Need for (the Right) Speed:

The Timing and Composition of Public Debt Deleveraging

JOB MARKET PAPER

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Abstract

This paper studies what is the optimal path for public debt deleveraging in a heterogeneous agents framework under incomplete financial markets. My analysis addresses two questions: What is the optimal fiscal instrument the government needs to use to reduce public debt? What is the optimal speed of public debt deleveraging? The main finding is that public debt should be reduced in a fast way and by cutting public expenditure. If the fiscal authority is forced to use income taxation, public debt deleveraging needs instead to be slow. Independently of fiscal instruments, the economy may end up in a liquidity trap. I show that, in my model, the zero lower bound has a redistributive effect. If the liquidity trap is very persistent, it can reallocate resources from financially constrained agents to financially unconstrained ones. Due to this mechanism, a very slow public debt reduction achieved by increasing income taxation is very costly in terms of aggregate welfare.

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1 Introduction

After the recent financial crisis, the United States have experienced an unprecedented rise in the size of public debt, as shown in Figure 1.[1] From a policy perspective, there are important concerns regarding a pressing need for the United States to reduce their stock of public liabilities. Indeed, recent experience in the European Union has highlighted the risks associated with large liability positions for the public sector, especially when sources of funding are abruptly reduced.

Moreover, a second fact has characterized recent developments in the US economy: Wealth inequality rose almost constantly in the last twenty years and it is now at one of the highest levels ever experienced in the last century. For example, the concentration of wealth of the top 10% and 1% agents rose from 1970 until 2010, both in the U.S. and in Europe, as shown in Figure 1, second panel. Policymakers in the United States and abroad have recently highlighted the concerns posed by rising income and wealth inequality.[2]

This paper studies what is the optimal way to implement austerity measures, in a context of heterogeneous agents in the spirit of Bewley (1977) and Aiyagari (1994). My analysis addresses two main questions: What is the optimal fiscal instrument the government needs to use to reduce public debt? What is the optimal speed of public debt deleveraging?

In my model, agents are heterogeneous along two dimensions. First, they differ in terms of employment status: they can be workers, either employed or unemployed, or they can be entrepreneurs. Secondly, they can endogenously accumulate assets, thus making agents heterogeneous in terms of wealth. Presence of heterogeneity with respect to the employment status allows me to match some key features of the wealth distribution in the United States.

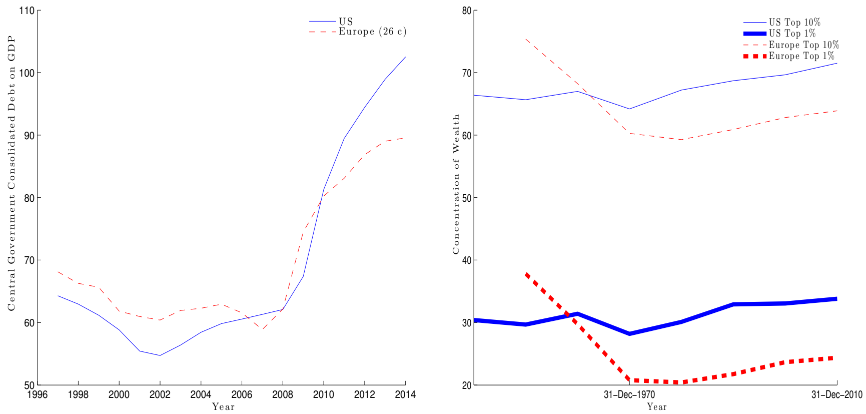
The stock of public debt can be reduced by the fiscal authority either by increasing revenue from distortionary income taxation or by reducing the flow of public expenditure. Public expenditure provides utility services to the agents, and it is an imperfect substitute for private consumption.[3]

A key result of my study, robust to different specifications, is that the optimal policy is to reduce public debt in a fast way, by cutting public ex-

[1]This figure shows public debt to GDP dynamic for the U.S. and for the European Union with 26 countries from 1996 to 2014.

[2]See, e.g. Yellen (2014).

[3]The case of welfare-increasing public expenditure that is imperfect complement with private consumption is analyzed in the appendix.



(a) Public Debt as Percentage of GDP **(b) Concentration of Wealth**

Figure 1: Motivating Evidence
Data for public debt are taken from the Federal Reserve Bank of St.Louis and from the European Commission AMECO. Data on wealth distribution are taken from Piketty (2014).

penditure. Behind this result, an important role is played by the dynamics of the real interest rate. By reducing public expenditure, the fiscal authority induces a demand shock on the economy. As a consequence, the real interest rate tends to be low precisely when the debt reduction is taking place. Such a phenomenon helps the government to achieve the public deleveraging, thus avoiding the need for larger cuts in public spending. Moreover, when the debt reduction takes place, the burden of interest payments faced by borrowers is reduced. It is important for public debt deleveraging to be fast, so as to have low real interest rates when debt is at its highest. Under this policy, however, some agents lose in terms of welfare: These are the wealthiest who experience, due to the fall in the interest rate, a decrease in the value of their assets.

If the fiscal authority is constrained in using income taxation, public debt must be reduced slowly. By increasing taxation, the fiscal authority now creates a negative supply shock: Differently than in the case of public expenditure, real interest rates are high when the bulk of public deleveraging occurs. Most agents want the real interest rate to be at its highest when debt is at its lowest, namely in the last quarter of deleveraging. This is only compatible with a slow debt reduction. However, agents do not postpone the debt deleveraging to an extreme degree, since they dislike fluctuations in tax rates. Again, under this policy, the wealthiest agents are the ones who lose the most in term of welfare.

Independently of the fiscal instrument used to achieve the public debt reduction, the economy may fall into a liquidity trap. The zero lower bound has, in the context of my model, a novel effect due to the presence of heterogeneous agents. This is additional to traditional aggregate effects that have been studied in detail in the literature. At the aggregate level, when the zero lower bound binds, the economy faces lower output and an inflation rate below the central bank's target.[4] Besides, the zero lower bound also has a redistributive effect. When the economy enters a liquidity trap, firms cut their labor demand, depressing real wages. This is due to the contemporaneous presence of nominal rigidities and inflation below the target of the central bank. Therefore, the labor income share falls and the profit share rises, meaning that income will be reallocated from the employed to the entrepreneurs. In my model, the financially-constrained agents are mostly among the employed, while entrepreneurs are, in general, financially unconstrained. Hence, the zero lower bound redistributes resources from agents whose marginal propensity to consume is high to the ones whose marginal propensity is low. This, in turn, exacerbates the output recession and enhances the welfare costs of the liquidity trap, thus affecting preferences regarding policy. In particular, this occurs when the fiscal authority reduces public debt slowly by increasing income taxation. Under this circumstance, the agents, aware of the future output recession, want to smooth consumption: Current savings increase, depressing current real interest rates and making the zero lower bound bind. As a consequence, these combinations become the costliest for the economy in terms of welfare. A policy implication is that the fiscal authority needs to avoid an excessive postponing of the debt reduction, when this is achieved by income taxation.

My research is closely related to two different strands of the literature: One that has focused on private debt deleveraging and its interaction with monetary policy, and another one on optimal fiscal policy under commitment. Papers in the literature on private debt deleveraging typically model such an event as an exogenous shock, then analyzing the impact of different monetary

[4]For a discussion of this mechanism, see Adam and Billi (2006, 2007), Eggertsson and Woodford (2003, 2004) and Werning (2011).

policies in that context. Some recent papers, such as Guerrieri and Lorenzoni (2010), Krugman and Eggertsson (2012), Philippon and Midrigan (2011) and Benigno and Eggertsson and Romei (2014) have studied debt deleveraging in a closed economy. Others, among which Fornaro (2012), Cook and Devereux (2012) and Benigno and Romei (2012), have focused on the consequences of private debt deleveraging in an international context. I depart from this literature in two important ways: First, I concentrate my analysis on a public debt deleveraging episode. Secondly, while this literature has focused on a particular deleveraging path, I analyze instead how different debt reduction paths can affect agents welfare.

Secondly, my paper is also related to the literature on optimal fiscal policy. In their seminal paper, Lucas and Stokey (1983) show how the public authority should react to a shock when it is possible to issue state contingent debt, in a representative-consumer framework. Aiyagari, Marcet, Sargent and Seppala (2002) analyze the same problem when the public authority cannot issue state contingent debt. More recently, many authors have proceeded to study how optimal fiscal policy changes once departing from the representative agent assumption, such as Werning (2007) and Bandhari, Evans, Golosov and Sargent (2013) among others. This literature aims to understand how a government can optimally react to a shock. My paper, instead, aims to understand what is the nature of the optimal government-induced shock in the economy.

Finally, my research is close to the work of Aiyagari and McGrattan (1997) and Röhrs and Winter (2014). Both papers study what is the optimal level of public debt in a model with heterogeneous agents where financial markets are incomplete. In particular, the novel contribution of Röhrs and Winter to the existing literature consists in the analysis of the consequences on aggregate welfare of a public debt reduction achieved by changes in income taxation. In reference to this literature, my contribution is to study the optimal composition and speed of fiscal consolidations, with particular focus on how these affect the distribution of wealth and welfare across agents. Moreover, I also focus on economies with relevant price frictions, in which the zero-lower bound can be an important constraint on the adjustment process.

2 Model

I consider a discrete-time closed-economy model inhabited by a continuum of consumers on a unit interval, interacting with final and intermediate good-producing firms, a fiscal authority, a social security scheme and a central bank. Consumers are subject to idiosyncratic uncertainty, but they have perfect foresight about the path of aggregate variables. The economy is not subject to aggregate uncertainty. Asset markets are incomplete in the spirit of Bewley (1977). I start by analyzing the steady state of the model. The next section studies the transition of the economy after the fiscal authority is forced to reduce the level of public debt. The following subsections describe the problem solved by each type of agent in detail.

2.1 Consumers

Consumers differ in terms of employment status and wealth. In each period an agent may be either unemployed, employed or an entrepreneur. The employment status of the agents evolves according to an exogenous Markov process. I denote this by the index s, which can take values in the set $\{U, L, E\}$ for unemployed, employed and entrepreneur, respectively. I divide the unit interval in three subsets, $\{\mathcal{U}, \mathcal{L}, \mathcal{E}\}$, the first referring to unemployed, the second to employed and the third to entrepreneurs respectively. The ergodic distribution of the Markov process implies a constant mass $\tilde{U}$ of unemployed agents, $\tilde{L}$ of employed agents and a mass $\tilde{E}$ of entrepreneurs. Agents' wealth evolves endogenously according to their borrowing and saving decision. This, in turn, is determined by their individual state variables, employment status and wealth, as well as by the path of the aggregate variables .

All agents have identical preferences over streams of consumption, c, government expenditure, G, and leisure. Employed agents can choose the numbers of hours, l, to supply to firms. Unemployed agents and entrepreneurs do not supply labor and they enjoy their whole endowment of leisure.[5] The expected lifetime utility of each individual agent i can be expressed as:

$$E_0 \left[\sum_{t=0}^{\infty} \beta^t \left(u(c_{it}, G_t) - \nu(l_{i,t}) \right) \right] \tag{1}$$

where $E[.]$ is the mathematical expectation operator, β is the subjective

[5] I assume that if an agent does not work $l = 0$.

discount factor and $u(.)$ is a concave, twice-differentiable function satisfying Inada conditions. The function $\nu(.)$ is convex and twice differentiable. Without loss of generality, I assume that $\nu(0) = 0$. The expectation operator $E_0[.]$ in equation (1) refers to the idiosyncratic uncertainty faced by agent i at time 0 regarding his future employment status.

Income differs across agents and it is endogenous. All the unemployed agents receive an unemployment benefit σ_t in each period. The employed agents receive an hourly real wage, w_t. They pay a lump sum social security contribution ρ_t that is time-varying. They also pay a proportional tax on their labor income, τ_t, accruing to the fiscal authority. The entrepreneurs receive profit from the firms, ϖ_t, and they pay the same tax rate τ_t on their income as the employed agents.

The agents, independently of their type, trade one-period riskless bonds, $b_{i,t+1}$, denominated in units of consumption goods, paying a real interest rate, r_t. They enter each period with a predetermined stock of debt, $b_{i,t}$. Negative values for b represent positive wealth. The individual budget constraint is as follows:

$$c_{i,t} = \frac{b_{i,t+1}}{1+r_t} - b_{i,t} + I_i^U \sigma_t + I_i^L \left(w_t l_{i,t}(1-\tau_t) - \rho_t\right) + (1 - I_i^U - I_i^L)\varpi_t(1-\tau_t) \quad (2)$$

where I_i^U is an indicator function equal to one if the agent is unemployed and I_i^L is an indicator function equal to one if the agent is employed. I assume that

$$\rho_t = \varrho w_t \int_{i \in \mathcal{L}} l_{i,t} di$$

where ϱ is a parameter governing the fraction of average labor income accruing to the unemployed, namely the gross replacement rate. As a consequence, each worker's contribution is independent of their individual labor income but it does depend on the aggregate labor income. Finally, all the agents face an exogenous borrowing constraint:

$$b_{i,t+1} \leq \psi \quad (3)$$

Consumers choose consumption, savings and labor supply by maximizing (1) subject to the budget constraint (2), and to the exogenous borrowing constraint (3). The first order conditions for the intertemporal choice are given by an Euler equation and by a complementary slackness condition:[6]

$$u_c(c_{i,t}, G_t) = \beta E_t \left[u_c(c_{i,t+1}, G_{t+1})(1 + r_t) \right] + \mu_{i,t} \tag{4}$$

$$\mu_{i,t}(b_{i,t+1} - \psi) = 0 \tag{5}$$

The employed agents' intratemporal first order condition for the labor supply is given by:

$$\nu_l(l_{i,t}) = u_c(C_{i,t}) w_t (1 - \tau_t). \tag{6}$$

2.2 Final Good Sector

The final good sector is populated by a continuum of firms producing under perfect competition. They have access to a constant returns to scale technology, allowing for aggregation into a representative firm. The representative firm transforms units of intermediate goods, $y(j)$, into final output, Y:

$$Y_t = \left[\int_0^1 y(j)^{\frac{\epsilon-1}{\epsilon}} dj \right]^{\frac{\epsilon}{\epsilon-1}} \tag{7}$$

where ϵ is the parameter governing the elasticity of substitution across intermediate goods.

Demand for intermediate inputs arising from profit maximization of the final good-producing firm is given by:

$$y(j) = \left(\frac{p(j)}{P} \right)^{-\epsilon} Y_t \tag{8}$$

where $p(j)$ is the nominal price of each intermediate input and P is the price of the final good, defined as:

$$P = \left(\int_0^1 p(j)^{1-\epsilon} dj \right)^{\frac{1}{1-\epsilon}}.$$

[6] I denote by $u_c(.)$ the partial derivative of the function $u(.)$ with respect to c. Consistently, I denote by $\nu_l(.)$ the derivative of the function $\nu(.)$ with respect to l.

2.3 Intermediate Good Sector

The intermediate good sector is populated by a continuum of identical firms of measure one. Each firm j has access to the following decreasing returns to scale technology:

$$y(j) = n(j)^{\omega}, \tag{9}$$

allowing it to transform labor input $n(j)$ into output of a differentiated good, $y(j)$. The parameter ω governs the elasticity of output with respect to labor input.

Each firm j competes under monopolistic competition with the other firms in its sector to meet the demand schedule (8).

Every firm is subject to a quadratic cost of adjusting nominal prices, which is measured in units of aggregate final output following Rotemberg (1982):

$$\phi(p_t(j), p_{t-1}(j), Y_t) = \frac{\varphi}{2}\left(\frac{p_t(j)}{p_{t-1}(j)\bar{\Pi}} - 1\right)^2 Y_t$$

where $\bar{\Pi}$ is the steady state inflation rate and $\varphi > 0$ determines the degree of nominal rigidity.

The firms maximize the present discounted sum of profits

$$\Xi(j) = \sum_{t=0}^{\infty} \beta^t \lambda_t \tilde{\varpi}_t(j) \tag{10}$$

subject to (8) and (9), where the per-period profit is

$$\tilde{\varpi}_t(j) \equiv \left[\frac{p_t(j)}{P_t} y_t(j) - w_t(j) n_t(j) - \phi(p_t(j), p_{t-1}(j), Y_t)\right]$$

and

$$\lambda_t \equiv \int_0^1 u_c(c_{i,t}, G_t) di$$

is a weighted average of agent-specific stochastic discount factors. [7] Due to the lack of aggregate uncertainty and firm-level idiosyncratic uncertainty I can neglect the expectation operator.

[7] Note that $\tilde{E}\varpi_t = \int_0^1 \tilde{\varpi}_t(j) dj$.

In this sector, all the firms, facing the same costs and demand schedule, will make identical choices. This allows me to omit the j index. In particular, they will all set the same price $p(j) = p = P$, and produce the same amount, $y(j) = y = Y$.

