Leverage, Value and Firm Scope*

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Abstract

This paper determines optimal capital structure together with the value of debt and equity claims in Holding-Subsidiary structures (HS), when there is a trade-off between bankruptcy costs and taxation. HS have higher firm value than their stand alone counterparts, because the holding provides a guarantee to its subsidiary's lenders. The guarantee - we argue - is conditional on the holding survival thanks to its separate incorporation. The guarantee lowers optimal debt of the H and increases that of its S wrt their stand-alone counterparts. We find conditions ensuring that HS have also higher debt capacity, which increases their value by reducing the tax burden and default costs below that of stand alone firms. HS value may also exceed the one of conglomerates, thanks to the guarantee conditionality together with diverse debt levels in affiliated activities. This preserves the holding ability to rescue its subsidiary even with perfect cash flow correlation.

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1 Introduction

Companies are often organized as holding-subsidiary structures (HS). These consist of activities which, while being separate entities, are connected by both ownership links and a common financial management. Like stand-alone companies, each HS affiliated firm can issue debt against its own cash flow and is not responsible for other affiliates' debts, being separately incorporated and therefore enjoying limited liability[1]. Contrary to stand alone firms, though, firms belonging to HS appear to assist each other in distress through either informal[2] or contractual (Deloof and Vershueren, 2006; Samson, 2001) guarantees. This paper studies how such guarantees and ownership links affect value creation in HS.

For this purpose, we model an entrepreneur who considers organizing her two activities taking into account that they can issue debt[3], which allows to deduct interest from taxes but increases bankruptcy probability. If the entrepreneur chooses the HS structure then one company - the holding - will support its insolvent subsidiary provided that both can survive. If she opts for stand alone organizations, then each activity will provide no guarantee to the other. Importantly, we focus on purely financial synergies arising from levered firm combinations as in Leland (2007): thus firm cash flows are exogenous.

In this setting, we show that the total value of an HS arrangement exceeds that of stand-alone companies provided that the guarantee is ex post enforceable. The guarantee - which permits to save on bankruptcy costs - has non negative value, because it is *conditional* on the survival of the holding company which enjoys limited liability.

The entrepreneur further increases such value through its choice of leverage. Indeed, we present conditions ensuring that total debt capacity in HS exceeds that of stand alone organizations, even though the optimal holding debt is lower than that of its stand alone counterpart . The guarantee reduces the subsidiary insolvency probability, at the optimal debt of two comparable stand alone firms, relative to the stand-alone arrangement without increasing the bankruptcy probability of the

[1] This is a common characteristic across major jurisdictions. See Blumberg (1989) for the US, and Hadden (1996) on Britain, France, Germany and the US.

[2] Khanna and Palepu (2000) observe that Indian group firms assist each other in times of financial distress, while Bertrand et al. (2002) document cash transfers in several forms - from asset sales to internal loans at subsidized rates. See also Chang and Hong (2000).

[3] One motivation for our focus on debt is the observation that HS rely extensively on debt rather than equity financing (Bae, Kang and Kim, 2002; Chang, 2003; Dewaelheyns et al., 2007).

holding. This implies that the subsidiary can increase its own debt financing, relative to the stand alone case, so that the higher interest payment reduces its tax burden. Thus value creation in HS structures results from an initial reduction in bankruptcy costs which boosts debt and the associated tax shield.

Several empirical studies focus on the value of equity in one type of HS organization - e.g. the traditional business group[4]. Group shares often trade at lower values than in stand-alone firms with comparable operating performance (Bennedsen and Nielsen, 2006; Claessens et al., 2002; Masulis et al, 2008). This evidence is puzzling - if one thinks of equity as the only corporate liability - because shareholders seem to give up value that can be simply created by spinning off the subsidiary. In our model equity values, averaged across holding and subsidiaries, are lower than in stand alone arrangements. Yet there is no puzzle as the entrepreneur, who originally owns the activities and issues corporate liabilities, gains with respect to the stand alone case by cashing in a higher market value of debt. Two effects generate the lower average value of HS equity in our model. On the one hand, the guarantee implies that the holding shareholders will transfer cash to the subsidiary lenders, should conditions for rescue hold. This reduces the value of equity in the holding below that of a stand alone with the same level of debt, leaving unaffected the value of subsidiary equity. On the other hand, both the holding and the subsidiary modify their leverage in order to optimize the tax-bankruptcy trade-off, leading to higher group debt than in the stand alone case. Since equity value is a call option, increasing debt reduces equity value. Thus, the equity value of two stand alone firms exceeds that of a holding and its subsidiary.

Previous theories of groups focus on fund provision by minority shareholders rather than by lenders. Groups emerge when entrepreneurs prefer to fund activities indirectly, through another company, rather than directly. This is the case when the present value of the activity, net of diversion, is negative: the equity discount thus reflects expropriation of minority shareholders (Almeida and Wolfenzon, 2006). This explanation for the existence of groups - despite their lower equity values - may be appropriate when affiliated firms are listed on public exchanges with loose enforcement of securities regulation. However, it remains unclear why groups thrive in strict-enforcement countries, such as Scandinavian

[4]This is common in both emerging markets (Khanna and Yafeh, 2007) and continental European countries (Barca and Becht, 2001). HS structures are present in innovative industries in the US and the UK (Allen, 1998; Sahlman, 1990; Mathews and Robinson, 2006), in the private equity industry (Jensen, 2007; Kaplan, 1989) as well as in the banking industry (Dell'Ariccia and Marquez, 2008).

ones, and why unlisted groups are quite common. By shifting the focus from minority shareholders onto lenders our model can account for these situations.

The above analytical results concerning leverage and value apply to the case of a guarantee which, consistent with our full information setting, is *ex-post* verifiable and enforceable in court. In such a case, numerical simulations of the Leland's case show that the holding company is unlevered - thus avoiding bankruptcy costs. Debt is shifted onto the subsidiary which - being burdened by the service of debt - is almost never able to distribute dividends. Thus, HS leverage and value are insensitive to the shareownership of the holding company into its subsidiary's equity.

We also numerically analyze a situation when the lender assigns a probability lower than one to the holding honouring the guarantee *ex-post*. This modelling captures the case of informal guarantees, such as comfort letters, which assure subsidiaries' lenders that the holding would assist them in distress but are legally unenforceable. This adds to our understanding of HS value differential relative to stand alone firms in two related respects. First, an informal guarantee lowers optimal subsidiary debt. This, in turn, makes the ownership stake of the holding into the subsidiary relevant for both the leverage choice of the holding and HS value: the higher the dividends that the holding receives thanks to its ownership stake, the higher is its optimal debt. Dividends are, in fact, just another form of guarantee - which is provided by the subsidiary in favour of H lenders. Second, our numerical exercise generates a "parent company discount" (Cornell and Liu, 2001), i.e. a situation where the equity value of the holding is lower than the value of its equity share in the subsidiary. The reason has to do with the higher debt burden onto the shareholders in the holding company. Our model thus hints at an arbitrage-free explanation of this valuation puzzle.

We then proceed to compare HS structures to the conglomerate merger (M), which the literature on firm combinations and internal capital markets has extensively investigated. For instance, Leland (2007) highlights conditions such that a merger has lower value than stand alone arrangements when cash-flows are normally distributed. Under the same conditions, we show that HS exceeds M value for all cash flows correlations. Such value differential is due to the joint incorporation of activities in conglomerates. This implies that M-divisions cannot have different debt levels, whereas the level of debt is specific to each activity in HS. Moreover, joint incorporation pools the activities' cash-flows, making them jointly liable vis-à-vis lenders: thus each offers an *unconditional guarantee* to the other, as opposed to a conditional one offered by H to S. It follows that one unprofitable activity may drag the other one into bankruptcy.

The possibility of choosing diverse debt levels in HS not only preserves but also enhances the ability to provide support. The easiest example refers to the case when cash flow correlation across two symmetric activities is perfect. In such a circumstances, the model by Leland (2007) - which we build upon - shows that diversification gains from a merger disappears. Accordingly, merging two stand alone firms produces no value gains. The reason is that both activities fail in the same contingencies, having the same level of debt and cash flows. In HS it is still possible for H to rescue S because debt from the holding is optimally lower than the subsidiary one: thus default costs fall, the optimal debt increases and the tax burden drops. Gains from combining firms as HS obtain irrespective of cash flow correlation thanks to debt optimal diversity across holdings and subsidiaries.

Numerical simulations also indicate that the value differential in favour of holding-subsidiary structures increases when activities differ in risk and bankruptcy costs. This suggests that constraining activities to unconditional reciprocal support in a conglomerate is suboptimal, when they bear diverse bankruptcy costs. Furthermore, conglomerates constrain activities to the same level of debt, while we know that shielding income from taxes through debt has higher value in the activity with riskier cash flow, because of the asymmetric nature of taxation. This reasoning also indicates which activity ought to provide - or receive - support in a HS, because we know that the supported one has higher leverage when the guarantee is *ex post* enforceable. Thus, a by-product of this analysis is the characterization of holding and subsidiary.

When we allow for informal guarantees in our comparison with mergers, HS may turn out to have lower values in our numerical exercise. The conditional guarantee in HS becomes less capable than the unconditional one in M in improving credit conditions, as lenders anticipate an uncertain ex post service of debt even when the holding cash flows are large enough to make rescue possible. As credit spreads widen, optimal debt in HS falls below the M one and HS value may accordingly fall below M value.

Our model extends Leland (2007) to the case of holding-subsidiary structures. In so doing, it borrows some key assumptions concerning operational cash flows, tax rates and bankruptcy costs with no *ad hoc* modification. Cash flows are exogenous and the firm receives no tax refunds when they are negative. In the real world, companies may carry forward some losses, in order to reduce the asymmetric nature of taxation - which however remains substantial. Bankruptcy costs, which the firm pays only when it does not meet its debt obligations, are proportional to cash-flows. This is a common assumption in structural models of credit risk. We also make our numerical results concerning HS comparable to those of Leland (2007) for stand alone and mergers by using the same calibrations for a BBB stand alone firm.

While our model posits exogenous operating cash flows, several papers study how agency problems in internal capital markets affect product market competition and investment choice. Most focus however on aspects, such as cash-flow pooling, that are typical of both conglomerates and groups without making any explicit distinction between the two organizations. See, among others, Rajan Servaes Zingales (2000), Inderst and Mueller (2003) and Faure Grimaud and Inderst (2005). Cestone and Fumagalli (2005) analyze instead the specificities of group internal capital market by both assuming limited liability of the holding and by allowing subsidiaries to raise their own debt. They study how transfers from the holding impact on the conditions obtained by subsidiaries from its outside financiers, when managerial effort cannot be observed. The benefit of HS relies on higher managerial effort, rather than a better tax-bankruptcy cost trade-off as in our paper.

Another related paper, Huizinga et al. (2008), studies tax arbitrage in multinational groups that is engineered by raising more debt in high-tax countries. In our model, groups minimize the tax burden through debt even if there is no tax rate differential between the holding and its subsidiary. Thus, we point out a powerful tax avoidance tool which, to our knowledge, has not yet been analyzed.

Last but not least, our results rely on the idea that the guarantee is conditional on the holding survival because of corporate limited liability. Blumberg (1989) reports that the legal theories of the separate legal personalities of corporations (entity law) and the limited liability of shareholders were applied to HS in the US during the twentieth century. Courts expanded the concept of limited liability to protect each layer in the HS from the liability of the junior company. Successive layers of limited liability were created on top of the primary limitation on the liability of the ultimate individual investor for the obligations of the parent company. In time, the law created a safety valve for judicial escape in exceptional cases, given the potential for abuse. This safety valve was the doctrine of "piercing of the corporate veil ". Many courts now impose liability on the parent when it is possible to prove the lack of separate existence of the subsidiary. However it must also be established that through the holding company's use of such dominated corporation, the plaintiff has been victimized by conduct "akin to fraud"

The paper is organized as follows. Section 2 analyzes the three organizational modes - stand alone, HS and conglomerate - for two activities, in the case of ex post enforceability. In particular, we indicate how the value of debt and equity (Propositions 2, **??** 11) and of optimal debt (Theorem 6) change due to the conditional guarantee. Section 3 presents numerical simulations allowing for a comparison of optimal leverage, value of debt and equity across the three organizations, for equally distributed cash flows, as the correlation between them varies. Section 4 examines activities differing in mean cash flow, volatility and bankruptcy costs. Section 5 numerically examines the interplay of informal guarantees and positive intercorporate ownership. Section 6 concludes.

2 The model

We consider a no arbitrage environment with two dates $t = \{0, T\}$. An entrepreneur owns two production units, and each activity i generates a random future operating (net) cash flow value X_i at time $t = T$. X_i is a continuous random variable that may take both negative and positive values: having denoted as F_i its distribution function, this means $F_i(0) < 1$. We assume that $X \in L^2$, namely that it admits at least the first two moments. The riskfree interest rate over the time period T is $r_T > 0$, and ϕ denotes the corresponding discount factor, $\phi = (1 + r_T)^{-1} < 1$.

