FIRM SPAWNING DYNAMICS*

Michel A. Habib† Ulrich Hege‡ Pierre Mella-Barral§

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Abstract

We analyze the consequences of organizational fit for firm size, focus, and profitability and for the extent of spawning of new firms by a parent firm. Organizational fit refers to the congruence between a firm's organizational form and the requirements for the successful management of the firm's product portfolio. We are able to replicate a number of important empirical regularities. These are the growth in firm size over time, the decline in spawning and in focus, the positive relation between focus and performance, and the decline in performance, as measured by ROA, over time. This last result is particularly noteworthy, for it shows that a decline in firm performance need not imply a decline in firm capabilities: performance declines not because existing products become less profitable, but because the firm introduces new products that are less fitting the firm's organizational form.

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†Swiss Banking Institute, University of Zurich, Plattenstrasse 14, 8032 Zurich, Switzerland; tel.: +41-(0)44-634-2507; e-mail: habib@isb.unizh.ch.

‡HEC School of Management Paris, Department of Finance and Economics, F-78351 Jouy-en-Josas Cedex, France; Tel. +33 (0)1-3967-7299; e-mail: hege@hec.fr.

§EDHEC Business School, 393 promenade des Anglais, F-06202 Nice Cedex 3, France; tel.: +33 (0)4-9318-7841; e-mail: mella@edhec.edu.

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1 Introduction

In a recent book, Roberts (2004, p.11) writes that "there needs to be a fit between strategy and organization and between these and the technological, legal, and competitive environment." We explore the implications of one such need—the fit between organization and environment—in the present paper. Specifically, we examine the implications of the fit between a firm's organization and the requirements for the successful management of the firm's product portfolio for a number of firm characteristics such as firm size, focus, and profitability and for the extent of spawning by the firm.

These characteristics are related to one another. The spawning of a new product by the firm that developed such product sees the management of the product delegated to a new firm spawned from the innovating firm. Spawning occurs where there is relatively little fit between the innovating firm's organization and the requirements for the successful management of the new product; the creation of a new firm to which the new product is spawned permits the adoption by the new firm of an organizational form more fitting the new product. The resulting benefit offsets the costs of setting up the new firm. Clearly, the lesser is the fit between the innovating firm's organization and the new product, the greater is the incentive to spawn the product to a new firm.

Spawning naturally constraints firm growth: products that are spawned do not contribute to the growth of the innovating firm. There is therefore a negative relation between the extent of spawning and firm size. Since it is those products that present the least fit with the innovating firm that are spawned, spawning contributes to the preservation of firm focus: the firm remains focused on those products that relatively closely fit the firm's organization. The preservation of firm focus in turn maintains firm performance, as measured by profitability for example: a more focused firm is one whose organizational form more closely fits the products in its portfolio; that closer fit makes it possible for the firm to operate more profitably.

We study the time dynamics of the aforementioned characteristics. We assume that the probability of successful introduction of a new product decreases over time: there is lesser need for new products in more crowded markets. This assumption applies to innovating and to spawned firms alike; yet, it decreases the attractiveness and thus the incidence of spawning over time. The reason is as follows. Start-up costs naturally are incurred only in case of spawning, for only in that case is there the creation of a new firm. As these costs must be incurred before success or failure can be ascertained, the probability that they will have been incurred in vain increases over time, in line with the decreased probability of successful new product introduction. This decreases the attractiveness of spawning over time.

The decrease in spawning over time has as direct counterpart the growth in firm size over time: fewer new products spawned means more products managed by the firm. More products—including some that present relatively little fit with firm organization—managed by the firm over time mean decreased firm focus over time. Finally, decreased firm focus over time implies decreased firm performance over time.

Our predictions appear to be consistent with the available empirical evidence. Gompers, Lerner, and Scharfstein (2005) find that older firms spawn less, Henderson (1999) that they are larger, Campa and Kedia (2002) that they are more diversified, Loderer and Waelchli (2009) that they are less profitable. This last result may be of particular interest, for it is difficult to reconcile with the view of firm growth as being driven by increased firm resources, skills, and/or knowhow (Penrose, 1959; Klette and Kortum, 2004). Loderer and Waelchli find a very strong negative relation between firm age and firm performance, which they attribute to what they refer to as 'corporate geriatrics.' What is striking about their result is that 'old age' follows 'birth' with barely an intervening period of 'youth:' their firms suffer not so much from corporate geriatrics as from 'corporate Progeria.'[1] Our analysis shows that the decline in firm performance over time need not be attributed to corporate geriatrics. Instead, it may be but the reflection of the decline in spawning, thereby focus, over time: as firms introduce progressively lesser fitting products, that is, products that present a lesser fit with firm organization, overall firm performance declines.

[1]Progeria is an illness that accelerates aging in humans by a factor of 6 to 8. Children born with Progeria die of heart failure around age 13, early in adolescence, before youth and adulthood.

Somewhat counter-intuitively perhaps, such decline is an indication of firm strength rather than weakness: overall performance declines over time because the range of products for which the innovating firm dominates spawned firms expands over time. Note that the decline is due entirely to the introduction of new products; there is no decline whatsoever in the profitability of each individual product over time.

Our results are predicated on the assumption that the probability of successfully introducing a new product decreases over time. This assumption can be justified by reference to the work of Klepper (1996) and Klepper and Simons (2000, 2005), who find that "the notion that the value of a unit cost reduction achieved through innovation is proportional to the level of output produced by the firm ... coupled with convex adjustment costs ... imparts an advantage to the earliest entrants which eventually causes a cessation in entry ..." (Klepper, p. 563). There is no cessation in entry—the introduction of new products—in our model (the probability of successfully introducing a new product is bounded away from zero), but the existence of start-up costs results in new products increasingly being introduced by existing firms, rather than new firms spawned specifically for the purpose of introducing these products.

Another important assumption, albeit one made primarily for modeling purposes, is that the ownership of a new product belongs an individual rather than a firm. This assumption is in line with much existing jurisprudence (Saxenian, 1996); it creates an externality to the spawning decision: the additional innovations made by the new firm in case there should be spawning are internalized only partially by the would-be spawning firm. We consider the implications of such externality and the partial remedy venture capital may provide.

Our work is related to various strands of the literature. It is related to the literature on firm size and firm growth, that part of the literature in particular that has attributed a firm's growth to the firm's entry into new submarkets (Klepper and Thompson, 2006; Klette and Kortum, 2004; Sutton, 1998): our products are these papers' submarkets.[2] It is also related to the literature on spawning, which has sought to explain when an innovation

[2] There is an extensive literature on firm size and growth; this literature is surveyed in Sutton (1997) and Coad (2007).

is implemented 'in-house' and when it is spawned to a new firm (Anton and Yao, 1995; Hellman, 2007; Klepper and Thompson, 2009). Finally, it is related to the literature on firm focus and diversification, in particular that part of the literature which has sought to explain why diversification might create value despite lowering average firm value and performance.[3]

Our work trades depth for breadth. Thus, we do not examine any of the issues mentioned above in as much detail as the work that has preceded ours; however, we do tie these issues together in one single model which, as noted above, accounts for firm size, focus, and performance and for the extent of spawning by the firm. What ties these issues together is organizational fit: fit i) determines the products the firm introduces itself and those the firm leaves to the firms it spawns, thereby determining size and ii) determines overall firm profitability through focus, that is, average organizational fit. Our focus on organizational fit i) accounts for the decision to spawn without appealing to additional considerations such as intellectual property rights, multi-task agency, or disagreements and ii) accounts for product diversification without the need for internal capital markets, winner-picking skills, decreasing returns to scale, or the distinction between general and specific firm skills.[4] We do not doubt the importance of these considerations, but believe there is a value to a common explanation with a single consideration.

Our work is perhaps most closely related to that of Lucas (1978), who relates the size distribution of firms to the distribution of talent across managers: organizational fit plays

[3]See for example Bernardo and Chowdhry (2002); Gomes and Livdan (2004); Maksimovic and Phillips (2002); Matsusaka (2001); and Matsusaka and Nanda (2002).

[4]Regarding i), Anton and Yao (1995) and Hellman (2007) make the decision to spawn revolve around the allocation of intellectual property rights and, in the case of Hellmann, the need to induce an employee to exert effort on a core task where the employee might otherwise be tempted to exert effort on another, innovative task; Klepper and Thompson (2009) attribute the decision to spawn to disagreements between firm and employee. Regarding ii), Fluck and Lynch (1999), Gertner, Scharfstein, and Stein (1994), Matsusaka and Nanda (2002), and Rajan, Servaes, and Zingales (2000) study internal capital markets in conglomerates, Stein (1997) studies winner-picking skills; Gomes and Livdan (2004)and Maksimovic and Phillips (2002) study decreasing returns to scale; and Bernardo and Chowdhry (2002) and Matsusaka (2001) study the implications of various forms of firm skills and capabilities.

in our model the role played by talent in Lucas's. The two models have very different focuses, however: Lucas's is the equilibrium size distribution of firms and the relation of firm size to employment, capital-labor ratio, and managerial rents; ours is time variation of firm size, and the relation of size to focus, profitability, and spawning.

