The Propensity to Save and Incentives to Reduce Debt

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Abstract

We explain large cash holdings in levered firms by low incentives to reduce debt. In the absence of frictions such as bankruptcy costs, transaction costs, dispersed debt ownership, financing constraints, and taxes, debt reduction cannot be rationalized. The minimum price at which lenders agree to sell the marginal dollar of risky debt is equal to its face value and the probability of bankruptcy remains the same. On the other hand, bankruptcy costs and dispersion of debt ownership increase incentives to repurchase debt. Although debt reduction in this case is beneficial to shareholders, they optimally delay the exercise of this option to obtain the best price. Finally, by introducing investment into the model, we find that financing constraints and the debt overhang problem accelerate debt repurchases only if low prices can be negotiated. Our analysis yields several important predictions concerning a firm's saving behavior that we test empirically.

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1. Introduction

When and why do corporations save out of their free cash flows? The question is economically interesting in light of the evidence that some U.S. firms have accumulated unusually high cash reserves (Bates, Kahle, and Stulz (2008)). From the point of view of agency theory and tax optimization, the level of cash beyond what is needed to provide essential liquidity would be suboptimal (see, e.g., Jensen (1986) and Auerbach (2001)). It is particularly puzzling that firms holding cash also have high outstanding debt. For example, in a 1970-2006 Compustat sample, profitable firms in the top decile by their cash holdings, that have cash reserves averaging almost 50% of the book assets, still maintain about 30% leverage. Since leverage in this case provides no (net) tax advantage but leaves the firm with an array of agency problems and restrictive covenants[1], shareholders could increase their value by buying back part or all of the outstanding debt. We attempt to shed light on this puzzle by studying the incentives to reduce debt using the firm's cash reserves.

Firms are regularly confronted with a choice between saving cash and reducing debt and the financial literature often assumes that this choice is irrelevant. However, the analysis in this paper recognizes that deviating from the idealistic assumptions changes this trade-off by affecting the price at which debt can be repurchased/retired and the potential of this transaction to reduce the expected costs of bankruptcy. Our discussion first focuses on the mechanics of the frictionless case, which develops the intuition for the main model. We then introduce and study the effects of bankruptcy costs, dispersion in debt ownership, transaction costs, financial constraints and taxation. The first two of these frictions increase the incentives to reduce debt. Transaction costs and financing constraints increase the value of the option to delay the reduction. Finally, taxation encourages debt reduction when the corporate tax rate exceeds individual tax rates.

[1] The covenants restrict many actions that a borrower may wish to do: increase capital expenditure, undertake an acquisition, increase dividends, liquidate assets, transfer money to subsidiaries, change the financial reporting procedure, alter collateral, consolidate assets, merge with another company, change lines of business, or modify the chapters and bylaws (Roberts and Sufi (2008)). The desire to remove restrictive covenants is the most common reason for calling debt (King and Mauer (2000)).

A perhaps counterintuitive result in the frictionless case, which is crucial to understanding the model, concerns the price at which debt can be repurchased. The minimum price at which lenders agree to sell the marginal dollar of risky debt is equal to its face value. It may be contrasted with a commonly used assumption in the literature that debt is repurchased at the current market price (Bulow and Rogoff (1991), Bulow, Rogoff, and Dornbusch (1988), and Gertner and Scharfstein (1991), and others). The key is to recognize that using cash to buy back debt adversely affects the value of the remaining debt claims. Therefore, the lenders need to be compensated with a higher repurchase price. To see that this price is equal to face value, note that debt is composed of a safe part, secured by cash inside the firm, and a risky part that is paid out of future risky cash flows. Absent deadweight bankruptcy costs, using cash to repurchase debt amounts to buying back the safe part of debt at its fair price, which is equal to the face value. At the same time, the per dollar risk of the remaining debt increases.

By a similar argument, reducing debt using cash fails to decrease the probability of bankruptcy. Intuitively, although repurchase of debt at face value reduces the outstanding liability, the firm's value is also reduced by the equivalent amount due to cash that the firm spends on the buyback. Savings, therefore, provide an equivalent hedging effect whether kept inside the firm or used to reduce debt. Accordingly, cash may be viewed in the frictionless environment as negative debt. Yet, since shareholders are likely to view paying a price above market value as gratuitous to lenders, and managers are likely to see cash in hand as a hedge against distress, we conjecture that incentives to reduce debt in this case are low.

In contrast, firms subject to expected distress and bankruptcy costs can reduce their debt liability at a lower price. This is possible because any reduction in deadweight costs creates a surplus that can be split between the firm's claimholders. In addition, the shareholders gain bargaining power when dividing this surplus because bondholders expect smaller debt recoveries after default—cash that is not spent on debt reduction is “taxed” in bankruptcy. Therefore shareholders can use this as a threat to negotiate a better price. Finally, reducing debt at a lower price than the face value decreases the likelihood of bankruptcy and further increases the incentive to repurchase. All else equal, we expect that firms subject to higher bankruptcy costs are more active in reducing their leverage.

Shareholders can also negotiate a better price in the absence of bankruptcy costs if they buy back diffused debt on the open market. It is almost universally assumed in the economics literature that the price of claims sold back to the company should be equal, in equilibrium, to the price of the remaining traded claims. However, as we have argued, a bondholder must not sell back the claims for anything less than their face value—this compensates the bondholder for a drop in the market price of the remaining bonds. Of course, if the bondholder should sell a fraction of the debt at a lower price, the price drop of the remaining debt will be smaller, but not sufficiently smaller to justify the lower sale price. In an open market transaction, the mechanism is similar, but dispersed bondholders are not in position to demand a higher price than is currently on the market. In fact, only initial sellers get the market price, and the rest have to surrender their claims at lower prices. Contrary to the initial intuition that a reduction in liabilities makes the outstanding debt safer, each bond that the company can purchase with cash reduces the value of the remaining bonds. A simple bank run argument suggests that bondholders will rush to sell their bonds first. When combined with the intuition in the frictionless case, the implication of this result is that debt can be reduced at prices below and above the market depending on whether the firm negotiates with dispersed or coordinated bondholders.

While it is clear from the above discussion that the option to reduce debt can be valuable for shareholders, we show that it may be optimal to delay its exercise. The equity-maximizing strategy entails holding on to cash and buying back debt only if the probability of bankruptcy becomes significant. The value of the option to delay increases in the transaction costs incurred from the repurchase. Intuitively, shareholders can negotiate the best price when the probability of bankruptcy is high. They should abandon the intent to repurchase debt when the probability of bankruptcy is negligible. Thus, we conjecture that companies contemplating debt reduction will favor saving unless the risk is sufficiently high. This observation sheds light on the fact that firms save a lot and hold their savings for long periods of time.

Naturally, if equity is in a position to negotiate a lower price, it derives most benefits from repurchasing the most junior debt. This is because cash serves as a guarantee of senior debt repayment. In contrast, the most junior debt is not likely to be repaid completely. Therefore, repurchasing junior debt at, say, market prices using cash benefits shareholders as much as using cash to repurchase some of their shares or paying a dividend prior to servicing debt (Bulow (1992)). In the absence of covenants that require the most senior debt to be repurchased first, we expect that the firms that issued debt with different seniority will have a stronger incentive to reduce debt.

Next, we introduce investment in the model and consider the role of financing constraints. Debt reduction can benefit shareholders of financially constrained firm by providing more cash for investment. The effect is similar to that in Acharya, Almeida, and Campello (2007)). They show that since the market value of risky debt is mainly supported by the states of the world where the firm is solvent, repurchasing debt at its market price increases the amount of cash available to a financially constrained firm in the good state and decreases the amount available in bankruptcy states. We contribute to their result by demonstrating that debt reduction can increase investment only if firms can buy back debt at the low price. For example, debt repuchases at the face value have no effect on investment incentives.[2]

Finally, we study how effective repurchasing debt is in reducing debt overhang (Myers

[2] By this argument, buying back debt has no effect on investment in the frictionless case. In addition, if bonholders realize that the value of the firm will increase due to investment, they will demand a higher price, reducing the benefits of debt repurchase.

(1977) and Hennessy (2004)). Our analysis contributes to the literature by showing that the debt overhang problem cannot be mitigated if equity cannot negotiate the low price. Intuitively, debt repurchase at face value cannot reduce the risk of outstanding debt but instead consumes cash that can be directly allocated to investment. We further show that unless the investment option has sufficiently large net present value, the price below the face value may not be feasible in debt reduction. This is because shareholders cannot commit to undertaking the investment after the debt repurchase.[3]

The framework developed in the paper produces a number of empirical hypotheses concerning the optimal amount of savings net of debt. Overall, we predict that the propensity to save should be higher and the propensity to reduce debt should be lower in firms where expected distress costs are small and the debt is held by a single bank or private investor. Further, we predict lower prices in repurchase transactions in firms with high bankruptcy costs and/or higher dispersion of debt ownership. Our model also allows us to link the propensity to save or reduce debt to maturity and risk.

