Competition for Attention*

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Abstract

We present a model of market competition and product differentiation in which consumers’ attention is drawn to the products’ most salient attributes. Firms compete for consumer attention via their choices of quality and price. Strategic positioning of a product affects how all other products are perceived. With this attention externality, depending on the cost of producing quality some markets exhibit “commoditized” price salient equilibria, while others exhibit “de-commoditized” quality salient equilibria. When the costs of quality change, innovation can lead to radical shifts in markets. In the context of financial innovation, the model generates the phenomenon of “reaching for yield”.
1 Introduction

In many markets, consumers’ attention to particular attributes of a product seems critical. In fashion goods, business class airline seats, and financial products, consumers focus on quality rather than price. In these markets, firms advertise quality to draw consumers’ attention. In fast food, economy air travel, or standard home goods, consumers seem much more attentive to prices. In these markets, firms typically advertise their low prices.

Scholars of strategy and marketing are keenly aware of these distinct modes of market competition, and tirelessly emphasize the importance of having differentiated attributes and drawing consumer attention to them (Levitt 1983, Rangan and Bowman 1992, Mauborgne and Kim 2005). Southwest wants to be known as “the low cost airline;” Singapore as the winner of prizes for luxury and comfort. Walmart touts its everyday low prices, Nordstrom’s its service. Successful firms “frame” competition by focusing consumers’ attention on their best attribute (quality or price). These mechanisms do not arise naturally in standard economic models, in which consumers attend to all product attributes equally.

This paper seeks to understand these phenomena. We take a standard model in which firms compete on quality and price, and add to it the mechanism of salience we developed elsewhere (Bordalo, Gennaioli, and Shleifer 2012, 2013). According to salience theory, the attention of decision makers is drawn to the most unusual, surprising, or salient attributes of the options they face, leading them to overweight these attributes in their decisions. Salience theory applied to consumer choice can shed light on a host of lab and field evidence on consumers’ context dependent behavior. Such context dependence is well established in experiments, including the well known decoy effects (Huber, Payne and Puto 1982) and compromise effects (Simonson 1989). More recently, Hastings and Shapiro (2013) show using field data that, after a parallel increase in the prices of all gas grades, the demand for premium gas drops to an extent that cannot be accounted for by standard income effects. The salience model accounts for this evidence by recognizing that surprising price hikes focus consumer attention on gas prices, rather than quality, favoring the choice of cheaper grades.

In this paper, we show that the influence of prices and qualities on consumer attention has significant implications for market competition. In competitive markets, the salience of
price and quality are endogenously determined by the firms’ strategic choices, and create an attention externality that lies at the heart of our model. A high quality good draws attention not only to its own quality, but also to the fact that the competitor product has lower quality, reducing the competitor’s relative valuation. A good with a low price draws attention to the competitor’s higher price, reducing the competitor’s relative valuation. When salience matters, firms compete for consumer attention via the choice of quality and price.

We show that, depending on the cost of producing quality, some markets exhibit price salient equilibria in which consumers are most attentive to prices and less sensitive to quality differences. In these markets firms compete on prices, and quality could be under-provided relative to the efficient level. Because consumers neglect quality upgrades, escaping such “commodity magnets” is difficult. Fast food and budget air travel can be described in this way.

In other markets, equilibria are quality salient in that consumers are attentive to quality and are to some extent insensitive to price differences. Firms compete on quality, which can be over-supplied relative to the efficient level. In these markets, it is again difficult to escape the high quality equilibrium because consumers neglect price cuts. We think of financial services or fashion as well-described by such equilibria.

We investigate how market equilibrium depends on the cost of providing quality. We explore the possibility of radical change in markets when the cost of producing quality changes dramatically. This can take the form of de-commoditization, whereby a firm acquires access to a technology of producing quality at a much lower cost than its competitor, and is able to change the market from a price-salient to a quality-salient equilibrium. Prices can then rise dramatically, but quality as perceived by consumers rises more. Market transformation can also take the form of commoditization, which arises when industry costs fall dramatically, so that large price cuts become possible. As price becomes salient, and quality differences are neglected, firms reduce quality in order to cut prices even more.

Some of these effects can also arise in a traditional model, under judicious assumptions about consumer heterogeneity. Section 5 describes similarities and differences between salience and the traditional approach to competition by using two real world examples: de-commoditization of the coffee market after the entry of Starbucks, and commoditization of
the U.S. air travel market after deregulation in the 1980’s. One key difference between the two approaches lies in the drivers of change. In standard models, it is typically the marginal consumers who shift in response to changes in quality or price. In our model, in contrast, the attention and thus the price-sensitivity of all consumers changes in response to significant innovation. As a consequence, shifts in demand and market structure can be massive.

We conclude the analysis by considering in detail the case of financial innovation in the form of new products with higher expected return and risk, such as mortgage backed securities (MBS). We show that such innovation is especially attractive in low interest rate environments, and when the innovation offers higher returns at a moderately higher risk. Higher returns are salient to investors when alternative yields are extremely low and the (small) extra risk of the new product is underweighted. The model generates the well documented phenomenon of “reaching for yield” in a psychologically intuitive way, based on the properties of salience.\(^1\)

Our paper is related to recent work on “behavioral industrial organization” (Ellison 2006, Spiegler 2011). In some models, consumers restrict their attention to a subset of available options, the consideration set, which can be manipulated by firms by expending a marketing cost (Spiegler and Eliaz 2011a,b and Hefti 2012), by setting a salient low price on some products (Ellison and Ellison 2009), or by setting an inconspicuous price (de Clippel, Eliaz and Rozen 2013). In our model, the attention externality operates within a given consideration set.

Another strand of the literature considers the working of market competition in settings in which some product attributes are “shrouded”, namely sufficiently obscured that consumers find it difficult to compare them across products (Gabaix and Laibson 2006, Ellison and Ellison 2009, Armstrong and Chen 2009, Spiegler and Piccione 2012). This literature takes as given the attributes that consumers pay attention to. In our analysis, consumers may pay differential attention to quality or price, but the neglect of one attribute or the other is endogenously determined by product design and market competition.

Azar (2008), Cunningham (2012), and Dahremöller and Fels (2012) explore models in

\(^1\)For explorations of the role of inattention in financial markets, see Barber and Odean (2008), DellaVigna and Pollet (2009), and Bordalo, Gennaioli and Shleifer (2013b).
which the relative weight that consumers put on different attributes depends on the choice context, and can thus be manipulated by firms. These papers model consumer attention by using approaches different from salience and explore a different set of issues, such as properties of markups or the monopolist problem. Finally, our analysis builds on recent work relating inattention to consumer demand. Some approaches – such as Gabaix (2012), Matějka and McKay (2012), and Persson (2012) – are grounded in the rational inattention framework, in which attention to different product features is efficiently allocated ex-ante. In our salience model consumer attention to different product attributes is drawn ex-post, depending on which attribute stands out. Koszegi and Szeidl (2013) follow a similar approach.

The paper is organized as follows. In section 2, we describe our basic model of competition and show how salience would influence product valuations by consumers. In section 3, we take qualities as fixed and examine the basic analytics of price competition and of price and quality salient equilibria. Section 4 focuses on the full model of quality competition, and derives our main results for markets for products where attribute salience matters. In section 5, we apply the model to discuss financial innovation. Section 6 concludes.

2 The Model

There are two firms, 1 and 2. Each firm \(k = 1, 2\) produces one unit of a good having quality \(q_k\) at cost \(c_k(q_k)\). Cost functions are common knowledge to firms (and consumers) and include a quality-dependent and a quality-independent component. Formally, \(c_k(q) = F_k + v_k(q)\), where \(v_k(q)\) is an increasing and convex function satisfying \(v_k(0) = 0\). Here \(F_k\) captures the cost of producing one unit of good \(k\) (not a fixed entry cost), so we refer to it as a unit cost. To obtain closed form solutions, we sometimes use the quadratic form:

\[
c_k(q) = F_k + \frac{v_k}{2} \cdot q^2, \text{ for } k = 1, 2. \tag{1}
\]

We assume that firm 1 has weakly lower total and marginal costs of quality than firm 2, namely \(c_1(q) \leq c_2(q)\) and \(c'_1(q) \leq c'_2(q)\) for all qualities \(q\). In the quadratic case, this implies \(F_1 \leq F_2\) and \(v_1 \leq v_2\).
There is a measure one of identical consumers, each of whom chooses one unit of one good from the choice set \( C \equiv \{(q_1, p_1), (q_2, p_2)\} \), where \((q_k, p_k)\) stand for the quality and price of the good produced by firm \( k \). Both qualities and prices are measured in dollars and assumed to be known to the consumer. Absent salience distortions, each consumer values good \( k = 1, 2 \) at:

\[
u(q_k, p_k) = q_k - p_k.\tag{2}\]

A salient thinker departs from (2) by inflating the weight attached to the attribute that he perceives to be more salient in the choice set \( C \equiv \{(q_1, p_1), (q_2, p_2)\} \).

For each good \( k \), its salient attributes are those whose levels are unusual or surprising, in the sense of being furthest from the reference attribute levels in the choice set \( C \). Following BGS (2013), we take the reference attribute levels to be the average levels in the choice set; thus, the reference good \((\bar{q}, \bar{p})\), has the average price \( \bar{p} = (p_1 + p_2)/2 \) and the average quality \( \bar{q} = (q_1 + q_2)/2 \) in \( C \).

We model salience using a salience function \( \sigma (x, y) \) that satisfies two main properties: ordering and homogeneity of degree zero. According to ordering, if an interval \([x, y]\) is contained in a larger interval \([x', y']\), then \( \sigma (x, y) < \sigma (x', y') \). According to homogeneity of degree zero, \( \sigma (\alpha x, \alpha y) = \sigma (x, y) \) for any \( \alpha > 0 \), with \( \sigma (0, 0) = 0 \). In the choice set \( C \), the salience of price for good \( k \) is \( \sigma (p_k, \bar{p}) \) while the salience of quality for good \( k \) is \( \sigma (q_k, \bar{q}) \). Good \( k \)'s quality is more salient than its price – or, for short, quality is salient – if and only if \( \sigma (q_k, \bar{q}) > \sigma (p_k, \bar{p}) \).

Ordering and homogeneity of degree zero of the salience function imply that the salience of a good’s quality is an increasing function of the percentage difference between the good’s quality and the average quality in the choice set, and similarly for price. In particular, consumers have diminishing sensitivity to attribute differences: increasing the prices of both goods by a uniform amount \( \epsilon \) makes prices weakly less salient, \( \sigma (p_k + \epsilon, \bar{p} + \epsilon) \leq \sigma (p_k, \bar{p}) \).

\(^2\)Although we consider the simplest setting in which there are only two firms, producing one good each, and there are no outside options, the analysis can be extended to include outside options. The role of the outside option is not critical and Online Appendix B.5 extends the model to include it. Several of our results continue to hold with more than two firms in the market; see Section 4.2. To apply the salience framework to a more general model of market competition, the relevant market should be taken as the definition of choice set. This process allows for some flexibility in defining what the alternatives of choice are, but this flexibility is similar to that involved in defining a specific competitive market context in conventional models of industrial organization.
for \( k = 1, 2 \), and strictly so when \( p_k \neq \bar{p} \). This property is consistent with Weber’s law of sensory perception.\(^3\)

In our main analysis, we assume that salience distorts consumer valuation by attaching a higher and fixed weight to the most salient attribute of a good. This “rank based weighting” allows a stark and intuitive characterisation of the central implications of salience.\(^4\) In the Online Appendix B.2 we analyze the model under a continuous weighting formulation, and show that our main results continue to hold. To keep the model tractable under both formulations, we impose an additional intuitive condition on the salience of attributes when the choice set has two goods, namely symmetry: \( \sigma(a_1, \bar{a}) = \sigma(a_2, \bar{a}) \) for \( a = p, q \). In words, each attribute is equally salient for both goods. As an example, the function \( \sigma(a, \bar{a}) = |a - \bar{a}|/\bar{a} \) laid out in BGS (2012), which measures attribute salience as the proportional difference from the average value of the attribute, satisfies this symmetry property in our two goods context.

Under rank based weighting, the salient thinker’s perceived utility from \((q_k, p_k)\) is:

\[
 u^{ST}(q_k, p_k) = \begin{cases} 
 q_k - \delta p_k & \text{if quality is salient} \\
 \delta q_k - p_k & \text{if price is salient} \\
 q_k - p_k & \text{if equal salience}
\end{cases},
\]

where \( \delta \in [0, 1] \) captures the extent to which valuation is distorted by salience (the above expression omits for simplicity the normalization factor \( 2/(1+\delta) \), see BGS (2013)). When \( \delta = 1 \), valuation coincides with (2) and the salient thinker behaves like a rational consumer. When \( \delta < 1 \), the salient thinker overweights the salient attribute. The competitive equilibrium then depends on \( \delta \), allowing us to study how salience affects market competition.

Under the continuous formulation of salience weighting explored in Appendix (B.2), the

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\(^3\)As in BGS (2013), homogeneity of degree zero balances the sometimes conflicting forces of ordering and diminishing sensitivity, but our main results depend only on the properties of ordering and diminishing sensitivity. Homogeneity of degree zero buys us tractability. While not necessarily always applicable, it is supported by an emerging paradigm in psychology stressing that people possess an innate “core number system” which compares magnitudes in terms of ratios, see Feigenson, Dehaene and Spelke (2004).

\(^4\)Rank based discounting also captures the idea that valuation can be drastically affected by introducing small differences in an attribute such as price (Tversky 1972, Kim, Novemsky and Dhar 2012).
perceived utility of \((q_k, p_k)\) is given by:

\[
\begin{align*}
    u^{\text{ST}}_{\text{cont}}(q_k, p_k) &= q_k \cdot \frac{e^{(1-\delta)\sigma(q_k, q_k)}}{e^{(1-\delta)\sigma(q_k, q_k)} + e^{(1-\delta)\sigma(p_k, p_k)}} - p_k \frac{e^{(1-\delta)\sigma(q_k, p_k)}}{e^{(1-\delta)\sigma(q_k, q_k)} + e^{(1-\delta)\sigma(p_k, p_k)}} + e^{(1-\delta)\sigma(q_k, q_k)} + e^{(1-\delta)\sigma(p_k, p_k)}; \\
\end{align*}
\]

Thus, valuation is rational for \(\delta = 1\) and fully distorted in favor of the salient attribute as \(\delta \to -\infty\). This model is less tractable than the rank based one, but the main effects of salience continue to hold.\(^5\)

Another simplifying assumption in our analysis is consumer homogeneity. In Appendix (B.3) we show that our main results extend to the case in which salience weighting varies across otherwise identical consumers. Integrating salience with heterogeneity in consumer tastes is an important topic for future research, particularly with regards to testing empirically the effect of salience on consumer demand.

Firms compete in two stages. In the first stage, each firm makes a costless commitment to produce quality \(q_k \in [0, +\infty)\). In the second stage, each firm competitively sets an optimal price \(p_k\) given the quality-cost bundle \((q_k, c_k)\) it committed to, where \(c_k \equiv c_k(q_k)\).\(^6\) To account for consumers’ exogenous budget constraints, we assume that possible prices are bounded above, i.e. \(p_k \leq p_{\text{max}} < +\infty\). With qualities and prices chosen, demand materialises and firms produce. Firm \(k\)’s payoff or profit is \(\pi_k = d_k \cdot [p_k - c_k(q_k)]\) where \(d_k \equiv d_k(q_k, q_{-k}, p_k, p_{-k})\) is the demand for good \(k\) at stage 2, such that \(d_k = 1 - d_{-k}\).

To map the consumer preferences in (3) into demand functions \(d_k, d_{-k}\), a “sharing” rule is required that specifies demand when ties arise in salience ranking or in valuation. Suppose that, at the strategy vector \((q_k, q_{-k}, p_k, p_{-k})\), good \(-k\) is weakly preferred to good \(k\), namely \(u^{\text{ST}}(q_k, p_k) \leq u^{\text{ST}}(q_{-k}, p_{-k})\), but that good \(k\) is strictly preferred if its price is slightly reduced:

\(^5\)As we discussed in BGS (2012, 2013), a full specification of salience also requires an editing stage in which dominated options are discarded (e.g. goods with very large negative attributes). This editing stage captures the notion that salience distorts the evaluation of genuine tradeoffs, while the consumer immediately notices the presence of a dominated good and does not choose it.

Editing is particularly important for rank based weighting. As we discuss in BGS (2013), valuation may in some cases be non-monotonic and a dominated good might be chosen (for instance the perceived utility of a good may increase in its price if the price of the good becomes non-salient). In the current setting with two goods, where salience is symmetric, this is not an issue because dominated goods are never chosen.

\(^6\)This game is similar to the one in Shaked and Sutton (1982), except that we abstract from the initial stage in which firms decide whether to enter the market. In our game, firms always post their quality choices at stage 1 and their prices at stage 2, and they only incur costs if there is demand for their products.
formally, there exists \( \epsilon \) such that \( u^{ST}(q_k, p_k - \epsilon) > u^{ST}(q_{-k}, p_{-k}) \) for any \( \epsilon \in (0, \bar{\epsilon}] \). We then specify the following sharing rule at vector \((q_k, q_{-k}, p_k, p_{-k})\): if firm \( k \) prices above cost, \( p_k > c_k \), while firm \( k \) prices at cost, \( p_{-k} = c_{-k} \), we set \( d_k = 1 \), \( d_{-k} = 0 \); in all other cases we set \( d_k = d_{-k} = 1/2 \). This sharing rule captures the idea that, at \((q_k, q_{-k}, p_k, c_{-k})\), firm \( k \) can infinitesimally reduce its price and capture the market with a profit.

We solve this game by finding subgame perfect equilibria. We restrict our attention to equilibria in pure strategies, but Online Appendix B.1 proves that mixed strategy equilibria do not exist in our model. We describe equilibria in two steps. First, in Section 3 we take each firm’s quality and cost \((q_k, c_k)\) as given and study price competition among firms. This price setting stage is of independent interest from endogenous quality choice because in the short run firms often take quality as given, and react to cost shocks only by adjusting their prices (in some settings firms may be unable to adjust quality, due to regulatory or technological constraints). The pricing game generally admits multiple equilibria, since the losing firm is indifferent between different strategy choices that yield zero market share (and thus zero profits). We restrict the equilibrium set by using the standard refinement that excludes equilibria in weakly dominated strategies. This refinement constrains firms to price weakly above cost so that, in equilibrium, the losing firm prices at cost.

In Section 4 we investigate the full game, endogenizing quality choice (given the refinement of the pricing game). Our main analysis deals with the case in which firms are symmetric, in the sense of having the same cost of quality \( c(q) \). However, we also consider what happens to the equilibrium when a shock occurs that reduces the cost of one of the

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7Such a shift in utility rankings can occur in three cases: i) at \( p_k \) the valuation constraint binds but the salience constraint is slack; ii) at \( p_k \) the salience constraint binds but valuation is slack; and iii) at \( p_k \) both the valuation and the salience constraint bind. In case i) it must be that \( u^{ST}(q_k, p_k) = u^{ST}(q_{-k}, p_{-k}) \) so when firm \( k \) lowers its price, the consumers goes from indifferece to a strict preference for good \( k \). In case ii), valuation jumps discontinuously at the salience bound. Because valuation is slack, it must be that \( u^{ST}(q_k, p_k) < u^{ST}(q_{-k}, p_{-k}) \). By lowering its price, firm \( k \) renders its advantage salient and strictly reverses the consumer’s preference ranking. Finally, case iii) occurs when the goods are identical.

8Alternatively, in these cases the sharing rule could be that demand is determined by the preference rankings that follow from Equation (4). Because these cases never arise in equilibrium except when firms both price at cost, and hence share the market, both specification lead to the same equilibria.

9This sharing rule is determined jointly with strategy selection in equilibrium. As in Reny (1999), we adopt this endogenous sharing rule to deal with discontinuities in the firms’ payoff functions. Another way to avoid discontinuities arising from ties is to discretize the set of prices firms can set. In this case, the firm delivering higher perceived surplus would set the highest price consistent with the consumer choosing its product, but this price will generally not leave the consumer indifferent between the two products.
two firms.

3 Price Competition

We begin with an analysis of price competition between firms 1 and 2, assuming that qualities \( q_1, q_2 \) and costs \( c_1, c_2 \) are fixed, and only prices are set by firms. Suppose that firm 1 chooses weakly higher quality than firm 2 and, as a consequence, incurs a weakly higher production cost, namely \( q_1 \geq q_2 \) and \( c_1 \geq c_2 \). In Section 4 we show that this is indeed the relevant case when quality and costs are determined endogenously. Before characterizing the outcome under salience, consider the rational benchmark that obtains when \( \delta = 1 \).