The paper proceeds as follows. Section 2 presents the model. Section 3 solves the model for firm value and the extent of spawning. Section 4 examines further firm characteristics such as size, focus, and profitability and derives their comparative statics. Section 5 considers the positive externalities associated with the process of new firm creation and Section 6 the role of venture capital. Section 7 derives a number of testable implications and provides supporting empirical evidence. Finally, Section 8 concludes.

2 The Model

2.1 The Firm

Consider a firm f set up in a period t_f at a cost $\kappa \in \mathbb{R}^+$; firm f has three characteristics that are central to our analysis: its organizational form, its human capital, and its portfolio of products.

A firm's organizational form, m, is represented by a point on the real line, $\mathbb{R}$; the choice of organizational form the firm makes at birth is irreversible.[5]

Agents who devise innovations make up the firm's human capital, $\mathcal{A}$. We assume the number $\lambda \equiv |\mathcal{A}|$ of agents is constant over time. In every period, an agent $a \in \mathcal{A}$ devises an innovation with probability p; the firm therefore generates λp innovations per period. An innovation provides the firm with the opportunity to introduce a new product.

As is the firm's organizational form, the firm's products are represented by points on the real line. The firm is born with a single product, $\theta_f \in \mathbb{R}$ for firm f. We denote $\mathcal{I}_t$ the

[5] See Carroll and Hannan (2002) for evidence of organizational imprint.

firm's portfolio of products in period t. That set is the cumulative outcome of products successfully introduced by the firm. Success in introducing a new product is random; it occurs with probability q_t. Only in case of success is selling the new product profitable, in which case the new product becomes an element of $\mathcal{I}_t$; otherwise, the new product is jettisoned. We denote $I_t \equiv |\mathcal{I}_t|$ the number of products in $\mathcal{I}_t$.

The 'distance' $|\theta_i - m|$ between a firm's organizational form, m, and a product, θ_i, defines the organizational fit between the firm and the product, ω_i. We write

$$\omega_i \equiv e^{-|\theta_i - m|} \ . \tag{1}$$

We have $\omega_i \in (0, 1]$: the fit between the firm and the product can be perfect, $m = \theta_i$ and $\omega_i = 1$, or non-existent, $\theta_i \to \pm\infty$ and $\omega_i \to 0$. Organizational fit in turn determines profitability: the per-period profit the firm obtains from selling product θ_i with organizational fit ω_i is

$$\pi_i = \alpha + \beta\, \omega_i \ , \tag{2}$$

The parameter $\beta \in \mathbb{R}^+$ indexes the importance of organizational fit to profits; the parameter $\alpha \in \mathbb{R}^+$ completes the profit function: it indexes other relevant dimensions of profitability. Per-period profits are discounted at the constant unit-period rate of interest $\rho \in (0, 1)$.

We let $\phi^*(\theta_i)$ denote the probability that an innovation provides the firm with the opportunity to introduce a new product θ_i. We assume that the corresponding probability density function is centered at the firm's organizational form, m: the innovations the agents of a firm devise are to a large extent determined by the firm's organizational form. We write $\phi^*(\theta) = \frac{1}{2}\, e^{-|\theta - m|}$ for $\theta \in \mathbb{R}$. Expressed as a function of the fit between firm organization and new product, $\omega = |\theta - m|$, that probability density function becomes $\phi(\omega) = 1$ for $\omega \in (0, 1)$: organizational fit is uniformely distributed over the entire range of possible fits with organizational form m.

Note that firm f's optimal organizational form equals θ_f. This is immediate from the symmetry of both the profit function and the distribution of innovations: both are functions of $|\theta - m|$. The firm therefore sets $m = \theta_f$.

2.2 Spawning

Not all new products need be introduced by the innovating firm: some products are best left to be introduced by new firms spawned from the innovating firm. Denote one such firm f^+. The advantage of leaving the introduction of a new product, θ_i, to the new firm is that the firm will have organizational form, m^+, perfectly fitting the product: $m^+ = \theta_i$. This increases the profits from selling the new product, in case its introduction should be successful, from $\alpha + \beta\,\omega_i$, $\omega_i = e^{-|\theta_i - m|}$, to $\alpha + \beta$. It does, however, involve the cost, κ, of setting up the new firm. There is therefore a trade-off between organizational fit and set-up cost, the former favoring spawning, the latter not. Such trade-off leads to the existence of a threshold fit ω_t^* such that the introduction of the new product is left to the spawned firm where $\omega_i \in (0; \omega_t^*)$ and to the innovating firm where $\omega_i \in [\omega_t^*; 1]$: a firm spawns those products that present a lesser fit with the firm's organizational form.

We assume that success of failure in introducing a new product, θ_i, can be ascertained only by that firm which can most profitably sell the product, the innovating firm if $\omega_i \in [\omega_t^*; 1]$ and the spawned firm if $\omega_i \in (0; \omega_t^*)$. It is therefore not possible to have the innovating firm introduce a product with fit $\omega_i \in (0; \omega_t^*)$ for the purpose of ascertaining success before spawning, in an attempt to limit the bearing of the set-up cost, κ, to those products that have been successfully introduced.

Spawning naturally results in the creation of new firms. We denote $\mathcal{F}_t$ the set of firms in existence in period t and $J_t \equiv \sum_{f \in \mathcal{F}_t} I_t$ the number of products sold by these firms at that period. We assume that the probability of successful product introduction, q_t, is a decreasing and concave function of J_t: selling a new product is more difficult where many products already are sold. We have

$$q_t \;\equiv\; q(J_t)\;, \quad \frac{d\,q(J_t)}{d\,J_t} \;<\; 0\;. \tag{3}$$

We assume $q_\infty \equiv \lim_{J_t \to \infty} q(J_t) > \frac{\rho\,\kappa}{\beta}$: this ensures that the threshold fit, ω_t^*, lies in the range $(0, 1)$ for all $t \geq 0$.

We allocate all property rights to an innovation to the agent who devised such innovation. As noted in the Introduction, this assumption is in line with much existing jurisprudence (Saxenian, 1996). In case the innovation should be implemented by the innovating firm rather than spawned, the agent bargains with his fellow agents: decision power within the firm belongs to those who make up the firm's human capital; only those with proper expertise—the firm's agents—can evaluate the trade-offs involved in spawning. Our firm is in many ways akin to a partnership, in which the agents are the partners, that is, they are owners and decision-makers. We assume costless bargaining.

We assume that the spawning of a new firm, f^+, generates a new stream of innovations: as does the firm f from which it was spawned, firm f^+ has agents that make up its human capital, $\mathcal{A}^+$. These generate as rich a stream of innovations as do their counterparts at firm f, albeit one centered at firm f^+'s organizational form, m^+, rather than at m: firm f^+ generates $\lambda\, p$ innovations per period. These additional innovations constitute an externality for the innovating firm and its agents: they do not take the additional innovations into account when deciding whether to spawn, for the value of these innovations does not accrue to them. The positive externality feature of spawning will be shown to have an important bearing on the welfare analysis of Section 5.

That existing and new firms generate an equal number, $\lambda\, p$, of innovations per period is a strong assumption, as is the assumption that the number of innovations is constant over periods. These assumptions are made for two reasons: i) they make possible the derivation of tractable formulas for the threshold fit, ω_t^*, and the firm characteristics that depend upon that fit and ii) they avoid imparting a 'built-in' advantage to existing or new firms, or to young or old firms. The latter reason underlies the assumption that the probability of successful product introduction, q_t, is the same for all firms: it depends only on the total number of products sold in period t, J_t; it does not depend on the number of products sold by the individual firm introducing the product, nor on the age of that firm.

We represent the sequence of events on the time-line in Figure 1. We divide period t into four subperiods, starting at dates t, $t+\epsilon$, $t+2\epsilon$, and $t+3\epsilon$, respectively. At date t, each of the λ agents of firm f engages in innovation. At date $t+\epsilon$, innovation succeeds with probability p; there are $\lambda\, p$ innovations. An innovation provides the opportunity to introduce a new product, θ_i. The fit, ω_i, between the new product and firm f's organizational form is known by all. At date $t + 2\epsilon$, agents bargain over whether to have the product introduced by the firm or by a new firm spawned from the innovating firm. In the latter case, agent a who devised the innovation leaves firm f to set up a new firm f^+ at cost κ; another agent takes a's place in $\mathcal{A}$, thereby keeping the number of agents that make up firm f's human capital constant at λ. At date $t + 3\epsilon$, success or failure in introducing the new product can be ascertained; in the former case, which occurs with probability q_t, profits accrue to the firm that introduced the product.