We document the following regularities that are consistent with these predictions. First, firms that are likely to have larger bankruptcy costs, such as small firms or firms with large growth options show larger propensity to reduce debt. Second, the propensity to reduce debt increases in the average maturity of debt and in the risk of bankruptcy while propensity to save decreases in these characteristics. The results are stronger for short-term debt. Third, the significant proportion of repurchases are conducted at the prices near the face value. The premium paid to reduce debt increases with maturity and decreases with probability of distress.

Our paper is related to the debt restructuring literature (Bulow and Rogoff (1991), Gertner and Scharfstein (1991), and Julio (2007)) and the strategic debt service literature (Mella-Barral and Perraudin (1997) and Hart and Moore (1998)). However, there is an

[3] At the same time, the large net present value of the investment project may be inconsistent with the presence of debt overhang problem on the first place.

important difference. The strategic debt service literature describes bargaining after the fact of default, when cash effectively already belongs to creditors. On the empirical side, our analysis is related to the literature on the savings/debt to cash flow sensitivity in financially constrained firms. For example, Almeida, Campello, and Weisbach (2004) measure the degree of financing constraints with propensities to save. Acharya, Almeida, and Campello (2007) focus on a saving versus reducing debt policy that maximizes the value of investment. Riddick and Whited (2008) discuss the interpretation of sensitivity regressions.

The remainder of this paper is organized as follows. Section 2 presents an overview of the related literature. Section 3 provides a series of simple examples and a bare bones model. Section 4 extends the analysis to a study of the timing of savings or debt reduction. Section 5 discusses investment and the role of financing constraints. Finally, Section 6 develops and tests the empirical hypotheses.

2. Background and Literature

Our paper fits into the literature on restructuring and reduction of debt. Surprisingly, very few studies discuss reductions in *corporate* debt. A notable exception is Gertner and Sharfstein (1991) who analyze renegotiation of bank debt and repurchase of public debt of financially distressed firms with valuable investment options.[4] Bulow and Rogoff (1991) study open market *sovereign* debt repurchases in the presence of the debt overhang problem. The major difference between corporate and sovereign debt pertains to the pledgeability assumption (Bulow (1992)).[5] Specifically, sovereign debt repurchases are done with debtor's funds, mainly by reducing consumption. In contrast, in the corporate case, repurchases are done with funds that would otherwise serve as collateral for creditors, essentially with creditors' own money.

[4] Gertner and Scharfstein (1991) discuss senior debt for juinior debt exchanges instead of cash repurchases (see also James (1996) for empirical evidence on debt exchanges). Debt reductions using cash and debt reductions using issuing a new most senior bond are equivalent if cash cannot be distributed.

[5] In contrast to assumptions in Bulow and Rogoff (1991) we do not require the market price of debt to be the same before and after the repurchase on the open market.

Our work is also related to the strategic debt service literature. Mella-Barral and Perraudin (1997) incorporate into their model the fact that firms facing financial distress can act strategically and force concessions from debtholders. Hart and Moore (1998) develop a model of a firm's financing when the information is symmetric but the repayment cannot be enforced. They show that a debt contract that requires the firm to make a fixed stream of payments and admits renegotiations in case of default is optimal. However, the similarity between our work and the strategic debt service literature does not go beyond the incentive to reduce the probability of bankruptcy. We study leverage reduction through repayments when there is a significant probability that the firm will not default and, in particular, we show that defaults can be avoided by reducing debt. In contrast, the debt renegotiation literature rules out levering down prior to default and considers debt restructuring when bankruptcy is imminent. Therefore, the role of cash as part of a firm's assets becomes irrelevant. For this reason, some of our predictions are contrary to those in the debt renegotiation literature. For example, we show that dispersion of debtholders that is commonly seen as an impediment to renegotiations actually helps to reduce leverage and the probability of bankruptcy when the firm uses cash for debt repurchase.

The early empirical literature focuses on financially constrained firms and studies the propensity to invest using free cash flows.[6] More recently, however, to reduce the endogeneity problem in the sensitivity regressions, a number of papers have shifted the discussion on the propensity to save. For example, Almeida, Campello and Weisbach (2004) emphasize precautionary saving motives and find that constrained firms display positive cash to cash flow sensitivity. Riddick and Whited (2008) reinvestigate and contrast the savings behavior of constrained and unconstrained firms by incorporating the firms' investment needs. Faulkender and Wang (2006) investigate the effect of savings

[6] Fazzari, Hubbard, and Petersen (1988) argue that if groups of firms are financially constrained, their investment is positively related to cash flows, holding the investment opportunities fixed. Kaplan and Zingales (1997) advance the argument that cash flow sensitivity does not have to be monotonic in the degree of financing constraints. Further, Erickson, and Whited (2000) explain that measurement error in the proxy for investment opportunities can account for the difference in the cash flow sensitivities across groups of firms.

on firm value by empirically checking stock price responses to savings.

Our work is also related to the literature that investigates investment and saving policies of financially constrained firms that have valuable real options. Acharya, Almeida and Campello (2007) describe the trade-off between saving cash and repurchasing risky debt when investment opportunities are correlated with cash flows. They show that debt adjustments lead to redistribution of cash flows from states where the firm is bankrupt or distressed to states where the firm is solvent. Higher cash flow in good states increases the value of investment (Froot, Scharfstein, and Stein (1993)). Therefore, firms with investment incentives prefer to reduce debt out of cash flows, but firms with hedging incentives prefer to increase savings. Julio (2007) studies the role of debt repurchase as a way to overcome debt overhang in financially constrained firms and as a way to increase profitable investment. In particular, he emphasizes the low value of cash held by the firm when the probability of bankruptcy is high.[7]

Finally, our paper is related to the literature on the determinants of cash holdings. Opler, Pinkowitz, Stulz, and Williamson (1999) relate savings to the properties of the firms. Bates, Kahle, and Stulz (2008) document the growth trends in cash savings and argue that U.S. companies savings' can be explained by hedging incentives. Foley, Hartzell, Titman, and Twite (2007) provide a tax explanation for why some firms are reluctant to spend cash if they earn profits abroad. Acharya, Davydenko, and Strebulaev (2008) link optimal cash holdings to bankruptcy risk by using a structural model of a risky firm with endogenous savings.

3. Debt Reduction

3.1. The frictionless case

[7] Julio's findings provide another way to see the argument in the frictionless case. In our model, the low value of cash to shareholders is the reason for why they pay a high price (face value) for risky debt.

Assume that the firm has cash C and outstanding debt with face value D. At time $t = 1$, the existing assets of the firm generate a cash flow x, distributed according to the probability distribution function $f(.)$ on non-negative support $[\underline{X}, \overline{X}]$. Relying on the observation that x can take very low and very high values we require that

$$C + \underline{X} < D \leq C + \overline{X}$$

That is, there are contingencies when debt is fully repaid and there are contingencies where debt is not fully repaid. Without this assumption, the case becomes trivial.

If the firm becomes bankrupt, the priority rule is observed and debtholders have first claim on the firm's assets. In the interest of clarity, this section assumes no costs associated with bankruptcy. Since the objective of this paper is to determine the impact of a firm's financial position on the incentive to increase or decrease leverage, we assume that the entrepreneur "inherits" debt and do not derive the optimal ex ante debt commitment.

Lenders assume equal seniority; however, future debt issues must be subordinate and will not affect the recovery claim of the senior lender in the event of default. We first restrict debt to be held by a single lender, such as a large bank or a private investor. This assumption can be easily extended to include several large lenders, who due to reputation or other concerns, collude when negotiating the sale price of debt. Later, we consider the possibility of repurchasing debt from dispersed bondholders.

The main objective of the manager is to maximize the market value of the firm with respect to his or her financing decisions. In particular, the manager considers two alternative strategies at t=0: saving amount C to be used in later period $t = 1$, or using cash to repurchase some amount of debt αD from the bondholders. Our analysis below focuses on the mechanism of this transaction and the identification of α. We make the important assumption that the manager is restricted from paying dividends or conducting share repurchases since distribution of cash would result in the value transfer from bondholders to shareholders. Provisions limiting distributions affecting debt repayments are commonly included in debt covenants (Smith and Warner (1979)).

