

Differences of Opinion and the Price Volume Relation *

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Abstract

For the first time in the asset pricing literature a direct link is drawn between asset prices and the financial volume of trade within a dynamic general equilibrium model with disagreement. The model exhibits two risk averse agents that hold heterogeneous beliefs about the conditional mean of the only source of macroeconomic risk, the aggregate consumption growth. The differences in opinions is supported by the fact that agents interpret public information differently. The model is able to explain a number of seemingly unrelated asset pricing facts namely the positive correlation between price changes and volume, the contemporaneous relation between volume and return volatility, the excess volatility, the volatility persistence and the negative correlation between price levels and volatility. The inability of existing models of trading to uncover such relations lies first on the simplistic preference assumptions and especially of risk-neutrality. When agents are sufficiently risk-averse prices become decreasing with the level of disagreement and trading is proportional to disagreement. Further, once we recognize that this disagreement is time-varying we are able to give an explanation to all the mentioned pricing and volume empirical evidence.

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1 Introduction

The amount of financial trading in any given day is extraordinary. In 2007 the monthly share turnover in the New York Stock Exchange (NYSE) ranged from 11% to 20% with an average of 14%.¹ Yet the asset pricing literature is unable to shed light on this fact. Understanding the amount of trading is quite important by itself. However, even from a purely asset pricing perspective understanding the determinants of financial trading is very important since prices and volume have been found to be amply related with each other.

This paper focuses on explaining the highly significant positive correlation between changes in the average level of prices and changes in the amount of trading. Figure 1 is similar to figure 3 of Hong and Stein (2007) and displays the real percentage change in the Standard and Poor's (S&P) composite index along with the percentage change in the NYSE share turnover from 1901 to 2003. The remarkable correlation between the two series is 0.52. The percentage changes were considered in order to remove time-trends from the data. Another way to remove the trend is by using the Hodrick-Prescott (HP) filter and the resulting plot is shown in figure 2. The correlation is virtually unaltered. About the same highly significant correlation is also exhibited by monthly data from January 1964 to August 2008. Figure 3 shows a scatter plot between real percentage changes in the monthly dollar volume of NYSE against real percentage changes on the S&P composite index. The statistical correlation is close to 0.3. When trend is removed using the HP filter the correlation between the two series increases to about 0.56 and the corresponding scatter plot is shown by figure 4.

The empirical regularity already mentioned along with the positive correlation between volume and volatility were the first and most notable regularities uncovered by the empirical literature. Karpoff (1987) offers an early literature overview that looks at these two effects.² Both of these regularities as well as other empirical facts like excess volatility and volatility persistence, are explained in a general equilibrium model of differences of opinions and risk averse agents. The main elements of our theoretical study is the risk-preference assumption that agents are more risk-averse than a logarithmic agent and a time-varying *disagreement factor*, which are new in this . In this paper we make a distinction between the belief dispersion and the level of disagreement in the market. While the first refers to the current beliefs about the future macroeconomic risk, the disagreement refers to how differently agents in-

¹New York Stock Exchange Factbook, (<http://www.nyxdata.com>).

²Further studies about the positive relation between volume and volatility are those of Schwert (1989) and Gallant, Rossi, and Tauchen (1992) . Other empirical studies that analyze the dynamic relation between trading and returns for both the cross-section and the aggregate stock market include Campbell, Grossman, and Wang (1993), Llorente et al. (2002), Chordia et al. (2007) and Griffin, Nardari, and Stulz (2007). Lo and Wang (2000) offers some further list of references.

interpret new information. The connection between the two is that a high disagreement is expected to lead to high dispersion in beliefs in the future.

The paper makes a number of contributions in the literature of dynamic models of trading activity as well as the pure asset pricing literature. First, we provide a natural general equilibrium framework that creates a continuous need for agents to trade. Secondly, with the introduction of what we call the *disagreement factor* and with our realistic preference assumption we create a direct link between price characteristics and volume, that are found to be strongly related by empirical studies, in a way consistent with many seemingly unrelated empirical regularities. Further, within this framework we give an argument that can explain the positive correlation between price changes and volume. Finally, even though it is not the focus of this paper, the model predicts that our disagreement factor is a significant risk asset pricing factor.

The seminal paper by Milgrom and Stokey (1982) shows that in a complete market economy with asymmetric information (and even different utility functions) agents do not trade beyond the first period.³ ⁴ The no-trade result has two main components. First, in a market with no frictions disparate information is aggregated and revealed by equilibrium prices. Secondly, the aggregated information is interpreted in the same way by agents. These two components have given birth to much of the literature that tries to explain trading. The first stream of research introduces frictions in the market through noise or liquidity traders whose action first impedes the aggregation of information and further introduces volatility and trading through their own non-informational motives.⁵ Wang (1994) explains the positive contemporaneous covariance between volume and absolute price changes in a model with asymmetric information where the informed investors enjoy private investment opportunities. For this reason the uninformed investors are unable to extract all the information from prices and require a significant drop in price for them to buy the stock. Therefore trading is accompanied by absolute price changes but the correlation per se is not examined. The predictions in the noise trading literature

³Another important no-trade theorem is that of Judd, Kubler and Schmedders (2003). They show that in an economy with preference heterogeneity and dynamic completeness agents do not trade beyond the first period. The required assumption is that the transition matrix of the exogenous state is of full rank. One could relax this assumption and predict trading in case for example when labor income risk is not perfectly correlated with aggregate endowment risk. In this case agents would need to adjust their portfolio because their income risk demands different hedging at different periods. However, the predicted trading is minimal and cannot be expected to be related to prices. A similar no-trade theorem in continuous time and the conditions for non-trivial trading volume are shown by Berrada, Hugonnier and Rindisbacher (2007)

⁴Blume et al. (2006) show that the no-trade theorem fails to hold when markets are incomplete since new arrival of information offers new opportunities for risk-sharing.

⁵See for example Grossman and Stiglitz (1980), Hellwig (1980), Diamond and Verrecchia (1981), Kyle (1985), Admati and Pfleiderer (1988), Grundy and McNichols (1989), Foster and Viswanathan (1990, 1993), De Long et al. (1990), Kim and Verrecchia (1991a, 1991b), Shalen (1993), and Wang (1994).

hold so much as there is non-informational trading. Additionally, Wang (1994) points out that it is unlikely that non-informational trading is the main driver of the trading volume we observe.

The second line of research in which this paper belongs, assumes difference in opinions that comes either explicitly or implicitly through relaxing the assumption of uniform interpretation of available information.⁶ Harris and Raviv (1993) and Scheinkman and Xiong (2003) offer a similar explanation to trading and volatility in models with risk-neutral agents and short selling constraints. Essentially, always the most optimistic agent holds the stock and the stock changes hands when the relative valuations reverse. Under their specific settings price changes are likely to be associated with trading since more valuation reversals will indicate volatility which will also be associated with trading. This explanation however seems unsatisfactory for the volume volatility relation firstly because the preference assumption is not realistic and the predicted trading behavior is too extreme.⁷ Further they produce a counterfactual prediction that stock ownership is concentrated. The remark of Gallant, Rossi and Tauchen (1992) that “(t)here seems to be no model with dynamically optimizing heterogeneous agents that can jointly account for major stylized facts - serially correlated volatility, contemporaneous volume-volatility correlation, and excess kurtosis of price changes,” appears to be a still fair description of the current state of research on trading.

The failure of this literature to uncover the connection between prices and volume seems to be firstly due to the simplistic preference assumptions. Despite the conventional wisdom, trading and especially pricing predictions are sensitive to the preference assumption when beliefs are heterogeneous. Whereas risk-neutrality predicts a bubble component when opinions differ, the more realistic assumption of a constant relative risk aversion higher than one predicts the exact opposite, i.e. that prices decrease when belief heterogeneity increases, especially when heterogeneity is persistent.⁸ Doukas, Kim and Pantzalis (2006) offer such evidence that stocks with higher dispersion in beliefs are priced at a discount. The other convenient preference assumption of constant absolute risk aversion (CARA) also exhibits major shortcomings in this context both in terms of prices as well as trading. The CARA utility predicts that the price impact only depends on the beliefs and not on the relative wealth of every agent because their positions are independent of their wealth. The reason we are able to use CRRA

⁶Examples of this literature are Harrison and Kreps (1978), Varian (1985, 1989), Harris and Raviv (1993), Kandel and Pearson (1995), Hong and Stein (2003), Scheinkman and Xiong (2003) and Cao and Yang (2008).

⁷In the continuous time setting of Scheinkman and Xiong (2003) without transaction costs trading volume is infinite.

⁸The discounting of prices due to belief heterogeneity has been noted in various settings by Varian (1985), Jouini and Napp (2006 a, b) and Dumas, Kurshev and Uppal (2009).

preferences is because the equilibrium is accurately approximated using a computational method developed to address complete market equilibria of heterogeneous agent economies.

In order to understand the pricing implication that prices are depressed with heterogeneity, consider a particular asset that has a positive payoff only in one future state. Let us further suppose that the opinions of two agents about the probability of this state start diverging. The one agent becomes more optimistic and the other agent more pessimistic by the same amount. Then the optimistic agent will want to buy more of the asset while the pessimistic agent to sell some of his current holdings. As they start exchanging the asset their positions become more risky. For each agent there are two competing pricing effects. The optimistic agent values the asset more now but the additional risk in his portfolio makes him value the asset less. If he is sufficiently risk-averse the additional riskiness effect is greater and causes a price decrease. The pessimist agent on the other hand values the asset less with his new opinion. As he starts giving up some of the asset his valuation increases but the additional riskiness causes a decrease in price. Eventually, the new equilibrium price will be lower if the agents are sufficiently risk-averse. These two effects cancel each other out for agents with logarithmic utility preferences and hence the price is unaffected by dispersion in beliefs.

The economic story that emerges from our model and is able to explain the positive correlation between price changes and volume relies on the mentioned pricing effect and on the assumption that disagreement in the market is either negatively or positively correlated with macroeconomic risk. Agents trade mainly for two reasons: (i) when their opinions change and (ii) when their relative wealth changes. When opinions diverge agents trade and prices decrease, while when opinions converge agents trade again but prices increase this time. These two effects on average cancel each other out and therefore the first reason does not predict a particular positive or negative correlation between price changes and volume. The positive or negative correlation comes from the second source of trading depending on how disagreement is related with macroeconomic risk. Without getting into the details of the argument we only mention that trade in this case happens when the realization of the uncertainty does not favor the agent who happens to load more heavily on the market portfolio due to the disagreement in the market and needs to unload his position. Whether prices will increase or decrease in this case depends on the specific assumption about the correlation between disagreement and macroeconomic risk. Two distinct cases are considered that both predict a positive correlation between price changes and volume and we choose between them using other pricing implications.

One significant contribution of the paper is that, irrespective of the correlation of disagreement with macroeconomic risk, it establishes a clear link between volume and prices through

the disagreement factor. In this way we are able to explain the connection between volatility and volume, the excess volatility, volatility persistence and the “leverage effect”, that is the negative correlation between price levels and conditional volatility of returns. The economic story behind the volatility volume relation is simple. When disagreement is high the expected reallocation in the economy as well as the expected change in disagreement is significant and this predicts a high trading activity. Further, this reallocation as well as changes to disagreement creates significant volatility in the market due to the discounting effect that we already mentioned. Direct evidences for the connection between disagreement and volume is offered by Ziebart (1990) that documents a positive relation between the volume and the absolute change in the mean forecast of analysts. Further, Bessembinder et al. (1995) find that increases in the S&P500 Index futures’ open interest which is considered to be caused by an increase in dispersion in beliefs is associated with higher trading volumes. In this paper we also offer empirical evidences that confirm all significant predictions of the model in relation to known empirical price and volume regularities. The model also generates some new predictions in relation to the connection between beliefs and prices for which we provide some supporting evidence.

The rest of the paper is structured as follows: Section 2 outlines the model economy and section 3 derives the equilibrium conditions and proves its Markovian structure. Section 4 looks at the impact of opinion differences on trading. With the inclusion of a varying disagreement factor we are able to explain the volatility volume relation, excessive volatility and volatility persistence. With a further assumption on the correlation of the disagreement factor with macroeconomic risk we are able to offer two distinct cases where price changes are positively correlated with volume. In section 5 we offer empirical evidence that support all significant predictions of the model. Section 6 concludes.

2 The Model

We consider a model of an endowment economy with a single consumption good where the aggregate consumption growth represents the only source of fundamental risk. Agents in this economy are risk-averse and differ in their opinions about the distribution of macroeconomic risk. The belief heterogeneity is supported by the assumption that agents differ in the way they interpret publicly available information. Kandel and Pearson (1995) offers such evidence. Classical economic thought through perfect rationality assumes that economic agents know the true data generating process and therefore common information does not allow them to differ in the way they assess future uncertainty. No matter how convenient as a modeling assumption it may be, it is hard to imagine anyone being able to establish this empirically.

The fact that we are unable to pin down the heterogeneity in beliefs should not prevent us from trying to see its economic implications. Already the literature on belief heterogeneity has been fruitful in addressing several asset pricing puzzles.

One way to rationally explain belief heterogeneity is by considering that agents in reality do not know the true data generating process. Then it would be natural to assume that agents can have a range of models of different types and complexities that are statistically indistinguishable. Then any agent in forming his beliefs about the future, needs to rely on one or maybe several of those models with certain weights. If we add to this decision process costs of collecting and processing all available information for picking the best possible model every period, we arrive at a point where the economic agent might rely randomly on one of competing models. Therefore, with this simple chain of thought we have arrived at our assumption concerning the differential interpretation of public information without even resorting to behavioral factors like varying confidence or sentiment that would affect how agents might transform information to beliefs.⁹

Uncertainty, Information and Beliefs

Time is discrete and infinite, $t = 0, 1, 2, \dots$. During every time step we introduce an intermediate point at which agents acquire relevant information and trade. The intermediate periods are denoted with $t.5$. It is assumed that agents do not consume during the intermediate periods. The reasons for the introduction of the intermediate period is for the agents to receive relevant information for the economy. At every intermediate period the underlying structure of the economy changes and beliefs may diverge due to different interpretations about available information concerning this change. We denote with \mathcal{F}_t all information about observable and unobservable quantities in the economy up and including period t . With \mathbb{P} we denote the true probability measure which will be assumed to be unknown.