The first order condition for the firms with respect to their individual price can be written as follows:

$$w_t = \frac{\omega}{\mu} Y_t^{1-\frac{1}{\omega}} + \frac{\varphi\omega}{\epsilon}\left(\frac{\Pi_t}{\bar{\Pi}} - 1\right)\frac{\Pi_t}{\bar{\Pi}} Y_t^{1-\frac{1}{\omega}} - \beta\frac{\lambda_{t+1}}{\lambda_t}\frac{\varphi\omega}{\epsilon}\left(\frac{\Pi_{t+1}}{\bar{\Pi}} - 1\right)\frac{\Pi_{t+1}}{\bar{\Pi}}\frac{Y_{t+1}}{Y_t^{\frac{1}{\omega}}} \tag{11}$$

where $\mu \equiv \frac{\epsilon}{\epsilon-1}$ is the steady state markup of prices over marginal costs and $\Pi_t \equiv \frac{P_t}{P_{t-1}}$ is the gross inflation rate.[8]

Equation (11) describes the firms' output supply: output responds negatively to an increase in the real wage. If inflation is close to the steady state, output responds positively to an increase in current inflation and negatively to an increase in future inflation.

2.4 Fiscal Authority, Social Security and Central Bank

The fiscal authority provides G_t units of a non-rival consumption good in every period. Public expenditure is financed either by charging taxes on the agents' income flows or by issuing a bond in terms of consumption good B_t^G, the return on which, also denominated in units of the consumption good, is the real interest rate, r_t. A positive value of B^G stands for public debt. The budget constraint of the fiscal authority is:

$$\tau_t w_t \int_{i\in\mathcal{L}} l_{i,t} di + \tau_t \tilde{\varpi}_t = G_t - \frac{B_{t+1}^G}{(1+r_t)} + B_t^G \tag{12}$$

The social security scheme runs a balanced budget. Aggregate unemployment benefits are financed in each period by the workers' contributions:

$$\tilde{L}\rho_t = \sigma_t \tilde{U}. \tag{13}$$

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[8] A full derivation of this expression is relegated to the Appendix.

[9] $\tilde{U}$ and $\tilde{L}$ have been defined in subsection 2.1 as the mass of unemployed and employed agents, respectively.

The real interest rate is determined by the Fisher equation:

$$(1 + r_t) = \frac{(1 + i_t)}{\Pi_{t+1}}$$

where i_t is the nominal interest rate at time t and Π_{t+1} is the gross inflation between time t and $t + 1$. Note that Π_{t+1} is in the information set of the agents due to the assumption of perfect foresight.

The policy is simple: the objective of the central bank is to maintain inflation on target, $\bar{\Pi}$, whenever possible. Otherwise, if this implies a negative nominal rate, it fixes the interest rate, i_t, at zero:

$$\begin{cases} \Pi_t = \bar{\Pi} & \text{if } i_t \geq 0 \\ i_t = 0 & \text{otherwise.} \end{cases} \tag{14}$$

Since the nominal interest rate cannot be negative, the real rate must be greater than the inverse of future inflation, i.e.

$$(1 + r_t) \geq \frac{1}{\Pi_{t+1}}. \tag{15}$$

2.5 Market Clearing Conditions

The goods, assets and labor markets clear. I will now describe in turn all the market clearing conditions.

The goods market clears:

$$\int_0^1 c_{i,t} di + G_t = Y_t - \phi(P_t, P_{t-1}, Y_t) \tag{16}$$

where I define

$$\phi(P_t, P_{t-1}, Y_t) \equiv \frac{\varphi}{2} \left(\frac{P_t}{P_{t-1}\bar{\Pi}} - 1 \right)^2 Y_t$$

as the quadratic price adjustment cost in terms of the consumption good as a function of aggregate current and past prices, P_t and P_{t-1} respectively, and aggregate production, Y_t.[10] Due to the assumption of Rotemberg costs, part of the production is purely wasted and does not enter the agents' consumption.

[10] I exploit the fact that all firms take the same decision in equilibrium.

The assets market clears:

$$B_{t+1}^{G} + \int_0^1 b_{i,t+1} di = 0. \tag{17}$$

By Walras's law, the labor market also clears:

$$n_t - \left(\int_{i \in \mathcal{L}} l_{i,t} di \right) = 0 \tag{18}$$

where I can write $n(j) = n$, exploiting the fact that in equilibrium all the firms make the same choices. I am also implicitly assuming that the hours supplied by different employed agents are perfect substitutes from the point of view of the firms. This is consistent with the fact that the employed agents only differ in terms of wealth and not productivity.

2.6 Equilibrium

Given a sequence of taxes, social security contributions, public expenditure $\{\tau_t, \varrho, G_t\}_{t=0}^{\infty}$, prices $\{r_t, i_t, w_t\}_{t=0}^{\infty}$, and the inflation targeting policy (14), let the policy rules for c and l at time t as a function of the individual state of the agents be $c_t(s, b)$ and $l_t(s, b)$ for a household with employment status $s_{i,t} = s$ and initial bond holding $b_{i,t} = b$. These two decision rules pin down the endogenous transition of the agents' bond holdings.

I define the joint distribution of assets and employment statuses at the beginning of the period as $\Psi_t(s, b)$. The transition for the agents' bond holdings, together with the exogenous Markov process for employment statuses determines the endogenous joint transition probability for agent-specific state variables. These are sufficient to characterize the next-period asset and employment status joint distribution,$\Psi_{t+1}(s, b)$. I can now define the equilibrium.

Definition 1. *Given a sequence of taxes, social security contributions, public expenditure* $\{\tau_t, \varrho, G_t\}_{t=0}^{\infty}$*, the inflation targeting policy* (14)*, an initial joint distribution of employment statuses and assets,* $\Psi_0(b, s)$*, and an initial price vector* $\{p_0(j)\}_{j=0}^{1}$*, an equilibrium is a sequence of prices* $\{r_t, i_t, w_t\}_{t=0}^{\infty}$*, allocations,* $\{c_t(s, b), l_t(s, b), b_{t+1}(s, b), Y_t\}_{t=0}^{\infty}$*, and a sequence of joint distributions for bond holdings and employment statuses,* $\{\Psi_t(s, b)\}_{t=0}^{\infty}$*, such that given* $\Psi_0(b, s)$ *and* $\{p_0(j)\}_{j=0}^{1}$ *:*

- $\{c_t(s,b), l_t(s,b), b_{t+1}(s,b), Y_t\}_{t=0}^{\infty}$ *are optimal given* $\{r_t, i_t, w_t\}_{t=0}^{\infty}$, $\{\tau_t, \varrho, G_t, \Pi_t\}_{t=0}^{\infty}$ *and the inflation targeting policy* (14)*;*
- $\{\Psi_t(s,b)\}_{t=0}^{\infty}$ *are consistent with the decision rule and the exogenous transition probability;*
- *The goods, asset and labor markets clear,* (16)*,* (17) *and* (18)*;*[11]
- *The fiscal authority budget constraint is satisfied,* (12)*;*
- *The social security budget constraint is satisfied,* (13)*;*
- *The nominal interest rate and inflation rate are consistent with* (14)*.*

3 Calibration

The model is simulated at quarterly frequency. I assume that $u(c, G)$ takes the following CRRA form:

$$u(c,G) \equiv \frac{f(c,G)^{(1-\gamma)}}{(1-\gamma)}$$

where γ is a parameter ruling the intertemporal elasticity of substitution and the degree of risk aversion. I set $\gamma = 1.5$ in line with the literature.

The agents' private consumption and public expenditure are aggregated by means of a CES function:

$$f(c,G) \equiv \left[\alpha^{\frac{1}{\chi}} c^{\frac{\chi-1}{\chi}} + (1-\alpha)^{\frac{1}{\chi}} G^{\frac{\chi-1}{\chi}}\right]^{\frac{\chi}{\chi-1}}$$

where χ is the parameter that governs the elasticity of substitution between public expenditure and private consumption and α is the parameter that defines the share of private consumption out of total consumption. I assume that $\chi = 3$, so that publicly provided goods and private consumption are imperfect substitutes and $\alpha = .9$. I will experiment in the Appendix with different values for χ in order to show how the results change when public expenditure and private consumption are imperfect complements.

[11]One of the three equations is redundant due to Walras's law.

In line with the literature on Real Business Cycles, I assume that labor disutility is of the type:

$$\nu(l) \equiv \frac{l^{1+\eta}}{1+\eta}$$

where η is the parameter governing the disutility of the labor supply. I set $\eta = 2$, which implies an average Frisch elasticity of the labor supply of .57, well in line with the admissible range of values in the macro literature.

In line with Quadrini (2000), I put the percentage of entrepreneurs at 14%.[12] I set $\tilde{U}$ to equal 0.0626 to match the average United States unemployment rate in 2013, 7.3%. I set $\tilde{L}$ residually.

Following Shimer (2005), I set the transition probability from the unemployment to the employment status to equal 0.882 and that from the employment to the unemployment status to equal 0.057. I put the transition probability from the employment to the entrepreneur status at 0.0830 and from entrepreneur status to employment at 0.403. In order to match a replacement rate of 40%, again in line with Shimer (2005), I choose the parameter value $\varrho = .4$.

I set ψ, the borrowing limit, equal to .95, allowing the agents to borrow annually up to 29% of the steady state total economy per capita GDP.

I set $\tau = 0.127$ to match the median payroll tax rate in the US in 2010, according to CBO (2012).[13] I put the real debt B^G at 0.6451 to match a value for annual public debt over GDP of 20%. I choose this value for initial debt to focus on that part of public debt that is held domestically by private agents. In 2013, the US debt to GDP held by the public excluding the Federal Reserve System equalled 55%. More than half of this was owned by foreign investors. Hence, considering a value of 20% does not seem implausible to describe US variables. Due to the non-linearities characterizing this model, it would be very difficult to find transition equilibria for all th exercises I consider with a higher level of public debt. Secondly, I want some agents to act as borrowers in this economy, and this would be not possible with very high government debt.

I set $\beta = 0.9807$ to match an annual real interest rate in the initial steady state of 2.6%, in line with historical US data. [14] The annual inflation rate is equal to 2%, in line with the average US inflation rate since the year 2000.

[12] One of the estimated values for the percentage of self-employed in that paper is 13.7%.

[13] Effective Marginal Tax Rates for Low-and Moderate-Income Workers.

[14] In the final steady state, to be defined in Section 5, the real interest rate equals 2.5%.

The intra-temporal elasticity of substitution among different inputs, ϵ, is put at 7, in line with the literature and implying a markup of 16.7%. I set ω, the parameter that governs the elasticity of output with respect to labor input, equal to 0.75, in line with the literature that poses this parameter between .66 and 1. Finally, I set the Rotemberg cost parameter at $\varphi = 68.2$. This matches, at first order approximation, price rigidities á la Calvo where only 25% of firms can adjust their prices in each quarter.[15]

Given the strong non-linearities present in the model, the solution method adopted is based on a global method. The details are provided in the Appendix.

[15]In line with Smets and Wouters (2003) and De Walque, Smets and Wouters (2005).

Table 1: Parameters

Parameter	Value	Source or Target
Risk Aversion	$\gamma = 1.5$	Standard Value
Elasticity of substitution c and G	$\chi = 3$	Imperfect Substitutability
CES parameter	$\alpha = .9$	Share of public expenditure out of total consumption 12%
Labor Disutility	$\eta = 2$	Frisch Elasticity 0.57
Share of entrepreneurs	$\tilde{E} = .14$	Quadrini (2000)
Share of unemployed	$\tilde{U} = .0626$	Unemployement Rate 2013
Transition from U to L	.882	Shimer (2005)
Transition from L to U	.057	Shimer (2005)
Transition from L to E	.0830	
Transition from E to L	.403	
Replacement Rate	$\varrho = .4$	Shimer (2005)
Debt Limit	$\psi = .95$	Household debt 29%
Tax rate	$\tau = .127$	Median payroll tax rate in the US in 2010
Public Debt	$B^G = 0.645$	Public Debt over GDP to equal 20%
Discount Factor	$\beta = .9807$	Real interest rate in the final steady state 2.5%
Inflation Target	$\Pi = 1.005$	Inflation target at 2%
Elasticity of substitution among intermediate inputs	$\epsilon = 7$	Standard Value
Elasticity of output with respect to labor input	$\omega = .75$	Standard Value
Rotemberg Costs parameter	$\varphi = 68.2$	First order Calvo model with 75% firms who do not move the price

4 Steady State

Let us now describe the initial steady state. First of all, I calibrate the model to match inequality in wealth distribution in the spirit of Castañeda et al. (2003). Figure 2 shows the policy function for asset accumulation, namely the difference between current and future debt as a ratio of per capita GDP, $b_t^{GDP} - b_{t+1}^{GDP}$, as a function of current assets as a ratio of per capita GDP. In this class of model, the borrowing-saving decisions of individual agents are endogenous functions of predetermined individual states and aggregate variables. Figure 2 shows whether an agent in a given employment status with a given predetermined asset is increasing or decreasing his stock of assets. A positive value for $b_t^{GDP} - b_{t+1}^{GDP}$ means that agents are accumulating assets, while a negative one means they are decumulating assets. Unemployed agents, if not close to the borrowing constraint, are the ones whose saving reduce the most. This is consistent with the fact that they are in the worst employment status. Thus, once an agent becomes unemployed, he begins to run down his assets. The employed also decumulate their assets, but at a lower pace than the unemployed. Entrepreneurs, instead, always accumulate bonds. This occurs since they are in the best employment status, consistent with the standard permanent income hypothesis.

Figure 3 shows the distribution of assets as a percentage of per capita GDP in the initial steady state by employment status. In equilibrium, the employed agents are the least wealthy. On the other hand, the entrepreneurs are net savers and the wealthiest agents in the model economy. Despite being the agents whose savings reduce the most, the wealth distribution of the unemployed lies slightly to the right of the wealth distribution of the employed. In fact, the distribution of assets depends on both the endogenous policy function and on the exogenous Markov chain. Data for the United States and evidence presented by Shimer (2005) suggest that there is more persistency in the employed status than in the unemployed one. An agent who is unemployed today, therefore, is likely to have been in a different employment status yesterday. Hence, despite the fact that, once unemployed, he will start to run down his assets, he is likely to have been subject to a different policy function in the past. As a consequence, in equilibrium, the unemployed agents turn out to be slightly wealthier than the employed.

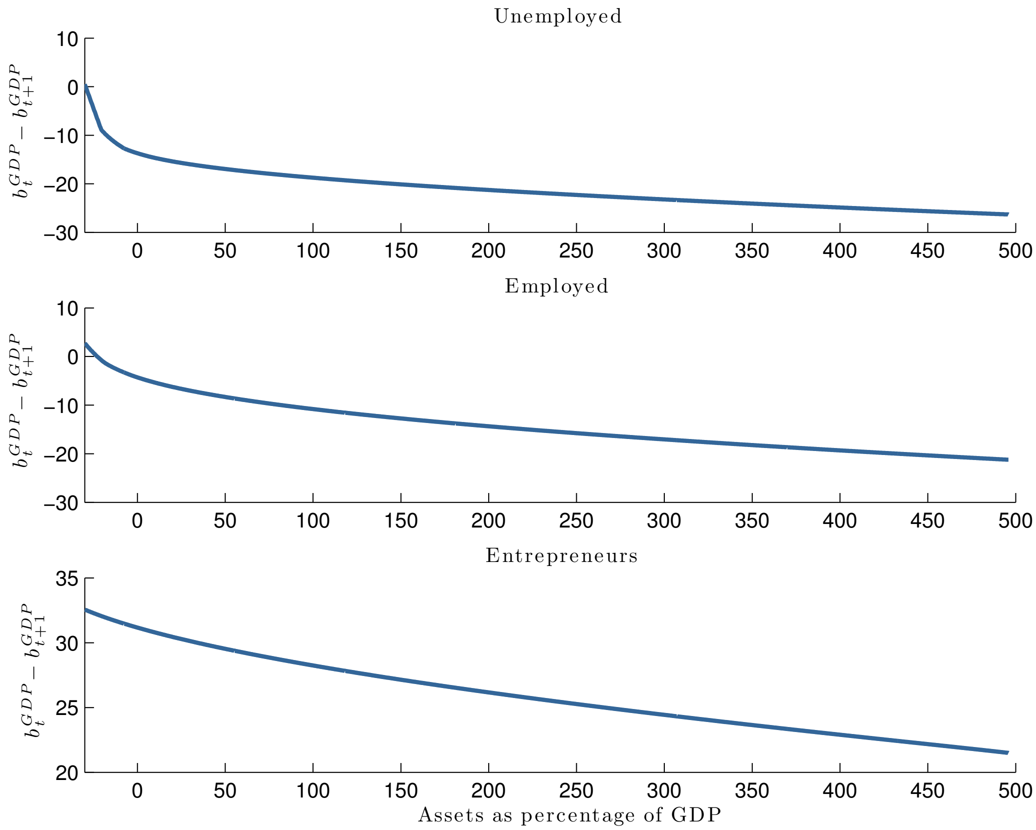


Figure 2: Policy Function for Asset Accumulation.
The figure shows the difference between current and future debt as a percentage of per capita GDP , $b_t^{GDP} - b_{t+1}^{GDP}$, as a function of current assets as a ratio of per capita GDP. All three employment statuses are represented: unemployed (first panel), employed (second panel) and entrepreneurs (third panel). Positive values are associated with asset accumulation, negative values with asset decumulation.

