Relative Performance Evaluation in Presence of Exposure Risk\textsuperscript{1}

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Abstract: I study the consequences of a random exposure to common risk factors for relative performance evaluation (RPE). I show analytically that the volatility of firms’ exposure to common risk precludes the perfect filtering of observable luck from firm performance. It also limits the ability to reduce the exposure to common risk and is a critical factor for the composition of peer groups. If the exposure risk becomes large, RPE becomes essentially useless. Simulated regression analyses of my model indicate that an increasing exposure risk can have the following consequences: First, it limits firms’ ability to correctly identify correlated performance signals and reduces the explanatory power of regressions of firm performance on peer performance. Second, it increases the likelihood of a type-II error in empirical compensation studies. Third, it constrains the correct identification of relevant performance peers and thereby biases the composition of peer groups. Overall, my analysis suggests that the randomness of firms’ exposure to common risk can be an important factor in determining the use(fulness) of RPE.

Keywords: executive compensation, relative performance evaluation, reward for luck, exposure to common risk

JEL Classification: D86, J33, M12, M41, M52
1 Introduction

1.1 Motivation and summary of main results

Agency theory recommends firms to control for exogenous risk factors in evaluating agents’ performance in order to shield them against the compensation risk arising from the volatility of variables beyond their control. The literature distinguishes two forms of control for common risk. If common risk factors are directly measurable, such as commodity prices or currency exchange rates, agents’ performance should be evaluated net of the influence of common risk to avoid reward for (observable) luck (henceforth RFL) (Bertrand and Mullainathan 2001). If common risk factors are not directly observable but other firms are exposed to the same risks, agents’ performance should be evaluated relative to the performance of their peers in order to remove the measurable part of common risk from their performance measures (Holmstrom 1982). Unless stated otherwise, I will subsequently refer to both policies as special cases of relative performance evaluation (henceforth RPE).

In this paper, I argue that firms’ exposure to common risk factors is typically volatile (e.g. Engle 2009) and find that the resulting exposure risk can significantly limit the ability to control the influence of common risk on firm performance. As a consequence, the random exposure to common risk limits the usefulness of RPE. To derive this result, I study the linear aggregation of performance measures in the context of a simple agency model. The problem consists of finding a linear aggregate of firm performance and an informative but uncontrollable measure of common risk. As in Holmstrom (1982), firm performance is a linear function of the agent’s effort, common and idiosyncratic risk. However, different from previous literature, I do not assume that the (marginal) exposure to common risk is a constant but consider it as a random variable.1

This seemingly marginal change of a standard assumption made in many agency models has important consequences for the analysis of the aggregation problem and its solution. Even with multivariate normality of all random variables and linear aggregation rules, the standard solution methods for the aggregation problem used in the literature cannot be applied because aggregate performance measures no longer satisfy the necessary distribu-

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1See section 2 for additional references and a detailed discussion of this central assumption.
tional assumptions. To deal with this problem, I derive the linear aggregation rules as best linear predictors of firm performance given the information contained in the available performance measures. This statistical solution approach to the aggregation problem replicates the relevant results found in the literature for a constant common risk exposure and can be reconciled with an optimal contracting approach under some restrictive conditions.

To analyze how the random exposure of performance measures affects the standard predictions of RPE models, I derive linear aggregation rules for the performance of a focal firm with three different signal structures that cover the most common problem settings considered in the literature. The first case examines the problem of filtering firm performance for an observable factor of common risk in order to avoid RFL. The second case considers the aggregation of firm performance with a single noisy measure of common risk. Finally, the third case considers the construction of a peer group and its weight in the aggregate performance index.

The randomness of the common risk exposure affects the covariance structure of performance measures. On the one hand, it increases the variance of a given performance measure by a constant term that I coin "exposure risk". This term is an increasing function of the exposure variance and the variance and expectation of common risk. Since the exposure risk contained in a given performance measure cannot be eliminated or reduced by RPE, it adds to the idiosyncratic risk contained in the performance signal and makes it less useful for the control of common risk. On the other hand, the covariance between any pair of performance measures prone to random exposure captures only the expected and not the actual exposure to common risk contained in the two performance measures.

My analysis of aggregation rules yields the following results: For the first case, I find that the random exposure to common risk precludes perfect filtering of observable luck because the firm can only remove the expected but not the actual impact of common risk from firm performance. Moreover, since the firm’s own exposure risk is not reduced by the use of RPE, the resulting variance reduction is smaller than in the standard case of a constant exposure to common risk. This observation holds for all three signal structures studied in this paper.

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2Existing studies require that performance measures are either jointly normal or that the joint distribution belongs to a particular class of exponential distributions as specified in Banker and Datar (1989). See section 3 for a detailed discussion.
For the second case, I find that the optimal weight on a noisy measure of common risk does not depend on the firms’ own exposure risk but is mainly determined by the exposure risk contained in the performance signal. The higher the exposure risk embedded in the performance signal, the lower its usefulness for reducing the impact of common risk on firm performance and the lower its weight in the aggregate performance index. A similar observation can be made for the third case, where I find that the relative weights of peer firms within the peer index are monotonically decreasing in the peers’ exposure risk. I also demonstrate that the weight of a peer index consisting of identical firms as in Holmstrom (1982) is a decreasing function of the standardized exposure risk.

At the limit, if the exposure risk of individual peer firms becomes large, the optimal weight of these firms within the peer group goes to zero. The same observation holds for the aggregate exposure risk contained in a given performance index or peer group. It follows that a large exposure risk can make RPE essentially useless.

To complement my theoretical analysis, I simulate regression models with randomly generated data for the signal structures used in the theoretical part of the paper. The first set of regressions explores the relation between focal firm performance and given signals of peer performance for various levels of exposure risk. The results indicate that a high exposure risk can constrain firms’ ability to identify a statistical significant relation between firm performance and measures of common risk even though these variables are correlated. In addition, the exposure risk can also significantly reduce the explanatory power of the regression. In a related test, I examine the efficiency of the aggregation rules derived in the main part of the paper and find that the presence of exposure risk increases the likelihood of a type-II error in empirical compensation studies. This error can lead to the wrong conclusion that firm’s using the aggregation rules derived in section 4 do not use RPE (or reward their executives for luck).

I also examine the impact of exposure risk on firms’ ability to determine their relevant set of performance peers and find that there is a significant risk of incorrectly excluding peers with correlated performance but high exposure risk. Since the weights of individual peer firms are determined simultaneously, the failure to identify a subset of relevant performance peers will also bias the weights put on properly identified peer firms.

The contribution of my analysis to the RPE literature is threefold. First, I identify the
size of the exposure risk as a relevant factor for determining the aggregation of performance measures and the composition of peer groups for RPE. Second, I provide a new explanation for the limited use of RPE. My results might help to understand why recent research on the explicit use of relative performance RPE in executive compensation contracts finds that only a small fraction of firms actually uses RPE to determine executive compensation (Bannister and Newman 2003; Gong et al. 2011). Third, my study provides a potential explanation for the fact that numerous implicit tests of RPE have failed to identify its use and why other studies have found evidence for RFL.\footnote{See Albuquerque (2009) and Dikolli et al. (2013) for recent summaries of implicit RPE tests. Empirical evidence for RFL can be found in Bertrand and Mullainathan (2001) and Garvey and Milbourn (2006).}

The rest of the paper is organized as follows. The next subsection provides a brief overview of the related literature. Section 2 explains the model assumptions. Section 3 reviews the existing benchmark solutions for the aggregation problem in the absence of exposure risk and derives the objective criterion for the aggregation of performance measures in presence of exposure risk. Section 4 derives linear aggregation rules in presence of exposure risk for three different signal structures. Section 5 provides simulated regression analyses based on the theoretical model and explores additional implications for empirical research. Section 6 concludes the analysis with a summary of the main findings and some suggestions for empirical research.

1.2 Related Literature

To my best knowledge, the present paper is the first to analyze the potential consequences of exposure risk on RPE. In the appendix of a recent empirical paper, Albuquerque (2014) sketches a related idea in the context of a LEN model. She assumes that the covariance between firm performance $\tilde{x}$ and peer performance $\tilde{y}$ is a random variable that must be estimated from observing a noisy signal $\tilde{s}_{xy}$ of the true covariance $\sigma_{xy}$. Without affecting the variance of the performance measures, this approach can yield more or less RPE depending on the signal realization $s_{xy}$.

Several other theoretical studies have proposed various theoretical arguments that could explain RFL or the lack and/or limited use of RPE. For example, Aggarwal and Samwick (1999) argue that firms in industries with a small number of competitors have an incentive to
put a positive weight on peer performance in order to soften product market competition and show that this incentive can outweigh the potential benefits of RPE. Garvey and Milbourn (2003) consider a setting where RPE takes the form of costly hedging a given market index. In their model, the CEO and the firm can both hedge the market index at different cost. The optimal level of RPE depends on the relative magnitude of hedging costs and decreases if hedging becomes relatively more costly for the firm. Gopalan et al. (2010) study a model where the CEO decides on a firm’s exposure to the performance of the industry sector in which it operates. Before deciding on the firm’s exposure, the CEO can exert costly effort to acquire a private signal about future sector performance. To motivate the CEO’s effort and exposure choices, the optimal contract exhibits a positive relation to sector performance.

Other studies provide rational explanations for the presence of RFL in executive compensation contracts. For example, Göx (2008) shows that RFL can be an optimal response to the million-dollar cap for the tax deductibility of non-performance-based pay in Section 162(m) of the Internal Revenue Code. In a related paper, Ferriozi (2011) studies a binary limited liability model with post-contract-pre-decision information and shows that RFL can improve the agent’s implicit incentives to avoid bankruptcy.

Two recent theoretical studies provide potential explanations for the lack of empirical support found in implicit tests of RPE. Dikolli et al. (2013) study the optimal composition of a peer index in the context of a LEN model. They show that implicit tests of RPE are potentially biased against finding support for the use of RPE if firms and empiricists use different aggregation rules for constructing the peer group. However, the authors only obtain this result if they allow firms to depart from optimal aggregation rules. In a related study, Dikolli et al. (2014) examine the consequences of CEO power on the optimal aggregation of peer groups. Assuming that the CEO can use her power to set the weight on a given peer index, they find that the resulting contract does not completely remove common risk from the agent’s compensation. As a consequence, implicit tests of the strong-form RPE hypothesis are unlikely to find evidence for the use of RPE.
2 Model

I consider the relative performance evaluation problem of a representative firm. A risk and effort-averse manager (the agent) runs the firm’s operations on behalf of a group of risk-neutral firm owners (the principal). As in Holmstrom (1982), firm performance \( x_i \) is a linear function of the agent’s effort \( a_i \) and the realizations of two different risk factors, a common risk factor \( \eta \) and an idiosyncratic risk factor \( \varepsilon_i \)

\[
x_i = a_i + c_i \cdot \eta + \varepsilon_i,
\]

where \( c_i = \frac{\partial x_i}{\partial \eta} \) is the firm’s (marginal) exposure to common risk, \( E[\tilde{\varepsilon}_i] = 0, VAR[\tilde{\varepsilon}_i], E[\tilde{\eta}], VAR[\tilde{\eta}] > 0, \) and \( COV[\tilde{\varepsilon}_i, \tilde{\eta}] = 0. \)

There are \( n \) firms in the economy having the output structure given in (1). In what follows, I use the index \( i = 0 \) for the focal firm to distinguish it from its peers. The common risk factor \( \tilde{\eta} \) in (1) represents economic variables that affect the performance of all firms in the economy, such as input prices, foreign exchange rates, interest rates, stock market conditions, and other factors determining the state of the economy. By contrast, \( \tilde{\varepsilon}_i \) represents factors that affect the performance of firm \( i \) only, such as the success of the firm’s R&D activities, the efficiency of its production processes, or idiosyncratic shocks in product demand.