There is one consumption good in the economy and y_t denotes the aggregate endowment of the consumption good in period t . Let y_0 be given. The aggregate endowment is paid by the only productive stock in the economy with price P_t and uncertain dividend growth $g_{t+1} = y_{t+1}/y_t$. The dividend growth which represents the macroeconomic uncertainty takes two possible values $\{g_h, g_l\}$ where $g_h > g_l$. We therefore consider no labor income in the economy. The probability of the high growth state, which is considered to represent the fundamentals of the economy, is assumed to be time varying. Let $\pi_{t.5} = \mathbb{P}(g_{t+1} = g_h | \mathcal{F}_{t.5})$ denote the true probability of the high growth state in period $t + 1$ given the path of the

⁹For models of overconfidence and market sentiment one can look at Barberis, Shleifer and Vishny (1998), Daniel, Hirshleifer and Subrahmanyam (1998), and Gervais and Odean (2001).

economy until and including the intermediate period $t.5$. We assume that the underlying probability changes in time according to the following autoregressive process,

$$\pi_{t.5} = \phi_1 \pi_{t-1.5} + (1 - \phi_1) \epsilon_{t.5}^\pi, \quad (1)$$

where $\phi_1 \in (0, 1)$ and ϵ^π is an *iid* process that takes values from $\{1, 0\}$ with equal probabilities. Note that the assumed process ensures that π will always be in the set $(0, 1)$ and has an unconditional average of 0.5. In fact the unconditional moments of dividend growth are independent of the process of the underlying probability due to the independence of the shocks ϵ^π and g . Let \mathbb{E} and \mathbb{V} denote the expectation and variance operators respectively under the probability measure \mathbb{P} .

There are two agents in the economy indexed by $i = 1, 2$. Agents do not observe the underlying probability π_t or the shocks ϵ^π . However, it is assumed that they know the form of the process (1) along with the parameter ϕ_1 . Their information set \mathcal{F}^a includes the past shocks g_t and information signals $\{\epsilon^i\}_{i=1,2}$ that are believed to carry information about the true shock ϵ^π . We denote with \mathbb{P}^i the probability measure of each agent i . Note that a probability measure different than the objective measure needs to be assumed in order to have heterogeneity of beliefs in the economy. This is because in the model the differences of opinions at any point in time is not assumed to be due to differences in information sets but due to differences in the interpretation of the same information \mathcal{F}^a . The signals ϵ^i are meant to represent different information about the economy, for example two macroeconomic indicators.¹⁰ Every agent then once he observes the new information will have to assess them in order to form his new opinion about the current state of the economy. The two agents differ in the weights they give to these two macroeconomic signals. The indexing i indicates that agent i believes that the macroeconomic signal ϵ^i on average carries more information than ϵ^j about the true change in the economy, which is represented in this model by the shock ϵ^π .

In order to formulate the way that agents form their opinions about macroeconomic uncertainty we need to specify their beliefs both about π and about the informativeness of macroeconomic signals $\{\epsilon^1, \epsilon^2\}$. In any whole period t we assume that agents hold beliefs about the current true probability of the high state, according to a beta distribution

¹⁰To be more precise the innovations ϵ^i should be regarded as changes to macroeconomic indicators.

$Beta(\pi_t^i \tau_t^i, (1 - \pi_t^i) \tau_t^i),$

$$d\mathbb{P}^i(\pi_{t-1.5} = x \mid \mathcal{F}_t^a) = \frac{x^{\pi_t^i \tau_t^i - 1} (1 - x)^{(1 - \pi_t^i) \tau_t^i - 1}}{\int_0^1 u^{\pi_t^i \tau_t^i - 1} (1 - u)^{(1 - \pi_t^i) \tau_t^i - 1} du} dx. \quad (2)$$

Therefore, the prediction of agent i is $\mathbb{E}^i(\pi_{t-1.5} \mid \mathcal{F}_t^a) = \pi_t^i$ and his uncertainty about this estimate is given by the variance $\mathbb{V}^i(\pi_{t-1.5} \mid \mathcal{F}_t^a) = \pi_t^i (1 - \pi_t^i) / (\tau_t^i + 1)$. The variable τ^i therefore is related to the precision of the belief π^i and indicates their confidence. Infinite confidence would imply that an agent is certain about the current true probability. This paper is not about confidence but about disagreement. The effect of confidence determines only the weight he puts on new information relative to his current beliefs. For simplicity and without loss of generality we assume that the initial beliefs for the two agents are identical.

We have already said that in every intermediate period when the true probability changes, the two agents observe new information in the form of two signals or macroeconomic indicators $\{\epsilon^1, \epsilon^2\}$. Each ϵ^i takes values from the set $\{0, 1\}$ just like ϵ^π . Each agent has his own beliefs as to the informativeness of these two pieces of information. In order to form their beliefs about the shock to π agents need to have certain beliefs about how these two pieces of information are generated given the true state of nature. In particular we assume that given some $\rho \in (0, 0.5)$ each agent i believes that,

$$\mathbb{P}^i(\epsilon^i, \epsilon^j \mid \epsilon^\pi) = \frac{1 - \rho}{2} \mathbb{1}\{\epsilon^i = \epsilon^\pi\} + \frac{\rho}{2} \mathbb{1}\{\epsilon^j = \epsilon^\pi\}, \quad (3)$$

where $\mathbb{1}\{A\}$ is the indicator function that takes the value of one if statement A is true and zero otherwise. ρ is related to the level of disagreement in the interpretation of new information and for now we assume it to be constant. Scheinkman and Xiong (2003) consider something similar but they interpret the differential interpretation of signals as overconfidence. Our approach and interpretation is different and this is justified by the specifics of assumption (3). For example in their model agents only consider one of the signals as they know that the other signal is uninformative. Here we assume that both pieces of information are relevant but since agents do not know their true informativeness they need to subjectively put relative weights. It is in this subjective approach that they disagree.

Let us examine closely assumption (3). First thing we need to note is that when the signals agree agents have no basis for disagreement no matter how they weight the signals. Further, they believe that there is zero probability for both signals to be wrong. When signals disagree ρ determines the disagreement between the agents when interpreting the new information. There is no disagreement only if ρ is equal to 0.5.

Lemma 1. *Each agent $i \in \{1, 2\}$ believes that signal i is more informative than the other signal by $(1/2 - \rho)$ in the sense that,*

$$\mathbb{P}^i(\epsilon^i = \epsilon^\pi | \epsilon^\pi) = 1 - \frac{\rho}{2}, \quad \text{and} \quad \mathbb{P}^i(\epsilon^j = \epsilon^\pi | \epsilon^\pi) = \frac{1 + \rho}{2}.$$

Further, the two signals are believed to be unconditionally independent,

$$\mathbb{P}^i(\epsilon^j | \epsilon^k) = \mathbb{P}^i(\epsilon^j) = \frac{1}{2},$$

where $i, j \in \{1, 2\}$ and $k \neq j$.

Quantity $(1/2 - \rho)$ indicates the level of disagreement between the agents. From lemma 1 we see that when ρ is (close to) zero, i.e. when disagreement is high, then each agent thinks that one of the pieces of information is (almost) fully informative while the other signal is (almost) not informative at all. If ρ is $1/2$ and therefore there is no disagreement then both agents consider both signals equally informative. The following lemma makes this point even clearer.

Lemma 2. *For each agent $i \in \{1, 2\}$ the expectation about the true shock ϵ^π given the signals $\{\epsilon^1, \epsilon^2\}$ and given $\rho \in (0, 0.5)$ is given by,*

$$\mathbb{E}^i(\epsilon^\pi | \epsilon^i, \epsilon^j) = (1 - \rho)\epsilon^i + \rho\epsilon^j.$$

The conditional variance is,

$$\mathbb{V}^i(\epsilon^\pi | \epsilon^i, \epsilon^j) = \rho(1 - \rho)\mathbb{1}\{\epsilon^i \neq \epsilon^j\}.$$

Now we see from lemma 2 how ρ plays the role of weights on the different pieces of information and through its deviation from $1/2$ how it generates disagreement. We only assume that the two agents put the opposite weights on the two pieces of macroeconomic information and further that ρ is time varying. The specific assumption about the time variation of ρ will be stated next but we would like first to clarify how the variable ρ affects individual beliefs through new information. We note that when the signals agree agents are certain about the true shock to the probability of the good state regardless of ρ and the uncertainty as given by the conditional variance is zero. When the signals disagree then the perceived informativeness depends on the value of ρ . When ρ has its maximum possible value $1/2$ and disagreement is at its lowest then the signals are believed to be equally informative and hence jointly non-informative since the variance attains its maximum value and the conditional expectation about ϵ^π is equal to its unconditional average.

We assume that the disagreement about new information as determined by ρ is time varying. In particular we assume that ρ follows a simple autoregressive process,

$$\rho_{t+1} = \phi_\rho \rho_t + (1 - \phi_\rho) \epsilon_{t+1}^\rho, \quad (4)$$

where $\phi_\rho \in (0, 1)$ and ϵ^ρ is an identically distributed process that takes values from the set $\{0, 0.5\}$. Further the shock ϵ^ρ is contemporaneous to and is allowed to be correlated with consumption growth g ,

$$\mathbb{P}(\epsilon^\rho = 0.5 | g = g_h) = \mathbb{P}(\epsilon^\rho = 0 | g = g_l) = \eta. \quad (5)$$

This time variability in disagreement can be thought of as capturing several things; (i) time variability in the uncertainty in the economy, (ii) variations in the amount of information available, (iii) sentimental factors and (iv) variations in the level of dispersion of information in the economy, that is variations in the degree to which the macroeconomic indicators disagree. Our two signals with a constant ρ cannot capture these issues and hence with the last assumption we are able to add another realistic and as it turns out an important dimension to disagreement. What this assumption does is to allow the sensitivity of beliefs to change through time and therefore control the speed by which beliefs either diverge or converge. With this assumption the quantity $(1/2 - \rho)$ becomes what we call the *disagreement factor* which connects asset pricing characteristics and trading volume. It will be the main factor driving the level of prices and the return volatility and the expected volume of trade. Finally, the assumption concerning the correlation of the innovations of ρ with consumption growth will be the one of two determinants for predicting a positive correlation between price changes and volume.

We have already assumed that the posterior distribution of each agent at the end of every whole period t is $Beta(\pi_t^i \tau_t^i, (1 - \pi_t^i) \tau_t^i)$. However, if we add to this distribution a discrete random variable as it is the case with the shock ϵ^π the resulting distribution does not belong to the family of beta distributions. For this reason we make the following simplifying assumption. After the signals are observed in period $t.5$ the posterior distribution of each agent concerning $\pi_{t.5}$ is $Beta(\pi_{t.5}^i \tau_{t.5}^i, (1 - \pi_{t.5}^i) \tau_{t.5}^i)$ where,

$$\pi_{t.5}^i = \phi_\pi \pi_t^i + (1 - \phi_\pi) \mathbb{E}^i(\epsilon_{t.5}^\pi | \mathcal{F}_{t.5}^a), \quad (6)$$

and $\tau_{t.5}^i$ is given by,

$$\frac{\pi_{t.5}^i(1 - \pi_{t.5}^i)}{\tau_{t.5}^i + 1} = \phi_\pi^2 \frac{\pi_t^i(1 - \pi_t^i)}{\tau_t^i + 1} + (1 - \phi_\pi)^2 \mathbb{V}^i(\epsilon_{t.5} | \mathcal{F}_{t.5}^a). \quad (7)$$

Essentially, what is assumed is that once the new information is available the posterior distribution remains to be beta and the uncertainty of the beliefs is determined by adding the uncertainty from the previous beliefs and the uncertainty of the new information.

The two variables π^i and τ^i that determine the belief of agent i and his confidence serve two different purposes. The probability π^i is used by the agent to form his expectations about the future. The precision variable τ^i determines how the new information will alter the mean estimate of the probability. In intermediate periods τ_t^i does not affect how $\pi_{t.5}^i$ is formed because the new posterior is about $\pi_{t.5}$ while the previous posterior was about $\pi_{t-1.5}^i$ and it only affects the new precision. The new precision $\tau_{t.5}^i$ is only used during the whole periods where the realization of aggregate consumption growth g_{t+1} is observed and agents update their beliefs about $\pi_{t.5}^i$ in Bayesian fashion. Therefore, since the beta distribution is a conjugate prior to a bernoulli random variable the posterior distribution is also beta with,

$$\pi_{t+1}^i = (1 - \kappa_{t+1}^i) \pi_{t.5}^i + \kappa_{t+1}^i \mathbb{1}\{g_{t+1} = g\}, \quad (8)$$

The precision variable is given by,

$$\tau_{t+1}^i = \tau_{t.5}^i + 1, \quad (9)$$

and the updating weight is hence given by $\kappa_{t+1}^i := 1/\tau_{t+1}^i$. Equation (7) can be modified and written in terms of κ ,

$$\kappa_{t+1}^i = \frac{\kappa_t^i}{1 + \kappa_t^i} \phi_\pi^2 \frac{\pi_t^i(1 - \pi_t^i)}{\pi_{t.5}^i(1 - \pi_{t.5}^i)} + (1 - \phi_\pi)^2 \frac{\mathbb{V}^i(\epsilon_{t.5} | \mathcal{F}_{t.5}^a)}{\pi_{t.5}^i(1 - \pi_{t.5}^i)}. \quad (10)$$

In summary the beliefs are updated in the intermediate periods once the signals are observed as given by (6) and during the whole periods according to (8) where κ is given by (10). The shocks that drive the economy are denoted by $z_{t+1} := (\epsilon_{t.5}^1, \epsilon_{t.5}^2, g_{t+1}, \epsilon_{t+1}^p)$ and let $\mathcal{Z} \equiv \{1, 0\}^2 \times \{g_h, g_l\} \times \{0.5, 0\}$. The information at any whole period t is the history of shocks z , $\mathcal{F}_t^a := (\{z_k, k = 1, \dots, t\})$ where σ denotes the sigma-algebra. In the intermediate periods we have $\mathcal{F}_{t.5}^a := (\{\epsilon_{t.5}^1, \epsilon_{t.5}^2\}, \mathcal{F}_t^a)$. We close this subsection by clarifying how an agent forms his expectation in time t about the next period.