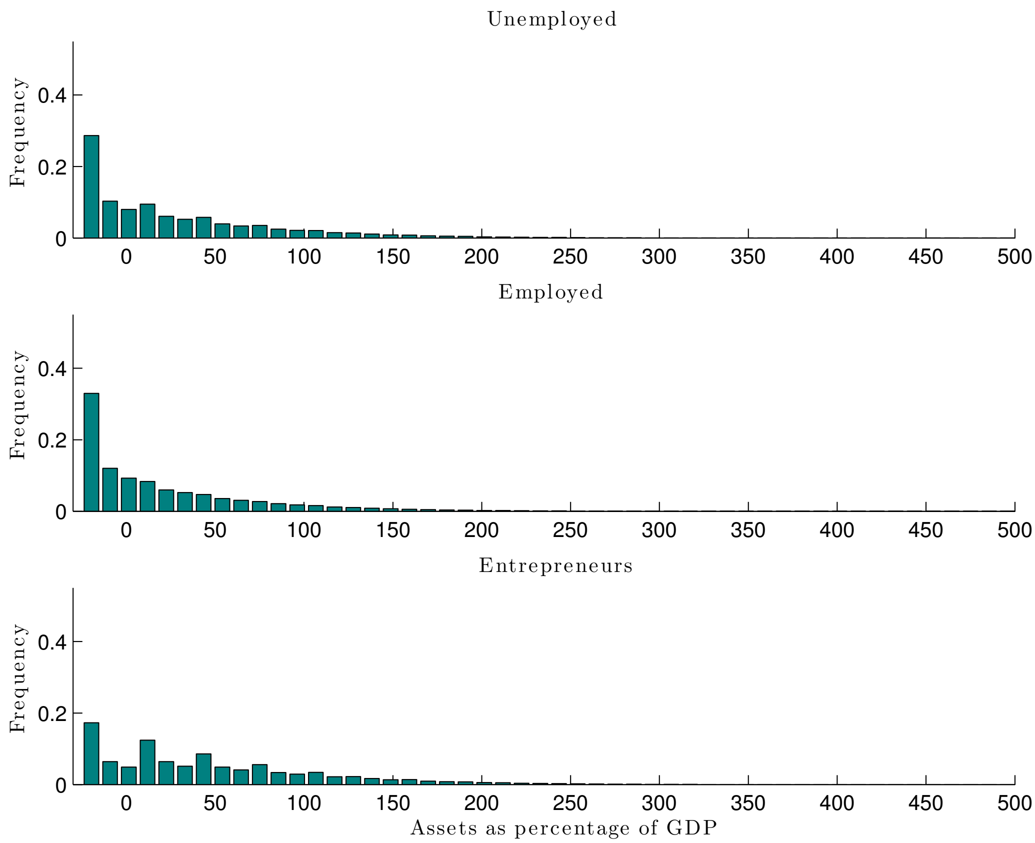


Figure 3: Wealth Distribution.
The figure shows the distribution of assets as a percentage of per capita GDP in the initial steady state by employment status: unemployed (first panel), employed (second panel) and entrepreneurs (third panel).

5 Quantitive Exercise

The goal of this paper is to understand what is the optimal way to implement an austerity plan, along two dimensions: timing of the debt reduction and fiscal instruments to adopt.

To address this question, I consider a scenario under which the fiscal authority is forced to bring down its debt from a high level, B_H^G, to a low one, B_L^G in a determined time span, T. Such a debt reduction episode does not need to be optimal. The reasons behind the implementation of the austerity plan are not object of discussion of this paper and are left unmodeled. Nevertheless, we can think of many recent cases where countries have been forced to implement debt reduction plans. For example, this path could be imposed by the existence of a supranational authority overseeing domestic fiscal policy or, implicitly, by international investors in financial markets.

Note that an infinite number of paths are available to the fiscal authority to converge to the new steady state. I restrict my analysis to a class of monotonic decreasing deleveraging paths. This seems consistent with casual empirical evidence: as southern European economies implemented austerity measures in response to the recent sovereign debt crisis, the proposed plans for public borrowing generally implied a monotonically decreasing path for public debt.

I model the path of public debt deleveraging as:

$$B_t^G = B_H^G + (B_L^G - B_H^G)\left(\frac{t}{T}\right)^{\iota}$$

where the parameter ι governs the concavity or convexity of the public debt reduction plan. I restrict ι to values in the positive subset of $\mathbb{R}$. As Figure 4 shows, ι is also a measure of the speed of public deleveraging. I define a path where $\iota < 1$ as a fast debt reduction episode. On the other hand, I define a path where $\iota > 1$ as a slow debt reduction episode. I refer to a deleveraging path where ι is around unity as a smooth one. Mapping the speed of debt reduction into a single parameter, ι, helps to clarify the analysis of the results.

As already described in Section 3, I set B_H^G equal to 0.6452 in order to match an initial public debt as a percentage of GDP of 20%. I reduce debt to B_L^G equalling .5809, implying a value for annual public debt over GDP of 18%. The 2 percentage point reduction may seem small. However, given my desire to study extremely slow and fast debt reduction paths, considering a larger debt deleveraging episode, even if realistic, would not allow a complete analysis of the results. For some values of ι, in fact, it would not to be possible for the economy to converge to an equilibrium. Deleveraging from high to low debt takes place in one year ($T = 4$). This choice mimics yearly implementation of fiscal plans.

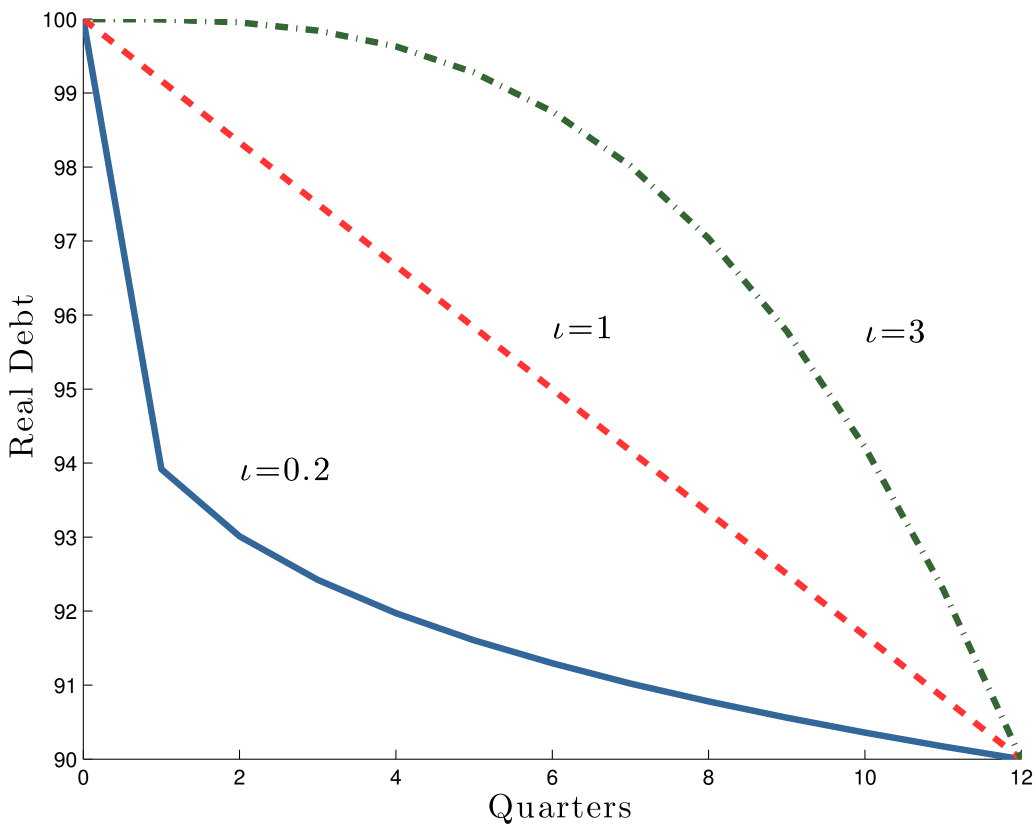


Figure 4: Deleveraging Paths According to Different Values of ι
A $\iota < 1$ (blue continuous line) represents a convex (fast) deleveraging, a $\iota = 1$ (red dotted line) stands for a smooth deleveraging, while a $\iota > 1$ (green line) represents a concave (slow) deleveraging.

I define the initial (final) steady state as an equilibrium of the economy where all the aggregate variables are constant, the agents are subject to idiosyncratic uncertainty, the assets and employment statuses distribution is the ergodic one and public debt is constant at B_H^G (B_L^G). At time $t = 1$ the economy begins the transition from the initial steady state to the final one. The agents become aware at time $t = 1$ of the path of public debt described above. From $t = 1$ onwards, they have perfect foresight again of the path of the aggregate variables.

I restrict the analysis to public debt deleveraging episodes that are brought about via one of the following fiscal instruments: income taxation or public expenditure. I choose to study only these two instruments due to their opposite impact on the real interest rate. I will show below how this variable plays a key role in determining many of the aggregate results. At the aggregate level, considering other types of taxation, such as consumption or wealth taxation, is unlikely to affect my conclusions significantly. At the individual level, it is possible that the introduction of different types of taxation may modify some of the agents' preferences. However, I choose to focus on a smaller set of fiscal instruments in order to simplify the analysis of the model.

I now turn to define the welfare measure I consider in the rest of the analysis. Let the initial steady state value function of agent i, with debt $b_{i,t}$ and employment status $s_{i,t}$, be:

$$\bar{J}(b_{i,0}, s_{i,0}) = E_0 \sum_{t=0}^{\infty} \beta^t \left(u(c_{i,t}(b_{i,t}, s_{i,t}), G_0) - \nu(l_{i,t}(b_{i,t}, s_{i,t})) \right)$$

where I define $c_{i,t}(b_{i,t}, s_{i,t})$ and $l_{i,t}(b_{i,t}, s_{i,t})$ as the optimal consumption and labor decisions of agent i at time t. I denote by G_0 public expenditure at the initial steady state. It is understood that the path of the aggregate variables is in the information set of the agents and is summarized, where relevant, by the time subscript t.

Let agent i's value function at the beginning of the transition be:

$$J(b_{i,0}, s_{i,0}, \iota, \zeta) = E_0 \sum_{t=0}^{\infty} \beta^t \left(u(c_{i,t}(b_{i,t}, s_{i,t}, \iota, \zeta), G_t) - \nu(l_{i,t}(b_{i,t}, s_{i,t}, \iota, \zeta)) \right)$$

where ζ is an indicator function taking value one if public debt reduction is achieved by a cut in public expenditure and zero otherwise. To evaluate this value function, the agents take into account the future path of the aggregate variables along the transition of debt towards its new value, B_L^G. The variables ζ and ι are sufficient statistics to characterize the path of all the aggregate variables, given B_L^G and B_H^G. I can now define the consumption equivalent as the permanent constant amount of consumption agent i would like to receive in the initial steady state to avoid the fluctuating path of consumption and labor associated with the transition, namely:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(u(c_{i,t}(b_{i,t}, s_{i,t}, X) + \xi_i \cdot c_{i,0}(b_{i,0}, s_{i,0}, X), G_t) - \nu(l_{i,t}(b_{i,t}, s_{i,t}, X)\right) = J(b_{i,0}, s_{i,0}, X, \iota, \zeta)$$

Note that, given my definition, a positive ξ_i stands for a welfare benefit associated with the transition, while a negative ξ_i stands for a welfare cost. I summarize the welfare of individual agents by their consumption equivalent, ξ_i.

6 Flexible Price Benchmark

I consider an economy where the central bank is always able to set the inflation rate at its target, $\bar{\Pi}$, even if this violates the non-negativity constraint for the nominal interest rate. This coincides with a model with flexible prices and it is the benchmark for my analysis.

6.1 Aggregate Variables in a Deleveraging Episode

In this subsection, I analyze how the aggregate variables react to a public debt reduction achieved by using different fiscal instruments, for different values of ι. As explained in Section 5, I assume that the fiscal authority can use, in turn, public expenditure or income taxation to deleverage.

In order to make the two exercises comparable, the initial and final steady states must be the same independently of the instrument used by the fiscal authority during the transition. I assume, then, that when the fiscal authority deleverages by increasing income taxation, public expenditure increases by a constant fraction for the first four quarters. Hence, in both exercises, at $t = 5$ the tax rate and public expenditure are at the final steady state level. This allows me to compare the two transitions. I show, later in the paper, that this assumption does not affect the results significantly in term of welfare.

Figure 5 shows the impulse response functions for the different exercises I consider.

Let us now analyze the effects of a public debt reduction achieved by cutting public expenditure, as shown in the first column of Figure 5. When most of the reduction in G occurs, the real interest rate decreases. Indeed, under this circumstance, the fiscal authority is creating a negative demand shock in the economy. Aggregate private consumption, instead, increases for two reasons: on the one hand, consumers are paid back part of the principal of the public debt; on the other hand, due to the assumption of imperfect substitutability, private consumption partly replaces public expenditure. As a consequence, output falls by less than G. Moreover, the decrease in the real interest rate has a feedback effect on public expenditure. The fiscal authority, indeed, being a borrower, has to finance lower interest payments. Hence, the response of the real interest rate and private consumption helps the deleveraging process.

The above description refers to a fast debt deleveraging where almost all of the debt reduction occurs in the first quarter (Figure 5, $\iota = 0.3$). If most of the debt reduction occurs instead in the future, the agents' expectations at $t = 1$ will adjust (Figure 5, $\iota = 4$). Consumers are aware that their consumption will increase in the future. They are therefore willing to borrow in the present to smooth consumption. Their borrowing exerts an upward pressure on the current real interest rates. The fiscal authority, facing a higher real interest rate, is forced to cut public expenditure even in the first quarter. The first-period increase in private consumption offsets the drop in public expenditure, causing an increase in output. Then, if the drop in public expenditure occurs in the future, the economy experiences a public expenditure contraction and a slight output boom in the present.

Let us now assume that the fiscal authority decides to reduce public debt by increasing income taxation. Under this circumstance, when most of the debt reduction occurs, the economy is hit by a supply shock. The employed agents, facing higher taxation, supply less labor. The firms, in response, cut production. The real interest rate will then increase, raising the interest payment for the fiscal authority. At the same time, due to the taxation increase and to the contraction in output, private consumption falls. Hence, when taxation increases the economy faces a recession in output as well as in private consumption.

Once again, the previous description fits well a public deleveraging that occurs mostly in the first quarter (Figure 5, $\iota = 0.2$). If, instead, the public deleveraging is expected to occur in the future (Figure 5, $\iota = 3$), the agents' expectations adjust accordingly. Consumers, aware of the consumption drop at $t = 4$, are willing to save at $t = 3$. As a consequence, in the third quarter the real interest rate falls, allowing the fiscal authority to decrease the tax rate. The workers supply more labor, expanding current output. Therefore, the economy faces an increase in output in $t = 3$ and a strong recession in

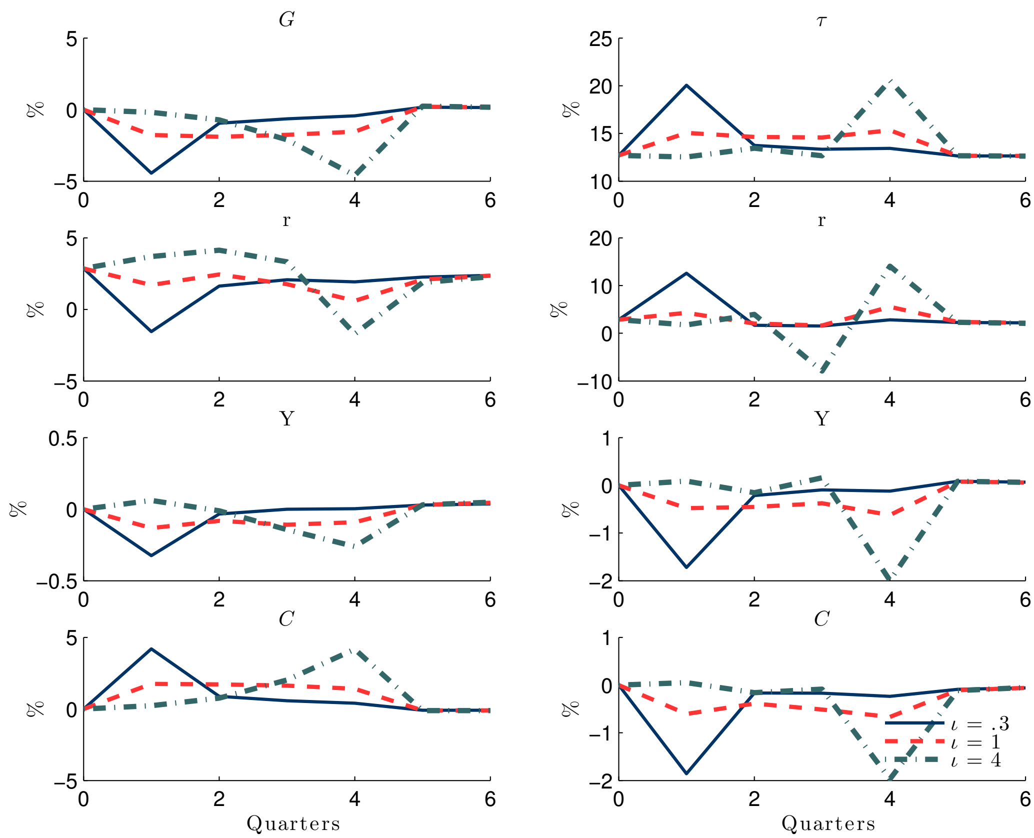


Figure 5: Impulse Response Functions under Different Deleveraging Speeds Impulse Response Functions for public expenditure, G, income taxation, τ, real interest rate, r, output, Y, and private consumption, C, according to different values for ι. $\iota = .3$ represents a fast debt reduction, $\iota = 1$ represents a smooth debt reduction and $\iota = 3$ represents a slow debt reduction. In the first column, the public debt reduction is financed by cutting public expenditure. In the second column, the public debt reduction is financed by increasing income taxation. All the variables, except for r and τ, are in log-deviations from the final steady state values. The values for r and τ are given in percentages.