We start by deriving the effect of debt repurchase on the value of equity. The shareholders' value S_0 if the firm holds cash is:

$$S_0 = \int_{D-C}^{\overline{X}} (x + C - D) f(x) dx. \tag{1}$$

If the firm buys back a fraction α of outstanding debt using the available cash C, the payoff to the shareholders is:

$$S_R = \int_{(1-\alpha)D}^{\overline{X}} [x - (1-\alpha)D] f(x) dx. \tag{2}$$

We show in the Appendix, that if

$$\alpha > C/D,$$

that is, if debt is purchased at the price lower than its face value, then, from (2) and (1), $S_R > S_0$. That is, if debt can be repurchased at a price lower than the face value and when there are no transaction costs, shareholders always benefit from repurchase.[8]

By the zero-sum argument, when shareholders are better off debtholders must be worse off. Specifically, debtholders will refuse to sell their debt to the firm at any price below the face value. To see this, compare the value of the debt contract before repurchase,

$$d_0 = \int_{\underline{X}}^{D-C} (x + C) f(x) dx + D \int_{D-C}^{\overline{X}} f(x) dx, \tag{3}$$

to the value of the debt contract after repurchase,

$$d_R = \int_{\underline{X}}^{D-\alpha D} x f(x) dx + (D - \alpha D) \int_{D-\alpha D}^{\overline{X}} f(x) dx. \tag{4}$$

To break even, debtholders must ensure that the sum of the value of debt after transaction, d_R, and the cash obtained in the transaction C, should not be lower than the

[8] See the discussion of the differences between sovereign and corporate debt by Jeremy Bulow in Newman, Milgate, and Eatwell (1992).

initial debt value d_0

$$d_R + C = d_0. \tag{5}$$

The unique solution to (5) subject to (3) and (4) is

$$\alpha = C/D. \tag{6}$$

That is, partial debt repurchase can only be accepted by bondholders at face value. The intuition for this result relies on a simple observation. All else equal, bondholders would benefit from selling a part of their claim at a price above the market price. However, the value of the remaining claim decreases due to the lower cash balance at the firm, and the original market price is no longer valid. Finally, note that this debt repurchase price is unique and is independent of the bargaining power between bondholders and shareholders. We show next that this is not the case when the firm's assets are subject to bankruptcy costs.

3.2. The role of deadweight bankruptcy costs

Next, we introduce bankruptcy costs into the model. We assume that the magnitude of these costs is known to both shareholders and creditors, and that the creditors take into account the possibility of renegotiations aimed at reducing these costs.[9] Following Leland (1994), we assume that in default lenders take over the firm and implement first-best policies subject to a fraction $\beta \in [0, 1]$ of the firm's assets being lost during the process.

The expected bankruptcy costs become smaller after debt reduction even if the probability of bankruptcy is unchanged. From (3) and (4), the expected bankruptcy costs if

[9] In support of the latter assumption, Alderson and Betker (1995) investigate the relation between liquidation cost and capital structure and find indirect evidence that debtholders are aware of the possibility of debt renegotiation in the future.

the manager prefers to hold cash:

$$BC_0 = \beta \int_{\underline{X}}^{D-C} (x + C) f(x) dx, \tag{7}$$

can be reduced to

$$BC_R = \beta \int_{\underline{X}}^{D-\alpha D} x f(x) dx, \tag{8}$$

if the manager repurchases fraction α of the debt. The deadweight costs are smaller for two reasons. First, cash C that was kept inside the firm is no longer threatened to be absorbed during bankruptcy since it is repaid to lenders. Second, as we show below, negotiations in the presence of bankrupcy costs may lead to a lower repurchase price. In this case the probability of bankruptcy decreases. These two effects result in higher firm value and therefore a positive surplus, the difference between (7) and (8).

The price of the repurchase depends on how this surplus is split between the claimholders.First, we assume that all bargaining power belongs to shareholders and derive the price from the break-even condition for debtholders (5).[10] To the extent that the surplus from the transaction is positive, this price is lower than the face value of debt. The Appendix derives the lower bound on the price (which is not restricted to be above the market) and proves the uniqueness of the solution.

In contrast, if bondholders are in position to make take-it-or-leave-it offer, they sell debt at the face value. Shareholders break even at this price just as in the frictionless case. This is because bankruptcy costs do not directly affect the shareholder's claim. It follows that prices at which debt can be repurchased in the presence of deadweight bankruptcy costs range from low (potentially below the market value) to high (as high as the face value) and depend on how the surplus is split in renegotiations between shareholders and creditors.

Our results lay the ground for several intuitive empirical predictions. First, the average price at which debt can be repurchased decreases in the deadweight costs associated

[10] In the case with bankruptcy costs, we tighten the upper bound on the amount of cash $C < d_0$. Otherwise the firm's optimal strategy is to retire all debt at once.

with bankruptcy. Second, since the incentives to repurchase increase in the negotiated price, we expect to see the higher propensity to reduce debt and lower leverage in the firms with large expected bankruptcy costs. Finally, bankruptcy costs fixed, the repurchase price increases in bargaining power of bondholders.

3.3. Dispersed debt

In many instances, managers must repurchase corporate debt from a number of dispersed bondholders. In these cases, two mechanisms are available—the open market repurchase and the tender offer. The latter is usually conducted by offering a single price to all bondholders. The former is generally executed over a period of time and allows for different prices. In the interest of clarity, we assume that bankruptcy costs are equal to zero and re-examine this assumption at the end of this section.

First, suppose the market value of debt is d_0 and the firm is able to repurchase a fraction α of debt through an open market transaction at this price[11]

$$C = \alpha \cdot d_0. \tag{9}$$

We have shown above that if the firm repurchases debt at a price lower than the face value D. Although the seller gets a fair price, the debtholders are (collectively) worse off:

$$d_R + C < d_0. \tag{10}$$

Then, from (9) and (10), dividing by the face value of date zero debt D, we obtain

$$\frac{d_R}{(1-\alpha)\,D} < \frac{d_0}{D}, \tag{11}$$

the market price of each bond should decrease.

[11] Although the results that follow hold for a wide range of prices, we choose to focus on the repurchase on the market price to facilitate the comparison with other models in the literature that use this assumption.

This result is interesting for the following reason. All else equal, reduction in total liability should make remaining debt safer and is therefore beneficial for holders of corporate bonds. However, this benefit is more than offset by the fall in firm's cash. Therefore, the market value of remaining bonds decreases. The resulting wealth transfer benefits shareholders at the expense of bondholders because marginal debt is bought back at the average (market) price instead of the fair (face value) price.

To summarize, the first seller(s) in the open market debt repurchase get market price, but those who hold to the bonds or sell them later get a lower price. Therefore, rational bondholders should sell as soon as possible in response to the offer from the firm.[12] Note that one assumption here—all bonds have equal seniority—is crucial for the result. The creditors can avoid this equilibrium by structuring debt in small batches, assigning a strict priority rule for each batch, and explicitely prohibiting the firm from buying back any part of debt other than the most senior one.

Now we turn to the case of the tender offer debt repurchase. In this case the equilibrium considerations require that the tender price of bonds must be equal to the expected price of bonds outstanding after the repurchase is complete (Bulow and Rogoff (1991)). Otherwise, if the expected price is higher, the bondholders will choose to hold out. We show below that in the tender offer to repurchase debt the managers can offer the price lower than the market price.

First, we require that the tender price (the tender offer is C dollars on the amount of debt αD) be equal to the price after the repurchase (d_R for the remaining amount of debt $(1-\alpha)\,D$, using our notation):

$$\frac{d_R}{(1-\alpha)\,D} = \frac{C}{\alpha D}. \tag{12}$$

Substituting the value of remaining debt from (4), we find the implicit expression for the

[12] This insight can shed light on the finding that market debt repurchase is associated with lower prices (Mann and Powers (2005)).

tender offer price (per unit of debt) $P = C/\alpha$

$$\frac{\int_{\underline{X}}^{D-CD/P} xf(x)dx + (D - CD/P) \int_{D-CD/P}^{\overline{X}} f(x)dx}{1 - C/P} = P. \tag{13}$$

Intuitively, the existence of the solution follows from considering the upper and lower limits for the price and noting that function (13) is continuous between these two limits. As evident from our basic argument, the market price is too high for the tender offer because it implies that the value of the remaining bonds must decrease—every bondholder will want to sell. At the same time, zero (or small $\varepsilon > 0$) price is too low because it means no cash leaves the firm and the liability decreases—the value of remaining bonds must increase after the repurchase and all bounholders will hold out. Therefore, there exists some intermediate price between zero and the initial market price

$$\varepsilon < P < \frac{d_0}{D}, \tag{14}$$

that satisfies (13). It is straightforward to show that P is below the current market price.