Lemma 1 When \( \delta = 1 \), the price competition subgame admits a unique pure strategy equilibrium under refinement, which satisfies:

i) If \( q_1 - c_1 > q_2 - c_2 \), then equilibrium prices are \( p_1 = c_2 + (q_1 - q_2) \) and \( p_2 = c_2 \). Demand satisfies \( d_1 = 1 \) and firm 1 makes positive profits \( \pi_1 = (q_1 - q_2) - (c_1 - c_2) \).

ii) If \( q_1 - c_1 < q_2 - c_2 \), then equilibrium prices are \( p_1 = c_1 \) and \( p_2 = c_1 - (q_1 - q_2) \). Demand satisfies \( d_2 = 1 \) and firm 2 makes positive profits \( \pi_2 = (c_1 - c_2) - (q_1 - q_2) \).

iii) If \( q_1 - c_1 = q_2 - c_2 \), then equilibrium prices are \( p_1 = c_1 \) and \( p_2 = c_2 \). Demand satisfies \( d_1 = d_2 = 1/2 \) and both firms make zero profits.

All proofs are in Appendix A. In the rational benchmark, the firm creating greater surplus \( q_k - c_k \) captures the entire market and makes a profit equal to the differential surplus created. When, as in case iii), the two goods yield the same surplus, firms share the market and make zero profits, as in standard Bertrand competition. The benchmark of fully homogeneous goods and zero profits corresponds to the special case \( q_1 = q_2 = q \), and \( c_1 = c_2 = c \).

To see how salience affects price competition, suppose that the firm producing the lower quality product 2 sets a lower price \( p_2 \leq p_1 \). The appendix proves that this always holds in equilibrium. Homogeneity of degree zero of the salience function then implies that the same attribute – either quality or price – is salient for both goods. In particular, quality is salient (that is, quality is more salient than price for both goods) provided:

\[
\frac{q_1}{q_2} > \frac{p_1}{p_2}
\]  (5)
Equivalently, quality is salient when the high quality good has a higher quality to price ratio than the low quality good (i.e., \(q_1/p_1 > q_2/p_2\)). Price is salient if and only if the low quality good has a better quality to price ratio than the high quality good (i.e., \(q_1/p_1 < q_2/p_2\)). When \(q_1/p_1 = q_2/p_2\), price and quality are equally salient. Because the good that fares better along the salient attribute is overvalued relative to the other good (equation (3)), salience tilts preferences in favor of the good that has the highest ratio of quality to price (BGS 2013).

This logic implies that the valuation of a good depends on the entire competitive context. In particular, by changing its price a firm imposes an “attention externality” on the competing good. To see this, suppose that \(q_1 > q_2\) and \(p_1 > p_2\), and the high quality firm reduces its price \(p_1\). This improves the consumer’s valuation of good 1 but, by making prices less salient, it also draws the consumer’s attention to the low quality of good 2. Suppose alternatively that the low quality firm reduces its price \(p_2\). This improves the consumer’s valuation of good 2, but by making prices more salient, it also draws his attention to the high price of good 1. Thus, by reducing price a firm draws the consumer’s attention to the attribute along which it fares better. This attention externality can either strengthen and dampen competitive forces, depending on the situation.

### 3.1 Salience and Competitive Pricing

When a firm sells to salient thinkers, it sets its price to render salient the advantage of its product relative to its competitor. To see how this affects competitive pricing, we examine price setting in two opposite situations, one in which quality is salient and firm 1 wins the market, another in which price is salient and firm 2 wins the market.

Consider first the optimal price set by the high quality firm 1 in order to win a quality-salient market (when it offers a higher perceived surplus to consumers). Suppose that firm 2 has set a price \(p_2\) for \(q_2\). The maximal price \(p_1\) at which firm 1 attracts consumers into
buying its product while keeping quality salient solves:

\[
\begin{align*}
\max_{p_1 \geq p_2} & \quad p_1 - c_1 \\
\text{s.t.} & \quad q_1 - \delta p_1 \geq q_2 - \delta p_2, \quad (6) \\
& \quad q_1/p_1 \geq q_2/p_2. \quad (7)
\end{align*}
\]

The “valuation constraint” in (6) ensures that the consumer prefers good 1 when quality is salient. The “salience constraint” in (7) ensures that quality is indeed salient. At this point, it is useful to illustrate the sharing rule (and the salience tie-breaking rule) assumed above. Firm 1 captures the entire market, \(d_1(q_1, q_2, p_1, p_2) = 1\), when both (6) and (7) hold – even if one constraint holds with equality – as long as \(p_1 > c_1\) and \(p_2 = c_2\). This is because only firm 1 can lower its price \(p_1\) and satisfy both the salience constraint and the valuation constraint strictly; it thus captures the market with a salient advantage and positive profits.

The optimisation problem above presents two departures from the rational case. On the one hand, firm 1 now has an additional reason to cut its price: by setting \(p_1\) low enough, it makes quality salient in (7), inducing the consumer to buy its high quality product. On the other hand, when quality is salient the high quality good is over-valued, which may allow firm 1 to hike its price \(p_1\) above the rational equilibrium level. This effect of salience is captured by Equation (6).

Consider next the optimal price set by the low cost firm 2 to win a price salient market when firm 2 offers a higher perceived surplus to consumers. The maximal price \(p_2\) at which firm 2 attracts consumers while keeping prices salient solves:

\[
\begin{align*}
\max_{p_2 \leq p_1} & \quad p_2 - c_2 \\
\text{s.t.} & \quad \delta q_2 - p_2 \geq \delta q_1 - p_1, \quad (8) \\
& \quad q_2/p_2 \geq q_1/p_1. \quad (9)
\end{align*}
\]

Once again, price setting is constrained by consumer valuation and salience. On the one hand, salience provides firm 2 with an additional incentive to cut its price. By lowering \(p_2\), firm 2 does not just make its product more attractive, it also makes its lower price salient,
inducing the consumer to buy its cheaper product. This effect is captured by (9). On the other hand, by causing an over-valuation of the cheap good, salience can allow firm 2 to charge a higher price than in the rational case. This effect is captured by (8).

This analysis suggests that, depending on the balance between the salience and valuation constraints, salient thinking may boost or dampen prices relative to a rational world. We now characterise equilibrium prices under salient thinking. We focus for simplicity on parameter configurations satisfying the restriction:

\[ A.1: \quad \delta(c_1 - c_2) < q_1 - q_2 < \frac{1}{\delta}(c_1 - c_2). \]

Assumption A.1 ensures that salience fully determines the preference of consumers among goods when prices are equal to production costs. If quality is salient, consumers prefer the high quality good 1; if price is salient they prefer the cheap good 2. This is akin to assuming that the two firms produce sufficiently similar surpluses \( q_k - c_k \) that changes in salience change the consumer’s preference ranking. In Appendix A, we extend the characterisation of equilibria to the full parameter space (not restricted to Assumption A.1).

**Proposition 1** For any parameter values \( \delta \in [0, 1] \) and \( q_1, q_2, c_1, c_2 \in \mathbb{R}_+ \) such that \( q_1 \geq q_2 \) and \( c_1 \geq c_2 \), the price competition subgame has a unique pure strategy equilibrium under refinement. Under A.1, this equilibrium satisfies:

i) if \( \frac{q_1}{c_1} > \frac{q_2}{c_2} \), prices are \( p_1 = \min\{q_1 \cdot \frac{c_2}{q_2}, c_2 + \frac{1}{\delta}(q_1 - q_2)\} \) and \( p_2 = c_2 \). Quality is salient, demand satisfies \( d_1 = 1 \) and firm 1 makes positive profits.

ii) if \( \frac{q_1}{c_1} < \frac{q_2}{c_2} \), prices are \( p_1 = c_1 \) and \( p_2 = \min\{q_2 \cdot \frac{c_1}{q_1}, c_1 - \delta(q_1 - q_2)\} \). Price is salient, demand satisfies \( d_2 = 1 \) and firm 2 makes positive profits.

iii) if \( \frac{q_1}{c_1} = \frac{q_2}{c_2} \), prices are \( p_1 = c_1 \) and \( p_2 = c_2 \). Quality and price are equally salient. Demand satisfies \( d_k = 1 \) if \( q_k - c_k > q_{-k} - c_{-k} \) and \( d_k = 1/2 \) if \( q_k - c_k = q_{-k} - c_{-k} \). Both firms make zero profits.

Under salience, the market equilibrium critically depends on the quality to cost ratios \( q_k/c_k \) of different products. A firm with a higher ratio \( q_k/c_k \) monopolizes the market and makes positive profits. When the two firms have identical quality to cost ratios, they earn zero profits in the competitive equilibrium.\(^{10}\)

\(^{10}\)As we show in Appendix A, in the full parameter space equilibria in pure strategies of the pricing game...
Proposition 1 holds because the firm having the highest quality to cost ratio can always engineer a price cut turning salience in its favor. When \( q_1/c_1 > q_2/c_2 \), the high quality firm can set a sufficiently low price that quality becomes salient, monopolizing the market. The low quality firm is unable to reverse this outcome: in fact, doing so would require it to cut price below cost. When instead \( q_1/c_1 < q_2/c_2 \), the low quality firm can set price sufficiently low so that price is salient, and it monopolizes the market. The high quality firm is unable to reverse this outcome: once again, doing so would require it to cut price below cost. Finally, consider the case in which \( q_1/c_1 = q_2/c_2 \). In this case, as soon as a firm tries to extract some consumer surplus by setting a price above cost, its disadvantage becomes salient and the price hike becomes self defeating. The equilibrium outcome is zero profits for both firms.

The central role of the quality to cost ratio is economically appealing because it pins down salience distortions in terms of average costs of quality \( c_k/q_k \). As we show when we endogenize quality, this feature allows our model to make tight predictions about how changes in cost structure affect salience and market outcomes. Before turning to that analysis, it is useful to look more closely at some implications of Proposition 1.

### 3.2 Price salient vs. Quality salient equilibria

Depending on the quality and cost parameters, salience leads to two types of equilibria: price salient and quality salient. In quality salient equilibria (case i of Proposition 1), consumers focus on quality for both goods. This resembles de-commoditized markets described in the marketing literature. In contrast, in price salient equilibria (case ii), consumers focus on prices but neglect quality differences among goods. This resembles the canonical description of commoditised markets (Rangan and Bowman 1992).

According to Proposition 1, in both types of equilibria the profits of the winning firm can be either lower or higher than in the rational benchmark. To see this, note that - due to the salience constraint - the equilibrium profits of the winning firm \( k \) (the one with lowest exist, are unique, and can also be characterized by the quality-cost ratios of the firms. When A.1 does not hold, a qualitatively new type of equilibrium in pure strategies arises in which a firm may win the market at equilibrium prices for which its advantage is not salient.
average cost) must satisfy:

\[
\pi^S_k \leq q_k \cdot \frac{c_k - c_{-k}}{q_k - q_{-k}} = q_k \left[ \frac{c_{-k}}{q_{-k}} - \frac{c_k}{q_k} \right],
\]  

(10)

where equality holds when the salience constraint binds. Equation (10) shows that equilibrium profits increase in the difference between the average cost of quality of the different firms. Consider the following special cases:

- The two goods yield different surpluses \( q_1 - c_1 \neq q_2 - c_2 \) but exhibit identical average costs of quality. Under rationality, the high surplus firm would make positive profits. Under salient thinking, in contrast, industry profits are zero. When average costs of quality are identical (similar), a firm can always undercut its competitor and render its advantage salient. Price cuts are very effective and profits are lower than under rationality.

- The two goods yield the same surplus \( q_1 - c_1 = q_2 - c_2 \), but differ in their average costs of quality. Here profits are zero under rationality but positive under salient thinking. The reason is that the firm with the lower average cost of quality can set a price above cost and still be perceived as offering a better deal than its competitor. Price cuts by the losing firm are ineffective, and salience dampens competitive forces.

Salience can create abnormal profits in both quality and price salient equilibria (this result extends to industry profits as a whole). In quality salient equilibria, consumers overvalue the high quality good. The high quality firm is then able to hike prices and earn high profits. Financial services and fashion may be examples of this type of competition. In price salient equilibria, consumers are attentive to prices and under-appreciate quality differences among products. This grants an extra advantage to the cheap (and low quality) firm, allowing it to raise the price above cost. Fast-food industry and low-cost airlines may be examples of this type of competition.
4 Optimal Quality Choice

We now examine endogenous quality choice. In the first stage of the game, each firm \( k = 1, 2 \) makes a costless commitment to produce quality \( q_k \in [0, +\infty) \), taking into account the price competition stage. In the second stage, firms compete in prices given the quality-cost attributes \((q_k, c_k(q_k))\), for \( k = 1, 2 \). The critical question is whether firm 1, which has lower costs, will choose to produce higher or lower quality than firm 2, and what this implies for the equilibrium market outcome.

The bulk of our analysis focuses on the symmetric case, in which firms have the same cost of producing quality, \( c_1(q) = c_2(q) \equiv c(q) \). We view this case as capturing the long run outcome arising when all firms, through imitation or entry, adopt the best available technology. Section 4.2 then considers how this equilibrium changes when one firm is hit by an asymmetric shock reducing its variable cost component.

4.1 Quality Choice in the Symmetric Cost Case

To fix ideas, consider the rational benchmark. Following Lemma 1, in stage 2 the market is monopolized by firm \( k \) producing the highest surplus \( q_k - c_k(q_k) \). Anticipating this, in stage 1 the two firms set their qualities as follows.

**Lemma 2** When \( \delta = 1 \), the full game admits a unique subgame perfect equilibrium in pure strategies, which is symmetric: both firms set quality \( q^* = \arg\max_q [q - c(q)] \) (i.e. such that \( c'(q^*) = 1 \)), price at cost, and make zero profits.

In the rational model, each firm produces the surplus maximizing quality. Quality provision decreases with the marginal cost of quality \( v(q) \) but is independent of the unit cost \( F \). Under the quadratic cost function of Equation (1), firms set:

\[
q_1^* = q_2^* \equiv q^* = \frac{1}{v},
\]

where \( v \) parameterizes the common marginal cost.

Consider now how salience affects quality choice. To build intuition, suppose that firms are at the “rational” quality level \( q^* \). If consumers are salient thinkers, would firm 1 have
an incentive to deviate to a different quality $q' \neq q^*$?

Consider the incentive of firm 1 to choose a marginally lower quality, cheaper, product. The new product has quality $q' = q^* - \Delta q$ and cost $c(q') = c(q^*) - \Delta c$. Whether this new product is successful or not against $q^*$ critically relies on salience. If the lower quality $q'$ is salient, the new product fails. If instead the lower price is salient, the new product may be successful. By Proposition 1, price is salient if and only if the quality to cost ratio of $q'$ is higher than that of product $q^*$:

$$\frac{q^* - \Delta q}{c(q^*) - \Delta c} > \frac{q^*}{c(q^*)} \Leftrightarrow \frac{\Delta c}{\Delta q} \frac{c(q^*)}{q^*} > \frac{c(q^*)}{q^*}. \quad (11)$$

A cost cutting deviation works if the marginal cost of quality $\Delta c/\Delta q$ is higher than the average cost $c(q^*)/q^*$ at the rational equilibrium. This is intuitive: when the marginal cost is high, a small quality reduction greatly reduces the cost of firm 1. This allows firm 1 to set a salient low price, and to win the market.

The attention externality plays a key role here. As prices become salient, consumers pay less attention to quality, which reduces consumer valuation of the quality $q'$ offered by the deviating firm. This effect may undermine the profitability of the new product. However, because price is now salient for both firms, the valuation by consumers of the competing product $q^*$ drops even more! This externality allows the quality reduction to be profitable for firm 1.

Consider the alternative move whereby firm 1 deviates to a marginally higher quality product $q' = q^* + \Delta q$, which entails a higher cost $c(q') = c(q^*) + \Delta c$. If the higher price of $q'$ is salient, the deviation fails. If however its higher quality is salient, the new product may be successful. This scenario occurs provided:

$$\frac{q^* + \Delta q}{c(q^*) + \Delta c} > \frac{q^*}{c(q^*)} \Leftrightarrow \frac{\Delta c}{\Delta q} < \frac{c(q^*)}{q^*}. \quad (12)$$

A quality improving deviation can work provided the marginal cost of quality is below the average cost at the rational equilibrium. Intuitively, if the marginal cost is low, a large quality improvement entails only a small price hike, making quality salient. Once again, the attention externality is at work. The salience of quality boosts consumer valuation of
the new product, but it also draws the consumer’s attention to the low quality \( q^* \) of the competing product. These effects cause a relative over-valuation of the high quality product \( q' \), allowing the deviating firm to make profits.

This discussion delivers two messages. First, salience creates incentives to deviate away from the rational equilibrium. Second, the deviation can be toward higher or lower quality depending on the relationship between marginal and average costs of quality. This suggests that, if an equilibrium exists, it is likely to entail inefficient quality provision.

Another way to see this is to note that, according to the salience constraint in (7) and (9), the maximum price per unit of quality that firm \( k \) can extract (while still having its advantage salient) is equal to the average cost \( c_j(q_j)/q_j \) of the competing firm \( j \). As a consequence, firm \( k \) has an incentive to raise quality when its marginal cost \( c'_k(q_j) \) is lower than the marginal benefit \( c_j(q_j)/q_j \), and to lower quality when the reverse is true. When the average cost of quality is high, the consumer pays a high price while still perceiving quality as salient. The equilibrium may feature quality over-provision. When the average cost of quality is low, the consumer notices even a slight price increase. Firm \( k \) now benefits from cutting both quality and price, so that quality under-provision may occur. The analysis of the model confirms that these conjectures are correct.

**Proposition 2** When \( \delta < 1 \) and firms have identical cost functions, the unique pure strategy equilibrium is symmetric. Denote by \( \bar{q} \) and \( q \) the quality levels such that \( c'(\bar{q}) = 1/\delta \) and \( c'(q) = \delta \), and by \( q(F) \) the quality level minimizing average cost, namely \( q(F) \equiv \arg \min c(q)/q \). Then, in equilibrium price and quality are equally salient, quality provision is given by:

\[
q^S_1 = q^S_2 = q^S \equiv \begin{cases} 
\bar{q} & \text{if } F > \bar{F} \equiv \bar{q}/\delta - v(\bar{q}) \\
q(F) & \text{if } F \in [\bar{F}, \bar{F}] \\
q & \text{if } F < \bar{F} \equiv q/\delta - v(q)
\end{cases},
\]

and firms price at cost, making zero profits.

This equilibrium has three main features. First, because costs are identical, firms produce the same quality, face the same production costs, and charge the same price. But then, because firms sell identical products, price and quality are equally salient in equilibrium, so
consumers value the products that are offered correctly (as in the case where $\delta = 1$), and firms make zero profits.

Second, although *in equilibrium* consumers correctly value the goods produced, there is inefficient provision of quality (and therefore lower consumer surplus) relative to the rational case. The reason is that salience makes the firms unwilling to deviate towards the socially efficient quality $q^*$. When quality is over-provided ($q^S > q^*$), reducing quality and price backfires because consumers’ attention is drawn to the quality reduction, rather than to the price cut. This sustains an equilibrium with high quality and high prices. Similarly, when quality is under-provided ($q^S < q^*$), increases in quality and price backfire because consumers focus on the price rather than the quality hike. This sustains an equilibrium with low quality and low prices. Although in equilibrium both attributes are equally salient, we refer to the equilibrium with quality over-provision as quality-salient and to the equilibrium with under-provision as price-salient. This terminology underscores which salience ranking constrains firms from deviating towards the efficient quality level.

The third key feature of the equilibrium is that – unlike in the rational case – quality provision increases in the unit cost $F$. Intuitively, $F$ affects average costs and thus the firms’ best responses. When $F$ is high, costs and thus prices are high. By the diminishing sensitivity property, the salience of prices is low. The firm has an incentive to boost quality because any small extra cost can be “hidden” behind the already high price. As a consequence, the small extra price is not salient and quality is over-provided. When in contrast $F$ is low, costs and thus prices are low. By diminishing sensitivity, prices are now very salient. In this case, any price cut is immediately noticed, encouraging firms to cut costs to an extent that quality is under-provided.

To see these effects clearly, consider the case of the quadratic cost function.

**Corollary 1** When $\delta < 1$ and firms have identical quadratic costs $c(q) = F + v \cdot q^2 / 2$, quality

---

11In fact, $\hat{q}(F)$ satisfies $v'(\hat{q}) \cdot \hat{q} - v(\hat{q}) = F$ and the left hand side increases in $q$ because $v(.)$ is convex.
provision in the symmetric equilibrium is given by:

\[ q_2^S = q_1^S = q^S = \begin{cases} 
\frac{1}{\delta v} & \text{if } F \cdot v > \frac{1}{2\delta^2} \\
\sqrt{\frac{2F}{v}} & \text{if } \frac{1}{2\delta^2} \leq F \cdot v \leq \frac{\delta^2}{2} \\
\frac{\delta}{v} & \text{if } F \cdot v < \frac{\delta^2}{2}.
\end{cases} \]  

(14)

Figure 1 below plots \( q^S \) as a function of the unit cost \( F \), and compares it to the surplus maximizing quality, given by \( q^* = \frac{1}{v} \). As evident from the figure, salience causes quality to be over-provided when the unit cost \( F \) is sufficiently high and under-provided otherwise. Recall that for \( \delta = 1 \), we have \( q^* = \frac{1}{v} \) and quality provision does not depend on \( F \).