$t = 4$.

To summarize, by using public expenditure, it is possible to reduce the real interest rate and prevent the output from falling much. Instead, by increasing income taxation the real interest rate rises and depresses the output significantly.

6.2 Aggregate Preferences

Egalitarian Aggregation and Voting Results

I analyze the aggregate preferences according to two different metrics: the aggregate consumption equivalent, $\xi \equiv \int_0^1 \xi_i di$, and the outcome of a voting process. The former solves the problem of an egalitarian fiscal authority which maximizes aggregate welfare behind the veil of ignorance, where the weights are given by the probability distribution of the agents. The latter corresponds, instead, to a decentralized voting process where the agents choose between two alternatives: a reduction in public expenditure or an increase in income taxation.

The results are plotted in Figure 6. The first panel shows the aggregate ξ under the public expenditure experiment and under income taxation, according to different speeds of debt reduction, $\xi_G(\iota)$ and $\xi_\tau(\iota)$, respectively. The second panel, instead, shows the percentage of agents who vote in favor of a public debt deleveraging achieved by reducing public expenditure.

I do not want to focus my analysis either on the magnitude of the welfare costs or on the sign, since in this class of models public debt, acting as insurance provider for the agents, is welfare-enhancing. My analysis focuses on the comparison between different instruments and different speeds of deleveraging. The differences in welfare, then, only depend on the transition path and they are independent of the final steady state.

Let us now analyze the results. First, the two criteria give similar answers: public expenditure cuts are preferred for almost all debt reduction speeds. This occurs since the real interest rate is relatively low during the transition, making the public debt deleveraging less costly. In addition, income taxation increases distortions in the economy, lowering the agents' welfare.

Second, in very slow debt reduction episodes, the income taxation experiment is preferred to the public expenditure one when measuring aggregate welfare by ξ. Under the voting criterion, the public expenditure option always beats the income taxation one. Nevertheless, the percentage of agents in favor of public expenditure drops considerably for extremely slow debt

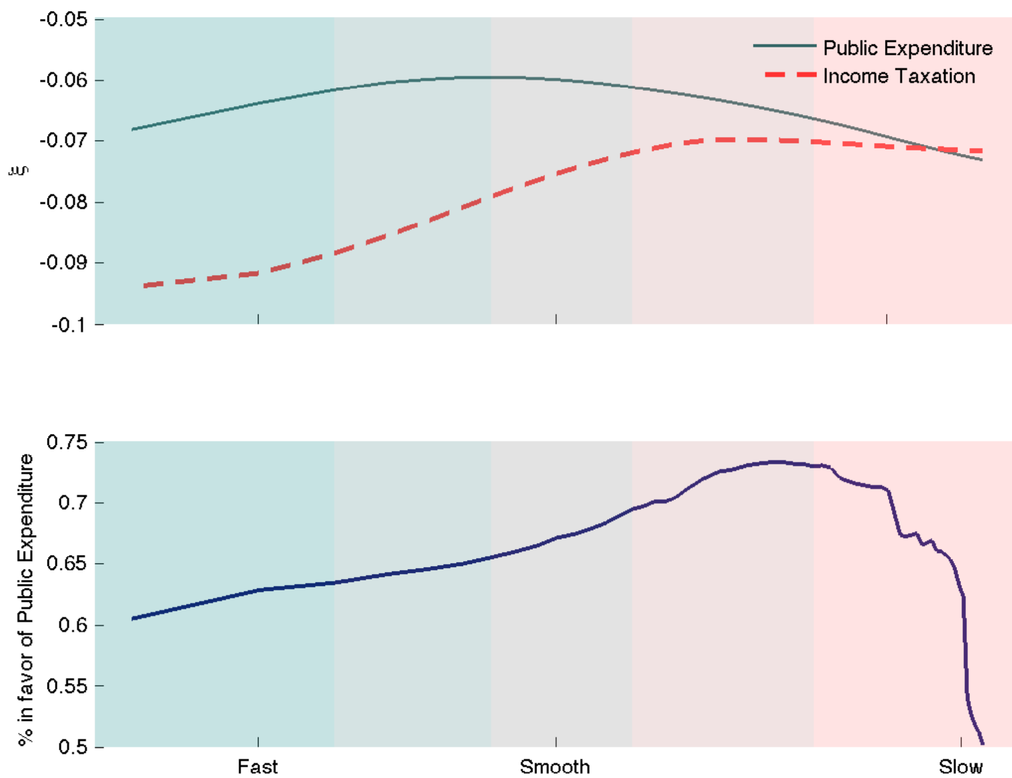


Figure 6: Aggregate Preferences
The first panel shows the aggregate consumption equivalent, ξ, as a function of public debt deleveraging speed, ι, both under the public expenditure experiment and under the income taxation one. The second panel shows the percentage of agents who vote in favor of the public expenditure option as a function of the public debt deleveraging speed.

reductions. Under high values of ι, the benefits of having low interest rates during the debt reduction will fall. Indeed, as shown in Figure 5, in the first three quarters the economy experiences high real interest rates and low public expenditure. These two factors together depress the welfare of the less wealthy agents, thus depressing aggregate welfare.

Finally, for low values of ι, even if the difference between $\xi_G(\iota)$ and $\xi_\tau(\iota)$ is large, the difference in voting is not. This means that fast debt reductions achieved by income taxation are very costly for some agents. The class disliking the most this option is that of the employed borrowers, due to the combination of high real interest rates and high taxes.

Voting among Four Alternatives

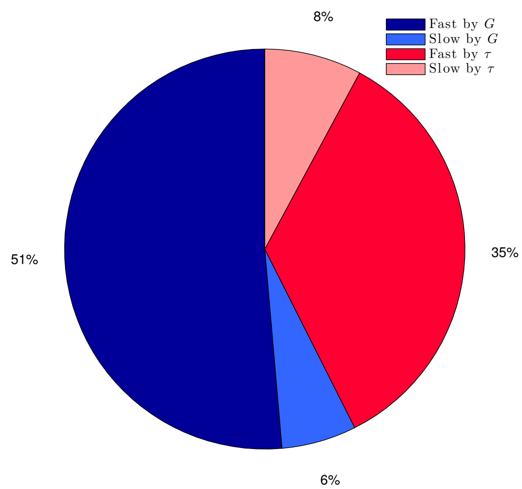
I will now describe a voting experiment. The Agents can choose among four options: a fast debt reduction achieved by income taxation ($\iota = .3$), a fast debt reduction achieved by public expenditure ($\iota = .3$), a slow debt reduction achieved by income taxation ($\iota = 4$) and a slow debt reduction achieved by public expenditure ($\iota = 4$).[16]

Figure 7.a shows the results of the vote: The winning option is fast debt reduction achieved by public expenditure. This occurs since it is the preferred option of the majority of the employed agents, who represent, in turn, the majority of the population in this economy. The second most voted option is the fast debt reduction achieved by income taxation. This is voted for by all the wealthier agents whose major source of income is financial wealth.

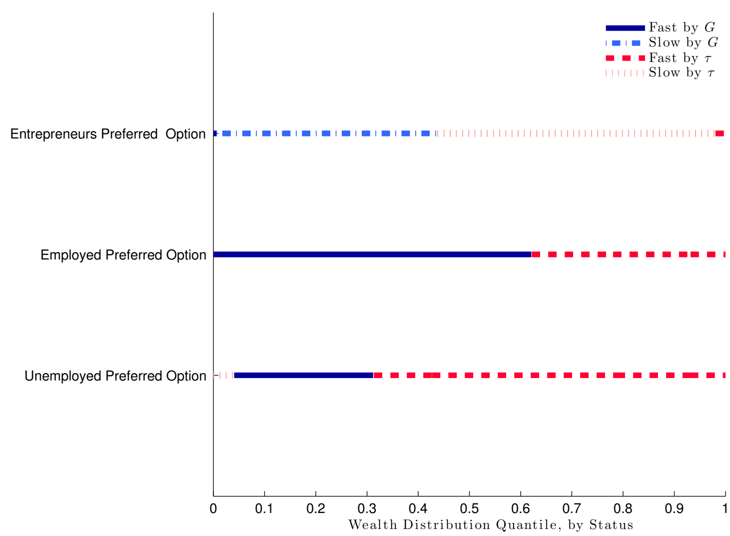
The agents' preferences, disaggregated by employment status and position in the wealth distribution within the employment statuses, are described in Figure 7.b. Notice that a general pattern emerges: agents below the median of the wealth distribution prefer a debt reduction achieved by public expenditure; agents above the median prefer, instead, a debt reduction achieved by income taxation.

The poorest unemployed prefer a slow debt reduction achieved by income taxation. These agents dislike low public expenditure and, being borrowers, high real interest rates. Hence, they do not want the fiscal authority to decrease public expenditure and they do not vote for fast debt reduction with income taxation, as this would increase real interest rates. The wealthiest, instead, prefer a fast debt reduction achieved by income taxation, since they mostly rely on financial wealth. Under this option, they can then enjoy a

[16]Figure 5 shows the impulse response functions for these options.



(a) Voting Outcome



(b) Agents' Preferences by Wealth Distribution Quintile

Figure 7: Voting Outcomes among Four Alternatives

high real interest rate without facing an increase in fiscal pressure. The unemployed agents between the 10th and 30th wealth percentile vote for a fast debt reduction under public expenditure. This occurs since they are borrowers and prefer to face low real interest rates.

The employed agents, if wealthy, vote for a fast debt reduction achieved by income taxation; if not, they prefer a fast debt reduction obtained by public expenditure. The wealthy employed, holding a large quantity of assets, prefer to face high real interest rates when their savings are high, namely, in the first quarter. Employed agents who are not very wealthy rely on labor income: they enjoy a low interest rate and a low distortion of the labor supply.

The entrepreneurs' preferences are, again, significantly affected by wealth. The less wealthy agents vote for a slow debt reduction achieved by public expenditure. Indeed, relying on profit income, they dislike high taxation and, being borrowers, they dislike high interest rates. The wealthier vote for a slow debt reduction via income taxation. Their motivations are very different from those of the poor unemployed. Under this option, as described in subsection 6.1 and shown in Figure 5, the movements in the real interest rate and profit income are negatively correlated, thus providing them with a favorably smooth path of income. The very wealthy among the entrepreneurs vote for a fast debt reduction achieved by an increase in the tax rate. Relying mostly on financial income, they like the real interest rate to be high in the first quarter.

Preferences Regarding Different Speeds of Deleveraging

I will now analyze whether the speed of debt reduction is an important variable in determing the agents' preferences. Figure 6 shows that this variable matters under the egalitarian welfare metric. An egalitarian fiscal authority prefers, indeed, a fast debt reduction, if it is constrained in using public expenditure, and a slow debt reduction, if it is constrained in using income taxation.

Let us now study if, under a voting experiment, the same result arises. I fix a fiscal instrument and I let the agents vote, in turn, between two alternatives: fast or slow public debt deleveraging.[17] Figure 8 shows the results of this vote. Not surprisingly,[18] under the public expenditure option, the

[17] I check that add a smooth debt reduction ($\iota = 1$) among the options does not change much the results. Hence for the sake of clarity I drop this option.

[18] A slow debt reduction achieved by public expenditure was preferred by more than 50% of the agents in the previous experiment, which included a larger set of options. Therefore,

majority of agents vote for a fast debt reduction. Under the income taxation option, the agents vote for a slow debt reduction, changing considerably the result of the voting outcome in the previous paragraph.[19] Indeed, most of the the agents who voted for a slow public debt deleveraging under public expenditure, will vote for a fast public debt deleveraging under income taxation. These two processes have something in common. They make it possible to have a low interest rate when public debt is high and a high interest rate when public debt is low, easing the costs of public deleveraging.

To conclude, both for the egalitarian fiscal authority and for the voting mechanism, it is optimal to perform a fast public debt deleveraging if it is achieved by public expenditure and a slow public debt deleveraging if achieved by income taxation. Therefore, it is possible to state that the public debt deleveraging speed plays an important role in determining the agents' preferences.

6.3 Disaggregated Preferences

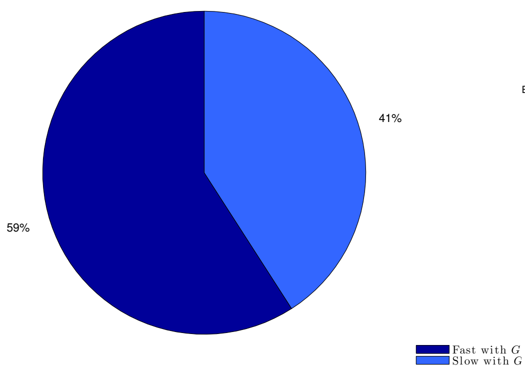
I now turn to study the disaggregated preferences of the agents. Figures 9, 10 and 11 show the aggregate consumption equivalent by quartile and by employment status, as a function of the public debt deleveraging speed.

Consider a debt reduction achieved by a cut in public expenditure. The position in the wealth distribution is the variable affecting the agents' preferences the most . The more an agent is wealthy the slower is his preferred public deleveraging path. Borrowers want a low interest rate when debt is high while savers prefer exactly the opposite. Moreover, the least wealthy unemployed agents face a trade-off: being unemployed, they want to prevent public expenditure from falling much; as borrowers, they want the real interest rate to fall the most. They choose a fast debt reduction, experiencing a substantial public expenditure reduction in order to gain a low interest rate.

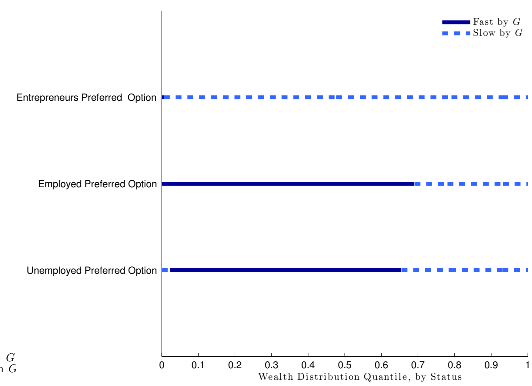
Consider a debt reduction achieved by an increase in income taxation. Under this circumstance, their employment status plays an important role in agents' preferences. The unemployed, if not close to the borrowing constraint, gain more utility in a fast debt reduction. Mostly relying on financial wealth, they prefer to have a high real interest rate in the first quarter. This is also optimal because they do not face any distortionary taxation. The employed,

once I restrict the set of possible choices, the same result needs to arise.

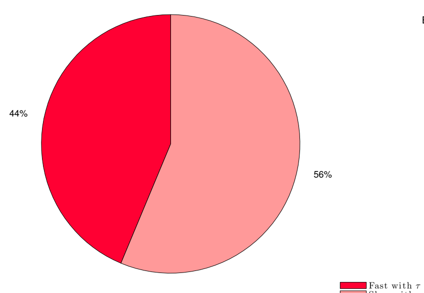
[19]The agents' second-best option was a fast debt reduction achieved by income taxation.



