totally non-credible and that the CRA cannot create value. In this subsection we derive an upper bound for the value created by a CRA in equilibrium. We then compare the value of this upper bound with the value created by a CRA under simultaneous ratings when the number of issuers is large.
Before presenting such an upper bound, it is instructive to briefly discuss how equilibria where the CRA creates value might emerge. The CRA can create value if its ratings are at least partially credible. This occurs when the CRA has an incentive not to rate every issuer as good and also an incentive to give good ratings to good issuers (as opposed to giving them to bad issuers). The following is an example of a situation where both of these incentives emerge. Suppose investors fully trust a rating given by the CRA if the CRA has not issued a good rating for a certain period of time, say in the last $l$ periods, and totally disregard a rating given by the CRA if otherwise. In such a setting, the CRA faces a cost when issuing a good rating: it has to wait $l$ periods until its ratings are trusted again and issuers are willing to pay for them. Because of this cost and because a good issuer can pay more for a good rating than a bad issuer, the CRA may prefer to wait until a good issuer appears in the market to issue a good rating, even if $l$ periods have elapsed since it last rated an issuer as good. If the CRA indeed prefers to do so, investors will be right to fully trust a good rating issued by the CRA after an abstinence in issuing good ratings for at least $l$ periods.

One can easily envision other situations where the credibility of the ratings given by the CRA evolves in a different way over time. An important observation though is that the efficiency in the allocation of funds (and therefore the value created by the CRA) may differ from one situation to the other. The next proposition presents an upper bound for the value created by the CRA across all possible equilibria. Note that the discounted expected first-best total surplus, namely, the value created in the case where in each period the project is financed if and only if it is good, is $pv_g/(1 - \beta)$. In what follows we let $V^F_B = pv_g/(1 - \beta)$.

**Proposition 4** The value created by the CRA under sequential rating is no larger than $(1 - q_b/q_g) \times V^F_B$.

In any equilibrium in which the CRA creates value, its ratings must create value in at least some periods. However, in any given period, a credit rating is informative about the issuer’s quality and creates value only if the CRA is better off by not giving a good rating to the issuer in the event the issuer is bad. That is, the CRA’s profit in that period from issuing a good rating to a bad issuer must not exceed the CRA’s future loss associated with issuing a good rating. We use this condition to obtain an upper bound for the value created by the CRA presented in Proposition 4. There are two potential sources for the “loss” in the value created by the CRA relative to first-best: the implementation of bad projects and the abandonment of some good projects. Interestingly, one can
show that the most efficient equilibria (those where the value created by the CRA is maximal) are of the sort described in the example given above.\textsuperscript{13} In that type of equilibria, only the second type of loss occurs. Once the CRA issues a good rating it has to wait $l$ periods until it can issue another good rating that will be trusted by investors. However, any good issuer arriving during this waiting period will not be financed and its project will be abandoned.

Proposition 4 states that at most a fraction $1 - q_b/q_g$ of the first-best total surplus can be realized under sequential rating. This result, which is independent of the value of the discount factor $\beta$, contrasts with that obtained in the case of simultaneous rating. As stated in Proposition 2, the value created by the CRA under simultaneous rating asymptotically approaches the first-best total surplus. Two factors explain the difference between the two cases.

First, the effect of the negative link is stronger under simultaneous rating than under sequential rating, and thus it is easier to provide the CRA with the incentives to limit the number of good ratings under simultaneous rating than under sequential rating. Under simultaneous rating, the CRA’s decision to issue one additional good rating lowers the value of all the ratings issued by the CRA. Under sequential rating, that decision can lower only the value of the ratings issued by the CRA in the future.

Second, good projects are more likely to be abandoned under sequential rating than under simultaneous rating. This is because the order with which good and bad projects appear in the market matters under sequential rating. To illustrate this point, consider the event where $k$ out of $N$ projects are good. Suppose again the situation under sequential rating where, after issuing a good rating, the CRA has to wait $l$ periods until it can issue another good rating. If the $k$ good projects appear in the market consecutively, there is a congestion of good projects and many will necessarily be abandoned. Under simultaneous rating, the order with which good projects appear is irrelevant, and there is no loss of surplus due to congestion of good projects.

5 Discussion and Extensions

The analysis above highlights the importance of simultaneous reporting of credit ratings for ratings credibility. In this section, we explore the robustness of our key findings to a set of alternative modeling assumptions and explore some possible generalizations of our environment.

\textsuperscript{13}This result is obtained as part of the proof of Proposition 4.
Competition in the market for credit ratings  We have considered so far the case in which only one CRA operates in the market for credit ratings. We have seen that under simultaneous rating credit ratings can be credible in equilibrium and provide valuable information to investors, and that this is because of the two following effects. First, a monopolist CRA has the incentive to limit the number of good ratings it issues, since the value of each good rating negatively depends on the total number of good ratings issued. Second, a monopolist CRA gains more by giving a good rating to a good issuer than by giving a good rating to a bad issuer. Here, we briefly discuss how competition between CRAs might impact the magnitude of these two effects.

We begin with the observation that under some extreme beliefs the equilibria obtained in the case of a monopoly continue to exist when two or more CRAs operate in market for credit ratings. Suppose, for example, investors (potentially) trust only one particular rating agency. That is, investors’ beliefs about issuers’ quality are never affected by the ratings given by any of the other CRAs. Given these beliefs, the trusted CRA can and will act as monopolist in the market. The ratings given by the other CRAs will be ignored in equilibrium and eventually will not even be issued. Another system of beliefs that sustains the monopoly outcomes is that where investors change their beliefs about an issuer’s quality only if the issuer obtains a rating of good from all the CRAs.14 In this case, as long as coordination between CRAs is feasible, CRAs will coordinate on the issuers to whom they give a good rating and (jointly) act as monopolist, obtaining each the respective share of the monopoly profits of the industry. This observation holds both in the case of simultaneous rating and in the case of sequential rating.