Lemma 3. *Let $f(z)$ be a function of the vector of shocks z . Then given the beliefs of an agent i in some period t ,*

$$\mathbb{E}^i [f(z)|\mathcal{F}_t^a] = \sum_{z \in \mathcal{Z}} \mathbb{P}^i(z|\mathcal{F}_t^a) f(z)$$

where,

$$\mathbb{P}^i(z|\mathcal{F}_t^a) = \mathbb{P}(\epsilon^\rho|g)\mathbb{P}^i(g|\mathcal{F}_{t.5}^a)\mathbb{P}(\epsilon^1, \epsilon^2).$$

Note that the conditional probability for g only depends on the prediction about the underlying probability; for example $\mathbb{P}^i(g_h|\mathcal{F}_{t.5}^a) = \pi_{t.5}^i$ and similarly for g_l and not on the confidence τ^i . Further note from lemma 3 that agents only disagree on macroeconomic risk and not on the non-fundamental uncertainty. Despite this it turns out that agents need to share the non-fundamental risks as well.

At this point we make a certain simplification in the way agents form their opinions in order to decrease the number of state variables and constitute the equilibrium computable. The beliefs of an agent can be summarized by π^i and κ^i where κ^i in any period depends on its previous value. Therefore, we need to include both in the state vector. Alternatively we remove the dependence on the previous value and substitute κ^i with a function $\kappa(\pi^i, \rho, \epsilon^i, \epsilon^j)$. For computations this functions is estimated by regressing κ^i linearly on the specified variables using simulated data with an R^2 higher than 90%. The quantitative results are insignificantly affected by this adjustment.

Financial Market

During every sub-period there are two shock realizations and each shock takes two possible values. In the second sub-period we have g and ϵ^ρ and in the first we have the signals $\{\epsilon^1, \epsilon^2\}$. Therefore, in order to have complete financial markets the economy needs to offer at least 4 independent financial assets during every sub-period. Out of these four shocks agents disagree only in terms of the probabilities of the fundamental shock g . One might think that since agents do not disagree about the non-fundamental (or extraneous) risks in the economy they have no need to share those risks and hence there is no need for more than two assets. It turns out that they do need to trade on these risks because they affect their wealth through their beliefs. These risks affect their wealth differently because their beliefs are different and hence it is optimal to share those risks differently every period. The reasons for this result will be discussed later once we derive the equilibrium variables.

The first and the most important asset in this study is the dividend paying stock whose price divided by the aggregate endowment is denoted by P and is in unit net supply. The other three assets are zero net supply contingent claims. In particular they are one sub-period

Arrow-Debreu securities that pay the aggregate endowment only in one of immediate future states. For the first sub-period the contingent claims pay the previous aggregate endowment. For example the claim for state $(1, 0)$ in period t pays y_t only if $(\epsilon_{t,5}^1, \epsilon_{t,5}^2) = (1, 0)$ and zero otherwise. For the first sub-period it is of no significance for which three states there are contingent claims since the two shocks are independent of each other and independent of anything else. However, the relation between trading and prices as we will see does depend on which states contingent claims exist for the second sub-period.

We consider two cases: (i) *case h* where there is no contingent claim for state $(g_h, 0.5)$ and (ii) *case l* where there is no contingent claim for state $(g_l, 0)$. The state of shock ϵ^ρ is immaterial. What matters is only the macroeconomic risk state. We call them *case h* or *l* because in each corresponding case the stock becomes a vehicle to invest in either the high growth state or the low growth state depending on the case. For example in *case h* when an agent is more optimistic he invests more in the stock whereas in *case l* the more optimistic agent invests less in the stock. The optimistic agent in a given state is the agent i such that $\pi^i > \pi^j$, i.e. the one who is more optimistic about high consumption growth. The trading behavior of these two cases is fundamentally different and we examine them both.

PREFERENCES AND ENDOWMENTS

Agents have standard time and state separable power utility preferences over consumption streams $c^i = (c_t^i, t \in \{0, 1, \dots\})$ with external habit similar to Galí (1994),

$$U(c^i, y) = \mathbb{E}^i \left\{ \sum_{t=0}^{\infty} \delta^t u(c_t^i, y_t) \middle| \mathcal{F}_0^a \right\}, \quad (11)$$

where $\delta \in (0, 1)$ is the subjective discount factor and

$$u(c, y) = y^{\gamma_1 - \gamma_2} \cdot \begin{cases} \ln(c), & \text{if } \gamma_1 = 1, \\ \frac{c^{1-\gamma_1}}{1-\gamma_1}, & \text{o/w.} \end{cases}, \quad (12)$$

where $\gamma_1 > 0$ is the coefficient of relative risk aversion and γ_2 is the habit parameter. When $\gamma_1 = \gamma_2$ then the preferences reduce to the usual power utility preferences without habit. The external habit is the contemporaneous aggregate consumption and it is introduced only for the following reason: When γ_2 is equal to one and beliefs are homogeneous then equilibrium prices are constant regardless of the value of γ_1 . Therefore, by forcing this parameter to be equal to one the resulting price volatility comes only from belief heterogeneity and in this way we can quantify its effect.

As it is made clear from the preference assumption, agents only consume during whole periods. In the intermediate periods preferences do not change apart from the exclusion of the utility of consumption in the previous whole period. Therefore in an intermediate period $t.5$ preferences are given by,

$$\mathbb{E}^i \left\{ \sum_{k=1}^{\infty} \delta^k u(c_{t+k}^i, y_{t+k}) \middle| \mathcal{F}_{t.5}^a \right\}. \quad (13)$$

Essentially, the intermediate period is considered to be a decision made by the agents once they have consumed which is when they receive the new information. This way we can examine the trading and asset pricing behaviors in the absence of changes in real allocation.

For simplicity and without loss of generality we assume that initially agents are endowed with equal proportion of the dividend paying stock. We denote portfolio holdings at the end of a period t with θ_t^i . Therefore at the beginning of time agents start with half of the dividend paying stock and no holdings of the contingent claims.

3 Equilibrium

This section is notational intensive and we hope the reader can bear with us. We need to first derive the conditions of a financial market sequential equilibrium and it is convenient at first to not consider the intermediate periods but instead a different asset structure that dynamically completes the market every whole period. We can do this since any sequential equilibrium with complete markets is also an Arrow-Debreu equilibrium. From the sequential equilibrium conditions we need to show that the equilibrium is time-homogeneous Markovian in terms of prices, consumption as well as holdings and derive the pricing conditions for the whole periods. Once the recursive equilibrium is in hand we derive the price conditions for the intermediate periods. We finally return to the assumed asset structure and derive the equilibrium portfolio holdings.

There are several important points to look for in this section. With respect to prices we show qualitatively why prices are discounted when opinions differ. In terms of trading we derive in closed form the portfolio allocations and trading for the special case of logarithmic utility. Further, we need to point out that both asset prices as well as portfolio holdings derive from one equilibrium function which is the individual wealth as a function of the state of the economy.

Financial Market Equilibrium

For this section it is easier to substitute the asset structure with only one period $z = (\epsilon^1, \epsilon^2, g, \epsilon^\rho)$ contingent claims with prices $p_t^*(z) = p(z|\mathcal{F}_t^a)$, $t = 0, 1, \dots$. An asset $p_t^*(z)$ is the price in terms of consumption in period t of the contingent claim that pays the entire wealth in period $t+1$ if the shock z realizes. This is equivalent to saying that a fraction of the contingent claim $p_t^*(z)$ pays only in the state z the same fraction of the aggregate endowment as well as the same fraction of all the contingent claims in that state. The contingent claims are in unit net supply.

Instead of 4 assets available every half period agents trade only when they consume on 16 contingent claims that are potentially required to complete the markets and allow agents to share the risks efficiently given their beliefs. The fact that we introduce all possible contingent claims does not mean that they are always necessary. In certain cases the equilibrium positions in some assets are always zero or some assets are redundant.

Further, let $p_t^* = (p^*(z|\mathcal{F}_t^a), z \in \mathcal{Z})$ denote the vector of prices of the contingent claims in period t . The budget constraint of agent i in period t is given by,

$$c_t^i + \theta_t^i p_t^* \leq \theta_{t-1}^i(z_t) W_t^* \quad (14)$$

where $\theta_t^i = (\theta^i(z|\mathcal{F}_t^a), z \in \mathcal{Z})$ is the portfolio holdings of contingent claims of agent i in the end of period t and W^* denotes the entire wealth in the economy. The entire wealth in any period t is given by the prices of the contingent claims and the aggregate endowment, $W_t^* = y_t + \sum_{z \in \mathcal{Z}} p_t^*(z)$. With the introduced notation we can define the equilibrium.

Definition 1. *A financial market equilibrium is a process of portfolio holdings $\{(\theta_t^1, \theta_t^2), t = 0, 1, \dots\}$ and a process of contingent claim prices $\{p_t^*, t = 0, 1, \dots\}$ such that:*

(i) *Financial markets clear every period $t = 0, 1, \dots$,*

$$\theta_t^1(z) + \theta_t^2(z) = 1, \quad \forall z \in \mathcal{Z}.$$

(ii) *For each agent i ,*

$$\begin{aligned} \theta^i &\in \arg \max_{\theta} U(c^i, y) \\ \text{s.t. } c_t^i &= \theta_{t-1}^i(z_t) W_t^* - \theta_t^i p_t^*, \quad t = 0, 1, \dots \end{aligned}$$

Speculative bubbles and Ponzi schemes are excluded from this equilibrium even though we have not assumed explicitly the required conditions. A financial equilibrium with complete

markets is well known to generically deliver equilibria with efficient allocations. It is equivalent to a social planner equilibrium with stochastic weights as for example in Basak (2005).

Before we characterize the equilibrium allocations and equilibrium prices we introduce two new quantities. We define $\alpha := c^1/y$ to be the consumption proportion of the first agent which means that in equilibrium the consumption proportion of the second agent is given by $(1 - \alpha)$. Further we define the following key variable,

$$q(z|\mathcal{F}_t^a) := \left[\alpha_t \mathbb{P}^1(z|\mathcal{F}_t^a)^{1/\gamma_1} + (1 - \alpha_t) \mathbb{P}^2(z|\mathcal{F}_t^a)^{1/\gamma_1} \right]^{\gamma_1}. \quad (15)$$

which is the *generalized weighted mean* of the individual beliefs about next period's shock realization. Note first that when agents have the same beliefs then $q(z|\mathcal{F}_t^a) = \mathbb{P}^i(z|\mathcal{F}_t^a)$ for $i = 1, 2$ and therefore it is independent of the risk-aversion parameter γ_1 . Otherwise, $q(z|\mathcal{F}_t^a) \gtrless \alpha_t \mathbb{P}^1(z|\mathcal{F}_t^a) + (1 - \alpha_t) \mathbb{P}^2(z|\mathcal{F}_t^a)$ when the risk aversion parameter $\gamma_1 \lesseqgtr 1$. The quantity $q(z)$ depends both on the level of heterogeneity of beliefs about the state z as well as the parameter γ_1 and we look at them in turn.

A known property (that can be shown using Jensen's inequality) is that given \mathcal{F}_t^a (i.e. fixing the set of beliefs and consumption allocations) in some period t , $\partial d(z|\mathcal{F}_t^a)/\partial \gamma_1 \leq 0$ with the equality holding only when agents agree on their beliefs. As a result the quantity q decreases as agents become more risk-averse given that they disagree. In particular, it is also known that the mentioned properties imply that $\lim_{\gamma_1 \rightarrow 0} q(z) = \max_i \{\mathbb{P}^i(z)\}$ and similarly $\lim_{\gamma_1 \rightarrow \infty} q(z) = \min_i \{\mathbb{P}^i(z)\}$. Therefore, when $\gamma_1 = 1$, q represents the average beliefs in the economy but as γ_1 decreases towards 0, q represents more and more the views of the most optimistic agents. The opposite happens when γ_1 increases from 1 to infinity. Note also that this happens state by state since the quantity $q(z)$ is state specific. This means for example, that as γ_1 increases, the economy becomes more and more pessimistic overall since for every state q leans towards the more pessimistic views for the given state.

The effect of the differences of opinions on the variable $q(z)$ is made through the following remark.

Remark 1. Let $\alpha \in (0, 1)$, $\pi \in (0, 1)$ and let the beliefs of the two agents be $\pi_1 = \pi + \Delta/\alpha < 1$ and $\pi_2 = \pi - \Delta/(1 - \alpha) > 0$ for some $\Delta > 0$, so that the weighted average of beliefs is constant and equal to π . Let then q be given by,

$$q = \left[\alpha \pi_1^{1/\gamma} + (1 - \alpha) \pi_2^{1/\gamma} \right]^\gamma$$

We then have that $\frac{\partial q}{\partial \Delta} \gtrless 0$ when $\gamma \lesseqgtr 1$.

For the typical case where agents are more risk-averse than a myopic logarithmic agent the level of heterogeneity of beliefs for a given state decreases the value of q and this decrease is greater the bigger is the value of γ_1 . The quantity q is a key variable because it affects directly the prices as we see from the following lemma.

Lemma 4. *In equilibrium the consumption allocation of the first agent follows the process,*

$$\alpha_{t+1} = \alpha_t \left(\frac{\mathbb{P}^i(z_{t+1}|\mathcal{F}_t^a)}{q(z_{t+1}|\mathcal{F}_t^a)} \right)^{1/\gamma_1}$$

and $\alpha_0 = 1/2$. The equilibrium prices are given by,

$$p_t^*(z_{t+1}) = \delta W_{t+1}^* g_{t+1}^{-\gamma_2} q(z_{t+1}|\mathcal{F}_t^a).$$

The prices $p_t^*(z)$ are linear in the quantity q . Hence, whatever applies to q in terms of how it is affected by the level of heterogeneity of beliefs or the risk-aversion parameter γ_1 also applies to the prices. This is the first element of the main argument of this paper and states that *ceteris paribus* at times of high dispersion in beliefs prices are depressed and therefore prices increase as the opinions converge.

Recursive Characterization

In order to characterize the equilibrium portfolio holdings we need to first formulate the equilibrium in recursive form. This means that the equilibrium must be Markovian in terms of a finite dimensional state vector with known law of motion. Looking at the equilibrium conditions in lemma 4 we see that α_{t+1} as well as the ratio $p_t^*(z_{t+1})/W_{t+1}^*$ are functions of the previous α_t and the beliefs of the agents, $\mathbb{P}^i(z_{t+1}|\mathcal{F}_t^a)$. Note that the ratio $p_t^*(z_{t+1})/W_{t+1}^*$, which we denote with $p_t(z_{t+1})$, is the price of an asset in period t that pays a unit of consumption in state z_{t+1} . From lemma 3 we see that these beliefs are a function of π_t^i and the external shock z_{t+1} . Therefore, α_{t+1} is a function of α_t the probabilities π_t^i , $i = 1, 2$, and the shock z_{t+1} . The state vector needs also to have the variables ρ_t and κ_t^i , $i = 1, 2$, since they are required for the law of motion of the beliefs. The state vector of the economy is hence given by $s := (\alpha, \pi^1, \pi^2, \rho)$ and let \mathcal{S} denote the set of all possible states. Therefore we can denote $q(z_{t+1}|\mathcal{F}_t^a)$ with $q(z_{t+1}|s_t)$ and $\pi_{t.5}^i = \pi^i(s, \epsilon^1, \epsilon^2)$. The next lemma looks at the recursivity of the equilibrium consumption allocations.