No arbitrage implies that the value of the operating cash flow at $t = 0$ is its discounted expected value:

$$X_{0i} = \phi \mathrm{E} X_i \tag{1}$$

where $\mathrm{E}X_i$ is evaluated under the risk neutral measure. The owner can "walk away" from negative cash flows thanks to limited liability. Thus the (pre-tax) value of each activity with limited liability is

$$H_{0i} = \phi \mathrm{E} X_i^+ \tag{2}$$

where $X_i^+ = \max(X_i, 0)$, and the pre-tax value of limited liability is

$$L_{0i} = H_{0i} - X_{0i} \geq 0 \tag{3}$$

Now consider a tax rate on future cash flows equal to τ_i. The aftertax value of the unlevered firm, which corresponds to its equity value, is

$$V_{0i} = (1 - \tau_i) H_{0i} \tag{4}$$

The present value of taxes it pays, named *tax burden* in the sequel, is

$$T_{0i}(0) = \tau_i H_{0i} \tag{5}$$

At time $t = 0$ the entrepreneur can lever the firm by issuing zero-coupon debt so as to maximize the value of his claims to the cash flows. Let its principal value be $P_i \geq 0$, and assume it is due, with absolute priority, at $t = T$. Let $D_{0i}(P_i)$ denote the value, at $t = 0$, of such debt, which is cashed-in by the entrepreneur at issuance. We assume that there is an incentive to issue debt, as interest is a deductible expense. The promised interest payment is equal to:

$$P_i - D_{0i}(P_i) \tag{6}$$

In turn, taxable income is the operating one net of interest payment:

$$X_i - (P_i - D_{0i}(P_i)) \tag{7}$$

and the zero-tax level of cash flow with positive leverage, X_i^Z, is

$$X_i^Z(P_i) = P_i - D_{0i}(P_i) \tag{8}$$

Hereafter the argument P_i of D_{0i} and X_i^Z is often suppressed.

We assume that no tax refunds are paid by the tax authority to the owners of the activity if $X_i < X_i^Z$. It follows that operating cash flows, net of tax payments, are[5]

$$X_i^n = X_i^+ - \tau_i(X_i - X_i^h)^+ = \begin{cases} 0 & X_i < 0 \\ X_i & 0 < X_i < X_i^Z \\ X_i(1-\tau) + \tau X_i^Z & X_i > X_i^Z \end{cases} \tag{9}$$

with $0 < \tau_i < 1$. The tax burden of the levered firm is equal to:

$$T_{0i}(P_i) = \tau_i \phi \mathrm{E}(X_i - X_i^Z)^+ \tag{10}$$

Clearly, some value gains obtain when (10) is lower than (5). However, issuing debt has costs as well. Similarly to Merton (1974), default occurs when net operating cash flow is smaller than the face value of the debt:

$$X_i^n < P_i \tag{11}$$

Having defined the default threshold X_i^d as

$$X_i^d(P_i) = P_i + \frac{\tau_i}{1-\tau_i} D_{0i}(P_i) = \frac{P_i - \tau_i X_i^Z}{1-\tau_i} \tag{12}$$

[5] Having assumed X_i continuous, we omit the boundary values in this and the following inequalities on payoffs.

the default triggering condition (11) can be written in terms of the pre tax cash flows as $X_i < X_i^d$. In the event of default, we assume that bondholders will receive a fraction $(1-\alpha_i)$ of operating cash flow, X_i, when this is positive; a fraction $\alpha_i, 0 < \alpha_i < 1$, of cash flows is instead lost upon liquidation. There is then a trade-off between the dissipative default costs, $\alpha_i X_i$, and the tax savings possibly generated by debt.

Let the levered value of equity be denoted as E_{0i}, and D_{0i} be the corresponding value of debt that is cashed in at time-0. The entrepreneur chooses the face value of debt, P_i, in the two activities, given the tax-bankruptcy cost trade-off, so as to maximize the time-zero combined value of the two units. The value of equity and debt is the expected present value of cash flows accruing to shareholders and lenders respectively, evaluated under the risk neutral measure. Such cash flows vary with the organization- specific guarantees, which we discuss below.

2.1 Levered firm value in the stand alone case

Stand alone firms - being separately incorporated and independently managed - are typically not liable for each others' debt. We therefore follow Leland (2007) in modelling stand-alone activities as never providing support to each other. Thus, the entrepreneur maximizes the levered firm value, $\nu_{0i}(P_i)$, of his two stand alone activities, $(i = 1, 2)$, with respect to the face values of debt:

$$\sum_{i=1}^{2} \nu_{0i}(P_i) = \sum_{i=1}^{2} [E_{0i}(P_i) + D_{0i}(P_i)] \tag{13}$$

We now determine the value of equity and debt, i.e. of the two elements on the right-hand side of equation (13), as a function of the payoff to financiers at time T. The cash flow to shareholders at $t = T$, E_i, is operating cash flow less taxes and the repayment of principal, when the difference is positive:

$$E_i(P_i) = (X_i^n - P_i)^+ \tag{14}$$

Indeed, limited liability ensures that shareholders bear no responsibility when the difference is negative. By no arbitrage the value of equity is simply[6]

$$E_{0i}(P_i) = \phi \mathrm{E}(X_i^n - P_i)^+ \tag{15}$$

[6]Notice that E_{0i} is a call option with underlying X_i^n and exercise price P_i. It depends on debt principal both directly and indirectly, through the tax shield X_i^Z that enters the underlying.

The cash flows D_i to lenders at time $t = T$ will equal P_i when the firm is solvent, i.e. $X_i > X_i^d$. When the firm is insolvent, debtholders become the residual claimants. They receive cash flows net of bankruptcy costs, $(1-\alpha_i)X_i$, if cash flows net of interests are lower or equal to zero ($X_i \leq X_i^Z$). Recalling that the government has priority for tax payments before lenders, debtholders will also bear a tax liability $\tau_i(X_i - X_i^Z)$ in default when $X_i^Z < X_i < X_i^d$. The payoff to lenders is therefore equal to:

$$D_i(P_i) = \begin{cases} (1-\alpha_i)X_i & 0 < X_i < X_i^Z \\ (1-\alpha_i)X_i - \tau_i(X_i - X_i^Z) & X_i^Z < X_i < X_i^d \\ P_i & X_i > X_i^d \end{cases} \quad (16)$$

and it can be represented as follows:

Insert here Figure 1

The present value of lenders' payoff (16), $D_{0i}(P_i)$, is the value of zero-coupon debt given the principal P_i:

$$D_{0i}(P_i) = \phi \mathrm{E} \begin{bmatrix} (1-\alpha_i)X_i \ \mathbf{1}_{\{0<X_i<X_i^Z\}} + \\ \left[(1-\alpha_i)X_i - \tau_i(X_i - X_i^Z)\right] \mathbf{1}_{\{X_i^Z<X_i<X_i^d\}} + \\ +P_i \ \mathbf{1}_{\{X_i>X_i^d\}} \end{bmatrix} \quad (17)$$

where $\mathbf{1}_{\{\bullet\}}$ is the usual indicator function.[7]

2.2 Holding-Subsidiary Structures: Conditional Guarantee

The two stand alone companies described in the previous section have no ownership links. Moreover, they do not guarantee each others' debt. In this section we model the case when one activity - the holding company ($i = h$) - supports its insolvent subsidiary ($i = s$) through a cash transfer. Such cash transfer is however conditional on the survival of the holding, which uses its limited liability otherwise. We will first assess how the presence of such conditional guarantee affects the value of the two activities to the entrepreneur, assuming no ownership links between the two

[7] Due to default costs and tax savings, debt $D_{0i}(P_i)$ is a portfolio of plain vanilla puts and the present value of the principal. Note that (17) is an implicit equation, since X_i^Z and X_i^d are themselves function of D_{0i} through (8) and (12). Numerical methods are necessary for its solution. Since D_{0i} determines the thresholds and the latter enter the equity value, the solution approach in (3) will consists in finding a fixed point for D_{0i} and then determine X_i^Z, X_i^d and E_{0i}.

companies. We will later generalize the setting. For the sake of simplicity, we also assume throughout that tax rates and default costs do not differ between the holding and its subsidiary ($\alpha_s = \alpha_h = \alpha, \tau_s = \tau_h = \tau$). Whenever we will assume equality between two cash flows (random variables), it will be equality in distribution.

We first determine the minimum amount of the transfer (or support) that allows to rescue an insolvent subsidiary, as well as the subset of states when the transfer occurs.

Lemma 1 *The conditional transfer from the holding to the subsidiary, associated with the guarantee, is equal to:*

$$(P_s - X_s^n)\mathbf{1}_{\left\{0<X_s<X_s^d, X_h>h(X_s)\right\}} \tag{18}$$

where $h(X_s)$ denotes the linear function:

$$h(X_s) = \begin{cases} X_h^d + \frac{P_s}{1-\tau} - \frac{X_s}{1-\tau} & X_s < X_s^Z \\ X_h^d + X_s^d - X_s & X_s > X_s^Z \end{cases} \tag{19}$$

Proof. A necessary condition for the transfer is that the subsidiary is unable to meet its debt obligations, $X_s < X_s^d$. Limited liability ensures that there is no rescue if the operating cash flows of the subsidiary are negative, as the holding would otherwise bear an operating loss that it could have avoided. Thus, the holding rescues its subsidiary if:

$$0 < X_s < X_s^d \tag{20}$$

The holding company enjoys limited liability, because - thanks to separate incorporation - is not responsible for its subsidiary's debt obligations.[8] Limited liability then implies that the transfer is conditional on the holding company ability to meet both its own and its subsidiary debt obligations. This is the case only if the holding after-tax cash flow, net of debt repayment, is positive and exceeds the corresponding difference for the subsidiary, i.e.

$$\begin{cases} X_h > X_h^d \\ X_h^n - P_h > P_s - X_s^n \end{cases} \tag{21}$$

If the second condition does not hold, the holding survives by letting the subsidiary default. Overall, a transfer occurs if and only if both (20)

[8] This assumption is consistent with legal texts (Blumberg, 1989; Hadden, 1996) as well as with some empirical literature. For instance, business groups appear to support struggling subsidiaries, but they tend to terminate such support once group profitability turns negative (Dewaelheyns and Van Hulle (2006)).

and (21) hold. In the proposition, we write the support conditions in compact notation as:

$$\begin{cases} 0 < X_s < X_s^d \\ X_h > h(X_s) \end{cases} \tag{22}$$

■

For the time being, assume that H exerts control over S with an infinitesimal intercorporate ownership. The entrepreneur maximizes levered firm value, $\nu_{0i}(P_h, P_s)$, of his holding and subsidiary ($i = h, s$) with respect to the face values of debt:

$$\nu_{0,HS}(P_h, P_s) = \sum_{i=h}^{s} \nu_{0i}(P_h, P_s) = \sum_{i=h}^{s} [E_{0i}(P_h, P_s) + D_{0i}(P_h, P_s)] \tag{23}$$

taking into consideration the conditional transfer from the holding to the subsidiary. Notice that, due to such transfer, we expect debt and equity of both firms to depend on both face values of debt: this explains why we use the notation $E_{0i}(P_h, P_s), D_{0i}(P_h, P_s)$ in (23).

The choice of debt that maximizes (23) subject to the existence of a transfer (18) will in general differ from the stand alone one. As a consequence, the default and tax shield thresholds of the holding and subsidiary will be different from their stand alone counterparts. We postpone the study of optimal debt until section 2.2.4.

In sections 2.2.1, 2.2.2 we compare the total value of HS and stand alone arrangements, when the face value of debt is exogenous. Specifically, we assume that it is equal for the holding and its stand alone counterpart (i.e. $P_h = P_1$) as well as for the subsidiary and its stand alone correspondent (i.e. $P_s = P_2$).

2.2.1 The value of equity in Holding Subsidiary structures

We now determine the payoff accruing to shareholders of the holding company at T. This is equal to operating cash flows net of taxes and the service of debt, less the conditional transfer from the holding to its subsidiary:

$$E_h(P_h, P_s) = (X_h^n - P_h)^+ - (P_s - X_s^n)\mathbf{1}_{\{0<X_s<X_s^d, X_h>h(X_s)\}} \tag{24}$$

Such payoff can be written as:

$$E_h(P_h, P_s) = E_1(P_h) - (P_s - X_s^n)\mathbf{1}_{\{0<X_s<X_s^d, X_h>h(X_s)\}} \tag{25}$$

where $E_1(P_h)$ is the cash flow accruing to shareholders of a comparable stand alone with the same nominal debt ($P_1 = P_h$). The last term highlights that the payoff to the holding shareholders never exceeds the payoff of the stand-alone with equal debt, because of the state-contingent support. It follows that also the equity value of a holding company:

$$E_{0h}(P_h,P_s) = \phi \mathrm{E}\left[(X_h^n - P_h)^+ - (P_s - X_s^n)\mathbf{1}_{\left\{0<X_s<X_s^d, X_h>h(X_s)\right\}}\right] \quad (26)$$

cannot exceed the one of a stand alone with the same capital structure. Equity holders of the subsidiary are unaffected, as the transfer occurs for the sake of servicing debt, and equation (14) still holds for $i = s$. An immediate consequence of the fact that $E_h < E_1, E_s = E_2$, is the following:

Proposition 2 *Consider a holding company, its subsidiary and two stand alone firms with the same face value of debt of H and S ($P_1 = P_h, P_2 = P_s > 0$) and the same operating profits ($X_1 = X_h, X_2 = X_s$). Then the average equity price of stand-alone firms exceeds that of HS affiliated counterparts, if the transfer occurs with positive probability.*

This is our first rationale for the observation that equity values are often lower in HS than in stand alone (SA) structures.