The beliefs considered above essentially rule out the possibility of competition between CRAs for issuers. In the perhaps more plausible environments where all or some CRAs are individually trusted (at least partially) by investors, some competition between CRAs is likely to emerge. If competition results in lower fees charged by CRAs to issuers, then it may weaken CRAs’ incentives to rate issuers well under simultaneous rating. The reason is the following. By reducing rating fees, competition weakens CRAs’ ability to extract surplus from issuers. In particular it weakens CRAs ability to extract surplus from good issuers whose projects generate more value, reducing CRAs’ incentive to give the good ratings to the good issuers (instead of the bad issuers). However, this effect of competition on CRAs’ incentives is also present under sequential rating with infinite issuers.14

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14The same applies if investors change their beliefs about an issuer’s quality only if the issuer is rated as good by a given group (subset) of rating agencies.
Hence, from this effect one cannot conclude that competition weakens the benefits of simultaneous rating relative to sequential rating.

Another implication of competition is that it may reduce the benefits of simultaneous rating by decreasing the magnitude of the economies of scale. Observe that if one CRA is asked to rate one issuer only, the rating cannot be informative in equilibrium. However, as shown in Section 3.1, credit ratings can be informative when one CRA is asked to rate two issuers. Also, as shown in Section 3.2, for a large number of issuers, the value created by a CRA asymptotically approaches the first-best total surplus. Both cases are reflections of the presence of economies of scale in the market for credit ratings. If competition between CRAs leads to a split of the market between them, some of the benefits of such economies of scale will not be materialized. Consider again the case of a large number $N$ of issuers studied in Section 3.2. In the case of a monopolist CRA, the loss in expected surplus (relatives to first-best) is $c\sqrt{N}$, for some $c > 0$. Suppose now there are two CRAs with market shares $\alpha$ and $1 - \alpha$, respectively. The surplus loss in this case is given by $c\sqrt{\alpha N} + c\sqrt{(1-\alpha)N}$. Since $\sqrt{N} < \sqrt{\alpha N} + \sqrt{(1-\alpha)N}$, such loss is clearly greater in the case of two CRAs than in the case of a monopolist CRA.

**Issuers with correlated qualities** We have assumed that issuers’ qualities are independent. In some cases this assumption may be unrealistic. Some exogenous factors may affect simultaneously the intrinsic quality of all issuers. For example, during a period of fast economic growth all projects may be more likely to succeed. Similarly, during a recession, all projects are less likely to succeed and their probability of default may increase. It is possible to extend the analysis in Section 3.1 to allow for intrinsic correlation between issuers’ qualities. Let $P_n$ denote the probability that exactly $n \in \{0, 1, 2\}$ issuers are of good quality. Clearly, $\sum_{n=0}^{2} P_n = 1$. Continue to assume that issuers are symmetric so that the probability that a given issuer is good is $P_1/2 + P_2$, and that this probability is low enough to discourage investors from financing an issuer at random, i.e. assume $[(P_1/2 + P_2)(q_g - q_b) + q_b]R < 1$. This condition, which can be written as $P_1/2 + P_2 < -V_b/\Delta R$, is the counterpart of (2). Observe that this formulation allows for any correlation between the qualities’ of the two issuers. Following the steps in Section 3.1, one can obtain a generalized version of Proposition 1. Specifically, if $P_1 + P_2 < -V_b/\Delta R$, then no issuer (even if with a good rating) is financed. However, if $P_1 + P_2 \geq -V_b/\Delta R$,

\[\text{See the proof of Proposition 2 for the complete mathematical expression of this loss and its derivation.}\]

\[\text{The proof of this more general version of Proposition 1 follows the exact same steps as those of Proposition 1 and are omitted.}\]
then the equilibria identified in Proposition 1 exist and these are the unique equilibria in which the CRA creates value. This result suggests that the advantages of simultaneous rating remain in more general environments where issuers’ qualities may be correlated.

One could expect the gains from simultaneous rating (relative to sequential rating) to vanish when issuers’ qualities are negatively correlated. This is so even in the setting with two issuers only. To see why, consider the limit case of negative correlation in which one issuer is necessarily bad when the other is good. In other words, there is always one (and only one) good issuer on the market. Investors know there is one good issuer, but fail to observe which of the two issuers is good. In this case, one could expect the CRA to create value under sequential rating by following a strategy where it gives a good rating to the first issuer if and only if the issuer is good. Under this strategy, by not giving a good rating to a bad issuer in period one, the CRA could credibly give a good rating to the second period issuer. The problem with this strategy is that a good rating given by the CRA in period two does not add any information about the quality of the issuer. By not giving a good rating in period one, the CRA reveals that the period one issuer is bad, which automatically reveals that the period two issuer is necessarily good. Hence, a good rating given to the period two issuer is worthless. It is easy to show that there is no equilibrium in which credit ratings create value under sequential rating even if issuers’ qualities are correlated.

