Taking Trends Seriously in DSGE Models:
An Application to the Dutch Economy

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This version: September 2013

Abstract

We construct a new-Keynesian DSGE model with a more careful stochastic specification than is standard in the literature. We emphasize the gains in including stochastic trends within the model structure itself and jointly estimating them with the cycles. We pay particular attention to common trends (or lack thereof) in the data and reverse-engineer them in our model. Our application suggests three such trends—in general technology, investment-specific technology, and labor supply. The trend-cycle decomposition captures the co-integrating properties of the data without which medium- to long-run analysis—whether scenario analysis or forecasting—would likely be misspecified. Our set-up produces better-behaved posteriors for parameters along decision margins where traditional modeling imposes highly persistent but temporary shocks. Furthermore, the co-existence of permanent and temporary disturbances along the same margin broadens the scope for counterfactuals. Specifically, our model extends the insights of the Permanent Income Hypothesis—that discounted valuation effects prompt consumers to respond differently to permanent and temporary income shocks—to the many other forward-looking decision margins characterized by smoothing motives.

JEL codes: C54, E27, E37, E47

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We thank seminar participants at the De Nederlandsche Bank and the Working Group on Econometric Modeling of the European Central Bank for useful comments and suggestions, and in particular Andreas Pick, Robert-Paul Berben, Adam Elbourne, Rob Loginbuhl, Filippo Ferroni, and Manu de Veirman. We thank in particular Vincent Sterk and Rasmus Kattai, who actively contributed to the project in its early and later stages, respectively. Views expressed in this paper are our own and do not necessarily reflect those of De Nederlandsche Bank or the Centraal Planbureau.
1 Introduction

This paper presents an estimated new-Keynesian Dynamic Stochastic General Equilibrium model through the lens of an unobserved components model. DSGE models are ubiquitous in policy and central banking spheres and need no further introduction beyond mentioning the seminal references (Christiano et al. (2005); Smets and Wouters (2007)). Such models appeal to policymakers and academics alike because they merge the versatility of state-space representations for time-series analysis—the ability to perform full-information system estimation, impulse response analysis and shock decompositions—with the rigor imposed by the internal consistency of the general equilibrium framework.

However, this appeal is not shared by all. In one particular line of criticism, Hoover et al. (2008) have argued that the current practice of DSGE modeling excessively emphasizes theorizing over what they call the “careful stochastic specification as necessary groundwork for econometric inference”. An important case in point is the treatment of trends. It has long been common practice to pre-filter a-theoretically the lower, ‘trend’ frequencies of the data—using band-pass variants, the Hodrick-Prescott filter, or simply linear de-trending—before analyzing the ‘cyclical’ remainder with a theoretical model. From the point of theory, trends are just an afterthought. Yet, a common model for both trend and cyclical components that is grounded in general equilibrium theory would be far more desirable.

The importance of trends becomes self-evident when DSGE models are viewed from the angle of a trend-cycle decomposition exercise. Clearly, trends and cycles are jointly determined, and re-balancing modeling efforts towards the former is a worthwhile venture. In addition, thirty years of co-integration theory and practice have emphasized that trends are jointly determined across variables, suggesting that univariate filtering techniques are inappropriate. Furthermore, a careful treatment of trends is crucial to policymakers because forecasting exercises require both trends and cycles, and many interesting counterfactuals in scenario analysis involve disturbances—whether policy based or not—that are permanent.

It is somewhat ironic that the tool best suited to deal with these issues —the state-space model—has always been available as the inherent form of linearized DSGE models, but that it has not, to date, been used to study trends and cycles within a self-contained theoretical
framework. This is what we remedy in this paper: we model and estimate trends and cycles jointly, on a sample of Dutch national accounts data. We do so by exploiting the structural time-series representation of the general equilibrium model to perform a multivariate unobserved components decomposition of the data with multiple stochastic trends. Using Robert Engle’s terminology, we view the DSGE model and its cross-equation restrictions through the lens of a VUCARIMA process (vector unobserved component autoregressive integrated moving average process) rather than the more standard VARMA.

Trends can be brought into a DSGE model in various ways. One approach is to introduce them as exogenous components in the observation equations of the model’s state-space form. Doing so, however, does not change the cross-equation restrictions that embody the general equilibrium dimension of the model, implying that trends remain purely statistical decomposition devices with no particular economic interpretation. In contrast, incorporating trends formally in a general equilibrium framework will generate cross-equation restrictions that link more tightly transition and observation equations, thus yielding a richer, theory-based correlation structure for the model objects.

In their seminal paper, Smets and Wouters (2007) take up the challenge and model trends formally. Specifically, they assume that all real variables in their model expand at the same deterministic rate, which reflects the assumption of a balanced-growth path. When taking the model to the data, this amounts to filtering variables with a common linear trend. In light of the versatility of the multivariate structural time-series framework and the characteristics of their data, this choice turns out to be relatively restrictive, with at least two important consequences.

First, observable variables are equal to model variables plus a common deterministic drift. From the point of view of an unobserved components model, all the dynamics of the model are forced into the cyclical component, which is the simplest use of an unobserved components representation and therefore unlikely to satisfy Hoover et al.’s criterion of “careful stochastic specification”. Second, a common growth rate for all real variables implies pairwise co-integration between them. Yet, US data displays clear evidence to the contrary (see Whelan (2003, 2004) for an early exposition of this fact). This suggests that Smets and Wouters’ estimation results likely suffer from mis-specification biases that arise from assuming more co-integrating relationships.
than warranted by the data.

These biases materialize in at least two instances in their model. The estimated investment shock is very persistent and takes up much of the long-run volatility decomposition of variables because it is incorrectly modeled as a stationary shock, while its observable counterpart in the data—the relative price of investment—actually displays a unit root. Similarly, the role of the wage mark-up shock is likely over-estimated because the model counterfactually assumes that the labor input and labor share are stationary (although, admittedly, the case is borderline). Recent work has dealt with these two points separately (Chang et al. (2007) for the labor input, Justiniano et al. (2011) for the relative investment price) but not jointly. More generally, these mis-specification issues suggest reverse-engineering whenever possible the trends into the models—as Hoover et al. recommend—and this requires careful consideration of which aspects of a model’s balanced-growth path are borne out in the data. This is the task we undertake in this paper, on national accounts data for the Netherlands.

Our approach to the “careful stochastic specification” for our data is as follows. Whenever feasible, we model the stochastic trends and related co-integrating relationships in a general equilibrium framework, so as to maximize theoretical content reflected in useful cross-equation and cross-frequency restrictions. When the theory hurdle is too high, we exploit the versatility of the state-space representation and relegate the ‘excess’ trends—trends we do not formally model—to the observation equations.

As we mentioned above, the common practice is to eliminate excess trends altogether in the data—‘pre-filter’ them—before estimating a model (see, for example, Adolfson et al. (2008), who do so in their trade-balance block). In contrast to this two-step approach, we estimate the filtering parameters for the excess trends jointly with the DSGE model’s structural parameters, including the parameters of the theoretically-founded stochastic trends. According to Ferroni (2011) and Canova (2012), who follow up on points made earlier by Cogley (2001), joint estimation is unambiguously preferable to the two-step approach, so as to avoid problems ranging from trend mis-specification to wrong cross-frequency correlations. Pre-filtering with the wrong trend specification can distort structural parameter estimates considerably.

Our work is closely related to these two recent contributions. In essence, Canova and Fer-
roni both recommend developing a flexible specification for trends in the observation equations and estimating them jointly with the (cyclical) theoretical model summarized in the transition equations. The upshot, as Canova suggests, is that his methodology “can be applied to models featuring (...) transitory and permanent shocks and only requires that interesting features of the data are left out from the model—these could be low frequency movements of individual series, different long run dynamics of groups of series, etc...” Yet, Canova also admits that the non-structural approach “does not substitute for theoretical efforts designed to strengthen the ability of DSGE models to account for all observable fluctuations. But it can fill the gap between what is nowadays available and such a worthy long run aspiration.”

In fact, this is precisely what we are striving for: instead of filling the gap with non-structural trends, we would rather close it with model-based ones. From a policy perspective, our approach provides a structural basis for permanent shocks which is particularly valuable. From the point of view of theory, we can analyze relevant data features such as low-frequency movements and common trends that would otherwise be left out in Canova’s methodology. Furthermore, we can impart flexibility to trend specifications in the transition equations, by allowing for any ARIMA process for trend shocks within the general equilibrium model structure.

As we will show, our approach with multiple, theoretically-founded trends requires lengthier steps towards implementing a log-linear version of our model because the trends affect the model along many dimensions and must be solved for jointly. However, the payoff is that we have a stochastic specification of the data that captures its co-integrating properties without which medium- to long-run scenario analysis would be misspecified. We can use the model for all frequencies—including the lower ones. Furthermore, we can exploit further explanatory power from the cross-frequency restrictions imposed by the data.

Some specificities of the data we use are worth summarizing at this point. We model the Netherlands as a small open economy within a monetary union which, given the realized pattern of trade, we take to represent the rest of the world. Thus, we view the Netherlands as a country with no currency of its own (a fixed exchange rate normalized to one) and facing an exogenous interest rate (set by the European Central Bank). As we will document below, the Dutch National Income and Product Accounts (NIPA) exhibit only a subset of the features of
a balanced-growth path. Specifically, no real ratio between GDP and any of its components is stationary, except government spending. Both the trade balance to GDP ratio and the labor input (full-time-equivalents per capita) display upward trends, running counter to standard assumptions of balanced-growth models. Nevertheless, the ratio of nominal consumption to nominal investment is stationary, as is the share of labor compensation in the sum of nominal consumption and investment. These two co-integrating relationships are the foundations of the restricted balanced-growth path around which we build our model. Our path is ‘restricted’ because not all our observable variables satisfy the requirements of a standard one. Such is the case with the trade-balance variables, whose trends we have found difficult to model in a general equilibrium setting. Recalling the discussion above, we capture these excess trends in the observation equations.

It is important to realize that, while our model is tailored to our data application, the philosophy of our approach and the methodology are general. An application to the Smets and Wouters model on US data would be straightforward, requiring the same common trends to account for overall growth, a secularly decreasing labor share, and a falling relative price of investment. In fact, we conjecture that our model approach with reverse-engineered multivariate de-trending could be fully automated in a package such as Dynare.

The road map of this paper is as follows. In the next section, we describe some crucial features of the Dutch data to support the case for our modeling choices. The essential ingredients of our model are presented in Section 3. In Section 4, we characterize the more complex set of trends that represents the value-added of our model. The full derivations and tedious algebra underpinning these two sections are relegated for reference purposes to the appendix, which will eventually be spun off to limit the length of this paper. We discuss calibration and prior assumptions and report our estimation results in Section 5, and Section 6 concludes.

2 Data description

We start by describing the data that our model should summarize, namely national product and income variables for the Netherlands and the subset of Eurozone data which affects the open-economy dimension of the model. The Dutch data are made available by the Central Bureau of
Statistics, while the Eurozone data are taken from the ECB’s Statistical Data Warehouse. We first run through some simple preliminaries to convey the intuition behind our approach to the data.

2.1 Some preliminaries

Our main objective in describing our data-set is simply to identify its underlying common trends. Recall the argument set forth in the previous section, that the state-space representation of a DSGE model enables one to reinterpret it as an unobserved components filter. The vector of (log-transformed) observable variables $y_t$ can be decomposed as

$$y_t = y_t^* + \hat{y}_t,$$

where trend $y_t^*$ and cycle $\hat{y}_t$ follow the (vector) processes

$$\Delta y_t^* = \mu + \varepsilon_t,$$

$$A(L) \hat{y}_t = u_t.$$

Smets and Wouters’ 2007 choice of a linear deterministic balanced-growth path for the model’s real variables implies that $\mu = 1\bar{\mu}$ with scalar $\bar{\mu}$, and $\varepsilon_t = 0$ for all $t$. Yet, as Whelan (2003, 2004) argued, these restrictions are not warranted in US data. Accordingly, we attempt to identify the likely common $\mu$’s and $\varepsilon$’s across variables in our own data. These co-integrating relationships connect the trends in the variables to the permanent shocks in the DSGE model. By identifying the potential error-correction mechanisms, we intend to build a non-stationary DSGE model with a co-integration structure consistent with the data. Such a model can then be used to analyze not only business cycle fluctuations but also fluctuations at lower frequencies.

Additive constraints in a model—such as the GDP identity—require that one conduct a preliminary search for common trends in the data because log-linearization is permissible only if all the pairwise ratios of the constraints’ subcomponents are stationary. Indeed, while the first-order Taylor approximation of

$$Y_t = X_t + Z_t$$
around a given point \( \frac{X^*_t}{Y_t} \) is always feasible

\[
\ln Y_t - \ln Y^*_t = \frac{X^*_t}{Y_t} (\ln X_t - \ln X^*_t) + \frac{Z^*_t}{Y_t} (\ln Z_t - \ln Z^*_t),
\]

the theoretical DSGE model requires this equation to be linear, because it imposes that \( \frac{X^*_t}{Y_t} \) be the constant steady-state value of the ratio \( \frac{X_t}{Y_t} \), which depends on time-invariant structural parameters. This tighter constraint on the first-order expansion implies that \( X_t \) and \( Y_t \) (and by implication \( Z_t \)) must have a common trend. The first check we perform, then, is whether such an assumption is warranted in the Dutch GDP identity.

To this end, we check visually whether the pairwise ratios in the GDP identity (the ‘great’ ratios) appear to fluctuate around a well-defined mean that we could interpret as the steady state in which the theoretical model settles. More generally, we check whether the main steady-state conditions of the standard DSGE model are validated as appropriately mean-reverting ratios in our data. Intuitively, multiple crossings of the sample mean is a good guide that out-of-sample forecasts will tend to the same mean in the long-run. The purpose of visual inspection is to inform our priors about where and how the common trends should be modeled in the theoretical framework; this is already a great step forward relative to common practice.\(^1\)

2.2 Features of the data

Figures 1 through 3 plot the data running from 1984Q1 to 2011Q3. When taking the model to estimation, we will end the sample in 2007Q4, depicted by the vertical line in the charts. We do so to see later on how the model interprets the crisis period (2008Q1 onwards for the Netherlands).

Figure 1 groups the relevant ratios in the GDP identity. The upper left-hand chart shows that the ratio of the sum of consumption and investment to output is not mean-reverting, whether in nominal or in real terms (when chain-aggregated). This reflects in part the secular increase

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\(^1\)We do not formally test for co-integration in the frequentist tradition because we later adopt the Bayesian perspective when moving to estimation. To side-step the issue, we appeal to the Sims and Uhlig (1991) argument that the connection between frequentist pre-sample confidence statements and Bayesian posterior probability statements—which we are ultimately interested in—breaks down when unit roots are involved, and all the more so with small samples such as ours.
in the ratio of the trade balance to output (upper right-hand chart), which is the flip-side of above-average private saving rates of Dutch households and firms. These two charts immediately highlight that a model with a typical balanced-growth path will be grossly inappropriate. The bottom chart shows the ratio of government spending to GDP. In the pre-crisis period, the ratio exhibits only minor deviations from stationarity. This suggests that modeling this ratio as an observed and exogenous first-order autoregressive process, as Smets and Wouters do, will have less critical mis-specification costs than if we were assuming the same for the ratio of the sum of consumption and investment to output.

The upper left-hand chart in Figure 2 depicts our first co-integrating relationship, the ratio of nominal consumption to investment, which seems to revert to a well-defined mean. The upper right-hand chart decomposes this ratio into its real component and the (inverse) relative price,
both of whom display the same trend to keep the nominal ratio stable. Our data therefore possesses the same stochastic particularities for these variables as US data, that is, a balanced-growth path in nominal terms for private domestic absorption. This implies two stochastic trends, which we model as a permanent general technology shock and a permanent investment-specific technology shock reflected in the relative investment price, along the lines of Justiniano et al. (2011).

On the income and input side, the lower left-hand chart in Figure 2 shows our measure of labor supply, which displays a positive trend that reflects the secular inflow of women into the labor force. The lower right-hand chart plots the share of labor compensation in either output or the sum of nominal consumption and investment. We argue that the latter makes a stronger case for mean-reversion and take it as the second of our crucial co-integrating relationships.
Figure 3. Searching for stationary in nominal variables and the foreign block

The consumption-investment ratio and the labor share are related by the optimal trade-off between consumption and leisure in a standard model of preferences. With log-utility of consumption and additively separable utility of leisure, this trade-off can be rewritten as follows:

\[ v'(L_t) = \frac{w_t L_t}{P_t C_t} = \frac{w_t L_t}{P_t C_t} \frac{P_t C_t I_t + P_t I_t}{P_t C_t I_t}. \]

The upper left-hand and lower right-hand charts in Figure 2 show that the two right-most terms in the equation are stationary. This implies a stationary left-hand side of the equation, such that the trending labor input must be offset by an opposite trend shift in the marginal utility function \( v' \), which we will formalize as a permanent preference shock (equivalently, a permanent mark-up shock), the third of our theory-based stochastic trends.
Figure 3 shows other important ratios for steady-state considerations. The upper left-hand chart depicts the annualized Eurozone short-term nominal interest rate (three-month Euribor) and the Dutch inflation rate (GDP deflator). Although we do not plot it, the Dutch short-term interest rate has tracked the German one very closely over our sample—for all intents and purposes, the Netherlands had already relinquished its monetary independence long before the advent of the euro. Given how closely the synthetic euro area rate has also tracked the German one, we treat the former as the relevant rate for the Netherlands. The short rate displays a distinct pattern of convergence to current low levels. In contrast, the domestic inflation rate remained stable throughout the sample period. The upper right-hand chart of Figure 3 depicts the ratio between the real interest rate and real domestic output growth. Most models require this ratio to equal the household discount rate in the long run, thus to be stationary, and this appears to be true in our data. As we will discuss below, we will offset the mean shift of this ratio relative to a standard calibration of the discount rate by assuming a non-zero risk premium.

The lower set of charts in Figure 3 compares Eurozone variables to domestic variables. The left-hand one shows that the real ratio of foreign to domestic output has a distinct downward trend, while the nominal ratio, after displaying a clear hump shape in the first half of the sample, appears to have stabilized. The gap between the two series implies an inflation differential over the sample. Thus, assuming a constant nominal exchange rate, we have no evidence of purchasing power parity, which is the standard long-run restriction needed to close the foreign block of the model (see Burriel et al. (2010) for a typical example). Instead, we will use the recent stability in the nominal ratio to close the foreign block accordingly. The right-hand chart shows that exports have grown faster than foreign output. Clearly, this reflects in large part our assumption that the Eurozone represents the rest of the world: exports may simply be expanding at the same rate as the faster growing, non-Eurozone rest of the world. Here again, the figure suggests we will have to handle extra trends in the estimation.

To repeat, eye-balling the data as we have just done is no substitute, from a frequentist perspective, for formal testing for stationarity and co-integration. However, our objective was to ensure that we identify the important trends in the data and make our assumptions in the model consistent with them. Bayesian estimation will reveal trend mis-specification in other
guises than frequentist tests.

3 Model

3.1 Outline of the model

The features of our model are similar to many existing open-economy models (in particular those developed at the Swedish Riksbank, an example of which is Adolfson et al. (2008)). As with most of these models, the core is a variant of Christiano et al. (2005) or Smets and Wouters (2007). The differences relate mainly to the open-economy aspects. First, we consider the euro area to be the rest of the world for the Netherlands. This is a reasonable approximation, given that the Eurozone accounts for roughly 80 percent of Dutch exports and imports. Second, we do not use a multi-country set-up but model instead the Eurozone separately with the basic ingredients of a New-Keynesian model—namely an IS curve, a Phillips curve, and a Taylor rule—with no feedback from Dutch domestic variables (which, given its relative size in the Eurozone, is also a passable approximation). There are two advantages to these assumptions. First, since the Netherlands is part of the monetary union, there is no nominal exchange rate to track, which is a blessing given how difficult exchange-rate modeling has proven to be. Second, euro area inflation, output, and interest rate are not affected by domestic developments and are therefore easily interpretable as observable shocks in our model. That is, we can treat and estimate the foreign block of the model independently of the domestic block (the transition matrix is lower block triangular). The drawback is that we experience ‘leakage’ in the trade block by assuming away the impact of the US, UK, and Asia as trading partners beyond their indirect effect via the Eurozone.

We now describe the essential components of the model. Its full description in finer detail is reported in Appendix A, mainly for reference purposes. Readers familiar with DSGE modeling will recognize many characteristics of a standard model and may wish to read diagonally until Section 4. If so, it is worth summarizing now the few non-standard features we bring into the model. First and foremost, shocks to labor supply, general purpose technology, and investment-specific technology have both a permanent and transitory component. This implies a more
involved process for de-trending the variables and finding the steady state of the model. It also implies that permanent shocks appear in many more instances in the log-linearized equations than otherwise—in fact, wherever an inter-temporal trade-off is involved.\(^2\) Second, following Adolfson et al. (2008), we introduce a smoothing filter from hours worked to employment—which is the observable measure of labor supply in the data—by means of a dynamic Euler equation (real ‘Calvo fairy’ with indexation) for employment. Third, we bundle consumption and investment baskets with constant elasticity of substitution (CES) aggregators in normalized form and with appropriate trends to maintain balanced-growth in nominal terms in the current account identities. As Cantore et al. (2010) argue, the normalized form is a much more sensible approach to CES aggregation and it is borne out in our derivations. Fourth, as we described above, we do not rely on purchasing power parity to close the foreign block but on the steady-state, constant-elasticity demand curve for exports.

Importantly, the convention we follow in the rest of this document is that all prices and wages (written in small letters) are expressed relative to the price of the final good.

### 3.2 Final good producers

Perfectly competitive firms produce the final good \(y_t\) by combining a continuum of intermediate goods \(y_t(i)\) according to the CES technology

\[
y_t = \left( \int_0^1 y_t(i) \frac{x_{t-1}^p}{x_t^p} \, di \right)^{\frac{1}{1-p}}.
\]

Individual profit maximization yields the demand curve for intermediate good \(i\)

\[
y_t(i) = p_t(i) - \varepsilon p_t y_t,
\]

\(^2\)Consider a generic inter-temporal Euler equation:

\[
F \left( \frac{Y_t}{Y_{t-1}}, X_t \right) = 0
\]

A log-linear approximation yields

\[
Y_t^{ cyc} - Y_{t-1}^{ cyc} + \varepsilon_t + \beta X_t^{ cyc} = 0
\]

where \(\varepsilon_t\) is the stochastic shock to the trend component of \(Y_t\).
where $p_t(i)$ is the relative price of the intermediate good $i$ in terms of the final good, i.e. $p_t(i) = \frac{p_t(i)}{P_t}$. The zero-profits requirement then yields the price index (again in relative terms)

$$1 = \int_0^1 p_t(i)^{1-\varepsilon_t^p} di.$$ 

The (log-)elasticity $\ln (\varepsilon_t^p)$, which we interpret as a price mark-up shock, follows the ARMA(1,1) process

$$\ln (\varepsilon_t^p) = (1 - \rho_p) \ln (\varepsilon_t^p) + \rho_p \ln (\varepsilon_{t-1}^p) + v_t^p + \theta_p v_{t-1}^p, v_t^p \sim N (0, \sigma_p^2).$$

### 3.3 Intermediate good producers

Monopolistically competitive firms produce intermediate goods $y_t(i)$ with Cobb-Douglas technology augmented with fixed costs

$$y_t(i) = a_t A_t K_t^s(i)^{\alpha} L_t(i)^{1-\alpha} - \bar{z}_t \Phi,$$

where $K_t^s(i)$ is effective capital, $L_t(i)$ is labor, $\Phi$ is a fixed cost, and $\bar{z}_t$ represents a composite stochastic trend term which will be defined in a later section. Total factor productivity is decomposed into a temporary shock $a_t$ and a permanent shock $A_t$, whose stochastic processes are respectively

$$\ln (a_t) = \rho_a \ln (a_{t-1}) + v_t^a, v_t^a \sim N (0, \sigma_a^2),$$

and

$$\ln (A_t) = \gamma_A + \ln (A_{t-1}) + v_t^A, v_t^A \sim N (0, \sigma_A^2),$$

where $\gamma_A$ is the drift in total factor productivity. Real profits of firms are given by

$$p_t(i) y_t(i) - w_t L_t(i) - r_t^k K_t^s(i),$$

where $w_t$ is the real wage and $r_t^k$ the real rental rate on effective capital.
3.3.1 Labor and capital decision

The cost-minimization problem of intermediate good producers yields the optimal capital-labor ratio

$$\frac{K_t(i)}{L_t(i)} = \frac{\alpha w_t}{1 - \alpha r_t^K},$$

and an expression for real marginal costs

$$mc_t(i) = (a_tA_t)^{-1} \alpha^{-\alpha} (1 - \alpha)^{-(1-\alpha)} w_t^{1-\alpha} (r_t^K)^\alpha.$$  

Intermediate good producers face a marginal cost curve that is independent of their level of production. Therefore, average variable cost equals marginal cost

$$avc_t = mc_t(i).$$

3.3.2 Price-setting decision

Price-setting in our model follows the standard Calvo set-up with indexation. This will yield in its log-linearized form a price Phillips curve with a backward-looking component. The fraction $\zeta$ of intermediate good producers that cannot reset prices freely indexes them instead to a weighted average of past inflation $\pi_{t-1}$ and steady-state inflation $\pi_s$. Let $\varrho_t(i)$ denote the optimal price for intermediate good firm $i$ in period $t$. Optimization yields the first-order condition

$$0 = E_t \sum_{s=0}^{\infty} \zeta_s \beta^s \Xi_{t+s} \left( (1 - \varepsilon_{t+s}) \varrho_t(i) X_{t,s}^p + \varepsilon_{t+s} avc_{t+s} \right) y_{t+s|t}(i),$$

where the individual demand curves and indexation terms are

$$y_{t+s|i}(i) = (\varrho_t(i) X_{t,s}^p)^{-\varepsilon_{t+s}} y_{t+s},$$

$$X_{t,s}^p = \frac{1}{x_t^p} \prod_{k=0}^{s} x_{t+k}^p,$$

$$x_t^p = \frac{\pi_{t-1} \pi_{s}^{1-\varepsilon_p}}{\pi_t}.$$
All intermediate good producers that can re-optimize will choose the same price, \( q_t = q_t(i) \), \( \forall i \).