3 Solving the Model

We denote $V_t^{\mathcal{A}}$ the value at date t, to the agents in $\mathcal{A}$, of the future innovations these agents are expected to devise. As all λ agents in $\mathcal{A}$ are identical, the value to a single agent $a \in \mathcal{A}$ equals $V_t^a = V_t^{\mathcal{A}}/\lambda$.

The values V_t^a and $V_t^{\mathcal{A}}$ clearly depend on whether a new product, θ_i, with organizational fit, ω_i, is introduced by the innovating firm at date $t + 2\epsilon$ or by a new firm spawned at a cost, κ, for the purpose of achieving perfect organizational fit with the new product. In the former case, the product has value

$$v_{t+2\epsilon}^{in}(\omega_i) = q_t \sum_{\tau=1}^{\infty} \frac{\alpha + \beta\,\omega_i}{(1+\rho)^\tau} = \frac{q_t}{\rho}\,(\alpha + \beta\,\omega_i)\ . \tag{4}$$

In the latter case, it has value

$$v_{t+2\epsilon}^{out} = q_t \sum_{\tau=1}^{\infty} \frac{\alpha + \beta}{(1+\rho)^\tau} - \kappa = \frac{q_t}{\rho}\,(\alpha + \beta) - \kappa\ . \tag{5}$$

The date-t value of future innovations therefore equals

$$V_t^{\mathcal{A}} = \lambda\, v_t + \frac{V_{t+1}^{\mathcal{A}}}{1+\rho}\ , \tag{6}$$

where v_t denotes the expected value of the date-t product; $v_t = p\, x_t$ with

$$x_t \equiv \int_0^{\omega_t^*} v_{t+2\epsilon}^{out}\, d\,\omega_i + \int_{\omega_t^*}^{1} v_{t+2\epsilon}^{in}(\omega_i)\, d\,\omega_i\ . \tag{7}$$

Note that, as the expected number of innovations is constant over time, the difference between $V_t^{\mathcal{A}}$ and $V_{t+1}^{\mathcal{A}}$ is due entirely the difference between q_t and q_{t+1}. Furthermore, as the expected number of innovations is constant across firms, there is no difference between $V_t^{\mathcal{A}}$ and $V_t^{\mathcal{A}^+}$.

The decision whether to spawn is made not at date t but at date $t+2\epsilon$. At that date, costless bargaining requires that

$$v_{i,t+2\epsilon}^{out} \; + \; \frac{V_{t+1}^{a\in\mathcal{A}^+} + V_{t+1}^{\mathcal{A}\setminus\{a\}}}{1+\rho} \quad = \quad v_{i,t+2\epsilon}^{in}(\omega_t^*) \; + \; \frac{V_{t+1}^{\mathcal{A}}}{1+\rho} \;, \tag{8}$$

The LHS of the preceding equation recognizes that the date-$t+1$ value of future innovations to the departing agent, $V_{t+1}^{a\in\mathcal{A}^+}$, will be determined by his being part of the set of agents $\mathcal{A}^+$ of the new firm f^+; the corresponding value to the remaining agents, $V_{t+1}^{\mathcal{A}\setminus\{a\}}$, likewise reflects agent a's departure. Note that neither the agent who would replace a in $\mathcal{A}$ in case of spawning nor the agents who would join a in $\mathcal{A}^+$ in that same case are involved in the decision to spawn.

Given the observation that the value of future innovations depends only on the number of innovations and is shared equally among agents, we have $V_{t+1}^{a\in\mathcal{A}^+} = \frac{1}{\lambda}\, V_{t+1}^{\mathcal{A}}$ and $V_{t+1}^{\mathcal{A}\setminus\{a\}} = \frac{\lambda-1}{\lambda} V_{t+1}^{\mathcal{A}}$. Combining, we have $V_{t+1}^{a\in\mathcal{A}^+} + V_{t+1}^{\mathcal{A}\setminus\{a\}} = V_{t+1}^{\mathcal{A}}$ in (8). The optimal choice of ω_t^* therefore depends only on the value attributable to the period-t innovations. Specifically, ω_t^* solves $v_{t+2\epsilon}^{out} = v_{t+2\epsilon}^{in}(\omega_t^*)$. We have

Lemma 1 *The threshold fit equals*

$$\omega_t^* = 1 - \frac{\rho\,\kappa}{\beta\, q_t} \;. \tag{9}$$

A new product with organizational fit $\omega_i < 1 - \frac{\rho\,\kappa}{\beta\, q_t}$ should be spawned to be introduced by a new firm; it should be introduced by the innovating firm otherwise.

Figure 2 shows the value attributable to the introduction of a new product, as a function of the product's fit with the innovating firm's organizational form. The solid line represents the value if the new product should be introduced by the innovating firm, the dotted line if it should be spawned to be introduced by a new firm; the threshold fit naturally is that at which the two lines intersect: low-fit products are spawned, high-fit products are not.

The threshold fit, ω_t^*, below which a new product is spawned decreases in the discount rate, ρ, and in the set up cost, κ; it increases in the importance of organizational fit, β, and in the probability of successful new product introduction, q_t. This is intuitive. A higher discount rate and a higher set-up cost increase capitalized set-up costs, $\rho\kappa$; higher capitalized set-up costs decrease the attractiveness of spawning, for they are borne only in case of spawning. A higher importance of fit naturally increases spawning, for only in case of spawning can there be perfect fit between firm organization and product. A higher probability of successful product introduction increases spawning, for it decreases the probability that the set-up cost will have been incurred in vain. An immediate implication of this last result is that spawning decreases over time: the probability of successful product introduction decreases in the number of products sold; that number increases over time.

Solving for $V_t^{\mathcal{A}}$, we can write

$$V_t^{\mathcal{A}} \;=\; \lambda\, p \sum_{\tau=t}^{\infty} \frac{x_\tau}{(1+\rho)^{\tau-t}} \;, \tag{10}$$

where x_t defined in (7) equals

$$x_t \;=\; q_t\,\frac{(\alpha+\beta)}{\rho} \;-\; \kappa \;+\; q_t^{-1}\,\frac{\rho\,\kappa^2}{2}\,\frac{(\beta-\alpha)}{\beta^2} \;. \tag{11}$$

We now add the value of those products that were successfully introduced by the firm prior to date t to obtain the date-t value of the firm. It is

$$W_t^{\mathcal{A}} \;=\; \sum_{i\in\mathcal{I}_t} \sum_{\tau=1}^{\infty} \frac{\alpha \,+\, \beta\,\omega_i}{(1+\rho)^{\tau}} \;+\; V_t^{\mathcal{A}} \;, \tag{12}$$

$$\;=\; I_t\,\frac{\alpha+\beta\,\bar{w}_t}{\rho} \;+\; V_t^{\mathcal{A}} \;, \tag{13}$$

where I_t will be recalled to denote the number of products in firm f's portfolio at date t and $\bar{w}_t$ represents the the average fit between firm f's organizational form and the products in its portfolio, $\mathcal{I}_t$, that is,

$$\bar{w}_t \;\equiv I_t^{-1}\; \textstyle\sum_{i\in\mathcal{I}_t}\omega_i \;. \tag{14}$$

4 Firm Characteristics

In this section, we consider a number of firm characteristics such as size, focus, the fit of products spawned, and profitability. We derive a number of comparative statics results; these will be seen to revolve around the extent of spawning.

Spawning Frequency. A fraction ω_t^* of the $\lambda\, p$ innovations firm f's agents expect to devise is spawned to new firms in period t; ω_t^* is the frequency of spawning expected of firm f in period $t > t_f$. it starts at $1 - \frac{\rho\,\kappa}{\beta\, q_{t_f+1}}$ and decrease over time to tend to the long-run level $1 - \frac{\rho\,\kappa}{\beta\, q_\infty}$.

Firm Size. We proxy firm size at date t by the number of product successfully introduced by the firm at that date, I_t. We show that the expected size of firm f at date $t > t_f$ equals

$$\mathbb{E}\left[I_t\right] \;=\; q_{t_f} + \frac{\lambda\, p\, \rho\, \kappa}{\beta}\left(t - 1 - t_f\right)\,. \tag{15}$$

Expected firm size starts at $\mathbb{E}\left[I_{t_f+1}\right] = q_{t_f}$; it increases by $\lambda\, p\, \rho\, \kappa/\beta$ every period. Note that only q_{t_f} appears in (15); later probabilities of successful product introduction, q_τ, $\tau > t_f + 1$ cancel out with their corresponding threshold fits, ω_τ^*: greater or lesser probabilities of successful product introduction are accommodated through the greater or lesser extent of spawning, respectively.