This analysis may help explain why sellers of expensive goods such as fancy hotel rooms or business class airplane seats compete mostly on the quality dimension, often providing customers with visible quality add-ons such as champagne, airport lounges, or treats. These visible quality add-ons help make overall product quality salient, and the profit margin associated with them can be hidden behind the high cost of the baseline good. In contrast, sellers of cheap goods such as low quality clothes or fast food compete on the price dimension. These firms cut product quality because it allows them to offer substantially lower prices. These cuts are proportionally larger in the price dimension, draw consumers’ attention to prices, and thus enable firms that supply these cheap goods to make abnormal profits. In both cases, equilibrium profits disappear as competing firms adopt the same add-on or quality.
cutting strategies, despite the fact that they are providing inefficient levels of quality.\footnote{The diminishing sensitivity property is also present in Prospect Theory (reviewed in Tversky and Kahneman, 1981). The distinctive feature of our model is the attention externality, namely the fact that changing attributes of one product alter the valuation of the competing product. This ingredient is important to generate strong reactions to price or quality changes. The benefit for a firm of increasing quality (and price) is particularly large when it induces the consumer to focus more on the full quality provided and on the lower quality of the competing product. In fact, this mechanism implies that there is a complementarity between an add-on quality and the baseline quality level.} \footnote{These examples illustrate how the results of this Section can be used to study markets with \( N > 2 \) firms. Although we do not formally analyze this case here, we note that the symmetric equilibrium that arises when firms are identical, described in Proposition 2, continues to hold for \( N > 2 \) identical firms (see the proof of Proposition 2). Unlike in the \( N = 2 \) case, however, the symmetric equilibrium of Proposition 2 may no longer be unique. In particular, firms’ products can in general have different salience rankings which may lead to distinct patterns of competition. We leave a detailed analysis of the \( N \) firm case for future study.}

Similar intuitions may help shed light on the technological and competitive forces leading product attributes to be “shrouded” (Gabaix and Laibson 2006) or to the introduction of “irrelevant” attributes (Carpenter, Glazer and Nakamoto, 1994). For instance, in a world where all hotels charge high prices for phone usage, it may be difficult for one hotel to cut phone charges and make that advantage salient to consumers (as opposed to other more important dimensions of hotel quality). In this sense, hotel charges for phone usage are shrouded. On the other hand, being the unique hotel that introduces a charge for pillows is a competitive disadvantage that is likely to draw consumers’ attention. In this sense, charging for pillows would not be shrouded. Such trade-offs can be analysed in a setting with horizontal differentiation. We leave this important topic to future work.

### 4.2 Innovation as a Cost Shock

We now use our model to explore the implications of salience for product innovation. We view innovation as a change in product characteristics and market equilibrium triggered by a cost shock. Such shock hits a market in the long run symmetric equilibrium of Proposition 2. We distinguish industry-wide cost shocks, such as those caused by deregulation or changes in input prices, and firm-specific shocks such as those stemming from the development of a new technology by an individual firm. This taxonomy illustrates the separate effects of the two key forces driving salience: diminishing sensitivity and ordering. Industry-wide shocks work mainly through diminishing sensitivity because they alter the average value of different attributes in the market. Firm-specific shocks instead work mostly through ordering: they...
allow one firm’s product to stand out against those of its competitors.

Real world innovation episodes often combine firm-specific and industry-wide factors. Initially only some firms discover new technologies or change their strategies in response to common shocks, so that the initial phase is effectively firm-specific. Subsequently, the new technologies or strategies spread to other firms, becoming industry-wide phenomena. One could view our analysis here as providing snapshots of short and long-run market adjustments to shocks. We leave the modelling of industry dynamics under salience to future research.

In what follows, we restrict attention to the case of quadratic costs, in which $c_k(q_k) = F_k + \frac{v_k}{2} \cdot q_k^2$, for $k = 1, 2$. We begin our analysis by considering industry-wide shocks to an industry in symmetric equilibrium.

**Proposition 3** Suppose that the market is in the equilibrium described by Equation (14). We then have:

i) A marginal increase (decrease) in the unit cost $F$ of all firms weakly increases (decreases) equilibrium quality provision under salient thinking ($\delta < 1$) while it leaves quality unaffected under rationality ($\delta = 1$).

ii) A marginal increase (decrease) in the marginal cost of producing quality $v$ of all firms strictly decreases (increases) equilibrium quality provision. Under salient thinking, the decrease (increase) in quality is larger than under rationality ($\delta = 1$) if and only if in the original equilibrium quality is sufficiently over-provided.

With rational consumers, changes in the unit cost $F$ do not affect quality provision. With salient thinkers, they do. This follows from the fact that a symmetric shock to the general level of costs shifts competition from quality to prices or vice-versa. A drastic increase in $F$ reduces, by diminishing sensitivity, the salience of price differences. This makes it very attractive for firms to upgrade their quality. Conversely, a drastic reduction in $F$ increases the salience of price differences. This makes it very attractive for firms to cut prices. Somewhat paradoxically, a drop in costs translates into lower quality provision.

As an example, the transportation costs involved in exporting German cars to the United States (akin to a rise in $F$ relative to the home market) may cause the car manufacturers to compete on quality provision in the US market, more than in the domestic market, by
adding quality add-ons to their cars. Similarly, truffles are served in omelettes in Provence, while truffle “shavings” are added to elegant dishes in the United States, where truffles are in relative terms much more expensive. Lobster is more likely to be served boiled in Boston than in Chicago. Conversely, a reduction in the tariffs on textile imports from China (akin to a drop in $F$) may induce clothing manufacturers in Europe to intensify price competition relative to the situation with higher tariffs.

The effect of a drop in the marginal cost of producing quality $v$ is more standard. As in the rational case, this shock increases quality provision. However, salience modulates the strength of this effect. The boost in quality provision is amplified at very high cost levels, when there is over-provision of quality, while it is dampened in all other cases. This effect is again due to diminishing sensitivity: by reducing the level of prices, reductions in $v$ render consumers more attentive to price differences, reducing firms’ incentive to increase quality.

Consider next the effect of a firm-specific shock. Suppose that, starting from a symmetric equilibrium, firm 1 acquires a cost advantage that enables it to monopolize the market. One complication is that in this asymmetric case there is typically a multiplicity of equilibria (both under rationality and salience): like in the price-competition subgame of Section 3, the losing firm is indifferent between choosing among quality levels leading to zero profits. To derive comparative statics, and compare the predictions of the salience model to those of the rational model, we introduce an intuitive equilibrium selection rule: we keep the quality of a firm fixed at the pre-innovation, “symmetric play”, unless it is strictly profitable for the firm to deviate from it (given the other firm’s best response to the original symmetric equilibrium). As we now show, this rule uniquely pins down the equilibrium both in the rational and the salience cases.\footnote{In this sense, equilibria are characterized by firms’ best response to each other’s “symmetric play”. This equilibrium selection rule is based on the idea that firms face some inertia in adjusting their quality level, and so they keep quality constant unless it is strictly beneficial for them to unilaterally deviate from the pre-shock symmetric play. In Online Appendix B.4 we perform a more detailed characterization of asymmetric equilibria that includes equilibria not satifying this equilibrium selection rule.}

For brevity, we report only the effects of reductions in the variable cost of quality. In the rational model, we find:

**Lemma 3** *Suppose that, starting from the symmetric equilibrium of Equation (14), the vari-


able cost of firm 1 drops to $v_1 < v_2 = v$. Then, when $\delta = 1$, in equilibrium firm 1 captures the market, $d_1 = 1$, and makes positive profits, $\pi_1 > 0$. Under the “symmetric play” selection rule, there is a unique subgame perfect equilibrium in pure strategies characterized by quality choices $q_1^*, q_2^*$, such that:

$$q_1^* = \frac{1}{v_1} > q_2^* = \frac{1}{v},$$ (15)

As a consequence, firm 1 increases quality provision, it wins the market ($d_1 = 1$), and makes positive profits. Equilibrium prices are $p_1^* = p_2^* + (q_1^* - q_2^*)$ and $p_2^* = c_2(q^*)$.

In the equilibrium pinned down by our selection rule, both firms choose the quality level that - given their own costs - maximizes social surplus. Relative to the symmetric benchmark in which both firms have marginal cost $v = v_2$, firm 1 increases quality provision, wins the market, and makes positive profits.

Consider now the case of salient thinking (namely $\delta < 1$):

**Proposition 4** Suppose that, starting from the symmetric equilibrium of Equation (14), the variable cost of firm 1 drops to $v_1 < v_2 = v$. Then, when $\delta < 1$ firm 1 monopolizes the market, $d_1 = 1$, and makes positive profits. Under the “symmetric play” selection rule, there are two cases:

i) The cost shock is large, $v_1 < v/2$. Then, firm 1 boosts both its quality and its price.

ii) The cost shock is small, $v_1 > v/2$. Then, there is a threshold $\hat{F} > 0$ such that firm 1 boosts its quality and price if and only if $F \geq \hat{F}$. If $F < \hat{F}$, firm 1 keeps its quality constant at the competitor’s level $\delta/v$ and wins the market.

The size of the cost shock plays a critical role. If the variable cost reduction is drastic, or if the unit cost is high (i.e. $F \geq \hat{F}$), firm 1 can win the market by boosting quality provision. In this case, prices tend not to be salient, because average costs are high, and therefore quality differences can be large. In this configuration, a substantial quality upgrading alters the market outcome, changing the equilibrium from price- to quality- salient: as firm 1 provides extra quality, the overall quality of its product becomes salient, and consumers’ willingness to pay rises even for infra-marginal quality units. In this sense, the quality add on acts as
a complement to baseline quality, greatly increasing the price that firm 1 can charge for its product. This logic provides the testable predictions: i) quality improving innovations regularly occur for goods that are already of high quality (and expensive), and ii) the level of such quality add-ons should respond positively to increases to the unit cost $F$, and to reductions of the marginal cost of quality.

Matters are different when the cost shock is small, $v_1 > v/2$, and the unit cost is low (i.e., $F < \hat{F}$). Now prices tend to be salient because of low average cost of quality, and the small cost advantage also makes it very costly for firm 1 to engineer a drastic increase in quality. In this case, quality upgrades make the associated price hikes salient, and thus backfire. As a consequence, it is optimal for firm 1 to keep its quality and price constant at the symmetric equilibrium level, since given the sharing rule firm 1 is then guaranteed to capture the market. This outcome, while puzzling in a rational model, is natural with salience: in a price-salient equilibrium quality upgradings are neglected, and firms exploit lower costs to cut prices.

An important implication of this analysis is that price-salient equilibria are very stable, particularly for low cost industries, and that in these industries quality upgrades are very hard to materialize. To escape a commoditized market, an individual firm must develop a drastic innovation that allows it to provide sufficiently higher quality than its competitors, and at such reasonable prices that quality becomes salient. Small cost reducing innovations neither beat the “commodity magnet”nor lead to marginal quality improvements. They just translate into lower prices.

This result more generally illustrates the working of our model when costs are asymmetric. The low cost firm wins the market, but whether it does so by setting higher quality or lower price depends on the extent of its cost advantage. If it has a large cost advantage, the low cost firm captures the market by setting a salient high quality. If the cost advantage is small, the low cost firm captures the market by setting a salient low price.

In Proposition 4, the strategy of the losing firm 2 is held at the quality it would set in a symmetric equilibrium where both firms have cost $c_2(q)$. This is a plausible refinement to study the effect of an innovation shock, but may be less appealing to study the equilibrium arising under permanently different cost functions. In Online Appendix 3, we describe
asymmetric equilibria more generally, where the high cost firm 2 is not constrained to the quality level given by (14). Although the results focus on the case where $F = 0$, they closely mirror Proposition 4. The only difference is that when the cost advantage of firm 1 is low, equilibria may arise in which – instead of producing the same quality – the low cost firm wins the market by providing less quality than the high cost firm. This is a further departure from the rational benchmark: the low cost firm may deliberately provide lower quality precisely to make its lower price salient to the consumer.

5 Applications

We now apply our model to describe three actual innovations. In Section 5.1 we discuss the innovations of Southwest in the airline market and of Starbucks in the coffee market. In Section 5.2 we show that our model can capture some features of recent financial innovations in securities markets.

5.1 Southwest and Starbucks

The rise of low cost airlines in the U.S. is directly linked to deregulation that started in the late seventies. Deregulation enabled carriers to freely set routes and prices, but also freed entry into the industry. In addition to the removal of price controls, these developments entailed a major reduction in the costs of operating an airline. New entrants such as Southwest implemented large price cuts. Traditional carriers, burdened with legacy costs, were unable to respond. Prices have declined steadily since deregulation, but some aspects of the quality of airline service have also declined.

These developments are consistent both with our model of salience and with a rational model with heterogeneous consumers. In the salience model, these events can be seen as a transition from a quality to a price salient equilibrium. In the pre-deregulation era, the equilibrium was quality salient: due to regulation, firms had little room to compete on prices, and the resulting small differences in airfares were rendered non-salient by high operating costs $F$. Airlines therefore competed by offering visible extra services to consumers. High quality and high prices went hand in hand. Deregulation shifted the equilibrium from quality
to price salient: the drop in operating costs $F$ created the opportunity for new entrants to implement large price cuts. In the new price salient equilibrium, firms cut their quality to reduce prices even further. The budget air travel market thus became commoditized, characterized by low prices and low quality.\footnote{Competition on quality in the pre-deregulation era may have also been driven by price controls, see Douglas and Miller (1974). However, in this regard two points should be noted. First, in the pre-deregulation era price controls merely specified a standard of “reasonableness” for prices. In principle, this rule did not prevent airlines to engage in small scale price competition (potentially affecting the standard price level itself). Of course, salience could be a reason why competition did not ignite a succession of “reasonable” price cuts: being small, such price cuts would not be noticed by consumers. In this sense, price controls and salience may be complementary forces.}

In a rational model with heterogeneous consumers, equilibrium fares are initially so expensive that only wealthy (and thus price inelastic) consumers can afford to fly. Airlines cater to these consumers by providing high quality, a tendency exacerbated by price controls (Douglas and Miller, 1974). As deregulation removes price controls and causes operating costs $F$ to fall, poorer (and thus price-elastic) consumers enter the market. This intensifies price competition. If the inflow of price elastic consumers is large, quality provision may also drop.

In both the rational and salience accounts, the drop in quality and price of airline service is accompanied by an increase in the average price sensitivity of the consumers. The central difference lies in where the change in average price sensitivity comes from. Only the salience model can explain why the market became commoditized, in the specific sense of inducing all consumers – even the wealthier ones – to become more price sensitive. In the standard model, original consumers continue to be price insensitive, and price competition intensifies just because price-sensitive consumers join the market.\footnote{Commoditization seems critical to create a strong incentive for companies to cut quality. If all legacy consumers were willing to buy expensive high-quality tickets, one might have expected deregulation to increase the size of business class or create a large market segment specialized in serving price insensitive flyers. If instead all consumers become relatively more price sensitive, the reduction in quality occurs across the board, sparing perhaps only existing business class seats, which remain characterized by a high unit cost $F$ of airplane space.} Consistent with this rational explanation, it is indeed the case that less affluent consumers increasingly used air travel after deregulation. This change in market composition makes it harder to appreciate the commoditizing role of salience.

A better market to distinguish the rational and salience models is one in which the
composition of consumers changes little over time, as innovations accumulate. To this end, consider the evolution of the coffee shop market. In the 1970’s, sellers of drip coffee at neighbourhood coffee-shops and fast food restaurants offered low quality coffee at low prices. In this initial regime, product innovations effectively translated into price cuts (e.g., free refills) instead of quality increases. In the 1980’s, firms such as Starbucks and Peet’s Coffee & Tea figured out how to deliver a much higher quality coffee at only a reasonable extra cost. This included serving expresso drinks but also training baristas to ensure consistency of the product, and providing a “cafe” experience through a comfortable in-shop environment. These firms gained market share by boosting the quality of coffee as well as prices.

Our model suggests the following account. The low quality and thus low prices prevailing in the 70’s locked coffee shops into a price salient equilibrium. The market was commoditized so that, consistent with Proposition 4, marginal innovations took the form of price reductions rather than quality upgrades. After inventing a way to drastically reduce the costs of producing quality, Starbucks could engineer a salient quality improvement. In the new quality salient equilibrium, much higher prices could be charged. Starbucks’ well-documented growth trajectory, while extreme, reflects the growth of the premium coffee market, which was successfully de-commoditized.

It is not easy to describe these events based solely on consumer heterogeneity. Arguably, the composition of coffee buyers has been stable over time. It is then difficult to explain why innovations in the coffee market changed from price-cutting to quality-improving. De-commodization in this market seems to require a generalized increase in the taste for quality, which is precisely what the salience model endogenously generates.

To take stock, the market wide predictions of the salience model are sometimes similar to those generated by a rational model of consumer heterogeneity, as in the case of airlines, and sometimes different, as in the case of the coffee market. Critically, in all applications the distinctive prediction of the salience model is that market changes trigger common shifts in the price sensitivity of all consumers. Such shifts in preferences can in principle be

\[17\] More broadly, salience and consumer heterogeneity may play complementary roles, in the sense that salience may amplify existing heterogeneity. If one firm reduces price to attract price sensitive consumers, and in the process makes price salient for all consumers, other firms face pressure to cut prices further. Similarly, salience amplifies the perceived differentiation in markets where products are very similar. By focusing the consumers’ attention on the (potentially small) differences between goods, salience allows for
observed as switches in the items bought by individual consumers and as changes in the sensitivity of a given consumer to given price differentials. Our model has novel predictions about such choice patterns, which ideally can be tested with individual-level data.

Direct tests of the comparative statics of Proposition 2 can be performed using natural variations in costs, e.g. due to changes in transportation costs, tariffs, or actual innovations, that occur across time or across otherwise similar consumer populations. To abstract from the role of price expectations, as we do here, it is important to examine the long term effects of cost changes.\footnote{Here we emphasize the long term response because, in the more general model of salience presented in BGS (2013), the salience of price also depends on how actual prices differ from price expectations. That model generates the novel prediction that surprising price increases make price salient in the short term, but in the long term (as expectations converge to actual price levels) higher price levels make price less salient. The long term mechanism is highlighted here. The short term mechanism, however, is also a powerful testing ground for the salience model, see Hastings and Shapiro (2013).}

For example, permanently higher taxes on cigarettes in some states in the US are associated with permanently higher demand for higher cigarette quality in those states (Sobel and Garret, 1997).

5.2 Financial Innovation

Our model can shed light on the working of financial innovation, and in the phenomenon of “reaching for yield.” We describe innovations that occurred in the safe (AAA) asset market, involving the creation of mortgage backed securities (MBS). A security $i$ is characterized by the expected return $R_i$ it yields to investors (net of intermediation fees), and its risk $\rho_i$. The investor’s “rational” valuation of asset $(R_i, \rho_i)$ is mean-variance, namely:

$$u_i(R_i; \rho_i) = R_i - \rho_i. \quad (16)$$

Under salient thinking, the investor overweights the more salient attribute, which can be either risk or return. We assume salience is determined by comparing only assets in the same AAA risk class.\footnote{This setting represents investors (possibly including fund managers) choosing which AAA securities to hold, rather than optimizing over the whole range of assets of different risk categories.} Suppose that the investor chooses between two assets $i = 1, 2$ and the salience function is $\sigma(\cdot, \cdot)$. The following cases can occur. If $\sigma(R_1, R_2) > \sigma(\rho_1, \rho_2)$, returns are salient and the investor values asset $i$ at $R_i - \delta \cdot \rho_i$. If $\sigma(R_1, R_2) < \sigma(\rho_1, \rho_2)$, risk is salient high markups which would otherwise require very high (even implausible) levels of consumer heterogeneity.
and the investor values asset $i$ at $\delta \cdot R_i - \rho_i$. Finally, if $\sigma(R_1, R_2) = \sigma(\rho_1, \rho_2)$, risk and return are equally salient and the investor’s valuation is rational.

Initially, there are two financial intermediaries, or brokers, $i = 1, 2$, each offering to the investor an identical asset, characterized by a gross expected return $\overline{R}$ and risk $\rho$.\footnote{The model works identically in the case where there are $N > 2$ intermediaries, as long as, as assumed here, only one intermediary has access to an innovation while the remaining $N - 1$ keep their traditional strategy. In this sense, our model captures the initial phases of innovation in which one firm introduce a new product, and studies the conditions that lead to the innovation’s success or failure.} In this pre-innovation benchmark, both intermediaries offer the “standard” asset, such as government bonds. We assume that intermediaries offer this asset to investors at some fee, which can be though of as brokerage of management fee.