The dynamics of the aggregate price level follow from

\[
1 = \zeta_p (x^p_t)^{1-\epsilon^p} + (1 - \zeta_p) q_t^{1-\epsilon^p}.
\]

### 3.3.3 Mapping labor into employment

The model concept of labor input developed above is hours worked, while our observable measure as described in Section 2 is full-time equivalents. We therefore use a similar trick to that described in Adolfson et al. (2008), which is to assume that firms are also Calvo-constrained in their choice of employment level. The optimality condition is

\[
0 = E_t \sum_{s=0}^{\infty} (\zeta_e \beta)^s X_{t-1,s} (n_e t (i) X_{t-1,s} - L_{t+s}),
\]

where \( \Psi_t \) is aggregate employment, \( \zeta_e \) is the fraction of firms that index employment to past aggregate employment growth, \( n_e \) is the fixed number of hours worked per employee, and the compound indexation term is

\[
X^e_{t,s} \equiv \frac{1}{x^e_t} \prod_{k=0}^{s} x^e_{t+k},
\]

\[
x^e_t = \left( \frac{\Psi_t}{\Psi_{t-1}} \right)^{\gamma_e} z^L 1^{-\epsilon_e}.
\]

All optimizing firm chooses the same level of employment \( e_t (i) = e_t \). Total employment is a weighted average of the constrained and unconstrained levels of employment, and the relationship can be written as follows

\[
1 = (1 - \zeta_e) \frac{e_t}{\Psi_t} + \zeta_e \Psi_{t-1} x^e_t \frac{\Psi_{t-1} x^e_t}{\Psi_t}.
\]

The interplay between the optimality and aggregation conditions will yield a double-sided smoothing filter from hours to employment.
3.4 Households

A continuum of households decide on consumption $C_t(j)$, hours worked $L_t(j)$, bonds $B_t(j)$, investment $I_t(j)$, capital $K_t(j)$, and capital utilization $U_t(j)$, so as to maximize the objective function

$$E_t \sum_{s=0}^{\infty} \beta^s \left( d_{t+s} \ln (C_{t+s}(j) - \lambda C_{t+s-1}(j)) - \varphi_{t+s} \frac{\psi}{1+\eta} L_{t+s}(j)^{1+\eta} \right),$$

where the degree of habit formation is captured by parameter $\lambda$ and $d_t$ is an inter-temporal preference shock which follows the AR(1) process

$$\ln(d_t) = \rho_d \ln(d_{t-1}) + v_t^d, \quad v_t^d \sim N(0, \sigma^2_d),$$

We assume the preference displacement $\varphi_t$ follows a random walk with drift,

$$\ln(\varphi_t) = \gamma_{\varphi} + \ln(\varphi_{t-1}) + v_t^\varphi, \quad v_t^\varphi \sim N(0, \sigma^2_{\varphi}),$$

where $\gamma_{\varphi}$ will eventually capture the upward drift in labor market participation. Households own raw capital $K_{t-1}(j)$, decide on the capital utilization rate $U_t(j)$, and rent effective capital $K_t^*(j)$ to firms at rate $r_t^k$. Effective capital is related to raw capital by

$$K_t^*(j) = U_t(j) K_{t-1}(j),$$

and real capital utilization costs are

$$p_t^i \Phi(U_t(j)) K_{t-1}(j),$$

where $\Phi(U_t(j))$ is an increasing and convex function which equals zero in the steady state—$\Phi'(\cdot) \geq 0$, $\Phi''(\cdot) \geq 0$ and $\Phi(U_*) = 0$. Note that we price utilization costs as investment instead of output. We have the freedom to normalize the steady-state utilization rate, so we set it to unity

$$U_* = 1.$$
Households face the following budget constraint in real terms,

\[(1 + \tau_c) p_{t+s}^c C_{t+s} (j) + \sigma_{t+s}^d I_{t+s} (j) + B_{t+s}^P (j) =

(1 - \tau_w) \omega_{t+s}^h L_{t+s} (j) + \left( (1 - \tau_k) r_{t+s}^k U_{t+s} (j) + \sigma_{t+s}^d (\delta \tau_k - \Phi (U_{t+s} (j))) \right) K_{t+s-1} (j) +

\frac{R_{t+s-1} H_{t+s-1}}{\pi_{t+s}} B_{t+s-1}^P (j) + T_{t+s} y_{t+s}^d + D_{t+s}^u + D_{t+s}^p + D_{t+s}^m + D_{t+s}^e ,

\]

where, again, small letters indicate relative prices and wages versus the domestic good, e.g. \( p_c^t = \frac{P_c^t}{P_t} \). Moreover, \( T_t \) is the ratio of lump-sum transfers to real domestic GDP and \( D_t^u, D_t^p, D_t^m \) and \( D_t^e \) are real profits from labor unions, intermediate good producers, importing firms, and exporting firms, respectively. \( B_t^P \) represents private sector assets—equal to net foreign assets \( B_t^F \) plus government debt \( B_t^G \)—and the risk premium \( H_t \) on bonds depends on the ratio of foreign assets to GDP, \(^3\)

\[ H_t = H \left( \frac{B_t^F}{y_t^d}, \varepsilon_t^b \right) , \]

where \( \varepsilon_t^b \) is a risk premium shock. We let \( \ln (\varepsilon_t^b) \) follow the AR(1) process

\[ \ln (\varepsilon_t^b) = \rho_b \ln (\varepsilon_{t-1}^b) + \upsilon_t^b, \quad \upsilon_t^b \sim N (0, \sigma_b^2) . \]

Households accumulate capital as follows,

\[ K_{t+s} (j) = (1 - \delta) K_{t+s-1} (j) + \mu_{t+s} \left( 1 - S \left( \frac{I_{t+s} (j)}{I_{t+s-1} (j)} \right) \right) I_{t+s} (j) , \]

where \( S (\cdot) \) is the investment adjustment cost function—with \( S (\cdot) = 0, S' (\cdot) = 0 \) along the balanced-growth path and \( S'' (\cdot) > 0 \)—and \( \mu_t \) is an investment-specific technology shock. We let \( \ln (\mu_t) \) follow the AR(1) process:

\[ \ln (\mu_t) = \rho_\mu \ln (\mu_{t-1}) + \upsilon_t^{\mu}, \quad \upsilon_t^{\mu} \sim N (0, \sigma_\mu^2) . \]

\(^3\)The exogeneity of the interest rate requires we close the model with a feedback mechanism such as a debt-dependent risk premium, à la Schmitt-Grohe and Uribe (2003).
Household maximization of utility subject to the budget constraint and capital accumulation yields the following first-order conditions:

\[
\frac{d_t}{C_t(j) - \lambda C_{t-1}(j)} - \beta \lambda E_t \left( \frac{d_{t+1}}{C_{t+1}(j) - \lambda C_t(j)} \right) = \Xi_t(j) p^c_t (1 + \tau_c),
\]

\[
\varphi_t \psi L_t(j) = \Xi_t(j) (1 - \tau_w) w^h_t,
\]

\[
\varphi_t \psi L_t(j) = \Xi_t(j) R_t H_t,
\]

\[
\varphi_t \psi L_t(j) = \Xi_t(j) \left( 1 - \tau_w \right) w_t,
\]

\[
\varphi_t \psi L_t(j) = \Xi_t(j) \left( 1 - \tau_c \right) w_t.
\]

where \(\Xi_t(j)\) and \(\Xi^k_t(j)\) are the Lagrange multipliers associated with the budget constraint—expressed in real terms—and the capital accumulation equation, respectively. Tobin’s \(Q_t\) is defined as the ratio of these multipliers

\[
Q_t(j) = \Xi^k_t(j) \Xi_t(j).
\]

In equilibrium households make the same choices. For this reason, we can drop the index \(j\).\(^4\)

### 3.5 Labor unions and packers

This section develops the ingredients for a wage Phillips curve with a backward-looking component. Formally, a continuum of monopolistically competitive labor unions intermediate between households and labor packers, buying homogenous labor from households at real wage \(w^h_t\), differentiating the labor one-to-one, and selling the differentiated labor \(L_t(l)\) to labor packers at real wage \(w_t(l)\). Perfectly competitive labor packers then create the composite labor bundle \(L_t\)

\(^4\)Nevertheless, we cannot drop the index of \(L_t(j)\), because \(L_t\) will already be used for the composite labor bundle. In the section about the aggregate resource constraints, we show that up to a log-linear approximation we can actually drop the index \(j\).
by combining the continuum of differentiated labor $L_t(l)$ and sell the bundle to intermediate good producers at real wage $w_t$.

### 3.5.1 Labor packers

The setup for the labor packers is similar to the setup for the final good producers. Profit maximization yields demand curves and a price index

$$L_t(l) = \left( \frac{w_t(l)}{w_t} \right)^{-\varepsilon^w_t} L_t,$$

$$w_t = \left( \int_0^1 w_t(l)^{1-\varepsilon^w_t} dl \right)^{-\frac{1}{1-\varepsilon^w_t}},$$

where we let the (log)-elasticity $\ln(\varepsilon^w_t)$ follow the ARMA(1,1) process

$$\ln(\varepsilon^w_t) = (1 - \rho_w) \ln(\varepsilon^w_{t-1}) + \nu^w_t + \theta^w_t \ln(\varepsilon^w_t - 1), \nu^w_t \sim N(0, \sigma^2_w).$$

and interpret this process as a wage mark-up shock.

### 3.5.2 Labor unions

Labor unions are restricted in the timing for re-optimization à la Calvo. Indexation is expressed as a function of nominal wages. Let $\omega_t(l)$ denote the optimal wage for labor union $l$ in period $t$. Optimization requires

$$0 = E_t \sum_{s=0}^{\infty} \zeta^s \beta^s \frac{\overline{x}_{t+s}}{x_t} \left( 1 - \varepsilon^w_{t+s} \right) \frac{\omega_t(l)}{w_t} X^w_{t,s} w_t + \varepsilon^w_{t+s} w_t \right) L_{t+s|t}(l),$$

where individual labor demand curves and indexation terms are

$$L_{t+s|t}(l) = \left( \frac{\omega_t(l)}{w_t} X^w_{t,s} \right)^{-\varepsilon^w_{t+s}} L_{t+s},$$

$$X^w_{t,s} = \frac{1}{x_t} \prod_{k=0}^{s} x^w_{t+k},$$

$$x_t^w = \frac{(\pi^w t-1) \gamma^w_t}{\pi^w t}.$$
and $z^w_t \equiv \frac{w_t}{w_{t-1}}$ is aggregate real wage growth and $\gamma_w$ is steady-state wage growth (which will be defined in a later section). All re-optimizing labor unions choose the same price, that is $\omega_t = \omega_t (l)$, $\forall l$. The dynamics of the aggregate wage level follow from
\[
1 = \zeta_w (x^w_t)^{1-\epsilon^w_t} + (1 - \zeta_w) \left( \frac{\omega_t}{w_t} \right)^{1-\epsilon^w_t}.
\]

3.6 Foreign block

The structure of the foreign block is simple and mirrors the pricing environment of goods and labor: in each sector, the interaction between pricing power and Calvo stickiness with indexation yields a demand curve and a Phillips curve with a backward-looking component for the relevant price index.

3.6.1 Import

The import channel consists of two types of firms for both the consumption and the investment goods. Consider the consumption good; relationships derived below are equivalent for the investment good. A continuum of monopolistically competitive importing firms buy the homogeneous foreign good in the world market at price $p^f_t$, differentiate the foreign good—one-to-one—, and sell the differentiated good $C^m_t (i)$ to good packers at price $p^c,m_t (i)$. Perfectly competitive good packers then create the composite good bundle $C^m_t$ by combining the continuum of differentiated goods $C^m_t (i)$ and sell the composite good bundle to households at price $p^c,m_t$. Again, profit maximization of packers yields demand curves and a price index:
\[
C^m_t (i) = \left( \frac{p^c,m_t (i)}{p^c,m_t} \right)^{\epsilon^c,m_t} C^m_t, \\
p^c,m_t = \left( \int_0^1 \frac{1}{p^c,m_t (i)^{1-\epsilon^c,m_t} di} \right)^{\frac{1}{1-\epsilon^c,m_t}},
\]
where the (log-)elasticity $\ln(\epsilon^c,m_t)$ follows the ARMA(1,1) process
\[
\ln(\epsilon^c,m_t) = (1 - \rho_{c,m}) \ln(\epsilon^c,m_{t-1}) + \rho_{c,m} \ln(\epsilon^c,m_t) + \nu^c,m_t + \theta_{c,m} \nu^c,m_{t-1}, \nu^c,m_t \sim N (0, \sigma^2_{c,m}).
\]
Let \( \varrho_{t}^{c,m}(i) \) denote the optimal price for importer \( i \) in period \( t \). Optimization requires

\[
0 = E_{t} \sum_{s=0}^{\infty} \zeta_{c,m}^{s} \beta^{s} \left( (1 - \varepsilon_{t+s}^{c,m}) \frac{\varrho_{t+s}^{c,m}}{p_{t+s}} X_{t,s}^{c,m} p_{t+s} + \varepsilon_{t+s} p_{t+s} \right) C_{t+s|t}^{m},
\]

where the compounded indexation term is defined as

\[
C_{t+s|t}^{m} = \left( \frac{\varrho_{t+s}^{c,m}}{p_{t+s}} X_{t,s}^{c,m} C_{t+s|t}^{m} \right)^{-\varepsilon_{t+s}^{c,m}},
\]

\[
X_{t,s}^{c,m} = \prod_{k=0}^{s} x_{t+k},
\]

\[
x_{t}^{c,m} = \frac{(\pi_{t-1}^{c,m})^{1-\varepsilon_{c,m}}}{\pi_{t}^{c,m}}.
\]

The dynamics of the consumption import price index follow from

\[
1 = \zeta_{c,m} \left( x_{t}^{c,m} \right)^{1-\varepsilon_{c,m}} + \left( 1 - \zeta_{c,m} \right) \left( \frac{\varrho_{t}^{c,m}}{p_{t}^{c,m}} \right)^{1-\varepsilon_{t}^{c,m}}.
\]

Identical equations hold for the investment import channel.

### 3.6.2 Export

A continuum of monopolistically competitive exporting firms buy the homogeneous domestic final good in the domestic market at unit (relative) price, differentiate the domestic final good—one-to-one—, and sell the differentiated good \( y_{t}^{x}(i) \) to good packers at price \( p_{t}^{x}(i) \). Perfectly competitive good packers then create the composite export bundle \( y_{t}^{x} \) by combining the continuum of differentiated goods \( y_{t}^{x}(i) \) and sell the composite export bundle in the world market at price \( p_{t}^{x} \).

To start with world trade, we assume that Dutch exporting firms—in contrast to Dutch importing firms—have pricing power in the world market. The market form is—again—monopolistic competition, and the foreign demand for export goods is thus

\[
y_{t}^{x} = n_{t}^{f} \left( \frac{p_{t}^{x}}{p_{t}} \right)^{-\varepsilon_{t}^{f}} y_{t}^{f},
\]
where world output is denoted by $y^f_t$, the world price is denoted by $p^f_t$, $\varepsilon^f$ is the elasticity of foreign demand, and $n^f_t$ is a demand-shifter (akin to a home-bias parameter) which follows the ARMA(1,1) process in logs

$$ \ln \left( n^f_t \right) = (1 - \rho^f) \ln \left( n^f \right) + \rho^f \ln \left( n^f_{t-1} \right) + v^f_t + \theta^f v^f_{t-1}, \quad v^f_t \sim N \left( 0, \sigma^2_f \right). $$

As above, profit maximization of packers yields

$$ y^x_t (i) = \left( \frac{p^x_t (i)}{p^f_t} \right)^{-\varepsilon^x_t} y^x_t, \\
 p^x_t = \left( \frac{1}{\int_0^1 p^x_t (i)^{1-\varepsilon^x_t} di} \right)^{\frac{1}{1-\varepsilon^x_t}}, $$

where the elasticity $\varepsilon^x_t$ follows the ARMA(1,1) process in logs

$$ \ln (\varepsilon^x_t) = (1 - \rho^x) \ln (\varepsilon^x) + \rho^x \ln (\varepsilon^x_{t-1}) + v^x_t + \theta^x v^x_{t-1}, \quad v^x_t \sim N \left( 0, \sigma^2_x \right). $$

Optimal pricing by exporters yields

$$ 0 = E_t \sum_{s=0}^{\infty} \zeta^x \beta^x \bar{\Xi}^{t+s}_t \left( (1 - \varepsilon^x_t) \frac{\partial^x}{p^x_t} X^x_{t,s} \bar{p}^{t+s}_t + \varepsilon^x_t \right) y^x_{t+s|t}, \\
y^x_{t+s|t} = \left( \frac{\partial^x}{p^x_t} X^x_{t,s} \right)^{-\varepsilon^x_{t+s}} y^x_{t+s}, \\
X^x_{t,s} = \frac{1}{x^x_t} \prod_{k=0}^{s} x^x_{t+k}, \\
x^x_t = \frac{(\pi^x_{t-1})^{\varepsilon^x} (\pi^x_t)^{1-\varepsilon^x}}{\pi^x_t}$$

The dynamics of the export price index follow from

$$ 1 = \zeta^x \left( x^x_t \right)^{1-\varepsilon^x} + (1 - \zeta^x) \left( \frac{\partial^x}{p^x_t} \right)^{1-\varepsilon^x}.$$
3.6.3 Net foreign asset accumulation

The net foreign asset accumulation follows from the balance of payments,

\[ B_t^F + p_t^f (C_t^m + I_t^m) = \frac{R_{t-1} H_{t-1}}{\pi_t} B_{t-1}^F + p_t^x y_t^x. \]

3.6.4 Foreign variables

Since the Dutch economy is small relative to the Eurozone—which we consider to be the rest of the world—we assume there is no feedback from domestic variables onto foreign variables. The foreign variables—world output \( y_t^f \), world price \( p_t^f \), and interest rate \( R_t^f \)—are modeled in an exogenous block consisting of a Eurozone IS curve, a Phillips curve and a Taylor rule. World output \( y_t^f \) enters the model via the export channel and the world price \( p_t^f \) via the import channel. We further assume that the Dutch interest rate is equal to the interest rate set by the European Central Bank (ECB), which affects every inter-temporal decision margin in the model via the stochastic discount factor. Since the three-equation system is standard, we relegate its description to Appendix F.

3.7 Bundling consumption and investment

The decision problem here is to divide consumption expenditures \( C_t \) into domestically produced goods \( C_t^d \) and imported goods \( C_t^m \). The equations below are derived for the consumption bundle and are similar for investment.

Bundling takes the form of a CES aggregator, which is written here in normalized form (see Cantore et al. (2010)):

\[ C_t = C_e \left( n_c \left( \frac{d.c}{z_t^d} \right)^{\frac{b.c - 1}{b.c}} \frac{z_t^c}{C_t^d} \right)^{\frac{b.c - 1}{b.c}} + (1 - n_c) \left( \frac{m.c}{z_t^m} \right)^{\frac{b.c - 1}{b.c}} \frac{z_t^c}{C_t^m} \right)^{\frac{b.c - 1}{b.c}}, \]

where \( n_c \) is a home-bias parameter, and \( z_t^d \) and \( z_t^m \) are stochastic processes (to be defined later in the section about detrending). Minimizing the expenditure index \( Z_t^c = C_t^d + p_t^c m C_t^m \) for
any given consumption level $C_t$ yields two demand functions for $C^m_t$ and $C^d_t$ are respectively

$$p_t^{c,m} = (1 - n_c) \left( z_t^{m,c} \frac{C^{m}_t}{C^{m}_x} \right)^{\frac{b,c - 1}{b,c}} \frac{1}{C^{m}_t},$$

$$1 = n_c \left( z_t^{d,c} \frac{C^{d}_t}{C^{d}_x} \right)^{\frac{b,c - 1}{b,c}} \frac{1}{C^{d}_t}.$$

After a lot of tedious algebra, we can express these two demand functions and the aggregate price definition as follows

$$\frac{C^{d}_t}{p_t C_t} = n_c \left( z_t^{d,c} \frac{C^{d}_t}{C^{d}_x} \right)^{1 - \frac{b,c - 1}{b,c}},$$

$$\frac{p_t^{c,m} C^{m}_t}{p_t^{c} C^{m}_t} = 1 - n_c \left( z_t^{d,c} \frac{C^{d}_t}{C^{d}_x} \right)^{1 - \frac{b,c - 1}{b,c}},$$

$$\left( z_t^{d,c} \frac{C^{d}_t}{p_t} \right)^{1 - \frac{b,c}{b,c}} = n_c + (1 - n_c) F^c_t,$$

with

$$F^c_t \equiv \left( z_t^{d,c} \frac{p_t^{c,m} C^{m}_t}{p_t^{c} C^{m}_t} \right)^{1 - \frac{b,c}{b,c}}.$$

Note the role of the home bias parameters and shift factors $z_t^{d,x}$ and $z_t^{m,x}$ in relative shares. Equivalent equations hold for the investment bundle.

### 3.8 Government

The budget constraint of the government is

$$b_t y_t^d + \tau_c p_t^c C_t + \tau_w w_t^d L_t + \left( R_k^k U_t - p_t^d \delta \right) \tau_k K_{t-1} = \frac{R_{t-1}}{\pi_t} b_{t-1} y_{t-1}^d + T_t y_t^d + g_t y_t^d,$$

where $b_t$ and $g_t$ are defined as the ratios to output of outstanding domestic debt and government expenditures, respectively. We let $g_t$ follow the AR(1) process

$$\ln (g_t) = (1 - \rho_g) \ln (g) + \rho_g \ln (g_{t-1}) + v_t^g, \ v_t^g \sim N(0, \sigma_g^2).$$

25
The tax (transfer) rule of the government reacts to deviations from the target debt-to-GDP ratio $b_\ast$ with a lag,

\[
\frac{T_t}{T_\ast} = \left( \frac{b_{t-1}}{b_\ast} \right)^{-\varepsilon_T}.
\]

### 3.9 Aggregate resource constraints

In practice, we do not have data on the composite labor bundle but on hours worked (in fact, on employment. See subsection 3.3.3). That is, we observe $\tilde{L}_t \equiv \int_0^1 L_t(j) \, dj$ rather than $L_t$. They are related as follows,

\[
\tilde{L}_t = L_t \int_0^1 \left( \frac{w_t(l)}{w_t} \right)^{-\varepsilon_w} \, dl. \tag{5}
\]

After log-linearizing, we obtain $\tilde{L}_t \simeq L_t$.

Similarly, observed output $y^d_t$ is related to the final good bundle $y_t$:

\[
y_t^d = \int_0^1 y_t(i) \, di = a_t A_t (K_t^{\alpha}) (L_t)^{1-\alpha} - z_t \Phi,
\]

Domestic good market clearing follows from

\[
(1 - g_t) y_t^d = C_t^d + I_t^d + X_t + p_t^d \Phi (U_t) K_{t-1},
\]

which is consistent with combining the household budget constraint—integrated over the continuum households—, the government budget constraint, and the net foreign asset accumulation equation. Observed export are denoted by $X_t$, which is—again—approximately equal to the composite export bundle $y_t^x$. For convenience, we already write

\[
X_t = y_t^x.
\]

Finally, we do not have separate data on consumption import and investment import. Since

---

5 Recall that households are indexed by $j$, labor unions by $l$ and firms by $i$. 

---
we will use import data in our estimation procedure, we introduce notation for total imports,
\[ M_t = C_t^m + I_t^m. \]

4 De-trending, solving, and log-linearizing the model

We now must determine how steady-state growth rates of all variables are related to the exogenous stochastic trends. This was a trivial exercise in Smets and Wouters’ model, where all trending variables expanded at the same deterministic rate, but it is more involved when multiple stochastic trends have to be accounted for.

The three exogenous domestic stochastic trends are the following:

\[
\begin{align*}
\Delta \ln A_t &= \gamma_A + v_t^A, \quad v_t^A \sim N (0, \sigma_A^2), \\
\Delta \ln \phi_t &= \gamma_\phi + v_t^\phi, \quad v_t^\phi \sim N (0, \sigma_\phi^2), \\
\Delta \ln \Gamma_t &= \gamma_\Gamma + v_t^\Gamma, \quad v_t^\Gamma \sim N (0, \sigma_\Gamma^2).
\end{align*}
\]

The first two processes \( A_t \) and \( \phi_t \) were introduced in the model description. The third is discussed below.

We consider the trend-cycle decomposition of all variables in the model as follows,

\[
\ln X_t = \ln \bar{X}_t + \ln \tilde{X}_t,
\]

for given variable \( X_t \), where trend and cycle follow an integrated and a stationary process, respectively,

\[
\begin{align*}
\Delta \ln \bar{X}_t &= \gamma_X + \epsilon_{xt}, \\
A(L) \left( \ln \bar{X}_t - \ln \bar{X}_t \right) &= u_{xt}, \quad A(1) \neq 0,
\end{align*}
\]

for some model-specific parameters and innovations. Steady-state paths of variables—denoted by \( X_{st} \), and around which we will log-linearize the model—are defined by setting both component
innovations to zero

\[ \Delta \ln \bar{X}_t = \gamma_X, \]
\[ \ln \bar{X}_t = \ln X_* . \]

The steps for determining the stochastic trend components are straightforward but lengthy, and are therefore relegated to various appendices. First, Appendix B.1 relates the \( \gamma_X \)'s to the exogenous drifts \( \gamma_A, \gamma_\varphi, \) and \( \gamma_\Gamma \). This yields a system of \( n \) equations in \( n \) unknowns. Appendix B.2 then solves this system of drifts recursively. In turn, Appendix B.3 decomposes every model variable in a trend and a cycle. The upshot is that the three exogenous processes \( A_t, \varphi_t, \) and \( \Gamma_t \) can be recovered easily with the trend components of wages \( \bar{w}_t \), labor \( \bar{L}_t \), and relative investment price \( \bar{p}_t \).

\[ \bar{p}_t = \Gamma_t \]
\[ \bar{L}_t = \varphi_t^{-\frac{1}{1+\alpha}} \]
\[ \bar{w}_t = A_t^{\frac{1}{1-\alpha}} \Gamma_t^{-\frac{\alpha}{1-\alpha}} \]

In assuming that \( \bar{p}_t = \Gamma_t \), we simply follow the lead of Justiniano et al.. Appendix B.3 discusses this choice more thoroughly. The second relationship is the formalization of the argument put forward when describing our data-set, namely that a stationary labor share imposes that the permanent preference shock offset the trend in labor supply. The third relationship reflects the fact that if the labor share is indeed stationary, real wages must grow at the same rate as labor productivity, which is a weighted average of general-purpose and investment-specific productivity. Equivalently, the relationship follows from the restriction that marginal costs be constant in the long run. The stochastic trend components of all the other domestic variables are then determined recursively from these three processes.

With all trends in hand, we can de-trend the model equations, solve for the steady state and log-linearize around it. These three steps involve yet more algebra and bring no particularly new insights. Therefore, we relegate the long pages of equations to Appendices C, D and E. Nevertheless, it is worth noting that the stochastic trend shocks appear in many instances in the
log-linearized equations—in fact, whenever an inter-temporal trade-off is involved. This enriches substantially the impulse and propagation mechanisms of the model.

5 Bayesian estimation

We assume the reader is familiar with Bayesian estimation of DSGE models. We use Dynare to solve and estimate the model as summarized by all the equations in Appendix E (see the reference to the latest Dynare manual in the bibliography). By solving, we mean finding the reduced form of the model and its state-space representation that makes it amenable to computing data log-likelihoods via the Kalman filter. By estimating, we mean using Markov-Chain Monte-Carlo (MCMC) techniques to obtain posterior information on a subset of model parameters.

5.1 Calibration and priors

We use a data-set of 14 variables to estimate the model: Dutch domestic GDP $y^{obs}$ and its demand components $C^{obs}$, $I^{obs}$, $G^{obs}$, $X^{obs}$, and $M^{obs}$; full-time employment equivalents per capita $E^{obs}$; deflator inflation for GDP $d\ln P^{obs}$, investment $d\ln P^{i,obs}$, exports $d\ln P^{x,obs}$ and wages $d\ln w^{obs}$; Eurozone GDP per capita $y^{f,obs}$, GDP deflator inflation $d\ln P^{f,obs}$ and short-term interest rate $R^{f,obs}$. All of these variables were plotted in Figures 1 to 3 in Section 2. We perform our estimation over the sample 1984Q1 to 2007Q4, that is, up to the onset of the financial crisis.