Commitment by investors Section 4.1 shows that a CRA cannot create value when issuers are rated sequential and the number of issuers is finite. Furthermore, we have argued in that section that such failure to create value persists even if the CRA could commit \textit{ex-ante} to issue only a limited number of good ratings. Interestingly, a CRA can create value under sequential rating if investors (instead of the CRA) could commit \textit{ex-ante} to finance only a limited number of issuers, provided these issuers have a good rating. In particular, some of the outcomes obtained under simultaneous rating can be achieved under sequential rating if such commitment by investors is possible.

Consider for example the outcome obtained under simultaneous rating (and identified in Proposition 1) where only one issuer receives a good rating regardless of the number of good issuers and that issuer is financed. This outcome can be replicated under sequential rating if investors commit \textit{ex-ante} to finance one and only one issuer (at a repayment identical to that in the outcome under simultaneous rating).\footnote{This repayment, which is equal to $1/(q_b + p(2-p)\Delta)$, is precisely the one under which investors (\textit{ex-ante}) break-even} Given this commitment by investors, the CRA will give the good rating to
the issuer in period one only if that issuer is good, and otherwise will wait until period two to give
the good rating. In equilibrium, one issuer will be financed irrespective of the total number of good
issuers. Commitment by investors is important because of the following. While investors are 

ex-ante
better off by making the commitment, they would prefer not to finance the issuer in period two in
the event that the period-one issuer does not receive the good rating. This is because investors know
that in such a situation, the period two issuer will receive a good rating regardless of its quality and
thus investing in it will generate a loss. However, a good rating issued in period one means that
the period-one issuer is good with probability one, and thus investing in it generates a net gain.
Obviously, 

ex-ante
the loss incurred by investors when they finance the period two issuer is offset by
the gain obtained when they finance the period one issuer, and investors break even.

Observe that not all the outcomes obtained under simultaneous rating can be achieved under
sequential rating when investors can commit 

ex-ante
to finance a fixed number of issuers. The
outcomes identified in Proposition 1 where the number of good ratings issued by the CRA depends
on the number of good issuers (i.e., equilibria where \( x < 1 \) and \( y < 1 \)) are examples of such outcomes.
To achieve such outcomes under sequential rating, investors would have to be able to commit 

ex-ante
to a repayment schedule that is contingent on the number of good ratings issued by the CRA over
time. This means, for example, that the terms of the contract offered by investors to one particular
issuer would have to depend on the number of ratings offered by the CRA to other issuers in the
future. While conceptually conceivable, this type of commitment requires substantial coordination
by investors and may be infeasible in practice.

6 Conclusion

Concerns that credit rating agencies have inflated the ratings of some financial products have in-
creased in recent years. This is in part because during the recent financial crisis, highly rated
projects performed very poorly and their ratings had to be significantly downgraded. We show in
this paper that requiring a rating agency to report the ratings of many issuers at once provides a
mechanism to discipline it against rating inflation and to increase the credibility of its ratings. The
simultaneity in the reporting of credit ratings disciplines rating agencies because it links the CRA’s
decisions regarding the ratings of different borrowers: by giving more good ratings, the rating agency

also in the sequential setting.

23
lowers the credibility of its ratings and the fee it can charge for a good rating. Moreover, it provides the rating agency with incentives to give good ratings to good projects, as these are the projects that can pay more for a good rating. We also show that our mechanism is asymptotically efficient. That is, when the number of borrowers is sufficiently large, the surplus generated in equilibrium (asymptotically) approaches the efficient total surplus.

This paper’s findings suggest an extra benefit to the idea of synchronizing bond issuance, which the practitioners have been considering mainly for the benefit of improving the liquidity of corporate bonds. Specifically, the synchronization of bonds issuance, may generate an opportunity for the mechanism highlighted in the paper to work, increasing the value created by rating agencies. There are, however, potential costs associated with the implementation of such a mechanism. One of such costs is the potential delay in the implementation of projects. Firms needing funds to implement a new project could have to wait longer to obtain a credit rating and, consequently, the needed funds. In this respect, the periodicity with which credit ratings can be reported seems important, as it affects the magnitude of both the benefits and the cost of the scheme. The implementation of the mechanism proposed in this paper in practice may require a more comprehensive analysis of all of its effects, including those on bond liquidity.

\(^{18}\)See the article “The Debt Penalty” in the *Financial Times*, 11 September 2013.
7 Appendix

Proof of Proposition 1. Let \( \phi_k \) denote the investors’ belief that an issuer with a good rating has a good project when the CRA issues \( k \in \{1, 2\} \) good ratings. Both \( \phi_k \) and \( C_k \) are determined in equilibrium. Whenever convenient, we denote the exogenous ex-ante probability that \( n \) projects are good by \( P_n \). The proof is given in the following steps.

Step 1: Characterization of CRA’s profit given repayment values \( C_k \). Because unrated projects are not financed, when offering a good rating to an issuer the CRA chooses the rating fee \( f \) so as to fully appropriate the profit generated by its project (net of any financial costs). Thus, conditional on issuing a total of \( k \) good ratings, the CRA’s expected profit from giving a good rating to an issuer of quality \( i \) is \( q_i \times \max \{ R - C_k, 0 \} \). Observe that the CRA’s profit of giving a good rating to a good issuer is greater than that of giving a good rating to a bad issuer, as \( q_g > q_b \). Hence, the CRA’s expected profit when it issues \( k \) good ratings and \( n \) issuers are good, denoted by \( \pi_k^n \), is as follows: \( \pi_0^0 = 0 \) for all \( n \in \{0, 1, 2\} \); \( \pi_1^0 = q_b \times \max \{ R - C_1, 0 \} \); \( \pi_1^1 = \pi_2^1 = q_b \times \max \{ R - C_1, 0 \} \); \( \pi_2^0 = 2q_b \times \max \{ R - C_2, 0 \} \); \( \pi_2^1 = (q_b + q_g) \times \max \{ R - C_2, 0 \} \); and \( \pi_2^2 = 2q_g(R - C_2) \).