2.2.2 The value of debt in Holding-Subsidiary structures

The value of subsidiary debt, $D_{0s}(P_h, P_s)$, is the present expected value of the following final payoffs:

$$\begin{aligned} &D_s(P_h, P_s) = \quad (27) \\ &= \left[X_s(1-\alpha) + \tau(X_s - X_s^Z)\mathbf{1}_{\{X_s>X_s^Z\}}\right]\mathbf{1}_{\left\{0<X_s<X_s^d,\ X_h<h(X_s)\right\}} + \\ &+P_s\left[\mathbf{1}_{\left\{0<X_s<X_s^d,\ X_h>h(X_s)\right\}} + \mathbf{1}_{\left\{X_s>X_s^d\right\}}\right] \end{aligned}$$

The first square bracket refers to the case when the subsidiary defaults and the holding does not support its subsidiary because its own cash flow is insufficient ($X_h < h(X_s)$). In this situation, lenders have to pay taxes only if cash flows exceed the tax shield ($X_s > X_s^Z$). The first term in the second square bracket refers to the case when the subsidiary, while defaulting if it were a stand alone firm, is able to reimburse its debt thanks to the holding transfer.

It is easy to show that the payoff to a subsidiary lender, relative to that of its stand alone counterpart with the same nominal debt ($P_2 = P_s$), is equal to:

$$\begin{aligned} D_s(P_h, P_s) = &D_2(P_s) + [P_s - X_s(1-\alpha) - \quad (28) \\ &-\tau(X_s - X_s^Z)\mathbf{1}_{\{X_s>X_s^Z\}}]\,\mathbf{1}_{\left\{0<X_s<X_s^d,\ X_h>h(X_s)\right\}} \end{aligned}$$

Subsidiary lenders obtain, on top of what accrues to stand-alone lenders, the nominal value of debt thanks to the guarantee (first term in square bracket) while losing the cash flow net of bankruptcy costs and taxes (second and third term). In other words, the payoffs to subsidiary lenders is the same as in the stand alone case, outside the states when a transfer takes place. It must instead be augmented by the transfer, as shown in Figure 2, when this occurs.

Insert here Figure 2.

The payoff to lenders of the holding does not change with respect to the stand alone case, as the transfer to the subsidiary occurs only after the service of the holding debt. Thus equation (17) holds for $i = h$.

It follows that the average value of debt is higher in a HS arrangement than in stand alone companies, given exogenous face levels of debt. Before studying the value differential between HS and the stand alone organizations, we generalize these expressions to the realistic case of ownership links between firms.

2.2.3 Value with Intercorporate Dividends

So far we maintained that H keeps control with a zero ownership share, ω, of subsidiary equity capital corresponding to a situation of extreme separation between ownership and control. This section generalizes the previous expressions to the case when $0 \leq \omega \leq 1$: $\omega = 1$ corresponds to a wholly owned subsidiary[9]. In the stand alone case we similarly posit that firm 1 owns a stake ω of firm 2, which remains independently managed. Thus both H and firm 1 are entitled to a share ω of the earnings of the subsidiary (firm 2 respectively) after interest and taxes.

We now determine the value differential between HS and two comparable stand alone firms, assuming no taxation of intercorporate dividends.[10] Denote as $D_{0i}(\bullet, \omega), E_{0i}(\bullet, \omega), \nu_{0,i}\ (\bullet, \omega)$ the debt, equity and total values when ownership ω is positive, keeping the notations in the above sections for the infinitesimal ownership case.

Proposition 3 *Consider the case of equal cash flows and debt principals across HS and stand alone* ($X_1 = X_h, X_2 = X_s,\ P_1 = P_h, P_2 = P_s$)*.The*

[9] When the HS structure is unlisted, the ownership share is usually close to 1. It often exceeds one half when the subsidiary is listed on a public exchange, but the entrepreneur wishes to keep majority control without using non-voting shares or other control enhancement mechanisms.

[10] When computing the total value of the two companies, however, we avoid double counting of dividends by considering only a share $1-\omega$ of the owned company. Other details are spelled out in Appendix A.

value differential for a holding (subsidiary) relative to a type-1 (type-2) company is unaffected by dividends:

$$\nu_{0,H}(P_h, P_s, \omega) - \nu_{01}(P_1, \omega) = \nu_{0,H}(P_h, P_s) - \nu_{01}(P_1)$$
$$\nu_{0,S}(P_h, P_s, \omega) - \nu_{02}(P_2, \omega) = \nu_{0,S}(P_h, P_s) - \nu_{02}(P_2)$$

Proof. Let d_s^+ denote dividends which are paid out by a solvent subsidiary to its holding. Observe first that the value of subsidiary debt is unaffected by the payment of dividends, because they are distributed only by a profitable and non defaulting subsidiary. Thus

$$D_{0s}(P_h, P_s, \omega) = D_{0s}(P_s, P_h)$$

Also the value of subsidiary equity is unchanged,because subsidiary profits do not vary:

$$E_s(P_s, \omega) = E_s(P_s)$$

with $E_s(P_s)$ defined by (14). The holding valuation changes, instead, because of two effects. On the one hand financiers enjoy higher gross cash flows, for any given level of H operating profits. On the other hand, subsidiary dividends can help service the holding debt. Denote as $X_{h,\omega}^d$ the new default thresholds for the holding, accounting for the fact the dividends can be used to service debt. Thus $X_{h,\omega}^d \leq X_h^d$, because dividends work as another type of conditional guarantee, allowing H to survive in a subset of states with low operating cash flows. H debt value with positive ownership ω can now be written as the corresponding zero-ω value plus larger recovery upon default of the holding (second term) and a higher expected value of debt service (third term):

$$D_{0h}(P_h, P_s, \omega) = D_{0h}(P_h) + \phi \mathrm{E}\left[(1-\alpha)\, d_s^+ 1_{\{X_h < X_{h,\omega}^d\}}\right] + \phi \mathrm{E}\left[P_h 1_{\{X_{h,\omega}^d < X_h < X_h^d\}}\right] \tag{29}$$

The cum-dividend equity value is equal to the zero-ω value plus the dividends, in all states when the holding survives thanks to its own operating profits, plus the dividends net of debt service when dividends prevent H default:

$$E_{0h}(P_h, P_s, \omega) = E_{0h}(P_h, P_s) + \phi \mathrm{E}\{d_s^+ 1_{\{X_h > X_h^d\}} + \left[d_s^+ + X_h - P_h\right] 1_{\{X_{h,\omega}^d < X_h < X_h^d\}}\} \tag{30}$$

The same type of reasoning allows to write equivalent expressions for the case of two stand alone firms:

$$D_{01}(P_1, P_2, \omega) = D_{01}(P_1) + \phi \mathrm{E}\left[(1-\alpha)\, d_2^+ 1_{\{X_1 < X_{1,\omega}^d\}}\right] + \phi \mathrm{E}\left[P_1 1_{\{X_{1,\omega}^d < X_1 < X_1^d\}}\right]$$

$$D_{02}(P_{2,}\omega) = D_{02}(P_2); E_{02}(P_{2,}\omega) = E_{02}(P_2)$$

$$E_{01}(P_{1,}P_2,\omega) = E_{01}(P_1)+\phi \mathrm{E}\left[\left(d_2^+ + X_1 - P_1\right) 1_{\left\{X_{1,\omega}^d<X_1<X_1^d\right\}}\right]+\phi \mathrm{E}\left[d_2^+ 1_{\left\{X_1>X_{1,\omega}^d\right\}}\right]$$

Some tedious algebra leads to the stated results. ■

It follows that the difference between the group and stand alone values with positive intercorporate dividends coincides with the same difference in the case of infinitesimal ownership:

$$\nu_{0,HS}(P_{h,}P_s,\omega)-\nu_{01}(P_1,\omega)-\nu_{02}(P_2,\omega) = \nu_{0,HS}(P_{h,}P_s)-\nu_{01}(P_1)-\nu_{02}(P_2)$$

This result is not surprising, because dividends are paid out from the subsidiary to the holding only if it is not defaulting and only if it is not receiving rescue funds. Thus, they are paid out to the holding in the same states in which a stand alone pays dividends to another stand-alone corporation which, while having a stake in the company, is independently managed. We can now use this result to determine the value of the conditional guarantee.

2.2.4 The value of the conditional guarantee

We can now measure the value of the guarantee G, defined as the value of the group less the value of two comparable stand alone units:

$$G(P_h, P_s) = \nu_{0,HS}(P_{h,}P_s,\omega) - \nu_{01}(P_1,\omega) - \nu_{02}(P_2,\omega)$$

Dropping the argument of the functions for brevity, this is equal to the discounted expected value of the following payoff:

$$D_s - D_2 + E_h - E_1 = \tag{31}$$

$$= [P_s-X_s(1-\alpha)-\tau(X_s-X_s^Z)\mathbf{1}_{\{X_s>X_s^Z\}}-(P_s - X_s^n))]\mathbf{1}_{\left\{0<X_s<X_s^d,\ X_h>h(X_s)\right\}} =$$

$$= \alpha X_s \mathbf{1}\ _{\left\{0<X_s<X_s^d,\ X_h>h(X_s)\right\}}$$

for any level of intercorporate dividends**.**

The value of the guarantee, when the debt burden is the same in the stand alone and in HS, is equal to discounted bankruptcy cost that is avoided:

$$G(P_h, P_s) = \alpha\phi \mathrm{E}\left[X_s \mathbf{1}\ _{\left\{0<X_s<X_s^d,\ X_h>h(X_s)\right\}}\right] \tag{32}$$

Equation (32) directly implies the following result:

Proposition 4 *Consider a holding-subsidiary structure and two corresponding stand alone firms, with the same operating profits* (X_1 =

$X_h; X_2 = X_s$) and any intercorporate ownership. If the conditional transfer has positive probability at $P_h = P_1^*, P_s = P_2^*$, then the value of the holding-subsidiary structure exceeds the value of two comparable stand-alone firms: $G(P_h, P_s) > 0$.

Proof. Since $\alpha > 0$ and the set of payoffs satisfying (22) has positive probability, the subsidiary is levered and the guarantee in (32) is positive. This directly implies that the value of holding-subsidiary structures exceeds the value of two comparable stand-alone firms with the same face value of debt outstanding ($P_1 = P_h; P_2 = P_s$) and the same distribution of operating profits ($X_1 = X_h; X_2 = X_s$). This holds for any fixed face values of debt, including the optimal ones for stand alone firms ($P_h = P_1^*, P_s = P_2^*$). ■

This proposition shows that there are value gains of HS relative to stand alone companies, because they are able to provide a conditional guarantee to the subsidiary lenders that has non-negative value. Thus our model fits into the literature highlighting the "bright side" of internal capital markets. Our emphasis is however on the possibility to optimize the trade off between bankruptcy risk and the tax burden thanks to the internal capital market, whereas existing research focuses on its ability in circumventing imperfections arising from asymmetric information (Stein, 1997; Gertner, Scharfstein and Stein, 1994). We are also highlighting conditions ensuring that subsidizing weaker firms is value increasing - namely the presence of bankruptcy costs and no endogenous investment choice [11]. Under these maintained assumption, Proposition 4 implies the following:

Corollary 5 *Assume that the conditional transfer has positive probability. An entrepreneur prefers to incorporate his activities as holding and subsidiary rather than as stand alone companies, irrespective of the ownership share ω .*

Proof. Denote the optimized value of the guarantee as $G(P_h^*, P_s^*)$. Since Proposition holds for any (P_h, P_s), it holds *a fortiori* in P_h^*, P_s^*. Thus the optimal HS value, corresponding to the optimal choice of debt for the holding and subsidiary, is higher than the sum of two stand alone values. ■

Remark 6 *In the sequel (proposition 9) we are going to show that at the optimum the support region is non-empty. The positive probability of*

[11] Such subsidization is value reducing in a setting with agency problems between managers and shareholders which lead to distorted investment choices across activities (Scharfstein and Stein, 2000).

the transfer, requested in both proposition 4 and its corollary, obtains if the distribution of cash flows has positive density on the positive orthant. This is the case, for instance, when cash flows are normally distributed, as in Leland (2007) and in our numerical examples below.

Observe that HS value exceeds that of comparable stand alone firms even if, from Proposition 2, its equity value is lower. These two propositions thus reconcile the paradoxical findings that holding-subsidiary organizations are common across the globe despite lower equity values associated to the same operating profits.

We now establish some properties of the guarantee, that will turn out to be useful in the analysis of debt capacity and optimal capital structure. We refer the reader to Appendix C for the Proof.

Lemma 7 *If proportional bankruptcy costs are positive ($\alpha > 0$), then a) the guarantee is non increasing in P_h :*

$$\frac{\partial G(P_h, P_s)}{\partial P_h} \leq 0$$

and has a null derivative if and only if $P_s = 0$:

$$\frac{\partial G(P_h, 0)}{\partial P_h} = 0$$

b) the guarantee has a null derivative wrt P_s at $P_s = 0$; c) the guarantee is decreasing in P_s when the latter diverges:

$$\lim_{P_s \to +\infty} \frac{\partial G(P_h, P_s)}{\partial P_s} < 0$$

The value of the guarantee is non-increasing in P_h: for any joint cash flow distribution and any capital structure, reducing debt in the holding enlarges - or at least does not reduce - the rescue area. The effect associated with changes in subsidiary debt is less obvious. Raising P_s on the one hand contributes to the value of the guarantee by increasing the value of subsidiary cash flows that are saved when the holding succeed in rescuing it. On the other hand it reduces the value of the guarantee by making it less likely that H cash flows will be sufficient to service S debt. When debt in the subsidiary diverges, the second effect dominates and the marginal value of the guarantee is negative.

We are now ready to assess the optimal choice of debt in Holding Subsidiary structures, by addressing the tax-bankruptcy trade-off.

2.3 Debt capacity: Holding-Subsidiary versus Stand Alone firms

In this section we show that optimal capital structure entails a shift of debt from the holding to the subsidiary and we present conditions ensuring that debt capacity in HS is higher than in stand alone firms with the same cash-flows distribution. Throughout, we focus on the simpler case of infinitesimal ownership.