5.1.1 Calibrated parameters

Calibrated parameters are listed in Tables 1 and 2. Table 1 reports parameters that can be easily traced back to first moments of variables, while Table 2 reports those that are less immediately identifiable from the set of observables.

The first subset of parameters in Table 1 consists of the drifts of the model’s stochastic processes, expressed in quarterly terms. We could estimate these drifts but choose not to in order to limit the space of parameters to estimate, which is already very large. We do not consider calibrating these parameters to be particularly objectionable because these first moments are the lowest hanging fruits when taking the model to the data and are in principle easily identifiable.
<table>
<thead>
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<th>Parameter</th>
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<th>Value</th>
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<td>$\gamma_I$</td>
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<tr>
<td>$\gamma_M$</td>
<td>import growth</td>
<td>0.0128</td>
</tr>
<tr>
<td>$\gamma_X$</td>
<td>export growth</td>
<td>0.0133</td>
</tr>
<tr>
<td>$\pi^M$</td>
<td>SS import price inflation</td>
<td>0.0015</td>
</tr>
<tr>
<td>$\pi^X$</td>
<td>SS export price inflation</td>
<td>0.0015</td>
</tr>
<tr>
<td>$1 - \alpha$</td>
<td>SS labor share</td>
<td>0.52</td>
</tr>
<tr>
<td>$I^<em>/Y^</em>$</td>
<td>SS investment-to-GDP ratio</td>
<td>0.21</td>
</tr>
<tr>
<td>$I^<em>/C^</em>$</td>
<td>SS investment-to-consumption ratio</td>
<td>0.43</td>
</tr>
<tr>
<td>$g^*$</td>
<td>SS government spending-to-GDP ratio</td>
<td>0.23</td>
</tr>
<tr>
<td>$b^*$</td>
<td>SS domestic debt-to-GDP ratio</td>
<td>0.60</td>
</tr>
<tr>
<td>$\pi_*$</td>
<td>target inflation rate</td>
<td>0.0048</td>
</tr>
<tr>
<td>$\beta$</td>
<td>discount factor</td>
<td>0.985</td>
</tr>
<tr>
<td>$R^*$</td>
<td>SS short-term nominal interest rate</td>
<td>0.0048</td>
</tr>
</tbody>
</table>

Table 1. Calibrated Parameter Values

The second subset consists of great ratios, which we use to solve for other parameters. Thus, we pin down the labor elasticity of output $1 - \alpha$ with the observable labor share. Similarly, we fix the steady-state nominal great ratios $I^*/Y^*$, $C^*/I^*$ and $G^*/Y^* (\equiv g^*)$ to the relevant sample averages and use them to solve for the depreciation rate $\delta$ and the foreign assets-to-GDP ratio $b^f$ with the steady-state relationships described in Appendix D. On the other hand, we benchmark the domestic debt-to-GDP ratio $b^*$ to the Maastricht criteria, which is not a restrictive choice since we have sufficient degrees of freedom in the fiscal block of the model to choose so.

The third subset consists of nominal variables, expressed in quarterly terms. The steady-state inflation rate $\pi_*$ is calibrated to the target rate of the ECB (so that $\pi_* = \pi^f$), and the steady-state interest rate $R^*$ to the mean of the post-convergence period (the second half of the sample), as depicted in Figure 3. Together with the steady-state growth rate of output, these parameterizations imply a particular value for the steady-state growth-adjusted real interest rate. Recalling Figure 3, we pick a standard value for the discount factor $\beta$ and reconcile it with the sample mean of the growth-adjusted real interest rate by solving for the steady-state bond
Table 2. Calibrated Parameter Values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\tau^c)</td>
<td>consumption tax</td>
<td>0.11</td>
</tr>
<tr>
<td>(\tau^w)</td>
<td>labor income tax</td>
<td>0.34</td>
</tr>
<tr>
<td>(\tau^k)</td>
<td>capital income tax</td>
<td>0.22</td>
</tr>
<tr>
<td>(\varepsilon^T)</td>
<td>debt elasticity of fiscal transfers</td>
<td>5.e-4</td>
</tr>
<tr>
<td>(n^c)</td>
<td>consumption home bias</td>
<td>0.3</td>
</tr>
<tr>
<td>(n^i)</td>
<td>investment home bias</td>
<td>0.4</td>
</tr>
<tr>
<td>(n^x)</td>
<td>share of domestic exports in foreign demand</td>
<td>0.4</td>
</tr>
<tr>
<td>(\varepsilon^w)</td>
<td>wage elasticity of labor supply</td>
<td>21</td>
</tr>
<tr>
<td>(\varepsilon^p)</td>
<td>price elasticity of output</td>
<td>11</td>
</tr>
<tr>
<td>(\varepsilon^X)</td>
<td>price elasticity of exports</td>
<td>11</td>
</tr>
<tr>
<td>(\varepsilon^{c,m})</td>
<td>price elasticity of consumption imports</td>
<td>11</td>
</tr>
<tr>
<td>(\varepsilon^{i,m})</td>
<td>price elasticity of investment imports</td>
<td>11</td>
</tr>
<tr>
<td>(\varepsilon^{b,c})</td>
<td>consumption bundle elasticity</td>
<td>6</td>
</tr>
<tr>
<td>(\varepsilon^{b,i})</td>
<td>investment bundle elasticity</td>
<td>6</td>
</tr>
<tr>
<td>(\eta)</td>
<td>labor supply elasticity</td>
<td>2</td>
</tr>
<tr>
<td>(\psi)</td>
<td>labor supply coefficient</td>
<td>7.5</td>
</tr>
<tr>
<td>(\iota_E)</td>
<td>employment indexation</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 2. Calibrated Parameter Values

premium \(H_*\) with equation (32) (referenced in Appendix C).

Table 2 first lists a number of parameters that are unidentifiable unless we add more relevant variables to the vector of observables. We thus calibrate all fiscal parameters (tax rates and transfer elasticity) and shares of imports (in consumption \(n^c\) and in investment \(n^i\)) from data provided by the Dutch statistical office. We also set the share of exports to Eurozone GDP \(n^x\) to the mean of the sample.

The next subset consists of price elasticities, which turn out to be structurally unidentified in the model, as can be seen from every log-linearized Phillips curve derived in Appendix E. The intuition is as follows. With decreasing-returns-to-scale technologies and pricing power, marginal costs are increasing in production, and thus in demand and relative prices. This feedback scales up the price elasticity of costs relative to revenue, so that any exogenous change to the optimality condition equalizing marginal cost to marginal revenue allows one to identify the slope of the demand curve. However, when returns to scale are constant, this feedback is
absent and the margins move one-for-one. Since all relevant technologies in the model in fact exhibit constant returns, the associated elasticities are unidentified. The exception is the price elasticity of exports, $\varepsilon^f$, because this demand function is directly observable (its determinants are all part of the vector of observable variables).

Furthermore, we fix the value of some parameters related to the labor supply block, which is over-parameterized in the model because of the mapping of hours $L$ into employment $E$. We estimate the stickiness parameter $\zeta_e$ and calibrate the labor supply elasticity $\eta$. We eliminate employment indexation ($\iota_e = 0$) because the filter from hours to employment is smooth enough without over-parameterizing it. Thus empirically, our employment Euler equation is identical to that in Adolfson et al. (2008). We also follow them in choosing the same value for the coefficient $\psi$, which cannot be identified from the dynamics of labor supply.

Lastly, since there is no feedback from domestic developments to the Eurozone block, we calibrate the parameters of the latter to the posterior means obtained by estimating the block separately. Priors, posteriors and estimation issues for this block are discussed in Appendix F.

### 5.1.2 Priors

Priors are described in Tables 3 and 4. Table 3 reports the priors attached to structural parameters affecting the endogenous propagation mechanisms of the model, while Table 4 lists priors for parameters related to the exogenous processes (autocorrelations and standard deviations).

The first set of distributions in Table 3 relates to parameters affecting the households’ real decision margins (consumption, investment, utilization and bond holdings). The choice for the habit formation parameter is standard. The other three priors are close to flat with wide support. The only elasticity we estimate is the export demand elasticity $\varepsilon^f$, for which we choose a prior centered on its least-squares estimate.

The second set of priors relate to the nominal margins. Calvo parameters $\zeta_i$ have relatively flat beta distributions centered on .5; they are restricted to lie between 0 and 1 because they represent probabilities. On the other hand—and we depart from existing literature on this point—we impose normal priors centered on zero and truncated between -1 and 1 for index-

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6Priors are depicted in Figures 4 to 7.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Prior distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda$</td>
<td>habit formation</td>
<td>$B(.5,.2)$</td>
</tr>
<tr>
<td>$\Phi''(1)$</td>
<td>utilization cost</td>
<td>$IG(.1,.1)$</td>
</tr>
<tr>
<td>$z_1S''(z_1)$</td>
<td>investment cost</td>
<td>$N_{[0,\infty]}(20,20)$</td>
</tr>
<tr>
<td>$a_{h,b}$</td>
<td>bond premium elasticity</td>
<td>$IG(.1,.1)$</td>
</tr>
<tr>
<td>$\varepsilon_f$</td>
<td>price elasticity of export demand</td>
<td>$N(1.5,.5)$</td>
</tr>
<tr>
<td>$\zeta_p$</td>
<td>price stickiness</td>
<td>$B(.5,.2)$</td>
</tr>
<tr>
<td>$\zeta_w$</td>
<td>wage stickiness</td>
<td>$B(.5,.2)$</td>
</tr>
<tr>
<td>$\zeta_e$</td>
<td>employment stickiness</td>
<td>$B(.5,.2)$</td>
</tr>
<tr>
<td>$\zeta_x$</td>
<td>export price stickiness</td>
<td>$B(.5,.2)$</td>
</tr>
<tr>
<td>$\zeta_{c,m}$</td>
<td>consumption import price stickiness</td>
<td>$B(.5,.2)$</td>
</tr>
<tr>
<td>$\iota_p$</td>
<td>price indexation</td>
<td>$N_{[-1,1]}(0,.5)$</td>
</tr>
<tr>
<td>$\iota_w$</td>
<td>wage indexation</td>
<td>$N_{[-1,1]}(0,.5)$</td>
</tr>
<tr>
<td>$\iota_x$</td>
<td>export price indexation</td>
<td>$N_{[-1,1]}(0,.5)$</td>
</tr>
<tr>
<td>$\iota_{c,m}$</td>
<td>consumption import price indexation</td>
<td>$N_{[-1,1]}(0,.5)$</td>
</tr>
</tbody>
</table>

Note: $B$: Beta; $IG$: Inverse Gamma; $N$: Normal. Arguments are mean, standard deviation, and support (when truncated).

Table 3. Prior distributions 1

The reason is that indexation schemes are weighted averages of relevant lagged and steady-state inflation, but mathematically, the weights are not restricted to lie on the unit interval. It is worth noting that, historically, indexation was introduced in DSGE modeling to account for the positive empirical autocorrelation of US inflation (the log-linear Phillips curves in Appendix E make this transparent). However, the US experience—a first-order AR coefficient of roughly .8 in the post-war period—does not carry over to the Netherlands, where various inflation measures show either no or slightly negative autocorrelation (.1 for GDP deflation inflation). Admittedly, indexation schemes with negative weights on lagged developments make for uneasy economic interpretation, but this implies we should look beyond these stylized indexation schemes to other micro-foundations for the observed autocorrelation of inflation.

Similarly, we choose centered normal priors for the autocorrelation parameters $\iota_i$. That is, we put more probability mass on the model having sufficiently strong internal propagation mechanisms that we need not ‘fill the gaps’ with exogenous persistence. Furthermore, the introduction of stochastic trends in the model implies that many
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Prior distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho_a$</td>
<td>AR(1) general technology</td>
<td>$N<a href="0,1">-1,1</a>$</td>
</tr>
<tr>
<td>$\rho_g$</td>
<td>AR(1) fiscal</td>
<td>$N<a href="0,1">-1,1</a>$</td>
</tr>
<tr>
<td>$\rho_\mu$</td>
<td>AR(1) investment-specific technology</td>
<td>$N<a href="0,1">-1,1</a>$</td>
</tr>
<tr>
<td>$\rho_b$</td>
<td>AR(1) bond premium</td>
<td>$N<a href="0,1">-1,1</a>$</td>
</tr>
<tr>
<td>$\rho_p$</td>
<td>AR(1) price mark-up</td>
<td>$N<a href="0,1">-1,1</a>$</td>
</tr>
<tr>
<td>$\rho_w$</td>
<td>AR(1) wage mark-up</td>
<td>$N<a href="0,1">-1,1</a>$</td>
</tr>
<tr>
<td>$\rho_x$</td>
<td>AR(1) export price mark-up</td>
<td>$N<a href="0,1">-1,1</a>$</td>
</tr>
<tr>
<td>$\rho_{c,m}$</td>
<td>AR(1) consumption import price mark-up</td>
<td>$N<a href="0,1">-1,1</a>$</td>
</tr>
<tr>
<td>$\rho_{nf}$</td>
<td>AR(1) export demand</td>
<td>$N<a href="0,1">-1,1</a>$</td>
</tr>
<tr>
<td>$\theta_f$</td>
<td>MA(1) export demand</td>
<td>$N<a href="0,0.3">-1,1</a>$</td>
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<tr>
<td>$\sigma_a$</td>
<td>s.e. general technology</td>
<td>$IG(0.01,1)$</td>
</tr>
<tr>
<td>$\sigma_g$</td>
<td>s.e. fiscal</td>
<td>$IG(0.01,1)$</td>
</tr>
<tr>
<td>$\sigma_\mu$</td>
<td>s.e. investment-specific technology</td>
<td>$IG(1,1)$</td>
</tr>
<tr>
<td>$\sigma_b$</td>
<td>s.e. bond premium</td>
<td>$IG(0.01,1)$</td>
</tr>
<tr>
<td>$\sigma_p$</td>
<td>s.e. price mark-up</td>
<td>$IG(0.01,1)$</td>
</tr>
<tr>
<td>$\sigma_w$</td>
<td>s.e. wage mark-up</td>
<td>$IG(0.01,1)$</td>
</tr>
<tr>
<td>$\sigma_x$</td>
<td>s.e. export price mark-up</td>
<td>$IG(0.01,1)$</td>
</tr>
<tr>
<td>$\sigma_{c,m}$</td>
<td>s.e. consumption import price mark-up</td>
<td>$IG(0.01,1)$</td>
</tr>
<tr>
<td>$\sigma_{nf}$</td>
<td>s.e. export demand</td>
<td>$IG(0.01,1)$</td>
</tr>
<tr>
<td>$\sigma_A$</td>
<td>s.e. trend general technology</td>
<td>$IG(0.001,0.01)$</td>
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<tr>
<td>$\sigma_\varphi$</td>
<td>s.e. trend labor supply</td>
<td>$IG(0.001,0.01)$</td>
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<td>$\sigma_\Gamma$</td>
<td>s.e. trend investment-specific technology</td>
<td>$IG(0.001,0.01)$</td>
</tr>
<tr>
<td>$\sigma_{y_f}$</td>
<td>s.e. trend EU output</td>
<td>$IG(0.001,0.01)$</td>
</tr>
</tbody>
</table>

Note: $B$: Beta; $IG$: Inverse Gamma; $N$: Normal. Arguments are mean, standard deviation, and support (when truncated).

Table 4. Prior distributions 2

of the shocks we normally perceive to be persistent have now been decomposed into trends and cycles, such that the cyclical part may exhibit much shorter decay because the low frequencies are explicitly captured by the trend components. Although the model in Section 3 described cyclical shocks in terms of ARMA processes, we shut down all the MA terms to avoid over-parameterization. The exception is the export demand shock $\varepsilon_{nf}^i$, because the degree of mis-specification of the export demand equation (there are no lags involved) requires an ARMA process to soak up the missing dynamics. We expect $\rho_f$ and $\theta_f$ to be easily identified from the estimation because, as argued above, the export demand equation is stand-alone and left- and

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right-hand-side variables are all included in the data vector. The same goes for $\rho_g$ because the government spending shock is directly observable.

Lastly, regarding the stochastic drifts, we impose priors on standard errors that are an order of magnitude smaller than for cyclical innovations, as we expect trend components to be smoother than cyclical components.

5.2 Results

We obtain the posterior distributions of the parameters on the sample 1984Q1 to 2007Q4 with Dynare’s MCMC algorithm. We run two chains of one million draws and retain the last fifth. Convergence of the algorithm is well-behaved according to the Brooks and Gelman diagnostics (not shown, but available upon request). Instead of producing a table cluttered with the prior and posterior moments, we prefer to show the results in figures. Figures 4 to 7 plot the prior and posterior distributions of the parameters of interest.

Some general comments are in order. It appears that the posterior distributions are well-behaved. Specifically, there is no evidence of pairwise multi-modality that would be a clear sign of multicollinearity. Furthermore, the draws for all parameters seem to travel sufficiently through the support of the distributions. In particular, there are no ‘spikes’, that is, no degenerate posteriors that would indicate a rank-deficient Hessian. Moreover, there are no ‘corner solutions’—posteriors with finite support that bunch at extremes—which would signal misspecification of the model along that particular dimension (for example, a Calvo parameter or the autoregressive parameter of a cyclical shock tending to one). Since we used relatively flat priors, this suggest that the model is not grossly misaligned with the true DGP of the data.

We now highlight the more interesting results from the estimation, with no attempt to be exhaustive.

Figure 4 and the top of Figure 5 shows that the data is relatively informative about most variances of shocks. The notable exception concerns the investment shock (and, to a lesser extent, the export price mark-up shock), which is most likely the symptom of identification issues inherent in trend-cycle decompositions. Furthermore, recall that we had imposed relatively tight priors on trend variances. Yet the posteriors are significantly different from the priors and
suggest trend shock volatilities of roughly the same order as the cyclical ones.

Regarding structural parameters per se, the habit formation parameter $\lambda$ is well-identified and conform to estimates in many other studies. The utilization parameter $\Phi'(1)$, on the other hand, is clearly unidentified, as one would expect from a model without utilization as an observable and with a highly parameterized labor margin (as is the case with Smets and Wouters’ model). The data is informative about the investment cost parameter $z^2 I S''(z_I)$, which is an order of magnitude larger than other studies have estimated because the investment data includes inventories, which are very volatile. The export demand elasticity $\varepsilon^f$ is pinned down accurately—a predictable result in that the export demand equation is stand-alone and could be estimated with simple least-squares (but for the ARMA form of the residual). In contrast, without an observable interest spread variable, the data has little to say about the bond elasticity $a_{h,b}$.
Figure 6 summarizes the information about the nominal dimensions of the model. Stickiness parameters are somewhat on the high side relative to estimates obtained from existing studies, such as Smets and Wouters’ estimates for the US. This result implies that the various inflation measures are not particularly sensitive to their exogenous drivers (the various measures of marginal costs). At the same time, indexation parameters are not significantly different from zero (in a Bayesian sense, of course), indicating that these measures display little autocorrelation. Combined, these posterior distributions suggest that inflation measures are close to exhibiting white noise behavior, which, as we argued above, is what we could expect from their univariate spectrum.

Figure 7 displays distributions for the autoregressive coefficients of the cyclical shocks. This is where the value-added of our approach is the most tangible. In particular, posteriors suggest that the cyclical productivity shock is only moderately persistent, and that the cyclical investment...
and labor supply shocks are essentially white noise. This is clearly the consequence of introducing along these margins stochastic trends that soak up the persistence usually attributed to the cycle. Given the tight link between decisions to work, invest and consume, this radical reshuffling of persistence from cycle to trend shocks also manifests itself along the consumption margin, where the bond premium shock—which is an inter-temporal consumption shifter—now exhibits negative autocorrelation.

In contrast, the price mark-up shock is highly persistent, with autocorrelation in the range obtained in many other studies. However, in light of our trend-cycle leitmotif, this degree of persistence most likely reflects the “great moderation” convergence in inflation and would disappear if we modeled the convergence formally with a stochastic trend, as we have done with productivity and labor supply.
5.3 Impulse responses

We can now calibrate our model with posterior means of the parameter distributions, to explore some of the model’s comparative dynamic properties. With 14 observable variables and 17 exogenous shocks (13 stationary and 4 integrated), we could plot and comment 238 impulse response functions (IRF) until the cows come home. Instead, it is more instructive to focus on our model’s value-added, namely the dual stationary-integrated characterization of certain shocks. Figures 8, 9, and 10 plot IRFs to innovations in the trend and cycle shocks to labor supply, general purpose technology and investment-specific technology. We disregard the fourth stochastic trend shock, the shock to Eurozone output, for the time being. The respondents are the major observable variables of interest expressed in levels. Most other IRFs are relatively standard and are not reported, but are available upon request.
The figures have two major common features. First is the mean-reversion of responses of non-stationary variables to the cyclical innovations $\varepsilon_t$ and the permanent deviation of responses to the trend innovations $\nu_t$—as one would expect. Second is the significant difference in short-run responses to permanent and cyclical shocks.

Consider Figure 8. By definition, the cyclical labor supply shock temporarily reduces real wages. Since these are sticky (because both prices and nominal wages are so), forward-looking, consumption-smoothing households offset some of the implied loss of labor income by supplying more hours and hold back temporarily on expenditures. Similarly, employers facing temporarily lower marginal costs increase employment, substitute away from capital (thus cutting down on investment expenses) and reduce prices. Lower domestic absorption coupled with greater labor input use implies that net exports increase in the short run (not shown)—the trade balance reconciles the impact response of output with the responses of consumption and investment in
the figure. The model exhibits significant internal persistence: although the cyclical shock decays quickly (recall the posterior mean of parameter $\rho_w$ in Figure 7), stickiness in wages, consumption, investment and employment imply its effects carry through for much longer before eventually dying out.

Turning to trend labor supply, a positive shock increases employment, output, consumption and investment permanently (relative to their original trend paths), while leaving real wages unaffected. In the short run, however, responses are radically different from responses to the cyclical shock because of a wealth effect: a permanent shock spurs households and firms to consume and invest immediately on the expectation of the future rewards to higher, permanent labor input use.

Figure 9 plots the IRFs to the innovations to the cyclical and trend components of general technology. A temporary productivity increase boosts output, consumption and investment and
reduces employment and prices on impact, as one would expect from a New-Keynesian model. Again, decay rates reflect the model’s internal persistence, as they are lower than what the AR coefficient of the shock would suggest (recall $\rho_a$ in Figure 7). In the long run, permanently higher productivity implies higher output, consumption, wages, investment and employment, as expected from the steady state of a standard RBC model. In the short run, however, the same shock reduces output, as the wealth effect of future, permanently higher output and income prompts households to cut down on hours worked and increase expenditures (part of which fall on imported goods). Obviously, productivity has no long-run effect on inflation, which is purely a nominal phenomenon. In the short-run, however, inflation is double-sided filter of marginal costs—which drop with productivity—and reacts accordingly.

Finally, Figure 10 depicts responses to innovations along the investment margin. A tempo-
rary exogenous increase in the relative price of investment, combined with sticky wages, implies that factor prices are too high for optimizing firms at their current level of production, so that they reduce investment expenditures and labor demand while increasing prices in line with (expected future) higher marginal costs. Households also cut on expenditures because of lower income. In the long run, the responses to both temporary and permanent shocks are similar on the real side: output, employment, investment and consumption drop in response to lower investment-specific productivity. However, in the short run, responses are quite different to responses to the cyclical disturbance. Faced with prospects of permanently less productive additions to capital, firms lock in profits today from the existing capital stock by pushing up employment and output. With sticky wages and a higher rate of return on capital, household income increases and sustains temporarily higher consumer expenditures. As the decrease in the overall productivity of the capital stock works its way through, output, employment, consumption and investment converge to their downshifted long-run trend paths.

What stands out of these figures is the power of the ‘wealth effect’— the interaction between smoothing motives and present discounted valuation that recalls the results of the Permanent Income Hypothesis, whereby permanent and transitory incomes shocks affect consumption differently. In the case of shocks to relative prices—prices of labor or investment in terms of consumption—short-run responses along the forward-looking consumption, investment and employment decision margins can have opposite signs because a permanent displacement carries so much more weight in present value terms than a quickly decaying one. In contrast, in the case of the technology disturbance, short-run responses have the same sign because the shock, whether permanent or temporary, is an outward shift in the production possibility frontier. With log-utility of consumption and homogenous production functions, this leaves relative prices and demand shares unaffected. These different patterns of short-run correlation between permanent and cyclical components have important consequences for the results in the next subsection.

5.4 Historical decompositions

Armed with the posterior means of these parameter distributions, we can obtain smoothed estimates of the shock innovations and back out a history of how these innovations have contributed
Figure 11. Structural decompositions: y-o-y GDP growth (top) and inflation (bottom)

to the developments of observable variables. Our interest is in showing how our unobserved components approach brings in a different, useful perspective on historical decompositions.

Since innovations are orthogonal, one can group them by adding them up according to an aggregation scheme of interest. Figure 11 depicts a standard decomposition for year-on-year domestic output growth and inflation, where the innovations are grouped into six categories: price mark-up, wage mark-up (including trend), fiscal spending, interest rate (monetary policy and bond premium), technology (general and investment-specific), and the open-economy block (Eurozone demand and supply, price and quantity margins in exports and imports).\footnote{The role of initial values in the decomposition will depend on the order of integration of the variable to be} These
decompositions show the extent to which, for example, the recent severe contraction and disinflation were largely imported from abroad, as the decomposition is dominated by the foreign block. More generally, it appears that domestic output is more highly correlated with foreign developments than domestic shocks over the sample. The decompositions also point to how little impact interest rates have on output and inflation, and how government spending helped offset some of the headwinds in the initial phase of the recent downturn but not in the later phase. Observed output growth is clearly a stationary process and the impact of initial conditions will decay at the rate of the variable’s underlying autoregressive process. In contrast, inflation over the sample is dominated by a very low frequency component and it is modeled with a very persistent process, implying that its initial conditions will decay only slowly.
Looking across the two charts, some shocks have clearly defined roles—the mark-up shock is a supply shifter, pushing inflation and output in opposite directions, while the government spending shock is a demand disturbance. In contrast, the foreign shock is a mongrel concept, as one would anticipate from aggregating mark-up and demand shocks in the same block. Its role in the recent downturn is that of a demand shock, but it switched from a demand disturbance in 2000-2001 (contributing positively both to output growth and inflation) to a supply shock in 2001-2002 (pushing growth and inflation in opposite directions). This split role highlights the care required in choosing and interpreting aggregation schemes.

Another way to decompose historical developments is to aggregate shocks according to their persistence, namely according to trend and cycle. This representation is less structural in terms of economic interpretation, but informative from an econometric point of view. Indeed, since trend shocks are random walks, the degree to which the historical decomposition is dominated by trend innovations reveals something about the predictability of the variable. At one extreme, all trend and no cycle implies little forecastability from the model beyond deterministic drifts. At the other, if all variation is cyclical, there is more scope for prediction via the autoregressive processes driving the propagation mechanisms.

With this in mind, Figure 12 depicts the trend-cycle decompositions of domestic output growth and inflation. Trend and cycle are negatively correlated over the sample (-.67 for y-o-y GDP growth), suggesting that deterministic de-trending would likely overstate the forecasting power of the cyclical component. Regarding specific historical episodes, output would have been significantly lower and inflation higher in the early 2000s had one not accounted for offsetting trend developments. In contrast, the recent recession is to a considerable extent a cyclical phenomenon. Over the sample, trend shocks account for roughly a third of the variance of both output growth and inflation.