Intermediaries compete by offering investors assets with net return $R_i \leq \overline{R}$ and risk $\rho$. Thus, $\overline{R} - R_i$ is the brokerage or management fee of intermediary $i$. Investors decide with which intermediary to invest. Competition then works as in Section 2, where quality and cost are fixed:\footnote{The only difference is that in the setting of Section 2, firms’ pricing strategies determine the cost for consumers to buy the good, while here the firms’ pricing strategies determine the “quality” of the asset for the investor (namely the investor’s return), while cost is exogenously given by the asset’s risk.} each intermediary offers a net of fee return $R_i$, which is analogous to product quality, at the cost to the investor of bearing risk $\rho$, which is analogous to price. As a consequence, the upside of the asset with the highest ratio of return to risk is salient, causing that asset to be overvalued relative to its competitor’s. Because firms are identical and returns and risk are given exogenously, the following equilibrium benchmark holds both in the rational case and with salient thinkers.

**Lemma 4** With no innovation, intermediaries charge zero fees, $R_1 = R_2 = \overline{R}$, and make zero profits, and the investor is indifferent between the two firms.

As in standard Bertrand competition, the two firms selling the same asset make zero profits, offering the full return $\overline{R}$ to the investor (under salient thinking, the logic is the same as that of Proposition 1 point $iii$).

Against this benchmark, we model financial innovation as the creation by one broker of a technology to generate excess return at only a moderate extra risk. The innovating broker, say broker 1, may for example find a way to better diversify the risks from the securities it
already manages and thus offer a different asset to investors. Formally, broker 1 invents a new asset in the same asset class, with the gross return:

$$\overline{R} + \alpha,$$

where $\alpha$ is the new asset’s excess return. The asset’s risk then increases to:

$$\rho + \frac{v}{2} \cdot \alpha^2,$$

where $v$ captures the marginal cost – in terms of added risk – of creating excess return $\alpha$. Broker 2 continues to offer the standard product with gross return $\overline{R}$. The no-innovation benchmark can be viewed as the extreme case where $v$ is prohibitively high for both firms.

With fully rational investors, the working of innovation is straightforward. In the spirit of Lemma 3, the innovating broker: i) captures the entire market by offering the investor a net return of $\overline{R} + (v/2) \cdot \alpha^2$ (which compensates the investor for bearing the extra risk), and ii) sets $\alpha$ to maximize its profit:

$$\max_\alpha \alpha - (v/2) \cdot \alpha^2$$

which implies $\alpha^* = 1/v$. The lower is the extra risk $v$, the greater is the excess return promised by the new financial product. As broker 1 manufactures an asset with a better return/risk combination, its profit and thus social welfare rise (the investor is left indifferent).

In the case of salient thinking, the critical question is whether, compared to the standard asset, the new asset’s risk or return is salient. Depending on which attribute is salient, the innovating broker will have an incentive to create a particular return vs. risk profile. The reason is that under salience the investor’s risk appetite endogenously depends on the salient features of the new asset. The new equilibrium is as follows.

**Proposition 5** *The innovating broker 1 captures the market and makes positive profits. The*
optimal excess return satisfies:

\[ \alpha^* = \begin{cases} 
\frac{1}{\delta - v} & \text{for } \bar{R} < \delta \cdot \rho \\
\frac{\rho}{\bar{R}} \cdot \frac{1}{v} & \text{for } \bar{R} \geq \delta \cdot \rho 
\end{cases} \]  

(18)

Relative to the rational benchmark, under salient thinking there is excessive risk taking if \( \bar{R} < \rho \) and too little risk taking if \( \bar{R} > \rho \).

The innovation is particularly successful when investors focus on the extra return offered by the new asset and underweight the extra risk that comes with it. As Proposition 5 illustrates, this is the case precisely when the net return \( \bar{R} \) of the standard asset is low. Diminishing sensitivity generates a “reach for yield” at low interest rates: an excess return of, say, 0.5% is much more salient when the baseline return is 1% than when the baseline return is 6%.\(^{22,23}\) Proposition 5 shows that in this case financial intermediaries have an incentive to offer excessively risky products. When investors focus on return, they underweight risk, enabling the broker to charge high fees.

An important implication of this analysis is that, when investors’ attention is drawn to returns, risks are relatively speaking neglected, and investors are disappointed when particularly bad returns render risk salient. Gennaioli, Shleifer, and Vishny (2012, 2013) modeled this neglect of risk as investors’ disregard of tail events. We also presented some evidence consistent with the prediction that downside risks were neglected in the period preceding the 2007 – 2008 financial crisis. The salience approach makes a similar point in a perhaps subtler way. During the “reach for yield” episodes, interest rates are low and investors are

\(^{22}\)If one views the baseline return \( \bar{R} \) as a net return generated after defraying the intermediary’s operating costs, the same intuition may explain why banks take more risk when their operating costs are higher. If intermediaries react to higher operating costs (i.e. a lower \( \bar{R} \)) by cutting the net return paid to investors (as the rational model would predict), investors would find this an unattractive deal, reducing their demand and the intermediary’s profit. If instead the intermediary takes more risk, investors focus on its excess return. Because the investors underweight the asset’s risk, the intermediary can increase fees. These fees allow the intermediary not only to cover its higher operating costs, but also to generate profits.

\(^{23}\)Proposition 5 also shows that financial innovations geared at creating excess returns are much less successful when net returns are already high. In this case, the investor is much less sensitive to a given increase in return, and the innovating firm must keep the risks of the new asset very low, lest the investors focus on them. In this case, there is too little risk taking, in the sense that the intermediary selects an excess return in (18) below its rational counterpart in (17). Although for simplicity we have not allowed for this possibility, here the intermediary may find it profitable to reduce excess returns and risks relative to the standard asset.
prone to be inattentive to risks. When investors underweight risks, they engage in too much risk taking. When bad states of the world materialize, these investors wish they had paid more attention.

6 Conclusion

We have shown how salience changes some of the basic predictions of a standard model of competition with vertical product differentiation. Yet the paper has only begun to explore the consequences of salience for market competition. Rather than summarizing our results, in conclusion we mention some issues we have not addressed, but which may be interesting to investigate. These include dynamics of competition, welfare, horizontal product differentiation, and advertising. We have not solved any of these problems, so the discussion here is strictly conjectural.

In a dynamic setting, the salience of a firm’s strategy is not only shaped by the background of its competitors, but also by past market outcomes. As we formalized in BGS (2013), the price of a product is salient not only if the product looks expensive relative to substitute goods available today, but also if it looks expensive relative to yesterday’s prices. This result has interesting implications for the dynamics of entry and imitation. In particular, these dynamics may be very different depending on whether the original innovation ultimately leads to quality-salient or price-salient long run equilibrium. If an innovator finds a way to escape the commodity magnet and produce higher quality at a higher price, the pace at which this change is implemented, and imitated, might be relatively slow. The reason is that firms need to keep quality rather than price salient, and prevent consumers from becoming focused on price increases. This slows down innovation. As an extreme example, if consumers are used to free education, as they are in Europe, charging for education might be extremely difficult even with significant quality improvements because the focus will be entirely on prices. (Of course, once prices are high enough, the pace of innovation and price increases accelerates.) In contrast, precisely because consumers are focused on prices and neglect quality, innovation that reduces price and quality will be extremely fast. The slide to the commodity magnet will be faster than in a rational model.
We have shown that – under the natural assumption that consumer welfare is measured by the undistorted utility – quality provision is generally inefficient in a duopoly, as a consequence of competition for attention between the two firms. An assessment of the welfare consequences of competition when consumers are salient thinkers would require a deeper understanding of the model with heterogeneous consumers, and in particular of monopoly and free entry.

Our approach might also be used to study horizontal differentiation, and to investigate the marketing dictum of “differentiate in any way you can” (Levitt 1983). If a firm horizontally differentiates its product from competitors, then differences along the differentiated attribute become salient, and will attract consumers’ attention. At the same time, differences in prices, which are similar across alternatives, will become non salient. In fact, firms might differentiate their products precisely to segment the market between consumers attracted to different attributes, and thus earn higher profits. This approach has clear applications to product markets, but it might also shed light on political competition, where it can reverse the median voter result in a plausible way. It would suggest that politicians might perhaps converge to the median voter viewpoint on some positions, but also seek to differentiate their views on dimensions that voters might find salient (and attractive). The two parties in the United States converge on their views on Social Security, for example, making sure that voters do not pay attention to that issue, but then seek to differentiate on the issues they choose, such as immigration or gay marriage.

Finally, salience may have significant implications for how we think about advertising, which deals precisely with drawing consumer attention to products and their attributes. Economists distinguish two broad approaches to advertising: informative and persuasive. The former focuses on provision of hard information about the product; the latter deals with its more emotional appeal. Salience suggests that in fact the two approaches are intimately related, and usually integrated: a key purpose of advertising is to inform about and thus draw attention to the attributes of the product that the seller wants the consumer to think about, but not others. Gas stations sell regular and super gasoline, even though the difference in octane content is only about 3%. Advertising of attributes is simultaneously informative (sometimes about prices, sometimes about quality, rarely both) and persuasive in that the
salience of the attributes being advertised is enhanced. The purpose of advertising is precisely to let some desirable attributes of the product stand out for the potential customers.

In all these situations, firms compete to attract attention to the attributes they want consumers to attend to, and to distract attention from their less attractive attributes.


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A Proofs

Lemma 1 (price competition under rationality). When $\delta = 1$ there are no salience distortions and utility is given by Equation (2). Recall that we exclude equilibria in weakly dominated strategies, which constrains firms to price weakly above costs, $p_k \geq c_k$.24

We analyse each case in turn. If $q_1 - c_1 > q_2 - c_2$ then firm 1 sets its price $p_1 = c_2 + (q_1 - q_2)$ and firm 2 sets its price $p_2 = c_2$. The sharing rule determines that at these prices firm 1 captures all demand (because only firm 1 can profitably reduce its price). Firm 2 has no incentive to increase its price, since it cannot capture demand by doing so. In turn, firm 1 has no incentive to increase its price, because it would violate the valuation constraint, nor to decrease its price, which would reduce profits. This demonstrates existence. To show uniqueness, assume by contradiction that firm 2 sets its price $p_2 > c_2$. Given the sharing rule, firm 1’s best response is to set its price arbitrarily close to, but less than, $p_2 + (q_1 - q_2)$. This cannot however be an equilibrium: firm 2 would have an incentive to lower its price below $p_2$ as that would capture demand and increase its profits.

If $q_1 - c_1 < q_2 - c_2$, the existence and uniqueness arguments carry through switching firms 1 and 2. Finally, if $q_1 - c_1 = q_2 - c_2$, then both firms price at cost and share the market. Neither firm has an incentive to deviate: increasing price would violate the valuation constraint and would not increase profits; decreasing price would lead to negative profits. Uniqueness follows as before: if one firm sets its price above cost, the other firm has an incentive to also set its price above its cost and capture the market. This cannot be an equilibrium because – at this configuration – the first firm has an incentive to slightly reduce its price. Thus, no equilibrium exists in which either firm prices above cost. ■

Proposition 1 (price competition under salient thinking). When $\delta < 1$, utility is given by Equation (3), where salience determines the relative weight of quality and price. We first characterise the equilibria in the full parameter space of exogenous qualities and

24This result is due to the following considerations. As we will see below, in the rational benchmark there cannot exist an equilibrium in which the firm producing the (strictly) lowest amount of surplus (say $k$) wins the market. Moreover, the best response correspondence of firm $k$ is a function of $q_{-k}$ equal to $p_k + q_k - q_{-k}$. In contrast, the best response correspondence of firm $k$ is represented by the interval $[c_{-k} + (q_k - q_{-k}), c_k]$. All strategies within this interval yield zero profit to firm $k$ (hence the multiplicity of equilibria), but only strategy $p_k = c_k$ is not weakly dominated.
costs satisfying $q_1 \geq q_2$ and $c_1 \geq c_2$, showing existence and uniqueness. This is illustrated in Figure 2. We then restrict the analysis to parameters satisfying Assumption A.1.

Let $q_1 \geq q_2$ and $c_1 \geq c_2$ be given. Note that it cannot be the case that in equilibrium $p_1 < p_2$. In fact, if either quality or price is salient (equally for both goods, due to symmetry of the salience function), the low quality firm 2 is certain to lose the market by setting a higher price than the high quality firm 1. Thus, in the remainder we focus on $p_1 \geq p_2$.

Accordingly, when $p_1 \geq p_2$ the valuation of a good always decreases in its price. This is obviously true within a salience ranking, but is also true across salience rankings. When firm 1 increases its price, it renders price salient, which reduces the consumer’s relative valuation of good 1. When firm 2 increases its price, it renders quality salient, which reduces the consumer’s relative valuation of good 2.

We now separately consider three cases: one in which firm 1 wins, another in which firms 2 wins, and a final one in which the two firms split the market. As a preliminary observation, note that – like in the rational case – the losing firm prices at cost. The winning firm maximizes its profit by setting a price that renders either the valuation or the salience constraint binding, given the losing firm’s price (see Equations (6) through (9) in the text). But this can only be an equilibrium if the losing firm prices at cost, because – by the assumed tie breaking rules – only in this case is the losing firm unable to win the market and make a profit by reducing its price.

- Suppose that in equilibrium firm 1 wins the market, so that $p_1 \geq c_1$ and $p_2 = c_2$. There are two cases:

  - Firm 1 wins with salient quality. This occurs when $p_1$ satisfies:

    $$\begin{cases} \frac{q_1}{p_1} \geq \frac{q_2}{c_2} & \text{(salience constraint)} \\ p_1 \leq c_2 + \frac{1}{\delta} (q_1 - q_2) & \text{(valuation constraint)} \\ p_1 \geq c_1 \end{cases}$$

    (19)

    The set of $p_1$ satisfying the above constraints is non empty when either: i) $\frac{q_1}{c_1} > \delta$ and $\frac{q_2}{c_2} \in \left[\delta, \frac{q_1}{c_1}\right]$, when the salience constraint binds; or ii) $\frac{q_2}{c_2} < \delta$ and $\frac{q_1}{c_1} \geq \delta - \frac{c_1}{c_2} \left(\delta - \frac{q_2}{c_2}\right)$ when the valuation constraint binds. The tie breaking rule assumed at
the outset guarantees that at the price \( p_1 \) at which either the salience or valuation constraint in (19) binds, firm 1 captures the entire market. This is because, when \( p_2 = c_2 \), only firm 1 can profitably reduce its price.

- Firm 1 wins with salient price. Then \( p_1 \) must satisfy:

\[
\begin{align*}
\frac{q_1}{p_1} &\leq \frac{q_2}{c_2} & \text{(salience constraint)} \\
p_1 &\leq c_2 + \delta(q_1 - q_2) & \text{(valuation constraint)} \\
p_1 &\geq c_1
\end{align*}
\]

Equation (20) identifies a set of prices \( p_1 \) that, if not empty, is bounded above by the valuation constraint. This set is non empty when: i) \( \frac{q_1}{c_1} > \frac{q_2}{c_2} \) and \( \frac{q_2}{c_2} > \frac{1}{\delta} \), when the salience constraint binds \( p_1 \) from below, and ii) \( \frac{q_1}{c_1} < \frac{q_2}{c_2} \) and \( \frac{q_1}{c_1} > \frac{1}{\delta} + \frac{c_2}{c_1} \left( \frac{q_2}{c_2} - \frac{1}{\delta} \right) \), which implies \( \frac{q_2}{c_2} > \frac{1}{\delta} \). This is the case in which the cost constraint binds prices from below. In particular, when \( \frac{q_1}{c_1} > \max \left\{ \frac{1}{\delta}, \frac{q_2}{c_2} \right\} \) firm 1 can win either with salient quality or with salient price. In either case, firm 2 sets its price equal to cost. Here firm 1 has an incentive to choose the price salience configuration as it allows it to set a higher price, and thus obtain a higher profit.

- Suppose that in equilibrium firm 2 wins the market. Then it must be that firm 2 sets its price \( p_2 \geq c_2 \) while firm 1 prices at cost. There are again two cases:

- Firm 2 wins with salient quality, so that \( \frac{q_2}{p_2} \leq \frac{q_1}{c_1} \). Salient quality implies that price satisfies \( p_2 \leq c_1 + \frac{1}{\delta}(q_1 - q_2) \), \( p_2 \geq c_1 \frac{q_2}{q_1} \) and \( p_2 \geq c_2 \). The set of \( p_2 \) satisfying these conditions, if non empty, is bounded above by the valuation constraint. The set is non empty when: i) \( \frac{q_2}{c_2} \leq \frac{q_1}{c_1} \) and \( \frac{q_1}{c_1} < \delta \), where the salience constraint provides a lower bound for price; or ii) \( \frac{q_2}{c_2} < \delta \) and \( \frac{q_2}{c_2} \geq \delta + \frac{c_1}{c_2} \left( \frac{q_1}{c_1} - \delta \right) \), where the cost constraint provides a lower bound, implying \( \frac{q_1}{c_1} < \delta \).

- Firm 2 wins with salient price, so that \( \frac{q_2}{p_2} \geq \frac{q_1}{c_1} \). In this case, price must satisfy \( p_2 \leq c_1 \frac{q_2}{q_1} \), \( p_2 \leq c_1 - \delta(q_1 - q_2) \), as well as \( p_2 \geq c_2 \). The set of \( p_2 \) satisfying these conditions is non empty when: i) \( \frac{q_1}{c_1} > \frac{1}{\delta} \) and \( \frac{q_2}{c_2} > \frac{1}{\delta} + \frac{c_1}{c_2} \left( \frac{q_1}{c_1} - \frac{1}{\delta} \right) \), in which case the valuation constraint provides an upper bound on price, and ii) \( \frac{q_1}{c_1} < \frac{1}{\delta} \).
and $\frac{q_2}{c_2} > \frac{q_1}{c_1}$, in which case the salience constraint provides an upper bound on price. In particular, when $\frac{q_2}{c_2} \in \left[ \frac{q_1}{c_1}, \frac{1}{\delta} \right]$ (for $\frac{q_1}{c_1} < \frac{1}{\delta}$), firm 2 can win either with salient quality or with salient price, while firm 1 always prices at cost. In this case, in equilibrium firm 2 sets its price such that quality is salient, since it can then obtain a higher profit by doing so.

The analysis above shows equilibria exist for any parameters satisfying $q_1 \geq q_2$ and $c_1 \geq c_2$. Furthermore, the equilibria are unique, since for every choice of quality and cost parameters, equilibrium prices are uniquely defined. While in some regimes the firm that wins the market makes its advantage salient (e.g. when $\frac{q_1}{c_1}, \frac{q_2}{c_2} \in \left[ \delta, \frac{1}{\delta} \right]$), in other regimes – namely when one firm’s quality cost ratio is extreme – a firm might win the market despite having increased its price to the point that its disadvantage (high price or low quality) is salient.

We now restrict the results to the case where Assumption A.1 holds. In equilibrium, the firm that wins the price competition sets its price so that its relative advantage is salient. Thus, if $q_1/c_1 > q_2/c_2$, firm 1 wins the market in equilibrium. Because the salience constraint binds, firm 1 sets $p_1 = \min \{ c_2 \cdot \frac{q_1}{q_2}, c_2 + \frac{1}{\delta} (q_1 - q_2) \}$ and firm 2 sets $p_2 = c_2$. It is instructive to analyse in more detail why firm 1 has no incentive to deviate from price $p_1$ (it is trivial to see that firm 2 has no incentive to deviate from $c_2$). Suppose the salience constraint is binding, $c_2 \cdot \frac{q_1}{q_2} < c_2 + \frac{1}{\delta} (q_1 - q_2)$. Then raising the price $p_1$ above the salience constraint makes price salient, and the valuation constraint becomes $p_1 < c_2 + \delta (q_1 - q_2)$. But then, because A.1 requires $c_2 + \delta (q_1 - q_2) < c_1$, this implies, together with $\frac{q_1}{c_1} > \frac{q_2}{c_2}$, that the valuation constraint is violated. By increasing its price to the point where it becomes salient, firm 1 shifts the consumer’s attention to its downside, and lowers the consumer’s valuation to the point where it loses the market. Suppose instead that the valuation constraint is binding. Then, by construction, any deviation in price leads to a decrease in profits. A similar argument shows that, if $q_2/c_2 > q_1/c_1$, firm 2 wins the market, and that equilibrium prices satisfy $p_2 = \min \{ c_1 \cdot \frac{q_2}{q_1}, c_1 + \delta (q_2 - q_1) \}$ and $p_1 = c_1$.

Finally, if the two firms have the same quality to cost ratio, namely $q_1/c_1 = q_2/c_2$, no firm can raise its price above cost without having its disadvantage salient. Given that, by A.1,
consumers do not buy a good whose disadvantage is salient, the only equilibrium is for the two firms to price at cost, setting $p_1 = c_1$, $q_2 = c_2$. Firms make zero profits, both attributes are equally salient and consumer select the good yielding higher (rational) surplus.