The fundamental difference between the two random factors for the purpose of incentive contracting is the fact that \( \tilde{\varepsilon}_i \) is not measurable whereas \( \tilde{\eta} \) can at least be measured with noise. More specifically, I assume that there is a publicly observable signal \( \tilde{y} \) that is informative about the common risk factor \( \tilde{\eta} \) but not controllable by the agent of the focal firm, i.e. the distribution of \( \tilde{y} \) does not depend on \( a_0 \). Since the agent is risk averse, the principal has an interest to shield him from the compensation risk caused by the common risk factor and to evaluate his performance relative to the realization of the informative signal \( \tilde{y} \). As in Holmstrom and Milgrom (1987), I assume that the focal firm evaluates the agent on the basis of a performance index that takes the form of a linear aggregation of realized firm performance and the value of the signal about the common risk factor

\[
z_0(x_0, y) = x_0 - \alpha \cdot y,
\]

\( ^4 \)Throughout the paper a “\( \sim \)” indicates a random variable, whereas the same letter without a “\( \sim \)” above denotes its realization.
where \( \alpha \) is the weight the focal firm puts on the realization of \( \tilde{y} \).

The key complication that distinguishes the present study from previous RPE models is the fact that the marginal exposure of firm \( i \) to common risk, \( \tilde{c}_i \), is a random factor on its own. This assumption reflects the well documented fact that firms’ exposure to common risk appears to vary over time (Engle 2009). For example, it is well documented in the asset pricing literature that firms’ exposure to market risk as measured by their beta factors exhibits a significant level of volatility (e.g. Jagannathan and Wang, 1996; Fama and French, 1997 and 2006; Lewellen and Nagel, 2006; Ang and Chen, 2007). Other studies find similar patterns for the exposure of U.S. firms to foreign exchange-rate risk (e.g. Allayannis and Ihrig 2001; Francis et al. 2008) or changes in commodity prices.\(^5\)

To capture the volatility of the firm’s exposure to common risk in my analysis, I model \( \tilde{c}_i \) as an independent random variable with \( E[\tilde{c}_i] > 0, \text{VAR}[\tilde{c}_i] > 0, \text{COV}[\tilde{c}_i, \tilde{c}_i] = 0, \text{COV}[\tilde{\eta}, \tilde{c}_i] = 0 \) for all \( i \), and \( \text{COV}[\tilde{c}_i, \tilde{c}_j] = 0 \) for \( i \neq j \). To keep the analysis tractable, I subsequently assume that all random variables are not only mutually independent but also drawn from a multivariate normal distribution. With this structure the overall performance risk of firm \( i \) takes the form (Bohrnstedt and Goldberger, 1969)

\[
\text{VAR}[\tilde{x}_i] = \text{VAR}[\tilde{\varepsilon}_j] + E[\tilde{c}_i]^2 \cdot \text{VAR}[\tilde{\eta}] + \text{VAR}[\tilde{c}_i] \cdot (\text{VAR}[\tilde{\eta}] + E[\tilde{\eta}]^2). \tag{3}
\]

The expression in (3) comprises three terms. The first term, \( \text{VAR}[\tilde{\varepsilon}_j] \), represents the idiosyncratic part of firm risk. The second term, \( E[\tilde{c}_i]^2 \cdot \text{VAR}[\tilde{\eta}] \), is the risk caused by the firm’s expected exposure to the common risk factor \( \tilde{\eta} \). Finally, the third term in (3),

\[
R_i = \text{VAR}[\tilde{c}_i] \cdot (\text{VAR}[\tilde{\eta}] + E[\tilde{\eta}]^2) \tag{4}
\]

is the part of the overall firm risk caused by the randomness of the firm’s exposure to the common risk factor. As indicated in the introduction, I subsequently refer to \( R_i \) as the exposure risk of firm \( i \) to distinguish this part of the firm’s performance risk from the risk caused by other factors.

It can be seen from the expression in (4) that the firm’s exposure risk is the product of two factors. The first factor, \( \text{VAR}[\tilde{c}_i] \), is the variance of the firm’s exposure to \( \tilde{\eta} \). The second

\(^5\) Comprehensive online data on intertemporal correlation patterns of prices for various asset classes including commodities is provided by The V-Lab (vlab.stern.nyu.edu) organized by The Volatility Institute at NYU.
factor is the second raw moment of $\eta$, i.e. $E[\eta^2] = \text{VAR}[\eta] + E[\eta]^2$. Thus, the exposure risk of firm $i$ is not only increasing in the variance of firm $i$’s own exposure $\tilde{c}_i$ but also in the variance and the expectation of the common risk factor $\tilde{\eta}$. The higher the value of each of these factors, the higher is the exposure risk of firm $i$. In other words, the volatility of the common risk factor and the volatility of firm $i$’s exposure to $\tilde{\eta}$, are mutually reinforcing each other in augmenting its overall exposure risk.

To analyze how the presence of exposure risk affects the standard predictions of RPE models, I derive linear aggregation rules for three different signal structures that cover the most common problem settings considered in the literature. The first case assumes that the common risk factor is directly observable, such as foreign exchange rates or commodity prices, so that

$$y = \eta. \quad (5)$$

The structure in (5) has been widely used as a benchmark model in studies analyzing the phenomenon of RFL (Bertrand and Mullainathan 2001, Garvey and Milbourn 2006). These studies present evidence suggesting that CEO pay is positively related to observable random factors beyond the CEO’s control. This observation contradicts the recommendation of agency theory to evaluate the performance of the CEO net of the impact of observable random factors. I also consider a second signal structure, where the signal itself is subject to exposure risk and measurement error

$$y = c_j \cdot \eta + \varepsilon_j, \quad j \neq 0. \quad (6)$$

The structure in (6) allows me to distinguish the firm’s own exposure risk $R_0$ from the exposure risk $R_j$ and the idiosyncratic risk embedded in the performance measure and to study the consequences of each of these risk factors for the composition of the performance index in (2). Finally, I consider a signal structure that has been frequently used in predicting the optimal aggregation of peer groups into a peer index for the purpose of filtering common shocks that affect the performance of all firms in a given peer group (Holmstrom 1982, Dikolli et al. 2013). For this case, $y$ takes the form of a weighted average of realized peer performance

$$y = \sum_{j=1}^n \gamma_j \cdot x_j, \quad (7)$$

where $\gamma_j$ and $x_j$ denote the index weight and the performance of peer firm $j$, respectively, and $x_j$ is defined in (1). Since $x_j$ is prone to both, measurement error and exposure risk,
the structure in (7) can also be interpreted as an extension of (6), i.e. a rule for filtering an index of \( n \) imperfectly measurable random factors beyond the agent’s control.

3  Linear aggregation of performance measures

To examine how the presence of exposure risk affects the linear aggregation of performance measures, I first review existing solutions to the aggregation problem and the conditions under which they have been derived. Holmstrom and Milgrom (1987) provide a first solution to the aggregation rule in (2). Assuming that \( \tilde{x}_0 \) and \( \tilde{y} \) are jointly normally distributed, they show that the optimal weight on realized performance \( y \) within the performance index \( z_0 \) equals\(^6\)

\[
\alpha = \frac{COV[\tilde{x}_0, \tilde{y}]}{VAR[\tilde{y}]}.
\] (8)

Bertrand and Mullainathan (2001) use the result in (8) to derive a theoretical prediction for their empirical test of RFL in executive pay. In their model, \( x_0 \) takes the form of (1) and \( y = \eta \) as in (5). However, in contrast to the present study, Bertrand and Mullainathan (2001) take \( \gamma_0 \) as a known constant. With these assumptions \( COV[\tilde{x}_0, \tilde{y}] = c_0 \cdot VAR[\tilde{y}] \) and \( \alpha = c_0 \). It follows that \( z_0 = a_0 + \varepsilon_0 \) so that the agent’s compensation does not depend on the realized value of the observable random factor \( \tilde{\eta} \).

Holmstrom (1982) first considers the problem of aggregating multiple peers into a performance index of the form given in (7). Assuming that \( c_i = 1 \) for all \( i \) and that \( \tilde{\eta} \) as well as all idiosyncratic noise terms \( \tilde{\varepsilon}_j \) are independent and normally distributed, he derives the optimal aggregation rule

\[
\gamma_j = \frac{\tau_j}{\sum_{j=1}^{n} \tau_j},
\] (9)

where \( \tau_j = 1/VAR[\tilde{\varepsilon}_j] \) is the precision of \( \tilde{\varepsilon}_j \). Extending Holmstrom (1979), he shows that the weighted average of peer firms in (9) is a sufficient statistic for the information contained in the output vector \( x = (x_0, \ldots, x_n) \) with respect to the agent’s effort \( a_0 \). More recently, Dikolli et al. (2013) study a variation of Holmstrom’s RPE model where \( c_i \) is a firm specific

\(^6\)Banker and Datar (1989) derive the same result assuming a general agency model and a restricted class of exponential joint density functions \( f(x_0, y; a_0) \) that includes the normal distribution as a special case.
constant and derive the aggregation rule

\[ \gamma_j = \frac{c_j \cdot \tau_j}{\sum_{j=1}^{\eta} c_j \cdot \tau_j}. \]  

(10)

Different from Holmstrom (1982), who finds his result in the context of a general agency model, Dikolli et al. (2013) determine the aggregation rule in (10) in the context of a LEN-model. However, despite the differences in the model details, the results in (8), (9), (10) are all based on the convenient analytical properties of normally distributed (aggregate) performance measures.

Unfortunately, this prerequisite is not given in the presence of exposure risk. The difficulty arises from the fact that the performance measure in (1) contains the product of the two random variables \( \tilde{c}_i \) and \( \tilde{\eta} \). This product is not normally distributed even if \( \tilde{c}_i \) and \( \tilde{\eta} \) follow a normal distribution as it is assumed here. Therefore, the performance evaluation problem in presence of exposure risk can neither be solved in the context of a LEN-model, nor can it be found by a rearrangement of probability density functions as in Holmstrom (1982).