Firm Focus. We wish to measure firm f's focus, that is, how similar—or dissimilar—are the products in the firm's portfolio. One possible measure of focus is the (inverse) average distance between the firm's products, another the distance between the firm's original product, θ_f, and all products subsequently introduced by the firm. Recalling that the firm's original product determines that firm's organizational form, $m = \theta_f$, that second measure is the (inverse) average distance between the firm's organizational form and the firm's products. A natural measure of focus is then average organizational fit, $I_t^{-1} \sum_{i \in \mathcal{I}_t} e^{-|\theta_i - \theta_f|}$, denoted $\bar{w}_t$ in (14). We show that, at a date $t > t_f$, firm f has expected focus

$$\mathbb{E}\left[\bar{\omega}_t\right] \;=\; 1 \;-\; \frac{1}{2}\,\frac{\left(\frac{\lambda\, p\, \rho\, \kappa}{\beta}\right)^2 \sum_{\tau=t_f+1}^{t-1} \frac{1}{q_\tau}}{q_{t_f} \;+\; \frac{\lambda\, p\, \rho\, \kappa}{\beta}\,(t-1-t_f)}\;. \tag{16}$$

Given that a new firm is born with a single product, firm focus starts at the maximum, $\bar{\omega}_{t_f+1} = 1$. It is expected to decrease over time and tend to the long run level $\lim_{t\to\infty}\mathbb{E}\left[\bar{\omega}_t\right] = 1 - \frac{1}{2}\,\frac{\lambda\, p\, \rho\, \kappa}{\beta}\,\frac{1}{q_\infty}$.

Fit of products spawned. We now measure the average organizational fit between firm f and the individual products it chooses to spawn rather than introduce itself. The average fit of products spawned by firm f in period $t > t_f$ equals

$$\mathbb{E}\left[\tilde{\omega}_t\right] \equiv \frac{\int_0^{\omega_t^*} \omega\, \phi(\omega)\, d\omega}{\int_0^{\omega_t^*} \phi(\omega)\, d\omega} \;=\; \frac{\omega_t^*}{2}\;. \tag{17}$$

The fit of products spawned in period t is simply one-half the spawning frequency in that period. It too decreases over time.

Return on Assets. We measure the profitability of firm f at date t as the total profit generated by all products in the firm's portfolio at that date divided by the number such products. This measure recalls a firm's return on assets. The expected ROA of firm f at date $t > t_f$ equals

$$\mathbb{E}\left[ROA_t\right] \;\equiv\; \mathbb{E}\left[\sum_{i\in\mathcal{I}_t} \pi_{i,t}\; I_t^{-1}\right] \;=\; \alpha \,+\, \beta\,\mathbb{E}\left[\bar{\omega}_t\right]\;. \tag{18}$$

Expected ROA starts at $ROA_{t_f+1} = \alpha + \beta$. It is expected to decrease over time and tend to the long run level $\lim_{t\to\infty}\mathbb{E}[ROA_t] = \alpha + \beta - \frac{1}{2}\,\frac{\lambda\, p\, \rho\, \kappa}{q_\infty}$. Note that this last result reproduces the findings of Loderer and Waelchli (2009) mentioned in the Introduction, without the need to appeal to corporate geriatrics. We return to this issue in Section ?? below.

We now show

Proposition 1 *The frequency of spawning increases in the importance of organizational fit, β, and decreases in time, t, and in capitalized set-up costs, $\rho\,\kappa$. Such property extends* *to firm focus, firm profitability, and the fit of products spawned by the firm. The opposite is true of firm size, which decreases in the importance of organizational fit and increases in time and in capitalized set-up costs. Firm focus decreases in the number of agents, λ, and the probability that an innovation be devised, p. Such property extends to firm profitability, and the fit of products spawned. The opposite is true of firm size. Finally, firm profitability naturally increases in the measure of the other determinants of profitability, α.*

The results of Proposition 1 are summarized in Table 1. Most results are quite intuitive. The comparative statics of the frequency of spawning are essentially identical to those of the threshold fit, ω_t^*, discussed after Lemma 1; the comparative statics of focus, profitability, the fit of products spawned, and size then stem naturally from those of the spawning frequency.

An increase in the threshold fit due a greater importance of fit, earlier time periods, or lower capitalized set-up costs implies that only those products that more closely fit the innovating firm's organizational form are introduced by that firm; this increases spawning frequency. Increased spawning frequency in turn implies higher firm focus: it is those products that present a lesser fit with firm organization that are spawned; spawning leaves those products that present a greater fit with firm organization within the firm; increased spawning increases average fit, i.e., focus. Products more fitting within the firm have as their counterparts products more fitting without the firm, too, in the sense that both sets of products present higher average fit with firm organization, m: increased spawning increases the fit of products spawned.[6]

Increased focus increases profitability, by virtue of the dependance of profit on organizational fit, which is greater on average for more focused firms. Size is in a sense the mirror image of spawning: a product that is spawned is not added to the firm's portfolio of products; it does not contribute to increasing firm size. Increased spawning frequency therefore decreases firm size.

[6]An increase in ω_t^* increases both the average of ω_t^* and one (a measure of the average fit of products within the firm) and the average of zero and ω_t^* (the average fit of products without).

The number of agents and the probability that an innovation be devised affect neither the frequency of spawning nor the fit of products spawned, for these two characteristics pertain only to individual products; they do affect the other characteristics of interest, for these pertain to the firm's portfolio of products. In particular, more numerous innovations per period, λp, decrease firm focus: every new product decreases firm focus, for no new product presents a perfect fit with the innovating firm's organizational form; more new products result in a greater decrease in firm focus. The greater decrease in focus is reflected in lower profitability. More numerous innovations combine with the constancy of spawning frequency with respect to the number of innovations to result in larger firm size.

Finally, an increase in the measure of the other determinants of profitability increases only profitability. That measure affects products spawned and products retained identically; it therefore has no impact on spawning, nor on the characteristics that depend on spawning.

5 The Choice of Resource Contributions and Social Welfare

In this section, we set out to broaden our objective function to include the true social effects of innovations. Any maximization carried out by the stakeholders of the firm, ignores the value of all the innovations expected to be made by future generations of agents joining subsequent firms that originate from the firm. It is therefore, from the point of view of social efficiency, biased.

To highlight these differences, we consider an endogenous choice of resource contributions in research: So far, at each date t that follows the firm creation at date $t_f + 2\epsilon$, each agent had an exogenous probability p of making an innovation at date $t + \epsilon$. Consider now that each date t that follows the firm creation, each agent makes a decision on the probability p_t of making innovation at date $t + \epsilon$. Each agent can dedicate more resources to enhance p_t, but more resources contributed has a resulting higher cost $c(p_t)$ at date t.

Take the simplest quadratic cost specification

$$c(p_t) \;=\; k\,\frac{p_t^2}{2} \quad . \tag{19}$$

Here, p_t is the extent of resource contributed by each one of the λ agents in $\mathcal{A}$. $k \in \mathbb{R}^+$ is a parameter that captures the difficulty with which innovations are found in the specific industry. We obtain that the resource contribution which each agent selects at date t to maximize his stakeholder value

$$\tilde{p}_t \;\equiv\; \arg\max_{p_t} V_t^a \;=\; \frac{x_t}{k} \; . \tag{20}$$

where x_t is given in (11).

In equilibrium, the resource allocation selected by the agents at each date is of similar form. Aggregating over the λ agents and over time, we obtain that the date-t value of the firm equals

$$W_t^{\mathcal{A}} \;=\; I_t\,\frac{\alpha+\beta\,\bar{w}_t}{\rho} \;+\; \frac{\lambda}{2\,k}\sum_{\tau=t_f+1}^{\infty}\frac{x_\tau^2}{(1+\rho)^{\tau-t_f}} \; . \tag{21}$$

Now, $W_t^{\mathcal{A}}$ is not the social value of firm f, for it only considers this perspective firm's current stakeholders or participants. It ignores that of future generations of potential innovators that are not currently agents of firm f and hence not current stakeholders, but will be associated in firms created in the future. Thus, their existence results indirectly from the formation of firm f. Consider therefore, not only all stakeholders in firm f, but also all agents in all subsequent spin-outs emanating directly or indirectly from firm f. Given that the founders of spawned firms are initially agents in firm f, the additional stakeholders we consider here are all the agents which will be associated to all newly created firms. These further agents will themselves become inventors and possibly create firms. Hence the newly created firms we consider here are not just first generation sons of firm f, but subsequent generation firms. The social value of firm f, that we denote by $W_t^{\mathcal{S}}$, is then the value generated by all the resulting innovation processes. It is the true social value generated by the creation of firm f, since it considers not only the participants in firm f, but also the welfare of those that participate in future firms resulting from firm f's activity.