Step 2: If \( p(2 - p) \leq -V_b/\Delta R \) then no equilibrium in which the CRA creates value exists. In what follows we consider that \( C_k < R \) for at least one \( k \in \{1, 2\} \), as this condition must hold in any equilibrium in which the CRA creates value. Given this condition, there are three possible cases regarding the values of \( C_1 \) and \( C_2 \), which are considered separately in the following steps.

Step 2.1: Suppose \( C_2 < R \) and \( R - C_1 < ((q_g + q_b)/q_g)(R - C_2) \). Then \( C_1 - R < 2(R - C_2) \), as \( q_g > q_b \). It follows that \( \pi_2^2 > \pi_1^1 \) and \( \pi_2^2 > \pi_0^1 \) for all \( n \in \{0, 1, 2\} \). Hence, in an equilibrium with such values of \( C_1 \) and \( C_2 \), if it exists, the CRA issues two good ratings regardless of the number of good issuers. Bayesian updating by investors implies that \( \phi_2 = p \). It follows from (3) and (2) that \( C_2 = 1/(pq_g + (1 - p)q_b) > R \), which contradicts that \( C_2 < R \). Hence, no equilibrium in which \( C_2 < R \) and \( C_1 - R < ((q_g + q_b)/q_g)(R - C_2) \) and the CRA creates value exists.

Step 2.2: Suppose \( C_2 < R \) and \((q_g + q_b)/q_g)(R - C_2) \leq R - C_1 < 2(R - C_2) \). In this case, \( \pi_0^2 > \pi_0^1 > \pi_0^0 \) for all \( n \in \{0, 2\} \) and and \( \pi_1^1 \geq \pi_2^2 > \pi_0^1 \). Hence, in such an equilibrium, if it exists, the CRA issues two good ratings both when \( n = 0 \) and \( n = 2 \), and the CRA either issues one good rating or randomizes between issuing one and two good ratings when \( n = 1 \). Let \( \mu \) denote the probability with which the CRA issues one good rating when \( n = 1 \). Bayesian updating by investors implies
that $\phi_1 = 1$ and
\[
\phi_2 = \frac{P_1(1 - \mu)/2 + P_2}{P_0 + P_1(1 - \mu) + P_2}.
\]

There are two possible cases depending on the relative values of $P_0$ and $P_2$. If $P_0 > P_2$, then $\phi_2$ decreases with $\mu$, which implies that $\phi_2 \leq P_1/2 + P_2 = p$. It follows from (3) and (2) that $C_2 > 1/(pq_g + (1 - p)q_b) > R$, which contradicts the fact that $C_2 < R$ obtained above. If instead $P_0 < P_2$, the $\phi_2$ increases with $\mu$ and $\phi_2 \leq P_2/(P_0 + P_2)$. From (3) it follows that $C_1 = 1/q_g$ and
\[
C_2 \geq 1/(P_2(P_0 + P_2)q_g + (1 - P_2(P_0 + P_2)q_b)).
\]

It is routine to show that these two conditions and the initial assumption that $C_1 - R < 2(R - C_2)$ cannot be simultaneously satisfied when $p < -V_b/\Delta R$.

**Step 2.3:** Suppose $C_1 < R$ and $R - C_1 \geq 2(R - C_2)$. Then, $R - C_1 > ((q_g + q_b)/q_g)(R - C_2)$. Hence, $\pi_n^1 \geq \pi_n^2$ and $\pi_n^1 > \pi_n^0 = 0$ for all $n \in \{0, 2\}$. Furthermore, $\pi_1^1 > \pi_2^1$ and $\pi_1^1 > \pi_0^1 = 0$. Hence, in such an equilibrium, if it exists, the CRA issues one good rating when $n = 1$, and either issues one good rating or randomizes between issuing one and two good ratings when $n = 0$ or $n = 2$. Let $\mu_0$ and $\mu_2$ denote the probability that the CRA issues one good rating when $n = 0$ and $n = 2$, respectively. Bayesian updating by investors implies
\[
\phi_1 = \frac{P_1 + \mu_2 P_2}{\mu_0 P_0 + P_1 + \mu_2 P_2} \quad \text{and} \quad \phi_2 = \frac{(1 - \mu_2)P_2}{(1 - \mu_0)P_0 + (1 - \mu_2)P_2}.
\]

If $R - C_1 > 2(R - C_2)$, then $\pi_1^0 > \pi_2^0$ for all $n \in \{0, 2\}$ and in such an equilibrium $\mu_0 = \mu_2 = 1$. This implies $\phi_1 = P_1 + P_2$. It follows from (3) that $C_1 = 1/((P_1 + P_2)q_g + (1 - P_1 + P_2)q_b)$, which is smaller than $R$ if and only if $P_1 + P_2 > -V_b/\Delta R$, or equivalently, $p(2 - p) > -V_b/\Delta R$.