We next note that P cannot be an equilibrium price. Conditional on the success of the repurchase, the bondholder expects to get price P. But if repurchase fails, no cash is paid and the bondholder expects to get the market price. Therefore, the equilibrium strategy, assuming that the strategy is the same for all homogeneous invesors, is not to tender and the repurchase cannot be completed. In sum, the feasible prices in tender offer repurchase are below the face value, however tender offer may fail at the prices below the face value. The optimal creditor's strategy entails participating in the tender offer at any price above the market price and participating at a lower price than the market price if the tender offer is believed to be successful.

Finally, we briefly discuss repurchasing debt from dispersed bondholders in the presence of bankruptcy costs. This assumption leads to the existence of positive surplus from the repurchase. The increase in firm value because the debt reduction removes cash from the firm prior to the bankruptcy and also reduces the probability of bankruptcy. The positive surplus creates an environment for the hold-up problem (see, e.g., Gertner and Scharfstein (1991) in relation to corporate debt repurchase or Shleifer and Vishny (1986) in the context of takeovers). Lenders may refuse to sell their bonds at the market price anticipating a value increase from the debt repurchase. Then, assuming that the surplus is significant, the repurchase price may potentially be higher than the market price.

3.4. The effect of a corporate tax

Cash saved inside the firm and cash saved by the firm's investors can receive differential tax treatment. Below, we briefly describe the fundamentals of this effect and relate it to the features of the tax code. First, suppose the firm saves amount C, and the saving accrues the interest rate r. The interest is taxable as a part of the firm's income and therefore is subject to a marginal corporate income tax that we denote as T_c. Since the proceeds that are paid to shareholders as dividends or share repurchases are also subject to the distribution tax, T_d, the after-tax amount that ends up in the pockets of shareholders is

$$\text{payoff}_1 = C\,(1 + r\,(1 - T_c))\,(1 - T_d). \tag{15}$$

If, instead, the company chooses to pay the amount C to investors who can invest at the same risk-free rate r and pay an individual tax T_i, the investors will collect the after-tax interest $r\,(1 - T_c)$ on the distributed amount $C(1 - T_d)$. Therefore, the net effect is

$$\text{payoff}_2 = C\,(1 + r\,(1 - T_i))\,(1 - T_d). \tag{16}$$

By comparing these two cases, we note that the after-tax payoffs are identical if the corporate tax rate T_c exceeds the individual tax rate T_i. The recent changes in corporate tax rates (2001 George W. Bush tax relief for the corporations) effectively set the top brackets for both corporate and individual tax rates at 33%.

Finally, if the firm chooses to use amount C to reduce its debt at face value, then the firm's outstanding liability decreases by $\Delta D = C(1 + r)$ but the firm loses a tax deduction of $(Cr)T_c$. Therefore the net effect, after the distribution tax, is identical to that with corporate savings,

$$\text{payoff}_3 = C\left(1 + r\left(1 - T_c\right)\right)\left(1 - T_d\right). \tag{17}$$

However, the effect is different if debt is repurchased at prices below the face value. In this case, there is a tax disadvantage to debt repurchase. The reductions in tax on savings interest do not offset the additional tax due to lower debt.

4. Timing of Debt Reduction

In the previous analysis, we have adopted the assumption that debt can be repurchased at a single date and the remaining cash flows are allocated to savings. This section extends the analysis by studying an intertemporal debt/cash policy. In particular, the firm gets "two chances" to reduce its leverage at $t = 0$ and $t = 1$.

4.1. The timing of debt reduction and transaction costs

Suppose the firm operates for two periods and the resulting payoff at the last date is equal to the product of two independent random variables $x \cdot y$. Accordingly, x captures the relevant uncertainty from $t = 0$ to $t = 1$, and y captures relevant uncertainty from $t = 1$ to $t = 2$. The manager maximizes the expected payoff of the firm with respect to saving/debt reduction decisions. Notation C_1 is for savings carried from $t = 0$ to $t = 1$, and C_2 denotes savings from $t = 1$ to $t = 2$. Similarly, D_1 denotes the face value of debt from $t = 0$ to $t = 1$, and D_2 denotes debt from $t = 1$ to $t = 2$. The payoff to shareholders is equal to cash flows net of debt payments; therefore they maximize $xy - D_2$ over the permissible domain $\{x, y\}$ plus the savings that remain at t=2

$$C_2 + \int_x \int_y (xy - D_2) f(x, y) dy dx, \tag{18}$$

where $f(x, y) = f(x) \cdot f(y)$ is a probability distribution function for x and y.

Next, we introduce the transaction costs. Roberts and Sufi (2008) document that although direct recontracting fees associated with changes in debt contracts are relatively small, the indirect costs may be significant. The latter take the form of time and effort spent by both borrower and lender on understanding the implications of the transaction, obtaining approval of ammendment and waivers in case of sindicated loans.

Denoting the transaction cost of the repurchase γ, we preserve all above derivations with one modification: the total price of the repurchase, e.g., at $t = 0$ increases from $C - C_1$ to

$$\frac{C - C_1}{1 + \gamma}, \tag{19}$$

and similarly for $t = 1$. The maximization in (18) is subject to two constraints. The first is the participation constraint for bondholders at $t = 0$:

$$d_0^R + \frac{C - C_1}{1 + \gamma} \geq d_0. \tag{20}$$

The inequality (20) implies that the value of bondholders' claims, including cash paid in repurchases, must be no less than the value prior to the repurchase. Since shareholders can remove any slack in (20) by paying less, this constraint must bind.

The second constraint is the break-even condition at $t = 1$:

$$d_1^R + \frac{C_1}{1 + \gamma} \geq d_1, \tag{21}$$

where d_1 and d_1^R is the value of debt before and after repurchase at $t = 1$, respectively. Again, from the equity value maximization (18), this constraint must be binding.

We discuss the solution to the maximization problem (18), subject to (20) and (21), in the Appendix. The solution proceeds in two steps. The first step demonstrates the indeterminacy of optimal savings C_1 in the frictionless case. Although, in general, the firm has incentives to buy back debt, it is indifferent between debt reduction at $t = 0$ and $t = 1$. The second step shows that equity maximization in the presence of (however small) transaction costs result in the corner solution. It follows that although the debt reduction option is valuable, there is value in delaying the exercise that increases in transaction costs.

The intuition is as follows. Naturally, the low purchase price translates into higher value to shareholders; therefore, they would benefit from buying back debt. It is also clear that the purchase price decreases in the probability of bankruptcy. For example, the wealth transfer to equity is larger if the risk of bankruptcy increases. Therefore, the presence of transaction costs makes the option to delay the repurchase valuable. The firm proceeds with repurchase if the risk of bankruptcy increases and it will abandon the intention to repurchase if the risk becomes smaller.

5. Investment and Financing Constraints

5.1. The role of debt overhang

Additionally, we discuss the effect of debt reductions on the debt overhang problem. The problem manifests itself in the prohibitively high cost of external capital for firms with risky debt. For example, Myers (1977) demonstrates that firms can pass on positive NPV opportunities since undertaking investment increases the value of debt and therefore decreases the value of equity. Building on this observation, the recent literature finds that firms can increase their investment by reducing debt. [13]

Although, all else equal, reducing the face value of debt in anticipation of a valuable investment clearly reduces the underinvestment problem, our analysis below casts doubt on the view that an equivalent result can be achieved if the firm uses internal liquid resources, such as cash, for debt reduction. Our result relies on the insight that reduction in liability may not reduce risk if it is accompanied by a simultaneous reduction in safe assets.

[13] Bulow and Rogoff (1991) show that the buyback of sovereign debt is a giveaway to creditors because the relief from debt overhang is expected to increase the market value of debt. Julio (2007) argues that repurchasing debt prior to investment can mitigate the problem.

Specifically, assume that the firm is contemplating an investment project at $t = 0$ that has cost I and generates cash flow i at $t = 1$. The distribution is governed by the continuous function $f_I(i)$. The NPV of the investment project is nonnegative and the proceeds from the investment are received at the liquidation date. The owners of the firm expect the payout to be equal to the sum of cash flows from the assets in place, x, and the investment proceeds, i, so that the resulting distribution of $(x+i)$ is $f(x,i)$.

Debt overhang prevents the firm from investing by raising new equity. Shareholders are worse off if they inject additional equity $(I - C)$ and exercise the investment option

$$\int_{D}^{\overline{X}+\overline{I}} [x+i-D] f(x+i) d(x+i) - (I-C) \leq \int_{D-C}^{\overline{X}} (x+C-D) f_X(x) dx. \tag{22}$$

Can debt repurchase change this condition? Three conditions must simultaneously hold to ensure that bonlders are willing to sell debt at some particular price, shareholders are willing to buy it at this price, and they are willing to invest after reducing the leverage.