Lemma 5. *In equilibrium the consumption allocation of the first agent is independent of the shock ϵ^p since*

$$\alpha(s, z) = \alpha \frac{1}{\alpha + (1 - \alpha)\xi(s, \epsilon^1, \epsilon^2, g)^{1/\gamma_1}},$$

where $\xi(s, \epsilon^1, \epsilon^2, g_h) := \pi^2(s, \epsilon^1, \epsilon^2)/\pi^1(s, \epsilon^1, \epsilon^2)$ and similarly for $\xi(s, \epsilon^1, \epsilon^2, g_l)$.

Lemma 5 states that agents change their consumption allocation in time only in states that they disagree on their probability. The shock ϵ^ρ changes only how the future beliefs will be affected. The variable ξ is the ratio of beliefs it depends on the state and the signals (ϵ^1, ϵ^2) since the signals affect the current beliefs.

Let further the function L denote the law of motion of the state vector, i.e. $s' = L(s, z')$. The law of motion of the state is given by the equilibrium relation for α in lemma 5 and the equations that describe how the individual beliefs change in time. Now let w_t^i be the wealth of agent i in some period t normalized by the aggregate endowment y_t . Then from the budget constraint (14) that is satisfied with equality in equilibrium and the equilibrium prices as given in lemma 4 we get the following equilibrium relation:

$$\alpha_t + \delta \sum_{z_{t+1} \in \mathcal{Z}} g_{t+1}^{1-\gamma_2} q(z_{t+1} | \mathcal{F}_t^a) w_t^1(z_{t+1}) = w_t^1.$$

The notation $w_t^1(z_{t+1})$ makes explicit the state we refer to in period $t + 1$. Hence, the equilibrium is Markovian if the wealth can be written as a function of the state vector which is the result of the next lemma.

Lemma 6. *If $\delta g^{1-\gamma_2} \leq 1 \forall g \in \{g_h, g_l\}$, then there exist a unique function $w(s)$ that satisfies*

$$w(s) = \alpha + \delta \sum_{z \in \mathcal{Z}} g^{1-\gamma_2} q(z|s) w(s'),$$

for all $s \in \mathcal{S}$, where $s' = L(s, z)$. It takes the form $w(s) = \alpha A(s)/(1 - \delta)$ and in the special case where $\gamma_1 = \gamma_2 = 1$ we have that $A(s) = 1 \forall s \in \mathcal{S}$. The price of a unit of consumption for each state next period is given by $p(s, z) = \delta g^{-\gamma_2} q(z|s)$.

Note that $w(s)$, where once more $s = (\alpha, \pi^1, \pi^2, \rho)$, is the equilibrium wealth (normalized by the aggregate endowment) of an agent with optimal consumption proportion α , and beliefs (π^1, κ^1) while the other agent's beliefs are given by (π^2, κ^2) . So in order to get the wealth of the other agent we only need to re-order the state variables. We denote the equilibrium aggregate wealth normalized by the aggregate endowment with W , which is given by,

$$W(s) = w(s) + w(\tilde{s}), \tag{16}$$

where $\tilde{s} = (1 - \alpha, \pi^2, \pi^1, \rho)$. Therefore W is derived by the fundamental equilibrium function w .

Since the individual wealth and the aggregate wealth are recursive in the state s then the portfolio holdings of the contingent claims are also recursive. For a given state s the portfolio holdings for the first agent are given by $\theta(s) = \{\theta(s, z), z \in \mathcal{Z}\}$, where,

$$\theta(s, z) = \frac{w(L(s, z))}{W(L(s, z))}.$$

From the equilibrium function $w(s)$ we can derive all equilibrium prices and the portfolio holdings that depend on the particular asset structure that we assume. But first we look at the agent decisions in the intermediate period and how they price assets.

Intermediate Period

We have proven that the financial equilibrium can also be represented as a recursive equilibrium. Thus, we can write the optimization problem of the individual as a stationary dynamic programming problem. We retain the assumption of 16 contingent claims which renders the intermediate trading period unnecessary. However, we can still derive the asset prices in the intermediate period and how they are related to the prices in the whole periods which is the goal of this sub-section.

Let $V_0(\theta|y, s)$ be the value function in the whole period given the state s and aggregate endowment y and given portfolio of contingent claims θ . We remind that agents trade on contingent claims that each pays the aggregate wealth in one specific state the next period. Let also $V_1(\theta|y, s, \epsilon^1, \epsilon^2)$ be the value function in the intermediate periods. The dynamic problem in whole periods is then given by,

$$\begin{aligned} V_0(\theta_0^i|y, s) &= \max_{\theta_1^i(s)} u(c^i, y) + \delta \mathbb{E}^i V_1(\theta_1^i(s)|y, s, \epsilon^1, \epsilon^2) \\ \text{s.t. } c^i &= \theta_0^i W(s)y - \sum_{z \in \mathcal{Z}} \theta_1^i(s, z) p(s, z) W(L(s, z)) yg. \end{aligned} \tag{\mathcal{P}_0}$$

Let us explain the optimization problem (\mathcal{P}_0) and its notation. First θ_0^i denotes the holdings of agent i of the contingent claim that pays the aggregate wealth in the current state s . Therefore, his current wealth is given by $\theta_0^i W(s)y$. Remember that W is the aggregate wealth normalized by the aggregate endowment. The control vector $\theta_1^i(s) = \{\theta_1^i(s, z), z \in \mathcal{Z}\}$ is the new portfolio to be acquired. The budget constraint (14) has the term $\sum_{z \in \mathcal{Z}} \theta_t^i(z) p_t^*(z)$. But each term $p_t^*(z)$ is equal to the price of the asset that pays a unit of consumption for state z , $p(s_t, z)$, times the aggregate wealth in that state, $W(L(s_t, z))yg$, and note that the aggregate endowment next period is yg .

The dynamic optimization problem in the intermediate periods is similarly given by:

$$\begin{aligned} V_1(\theta_1^i(s)|y, s, \epsilon^1, \epsilon^2) &= \max_{\theta_2^i(s)} \delta \mathbb{E}^i \left\{ V_0 \left(\theta_2^i(s, z) \middle| yg, L(s, z) \right) \middle| \epsilon^1, \epsilon^2 \right\} \\ \text{s.t. } 0 &\leq \sum_{z \in \mathcal{Z}} \left[\theta_1^i(s, z) - \theta_2^i(s, z) \right] p(s, \epsilon^1, \epsilon^2, z) W(L(s, z)) yg. \end{aligned} \quad (\mathcal{P}_1)$$

The new portfolio to be composed is denoted by the vector $\theta_2^i(s)$ while $p(s, \epsilon^1, \epsilon^2)$ is the vector of contingent claim prices in the intermediate period.

The budget constraints of both optimization problems are satisfied with equality at the optimal solution. However, for problem (\mathcal{P}_0) consumption c^i is used as a substitution variable while for problem (\mathcal{P}_1) we use the Lagrange multiplier approach. The next lemma gives the equilibrium prices.

Lemma 7. *Given state s , the equilibrium Lagrange multiplier of agent i 's intermediate period problem is given by $u_1(\alpha^i y, y)/\lambda(s, \epsilon^1, \epsilon^2)$ where $\lambda(s, \epsilon^1, \epsilon^2) > 0$. The equilibrium price of a unit of consumption for each state next period is given by:*

$$p(s, \epsilon^1, \epsilon^2, z) = \delta \lambda(s, \epsilon^1, \epsilon^2) g^{-\gamma_2} q(z|s, \epsilon^1, \epsilon^2).$$

In lemma 7 the variable $q(z|s_t, \epsilon_{t,5}^1, \epsilon_{t,5}^2)$ is the generalized weighted average of the individual beliefs as defined in (15) but in the intermediate state, i.e. $q(z|\mathcal{F}_{t,5}^a)$. The equilibrium variable $\lambda(s, \epsilon^1, \epsilon^2)$ is a “free” variable that only determines the unit of prices and it does not affect either real allocations or equilibrium portfolios. In the whole periods the numeraire good is consumption but since in the intermediate periods there is no consumption the level of prices is undetermined. However, for our analysis it is natural to set $\lambda(s, \epsilon^1, \epsilon^2)$ such that the prices change only if the beliefs change and hence the variable q changes. Therefore we set $\lambda(s, \epsilon^1, \epsilon^2) = 1 \forall (s, z) \in \mathcal{S} \times \mathcal{Z}$.

Even though there are 16 assets in the intermediate period only 4 of them have positive value since $q(z|s, \epsilon^1, \epsilon^2)$ is equal to zero for all z that the shocks (ϵ^1, ϵ^2) are different. With the equilibrium prices and using the budget constraint in the intermediate period we can express the wealth of the first agent standardized by the aggregate endowment y of the previous whole period:

$$w(s, \epsilon^1, \epsilon^2) = \delta \sum_{z \in \mathcal{Z}} g^{1-\gamma_2} q(z|s, \epsilon^1, \epsilon^2) w(s'), \quad (17)$$

where $s' = L(s, z)$. In the special case when $\gamma_1 = \gamma_2 = 1$ and heterogeneity of beliefs does not matter for q , the wealth of an agent in the whole periods is simply given by $\alpha/(1 - \delta)$

as derived in lemma 4. As a result in the intermediate periods the wealth is given by $w(s, \epsilon^1, \epsilon^2) = \delta\alpha/(1 - \delta)$. This decrease in wealth is only to account for the consumed good, otherwise it is unaffected by any change in beliefs in the economy.

Asset Prices and Portfolio Holdings

We return to our originally assumed asset structure to look at the equilibrium portfolio holdings. We remind that every period or sub-period agents are able to invest in the dividend paying stock which is in unit net supply and in three zero net supply contingent claims, that pay the aggregate endowment in the next period or intermediate period, in only one state and in different states each. The contingent claims available in the whole periods that expire in the sub-periods pay the aggregate endowment of the previous period. The ordering of the shocks as well as the ordering of the holdings is $(\epsilon^1, \epsilon^2) \in \{(1, 1), (1, 0), (0, 1), (0, 0)\}$ for the intermediate periods and $(g, \epsilon^p) \in \{(g_h, 0.5), (g_h, 0), (g_l, 0.5), (g_l, 0)\}$ for the whole periods. We also remind that we consider two different asset structures in the second sub-period that we call *case h* and *case l*. In *case h* the three contingent assets are for the last three states, i.e. $(g_h, 0)$, $(g_l, 0.5)$ and $(g_l, 0)$ and in *case l* the three contingent assets are for the first three states.

The price of the stock in state s normalized by the aggregate endowment (i.e. the price-dividend ratio) is denoted by $P(s)$ and in sub-periods by $P(s, \epsilon^1, \epsilon^2)$. Henceforth we refer to the price dividend ratio simply as stock price. We start with an important result that shows the significance of preference assumptions in explaining the relation between price changes and the volume of trade.

Proposition 1. *When agents have logarithmic preferences with no external habit, i.e. $\gamma_1 = \gamma_2 = 1$, then the stock price is constant,*

$$P(s) = P(s, \epsilon^1, \epsilon^2) = \frac{\delta}{1 - \delta}, \quad \forall (s, z) \in \mathcal{S} \times \mathcal{Z}.$$

In whole periods only the stock is required for trading since the equilibrium holdings for the first agent are,

$$\theta^1(s) = (\alpha, 0, 0, 0), \quad \forall s \in \mathcal{S}.$$

In the intermediate periods only one other asset is required that pays in state g_l since the equilibrium holdings of the first agent in case h are,

$$\theta^1(s, \epsilon^1, \epsilon^2) = (\alpha(s, \epsilon^1, \epsilon^2, g_h), 0, \vartheta, \vartheta), \quad \forall (s, z) \in \mathcal{S} \times \mathcal{Z},$$

where $\vartheta = \alpha(s, \epsilon^1, \epsilon^2, g_l) - \alpha(s, \epsilon^1, \epsilon^2, g_h)$. In case l the holdings are $(-\vartheta, -\vartheta, 0, \alpha(s, \epsilon^1, \epsilon^2, g_l))$.

To understand why prices are constant even with heterogeneous beliefs in the special case of proposition 1 we need to remember the behavior of the variable q . When γ_1 is equal to one the heterogeneity of beliefs does not affect the variable q that determines the state prices. What matters is only the weighted average of beliefs. But when we additionally assume that γ_2 is also one then average beliefs do not matter either because the intertemporal marginal utility term $g^{-\gamma_2}$ is offset by the increase in aggregate endowment g which is paid by the stock.

In proposition 1 consumption proportion $\alpha(s, \epsilon^1, \epsilon^2, g)$ refers to the next state of s when the shocks realized are $(\epsilon^1, \epsilon^2, g)$. The result that only the stock is necessary for trade in the whole periods is not at all surprising after seeing that the the stock price is constant and therefore unaffected by beliefs. In the whole periods agents would need to trade if the endogenous risk arising from the release of new information in the market (ϵ^1, ϵ^2) needed to be shared. This risk refers to the variation in the agents' portfolio wealth from the release of new information, but since such variation does not occur in this special case then there is no risk to be shared. The trading that occurs during the sub-periods is to finance their new investment plans once the new beliefs are formed. These new investment plans in turn since the beliefs do not affect their wealth apart from through their consumption ratio, involve only their consumption plan.

Let us now examine for each of the two cases h and l the trading that takes place throughout a single period, from a state s to the next.

- h*: When the new information arrives in the intermediate period agents update their beliefs. If opinions differ the more optimistic agent will increase his holdings of the stock by as much as he wants to increase his consumption in the high state from the previous period. The next period there is only trade if the low state arrives in which case the optimistic agents loses and has to give up part of his holdings in order to eat. Being risk-averse the agent does not want to decrease his consumption as much as the wealth in the economy decreased.
- l*: When the new information arrives and agent opinions differ the more pessimistic agent will increase his stock holdings as much as he wants to increase his consumption in the low state. When growth realizes trade occurs only in the high state since then the previously pessimistic agent needs to give up some of his financial wealth to eat.