Figure 2: Equilibria of the price competition game when $q_1 > q_2, c_1 > c_2$.

Lemma 2 (quality competition under rationality with symmetric costs). We proceed by backward induction. By Lemma 1, the firm that produces higher surplus wins the market and sets its price equal to the differential surplus, while the other firm prices at cost. At the quality choice stage, firm $k$’s optimisation problem is

$$\max_{q_k} \left( [q_k - c_k(q_k)] - [q_{-k} - c_{-k}(q_{-k})] \right) \cdot d_k(q_k, q_{-k})$$

(21)

where $d_k(q_k, q_{-k}) = \begin{cases} 1 & \text{if } q_k - c_k(q_k) > q_{-k} - c_{-k}(q_{-k}) \\ 1/2 & \text{if } q_k - c_k(q_k) = q_{-k} - c_{-k}(q_{-k}) \\ 0 & \text{if } q_k - c_k(q_k) < q_{-k} - c_{-k}(q_{-k}) \end{cases}$
The maximum surplus produced by firm $k$ is equal to $q_k^* - c_k(q_k^*)$ where $q_k^*$ satisfies $c'_k(q_k^*) = 1$, and similarly for firm $-k$.

Consider now the symmetric case where firms have the same cost function $c(\cdot)$. In equilibrium both firms maximize surplus, $q_k^* = q_{-k}^* = q^*$. Neither firm has an incentive to deviate, as doing so implies losing demand in the pricing stage and is thus not profitable. This equilibrium is unique; suppose for contradiction that firm $k$ chooses quality $q_k'$. Then firm $-k$ can win the market, e.g. by setting quality $q^*$ and price $p_{-k} = q^* - q_k' + c(q_k')$. But this is not an equilibrium, since firm $k$ can capture the market by setting the same quality $q^*$ and a slightly lower price. ■

**Proposition 2 (symmetric equilibrium under salient thinking).** We begin by showing that any pure strategy subgame perfect equilibrium (if it exists) is symmetric: both firms set the same quality, price at cost, and split the market ($d_k = d_{-k} = 1/2$). Suppose by contradiction that firms set different qualities. Then there are two cases: i) firm $k$ wins the market with quality $q_k$ and makes positive profits. In this case, firm $-k$ can capture the market by also choosing quality $q_k$ and setting a slightly lower price; ii) firms set different qualities and price at cost, making zero profits. There are two subcases: ii.a) if the valuation constraint binds, so that consumers are indifferent between the two firms, then no firm is providing the quality level maximizing social surplus at the given salience ranking (because the two firms provide different qualities and the quality level that maximizes social surplus is unique, because costs are strictly convex). As a result, each firm has an incentive to change its quality in the direction of the level that maximizes social (at the given salience ranking) while keeping the salience ranking constant. This deviation increases consumer valuation, attracting all consumers; ii.b) the salience constraint binds. In this case, we have both $q_k/c(q_k) = q_{-k}/c(q_{-k})$ and (given that firms are ex ante equally valued) $q_k - c_k = q_{-k} - c_{-k}$. Together, these conditions imply $q_k = q_{-k}$, so that the firms are indeed pursuing a symmetric strategy.

We now show that a symmetric equilibrium always exists, and is unique, for any cost function $c(q) = v(q) + F$. Suppose both firms choose quality $q^*$; when is it profitable to deviate to $q' \neq q^*$? From Proposition 1, we know that when the average cost $c(q^*)/q^*$ lies
in the interval \((\delta, \frac{1}{\delta})\), the salience constraint binds: by deviating to a quality level \(q'\) with a lower average cost, \(c(q')/q' < c(q^*)/q^*\), a firm obtains a salient advantage and captures the market. In particular, a firm \(k\) deviating locally from the symmetric equilibrium \(q^*\) obtains a marginal benefit (i.e. a price) equal to \(c(q^*)/q^*\). Therefore, the firm has no incentive to deviate from \(q^*\) provided:
\[
c'(q^*) = c(q^*)/q^*. \]

This equation says that the equilibrium quality \(q^*\) should be equal to the quality level \(\hat{q}\) minimizing average costs (i.e. \(\hat{q} = \arg\min_q \frac{c(q)}{q}\)). Intuitively, when the salience constraint is binding, firms compete by trying to render their advantage more salient, which induces them to minimize their average cost. Thus, for \(\frac{c(\hat{q})}{\hat{q}} \in (\delta, \frac{1}{\delta})\), the quality provision in the symmetric equilibrium is given by \(\hat{q}\).

To see why this equilibrium is unique note that, because deviating from \(\hat{q}\) implies a higher average cost, such a deviation would slacken the salience constraint for the firm that stays at \(\hat{q}\). The latter can then capture the market with a strict salient advantage; it is then optimal to deviate from \(\hat{q}\), in order to produce (and capture) a higher social surplus (given the salience ranking). As a consequence, no \(q' \neq \hat{q}\) can be a symmetric equilibrium. To rewrite the condition \(\frac{c(\hat{q})}{\hat{q}} \in (\delta, \frac{1}{\delta})\) in terms of the cost parameters, note that the average cost minimizing quality satisfies \(c'(\hat{q}) = c(\hat{q})/\hat{q}\). Because the cost function is convex, this implies that i) \(\hat{q}\) is restricted to the range \((\bar{q}, \tilde{q})\) where \(c'(\bar{q}) = \delta\) and \(c'(\tilde{q}) = \frac{1}{\delta}\), and ii) i) as the unit cost \(F\) increases, \(\hat{q}\) strictly increases (unbounded by the range \((\bar{q}, \tilde{q})\)). Therefore, \(\frac{c(\hat{q})}{\hat{q}}\) is in \((\delta, \frac{1}{\delta})\) if and only if the unit cost \(F\) lies in the interval \((\bar{F}, \tilde{F})\), where \(\bar{F} = \bar{q}\delta - v(\bar{q})\) and \(\tilde{F} = \tilde{q}/\delta - v(\tilde{q})\), and \(\bar{q}, \tilde{q}\).

Consider now the case where the minimum average cost satisfies \(c(\hat{q})/\hat{q} \leq \delta\), or equivalently \(F \leq \bar{F}\). In this case, the average-cost minimizing quality satisfies \(\hat{q} < \bar{q}\). If firms choose quality \(q > \bar{q}\), then salience is binding (as in the previous case), so that firms have the incentive to reduce quality in order to reduce average costs. Otherwise, price would be salient. If instead \(q < \bar{q}\), then valuation is binding, so that firms have an incentive to increase quality from any \(q < \bar{q}\) in order to capture consumer surplus. Recall that \(\bar{q}\) satisfies \(c'(\bar{q}) = \delta\), and maximizes surplus when price is salient. As a consequence, in this parameter...
range $F \leq \mathcal{F}$ the equilibrium quality is $q = \bar{q}$.

Finally, consider the case where $c(q)/q \geq 1/\delta$, or equivalently $F \geq \mathcal{F}$. An argument similar to the above shows that the equilibrium quality provision is $\bar{q}$ where $\bar{q}$ satisfies $c'(\bar{q}) = 1/\delta$. We have thus recovered the equilibrium in Equation (13). To see this equilibrium is unique, note that $q, \hat{q}$ and $\bar{q}$ are unique due to the convexity of the cost function. ■

**Corollary 1 (symmetric equilibrium for quadratic costs).** Consider quadratic costs, where $v(q) = \frac{v}{2}q^2$. Then $c'(q) = v \cdot q$ so that $\bar{q} = \frac{1}{\delta v}$, and $\bar{q} = \frac{\delta}{v}$. Moreover, $\mathcal{F} = \frac{1}{2\delta^2 v}$ and $\bar{F} = \frac{\delta^2}{2\delta v}$. Finally, $\hat{q}$ satisfies $c'(\hat{q}) = c(\hat{q})/\hat{q}$, which yields $\hat{q} = \sqrt{2F/v}$. ■

**Lemma 3 (quality competition under rationality with asymmetric costs).** Once again, consider Equation (21), and now consider the asymmetric case in which firm 1 has lower marginal cost than firm 2. Because firm 1 can always produce the quality of firm 2 at lower cost, in equilibrium it wins the market with certainty. Equation (21) then indicates that to maximize its profit, firm 1 chooses the surplus maximizing quality, that solves $c'_1(q^*_1) = 1$. Under quadratic costs, we have $q^*_1 = 1/v_1$. Our “symmetric play” equilibrium selection rule then implies that the losing firm 2 sets quality as in the symmetric equilibrium in which both firms have the same cost function $c_2(q)$. Thus, firm 2 sets $c'_2(q^*_2) = 1$, namely $q^*_2 = 1/v$. Equilibrium prices are $p_2 = c_2(q^*_2)$ and $p_1 = c_2(q^*_2) + (q^*_1 - q^*_2)$. ■

**Proposition 3 (industry wide cost shocks).** Under the symmetric equilibrium of Equation (14), consider an increase in the unit cost of all firms, from $F_0$ to $F_1 > F_0$. If the interval $[F_0, F_1]$ has a non-empty overlap with the interval $[F, \mathcal{F}]$, then equilibrium quality strictly increases from $\max\{\delta/v, \sqrt{2F_0/v}\}$ to $\min\{\sqrt{2F_1/v}, 1/(\delta v)\}$. Otherwise, equilibrium quality provision does not change, staying at $\delta/v$ if $F_1 < F$ or at $1/(\delta v)$ if $F_0 > \mathcal{F}$.

Note that, when $\delta < 1$, the equilibrium quality can be written as $\frac{1}{\delta} \cdot A(v, F)$, where $A(v, F) = \max\{\delta, \min\{\sqrt{2F/v}, 1/\delta\}\}$. As a consequence, following an increase in the marginal cost of producing quality for all firms, quality provision strictly decreases. Formally, $\partial_v \frac{1}{\delta} \cdot A(v, F) < \partial_v \sqrt{\frac{2F}{v}} < 0$.

We can also ask when is the change in quality provision in reaction to a marginal increase in $v$ larger than in the rational case? When $\delta = 1$, quality provision equals $1/v$. Therefore,
the change in quality provision increases when \( \delta < 1 \) if and only if \( A(v, F) > 1 \), namely when quality is over provided to begin with (i.e. if \( F > \frac{1}{2v} \)).

**Proposition 4 (firm specific cost shocks).** Starting from the symmetric equilibrium of Equation (14), let the marginal cost of firm 1 drop to \( v_1 < v_2 = v \). This implies that firm 1 will win the market and firms 2 will lose it, making zero profits. Suppose in fact that this was not the case. Then, firm 1 could adopt the same quality of firm 2, produce it at lower cost, and win the market.

To work out the equilibrium in which the low cost firm 1 wins we proceed in two steps: i) we first compute firm 1’s best response from the symmetric equilibrium quality provision \( q^s \) of firm 2; ii) we then show that firm 2 has no incentive to deviate; the resulting configuration is thus an equilibrium. Because the losing firm plays the “symmetric” quality strategy, this is the equilibrium selected by our refinement.

When the unit costs are sufficiently high, \( F > \frac{\delta^2}{2v} \), the average costs of firm 2 satisfy \( c(q^s)/q^s > \delta \). It then follows from the analysis in Proposition 2 that firm 1’s best response is to engineer a salient quality increase: i) when \( c(q^s)/q^s \in [\delta, 1/\delta] \), firm 1 sets \( q_1^* \) satisfying \( c_1'(q_1^*) = c(q^s)/q^s \). With quadratic costs, this reads \( q_1^* = q^s \cdot \frac{v}{v_1} > q^s \). Firm 2 has no incentive to deviate: in this parameter range, it is already minimizing average cost, so it cannot engineer a quality innovation that gives it a salient advantage. Together with the fact that it has higher costs, this precludes any profitable deviation. ii) when the average costs of firm 2 exceed \( 1/\delta \), the quality provision in the symmetric equilibrium satisfies \( c'(q^s) = 1/\delta \).

In this case, firm 1 boosts quality to \( q_1^* \) satisfying \( c_1'(q_1^*) = 1/\delta \), so that \( q_1^* > q^s \). Firm 2 again has no incentive to deviate, since increasing quality (thereby diminishing average costs, if possible below that of firm 1) is never profitable: if firm 2 engineers a salient quality advantage then it decreases its valuation, while if it creates a salient price advantage it cannot price above cost.

Consider now the case where \( F < \frac{\delta^2}{2v} \). While firm 2 sets \( q^s \) such that \( c'(q^s) = \delta \), firm 1’s best response is to set \( q_1^* \) satisfying \( c_1'(q_1^*) = c(q^s)/q^s \), provided \( q_1^* > q^s \). With quadratic costs, this reads \( q^s = \delta/v \) and \( q_1^* = c(q^s)/q^s \). Thus, \( q_1^* > q^s \) requires \( F > \frac{\delta^2}{v} (\frac{v_1}{v} - \frac{1}{2}) \). If firm 1’s cost advantage is sufficiently large, namely \( v_1 < v/2 \), then firm 1 strictly increases quality
provision. If instead firm 1’s cost advantage is small, \( v_1 > v/2 \), then for low enough levels of the unit cost \( F \), it is optimal for firm 1 to keep quality provision at the equilibrium level prior to the shock, \( q^*_1 = \delta/v \), and translate its cost advantage into profits by setting price \( p_1 = c(\delta/v) \). Finally, firm 2 has no incentive to deviate because decreasing quality (thereby diminishing average costs) also decreases perceived surplus. ■

**Lemma 4 (returns competition under rationality).** This setting is similar to the price competition game of Lemma 1. While the costs facing investors are fixed at \( v \) (the security’s risk), intermediaries compete in terms of the return they provide investors. Since intermediaries provide identical securities, this competition game only admits symmetric equilibria. In particular, both firms offer the maximum return to investors, \( R_i - F = \overline{R} - F \), and share the market. No intermediary has an incentive to deviate from this configuration: increasing the returns offered to investors would lead to negative profits, while decreasing the returns would lead to the loss of the market share. ■

**Proposition 5 (financial innovation under salient thinking).** Suppose firm 2 creates a security of fixed total return and cost, \((R - F, \rho)\). Firm 1 develops a financial innovation and can create a family of securities \((R + \alpha - F, \rho + \frac{v}{2} \cdot \alpha^2)\), indexed by \( \alpha \), the increase in returns relative to the competition. The firms play a two stage game: in the first stage firm 1 chooses \( \alpha \), and in the second stage both firms choose how big a return to pledge to investors. Firm 1 pledges return \( R_\alpha - F \) where \( R_\alpha \in [\overline{R}, \overline{R} + \alpha] \) so that in the return competition stage it sells security \((R_\alpha - F, \rho + \frac{v}{2} \cdot \alpha^2)\) and maximizes profits \( \overline{R} + \alpha - R_\alpha \).

To determine the optimal choice of \( \alpha \), we begin by noticing that, for \( \alpha \) sufficiently small, the marginal cost of quality for firm 1 is lower than its average cost. This is because returns increase linearly in \( \alpha \), while risk increases quadratically. As a result, firm 1 finds it optimal to provide a salient increase in returns. The pledged returns \( R_\alpha \) must satisfy both the constraint that returns are salient, and the valuation constraint. The salience constraint reads \( R_\alpha - F > (\overline{R} - F) \cdot \frac{\frac{\rho + \frac{v}{2} \cdot \alpha^2}{\rho}}{\rho} \) (recall that firm 1 provides higher returns at a higher risk), while the valuation constraint reads \( R_\alpha > \overline{R} + \delta \frac{v}{2} \alpha^2 \). The valuation constraint is binding when \( \overline{R} > F + \delta \rho \). In this case, firm 1 must provide at least \( R_\alpha = \overline{R} + (\overline{R} - F) \frac{\delta \rho}{\delta v} \alpha^2 \). To maximize profits \( \overline{R} + \alpha - R_\alpha \), firm 1 sets \( \alpha = \frac{1}{\delta v} \).
The salience constraint is binding when $\bar{R} \geq F + \delta \rho$. In this case, firm 1 must provide at least $R_\alpha = F + (\bar{R} - F) \left( 1 + \frac{\alpha}{2 \rho} \alpha^2 \right)$. To maximize profits $\bar{R} + \alpha - R_\alpha$, firm 1 sets $\alpha = \frac{1}{\bar{R} - F} \cdot \frac{1}{\rho}$.


B Appendices for Online Publication

B.1 Equilibria in Mixed Strategies

In this appendix we show that there are no equilibria for the two stage game in mixed strategies. We start by showing that the pricing game does not admit mixed strategies.

B.1.1 Pricing Game

We first study the rational case and then move on to salience thinking.

1) Rational case, $\delta = 1$. For $k = 1, 2$, let firm $k$ produce quality $q_k$ at cost $c_k$, given exogenously. As a preliminary step, suppose that the lowest and highest prices in the support of the equilibrium strategy of firm $k$ are given by $p_k$ and $\bar{p}_k$, respectively. Then, note that $p_k \geq c_k$ (pricing below cost is never optimal) and $\bar{p}_k \geq c_{-k} + (q_k - q_{-k})$. To see the latter, suppose that $\bar{p}_k < c_{-k} + (q_k - q_{-k})$. Then, there are two cases. First, if firm $k$ generates lower surplus than firm $-k$, then $\bar{p}_k < c_k$ which cannot hold. Second, if firm $k$ generates higher surplus than $-k$, then it is for sure profitable to set price equal to $c_{-k} + (q_k - q_{-k})$, which (by the tie breaking rule assumed above) would increase its profits with probability one. Finally, it is easy to see that $\bar{p}_k = p_{-k} + (q_k - q_{-k})$. If $\bar{p}_k > p_{-k} + (q_k - q_{-k})$, then firm $k$ is certain to lose when playing $\bar{p}_k$, and vice versa for firm $-k$.

It is easy to see that any equilibrium in mixed strategies cannot include mixing over a discrete set of prices. More generally, let a mixed strategy for firm $k$ be represented by a union of disjoint intervals $\bigcup_{i=1,...,N} [p_{k,i}, p_{k,i+1}]$, where $p_{k,i} = p_{k,i+1}$ implies that $[p_{k,i}, p_{k,i+1}] = \{p_{k,i}\}$. Consider the case where one interval for firm $k$ is a singleton, say $\{p_{k,i}\}$, which firm $k$ plays with positive probability. Then there are two cases: if the price $p_{k,i} - q_k + q_{-k}$ is in the support of firm $-k$’s strategy with positive probability, then it is profitable for firm $k$ to replace $p_{k,i}$
with $p_{k,i} - \epsilon$. Otherwise, it is profitable for firm $-k$ to shift its own price distribution by increasing probability weight on prices just below $p_{k,i} - q_k + q_{-k}$. Intuitively, putting a positive probability in a singleton cannot occur in equilibrium.$^{25}$

Consider now the case in which mixed strategies include no singletons, and focus on the randomization occurring within the highest price interval $i = N$ for firm $k$. In particular, let $p_{k,N} < \underline{p}_k \equiv p_{k,N+1}$ and $p_{-k,N} < \underline{p}_{-k} \equiv p_{-k,N+1}$ (note that for $N = 1$ these are compact intervals). We now show that there are no cumulative price distributions for firms $k, -k$ such that it is an equilibrium for firms to mix within this range. Denote by $F_{-k}$ the cumulative distribution of prices set by firm $-k$. The expected profit $E[\pi_k | p]$ of firm $k$ from choosing a price $p$ is given by $\Pr(q_k - p_k > q_{-k} - p_{-k}) \cdot (p_k - c_k)$, which equals $(1 - F_{-k}(p_k + q_{-k} - q_k)) \cdot (p_k - c_k)$. A necessary condition for firm $k$ to play a mixed strategy is that $\frac{\partial E[\pi_k]}{\partial p} = 0$ for all $p \in (\underline{p}_k - \epsilon, \underline{p}_k]$ (recall that this interval is in the support of firm $k$’s prices). This condition reads $(1 - F_{-k}(x)) - (x + q_k - q_{-k} - c_k)F'_{-k}(x) = 0$, where $x = p_k + q_{-k} - q_k$. This differential equation has solution $F_{-k}(x) = \frac{x + Z}{x + q_k - q_{-k} - c_k}$ where $Z \in \mathbb{R}$. At the supremum price $\underline{p}_{-k}$ in the support of the price distribution of firm $-k$, we have $1 = \frac{\underline{p}_{-k} + Z}{\underline{p}_{-k} + q_k - q_{-k} - c_k}$, which implies $Z = q_k - q_{-k} - c_k$. In turn, this value of $Z$ would imply that $F_{-k}(p) = 1$ for some $p < \underline{p}_{-k}$. This contradicts the fact that $\underline{p}_{-k}$ is the least upper bound of the support of firm $-k$’s price distribution, so the necessary condition on $F_k$ near $\underline{p}_k$ is never fulfilled.