The first condition is the participation constraint for the shareholders; it ensures that bondholders are at least as good as they were before debt repurchase and investment:

$$\int_{\underline{X}+\underline{I}}^{D-\Delta D} (x+i) f(x,i) dx di + \int_{D-\Delta D}^{\overline{X}+\overline{I}} (D-\Delta D) f(x+i) d(x+i) \geq \tag{23}$$

$$\int_{\underline{X}}^{D-C} (x+C) f_X(x) dx + \int_{D-C}^{\overline{X}} D f_X(x) dx. \tag{24}$$

This condition defines the lower boundary for the price at which bondholders agree to sell their debt. Two additional conditions are for equity. One requires that shareholders are willing to repurchase:

$$\int_{D-\Delta D}^{\overline{X}} [x - (D - \Delta D)] f_X(x) dx \geq \int_{D-C}^{\overline{X}} (x+C-D) f_X(x) dx, \tag{25}$$

The final condition ensures that after reducing debt shareholders commit to investment. Shareholders can renege on the promise to invest after negotiating the low repurchase price. Therefore, we also require that, conditional on debt reduction, the shareholders are better off if they invest:

$$\int_{D-\Delta D}^{\overline{X}+\overline{I}} [x+i-(D-\Delta D)]f(x+i)d(x+i) - I \geq \int_{D-\Delta D}^{\overline{X}} [x-(D-\Delta D)]f_X(x)dx, \quad (26)$$

where the investment cost on the left-hand side is I, because shareholders spend cash C on reducing debt and must raise the full amount I in order to invest.

Two conditions for equity (25) and (26) effectively define the maximum price at which shareholders agree to buy back debt.

With those conditions in mind, we first consider debt reductions at or above the face value of debt, $C \geq \Delta D$. As we have argued, keeping investment fixed and reducing debt at exactly face value leaves shareholders indifferent. Therefore, condition (26) is satisfied at prices of debt below the face value.

We therefore need to check that shareholders retain the incentive to invest after debt reduction (condition (26)), subject to the assumption that shareholders refuse the investment without debt reduction (condition (22)). Note that the right-hand sides of (26) and (22) are the same.

But comparing the left-hand sides, we find that the debt overhang assumption (22) implies that (26) does not hold and shareholders will not invest after debt reduction. Specifically,[14]

$$\int_{D-\Delta D}^{\overline{X}+\overline{I}} [x+i-(D-\Delta D)]f(x+i)d(x+i) - I > \int_{D}^{\overline{X}+\overline{I}} [x+i-D]f(x+i)d(x+i) - (I-C), \quad (27)$$

Debt reduction does not create additional incentives to repurchase in the presence of debt overhang. We further show that repurchasing at lower prices in the presence of friction can help to mitigate the problem.

5.2. Investment and hedging incentives of a financially constrained firm

[14] It follows from $\int_{D-\Delta D}^{D} [x+i-(D-\Delta D)]f(x+i)d(x+i) > \int_{\underline{X}+\underline{I}}^{D} C$.

The appendix considers the case when the investment function is concave, $0 < B < 1$ and the firm is financially constrained. The firm is able to invest more when receiving higher cash flows or having to repay less debt, although the marginal benefit of the investment becomes progressively small. In this case, as the Appendix demonstrates, the firm saves even in the absence of transaction costs. The intuition relies on the observation that the firm has an incentive to hedge when the payoff function is concave. Savings at date $t = 0$ allow the firm to smooth the investment across the states of the world, while repurchasing debt effectively shifts the contingent cash flows to the states with the highest realization of $x \cdot y$, where the marginal investment is less profitable.

6. Empirical Tests

The framework developed in this paper produces several empirical hypotheses concerning the optimal amount of savings net of debt. We first briefly discuss the data and the methodology.

6.1. Description of the data

The data for our empirical analysis comes from the *Compustat* tapes for 1970-2006. As is standard in the literature (e.g., Erickson and Whited (2000), Hennessy (2004), Acharya, Almeida, and Campello (2008), and Dasgupta, Noe, and Wang (2008)), we exclude all non-manufacturing firms by confining our sample to SIC codes 2000-3999.

We construct the market value of assets as book assets (item 6) plus the market capitalization (item 24 multiplied by item 25) minus the book value of common equity (item 60), minus the deferred tax (item 74). Cash holdings equals cash plus marketable securities (item 1). Following Faulkender and Wang (2006), we define total debt as the sum of short-term debt (item 34) and long-term debt (item 9). We define cash flows CF as the income before extraordinary items (item 19) plus depreciation (item 14).[15] All

[15] As a robustness check, we followed literature in subtracting the working capital accruals from cash flows. Working capital accrual is calculated as the change in current assets (item 4) minus the change in cash and equivalents (item 1) minus the change in current liabilities (item 5) plus the change in

firm-specific variables are winsorized at the 1% and 99% tails to reduce the influence of outliers.

In addition, to investigate the repurchase prices we also obtain the sample of firms that participated in bond issuance, redemption or tender offers. These data come from the 2008 version of the *Fixed Income Securities Database (FISD)*, which provides information for issues maturing after 1989 but before 2008. The information includes a detailed history of changes in the amount of debt. The information for new issues includes issuer identifier, offering date, maturity date, coupon type, offering yield to maturity, seniority level, and any applicable coupon covenants, such as call, put and convertibility provisions. The information on redemptions and tender offers includes the effective date, the amount of the transaction, the outstanding debt after the transaction is complete, and the price.

6.2. Evidence on cash policy

To motivate the analysis, we first provide evidence on cash and debt policy. Extending the findings of Bates, Kahle, and Stulz (2008), we document that firms not only tend to carry significant cash savings, but that their savings are, on average, large enough to entirely repurchase or significantly reduce debt. In Table I we sort the firms according to their debt holdings and evaluate their cash holdings and leverage. We find that even the firms in the top decile by cash holdings (cash for such firms is, on average, approximately 69% of total assets) maintain 10.6% in short-term debt, 6.2% of which is short term debt. That is, in a large fraction of firms, cash holdings are sufficiently large to reduce or even completely eliminate debt.

These new finds are puzzling for two reasons.[16] First, assuming that the marginal corporate tax rate is not lower than the individual tax rate, the tax advantage of debt in

short-term debt (item 34), and plus the change in tax payable (item 71). This definition follows the one by Bushman, Smith, and Zhang (2007). We found that results remain qualitatively and quantitatively similar.

[16] It is difficult to argue that the presence of cash reserves in levered firms may be simply explained by the fact that proceeds from issuing debt are saved. In most cases debt is issued by the financially constrained firms that allocate the proceeds to investment.

this case is more than offset by taxes paid on corporate savings—the absence of *net* tax advantage eliminates the most compelling argument for keeping debt in capital structure. Second, the accumulation of cash inside the company is likely to exacerbate the agency problems (Jensen (1986)). Although the requirement to service debt at the due date will reduce the incentives to steal cash from the company, these incentives nevertheless are going to remain much stronger than in the case when the company does not have cash in the first place. However the most important reason is that shareholders could achieve much better control over the firm if they could eliminate the claims held by the lenders.

Once we establish that companies have both high cash and leverage, the question arises about the motivation of these policies. We conjecture that there are two reasons for this. First, for some firms that cannot negotiate a low price on debt repurchases, the incentives to repurchase debt are small. Second, for other firms that would benefit from the debt reduction, there is a value in delaying the repurchase until the price is the lowest. Our empirical analysis attempts to identify the types of firms and to investigate their propensity to reduce leverage as well as the incentives to delay the reduction.

6.3. Propensity to reduce debt or increase savings

Since model predicts that firms benefit from debt reduction in the presence of bankruptcy costs, we examine this question using several common proxies of high bankruptcy or distress costs: small size and large growth options.

Table II reports the results. Consistent with our prediction, the coefficient on cash flows is negative and significant for firms with high bankruptcy costs. In contrast, in firms with low expected bankruptcy costs, the coefficient is insignificantly different from zero. Results in Table II also show that risk increases the incentives to reduce debt. The risk of bankruptcy is proxied by Z-score. This result is consistent with the idea that firms delay repurchase until the risk is significant. Finally, Table II reports that incentives to decrease debt increase in maturity.

Two additional panels (Panel B and Panel C in Table II) show the results separately for short-term and long-term debt. The results are much more pronounced for long-term debt. The explanation lies in the fact that Compustat data may contain debt reductions that are simply a result of the firm's policies not to extend their existing debt contracts. The short-term debt is more subject to this concern.

6.3. Debt Reduction Prices

In addition, we investigate the prices of the transaction using a small sample of tender offers in the *Mergent* database. We select only debt reduction transaction with code "T" (tender offer) or code "IRP" (open market repurchase). We remove all observations with missing prices, convertible bonds transaction and debt issues that were in bankruptcy since our predictions do not apply to those cases. Most of the bond issues in the sample tend to be senior.