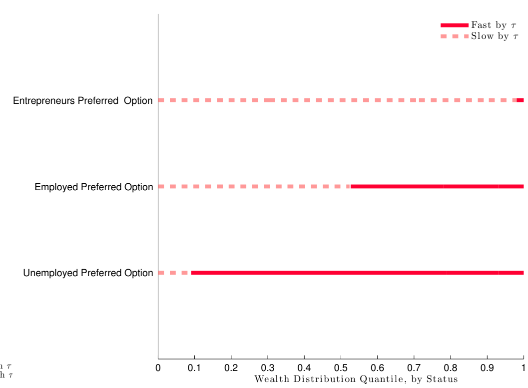
(a) Voting Outcome



(b) Agents' Preferences by Wealth Distribution Quintile



(c) Voting Outcome



(d) Agents' Preferences by Wealth Distribution Quintile

Figure 8: Voting Outcomes for Deleveraging Speed

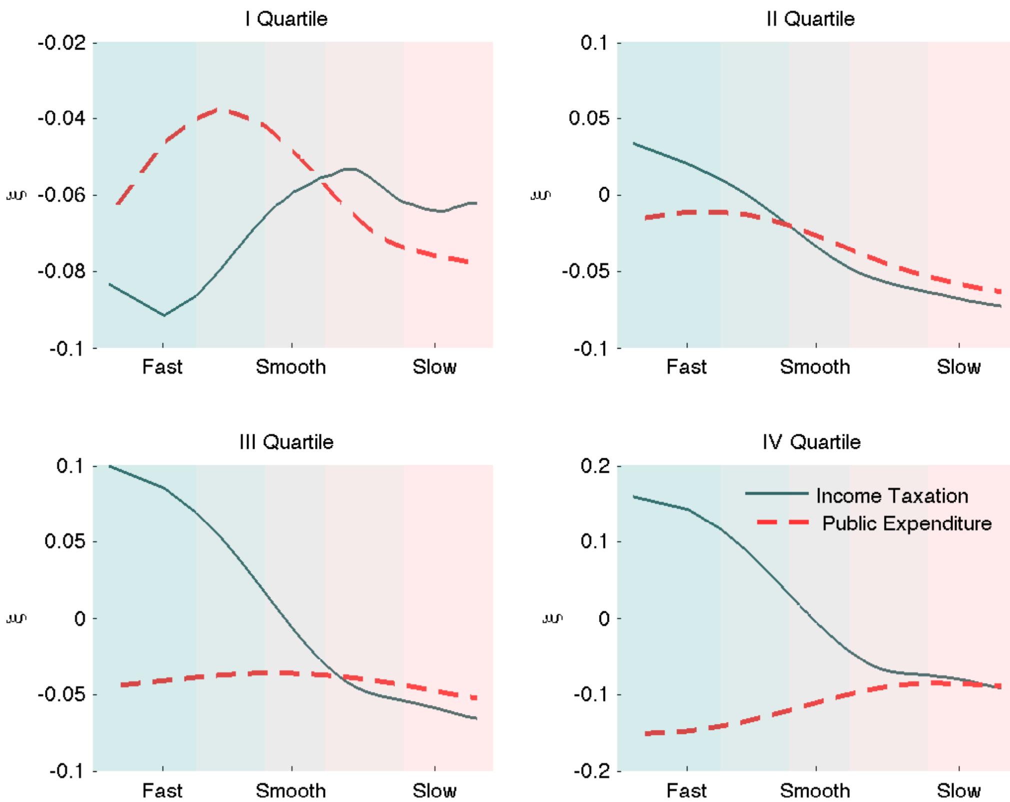


Figure 9: Unemployed Consumption Equivalent
Mean of the average consumption by quartile as a function of the speed of debt reduction for the unemployed. The dark green continuous line represents the income taxation experiment, the red dotted line the public expenditure experiment.

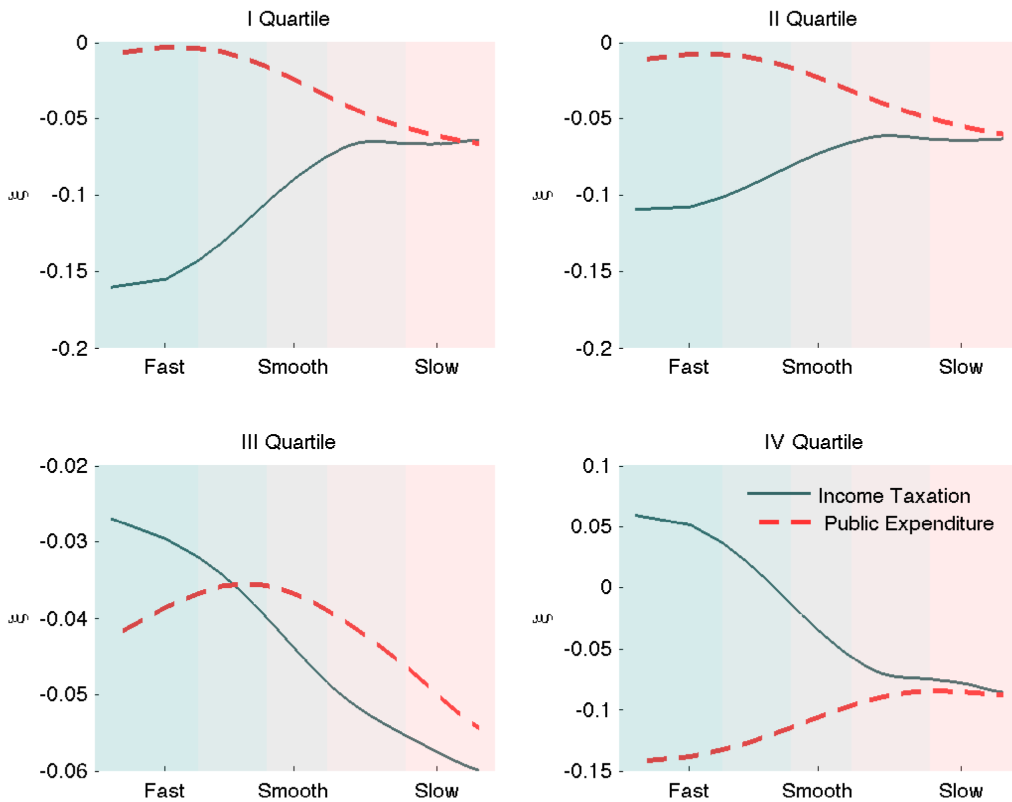


Figure 10: Employed Consumption Equivalent
Mean of the average consumption by quartile as a function of the speed of debt reduction for the employed.The dark green continuous line represents the income taxation experiment, the red dotted line the public expenditure experiment.

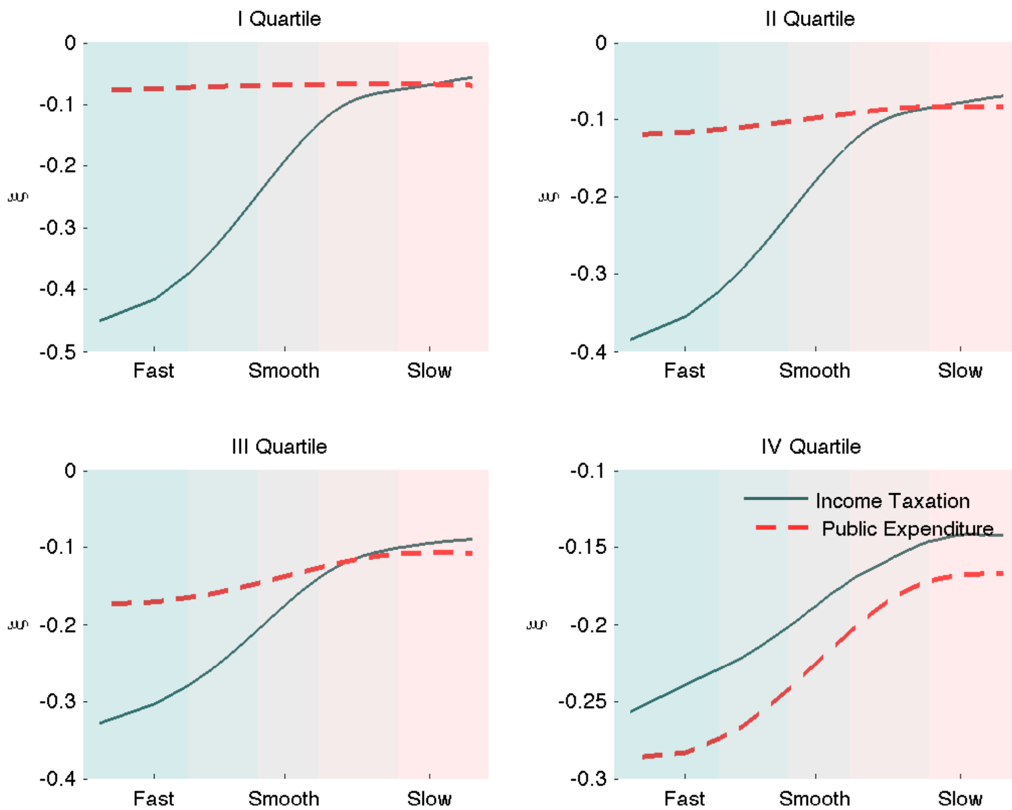


Figure 11: Entrepreneurs Consumption Equivalent
Mean of the average consumption by quartile as a function of the speed of debt reduction for the entrepreneurs. The dark green continuous line represents the income taxation experiment, the red dotted line the public expenditure experiment.

if below the median of wealth distribution, prefer a slow debt reduction; otherwise, a fast one. Relying on labor income, the former want to minimize the distortion due to taxation. Relying mostly on financial income, the latter want to maximize the pressure on the real interest rate. The entrepreneurs are better off under an extremely slow debt reduction. As Figure 5 shows, when a debt reduction occurs slowly, the economy faces a cycle of high taxes under high real interest rates and low taxes under low real interest rates. The entrepreneurs, being savers, gain from high real interest rates and, being tax-payers, gain from low taxation. Thus, they get more utility by having an extremely slow debt reduction that exacerbates this process. Note that the reason why they prefer a slow debt deleveraging is different from the one for the employed. The latter minimize tax distortion, while the former maximize the taxes-real interest rate cycle.

To conclude, if the central bank is always able to targeting the inflation, public debt reduction needs to be achieved by public expenditure and needs to be fast. If it is not possible to cut public expenditure, the fiscal authority must slowly reduce public debt using income taxation. These results are robust to different welfare metrics. Nevertheless, note that in some equilibria, the nominal interest rate violates the non-negativity constraint implied by equation (14). I will now turn to analyze how the presence of the zero lower bound affects the results.

7 Nominal Price Rigidities and the Zero Lower Bound

In this section, I assume that the central bank needs to satisfy the non-negativity constraint on the nominal interest rate, namely equation (14). Note that if the constraint does not bind, the central bank targets the inflation rate and the results are those described in Section 6. If, instead, the constraint binds, some of the equilibria described in Section 6 are not feasible. Then, the central bank sets the nominal interest rate equal to zero, leaving the inflation rate free to float.

Under this circumstance, the economy is subject to two different effects: one at the aggregate and one at the individual level. At the aggregate level, if the zero lower bound binds, output falls and inflation is below the target.

This occurs because of a known mechanism in the literature, described by Eggertsson and Woodford (2003) among others.[20] Let us suppose that the equilibrium nominal interest rate violates the non-negativity constraint only at time t. The real interest rate is determined by the Fisher equation:

$$(1+r_t) = \frac{(1+i_t)}{(1+\Pi_{t+1})}$$

where Π_{t+1} is the inflation rate at time $t+1$, set by the central bank at the target $\bar{\Pi}$. I define r_t^B as the real interest rate under the benchmark case and r_t^{ZLB} the real interest rate under the zero lower bound. Then:

$$(1+r_t^B) = \frac{(1+i_t)}{(1+\bar{\Pi})} \leq \frac{1}{(1+\bar{\Pi})} = (1+r_t^{ZLB})$$

Consequently, once the non-negativity constraint on the nominal interest rate is in place, the agents face a higher real interest rate. They choose, therefore, to save more, depressing the aggregate demand. The equilibrium price level decreases, pushing inflation below the target.

In this model, a second effect is present. At the individual level, the zero lower bound indirectly redistributes resources from the employed to the entrepreneurs or vice-versa. This mechanism is triggered by the contemporaneous presence of below-target inflation and Rotemberg costs. Rearranging equation (11) and the per-period profits, I obtain expressions for the average labor income as a share of GDP and profits as a share of GDP:

$$\frac{w_t n_t}{Y_t} = \left(\frac{\omega}{\mu} + \frac{\phi\omega}{\epsilon}(\tilde{\Pi}_t - 1)\tilde{\Pi}_t - \beta\frac{\lambda_{t+1}Y_{t+1}}{\lambda_t Y_t}\frac{\phi\omega}{\epsilon}(\tilde{\Pi}_{t+1} - 1)\tilde{\Pi}_{t+1} \right) \tag{19}$$

and

$$\frac{\varpi_t}{Y_t} = \left(1 - \frac{\omega}{\mu} - \frac{\phi\omega}{\epsilon}(\tilde{\Pi}_t - 1)\tilde{\Pi}_t - \frac{\phi}{2}(\tilde{\Pi}_t - 1)^2 + \beta\frac{\lambda_{t+1}Y_{t+1}}{\lambda_t Y_t}\frac{\phi\omega}{\epsilon}(\tilde{\Pi}_{t+1} - 1)\tilde{\Pi}_{t+1} \right) \tag{20}$$

where $\tilde{\Pi} \equiv \frac{\Pi_t}{\bar{\Pi}}$. The current below-target inflation, $\tilde{\Pi}_t \leq 1$, depresses the labor income share and, under certain conditions, it increases the profits

[20] Adam and Billi (2006),(2007), Eggertsson and Woodford (2004) and Werning (2011).

share. [21] Indeed, facing low prices, the firms cut their labor demand and depress real wages. Conversely, below-target future inflation, $\tilde{\Pi}_{t+1} \leq 1$, is positively correlated with the labor income share and it decreases the profits share. The firms, aware that prices will decrease in the future, want to increase current production. They demand more labor, exerting an upward pressure on the real wage. The net redistributitive effect of current and future inflation on the labor income share is given by:

$$h_t \equiv (\tilde{\Pi}_t - 1)\tilde{\Pi}_t - \beta \frac{\lambda_{t+1} Y_{t+1}}{\lambda_t Y_t} (\tilde{\Pi}_{t+1} - 1)\tilde{\Pi}_{t+1}$$

If, indeed, $h_t \geq 0$ ($h_t < 0$), resources will be redistributed from the entrepreneurs (employed) to the employed (entrepreneurs).

Note that this income reallocation has a strong consequence. Most of the entrepreneurs in the model are financially-unconstrained agents, while some of the employed agents are financially-constrained. The inflation path can, then, move resources from the constrained to the unconstrained agents or vice-versa, making, in turn, the borrowing constraint more or less stringent. As a consequence, it is not trivial to state ex-ante whether the presence of the zero lower bound will have a negative or positive effect on the economy.

I now turn to analyze the aggregate variable dynamics during a public deleveraging episode, when considering the presence of the zero lower bound on the nominal interest rate. Then, I will study how the presence of the non-negativity constraint on the nominal interest rate affects the welfare and preferences of the agents.

7.1 Aggregate Variables under Zero Lower Bound

The zero lower bound may bind both when the fiscal authority reduces public debt by increasing income taxation and by cutting public expenditure. Let us analyze what occurs when public debt deleveraging is achieved by cutting public expenditure and the non-negativity constraint on the nominal interest rate may bind. First, the economy faces the zero lower bound only under an over-fast or an over-slow debt reduction. Indeed, in this experiment, the real interest rate is low only when most of the deleveraging shock occurs. Secondly, despite the fact that the non-negativity constraint may bind, the economy will never enter a prolonged liquidity trap. In effect, if the debt

[21] $(\frac{\omega}{\epsilon} + \frac{1}{2})\tilde{\Pi}_t > \frac{1}{2}$.

reduction is too fast, from the second quarter onwards it converges to the new steady state. If, instead, debt reduction is too slow, the agents' expectation will adjust accordingly. Since they know that output recession will be mild and private consumption will be high in the last quarter, they will be willing to borrow more in the first three quarters. This puts an upward pressure on the real interest rate during these quarters, preventing the economy from entering a prolonged liquidity trap. Therefore, the zero lower bound binds, if it does, either in the first or in the last quarter.