As a result, an equilibrium cannot entail randomization within a compact price interval. We do not go further in the analysis, but similar arguments can be used to show that there are no mixed strategy equilibria when different firms mix over different types of sets (including open intervals and discrete sets with accumulation points). This material is available upon request. As a result, there are no mixed strategy equilibria in the model.

2) Salient thinking, $\delta < 1$. In any mixed strategy equilibrium, it must be that $u^{ST}(q_k, \underline{p}_k) = u^{ST}(q_{-k}, \underline{p}_{-k})$. In fact, should $u^{ST}(q_k, \underline{p}_k) > u^{ST}(q_{-k}, \underline{p}_{-k})$, then firm $-k$ would have an incentive to deviate to prices lower than $\underline{p}_{-k}$. If that is not possible, namely if $\underline{p}_{-k} = c_{-k}$, then firm $-k$ is effectively playing a pure strategy, namely pricing at cost (recall that we exclude weakly dominated strategies in the pricing game). At this stage, the arguments for

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$^{25}$The same reasoning allows us to exclude price strategies with discontinuous density functions, e.g. where firms put a positive probability on a given price in their compact support. Slightly reducing the price to which positive probability is allocated results in a first order gain in expected profits.
the rational case follow through: if \( \bar{p}_k \) and \( \bar{p}_{-k} \) are singletons, then firm \( k \) has an incentive to lower its maximum price to \( \bar{p}_k - \epsilon \), even if (or especially when) doing so makes price salient.

Consider now the case where the support of the price distributions includes non-singleton intervals \( [p^N_k, \bar{p}_k] \) and \( [p^N_{-k}, \bar{p}_{-k}] \), with cumulative distributions \( F_k \) and \( F_{-k} \) respectively. Then the expected profit of firm \( k \) choosing a price \( p \) is \( \mathbb{E}[\pi_k | p] = \Pr(\text{consumers choose good } k) \cdot (p_k - c_k) \) where the event that consumers choose good \( k \) is given by the following conditions

\[
\begin{cases}
q_k - \delta p_k > q_{-k} - \delta p_{-k} & \text{if } p_{-k} > \frac{p_k}{q_k} q_{-k}, \text{ or } \\
\delta q_k - p_k > \delta q_{-k} - p_{-k} & \text{if } p_{-k} < \frac{p_k}{q_k} q_{-k} \geq 0.
\end{cases}
\]

Note that the case where good \( k \) dominates good \(-k\) is included in the conditions above.

We then have

\[
\Pr(\text{consumers choose firm } k) = \begin{cases}
1 - F_{-k} (p_k - \delta(q_k - q_{-k})) & \text{if } p_k \leq q_k \delta \\
1 - F_{-k} \left( \frac{q_{-k}}{q_k} p_k \right) & \text{if } p_k \in (q_k \delta, \frac{q_k}{\delta}] \\
1 - F_{-k} (p_k - \frac{q_k - q_{-k}}{\delta}) & \text{if } p_k > \frac{q_k}{\delta}
\end{cases}
\]

Firm \( k \) is willing to play a mixed strategy only if in a neighborhood of \( \bar{p}_k \) we have that \( \frac{\partial E \pi_k}{\partial p} = 0 \). We can now apply the same logic as in the \( \delta = 1 \) case: for sufficiently small \( \epsilon \), the condition \( \frac{\partial E \pi_k}{\partial p} = 0 \) implies that \( F_{-k} = 1 \) in \( (\bar{p}_{-k} - \epsilon, \bar{p}_{-k}) \), regardless of which of the cases above hold. This contradicts the assumption that \( \bar{p}_{-k} \) is the least upper bound of the support of firm \(-k\).

### B.1.2 Quality Choice

We consider only the case where firms have identical cost functions, and mix over a bounded set \( [q_l, q_h] \) (it is suboptimal to mix over a finite set of quality levels, as it is always profitable to deviate from at least one of the extremes). Note that in equilibrium any randomizing set of quality choices \( [q_l, q_h] \) must fulfils that \( [c'(q_l), c'(q_h)] \) is a subset of \( [\delta, 1/\delta] \). Setting a quality level outside of this interval would increase average costs (thereby hurting salience) and would reduce the surplus that the firm can extract though the valuation constraint. As
before, let $\hat{q}$ denote the average-cost-minimizing quality.

We first note that choosing $q_l, q_h$ such that $q_h < \hat{q}$ cannot be an equilibrium. In fact, each firm would have an incentive to drop qualities close to $q_l$ and increase probability mass near $q_h$. Doing so increases the average quality-cost ratio of the firm’s play and increases its chance to win the market. Moreover, because $c'(q_h) < 1/\delta$, perceived surplus increases as quality gets closer to $q_h$, thus allowing for larger profits for the firm that wins the market. Similarly, $q_l > \hat{q}$ cannot be an equilibrium; each firm would increase probability mass near $q_l$ and away from $q_h$, as that increases the quality-cost ratio of the firm’s play, and also increases profits when the firm does win the market.

Finally, consider the case $q_l < \hat{q} < q_h$. By choosing a quality away from $\hat{q}$, say close to $q_h$, a firm increases its average cost and reduces its chances to win the market. Furthermore, by making its disadvantage salient (in this case its higher price) the firm reduces the perceived surplus it can extract in case it does win the market. As a consequence, firms have an incentive to put higher probability mass closer to $\hat{q}$, so this configuration is also not an equilibrium.

B.2 Competition with Continuous Salience Weights

B.2.1 Price Competition

Firm $k = 1, 2$ produces a good of quality $q_k$ at cost $c_k$, where we assume $q_1 \geq q_2$ and $c_1 \geq c_2$. If the two firms set prices $(p_1, p_2)$, the salient thinker’s valuation of good $k = 1, 2$ is given by:

$$u^{ST}(q_k, p_k) = q_k e^{(1-\delta)\sigma(q_k, q)} - p_k e^{(1-\delta)\sigma(p_k, p)}$$

where according to the previous definitions $q \equiv (q_1 + q_2)/2$ and $p \equiv (p_1 + p_2)/2$. In Equation (22), salience weights are continuous in product attributes. Given the assumed symmetry of the salience function, namely the fact that $\sigma(a_1, q) = \sigma(a_2, q)$, for $a = q, p$, we have that at prices $(p_1, p_2)$, the higher quality good $1$ is chosen when

$$(q_1 - q_2) \cdot e^{(1-\delta)\sigma(q, q)} \geq (p_1 - p_2) \cdot e^{(1-\delta)\sigma(p, p)}$$  (23)
As long as \( p_1 > p_2 \), this condition is less likely to be satisfied when \( p_1 \) is higher or when \( p_2 \) is lower (i.e., the right hand side increases in \( p_1 \) and decreases in \( p_2 \)).

We consider pure strategy equilibria of the model, ruling out those holding in weakly dominated strategies. The optimal price at which firm 1 attracts all consumers is the value of \( p_1 \) that maximizes the firm’s profit \((p_1 - c_1)\) subject to Equation (23). A similar characterization holds for firm 2’s optimal price \( p_2 \). We apply the endogenous sharing rule introduced in Section 2. As a preliminary step, note that in any equilibrium of the model the firm losing the market will price at cost. Indeed, suppose that the losing price was above cost. Then, since in equilibrium the consumer is indifferent between the two goods, the losing firm would have the incentive to cut its price and attract all consumers and make a profit.

Pure strategy equilibria can be characterized as follows.

**Proposition 6** A pure strategy equilibrium always exists and is unique. There are three cases:

1) If \((c_1 - c_2)\cdot e^{(1-\delta)\cdot \sigma(c_k, \bar{\sigma})} < (q_1 - q_2)\cdot e^{(1-\delta)\cdot \sigma(q_k, \bar{\sigma})}\), the high quality firm 1 wins the market and equilibrium prices \((p_1^*, p_2^*)\) satisfy:

\[
(p_1^* - c_2) \cdot e^{(1-\delta)\cdot \sigma\left(p_1^*, \frac{p_1^* + c_2}{2}\right)} = (q_1 - q_2) \cdot e^{(1-\delta)\cdot \sigma(q_k, \bar{\sigma})},
\]

\[p_2^* = c_2.\]

2) If \((c_1 - c_2)\cdot e^{(1-\delta)\cdot \sigma(c_k, \bar{\sigma})} > (q_1 - q_2)\cdot e^{(1-\delta)\cdot \sigma(q_k, \bar{\sigma})}\), the low quality firm 2 wins the market and equilibrium prices \((p_1^*, p_2^*)\) satisfy:

\[p_1^* = c_1,\]

\[
(c_1 - p_2^*) \cdot e^{(1-\delta)\cdot \sigma\left(p_2^*, \frac{p_2^* + c_1}{2}\right)} = (q_1 - q_2) \cdot e^{(1-\delta)\cdot \sigma(q_k, \bar{\sigma})}.
\]

3) If \((c_1 - c_2)\cdot e^{(1-\delta)\cdot \sigma(c_k, \bar{\sigma})} = (q_1 - q_2)\cdot e^{(1-\delta)\cdot \sigma(q_k, \bar{\sigma})}\), the two firms split the market and equilibrium prices \((p_1^*, p_2^*)\) satisfy:

\[p_1^* = c_1,\]

\[p_2^* = c_2.\]
Proof. Start by showing existence of equilibria. In case 1), at equilibrium prices goods 1 and 2 provide the same utility, and the sharing rule determines that all consumers choose good 1. Firm 2 has no incentive to increase price (as it would not win the market)\(^{26}\), neither does it have an incentive to cut price (a weakly dominated strategy that might lead to negative profits). Firm 1 has no incentive to lower its price \(p_1^*\) as that would strictly reduce profits (recall that under the sharing rule, firm 1 gets all the demand at price \(p_1^*\)), and even less incentive to increase price as that would cause it to lose the market. A similar argument holds for case 2). Case 3) is the limit of the previous two cases: the consumer is now strictly indifferent between the two goods. No firm has an incentive to increase prices (as that would not increase profits) or to cut prices (as that would entail a negative profit).

To show uniqueness, consider case 1) and suppose, by contradiction, that an equilibrium exists where firm 2 sets price \(p_2^* > c_2\). Then, if in this equilibrium firm 1 wins the market and sets price to maximize its profits, firm 2 can reduce its price arbitrarily close to cost and capture the market at a profit. If instead in this equilibrium firm 2 wins the market and sets price to maximize its profits, then firm 1 can reduce its price arbitrarily close to cost and make a profit. Thus, any equilibrium in case 1) must have \(p_2^* = c_2\), which determines it uniquely. A similar reasoning applies to cases 2) and 3). ■

This equilibrium shares the main properties of the discrete case (see Lemma 1 in the text). In particular the following properties hold.

**Corollary 2** The equilibrium described in proposition 1 implies that:

i) The firm with highest quality to price ratio can win the market even if it delivers lower rational surplus than its competitor. Formally, if firm \(k\) has higher quality to price ratio than firm \(-k\), then firm \(k\) wins the market if and only if \(q_k - c_k > u\), where \(u\) is a threshold fulfilling \(u < q_{-k} - c_{-k}\).

ii) Salient quality increases the profits of firm 1 relative to the profits the same firm would make in the rational case when \(q_2 < c_2\). Salient price increases the profits of firm 2 relative to the positive profits the same firm would make in the rational case when \(c_1 < p_1\).

\(^{26}\)Note that firm 2 never has an incentive to set its price above that of good 1.
Proof. i) Assume $q_1 - c_1 = q_2 - c_2 - t$ for some $t > 0$. From Equation (23), if firms price at cost, firm 1 wins the market when

$$(c_1 - c_2) \cdot e^{(1-\delta) \cdot \sigma(c_k,\bar{c})} < (c_1 - c_2 - t) \cdot e^{(1-\delta) \cdot \sigma(q_k,\bar{q})}$$

which is equivalent to $1 - t/(c_1 - c_2) > e^{(1-\delta) \cdot [\sigma(c_k,\bar{c}) - \sigma(q_k,\bar{q})]}$. If firm 1 has a higher quality to price ratio, then $\sigma(c_k,\bar{c}) < \sigma(q_k,\bar{q})$ so that, for $t$ small enough, firm 1 is chosen in equilibrium. A similar argument shows that firm 2 wins the market when it has a higher quality price ratio, provided $t$ is not too negative.

ii) Suppose we are in case 1) of Proposition 6, and that firm 1 also generates higher (rational) surplus than firm 2. To see when firm 1 makes higher profits than in a rational world, insert the rational prices $c_2 + q_1 - q_2$, $c_2$ into Equation (23). At these prices, the consumer strictly prefers good 1 if and only if its higher quality is salient, $\sigma(q_2,\bar{q}) > \sigma(c_2 + q_1 - q_2,\bar{p}^r)$, where $\bar{p}^r = (2c_2 + q_1 - q_2)/2$. This implies that $p_1^* > c_2 + q_1 - q_2$, and firm 1 makes higher profits than in the rational case, if and only if $q_1/q_2 > (c_2 + q_1 - q_2)/c_2$, which reads $q_2 < c_2$.

Suppose now we are in case 2) of Proposition 6, and that firm 2 also generates higher (rational) surplus than firm 1. Following the same reasoning as above, at the (rational) prices $c_1$, $c_1 - (q_1 - q_2)$ the consumer strictly prefers good 2 if and only if its lower price is salient, $\sigma(c_1 - (q_1 - q_2),\bar{p}^r) > \sigma(q_2,\bar{q})$, with $\bar{p}^r = (2c_1 - (q_1 - q_2))/2$. This happens if and only if $c_1 < p_1^*$.

B.2.2 Endogenous Quality

To obtain insight into endogenous choice of quality when salience weighting is continuous, we consider the simplest setting in which firms can choose among two quality levels. Specifically, suppose that quality can take values in \{q_1, q_2\} where $q_1 > q_2$ and the respective costs satisfy $c_1 > c_2$. It follows from Equation (23) that if

$$\frac{q_1 - q_2}{c_1 - c_2} \geq \frac{e^{(1-\delta) \cdot \sigma(c_1,\frac{c_1 + c_2}{2})}}{e^{(1-\delta) \cdot \sigma(q_1,\frac{q_1 + q_2}{2})}}$$

(24)
then in equilibrium both firms produce $q_1$ and price at cost, setting $p = c_1$. When condition (24) holds, any firm that tries to cut quality to $q_2$ renders quality salient, even when the firm cuts its price to the new cost level $c_1$. As a result, deviating to a lower quality level backfires.

Critically, note that (24) holds provided the quality difference $q_1 - q_2$ is more salient than the cost difference $c_1 - c_2$, namely provided $q_1/c_1 > q_2/c_2$. As a result, (24) can be satisfied, and thus high quality may be a symmetric Nash equilibrium, even if higher quality is inefficient in a rational world, namely $q_1 - q_1 < c_1 - c_2$. As in the baseline rank-based discounting model in the text, salience favors quality provision that minimizes average costs, even if such quality provision is excessive (or insufficient).

B.3 Heterogeneity in Salience

We now introduce consumer heterogeneity in individual perceptions of salience. Formally, for given qualities $q_1 \geq q_2$, we assume that the salience of quality is a stochastic function $\sigma(q_k, q_{\Delta \epsilon})$, where $\Delta \epsilon$ is a random shock that varies across consumers. This captures the idea that – holding the quality of different goods constant – some consumers may focus on quality differences more than others, due for instance to their habits.

Introducing heterogeneity generates “smooth” demand functions, and allows both firms to earn some profits in equilibrium. These features render the model more suitable to systematic empirical analysis. Heterogeneous salience also allows us for smoothen the effect of product attributes on the overall salience ranking, providing a way to assess the robustness of our findings to the case in which the salience weighting is continuous (rather than rank-based). An alternative approach would be to model consumer heterogeneity as affecting utility. This formulation yields similar results but the analysis becomes less tractable.

As in Section 2, we assume that the objective utility provided by goods 1 and 2 is sufficiently similar and non-salient dimensions are sufficiently discounted ($\delta$ is sufficiently low) that each consumer chooses the good whose advantage he perceives to be more salient. That is, a consumer receiving a perceptual shock $\Delta \epsilon$ inducing him to view quality as salient chooses the high quality good 1, while a consumer receiving a perceptual shock $\Delta \epsilon$ inducing him to view price as salient chooses the low quality good 2. Formally, denoting by $(p^*_1, p^*_2)$ equilibrium prices, we assume that $\delta$ is sufficiently small that $q_1 - q_2 > \delta(p^*_1 - p^*_2)$ and
\( \delta(q_1 - q_2) < p_1^* - p_2^* \). As we will see, optimal prices \((p_1^*, p_2^*)\) are independent of \(\delta\), so it is always possible to find values of \(\delta\) such that the above conditions hold at equilibrium.

To attain tractability, we model the shock \(\Delta \epsilon\) as affecting salience through the consumer’s focus on the ratio \(q_1/q_2\) among the quality of the goods. Technically, this ensures that - as in our main analysis - the two goods have the same salience ranking (i.e., quality or price is salient for both good). In particular, we assume that the perceptual shock transforms the ratio \(q_1/q_2\) into \(\frac{q_1/q_2(2+\Delta \epsilon)+\Delta \epsilon}{2-\Delta \epsilon(q_1/q_2+1)}\), where \(\Delta \epsilon = \epsilon_1 - \epsilon_2\) and \(\epsilon_1, \epsilon_2\) are iid from a Gumbel distribution with scale \(\beta > 0\) and location \(\mu = 0\). As a result of this transformation, the salience of quality for goods 1 and 2 depends on \(\Delta \epsilon\). It is easy to show that quality is salient for good 1 when
\[
\frac{q_1}{(q_1 + q_2)/2} + \Delta \epsilon > \frac{p_1}{(p_1 + p_2)/2}
\]
while quality is salient for good 2 when
\[
\frac{q_1 + q_2}{2q_2} \cdot \frac{2q_2}{2q_2 - \Delta \epsilon(q_1 + q_2)} > \frac{p_1 + p_2}{2p_2}
\]
(Taking \(\Delta \epsilon = 0\) in either of the above equations yields condition (3) in the text). By construction, we find that quality is salient for each good if and only if:
\[
\Delta \epsilon \geq 2 \cdot \frac{(r_p - r_q)}{(r_p + 1)(r_q + 1)},
\]
where we denote, for simplicity, \(r_q = q_1/q_2\) and \(r_p = p_1/p_2\).

The assumed structure for stochastic disturbances to salience yields a simple equation for demand. Because the shock \(\Delta \epsilon\) is distributed according to a logistic function, the underlying demand structure is akin to a simple modification of the conventional multinomial logit model.\(^{27}\)

\(^{27}\)Consider alternatively the case where noise enters through independent shocks to the perception of (or tastes for) qualities, \(u_i = q_i + \epsilon_i - p_i\) for \(i = 1, 2\). Here the \(\epsilon_i\) are taken (independently) from Gumbel distributions. Then good 1 is chosen iff \(q_1 + \epsilon_1 > q_2 + \epsilon_2\) and \(\frac{q_1 + \epsilon_1}{q_2 + \epsilon_2} > \frac{p_1}{p_2}\), in other words, if and only if
\[
\epsilon_1 - \epsilon_2 > q_2 - q_1, \quad \epsilon_1 - \epsilon_2 \cdot \frac{p_1}{p_2} > q_2 \cdot \left(\frac{p_1 - q_1}{p_2 - q_2}\right)
\]
If either of these conditions fail, then good 2 is chosen. Good 2 has an advantage in this setting because its price is always perceived (correctly) to be lower, while the quality ranking may be affected by noise. Because
Lemma 5  Firms $i = 1, 2$ face demand $D_i(p_1, p_2)$ given by

$$\begin{align*}
D_1 &= \frac{1}{1 + e^{\frac{1}{\beta} K\cdot|r_p - r_q|}}, \\
D_2 &= \frac{1}{1 + e^{-\frac{1}{\beta} K\cdot|r_p - r_q|}}
\end{align*}$$

(25)

where $K = 2/[(r_q + 1)(r_p + 1)]$.

Proof. The probability that good 1 is chosen is

$$Pr(u_1 > u_2) = Pr\left(\frac{q_1}{(q_1 + q_2)/2} + \Delta \epsilon > \frac{p_1}{(p_1 + p_2)/2}\right)$$

$$= Pr(\Delta \epsilon > K\cdot[r_p - r_q])$$

(26)

where $r_p = \frac{p_1}{p_2}$, $r_q = \frac{q_1}{q_2}$ and $K = \frac{2}{(r_p + 1)(r_q + 1)}$. To compute this expression, we first integrate over $\epsilon_2$ keeping $\epsilon_1$ fixed, and then integrate over all $\epsilon_1$. The first integration is written in terms of the CDF of the Gumbell distribution, which is $CDF(x) = e^{-e^{-x}}$. To integrate over $\epsilon_1$ we use the Gumbel PDF, which is $PDF(x) = e^{-x}e^{-e^{-x}}$. Therefore, equation (26) becomes

$$Pr(u_1 > u_2) = \int \left(e^{-e^{-\epsilon_1 + K\cdot|r_p - r_q|}}\right) \cdot e^{-\epsilon_1}e^{-e^{-\epsilon_1}} d\epsilon_1$$

(27)

We find $Pr(u_1 > u_2) = \frac{1}{1 + e^{-K\cdot|r_q - r_p|}}$ from which the result follows. □

This demand structure has some very intuitive properties. First, good 1 has a larger market share than good 2 if and only if quality is salient, namely if $r_q > r_p$, which is equivalent to the same condition $q_1/p_1 > q_2/p_2$ of Section 2. The scale parameter $1/\beta$ measures how sensitive demand is to the difference $\Delta r$ between the salience of quality and price: for large $1/\beta$, demand is extremely sensitive to any deviations from equal salience, thus implying that providing a higher quality to price ratio is critical to attracting a large share of consumers. For low $1/\beta$ consumers effectively choose randomly between the two options.