First, following (Leland, 2007), we rewrite the levered firm value in a stand-alone as unlevered firm value, V_{0i}, plus tax savings from interest deduction less default costs:

$$\nu_{0i}(P_i) = V_{0i} + TS_i(P_i) - DC_i(P_i) \tag{33}$$

where $TS_i(P_i)$ is the present value of tax savings, equal to the differential tax burden of the unlevered and the levered firm:

$$TS_i(P_i) = T_i(0) - T_i(P_i) = \tau_i \phi \left[\mathrm{E} X_i^+ - \mathrm{E}(X_i - X_i^Z)^+ \right] \tag{34}$$

and $DC_i(P_i)$ is the present value of the default costs incurred in because of leverage:

$$DC_i(P_i) = \alpha_i \phi \mathrm{E} \left[X_i \ \mathbf{1}_{\{0 < X_i < X_i^d\}} \right] \tag{35}$$

Tax savings are increasing in the face value of debt, since the latter enlarges interest deductions and the associated tax shield. Default costs too increase in the face value of debt, because the set of default states gets larger.[12] The optimal stand alone debt results from trading off these effects, as well known from seminal results by Kim (1978), among others. Appendix B gives necessary and sufficient conditions for the solution of the stand alone problem:

$$\min_{P_i \geq 0} \left[T_i(P_i) + DC_i(P_i) \right] \tag{36}$$

Moreover, it shows that the optimal leverage of a stand alone is positive, if an optimum exists. A sufficient condition for existence, that we maintain below, is that the sum of the tax burden and default costs in convex in debt. Results in Appendix B also show that the stand alone is unlevered when the tax rate is equal to zero.

[12] Tax savings are short a call option on X_i with strike X_i. The call is decreasing in debt, since the strike is increasing in it. Default costs are a barrier call option on X_i with zero strike and barriers equal to zero and X_i^d. The call is increasing in debt, since the upper barrier is increasing in it.

Second, we observe that the tax burden on HS affiliates coincides with the tax burden of their stand-alone counterparts with equal debt outstanding ($P_1 = P_h, P_2 = P_s$):

$$T_{HS}(P_h, P_s) = \tau\phi \left[\mathrm{E}(X_s - X_s^Z)^+ + \mathrm{E}(X_h - X_h^Z)^+\right] = \\ = T_1(P_h) + T_2(P_s) \tag{37}$$

On the contrary, default costs in HS can be written as:

$$DC_{HS}(P_h, P_s) = \alpha\phi\mathrm{E}\left[X_s \mathbf{1}_{\{0<X_s<X_s^d,\ X_h<h(X_s)\}} + X_h \mathbf{1}_{\{0<X_h<X_h^d\}}\right] = \tag{38}$$

$$= DC_1(P_h) + DC_2(P_s) - G(P_h, P_s)$$

When the affiliated firms have the same capital structure as their stand alone counterparts, subsidiary default costs are lower than the sum of the stand alone costs, the difference arising from the guarantee.

Last, turning to the entrepreneur's choice of HS leverage, it can be shown that maximizing HS value is equivalent to minimizing the sum of tax burdens and default costs for the stand alone companies, net of the guarantee. It corresponds to solving the following problem with respect to (non negative) face values of debt, P_h, P_s:

$$\min_{P_h \geq 0, P_s \geq 0} \left[T_{HS}(P_h, P_s) + DC_{HS}(P_h, P_s)\right] = \tag{39}$$

$$= \min_{P_h \geq 0, P_s \geq 0} \left[T_1(P_h) + T_2(P_s) + DC_1(P_h) + DC_2(P_s) - G(P_h, P_s)\right]$$

In order to solve (39), we do not introduce specific assumptions on the joint distribution function of the operational cash flows[13]. Consistently with the stand alone case of Leland, we study it - globally - for the case in which the objective function is convex, so as to ensure the existence of an optimum.

The intuition is straightforward. Imagine first transferring one unit of debt from the holding to the subsidiary. Lowering the holding debt has

[13] We simply assume that a technical, but innocuous condition holds: the function $xf(x, y)$, where $f(x, y)$ is the joint cash flow density, satisfies the dominated convergence condition when x diverges and $y > 0$.This allows us to exchange limits and integration in the proofs of the Appendices.

the same effect on its marginal tax burden and default costs than increasing the subsidiary's debt, apart from the guarantee. Overall, however, at positive debt levels, P_1^*, P_2^*, that maximize the value of two stand-alone firms, the marginal default cost in the subsidiary should be lower than the marginal default cost in the corresponding stand alone activity, thanks to the guarantee, while tax savings are the same. Since the guarantee is decreasing in the holding debt, it is convenient to decrease the latter. Debt in the subsidiary must be larger than in the corresponding stand alone in order to re-establish equality with tax savings. How large? If the guarantee is valid enough, we can probably do better than transferring one unit of debt from the holding to the subsidiary. It may even happen that we can increase the subsidiary debt by more than one unit. In this case total debt capacity is higher in HS than in stand alone firms. When will this be the case? When the subsidiary has low initial tax shield, or when the negative impact of increasing overall debt on default costs is mitigated by the guarantee. Proving this result is less straightforward.

We give first a local result, then a global one. The local result imposes no condition for the debt shift from the holding to the subsidiary; it ties the increase of the overall debt capacity to its net effect on the guarantee:

Proposition 8 *Let P_1^*, P_2^* be the optimal debt levels for two stand alone companies. Then, locally, (i) the holding debt can be decreased - and the subsidiary increased - so as to increase the overall group value; (ii) provided that the value of the guarantee is increasing in P_2 at P_2^* , and*

$$\left(- \left. \frac{dX_1^d}{dP_1} \right|_{P_1^*} + \frac{1}{1-\tau} \right) \int_0^{X_2^{Z*}} xf\left(x, h(x)\right) dx - \left. \frac{dX_2^d}{dP_2} \right|_{P_2^*} \int_{X_1^{d*}}^{+\infty} X_2^{d^*} f(X_2^{d^*}, y) dy > 0$$

then the overall debt capacity increases with respect to $P_1^* + P_2^*$.

Proof. In order to increase value, we need to decrease the following function:

$$T_{HS}(P_h, P_s) + DC_{HS}(P_h, P_s) =$$

$$= T_1(P_h) + T_2(P_s) + DC_1(P_h) + DC_2(P_s) - G(P_h, P_s)$$

Since the derivative of both $T_1 + DC_1$ and $T_2 + DC_2$ wrt their own arguments is null at the optimum of the stand alone leverage, P_1^*, P_2^*, then the impact of a local variation depends on the sign of the derivatives of the guarantee.

i) We know from lemma 7 that decreasing the holding debt increases the guarantee (at any positive leverage of the subsidiary, including the optimal stand alone one), and therefore reduces $T_{HS}+DC_{HS}$, as needed. Given this, one can reduce the holding debt and increase the subsidiary one so that $T_{HS}+DC_{HS}$ decreases, as follows. We have:

$$d\left(T_{HS}\left(P_1^*,P_2^*\right)+DC_{HS}\left(P_1^*,P_2^*\right)\right)=-\frac{\partial G\left(P_1^*,P_2^*\right)}{\partial P_1}dP_1^*-\frac{\partial G\left(P_1^*,P_2^*\right)}{\partial P_2}dP_2^* \tag{40}$$

and the differential is negative if and only if

$$+\frac{\partial G\left(P_1^*,P_2^*\right)}{\partial P_1}dP_1^*+\frac{\partial G\left(P_1^*,P_2^*\right)}{\partial P_2}dP_2^*>0$$

Consider expression (67) for $\frac{\partial G\left(P_1^*,P_2^*\right)}{\partial P_2}$, neglect $\alpha\phi$ and recognize that such derivative has a negative and a positive part. Define them as follows:

$$\begin{aligned}G_2^n &:= -\frac{1}{1-\tau}\int_0^{X_s^Z} xf\left(x,h(x)\right)dx+\\ &\quad -\frac{dX_2^d}{dP_2}\int_{X_s^Z}^{X_s^d} xf(x,h(x))dx\\ G_2^p &:= \frac{dX_2^d}{dP_2}\int_{X_h^d}^{+\infty} X_s^d f(X_s^d,y)dy\end{aligned}$$

Then the differential (40) is negative **if** there exists a couple $dP_1^*<0, dP_2^*>0$ such that

$$\frac{\partial G\left(P_1^*,P_2^*\right)}{\partial P_1}dP_1^*+G_2^n dP_2^*=0$$

For such couple (in particular, for $dP_2^*>0$), indeed, $G_2^p dP_2^*>0$. It suffices to take

$$dP_1^*<0, dP_2^*=-\frac{\partial G\left(P_1^*,P_2^*\right)}{\partial P_1}\frac{1}{G_2^n}dP_1^*$$

since the last differential is positive.

ii) provided that $\frac{\partial G\left(P_1^*,P_2^*\right)}{\partial P_2}>0$, in order to demonstrate the assert we need to show that there exists a variation in P_2^*, of the type $dP_2^*=-dP_1^*+\varepsilon,\varepsilon>0, dP_1^*<0$, such that the differential (40) is negative.

Indeed, the differential is negative, for $dP_1^* < 0$ and $\frac{\partial G\left(P_1^*,P_2^*\right)}{\partial P_2} > 0$, if and only if

$$\varepsilon > -\frac{\frac{\partial G\left(P_1^*,P_2^*\right)}{\partial P_1} - \frac{\partial G\left(P_1^*,P_2^*\right)}{\partial P_2}}{\frac{\partial G\left(P_1^*,P_2^*\right)}{\partial P_2}} dP_1$$

The right hand side of the last expression is positive, as required, as soon as the net effect of the compensated ($dP_1 = -dP_2$) change in the guarantee is positive. The condition in the proposition statement is indeed equivalent to having $\partial G/\partial P_1 - \partial G/\partial P_2 > 0$, since the difference gives

$$\left(-\left.\frac{dX_1^d}{dP_1}\right|_{P_1^*} + \frac{1}{1-\tau} \right) \int_0^{X_2^{Z*}} xf\left(x, h(x)\right) dx - \left.\frac{dX_2^d}{dP_2}\right|_{P_2^*} \int_{X_1^{d*}}^{+\infty} X_2^{d^*} f(X_2^{d^*}, y) dy$$

■

The global result excludes zero leverage for the subsidiary and sets an upper bound on the ratio α/τ, below which the marginal process of transferring debt is preserved at the final optimum and the final debt capacity increases. Denote as X_s^{Z**}, X_s^{d**} the tax shield and default threshold of a subsidiary with debt $P_1^* + P_2^*$.

Theorem 9 *If tax burdens and default costs net of the guarantee are convex in debts (i) There cannot be a local minimum for the HS problem in which the subsidiary is unlevered (ii) if the ratio of default costs to the tax rate is bounded above by*

$$\frac{\Pr(X_s > X_s^{Z**}) \frac{dX_2^{Z**}}{dP_s}}{X_s^{d**} \left(\frac{dX_s^{d**}}{dP_s} \Pr\left(X_s = X_s^{d**}, X_h < X_h^{d^*}\right) + \frac{dh}{dP_s} \Pr(0 < X_s < X_s^{d**}, X_h = h(X_s) \right)}$$

then the holding is optimally less levered than a stand alone ($P_h^* < P_1^*$) and total debt in the holding-subsidiary organization - which coincides with the subsidiary one - is higher than in two stand alone companies ($P_s^* > P_1^* + P_2^*$).

Proof. See Appendix C[14]. ■

[14] As mentioned above, convexity in the hypothesis is concavity of HS value, for well posedness of the problem. Under convexity, the conditions given in the theorem are necessary and sufficient. Necessity follows from the fact that the constraint qualification condition holds.

Remark 10 *When the theorem holds, the support region is non-empty. Proposition 4 and its corollary apply under the conditions of remark 6.*

The upper bound in the theorem can be written as:

$$\alpha X_s^{d**}\left(\frac{dX_s^{d**}}{dP_s}\Pr\left(X_s = X_s^{d**}, X_h < X_h^{d^*}\right) + \frac{dh}{dP_s}\Pr(0 < X_s < X_s^{d**}, X_h = h(X_s)\right)$$

$$< \tau \Pr(X_s > X_s^{Z**})\frac{dX_s^{Z**}}{dP_s}$$

The left hand side represents (is actually an upper estimate of) the impact of changing the subsidiary debt on default costs, taking the guarantee mitigation into account. It indeed considers the impact only over the boundary of the rescue region. The right hand side represents the impact on the tax burden. Since the tax burden is a call option with strike X_s^{Z**}, this is nothing else than the variation in a call when you change its strike. Thus, debt in the subsidiary is higher than debt in the two stand alone firms if - at $P_s = P_1^* + P_2^*$ - the marginal increase in default costs on the left hand side is lower than the marginal savings in taxes on the right hand side. When the subsidiary has low initial tax shield, the right hand side is high, and the constraint is likely to be satisfied. The same happens when the lhs is low, namely when the negative impact of increasing overall debt on default costs is mitigated by the guarantee.

We will see that this condition is met - in the base case of Leland - when firms cash flows are equal in distribution. Not only it is satisfied, but the optimal debt in the holding is zero. Only when the holding company is much larger than its subsidiary, we find positive - but almost negligible - leverage for the holding. However, we first complete our analysis of firm scope by comparing HS to mergers.