If instead $R - C_1 = 2(R - C_2)$, then $C_1 < C_2 < R$. Solving the simultaneous equations in (4) with respect to $\mu_0$ and $\mu_2$, we obtain
\[
\mu_0 = \frac{(P_1 + P_2 - \phi_2)(1 - \phi_1)}{(1 - P_2 - P_1)(\phi_1 - \phi_2)} \quad \text{and} \quad \mu_2 = \frac{(P_1\phi_2 + P_2\phi_1 - \phi_1\phi_2)}{(\phi_1 - \phi_2)P_2}.
\]

Since by (3) $C_2 = 1/(q_L + \phi_2\Delta)$ and $C_2 < R$, then $\phi_2 > (-V_L)/\Delta R$. Moreover, because $C_1 < C_2$, $\phi_1 > \phi_2$. From direct inspection of (5) it follows that $\mu_0 \geq 0$ only if $P_1 + P_2 \geq \phi_2$, which implies that $P_1 + P_2 \geq \phi_2 > (-V_L)/\Delta R$. 

26
Step 3: Existence and characterization of equilibria in which the CRA creates value when \( p(2 - p) > -V_b/\Delta R \). Any of such equilibria must be of the type of equilibrium considered in step 2.3, as steps 2.1-2.2 do not require the assumption \( p(2 - p) \leq -V_b/\Delta R \). It is clear that there exists an equilibrium in which the CRA issues one good rating for all \( n \in \{0, 1, 2\} \), i.e. in which \( \mu_0 = \mu_2 = 1 \). In this case, as shown in step 2.3, \( \phi_1 = P_1 + P_2 \) and \( C_1 < R \) if \( p(2 - p) > -V_b/\Delta R \). The event where the CRA issues two good ratings is off-the-equilibrium path. Hence, we can choose \( \phi_2 \) so that \( R - C_1 \geq 2(R - C_2) \), which makes issuing one good rating for all \( n \in \{0, 1, 2\} \) optimal for the CRA.

Let us now consider equilibria where \( \mu_n < 1 \) for some \( n \in \{0, 2\} \). In this type of equilibrium, \( \pi_1^n = \pi_2^n \), which is equivalent to

\[
R - C_1 = 2(R - C_2). \tag{6}
\]

Hence, such an equilibrium exists if and only if there exists \( \mu_0 \) and \( \mu_2 \) such that: (i) \( \phi_1 \) and \( \phi_2 \) satisfy (4); and (ii) \( C_1 \) and \( C_2 \) satisfy (6), and

\[
C_i = \frac{1}{\phi_i q_b + (1 - \phi_i) q_g} \tag{7}
\]

and \( C_i \in [1/q_g, R] \) for all \( i = 1, 2 \). We next show that for each \( \mu_2 \geq \max\{0, 1 - \frac{(1-p)^2}{p^2} \frac{1 - q_b/q_g - V_b}{V_g}\} \) such an equilibrium exists. By (4),

\[
\mu_0 = \frac{1 - \phi_1}{\phi_1} \cdot \frac{P_1 + P_2 \mu_2}{P_0} \quad \text{and} \quad 1 - \mu_0 = \frac{1 - \phi_2}{\phi_2} \cdot \frac{P_2(1 - \mu_2)}{P_0}. \tag{8}
\]

It follows that

\[
\frac{1 - \phi_1}{\phi_1} \cdot \frac{P_1 + P_2 \mu_2}{P_0} + \frac{1 - \phi_2}{\phi_2} \cdot \frac{P_2(1 - \mu_2)}{P_0} = 1. \tag{9}
\]

From (7) it follows that for all \( i = 1, 2 \),

\[
\frac{1 - \phi_i}{\phi_i} = \frac{q_g C_i - 1}{1 - q_b C_i}. \tag{10}
\]

Substituting it into (9) and rearranging, we obtain

\[
\frac{q_g C_1 - 1}{1 - q_b C_1} \cdot (P_1 + P_2 \mu_2) + \frac{q_g C_2 - 1}{1 - q_b C_2} \cdot P_2(1 - \mu_2) = P_0. \tag{11}
\]
Observe that for any \((C_1, C_2) \in [1/q_b, R]^2\), by (10) \((1 - \phi_i)/\phi_i \geq 0\) for \(i = 1, 2\), which by (8) implies that \(\mu_0 \geq 0\) and \(1 - \mu_0 \geq 0\). This means that given \(\mu_2 \in [0, 1]\), if the simultaneous equations (6) and (11) have a solution for \((C_1, C_2)\) in \([1/q_b, R]^2\), then there exists \(\mu_0 \in [0, 1]\) such that \(\mu_0\) and \(\mu_2\) constitute an equilibrium. Using (6) to eliminate \(C_2\) in (11), we obtain

\[
g(C_1) = \frac{q_g C_1 - 1}{1 - q_b C_1} \cdot (P_1 + P_2 \mu_2) + \frac{q_g (C_1 + R)/2 - 1}{1 - q_b (C_1 + R)/2} \cdot P_2 (1 - \mu_2) = P_0. \tag{12}
\]

Observe that \(g\) is continuous and \(g' > 0\) for all \(C_1 \in (1/q_b, R]\). Moreover,

\[
g(R) = \frac{V_g}{-V_b} \cdot (P_1 + P_2 \mu_2) + \frac{V_g}{-V_b} \cdot P_2 (1 - \mu_2) = \frac{V_g}{-V_b} \cdot (P_1 + P_2),
\]

which means that \(g(R) > P_0\) when \(P_1 + P_2 > (V_g/ - V_b)\), or equivalently, when \(p(2 - p) > (V_g/ - V_b)\).

Hence, equation (12) has a solution in \(C_1 \in [1/q_g, R]\) (and that solution is unique) iff \(g(1/q_g) \leq P_0\).