What is behind this negative correlation? One may be tempted to view it as an artifact of the structural decomposition. It is well-known, for example, that trend and cycle components from the Beveridge-Nelson decomposition on a univariate ARIMA process are perfectly negatively correlated (see Favero (2001, pg. 54), for example). Since we are essentially performing a similar decomposition in a multivariate setting, the negative correlation between the two components
may just reflect the same mechanical feature. However, the analysis of IRFs in the previous section provides a theoretical rationale for this correlation: permanent and transitory shocks elicit responses that are often of opposite sign because of wealth effects.

Mapping IRFs into historical contributions is not straightforward because roughly, the latter capture the integral of the former. Nevertheless, some information can be gained by restricting the analysis to the three trend-cycle margins (labor supply, investment and production). Table 5 reports on the left-hand side the correlation matrix between the smoothed innovations along these margins (in rows and columns respectively). There is roughly very little significant correlation between trend and cycle innovations. On the other hand, the historical contributions to GDP growth reported on the right-hand side of the table exhibit greater correlation patterns. In particular, the trend and cycle contributions of labor supply are highly negatively correlated, although the underlying innovations are not. The same goes, to a lesser extent, for the trend and cycle productivity contributions. When summing across trend and cycle, the resulting aggregate innovations are orthogonal in sample, while the corresponding historical contributions correlate with a coefficient of -0.4.

### Table 5. Correlation matrix of selected innovations and contributions to GDP growth

<table>
<thead>
<tr>
<th></th>
<th>innovations</th>
<th>hist. contributions</th>
</tr>
</thead>
<tbody>
<tr>
<td>(v^\varphi)</td>
<td>0.24</td>
<td>0.52</td>
</tr>
<tr>
<td>(v^G)</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>(v^A)</td>
<td>0</td>
<td>-0.38</td>
</tr>
</tbody>
</table>

Note: 0 implies insignificance at 5% level.

6 Concluding remarks

As the length of this paper will attest, setting up, solving, and estimating a DSGE model that includes multiple theoretically- and empirically-founded trends is a hefty exercise. We emphasized—perhaps to the point of belaboring—the value of including these stochastic trends by describing every step from their inclusion in the non-linear theoretical model to their place...
in the resulting linearized state-space form. These steps, however, are not tailored to the particulars of our application; there is a strong sense in which the inclusion of trends in a model is mechanical—that it could be reverse-engineered from the steady-state conditions of the model, which impose the co-integrating relationships expected in the data. Although we have not explored this road yet, we conjecture that these steps could be fully automated in a DSGE solver program such as Dynare.

In this paper, we have only discussed the 'point estimates' of our model. Yet, the payoff for all this work is already tangible, in that the model produces output that differs substantially from the large body of existing literature yet remains sensible ex-post. Our “careful stochastic specification“enables us to distinguish between permanent and transitory shocks along the same margin that capture the right co-integrating relationships in the data. This provides us with a very different view of exogenous and endogenous persistence in the model, with a richer set of impulse responses to perform scenario analysis, and with a better understanding of the temporary and permanent drivers behind historical developments.

The next step is to deal with the uncertainty about and robustness of our model. We plan to tackle identification issues that are pervasive both in highly-parameterized models (see Canova and Sala (2009) and Iskrev (2010) for the theoretical and practical aspects of this issue in the context of DSGE models) and in unobserved components models. Furthermore, we plan to run out-of-sample forecasting horse-races against competing models (as in Smets and Wouters (2007)), with the expectation that our stochastic specification should perform well. These are the topics of a follow-up paper.
Appendices

A Model

A.1 Final good producers

Perfectly competitive firms produce the final good $y_t$ by combining a continuum over $[0,1]$ of intermediate goods $y_t(i)$ according to the constant elasticity of substitution (CES) technology

$$y_t = \left( \int_0^1 y_t(i)^{-1/\eta} \frac{dp_t}{y_t(i)^{-1/\eta}} di \right)^{-\eta}. $$

Final good producers maximize real profits,

$$\max_{y_t(i), \ i \in [0,1]} y_t - \int_0^1 p_t(i) y_t(i) di,$$

where $p_t(i)$ is the relative price of the intermediate good $i$ in terms of the final good, i.e. $p_t(i) = \frac{P_t(i)}{P_t}$. The first-order condition with respect to $y_t(i)$ is

$$0 = \left( \int_0^1 y_t(i)^{-1/\eta} \frac{dp_t}{y_t(i)^{-1/\eta}} di \right)^{-\eta} y_t(i)^{-1} - p_t(i)$$

$$= \left( \frac{y_t}{y_t(i)} \right)^{1/\eta} - p_t(i).$$

Hence, the demand for intermediate good $i$ is

$$y_t(i) = p_t(i)^{-\eta} y_t.$$

The zero-profits condition implied by perfect competition requires that $y_t = \int_0^1 p_t(i) y_t(i) di$. Combining with the demand for intermediate good $i$ gives us the price index (again in relative
1 = \int_0^1 p_t(i)^{1-\varepsilon_t^i} di.

We let the (log-)elasticity $\ln (\varepsilon_t^p)$ follow the ARMA(1,1) process

$$\ln (\varepsilon_t^p) = (1 - \rho_p) \ln (\varepsilon_t^p) + \rho_p \ln (\varepsilon_{t-1}^p) + v_t^p + \theta_p v_{t-1}^p, \quad v_t^p \sim N (0, \sigma_p^2).$$

This process is interpreted as a price mark-up shock since the elasticity relates to the price mark-up via $\frac{1}{\varepsilon_t^p}$ (see the price-setting decision of the intermediate good producers below).

Summarizing,

$$y_t = \left( \int_0^1 y_t(i)^{\varepsilon_{t-1}^p} \frac{\varepsilon_t^p}{\varepsilon_{t-1}^p} di \right)^{\varepsilon_t^p}, \quad \text{(1)}$$

$$y_t(i) = p_t(i)^{-\varepsilon_t^p} y_t, \quad \text{(2)}$$

$$1 = \int_0^1 p_t(i)^{1-\varepsilon_t^p} di.$$

### A.2 Intermediate good producers

There is a continuum of monopolistically competitive intermediate good producers, indexed by $i \in [0, 1]$, whose individual production functions are given by

$$y_t(i) = a_t A_t K_t^s(i)^{\alpha} L_t(i)^{1-\alpha} - \bar{z}_t \Phi, \quad \text{(3)}$$

where $K_t^s(i)$ is effective capital, $L_t(i)$ is labor, $\Phi$ is a fixed cost, and $\bar{z}_t$ represents a composite stochastic trend term which will be defined in a later section. Total factor productivity is decomposed into a temporary shock $a_t$ and a permanent shock $A_t$, whose stochastic processes are respectively

$$\ln (a_t) = \rho_a \ln (a_{t-1}) + v_t^a, \quad v_t^a \sim N (0, \sigma_a^2) .$$

---

8 The fixed costs are included in the production function instead of the cost function.
\[ \ln(A_t) = \gamma_A + \ln(A_{t-1}) + v_t^A; \quad v_t^A \sim N(0, \sigma_A^2), \]

where \( \gamma_A \) is the drift in total factor productivity. Real profits of firms are given by

\[ p_t(i) y_t(i) - w_t L_t(i) - r_k^t K_t^s(i), \]

where \( w_t \) is the real wage and \( r_k^t \) the real rental rate on effective capital.

**A.2.1 Labor and capital decision**

We formalize the cost-minimization problem \( w_t L_t(i) + r_k^t K_t^s(i) \) for any given output level \( y_t(i) \) with the Lagrangian

\[ L = -w_t L_t(i) - r_k^t K_t^s(i) + \Theta_t(i) \left( a_t A_t K_t^s(i)^\alpha L_t(i)^{1-\alpha} - z_t \Phi - y_t(i) \right), \]

where \( \Theta_t(i) \) is the Lagrange multiplier associated with the production function and reflects the real marginal cost of production. We define \( mc_t(i) \equiv \Theta_t(i) \) for future reference. The first-order conditions with respect to \( L_t(i) \) and \( K_t^s(i) \) are respectively

\[ \Theta_t(i) a_t A_t (1 - \alpha) \left( \frac{K_t^s(i)}{L_t(i)} \right)^\alpha = w_t, \]

\[ \Theta_t(i) a_t A_t \alpha \left( \frac{K_t^s(i)}{L_t(i)} \right)^{\alpha - 1} = r_k^t. \]

Dividing one by the other yields the optimal capital-labor ratio

\[ \frac{K_t^s(i)}{L_t(i)} = \frac{\alpha \ w_t}{1 - \alpha \ r_k^t}. \quad (4) \]

Substituting this ratio into the first-order condition for capital yields an expression for real marginal costs

\[ mc_t(i) = (a_t A_t)^{-1} \alpha^{-\alpha} (1 - \alpha)^{-(1-\alpha)} w_t^{1-\alpha} \left( r_k^t \right)^{\alpha}. \quad (5) \]
Intermediate good producers face a marginal cost curve that is independent of their level of production. Therefore, average variable cost equals marginal cost

\[ \text{avc}_t = mc_t (i). \] (6)

A.2.2 Price-setting decision

Intermediate good producers face downward-sloping demand curves (2) and thus exert pricing power. However, every period, only a fraction \(1 - \zeta_p\) of them may re-optimize their prices. The fraction \(\zeta\) that cannot indexes instead prices to a weighted average of past inflation \(\pi_{t-1}\) and steady-state inflation \(\pi_s\):

\[ \frac{p_t (i)}{p_{t-1} (i)} = \frac{\pi^{ip}_{t-1} \pi^1_{t-1}}{\pi_t} \cdot \]

Firms that are free to reset their prices maximize expected discounted profits generated up to the next re-optimization. Firms are owned by households and accordingly discount real payoffs using the households’ stochastic discount factor

\[ \beta^s \Xi_{t+s} / \Xi_t, \]

where \(\Xi_t\) is the Lagrange multiplier associated with the households’ budget constraint—expressed in real terms—and \(\beta\) is the households’ discount factor for utility (see the household optimization problem below). Let \(q_t (i)\) denote the optimal price for intermediate good firm \(i\) in period \(t\). Thereafter, if firm \(i\) is not allowed to re-optimize up to and including period \(t + s\), its period \(t + s\) price will be

\[ p_{t+s|t} = q_t (i) X_{t,s}^p, \]

where the compounded indexation term can be written as follows

\[ X_{t,s}^p \equiv \frac{1}{x_t^p} \prod_{k=0}^{s} x_{t+k}^p, \]

with

\[ x_t^p = \frac{\pi^{ip}_{t-1} \pi^{1-\text{i}p}_t}{\pi_t}. \]
We use the subscript $t$ to condition on the timing of the last price reset. Collecting the pieces, re-optimizing firms face the following dynamic problem

$$\max_{\varrho_t(i)} \mathbb{E}_{t} \sum_{s=0}^{\infty} \zeta_s \beta^s \Xi_{t+s} \left( \varrho_t(i) X_{t,s}^p - \text{avc}_{t+s} \right) y_{t+s|t}(i),$$

subject to

$$y_{t+s|t}(i) = \left( \varrho_t(i) X_{t,s}^p \right)^{1-\varepsilon_p} y_{t+s}. \quad \text{(7)}$$

The first-order condition with respect to $\varrho_t(i)$ is

$$0 = \mathbb{E}_{t} \sum_{s=0}^{\infty} \zeta_s \beta^s \Xi_{t+s} \left( \left( X_{t,s}^p \right)^{1-\varepsilon_p} y_{t+s} \left( 1 - \varepsilon_{t+s} \right) \varrho_t(i)^{-\varepsilon_p} - \left( X_{t,s}^p \right)^{-\varepsilon_p} y_{t+s} \left( -\varepsilon_{t+s} \right) \varrho_t(i)^{-\varepsilon_p} \text{avc}_{t+s} \right).$$

Simplifying,

$$0 = \mathbb{E}_{t} \sum_{s=0}^{\infty} \zeta_s \beta^s \Xi_{t+s} \left( \left( 1 - \varepsilon_{t+s} \right) \varrho_t(i) X_{t,s}^p + \varepsilon_{t+s} \text{avc}_{t+s} \right) y_{t+s|t}(i).$$

Since indexation and marginal costs are independent of individual variables, all intermediate good producers that can re-optimize will choose the same price, that is $\varrho_t = \varrho_t(i)$, $\forall i$. Let $S_t^p \subset [0,1]$ represent the set of firms that are not allowed to re-optimize in period $t$. The dynamics of the aggregate price level follow from

$$1 = \int_{S_t^p} (x_t p_{t-1}(i))^{1-\varepsilon_p} \, di + \left( 1 - \zeta_p \right) \varrho_t^{1-\varepsilon_p} \quad \text{(7)}$$

$$= \zeta_p (x_t^p)^{1-\varepsilon_p} + \left( 1 - \zeta_p \right) \varrho_t^{1-\varepsilon_p},$$

where the second line follows from a law of large numbers.

### A.2.3 Mapping labor into employment

The model concept of labor input developed above is hours worked, while our observable measure as described in Section 2 is full-time equivalents. We therefore use a similar trick to that
described in Adolfson et al. (2008), which is to assume that firms are also Calvo-constrained in their choice of employment level. To be specific, in a frictionless world where hours worked per employee are fixed, firm $i$ would choose its employment level to match its optimal level of hours, that is, it would choose employment level $e(i)$ such that

$$n_* e(i) = L,$$

where $n_*$ is the fixed number of hours worked per employee. However, when the Calvo fairy comes into play, only a fraction $1 - \zeta_e$ of firms may adjust individual employment levels to target levels of hours, while the remainder $\zeta_e$ indexes employment to past aggregate employment growth. Formally, consider the optimization problem

$$\min_{e_t(i)} \mathbb{E}_t \sum_{s=0}^{\infty} \left( \zeta_e \beta \right)^s (n_* e_t(i) X^{e}_{t-1,s} - L_{t+s})^2,$$

where the compound indexation term is

$$X^{e}_{t,s} = \frac{1}{x^e_t} \prod_{k=0}^{s} x^e_{t+k},$$

with

$$x^e_t = \left( \frac{\Psi_t}{\Psi_{t-1}} \right)^{i_e} z^{1-i_e} L,$$

and $\Psi_t$ is aggregate employment. The optimality condition is

$$0 = \mathbb{E}_t \sum_{s=0}^{\infty} \left( \zeta_e \beta \right)^s X^{e}_{t-1,s} (n_* e_t(i) X^{e}_{t-1,s} - L_{t+s}),$$

implying that each optimizing firm chooses the same level of employment

$$e_t(i) = e_t.$$

Following a similar argument to the derivation of the aggregate price level, we can express total employment as the weighted average of the constrained and unconstrained levels of employment,
or equivalently,

\[ 1 = (1 - \zeta_e) \frac{\zeta_t}{\Psi_t} + \zeta_e \frac{\Psi_{t-1}}{\Psi_t}. \]

The interplay between the optimality and aggregation conditions will yield a smoothing filter from hours to employment.

### A.3 Households

There is a continuum of identical households, indexed by \( j \in [0, 1] \), who decide on consumption \( C_t (j) \), hours worked \( L_t (j) \), bonds \( B_t (j) \), investment \( I_t (j) \), capital \( K_t (j) \), and capital utilization \( U_t (j) \), so as to maximize the objective function

\[
E_t \sum_{s=0}^{\infty} \beta^s \left( d_{t+s} \ln (C_{t+s} (j) - \lambda C_{t+s-1} (j)) - \varphi_{t+s} \frac{\psi}{1 + \eta} L_{t+s} (j)^{1+\eta} \right),
\]

where the degree of habit formation is captured by parameter \( \lambda \) and \( d_t \) is an inter-temporal preference shock which follows the AR(1) process

\[
\ln (d_t) = \rho_d \ln (d_{t-1}) + v_t^d, \quad v_t^d \sim N (0, \sigma_d^2).
\]

We assume \( \varphi_t \) follows a random walk with drift,

\[
\ln (\varphi_t) = \gamma_\varphi + \ln (\varphi_{t-1}) + v_t^\varphi, \quad v_t^\varphi \sim N (0, \sigma_\varphi^2),
\]

where \( \gamma_\varphi \) will eventually capture the upward drift in labor market participation. Households own raw capital \( K_{t-1} (j) \), decide on the capital utilization rate \( U_t (j) \), and rent effective capital \( K_t^* (j) \) to firms at rate \( r_k^t \). Effective capital is related to raw capital by

\[
K_t^* (j) = U_t (j) K_{t-1} (j),
\]

and real capital utilization costs are

\[
p_t^l \Phi (U_t (j)) K_{t-1} (j),
\]
where \( \Phi (U_t(j)) \) is an increasing and convex function which equals zero in the steady state—\( \Phi' (\cdot) \geq 0, \Phi'' (\cdot) \geq 0 \) and \( \Phi (U_*) = 0 \). Note that we price utilization costs as investment instead of output. We have the freedom to normalize the steady-state utilization rate, so we set it to unity

\[ U_* = 1. \]

Households face the following budget constraint in real terms,

\[
(1 + \tau_c) p^c_{t+s} C_{t+s} (j) + p^I_{t+s} I_{t+s} (j) + B^P_{t+s} (j) = \\
(1 - \tau_w) w^h_{t+s} L_{t+s} (j) + \left( (1 - \tau_k) r^k_{t+s} U_{t+s} (j) + p^I_{t+s} (\delta \tau_k - \Phi (U_{t+s} (j))) \right) K_{t+s-1} (j) + \\
\frac{R_{t+s-1} H_{t+s-1}}{\pi_{t+s}} B^P_{t+s-1} (j) + T_{t+s} y^d_{t+s} + D^u_{t+s} + D^P_{t+s} + D^m_{t+s} + D^l_{t+s},
\]

where, again, small letters indicate relative prices and wages versus the domestic good, e.g. \( p^c_t = \frac{P^c_t}{P_t} \). Moreover, \( T_t \) is the ratio of lump-sum transfers to real domestic GDP and \( D^u_t, D^P_t, D^m_t \) and \( D^l_t \) are real profits from labor unions, intermediate good producers, importing firms, and exporting firms, respectively. \( B^P_t \) represents private sector assets—equal to net foreign assets \( B^F_t \) plus government debt \( B^G_t \)—and the risk premium \( H_t \) on bonds depends on the ratio of foreign assets to GDP,\(^9\)

\[ H_t = H \left( \frac{B^F_t}{y^d_t}, \varepsilon^b_t \right), \]

where \( \varepsilon^b_t \) is a risk premium shock. We let \( \ln (\varepsilon^b_t) \) follow the AR(1) process

\[ \ln (\varepsilon^b_t) = \rho_b \ln (\varepsilon^b_{t-1}) + \nu^b_t, \nu^b_t \sim N (0, \sigma^2_b) . \]

Households accumulate capital as follows,

\[
K_{t+s} (j) = (1 - \delta) K_{t+s-1} (j) + \mu_{t+s} \left( 1 - S \left( \frac{I_{t+s} (j)}{I_{t+s-1} (j)} \right) \right) I_{t+s} (j), \tag{8}
\]

\(^9\)The exogeneity of the interest rate requires we close the model with a feedback mechanism such as a debt-dependent risk premium, à la Schmitt-Grohe and Uribe (2003).
where $S(\cdot)$ is the investment adjustment cost function—with $S(\cdot) = 0$, $S'(\cdot) = 0$ along the balanced-growth path and $S''(\cdot) > 0$—and $\mu_t$ is an investment-specific technology shock. We let $\ln(\mu_t)$ follow the AR(1) process:

$$\ln(\mu_t) = \rho_\mu \ln(\mu_{t-1}) + \nu^\mu_t, \nu^\mu_t \sim N(0, \sigma^2_\mu).$$

The maximization problem of the households can be formalized by means of the Lagrangian,

$$\mathcal{L} = E_t \sum_{s=0}^\infty \beta^s \Xi_{t+s}(j) \begin{pmatrix} d_{t+s} \ln(C_{t+s}(j) - \lambda C_{t+s-1}(j)) - \varphi_{t+s} \psi_{t+s+1} L_{t+s}(j)^{1+\eta} + \\
(1 - \tau_w) w^h_{t+s} L_{t+s}(j) \\
\sum_{j} \left( (1 - \tau_k) r^k_{t+s} U_{t+s}(j) + p^i_{t+s} (\delta \tau_k - \Phi(U_{t+s}(j))) K_{t+s-1}(j) \\
+ R_{t+s+1} H_{t+s+1} B^p_{t+s-1}(j) + T_{t+s} \eta^h_{t+s} + D^p_{t+s} + D^m_{t+s} + D^c_{t+s} \\
- (1 + \tau_c) p^c_{t+s} C_{t+s}(j) - p^i_{t+s} I_{t+s}(j) - B^c_{t+s}(j) \\
+ \Xi^k_{t+s}(j) (1 + \delta) K_{t+s-1}(j) + \mu_{t+s} (1 - S(\frac{I_{t+s}(j)}{I_{t+s-1}(j)}) I_{t+s}(j) - K_{t+s}(j)) \end{pmatrix},$$

where $\Xi_t(j)$ and $\Xi^k_t(j)$ are the Lagrange multipliers associated with the budget constraint—expressed in real terms—and the capital accumulation equation, respectively. In a marginal sense, Tobin’s $Q_t$ is defined as the ratio between the marginal market value of capital and the marginal replacement costs of capital (investment adjustment costs not included)

$$Q_t(j) \equiv \frac{\Xi^k_t(j)}{\Xi_t(j)}.$$

The first-order conditions are:

$$\begin{align*}
(\partial C_t(j)) & : \frac{d_t}{C_t(j) - \lambda C_{t-1}(j)} - \beta \lambda E_t \left( \frac{d_{t+1}(j) - \lambda C_t(j)}{C_{t+1}(j) - \lambda C_t(j)} \right) = \Xi_t(j) \frac{p^c_t}{(1 + \tau_c)} , \\
(\partial L_t(j)) & : \varphi_t \psi L_t(j) = \Xi_t(j) (1 - \tau_w) w^h_t, \\
(\partial B_t(j)) & : 1 = \beta E_t \left( \frac{\Xi_{t+1}(j) R_t H_t}{\Xi_t(j) \pi_{t+1}} \right), \\
(\partial I_t(j)) & : \frac{p^i_t}{Q_t(j) \mu_t} = 1 - S \left( \frac{I_t(j)}{I_{t-1}(j)} \right) - S' \left( \frac{I_t(j)}{I_{t-1}(j)} \right) \frac{I_t(j)}{I_{t-1}(j)} \\
& + \beta E_t \left( \frac{\Xi_{t+1}(j) Q_{t+1} \mu_{t+1}}{Q_t \mu_t} S' \left( \frac{I_{t+1}(j)}{I_t(j)} \right) \left( \frac{I_{t+1}(j)}{I_t(j)} \right)^2 \right),
\end{align*}$$

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\[(\partial K_t (j)) : Q_t (j) = \beta E_t \left( \frac{\Xi_{t+1} (j)}{\Xi_t (j)} \left( \begin{array}{c} Q^k_{t+1} (j) (1 - \delta) + (1 - \tau_k) r^k_{t+1} U_{t+1} (j) \\ + p^l_{t+1} (\delta \tau_k - \Phi (U_{t+1} (j))) \end{array} \right) \right), \] (13)

\[(\partial U_t (j)) : (1 - \tau_k) r^k_t = p^l \Phi' (U_t (j)). \] (14)

In equilibrium households make the same choices. For this reason, we can drop the index \(j\).\(^ {10}\)

### A.4 Labor unions and packers

There is a continuum of monopolistically competitive labor unions, indexed by \(l \in [0, 1]\). Labor unions are intermediaries between households and labor packers, buying homogenous labor from households at real wage \(w^h_t\), differentiating the labor one-to-one, and selling the differentiated labor \(L_t (l)\) to labor packers at real wage \(w^l_t (l)\). Thereafter, perfectly competitive labor packers create the composite labor bundle \(L_t\) by combining the continuum of differentiated labor \(L_t (l)\), with \(l \in [0, 1]\) and sell the bundle to intermediate good producers at real wage \(w_t\).

#### A.4.1 Labor packers

The setup for the labor packers is similar to the setup for the final good producers. For this reason, we present the results without derivations,

\[L_t = \left( \frac{1}{w_t} \int_0^{w^w} L_t (l) \frac{\epsilon^w}{\epsilon^w + 1} \frac{dl}{\epsilon^w + 1} \right),\]

\[L_t (l) = \left( \frac{w_t (l)}{w_t} \right)^{1 - \epsilon^w} L_t,\]

\[w_t = \left( \frac{1}{w_t} \int_0^{w_t (l)^{1 - \epsilon^w}} dl \right)^{\frac{1}{1 - \epsilon^w}},\]

\(^ {10}\)Nevertheless, we cannot drop the index of \(L_t (j)\), because \(L_t\) will already be used for the composite labor bundle. In the section about the aggregate resource constraints, we show that up to a log-linear approximation we can actually drop the index \(j\).
where we let the (log)-elasticity $\ln (\varepsilon^w_t)$ follow the ARMA(1,1) process

$$\ln (\varepsilon^w_t) = (1 - \rho_w) \ln (\varepsilon^w_t) + \rho_w \ln (\varepsilon^w_{t-1}) + \upsilon^w_t + \theta_w \upsilon^w_{t-1}, \upsilon^w_t \sim N (0, \sigma^2_w).$$

This process is interpreted as wage mark-up shock since the elasticity relates to the wage mark-up via $\frac{1}{\varepsilon^w_{t-1}}$, as the wage-setting decision of the labor unions makes clear in the next subsection.