2.4 The Conglomerate Merger Case: Unconditional Guarantee

In a conglomerate-merger case, the two activities are incorporated as one firm - with cash flow $X_m = X_1 + X_2$ - and are jointly liable vis-à-vis lenders. Leland (2007) finds the solution to the maximization of the merger value, relative to its debt P_m:

$$\nu_0(P_m) = E_{0m}(P_m) + D_{0m}(P_m) \tag{41}$$

where $E_0(P_m)$ and $D_{0m}(P_m)$ are computed according to (14) and (16) with $i = m$.

As in the case of other organizations, the problem of value maximization can be equivalently stated as the minimization of tax burden:

$$T_m = \tau\phi\left[\mathrm{E}(X_m - X_m^Z)^+\right] \tag{42}$$

plus default costs:

$$DC_m = \alpha\phi\mathrm{E}\left[X_m \mathbf{1}_{\{0<X_m<X_m^d\}}\right] \tag{43}$$

where X_m^Z and X_m^d are defined as in (8) and (12).

Merging two activities may allow the resulting conglomerate to increase debt capacity above the one of the two stand alone firms, when cash flow pooling between the two activities reduces the probability of default. This brings enhanced tax advantages because of interest deductions, as predicted by Lewellen (1971). It also allows to use the losses from one unit to offset taxable income from the other unit, thus reducing the negative impact of tax asymmetries (Majd and Myers, 1987). However, one unprofitable conglomerate division may absorb the cash flows of a profitable one, reducing the value of limited liability for the unlevered conglomerate - the "Sarig effect". Leland (2007) shows that the Sarig effect tend to dominate when cash flow volatility differs, or when the correlation between activities' cash flow is high, so that diversification opportunities are limited. Below we compare the merger to HS structures.

2.5 Comparison with HS

The value differential between HS and a conglomerate merger is given by:

$$\Delta\nu_{0HS}(P_h, P_s, P_m) = \nu_{0,HS}(P_h, P_s) - \nu_{0m}(P_m) = -\Delta V_{0m} - \Delta T_{HS} - \Delta DC_{HS} \tag{44}$$

where

$$\Delta V_{0m} = \phi\left[\mathrm{E}(X_m)^+ - \mathrm{E}X_h^+ - \mathrm{E}X_s^+\right] \tag{45}$$

is the Sarig effect, i.e. the loss to the unlevered conglomerate due to the pooling of cash flows which reduces merger value when one activity is unprofitable. Such loss of value does not occur in HS, because activities are separately incorporated and limited liability allows to abandon an unprofitable activity. The second term, ΔT_{HS}, is the differential tax burden, i.e. equation (37) minus equation (42), while the third term is the differential default cost, i.e. equation (38) minus equation (43).

These two effects, due to leverage, are respectively equal to:

$$\Delta T_{HS} = T_h + T_s - T_m = \tau\phi[E(X_h - X_h^Z)^+ + \mathrm{E}(X_s - X_s^Z)^+ - \mathrm{E}(X_m - X_m^Z)^+] \tag{46}$$

$$\Delta DC_{HS} = DC_h + DC_s - DC_m =$$

$$= \alpha\phi[\mathrm{E}\left(X_h \mathbf{1}_{\{0<X_h<X_h^d\}}\right) + \mathrm{E}\left(X_s \mathbf{1}_{\{0<X_s<X_s^d, X_h<h(X_s)\}}\right) - \mathrm{E}(X_m 1_{\{0<X_m<X_m^d\}})] \tag{47}$$

Substituting from (45, 46, 47) in the difference (2.5), we get:

$$\frac{1}{\phi}\Delta\nu_{0,HS} =$$

$$-\mathrm{E}(X_m)^+ + \mathrm{E}X_h^+ + \mathrm{E}X_s^+ +$$
$$+\tau[\mathrm{E}(X_m - X_m^Z)^+ - \mathrm{E}(X_h - X_h^Z)^+ - \mathrm{E}(X_s - X_s^Z)^+] +$$
$$+\alpha\left[\mathrm{E}(X_m 1_{\{0<X_m<X_m^d\}}) - \mathrm{E}\left(X_h \mathbf{1}_{\{0<X_h<X_h^d\}}\right) - \mathrm{E}\left(X_s \mathbf{1}_{\{0<X_s<X_s^d, X_h<h(X_s)\}}\right)\right] \tag{48}$$

In the absence of any parametric or distributional restriction (on taxes or cash flows respectively) this value differential cannot be signed, even when we assume comparable debt levels in the two organizations, i.e. $P_h + P_s = P_m$. Using results in Denuit, Genest and Marceau (1999) we can see that at least the first and second term in the sum have opposite signs: limited liability favours HS over mergers, while the tax burden is higher in HS, depressing its value, because loss offsetting is imperfect.

We can however establish the following results concerning $\Delta\nu_{0,HS}$ when computed at the optimum of both the HS and merger principal, $\Delta\nu_{0,HS}(P_h^*, P_s^*, P_m^*)$**:**

Proposition 11 *Assume that the conditional transfer has positive probability. Then the value of holding-subsidiary structures exceeds the value of the merger of two comparable activities with the same operating profits ($X_m = X_h + X_s$), if either (i) activities cash flows are equal and perfectly correlated, or (ii) they are normal, with $\rho^Q < \rho \leq 1$ and (common) volatility $\sigma > \sigma_L$, where*

$$\begin{cases} \rho^Q < 1 \\ \sigma_L = \arg\min \nu^*(P_m) \end{cases}$$

or (iii) they are normal, with $\rho^R < \rho \leq 1$ and distinct volatilities: $\sigma_h \neq \sigma_s$. In both cases $\Delta\nu_{0,HS}(P_h^*, P_s^*, P_m^*) > 0$.

Proof. Let us ad and subtract to the value differential (2.5) the value of two stand alone firms with cash flows $X_h = X_1, X_s = X_2$:

$$\Delta\nu_{0,HS} = [\nu_{0,HS}(P_h, P_s) - \nu_{01}(P_1) - \nu_{02}(P_2)] - [\nu_{0m}(P_m) - \nu_{01}(P_1) - \nu_{02}(P_2)] \tag{49}$$

We know that the first is always positive, when the transfer event has positive probability (for instance, when theorem 9 holds and remark 6 apply). The second differential

$$\begin{gathered}
\nu_{0m}(P_m) - \nu_{01}(P_1) - \nu_{02}(P_2) = \\
\phi\left[-\mathrm{E}(X_m)^+ + \mathrm{E}X_1^+ + \mathrm{E}X_2^+\right] + \\
+\tau\phi[\mathrm{E}(X_m - X_m^Z)^+ - \mathrm{E}(X_1 - X_1^Z)^+ - \mathrm{E}(X_2 - X_2^Z)^+] + \\
+\alpha\phi\left[\mathrm{E}(X_m 1_{\{0<X_m<X_m^d\}}) - \mathrm{E}\left(X_1\mathbf{1}_{\{0<X_1<X_1^d\}}\right) - \mathrm{E}\left(X_2\mathbf{1}_{\{0<X_2<X_2^d\}}\right)\right]
\end{gathered} \tag{50}$$

cannot be signed without additional assumptions. We are going to show that it is null under (i) and non-positive under (ii) and (iii).

(i) Consider the case of equal cash flows for merged activities, and denote their common value as X. Notice that (a) the corresponding stand alone firms have the same optimal debt and cash-flow thresholds, namely $P_i = P^*, X_i^d = X^d, X_i^Z = X^Z, i = 1, 2$ (b) the volatility of $2X$ is twice the one of X, if $\rho = 1$. Since the merger can thus be thought of as a stand alone with double cash flow and double volatility, homogeneity of degree one of optimal debt applies, and we have $P_m^* = 2P^*, X_m^d = 2X^d, X_m^Z = 2X^Z$.It follows that the unlevered-value differential is null:

$$\phi\left[-\mathrm{E}(X_m)^+ + \mathrm{E}X_1^+ + \mathrm{E}X_2^+\right] = \phi\left[\mathrm{E}(2X)^+ - 2\mathrm{E}X^+\right] = 0.$$

The differential tax burden is null too:

$$\tau\phi[\mathrm{E}(X_m - X_m^Z)^+ - \mathrm{E}(X_1 - X_1^Z)^+ - \mathrm{E}(X_2 - X_2^Z)^+] =$$

$$= \tau\phi[\mathrm{E}(2X - (P_m - D_m))^+ - 2\mathrm{E}(X - (\frac{P_m}{2} - \frac{D_m}{2}))^+] = 0 \tag{51}$$

and differential default costs are null too:

$$\begin{gathered}
\alpha\phi\left[\mathrm{E}(X_m 1_{\{0<X_m<X_m^d\}}) - \mathrm{E}\left(X_1\mathbf{1}_{\{0<X_1<X_1^d\}}\right) - \mathrm{E}\left(X_2\mathbf{1}_{\{0<X_2<X_2^d\}}\right)\right] \\
= \alpha\phi[\mathrm{E}(2X\mathbf{1}_{\{0<2X<X_m^d\}}) - 2\mathrm{E}X\mathbf{1}_{\{0<X<\frac{X_m^d}{2}\}}] = 0
\end{gathered} \tag{52}$$

As a result, the value of merger and stand alone coincide[15]. Summing up, in this case we have

$$\nu_{0,HS}(P^*_{h,}P^*_s) - \nu_{01}(P^*_1) - \nu_{02}(P^*_2) > 0,$$
$$\nu_{0m}(P^*_m) - \nu_{01}(P^*_1) - \nu_{02}(P^*_2) = 0$$

Thus $\nu_{0HS}(P^*_h, P^*_s, P^*_m) > 0$.

(ii) Let activities' cash flows be normally distributed with the same volatility (even with different means), and let their volatility satisfy the condition stated in the theorem. Then Proposition 2 in Leland (2007) indicates that there exists a correlation coefficient ρ^Q such that the second term in (49) is negative, when evaluated at P^*_m, provided that correlation is greater than ρ^Q and less than perfect. For perfect correlation the second term is null again.

(iii) If cash flows are normally distributed with different volatilities, then we know from Proposition 4 in Leland (2007) that there exists a correlation coefficient ρ^R such that the second term in (49) is negative, when evaluated at P^*_m, provided that correlation is greater than ρ^R and less than perfect. For perfect correlation the second term is null again.

In cases ii) and iii) it follows that $\Delta\nu_{0,HS}(P^*_h, P^*_s, P^*_m) > 0$. ■

Case (i), when activities are equal with perfectly correlated cash flows, highlights another important feature of HS, namely debt diversity. This is allowed for by separate incorporation. If cash flows and debt in the two activities were the same, the holding would not be able to rescue its subsidiary ever. This is precisely the situation in a merger, because activities are incorporated as one firm and cannot have different debt levels. As Leland (2007) shows, the value of a merger coincides with the value of two stand alone firms when diversification opportunities vanish. In HS, it becomes possible for H to rescue S as one unit of debt gets transferred from the holding onto its subsidiary: thus default costs fall, the optimal debt increases and the tax burden drops. Thus debt diversity preserves the value of the guarantee when diversification opportunities vanish.

In the other two cases, the Sarig effect - due to unconditional cash flow pooling - is large enough to make a merger less desirable than two stand alone firms. The value of HS is larger because cash flow pooling - i.e. the guarantee - is conditional.

We now turn to a numerical study of optimal firm scope with endogenous leverage. Our numerical exploration follows Leland's main assumptions, including normality of cash flows. The above theorem,

[15] This is recognized in the Gaussian case by Leland (2007).

part (ii) and (iii), explains why we will always find with higher value for HS than for mergers - unless we abandon the maintained assumption of .an ex post enforceable guarantee.

3 Numerical analysis

This section analyzes the properties of different organizational modes through numerical methods, assuming that the annual cash flow distribution is Normal. The parameters are equal to the base case in Leland (2007), which is consistent with a typical firm issuing BBB-rated unsecured debt. Table 1 reports parameter values. Expected operating cash flow for each activity, $Mu = 127.6$, is chosen such that its present value is $X_0 = 100$. Operating cash flow at the end of 5 years has standard deviation (Std) of 49.2, consistent with an annual standard deviation of cash flows equal to 22.0 ($= 49.2/\sqrt{5}$) if annual cash flows are independently distributed in time. Henceforth we express volatility σ as an annual percent of initial activity value X_0, e.g. $\sigma = 22\%$. The tax rate $\tau = 20\%$ and the default cost parameter $\alpha = 23\%$ are chosen so as to generate optimal leverage and recovery rates consistent with the BBB choice.

Insert here Table 1

Table 2 shows the optimal capital structure and value for a firm with base-case parameters. The first column reports results for a stand alone. The second, third and fourth columns refer to holding, subsidiary and average affiliated company respectively, while the last column to half of a conglomerate. The correlation coefficient between the units cash flows is set equal to 0.2, as in Leland, so as to allow for comparison. We checked that results are insensitive to intercorporate dividends ω, as implied by our analytical results.

Insert here Table 2

3.1 HS versus stand alone

The optimal face value of debt, which is equal to 57.2 for a stand alone company, reaches 110 in the "average" HS affiliate, consistent with our analytical results establishing higher debt capacity for HS. Accordingly, both expected tax savings and default costs are smaller in the former (2.33 and 0.90) than in the latter case (7.31 and 4.07). The value increase due to leveraging jumps from 1.43 for the stand alone firm to 3.24 for the representative HS affiliate. This jump is the effect of differential tax savings and differential default costs, and captures the value of the internal capital market in groups. As a result, HS affiliate value (83.29) exceeds that of a SA (81.23). The beneficiary of the internal capital market is the initial owner of the two activities, who sells them for more.