To deal with this problem, I do not derive the aggregation rule in (2) from an optimal contracting model but from a statistical objective criterion that yields the results in (8), (9), and (10) without requiring normally distributed performance measures. Since \( \tilde{y} \) is not controllable but informative, its sole purpose is to improve the principal’s inference drawn from a given realization of \( \tilde{y} \) on the realization of the common risk factor in \( \tilde{x}_0 \) and thereby on the agents’ effort level \( a_0 \). This statistical measurement problem can be represented in terms of finding the best linear predictor (BLP) for \( \tilde{x}_0 \) given \( \tilde{y} \). If \( \tilde{y} \) is a scalar, the BLP takes the form

\[ h(\tilde{y}) = \beta_0 + \beta_1 \cdot \tilde{y}, \]

where the parameters \( \beta_0 \) and \( \beta_1 \) minimize the mean square error \( E[(\tilde{x}_0 - h(\tilde{y}))^2] \). The solution of this estimation problem is equivalent to the result of a linear population regression of \( \tilde{x}_0 \) on \( \tilde{y} \). Moreover, given the linearity of the aggregation rule in (2), the BLP is equivalent to

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7As Craig (1936) and Epstein (1948) show, the product of two normally distributed random variables with mean \( \mu_i \) and variance \( \sigma_i^2 \) takes the form of a Bessel function. As shown by Aroian (1947) and discussed in further detail by Hayya and Ferrara (1972), this function can only be approximated by a normal distribution if the ratios \( \mu_i / \sigma_i \) approach infinity for both random variables. However, in the context of my model, this feature is not a convenient assumption because it implies that the exposure risk is negligible.
the conditional expectation function $E[\tilde{x}_0|\tilde{y}]$ (e.g. Goldberger, 1991, ch. 5; Amemiya 1994, ch. 4). The following proposition defines the objective criterion for the case of multiple signals and relates the result to the aggregation rules in (8), (9), and (10).

**Proposition 1:** Let $\tilde{x}_0$ be a random variable and $\tilde{y}' = (\tilde{y}_1, ..., \tilde{y}_n)$ the transpose of a $n \times 1$ random vector with $n \times n$ covariance-matrix $V$ and a $n \times 1$ covariance vector $d$, where

$$
V = \begin{pmatrix}
VAR[\tilde{y}_1], & \ldots & COV[\tilde{y}_1, \tilde{y}_n] \\
\vdots & \ddots & \vdots \\
COV[\tilde{y}_n, \tilde{y}_1] & \ldots & VAR[\tilde{y}_n]
\end{pmatrix},
d = \begin{pmatrix}
COV[\tilde{x}_0, \tilde{y}_1] \\
\vdots \\
COV[\tilde{x}_0, \tilde{y}_n]
\end{pmatrix}.
$$

With this structure, the BLP takes the form $\beta_0 + \beta' \tilde{y}$, where $\beta' = (\beta_1, ..., \beta_n)$ is the transpose of a $n \times 1$ parameter vector satisfying

$$
\beta = V^{-1}d.
$$

(11)

The aggregation rules in (8), (9), and (10) are special cases of (11). **Proof:** see appendix.

According to proposition 1, the BLP in (11) reproduces the results in (8), (9), and (10). More importantly, since information on the structure of the covariance-matrix $V$ and the covariance vector $d$ is sufficient to determine the BLP, a solution to the aggregation problem can also be found if $c_i$ is a random variable. This fact is particularly convenient for two reasons: First, it permits to examine how the presence of exposure risk affects the standard rules for the linear aggregation of performance measures. Second, since the BLP can be interpreted as the expected result of a linear population regression, it is an appropriate benchmark for evaluating the consequences of exposure risk for the results of empirical RFL and RPE studies. However, the convenient properties of the BLP do not imply that the aggregation rule in (11) is generally part of the optimal solution of the underlying agency problem. Corollary 1 identifies conditions for which the BLP optimally solves the agency problem.

**Corollary 1:** If the focal firm offers the agent a linear compensation contract $s(z_0) = w_0 + v_0 \cdot z_0$ and the agent has mean variance-preferences $U_A = E[s(\tilde{z}_0)] - C(a_0) - h(r) \cdot VAR[s(\tilde{z}_0)]$ as well as strictly convex effort cost $C(a_0)$, the aggregation rule in (11) optimally solves the agency problem. **Proof:** see appendix.

Intuitively, corollary 1 relies on the fact that the BLP is found by minimizing the mean square error. As shown in the appendix, this objective criterion can be reformulated in a
way that it is equivalent to the minimization of the variance of the performance measure, \( \text{VAR}[^x_0] \). If the contract is linear and the agent has mean-variance preferences, the optimal aggregation rule serves the same purpose, that is, the optimal weight \( \alpha \) in (2) is set so that it minimizes the variance of the agent’s performance measure for an arbitrary effort level \( a_0 \). Therefore, the BLP optimally solves the agency problem under the conditions given in corollary 1.

4 Linear aggregation with exposure risk

4.1 Filtering observable luck

I begin the analysis of the consequences of exposure risk for the linear aggregation of performance measures with the simplest case in (5), where the realization of the common risk factor is perfectly measurable. As explained in section 3, Bertrand and Mullainathan (2001) employ this signal structure as a theoretical benchmark for their empirical RFL test. With constant exposure to the common risk factor, the optimal aggregation rule sets the signal weight in the performance index equal to the marginal exposure parameter \( \alpha = c_0 \). The corresponding aggregation rule in the presence of exposure risk is given in proposition 2.

**Proposition 2:** If the common risk factor is perfectly measurable, the aggregation rule in (11) becomes

\[
\alpha^* = E[^c_0].
\]  

**Proof:** Under multivariate normality and independence of all random variables, \( \text{COV}[^x_0, \widetilde{y}] = E[^c_0] \cdot \text{VAR}[\widetilde{y}] \) (Bohrnstedt and Goldberger, 1969). With this covariance, the BLP in (11) becomes (12).

Different from the case of a known exposure to common risk, the aggregation rule in (12) removes only the expected marginal exposure to the common risk factor \( \widetilde{y} \) from the performance measure in (2) even though the realization of \( \widetilde{y} \) is perfectly observable. This policy has important consequences for the accuracy of filtering performance measures for observable random factors.

**Corollary 2:** The presence of exposure risk precludes the complete filtering of observable
luck. **Proof:** Substituting for $\alpha$ from (12) into (2) yields the aggregate performance index

$$z_0(\alpha^*) = a_0 + \varepsilon_0 + (c_0 - E[\bar{c}_0]) \cdot \eta. \quad (13)$$

The expression in (13) is a function of $\eta$.

According to Corollary 2, the aggregate firm performance varies with the realization of the common risk factor $\bar{\eta}$ although the firm removes the expected impact of observable luck from the performance index. The aggregate performance measure in (13) is only independent of $\eta$ if the realized exposure $c_0$ happens to equal the expected exposure $E[\bar{c}_0]$. However, this hairline case is a zero probability event ex ante. Thus, depending on the value of the realized marginal exposure $c_0$, aggregate firm performance can either be positively ($c_0 > E[\bar{c}_0]$) or negatively ($c_0 < E[\bar{c}_0]$) related to the realization of the common risk factor $\bar{\eta}$.

In any case, the presence of exposure risk excludes a complete removal of the common risk factor from the performance measure in (2). In fact, the filtering rule in (12) reduces the risk of the focal firm’s performance measure for a given effort level $a_0$ by the expected contribution of the common risk factor to the firm’s performance risk,

$$VAR[\bar{x}_0] - VAR[\bar{z}_0(\alpha^*)] = E[\bar{c}_0]^2 \cdot VAR[\bar{\eta}]. \quad (14)$$

Accordingly, the resulting variance of the aggregate performance measure in (13),

$$VAR[\bar{z}_0(\alpha^*)] = VAR[\bar{\varepsilon}_0] + R_0, \quad (15)$$

comprises the idiosyncratic performance risk and a second term, $R_0 = VAR[\bar{c}_0] \cdot E[\bar{\eta}^2]$, representing the firm’s own exposure risk as defined in (4). The expression in (15) indicates that the optimal aggregation rule in (12) does not allow the firm to remove its own exposure risk from its performance measure. Thus, the randomness of the firm’s exposure to the common risk factor essentially augments the undiversifiable part of the firm’s performance risk and thereby makes the performance measure riskier as for the case of a constant exposure to common risk.

### 4.2 Aggregating a noisy measure of common risk

In this subsection, I consider the aggregation rule for the second signal structure in (6) where the common risk is not perfectly measurable. The signal structure in (6) exhibits two
different distortions that prevent the firm from perfectly measuring the realized value of the common risk factor $\tilde{\eta}$. The first distortion is a standard measurement error represented by the additive noise term $\tilde{\varepsilon}_j$. The second distortion is the random exposure to the common risk factor embedded in the performance measure, represented by the random variable $\tilde{c}_j$, where $j \neq 0$. The aggregation rule for this signal structure is given in proposition 3.

**Proposition 3:** If the common risk factor is imperfectly measurable, the aggregation rule in (11) becomes

$$
\alpha^o = \frac{E[\tilde{c}_0] \cdot E[\tilde{c}_j] \cdot \text{VAR}[\tilde{\eta}]}{\text{VAR}[\tilde{\varepsilon}_j] + E[\tilde{c}_j]^2 \cdot \text{VAR}[\tilde{\eta}] + R_j}
$$

where $R_j = \text{VAR}[\tilde{c}_j] \cdot (\text{VAR}[\tilde{\eta}] + E[\tilde{\eta}]^2)$ is the exposure risk embedded in the performance measure as defined in (4). The performance measure weight in (16) is decreasing in $R_j$, $\text{VAR}[\tilde{c}_j]$ and $\text{VAR}[\tilde{\varepsilon}_j]$. It is increasing in $\text{VAR}[\tilde{\eta}]$ and $E[\tilde{c}_0]$. The effect of $E[\tilde{c}_j]$ is ambiguous. **Proof:** see appendix.

A comparison of the aggregation rules in (12) and (16) shows that the presence of measurement distortions embedded in the performance signal $\tilde{y}$ can have a significant impact on the aggregation of the performance index. Since $R_j$ as well as the variances of $\tilde{c}_j$ and $\tilde{\varepsilon}_j$ increase the variance of $\tilde{y}$ without affecting the covariance between $\tilde{x}_0$ and $\tilde{y}$, the optimal performance measure weight $\alpha^o$ is monotonically decreasing in $R_j$, $\text{VAR}[\tilde{c}_j]$ and $\text{VAR}[\tilde{\varepsilon}_j]$. Thus, the additional risk factors embedded in the performance measure decrease the precision of $\tilde{y}$ and thereby reduce the relative usefulness of $\tilde{y}$ for protecting the agent’s compensation against variations of the common risk factor $\tilde{\eta}$.