Since

\[
g(1/q_g) = \frac{V_g}{1 - q_b/q_g - V_b} \cdot P_2 (1 - \mu_2),
\]

this condition is satisfied if and only if \(\mu_2 \geq \max\{0, 1 - \frac{p_b 1 - q_b/q_g - V_b}{V_g}\} = \max\{0, 1 - \frac{(1-p)^2 1 - q_b/q_g - V_b}{V_g}\}\).

Since when (6) is satisfied, \(C_2 \in [1/q_g, R]\) whenever \(C_1 \in [1/q_g, R]\), we obtain the existence of the equilibrium. Next, observe that by (8), \(\mu_0 = 1\) when \(\mu_2 = 1\). To obtain the result stated in the proposition, simply let \(x = \mu_2\) and \(y = \mu_0\). \(\blacksquare\)

**Proof of Proposition 2.** Given the number of issuers \(N\), the value created (total surplus generated) in a pooling equilibrium where \(k\) good ratings are issued when \(n \in \{0, 1, \ldots, N\}\) issuers are good is \(kV_g\) if \(n \geq k\) and \(nV_g + (k - n)V_b\) if \(n < k\). Thus, the expected total surplus (before \(n\) is realized) is given by

\[
V(k, N) = \Pr(n \geq k) \times kV_g + \sum_{n=0}^{k-1} P_n (nV_g + (k-n)V_b)
\]

\[
= \Pr(n \geq k) \times kV_g + \Pr(n < k) \times kV_b + (V_g - V_b) \times \sum_{n=0}^{k-1} P_n n
\]

\[
= kV_g - \Pr(n < k) k \times (V_g - V_b) + (V_g - V_b) \times \sum_{n=0}^{k-1} P_n n
\]

\[
= kV_g - (V_g - V_b)[\Pr(n < k) k - \sum_{n=0}^{k-1} P_n n],
\]

where \(P_n\) denotes the *ex-ante* probability that \(n\) issuers are good. The optimal \(k\) maximizes \(V(k, N)\).
Finding the optimal $k$ is not a trivial problem. Therefore, we next obtain an asymptotic approximation to $V(k, N)$.

Observe that
\[
\frac{V(k, N)}{N} = \frac{k}{N} V_g - (V_g - V_b) \frac{\Pr(n < k) k - \sum_{n=0}^{k-1} P_n n}{N}.
\] (13)

In what follows, it is convenient to write $k$ and $n$ as $k = Np + \lambda_N \sqrt{Np(1-p)}$ and $n = Np + t \sqrt{Np(1-p)}$, where $\lambda_N, t \in R$. These are just variable transformations. Hence, we can write
\[
\frac{k}{N} = p + \lambda_N \sqrt{\frac{p(1-p)}{N}}.
\] (14)

By the Central Limit Theorem, asymptotically, $t \sim N(0, 1)$ with density $\phi(t) = (\sqrt{2\pi})^{-1} e^{-\frac{t^2}{2}}$ and c.d.f. $\Phi(t) = \int_{-\infty}^{t} \phi(s) ds$. By the Barry-Esseen Theorem, for $N$ large,
\[
\Pr(n < k) = \Phi(\lambda_N) + O(\frac{1}{\sqrt{N}}).
\] (15)

Observe also that
\[
\sum_{n=0}^{k-1} P_n \frac{n}{N} = \Pr(n < k) E\left[\frac{n}{N} \mid n < k\right]
\]
\[
= \Pr(n < k) E\left[\frac{Np + t \sqrt{Np(1-p)}}{N} \mid n < k\right]
\]
\[
= \Pr(n < k) E[p + t \sqrt{\frac{p(1-p)}{N}} \mid n < k]
\]
\[
= \Pr(n < k) p + \sqrt{\frac{p(1-p)}{N}} \times \Pr(n < k) E[t \mid n < k]
\]
\[
= \Pr(n < k) p + \sqrt{\frac{p(1-p)}{N}} \int_{-\infty}^{\lambda_N} t \phi(t) dt + o\left(\frac{1}{\sqrt{N}}\right).
\]

Using the fact that $\int_{-\infty}^{\lambda_N} \phi(t) dt = \int_{-\infty}^{\lambda_N} \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} dt = -\phi(\lambda_N)$, we can write
\[
\sum_{n=0}^{k-1} P_n \frac{n}{N} = \Pr(n < k) p - \sqrt{\frac{p(1-p)}{N}} \phi(\lambda_N) + o\left(\frac{1}{\sqrt{N}}\right).
\] (16)

Using (14), (15) and (16) into (13), we obtain

\[
\frac{V(k, N)}{N} = V_g \times (p + \lambda_N \sqrt{\frac{p(1-p)}{N}}) - (V_g - V_b) \times \{[\Pr(n < k) \times (p + \lambda_N \sqrt{\frac{p(1-p)}{N}})] - [\Pr(n < k) \times p - \sqrt{\frac{p(1-p)}{N}} \phi(\lambda_N) + o\left(\frac{1}{\sqrt{N}}\right)]\}
\]

\[
= pV_g + \sqrt{\frac{p(1-p)}{N}} \lambda_N V_g - \sqrt{\frac{p(1-p)}{N}} (V_g - V_b) [\Phi(\lambda_N) \lambda_N + \phi(\lambda_N)] + o\left(\frac{1}{\sqrt{N}}\right)
\]

\[
= pV_g + \sqrt{\frac{p(1-p)}{N}} \{\lambda_N V_g - (V_g - V_b) [\Phi(\lambda_N) \lambda_N + \phi(\lambda_N)]\} + o\left(\frac{1}{\sqrt{N}}\right).
\]