The significant difference between the two cases is in the second round of trading. In the first case trade occurs in the low state whereas in the other case trade occurs in the high state. The specific trading behavior of the two cases will be the basis for predicting a positive correlation between trading and price changes once we allow agents to be more risk-averse than a logarithmic utility agent.

Once we relax the assumptions of proposition 1 and allow the preference parameters γ_1 and γ_2 to be different than one, then all assets become necessary for trade in all periods. Both the individual beliefs as well as the heterogeneity of beliefs affects the wealth of the agents and hence agents need to hedge against the wealth risk they face. With the term wealth risk we mean the variation in wealth that is uncorrelated with consumption. It is driven by the term $A(s)$. This is an important implication of a model with both belief heterogeneity and preferences other than logarithmic and is summarized in the following proposition. For the next proposition we define the payoff matrices $\mathbf{R}(s)$ for the first sub-period and $\mathbf{R}_k(s, \epsilon^1, \epsilon^2)$ for the second sub-period where $k = \{h, l\}$ refers to the two different cases of asset structures considered. These payoff matrices are normalized with the aggregate endowment. In the column of the stock price it has the price-endowment ratio P for the different states (ϵ^1, ϵ^2) for the first sub-period and $P + 1$ in $R_k(s, \epsilon^1, \epsilon^2)$ again for the four different states of (g, ϵ^p) . For the contingent claims we only have 1 in the corresponding state and zero otherwise. We denote also with $\mathbf{w}(s)$ and $\mathbf{w}(s, \epsilon^1, \epsilon^2)$ the vector of wealth of the first agent after the corresponding state.

Proposition 2. *When $\gamma_1 \neq 1$ then agents need to trade in all securities to share efficiently the non-fundamental wealth risk that arises. The equilibrium portfolio holdings of the first agent are given by,*

$$\theta^1(x) = \mathbf{R}(x)^{-1}\mathbf{w}(x)$$

for $x \in \{(s), (s, \epsilon^1, \epsilon^2)\}$ and $\forall (s, z) \in \mathcal{S} \times \mathcal{Z}$.

There are two distinct effects, one due to belief heterogeneity when $\gamma_1 \neq 1$ and one due to variations in the average beliefs when $\gamma_2 \neq 1$. We will restrict to cases where $\gamma_2 = 1$ in order to see the effect from belief heterogeneity in the absence of any additional effect due to variations in the average beliefs. Another reason why we restrict to γ_2 being one is to isolate the effect of belief heterogeneity on stock return volatility.

The trading behavior on the stock does not change much when we move from the special case of proposition 1 to the general case of proposition 2. The most notable difference is

that when agents are more risk-averse they invest less aggressively and therefore the overall trading decreases. What changes significantly is the behavior of the stock price since when γ_1 increases the stock becomes more volatile with the variation in the dispersion in beliefs. We remind that when opinions differ the state prices are depressed and hence the same happens with the stock price. What remains to be seen is whether quantitatively this variation is important and in what cases we can predict a positive correlation between trading and price changes.

4 Asset Price Variation and Trading

We have so far analyzed the equilibrium theoretically and derived the trading dynamics when agents have logarithmic preferences. In this section we will analyze quantitatively the asset pricing behavior of this model and its relation with trading volume. We will also look at the return volatility and its relation with trading volume as well. For this analysis we have chosen a certain parameter configuration as shown in table 1. The preference parameter γ_1 was chosen to be take the generally acceptable value of 5. A lower value would produce more trading due to more aggressive behavior and less volatility for stock returns since prices become less sensitive to belief heterogeneity. The subjective discount factor δ was set to 0.99 only to control the level of prices. The second preference parameter γ_2 was set to one in order to remove any price volatility due to other factors apart from heterogeneity. The autocorrelation of the true probability ϕ_π was chosen to be $1/2$. This value does not affect the two first moments of consumption growth. Its pricing effects will be discussed later. The two consumption growth states were chosen to match the mean and volatility of real quarterly consumption growth as obtained from the Bureau of Economic Analysis NIPA tables for the period between 1947 to the end of 2007. The autocorrelation of the process that determines disagreement ϕ_ρ was set to 0.95 in order to make the prices highly autocorrelated as observed in the data. The quarterly price-dividend ratio obtained from the CRSP time-series of value weighted returns including and excluding dividends or the quarterly price earnings ratio as obtained from Shiller's data exhibit an autocorrelation slightly higher than 0.95. About the same autocorrelation is obtained in this model. Finally the correlation between the shocks to ρ and consumption growth will be let to vary across the entire region of $(0, 1)$. The quantities that will be shown are quarterly.

The Stock Price

In this model the stock price is affected by the belief heterogeneity through three distinct avenues. First it is affected by the contemporaneous disagreement, i.e. the difference between

π^1 and π^2 , that directly affects current state prices. Secondly, it is affected by the wealth dispersion across different agents as given by α . Even if agents do not disagree in a given state prices are still affected because the wealth dispersion allows for future disagreement. Consequently the wealth dispersion depresses current prices through decreased future prices. Finally, the stock price is affected by the variable ρ . When ρ is high it implies that agents will interpret the new information similarly and as a consequence either will make beliefs converge or will not allow them to diverge. The opposite happens when ρ is low which predicts a high future dispersion in beliefs. Effectively the stock price is decreasing in the variable ρ . Further, the persistence of ρ will determine how sensitive the stock price is to ρ .

Figure 5 shows the logarithm of the price of the stock after being divided by the aggregate endowment for various combinations of states of the economy. The price shown corresponds to either the log price-dividend ratio or more likely to the log price-earnings ratio of an economy. We remind that the equilibrium price is a function of four state variables the consumption proportion of the first agent α which is close to his wealth proportion. π_1 and π_2 are the current beliefs of the two agents about the probability of the high growth state g_h . Finally, ρ indicates the current level of the factor that determines disagreement in the interpretation of public information. In the left top panel we keep $\alpha = 0.5$, $\rho = 0.5$ and we vary π_1 and π_2 . We see that when these beliefs diverge the equilibrium price becomes depressed but the effect is not particularly strong. In the left bottom panel we see the effect of ρ and the beliefs of the agents when they agree while we keep $\alpha = 0.5$. We see that ρ affects the price greatly as the price ranges from 3 to about 4 when the disagreement in the economy decreases (i.e. the factor ρ increases). The effect of the beliefs, i.e. how optimistic the economy is, is very small firstly due to the fact that the correlation η is set to 0.5. This means that how optimistic the economy is does not affect how they form their views about future ρ . If η is close to 1 which means that an optimism is accompanied by expectations for high ρ then this would increase the prices. Even though this effect is not shown here, in reality it is very small compared to the other effects. The other reason why optimism does not affect much prices is because the beliefs are mostly affected by the information $\{\epsilon_1, \epsilon_2\}$ and very little by the realization g . If it was the opposite then the optimism would be accumulative and would have a more significant impact on prices.

The two right panels show the effect of dispersion in wealth in the economy even when agents agree. For example in the lower right panel we see that even if π_1 is equal to π_2 and regardless of their common belief, higher dispersion in wealth causes prices to be depressed simply because the wealth dispersion allows for future disagreement and therefore future depressed prices. This effect becomes stronger when the disagreement factor ρ decreases,

i.e. potential disagreement increases. We need to note here even if we do not provide the visual confirmation the effect of a different ϕ_π . If it was higher than 0.5 then the effect of the differences in beliefs π_1 and π_2 would be greater and the effect of α and ρ smaller. By smaller or greater effect we only mean the sensitivity or variability of the price due to variations in these factors. Now, these effects are naturally due to the fact that if ϕ_π were chosen to be higher then the individual beliefs would be more persistent and the effect of the public information $\{\epsilon_1, \epsilon_2\}$ on shaping beliefs would be smaller.

Figure 6 shows the one period expected return of the stock for the same combinations of states. Looking at this figure along with figure 5 we see that naturally the expected return on the stock is decreasing in the level of the price. In particular lower prices and high expected returns are associated with states where beliefs are disperse, wealth is disperse and the disagreement through the factor ρ is high. We only feel the need to note the reason behind the not so obvious relation between high disagreement and high expected return. This result is simply a result of the mean reversion in the variable ρ . So when disagreement is high agents believe that future disagreement will be lower and therefore returns will be high. Similarly, the dispersion in beliefs affects expected returns. When there is high dispersion in beliefs, beliefs tend to converge and therefore increase prices. We next look at trading and its connection with the level of prices and the conditional return volatility.

Price, Return Volatility and Volume

Let us first look at the behavior of return volatility. We have said before that the stock price is affected particularly by the disagreement factor ρ and the dispersion in wealth as indicated by α . It is therefore natural to expect that the return volatility will be greatly determined by these two factors. Looking at the right top panel of figure 7 we see that when disagreement increases through the decrease of ρ from $1/2$ to zero, the conditional volatility of returns moves from less than 1% which is the fundamental volatility to a value almost fifteen-fold when the disagreement factor is at its highest. This result addresses the volatility puzzle which states that the aggregate stock price and market returns should not be more volatile than the fundamental risk that they carry. However, the endogenous risk of belief heterogeneity which would be natural to expect to be time-varying along with a realistic preference assumption leads to such an increase in price and return volatilities. Further, the assumption that the factor ρ that determines disagreement is persistent increases its impact and therefore the volatility of prices and additionally lends its persistence both to the prices as well as the volatilities. Summing up, this disagreement factor in this model is able to produce time variability in prices, high volatility and volatility variation, associate

high volatility with low prices and finally lend its persistence to both of these quantities. All these predictions are well known to hold empirically.

The dispersion in wealth α has also a significant impact on volatility and the higher is the level of disagreement. Again the reason is that even if agents agree currently when in general disagreement is potentially high in the future, prices are sensitive and therefore returns volatile with higher dispersion in wealth. The non-monotonic behavior observed in the left two panels of figure 7 is due to the two agent assumption. When α is equal to 0.5 then the dispersion in wealth can only decrease and the prices become more sensitive around 0.25 when the wealth dispersion can both increase and decrease substantially. Finally, the disagreement in current beliefs does not affect volatility that much unless it is extreme in which case it is expected with great certainty that this disagreement can only decrease and prices increase substantially.

The basic story behind the volume of trade is quite simple. Agents are expected to trade more the higher the probability that this dispersion in beliefs will change in the future. We have to note that trade does not happen because beliefs are heterogeneous but because this heterogeneity changes, either converges, diverges or beliefs reverse. The current positions held by the agents reflect their current beliefs and these will not change and will not produce any trading unless the beliefs change and effectively their heterogeneity as well. Therefore, we have to look for variables or states of the economy where the expected change in belief heterogeneity is high and these will be the states of high expected volume of trade. Figure 8 shows the expected turnover on the stock within the next period for the same combinations of states that we have considered before for the price and return volatility. The plots shown are for the *case l* where the stock becomes a vehicle for investing in the low growth state. By this we mean that if one is pessimistic in relation to the other and wants to increase his wealth in the low state and decrease in the high state then he will invest more in the stock. We do not provide the figures for the other case but we explain the effect which is minimal and does not alter the conclusions outlined here. The different cases will be needed for the two different stories that can explain the positive correlation between price changes and volume.

The most important plot is the right top panel where we see the effect of the disagreement factor ρ . When ρ is low and disagreement high then we expect the new information in the economy to cause the beliefs of the agents to change significantly and in particular diverge when the signals in the economy do not agree and converge if they agree and in both cases it will cause trade. What is important is to note that the higher the disagreement the higher the expected trade. The expected turnover is substantial and it goes up to 10% even in the

case when currently agents agree. If we see this plot along with the corresponding plot of the conditional volatility of returns we observe a high correlation between the two. This is because high potential disagreement implies that it is expected that belief heterogeneity will change and through it prices as well. A clear and strong link is therefore established between these two quantities using an element that was missing from the asset pricing literature. Namely, that the disagreement which is the rate by which beliefs change in the economy is time-varying and this leads to periods of high and low volume along with high and low volatility. This effect now becomes quantitatively important once we take into account something that has also been missing from the asset pricing literature on trading namely the risk aversion.

The connection between volatility and volume can also be seen through the rest of the factors. When dispersion in wealth is high we also observe high expected turnover for the simple reason that holdings are disperse and all agents have the ability to change substantially their position. Finally, when beliefs are disperse we also expect high turnover because they are expected to converge. We see that the connection is very clear and strong, in that both volume and volatility are always driven by the same factor whatever this is that can cause the heterogeneity of beliefs to change. We also see that the periods of high volatility and high expected turnover are also periods of low prices and high expected return.

As we have said figure 8 shows the expected turnover for *case l*. In the other case where the stock is a vehicle for investing in the low growth state then we have the following changes. We have analyzed before after solving for the equilibrium holdings for the logarithmic utility case, there is more trading in the second sub-period in the low growth state for *case h* and in the high growth state in the *case l*. This is because in each case the agent who loads on the stock market needs to unload his position when the opposite happens of what he expects. So in the results of figure 8 we see higher expected turnover when the beliefs are higher for state g_h . Therefore the same figures appear for *case h* when we reverse the axis of π_1 and π_2 and hence there is higher expected turnover when the beliefs about the high state are lower.

For these results of turnover we have also set parameter η which is the correlation between the shocks to the disagreement factor and consumption growth to 0.5 which means no correlation. The results presented however are negligibly affected by this parameter. This correlation parameter only helps to explain the time-series joint behavior between price and volume which we do next.

Price Changes and Volume

The main focus of this paper is to explain the pervasive time-series positive correlation between price changes in the aggregate and overall turnover. As we saw from the empirical evidence shown in figures 1 through 4 higher volume or an increase in volume is associated with increases in prices. The model is flexible enough to provide two different possible stories. We choose between the two on the basis of additional pricing implications and in particular the correlation between stock returns and consumption growth.