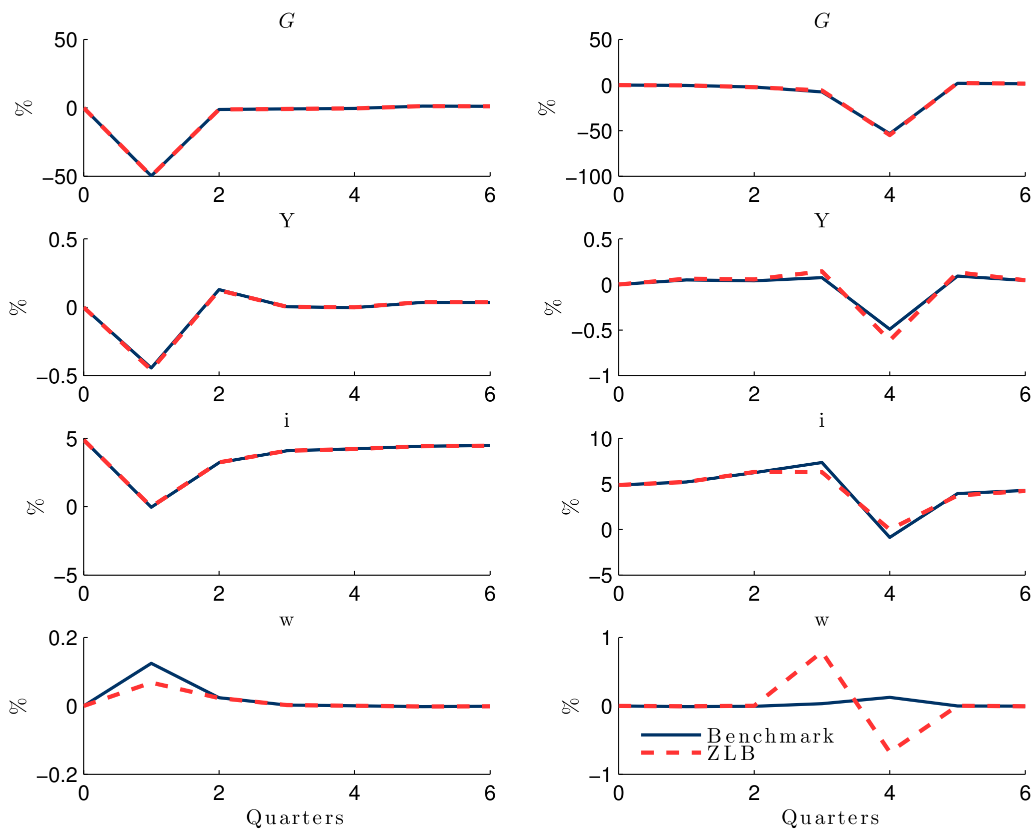


Figure 12: Impulse Response Functions
Impulse Response Functions for public expenditure, G, output, Y, real wage, w, and nominal interest rate under the benchmark exercise (continuous blue line) and the zero lower bound (red dashed line). I set $\iota = .1$ in the first column and $\iota = 10$ in the second column. All the variables except for i are in log-deviations from the final steady state values. The values for i are given in percentages.

Figure 12 shows the impulse response functions under $\iota = .1$ (first column) and $\iota = 10$ (second column). The presence of the zero lower bound does not significantly change the behavior of the aggregate variables for two reasons. First, under this deleveraging shock the nominal interest rate is slightly below zero. Second, since the non-negativity constraint binds only for one quarter, it does not have big effect on the economy. When the zero lower bound binds, the fiscal authority must pay a higher interest rate than under the flexible price experiment. As a consequence, public expenditure drops. Due to the imperfect substitutability, the fall in public expenditure induces an increase in private consumption, preventing the aggregate demand from falling. The output recession will be very mild. Hence, at the aggregate level there is almost no effect. Instead, at the individual level, resources are reallocated according to the speed of debt reduction. Under low values of ι, inflation is below the target in the first quarter. The labor income share decreases since firms hire fewer workers. If, on the other hand, the value of ι is very high, inflation is below the target at $t = 4$. At $t = 3$, the firms increase their labor demand, raising real wages and inducing an increase in the labor income share.

Let us now study the effect of public debt deleveraging when it is achieved by increasing income taxation. The zero lower bound binds only if the debt reduction is too slow (for high values of ι). Under this circumstance, indeed, the agents are aware that, in the last quarter, output will fall considerably. In order to smooth consumption, they want to save in the second-to-last quarter. This exerts a downward pressure on real interest rates, pushing the nominal interest rate below zero at $t = 3$. Moreover, once the zero lower bound binds in the third quarter, the economy may enter a prolonged liquidity trap. Indeed, the agents know that there will be an output recession even in the third quarter. As a reaction, they want to save at $t = 2$, exerting a downward pressure on the nominal interest rate in this quarter as well. So, it could be possible that the zero lower bound binds even in the second quarter, depressing the output again. The same recursive reasoning can be applied to the first quarter. Thus, depending on the magnitude of the output recession in the fourth quarter, the economy can enter a long or short liquidity trap. Remember that the slower the public debt reduction, the higher will be the magnitude of the output recession in the last quarter. Consequently, in the event of a very slow public debt reduction, the zero lower bound binds from the first quarter. This is analyzed in Figure 13, where I plot the impulse response functions under the benchmark case and under the zero lower bound when the value of $\iota = 5$. The firms, in response to the deflation, cut their production from the first quarter. Real wages fall from the first quarter while the firms' profits increase. The economy experiences a long and deep recession where resources are redistributed from the employed to the entrepreneurs.

If the debt reduction in the last quarter is not too slow, namely if the value of ι is not too high, the economy enters a liquidity trap from the second quarter. This is shown in Figure 14, where I set the value of ι to equal 3. The nominal interest rate is equal to zero and the inflation rate is below the target at $t = 2$ and $t = 3$. Aware of the path of the inflation rate, the firms increase their production in the first quarter. Then, at $t = 1$ real wages, w, increase and profits, ϖ, fall. Moreover, the firms demand more labor at $t = 2$, exerting upward pressure on the real wage, w, even in this quarter. Hence, if the debt reduction is not too slow, the output recession occurs only in the last quarter and the employed agents gain while the entrepreneurs lose in term of income.

The zero lower bound is more detrimental and influential under the income taxation case than under the public expenditure one. This mainly depends on the fact that in the former case it binds when private consumption is low, while in the latter it binds when private consumption is high. Anyhow, it has some consequences for the resource allocation in the economy in both experiments. Although it is clear who gains and loses in terms of income, it is not clear who are the winners and losers in terms of welfare. Therefore, I now turn to analyze how the introduction of the zero lower bound can affect the preferences of the agents.

7.2 Aggregate Preferences and the Zero Lower Bound

Egalitarian Aggregation and Voting Results

Let us now study if the presence of the zero lower bound can affect the agents' preferences regarding the speed of deleveraging and the fiscal instrument to use. The first panel in Figure 15 shows the aggregate consumption equivalent ξ, defined in subsection 6.2, under the public expenditure benchmark-experiment, under the income taxation benchmark-experiment, under the public expenditure and income taxation experiment in the presence of the zero lower bound. The second panel in Figure 15 shows the percentage of agents who vote in favor of public expenditure against income taxation for a given value of ι. The dark blue continuous line represents the voting outcome under the benchmark, while the light blue dashed line represents

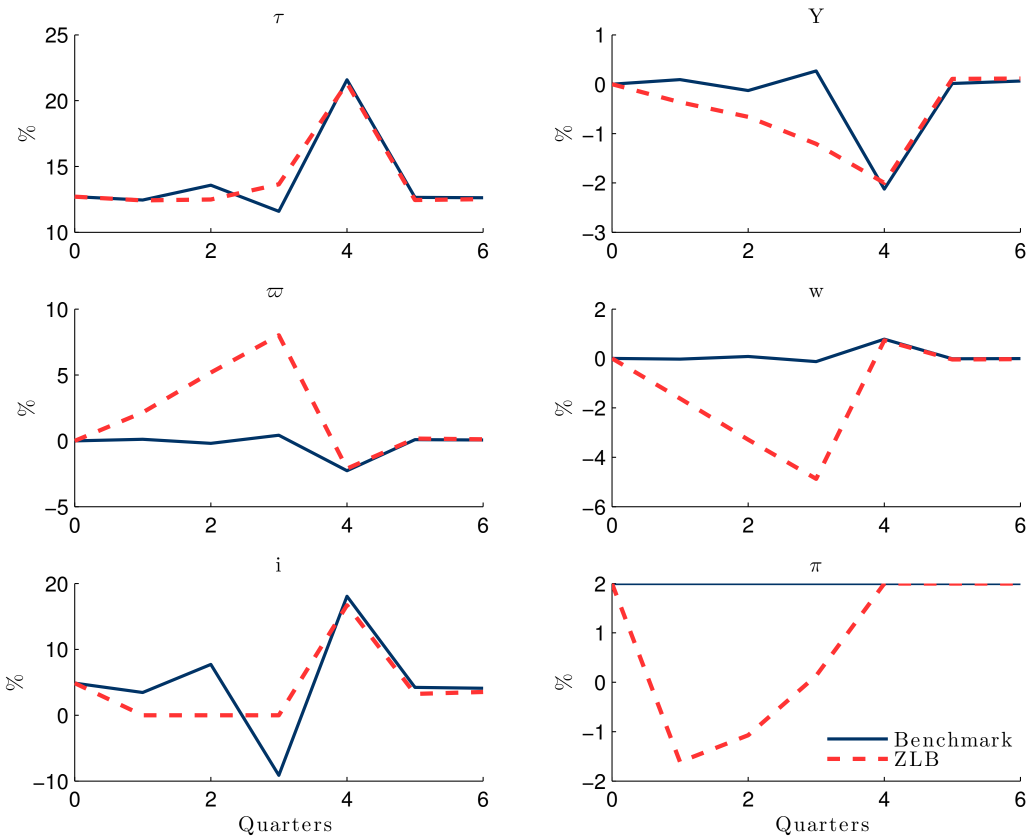


Figure 13: Impulse Response Functions
Impulse Response Functions for income taxation, τ, output, Y, profits, ϖ, real wage, W, nominal interest rate, i, and inflation rate, $\pi = \Pi - 1$ under the benchmark exercise (continuous blue line) and the zero lower bound (red dashed line). I set $\iota = 5$ to represent a fast debt reduction. All the variables except for τ, i and π are in log-deviations from the final steady state values. The values for τ, i and π are given in percentages.

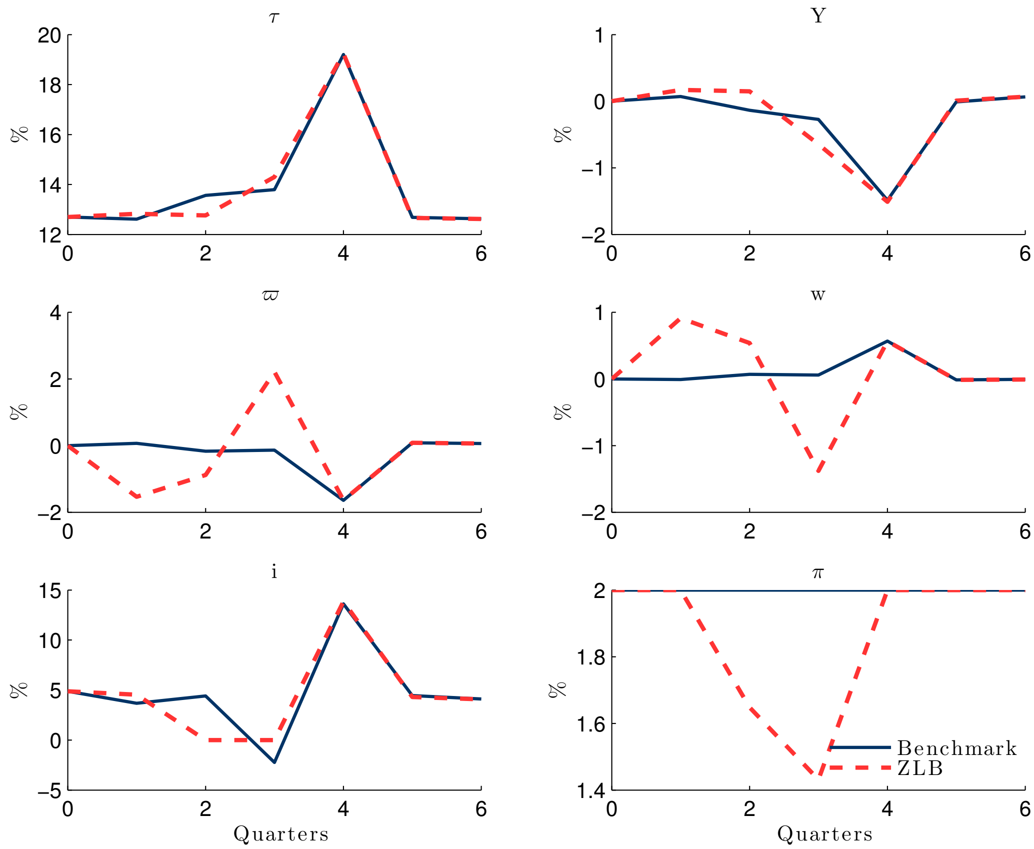


Figure 14: Impulse Response Functions
Impulse Response Functions for income taxation, τ, output, Y, profits, ϖ, real wage, w, nominal interest rate, i, and net inflation rate, $\pi = \Pi - 1$ under the benchmark exercise (continuous blue line) and the zero lower bound (red dashed line). I set $\iota = 3$ to represent a fast debt reduction. All the variables except for τ, i and π are in log-deviations from the final steady state values. The values for τ, i and π are given in percentages.

the voting outcome under the presence of the zero lower bound.

Once the non-negativity constraint on the nominal interest rate binds, the preferences of the agents change considerably under the income taxation experiment but not under the public expenditure one. From the aggregate consumption equivalent, it is possible to infer that the egalitarian social planner always finds it optimal to reduce public debt by cutting public expenditure. Remember that in the benchmark experiment, the social planner finds it optimal for high values of ι to use income taxation. Under the zero lower bound, instead, in the occurrence of an extremely slow debt reduction, the economy enter a long and detrimental liquidity trap that depresses the aggregate consumption equivalent. Moreover, under this circumstance resources are reallocated from the employed to the entrepreneurs. The former are agents close to the borrowing constraint while the latter are not. Thus, the zero lower bound redistributes resources from the financially-constrained agents to the financially-unconstrained ones. This, making the borrowing constraint more stringent, depresses aggregate welfare even more.

Note that there is a small range of values of ι under which the aggregate consumption equivalent is higher under the zero lower bound than in the benchmark exercise. This depends crucially on the redistributive effects of the zero lower bound. Indeed, if the economy enters a liquidity trap from the second quarter onward, income is reallocated from the entrepreneurs to the employed, namely from the financially-unconstrained agents to the financially-constrained ones. Then, receiving more resources, the agents who were close to the borrowing constraint consume more. The additional consumption and the possibility of relaxing the borrowing constraint increase the aggregate consumption equivalent. This result is new in the literature.

The voting outcome is shown in the second panel of Figure 15. Unlike in the benchmark exercise, there exists a small interval of values of ι under which the majority of agents vote for the income taxation option rather than for the public expenditure option. This is the same interval under which the share of output accruing to labor income increases. Then, if ι belongs to that interval, all the employed agents, who represent the majority of the population, vote for the income taxation option. If the value of ι is very high, namely if the public debt reduction is slow, the majority of agents will vote for the public expenditure option. Indeed, under these values the economy enters a prolonged and detrimental liquidity trap where resources are transferred from the employed to the entrepreneurs. Consequently, the employed, who represent the majority of the agents, vote for public expenditure.

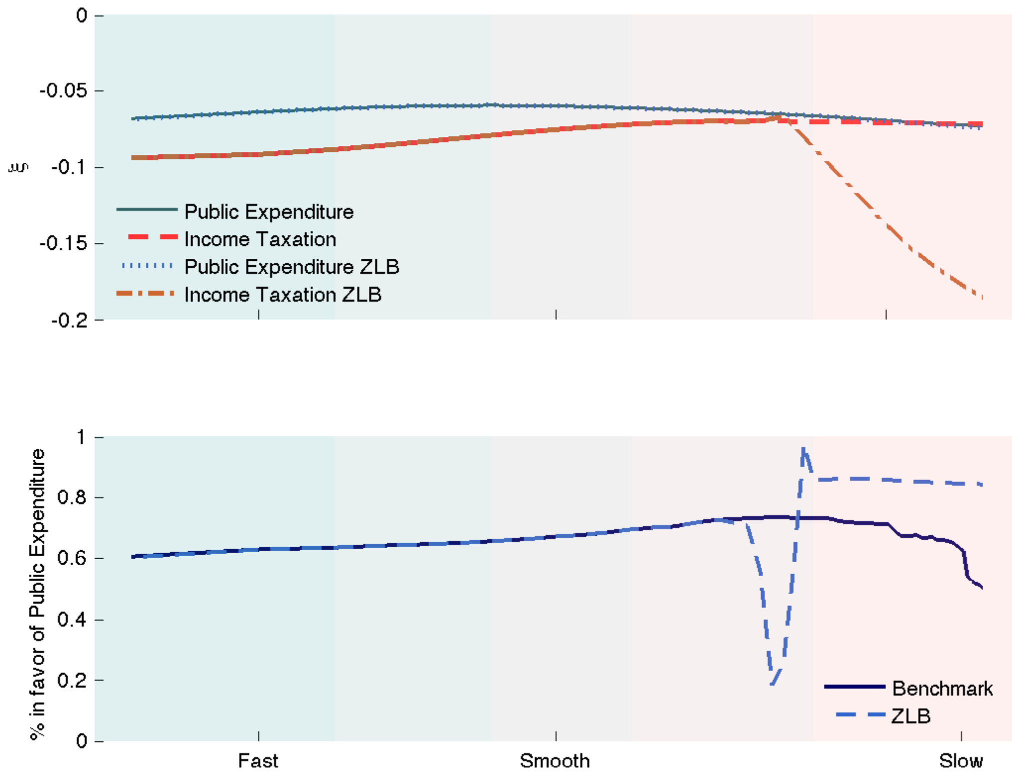


Figure 15: Aggregate Preferences
The first panel shows the aggregate consumption equivalent, ξ, as a function of the public debt deleveraging speed, ι, under the public expenditure-benchmark experiment, income taxation-benchmark experiment, under the public expenditure and the income taxation experiment in the presence of the zero lower bound. The second panel shows the percentage of agents who vote in favor of the public expenditure option as a function of the public debt deleveraging speed, in the benchmark (continuous dark blue line) and under the zero lower bound (dashed light blue line).