They benefit despite the fact that intercorporate guarantees reduce the market value of equity below that of stand alone counterparts - namely, $E_h^* + E_s^* < E_1^* + E_2^*$. Proposition 2 emphasizes the transfer from H to S, while in this exercise we appreciate the interplay between the transfer and the level of debt. Specifically, in Table 2 we see that the value of equity in the stand alone is larger than in the subsidiary (39.01 instead of 0.07), because of its much lower level of debt (57.2 versus 220). On the contrary, the value of equity in the holding is larger than the stand alone one (49.46 versus 39.01), even if part of its cash flow is being transferred to the subsidiary lenders, because its debt is lower (0 versus 57.2).[16] This fact could mistakenly be interpreted, by an outside observer such an econometrician, as the consequence of HS inefficiency.

Figure 2 helps understanding the optimal leverage strategy of the HS in this case. By setting debt to zero in the holding, its default threshold coincides with the horizontal axis. This maximizes the transfer area A, given P_s and X_s^d. Raising more debt in the subsidiary ensures that its tax threshold moves to the right - making it less likely that taxes will be paid both when the firm is bankrupt and when it is not. Clearly also the subsidiary default threshold, X_s^d, moves to the right, but the guarantee will often make sure that the subsidiary does not default - while enjoying tax privileges.

It is worth noting the similarity between this type of HS and leveraged buy-outs. The tax burden of debt drops from 17.62% of operating cash-flow in a stand alone to 12.70% on average for HS and 5.39% for the subsidiary alone. In firms taken private through LBOs, the tax burden dropped from 20% to 1% in the first two years and to 4.8% in the third year (Kaplan, 1989). In Table 2, the default threshold for a subsidiary is 249, way above the mean operating income. In a sample of distressed highly leverage transactions, all sample firms had operating margins in excess of the industry median (Andrade and Kaplan, 1998). Last but not least, the model implies leverage in excess of 95% in subsidiaries. This is also observed in the first wave of LBOs, where our assumption of no agency costs applies well.[17]

[16] We will see in later sections that the opposite may happen, i.e. $E_h^* < E_1^* < E_s^*$. However, it will still be the case that the average value of SA equity exceeds that of HS.

[17] Private equity partners often need to raise new funds in the market because of the limited temporal commitments of financiers, and this is possible only if their

3.2 HS versus conglomerate mergers

In Leland (2007), the value of a conglomerate merger exceeds that of two stand alone activities - for the base case parameter - due to the benefits of cash flow pooling which increase its debt capacity. His results emerge from a comparison between the first and the last column of Table 2.

We now turn to the numerical comparison between half of a conglomerate and a representative affiliate of a holding-subsidiary structure. The optimal debt in HS is far larger than in conglomerates (110 versus 58.5). As a consequence of higher debt, expected default costs rise to 4.07 as opposed to 0.61 but the tax burden in HS falls to 12.70 as opposed to 17.77 in the conglomerate. As a consequence, the value of the average HS affiliate, 83.29, exceeds that of half a conglomerate, 81.57.

The possibility of raising different debt levels in the two affiliates (0 and 220) versus an equal debt level of 58.5 in merger divisions enhances the value of HS, by allowing it to better exploit both conditional support and the asymmetry of taxation. Indeed, raising more debt from the subsidiary increases its no tax threshold, which reaches 102.93, against a mean cash flow of 127.63. Its tax savings are as large as 14.62, which compare to 2.18 in the conglomerate division. However, the default threshold of the subsidiary is pushed up to 249. Such a burdensome debt service can be sustained because of conditional transfers from the holding company.

In the following section we assess whether these patterns hold when diversification opportunities change.

3.3 Capital structure and value with changing correlation

Leland (2007) shows that gains from a merger (M) disappear together with diversification: as the correlation coefficient between activities' cash flows tends to 1, the default costs of the merger converge to those of stand alone firms, so does its debt level and overall value. The HS structure also exploits diversification. One may thus expect that, as correlation among cash flows increases, the transfers from the H to S will become less likely and the optimal face value of debt will converge to the stand alone level.

The first part of this reasoning is correct: as the correlation coefficient ρ increases from -0.8 to 0.8, the (unreported) probability of a transfer from H to S halves. The second part of the argument is however incorrect: debt in HS continues to be much larger than in SA, and very

reputation is good. Moreover, subsidiary managers receive bonuses only when they repay their debt obligations. See Jensen (2007).

diverse between H and S. Figure 3 displays such result in the bottom right panel.[18]

Insert here Figure 3

The optimal face value of debt in HS actually increases in ρ. In conglomerates the opposite holds: debt falls as the probability of paying twice bankruptcy costs increases. In other words, the unconditional guarantee has no value in the conglomerate at $\rho = 1$, while the conditional guarantee still works in HS thanks to debt diversity which ensures that H and S have different levels of earnings after interest. Thus, lower debt in H ensures that H can still rescue S even if they have the same operating cash flows - the reason being that H has larger earnings after interests. Furthermore, H never incurs into bankruptcy costs having zero optimal leverage. Thus, debt diversity enhances the value of the conditional guarantee, the more so the larger is ρ. Consider in fact that expected default costs $E\left[\alpha X_s \mathbf{1}_{\ 0<X_s<X_s^d}\right]$ are increasing in X_s^d, not only because the probability of default increases but also because conditional default costs, αX_s, are larger. The larger is ρ, the likelier it is that H cash flow suffices to rescue S, i.e. $X_h > h(X_s)$, when conditional default costs are also large.

Consistent with implications of our Proposition 2, HS **equity value** $(E_1^* + E_2^*)$ is lower than in the case of stand alone firms for all correlation coefficients (upper right panel of Figure 3). The wedge between conglomerate and HS equity value increases in ρ. Transfers from stockholders to lenders disappear in the merger with diversification benefits, reaching zero at $\rho = 1$. On the contrary, they are still positive in HS thanks to debt diversity across affiliates.

Finally, and most importantly, the **total value** differential between HS and M - and stand alone firms - is always positive (top left panel of Figure 3), and achieves a maximum for $\rho = 1$. Cash flows after interest are reduced in the subsidiary thanks to its high debt level. On the contrary, tax asymmetries hit the conglomerate the most because there is no profit smoothing between the two activities, which obtain the same operating cash flows and the same after tax cash flows.

4 HS, M and SA when activities differ

Numerical results so far refer to two symmetric activities, that differ only - in the HS structure - because one is assumed to support the other. The

[18] Contrary to the face value, the market value of S debt falls as correlation increases (see upper right panel of Figure 3): lenders required spread grows in ρ in anticipation of reduced support by H.

analysis below refers to cases when activities differ in either cash flow volatility (σ), or in size (Mu) or in proportional bankruptcy costs (α).

This investigation deserves attention for two reasons. First, Rajan et al. (2000) point to potential inefficiencies stemming from diversity - in size and investment opportunities - across conglomerate activities. While they focus on inefficiencies arising from capital budgeting, Leland (2007) highlights that cash flow diversity has a cost in conglomerates because of the larger foregone value of limited liability: conglomerate may turn out to have lower value than stand alone organizations only when activities are asymmetric. We now assess whether HS are more or less valuable when activities are diverse, in the BBB case of Leland.

Second, we observe that strategic alliances (Robinson, 2008), venture capital funds (Sahlman, 1990) and innovative firms (Allen, 1998) often adopt a HS structure, with riskier ventures incorporated as subsidiaries. The same is true in traditional business groups (Bianco and Nicodano, 2006; Masulis et al., 2008). This suggests that HS value is not invariant to the relative features of H and S. Below, we endogenously derive the characteristics of holding and subsidiaries that maximize HS value, which is a further contribution of this paper relative to prior literature.

The following proposition summarizes our main numerical findings, under our parametric assumptions:

Conclusion 12 *Assume asymmetric cash-flow distributions for BBB calibrated companies. Then (i) the optimal HS structure has higher value than competing organizations, and value gains increase with risk and bankruptcy costs asymmetries between activities; (ii) in the optimal HS structure, the exogenous default cost parameter and the size of the holding are at least as large as those of its subsidiary. The subsidiary, in turn, has at least as risky a cash-flow as the holding.*

These results hold for the case, displayed in Tables 3 to 5, when correlation between cash-flows is equal to 0.2, as well as for the unreported range {-0.8, +0.8}.

Table 3 displays numerical results when the two activities have proportional bankruptcy costs respectively equal to 23%, as in the base case, and 75%. With larger bankruptcy costs, the optimal value of a stand alone firm drops from 81.23 (see the second column) to 80.83 (first column) as its face value of debt reduces from 57.2 to 33. In the case of HS, by contrast, it turns out that the activity with larger bankruptcy costs should be the holding company, because - under the optimal capital structure - H never pays them. Both the optimal capital structure and group value do not change as default costs in the subsidiary increase from 23% to 75%. It follows that value gains from group structure increase (from 3.24 in Table 2 to 3.15 in Table 3) with asymmetries in default costs across activities.

Insert here Table 3

The value of a conglomerate merger case also falls from 163.14 to 162.47 with asymmetric default costs, as joint incorporation of activities constrains divisions with diverse bankruptcy costs to the same face value of debt. The value gains from a HS, relative to the M, structure grow from 3.34 in the symmetric case to 3.94 in the current, asymmetric one.

Unreported optimizations assess the cost of a suboptimal HS structure, associated with the subsidiary bearing higher proportional default costs than its holding. Its face value of debt falls to 107 (as opposed to 220). Due to a reduced tax shield, HS value is now lower (162.37) than the one of the merger (162.47), and this holds for all correlations between -0.8 and +0.8. However, even a suboptimal HS dominates the stand alone organization, consistent with our analytical results.

Table 4 concerns the case of different risk, with one unit having an annualized cash flow volatility of 44% as opposed to 22% of the other. We know that the tax shield has higher value with a riskier cash flow, because the firm pays taxes when earnings after interests are positive, but does not get a comparable tax refunds in the opposite situation. It is therefore unsurprising to find higher optimal debt, and associated tax shield, in all organizations. A comparison between the first and the second column reveals that the higher volatility unit, when incorporated as a stand alone, has higher face value of debt (83 instead of 57), and is accordingly charged a much higher spread (6.2% as opposed to 1.26%) by lenders. This implies increased tax savings (4.66 versus 2.33), and a higher value of the riskier stand alone (84.84 versus 81.23) - even though its equity value drops from 39.01 to 36.1. The total value of these two stand alone is equal to 166.07.

Insert here Table 4

The conglomerate value is 163.24, only marginally higher than in the base case of equal risk across divisions, and lower than the stand alone value. This result echoes Leland (2007) observation that merging two diverse activities may reduce firm value, despite diversification gains. On the one hand the riskier division is more likely to drag the safer one in default, while on the other the tax shield is constrained to be equal across the two diverse activities.

This is not a problem for the HS structure, as the holding can use its limited liability to avoid joint default and diverse debt levels to tailor the tax shields to each activity needs. Consistent with empirical evidence, we find that subsidiaries are riskier than their holding companies in the optimal group structure. A riskier holding incurs into larger losses more often than a safer one. Hence it would not be able to rescue its subsidiary as often as a safer one. Moreover, a riskier holding would suffer more than a riskier subsidiary from the asymmetric nature of taxation, as it uses less the tax shield of debt. Going back to Table 4, subsidiary debt reaches 223 (up from 220 in the base volatility case), ensuring that interests shield the larger profits - which are now more likely - from taxes. Total group value is now equal to 170.12, exceeding the value of the group in the base case (166.58), when the subsidiary is less risky. Importantly, value gains relative to SA (M) increase from 3.24 (3.34) to 5.26 (7.36). [19]

The last case we examine is the one of differing size. Simulation results in Table 5 refer to a situation where the expected cash flow of one activity is five times the other's. In all the organizations, value falls relative to the symmetric case. For a SA and HS structures, the reduction is small (SA from 162.46 to 162.44; HS from 166.58 to 165.66) provided that the smaller activity is the subsidiary. On the contrary, M value drops from 163.14 to 162.98. Once again, the larger company is more likely to drag the smaller, healthy one into insolvency; moreover both are constrained to the same tax shield.

Insert here Table 5

In the HS case, smaller activities should be the ones that receive conditional support. The smaller subsidiary - see the fourth column - raises less debt than the one of the base case (121.33 as opposed to 220), with two consequences. On the one hand the holding company wants to raise debt as well so as to reduce taxation, on the other it can do so without compromising the provision of support to its subsidiary- thanks to its size. Debt appears in the holding company (63.33). A HS with a large subsidiary would be suboptimal, as the large subsidiary could hardly be rescued by a small holding. Consequently, it would raise less debt with a corresponding reduction in both the tax shield and value (163.25). This is however still higher than in the competing organizations.

[19] The size of these gains makes it more likely that they will outweigh costs associated with HS structures, which are not included in the present analysis. This may explain why HS structures are widespread in innovative industries, where the risk of subsidiary activities is large relative to that of holding companies.

All the previous qualitative results under asymmetry hold true when correlation varies: we conclude that - at least in Leland's set up - the ability of groups to create value by optimally trading off taxes and bankruptcy costs is a strikingly robust result - absent incentive problems.

5 Informal guarantees

So far we assumed the guarantee to be contractual and enforceable in court. Often guarantees are not legally binding, though: comfort letters, for instance, are legally unenforceable promises of rescue sent by the parent to its subsidiary's lenders. Subsidiary lenders may nonetheless expect support from the holding company with some positive probability Boot et al. (1993) argue that the holding trades-off the benefits from improved future credit conditions with the costs of reduced financial integrity in deciding whether to honor comfort letters.[20] In this section, we will assume that lenders attribute an exogenous probability $\pi < 1$ to ex post rescue, whereas we had implicitly set such probability to 1 in the previous ones. This is a short-cut for modelling an informal guarantee without resorting to a dynamic setting, where such probability would become endogenous.