For a given variance of the signal $\tilde{y}$, its optimal weight in the aggregated performance index is increasing in the expected exposure of the focal firm to the common risk factor, $E[\tilde{c}_0]$. Intuitively, $\alpha^o$ is positively related to the firm’s expected exposure to common risk because an increasing exposure implies a stronger correlation between $\tilde{x}_0$ and $\tilde{\eta}$ and a higher correlation makes the use of $\tilde{y}$ more valuable for contracting.

Unlike the other factors, a higher variance of the common risk factor $\tilde{\eta}$ and an increase of $E[\tilde{c}_j]$ positively affect $\text{COV}[\tilde{x}_0, \tilde{y}]$ and $\text{VAR}[\tilde{y}]$. However, an increase of $\text{VAR}[\tilde{\eta}]$ has a positive net effect on $\alpha^o$ because for any given level of the ratio between the two measures the relative increase of the numerator exceeds the relative change of the denominator. By

\[ \frac{\partial \text{COV}[\tilde{x}_0, \tilde{y}]}{\partial \text{VAR}[\tilde{\eta}]} / \text{COV}[\tilde{x}_0, \tilde{y}] > \frac{\partial \text{VAR}[\tilde{y}]}{\partial \text{VAR}[\tilde{\eta}]} / \text{VAR}[\tilde{y}] \]
contrast, an increase of the expected signal exposure to the common risk factor, \( E[\tilde{c}_j] \), can reduce or increase \( \alpha^o \). The effect is positive, whenever

\[
V_j = VAR[\tilde{x}_j] + R_j > E[\tilde{c}_j]^2 \cdot VAR[\tilde{\eta}] \tag{17}
\]

where \( V_j \) is the total error of \( \tilde{y} \) in measuring \( \tilde{\eta} \). It comprises the idiosyncratic risk \( VAR[\tilde{c}_j] \) and the exposure risk \( R_j \) contained in \( \tilde{y} \). That is, whenever the expected contribution of the common risk factor to the variance of \( \tilde{y} \) on the right hand side of (17) is smaller (larger) than the total measurement error \( V_j \), an increase of \( E[\tilde{c}_j] \) has a positive (negative) impact on \( \alpha^o \).

Finally, it can be seen that the firm’s own exposure risk, \( R_0 \), or equivalently \( VAR[\tilde{c}_0] \), does not affect the aggregation rules in (12) and (16) because it is idiosyncratic to the focal firm and independent of the weight placed on the performance measure \( \tilde{y} \). Corollary 3 summarizes the overall effect of the aggregation rule in (16) on the risk of the aggregated performance measure \( \tilde{z}_0 (\alpha^o) \).

**Corollary 3:** If the common risk factor is imperfectly measurable, the optimal aggregation rule reduces the risk of the performance measure by

\[
VAR[\tilde{x}_0] - VAR[\tilde{z}_0 (\alpha^o)] = E[\tilde{c}_0]^2 \cdot VAR[\tilde{\eta}] \cdot q^o, \quad q^o = \frac{E[\tilde{c}_j]^2 \cdot VAR[\tilde{\eta}]}{VAR[\tilde{y}]} < 1. \tag{18}
\]

The risk reduction is strictly smaller than the risk reduction with a perfectly measurable common risk factor in (14) and monotonically decreasing in \( VAR[\tilde{y}] \). **Proof:** Follows from the fact that \( VAR[\tilde{y}] = V_j + E[\tilde{c}_j]^2 \cdot VAR[\tilde{\eta}] \) and \( \partial q^o / \partial VAR[\tilde{y}] < 0 \).

According to corollary 3, the presence of exposure risk and measurement error in the performance signal \( \tilde{y} \) lowers its usefulness for reducing the variance of the performance index and thereby its desirability for contracting. The higher the exposure risk and the higher the measurement noise embedded in the performance signal, the lower the scaling factor \( q^o \) in (18) that determines the level of risk reduction relative to the benchmark case of a perfectly measurable common risk factor in (14). The scaling factor \( q^o \) has an intuitive interpretation. It is defined as the ratio of the expected amount of common risk to the total risk contained in the signal \( \tilde{y} \) and represents the expected percentage of common risk contained in the performance measure. As shown in (3), \( VAR[\tilde{y}] \) is the sum of three factors, the expected common risk, the idiosyncratic risk and the exposure risk. Therefore, the scaling factor
$q^o$ is increasing in the expected amount of common risk and decreasing in the amounts of idiosyncratic and exposure risk. The consequences of the last observation are summarized below:

**Corollary 4:** If the exposure risk become very large, the signal weight $\alpha^o$ in (17) and the scaling factor $q^o$ in (18) both go to zero. **Proof:** It holds that $\lim_{R_j \to \infty} \alpha^o = \lim_{R_j \to \infty} q^o = 0$.

Thus, at the limit a substantial exposure risk can render the performance signal $\tilde{y}$ useless for shielding the agent’s compensation against common risk.

### 4.3 Aggregation of a peer index

The last case of my analysis considers the aggregation rule for the third signal structure in (7) where the performance index $\tilde{y}$ comprises a group of peer firms with a similar performance structure. That is, the performance of each of the $n$ potential peer firms is affected by an idiosyncratic shock $\tilde{e}_j$ and each peer faces a random exposure to the common risk factor $\tilde{\eta}$ represented by the random variable $\tilde{c}_j$. The optimal composition of the peer index in (7) is summarized in proposition 4.

**Proposition 4:** If the performance index comprises a weighted average of peer firms with firm performance $\tilde{x}_j$ as defined in (1), the optimal weight of firm $j$ equals

$$\gamma^+_j = \frac{E[\tilde{c}_j] \cdot \delta_j(R_j)}{\sum_{j=1}^{n} E[\tilde{c}_j] \cdot \delta_j(R_j)} \tag{19}$$

where $\delta_j(R_j) = 1/V_j$ is the total precision of the peer signal $\tilde{x}_j$ in measuring $\tilde{\eta}$ and $V_j = \text{VAR}(\tilde{c}_j) + R_j$ as defined in (17). The following observations can be made:

1) The optimal peer weight in (19) is increasing in $E[\tilde{c}_j]$ and $\delta_j(R_j)$ and decreasing in $E[\tilde{c}_k]$ and $\delta_k(R_k)$ for $k \neq j$.

2) The optimal weight of peer $j$ is monotonically decreasing in its own exposure risk. It approaches zero as $R_j$ approaches infinity.

**Proof:** see appendix.

A comparison of the aggregation rules in the absence and the presence of exposure risk in (10) and (19) shows that in both cases the index weight of peer firm $j$ is increasing in its (expected) exposure and in its precision in measuring the peer’s exposure to the common risk factor $\tilde{\eta}$. However, since the signal precision in the presence of exposure risk, $\delta_j(R_j)$,
is decreasing in $R_j$, a higher exposure risk reduces the usefulness of firm $j$ for the purpose of RPE. At the limit, if the exposure risk of firm $j$ becomes very large, the firm’s optimal weight in the peer index goes to zero. Likewise, for a given exposure and precision of peer firm $j$, an increase of the exposure and/or precision of other firms within the peer index reduces the optimal weight of firm $j$. The opposite is true for an increase in the exposure risk of firm $k$. Because $\delta_k(R_k)$ is decreasing in $R_k$, a higher exposure risk of firm $k$ increases the optimal weight of all other firms in the performance index.

These considerations show that the exposure risk of peer firms is a critical factor in determining their usefulness for the purpose of relative performance evaluation. The higher the exposure risk of a potential peer firm, the lower its relevance for RPE and for the composition of the peer index. Moreover, since the optimal index weight of each peer firm depends on its own exposure risk as well as on the exposure risk of the other firms in the performance index, a change of the exposure risk at one firm affects the optimal index weights of all other firms. Corollary 5 summarizes the impact of exposure risk on the optimal weight of the peer group in the aggregate performance index and the variance of the performance measure.

**Corollary 5**: With the peer index in (7) and the peer weights in (19), the optimal index weight becomes

$$
\alpha^+ = \frac{E[\tilde{c}_0] \cdot VAR[\tilde{\eta}] \cdot \sum_{j=1}^{n} E[\tilde{c}_j] \cdot \delta_j(R_j)}{1 + \sum_{j=1}^{n} E[\tilde{c}_j]^2 \cdot \delta_j(R_j)}.
$$

Using the index weight in (20) reduces the risk of the focal firm’s performance measure by

$$
VAR[\tilde{x}_0] - VAR[\tilde{z}_0(\alpha^+)] = E[\tilde{c}_0]^2 \cdot VAR[\tilde{\eta}] \cdot q^+, \quad q^+ = \frac{z^+}{1 + z^+} < 1
$$

where $z^+ = VAR[\tilde{\eta}] \cdot \sum_{j=1}^{n} E[\tilde{c}_j]^2 \cdot \delta_j(R_j)$. The risk reduction is proportional to the scaling factor $q^+ \in (q^o, 1)$ and monotonically increasing in $z^+$. **Proof**: see appendix.

A comparison of the risk reductions achieved with a perfectly measurable common risk factor and the peer index in (14) and (21) shows that the exposure risk contained in the measure of peer performance significantly affects the usefulness of the peer index in reducing the focal firm’s compensation risk. Similar to $q^o$ in (18), the scaling factor $q^+$ determines the level of risk reduction relative to the benchmark case of a perfectly measurable common risk factor in (14). Its size is determined by the factor $z^+$ representing a weighted sum of the expected common risks contained in the performance measures of the firms within the
peer index. Using the definition of $\delta_j(R_j)$, the contribution of firm $j$ to the factor $z^+$ can be expressed as:

$$z^+_j = \frac{VAR[\bar{\eta}] \cdot E[\bar{c}_j]^2}{VAR[\bar{\xi}_j] + R_j},$$

where $z^+ = \sum_{j=1}^n z^+_j$. Thus, $z^+_j$ is the ratio of the expected common risk contained in $\bar{\xi}_j$ to the sum of the idiosyncratic risk and the exposure risk in $\bar{x}_j$. The higher the expected common risk and the lower the idiosyncratic risk and the exposure risk, the higher is the incremental contribution of peer firm $j$ to the scaling factor $z^+$ and thereby to the overall reduction of the performance measure risk. Intuitively, a large expected exposure to common risk increases the usefulness of firm $j$ for RPE, whereas a high exposure risk reduces its desirability for RPE.