Moreover, both welfare metrics show a point of discontinuity. This is the value of ι that generates an inflation path under which h_1 becomes negative. Then, under this value, income is reallocated from the employed to the entrepreneurs, making the zero lower bound very costly for the economy.

To summarize, by introducing the zero lower bound, the consumption equivalent under the public expenditure experiment will always be higher than under the income taxation one, independently of the speed of public debt reduction. As a consequence, an egalitarian social planner finds it always optimal to reduce public debt using public expenditure instead of income taxation. On the contrary, if the agents can vote, there is a small interval of values of ι under which the income taxation option will be preferred to the public expenditure one. Thus, it is still not clear whether one instrument dominates the other.

Voting among Four Options

Let us now understand if the introduction of the zero lower bound affects the results of the four-options vote. I will show that the previous results about the first-best and second-best option remain unchanged.

The agents can vote, as before, among four options: a fast debt reduction achieved by income taxation ($\iota = .3$), a fast debt reduction achieved by public expenditure ($\iota = .3$), a slow debt reduction achieved by income taxation and a slow debt reduction achieved by public expenditure. To check the robustness of the results, I define the slow debt reduction using two values of ι, one in the interval within which the majority of agents prefers income taxation to public expenditure, $\iota = 3$, and one outside this interval, $\iota = 5$.[22]

Figure 16 shows the results. The unemployed and the employed, even when $\iota = 3$, continue to prefer a fast debt reduction to a slow one. Moreover, their preferences regarding the fiscal instrument do not change with respect to the benchmark experiment. The only difference is that, if ι is very large, the unemployed agents close to the borrowing constraint prefer a slow debt reduction achieved by public expenditure to one achieved by income taxation. Indeed, under the latter option, labor income drops dramatically, also depressing unemployment benefits. Hence, the preferences of the employed and unemployed will not change with respect to the benchmark exercise.

[22] I also check for values of ι under which the zero lower bound binds in the public expenditure experiment. Since the results do not change at all, I do not report these voting experiments.

On the contrary, the entrepreneurs change their preferences according to the value of ι: they prefer a slow debt reduction achieved by public expenditure if $\iota = 3$ and by income taxation if $\iota = 5$. If $\iota = 3$ and the fiscal authority chooses to deleverage by increasing income taxation, profits decrease. Therefore, the entrepreneurs prefer a slow debt reduction under public expenditure, since it keeps the real interest rate relatively high during the transition. If, instead, $\iota = 5$ and the fiscal authority chooses to deleverage by increasing income taxation, profits increase significantly as shown in Figure 13. Consequently, all the entrepreneurs will vote for a slow-debt reduction income-taxation option.

Note that the new preferences of entrepreneurs do not affect the first-best and second-best vote outcome. Indeed, if the agents must decide among these four options, they still prefer a fast debt reduction: 51% of agents want it to be achieved by public expenditure, while 34% of agents want it to be achieved by income taxation. Hence, under this experiment, the presence of the zero lower bound does not affect the outcomes of this voting process.

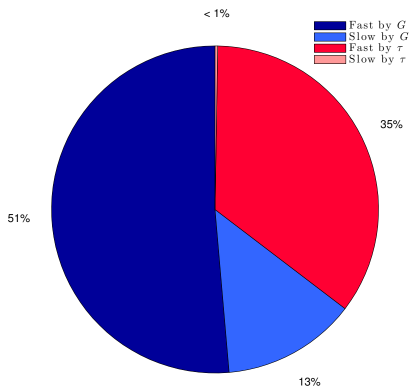
Preferences Regarding Different Speeds of Deleveraging

From the previous paragraphs it is possible to conclude that the debt reduction needs to be fast and achieved by public expenditure. Let us assume that this is not be feasible and that the fiscal authority is constrained to use income taxation.[23] Once the fiscal instrument is fixed it is possible to build the following voting experiment: the agents can vote for a fast or slow debt reduction. Again, I define the slow debt reduction using two values of ι: $\iota = 3$ and $\iota = 5$.

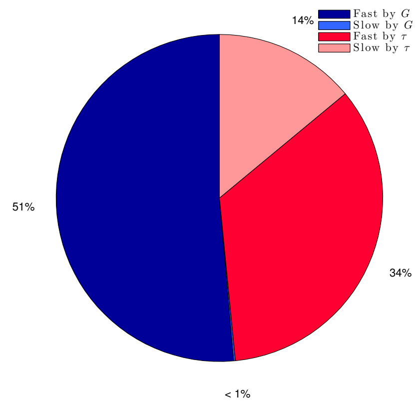
Under this voting experiment, the value of ι considerably changes the outcomes of the voting process. Figure 17.a and Figure 17.b show the outcomes of the voting process under $\iota = 3$ and $\iota = 5$, respectively. Figure 17.c shows agents' preferences under the two experiments, by employment status and by position in the wealth distribution. If the liquidity trap is not too severe, the majority of the agents would prefer a slow debt reduction to a fast one. This result is not significantly different to that under the benchmark experiment.

If, instead, the debt reduction is too slow, the economy enters a very costly liquidity trap where the employed agents face a considerable reduction

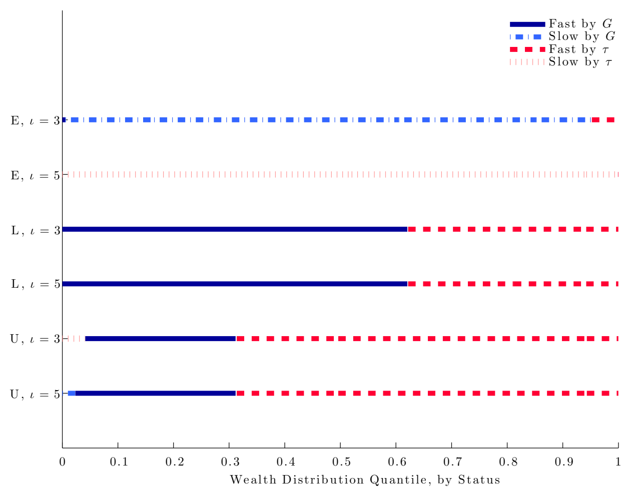
[23] Once again, I do not repeat the same experiment for public expenditure, since the results do not change with respect to the benchmark case.



(a) Results, $\iota = 3$

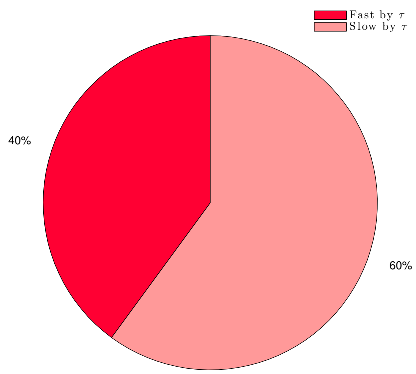


(b) Results, $\iota = 5$

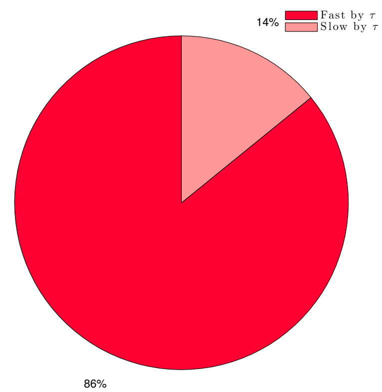


(c) Agents' Preferences by Wealth Distribution Quintile

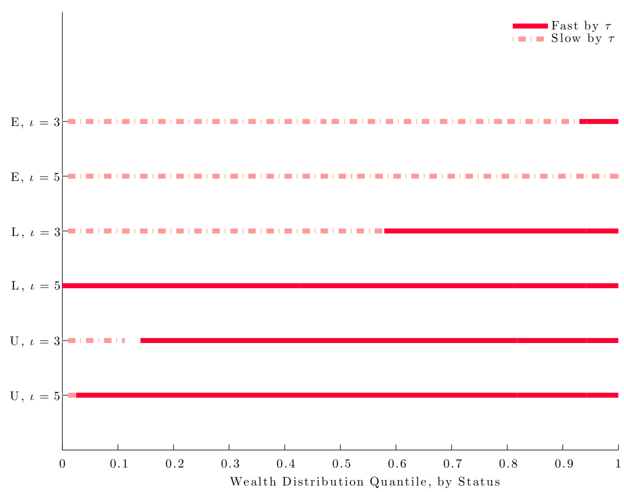
Figure 16



(a) Results, $\iota = 3$



(b) Results, $\iota = 5$



(c) Agents' Preferences by Wealth Distribution Quintile

Figure 17

in their labor income. Then, independently of their wealth, they vote for a fast debt reduction. This changes the voting outcome, letting the fast debt reduction option win.

To summarize, the presence of the zero lower bound has important consequences only if debt deleveraging is achieved by increasing income taxation. Depending on the speed of debt reduction, the economy can enter a detrimental liquidity trap. This new scenario does not change the agents' first-best option, this being a fast debt reduction achieved by cutting public expenditure. It changes, however, the preferred speed of deleveraging if the fiscal authority is forced to use income taxation. Indeed, the majority of the agents prefer a slow debt reduction only if, under this option, the economy does not enter a deep and prolonged liquidity trap. If this occurs, they will prefer a fast debt reduction. A policy implication is, thus, to not postpone the debt reduction, excessively, as it can be extremely costly for the economy.

8 Conclusions

I study what is the optimal public debt deleveraging in a context of heterogenous agents and incomplete markets. I analyze what is the optimal fiscal instrument to be used, as well as the optimal speed of debt reduction.

The central result of the paper is that it is always optimal to reduce debt in a fast way by cutting public expenditure. Indeed, by using this instrument, the fiscal authority is able to keep the real interest rate low during the public debt deleveraging process. A low interest rate helps the fiscal authority and borrowers, who are the closest to the borrowing constraint, when the debt reduction takes place. On the other hand, if the fiscal authority is constrained in increasing income taxation, the public debt reduction needs to be slow. Again, it is the dynamics of the real interest rate that play a central role in delivering this result.

A second result is that, independently of the fiscal instrument used, the economy may enter a liquidity trap during the deleveraging process. In my model, the zero lower bound has a new effect, beyond the classical low-output low-inflation one. It can reallocate resources from the employed to the entrepreneurs. Since the former are the closest to the borrowing constraint, this corresponds to a redistribution of resources from the financially constrained to the financially unconstrained, making in turn the borrowing constraint more stringent. Due to these effects, the presence of the zero lower bound affects the preferences of some agents: An extremely slow debt reduction, achieved by increasing income taxation, becomes the most costly combination.

There are two main limitations of the model. First, agents can invest only in one type of asset, risk-free bonds, while, in the real world, they have access to a wide range of financial assets. The possibility to invest in different types of assets may dampen the fluctuations in real interest rates. This can, in turn, affect the dynamics implied by the different fiscal instruments, modifying agents' preferences. Notwithstanding the large variety of assets traded in financial markets, it has been shown that agents at the bottom of wealth distribution tend to invest, when they do, mainly in safe assets.[24] Wealthier agents diversify their portfolio more, but participation in equity market is still "far from universal".[25] For example, only half of median-wealth agents participate in public equity markets. Hence, the assumption of a unique type of asset, despite being restrictive, does not appear to be implausible.

Secondly, in my model, only domestic agents can invests in domestic bonds. This assumption is slightly unrealistic, especially when considering the United States where much of government debt is held by foreign agents.[26] The presence of international investors can affect some of the results in my model. During the deleveraging process, the net foreign asset position of the economy remains unchanged in a close economy. This can reduce the welfare cost of deleveraging. Moreover, real interest rate dynamics are driven by fluctuations in aggregate domestic consumption. Assuming the presence of internationally held debt can *in primis* decrease agents' welfare during the deleveraging and *in secundis* weaken the response of the real interest rate. Introducing this feature in my model represents a promising avenue for future research.

This paper highlights some important concerns that a fiscal authority may face when compelled to reduce public debt. My model has strong policy implications, especially for countries that are currently undertaking austerity measures. Indeed, the model suggests that some policies adopted by European countries to manage the reduction of debt were suboptimal. Italy, for example, has announced the implementation of future austerity measures

[24]Survey of Consumer Finances (2001).
[25]Campbell (2006).
[26]Gourinchas, Rey, Govillot (2010).

consisting in tax rate increases.[27] Following the model, announcements of increases in the tax rate that take place far in the future can make the deleveraging process very costly. My analysis then also suggests that the liquidity trap that European economies are currently experiencing can partly be explained by the management of fiscal policy in some Euro Area countries.

[27] Budget laws presented in the summer of 2011 projected large increases in tax revenues for the years 2012, 2013 and 2014, see Ministero dell'Economia e delle Finanze, Press Release, "Italy Delivers", 16 September 2011.

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A Numerical Method

In order to solve the model I use the method described by Guerrieri and Lorenzoni (2010).

First, in the steady state I find the optimal policy for consumption, $c(s,b)$, and labor supply, $l(s,b)$ given the real interest rate, r, and the real wage, w. I discretize the endogenous state variables, b, using a grid of 2500 points and then I iterate the intertemporal Euler equation (4) and the intratemporal one (6) using endogenous gridpoint method by Carroll(2006). I defined $g(s,b)$ as the inverse of bond accumulation policy.[28] I compute the invariant distribution, $\Psi(s,b)$, updating the conditional distribution function, $\Psi_k(s,b) = \sum_{\tilde{s}} \Psi_k(\tilde{s}, g(\tilde{s}, b))P(s|\tilde{s})$ where k refers to the iteration step, $P(s|\tilde{s})$ is the transitional probability from state $\tilde{s} = s_{t-1}$ to state $s = s_t$. The bond accumulation formula is not invertible for $b = \psi$. Nevertheless, it is possible to prove that the above formula hold, once defining $g(s,\psi)$ as the largest b such that $b' = \psi$. After finding the invariant distribution, I check that the bonds and labor market clear. If they do not, I updated the guess of r and w and I iterate again till convergence.

For the transition, I guess a path of real interest rate,$\{r_t\}_{t=1}^{T_1}$, and real wages,$\{w_t\}_{t=1}^{T_1}$ where I set $T_1 = 200$. I find, backward, the optimal policy functions for consumption $\{c_t(s_t,b_t)\}_{t=1}^{T_1}$ and labor supply, $\{l_t(s_t,b_t)\}_{t=1}^{T_1}$. Again, in order to find the optimal policy functions I use the intertemporal Euler equation (4) and the intratemporal one (6). Then, exploiting the fact that I know the initial steady-state distribution, I compute $\Psi_t(s_t,b_t)$ from $t = 1$ to $t = T_1$. Lastly, I check that the bonds and labor market clear every quarter t. If they do not, I update the path of the real interest rate and real wage, accordingly.

Under the Rotemberg costs assumption, I use the same algorithm described above. The only difference is in that I guess and update also the inflation path.

[28]This can be derived by the optimal policy functions.

B Complementarity and Substitutability

In this section, I study whether the elasticity of substitution among private consumption and public expenditure affects the results. I set $\chi = 0.9$ and in order to keep the previous calibration unchanged I set $\beta = 0.979$.

B.1 Flexible Price

I assume that the nominal interest rate can go below zero. I study how the aggregate variables' impulse response functions change with respect to the benchmark exercise. Then I analyze whether the value of χ changes aggregate preferences.

Aggregate Variables Impulse Response Functions

Figure 18 shows the impulse response functions under a debt reduction achieved by public expenditure (first column) and income taxation (second column). The values of ι are set equal to 0.2, 1 and 3, as in the benchmark case, to make the two exercises comparable.

If the fiscal authority reduces public debt by increasing income taxation, the value of χ is irrelevant to determining the impulse response functions of the aggregate variables. Instead, if the public debt deleveraging occurs by cutting public expenditure, the value of χ has a non-negligible effect on them. The lower is χ, the lower will be the fluctuation in private consumption and public expenditure and the higher will be those in output and the real interest rate. Let us explain why. Once the fiscal authority cuts the public expenditure, private consumption is subject to two forces: On the one hand it will increase since the agents receive the back public principal, on the other hand it will decline since public-provided services are complemented by private consumption. Even if the first effect dominates the second, private consumption increases much less than under the assumption of imperfect substitutability. This will have a negative impact on output. Consequently, the nominal interest rate declines much more per given deleveraging shock as it has been shown in Figure 18. Due to the deeper fall in the nominal interest rate, public expenditure declines less under imperfect complementarity than under imperfect substitutability.