Hence, asymptotically, the optimal \(k\) for each \(N\) can be obtained by solving:

\[
\max_{\lambda_N} \lambda_N V_g - (V_g - V_b) [\Phi(\lambda_N) \lambda_N + \phi(\lambda_N)].
\]

Observe that \(\Phi'(\lambda_N) = \phi(\lambda_N)\) and \(\phi'(\lambda_N) = -\lambda_N \phi(\lambda_N)\). The first-order condition associated with this maximization problem is

\[
V_g - (V_g - V_b) \Phi(\lambda_N) = 0,
\]

from which it follows that the optimal \(\lambda_N\) does not depend on \(N\) and is implicitly defined by the condition

\[
\Phi(\lambda^*) = \frac{V_g}{V_g - V_b}.
\]

Hence, asymptotically, the optimal \(k\) is \(k^* = Np + \lambda^* \sqrt{Np(1-p)}\) and the expected total surplus evaluated at the optimal pooling equilibrium is

\[
V(k^*) = N(pV_g - \sqrt{\frac{p(1-p)}{N}} (V_g - V_b) \phi(\lambda^*)),
\]

Since the first-best expected total surplus is \(V^{FB}(N) = NpV_g\), we obtain that

\[
\frac{V^{FB}(N) - V(k^*, N)}{V^{FB}(N)} = \sqrt{\frac{1-p}{Np}} (V_g - V_b) \phi(\lambda^*),
\]

which means that \(V(K^*, N)\) asymptotically approaches \(V^{FB}(N)\) in the order of \(N^{-1/2}\). Finally, observe that under the optimal \(k\), the probability that a project with a good rating is indeed good
when $N$ is sufficiently large is given by

\[
\phi_k^N = \Pr(n \geq k^*) \times \left(1 + \sum_{n=0}^{k^*-1} P_n \frac{n}{k^*} \right)
\]

\[
= 1 - \Phi(\lambda^*) + \frac{N}{k^*} \times \sum_{n=0}^{k^*-1} P_n \frac{n}{N} + O\left(\frac{1}{\sqrt{N}}\right)
\]

\[
= 1 - \Phi(\lambda^*) + \frac{1}{p + \lambda^* \sqrt{p(1-p)/N}} \times [p\Phi(\lambda^*) - \sqrt{p(1-p)/N} \phi(\lambda^*)] + O\left(\frac{1}{\sqrt{N}}\right)
\]

\[
= 1 - \Phi(\lambda^*) + \frac{1}{p} \times (1 - \lambda^* \sqrt{1-p/N}) \times [p\Phi(\lambda^*) - \sqrt{p(1-p)/N} \phi(\lambda^*)] + O\left(\frac{1}{\sqrt{N}}\right)
\]

\[
= 1 - \Phi(\lambda^*) + \Phi(\lambda^*) + O\left(\frac{1}{\sqrt{N}}\right)
\]

\[
= 1 + O\left(\frac{1}{\sqrt{N}}\right).
\]

It converges to one in the order of $N^{-1/2}$. Finally, the fact that this probability converges to one ensures that all projects with a good rating are financed in equilibrium, justifying the initial assumption about the equilibrium considered.

Proof of Proposition 4. Let $E$ denote the set of Perfect Bayesian Equilibria of the game, $H_t$ the set of histories until the beginning of period $t$, $H = \cup_{t=1}^{\infty} H_t$ the set of all histories of the game, and $v_{e,h}$ the continuation value to the CRA in equilibrium $e$ after history $h$. We can define the set $\mathcal{V} = \{v = v_{e,h}|e \in E \text{ and } h \in H\}$. This is the set of all the continuation values to the CRA in any Perfect Bayesian equilibrium. $\mathcal{V}$ is non-empty, as there is always an equilibrium in which investors never trust a good rating given by the CRA and the CRA abstains from issuing any good rating. The continuation value to the CRA in this equilibrium is always zero. Moreover, $\mathcal{V}$ is bounded from above, as $v_{e,h} \leq pv_{g}/(1-\beta) = V_s^{FB}$ for all $e \in E$ and $h \in H$, which follows from the fact that investors always break-even and in a Perfect Bayesian Equilibrium investors’ beliefs must be consistent with the CRA’s rating strategy both on and off-the-equilibrium path. Let $\overline{v} = \sup \mathcal{V}$, which exists and is finite, since $\mathcal{V}$ is non-empty and bounded from above. If $\overline{v} = 0$, then the proposition trivially holds. Hence, the remainder of the proof supposes the case where $\overline{v} > 0$ and consists of showing that in this case $\overline{v} \leq (1 - q_h/q_g)V_s^{FB}$.