To gain additional insights on the impact of the aggregate exposure risk on the optimal level of RPE, assume that all peer firms face identically distributed risk factors. With $\bar{\xi}_j = \bar{\xi}, \bar{c}_j = \bar{c}$, and $R_j = R$, the optimal index weight in (20) becomes

$$\alpha^+ = \frac{n \cdot E[\bar{c}_0] \cdot E[\bar{c}] \cdot VAR[\bar{\eta}]}{VAR[\bar{\xi}] + n \cdot E[\bar{c}]^2 \cdot VAR[\bar{\eta}] + R}. \quad (22)$$

A closer inspection of the expression in (22) shows to facts. First, for a given value of the standardized exposure risk, $R = VAR[\bar{c}] \cdot E[\bar{\eta}^2]$, the usefulness of RPE is increasing in the number of peer firms because $\partial \alpha^+ / \partial n > 0$. Second, and more important, the standardized exposure risk has a negative impact on the optimal weight of the performance index. The second relation implies the following observation:

**Corollary 6:** If $R$ approaches infinity, the optimal index weight in (22) approaches zero.

**Proof:** Evident from the structure of the expression in (22).

Thus, at an aggregate level, a high exposure risk can render peer groups completely useless for the purpose of relative performance evaluation.

## 5 Practical and empirical implications of exposure risk

The results of the theoretical analysis in section 4 indicate that the presence of exposure risk can have a significant impact on the linear aggregation of performance measures. To illustrate the potential economic consequences of varying degrees of exposure risk and to
provide additional insights regarding the practical implementation of the theoretical solutions and their implications or empirical research, this section provides a comprehensive set of simulation results for the three different types of performance measures analyzed in sections in section 4.

[please insert table 1 about here]

Table 1 shows simulation results for the problem of filtering observable luck from firm performance. The simulation compares the results of six different regressions of firm performance \( x_0 \) on common risk \( \eta \). Holding all else constant, the regressions are performed on 1,000 random samples of 1,000 observations using parameters drawn from six different distributions of the firm’s exposure to the common risk factor \( (\bar{c}_0) \). To isolate the consequences of an increasing exposure risk from changes in other variables, I distinguish the distributions by varying the standard deviation \( \sigma_0 \) from 0 (i.e. a constant exposure) to 12 as shown in panel A.

Panel B shows the results from 1,000 linear regressions of \( x_0 \) on \( \eta \) for each of the six exposure distributions in panel A. The first four rows report the summary statistics of the estimated regression coefficients of \( \eta \). In line with equation (12), the mean estimates are close to \( E[\bar{c}_0] = 1 \) for all 6 cases. However, as indicated by the standard deviations and the range of the estimated regression coefficients, exposure risk introduces a substantial degree of variation among the regression coefficients estimated for different random samples.

More importantly, the distribution of \( t \)-values in rows 5 and 6 as well as the share of significant regression coefficients in row 7 indicate that a high exposure risk increases the likelihood that a regression does not find a statistically significant relation between firm performance and common risk although both variables are correlated. In fact, for a constant exposure (model 1) and a moderate exposure risk (model 2) the regression coefficients are found to be significant at the 95% level for all random samples, whereas only 46.6% of the estimates for the highest exposure risk (model 6) yield regression coefficients that are significant at the 95% level. The summary statistics of \( R^2 \) in rows 7 and 8 of panel B with mean values ranging from 0.4997 (case 1) to 0.0055 (case 6) indicate that the variation in firm performance that can be attributed to changes of the common risk factor is decreasing in the firm’s exposure risk.
As a practical matter, these findings indicate that a high exposure risk might prevent firms from filtering observable luck from their own performance. In the absence of precise information on the theoretical distributions of $\xi_0$ on $\eta$, firms must assess the potential relation between firm performance and observable measures of common risk by means of regression models similar to the model used in the simulation. Accordingly, the presence of exposure risk impedes the identification of a statistically significant relation between firm performance and common risk. Likewise, exposure risk can render the filtering of observable luck economically insignificant because a low $R^2$ indicates a negligible reduction of the firm’s overall performance risk.

Panel C provides the summary statistics of an additional test for the efficiency of the optimal filtering rule proposed in equation (12). The test reports simulated regressions for the aggregated performance measure $z_0(\alpha^*)$ in equation (13) on common risk $\eta$ using the data generated for the first test in panel B. The results indicate that exposure risk increases the likelihood of finding a significant statistical relation between $z_0(\alpha^*)$ and $\eta$ even though the expected contribution of luck has been removed from the firm’s performance measure. The shares of significant regression coefficients at the 95 % level with exposure risk are in the range between 14.3% (model 2) and to 18.2% (model 6) and significantly higher than in the constant exposure case (model 1) with 5.7%. These findings suggest that the presence of exposure risk increases substantially the likelihood of a type-II error (a false rejection of the hypothesis that firms optimally filter for observable luck) in empirical compensation studies leading to the wrong conclusion that firms reward their executives for observable luck.9

Table 2 shows equivalent simulation results for the problem of aggregating firm performance and a noisy measure of common risk as studied in section 4.2. The simulation compares the results of six different regressions of firm performance $x_0$ on a given measure $y$ of common risk $\eta$. Since the optimal aggregation rule in (16) is not affected by the focal firm’s own exposure risk but by the exposure risk contained in the performance signal $y$ (henceforth "index exposure risk"), the standard deviation of $\tilde{\xi}_0$ is held constant at $\sigma_0 = 1$.

---

9In fact, if the agent’s compensation $s(\cdot)$ is a linear function of $z_0(\alpha^*)$, a regression of $s(\cdot)$ on $\eta$ will find evidence for RFL.
To study the consequences of varying index exposure risk, I use the same simulation approach as in table 1 but allow the standard deviation of the index exposure, $\sigma_1$, to take 6 different values ranging from 0 to 12.

The results of the second simulation are in line with those of the first in table 1. However, since the optimal performance measure weight is negatively related to index exposure risk, the mean values of the regression coefficients are decreasing in $\sigma_1$. All other results are similar to those found for varying degrees of the focal firm’s own exposure risk in panels B and C of table 1. A higher index exposure risk increases the standard deviation and the range of the regression coefficients relative to the mean and thereby reduces the mean of the $t$-values. As a consequence, the share of regression coefficients that are significant at the 95% level drops from 100% in the absence of index exposure risk (model 1) to 26.2% for the highest index exposure risk (model 6). The same trend can be observed for the distribution of the $R^2$-values ranging from 0.1271 (model 1) to 0.0029 (model 6).

The additional regression of the aggregate performance measure $z_0(\alpha^\circ)$ on $y$ reported in panel C of table 2, yields similar results as the regression of $z_0(\alpha^*)$ and $\eta$ reported in panel C of table 1. An overall comparison of tables 1 and 2 suggests that, ceteris paribus, the consequences of an increasing index exposure risk are similar to those of an increase in the focal firm’s own exposure risk but most effects found in table 2 are slightly more pronounced than those reported in table 1.

[please insert tables 3 and 4 about here]

Tables 3 and 4 show a numerical example and related simulation results to study the consequences of varying peer exposure risk for the aggregation of a peer index as analyzed in section 4.3. Holding all other parameters constant, the example considers a sample of ten potential peers with identically structured performance signals $x_j$ but different degrees of exposure risk. The exposure risk of peer $j, j = 1, \ldots, 10$, is determined by the standard deviation $\sigma_j$ of its marginal exposure to common risk, $\tilde{c}_j$. To simplify notation, I let $\sigma_j = j$.

As a benchmark for the simulation reported in table 4, table 3 summarizes the theoretically optimal weights for the ten peer firms $\gamma_j^+$ and the index $\alpha^+$ as defined in equations (19) and (20) and compares the results to the optimal weights in the absence of exposure risk. The optimal peer weights reported in the second row of table 3 illustrate the importance
of exposure risk for the composition of the peer index. The lower the peer exposure risk, the higher the relative weight of a given peer firm within the aggregate performance index. While the performance of the lowest exposure risk firm enters the peer group with a weight of 56.53%, the firm with the highest exposure risk has a weight of 0.84% only. Multiplying the relative weights with the optimal index weight of 0.3709 yields the effective weight that the focal firm puts on an individual peer firm relative to its own performance.

The comparison of the optimal weights with and without exposure risk in table 3 exhibits significant differences for both, the relative weights of individual peers and the aggregate peer index. While the relative weights of individual peer firms are higher (lower) for peers with low (high) exposure risk, the optimal index weight unambiguously declines by 49% from 0.7273 to 0.3709 due to the presence of exposure risk. Taken together, the example firm optimally puts less effective weights on all firms except for the firm with the lowest exposure risk for which the effective weight rises from 0.0727 to 0.2097. Overall, the comparison of RPE policies in table 3 indicates that ignoring the presence of peer exposure risk can cause a significant deviation from optimal aggregation rules.

Table 4 shows the summary statistics of 1,000 multiple regressions for simulated random samples of focal firm and peer performance using the parameters of the example in table 3. The mean values of the regression coefficients in the first row of table 4 are close to the effective peer weights reported in row 4 of table 3. However, as for a given peer index in table 2, a higher peer exposure risk increases the standard deviation and the range of the regression coefficients relative to their mean. As a consequence, the mean $t$-values are significantly declining as the peer exposure risk increases. Likewise, the share of regression coefficients that are significant at the 95% level drops from 100% for the lowest peer exposure risk (firm 1) to 20.4% for the highest peer exposure risk (firm 10).

These findings suggest that a high exposure risk can restrain firms’ ability to efficiently identify and compose optimal peer groups for the purpose of RPE. If precise information on the theoretical distributions of $\bar{x}_0$ on $\bar{x}_j$ is lacking, firms need to assess the potential relation between their own performance and the performance of potential peers by means of a regression analysis similar to the model used in table 4. The low number of significant regression coefficients for peer firms with a sufficiently high exposure risk indicates that this procedure bears the risk of failing to identify the relevant set of peer firms. Moreover, since
the weights of individual peer firms are determined simultaneously, the failure to identify a subset of relevant performance peers will necessarily bias the weights put on the peer firms that were properly identified.

6 Summary and suggestions for future research

I study the consequences of a random exposure to common risk factors for relative performance evaluation. I find that the volatility of firms’ exposure to common risk can significantly limit their ability to control its influence on firm performance. As a consequence, the random exposure to common risk limits the usefulness of RPE.

I derive linear aggregation rules for the performance of a focal firm with a non-controllable but informative signal of common risk and find the following results. First, the random exposure to common risk precludes the perfect filtering of observable luck from measures of firm performance. Second, it reduces the effectiveness of noisy measures of common risk in reducing the exposure of firm performance to common risk. As a consequence, a high exposure risk implies a lower weight on given signals of peer performance. Third, the presence of exposure risk affects the optimal composition of peer groups because a firm’s relative weight within the peer index is a decreasing function of its exposure risk. Fourth, if the exposure risk becomes large, firms put zero weight on noisy measures of common risk and RPE becomes essentially useless.