The model provides two rounds of trade in order to be able to separate the trading due to merely changes in beliefs and due to wealth allocation changes. This distinction was not made when we looked at the relation of volume with volatility because these two affects are highly correlated in magnitude and magnitude was the scope of that analysis. Here however we need to look at the correlation of these effects with price changes. We start with the first sub-period where we only have changes in beliefs due to the new arrival of information in the form of the signals $\{\epsilon_1, \epsilon_2\}$. We have noted after lemma 7 that we assume prices to change in the intermediate period only when beliefs change. This and due to the fact that $q(z|s, \epsilon_1, \epsilon_2) = \mathbb{P}(\epsilon_1, \epsilon_2)q(z|s)$ we have that the expected change of prices for the first sub-period is zero:

$$P_t = \mathbb{E}(P_{t.5}).$$

We note once more that the above expectation is the same for all agents because they agree on the joint unconditional distribution of the two signals. They only disagree on how to interpret these signals when they differ. In the two states that the two signals agree and hence their interpretation is the same their beliefs change towards the same direction and the price more typically increases. The trading that results in these two states is minimal because essentially their heterogeneity does not change. They add or subtract the same quantity from their previous beliefs. In the other two states where they interpret the signals differently their belief heterogeneity changes and typically increases and this causes trading. The increase in belief heterogeneity results into a decrease in price which is associated therefore with price decreases. The first round of trade exhibits a high negative correlation between price changes and volume. In order now the effect to be overall positive we need the second round of trading to entail a positive correlation with greater magnitude.

For the second sub-period we need to distinguish between the two different cases of asset structure that we consider. We remind however that regardless of the structure, significant trading happens only in one of the growth states and in particular in the opposite state that is expected by the agent who loads on the stock relative to his wealth. Let us first take *case*

h where the optimistic agent will always hold more stock than what his wealth commands. If now the high growth state realizes then the agent will arrive at a state where he will have increased his wealth. This will result in holding more of the stock but he has already bought that part of the stock in the previous intermediate period. Therefore, the trading that occurs is minimal as compared to the other state and is only due to the fact that his holdings on the stock are not perfectly correlated with his wealth. Consumption and wealth are only perfectly correlated with logarithmic utility. In the other state now, i.e. of low consumption growth, the optimistic agent loses part of his wealth and he needs to unload his holdings on the stock in order to eat. This results into trading. Therefore, higher turnover happens in this case in low growth states. In order finally to have a positive correlation between turnover and price changes we need to assume that the factor ρ is negatively correlated with consumption growth. A decrease in aggregate consumption growth would be associated with a decrease in disagreement and therefore an increase in prices. Such an assumption however results into a counterfactual prediction namely the negative correlation between consumption growth and stock returns.

The second story assumes the asset structure of *case l*. Under this case we assume that if an agent is optimistic about the economy relative to other agents will choose to invest in individual stocks or derivatives rather than the stock market. Therefore an optimistic agent will shift his holdings from the market towards individual components that are more correlated with consumption growth or for example call options on the market. If now the good consumption state arrives the optimistic agent will have increased his wealth and this will lead him to go back to holding the market and in higher proportions than before. This leads to significant turnover on the stock. If further the high growth state is correlated with an increase in ρ , i.e. the parameter η being higher than $1/2$, then this turnover is associated with price increases. This is a more realistic assumption first because it predicts a positive correlation between consumption growth and returns. Under this story factor ρ starts to look like sentiment in the market that typically increases when the market goes up and it is likely associated with good states of nature. Further, it is not hard to imagine that an increase in sentiment would at the same time decrease disagreement in the market and increase prices.

From the data we have obtained a correlation between price changes and volume that ranges from around 0.3 to about 0.5. In order to see what the model predicts about this quantity we run simulations of the model for each case and for different values for η . For each parametrization we run 100 simulations of 200 whole periods each. We always start the economy from the state $(0.5, 0.5, 0.5, 0.25)$. Figure 9 shows the correlation between whole period price changes and volume changes. By whole period we mean the price change from

t to $t + 1$ and for each whole period the volume is given by adding the turnover on the stock in both sub-periods. Consistent with our explanation we see that for *case h* the correlation between price changes and volume changes is decreasing in η and increasing in η for *case l*.

The mode correlation however does not become very positive but only up to slightly greater than 0.1 in either case. If we look at the correlation between price changes and volume per se which we do in figure 10 then these correlations increase up to 0.3 which falls within the region observed in the data. We believe that this measure corresponds better to the empirical evidence because the only reason that we used differences in the volume data was in order to remove low frequency trends that is driven by other factors such as trading technology, liquidity or other exogenous factors like wars. Similar correlations were obtained also when we removed the trend using the Hodrick-Prescott filter. Neither of these procedures is needed for the simulated data since the model equilibrium is stationary.

In order to choose one case as more plausible for the explanation behind this evidence we use the pricing implication about the correlation of consumption growth with stock returns. In figure 11 we plot the model correlation as we vary η . As we see this correlation is increasing in η since an increase in the disagreement factor increases prices. The post-war correlation between the quarterly return on the value weighted Center of Research in Security Prices (CRSP) index and the quarterly real consumption growth as obtained from NIPA tables is positive and close to 0.25. It is interesting to note that the model does not predict a high correlation between consumption growth and asset returns which is typically the case with asset pricing models. This is obviously due to the fact that the model has other factors unrelated to consumption growth like disagreement, dispersion in beliefs and dispersion in wealth that affect the stock price. In order to predict the observed pro-cyclicality of equity prices we need the assumption that disagreement is counter-cyclical. This could be due to behavioral/sentimental factors as the model requires agents to become more critical and more sensitive to new information during bad times. This could be in turn due to issues related to nature that beings become more alert during bad times or under danger.

5 Empirical Evidence

Besides the positive correlation between price changes and volume, the model developed in this paper has certain strong asset pricing predictions. These predictions are independent of the correlation parameter η and are driven by the two main elements of this model, namely risk-aversion of the investors and the time-varying and relatively persistent disagreement factor. The predictions are:

- i. Volume and volatility are positively correlated.
- ii. The level of prices is negatively correlated with volume.
- iii. Volatility and prices are inversely related.

In this section we examine these predictions empirically.

The main difficulty of this empirical analysis is to obtain a stationary measure of trading activity. We use from CRSP monthly data of turnover on all common stock traded at the NYSE, NASDAQ and ASE for the period 1926 to 2008. The raw data, which is not shown here, exhibits high variability in its low frequency trend. The overall trading activity decreases substantially from around 20% per month to around 5% during and after WW II. It starts a steady increase around the beginning of the 1980's and reaches again levels above 20% by 2008. We then first remove the trend with the Hodrick-Prescott filter, using a typical monthly smoothing parameter of 1440. However, the resulting time-series is still highly non-stationary due to its changing variance. This variance is naturally connected to the average level and, therefore, we divide the resulting series by the extracted trend. The final series looks like white noise centered around zero. The measure for trading activity is then taken to be the volatility of the final series at the given point. Higher variability indicates higher trading activity during that period. The trading activity is estimated by fitting a GARCH(1,1) process to the final series.

For the same period we also obtain the market S&P composite index log price-earnings ratio from Robert Shiller's web-page.¹¹ A time-series of return volatility is obtained using the return on the value-weighted CRSP index and fitting an ARMA(1,0) model for the mean and a GARCH(1,1) model for the volatility. We first compute the cross-correlations between the price-earnings ratio, the conditional return volatility and the turnover measure we just discussed. This is shown in table 2. The results confirm all the predictions of the model listed i. to iii. at the beginning of this section. It is interesting to note that not only the predictions are confirmed with respect to the sign of the correlations but they are also significant with respect to their magnitude. Figures 12 and 13 plot our turnover measure compared to the price-earnings ratio and the return volatility, respectively. Looking at figure 12 we note the particularly strong negative correlation between trading and the level of prices from the beginning of the 1960's until today. The correlation during this period is close to -0.7 . Furthermore, the positive correlation between trading and volume is evident from figure 13, in particular during periods of high volatility.

¹¹<http://www.econ.yale.edu/~shiller/>

The evidence offered so far confirms the empirical predictions of the model about the relation between prices and volume. In addition, we would like to study empirically the connection the model establishes between some fundamentals and equilibrium prices. In particular we would like to see if in fact belief dispersion and disagreement, as determined by the variable ρ , affects prices in the way predicted by the model. For a proxy of the dispersion in beliefs about macroeconomic fundamentals, we chose the cross-sectional standard deviation of individual mean forecasts of quarterly Survey of Professional Forecasters provided by the Philadelphia Fed. The longer continuous series available is forecasts about the nominal Gross Domestic Product -Gross National Product, prior to 1992-.

We plot the raw series of our measure of belief dispersion along with the S&P composite index log price-earnings ratio. The plotted series offers strong support for the model predictions. From the figure we see clearly that the level of belief dispersion is negatively correlated with the level of prices with a correlation of -0.60 . The data also offers evidence that the disagreement factor ρ affects prices in the way predicted by the model: In particular, a high level of disagreement (i.e. a low value for ρ) should be detected in the data by radical changes in the level of heterogeneity. Therefore the volatility of the belief dispersion series should be an indication for what we call disagreement in the model and is controlled by the factor ρ . High disagreement should be associated with low level of prices, and this is exactly what we observe in the data. Periods where the belief heterogeneity is volatile, are also periods of low prices. For example this relations is evident in the period from 1975 to 1990. After 1990 we see low volatility in belief dispersion and high prices.

6 Conclusion

The classical asset pricing paradigm even though particularly successful in offering a workhorse with which we progressively understand the interrelation between prices and their evolution in time, this same framework has shed no light on trading volume. Consequently, the theoretical asset pricing literature lacks a general framework within which we can study and understand the crucial element of trading which appears to be so intimately related with asset prices. Understanding the determinants of volume which is so pertinent to asset prices will not only be important in understanding in general the workings of the financial markets but it will also offer a new aspect of how equilibrium prices are shaped.

This paper is an attempt in this direction. Past literature has indicated that the most promising modeling element towards explaining volume is belief heterogeneity which can be generated due to opinion differences about common information as initially assumed by

Harris and Raviv (1993) and further developed by Scheinkman and Xiong (2003). With this paper we show that not only substantial volume can be generated in an economy where agents hold different beliefs about the relevant macroeconomic risk but also that volume and its time-series properties are directly connected to price variations. The economic reasoning put forward by this paper relies first on the asset pricing prediction that variations in belief dispersion causes variations in the level of prices due to differences in individual consumption processes. This prediction is a result of a realistic preference assumption of power utility which also affects how agents trade. This results goes against the general practice of using simplistic preference assumptions and points to the importance of the preference assumption in understanding both prices and volume.

Further, the success of this model in explaining a range of asset pricing and volume regularities depends on the assumption of a time-varying disagreement factor that determines how different agents interpret the same public information. This factor is behind the time-variation in prices because it predicts future belief dispersion as well as trading because it predicts changes in the level of dispersion. It is made clear in this paper that it is not the belief heterogeneity that causes trade but changes to it and for this reason the disagreement factor is so important.

Due to the positive results of this paper, further empirical and theoretical examination of the constituents of this model needs to be conducted. For example, even though we offer some preliminary evidence that possible time-variation in disagreement is associated with variation in the level of prices a much deeper and more sophisticated study is required. On the theoretical front an attempt should be made to endogenize this disagreement factor and of-course to try to understand its origin. Finally, an interesting question that arises is how the cross-section of prices as well as trading behaves in this context once we assume multiple dividend paying assets.

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A Tables and Figures

Table 1: Model Configuration

<i>Preferences</i>	γ_1	γ_2	δ	g_t	$\log(g_h)$	$\log(g_l)$	ϕ_π	ρ_t	ϕ_ρ	η
	5.00	1.00	0.99		1.31%	0.34%	0.50		0.95	(0, 1)

Table 2: Contemporaneous Correlations

	<i>PE ratio</i>	<i>Return vol.</i>	<i>Turnover</i>
<i>PE ratio</i>	1.0000		
<i>Return vol.</i>	-0.2457	1.0000	
<i>Turnover</i>	-0.4296	0.4605	1.0000

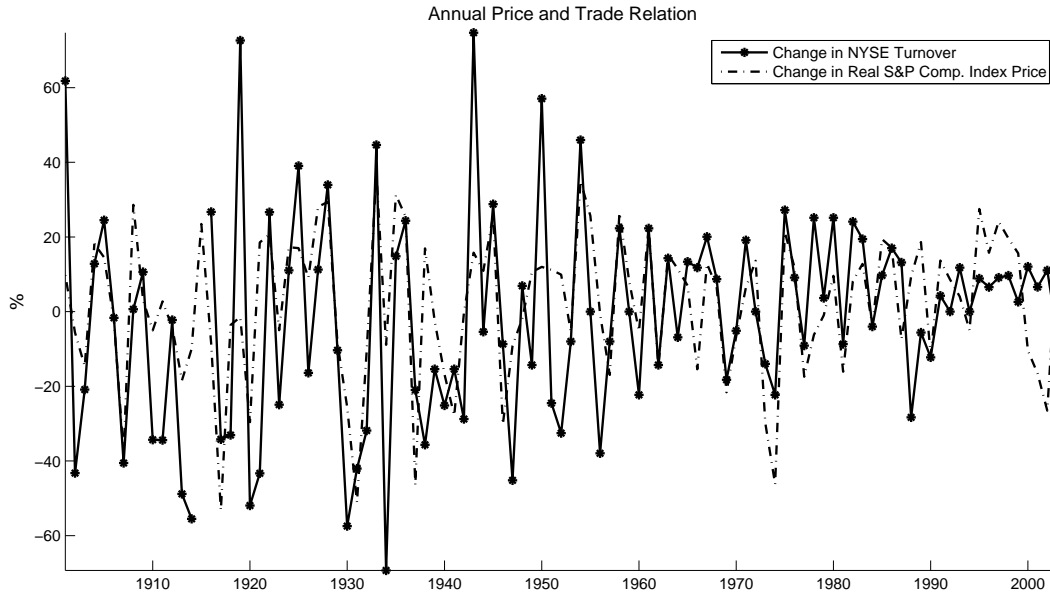


Figure 1: The plot shows the annual percentage changes in the S&P Composite index level adjusted for inflation along with the annual percentage changes in the NYSE turnover. The correlation between the series is 0.5257. The data cover the period from 1900 to 2003. Price date were obtained from Rober Shiller’s website and turnover data from the NYSE factbook.