### A.4.2 Labor unions

The setup for the labor unions is similar to that of intermediate good producers: they are restricted in the timing for re-optimization à la Calvo. Indexation is expressed as a function of nominal wages. Namely, if $W_t$ is the nominal wage, the indexation is a weighted average of past and steady-state aggregate nominal wage growth,

$$\frac{W_t(l)}{W_{t-1}(l)} = \left( \frac{W_{t-1}}{W_{t-2}} \right)^{\upsilon^w} \left( \pi^*_s \gamma^w \right)^{1-\upsilon^w}.$$

In real terms, this reads as

$$\frac{w_t(l)}{w_{t-1}(l)} = \frac{\left( \pi^*_{t-1} z^w_{t-1} \right)^{\upsilon^w} \left( \pi^*_s \gamma^w \right)^{1-\upsilon^w}}{\pi^*_t},$$

where $z^w_t \equiv \frac{w_t}{w_{t-1}}$ is aggregate real wage growth and $\gamma^w$ is steady-state wage growth (which will be defined in a later section). It turns out that it will be more instructive to work with an expression for the ratio of the indexed wage to the aggregate wage (the indexation term will drop out in the steady state). Denoting aggregate wage growth as $\frac{w_t}{w_{t-1}} \equiv z^w_t$, we can consider instead the indexation term

$$\frac{w_t(l)}{w_{t-1}(l)} = \frac{\left( \pi^*_{t-1} z^w_{t-1} \right)^{\upsilon^w} \left( \pi^*_s \gamma^w \right)^{1-\upsilon^w}}{\pi^*_t z^w_t},$$
which measures the growth of the wage reset margin. The dynamic optimization problem for labor union \( l \) now reads as

\[
\max_{\omega_t(l)} \mathbb{E}_t \sum_{s=0}^{\infty} \zeta_w^s \beta^s \left( \frac{\omega_t(l)}{w_t} X_{t,s}^w w_{t+s} - w_{t+s}^h \right) L_{t+s|t}(l),
\]

s.t. \( L_{t+s|t}(l) = \left( \frac{\omega_t(l)}{w_t} X_{t,s}^w \right)^{-\varepsilon_{t+s}^w} L_{t+s}, \)

where the compounded indexation term can be written as follows

\[
X_{t,s}^w = \frac{1}{x_t^w} \prod_{k=0}^{s} x_{t+k}^w,
\]

with

\[
x_t^w = \frac{(\pi_{t-1}^w \pi_t^w)}{\pi_t^w} \gamma_w^{1-w}.
\]

The first-order condition with respect to \( \omega_t(l) \) is:

\[
0 = \mathbb{E}_t \sum_{s=0}^{\infty} \zeta_w^s \beta^s \left( \frac{X_{t,s}^w}{w_t} \right)^{-\varepsilon_{t+s}^w} L_{t+s|t}^{\varepsilon_{t+s}^w} \omega_t(l)^{-1-\varepsilon_{t+s}^w} w_{t+s}^h - \left( X_{t,s}^w \right)^{-\varepsilon_{t+s}^w} L_{t+s|t}^{\varepsilon_{t+s}^w} \omega_t(l)^{1-\varepsilon_{t+s}^w} w_{t+s}^h.
\]

Simplifying,

\[
0 = \mathbb{E}_t \sum_{s=0}^{\infty} \zeta_w^s \beta^s \left( 1 - \varepsilon_{t+s}^w \right) \omega_t(l)^{-1-\varepsilon_{t+s}^w} X_{t,s}^w w_{t+s}^h + \varepsilon_{t+s}^w w_{t+s}^h - \left( X_{t,s}^w \right)^{-\varepsilon_{t+s}^w} L_{t+s|t}^{\varepsilon_{t+s}^w} \omega_t(l)^{1-\varepsilon_{t+s}^w} w_{t+s}^h.
\]

from which we can see that all labor unions that can re-optimize choose the same price, that is \( \omega_t = \omega_t(l), \forall l \). The dynamics of the aggregate wage level follow from

\[
1 = \zeta_w (x_t^w)^{1-\varepsilon_t^w} + (1 - \zeta_w) \left( \frac{\omega_t}{w_t} \right)^{1-\varepsilon_t^w}.
\]

A.5 Foreign block

A.5.1 Import

The import channel consists of two types of firms for both the consumption and the investment goods. Consider the consumption good; relationships derived below are equivalent for the in-
vestment good. There is a continuum of monopolistically competitive importing firms, indexed
by \( i \in [0, 1] \). Importing firms buy the homogeneous foreign good in the world market at price \( p_f^t \), differentiate the foreign good—one-to-one—, and sell the differentiated good \( C^m_i \) to good packers at price \( p_{c,m}^i \). Thereafter, perfectly competitive good packers create the composite good bundle \( C^m_t \) by combining the continuum of differentiated goods \( C^m_i \) and sell the composite good bundle to households at price \( p_{c,m}^t \).

The setup for the imported good packers is analogous to the setup for the final good producers. For this reason, we present the results without derivations,

\[
C^m_t = \left( \int_0^1 C^m_i (i) \frac{\varepsilon^c_{m-1}}{\varepsilon_{m-1}} \, di \right),
\]

\[
C^m_t (i) = \left( \frac{p_{c,m}^i (i)}{p_t} \right)^{-\varepsilon^c_m} C^m_t,
\]

\[
p_{c,m}^t = \left( \int_0^1 p_{c,m}^i (i) (1 - \varepsilon^c_m) \, di \right)^{\frac{1}{1 - \varepsilon^c_m}},
\]

where the (log-)elasticity \( \ln(\varepsilon^c_m) \) follows the ARMA(1,1) process

\[
\ln(\varepsilon^c_m) = (1 - \rho_{c,m}) \ln(\varepsilon^c_m) + \rho_{c,m} \ln(\varepsilon^c_{t-1}) + \nu_t^c + \theta_{c,m} \nu_{t-1}^c, \quad \nu_t^c \sim N(0, \sigma_{c,m}^2).
\]

The setup for the importing firms is analogous to the setup for the labor packers. The dynamic optimization problem is thus written in terms of the ratio between the indexed import price and the aggregate import price. We present the price-setting equation without derivations

\[
0 = E_t \sum_{s=0}^{\infty} \frac{\varepsilon^c_s}{\varepsilon^c_t} \left( (1 - \varepsilon^c_{t+s}) \frac{\partial^c_m}{p_t} X^c_{t,s} p_{t+s} + \varepsilon^c_{t+s} \right) C^m_{t+s | t},
\]

\[
C^m_{t+s | t} = \left( \frac{\partial^c_{m} X^c_{t,s}}{p_t} \right)^{-\varepsilon^c_{t+s}} C^m_{t+s},
\]

where the compounded indexation term is defined as

\[
X^c_{t,s} = \frac{1}{\epsilon^c_{t,s}} \prod_{k=0}^{s} x^c_{t+k},
\]

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with
\[ x_{c,m} = \left( \frac{\pi_{c,m}^{t-1} \pi_{c,m}^{t}}{\pi_{c,m}^{t}} \right)^{1-\epsilon_{c,m}}. \]

The dynamics of the consumption import price index follow from
\[ 1 = \zeta_{c,m} \left( x_{c,m} \right)^{1-\epsilon_{c,m}} + \left( 1 - \zeta_{c,m} \right) \left( \frac{\epsilon_{c,m}}{\pi_{c,m}} \right)^{1-\epsilon_{c,m}}. \]

The same equations are used for the investment import channel. Hence, for the good packers we have
\[ I_{t}^{i,m} = \left( \int_{0}^{1} I_{t}^{i,m}(i)^{1-\epsilon_{i,m}} \, di \right)^{1-\epsilon_{i,m}}. \]
\[ I_{t}^{i,m}(i) = \left( \frac{p_{t}^{i,m}(i)}{p_{t}^{i,m}} \right)^{-\epsilon_{i,m}} I_{t}^{i,m}, \]
\[ p_{t}^{i,m} = \left( \int_{0}^{1} p_{t}^{i,m}(i)^{1-\epsilon_{i,m}} \, di \right)^{1-\epsilon_{i,m}}. \]

where the elasticity \( \epsilon_{i,m} \) follows the ARMA(1,1) process in logs
\[ \ln \left( \epsilon_{i,m} \right) = (1 - \rho_{i,m}) \ln \left( \epsilon_{i,m-1} \right) + \rho_{i,m} \ln \left( \epsilon_{i,m-1} \right) + \nu_{i,m}^{i,m} + \epsilon_{i,m} + \sigma_{i,m}^{2}, \]

and for the importing firms we have
\[ 0 = E_{t} \sum_{s=0}^{\infty} \zeta_{i,m,s} \beta^{s} \left( 1 - \epsilon_{i,m} \right) \left( \frac{\theta_{t}^{i,m}}{p_{t}^{i,m}} X_{i,s}^{i,m} p_{t+s}^{i,m} + \epsilon_{i,m} p_{t+s}^{i,m} \right) I_{t+s}^{i,m}, \]
\[ I_{t+s}^{i,m} = \left( \frac{\theta_{t}^{i,m}}{p_{t}^{i,m}} X_{i,s}^{i,m} \right)^{1-\epsilon_{i,m}} I_{t+s}^{i,m}, \]
\[ 1 = \zeta_{i,m} \left( x_{t}^{i,m} \right)^{1-\epsilon_{i,m}} + \left( 1 - \zeta_{i,m} \right) \left( \frac{\epsilon_{i,m}}{\pi_{i,m}} \right)^{1-\epsilon_{i,m}}. \]
with compound indexation term

\[ X_{t,s}^{i,m} = \frac{1}{x_t^{i,m}} \prod_{k=0}^{s} x_{t+k}^{i,m} \]

and

\[ x_t^{i,m} = \left( \frac{\pi_t^{i,m}^{\tau_{i,m}}}{\pi_t^{i,m}} \right) \left( \frac{\pi_t^{i,m}}{\pi_t^{i,m}} \right)^{1-\tau_{i,m}}. \]

### A.5.2 Export

There is a continuum of monopolistically competitive exporting firms, indexed by \( i \in [0,1] \). Exporting firms buy the homogeneous domestic final good in the domestic market at price 1, differentiate the domestic final good—one-to-one—, and sell the differentiated good \( y_t^x(i) \) to good packers at price \( p_t^x(i) \). Thereafter, perfectly competitive good packers create the composite export bundle \( y_t^x \) by combining the continuum of differentiated goods \( y_t^x(i) \), with \( i \in [0,1] \), and sell the composite export bundle in the world market at price \( p_t^x \).

To start with world trade, we assume that Dutch exporting firms—in contrast to Dutch importing firms—have pricing power in the world market. The market form is—again—monopolistic competition, and the foreign demand for export goods is thus

\[ y_t^x = n_f^l \left( \frac{p_t^f}{p_t^x} \right)^{-\varepsilon_f} y_t^f, \tag{17} \]

where world output is denoted by \( y_t^f \), the world price is denoted by \( p_t^f \), \( \varepsilon_f \) is the elasticity of foreign demand, and \( n_f^l \) is a demand-shifter (akin to a home-bias parameter) which follows the ARMA(1,1) process in logs

\[ \ln \left( n_f^l \right) = (1 - \rho_f) \ln \left( n_f^l \right) + \rho_f \ln \left( n_{f-1}^l \right) + v_t^f + \theta_f v_{t-1}^f, \quad v_t^f \sim N \left( 0, \sigma_f^2 \right). \]

The setup for the good packers is similar to the setup for the final good producers. For this
reason, we present the results without derivations,

\[
y_t^x = \left( \int_0^1 y_t^x(i) \frac{\varepsilon_t^{-1}}{\varepsilon_i^{-1}} \, di \right)^{\varepsilon_t^{-1}}
\]

\[
y_t^x(i) = \left( \frac{p_t^x(i)}{p_t^x} \right)^{-\varepsilon_t} y_t^x,
\]

\[
p_t^x = \left( \int_0^1 p_t^x(i)^{1-\varepsilon_t} \, di \right)^{1-\varepsilon_t}
\]

where the elasticity \( \varepsilon_t^x \) follows the ARMA(1,1) process in logs

\[
\ln \left( \varepsilon_t^x \right) = (1 - \rho_x) \ln \left( \varepsilon_x^t \right) + \rho_x \ln \left( \varepsilon_{t-1}^x \right) + \nu_t^x + \theta_x \nu_{t-1}^x, \quad \nu_t^x \sim N \left( 0, \sigma_x^2 \right).
\]

The setup for the exporting firms is analogous to the setup for the importing firms. For this reason, we present the price-setting equation without derivations

\[
0 = E_t \sum_{s=0}^{\infty} \zeta_{t+s} \left( 1 - \varepsilon_{t+s}^x \right) \frac{\partial t^x}{p_t^x} X_{t,s}^x p_{t+s}^x \varepsilon_{t+s}^x y_{t+s\mid t}^x,
\]

\[
y_{t+s\mid t}^x = \left( \frac{\partial t^x}{p_t^x} X_{t,s}^x \right)^{-\varepsilon_{t+s}} y_{t+s}^x,
\]

where the compounded indexation term is defined as

\[
X_{t,s}^x = \frac{1}{x_t^x} \prod_{k=0}^{s} x_{t+k}^x,
\]

with

\[
x_t^x = \frac{\left( \pi_{t-1}^x \right)^{1-x} \left( \pi_{t}^x \right)^{1-t_x} \gamma_t^x}{\pi_t^x}.
\]

The dynamics of the export price index follow from

\[
1 = \zeta_x \left( x_t^x \right)^{1-\varepsilon_t^x} + (1 - \zeta_x) \left( \frac{\partial t^x}{p_t^x} \right)^{1-\varepsilon_t^x}.
\]
A.5.3 Net foreign asset accumulation

The net foreign asset accumulation follows from the balance of payments,

\[ B_t^F + p_t^f (C_t^m + I_t^m) = \frac{R_{t-1}H_{t-1}}{\pi_t} B_{t-1}^F + p_t^x y_t^x. \] (18)

A.5.4 Foreign variables

Since the Dutch economy is small relative to the Eurozone—which we consider to be the rest of the world—we assume there is no feedback from domestic variables onto foreign variables. The foreign variables—world output \( y_t^f \), world price \( p_t^f \), and interest rate \( R_t^f \)—are modeled in an exogenous block consisting of a Eurozone IS curve, a Phillips curve and a Taylor rule. World output \( y_t^f \) enters the model via the export channel and the world price \( p_t^f \) via the import channel. We further assume that the Dutch interest rate is equal to the interest rate set by the European Central Bank (ECB), which affects every inter-temporal decision margin in the model via the stochastic discount factor. Since the three-equation system is standard, we relegate its description to Appendix F.

A.6 Bundling consumption and investment

The decision problem here is to divide consumption expenditures \( C_t \) into domestically produced goods \( C_t^d \) and imported goods \( C_t^m \). The equations below are derived for the consumption bundle and are similar for investment.

Bundling takes the form of a CES aggregator, which is written here in normalized form (see Cantore et al. (2010)). The problem of minimizing the expenditure index \( Z_t^c = C_t^d + p_t^c m C_t^m \) for any given consumption level \( C_t \) can be formalized by means of the Lagrangian

\[
\mathcal{L} = -C_t^d - p_t^c m C_t^m + \Upsilon_t \left( C_t \left( n_c \left( z_t^{d,c} C_t^d \right)^{\frac{b_c}{\varphi_c-1}} + (1-n_c) \left( z_t^{m,c} C_t^m \right)^{\frac{b_c}{\varphi_c-1}} \right) - C_t \right),
\]

where \( n_c \) is a home-bias parameter, \( \Upsilon_t \) is the Lagrange multiplier on the CES aggregator, and \( z_t^{d,c} \) and \( z_t^{m,c} \) are stochastic processes (to be defined later in the section about detrending).
The first-order condition for $C^m_t$ and $C^d_t$ are respectively

\[
p_{t}^{c,m} = (1 - n_c) \left( z_t^{m,c} C^m_t \right)^{\theta_{c-1}} \frac{1}{C^m_t},
\]

\[
1 = n_c \left( z_t^{d,c} C^d_t \right)^{\theta_{c-1}} \frac{1}{C^d_t}.
\]

Divide the former by the latter and re-arrange,

\[
\frac{n_c}{1 - n_c} p_{t}^{c,m} C^m_t = \left( \frac{z_t^{m,c}}{z_t^{d,c}} \frac{C^m_t}{C^d_t} \right)^{\theta_{c-1}} \left( \frac{C^d_t}{C^m_t} \right)^{\theta_{c-1}}.
\]

We could work with this expression, but we rewrite it in order to facilitate interpretation. First, evaluate this expression in the steady state

\[
\frac{n_c}{1 - n_c} p_{s}^{c,m} C^m_s = 1.
\]

Second, substitute out the steady-state import-domestic ratio $\frac{C^d_t}{C^m_t}$ to obtain the relative demand function

\[
\frac{n_c}{1 - n_c} C^d_t = \left( \frac{z_t^{d,c}}{z_t^{m,c}} \frac{p_{t}^{c,m}}{p_{s}^{c,m}} \right)^{1 - \theta_{c}}.
\]

Third, use this expression to replace the components in the expenditure index $Z^c_t = C^d_t + p_{t}^{c,m} C^m_t$. This gives us the demand equations for domestically produced and imported consumption goods, respectively,

\[
C^d_t = \frac{n_c}{n_c + (1 - n_c) F^c_t} Z^c_t,
\]

\[
p_{t}^{c,m} C^m_t = \frac{(1 - n_c) F^c_t}{n_c + (1 - n_c) F^c_t} Z^c_t,
\]

where for notational convenience, we have defined

\[
F^c_t \equiv \left( \frac{z_t^{d,c}}{z_t^{m,c}} \frac{p_{t}^{c,m}}{p_{s}^{c,m}} \right)^{1 - \theta_{c}}.
\]
The consumption price index $p^c_t$ is defined via the expenditure index

$$p^c_t C_t \equiv Z^c_t,$$

That is, the price of the consumption bundle should be such that total expenditures on the consumption bundle are consistent with the expenditure index. We can substitute out the expenditure index by its components and re-arrange,

$$p^c_t = \frac{C^d_t + p^c_{m} C^m_t}{C_t}.$$

To derive the consumption price definition, substitute out the various terms in the previous expression—use the CES aggregator and the relative demand function. A lot of tedious algebra then yields

$$(z^d_{t} p^c_t)^{1-\epsilon_{b;c}} = n_c + (1 - n_c) F^c_t.$$

This expression allows us to rewrite the two demand equations

$$\frac{C^d_t}{p^c_t C_t} = n_c \left(z^d_{t} p^c_t\right)^{\epsilon_{b;c} - 1},$$

$$\frac{p^c_{m} C^m_t}{p^c_t C_t} = \left(1 - n_c\right) F^c_t \left(z^d_{t} p^c_t\right)^{\epsilon_{b;c} - 1}.$$

The latter demand function can further be simplified as

$$\frac{p^c_{m} C^m_t}{p^c_t C_t} = 1 - n_c \left(z^d_{t} p^c_t\right)^{\epsilon_{b;c} - 1}.$$

The demand functions are now easy to interpret. The roles of the home-bias parameter and shocks—in favor of domestically produced goods $z^d_{t}$ or imported goods $z^m_{t}$—are now (relatively) clear.

Summarizing, we have two demand functions and an aggregate price definition for the con-
sumption bundling,

\[
\begin{align*}
\frac{C^d_t}{p_t^C} &= n_c \left( z_{t}^{d,c} p_t^C \right)^{\epsilon^{b,c}-1}, \\
\frac{p_t^{c,m} C^m_t}{p_t^C} &= 1 - n_c \left( z_{t}^{d,c} p_t^C \right)^{\epsilon^{b,c}-1}, \\
\left( z_{t}^{d,c} p_t^C \right)^{1-\epsilon^{b,c}} &= n_c + (1 - n_c) F^c_t,
\end{align*}
\]

Equations for investment bundling are analogous,

\[
\begin{align*}
\frac{I^d_t}{p_t^I I_t} &= n_i \left( z_{t}^{d,i} p_t^I \right)^{\epsilon^{b,i}-1}, \\
\frac{p_t^{i,m} I^m_t}{p_t^I I_t} &= 1 - n_i \left( z_{t}^{d,i} p_t^I \right)^{\epsilon^{b,i}-1}, \\
\left( z_{t}^{d,i} p_t^I \right)^{1-\epsilon^{b,i}} &= n_i + (1 - n_i) F^i_t,
\end{align*}
\]

with

\[
\begin{align*}
F^c_t &= \left( \frac{z_{t}^{d,c} p_t^{c,m}}{z_{t}^{m,c} p_t^{c,m}} \right)^{1-\epsilon^{b,c}}, \\
F^i_t &= \left( \frac{z_{t}^{d,i} p_t^{i,m}}{z_{t}^{m,i} p_t^{i,m}} \right)^{1-\epsilon^{b,i}}.
\end{align*}
\]

A.7 Government

The budget constraint of the government is

\[
b_t y_t^d + \tau_c p_t C_t + \tau_w w_t^b L_t + \left( r_k U_t - p_t^I \right) \tau_k K_{t-1} = \frac{R_{t-1} b_{t-1} y_{t-1}^d}{\pi_t} + T_t y_t^d + g_t y_t^d,
\]

where \( b_t \) and \( g_t \) are defined as the ratios to output of outstanding domestic debt and government expenditures, respectively. We let \( g_t \) follow the AR(1) process

\[
\ln (g_t) = (1 - \rho_g) \ln (g) + \rho_g \ln (g_{t-1}) + \nu_t^g, \quad \nu_t^g \sim N \left( 0, \sigma_g^2 \right).
\]
The tax (transfer) rule of the government reacts to deviations from the target debt-to-GDP ratio $b_*$ with a lag,

$$\frac{T_t}{T_*} = \left( \frac{b_{t-1}}{b_*} \right)^{-\varepsilon_T}.$$

### A.8 Aggregate resource constraints

In practice, we do not have data about the composite labor bundle but on hours worked. That is, we observe $L_t \equiv \int_0^1 L_t (j) \, dj$ rather than $L_t$. They are related as follows,

$$\tilde{L}_t = \int_0^1 L_t (j) \, dj = \int_0^1 L_t (l) \, dl = L_t \int_0^1 \left( \frac{w_t (l)}{w_t} \right)^{-\varepsilon_t^w} \, dl, \quad \text{where the second line follows from the fact that labor unions transform the homogenous labor on an one-to-one basis into differentiated labor and the third line follows from the demand function of the labor packers. After log-linearizing, we obtain } \tilde{L}_t \simeq L_t.$$

Similarly, observed output $y_t^d$ is related to the final good bundle $y_t$: output $y_t^d$ is approximately equal to the integral over all intermediate good products $y_t (i)$, with $i \in [0, 1]$. This yields the following aggregate production

$$y_t^d = \int_0^1 y_t (i) \, di = \int_0^1 \left( a_t A_t K_t^\gamma (i)^\alpha L_t (i)^{1-\alpha} - z_t \Phi \right) \, di = a_t A_t (K_t^\gamma)^\alpha (L_t)^{1-\alpha} - z_t \Phi,$$

where the last line follows by exploiting the constant-returns-to-scale property. Aggregate pro-

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11Recall that households are indexed by $j$, labor unions by $l$ and firms by $i$.  

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duction is divided over consumption, investment, government expenditures, export, and utilization. Domestic good market clearing follows from

\[(1 - g_t) y^d_t = C^d_t + I^d_t + X_t + p_t^j \Phi (U_t) K_{t-1}, \tag{22}\]

which is consistent with combining the household budget constraint—integrated over the continuum households—, the government budget constraint, and the net foreign asset accumulation equation. Observed export are denoted by \(X_t\), which is—again—approximately equal to the composite export bundle \(y^e_t\). For convenience, we already write

\[X_t = y^e_t.\]

Finally, we do not have separate data on consumption import and investment import. Since we will use import data in our estimation procedure, we introduce notation for total imports,

\[M_t = C^m_t + I^m_t.\]

B Determining trends

B.1 Steady-state growth rates

Recall that we consider the trend-cycle decomposition of all variables in the model as follows,

\[
\ln X_t = \ln \bar{X}_t + \ln \tilde{X}_t, \\
\Delta \ln \bar{X}_t = \gamma_X + e_{xt}, \\
A(L) \left( \ln \bar{X}_t - \ln X^* \right) = u_{xt}, A(1) \neq 0,
\]
for some model-specific parameters and innovations. Steady-state paths of variables—denoted by $X_{st}$, and around which we will be log-linearizing the model later on—are defined by setting both component innovations to zero

$$
\Delta \ln X_t = \gamma X,
$$

$$
\ln X_t = \ln X_s.
$$

The goal of this section is to relate the $\gamma X$'s to the exogenous drifts $\gamma_A$, $\gamma_\varphi$, and $\gamma_\Gamma$. We now roll through the model to collect the relevant relationships.

### B.1.1 Prices

Recall the dynamics of the aggregate price index (7) and evaluate the expression in the steady state, where inflation is constant at $\pi_s$,

$$
1 = \zeta_p \left( \frac{\pi_s^{i_p}}{\pi_s} \right)^{1-\varepsilon^p} + (1 - \zeta_p) \varrho_{st}^{1-\varepsilon^p}.
$$

It immediately follows that the steady-state reset price equals one,

$$
\varrho_{st} = 1.
$$

In the steady state, all intermediate good prices are the same: there is no price dispersion. It follows from the demand function for intermediate goods (2) that demand is symmetric. Hence, from the CES aggregator (1) we obtain

$$
y_{st} = y_{st} (i), \forall i.
$$
B.1.2 Wages

Recall the dynamics of the aggregate wage index (16) and evaluate the expression in the steady state, where real wage growth and inflation are constant at $\gamma_w$ and $\pi$, respectively,

$$1 = \zeta_w \left( \left( \frac{\pi \gamma_w}{\pi^* \gamma_w} \right)^{t-w} \left( \frac{\pi^* \gamma_w}{\pi^* \gamma_w} \right) \right)^{1-\epsilon_w} + (1 - \zeta_w) \left( \frac{\omega}{\omega^*} \right)^{1-\epsilon_w}.$$  

Since by definition, $z^w = \gamma_w$, this simplifies to

$$1 = \zeta_w + (1 - \zeta_w) \left( \frac{\omega}{\omega^*} \right)^{1-\epsilon_w}.$$  

It immediately follows that the steady-state wage reset margin equals one,

$$\frac{\omega}{\omega^*} = 1.$$

In the steady state, all wages are the same: there is no wage dispersion. A quick corollary is that the compounded indexation term disappears along the balanced-growth path,

$$X^{w}_{st,s} = 1,$$

implying that demand for labor is symmetric,

$$L_{st} (l) = L_{st}.$$  

Furthermore, evaluate the first-order condition for labor unions (15) in the steady state and re-arrange,

$$0 = E_t \sum_{s=0}^{\infty} \zeta^s \beta^s \frac{\Xi^{st+s}_{st+t}}{\Xi^*_{st}} \left( (1 - \epsilon^w) \omega^{st+s} + \epsilon^w \omega^h_{st+s} \right).$$  

For this expression to hold for all $t$, it must be that the term in the parenthesis is equal to zero, implying that

$$w_{st} = \frac{\epsilon^w}{\epsilon^w - 1} w^h_{st}.$$  

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so that wages are marked up over the workers’ marginal rate of substitution between consumption and leisure. Along the balanced-growth path, mark-ups are constant, so that wages grow at the same rate as the workers’ marginal rate of substitution,

\[ \gamma_w = \gamma^h. \]

### B.1.3 Intermediate good production

First, recall the intermediate good production function (3) and evaluate it in the steady state,

\[ y_t(i) = A_t K_t^s(i)^\alpha L_t(i)^{1-\alpha} - \bar{z}_t \Phi. \]

Since demand for intermediate goods is symmetric in the steady state, we can drop the index \( i \) from the left-hand side. The right-hand side must be independent of index \( i \) as well, which is the case since the optimal capital-labor ratio (4) is the same for all intermediate good producers. Therefore, we can write

\[ y^*_t = A^*_t (K^*_t)^\alpha L^*_t^{1-\alpha} - \bar{z}_t \Phi. \]

The second term on the right-hand side grows at rate \( \gamma_z \), so the left-hand side and the first term on the right-hand side must have the same steady-state trend. That is,

\[ \gamma_y = \gamma_z = \gamma_A + \alpha \gamma_K + (1 - \alpha) \gamma_L. \]  

(23)

Note that for the time being, we make no assumptions about what determines growth in the labor input.