Firms $i = 1, 2$ sets price $p_i$ to maximize profits $\pi_i = D_i \cdot (p_i - c_i)$. We focus on pure strategy equilibria. We prove: there are two conditions, in this model it is difficult to compute the probability that 1 gets chosen.
Proposition 7 In equilibrium, firms set prices $p_1, p_2$ satisfying

$$\frac{p_1 - c_1}{p_1} = \frac{p_2 - c_2}{p_2} e^{-K\Delta r}$$  \hspace{1cm} (28)

As a result, firm $i$ with the lowest average cost: i) sets the highest markup $p_i/c_i > p_{-i}/c_{-i}$, ii) captures the highest market share $D_i > D_{-i}$, and iii) makes the highest profit $\pi_i > \pi_{-i}$.

Proof. Denote $\Delta r = r_p - r_q$. Optimal prices satisfy:

$$FOC_1 : \quad D_1 e^{K\Delta r} \cdot \frac{r_q + 1}{r_p + 1} \cdot \frac{p_1 - c_1}{p_2} = 1$$

$$FOC_2 : \quad D_2 e^{-K\Delta r} \cdot \frac{r_q + 1}{r_p + 1} \cdot \frac{p_1(p_2 - c_2)}{p_2^2} = 1$$

Since $D_1 e^{K\Delta r} = D_2$, together these imply condition (28). This captures several properties of equilibrium prices:

- In the symmetric case, $c_1 = c_2 = c$, firms price at cost, $p_1 = p_2 = c$.

- The good with the larger quality price ratio also has the larger markup. Suppose $\Delta r > 0$ so that $q_2/p_2 > q_1/p_1$ and price is salient. Then (28) implies that $(p_1 - c_1)/p_1 < (p_2 - c_2)/p_2$ so that $p_2/c_2 > p_1/c_1$. The reverse conditions hold when $\Delta r < 0$.

- The good with the highest quality price ratio is the good with the highest quality to cost ratio (or the lowest average cost). To see that, rewrite (28) as

$$1 - \frac{c_1}{q_1} \cdot \frac{q_1}{p_1} = \left(1 - \frac{c_2}{q_2} \cdot \frac{q_2}{p_2}\right) e^{-\frac{K q_2}{p_2} \left[\frac{q_1}{q_1} - \frac{q_2}{q_2}\right]}$$  \hspace{1cm} (29)

This implies that $q_1/p_1 > q_2/p_2$ if and only if $q_1/c_1 > q_2/c_2$. In particular, if firms have equal average costs, they both price at cost and make zero profits.

- Finally, this implies that the firm with lower average cost makes higher profits (equivalently, it extracts higher total surplus). It is clear that if quality is salient the higher quality firm makes higher profits. It is also straightforward to see that if the low quality firm has sufficiently lower average costs, it makes higher profits. By continuity, and
by the fact that both firms make zero profits when average costs are equal, the result follows.

Adding heterogeneity in consumers’ salience rankings preserves the key result of our basic model, namely that under salient thinking quality cost ratios are critical to determine the outcome of price competition.

Consider now the implications of Proposition 7 for the symmetric case where both firms produce the same quality \( q_1 = q_2 = q \) at identical costs, \( c_1 = c_2 = c \). Condition (28) then implies that firms set equal prices \( p_1 = p_2 = p \). Inserting this condition into the first order conditions, we find

\[
p = c \cdot \frac{1/\beta}{1/\beta - 4}
\] (30)

When consumers are sufficiently sensitive to salient advantages (namely \( 1/\beta > 4 \)), there exists a symmetric equilibrium with prices above costs.\(^{28}\) When consumers are infinitely sensitive to differences in quality to price ratios, namely \( \beta \to 0 \), equilibrium prices fall to cost and the model boils down to the standard Bertrand competition case.

We can now study the endogenous quality case when firms have identical cost of quality structures, \( c_1(q) = c_2(q) = c(q) \). We find:

**Proposition 8** The unique pure strategy subgame perfect equilibrium with identical firms is symmetric. Firms provide quality \( q^* \) satisfying

\[
c'(q^*) = \frac{1}{1-\beta} \cdot \frac{c(q^*)}{q^*}
\] (31)

**Proof.** At the quality selection stage, firms take into account the outcome of the price competition stage, where they implement a price schedule given by \( p(q) = c(q)/(1-4\beta) \). At

\(^{28}\)The fact that prices are above costs mirrors Anderson and de Palma (1992)’s description of imperfect competition under logit demand.
the first stage, firm 1’s optimisation problem is then

$$\max_{q_1} \frac{1}{1 + e^{\frac{1}{\beta} K \Delta r}} \cdot [p(q_1) - c(q_1)]$$

Notice that \( r_p = c(q_1)/c(q_2) \) and \( r_q = q_1/q_2 \). The first order condition then reads

$$c'(q_1) = \frac{c(q_1)}{1 + e^{\frac{1}{\beta} K \Delta r}} \frac{2}{\beta} \partial_{q_1} \left[ \frac{c(q_1)/c(q_2) - q_1/q_2}{(c(q_1)/c(q_2) + 1)(q_1/q_2 + 1)} \right]$$

This is evaluated at the symmetric equilibrium condition \( q_1 = q_2 \), so that \( \Delta r = 0 \). The factor multiplying the derivative term then simplifies to \( \frac{c(q_1)}{\beta} \). Developing and simplifying the expression above gives the result (31). 

Propositions 7 and 8 extend essentially all our results for discrete salience in the symmetric case. In particular, as \( \beta \) approaches zero and consumers are infinitely attuned to salience ranking, expression (31) states that firms choose quality that minimizes average cost.

### B.4 Asymmetric Equilibria

This appendix characterizes the set of equilibria that arise when firms have different cost functions. We focus on pure strategy equilibria, and restrict our analysis to the quadratic cost structure:

$$c_1(q) = \frac{c_1}{2} q^2 + F, \quad c_2(q) = \frac{c_2}{2} q^2 + F, \quad 0 < c_1 < c_2$$

in which the unit cost \( F \) is the same for both firms, but firm 1 has lower marginal cost than firm 2. We begin by describing general properties of the model for arbitrary unit cost \( F \geq 0 \) (Lemmas 1 and 2). We then fully characterize equilibria for the special case \( F = 0 \), and comment on how the results extend to the case \( F > 0 \).

In this section, we denote the equilibrium quality provided by firm \( k \) by \( q_k^* \). When we examine quality deviations by firm \( k \), we denote them by \( \hat{q}_k \). We let \( A_k = c_k(q_k)/q_k \) denote the average cost of quality \( q_k \) to firm \( k \). We write \( A_k^* = A_k(q_k^*) \) and \( \hat{A}_k = A_k(\hat{q}_k) \).

**Lemma 6** In all pure strategy equilibria, firm 1 captures the market.
Proof. Suppose, by contradiction, that for some quality choice \((q_1^*, q_2^*)\) firm 2 would capture the market with non-negative profits, \(p_2 \geq c_2(q_2^*)\). Then firm 1 can shift quality to mimic firm 2 and lower its price to capture the market: \(q_1' = q_2^*\) and \(p_1' = c_2(q_2^*)\). Since the good \((q_1', p_1')\) offered by firm 1 dominates any good offered by firm 2 at quality \(q_2^*\), and since it is priced above cost \((c_1(q_2^*) > c_2(q_2^*))\), this deviation is profitable for firm 1. Thus, there is no pure strategy equilibrium where firm 2 captures the market. The same logic shows that there is no pure strategy equilibrium where both firms share the market. ■

Intuitively, firm 1 can always mimic firm 2’s quality provision and exploit its cost advantage. To identify the equilibria of the model, it is useful to characterize what kind of deviations may be profitable for firm 2. We find:

Lemma 7 Let firm 1 produce a quality level \(q_1^*\) that is a best response to firm 2’s quality \(q_2^*\), which can be arbitrary. Then:

i) firm 2 can never profitably deviate to configuration \(\hat{q}_2\) where it has a salient quality advantage, namely \(\hat{q}_2 > q_1^*\) and \(c_2(\hat{q}_2)/\hat{q}_2 \leq c_1(q_1^*)/q_1^*\).

ii) a price-salient and profitable deviation \(\hat{q}_2\) (where \(\hat{q}_2 < q_1^*\) and \(c_2(\hat{q}_2)/\hat{q}_2 \leq c_1(q_1^*)/q_1^*\)) exists for firm 2 only if firm 1’s quality is sufficiently high, namely \(q_1^* > \delta/c_1\).

Proof. To prove point i) of the Lemma, note that all deviations by firm 2 to a profitable quality salient configuration \(\hat{q}_2\) require the competing conditions of providing high quality \(\hat{q}_2 > q_1^*\) at relatively low average cost \(A_2(\hat{q}_2) < A_1(q_1^*)\). We first show, case by case, that these conditions cannot be simultaneously satisfied.

To simplify notation, write \(A_k^* = A_k(q_k^*)\) for \(k = 1, 2\). Also, a configuration where quality is salient and the valuation constraint is binding is referred to as a QV configuration, whereas QS refers to a quality salient configuration where the salience constraint is binding. Similarly, we use the terms PV and PS to refer to price salient configurations where the valuation constraint, or the salience constraint, is binding.

- Consider a configuration where firm 1 has a salient quality advantage, and the valuation constraint is binding. Then \(q_1^* = \frac{1}{\delta c_1}\) and \(p_1^* = \frac{1}{\delta}(q_1^* - q_2^*) + c_2(q_2^*)\), for some \(q_2^*\) satisfying \(A_2^* > 1/\delta\).
If \( A_1^* > 1/\delta \), firm 2 deviates to a QV configuration, \( \hat{p}_2 = \frac{1}{\delta} (\hat{q}_2 - q_1^*) + c_1(q_1^*) \) so \( \hat{q}_2 = \frac{1}{c_2} \). Then \( \hat{q}_2 < q_1^* \) (since \( c_2 > c_1 \)), so the quality ranking condition is violated.

If \( A_1^* < 1/\delta \), firm 2 deviates to a QS configuration, \( \hat{p}_2 = \hat{q}_2 A_1^* \) so \( \hat{q}_2 = \frac{1}{c_2} A_1^* \). Then the quality constraint \( \hat{q}_2 > q_1^* \) is \( F > q_1^{*^2} [c_2 - \frac{1}{\delta} c_1] \), while the salience constraint \( A_1^* < \frac{1}{\delta} \) reads \( F < \frac{1}{2\delta^2 c_1} \). Inserting \( q_1^* = 1/\delta c_1 \), and using \( c_2 > c_1 \) we find that these conditions are incompatible.

- Consider a configuration where firm 1 has a salient price advantage and the salience constraint is binding. Then \( q_1^* = \frac{1}{c_1} A_1^* \) and \( p_1^* = \frac{1}{c_1} A_2^* \) for some \( q_2^* \) satisfying \( A_2^* < 1/\delta \).

- We have \( A_1^* < A_2^* < 1/\delta \), so firm 2 deviates to a QS configuration, where \( \hat{p}_2 = \hat{q}_2 A_1^* \) so \( \hat{q}_2 = \frac{1}{c_2} A_1^* \). Then \( \hat{q}_2 > q_1^* \) if and only if \( \frac{1}{c_2} A_1^* > \frac{1}{c_1} A_2^* \) which again is incompatible with \( A_1^* < A_2^* \), and \( c_2 > c_1 \).

- Consider a configuration where firm 1 has a salient quality advantage and the valuation constraint is binding. Then \( q_1^* = \frac{1}{c_1} A_1^* \) and \( p_1^* = \frac{1}{c_1} A_2^* \), where \( A_2^* > \max \{ A_1^*, \delta \} \).

- If \( A_1^* > 1/\delta \) firm 2 deviates to a QV configuration, and we have \( \hat{p}_2 = \frac{1}{\delta} (\hat{q}_2 - q_1^*) + c_1(q_1^*) \) and \( \hat{q}_2 = \frac{1}{c_2} \). Then \( \hat{q}_2 > q_1^* \) if and only if \( A_2^* < \frac{c_1}{\delta c_2} \), which is inconsistent with the requirement that \( A_2^* > A_1^* > \frac{1}{\delta} \).

- If \( A_1^* < 1/\delta \), firm 2 tries deviates to a QS configuration. This is not profitable, for identical reasons to the case above where the ex ante regime was quality salient (only difference was ex ante ranking of qualities \( q_1^* \) and \( q_2^* \) which does not matter for the deviation).

- Consider a configuration where firm 1 has a salient price advantage and the valuation constraint is binding. Then \( q_1^* = \frac{\delta}{c_1} \) and \( p_1^* = \delta(q_1^* - q_1^*) + c_2(q_2^*) \). Moreover, \( A_2^* < \delta \).

- We have \( A_1^* < A_2^* < \delta < 1/\delta \), so firm 2 deviates to a QS configuration. We have \( \hat{p}_2 = \hat{q}_2 A_1^* \) and \( \hat{q}_2 = \frac{1}{c_2} A_1^* \). However, the salience constraint on \( A_1^* \) implies \( \hat{q}_2 < \frac{\delta}{c_2} < q_1^* \) so the quality ranking is violated.
We now turn to point ii) of the lemma. Since \( q_1^* \) is a best response to some \( q_2 \), we can distinguish two cases: either \( q_1^* = \delta/c_1 \) and \( A_2 \leq \delta \), or \( q_1^* = A_2/c_1 \) and \( A_2 > \delta \). To prove point ii), therefore, it is sufficient to show that a price-salient and profitable deviation \( \hat{q}_2 \) does not exist for firm 2 if firm 1 sets quality \( q_1^* = \delta/c_1 \). (As we show below, for higher values of \( q_1 \) such profitable deviations do exist for firm 2).

In this case, since firm 1’s quality setting is optimal, we have \( A_1^* < A_2^* < \delta \). As a consequence, only deviations to price salient configurations where the valuation constraint binds are available to firm 2. Firm 2 thus sets \( \hat{q}_2 = \frac{\delta}{c_2} \) which satisfies \( \hat{q}_2 < q_1^* \). However, this move does not satisfy the salience constraint, since \( \hat{A}_2 = \frac{\delta}{2} + F c_2/\delta \) and \( A_1^* = \frac{\delta}{2} + F c_1/\delta \) so \( \hat{A}_2 > A_1^* \) for any positive \( F \). Even for \( F = 0 \) the cost structure implies that this move cannot be profitable. In fact, the valuation constraint on firm 2’s price is \( \hat{p}_2 = \delta(\hat{q}_2 - q_1^*) + c_1(q_1^*) \). Inserting the values for \( \hat{q}_2 \) and \( q_1^* \) it is easy to show that \( \hat{p}_2 < c_2(\hat{q}_2) \).

Intuitively, if the high cost firm 2 tries to provide more quality than firm 1, it will face much higher costs. Prices will be salient, and quality improvements will backfire. As a consequence, optimal deviations by firm 2 consist of salient price cuts. This suggests that equilibria in the model occur when quality levels \( (q_1^*, q_2^*) \) are optimal for both firms and firm 2 is unable to undercut firm 1.

In light of these results, we describe the equilibria arising when \( F = 0 \). In this case, pure strategy equilibria can be characterized in the following intuitive way:

**Lemma 8** Let \( F = 0 \). For any \( c_1 < c_2 \), there exist equilibria \( (q_1^*, q_2^*) \). Pure strategy equilibria fall into three cases:

i) the cost advantage of firm 1 is large, \( \frac{c_1}{c_2} < \frac{1}{2} \), equilibria are quality-salient with \( q_1^* > q_2^* \).

ii) the cost advantage of firm 1 is small, \( \frac{c_1}{c_2} \geq \frac{1}{2} \), equilibria are price-salient with \( q_1^* < q_2^* \).

iii) the cost advantage of firm 1 is small, \( \frac{c_1}{c_2} \geq \frac{1}{2} \), equilibria feature homogeneous qualities, \( q_1^* = q_2^* < \delta/c_1 \). Firm 1 charges at \( c_2(q_2^*) \) and price is salient.

**Proof.** We proceed in two steps: in the first step, we show that the conditions on the cost advantage \( c_1/c_2 \) are necessary for the equilibria. In the second step, we derive the full equilibrium conditions, including all constraints on qualities and costs.
We adopt the notation of Lemma 7, where $A^*_k$ stands for $A_k(q^*_k)$ for $k = 1, 2$, $QV$ and $QS$ refer to quality salient configurations where the valuation and the salience constraints bind respectively, while $PV$ and $QS$ refer to price salient configurations where the valuation and the salience constraints are binding.

- **Step 1: necessity of constraints on $c_1/c_2$.**

I. In equilibrium, firm 1 chooses a high quality strategy only if it satisfies the competing constraints of higher quality, $q^*_1 > q^*_2$, and lower average cost, $p^*_1/q^*_1 < A^*_2$ when the valuation constraint is binding. The two constraints are compatible if and only if the cost advantage $c_1/c_2$ is sufficiently large, as we now show.

- Consider equilibria where quality is salient and the valuation constraint is binding ($QV$). Then $p^*_1 = \frac{1}{\delta}(q^*_1 - q^*_2) + c_2(q^*_2)$ and $q^*_1 = \frac{1}{\delta c_1}$. The higher quality constraint implies $q^*_2 < \frac{1}{\delta c_1}$. In turn, the valuation constraint is binding if and only if $A^*_2 > \frac{1}{\delta}$, which reads $q^*_2 > \frac{2}{\delta c_2}$, which requires $c_1/c_2 < 1/2$.

- Consider now a configuration where quality is salient and the salience constraint is binding ($QS$). In this case, $p^*_1 = q^*_1 A^*_2$ and $q^*_1 = \frac{1}{c_1} A^*_2$. The higher quality constraint reads $1/c_1 A^*_2 > q^*_2$, or equivalently $q^*_2 > 1/c_1 A^*_2 > q^*_2$, which again requires $c_1/c_2 < 1/2$.

II. Firm 1 chooses a low price strategy only if it provides weakly lower quality, $q^*_1 \leq q^*_2$ and lower average cost.

- In a configuration where price is salient and the salience constraint is binding ($PS$), we have $p^*_1 = q^*_1 A^*_2$ and again $q^*_1 = \frac{1}{c_1} A^*_2$. The lower quality constraint reads $1/c_1 A^*_2 < q^*_2$, or equivalently $q^*_2 > 1/c_1 A^*_2 > q^*_2$, which now requires $c_2 < 2c_1$.

- Finally, in a configuration where price is salient and the valuation constraint is binding ($PS$), we have $p^*_1 = \delta(q^*_1 - q^*_2) + c_2(q^*_2)$ and $q^*_1 = \frac{\delta}{c_1}$. The lower quality constraint implies $q^*_2 > \frac{\delta}{c_1}$. In turn, the valuation constraint is binding if and only if $A^*_2 < \delta$, which reads $q^*_2 < \frac{2\delta}{c_2}$. Compatibility of the two conditions again requires $c_2 < 2c_1$. 

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Step 2a: equilibrium conditions when $c_1/c_2 < 1/2$. From the previous step, we know that any equilibrium must be quality salient. Under what conditions does firm 2 have no incentive to deviate? We now show that firm 2 can profitably deviate whenever the quality offered by firm 1 (and hence its average cost) is high. However, the equilibrium is sustained for lower quality levels, namely when $q_1^* < 2\delta/c_1$ and $A_1^* < \delta$, since then firm 2’s deviation to a price salient, salience constrained configuration is not profitable.