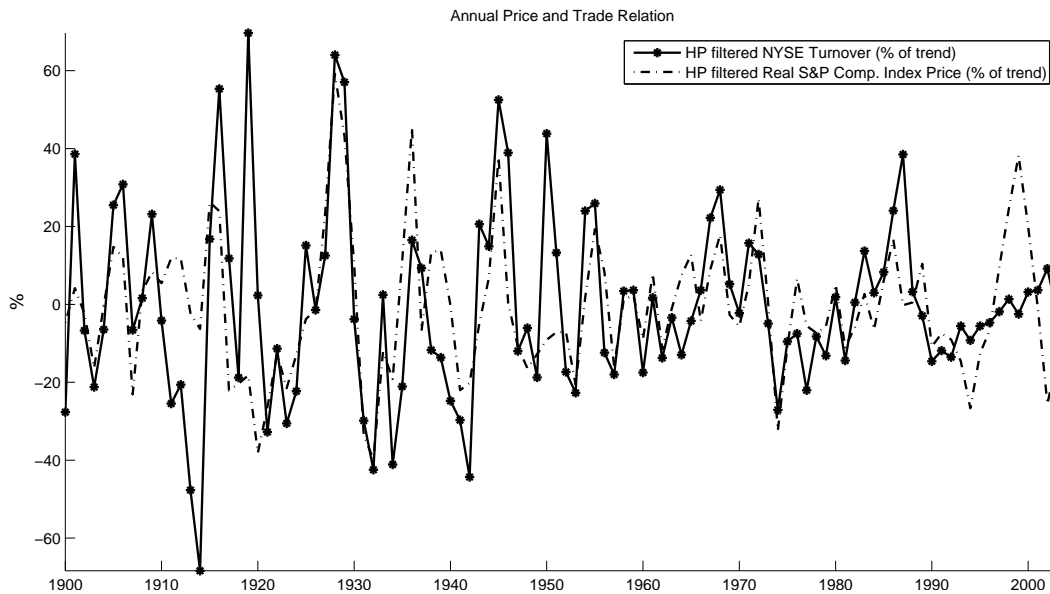


Figure 2: The plot shows the standardized deviations from a trend of the S&P Composite index level adjusted for inflation along with standardized deviations from a trend of the NYSE turnover. The trend was estimated using the Hodrick-Prescott filter and the deviations were standardized with the trend. The correlation between the two series is 0.5269. The data is annual cover the period from 1900 to 2003.

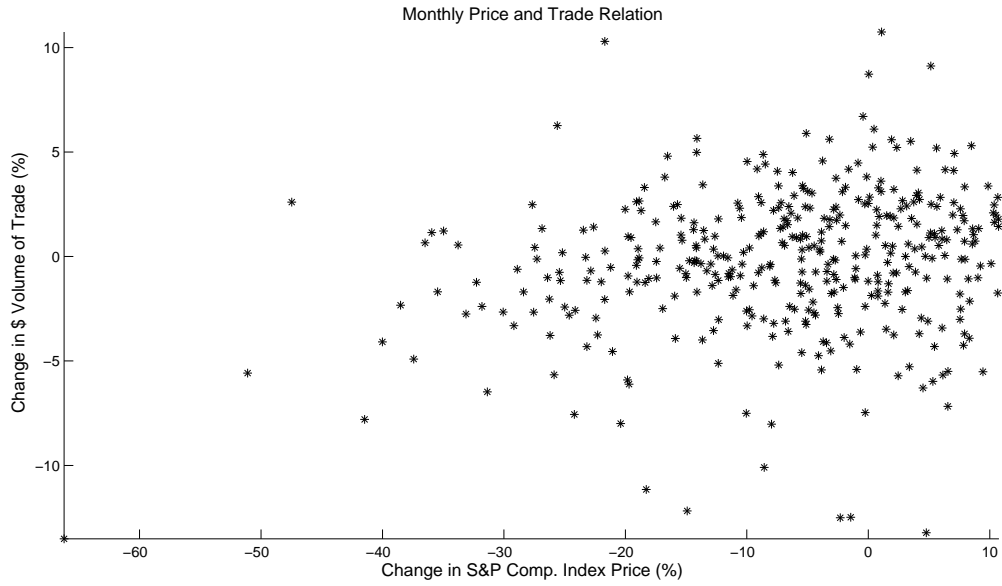


Figure 3: This is a scatter plot of percentage changes in the level of the S&P Composite index adjusted for inflation against real percentage changes in the dollar volume of trade in NYSE. The correlation is 0.2858 and the data is monthly and cover the period from January 1965 to May 2008.

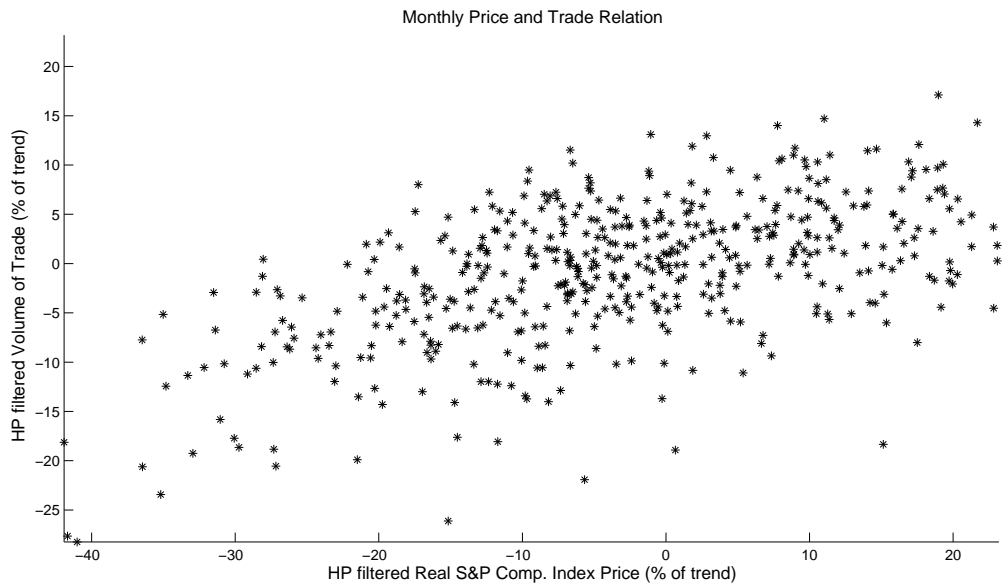


Figure 4: The scatter plot shows the standardized deviations from an estimated trend of the real level of the S&P Composite index against the standardized deviations from an estimated trend of the NYSE dollar volume. The trends were estimated using the Hodrick-Prescott filter. The correlation is 0.5625. Data is monthly and cover the period from January 1965 to May 2008.

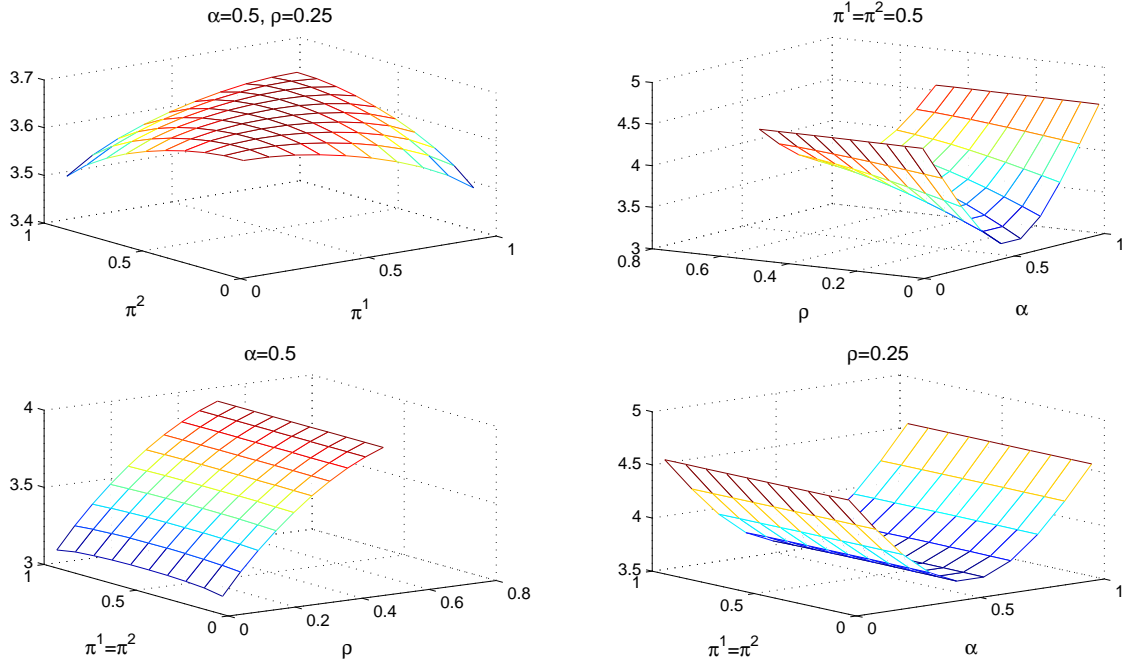


Figure 5: Log-price divided by the aggregate endowment ($\eta = 0.5$).

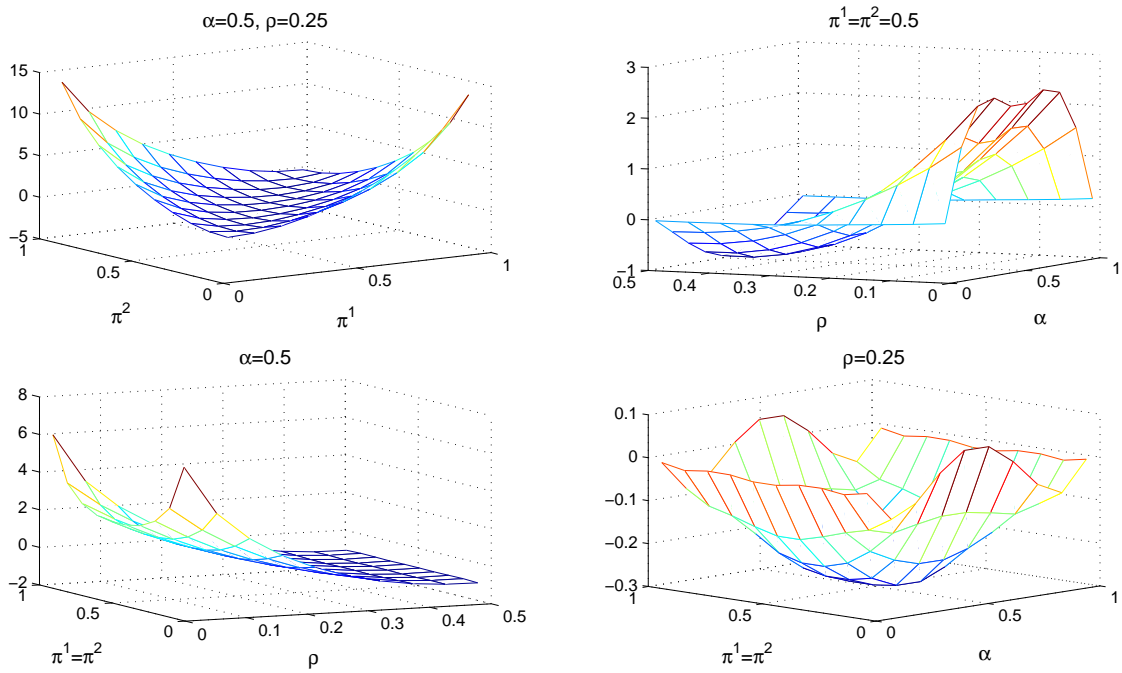


Figure 6: Expected stock return ($\eta = 0.5$).

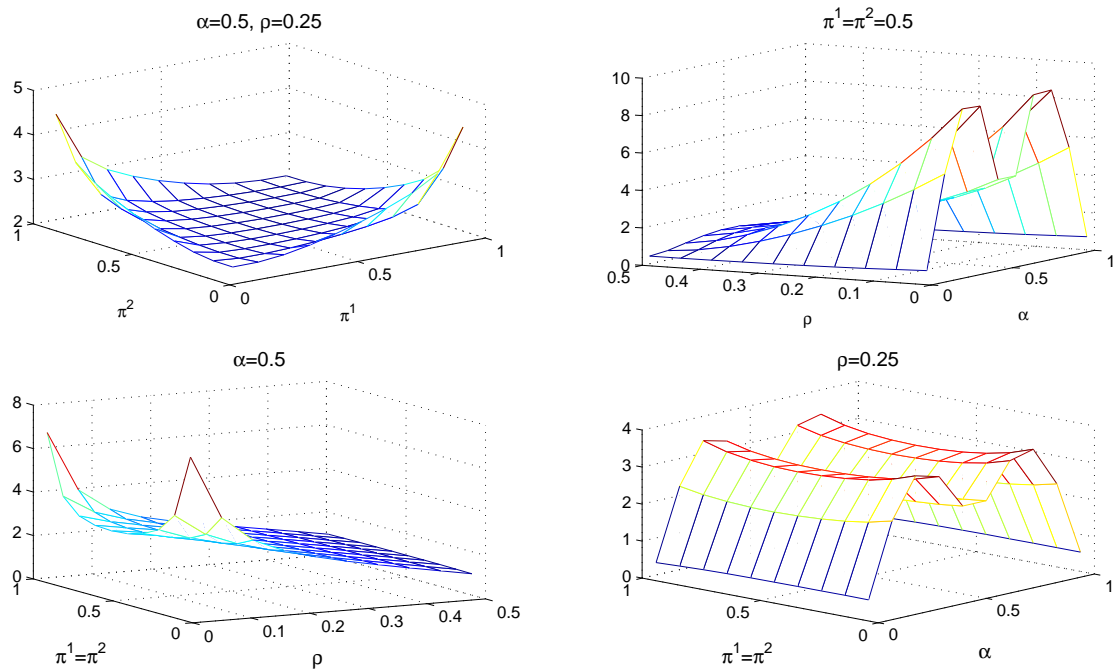


Figure 7: Conditional return volatility ($\eta = 0.5$).