Second, recall the optimal capital-labor ratio (4) and evaluate it in the steady state,

\[ \frac{K^*_t}{L^*_t} = \frac{\alpha w_{st}}{1 - \alpha r^*_t}. \]

We know that wages grow at rate \( \gamma_w \). This implies the following relationship between the growth
rates of the input factors and their factor prices,
\[ \gamma_K - \gamma_L = \gamma_w - \gamma_r. \]

Third, recall the intermediate good producers’ average variable cost expression (5) and evaluate it in the steady state,
\[ avc_{st} = (A_{st})^{-1} \alpha^{-\alpha} (1 - \alpha)^{-(1-\alpha)} w_{st}^{1-\alpha} \left( \frac{r_{st}}{K_{st}} \right)^{\alpha}, \]
from which it follows that
\[ \gamma_{avc} = (1 - \alpha) \gamma_w + \alpha \gamma_r - \gamma_A. \tag{24} \]

Fourth, the steady-state zero-profit condition for intermediate good producers implies that total costs equal total output,
\[ w_{st} L_{st} + r_{st} K_{st} = y_{st}, \]
from which it follows that
\[ \gamma_w + \gamma_L = \gamma_r + \gamma_K = \gamma_y. \tag{25} \]

\textbf{B.1.4 Households}

We now roll through the first-order conditions of the households. First, recall the consumption first-order condition (9) and evaluate it in the steady state,
\[ \frac{1}{C_{st} - \lambda C_{st-1}} - \frac{1}{C_{st+1} - \lambda C_{st}} = \Xi_{st} p_{st}^c (1 + \tau_c), \]
which we can rewrite as
\[ \frac{\gamma_c - \beta \lambda}{\gamma_c - \lambda} = \Xi_{st} p_{st}^c C_{st} (1 + \tau_c). \]
The left-hand side being constant implies the following relationship between the steady-state growth rates of the right-hand side variables,
\[ \gamma_{\Xi} + \pi_{sc} - \pi_s + \gamma_G = 0. \tag{26} \]
Second, recall the labor first-order condition (10) and evaluate it in the steady state (recalling that \( w_{st} = \frac{\varepsilon^w}{\varepsilon^w - 1} w_{ht} \)):
\[
\varphi_{st} L_{st}^{\theta} = \Xi_{st} (1 - \tau_w) \frac{\varepsilon^w - 1}{\varepsilon^w} w_{st}.
\]
Again, this implies the following relationship between the steady-state growth rates,
\[
\gamma_{\varphi} + \eta \gamma_L = \gamma_\Xi + \gamma_w. \tag{27}
\]

Third, evaluate the investment first-order condition (12) at the steady state (recalling that along the steady-state growth path \( S(\gamma_I) = 0 \) and \( S'(\gamma_I) = 0 \)),
\[
\frac{p_{st}^i}{Q_{st}} = 1,
\]
implying that
\[
\gamma_Q = \pi_{si} - \pi_s.
\]

Fourth, recall the capital first-order condition (13) and evaluate it along the steady-state growth path. Together with \( U_s = 1 \) and \( \Phi (U_s) = 0 \), this yields
\[
Q_{st} = \beta \gamma_\Xi \left( Q_{st+1} (1 - \delta) + (1 - \tau_k) r_{st+1}^k + \delta \tau_k p_{st+1}^i \right),
\]
which combined with the utilization first-order condition (14), produces
\[
(1 - \tau_k) r_{st}^k = p_{st}^i \Phi'(1),
\]
implying that
\[
\gamma_r = \gamma_Q = \pi_{si} - \pi_s. \tag{28}
\]

Fifth, recall the capital accumulation equation (8) and evaluate it along the steady-state growth path,
\[
K_{st} = (1 - \delta) K_{st-1} + I_{st},
\]

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from which it immediately follows that

$$\gamma_k = \gamma_I.$$  \hspace{1cm} (29)

**B.1.5 Aggregate resource constraint**

Evaluate the domestic good market clearing equation (22) along the steady-state growth path (recalling that $\Phi(U_*) = 0$),

$$\left(1 - g_*\right) y^d_{st} = C^d_{st} + I^d_{st} + X_{st}.$$  

This additive constraint implies that the various components of GDP grow at the same rate as GDP,

$$\gamma_y = \gamma_{C^d} = \gamma_{I^d} = \gamma_X,$$  \hspace{1cm} (30)

where we simply dropped the superscript $d$ from $\gamma^d_y$ because $y^d_t \simeq y_t$.

**B.1.6 Consumption and investment bundles**

Recall the demand function for domestic investment goods (19) and evaluate it in the steady state,

$$\frac{I^d_{st}}{p^d_{st} I_{st}} = n_i \left( z^{d,i}_i p_{st} \right)^{\varepsilon_{b;i} - 1}.$$  

Along the steady-state growth path, we have

$$\gamma_{I^d} - \gamma_I - \pi_{s_i} + \pi_* = \left(\varepsilon_{b;i} - 1\right) \left(\gamma_{z^{d,i}_i} + \pi_{s_i} - \pi_*\right).$$

We assume that the home-bias shock has a stochastic trend equal to the inverse of the relative price of investment,

$$\gamma_{z^{d,i}_i} = \pi_* - \pi_{s_i},$$

so that

$$\gamma_{I^d} = \gamma_I + \pi_{s_i} - \pi_*.$$
An equivalent assumption for the consumption bundle implies

\[ \gamma_{C^d} = \gamma_C + \pi_{sc} - \pi_s. \quad (31) \]

### B.2 Solving for the steady-state growth rates

Collecting all the relevant relationships derived in the previous subsection, we have a system of \( n \) equations in \( n \) unknowns which we solve recursively. Note that for now, we consider \( \gamma_Q \) to be exogenous instead of \( \gamma_T \), a point we come back to below.

Start from inputs to production. From the households’ steady-state capital first-order condition (28), the trend in the factor price for capital simply relates to one of the exogenous stochastic trends,

\[ \gamma_r = \gamma_Q. \]

Thus, the user cost grows at the rate of relative investment price inflation. The trend in the factor price for labor ensues from the zero-profit condition (25) and the intermediate good production function (23), respectively,

\[ \gamma_y = \gamma_A + \alpha \gamma_K + (1 - \alpha) \gamma_L. \]

Solving out for \( \gamma_w \) in terms of exogenous drifts, we obtain

\[ \gamma_w = \frac{1}{1 - \alpha} \gamma_A - \frac{\alpha}{1 - \alpha} \gamma_Q. \]

With factor price trends in hand, we can now compute the drift in average variable costs via the expression (24), yielding

\[ \gamma_{avc} = 0. \]

so that marginal cost is constant in the long-run.

Next, we back out the trend in labor via the steady-state households’ first-order conditions for consumption (26) and labor (27). In the condition for labor, \( \gamma_\varphi \) is known—it is one of the
exogenous stochastic processes—and $\gamma_w$ was solved for just above. Hence, we substitute out $\gamma_\Xi$ using the condition for consumption to solve for $\gamma_L$, yielding

$$\gamma_\varphi + \eta\gamma_L = -(\pi^*_c - \pi^*_s + \gamma_C) + \gamma_w.$$ 

Using the steady-state condition for the consumption bundle (31) and the aggregate resource constraint (??), we can replace the term in brackets by $\gamma_y$, which in turn we can substitute out using the steady-state zero profit condition (25) as above. After canceling out terms, we obtain

$$\gamma_L = -\frac{1}{1 + \eta}\gamma_\varphi.$$ 

From the zero profit condition (25) and the capital accumulation equation (29), we can now recursively solve for the trends in output, capital, and investment,

$$\gamma_y = \gamma_w + \gamma_L,$$
$$\gamma_K = \gamma_y - \gamma_Q,$$
$$\gamma_I = \gamma_K.$$ 

Recall that the trend in the Lagrange multiplier on the households' budget constraint negatively relates to the trend in output,

$$\gamma_\Xi = -\gamma_y.$$ 

To finish with domestic drifts, we must deal with two issues. First, it is clear that we cannot separately pin down $\gamma_C$ and $\pi^*_c - \pi^*_s$ without further assumptions. This is where we bring in the hypothesis of balanced-growth in nominal terms between consumption and investment, which we showed in Section 2 holds in the data. Specifically, we assume that along the steady-state path, we have a constant ratio of nominal consumption to nominal investment,

$$\rho = \frac{p^c_v I^* c}{p^c_v C^* c},$$

Furthermore, we assume that—consistent with the use of chain-aggregated data in the Dutch
national product accounts—output growth and inflation are separately equal to weighted averages. These assumptions imply that (recalling that $\gamma_X = \gamma_y$ and anticipating $\pi_{*x} = \pi_*$ as explained below)

$$\gamma_y = \frac{1}{1 + \rho} \gamma_C + \frac{\rho}{1 + \rho} \gamma_I,$$

$$\pi_* = \frac{1}{1 + \rho} \pi_{*c} + \frac{\rho}{1 + \rho} \pi_{*i}.$$

Re-arranging yields

$$\gamma_C = \gamma_y + \rho \gamma_Q,$$

$$\pi_{*c} = \pi_* - \rho \gamma_Q.$$

Second, the discussion remains why we have considered $\gamma_Q$ as exogenous stochastic trend instead of $\gamma_T$. The reason is simply that, following Justiniano et al. (2011), we assume that they are the same,

$$\gamma_Q = \gamma_T.$$

That is, we assume in very standard fashion that the sole determinant of the steady-state relative investment price inflation is relative investment technology growth. This completes the search for a solution to common stochastic trends for the domestic components of the model.

The foreign block yields another set of relationships. First, consider the world demand for exports (17) and evaluate it along the steady-state growth path,

$$y^x_{st} = n^f \left( \frac{p_{st}^x}{p_{st}^f} \right)^{-\varepsilon^f} y^f_{st}.$$

Recalling that we cannot distinguish between $y^f_t$ and $X_t$ (since we log-linearize our model), we obtain

$$\gamma_X = -\varepsilon^f (\pi_{*x} - \pi_{*f}) + \gamma_y.$$

Concerning the first two terms, recall that $\gamma_X = \gamma_y$ and $\pi_{*x} = \pi_*$ as explained below. Concerning
the second two terms, we assume that $p^{f}_t$ and $y^{f}_t$ follow two exogenous—foreign—stochastic processes; hence, $\pi^{*}_f$ and $\gamma^{y}_f$ are two parameters on which the above expression puts a cross-equation restriction. Assuming the foreign processes to be exogenous makes sense because we expect the Dutch economy to be too small to have an effect on the foreign economy. But this is in contrast to many other papers—e.g. Adolfson et al. (2008)—that assume that the law of one price holds (which would imply $\pi^{*}_f = \pi_*$ and $\gamma^{y}_f = \gamma_y$). In our setup, we do not need the law of one price to hold, and without this assumption, we can actually identify the stochastic shock $\varepsilon^{f}_t$ in the world demand for exports (17) and—more importantly—do not need to pre-filter the data (to eliminate excess trends as in Adolfson et al.).

Second, price setting in the foreign block imposes a couple of long-run restrictions. For the import and export sector we get, respectively,

\[
\begin{align*}
\pi^{*}_f &= \pi^{*}_{cm} = \pi^{*}_{im} (\equiv \pi^{*}_m), \\
\pi^{*}_x &= \pi_*.
\end{align*}
\]

The derivations leading to these equalities are similar to the derivations for the wage Phillips curve. The intuition is simple: along the balanced growth path, mark-ups are constant so that prices (wages) must grow at the same rate as marginal costs (marginal rate of substitution). In the case of the foreign block, these marginal costs are themselves purchase prices (on the foreign market for imports, on the domestic market for exports).

Third, the net foreign asset accumulation equation (18) imposes that:

\[
\gamma_M + \pi^{*}_m = \gamma_X + \pi^{*}_x.
\]

Notice that the nominal shares of imports and exports in GDP are constant along the balanced-growth path,

\[
\gamma_M + \pi^{*}_m = \gamma_X + \pi^{*}_x = \gamma_y + \pi_*,
\]

but—for imports—the real shares are not (since we do not assume the law of one price to hold),

\[
\gamma_M = \gamma_{Cm} = \gamma_{1m} \neq \gamma_y.
\]
Finally, it should be clear that the model will miss important trends in the foreign block, as we assume co-integrating relations in the model that do not hold in the data. Our solution to this—described below—will be to capture the missing trends in the observation equations of the state-space representation of the model. Therefore, the model imposes cross-equation restrictions on the cyclical components of the foreign block (the current account), but treats the trends purely as a statistical decomposition device.

B.3 Determining the common trends

With all individual drifts in hand, we now proceed with the trend-cycle decomposition of the model variables. Recall the three exogenous domestic stochastic trends,

\[
\Delta \ln A_t = \gamma_A + v_t^A, \quad v_t^A \sim N(0, \sigma^2_A),
\]

\[
\Delta \ln \varphi_t = \gamma_\varphi + v_t^\varphi, \quad v_t^\varphi \sim N(0, \sigma^2_\varphi),
\]

\[
\Delta \ln \Gamma_t = \gamma_\Gamma + v_t^\Gamma, \quad v_t^\Gamma \sim N(0, \sigma^2_\Gamma).
\]

In general, the stochastic trend components of all the domestic variables must be linear combinations of these processes. We start with the one-to-one mappings. First, we decompose the relative price of investment as

\[
\ln p^i_t = \ln \bar{p}^i_t + \ln \tilde{p}^i_t,
\]

where \(\ln \bar{p}^i_t\) represents the stochastic trend and \(\ln \tilde{p}^i_t\) the cycle. We follow Justiniano et al. in assuming that \(\bar{p}^i_t = \Gamma_t\) but leave the cyclical component unrestricted for the time being (although we will indeed implement their assumption that \(\tilde{p}^i_t = 1\) in the Bayesian estimation). Thus, the decomposition re-writes as

\[
\ln p^i_t = \ln \Gamma_t + \ln \tilde{p}^i_t.
\]

Recalling that \(\gamma_r = \gamma_Q = \gamma_\Gamma\), it follows that the stochastic trend components of the factor price for capital and Tobin’s Q must be restricted to

\[
\tilde{r}^k_t = Q_t = \Gamma_t.
\]
so that the trend-cycle decompositions of the factor price for capital and Tobin’s Q are

\[ \ln r_t^k = \ln \Gamma_t + \ln r_t^k, \]
\[ \ln Q_t = \ln \Gamma_t + \ln \tilde{Q}_t. \]

Second, we decompose the labor input as

\[ \ln L_t = \ln \bar{L}_t + \ln \tilde{L}_t, \]

where—again—a bar indicates trend and a tilde indicates cycle. Recall that the solution to the long-run growth rate of labor input is \( \gamma_L = -\frac{1}{1+\eta} \gamma_A \), which suggests that we can identify the stochastic trend component of labor input as

\[ \bar{L}_t = \varphi_t \frac{1}{1+\eta}, \]

which implies that

\[ \ln L_t = -\frac{1}{1+\eta} \ln \varphi_t + \ln \tilde{L}_t. \]

Third, we decompose wages as

\[ \ln w_t = \ln \bar{w}_t + \ln \tilde{w}_t, \]

Recall that the solution to the long-run growth rate of wages is \( \gamma_w = \frac{1}{1-\alpha} \gamma_A - \frac{\alpha}{1-\alpha} \gamma_Q \), which suggests that we can identify the stochastic trend component of wages as

\[ \bar{w}_t = A_t^{\frac{1}{1-\alpha}} \Gamma_t^{\frac{\alpha}{1-\alpha}}, \]

which implies that

\[ \ln w_t = \frac{1}{1-\alpha} \ln A_t - \frac{\alpha}{1-\alpha} \ln \Gamma_t + \ln \tilde{w}_t. \]

Next, just as in the case of solving for the steady-state growth rates, the stochastic trend components of all the domestic variables must be (log-)linear combinations of the stochastic
trends of the relative price of investment, labor input, and wages. Thus, following the same steps, we can recursively roll through the various domestic variables. This gives us the following (recursive) relationships,

\[
\bar{y}_t = \bar{w}_t \bar{L}_t = A_t^{1-\alpha} \Gamma_t^{-\frac{\alpha}{1-\alpha}} \varphi_t^{-\frac{1}{1-\alpha}},
\]

\[
\bar{K}_t = \frac{\bar{y}_t}{\Gamma_t} = A_t^{1-\alpha} \Gamma_t^{-\frac{1}{1-\alpha}} \varphi_t^{-\frac{1}{1-\alpha}},
\]

\[
\bar{I}_t = \bar{K}_t = A_t^{1-\alpha} \Gamma_t^{-\frac{1}{1-\alpha}} \varphi_t^{-\frac{1}{1-\alpha}},
\]

\[
\bar{\Xi}_t = \frac{1}{\bar{y}_t} = A_t^{-\frac{1}{1-\alpha}} \Gamma_t^{\frac{\alpha}{1-\alpha}} \varphi_t^{\frac{1}{1-\alpha}},
\]

\[
\bar{C}_t = \bar{y}_t \Gamma_t^\rho = A_t^{1-\alpha} \Gamma_t^{-\frac{\alpha}{1-\alpha}+\rho} \varphi_t^{-\frac{1}{1-\alpha}},
\]

\[
\bar{p}_t^C = \Gamma_t^{-\rho},
\]

which identify the common stochastic trends (hence the co-integrating relationships) of the model. There are a few more variables in the model—such as average variable costs, marginal rate of substitution, compounded indexation, domestic consumption, domestic import, etc.—for which the stochastic trend components immediately follow from what is already derived. They are not listed here for notational convenience.

Finally, we must also consider two exogenous stochastic processes from the foreign block. For world output and world price, we use the following trend-cycle decompositions, respectively,

\[
\ln y_t^f = \ln \bar{y}_t^f + \ln \hat{y}_t^f,
\]

\[
\ln p_t^f = \ln \bar{p}_t^f + \ln \hat{p}_t^f,
\]

and we assume the following foreign stochastic trends,

\[
\Delta \ln \bar{y}_t^f = \gamma_{yf} + v_t^{yf}, \quad v_t^{yf} \sim N\left(0, \sigma_{yf}^2\right),
\]

\[
\Delta \ln \bar{p}_t^f = \pi_{sf} - \pi_\ast + v_t^{pf}, \quad v_t^{pf} \sim N\left(0, \sigma_{pf}^2\right),
\]

where—as noted before—the world demand for exports (17) puts the following cross-equation
restriction on the parameters $\pi_{sf}$ and $\gamma_{yf}$,

$$
\gamma_y = -\varepsilon_f (\pi_* - \pi_{sf}) + \gamma_{yf},
$$

where we have used that export and output have the same trend, both in quantities and prices. This cross-equation restriction suggests that we can express the stochastic trend component of the world price in terms of the stochastic trend component of world output relative to domestic output (or the other way around),

$$
\bar{p}_f t = \left( \frac{y_f t}{y_t} \right)^{-\frac{1}{\gamma_f}}.
$$

which, in fact, means that there is only one underlying stochastic trend in the foreign block and one corresponding co-integrating relationship. The underlying foreign stochastic trend enters the foreign block in various ways. First, consider the price setting in the foreign block. Recall that consumption import inflation and investment import inflation are equal to foreign inflation, that is $\pi_{sf} = \pi_{scm} = \pi_{sim} (\equiv \pi_{sm})$, and that export inflation equals domestic inflation, that is $\pi_{sx} = \pi_*$. This gives us the following relationships,

$$
\bar{p}_{c,m} t = \bar{\rho}_{c,m} t = \bar{p}_{i,m} t = \bar{\rho}_{i,m} t = \bar{p}_f t, \\
\bar{p}_x t = 1.
$$

Second, consider the quantities in the foreign block. Recall that the nominal shares of imports and exports in GDP are constant along the balanced growth path, that is $\gamma_M + \pi_{sm} = \gamma_X + \pi_{sx} = \gamma_y + \pi_*$, which suggests that we can identify the stochastic trend components in imports and exports as follows,

$$
\bar{p}_t M_t = \bar{p}_{c,m} C_t = \bar{p}_{i,m} I_t = X_t = \bar{y}_t.
$$

Concerning the weights of consumption and investment goods in imports, the real shares are constant along the balanced-growth path, yielding

$$
\bar{M}_t = \bar{C}_t^m = \bar{I}_t^m.
$$
C De-trending the model equations

We roll through all of the equations derived in Section 3 to de-trend them with the trends obtained in Section 4, using the same trend-cycle notation \( X_t = \tilde{X}_t \bar{X}_t \) and writing the gross growth rate of components as \( z_{X_t} = \frac{X_t}{X_{t-1}} \) (with \( \bar{z}_{X_t} \) and \( \tilde{z}_{X_t} \) defined accordingly). As was described in Section 4, the steady state is obtained by setting \( \bar{z}_{X_t} = z_X \equiv e^{\gamma X} \) and \( \tilde{z}_{X_t} = 1 \), so that \( \tilde{X}_t = X_s \).

C.1 Households

- First, rewrite the capital accumulation equation in trend-cycle variables

\[
\tilde{K}_t K_t = (1 - \delta) \tilde{K}_{t-1} K_{t-1} + \mu_t \left( 1 - S \left( \frac{\bar{I}_t I_t}{I_{t-1} I_{t-1}} \right) \right) \bar{I}_t I_t
\]

and re-arrange

\[
\tilde{K}_t = \frac{(1 - \delta)}{\bar{z}_{K_t}} \tilde{K}_{t-1} + \mu_t (1 - S (\bar{z}_{I_t} \tilde{I}_t)) \bar{I}_t
\]

The steady-state is

\[
1 = \frac{(1 - \delta)}{z_K} + \frac{I_s}{K_s}
\]

- Second, rewrite the consumption Euler equation in trend-cycle variables

\[
\frac{d_t}{1 - \lambda \frac{C_{t-1}}{C_t}} \frac{1}{\bar{C}_t C_t} - \beta \lambda E_t \left( \frac{d_{t+1}}{C_{t+1}} \frac{1}{C_t} - \lambda \right) \bar{C}_t C_t = \tilde{\Xi}_t \bar{p}_t^c \tilde{p}_t^c \left( 1 + \tau_c \right)
\]

From the definitions of the balanced-growth trends above, we have

\[
\tilde{\Xi}_t \bar{p}_t^c \bar{C}_t = 1
\]

Therefore, the equation simplifies to

\[
\frac{d_t \bar{z}_{Ct} \tilde{C}_t}{\bar{z}_{Ct} \tilde{C}_t} - \bar{\lambda} \frac{C_{t+1}}{\bar{C}_{t+1}} \frac{1}{\bar{z}_{Ct} \tilde{C}_{t+1} - \lambda} = (1 + \tau_c) \tilde{\Xi}_t \bar{p}_t^c \tilde{C}_t
\]
and the steady state is
\[
\frac{zC - \beta \lambda}{zC - \lambda} = (1 + \tau_c) \Xi_p p^c C^c
\]

- Third, applying the same trick to the Euler equation for bonds yields
\[
1 = \beta E_t \left( \frac{\bar{z}_{\pi_{t+1}} \bar{z}_{\pi_{t+1}} R_{t+1} H_t}{\pi_{t+1}} \right)
\]
for which the steady state is (recalling that \( z = \frac{1}{z_y} = \frac{1}{\bar{z}} \))
\[
1 = \frac{\beta}{\bar{z}} \frac{R_{s} H_s}{\pi_s}
\]
(32)

The bond premium is
\[
H_t = H \left( b^t, \epsilon^t \right)
\]
with steady state
\[
H^*_s = H \left( b^*, 1 \right)
\]

- Fourth, the investment Euler equation can be written as follows:
\[
\frac{\bar{p}_t}{Q_t \mu_t} = 1 - S' \left( \bar{L}_t \bar{z}_t \right) - S' \left( \bar{L}_t \bar{z}_t \right) \bar{L}_t \bar{z}_t + \beta E_t \left( \bar{z}_{\pi_{t+1}} \bar{z}_{\pi_{t+1}} \bar{z}_{\mu_{t+1}} \bar{z}_{\pi_{t+1}} \bar{z}_{\pi_{t+1}} \bar{z}_{\pi_{t+1}} S' \left( \bar{z}_{\pi_{t+1}} \bar{z}_{\pi_{t+1}} \right) (\bar{z}_{\pi_{t+1}} \bar{z}_{\pi_{t+1}})^2 \right)
\]
and because \( S'(z) = 0 \), the steady state simplifies considerably to
\[
\frac{\bar{p}^*_t}{Q_s} = 1
\]

- Fifth, the consumption-leisure trade-off is detrended as follows:
\[
\phi_t \psi \bar{L}_t \bar{L}_t^n = (1 - \tau_w) \Xi_t \Xi_t \bar{w}_t^h \bar{w}_t^h
\]
yet the trends relate as
\[
\Xi_t \bar{w}_t^h \bar{L}_t = \phi_t \bar{L}_t^{1+n} = 1
\]
\[ \psi L_t^q = (1 - \tau_w) \Xi_t w_t^h \]

and the steady-state relationship is
\[ \psi L_s^q = (1 - \tau_w) \Xi_s w_s^h \]

- Sixth, the optimal utilization condition becomes
\[ (1 - \tau_k) \bar{r}_t^k = \bar{p}_t^i \Phi' \left( U_t \right) \]

for which the steady state is
\[ (1 - \tau_k) r_s^k = \bar{p}_s^i \Phi' (1) \]

- Finally, the optimal capital decision becomes
\[
\tilde{Q}_t \tilde{Q}_t = \beta E_t \tilde{z}_{t+1} \tilde{z}_{t+1} \left( \tilde{Q}_{t+1} \tilde{Q}_{t+1} \left( 1 - \delta \right) + (1 - \tau_k) \bar{r}_{t+1}^k \tilde{r}_{t+1}^k U_{t+1} + \bar{p}_{t+1}^i \bar{p}_{t+1}^i \left( \delta \tau_k - \Phi \left( U_{t+1} \right) \right) \right)
\]

Re-arranging yields
\[
\tilde{Q}_t = \beta E_t \tilde{z}_{t+1} \tilde{z}_{t+1} \tilde{z}_{t+1} \left( \tilde{Q}_{t+1} \left( 1 - \delta \right) + (1 - \tau_k) \bar{r}_{t+1}^k U_{t+1} + \bar{p}_{t+1}^i \left( \delta \tau_k - \Phi \left( U_{t+1} \right) \right) \right)
\]

and recalling that \( \Phi (1) = 0 \)
\[
\frac{z}{\beta z_1} - 1 = (1 - \tau_k) \left( \frac{r_s^k}{\bar{p}_s^i} - \delta \right)
\]

**C.2 Labor**

To repeat, the optimality condition for labor packers is:
\[
0 = E_t \sum_{s=0}^{\infty} \frac{\zeta_{w/s} \beta^s \zeta_t + s}{\bar{z}^t} \left( \left( 1 - \varepsilon_{t+s}^w \right) \omega_t X_{t,s}^w + \varepsilon_{t+s}^w w_{t+s}^h \right) L_{t+s|t}
\]
and the aggregate wage definition is

\[ 1 = \zeta_w \left( \frac{x^w_{t+1}}{x^w_t} \right)^{1-\varepsilon^w_t} + (1 - \zeta_w) \left( \frac{\omega_t}{w^h_t} \right)^{1-\varepsilon^w_t} \]

with demand and indexation components

\[
L_{t+s|t} = \left( \frac{\omega_t X^w_{t,s}}{w_{t+s}} \right)^{1-\varepsilon^w_{t+s}} L_{t+s} \\
X^w_{t,s} = \frac{1}{x^w_t} \prod_{k=0}^{s} x^w_{t+k} \\
x^w_t = \frac{(\pi^s_t - 1)^{1-\varepsilon^w_t} (\pi^* s) \gamma_w}{\pi_t} \]