Table III reports summary statistics. We attribute junior, subordinate, junior subordinate, and no security to unsecured bonds, and senior, senior secured, senior subordinate qualify as senior bonds. Since some bonds are of discount type, we also calculate the premium relative to issuance price, defined as "Action price" divided by the "Offering Price." The bonds are repurchased at the average 7.6% premium, and large part of repurchases are carried at prices close to face value. The average maturity the issues in this sample is 7.87 years, suggesting that long-maturity bonds are more likely to be bought back. Most transactions in this sample (82%) are of the tender offer type.

Table IV offers the results of the regression of price and premium on the issue's characteristics. Notably, the market repurchase transactions are done at smaller prices than the tender offer transactions. Price used in transaction decreases with maturity. However, we have to be careful about the interpretation of the last result. It may be driven by the change in the interest rates relative to the fixed coupon in the contract. In addition, the result may be explained by firms trying to get rid of the restrictive covenants. In both cases, the incentive to repurchase increase in the remaining maturity. Finally, as Table IV shows, the firms in distress negotiate lower prices.

7. Conclusions

This paper contributes to the literature on debt reduction and debt renegotiations and develops the new intuition and empirical predictions. First, we showed that, in the absence of distress/bankruptcy costs, dispersion of debt ownership and other frictions, debt reductions are undertaken at the face value and they do not reduce the probability of bankruptcy. Additionally, we showed that both bankruptcy costs and dispersion of bondholders create incentives to repurchase debt by handing the larger bargaining power to shareholders in the negotiations. Finally, we showed that firms, even when the option to reduce debt is highly valuable, have incentives to delay the exercise to get the best price in the repurchase.

Empirically, we document the following regularities. The propensity to repurchase debt out of cash flows is partially responsible for the large cash holdings of U.S. firms. Firms with larger expected distress costs show higher propensities to reduce leverage. The longer time to maturity increases the price of debt repurchase.

Appendix A. Calculations for Debt Repurchase

The basic mechanism of buying back debt:

We can show that

$$\int_{D-C}^{\overline{X}} (x + C - D) f(x) dx \; < \int_{D-\alpha D}^{\overline{X}} [x + \alpha D - D] f(x) dx \tag{28}$$

by defining the function of $G(y)$ as

$$G(y) = \int_{D-y}^{\overline{X}} [x + y - D] f(x) dx, \; y \in [0, D]. \tag{29}$$

and showing that it increases in argument. To see this, note that

$$\begin{aligned} G'(y) &= (D-y) f(D-y) + \int_{D-y}^{\overline{X}} f(x) dx - (D-y) f(D-y) \\ &= \int_{D-y}^{\overline{X}} f(x) dx > 0 \end{aligned} \tag{30}$$

So if, $C < \alpha D$, then

$$G(C) < G(\alpha D), \tag{31}$$

which directly implies the inequality above.

The case with bankruptcy costs

We rely on the following condition that requires that bondholders must be at least as well off as after debt reduction:

$$d_0 \leq d_0^R + C. \tag{32}$$

When shareholders fully exploit their threat point, they can drive the lenders to the reservation value and therefore this condition becomes equality. Substituting the expressions for the market value of debt, we can write it as

$$\int_{D-\alpha D}^{D-C}(x-D+\alpha D)f(x)dx+\int_{D-C}^{\overline{X}}(\alpha D-C)f(x)dx=\beta\left[\int_{D-\alpha D}^{D-C}xf(x)dx+\int_{\underline{X}}^{D-C}Cf(x)dx\right]. \tag{33}$$

Given that the support of x, $\left[\underline{X},\overline{X}\right]$, is bounded and $f(x)$ is continuous, the left-hand side of the inequality is a continuous increasing function of αD and has a minimum of zero when $\alpha D=C$. Therefore, there is some $\alpha^*\in(0,1)$ such that there is a unique solution and

$$C<\alpha^* D, \tag{34}$$

which gives the upper bound for α^* (lower bound for the repurchase price).

Appendix B. Existence of the interior solution for the firm with hedging incentives

The expression (18) can be written explicitly as

$$\begin{aligned} C_2 + \int_{\underline{X}}^{\overline{X}} F(x) f(x) dx &= C_2 + \int_{\underline{X}}^{\overline{X}} \left(\int_{\frac{D_2}{x}}^{\overline{Y}} (xy - D_2)^B f_{y|x}(y) f(x) dy \right) dx, \quad (35) \\ \text{where } F(D_2, x) &= \int_{\frac{D_2}{x}}^{\overline{Y}} (xy - D_2)^B f_{y|x}(y) dy, \end{aligned}$$

where the maximization is with respect to C_1, C_2, D_2, and D_0^1, the amount of debt outstanding after the first repurchase. Note that $F(D_2, x)$ does not explicitly depend on C_1.

In (20), d_0 denotes the market value of debt conditional on no repurchase

$$d_0 = \int_{\underline{X}}^{\overline{X}} \left[(1-\beta) \int_{\underline{Y}}^{\frac{D-C}{x}} (xy + C) f_{y|x}(y) dy + D \int_{\frac{D-C}{x}}^{\overline{Y}} f_{y|x}(y) dy \right] f(x) dx, \qquad (36)$$

and d_0^R is the value of debt conditional on repurchase using $C - C_1$ in cash

$$d_0^R = \int_{\underline{X}}^{\overline{X}} \left[(1-\beta) \int_{\underline{Y}}^{\frac{D_0^R - C_1}{x}} (xy + C_1) f_{y|x}(y) dy + D_0^R \int_{\frac{D_0^R - C_1}{x}}^{\overline{Y}} f_{y|x}(y) dy \right] f(x) dx. \qquad (37)$$

One point requires justification. Note that the valuation expressions for d_0 and d_0^R use the fact that the expected debt value at $t = 0$ is not changed if the firm plans additional repurchase at $t = 1$.

Using normalization for the probability distribution function,

$$\int_x \int_y f(x, y) dy dx = 1,$$

we can rewrite the first constraint (20) by substituting the value of debt (36) and (37) as

$$\int_{\underline{X}}^{\overline{X}} v(C_1, D_0^R, x) f(x) dx = 0, \tag{38}$$

where we have introduced a function $v(C_1, D_0^R, x)$,

$$v(C_1, D_0^R, \overline{X}) = \frac{\partial \left(d_0^R + C - C_1 - d_0\right)}{\partial \overline{X}}. \tag{39}$$

Now, discuss the second constraint. Analogously to above, the market value of debt before and after repurchase is respectively given by

$$d_1 = (1-\beta) \int_{\underline{Y}}^{\frac{D_0^R - C_1}{x}} (xy + C_1) f_{y|x}(y) dy + D_0^R \int_{\frac{D_0^R - C_1}{x}}^{\overline{Y}} f_{y|x}(y) dy, \tag{40}$$

$$d_1^R = (1-\beta) \int_{\underline{Y}}^{\frac{D_2}{x}} (xy) f_{y|x}(y) dy + D_2 \int_{\frac{D_2}{x}}^{\overline{Y}} f_{y|x}(y) dy. \tag{41}$$

Note that d_0^R, the market value of debt after the repurchase at $t = 0$, does not have to be equal to the value of debt at $t = 1$, which we denote as d_1. This is because there is additional news released between $t = 0$ and $t = 1$ via the realization of variable y.

Again, we can rewrite constraint (21) after substituting (40) and (41) as

$$w(C_1, D_0^R, D_2(x), x) = d_1^R + C_1 - d_1 = 0. \tag{42}$$

In addition to these two constraints, we require that $C - C_1 \geq 0$, which is the amount repurchased at $t = 0$ must be nonnegative and that the amount repurchased at $t = 0$, C_1, cannot exceed the total available cash reserves.