I also conduct simulated regression analyses with randomly generated data. The first set of regressions explores the relation between focal firm performance and given signals of peer performance for different levels of exposure risk. I find that an increasing exposure risk limits firms’ ability of correctly identifying correlated performance signals and reduces the explanatory power of the regression. In additional tests, I find that the presence of exposure risk increases the likelihood of a type-II error in empirical compensation studies. Finally, I find that exposure risk constrains the correct identification of relevant performance peers because there is a substantial risk of incorrectly excluding peers with a high exposure risk. Since the weights of individual peer firms within the performance index are determined simultaneously, the failure to identify the relevant performance peers implies a biased composition of the remaining peer group.
Overall, my analysis suggests that the randomness of firms’ exposure to common risk can be an important factor in determining whether or not and how firms practice RPE. Future empirical studies could measure the exposure risk implied by various risk factors and analyze its importance for the explicit use of RPE, the choice of peer firms or changes of peer groups over time and thereby provide evidence for the empirical relevance of exposure risk for the practice of RPE.
Appendix

Proof of Proposition 1

Part 1: The BLP is found by minimizing the mean squared error

\[ MSE = E[(\bar{x}_0 - \beta_0 - \beta\bar{y})^2] \]  

with respect to \( \beta_0 \) and \( \beta \). Rearranging terms

\[ MSE = \beta_0^2 - 2 \cdot \beta_0 \cdot (E[\bar{x}_0] - \beta' E[\bar{y}]) + E[(\bar{x}_0 - \beta' \bar{y})^2] \]  

and taking the derivative w.r.t. \( \beta_0 \) yields

\[ \beta_0^* = E[\bar{x}_0] - \beta' E[\bar{y}] \].  

Substituting for \( \beta_0 \) from (25) into (24) yields

\[ MSE = E[(\bar{x}_0 - \beta' \bar{y})^2] - (E[\bar{x}_0] - \beta' E[\bar{y}])^2 \]  

Taking the first derivative of the last expression in (26) yields the system of first order conditions

\[ V\beta = d. \]  

The solution of this system is given in (11).

Part 2a: To reproduce the result in (8), note that for \( n = 1, \beta = \beta_1, V = VAR[\bar{y}_1] \) and \( d = COV[\bar{x}_0, \bar{y}_1] \). It follows that

\[ \beta_1 = \frac{COV[\bar{x}_0, \bar{y}_1]}{VAR[\bar{y}_1]} \]

Part 2b: Since (10) is a generalization of (9), it suffices to verify that (10) is a special case of (11). Let \( \bar{x}_0 \) and \( \bar{y}_i \) have the structure given in (1) and let \( c_i \) be a firm specific constant. It follows that

\[ VAR[\bar{y}_i] = c_i^2 \cdot VAR[\bar{y}] + VAR[\bar{e}_i] \]

\[ COV[\bar{y}_i, \bar{y}_j] = c_i \cdot c_j \cdot VAR[\bar{y}] \]

\[ COV[\bar{x}_0, \bar{y}_i] = c_0 \cdot c_i \cdot VAR[\bar{y}] \]
With this structure, the equation system in (27) takes the form
\[
\begin{pmatrix}
c_1^2 \cdot VAR[\bar{\eta}] + VAR[\bar{\zeta}_1] & \cdots & c_1 \cdot c_n \cdot VAR[\bar{\eta}]
\vdots & \ddots & \vdots
\end{pmatrix}
\begin{pmatrix}
\beta_1 \\
\vdots \\
\beta_n
\end{pmatrix}
= 
\begin{pmatrix}
c_0 \cdot c_1 \cdot VAR[\bar{\eta}]
\vdots \\
c_0 \cdot c_n \cdot VAR[\bar{\eta}]
\end{pmatrix},
\]

dividing each row of this system by \( c_0 \cdot c_i \cdot VAR[\bar{\eta}] \) yields the simplified system
\[
H\beta = 1,
\]
where \( 1 \) is the \( n \times 1 \) ones vector,
\[
H = \begin{pmatrix}
k_1 + h_1 & \cdots & k_n \\
\vdots & \ddots & \vdots \\
k_1 & \cdots & k_n + h_1
\end{pmatrix}
\]
and
\[
k_i = \frac{c_i}{c_0}, \quad h_i = \frac{VAR[\bar{\zeta}_i]}{c_0 \cdot c_i \cdot VAR[\bar{\eta}]} = \frac{1}{c_0 \cdot c_i \cdot \tau_i \cdot VAR[\bar{\eta}]}.
\]

Applying Cramer’s rule, yields the solution
\[
\beta_i = \frac{|H_i|}{|H|} = \frac{\prod_{j=1, j\neq i}^n h_j}{\prod_{j=1}^n h_j + \sum_{j=1}^n k_j \cdot \prod_{l=1, l\neq i}^n h_l},
\]
where \( H_i \) is the matrix \( H \) with the \( i \)-th column replaced by the ones vector \( 1 \). From (31) and the fact that \( \beta_i = \alpha \cdot \gamma_i \), the ratio of any two peer weights in (7) equals
\[
\frac{\gamma_i}{\gamma_j} = \frac{h_j}{h_i} = \frac{c_i \cdot \tau_i}{c_j \cdot \tau_j}.
\]
Solving (32) for all \( \gamma_j \) and substituting the resulting expression for all \( j \neq i \) into the index equation
\[
\sum_{i=1}^n \gamma_i = 1
\]
yields the aggregation rule in (10). Setting \( c_i = 1 \) for all \( i \) yields Holstrom’s aggregation rule in equation (9).
Proof of Corollary 1

The optimal solution of the agency problem solves the following maximization problem:

\[
\max_{U_P} U_P = E[\tilde{x}_0] - E[s(\tilde{z}_0)]
\]

subject to

\[
\begin{align*}
U_A &= E[s(\tilde{z}_0)] - h(r) \cdot \text{VAR}[s(\tilde{z}_0)] \geq U \\
\frac{\partial U_A}{\partial a_0} &= \frac{\partial E[s(\tilde{z}_0)]}{\partial a_0} - C(a_0) = 0
\end{align*}
\]

where

\[
\begin{align*}
E[\tilde{x}_0] &= a_0 + E[\tilde{c}_0] \cdot E[\tilde{\eta}] \\
E[s(\tilde{x}_0)] &= w_0 + v_0 \cdot (E[\tilde{x}_0] - \alpha \cdot E[\tilde{\eta}]) \\
\text{VAR}[s(\tilde{z}_0)] &= v_0^2 \cdot \text{VAR}[\tilde{z}_0] = v_0^2 \cdot (\text{VAR}[\tilde{x}_0] + \alpha^2 \text{VAR}[\tilde{\eta}] - 2 \cdot \alpha \cdot \text{COV}[\tilde{x}_0, \tilde{\eta}])
\end{align*}
\]

Let \( L \) denote the Lagrangian of the principal’s constrained maximization problem and let \( \lambda \) be the multiplier of the participation constraint in (34). Taking the derivatives of \( L \) with respect to \( w_0 \) and \( \alpha \) yields:

\[
\begin{align*}
\frac{\partial L}{\partial w_0} &= -1 + \lambda = 0 \\
\frac{\partial L}{\partial \alpha} &= -v_0(1 - \lambda) + \lambda \cdot h(r) \cdot v_0^2 \cdot \frac{\partial \text{VAR}[\tilde{z}_0]}{\partial \alpha} = 0.
\end{align*}
\]

It follows that \( \alpha \) is set so that the variance of the agent’s pay is minimized. This solution is independent of the optimal incentive rate \( v_0 \). An analogous result can be found if the performance measure explicitly aggregates multiple peers into a performance index and takes the form \( \tilde{z}_0 = \tilde{x}_0 - \beta' \tilde{y} \). Taking into account that \( \lambda = 1 \), the optimal aggregation rule for this case must satisfy

\[
\frac{\partial L}{\partial \beta} = \frac{\partial \text{VAR}[\tilde{z}_0]}{\partial \beta} = 0.
\]

The condition in (38) is equivalent to the optimality condition for the BLP in (27) and solved by (11) because essentially, the MSE criterion minimizes \( \text{VAR}[\tilde{z}_0] = \text{VAR}[\tilde{x}_0 - \beta' \tilde{y}] \).

Proof of Proposition 3

As shown in the proof of proposition 1, the BLP for a single performance signal is given by (8). From equation (13) in Bohrnstedt and Goldberger (1969), the covariance between two products of random variables under multivariate normality equals:

\[
\text{COV}[\tilde{x}, \tilde{y}] = E[\tilde{c}_0] \cdot E[\tilde{c}_j] \cdot \text{VAR}[\tilde{\eta}].
\]
From (3), the variance of $\tilde{y}$ equals

$$VAR[\tilde{y}] = VAR(\tilde{\varepsilon}_j) + E[\tilde{c}_j]^2 \cdot VAR[\tilde{\eta}] + R_j,$$

where $R_j$ is the exposure risk as defined in (4)

$$R_j = VAR[\tilde{c}_j] \cdot (VAR[\tilde{\eta}] + E[\tilde{\eta}]^2).$$

With this structure, the optimal aggregation rule becomes

$$\alpha^* = \frac{E[\tilde{c}_0] \cdot E[\tilde{c}_j] \cdot VAR[\tilde{\eta}]}{VAR(\tilde{\varepsilon}_j) + E[\tilde{c}_j]^2 \cdot VAR[\tilde{\eta}] + R_j}. \quad (39)$$

It holds that

$$\frac{\partial \alpha^*}{\partial R_j} = - \frac{COV[\tilde{x}, \tilde{y}]}{VAR[\tilde{y}]^2} < 0 \quad (40)$$

$$\frac{\partial \alpha^*}{\partial VAR[\tilde{c}_j]} = (VAR[\tilde{\eta}] + E[\tilde{\eta}]^2) \cdot \frac{\partial \alpha^*}{\partial R_j} < 0 \quad (41)$$

$$\frac{\partial \alpha^*}{\partial VAR[\tilde{\eta}]} = \frac{E[\tilde{c}_0] \cdot E[\tilde{c}_j] \cdot (VAR(\tilde{\varepsilon}_j) + VAR[\tilde{c}_j] \cdot E[\tilde{\eta}]^2)}{VAR[\tilde{y}]^2} > 0 \quad (42)$$

$$\frac{\partial \alpha^*}{\partial E[\tilde{c}_0]} = \frac{E[\tilde{c}_j] \cdot VAR[\tilde{\eta}]}{VAR[\tilde{y}]} > 0 \quad (43)$$

$$\frac{\partial \alpha^*}{\partial E[\tilde{c}_j]} = \frac{E[\tilde{c}_0] \cdot VAR[\tilde{\eta}]}{VAR[\tilde{y}]^2} \cdot \Delta_j \quad (44)$$

where $\Delta_j$ in (44) is given by the following expression.

$$\Delta_j = V_j - E[\tilde{c}_j]^2 \cdot VAR[\tilde{\eta}]. \quad (45)$$

The expression in (45) can be positive or negative. It is strictly positive if $V_j > E[\tilde{c}_j]^2 \cdot VAR[\tilde{\eta}]$ where $V_j = VAR(\tilde{\varepsilon}_j) + R_j$ is the sum of the idiosyncratic risk $VAR(\tilde{\varepsilon}_j)$ and the exposure risk $R_j$ contained in the performance measure $\tilde{y}$ as defined in (4).