Recalling that \( \bar{\omega}_t = \bar{w}_t = \bar{w}^h_t \), the aggregate wage definition can be rewritten in detrended form as

\[ 1 = \zeta_w \left( \frac{\bar{x}^w_t \bar{z}^w_t}{\bar{x}^w_t \bar{z}^w_t} \right)^{1-\varepsilon^w_t} + (1 - \zeta_w) \left( \frac{\bar{\omega}_t}{\bar{w}_t} \right)^{1-\varepsilon^w_t} \]

note that

\[ \frac{\bar{x}^w_t}{\bar{z}^w_t} = \left( \frac{\bar{x}^w_t \bar{z}^w_t}{\bar{x}^w_t \bar{z}^w_t} \right)^{1-\varepsilon^w_t} \]

and in steady state, \( \bar{z}^w_t = \gamma_w \), meaning that

\[ \frac{\bar{x}^w_t}{\bar{z}^w_t} = \frac{x^w_s}{z^w_s} = \frac{\omega_s}{w_s} = 1 \]

Now, call

\[ Z^w_{t,s} = \frac{\omega_t X^w_{t,s}}{w_{t+s}} = \frac{\omega_t \bar{x}^w_t \bar{z}^w_t \prod_{k=0}^{s} \bar{z}^w_{t+k}}{\bar{x}^w_t \bar{z}^w_t} \]

\[ \Omega^w_{t,s} = \frac{w_{t+s} L_{t+s}}{\bar{w}_t L_t} = \frac{1}{\bar{z}_t \bar{z}_w \bar{z}_{t+k} \bar{z}_t+k} \prod_{k=0}^{s} \bar{z}_{t+k} \bar{z}_{t+k} \bar{z}_{t+k} \]

and

\[ 0 = E_t \sum_{s=0}^{\infty} \beta^s \Omega^w_{t,s} \left( Z^w_{t,s} \right)^{1-\varepsilon^w_{t+s}} \left( 1 - \varepsilon^w_{t+s} \right) Z^w_{t,s} + \varepsilon^w_{t+s} \frac{w^h_{t+s}}{w_{t+s}} \]

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so that, in detrended form,

\[
0 = E_t \sum_{s=0}^{\infty} \zeta^w \beta^s \Omega^w_t, \Omega^w_t \left( Z^w_t \tilde{Z}^w_t \right)^{-\varepsilon^w} \left( 1 - \varepsilon^w \tilde{Z}^w_{t,s} \right) \left( \tilde{Z}^w_{t,s} \tilde{Z}^w_{t,s} + \varepsilon^w \frac{\tilde{w}^h_{t+s}}{\tilde{w}^h_{t+s}} \right)
\]

Since \( \bar{\Xi}^w_t = 1 \), we have \( \Omega^w_{t,s} = \Omega^w_{st,s} = 1 \), which simplifies the expression to

\[
0 = E_t \sum_{s=0}^{\infty} \zeta^w \beta^s \Omega^w_t \left( \tilde{Z}^w_t \right)^{-\varepsilon^w} \left( 1 - \varepsilon^w \tilde{Z}^w_t \right) \left( \tilde{Z}^w_t \tilde{Z}^w_t + \varepsilon^w \frac{\tilde{w}^h_{t+s}}{\tilde{w}^h_{t+s}} \right)
\]

Furthermore,

\[
Z^w_{t,s} = 1
\]

implying in the steady state that

\[
w^*_s = \omega^*_s = \frac{\varepsilon^w}{\varepsilon^w - 1} \tilde{w}^h
\]

C.3 Domestic sector

- First, consider the production function in trend-cycle mode and, recalling the relationships between trends, re-arrange accordingly

\[
\tilde{y}_t = a_t A_t \bar{K}^{sa} \tilde{L}^{1-\alpha}_t \bar{K}^{sa} \tilde{L}^{1-\alpha}_t - \Phi
\]

\[
= a_t A_t \left( \frac{\tilde{L}_{t-1}}{\tilde{y}_t} \right)^{\alpha} \left( \frac{\tilde{L}_t}{\tilde{y}_t} \right)^{1-\alpha} \bar{K}^{sa} \tilde{L}^{1-\alpha}_t - \Phi
\]

\[
= a_t \bar{K}^{sa} \tilde{L}^{1-\alpha}_t - \Phi
\]

The steady state is then

\[
y^*_s = \bar{K}^{sa} \tilde{L}^{1-\alpha}_s - \Phi
\]

- Next, consider the optimal capital-labor ratio in trend-cycle format

\[
\bar{K}^s \bar{K}^s \bar{L}^s \bar{L}^s = \frac{\alpha \tilde{w}_t \tilde{w}_t}{1 - \alpha \tilde{w}_t \tilde{w}_t}
\]
\[
\frac{\hat{r}_t^k \hat{K}_t^s}{\hat{w}_t L_t} = \frac{\alpha}{1 - \alpha} \hat{z}_K
\]
for which the steady state is
\[
\frac{r_t^k K_s^s}{w_s L_s} = \frac{\alpha}{1 - \alpha} \hat{z}_K
\]

**Third, re-arrange the definition of marginal cost**
\[
m_c_t = \left( (a_t A_t)^{-1} \alpha^{-\alpha} (1 - \alpha)^{-(1-\alpha)} \frac{1}{\bar{w}_t^{1-\alpha}} \bar{w}_t \bar{r}_t^k \right) \left( \frac{r_t^k}{\bar{r}_t^k} \right)^\alpha
\]
for which the steady state is
\[
m_c_s = \left( \frac{w_s}{1 - \alpha} \right)^{1-\alpha} \left( \frac{r_s^k}{\bar{r}_s^k} \right)^\alpha
\]

**Fourth, to repeat, the optimality condition for intermediate producers is**
\[
0 = E_t \sum_{s=0}^{\infty} \zeta_p^s \beta^p \sum_{l+s}^{\infty} ( (1 - \varepsilon_{t+s}^p) \theta_t X_{t,s}^p + \varepsilon_{t+s}^p avc_{t+s} ) y_{t+s|t}
\]
and the aggregate price definition is
\[
= \zeta_p (x_t^p)^{1-\varepsilon_t^p} + (1 - \zeta_p) \theta_t^{1-\varepsilon_t^p}
\]
with demand and indexation components
\[
y_{t+s|t} = \left( \theta_t X_{t,s}^p \right)^{-\varepsilon_{t+s}^p} y_{t+s}
\]
\[
X_{t,s}^p = \frac{1}{x_t^p} \prod_{k=0}^{s} x_{t+k}^p
\]
\[
x_t^p = \frac{\pi_{t-1}^{s-p} \pi_s^{1-\varepsilon_t^p}}{\pi_t}
\]
With $\bar{\pi}_t = \pi_*$, we have that $\bar{x}_t^p = 1$, so we can write the aggregate price definition in detrended form as

$$1 = \zeta^p (\bar{x}_t^p)^{1-\varepsilon^p} + (1 - \zeta^p) \bar{\theta}_t^{1-\varepsilon^p}$$

and in steady state

$$\theta_* = x_*^p = 1$$

Call

$$Z_{t,s}^p = \theta_t X_{t,s}^p$$

and

$$\Omega_{t,s}^p = \Xi_{t} \delta_t y_{t+s}^{\varepsilon_p} = \frac{1}{z_{t} \Xi_{t}^{\varepsilon_p}} \prod_{k=0}^{s} z_{t+k} \Xi_{t}^{\varepsilon_p}$$

We can then simplify the optimality condition to

$$0 = E_t \sum_{s=0}^{\infty} \zeta^p \beta^s \Omega_{t,s}^p (Z_{t,s}^p)^{1-s} \left( (1 - \varepsilon_{t+s}^p) Z_{t,s}^p + \varepsilon_{t+s}^p \text{avc}_{t+s} \right)$$

Since $\Xi_t \bar{\gamma}_t = 1$, we have $\bar{\Omega}_{t,s}^p = \Omega_{*t,s}^p = \bar{Z}_{t,s}^p = 1$, so that in detrended form:

$$0 = E_t \sum_{s=0}^{\infty} \zeta^p \beta^s \bar{\Omega}_{t,s}^p (\bar{Z}_{t,s}^p)^{1-s} \left( (1 - \varepsilon_{t+s}^p) \bar{Z}_{t,s}^p + \varepsilon_{t+s}^p \text{avc}_{t+s} \right)$$

implying in the steady state that

$$1 = \frac{\varepsilon_p}{\varepsilon_p - 1} mc_*$$

- Fifth, we consider the fact that in the steady state, pure profits are zero. This implies the following relationship between fixed and marginal cost:

$$\Phi = \left( \frac{1}{mc_*} - 1 \right) y_s$$

so that we can rewrite steady state output as

$$y_s = mc_* z_K^{-\alpha} K_s^{\alpha} L_s^{1-\alpha}$$
C.4 Foreign block

- First, consider imports of consumption goods. To repeat, the optimality condition is

$$0 = E_t \sum_{s=0}^{\infty} \varepsilon_{t+s} \Xi_t \left( (1 - \varepsilon_{t+s}) \varrho_t X_{t,s} + \varepsilon_{t+s} p_{t+s} \right) C_{t+s}^m,$$

and the aggregate consumption import price definition is

$$p_t^c = \left( \zeta_{t}^c (x_t^c p_{t-1}^{c,m})^{1-\varepsilon_t^c} + (1 - \zeta_t^c) (\varrho_t^c)^{1-\varepsilon_t^c} \right) \frac{1}{1-\varepsilon_t^c}$$

with demand and indexation components:

$$C_{t+s|t}^m = \left( \varrho_t^c X_{t,s} \right)^{1-\varepsilon_t^c} C_{t+s}^m$$

$$X_{t,s} = \frac{1}{x_t^c} \prod_{k=0}^{s} x_{t+k}^c$$

$$\tilde{x}_t^c = \frac{(\pi_{t-1}^c)^{1-\varepsilon_t^c}}{\pi_t}$$

Recalling that $\bar{p}_t^c = \bar{\varrho}_t^c = \bar{p}_t^f$ and $\bar{\pi}_t^c = \bar{\pi}_t^c$ so that $\bar{x}_t^c = \tilde{x}_t^c$, the aggregate price index definition can be written in detrended form as

$$1 = \zeta_{t}^c \left( \tilde{x}_t^c \frac{\tilde{\pi}_t^c}{\pi_t} \right)^{1-\varepsilon_t^c} + (1 - \zeta_t^c) \left( \frac{\varrho_t^c}{p_t^c} \right)^{1-\varepsilon_t^c}$$

and in the steady state

$$\varrho_{t}^c = p_t^c$$

Now, call

$$Z_{t,s}^c = \frac{\varrho_t^c X_{t,s}^c}{p_{t+s}}$$

and

$$\Xi_{t,s}^c = \bar{z}_{t}^c \frac{C_{t+s}^m p_{t+s}^c}{C_t^m p_t^c} = \frac{\pi_t}{\bar{z}_{t}^c \bar{\pi}_t^c \bar{\pi}_t^c} \prod_{k=0}^{s} \bar{z}_{t+k} \bar{z}_{cmt+k} \bar{\pi}_{t+k}$$

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to rewrite the optimality condition as

$$0 = E_t \sum_{s=0}^{\infty} \zeta_{c,m} \beta^s \Omega_{t,s} (Z_{c,m}^{t,s})^{-\varepsilon_{t,s}} (1 - \varepsilon_{t,s}^{c,m}) Z_{t,s}^{c,m} + \varepsilon_{t,s}^{c,m} \frac{p_{t+s}^{f,c,m}}{p_{t+s}^{c,m}})$$

Recall that $\bar{p}_{c,m} = \bar{\rho}_{c,m} = \bar{p}_{f,t}$ and $\bar{p}_{c,m} \bar{C}_{t} = 1$, implying $\bar{\Omega}_{c,m}^{t,s} = \Omega_{c,m}^{t,s} = \bar{Z}_{c,m}^{t,s} = Z_{c,m}^{t,s} = 1$, which simplifies the expression to

$$0 = E_t \sum_{s=0}^{\infty} \zeta_{c,m} \beta^s \bar{\Omega}_{t,s} (\bar{Z}_{t,s}^{c,m})^{-\varepsilon_{t,s}} (1 - \varepsilon_{t,s}^{c,m}) \bar{Z}_{t,s}^{c,m} + \varepsilon_{t,s}^{c,m} \frac{p_{t+s}^{f,c,m}}{p_{t+s}^{c,m}})$$

and in the steady state

$$\bar{q}_{c,m}^{t,s} = p_{c,m}^{t,s} = \frac{\varepsilon_{c,m}}{1 - \varepsilon_{c,m}} p_{f,t}^{c,m}$$

• Similar equations hold for the importers of investment goods:

$$0 = E_t \sum_{s=0}^{\infty} \zeta_{i,m} \beta^s \bar{\Omega}_{t,s} (\bar{Z}_{t,s}^{i,m})^{-\varepsilon_{t,s}} (1 - \varepsilon_{t,s}^{i,m}) \bar{Z}_{t,s}^{i,m} + \varepsilon_{t,s}^{i,m} \frac{p_{t+s}^{f,i,m}}{p_{t+s}^{i,m}})$$

$$1 = \zeta_{i,m} \left( \bar{x}_{i,m} \frac{\bar{\pi}_{t}^{i,m}}{\bar{\pi}_{t}^{i,m}} \right)^{1-\varepsilon_{t,s}^{i,m}} + (1 - \zeta_{i,m}) \left( \frac{\bar{\rho}_{i,m}^{i,m}}{\bar{p}_{i,m}^{i,m}} \right)^{1-\varepsilon_{t,s}^{i,m}}$$

with steady-state relationships

$$\bar{q}_{i,m}^{t,s} = p_{i,m}^{t,s} = \frac{\varepsilon_{i,m}}{1 - \varepsilon_{i,m}} p_{f,t}^{i,m}$$

• Next, consider exporters. To repeat, the optimality condition is

$$0 = E_t \sum_{s=0}^{\infty} \zeta_{x} \beta^s \bar{Z}_{t,s}^{x} (1 - \varepsilon_{t+s}^{x}) \bar{g}_{t,s}^{x} X_{t,s}^{x} + \varepsilon_{t+s}^{x} y_{t+s}^{x})$$

and the aggregate export price definition is

$$p_{t}^{x} = \left( \zeta_{x} \left( x_{t}^{x} p_{t-1}^{x} \right)^{1-\varepsilon_{t}} + (1 - \zeta_{x}) (\bar{q}_{t}^{x})^{1-\varepsilon_{t}} \right)^{1-\varepsilon_{t}}$$

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with demand and indexation components:

\[ y_{t+s|t} = \left( \frac{\varrho_t X^x_{t,s}}{p_{t+s}} \right)^{-\varepsilon^x_t} y_{t+s} \]

\[ X^x_{t,s} = \frac{1}{x_t^s} \prod_{k=0}^{s} x_{t+k}^s \]

\[ x_t^x = \frac{\left( \pi_t^x \right)^{1-x_t^s} (\pi_t^x)^{1-t^x}}{\pi_t} \]

Recalling that \( \bar{\varrho} = \bar{\pi} = 1 \) and \( \bar{\pi}^x = \pi^x \) so that \( \bar{x}^x_t = 1 \), the aggregate price index definition can be written in detrended form as

\[ 1 = \zeta_x \left( \frac{\bar{x}^x_t \bar{\pi}_t}{\bar{x}^x_t \bar{\pi}_t} \right)^{1-\varepsilon^x_t} + (1 - \zeta_x) \left( \frac{\bar{\varrho}_t}{\bar{p}_t} \right)^{1-\varepsilon^x_t} \]

and in the steady state

\[ \bar{u}^*_s = \bar{p}^*_s \]

Now, call

\[ Z^x_{t,s} = \frac{\theta^x_t X^x_{t,s}}{p_{t+s}} \]

and

\[ \Omega^x_{t,s} = \frac{\Xi_{t+s} y_{t+s}^x p_{t+s}^x}{y_t^x p_t^x} = \frac{\pi_t}{\bar{\Xi}_t} \prod_{k=0}^{s} \bar{z}_{g_{x+k}} \frac{\pi_{x+k}}{\pi_{t+k}} \]

to rewrite the optimality condition as

\[ 0 = E_t \sum_{s=0}^{\infty} \zeta_x \beta^s \Omega^x_{t,s} \left( Z^x_{t,s} \right)^{-\varepsilon^x_t} \left( 1 - \varepsilon^x_{t+s} \right) Z^x_{t,s} + \varepsilon^x_{t+s} \frac{1}{p_{t+s}} \]

Recall that \( \bar{p}_t \bar{y}_t^x \bar{\Xi}_t = 1 \), implying \( \Omega^x_{t,s} = \Omega^x_{s,s} = \bar{Z}^x_{t,s} = Z^x_{s,s} = 1 \), which simplifies the expression to

\[ 0 = E_t \sum_{s=0}^{\infty} \zeta_x \beta^s \Omega^x_{t,s} \left( \bar{Z}^x_{t,s} \right)^{-\varepsilon^x_t} \left( 1 - \varepsilon^x_{t+s} \right) \bar{Z}^x_{t,s} + \varepsilon^x_{t+s} \frac{1}{\bar{p}^x_{t+s}} \]

and in the steady state,

\[ \bar{u}^*_s = \bar{p}^*_s = \frac{\varepsilon_x}{1 - \varepsilon_x} \]
• Detrended demand for exports is

\[ \tilde{y}_t = n_t \left( \frac{\tilde{p}_t}{p_t} \right)^{-\varepsilon f} \tilde{y}_t, \]

and along the steady state, we have

\[ y^*_t = n^f \left( \frac{p^*_t}{p_t} \right)^{-\varepsilon f} y^*_t. \]

**C.5 Net foreign asset accumulation**

Rewrite the net foreign asset accumulation equation as:

\[ b^f_t = \frac{R_{t-1} H_{t-1}}{\pi_t z_{yt}} b^f_{t-1} + \frac{p^*_t y^*_t}{y_t} - \frac{p^*_t M^*_t}{y_t} \]

where

\[ b^f_t = \frac{B^f_t}{y_t} \]

Recalling that \( \bar{p}_f \bar{M}_t = \bar{X}_t = \bar{y}_t \), \( \bar{p}_x = 1 \) and assuming no trend in the debt to output ratio, \( \bar{b}^f_t = 1 \), this simplifies to

\[ \tilde{b}^f_t = \frac{R_{t-1} H_{t-1}}{\pi_t \bar{z}_{yt}} \tilde{b}^f_{t-1} + \frac{\tilde{p}_t \tilde{y}_t}{\tilde{y}_t} - \frac{\tilde{p}_t \tilde{M}_t}{\tilde{y}_t} \]

and in the steady state

\[ (1 - \frac{1}{\beta}) b^*_t y^*_t = p^*_t X^*_t - p^*_t M^*_t \]

**C.6 Bundling**

Using the definitions of the trends, we can rewrite the relative demand for domestic consumption goods as

\[ \frac{\tilde{C}^{d}_{t}}{\tilde{p}^{c}_{t} \tilde{C}_{t}} = n_c \left( z^{d,c}_{t} z^{d,c}_{t} p^{d,c}_{t} p^{d,c}_{t} \right)^{e^{h,c}-1} \]
The constraint that the nominal share of domestic goods in consumption expenditures be constant in the long run requires that \( \bar{z}_{d,c} t \bar{p}_c t = 1 \), in which case

\[
\frac{\bar{C}_d t}{\bar{p}_t \bar{C}_t} = n_c \left( \bar{z}_{d,c} t \bar{p}_t \right)^{\varepsilon_{b,c} - 1}
\]

and in the steady state

\[
\frac{C^d_s}{p^c_s C^s} = n_c \left( z_{d,c}^* p_{c_s}^* \right)^{\varepsilon_{b,c} - 1}
\]

Similarly for the relative demand for imported consumption goods:

\[
\frac{\tilde{p}^c_m \tilde{C}_m}{\tilde{p}_t \tilde{C}_t} = 1 - n_c \left( \tilde{z}_{d,c} t \tilde{p}_t \right)^{\varepsilon_{b,c} - 1}
\]

implying

\[
\frac{p^c_m C^m}{p^c_s C^s} = 1 - n_c \left( z_{d,c}^* p_{c_s}^* \right)^{\varepsilon_{b,c} - 1}
\]

Furthermore, using the relevant trend relationships in the definition of the aggregate consumption price index, we obtain

\[
\left( \frac{\tilde{z}_{d,c} t \tilde{p}_t}{\tilde{p}_t} \right)^{1 - \varepsilon_{b,c}} = n_c + \left( 1 - n_c \right) \left( \frac{\tilde{p}^c_m \tilde{z}_{d,c} m \tilde{p}_t^c}{\tilde{p}_t \tilde{z}_{d,c} m \tilde{p}_t} \right)^{1 - \varepsilon_{b,c}}
\]

Thus, for the left-hand side to be stationary, we must impose that \( \tilde{z}_{m,c} t = \frac{\tilde{p}^c_m}{\tilde{p}_t} \), in which case

\[
\left( \frac{\tilde{z}_{d,c} t \tilde{p}_t}{\tilde{p}_t} \right)^{1 - \varepsilon_{b,c}} = n_c + \left( 1 - n_c \right) \left( \frac{\tilde{z}_{d,c} m \tilde{p}_t^c}{\tilde{z}_{d,c} m \tilde{p}_t^c} \right)^{1 - \varepsilon_{b,c}}
\]

and in the steady state

\[
\left( \frac{\tilde{z}_{d,c}^* p_{c_s}^*}{\tilde{p}_s^*} \right)^{\varepsilon_{b,c} - 1} = n_c + \left( 1 - n_c \right) \left( \frac{\tilde{z}_{d,c}^*}{\tilde{z}_{m,c}^*} \right)^{1 - \varepsilon_{b,c}}
\]
We have the degree of freedom to normalize $z_{d,c} = z_{m,c} = 1$, so that the steady state simplifies to

\[
\begin{align*}
    p^c_s &= 1 \\
    C^d_s &= n_c \\
    p^c_m C^m_s &= 1 - n_c
\end{align*}
\]

Equations are analogous for the investment bundle, but for the fact that we must assume that $\bar{z}_t \bar{p}_t = 1$ and $\bar{z}_t^{m,i} = \bar{p}_t^{c,i} \bar{p}_t^{i,m}$. Thus

\[
\begin{align*}
    \frac{\bar{I}_d}{\bar{p}_t I_t} &= n_i \left( \frac{\bar{z}_t^{d,i}}{\bar{p}_t} \right)^{\bar{b}^{i} - 1} \\
    \frac{\bar{I}_t^{m,i}}{\bar{p}_t I_t} &= 1 - n_i \left( \frac{\bar{z}_t^{d,i}}{\bar{p}_t} \right)^{\bar{b}^{i} - 1} \\
    \left( \frac{\bar{z}_t^{d,i}}{\bar{p}_t} \right)^{1 - \bar{b}^{i}} &= n_i + (1 - n_i) \left( \frac{\bar{z}_t^{d,i}}{\bar{z}_t^{m,i}} \right)^{1 - \bar{b}^{i}}
\end{align*}
\]

and in the steady state

\[
\begin{align*}
    p^i_s &= 1 \\
    I^d_s &= n_i \\
    p^{i,m}_s I^m_s &= 1 - n_i
\end{align*}
\]

**C.7 Policy**

Again, using the relationships between the trends and assuming a trend-less debt to GDP ratio, we can write the government budget constraint as

\[
\tilde{b}_t - \frac{R_{t-1} H_{t-1} \bar{b}_{t-1}}{\bar{p}_t \bar{z}_{y,t} \bar{z}_{g,t} - \bar{y}_t} = \bar{T}_t + \bar{g}_t - \tau_c \bar{p}_t^c \bar{C}_t \bar{y}_t - \tau_w \bar{u}_t^w \bar{L}_t \bar{y}_t - \left( \bar{r}_t^k \bar{U}_t - \bar{p}_t^k \bar{\delta} \right) \tau_k \frac{\bar{z}_t \bar{K}_{t-1}}{\bar{z}_{y,t} \bar{y}_t}
\]

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with the tax rule:

$$\tilde{T}_t = T_s \left( \frac{b_{t-1}}{b_s} \right)^{-\varepsilon_T}.$$  

Thus, in the steady state

$$\left( 1 - \frac{1}{\beta} \right) b_s = T_s + g_s - \tau_c \frac{p_f^c C_s}{y_s} - \tau_w \frac{w_s L_s}{y_s} - \left( \tau_{k}^s - \delta \right) \frac{z_{yt}}{z_{yt}} T_s K_s$$

C.8 Market clearing

Exploiting the trend relationships, we can write the domestic market clearing constraint as

$$(1 - \tilde{g}_t) \tilde{y}_t = \tilde{C}_t^d + \tilde{I}_t^d + \tilde{X}_t + \tilde{p}_t \bar{z}_{yt} \Phi (U_t) \tilde{K}_{t-1}$$

which reads in the steady state as

$$(1 - g) y_s = C_s^d + I_s^d + X_s$$

since $\Phi (U_s) = 0$.

Furthermore, imports and exports are

$$\tilde{M}_t = \tilde{C}_t^m + \tilde{I}_t^m$$

$$\tilde{X}_t = \tilde{y}_t$$

D Solving for the steady state of the de-trended model

We now pick out the important block that pins down the steady state. From domestic market clearing and the net financial assets position

$$(1 - g) y_s = C_s^d + I_s^d + \frac{1}{p^d_s} \left( p^f_s M_s + \left( 1 - \frac{1}{\beta} \right) b^f_s y_s \right)$$

call

$$\kappa = \frac{1}{p^d_s} \left( 1 - \frac{1}{\beta} \right) b^f_s$$
so the aggregate domestic constraint can be written as (recalling \( p_s^i = p_s^c = 1 \))

\[
(1 - g - \varepsilon) y_s = C^d_s + I^d_s + \frac{p_s^f}{p_s^c} (C^m_s + I^m_s)
\]

\[
= \left( n_c + \frac{p_s^f}{p_s^c p_s^f p_i^m} (1 - n_c) \right) C_s + \left( n_i + \frac{p_s^f}{p_s^c p_i^m} (1 - n_i) \right) I_s
\]

\[
= P^c C_s + P^i I_s
\]

We defined previously the nominal investment-to-consumption ratio

\[
\rho = \frac{p_s^i I_s}{p_s^c C_s} = \frac{I_s}{C_s}
\]

So we can rewrite the aggregate domestic constraint as

\[
(1 - g - \varepsilon) y_s = \left( \frac{P^c}{\rho} + P^i \right) I_s
\]

Furthermore, from production, the zero profit condition and the optimal capital-labor ratio, we have

\[
y_s = \frac{w_s L_s}{1 - \alpha} = \frac{1}{\alpha} \frac{r_s^k K_s}{z_K}
\]

and the capital accumulation equation is

\[
I_s = \left( 1 - \frac{(1 - \delta)}{z_K} \right) K_s
\]

Therefore, we solve for the investment-output ratio in two ways:

\[
\frac{I_s}{y_s} = \frac{(z_K - (1 - \delta)) \alpha}{r_s^k} = \frac{1}{\frac{P^c}{\rho} + P^i}
\]

which imposes a parameter restriction (see below in the calibration section).