To summarize, we maximize

$$\max_{C_1, D_0^R, D_2} \int_{\underline{X}}^{\overline{X}} F(C_1, D_0^R, D_2(x), x) f(x) dx \tag{43}$$

$$= \max_{C_1, D_0^R, D_2} \int_{\underline{X}}^{\overline{X}} \int_{\frac{D_2}{x}}^{\overline{Y}} (xy - D_2)^B f_{y|x}(y) f(x) dy dx,$$

subject to

$$
\begin{aligned}
\int_{\underline{X}}^{\overline{X}} v(C_1, D_0^R, x) f(x) dx &= 0, && (44)\\
\int_{\underline{X}}^{\overline{X}} w(C_1, D_0^R, D_2(x), x) dx &= 0, && (45)\\
C - C_1 &\geq 0, && (46)\\
C_1 &\geq 0. && (47)
\end{aligned}
$$

Since $0 < B < 1$, the existence of a solution to this problem is guaranteed by the concavity of the maximized expression. Thus, we can apply the standard maximization method as follows. From the Lagrangian, dropping the integrals over x yields

$$
\begin{aligned}
\mathcal{L} = \; & F(C_1, D_0^R, D_2(x), x) f(x) + \pi(x) v(C_1, D_0^R, x) f(x) && (48)\\
& + \lambda(x) w(C_1, D_0^R, D_2(x), x) + \delta_1 (C - C_1) + \delta_2 C_1,
\end{aligned}
$$

where $\pi(x)$, $\lambda(x)$, δ_1, δ_2 are nonnegative multipliers on the constraints[17]

$$
\begin{aligned}
\pi(x) &= \pi > 0 && (49)\\
\lambda(x) &> 0 && (50)\\
\delta_1 &\geq 0 && (51)\\
\delta_2 &\geq 0. && (52)
\end{aligned}
$$

The proof that there exists an internal solution to the optimal savings problem uses the following argument. If we could show that $\delta_1 - \delta_2 = 0$, it would imply that $\delta_1 = 0$ and $\delta_2 = 0$. This is because conditions $C_1 \geq 0$ and $C - C_1 \geq 0$ cannot bind simultaneously, they only define upper and lower bounds on savings C_1. By this fact we know that δ_1 and δ_2 cannot both be positive. Therefore, $\delta_1 = 0$ and $\delta_2 = 0$. Finally, since both constraints do not bind, $0 < C_1 < C$.

[17] Why did we need to show that $\dot{\pi}(x) = -\frac{\partial \mathcal{L}}{\partial k} = 0$?

Now we demonstrate that $\delta_1 - \delta_2 = 0$. The first order conditions (FOC) are:

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial C_1} &= 0 && (53)\\ \frac{\partial \mathcal{L}}{\partial D_0^R} &= 0 && (54)\\ \frac{\partial \mathcal{L}}{\partial D_2(x)} &= 0. && (55) \end{aligned}$$

The first condition yields:

$$\begin{aligned} 0 &= \pi \frac{\partial v}{\partial C_1} + \lambda(x) \frac{\partial w}{\partial C_1} - \delta_1 + \delta_2 && (56)\\ &= \pi \left[\frac{\partial d_1}{\partial C_1} - 1 \right] f(x) + \lambda(x) \left[1 - \frac{\partial d_1}{\partial C_1} \right] - \delta_1 + \delta_2. \end{aligned}$$

This implies that if

$$\frac{\partial d_1}{\partial C_1} - 1 \neq 0, \quad (57)$$

then

$$-\delta_1 + \delta_2 = 0 \Leftrightarrow \lambda(x) = \pi f(x). \quad (58)$$

Now, the following holds:

$$\begin{aligned} \text{if } \frac{\partial d_1}{\partial C_1} - 1 &\neq 0, \text{then} && (59)\\ 0 &< C_1 < C \Leftrightarrow -\delta_1 + \delta_2 = 0 \Leftrightarrow \lambda(x) = \pi f(x). \end{aligned}$$

The second condition ensures that if $\frac{\partial d_1}{\partial D_0^R} \neq 0$, $\lambda(x) = \pi f(x)$:

$$\begin{aligned} 0 &= \pi \frac{\partial v}{\partial D_0^R} f(x) + \lambda(x) \frac{\partial w}{\partial D_0^R} && (60)\\ &= \pi \frac{\partial d_1}{\partial D_0^R} f(x) - \lambda(x) \frac{\partial d_1}{\partial D_0^R}. \end{aligned}$$

This directly implies that, if

$$\frac{\partial d_1}{\partial D_0^R} \neq 0, \quad (61)$$

then

$$\lambda(x) = \pi f(x). \tag{62}$$

Now, the conclusion is:

$$\begin{aligned} \text{if } \frac{\partial d_1}{\partial C_1} - 1 &\neq 0 \text{ and } \frac{\partial d_1}{\partial D_0^R} \neq 0, \text{ then} \\ \lambda(x) &= \pi f(x)(\text{FOC2}) \Rightarrow -\delta_1 + \delta_2 = 0(\text{FOC1}) \Rightarrow 0 < C_1 < C. \end{aligned} \tag{63}$$

The firm thus saves some cash at time $t = 0$.

The following regularity conditions are required:

1) The solution exists (this is ensured by some B such that $0 < B < 1$);

2) $\frac{\partial d_1}{\partial D_0^R} \neq 0$ for every x;

3) $\frac{\partial d_1}{\partial C_1} - 1 \neq 0$ for every x;

The latter two conditions mean that the debt should not be either defaulted with probability 1 or totally risk free. If X and Y are log-normally distributed, then these conditions are satisfied.

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Table I. Cash Holdings and Debt.

The table presents the full sample of Compustat firms sorted by their cash holdings. Cash holdings equals cash plus marketable securities. We define the market value of assets as book assets plus market capitalization minus common equity minus the deferred tax. We define total debt as the sum of short-term debt and long-term debt. Book leverage is the ratio of total debt to book assets. Market Leverage is the ratio of total debt to market value of assets. Q is the ratio of book assets to market value of assets. We only include in the sample the firm-years with non-missing book assets and non-missing market values of assets, and data are winsorized as 1 and 99 percentage level.

Cash Deciles	Cash	LT Debt	ST Debt	Book Lev	Market Lev
0	0.0043	0.2412	0.1466	0.3990	0.2985
1	0.0142	0.2297	0.1120	0.3458	0.2882
2	0.0400	0.2026	0.0953	0.3016	0.2599
3	0.0612	0.1866	0.0852	0.2763	0.2283
4	0.0677	0.1769	0.0701	0.2385	0.1854
5	0.0932	0.1644	0.0617	0.2155	0.1828
6	0.1423	0.1364	0.0628	0.2043	0.1447
7	0.2215	0.1044	0.0492	0.1569	0.0994
8	0.3645	0.0791	0.0389	0.1246	0.0582
9	0.6972	0.0615	0.0358	0.1066	0.0312
Total	0.1664	0.1618	0.0801	0.2475	0.1872

Table II. Propensity to Reduce Debt or Increase Cash Holdings.

Dependent variables are Δ *Total (Long Term, Short Term) Debt* defined as the annual change in total (short-term, long-term) debt, and Δ *cash holdings*, defined as the annual change in cash holdings. *Cash Flow* is the free cash flow net of the change in working capital, normalized by the book value of assets. *Firm Size* is natural logarithm of book value of assets. *Q* is measured as book value of assets plus market capitalization minus deferred tax, all divided by book value of assets. Standard errors of estimates are listed in parentheses. Significance at the 1%, 5%, and 10% levels is denoted by ***, **, and *, respectively. *Small (Large) firms* have lagged book value below (above) median size. *Low (High) Z-score* firms have Z-score below (above) median score. *High (Low) LTD proportion* firms have proportion of long term to short term debt in Compustat above (below) median proportion. *Low (High) Z-score* firms have Z-score below (above) median score. Other definitions are given below.
Average_Maturity=1·DebtDueInOneYear+2·DebtDueInTwoYears+...
+5·DebtDueInFiveYears+10·RemainingDebt.
Z_Score=1.2·WorkingCapital/Assets+1.4·RetainedEearnings/Assets+
3.3·EBIT/Assets+0.6·MarketValue/TotalLiabilities+.999·NetSales/Assets.

Panel A. *Dependent Variable is* Δ*Total Debt*					
Financing Constraints	Independent variables				
	Cash Flow	Q	Firm Size	Obs.	Adj.-R^2
1. Firm Size					
Small Firms	-0.074***	0.001**	0.021***	43,402	0.039
	(-13.62)	(2.20)	(15.52)		
Large Firms	-0.003	-0.001	0.009***	45,038	0.023
	(-0.34)	(-0.72)	(16.68)		
2. Growth Options					
Q Above Median	-0.110***	-0.004***	0.008***	41,237	0.049
	(-17.81)	(-5.58)	(12.45)		
Q Below Median	-0.004	0.003	0.004***	41,430	0.019
	(-0.30)	(1.32)	(10.87)		
3. Probability of Default					
Low Z-score	-0.111***	-0.001	0.006***	14,280	0.059
	(-9.11)	(-0.87)	(5.98)		
High Z-score	-0.075***	-0.002***	0.011***	14,489	0.029
	(-7.69)	(-2.95)	(10.35)		
4. Proportion of LTD					
High LTD Proportion	-0.109***	0.006***	0.004***	40,271	0.050
	(-10.41)	(4.18)	(6.92)		
Low LTD Proportion	-0.089***	0.001	0.004***	38,381	0.053
	(-12.74)	(0.68)	(7.24)		
5. Debt Maturity					
Average Maturity Above Median	-0.103***	0.002*	0.003***	29,297	0.056
	(-9.52)	(1.64)	(4.11)		
Average Maturity Below Median	-0.065***	0.000	0.006***	30,001	0.025
	(-7.88)	(0.11)	(9.15)		