**Proof of Proposition 4**

The proof of proposition 4 is similar to the second part of the proof of proposition 1. In the presence of exposure risk, the elements of the covariance matrix $\mathbf{V}$ and the covariance vector $\mathbf{d}$ in (27) become:

$$VAR[\tilde{y}_i] = VAR(\tilde{\varepsilon}_i) + E[\tilde{c}_i]^2 \cdot VAR[\tilde{\eta}] + R_i$$

$$COV[\tilde{y}_i, \tilde{y}_j] = E[\tilde{c}_i] \cdot E[\tilde{c}_j] \cdot VAR[\tilde{\eta}]$$

$$COV[\tilde{x}_0, \tilde{y}_i] = E[\tilde{c}_0] \cdot E[\tilde{c}_i] \cdot VAR[\tilde{\eta}]$$
where $R_i = VAR[\tilde{c}_i] \cdot (VAR[\tilde{y}] + E[\tilde{y}]^2)$ is the exposure risk embedded in $\tilde{y}_i$. With these definitions, the matrix $H$ in (29) that determines the solution of the modified system in (28) has entries

$$k_i = \frac{E[c_i]}{E[c_0]}, h_i = \frac{VAR[\tilde{c}_i] + R_i}{E[c_0] \cdot E[\tilde{c}_i] \cdot VAR[\tilde{y}]}.$$  

(46)

Now, let

$$\delta_i(R_i) = \frac{1}{V_i} = \frac{1}{VAR[\tilde{c}_i] + R_i}$$

(47)

where $V_j$ is the sum of the idiosyncratic risk $VAR[\tilde{c}_j]$ and the exposure risk $R_i$ embedded in $\tilde{y}_i$ as defined in equation (4). It follows that

$$\beta_k \gamma_i = \frac{h_j}{E[c_j]} \cdot \delta_i(R_i).$$

(48)

Solving (48) for all $\gamma_j$ and substituting the resulting expression for all $j \neq i$ into the index equation $\sum_{i=1}^{n} \gamma_i = 1$, yields the aggregation rule

$$\gamma^+ = \frac{E[\tilde{c}_j] \cdot \delta_j(R_j)}{\sum_{j=1}^{n} E[\tilde{c}_j] \cdot \delta_j(R_j)}$$

(49)

1) It holds that

$$\frac{\partial \gamma^+}{\partial \delta_j} = \frac{E[\tilde{c}_j] \cdot \sum_{k=1,k \neq j}^{n} E[\tilde{c}_k] \cdot \delta_k}{(\sum_{j=1}^{n} E[\tilde{c}_j] \cdot \delta_j)^2} > 0$$

(50)

$$\frac{\partial \gamma^+}{\partial E[\tilde{c}_j]} = \frac{\delta_j \cdot \sum_{k=1,k \neq j}^{n} E[\tilde{c}_k] \cdot \delta_k}{(\sum_{j=1}^{n} E[\tilde{c}_j] \cdot \delta_j)^2} > 0$$

(51)

and, for any peer with $k \neq j$

$$\frac{\partial \gamma^+}{\partial \delta_k} = -\frac{\delta_j \cdot E[\tilde{c}_j] \cdot E[\tilde{c}_k]}{(\sum_{j=1}^{n} E[\tilde{c}_j] \cdot \delta_j)^2} < 0$$

$$\frac{\partial \gamma^+}{\partial E[\tilde{c}_k]} = -\frac{\delta_j \cdot E[\tilde{c}_j] \cdot \delta_k}{(\sum_{j=1}^{n} E[\tilde{c}_j] \cdot \delta_j)^2} < 0$$

2) Since $\delta_j(R_j) = \frac{1}{VAR[\tilde{c}_j] + R_j}$, it holds that

$$\frac{\partial \delta_j(R_j)}{\partial R_j} = -\frac{1}{(VAR[\tilde{c}_j] + R_j)^2}$$

and

$$\frac{\partial \gamma^+}{\partial R_j} = \frac{\partial \gamma^+}{\partial \delta_j} \frac{\partial \delta_j(R_j)}{\partial R_j} < 0.$$

Moreover,

$$\lim_{R_j \to \infty} \gamma^+_j = \lim_{R_j \to \infty} \frac{E[\tilde{c}_j]}{E[\tilde{c}_j] + (VAR[\tilde{c}_j] + R_j) \cdot \sum_{k=1,k \neq j}^{n} E[\tilde{c}_k] \cdot \delta_k} = 0.$$
Proof of Corollary 5

Plugging the definitions from (46) into (31) yields

$$\beta_j^+ = \frac{E[\bar{c}_0] \cdot VAR[\bar{\eta}] \cdot E[\bar{c}_j] \cdot \delta_j(R_j)}{1 + VAR[\bar{\eta}] \cdot \sum_{j=1}^{n} E[c_j]^2 \cdot \delta_j(R_j)}.$$  \hfill (52)

Using the result in (49), the optimal index weight becomes

$$\alpha^+ = \frac{\beta_j^+}{\gamma_j^+} = \frac{E[\bar{c}_0] \cdot VAR[\bar{\eta}] \cdot \sum_{j=1}^{n} E[\bar{c}_j] \cdot \delta_j(R_j)}{1 + VAR[\bar{\eta}] \cdot \sum_{j=1}^{n} E[c_j]^2 \cdot \delta_j(R_j)}.$$  \hfill (53)

Substituting the expressions for $\gamma_j^+$ and $\alpha^+$ into the $VAR[\bar{z}_0]$ yields (21). The expression in (21) is increasing in $z^+$ because $\partial q^+ / \partial z^+ > 0$.

The limits of the scaling factor are determined as follows:

1) Since $z^+ = VAR[\bar{\eta}] \cdot \sum_{j=1}^{n} E[\bar{c}_j]^2 \cdot \delta_j(R_j) > 0$ it must be that $q^+ < 1$.

2) To show that $q^+ > q^0$, recall first from (18) that

$$q^0 = \frac{VAR[\bar{\eta}] \cdot E[\bar{c}_j]^2}{VAR[\bar{\eta}] + VAR[\bar{\epsilon}_j] + \sum_{j=1}^{n} E[c_j]^2 + R_j},$$  \hfill (54)

using the definition of $\delta_j(R_j)$ and rearranging terms yields

$$q^0 = \frac{z^0}{1 + z^0}$$  \hfill (55)

where $z^0 = VAR[\bar{\eta}] \cdot E[\bar{c}_j]^2 \cdot \delta_j(R_j)$. If firm $j$ is part of the index, it must be that

$$E[\bar{c}_j]^2 \cdot \delta_j(R_j) < \sum_{j=1}^{n} E[c_j]^2 \cdot \delta_j(R_j).$$

It follows that $z^+ > z^0$ and that $q^+ > q^0$.

3) Using the expressions in (14), (18), and (21), the range of $q^+$ implies that

$$VAR[\bar{z}_0(\alpha^0)] > VAR[\bar{z}_0(\alpha^+)] > VAR[\bar{z}_0(\alpha^*)].$$

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References


Table 1

Panel A shows the relevant parameters used for simulating firm performance $\tilde{x}_0 = k_0 + \tilde{c}_0 \cdot \tilde{\eta} + \tilde{\epsilon}_0$ and common risk $\tilde{\eta}$. For each regression 1,000 observations were randomly generated for each variable using the parameter $k_0 = 1$ and independent random numbers generated from the distributions: $\tilde{\eta} \sim \mathcal{N}(1,1), \tilde{c}_0 \sim \mathcal{N}(1, \sigma_{\tilde{c}_0}^2), \tilde{\epsilon}_0 \sim \mathcal{N}(0,1)$. Panel B presents the summary statistics of 1,000 estimations of the regression model $x_{0i} = a_0 + a_1 \cdot \eta_i + \epsilon_i$ using simulated data for the 6 parameter sets and random numbers as defined in Panel A. Panel C presents the summary statistics of 1,000 estimations of the regression model $z_{0i} = b_0 + b_1 \cdot \eta_i + \epsilon_i$ where $z_{0i} = x_{0i} - E[\tilde{c}_0] \cdot \eta_i$ and $E[\tilde{c}_0] = 1$. All regressions are estimates using the same data as for the regressions displayed in Panel B.

**Panel A:** Parameters for the standard deviation of $\tilde{c}_0$ used for simulating firm performance

<table>
<thead>
<tr>
<th>Regression model</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_0$</td>
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<td>3</td>
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<td>9</td>
<td>12</td>
</tr>
</tbody>
</table>

**Panel B:** Simulated regressions of unfiltered firm performance on common risk

<table>
<thead>
<tr>
<th>Risk exposure ($\tilde{a}_i$)</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
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<td>0.9976</td>
<td>1.0015</td>
<td>0.9939</td>
<td>0.9978</td>
<td>0.9964</td>
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<tr>
<td>Standard deviation</td>
<td>0.0320</td>
<td>0.0723</td>
<td>0.1935</td>
<td>0.3661</td>
<td>0.5756</td>
<td>0.7686</td>
</tr>
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<td>1.2850</td>
<td>1.5708</td>
<td>2.0242</td>
<td>2.6055</td>
<td>3.9774</td>
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<td>Share of $\tilde{a}_i$-coefficients significant at 95%-level</td>
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<tr>
<td>Mean</td>
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<td>0.0151</td>
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<td>0.0055</td>
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<td>Standard deviation</td>
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**Panel C:** Simulated regressions of filtered firm performance on common risk

<table>
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<tr>
<th>Risk exposure ($\tilde{b}_i$)</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.0002</td>
<td>-0.0024</td>
<td>0.0015</td>
<td>-0.0061</td>
<td>-0.0022</td>
<td>-0.0036</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>0.0320</td>
<td>0.0723</td>
<td>0.1935</td>
<td>0.3661</td>
<td>0.5756</td>
<td>0.7686</td>
</tr>
<tr>
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<td>-0.7498</td>
<td>-1.1868</td>
<td>-2.2238</td>
<td>-2.2425</td>
</tr>
<tr>
<td>Max</td>
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<td>0.2850</td>
<td>0.5708</td>
<td>1.0242</td>
<td>1.6055</td>
<td>2.9774</td>
</tr>
<tr>
<td>t-values</td>
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</tr>
<tr>
<td>Mean</td>
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<td>-0.0204</td>
<td>-0.0052</td>
<td>-0.0039</td>
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<tr>
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</tr>
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</tbody>
</table>
Table 2

PANEL A shows the relevant parameters used for simulating firm performance $\bar{x}_0 = k_0 + \hat{\eta} + \hat{\varepsilon}_0$ and a noisy measure of common risk $\bar{y} = k'_1 + \hat{\varepsilon}_1 \cdot \hat{\eta} + \hat{\varepsilon}_1$. For each regression 1'000 observations were randomly generated for each variable using independent random numbers generated from the distributions $\hat{\eta} \sim N(1,1), \hat{\varepsilon}_0, \hat{\varepsilon}_1 \sim N(0,1)$, $\hat{\varepsilon}_0 \sim N(1,1), \hat{\varepsilon}_1 \sim N(1, \sigma^2_{\varepsilon})$ with parameters $k_0 = k_1 = 1$, and the values for $\sigma_{\varepsilon}$ given below.