At any future date $\tau > t$, a fraction $\int_0^{\omega_\tau^*} \phi(\omega_i)\, d\,\omega_i = \omega_\tau^*$ of the λ innovations expected to be made in that unit-period are expected to result in firm creations. Now, each firm creation leads to an increase in the generation of new ideas. Each new firm is expected to generate $\lambda\, p_\tau$ new innovations per period. Therefore $\lambda\, p_\tau\, \omega_\tau^*$ new innovation processes are expected to be started in period τ, and each of these new innovation processes has then a social value equal to firm f social value at date $\tau + 1$, $W_{\tau+1}^{\mathcal{S}}$. At date t, we then have

$$W_t^{\mathcal{S}} \;=\; W_t^{\mathcal{A}} \;+\; \sum_{\tau=t}^{\infty} \frac{\lambda\, p_\tau\, \omega_\tau^*}{(1+\rho)^{\tau+1-t}}\, W_{\tau+1}^{\mathcal{S}} \;. \tag{22}$$

The resource contribution per agent which is optimal from society's perspective at date t

$$\tilde{p}_t^{\mathcal{S}} \;\equiv\; \arg\max_e W_{t_f+2\epsilon}^{\mathcal{S}} \;=\; \frac{1}{k}\left(x_t \;+\; \left(1 \;-\; \frac{\rho\,\kappa}{\beta\, q_t}\right) \frac{W_{t+1}^{\mathcal{S}}}{1+\rho} \right) \;. \tag{23}$$

We therefore have

$$\tilde{p}_t \;<\; \tilde{p}_t^{\mathcal{S}} \;, \tag{24}$$

for all date $t \geq t_f + 1$. Consequently,

Proposition 2 *The resource contribution in research that maximizes stakeholders' value is insufficient from society's perspective, because the resulting number of created firms is insufficient.*

6 Venture Capital

We haven't yet discussed how the endogenous levels of α and β are actually chosen. The perspective that we will pursue in this subsection is to consider how institutions are likely to influence the choices of α and β. We consider in particular venture capital as a financing mode that plays an essential role in the financing of innovation. A unique defining characteristic of venture capitalists is that they provide monitoring and advisory services to firm management (Casamatta, 2003). Translated into our context, venture capitalists are capable of observing ex-post distortions in resource allocations and ensuring that a specific breakdown is implemented. Therefore, if a founding owner receives VC-backing, we interpret this as a decision that leads to an increase in β (for simplicity, we omit the effect on α which is similar). More precisely, we consider a change in the financial system towards a system with more abundant sources of VC-funding. In this environment, the founder is more likely to be VC-backed, which increases his productivity β. Moreover, new firms spawned from this founding owner's firm are also more likely to receive VC-backing which increases their expected β. Thus, the comparative statics that we consider is one of an increase in the offer of VC funding and hence a monotonic increase in the likelihood for any start-up to receive VC-funding, both leading to a permanent increase in that firm's β.[7]

Since VC-backed firms will then exhibit higher levels of organizational capital, β, than non VC-backed firms, The spin-out decision threshold central to our analysis, ω_t^* in (9), is then *higher* for VC-backed firms than for non VC-backed firms. We conduct this analysis in the case in whcih the ressource contributions determining p_t are endogenously chosen, so that $p_t = \frac{x_t}{\kappa}$. We then obtain:

Proposition 3 *In industries with higher presence of venture capitalists, firms (i) are more focused, (ii) have higher return on assets, (iii) spawn more and (iv) spawn firms that are more related to their operations. The impact on the physical growth rate is ambiguous, but initially VC-backed firms are likely to grow faster.*

VC-backing increases the value of assets in place. However, the enhanced β increases the sensitivity of firm profits to proximity. Both effects increase the proportion of future

[7]Another possible perspective would be to consider that one of various possible constituencies - the founder, the current set of owners, current workers or current stakeholders, or even current stakeholders and future workers combined - endogenously decides on α and/or β bas an investment decision, by fixing the resource contributions that yield the optimal value. Presumably, the most relevant constituency is the founder of the firm, since he can fix the resource contributions at the outset of the firm. We have characterized the founder's preferences for resource contributions above. As noted, the founder will tend to underinvest in resource contributions that increase α and β. Hence, our argument about venture capital can also be regarded as beign a partial remedy in view of this underinvestment problem.

innovations that will be spun out. Essentially, VC-backing leads to more profitable firms as it enhances the frequency of spawning, and hence firms that will generate more spin-outs.

At the same time, the higher β means also that workers at the firm will contribute larger resource contributions p_t, this increasing the rate of innovation λp. These two effects generate conflicting effects on the growth rate of the firm: on one hand, more of its innovation end in spawned firms, which potentially decreases firm growth. At the same time, the resource contributions p_t will be larger, which leads to a higher innovation rate λp_t and hence to more growth potential. A detailed analysis shows that, initially, the second effect is prevailing. An interesting observation is that the usage of the venture capital will tend to influence the resource allocation in a direction that is socially efficient: in this view, the monitoring and advising ability of venture capitalists pushes the resource allocations away from that preferred by owners, towards an allocation that is more preferably from the point of view of stakeholder value or social value. In other words, we view VC-backing as an institutional that pushes the resource allocations in the direction of the socially optimal ones. Essentially, VC-backing is socially valuable because it enhances innovation rates and at the same time also spawning.

7 Testable Implications and Empirical Evidence

We derive a number of testable implications and discuss their relation to existing empirical evidence. We present these implications under four main headings, relating to firm growth and industry dynamics, spawning, focus, and profitability.

Firm growth and industry dynamics. A firm's size is proxied by the number of products successfully introduced by the firm, I_t. We define firm growth at date t to be the ratio of expected firm size at dates $t+1$ and t, $g_t \equiv \mathbb{E}\left[I_{t+1}\right]/\mathbb{E}\left[I_t\right]$. It is easy to obtain the testable implication

Implication 1 *Older firms grow more slowly; more innovative firms grow faster; firms active in industries in which organizational fit is less important grow faster.*

The first prediction is the trajectory of the growth rate over time which follows immediately from expression (15). It is empirically confirmed by the findings of, e.g., Evans (1987). The second prediction, which refers to the comparative statics of the growth rate with respect to λ and p, is consistent with findings of Audretsch (1995).[8]

We now turn from firm to industry. We consider in particular the fraction of new products introduced by new firms as opposed to existing firms. This fraction equals $\sum_{f \in \mathcal{F}_t} \omega_t^*$. The result $\partial \omega_t^* / \partial t$ then implies

Implication 2 *The fraction of new products introduced by new firms decreases over time in a given industry.*

This implication is confirmed by Audretsch and Feldman (1995), who finds that most new products are introduced by start-ups or small firms in the early years of a new industry; as the industry matures, most new products eventually are developed by large, established firms.

Our model also has implications for the time-variation in the number of start-ups in a given industry. We denote N_t the number of start-ups in period t; we show that

$$\mathbb{E}\left[N_t\right] \;=\; \lambda\, p\, \omega_{t-1}^* \prod_{\tau=1}^{\tau=t-2} \left(1 + \lambda\, p\, \omega_\tau^*\right) . \qquad (25)$$

The expected change in the number of start-ups between periods $t-1$ and t is then

$$\Delta \mathbb{E}\left[N_t\right] \;\equiv\; \mathbb{E}\left[N_t\right] - \mathbb{E}\left[N_{t-1}\right] \;=\; \lambda\, p \left[\omega_{t-1}^* (1 + \lambda\, p\, \omega_{t-2}^*) - \omega_{t-2}^*\right] \prod_{\tau=1}^{\tau=t-3} \left(1 + \lambda\, p\, \omega_\tau^*\right) . \qquad (26)$$

For q_t concave in t, $\Delta \mathbb{E}\left[N_t\right]$ decreases in t, from positive to negative: there is an increase in the number of new firms in the early stages of a new industry; such increase eventually reverses to become a decrease. We thus have

[8] Our model predicts that growth increases in firm size keeping firm age constant: this is because higher λ, p, and κ and lower β increase both size and growth. This prediction is contradicted by existing empirical evidence (Evans, 1987); it points to a limitation of our model.