Now, the consumption and labor supply Euler equations require

\[
\Xi_s C_s = \frac{1}{1 + \tau_c} \frac{z C(\rho) - \beta \lambda}{z C(\rho) - \lambda} = F_1
\]
and

$$L_s^{\eta} = \frac{\varepsilon_w - 1}{\varepsilon_w} \frac{1 - \tau_w}{\psi} \Xi_s w_s = F_2 \Xi_s w_s$$

so we can solve for the labor input

$$L_s^{1+\eta} = F_2 \Xi_s (1 - \alpha) y_s = (1 - \alpha) F_1 F_2 \frac{y_s}{C_s} = \rho (1 - \alpha) F_1 F_2 \frac{y_s}{I_s}$$

Then, we unravel the other variables

$$K_s = \frac{\alpha}{1 - \alpha} z_K \frac{w_s}{r_s} L_s$$

$$y_s = \frac{w_s L_s}{1 - \alpha}$$

$$I_s = \left(1 - \frac{(1 - \delta)}{z_K}\right) K_s$$

$$C_s = \frac{I_s}{\rho}$$

$$\Xi_s = \frac{F_1}{C_s}$$

### E Log-linearized equations

We now roll through the model equations to log-linearize them. We use the following notation and relationships to derive the equations:

$$\dot{X}_t = \ln \dot{X}_t - \ln X_s$$

$$z_{X_s} = z_X = e^{\gamma_X}$$

$$\dot{z}_{X_s} = 1$$

$$d \ln \tilde{z}_{Xt} = \dot{X}_t - \dot{X}_{t-1}$$

$$E_t \ln \tilde{z}_{Xt+1} = 0$$
where the last line comes from the fact that the stochastic trends are random walks with drifts. Equation numbers from here onwards are matched with the equation numbers in the Dynare model file, so that the reader can follow the latter easily (the file is available upon request).

### E.1 Households

- **capital accumulation:**
  \[
  K_t = \left(1 - \frac{I_s}{K_s}\right) K_{t-1} + \frac{I_s}{K_s} \left(\hat{\mu}_t + \hat{I}_t\right)
  \]  
  (33)

- **consumption Euler (introduce intermediate variable \(\hat{x}\) to improve readability)**
  \[
  \hat{\xi}_t + \hat{\beta}^C_t = \frac{z_C}{z_C - \beta \lambda} \hat{x}_t - \frac{\beta \lambda}{z_C - \beta \lambda} E_t \hat{x}_{t+1}
  \]  
  (34)

  \[
  \hat{d}_t - \hat{x}_t = \frac{z_C}{z_C - \lambda} \hat{C}_t - \frac{\lambda}{z_C - \lambda} \left(\hat{C}_{t-1} - \hat{z}_C\right)
  \]  
  (35)

- **Bond Euler:**
  \[
  E_t \left(\hat{\xi}_t - \hat{\xi}_{t+1}\right) = \hat{R}_t - E_t \hat{x}_{t+1} + \hat{H}_t
  \]  
  (36)

- **Bond premium:**
  \[
  \hat{H}_t = a_{h,b} \hat{b}_t^b + a_{h,c} \hat{c}_t^b
  \]  
  (37)

- **investment Euler:**
  \[
  \hat{Q}_t + \hat{\mu}_t - \hat{p}_t^i = z_t^2 S''(z_t) \left(\hat{I}_t - \hat{I}_{t-1} + \hat{z}_I - \beta E_t \left(\hat{I}_{t+1} - \hat{I}_t\right)\right)
  \]  
  (38)

- **consumption-leisure trade-off:**
  \[
  \eta \hat{L}_t = \hat{\xi}_t + \hat{w}^h_t
  \]  
  (39)

- **optimal utilization**
  \[
  (1 - \tau_k) r^k_t \left(r^k_t - \hat{p}_t^k\right) = \Phi''(1) \hat{U}_t
  \]  
  (40)
• optimal capital

\[ E_t \left( \hat{\Xi}_t - \hat{\Xi}_{t+1} \right) = \beta z E_t \left( (1 - \delta) \hat{Q}_{t+1} + \Phi' (1) \hat{r}^k_{t+1} + \delta \tau_k \hat{p}_{t+1} \right) - \hat{Q}_t \]  

(41)

E.2 Labor

• First-order condition for wages

\[ E_t \sum_{s=0}^{\infty} \zeta_w^s \beta^s \left( \hat{Z}_{t,s}^w - \hat{w}_{t+s}^h + \hat{w}_{t+s} + \frac{1}{\varepsilon_w} \hat{w}_{t+s} \right) = 0 \]

• compound indexation

\[ \hat{Z}_{t,s}^w = \hat{\omega}_t - \hat{w}_t - \hat{x}_t^w + \sum_{k=0}^{s} \hat{x}_{t+k}^w \]

where

\[ \hat{x}_t^w = - (1 - \tau_w) (\hat{\pi}_t + \hat{w}_t - \hat{w}_{t-1} + \hat{z}_{wt}) \]

• aggregate wage definition

\[ \zeta_w \hat{x}_t^w + (1 - \zeta_w) (\hat{\omega}_t - \hat{w}_t) = 0 \]

Combining these three ingredients yields the wage Phillips curve:

\[ E_t (1 - \beta F) (1 - \tau_w) (\hat{\pi}_t + \hat{w}_t - \hat{w}_{t-1} + \hat{z}_{wt}) = \frac{(1 - \zeta_w)(1 - \zeta_w) \beta}{\zeta_w} \left( \hat{w}_t^h - \hat{w}_t - \frac{1}{\varepsilon_w} \hat{w}_{t}^w \right) \]  

(42)

• First-order condition for employment

\[ E_t \sum_{s=0}^{\infty} \left( \zeta_e \beta z_L^2 \right)^s \left( \hat{Z}_{t,s}^L + \hat{e}_t - \hat{L}_{t+s} \right) = 0 \]

• compound indexation

\[ \hat{Z}_{t,s}^L = \left( \sum_{k=0}^{s} F^k - 1 \right) \left( \hat{x}_t^L + \hat{\Psi}_t - \hat{\Psi}_{t-1} \right) \]

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where
\[ \hat{x}_t^L = -(1 - \iota e L) \left( \hat{\Psi}_t - \hat{\Psi}_{t-1} + \hat{z}_{Lt} \right) \]

- aggregate employment definition

\[ (1 - \zeta_e) \left( \hat{e}_t - \hat{\Psi}_t \right) + \zeta_e \hat{x}_t^L = 0 \]

Combining these equations yields a dynamic Euler equation for employment as a function of hours:

\[ E_t (1 - \beta F) (1 - \iota e L) \left( \hat{\Psi}_t - \hat{\Psi}_{t-1} + \hat{z}_{Lt} \right) = \frac{(1 - \zeta_e \beta) (1 - \zeta_e)}{\zeta_e} \left( \hat{L}_t - \hat{\Psi}_t \right) \quad (43) \]

### E.3 Domestic sector

- production function

\[ mc_s \hat{y}_t = \hat{a}_t - \alpha \hat{z}_K t + \alpha \hat{K}_s t + (1 - \alpha) \hat{L}_t \quad (44) \]

- optimal capital-labor ratio

\[ \hat{r}_t^k + \hat{K}_s t - \hat{w}_t - \hat{L}_t = \hat{z}_K t \quad (45) \]

- marginal cost

\[ \hat{m}_c_t = \alpha \hat{r}_t^k + (1 - \alpha) \hat{w}_t - \hat{a}_t \quad (46) \]

- Pricing first-order condition

\[ E_t \sum_{s=0}^{\infty} \zeta_p^s \beta^s \left( \hat{Z}_{t,s}^p - \hat{m}_c_{t+s} + \frac{1}{\varepsilon_p - 1} \hat{z}_{t+s}^p \right) = 0 \]

- Compound indexation

\[ \hat{Z}_{t,s}^p = \hat{\theta}_t - \hat{x}_t^p + \sum_{k=0}^{s} \hat{x}_{t+k}^p \]

where

\[ \hat{x}_t^p = -(1 - \iota_p L) \hat{\pi}_t \]
aggregate price definition

\[ \zeta p \hat{\pi}^P_t + (1 - \zeta_p) \hat{\pi}_t = 0 \]

Combining the three pricing equations yields the price Phillips curve:

\[ (1 - \beta F) (1 - \iota_p L) \hat{\pi}_t = E_t \left( 1 - \zeta_p \beta \right) \left( 1 - \zeta_p \right) \left( \hat{m} c_i - \frac{1}{\varepsilon_p - 1} \hat{\pi}^P_t \right) \]

(47)

E.4 Open-economy block

E.4.1 imports

- Pricing first-order condition for imports of consumption goods

\[ E_t \sum_{s=0}^{\infty} \zeta_{c,m} \beta^s \left( \hat{Z}_{t,s} - \left( \hat{p}_{t+s}^f - \hat{p}_{t+s}^{c,m} \right) + \frac{1}{\varepsilon_{c,m} - 1} \hat{\varepsilon}_{t+s}^{c,m} \right) = 0 \]

- Compound indexation

\[ \hat{Z}_{t,s}^{c,m} = \hat{\theta}_t^{c,m} - \hat{p}_t^{c,m} - \hat{x}_t^{c,m} + \sum_{k=0}^{s} \hat{x}_{t+k}^{c,m} \]

where

\[ \hat{x}_t^{c,m} = - (1 - \iota_{c,m} L) \hat{\pi}_t^{c,m} \]

- aggregate import consumption price definition

\[ \zeta_{c,m} \hat{x}_t^{c,m} + (1 - \zeta_{c,m}) \left( \hat{p}_t^{c,m} - \hat{p}_t^{c,m} \right) = 0 \]

Combining these equations yields the import consumption price Phillips curve:

\[ E_t (1 - \beta F) (1 - \iota_{c,m} L) \hat{\pi}_t^{c,m} = \left( 1 - \zeta_{c,m} \beta \right) \left( 1 - \zeta_{c,m} \right) \left( \hat{p}_t^f - \hat{p}_t^{c,m} - \frac{1}{\varepsilon_{c,m} - 1} \hat{\varepsilon}_t^{c,m} \right) \]

(48)

- A similar Phillips curve holds for the importers of investment goods:

\[ E_t (1 - \beta F) (1 - \iota_{i,m} L) \hat{\pi}_t^{i,m} = \left( 1 - \zeta_{i,m} \beta \right) \left( 1 - \zeta_{i,m} \right) \left( \hat{p}_t^f - \hat{p}_t^{i,m} - \frac{1}{\varepsilon_{i,m} - 1} \hat{\varepsilon}_t^{i,m} \right) \]

(49)
E.4.2 exports

- World demand for exports

\[ \dot{X}_t = -\varepsilon_f \left( \hat{p}_t - \hat{p}_t^f \right) + \hat{y}_t + \hat{\varepsilon}_t^n f \]  
(50)

where \( \hat{\varepsilon}_t^n f \equiv \ln n_t^f - \ln n_f^t \).

- Pricing first-order condition for exports

\[ E_t \sum_{s=0}^{\infty} \zeta_x s \beta_s \left( \hat{Z}_{t,s}^x + \hat{p}_t^x + s + 1 \varepsilon_x - 1 \hat{\varepsilon}_t^x \right) = 0 \]

- Compound indexation

\[ \hat{Z}_{t,s}^w = \hat{\xi}_t^x - \hat{p}_t^x - \hat{x}_t^x + \sum_{k=0}^{s} \hat{x}_{t+k}^x \]

where

\[ \hat{x}_t^x = - (1 - \iota_x L) \hat{\pi}_t^x \]

- Aggregate export price definition

\[ \zeta_x \hat{x}_t^x + (1 - \zeta_x) (\hat{\xi}_t^x - \hat{p}_t^x) = 0 \]

Combining the pricing equations yields the export price Phillips curve:

\[ E_t (1 - \beta F) (1 - \iota_x L) \hat{\pi}_t^x = \frac{(1 - \zeta_x \beta) (1 - \zeta_x)}{\zeta_x} \left( -\hat{p}_t^x - \frac{1}{\varepsilon_x - 1} \hat{\varepsilon}_t^x \right) \]  
(51)

E.4.3 Net foreign asset accumulation

- Balance of payments

\[ \dot{b}_t^f + \dot{y}_t = \frac{1}{\beta} \left( \dot{b}_{t-1}^f + \dot{y}_{t-1}^f + \dot{\hat{r}}_{t-1} - \hat{\pi}_t - \dot{z}_y t + \dot{H}_{t-1} \right) \]

\[ + \frac{p_t^x X_t}{b_t y_t} (\hat{\xi}_t^x + \hat{X}_t) - \frac{p_t^f M_t}{b_t y_t} (\hat{p}_t^f + \hat{M}_t) \]  
(52)
E.4.4 Eurozone block

The equations describing developments in Eurozone output, inflation and interest rates are described in appendix bla.

E.5 Bundling

- domestic investment demand

\[ \hat{I}_t^d - \left( \hat{p}_t^i + \hat{I}_t \right) = (1 - n_i) \left( \varepsilon_{b,i} - 1 \right) \left( \hat{z}_t^{d,i} - \hat{z}_t^{m,i} + \hat{p}_t^{i,m} \right) \]  

(53)

- import investment demand

\[ \hat{I}_t^m + \hat{p}_t^{i,m} - \left( \hat{p}_t^i + \hat{I}_t \right) = -n_i \left( \varepsilon_{b,i} - 1 \right) \left( \hat{z}_t^{d,i} - \hat{z}_t^{m,i} + \hat{p}_t^{i,m} \right) \]  

(54)

- aggregate investment price definition

\[ \hat{z}_t^{d,i} + \hat{p}_t^i = (1 - n_i) \left( \hat{z}_t^{d,i} - \hat{z}_t^{m,i} + \hat{p}_t^{i,m} \right) \]  

(55)

- domestic consumption demand

\[ \hat{C}_t^d - \left( \hat{p}_t^c + \hat{C}_t \right) = (1 - n_c) \left( \varepsilon_{b,c} - 1 \right) \left( \hat{z}_t^{d,c} - \hat{z}_t^{m,c} + \hat{p}_t^{c,m} \right) \]  

(56)

- import consumption demand

\[ \hat{C}_t^m + \hat{p}_t^{c,m} - \left( \hat{p}_t^c + \hat{C}_t \right) = -n_c \left( \varepsilon_{b,c} - 1 \right) \left( \hat{z}_t^{d,c} - \hat{z}_t^{m,c} + \hat{p}_t^{c,m} \right) \]  

(57)

- aggregate consumption price definition

\[ \hat{z}_t^{d,i} + \hat{p}_t^c = (1 - n_c) \left( \hat{z}_t^{d,c} - \hat{z}_t^{m,c} + \hat{p}_t^{c,m} \right) \]  

(58)
E.6 Policy

- Government budget constraint

\[
b_\nu \left( \hat{b}_t + \hat{y}_t \right) = \frac{1}{\beta} \left( \hat{b}_{t-1} + \hat{y}_{t-1} + \hat{R}_{t-1} - \hat{\pi}_t - \hat{\pi}^i_t + \hat{\pi}^c_t - \hat{\pi}^x_t + \hat{H}_{t-1} \right)
\]

\[
= g_\nu \left( \hat{y}_t + \hat{y}_t \right) + T_\nu \left( \hat{y}_t + \hat{T}_t \right) - \left( r^k_s - \delta \right) \tau_k \frac{z_T}{z_y} K_s \left( r^k_s - \delta \hat{\pi}^i_t + \hat{K}_{t-1} - \hat{\pi}^i_t - \hat{\pi}^c_t \right)
\]

\[
- \tau_w \frac{w_s L_s}{y_s} \left( \hat{\omega}_t + \hat{L}_t \right) - \tau_c \frac{p^c_s C_s}{y_s} \left( \hat{\pi}^c_t + \hat{C}_t \right)
\]

- Transfer rule

\[
\hat{T}_t = -\varepsilon_T \hat{b}_{t-1}
\]

E.7 Market clearing

- domestic

\[
(1 - g_\nu) y_s \hat{y}_t = C^d_s \hat{C}^d_t + I^d_s \hat{I}^d_t + g_\nu y_s \hat{y}_t + X_s \hat{X}_t + (1 - \tau_k) \frac{z_T}{z_y} r^k_s K_s \hat{u}_t
\]

- imports

\[
M_s \hat{M}_t = C^m_s \hat{C}^m_t + I^m_s \hat{I}^m_t
\]

E.8 Definitions

- inflation rates

\[
\hat{\pi}_t^i - \hat{\pi}_t = (1 - L) \hat{\pi}^i_t + \hat{\pi}^i_t
\]

\[
\hat{\pi}_t^c - \hat{\pi}_t = (1 - L) \hat{\pi}^c_t - \hat{\pi}^c_t
\]

\[
\hat{\pi}_t^{i,m} - \hat{\pi}_t = (1 - L) \hat{\pi}^{i,m}_t + \hat{\pi}^{i,m}_t
\]

\[
\hat{\pi}_t^{c,m} - \hat{\pi}_t = (1 - L) \hat{\pi}^{c,m}_t + \hat{\pi}^{c,m}_t
\]

\[
\hat{\pi}_t^f - \hat{\pi}_t = (1 - L) \hat{\pi}^f_t + \hat{\pi}^f_t
\]

\[
\hat{\pi}_t^x - \hat{\pi}_t = (1 - L) \hat{\pi}^x_t
\]
• growth rates

\[
\hat{z}_{wt} = \frac{1}{1-\alpha} (\hat{z}_{At} - \alpha \hat{z}_{\Gamma t}) \tag{69}
\]
\[
\hat{z}_{Lt} = -\frac{1}{1+\eta} \hat{z}_{\varphi t} \tag{70}
\]
\[
\hat{z}_{yt} = \hat{z}_{Lt} + \hat{z}_{wt} \tag{71}
\]
\[
\hat{z}_{Kt} = \hat{z}_{yt} - \hat{z}_{\Gamma t} \tag{72}
\]
\[
\hat{z}_{It} = \hat{z}_{Kt} \tag{73}
\]
\[
\hat{z}_{Ct} = \hat{z}_{yt} + \rho \hat{z}_{\Gamma t} \tag{74}
\]
\[
\hat{z}_{Mt} = \hat{z}_{yt} - \hat{z}_{ppt} \tag{75}
\]
\[
\hat{z}_{ppt} = \frac{1}{\varepsilon_f} (\hat{z}_{yt} - \hat{z}_{yft}) \tag{76}
\]

E.9 Stochastic processes

• trend

\[
\hat{z}_{At} = v^A_t \sim N(0, \sigma^2_A) \tag{77}
\]
\[
\hat{z}_{\varphi t} = v^\varphi_t \sim N(0, \sigma^2_\varphi) \tag{78}
\]
\[
\hat{z}_{\Gamma t} = v^\Gamma_t \sim N(0, \sigma^2_\Gamma) \tag{79}
\]
\[
\hat{z}_{yft} = v^{\gamma y}_t \sim N\left(0, \sigma^2_y\right) \tag{80}
\]
• cyclical

\[
\hat{a}_t = \rho_a \hat{a}_{t-1} + v^a_t, \quad v^a_t \sim N(0, \sigma^2_a) \tag{81}
\]
\[
\hat{d}_t = \rho_d \hat{d}_{t-1} + v^d_t, \quad v^d_t \sim N(0, \sigma^2_d) \tag{82}
\]
\[
\hat{g}_t = \rho_g \hat{g}_{t-1} + v^g_t, \quad v^g_t \sim N(0, \sigma^2_g) \tag{83}
\]
\[
\hat{\mu}_t = \rho_{\mu} \hat{\mu}_{t-1} + v^\mu_t, \quad v^\mu_t \sim N(0, \sigma^2_{\mu}) \tag{84}
\]
\[
\hat{\epsilon}^b_t = \rho_b \hat{\epsilon}^b_{t-1} + v^b_t, \quad v^b_t \sim N(0, \sigma^2_b) \tag{85}
\]
\[
\hat{\epsilon}^p_t = \rho_p \hat{\epsilon}^p_{t-1} + v^p_t + \theta_p v^p_{t-1}, \quad v^p_t \sim N(0, \sigma^2_p) \tag{86}
\]
\[
\hat{\epsilon}^w_t = \rho_w \hat{\epsilon}^w_{t-1} + v^w_t + \theta_w v^w_{t-1}, \quad v^w_t \sim N(0, \sigma^2_w) \tag{87}
\]
\[
\hat{\epsilon}^r_t = \rho_r \hat{\epsilon}^r_{t-1} + v^r_t + \theta_r v^r_{t-1}, \quad v^r_t \sim N(0, \sigma^2_r) \tag{88}
\]
\[
\hat{\epsilon}^{nf}_t = \rho_{nf} \hat{\epsilon}^{nf}_{t-1} + v^{nf}_t + \theta_{nf} v^{nf}_{t-1}, \quad v^{nf}_t \sim N(0, \sigma^2_{nf}) \tag{89}
\]
\[
\hat{\epsilon}^{c,m}_t = \rho_{c,m} \hat{\epsilon}^{c,m}_{t-1} + v^{c,m}_t + \theta_{c,m} v^{c,m}_{t-1}, \quad v^{c,m}_t \sim N(0, \sigma^2_{c,m}) \tag{90}
\]
\[
\hat{\epsilon}^{i,m}_t = \rho_{i,m} \hat{\epsilon}^{i,m}_{t-1} + v^{i,m}_t + \theta_{i,m} v^{i,m}_{t-1}, \quad v^{i,m}_t \sim N(0, \sigma^2_{i,m}) \tag{91}
\]

E.10 Observation equations

The observations equations map model variables into observables in growth rates.

• for the non-stationary variables \( x_t = C_t, I_t, E_t, w_t, y_t, y^{f}_t, M_t \) and \( X_t \), we have

\[
\Delta \ln x^{obs}_t = \hat{x}_t - \hat{x}_{t-1} + \hat{\epsilon}_{xt} + \gamma_x
\]

where the shocks \( \hat{\epsilon}_{xt} \) and drifts \( \gamma_x \) were defined above and are subject to the cross-equation restrictions ensuring the correct co-integrating relationships. Section bla discusses how we implement these restrictions.

• For the stationary variables \( x_t = g_t, \pi_t, \pi^{i}_t, \pi^{f}_t, \pi^{x}_t, \pi^{m}_t \) and \( R_t \), we have

\[
x^{obs}_t = \hat{x}_t + x_\ast
\]

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We describe the Eurozone block directly in log-linearized form. The block consists of an IS curve for the output gap, \( \hat{y} \), Phillips curve for inflation, \( \hat{\pi} \), and a Taylor type empirical monetary policy rule for nominal interest rate, \( \hat{R} \). The output gap is the difference between actual output, \( \hat{y}_t \), and potential output, \( \hat{y}_f \), both given in terms of deviations from their common trend.

\[
\hat{y}_t = \alpha_y E_t \hat{y}_{t+1} + (1 - \alpha_y) \hat{y}_{t-1} - \phi_y (\hat{R}_t - E_t \hat{\pi}_f) + \varepsilon^y_t, \quad (92)
\]

\[
\hat{\pi}_f = \alpha_\pi E_t \hat{\pi}_{f,t+1} + (1 - \alpha_\pi) \hat{\pi}_{f,t-1} + \phi_{y} \hat{y}_t + \varepsilon^\pi_t, \quad (93)
\]

\[
\hat{R}_t = \rho_r \hat{R}_{t-1} + (1 - \rho_r) (\kappa E_t \hat{\pi}_{f,t+1} + \gamma E_t \hat{y}_{t+1}) + \varepsilon^r_t, \quad (94)
\]

\[
\hat{y}_f = \hat{y}_p + \hat{\pi}_f. \quad (96)
\]

Demand and supply shocks are autoregressive processes, while the monetary policy and potential output shocks are white noise.

\[
\varepsilon^y_t = \rho_y \varepsilon^y_{t-1} + v^y_t, \quad v^y_t \sim N(0, \sigma^2_y) \quad (97)
\]

\[
\varepsilon^\pi_t = \rho_\pi \varepsilon^\pi_{t-1} + v^\pi_t, \quad v^\pi_t \sim N(0, \sigma^2_\pi) \quad (98)
\]

\[
\varepsilon^r_t = v^r_t, \quad v^r_t \sim N(0, \sigma^2_r) \quad (99)
\]

\[
\varepsilon^{y_p}_t = v^{y_p}_t, \quad v^{y_p}_t \sim N(0, \sigma^2_{y_p}) \quad (100)
\]

Model parameters are estimated with Dynare’s MCMC routine, with two chains of 1 million draws, of which we retain the last 4. We limit the sample to 1995Q1 to 2011Q2, to ensure series are stationary. This sample corresponds roughly to the period of the existence of the Eurozone so as to have a clear economic interpretation of the model parameters, since the model contains monetary policy rule with the ECB’s monetary policy objective. Increasing the sample backwards an additional four years before the launch of the euro can be justified by the anticipated formation of the Eurozone.

Table 6 summarizes the results. The estimation is well-behaved according to the Brooks
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Prior Distribution</th>
<th>Posterior mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_y$</td>
<td>Weight on expected output gap</td>
<td>$N(0.5, 0.2)$</td>
<td>0.39</td>
</tr>
<tr>
<td>$\alpha_\pi$</td>
<td>Weight on expected inflation</td>
<td>$N(0.5, 0.2)$</td>
<td>0.74</td>
</tr>
<tr>
<td>$\phi_y$</td>
<td>Real interest rate elasticity of output gap</td>
<td>$N(-2, 1)$</td>
<td>0.23</td>
</tr>
<tr>
<td>$\phi_\pi$</td>
<td>Slope of Phillips curve</td>
<td>$N(0.03, 0.01)$</td>
<td>0.02</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>Weight of inflation in MP rule</td>
<td>$N(2, 5)$</td>
<td>1.85</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Weight of output gap in MP rule</td>
<td>$N(1, 4)$</td>
<td>0.45</td>
</tr>
<tr>
<td>$\rho_r$</td>
<td>AR(1) interest rate</td>
<td>$N(0.8, 1)$</td>
<td>0.79</td>
</tr>
<tr>
<td>$\rho_y$</td>
<td>AR(1) demand shock</td>
<td>$N(0, 4)$</td>
<td>0.53</td>
</tr>
<tr>
<td>$\rho_\pi$</td>
<td>AR(1) supply shock</td>
<td>$N(0, 4)$</td>
<td>-0.35</td>
</tr>
<tr>
<td>$\varrho$</td>
<td>AR(1) potential output</td>
<td>$B(0.95, 0.03)$</td>
<td>0.98</td>
</tr>
<tr>
<td>$\sigma_r$</td>
<td>s.e. monetary shock</td>
<td>$IG(0.001, \infty)$</td>
<td>0.04</td>
</tr>
<tr>
<td>$\sigma_y$</td>
<td>s.e. demand shock</td>
<td>$IG(0.001, \infty)$</td>
<td>0.02</td>
</tr>
<tr>
<td>$\sigma_\pi$</td>
<td>s.e. supply shock</td>
<td>$IG(0.001, \infty)$</td>
<td>0.04</td>
</tr>
<tr>
<td>$\sigma_{yp}$</td>
<td>s.e. potential output shock</td>
<td>$IG(0.001, \infty)$</td>
<td>0.06</td>
</tr>
</tbody>
</table>

Note: $B$: Beta; $IG$: Inverse Gamma; $N$: Normal. Arguments are mean, standard deviation, and support (when truncated).

Table 6. Prior distributions and posterior means

and Gelman diagnostics. The data appear informative for most parameters, in that posteriors usually differ significantly from priors. The exceptions are the interest elasticity of the output gap $\phi_y$ and the coefficient of inflation in the policy rule $\kappa$, where priors and posteriors coincide.
References


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