Panel B. *Dependent Variable is* Δ*Short Term Debt*					
Financing Constraints	Independent variables				
	Cash Flow	Q	Firm Size	Obs.	Adj.-R^2
1. Firm Size					
Small Firms	-0.025***	0.000	0.005***	43,441	0.015
	(-8.51)	(0.90)	(6.78)		
Large Firms	-0.025***	-0.000	0.002***	45,149	0.013
	(-6.68)	(-0.02)	(13.98)		
2. Growth Options					
Q Above Median	-0.043***	-0.001***	0.001***	41,346	0.030
	(-13.03)	(-3.93)	(3.74)		
Q Below Median	-0.044***	-0.002	0.001***	41,465	0.014
	(-6.08)	(1.58)	(6.01)		
3. Probability of Default					
Low Z-score	-0.046***	-0.001	0.001**	14,284	0.024
	(-6.49)	(-1.05)	(2.32)		
High Z-score	-0.032***	-0.001***	0.001	14,504	0.031
	(-6.97)	(-2.84)	(1.18)		

Panel C. *Dependent Variable is* Δ*Long Term Debt*					
Financing Constraints	Independent variables				
	Cash Flow	Q	Firm Size	Obs.	Adj.-R^2
1. Firm Size					
Small Firms	-0.035***	-0.000	0.017***	43,413	0.019
	(-11.36)	(-0.20)	(17.93)		
Large Firms	0.024***	-0.001*	0.009***	45,070	0.017
	(3.08)	(-1.77)	(13.95)		
2. Growth Options					
Q Above Median	-0.046***	-0.003***	0.005***	41,245	0.018
	(-13.17)	(-7.45)	(12.09)		
Q Below Median	0.036***	0.005***	0.003***	41,457	0.014
	(3.23)	(2.73)	(10.36)		
3. Probability of Default					
Low Z-score	-0.038***	-0.002	0.004**	14,280	0.013
	(-5.62)	(-1.53)	(5.39)		
High Z-score	-0.037***	-0.002***	0.010***	14,489	0.021
	(-5.46)	(-2.80)	(11.69)		

Panel D: *Dependent Variable is* Δ*Cash Holdings*					
Financing Constraints	Independent variables				
	Cash Flow	Q	Firm Size	Obs.	Adj.-R^2
1. Firm Size					
Small Firms	-0.370***	0.001**	0.136***	43,445	0.203
	(-31.50)	(0.30)	(37.50)		
Large Firms	0.085***	0.026***	0.002***	45,175	0.063
	(4.29)	(12.54)	(3.67)		
2. Growth Options					
Q Above Median	-0.187***	-0.001	0.019***	41,348	0.087
	(-15.06)	(-0.80)	(16.32)		
Q Below Median	0.143***	0.020***	0.001*	41,487	0.044
	(5.96)	(5.08)	(1.90)		
3. Probability of Default					
Low Z-score	-0.249***	-0.005*	0.016***	14,281	0.143
	(-10.68)	(-1.74)	(10.20)		
High Z-score	-0.121***	0.010***	0.024***	14,508	0.061
	(-3.97)	(4.25)	(11.00)		
4. Proportion of LTD					
High LTD Proportion	-0.251***	0.005***	0.004***	40,286	0.107
	(-11.24)	(1.89)	(5.09)		
Low LTD Proportion	-0.268***	-0.005***	0.017***	38,399	0.116
	(-17.97)	(-2.76)	(14.28)		
5. Debt Maturity					
Average Maturity Above Median	-0.191***	-0.005**	0.006***	29,300	0.077
	(-9.40)	(-2.10)	(6.29)		
Average Maturity Below Median	-0.336***	0.003	0.021***	30,010	0.146
	(-17.62)	(0.98)	(14.19)		

Table III. Summary Statistics for Firms with Debt Tender Offers or Issues Repurchase.

Data is from the 2008 version of the Fixed Income Securities Database (FISD) available through the Mergent Database; we include only "Tender Offer" or "Issues Repurchase" transactions. *Repurchase Price* is defined as the price of tender or repurchase as the percentage of the face value ($100). *Premium* is defined as the ratio of repurchase price to the offering price. *Time to maturity* is the number of years between the transaction to the maturity date of the issue. *Tender Offer Dummy* equals to one if the transaction is of the tender offer type and equals to zero is it the transaction is an issues repurchase. *Bond Seniority* measures debt seniority; it is equals to 6 if the issue is senior secured, 5 if the issue is senior, 4 if the issue is senior subordinate, 3 if the issue is subordinate, 2 if the issue is junior, 1 if the issue is junior subordinate, 0 if the issue is not secured. *Amount Repurchased* and *Amount Outstanding* are transaction amount and the amount of debt outstanding, respectively, in $ millions. *Distress dummy* equals to one if the issue has already defaulted and equals to zero if it has not defaulted. *Coupon Rate* is annualized coupon rate (most bonds pay interest semi-annually or quarterly). All data except dummies are winsorized as 1 and 99 percent.

Tender Offers or Issue Repurchase (Obs. 1,254)					
Variable	Mean	Std. Dev.	25%	50%	75%
Repurchase Price	104.99	9.36	100.00	102.16	109.42
Premium	107.60	16.03	100.00	102.80	110.85
Time to Maturity	7.87	6.69	3.59	5.79	9.00
Tender Offer Dummy	0.823	0.381	1.00	1.00	1.00
Bond Seniority (0 to 6)	4.20	1.76	4.00	5.00	5.00
Amount Repurchased ($M)	0.302	5.032	0.015	0.100	0.203
Amount Outstanding ($M)	1.337	30.231	0	0.002	0.065
Distress Dummy	0.006	0.074	0	0	0
Coupon Rate	7.44	2.83	6.00	7.50	9.25

Table IV. Maturity and the Tender Offer Price.

Data is from the 2008 version of the Fixed Income Securities Database (FISD) available through the Mergent Database; we include only “Tender Offer” or “Issues Repurchase” transactions. *Repurchase Price* is defined as the price of tender or repurchase as the percentage of the face value ($100). *Premium* is defined as the ratio of repurchase price to the offering price. *Time to maturity* is the number of years between the transaction to the maturity date of the issue. *Tender Offer Dummy* equals to one if the transaction is of the tender offer type and equals to zero is it the transaction is an issues repurchase. *Bond Seniority* measures debt seniority; it is equals to 6 if the issue is senior secured, 5 if the issue is senior, 4 if the issue is senior subordinate, 3 if the issue is subordinate, 2 if the issue is junior, 1 if the issue is junior subordinate, 0 if the issue is not secured. *Amount Repurchased* and *Amount Outstanding* are transaction amount and the amount of debt outstanding, respectively, in $ millions. *Distress dummy* equals to one if the issue has already defaulted and equals to zero if it has not defaulted. *Coupon Rate* is annualized coupon rate (most bonds pay interest semi-annually or quarterly). *Covenants Dummy* equals to one if the database contains a record of covenants for this issue, and zero otherwise. All data except dummies are winsorized as 1 and 99 percent. Standard errors are reported in the parentheses. *** indicates significance level of 1%, ** indicates significance level of 5%, * indicates significance level of 10%.

	Dependent Variables				
	Repurchase Price	Premium	Premium	Premium	Premium
Constant	96.333*** (33.05)	102.001*** (19.81)	98.088*** (43.08)	90.661*** (31.46)	97.931*** (43.04)
Time to Maturity	0.244*** (2.74)	0.312*** (2.70)	0.284*** (3.00)	0.281*** (3.11)	0.275*** (3.02)
Tender Offer	4.512*** (5.31)	5.347*** (4.82)	4.632*** (5.43)	3.361*** (3.77)	4.432*** (4.69)
Bond Seniority	0.397* (1.74)	0.506** (2.15)	0.472** (2.16)	0.358* (1.87)	0.404* (1.71)
Amount Rep. ($M)	0.023*** (3.27)	0.016 (1.10)	0.020*** (2.75)	0.000 (0.06)	0.017*** (3.19)
Distress Dummy	-3.616*** (-3.52)	-6.443*** (-4.15)	-3.476*** (-3.62)	-5.918*** (-4.63)	-3.097** (-2.46)
Coupon Rate				0.800*** (4.13)	
Covenants Dummy					0.826 (0.70)
Adjusted-R^2	0.146	0.071	0.141	0.170	0.141
Observations	1,254	1,254	1,190	1,189	1,190