Implication 3 *Over the life cycle of an industry, the number of start-ups first increases, then decreases.*

The intuition is a follows: the spawning rate is high in the early stages of a new industry, but there are few firms that would spawn new firms; there are therefore few start-ups. As time passes, the increase in the number of firms due spawning dominates, under some conditions, the decrease in the spawning rate: there are more new firms. Eventually, however, the concavity of the probability of successful product introduction decreases spawning to such an extent as to dominate the increase in the number of firms; the number of start-up decreases. Implication 3 is consistent with the findings of Klepper and Graddy (1990) and Klepper and Simons (1993) who show that this time pattern holds for the number of firms active in an industry, and argue that it is driven by market entry as well by exit decisions.

Spawning. A key element of our model is its emphasis on the spawning of new firms. The number of start-ups spawned by a given firm over period t is $s_t \equiv \lambda p \omega_t^* q_t = \lambda p(q_t - \rho\kappa/\beta)$. The Analysis of this expression shows:

Implication 4 *Older firms spawn less; more innovative firms spawn more; firms active in industries in which organizational fit is more important spawn more.*

The predictions are derived with the same comparative statics and dynamics as those in Implication 1. The first and second prediction are confirmed by the findings of Gompers et al. (2005) and Gort and Klepper (1982). In Section 6, we argue that venture capital financing leads to an increase in the organizational fit β. The third prediction, too, is then confirmed by the findings of Gompers et al. (2005).

The common dependency of spawning, focus, and the fit of products spawned on the parameters of the model implies, for changes in β, κ, and ρ:

Implication 5 *More focused firms spawn more, and they spawn firms that are more related to them.*

This implication again is consistent with the empirical findings in Gompers et al. (2005). Similar to Implication 4, it follows immediately from the observation that changes in β and κ have identical effects on focus, the spawning frequency, and the fit of products spawned. The implication does not, however, hold true for changes in λ and p, which have opposite effects on focus and the number of products spawned.

Firm Focus. Regarding firm focus, we have from Section 4:

Implication 6 *Firm focus decreases in firm age.*

Implication 6 closely matches the empirical findings of Denis, Denis and Sarin (1997).

Firm Profitability. The observation in Section 4 that changes in t , β, κ, λ, p, and ρ have similar effects on ROA and focus and opposite effects on ROA and size implies:

Implication 7 *ROA increases in focus and decreases in age and in size.*

The second prediction is confirmed by the findings in Loderer and Waelchli (2009). The first prediction is confirmed by the empirical findings in the seminal papers by Berger and Ofek (1995) and Lang and Stulz (1995) and in much of the subsequent work on conglomerates. Note that firms that diversify relatively early on differ from those that remain focused for longer: the former have low β and low q_{t_f}, they therefore have low ROA to begin with. That low β and low q_{t_f} induce diversification creates what may appear to be a vicious circle, whereby low profitability indices diversification which further lowers profitability. Yet, such diversification is optimal, and firm value would be lower if diversification were eschewed.

8 Conclusion

As noted in the Introduction, our paper is an attempt at examining the implications of organizational fit for a number of firm characteristics such as spawning frequency, size, focus, and profitability. There are three basic components to our model: organizational form, human capital, and product portfolio. The individuals that make up the firm's human capital devise innovations; these provide the firm with the opportunity to introduce new products. The fit between firm organization and product, along with the cost of setting up a new firm determine whether a new product should be introduced by the innovating firm or by a new firm set-up specifically for the purpose of introducing the new product, with organizational form dedicated to that product.

From that basic mechanism, we are able to draw a number of conclusions regarding spawning frequency, firm size, focus, and profitability, as well as what we call the fit of product spawned, that is, the average organizational fit between the firm and the individual products the firm chooses to spawn rather than itself introduce. We relate these characteristics to the basic parameters of the model, namely the importance of organizational fit, the value of human capital, the rate of innovation, the set-up cost of new firm and the discount rate. We study the dynamics of the characteristics of interest under the assumption that the probability of successful product introduction decreases in the number of products sold, that is, it decreases over time. We provide supporting empirical evidence.

We believe the phenomenon we analyze, whereby a single firm or a handful of firms indirectly give rise to a large fraction of firms active in a given industry or a set of industries is one that is quite widespread. Buenstorf and Klepper (2009), Klepper (2007) and Klepper and Sleeper (2005) have documented such phenomena for tires in Akron, cars in Detroit,and lasers across the United States, respectively. Hall (1998) has documented that same phenomenon in a set of settings as diverse as semiconductors in Silicon Valley during the second half of the twentieth century, heavy electrical machinery in late nineteenth century/early-twentieth century Berlin, ship- and marine engine-building in the nineteenth century Clyde Estuary, and textiles and textile machinery in eighteenth century Lancashire. Indeed, Hall argues that the phenomenon is not limited to industry in its classical sense but extends, in somewhat different guises to the entertainment industry and to art proper.[9]

[9]Hall (1998) gives the examples of early-twentieth century Hollywood and the motion picture industry; nineteen-fifties and sixties Memphis and the music industry; painting in Paris in the seventy or so years

We believe the wide range of the phenomenon we analyze make it a promising topic for further research.

starting in the mid-nineteenth century; and drama in Elizabethan London.

Appendix

Proof of (15): At the firm creation date $t_f + 2\epsilon$, the initially implemented innovation has a probability q_{t_f} of being successful. Out of the $\lambda\, p$ innovations the agents of firm f are expected to make at an interim date $\tau \in (t_f+1+\epsilon, t-1+\epsilon)$, only $\lambda\, p\,(1-\omega^*_\tau)$ are expected to be implemented in the parent firm. Out of these, only $q_\tau\, \lambda\,(1-\omega^*_\tau)$ are expected to turn out to be successful at date $\tau+1$. Aggregating over $t_f + 2\epsilon$ and interim dates yields $\mathbb{E}\,[I_t] = q_{t_f} + \sum_{\tau=t_f+1}^{t-1} q_\tau\, \lambda\, p\,(1-\omega^*_\tau)$. Using ω^*_t in (9) gives (15).

Proof of (16): We have

$$\mathbb{E}\,[\bar{\omega}_t] \;=\; \frac{q_{t_f} \,+\, \lambda\, p \sum_{\tau=t_f+1}^{t-1} q_\tau \int_{\omega^*_\tau}^{1} \omega_i\, d\omega_i}{q_{t_f} \,+\, \lambda\, p \sum_{\tau=t_f+1}^{t-1} q_\tau \int_{\omega^*_\tau}^{1} d\omega_i} \;=\; \frac{q_{t_f} \,+\, \lambda\, p \sum_{\tau=t_f+1}^{t-1} q_\tau\,(1-\omega^*_\tau)\frac{1}{2}\,(1\,+\,\omega^*_\tau)}{q_{t_f} \,+\, \frac{\lambda\, p\, \rho\, \kappa}{\beta}\,(t-1-t_f)} \tag{27}$$

$$=\; \frac{q_{t_f} \,+\, \frac{\lambda\, p\, \rho\, \kappa}{\beta} \sum_{\tau=t_f+1}^{t-1}\left(1 \,-\, \frac{(1-\omega^*_\tau)}{2}\right)}{q_{t_f} \,+\, \frac{\lambda\, p\, \rho\, \kappa}{\beta}\,(t-1-t_f)} \;=\; 1 \;-\; \frac{1}{2}\, \frac{\left(\frac{\lambda\, p\, \rho\, \kappa}{\beta}\right)^2 \sum_{\tau=t_f+1}^{t-1} \frac{1}{q_\tau}}{q_{t_f} \,+\, \frac{\lambda\, p\, \rho\, \kappa}{\beta}\,(t-1-t_f)}\;, \tag{28}$$

which gives (16). Then, $\lim_{t\to\infty} \mathbb{E}\,[\bar{\omega}_t] = 1 - \frac{1}{2}\,\frac{\lambda\, p\, \rho\, \kappa}{\beta\, q_\infty}$.