By definition of $\overline{v}$, it follows that for any given $\varepsilon > 0$, there exist an equilibrium $e$ and a history
such that \( v_{e,h} > \overline{v} - \varepsilon \). Consider the continuation game at this equilibrium and history. Let \( \gamma_g \) (resp. \( \gamma_b \)) be the probability that in the first period (of the continuation game) the CRA rates the issuer as good in the event the issuer is good (resp. bad). Let also \( \gamma := p\gamma_g + (1 - p)\gamma_b \), which is the \textit{ex ante} probability that the CRA rates the issuer as good in that period. Similarly, let \( v_0 \) denote the continuation value to the CRA at the beginning of the second period if the CRA abstains from giving a good rating in the first period, and \( v_1 \) denote that value if the CRA issues a good rating in the first period. Given the CRA’s rating strategy, Bayesian updating by investors implies that their belief that the issuer in the first period (of the continuation game) is good if the issuer has a good rating is \( \phi = p\gamma_g / (p\gamma_g + (1 - p)\gamma_b) \). This implies, by (3), that the repayment required by investors to finance the issuer is

\[
C = \frac{p\gamma_g + (1 - p)\gamma_b}{p\gamma_g q_g + (1 - p)\gamma_b q_b}.
\]

Hence, we can write

\[
v_{e,h} = p \cdot \left\{ \gamma_g [q_g (R - C) + \beta v_1] + (1 - \gamma_g) [0 + \beta v_0] \right\} + (1 - p) \cdot \left\{ \gamma_b [q_b (R - C) + \beta v_1] + (1 - \gamma_b) [0 + \beta v_0] \right\}
= [p\gamma_g q_g + (1 - p)\gamma_b q_b] \cdot (R - C) + \gamma \cdot \beta v_1 + (1 - \gamma) \cdot \beta v_0
= [p\gamma_g q_g + (1 - p)\gamma_b q_b] \cdot R - [p\gamma_g + (1 - p)\gamma_b] \cdot \gamma \cdot \beta v_1 + (1 - \gamma) \cdot \beta v_0
= p\gamma_g V_g + (1 - p)\gamma_b V_b + \gamma \cdot \beta v_1 + (1 - \gamma) \cdot \beta v_0.
\]  

By definition,

\[
v_0 \leq \overline{v}.
\]

For \( \varepsilon \) sufficiently small (i.e., for equilibria and history where the CRA’s continuation value is sufficiently close to \( \overline{v} \)), the CRA must create value in the first period of the continuation game (otherwise the continuation value to the CRA at the beginning of the next period would exceed \( \overline{v} \), which is impossible by definition of \( \overline{v} \)). Hence, the CRA must not be strictly better off giving a good rating to a bad issuer than abstaining from doing it, or

\[
0 + \beta v_0 \geq q_b \cdot (R - C) + \beta v_1 \iff \\
\beta v_0 - q_b \cdot (R - C) \geq \beta v_1.
\]
Using inequalities (18) and (19) and equation (17), we obtain

\[ v_{e,h} = p \gamma_g V_g + (1 - p) \gamma_b V_b + \gamma \cdot \beta v_1 + (1 - \gamma) \cdot \beta v_0 \]
\[ \leq p \gamma_g V_g + (1 - p) \gamma_b V_b - \gamma q_b (R - C) + \beta v_0 \]
\[ = f(\gamma_g, \gamma_b) + \beta v_0 \]
\[ \leq f(\gamma_g, \gamma_b) + \beta \overline{v}. \]

From this inequality and the fact that \( \overline{v} - \epsilon < v_{e,h} \), it follows that \( \overline{v} - \epsilon/(1 - \beta) < f(\gamma_g, \gamma_b)/(1 - \beta) \), which implies that \( \overline{v} < \max_{0 \leq \gamma_g, \gamma_b \leq 1} f(\gamma_g, \gamma_b)/(1 - \beta) + \epsilon/(1 - \beta) \), and since this inequality holds for any \( \epsilon > 0 \), we can write that

\[ \overline{v} \leq \max_{0 \leq \gamma_g, \gamma_b \leq 1} f(\gamma_g, \gamma_b)/(1 - \beta). \quad (20) \]

We next calculate \( \max_{0 \leq \gamma_g, \gamma_b \leq 1} f(\gamma_g, \gamma_b) \). Recall that, as defined above,

\[ f(\gamma_g, \gamma_b) = p \gamma_g V_g + (1 - p) \gamma_b V_b - \gamma q_b (R - C), \]

and observe that we can write

\[ C = \frac{p + (1 - p) t}{pq_g + (1 - p) q_b t} \]

where \( t \equiv \gamma_b/\gamma_g \). Thus,

\[ f(\gamma_g, \gamma_b) = g(\gamma_g, t) \equiv \gamma_g \cdot \left\{ pv_g + (1 - p) v_b t - \left[ p + (1 - p) t \cdot q_b[R - \frac{p + (1 - p) t}{pq_g + (1 - p) q_b t}] \right] \right\}. \quad (21) \]

Using the fact that \( V_b - q_b (R - C) = q_b R - 1 - q_b (R - C) = q_b C - 1 \), we obtain that

\[ \frac{\partial g}{\partial t} = -\gamma_g (1 - p) \left( \frac{p(q_g - q_b)}{pq_g + (1 - p) q_b t} \right)^2 < 0. \]

Therefore, to maximize \( g(\gamma_g, t) \), we set \( t = 0 \). It follows that to maximize \( f(\gamma_g, \gamma_b) \), we set \( \gamma_b = 0 \).

Using (21), we obtain that \( f(\gamma_g, 0) = \gamma_g \cdot \{ pv_g - p \cdot q_b [R - 1/q_g] \} = \gamma_g \cdot p V_g (1 - q_b/q_g) \). Therefore, to maximize \( f \), we set \( \gamma_g = 1 \) and \( \max_{0 \leq \gamma_g, \gamma_b \leq 1} f(\gamma_g, \gamma_b) = p V_g (1 - q_b/q_g) \). It follows by (20) that \( v \leq (1 - q_b/q_g) \cdot pv_g/(1 - \beta) \). This completes the proof. \( \blacksquare \)
References