Table 6 reports numerical results when the probability, π, is equal to 0.5 instead of 1, which is the value implicit in Tables 2 to 5.[21] Intercorporate dividends are equal to zero ($\omega = 0$).

Insert here Table 6

The observations concerning the comparison between stand alone firms and HS hold true (and carry over to other positive values of the probability π). The subsidiary is still able to raise a larger amount of debt (69) relative to its stand alone counterpart (57.2). Total debt capacity also increases in the HS (124) relative to the case of two stand alone firms (114.4) thanks to its lower default probability at a level of debt equal to 57.2. This still allows to obtain proceeds from a group sale (163.24) higher than those from the sale of two stand alone firms (162.46).

However, total debt capacity is drastically smaller than in the contractual case (124 instead of 220) and its distribution between the holding and its subsidiary is more balanced (55 as opposed to zero; 69 as opposed to 220), due to the uncertainty regarding the actual rescue.

[20] In a similar vein, rating agencies consider the strategic importance of the subsidiary, the percentage ownership of the holding, shared names and common sources of capital as relevant factors is assessing the probability of *ex post* rescue (Samson, 2001).

[21] We do not allow π to change with corporate organization, as modelled by Inderst and Mueller (2003).

Correspondingly, total value falls from 166.58 to 163.24. The holding raises more debt than in the contractual case: lenders would charge too high a spread if the entrepreneur shifted all of HS debt onto the subsidiary[22]. Finally, it is still the case that average equity prices in group affliated firms (35.80) fall short of stand-alone equity valuation (39.01).

We now allow for intercorporate dividends ($0 < \omega < 1$).Recall that intercorporate dividends do not affect HS value (see proposition 2) and debt capacity (see theorem 9) relative to SA when the guarantee is legally binding. Results change when the guarantee is informal. For $\pi = 0.5$, Table 7 reveals that the larger is ω, the larger is total group value. For $\rho = 0.2$, total value grows from 163.23 (for $\omega = 0$) to 163.71 (for $\omega = 1$). This value gain stems from a reduction in the probability that the holding defaults, at any given level of its debt, thanks to the transfer from its subsidiary. Consistent with this conjecture, we observe an increase in the optimal face value of debt in the holding - which restores equality between the marginal tax benefit and the marginal bankruptcy cost. P_h now ranges from 55 at $\omega = 0$ to 83 at $\omega = 1$.

Insert here Table 7

Total group debt increases at a slower pace (from 124 to 135), as the level of debt in the subsidiary falls (from 69 to 52). This reduction, in turn, stems from the higher holding leverage which reduces both its net cash flow after interest and its ability to rescue its subsidiary.

The behavior of tax savings, default costs and equity prices for the subsidiary are non-linear. However, tax savings in S are lower at $\omega = 1$ than at $\omega = 0$ because of lower optimal debt which translates into a reduced tax shield. This holds true for all values of ρ. The opposite occurs for tax savings in the holding.

Perhaps counterintuively, default costs in the holding are higher at $\omega = 1$ than at $\omega = 0$ if $\rho > -0.8$. This is because the holding has higher debt now; at the same time, H and S have similar debt levels, implying that dividends are of little help in avoiding H insolvency unless cash flows are highly negatively correlated. On the contrary, default costs in the holding are lower at $\omega = 1$ than at $\omega = 0$ for $\rho = -0.8$. In other words, the capital structure in HS becomes more similar to the conglomerate one when $\omega > 0$ and $\pi < 1$. Gains from debt diversity, that show up in Tables 2 through 5, are reduced and the typical reasoning relating to diversification dominates again.

Importantly, unreported results show that conglomerate value (163.15) exceeds HS value in the base case when $\omega = 0$, for $\pi = 0.1$.The optimal

[22] This may explain why both holding and subsidiaries raise debt in Belgian and Italian groups (Dewaelheyns and Van Hulle, 2007; Bianco and Nicodano, 2006).

HS debt (116) is now lower than the merger debt (117.4), as lenders - anticipating uncertain support by H - charge higher spreads to both the holding (1.22%) and its subsidiary (1.24%) relative to the merger (0.6%). The tax burden is almost equal (35.31 versus 35.61) but the expected default cost are higher in HS (1.81 versus 1.24) because the unconditional guarantee in mergers always works while the conditional one in HS is less reliable.

We can summarize these numerical findings as follows.

Conclusion 13 *Consider a BBB-calibrated stand alone company. (a) Let* $\pi = 1$*. Then HS value achieves its maximum at* $\rho = 1$*, for any admissible* ω *(b) Now let* $0 < \pi < 1$*. Then HS achieves its maximum value, which is decreasing in* ρ*, for* $\omega = 1$*. (c)The entrepreneur always prefers HS over SA. (d) There exists a* $\pi'(\omega = 0, \rho)$ *such that the entrepreneur prefers M to HS for all* $\pi < \pi'$*, when activities cash flows are equal in distribution.*

One last remark concerns the value of the holding company. Higher dividend transfers, which range from 0 to 38.97 as ω varies between 0 and 1, keeping $\rho = 0.2$, translate into higher equity value for the holding, which varies from 30.91 to 62.30. Note, however, that the equity appreciation is less than one for one, because of the higher debt burden onto the holding shareholders. This opens up the possibility to explain the holding company discount (Cornell and Liu, 2001). Observe in fact that the value of equity in a stand alone is 39.01.

One may expect the equity value of a firm that owns 100% of a clone to be twice as much, i.e. 78.02. We see instead that, in the base case $\rho = 0.2$, the equity value of the holding company with $\omega = 1$ is only 62.30, only 1.60 times the capitalization of one stand-alone. The comparison fares even worse when the benchmark is the value of the subsidiary equity, which is 42.56, leading to a ratio of 1.1 instead of 2. Such "holding company discount" gets worse for more extreme cash flow correlations. The relative values of equity in the holding and in the subsidiary, given 100% ownership, is 1.4 for $\rho = 0.8$ but falls to 0.8 when $\rho = -0.8$, when share prices in the holding are lower than in the stand alone firms. This is due to the combined effect of the guarantee, that implies a transfer from holding shareholders to subsidiary lenders, and of its high level of debt (119). Such equity discounts in the holding company are often documented in the literature (Khanna and Yafeh, 2007), and they are interpreted as the outcome of inefficiencies or moral hazard in complex organizations. Here, lower equity valuations in the holding are the counterpart of higher obligations vis-à-vis both its own and its subsidiary lenders, absent any inefficiency.

6 Concluding comments

This paper contributes to our understanding of firm scope, by clarifying the role of intercorporate guarantees and ownership in affecting capital structure and value creation. Guarantees determine the overall debt capacity of the organization, its tax burden and expected default costs. These, in turn, affect the value of lenders' and shareholders' claims to cash-flows, which add up to the total value of the organization to the entrepreneur.

Our paper shows, in particular, that holding- subsidiary structures are value maximizing arrangements because they allow for the provision of a conditional guarantee which enhances debt capacity - and reduces their tax burden - relative to stand alone counterparts.

Furthermore, it clarifies that HS have higher total value than conglomerates, when they provide an ex-post enforceable guarantee, because they may choose diverse levels of debt in the affiliated activities. This feature preserves the value of the guarantee, and actually enhances it, when diversification opportunities vanish. Numerical examples also indicate that debt diversity proves especially valuable in contexts with asymmetric cash flows deriving from the activities.

Despite these strength, a holding-subsidiary structure may turn out to have lower values than conglomerate mergers when the guarantee is not enforceable in court and subsidiary's lenders attribute a sufficiently low probability to *ex post* rescue by the holding shareholders. The unconditional guarantee that binds activities inside a conglomerate may prove superior to a conditional, but uncertain, one provided by a HS structure.

One striking result concerns bankruptcy costs: numerical simulations indicate that HS appear to incur into higher large bankruptcy costs than competing organization, despite the holding providing a guarantee to subsidiary lenders. Future work ought to investigate their comparative welfare properties: the HS appears to be value maximizing, but socially wasteful. Moreover, it derives value gains from tax avoidance.

Importantly, our model is just a first step towards a better understanding of intercorporate guarantees in holding- subsidiary structures, as it relies on a simple static setting with two activities, no agency problems and exogenous contracts. Developments relying on endogenous contracts in a dynamic setting are postponed to further work.

Appendix A - Intercorporate dividends

6.1 Holding-Subsidiary Structures

This Appendix generalizes the expressions for debt, equity and total value - given debt principal - to allow for intercorporate ownership We start with the HS case.

Let ω be the ownership share of the holding in the subsidiary ($\omega \in [0,1]$). The cash flows of the holding must be augmented for dividends:[23]

$$d_s = \omega(X_s^n - P_s) \tag{53}$$

Dividends are positive only when the subsidiary is not in default, i.e. when $X_s < X_s^d$, or equivalently $X_s^n < P_s$. We will thus indicate with d_s^+ this contingency

Thanks to dividends, the holding may avoid defaulting in a subset of cases when its own operational cash flow is insufficient to service debt. Thus, its generalized default threshold is a function of the dividend level, and ultimately of the ownership share and subsidiary profits $X_{h,\omega}^d = X_{h,\omega}^d(X_s)$. This coincides with our previous default threshold for zero ownership, i.e. $X_{h,0}^d = X_h^d$.

The payoff to H lenders is accordingly equal to:

$$\begin{cases} (1-\alpha)\left(X_h + d_s^+\right) \; - \tau\left(X_s - X_s^Z\right)^+ & 0 < X_h < X_{h,\omega}^d \\ P_h & X_h > X_{h,\omega}^d \end{cases} \tag{54}$$

In the first case the holding defaults and lenders receive all cash flows by absolute priority. When the holding is solvent, they receive the face value of debt. This expression thus coincides with (16), with cash flows gross of dividends, $X_{h,\omega} \equiv X_h + d_s^+$,replacing the ones of the holding company, X_h.

Correspondingly, the payoff to H shareholders is

$$X_h^n + d_s^+ - P_h \qquad\qquad X_h > X_{h,\omega}^d \tag{55}$$

As usual, they receive cash flows gross of any dividends and net of debt repayment only if H does not default.

It is convenient to write the lenders' and shareholders' payoffs in differential terms with respect to the $\omega = 0$ case:

[23] We are assuming zero intercorporate taxation of dividends.

$$D_h(P_{h,}P_s,\omega) = D_h(P_h) + (1-\alpha)\, d_s^{+} 1_{\left\{X_h < X_{h,\omega}^d\right\}} + P_h 1_{\left\{X_{h,\omega}^d < X_h < X_h^d\right\}}$$

$$E_h(P_{h,}P_s,\omega) = E_h(P_h, P_s) + \left[d_s^{+} + X_h - P_h\right] 1_{\left\{X_{h,\omega}^d < X_h < X_h^d\right\}} + d_s^{+} 1_{\left\{X_h > X_{h,\omega}^d\right\}}$$

Their corresponding values obtain by discounting the expectation of these new cash flows, as in 29 and in 30.

The values of subsidiary debt and equity are instead unaffected by the payment of dividends to the holding: therefore, they can be represented as in (28) and (14).

The generalized group value is given by:

$$\nu_{0,HS}(P_{h,}P_s,\omega) = D_{0h}(P_{h,}P_s,\omega) + E_{0h}(P_{h,}P_s,\omega) + D_{0s}(P_s) + (1-\omega)E_{0s}(P_s, P_h)$$

where, to avoid double counting, we consider only the fraction of subsidiary equity which is not property of the holding, namely $(1-\omega)E_s(P_s, P_h)$, with $E_s(P_s, P_h)$ defined by (14).

6.2 Two stand alone firms

Consider now two stand alone companies which replicate the same ownership patterns of the HS just described: Firm 1 (corresponding to H) owns a fraction ω of the equity of firm 2 (corresponding to S).

The payoffs to lenders and equity holders of firm 1 become respectively:

$$D_1(P_{1,}P_2,\omega) = D_1(P_1) + (1-\alpha)\, d_2^{+} 1_{\left\{X_1 < X_{1,\omega}^d\right\}} + P_1 1_{\left\{X_{1,\omega}^d < X_1 < X_1^d\right\}}$$

$$E_1(P_{1,}P_2,\omega) = E_1(P_1) + \left[d_2^{+} + X_1 - P_1\right] 1_{\left\{X_{1,\omega}^d < X_1 < X_1^d\right\}} + d_2^{+} 1_{\left\{X_1 > X_{1,\omega}^d\right\}}$$

The corresponding quantities for firm 2 are unaltered.

It follows that the difference between the group and stand alone values is

$$\nu_{0,HS}(P_{h,}P_s,\omega) - \nu_{01}(P_1,\omega) - \nu_{02}(P_2,\omega) =$$

$$\begin{aligned} &D_{0h}(P_{h,}P_s,\omega) + E_{0h}(P_{h,}P_s,\omega) + D_{0s}(P_h, P_s) + (1-\omega)E_{0s}(P_s) + \\ &-D_{01}(P_{1,}P_2,\omega) - E_{01}(P_{1,}P_2,\omega) - D_{02}(P_{2,}\omega) - (1-\omega)E_{02}(P_{2,}\omega) \end{aligned}$$

Let $P_1 = P_h, P_2 = P_s$. Substituting in the above expressions for debt and equity values one can verify that the difference corresponds to the one for the $\omega = 0$ case. In other words, the modifications to payoffs of firm 1 and firm H coincide if $P_1 = P_h, P_2 = P_s$ and cash flows equal in distribution: they therefore cancel out.