Proof of Time Dynamics and Comparative Statics Results in Table 1: We have $\Delta\omega^*_t \equiv \omega^*_t - \omega^*_{t-1} = \frac{\rho\,\kappa}{\beta}\,\frac{\Delta q_t}{q_t\, q_{t-1}} \le 0$, given that $\Delta q_t \equiv q_t - q_{t-1} \le 0$.
Denote $L \equiv \frac{\lambda\, p\, \rho\, \kappa}{\beta}$, $M_t \equiv \sum_{\tau=t_f+1}^{t-1} (q_\tau)^{-1}$ and $N_t \equiv \sum_{\tau=t_f+1}^{t-1} 1 = t-1-t_f$, so $\mathbb{E}\,[\bar{\omega}_t] = 1 - \frac{1}{2}\,\frac{L^2 M_t}{q_{t_f} + L\, N_t}$. Denoting $\Delta\mathbb{E}\,[\bar{\omega}_t] \equiv \mathbb{E}\,[\bar{\omega}_t] - \mathbb{E}\,[\bar{\omega}_{t-1}]$, given that $M_t = M_{t-1} + (q_t)^{-1}$ and $N_t = N_{t-1} + 1$, we obtain

$$\Delta\mathbb{E}\,[\bar{\omega}_t] \;=\; -\,\frac{L^2\,\left(q_{t_f} + L\left[N_{t-1}\,(q_t)^{-1} \,-\, M_{t-1}\right]\right)}{2\,(q_{t_f} + L\, N_t)\,(q_{t_f} + L\, N_{t-1})}\;. \tag{29}$$

Now, $M_{t-1} = \sum_{\tau=t_f+1}^{t-2} (q_\tau)^{-1} < \sum_{\tau=t_f+1}^{t-2} (q_t)^{-1} = (q_t)^{-1}\, N_{t-1}$. So $\Delta\mathbb{E}\,[\bar{\omega}_t] \le 0$.
Denoting $\Delta\mathbb{E}\,[ROA_t] \equiv \mathbb{E}\,[ROA_t] - \mathbb{E}\,[ROA_{t-1}] = \beta\, \Delta\mathbb{E}\,[\bar{\omega}_t]$, we have $\Delta\mathbb{E}\,[ROA_t] \le 0$.

For all $\Xi \in \{\beta, \kappa, \lambda, p, \rho\}$, we have

$$\frac{\partial \mathbb{E}\,[\bar{\omega}_t]}{\partial \Xi} \;=\; -\,\frac{L\, M_t\,\left(2q_{t_f} + L\, N_t\right)}{2\left(q_{t_f} + L\, N_t\right)^2}\,\frac{\partial L}{\partial \Xi}\;. \tag{30}$$

So $\frac{\partial \mathbb{E}[\bar{\omega}_t]}{\partial \Xi}$ and $\frac{\partial L}{\partial \Xi}$ are of opposite sign, for all $\Xi \in \{\beta, \kappa, \lambda, p, \rho\}$. Then, for all $\Xi' \in \{\kappa, \lambda, p, \rho\}$, we have

$$\frac{\partial \mathbb{E}\,[ROA_t]}{\partial \Xi'} \;=\; \beta\, \frac{\partial \mathbb{E}\,[\bar{\omega}_t]}{\partial \Xi'}\;. \tag{31}$$

So $\frac{\partial \mathbb{E}[ROA_t]}{\partial \Xi'}$ and $\frac{\partial \mathbb{E}[\bar{\omega}_t]}{\partial \Xi'}$ are of same sign, for all $\Xi' \in \{\kappa, \lambda, p, \rho\}$. Finally,

$$\frac{\partial \mathbb{E}\left[ROA_t\right]}{\partial \beta} \;=\; \beta \, \frac{\partial \mathbb{E}\left[\bar{\omega}_t\right]}{\partial \beta} \;+\; \mathbb{E}\left[\bar{\omega}_t\right] \;>\; 0 \;. \tag{32}$$

Proof of (11): From x_t defined in (7), we have

$$x_t \;=\; q_t \, \frac{(\alpha+\beta)}{\rho} \;-\; \kappa \;-\; (1-\omega_t^*) \left[q_t \, \frac{(\alpha+\beta)}{\rho} \left(\frac{1-\omega_t^*}{2} \right) \;-\; \kappa \right] \;. \tag{33}$$

Replacing the optimal threshold level ω_t^* given by (9) in (33) gives (11).

Proof of (20) and (21): Each stakeholder calculates at date t (where $t > t_f + 2\epsilon$) his optimal resource allocation p_t considering the resource allocation selected by other agents at this and future dates as well as himself at later dates as given. We then have

$$V_t^a \;=\; -k \, \frac{p_t^2}{2} \;+\; v_t \;+\; V_{t+1}^a \;, \tag{34}$$

where $v_t = p_t \, x_t$ with x_t defined in (7). The shareholders' optimal threshold level ω_t^* is still as in (9), hence x_t is still given by (11). We then have $\frac{\partial V_t^a}{\partial p_t} = x_t - p_t \, k$. The f.o.c. $\frac{\partial V_t^a}{\partial p} = 0$ gives (20).

Proof of (23): The social planner calculates the date t (where $t > t_f + 2\epsilon$) optimal resource allocation, p_t, maximizing the social value

$$W_t^{\mathcal{S}} \;=\; W_t^{\mathcal{A}} \;+\; \frac{\lambda \, p_t \, \omega_t^*}{1+\rho} \, W_{t+1}^{\mathcal{S}} \;+\; \sum_{\tau=t+1}^{\infty} \frac{\lambda \, p_\tau \, \omega_\tau^*}{(1+\rho)^{\tau+1-t}} \, W_{\tau+1}^{\mathcal{S}} \;. \tag{35}$$

Now,

$$\frac{\partial \, W_{t+1}^{\mathcal{S}}}{\partial \, p_t} \;=\; 0 \quad \text{and} \quad \frac{\partial}{\partial \, p_t} \left[\sum_{\tau=t+1}^{\infty} \frac{\lambda \, p_\tau \, \omega_\tau^*}{(1+\rho)^{\tau+1-t}} \, W_{\tau+1}^{\mathcal{S}} \right] \;=\; 0 \;. \tag{36}$$

The social optimal threshold level $\omega_\tau^* =$ is still as in (9). We then have

$$\frac{\partial \, W_t^{\mathcal{S}}}{\partial \, p_t} \;=\; \lambda \, (x_t \;-\; p_t \, k) \;+\; \frac{\lambda \, \omega_t^*}{1+\rho} \, W_{t+1}^{\mathcal{S}} \;. \tag{37}$$

The f.o.c. $\frac{\partial \, W_t^{\mathcal{S}}}{\partial \, p} = 0$ gives (23). Aggregating over over time, we obtain that the social value of the firm upon creation at date $t_f + 2\epsilon$ equals

$$W_{t_f+2\epsilon}^{\mathcal{S}} \;=\; \frac{q_{t_f} \, \beta}{\rho} \;+\; \frac{\lambda \, k}{2} \, \sum_{\tau=t_f+1}^{\infty} \frac{(\tilde{p}_\tau^{\mathcal{S}})^2}{(1+\rho)^{\tau-t_f}} \;. \tag{38}$$

Proof of (25) and (26): Denote $F_t \equiv |\mathcal{F}_t|$, the number of firms in existence at date t. We have $F_1 = 1$ and $\mathbb{E}[F_{t+1}] = \mathbb{E}[F_t\,(1 + \lambda\, p\, \omega_t^*)]$. Hence

$$\mathbb{E}[F_t] \;=\; \prod_{\tau=1}^{\tau=t-1} (1 + \lambda\, p\, \omega_\tau^*) \;, \tag{39}$$

and the expected growth in the number of firms

$$\mathbb{E}\left[\frac{F_{t+1} \,-\, F_t}{F_t}\right] \;=\; \lambda\, p\, \omega_t^* \;. \tag{40}$$

The number of products sold by these firms at date t is $J_t \equiv \sum_{f\in\mathcal{F}_t} I_t$. We have $\mathbb{E}[J_1] = q_0$ and $\mathbb{E}[J_{t+1}] = \mathbb{E}[J_t + F_t\,\lambda\, p\, q_t]$. Hence

$$\mathbb{E}[J_t] \;=\; q_0 \;+\; \sum_{s=1}^{s=t-1} \left(\lambda\, p\, q_s \prod_{\tau=1}^{\tau=s-1} (1 + \lambda\, p\, \omega_\tau^*) \right) \;. \tag{41}$$

Denote $N_t \equiv F_t - F_{t-1}$, the number of firms created per period at date t only.

$$\mathbb{E}\,[N_t] \;=\; \lambda\, p\, \omega_{t-1}^* \prod_{\tau=1}^{\tau=t-2} (1 + \lambda\, p\, \omega_\tau^*) \;. \tag{42}$$

The change in expected number of firms created per period at date t,

$$\Delta\mathbb{E}\,[N_t] \;\equiv\; \mathbb{E}\,[N_t] - \mathbb{E}\,[N_{t-1}] \;=\; \lambda\, p \left[\omega_{t-1}^*(1 + \lambda\, p\, \omega_{t-2}^*) \;-\; \omega_{t-2}^*\right] \prod_{\tau=1}^{\tau=t-3} (1 + \lambda\, p\, \omega_\tau^*) \;. \tag{43}$$

The sign of $\Delta\mathbb{E}\,[N_t]$ is that of $s_t \equiv \lambda\, p\, \omega_{t-1}^*\, \omega_{t-2}^* \;-\; \left(\omega_{t-2}^* \;-\; \omega_{t-1}^*\right)$, where $\omega_{t-2}^* \;-\; \omega_{t-1}^* > 0$.