Note first that the condition $c_1/c_2 < 1/2$ implies (in fact is equivalent to) the condition $q_2^* < A_2^*/c_1$. This means that firm 1 can set quality at the salience constraint, $q_1^* = A_2^*/c_1$ so that $q_2^* < q_1^*$ and a quality salient configuration ensues. Whether the salience constraint or the valuation constraint binds, and thus the type of equilibrium that arises, then depend on the ranking of $1/\delta c_1$ relative to $q_2^*$ and $A_2^*/c_1$. We now examine all three possible cases:

- When $q_2^* < 1/\delta c_1 < A_2^*/c_1$, the valuation constraint binds (QV configuration). The valuation constraint binds because $A_2^* > 1/\delta$, so firm 1 sets $q_1^* = 1/\delta c_1$. Moreover, the inequalities above restrict $q_2^*$ to the interval $q_2^* \in [2/\delta c_2, 1/\delta c_1]$. We now show that these configurations are equilibria if and only if firm 1’s average cost is low, $A_1^* < \delta$ (namely, $\delta^2 > 1/2$), and its cost advantage is sufficiently large, $c_1/c_2 < \frac{1}{2\delta^2}$. To start, note that $A_1^* = 1/2\delta$, so there are two cases for firm 2’s deviations:

  * if $A_1^* > \delta$, namely if the average cost of $q_1^*$ is large (i.e. iff $\delta^2 < 1/2$), firm 2 can always deviate to a PS configuration. In this case, firm 2 sets $\hat{q}_2 = \frac{A_1^*}{c_2}$ which satisfies $\hat{q}_2 < q_1$. Moreover, it is easy to check that $\hat{A}_2 = A_1^*/2$ and $\hat{p}_2 = \delta(\hat{q}_2 - q_1^*) + c_1(q_1^*)$ satisfies $\hat{p}_2 = 1/4\delta^2 c_2 > c_2(\hat{q}_2)$. By construction, the good $(\hat{q}_2, -\hat{p}_2)$ is chosen (satisfies the valuation constraint) when compared to $(q_1^*, -c_1(q_1^*))$.

  * if $A_1^* < \delta$ (namely, $\delta^2 > 1/2$) firm 2 may deviate to a PV configuration, setting $\hat{q}_2 = \delta/c_2$. This satisfies $\hat{q}_2 < q_1^*$, as well as $\hat{A}_2 = \delta/2 < A_1^*$. The profitability condition $\hat{p}_2 > c_2(\hat{q}_2)$ reads $\frac{c_1}{c_2} > \frac{\delta^2}{\delta^2} (1 - \frac{1}{2\delta^2})$. The constraint on $A_1^*$ ensures the right hand side is always positive, and ranges from 0 to 65.
1. So this configuration is an equilibrium only if $c_1/c_2$ is sufficiently low, $\frac{c_1}{c_2} < \frac{2}{\delta^2} (1 - \frac{1}{2\sigma^2})$. This condition requires $\delta^2 > 1/2$, which — as noted above — is satisfied exactly when the relevant constraint $A_1^* < \delta$ holds.

* because of Lemma 3 we need not check deviations to quality salient equilibria.

We need also not check for deviations to homogeneous quality goods, since in the price setting stage the low cost firm would always win the competition.

- When $q_2^* < A_2^*/c_1 < 1/\delta c_1$, the salience constraint binds (QS configuration), because $A_2^* < 1/\delta$, so firm 1 sets $q_1^* = A_2^*/c_1 = \frac{c_2}{2c_1} q_2^*$. We now show that such equilibria always exist, and are indexed by $q_2^*$ in the interval $[\frac{2\delta}{c_2}, \frac{2\delta}{c_1} (1 + \sqrt{1 - \frac{c_1}{c_2}})]$. To see this, note that $A_1^* = A_2^*/2 < 1/2\delta$, so there are again two cases for firm 2’s deviations:

* if $A_1^* > \delta$, firm 2 can again always deviate to a PS configuration. It sets $\hat{q}_2 = \frac{A_1^*}{c_2}$ which satisfies $\hat{q}_2 < q_1$ as well as $A_2 = A_1^*/2$ and $\hat{p}_2 = 2c_2(\hat{q}_2)$. We also check that $\hat{p}_2 = \frac{A_1^* c_2}{2c_1} < \frac{A_1^* c_2}{2c_1} < c_1(q_1^*)$;

* $A_1^* < \delta$ if and only if $A_2^* < 2\delta$. Firm 2 may deviate to a PV configuration, setting $\hat{q}_2 = \delta/c_2$. This satisfies $\hat{A}_2 < A_1^*$ if and only if $A_2^* > \delta$ (this condition also guarantees $\hat{q}_2 < q_1^*$). The profitability condition $\hat{p}_2 > c_2(\hat{q}_2)$ then reads $c_1 \gamma < \frac{2A_2^*}{\delta^2} \left( \delta - \frac{A_2^*}{2} \right)$. The constraint on $A_1^*$ ensures the right hand side is always positive, and in fact it ranges from 0 to 1. Replacing the $A_2^* = c_2q_2^*/2$ we find that the pricing constraint is satisfied if and only if $q_2^*$ lies outside the interval $[q_2^{-}, q_2^{+}]$, where $q_2^{\pm} = \frac{2\delta}{c_1} \left( 1 \pm \sqrt{1 - \frac{c_1}{c_2}} \right)$. Recall, however, that the salience constraint requires $A_2^* > \delta$ namely $q_2^* > \frac{2\delta}{c_2}$. Since $\frac{2\delta}{c_2} > q_2^{-}$, we get the result.

- Finally, consider the case where $1/\delta c_1 < q_2^* < A_2^*/c_1$. Then, the best response of firm 1 featuring a salient quality advantage (namely $q_1^* = 1/\delta c_1$) pushes firm 1 to a lower quality, while the best response featuring a salient price advantage (namely $q_1^* = A_2^*/c_1$) pushes firm 1 to a higher quality: firm 1 cannot optimise the first order conditions within a strict salience ranking. Instead, it can set $q_1^* = q_2^*$ and $p_1^* = c_2(q_2^*)$. Because in this case the goods are homogeneous, quality and price are equally salient. When is this an equilibrium? If $A_1^* > \delta$, firm 2 can
deviate, as in the cases above. Consider the case $A_1^* < \delta$, which requires $\delta^2 > 1/2$. Then firm 2 deviates to a PV configuration, $\hat{q}_2 = \delta/c_2$, which trivially satisfies $\hat{q}_2 < q_1^*$ and $\hat{A}_2 = \delta/2 <$. Profitability of this deviation requires $\hat{p}_2 > c_2(\hat{q}_2)$, which reads $\frac{\delta^2}{2c_2} > q_1^* \left( \delta - \frac{c_1 q_1^*}{2} \right)$. The right hand side is maximised at $q_1^*$’s lower bound $1/\delta c_1$. Then the condition reads $\frac{\delta}{c_2} > \frac{2}{\delta^2} \left( 1 - \frac{1}{2\delta^2} \right)$. By continuity with the QV case above, if this condition holds there exist equilibria with homogeneous goods. Such equilibria exist only under some ranges of the model’s parameters: it requires both $\delta^2 > 1/2$ and $\frac{\delta}{c_2} < \frac{2}{\delta^2} \left( 1 - \frac{1}{2\delta^2} \right)$.

• Step 2b: equilibrium conditions when $c_1/c_2 > 1/2$. According to step 1 above, any equilibrium must be price salient. Under what conditions does firm 2 have no incentive to deviate? Similar to the above, we now show that firm 2 can profitably deviate whenever the quality provision of firm 1 (and hence its average cost) is large. However, the equilibrium is sustained for lower quality levels, when firm 2’s deviation to price salient, salience constrained configuration is not profitable.

Note that the condition $c_1/c_2 > 1/2$ implies (in fact it is equivalent to) the condition $q_2^* > A_2^*/c_1$. Therefore, firm 1 can set quality at the salience constraint, $q_1^* = A_2^*/c_1$ so that $q_1^* < q_2^*$ and a price salient configuration ensues. Whether the salience constraint or the valuation constraint binds then depends on the ranking of $\delta/c_1$ relative to $A_2^*/c_1$ and $q_2^*$. We now examine all three possible cases:

The condition $c_1/c_2 > 1/2$ holds if and only if $q_2^* > A_2^*/c_1$, so that if salience binds firm 1’s best response, then $q_2^* > q_1^*$. The binding constraints, and thus the type of equilibrium that arises, then depend on the ranking of $\delta/c_1$ relative to $q_2^*$ and $A_2^*/c_1$.

- When $\delta/c_1 < A_2^*/c_1 < q_2^*$, the salience constraint binds (PS configuration), because $A_2^* > \delta$, so firm 1 sets $q_1^* = A_2^*/c_1$. As before, there are two cases for firm 2’s deviations:

  * if $A_1^* > \delta$, firm 2 can always deviate to a PS configuration. In fact, firm 2 sets $\hat{q}_2 = \frac{A_1^*}{c_2}$ which satisfies $\hat{q}_2 < q_1$ provided $c_1/c_2 < 1/2$. Moreover, it is easy to check that $\hat{A}_2 = A_1^*/2$ and $\hat{p}_2 = 2c_2(\hat{q}_2)$. Finally, we check that $\hat{p}_2 = \frac{A_2^*}{4c_2} < \frac{A_2^*}{2c_1} = c_1(q_1^*)$. 

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* if $A_1^* < \delta$, firm 2 can never deviate to a PV configuration. Setting $\hat{q}_2 = \delta / c_2$ satisfies $\hat{q}_2 < q_1^*$ provided $q_2^* > \frac{\delta}{c_2} \cdot \frac{2c_1}{c_2}$. The profitability condition $\hat{p}_2 > c_2(\hat{q}_2)$ reads, as above, $\frac{\alpha}{c_2} > \frac{2A_2^*}{\delta^2} \left( \delta - \frac{A_2^*}{\delta} \right)$. However, given the lower bound on $q_2^*$ and thus on the average cost $A_2^* > \delta c_1 / c_2$, the deviation is never profitable while $c_1 < c_2$.

- When $A_2^* / c_1 < \delta / c_1 < q_2^*$, the valuation constraint binds (PV configuration), because $A_2^* < \delta$, so firm 1 sets $q_1^* = \delta / c_1$. Lemma 7 shows that in this case no profitable deviations exist for firm 2. As a consequence, such equilibria always exist.

- Finally, consider the case where $A_2^* / c_1 < q_2^* < \delta / c_1$. In this case, firm 1 cannot optimise the first order conditions within a strict salience ranking. Instead, it can opt for homogeneous goods, $q_1^* = q_2^*$ and $p_1^* = c_2(q_2^*)$, such that quality and price are equally salient. To see that firm 2 has no incentive to deviate, note that $A_1^* < \delta / 2$. This means that firm 2 tries to deviate to a PV configuration with $\hat{q}_2 = \delta / c_2$. Even assuming that $\hat{q}_2 < q_2^*$ (which is not guaranteed), we find that $\hat{A}_2 = \delta / 2 > A_1^*$ so this move backfires. As a consequence, such equilibria always exist.

As in our characterization of equilibria under unilateral cost shocks (Proposition 4), the low cost firm 1 wins the market by providing salient high quality if its marginal cost advantage is sufficiently large, namely when $c_1 < c_2 / 2$. If the cost advantage is small, the low cost firm 1 wins a price-salient market by undercutting quality relative to its competitor. This includes case iii) where both firms provide the same quality level, effectively offering homogeneous goods.\(^{29,30}\)

We conclude this Appendix by briefly discussing how the analysis might extend to a

\(^{29}\)In Lemma 8, we consider only equilibria that arise for any $\delta \in [0, 1)$. More details can be found in the proof.

\(^{30}\)One unappealing aspect of this model is that it features many equilibria. This is because: i) firm 2 is certain to lose and so it is indifferent between setting several quality levels, and ii) the equilibrium quality that firm 1 optimally provides sometimes depends on the quality chosen by firm 2. One way to tackle equilibrium multiplicity would be to provide a refinement criterion. This is beyond the scope of our current analysis, which simply seeks to characterize the broad properties of asymmetric equilibria as stated in Lemma 8.
positive (and common across firms) unit cost $F$. Although the analysis is more complicated, some effects are intuitive. Now the average cost of firm $k$ to produce quality $q$ becomes $A_k(q) = c_k \cdot q/2 + F/q$. A positive $F$ has two effects: i) it increases the average costs for both firms and for any quality level, but ii) this effect is particularly large for low quality levels, so that $A_k(q)$ is U-shaped. The quality minimizing average cost for firm $k$ is now $\sqrt{2F/c_k}$, which increases with $F$. This has two implications. First, since cost levels are higher, it is more likely that a quality salient equilibrium obtains for a given level of $c_1, c_2$. Second, it is harder for firm 2 to profitably undercut a given quality provision of firm 1. This last feature expands the set of possible equilibria, particularly quality salient equilibria. In particular, price salient equilibria now require not only that firm 1’s cost advantage is small, $c_1/c_2 < 1/2$, but also that unit costs are small. In contrast, quality salient equilibria arise when either firm 1’s cost advantage is large, or when $F$ – and thus cost levels – is high.

B.5 Competition with an outside option

We extend the analysis of the paper to the case where each consumer chooses one unit of one good from the choice set $C_O \equiv \{(q_2, -p_2), (q_1, -p_1), (0, 0)\}$, where $(0, 0)$ represents the outside option of buying neither good 1 nor good 2. Including an outside option has two effects on market competition: first, as in standard models, it makes the valuation constraint weakly stronger; second, by shaping the reference good, the outside option also influences the attention externality each good exerts on the other. We now show, however, that the fundamental role of the quality cost ratio in determining who wins the market survives in this setting as well.

Consider first the price competition stage, where qualities $q_1, q_2$ and costs $c_1, c_2$ are given. As in section 3, assume that $q_1 > q_2$ and $c_1 > c_2$. In the choice set $C_O$, the reference good has $\bar{q} = (q_1 + q_2)/3$ and $\bar{p} = (p_1 + p_2)/3$. For good 1, its advantage – namely, quality – is salient when $q_1/\bar{q} > p_1/\bar{p}$, and it is easy to see this condition is equivalent to

$$q_1/p_1 > q_2/p_2$$

Thus, assumption A.1 implies that good 1 is preferred to good 2 if and only if the salience
constraint (32) holds, independently of the salience ranking of good 2. The preference ranking between the goods, under assumption A.1, is thus invariant to the inclusion of the outside option \((0, 0)\).

We now consider the impact of the outside option on the valuation constraint. When (32) holds, the consumer buys good 1 provided the valuation constraint \(q_1 - \delta p_1 > 0\) is satisfied. When (32) does not hold, the consumer buys good 2 provided its valuation is positive. To determine good 2’s valuation, we now characterise its salience ranking. To do so, note that \(q_2 > q_1/2\) iff \(q_2 > q_1/2\), and \(p_2 > p_1\) iff \(p_2 > p_1/2\). As a consequence, there are four cases to consider, depending on whether good 2’s quality and price are above or below the reference good’s quality and price. Here we focus on the cases where good 2 is preferred to good 1, namely \(q_2/p_2 > q_1/p_1\). Consider first the case where good 2 is close to good 1 along both dimensions, namely \(q_2 > q_1/2, p_2 > p_1/2\). In this case, the two goods have opposite salience ranking: good 2’s quality is salient and good 1’s price is salient. When good 2 is distant from good 1, namely \(q_2 < q_1/2, p_2 < p_1/2\), the two goods have the same salience ranking (as in the absence of the outside option), so price is salient for both goods. Finally, when \(q_2 > q_1\) and \(p_2 < p_1\) good 2 has a high quality-price ratio, so good 1 is price salient, while good 2 is quality salient provided \(q_2p_2 > q_1p_1\).

At the price competition stage, we show that the good with the highest quality price ratio captures the market, provided the costs of quality are not too high.

**Lemma 9** Under A.1, pure strategy equilibria under price competition in the choice set \(C_O\) satisfy:

i) if \(q_1/c_1 > q_2/c_2\), the consumer buys good 1 provided costs are not too high, \(c_1 < q_1/\delta\).

ii) if \(q_2/c_2 > q_1/c_1\), the consumer buys good 2 provided the costs are not too high. A sufficient condition for that is \(c_2 < \delta q_2\).

iii) if \(q_1/c_1 = q_2/c_2\), quality and price are equally salient, the consumer buys the good yielding the highest (rational) surplus \(q_k - c_k\), if the latter is positive. Prices are \(p_1 = c_1, p_2 = c_2\).

**Proof.** From the analysis in the text, we know that the high quality good 1 is quality salient,

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\(^31\)This is not generally the case with an arbitrary outside option, so the reasoning may have to be adjusted in situations where there is a clear default option. However, in a choice set defined only by goods 1 and 2, it is natural to represent the alternative of not buying a good as \((0, 0)\), which can be thought of as narrow framing.
and thus is preferred to good 2, if and only if $q_1/p_1 > q_2/p_2$. As in the case without the outside option, then, in a market equilibrium (where firm 1 sells the good) this condition is satisfied if and only if $q_1/c_1 > q_2/c_2$. In this case, firm 2 sets price $p_2 = c_2$ and firm 1 sets price satisfying $p_1 \leq \min \{ q_2 \cdot \frac{c_2}{q_2}, \frac{1}{\delta}(q_1 - q_2) + c_2 \}$. The market equilibrium exists (namely, firm 1 sells its good) if and only if good 1 is preferred to the outside option, namely it has positive valuation, $q_1 - \delta p_1 > 0$. Firm 1 can set a price satisfying this condition whenever $c_1 < \delta q_1$. Consider now the case where $c_1 > \delta q_1$. Firm 1 cannot sell its good and, as before, we assume it sets price at $p_1 = c_1$. In this case, no good is sold. In particular, under assumption A.1, firm 2 is also unable to sell its good, regardless of its salience ranking, since $q_2 - \delta c_2 < q_1 - \delta c_1 < 0$.

A similar analysis carries through if $q_2/c_2 > q_1/c_1$. In this case, in any market equilibrium firm 1 sets price $p_1 = c_1$ while firm 2 sets price $p_2 \leq q_2 \frac{c_1}{q_1}$. In order to sell its good, firm 2’s good must be preferred both to good 1 and to the outside option. Both valuation constraints depend on the salience ranking of good 2 (note that good 1 is always price salient). A sufficient condition for the market to exist is that good 2 is chosen when it is price salient, namely $c_2 < \delta q_2$.

We now turn to the endogenous quality case when firms have identical costs. For simplicity, we consider the case of quadratic costs, $c(q) = \frac{\delta}{2} q^2 + F$. As before, pure strategy equilibria entail symmetric quality production, where price and quality are equally salient (we assume that firms strictly prefer to share the market with zero profits than being out of the market). Analogously to proposition 2, we derive the equilibrium quality provision $q^S$ as a function of the unit cost $F$. Because the quality price ratio is a necessary statistic for winning the market, proposition 2 carries through in the range of intermediate $F \in [F, \overline{F}]$ where the salience constraint binds. In this range, firms set $q^S = \sqrt{2F/c}$. Participation in the market requires that $q^S - c(q^S) \leq 0$, namely $F < 1/2c$. So the outside option truncates the symmetric equilibria for $F$ larger than $1/2c$, excluding equilibria where quality is over provided relative to the efficient level. Consider now the range $F < F$, where in the absence of the outside option the equilibrium quality provision is $q^S = \delta/c$. With an outside option, there might in principle be profitable deviations from this solution because a firm that cuts
quality slightly in order to minimise its average cost renders its good quality salient, while rendering its competitor price salient. This stronger salience advantage would push firms to always minimise average costs. We find:

**Lemma 10** When firms have identical cost functions, the unique pure strategy subgame perfect equilibrium in the choice set \( C_O \) is symmetric. Equilibria exists provided costs are not too high, \( F \leq \frac{1}{2c} \). In equilibrium price and quality are equally salient, firms make no profits, and quality provision is given by: \( q_2^S = q_1^S = q^S \equiv \hat{q}(F) \).

**Proof.** Given the discussion in the text, it is sufficient to check that \( q^S = \sqrt{2F/c} \) is the unique (symmetric) equilibrium for \( F < F^* \). We first show that there are profitable deviations from the two-good symmetric equilibrium quality \( \delta/c \) in this range. Suppose firm 1 cuts quality slightly to \( \delta/2c < q_1 < \delta/c \), while firm 2 keeps quality at \( q_2 = \delta/c \). As a consequence good 1 becomes quality salient while good 2 becomes price salient. This is a profitable move when firm 1 can sell its good with a positive profit. To see that is the case, note that \( q_1 - \delta c(q) > 0 \) for \( F < F^* \) and also that \( q_1 - \delta c(q) > \frac{\delta^2}{c} - c(\delta/c) \) for any \( \delta < 1 \). The same logic implies that \( q^S = \sqrt{2F/c} \) is an equilibrium for any \( F < F^* \): deviating from the average-cost minimizing quality makes the deviating firm’s good price salient while its competitor becomes quality salient. Therefore, such deviations are never profitable.

Finally, we turn to innovation, where firm 1 has a positive shock to the marginal cost of quality. Recall that in this case firm 1 always weakly increases quality provision, so that quality is (weakly) salient. As a consequence, firm 1 captures the market, provided good 1 satisfies the valuation constraint \( q_1 > \delta c_1(q_1) \). Because the valuation constraint is strictly weaker than in the symmetric case, such innovation equilibria exist for all \( F \leq \frac{1}{2c} \). In fact, for the same reason, such equilibria exist even for higher levels of \( F \) where quality is over provided relative to the efficient level. However, such equilibria do not preserve the interpretation as an innovation relative to the symmetric case.