PANEL B presents the summary statistics of 1,000 estimations of the regression model $x_{0i} = a_0 + a_1 \cdot y_i + \varepsilon_i$ using simulated data for the 6 parameter sets and random numbers as defined in PANEL A.

PANEL C presents the summary statistics of 1,000 estimations of the regression model $z_{0i} = b_0 + b_1 \cdot y_i + \varepsilon_i$ where $z_{0i} = x_{0i} - \hat{\alpha} \cdot y_i$ where $\hat{\alpha}$ is the optimal weight of $\bar{y}$ as defined in equation (16).

### PANEL A: Parameters for the standard deviation of $\hat{\varepsilon}_1$ used for simulating the noisy signal of common risk

<table>
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<th>Regression model</th>
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<td>$\sigma_{\varepsilon}$</td>
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</table>

### PANEL B: Simulated regressions of unfiltered firm performance on noisy signal of common risk

<table>
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<tr>
<th>Risk exposure ($\hat{\alpha}_i$)</th>
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</table>

### PANEL C: Simulated regressions of filtered firm performance on noisy signal of common risk

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<tr>
<th>Risk exposure ($\hat{\beta}_i$)</th>
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<th>(2)</th>
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<tbody>
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<tr>
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<td>-0.0214</td>
<td>-0.0161</td>
</tr>
<tr>
<td>Max</td>
<td>0.1494</td>
<td>0.1279</td>
<td>0.0679</td>
<td>0.0314</td>
<td>0.0237</td>
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<tr>
<td>t-values</td>
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</tr>
<tr>
<td>Mean</td>
<td>0.0711</td>
<td>0.0013</td>
<td>0.0453</td>
<td>0.0974</td>
<td>-0.0210</td>
<td>0.0410</td>
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<tr>
<td>Standard deviation</td>
<td>1.1116</td>
<td>1.2188</td>
<td>1.4624</td>
<td>1.3869</td>
<td>1.3986</td>
<td>1.3935</td>
</tr>
<tr>
<td>Share of $\hat{\beta}_i$-coefficients significant at 95%-level</td>
<td>8.3%</td>
<td>10.3%</td>
<td>18.1%</td>
<td>15.8%</td>
<td>15.7%</td>
<td>16.6%</td>
</tr>
<tr>
<td>R-squared</td>
<td></td>
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</tr>
<tr>
<td>Mean</td>
<td>0.0012</td>
<td>0.0015</td>
<td>0.0021</td>
<td>0.0019</td>
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<tr>
<td>Standard deviation</td>
<td>0.0018</td>
<td>0.0021</td>
<td>0.0030</td>
<td>0.0028</td>
<td>0.0027</td>
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<tr>
<td>Observations</td>
<td>1,000</td>
<td>1,000</td>
<td>1,000</td>
<td>1,000</td>
<td>1,000</td>
<td>1,000</td>
</tr>
</tbody>
</table>
Table 3

The table presents the exposure risk $R_j$, optimal peer weights $\gamma_j^+$ and the index weight $\alpha^+$ as defined in equations (4), (19), and (20) for the peer index $y = \sum_{j=1}^{10} x_j$ for a set of 10 peer firms with firm performance $\bar{x}_j = k_j + \tilde{c}_j \cdot \tilde{\eta} + \epsilon_j$ and a focal firm with performance $\bar{x}_0 = k_0 + \tilde{c}_0 \cdot \tilde{\eta} + \epsilon_0$. The solutions are determined assuming the probability distributions $\tilde{\eta} \sim N(1,1)$, $\tilde{\epsilon}_0, \epsilon_j \sim N(0,1)$, $\tilde{c}_0 \sim N(1,1), \tilde{c}_j \sim N(1,\sigma^2_j)$ with parameter values $k_0 = k_j = 1$, and $\sigma_j = j$ for $j=1,\ldots,10$. The second set of entries compares the results to the same set of peers in the absence of exposure risk, i.e. $\sigma_j = 0$ for $j=1,\ldots,10$.

<table>
<thead>
<tr>
<th>Peer firm /$\sigma_j$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>RPE with exposure risk</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Exposure risk ($R_j$)</td>
<td>2</td>
<td>8</td>
<td>18</td>
<td>32</td>
<td>50</td>
<td>72</td>
<td>98</td>
<td>128</td>
<td>162</td>
<td>200</td>
</tr>
<tr>
<td>Peer weight ($\gamma_j^+$)</td>
<td>0.5653</td>
<td>0.1884</td>
<td>0.0893</td>
<td>0.0514</td>
<td>0.0333</td>
<td>0.0232</td>
<td>0.0171</td>
<td>0.0131</td>
<td>0.0104</td>
<td>0.0084</td>
</tr>
<tr>
<td>Index weight ($\alpha^+$)</td>
<td>0.3709</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Effective weight ($\gamma_j^+ \cdot \alpha^+$)</td>
<td>0.2097</td>
<td>0.0699</td>
<td>0.0331</td>
<td>0.0191</td>
<td>0.0123</td>
<td>0.0086</td>
<td>0.0064</td>
<td>0.0049</td>
<td>0.0039</td>
<td>0.0031</td>
</tr>
<tr>
<td><strong>RPE absent exposure risk</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Peer weight ($\gamma_j^n$)</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>Index weight ($\alpha^n$)</td>
<td>0.7273</td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Effective weight ($\gamma_j^n \cdot \alpha^n$)</td>
<td>0.0727</td>
<td>0.0727</td>
<td>0.0727</td>
<td>0.0727</td>
<td>0.0727</td>
<td>0.0727</td>
<td>0.0727</td>
<td>0.0727</td>
<td>0.0727</td>
<td>0.0727</td>
</tr>
<tr>
<td>% Differences</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$(\gamma_j^+ - \gamma_j^n) / \gamma_j^n$</td>
<td>465.31%</td>
<td>88.44%</td>
<td>-10.74%</td>
<td>-48.61%</td>
<td>-66.75%</td>
<td>-76.77%</td>
<td>-82.87%</td>
<td>-86.85%</td>
<td>-89.60%</td>
<td>-91.56%</td>
</tr>
<tr>
<td>$(\alpha^+ - \alpha^n) / \alpha^n$</td>
<td>-49.00%</td>
<td></td>
<td></td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>
Table 4
The table presents the summary statistics of 1,000 estimations of the multiple regression model $x_{0i} = a_0 + \sum_{j=1}^{10} a_j \cdot x_{ji} + \epsilon_i$ using simulated data for focal firm performance $\bar{x}_0 = k_0 + \tilde{c}_0 \cdot \tilde{\eta} + \tilde{\varepsilon}_0$ and peer performance $\bar{x}_j = k_j + \tilde{c}_j \cdot \tilde{\eta} + \tilde{\varepsilon}_j$ for peer firm $j$, $j = 1, \ldots, 10$. For each regression 1'000 observations for each variable were randomly generated using independent random numbers generated from the distributions $\tilde{\eta} \sim N(1,1)$, $\tilde{\varepsilon}_0, \tilde{\varepsilon}_j \sim N(0,1)$, $\tilde{c}_0 \sim N(1,1)$, $\tilde{c}_j \sim N(1, \sigma_j^2)$ with parameters $k_0 = k_j = 1$, and $\sigma_j = j$ for $j = 1, \ldots, 10$.

<table>
<thead>
<tr>
<th>Peer firm /$\sigma_j$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Firm to peer performance exposure ($\hat{a}_j$) Mean</td>
<td>0.2069</td>
<td>0.0716</td>
<td>0.0327</td>
<td>0.0196</td>
<td>0.0127</td>
<td>0.0089</td>
<td>0.0066</td>
<td>0.0046</td>
<td>0.0038</td>
<td>0.0028</td>
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<tr>
<td>Standard deviation</td>
<td>0.0406</td>
<td>0.0276</td>
<td>0.0193</td>
<td>0.0147</td>
<td>0.0115</td>
<td>0.0096</td>
<td>0.0085</td>
<td>0.0073</td>
<td>0.0067</td>
<td>0.0060</td>
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<tr>
<td>Min</td>
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<td>-0.0231</td>
<td>-0.0209</td>
<td>-0.0238</td>
<td>-0.0253</td>
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<td>-0.0249</td>
<td>-0.0201</td>
<td>-0.0162</td>
<td>-0.0200</td>
</tr>
<tr>
<td>Max</td>
<td>0.3297</td>
<td>0.1559</td>
<td>0.1025</td>
<td>0.0730</td>
<td>0.0458</td>
<td>0.0472</td>
<td>0.0350</td>
<td>0.0283</td>
<td>0.0271</td>
<td>0.0221</td>
</tr>
<tr>
<td>t-values Mean</td>
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<td>3.6747</td>
<td>2.3908</td>
<td>1.8763</td>
<td>1.5031</td>
<td>1.2638</td>
<td>1.0772</td>
<td>0.8587</td>
<td>0.8084</td>
<td>0.6626</td>
</tr>
<tr>
<td>Standard deviation</td>
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<td>1.4086</td>
<td>1.4070</td>
<td>1.3587</td>
<td>1.3664</td>
<td>1.3945</td>
<td>1.3744</td>
<td>1.3998</td>
<td>1.3935</td>
</tr>
<tr>
<td>Share of $\hat{a}_j$-coefficients significant at 95%-level</td>
<td>100.0%</td>
<td>88.0%</td>
<td>62.2%</td>
<td>47.7%</td>
<td>37.0%</td>
<td>31.8%</td>
<td>28.4%</td>
<td>22.1%</td>
<td>22.7%</td>
<td>20.4%</td>
</tr>
<tr>
<td>Adjusted R-squared Mean</td>
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<td>0.0236</td>
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<tr>
<td>Observations</td>
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