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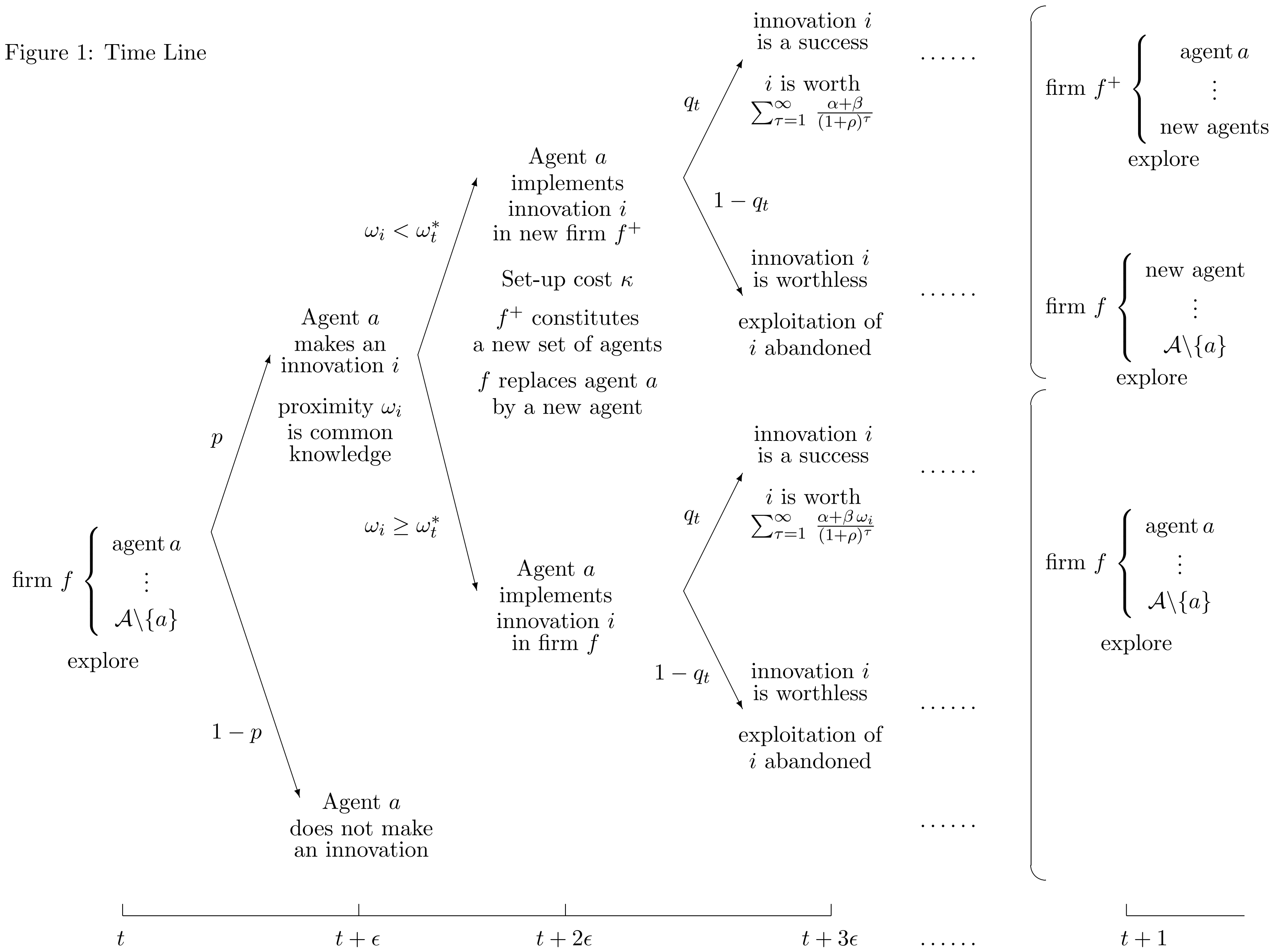
Figure 1: Time Line



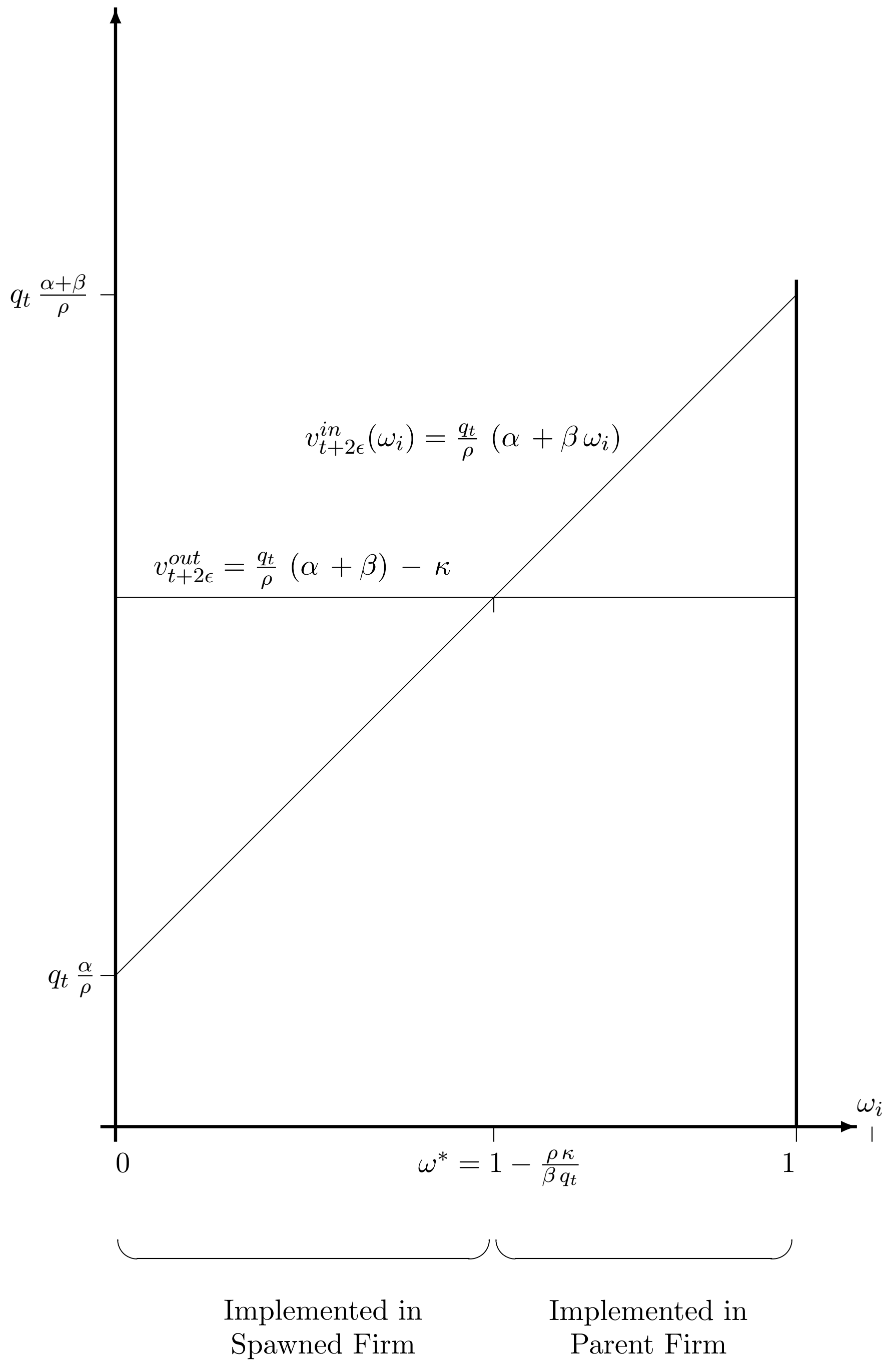


Figure 2: Choice of Implementation of an Innovation at date $t + 2\epsilon$

Table 1: Comparative Statics

	Firm Size $\mathbb{E}[I_t]$	Firm Focus $\mathbb{E}[\bar{\omega}_t]$	Spawning Frequency ω_t^*	Spin-outs Proximity $\mathbb{E}[\tilde{\omega}_t]$	Return on Assets $\mathbb{E}[ROA_t]$
t	$+$	$-$	$-$	$-$	$-$
α	0	0	0	0	$+$
β	$-$	$+$	$+$	$+$	$+$
κ	$+$	$-$	$-$	$-$	$-$
λ	$+$	$-$	0	0	$-$
p	$+$	$-$	0	0	$-$
ρ	$+$	$-$	$-$	$-$	$-$