Despite the value of χ affecting the impulse response functions of the aggregate variables under some cases, it is not clear whether this changes the agents' preferences. I now turn to analyze whether it does and how.

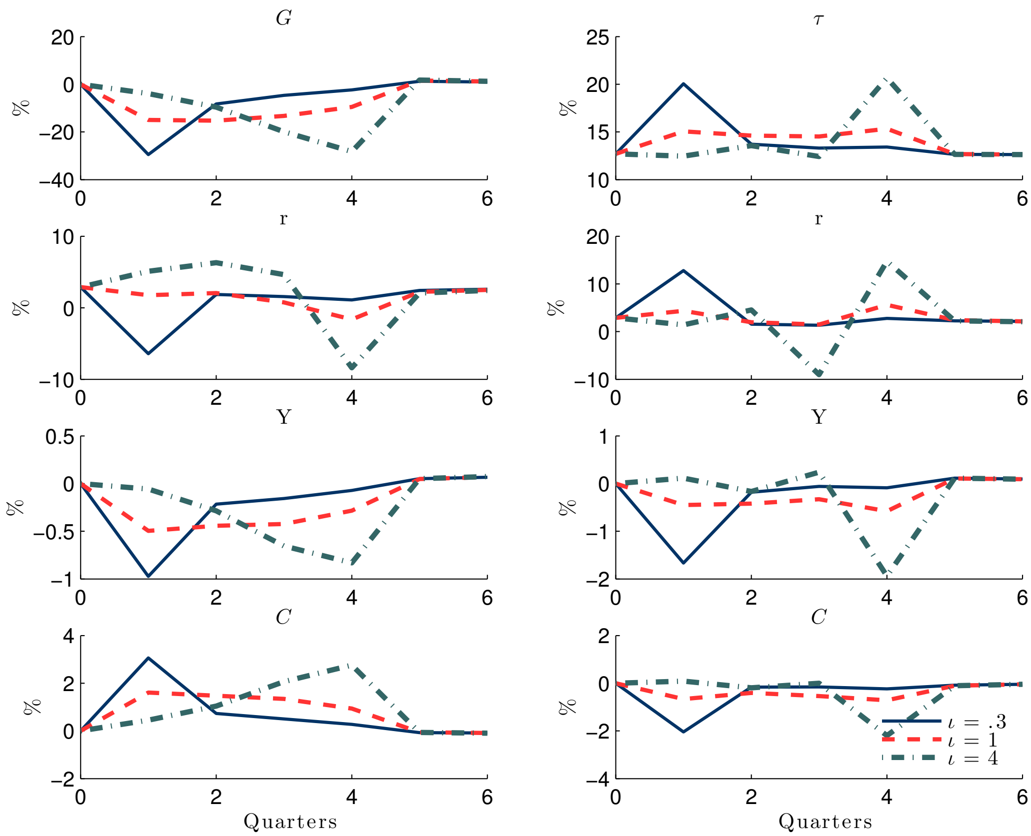

Figure 18: Impulse Response Functions under Different Deleveraging Speeds, $\chi = 0.9$

Impulse Response Functions for public expenditure, G, income taxation, τ, real interest rate, r, output, Y, and private consumption, C, according to different values for ι. $\iota = 0.3$ represents a fast debt reduction, $\iota = 1$ represents a smooth debt reduction and $\iota = 3$ represents a slow debt reduction. In the first column, the public debt reduction is financed by cutting public expenditure. In the second column, the public debt reduction is financed by increasing income taxation. All the variables, except for r and τ are in log-deviations from the final steady state values. The values for r and τ are given in percentages.

Aggregate Welfare

I study the aggregate preferences using two metrics: the aggregate consumption equivalent, ξ, and the voting experiment, both described in subsection 6.2. Figure 19 shows the results: the first panel represents the aggregate consumption equivalent, ξ, as a function of the public debt deleveraging speed, ι, both under the public expenditure experiment and under the income taxation one. In the second panel, I plot the percentage of agents who vote in favor of the public expenditure option as a function of the public debt deleveraging speed.

Under imperfect complementarity of public expenditure and private consumption, it is always optimal to reduce public debt by cutting public expenditure rather than by increasing income taxation. This result arises for different reasons that mainly depend on the behavior of the real interest rate when the fiscal authority cuts the public expenditure. First, since the real interest rate falls significantly, financially-constrained agents would be better-off. Second, facing a low interest rate, the fiscal authority cuts the public expenditure much less than under the imperfect substitutability exercise. Therefore, less wealthy agents, who mainly rely on this source of consumption, are better off. Moreover, since there is a smaller fall in public expenditure, the economy faces less distortion. As a result, independently of the metric I use to evaluate welfare, public expenditure is always better than income taxation.

B.2 Nominal Rigidities and the Zero Lower Bound

I do not describe the consequences of the zero lower bound when the fiscal authority reduces public debt by increasing income taxation. Indeed, these are exactly the same as under the imperfect substitutability experiment.

I focus my analysis on debt reduction via public expenditure. Since under the flexible price experiment the real interest rate fluctuates considerably, the non-negativity constraint binds more frequently. Moreover,under a slow debt reduction, the economy may enter a liquidity trap. Let us understand why. Figure 20 shows the impulse response functions for the flexible price exercise and the zero lower bound when debt reduction occurs via public expenditure. I use the same values of ι as in Figure 12 to make the two exercises comparable. In the first column, I plot $\iota = 0.1$ and in the second $\iota = 10$. If the debt reduction is fast (first column), then the nominal interest rate falls in the first quarter. When the non-negativity constraints binds, output

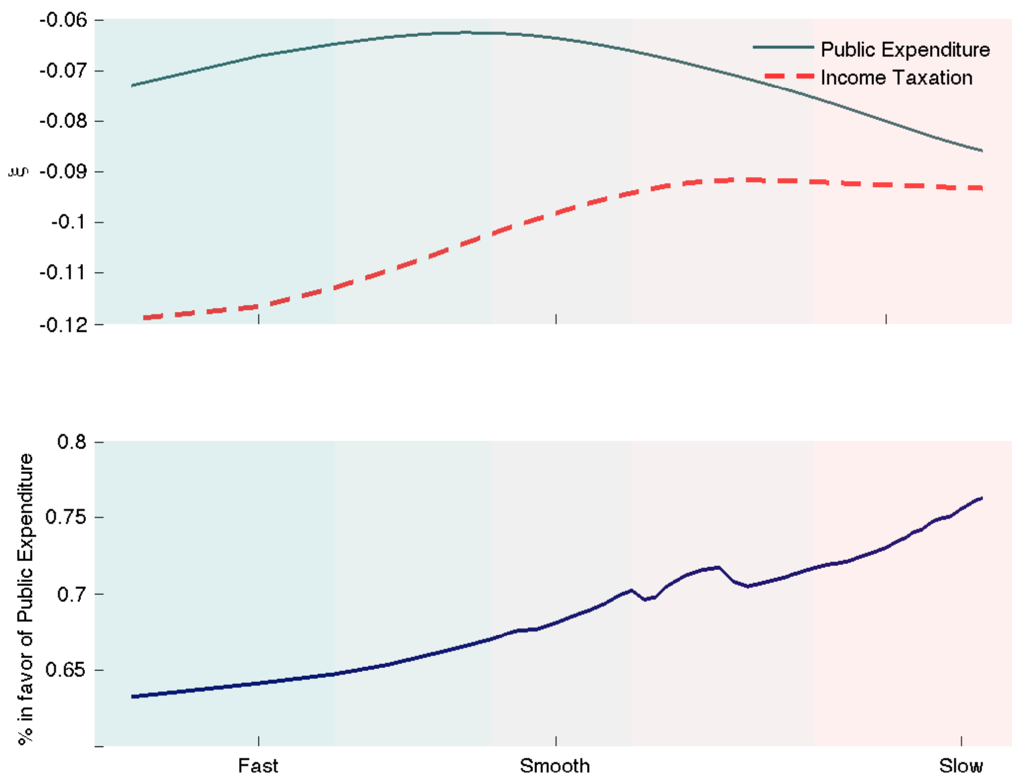

Figure 19: Aggregate Preferences, $\chi = .9$
The first panel shows the aggregate consumption equivalent, ξ, as a function of the public debt deleveraging speed, ι, both under the public expenditure experiment and under the income taxation one. The second panel shows the percentage of agents who vote in favor of the public expenditure option as a function of the public debt deleveraging speed.

decreases significantly as well as public expenditure. Since the firms cut their production, decreasing their labor demand, real wages fall considerably. As a consequence, output is redistributed from the employed to the entrepreneurs, enhancing the output recession.[29]

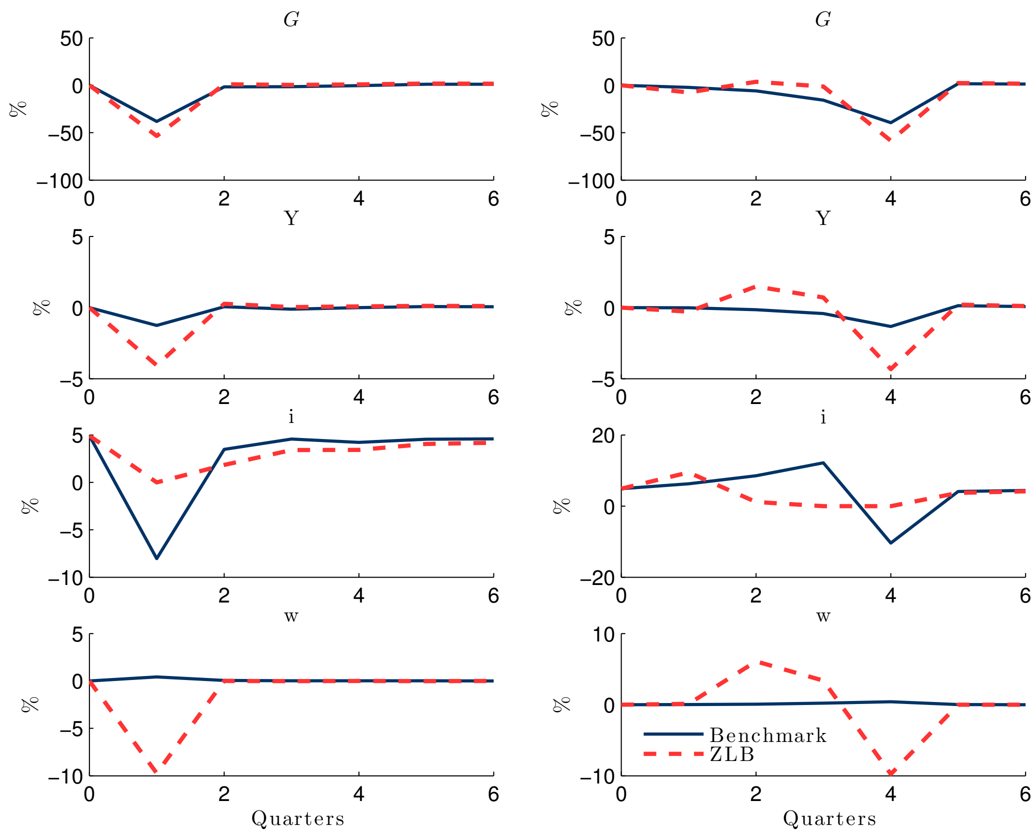

Figure 20: Impulse Response Functions and the Zero Lower Bound, $\chi = .9$
Impulse Response Functions for public expenditure, G, output, Y, real wage, w, and the nominal interest rate under the benchmark exercise (continuous blue line) and the zero lower bound (red dashed line). I set $\iota = 0.1$ in the first column and $\iota = 10$ in the second column. All the variables except for i are in log-deviations from the final steady state values. The values for i are given in percentages.

On the other hand, under a slow debt reduction, the zero lower bound binds in the last quarter. This is represented in the second column of Fig-

[29]As explained in section 7, under this circumstance, resources are reallocated from the financially-constrained agents to the financially unconstrained ones.

ure 20. Under imperfect complementarity, the presence of the zero lower bound can lead to a deep output recession in the last quarter. Then, if this happens, in the third quarter the agents are willing to save more, exerting a downward pressure on the nominal interest rate even in this quarter: the economy may enter a prolonged liquidity trap. Note, by the way, that the liquidity trap is not so detrimental for the economy. Indeed, in the third quarter, since the real rate is lower, public expenditure increases (the fiscal authority is a borrower). Then, due to the imperfect complementarity, private consumption also increases, leading to an output expansion. Since the firms demand more labor, the real wage increases and profits decrease. Then, resources are redistributed from the entrepreneurs to the employed agents.

The presence of the zero lower bound, under the imperfect complementarity assumption, also influences the agents' preferences for public expenditure as shown in Figure 21. In the first panel, I plot the aggregate consumption equivalent, ξ defined in subsection 6.2, under the public expenditure benchmark experiment, under the income taxation benchmark experiment, under the public expenditure and the income taxation experiment with the presence of the zero lower bound. The second panel in Figure 21 shows the percentage of agents who vote in favor of public expenditure against income taxation for a given value of ι. The dark blue continuous line represents the voting outcome under the benchmark, while the light blue dashed line represents the voting outcome under the presence of the zero lower bound.

Differently to before, under a very fast debt reduction the agents prefer the income taxation instrument, independently of the welfare criteria considered. This is due to the deep recession caused by the zero lower bound which redistributes resources from financially constrained agents to financially unconstrained ones. Note that under a slow debt reduction the economy prefers the public expenditure instrument to the income taxation one. The policy implication is that, if the public and private consumption are imperfect complements, the fiscal authority must reduce the debt by cutting public expenditure, except if the debt deleveraging is too fast.

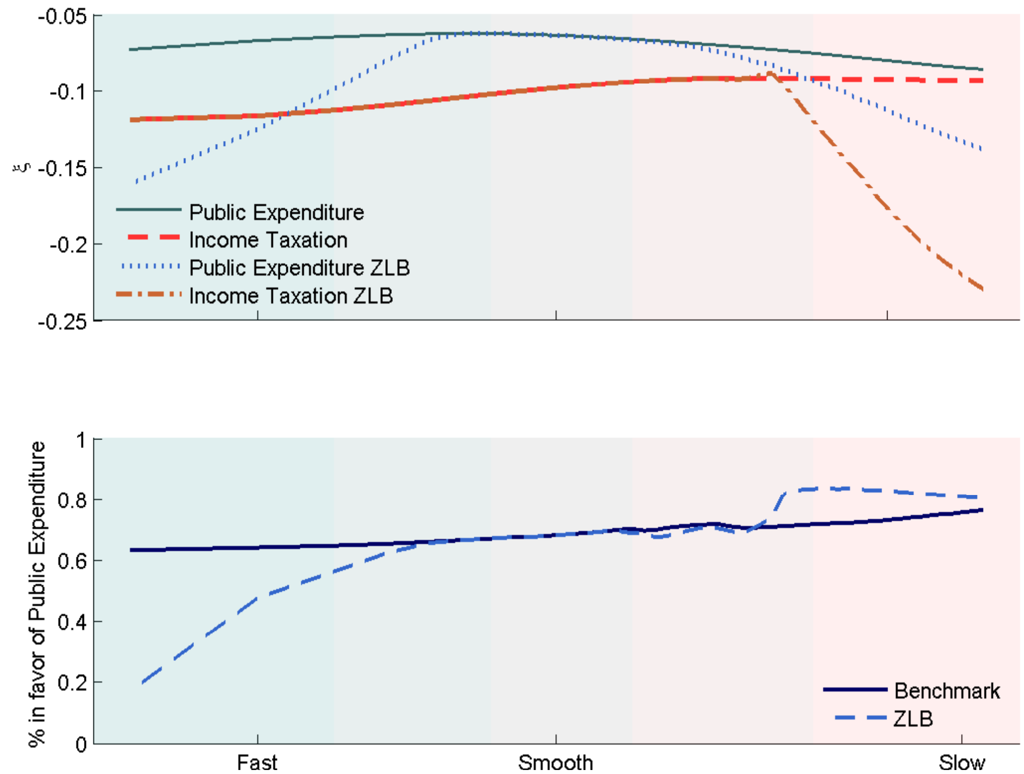

Figure 21: Aggregate Preferences and the Zero Lower Bound, $\chi = .9$
The first panel shows the aggregate consumption equivalent, ξ, as a function of public debt deleveraging speed, ι, both under public expenditure experiment, under the income taxation-benchmark experiment, under the public expenditure and the income taxation experiment with the presence of zero lower bound. The second panel shows the percentage of agents who vote in favor of the public expenditure option as a function of public debt deleveraging speed in the benchmark (continuous dark blue line) and under the zero lower bound (dashed light blue line).