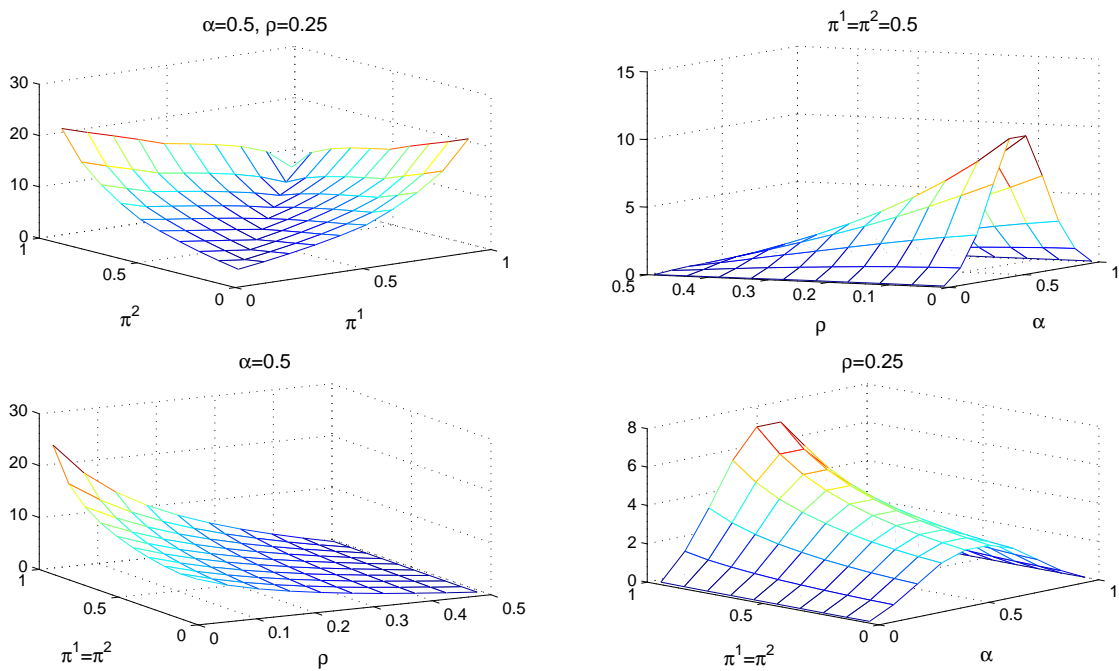


Figure 8: Expected turnover ($\eta = 0.5$).

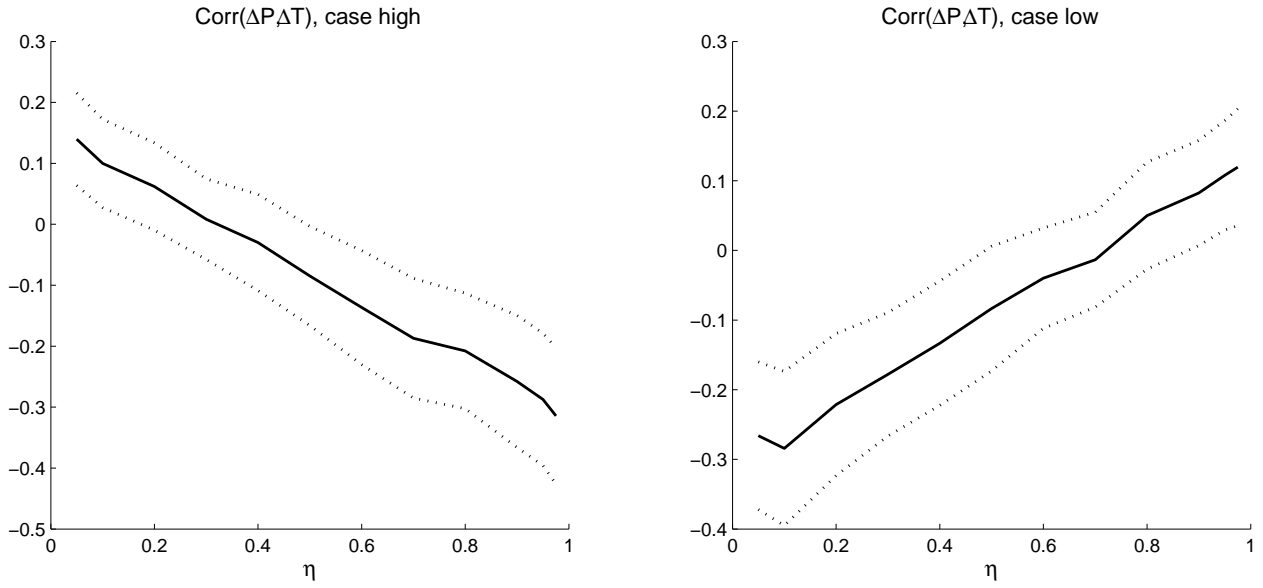


Figure 9: Model correlation between price changes and turnover changes. Data were generated using 100 simulations of 200 periods each. The dotted line shows the one-standard deviation bounds.

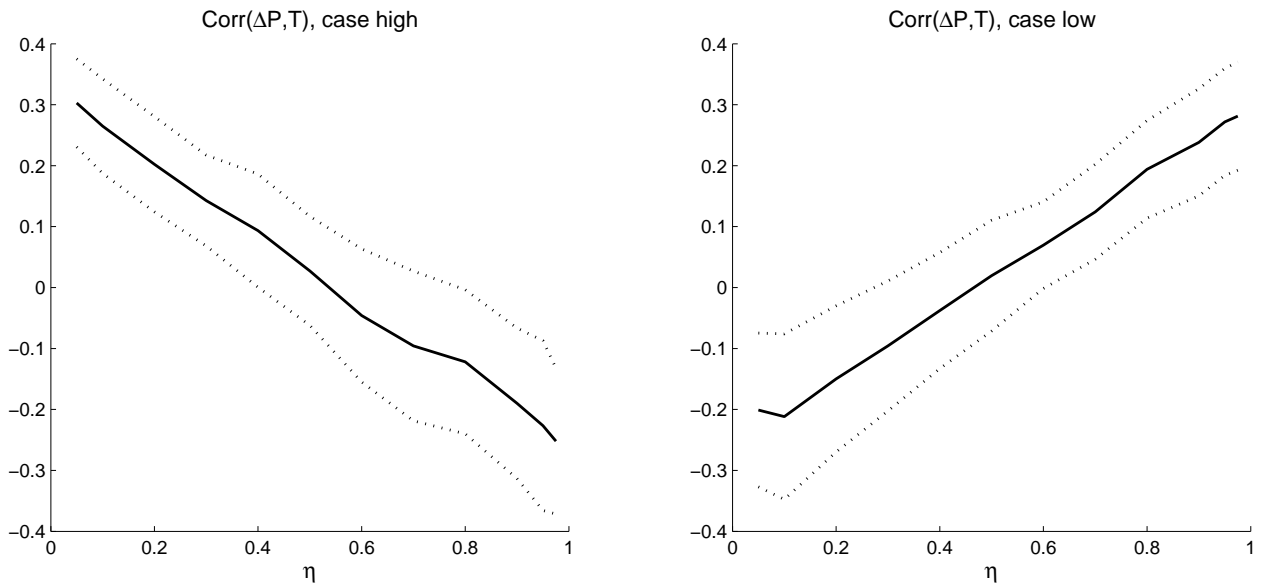


Figure 10: Model correlation between price changes and turnover. Data were generated using 100 simulations of 200 periods each. The dotted line shows the one-standard deviation bounds.

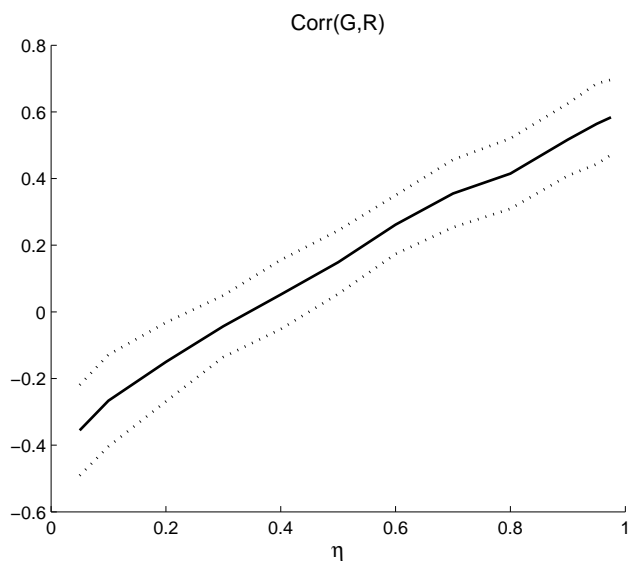


Figure 11: Model correlation between stock return and consumption growth. Data were generated using 100 simulations of 200 periods each. The dotted line shows the one-standard deviation bounds.

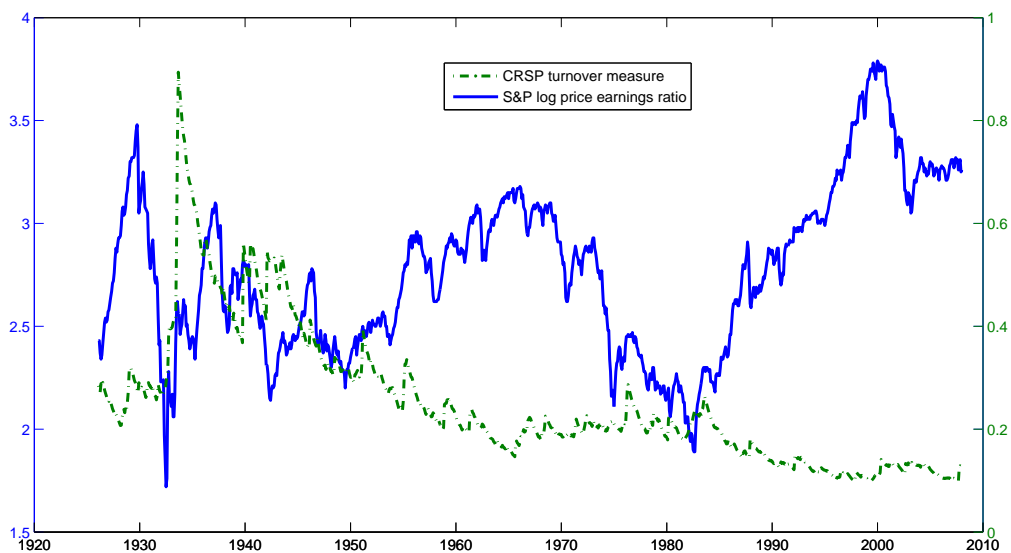


Figure 12: The plot shows the log price-earnings ratio of the S&P composite index along with the turnover measure constructed from the monthly turnover series obtained from CRSP. The data is monthly and cover the period from 1926 to 2008. The turnover measure is the conditional volatility of the standardized deviations from the estimated trend of the raw data. The trend was estimated using the Hodrick-Prescott filter.

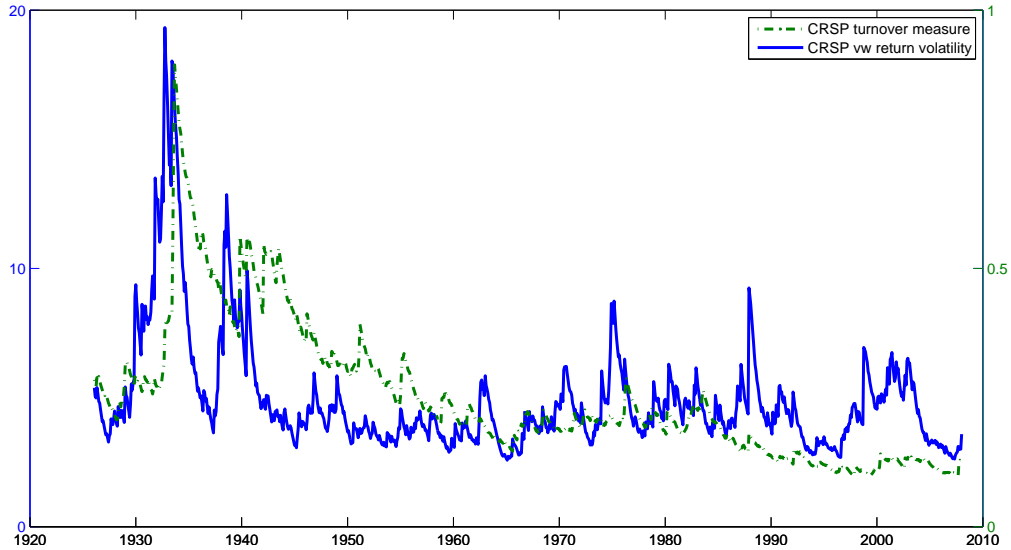


Figure 13: The plot shows the fitted time series of the conditional volatility of returns on the value-weighted CRSP index along with the turnover measure constructed from the monthly turnover series obtained from CRSP. The data is monthly and cover the period from 1926 to 2008. The turnover measure is the conditional volatility of the standardized deviations from the estimated trend of the raw data. The trend was estimated using the Hodrick-Prescott filter. The fitted conditional volatility series was obtained after estimated an ARMA(1,0) for the means and GARCH(1,1) for the volatilities model.

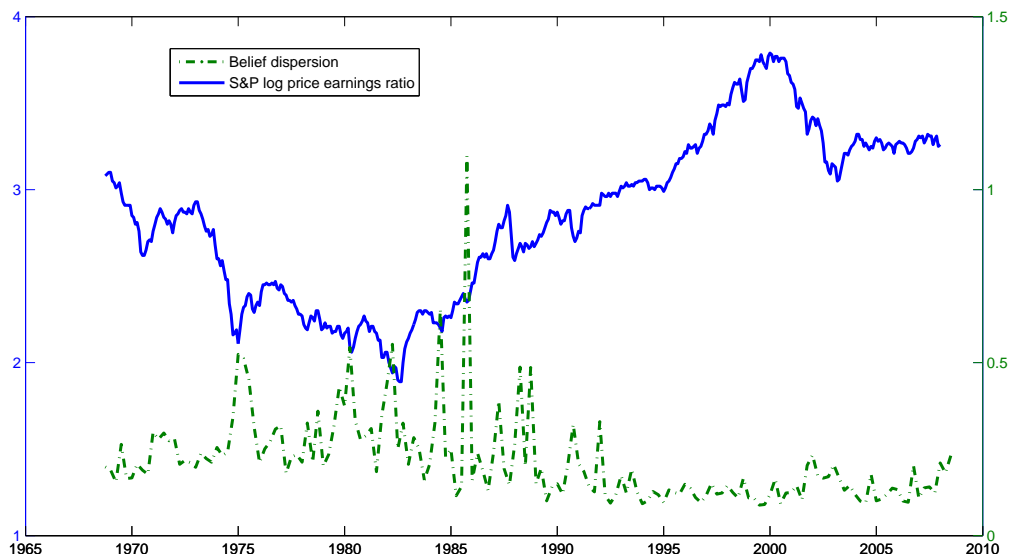


Figure 14: The plot shows the log price-earnings ratio of the S&P composite index along with the dispersion in professional macroeconomic forecasts of one quarter ahead nominal GDP. The data was obtained from the Survey of Professional Forecasters provided by the Philadelphia Fed. The correlation between the two series is -0.60 . The data cover the period from the third quarter of 1968 until the last quarter of 2007.