6.3 Generalized default threshold for H: characterization

We now conclude by characterizing the new default threshold, $X^d_{h,\omega}$.To this end, we highlight the cash flow level in four contingencies, depending on whether the holding pays taxes $(X_h > X^Z_h)$ and on whether it receives dividends from its subsidiary:

$$\begin{cases} X_h & X^n_s < P_s, X_h < X^Z_h \\ X_h - \tau(X_h - X^Z_h) & X^n_s < P_s, X_h > X^Z_h \\ X_h + d_s & X^n_s > P_s, X_h < X^Z_h \\ X_h - \tau(X_h - X^Z_h) + d_s & X^n_s > P_s, X_h > X^Z_h \end{cases} \tag{56}$$

By definition, $X^d_{h,\omega}$ is the level of operating cash flows, net of taxes but gross of dividends, that equals P_h:

$$X^d_{h,\omega} - \tau(X^d_{h,\omega} - X^Z_h)^+ + d_s = P_h \tag{57}$$

Solving for $X^d_{h,\omega}$ we obtain the generalized default threshold for the holding company:

$$X^d_{h,\omega} = \begin{cases} X^d_h & 0 < X_s < X^d_s \\ X^d_h - \omega\left(X_s - X^d_s\right) & X^d_s < X_s < X^\circ_s \\ P_h - \omega\left(1-\tau\right)\left(X_s - X^d_s\right) & X^\circ_s < X_s \end{cases}$$

where

$$X^\circ_s = X^d_s - \frac{X^Z_h - P_h}{\omega\left(1-\tau\right)}$$

In order to represent the new situation in the cash flow planes, and be able to integrate over the appropriate sets in the numerical optimization, we highlight the cash flow combinations $\{X_s, X_h\}$, such that the holding defaults - given $\{P_h, P_s\}$:

$$\begin{cases} X_h < X^d_h,\ X_s < X^d_s \\ X_h < X^d_h - \omega\left(X_s - X^d_s\right),\ X^d_s < X_s < X^\circ_s \\ X_h < P_h - \omega\left(1-\tau\right)\left(X_s - X^d_s\right),\ X_s > X^\circ_s \end{cases} \tag{58}$$

We visualize the default threshold in Figure 4.

Insert here Figure 4

7 Appendix B - The stand alone optimization problem

This Appendix studies the stand alone firm optimization problem, namely the maximization of levered value (33) with respect to non-negative debt levels, $P_i \geq 0$, with $i = 1, 2$, through its equivalent problem, namely the minimization of the tax burden plus default costs (36).

We first establish some properties of the market value of debt for stand alone companies.

Lemma 14 *Debt is increasing less than proportionally in the face value of debt:*

$$0 \leq dD_{0i}(P_i)/dP_i < 1$$

with

$$\lim_{P_i \to 0+} \frac{dD_{0i}(P_i)}{dP_i} > 0$$

Proof. As default costs and taxes approach zero:

$$0 \leq dD_{0i}(P_i)/dP_i = (1 - F_i(P_i))\phi \leq \phi < 1$$

In particular, we have

$$\lim_{P_i \to 0+} \frac{dD_{0i}(P_i)}{dP_i} = (1 - F_i(0))\,\phi > 0$$

since the probability that X_i is positive is positive ($F_i(0) < 1$). In the case of costly bankruptcy and taxation, when closed form expressions for $D_{0i}(P_i)$ do not obtain, we will use the fact that risky debt D_{0i} can be written as the difference between the corresponding riskless debt, $P_i\phi$, and the discounted expected loss.

The first part of the proof proceeds by contradiction. Thus assume instead $dD_{0i}(P_i)/dP_i \geq 1$. The derivative of riskless debt wrt the face value of debt (ϕ) is smaller than one. In order for the risky debt to have a derivative not smaller than one, the discounted expected loss should have a derivative smaller than zero. It would then decrease in the face value of debt, contradicting the fact that both default probability and expected default costs increase in the face value of debt.

As for the other property, let $\lim_{P_i \to 0+} \frac{dD_{0i}(P_i)}{dP_i} \leq 0$, when default costs and taxes are finite. This implies that the discounted expected loss has a derivative, when $P_i \to 0+$, positive and not smaller than ϕ. This implies that lenders' expected loss has a derivative greater than one with respect to debt , which is absurd. ■

This Lemma implies that both the tax shield and the default threshold are increasing in the face value of debt:

$$\frac{dX_i^Z}{dP_i} = 1 - \frac{dD_{0i}(P_i)}{dP_i} > 0; \frac{dX_i^d}{dP_i} = 1 + \frac{\tau_i}{1 - \tau_i} \frac{dD_{0i}(P_i)}{dP_i} > 0 \qquad (59)$$

but:

$$\frac{dX_i^Z}{dP_i} \leq 1, \frac{dX_i^d}{dP_i} \geq 1 \tag{60}$$

We are now able to show that the stand alone is optimally unlevered, when there are no taxes. And that, with taxes, the minimum of the tax burden plus defalt costs entails positive leverage for a stand alone firm. These results will be used in Appendix C in the proof of our main theorem.

The Kuhn-Tucker (KT) conditions for the above problem are

$$\begin{cases} \frac{dT_i(P_i^*)}{dP_i} + \frac{dDC_i(P_i^*)}{dP_i} \geq 0 \\ P_i^* \geq 0 \\ \left[\frac{dT_i(P_i^*)}{dP_i} + \frac{dDC_i(P_i^*)}{dP_i}\right] P_i^* = 0 \end{cases} \tag{61}$$

If $T_i + DC_i$ is convex in $P_i \geq 0$, as assumed in the text, the above conditions are necessary and sufficient.

The derivative of tax burdens and default costs is equal to:

$$\frac{dT_i(P_i)}{dP_i} + \frac{dDC_i(P_i)}{dP_i} =$$

$$= -\tau_i(1 - F_i(X_i^Z)) \left[1 - \frac{dD_{0i}(P_i)}{dP_i}\right] \phi +$$

$$+\alpha X_i^d f_i(X_i^d) \left[1 + \frac{\tau_i}{1-\tau_i} \frac{dD_{0i}(P_i)}{dP_i}\right] \phi \tag{62}$$

where f_i is the density of X_i.

If $\tau_i = 0$,a minimum exists, with

$$\lim_{\tau \to 0+} P_i^* = 0+ \qquad i = 1, 2 \tag{63}$$

which implies

$$\lim_{\tau \to 0+} X_i^d = \lim_{\tau \to 0+} X_i^Z = 0+ \qquad i = 1, 2 \tag{64}$$

When there is taxation ($\tau_i > 0$), then a minimum at $P_i = 0$ cannot exist, since

$$\frac{dT_i(0)}{dP_i} + \frac{dDC_i(0)}{dP_i} = \tag{65}$$

$$= -\tau_i(1 - F_i(0)) \left[1 - \frac{dD_{0i}(0)}{dP_i}\right] \phi < 0$$

and the KT conditions are violated. At the optimum, the first KT condition is satisfied as an equality, namely (62) is null.

8 Appendix C - proof of theorem 9

We first prove Lemma 7 which characterizes the guarantee G. Letting $f(x,y)$ be the joint density of the cash flows (X_s, X_h), we can write the guarantee as

$$G(P_h, P_s) = \alpha\phi \int_0^{X_s^d} \int_{h(x)}^{+\infty} x f(x,y) dx dy =$$

$$= \alpha\phi \left[\int_0^{X_s^Z} x \int_{X_h^d + \frac{P_s}{1-\tau} - \frac{x}{1-\tau}}^{+\infty} f(x,y) dy dx + \int_{X_s^Z}^{X_s^d} x \int_{X_h^d + X_s^d - x}^{+\infty} f(x,y) dy dx \right]$$

Proof. of Lemma7: Part (a) follows from the fact that

$$\frac{\partial G}{\partial P_h} = -\alpha\phi \times$$

$$\times \left[\int_0^{X_s^Z} x f\left(x, X_h^d + \frac{P_s}{1-\tau} - \frac{x}{1-\tau}\right) dx + \int_{X_s^Z}^{X_s^d} x f(x, X_h^d + X_s^d - x) dx \right] \times$$

$$\times \left[1 + \frac{\tau}{1-\tau} \frac{dD_{01}(P_1)}{dP_1} \right] \leq 0 \qquad (66)$$

since - according to (59) - we have

$$1 + \frac{\tau}{1-\tau} \frac{dD_{01}(P_1)}{dP_1} > 0$$

Equality in (66) holds if and only if

$$X_s^d = X_s^Z = 0$$

which in turn happens if and only if $P_s = 0$. As concerns part (b), we compute:

$$\frac{\partial G}{\partial P_s} = \alpha\phi \times \qquad (67)$$

$$\times \left\{ -\frac{1}{1-\tau} \int_0^{X_s^Z} x f\left(x, X_h^d + \frac{P_s}{1-\tau} - \frac{x}{1-\tau}\right) dx + \right.$$

$$\left. -\frac{dX_2^d}{dP_2} \int_{X_s^Z}^{X_s^d} x f(x, X_h^d + X_s^d - x) dx + \frac{dX_2^d}{dP_2} \int_{X_h^d}^{+\infty} X_s^d f(X_s^d, y) dy \right\} =$$

$$= \frac{\alpha}{(1-\tau)(1+r_T)} \times \left\{ - \int_0^{X_s^Z} x f\left(x, X_h^d + \frac{P_s}{1-\tau} - \frac{x}{1-\tau}\right) dx + \right.$$

$$+\left(1-\tau+\tau\frac{dD_{02}(P_2)}{dP_2}\right)\left[-\int_{X_s^Z}^{X_s^d} xf(x, X_h^d+X_s^d-x)dx+\int_{X_h^d}^{+\infty} X_s^d f(X_s^d, y)dy\right]\Bigg\}$$

When $P_s = 0$, then $X_s^d = X_s^Z = 0$, all the integrals vanish and the previous derivative is null.
As concerns part (c), when $P_s \to +\infty$, definition (12) implies that

$$\lim_{P_s\to+\infty} X_s^d = +\infty$$

For fixed y, the convergence condition

$$\lim_{x\to+\infty} xf(x,y) = 0 \tag{68}$$

- which follows from the fact that f is a density - implies that, for any sequence x_n which goes to $+\infty$, then

$$f_n(y) := x_n f(x_n, y)$$

converges to zero. We supposed in the text that the function $f_n(y)$ satisfies the dominated convergence property. This allows us to exchange integration and limit:

$$\lim_{n\to+\infty}\int_{X_h^d}^{+\infty} x_n f(x_n, y)dy =$$
$$= \lim_{n\to+\infty}\int_{X_h^d}^{+\infty} f_n(y)dy =$$
$$= \int_{X_h^d}^{+\infty} \lim_{n\to+\infty} f_n(y)dy = 0$$

and, as a consequence,

$$\lim_{X_s^d\to+\infty}\int_{X_h^d}^{+\infty} X_s^d f(X_s^d, y)dy = 0$$

Together with (59) - which signs the derivative of X_s^d wrt debt - this entails

$$\lim_{P_s\to+\infty}\frac{\partial G}{\partial P_s} =$$

$$= \lim_{P_s\to+\infty}\frac{\alpha}{(1-\tau)(1+r_T)}\times\left\{-\int_0^{X_s^Z} xf\left(x, X_h^d+\frac{P_s}{1-\tau}-\frac{x}{1-\tau}\right)dx-\right.$$

$$\left.-\left(1-\tau+\tau\frac{dD_{02}(P_2)}{dP_2}\right)\int_{X_s^Z}^{X_s^d} xf(x, X_h^d+X_s^d-x)dx\right\} < 0$$

and proves part (c). ■

We are now ready for the proof of the main theorem.

Proof. Part i): let us examine the Kuhn Tucker conditions for a

minimum of $T_{HS} + DC_{HS}$ with respect to subsidiary debt, considering that the latter cannot be negative. Remind that such conditions are necessary and sufficient, under the convexity assumption of the theorem.

$$\begin{cases} \frac{\partial T_{HS}(P_h^*,P_s^*)}{\partial P_s} + \frac{\partial DC_{HS}(P_h^*,P_s^*)}{\partial P_s} = \frac{dT_2(P_s^*)}{dP_s} + \frac{dDC_2(P_s^*)}{dP_s} - \frac{\partial G(P_h^*,P_s^*)}{\partial P_s} \geq 0 \\ P_s^* \geq 0 \\ \left[\frac{dT_2(P_s^*)}{dP_s} + \frac{dDC_2(P_s^*)}{dP_s} - \frac{\partial G(P_h^*,P_h^*)}{\partial P_s}\right] P_s^* = 0 \end{cases} \tag{69}$$

According to 65, the derivative of tax burdens and default costs paid by the subsidiary wrt its own debt, which appears in the first and third condition above is equal to:

$$\frac{dT_2(P_s)}{dP_s} + \frac{dDC_2(P_s)}{dP_s} = \tag{70}$$

$$= -\tau(1 - F_2(X_s^Z)) \left[1 - \frac{dD_{02}(P_s)}{dP_2}\right] \phi +$$

$$+\alpha X_s^d f_2(X_s^d) \left[1 + \frac{\tau}{1-\tau} \frac{dD_{02}(P_s)}{dP_2}\right] \phi$$

where F_2, f_2 are respectively the distribution and density functions of $X_2 = X_s$.

Let us examine whether the KT conditions are satisfied at $P_s = 0$. As a consequence of part (b) of the previous lemma, if $P_s^* = 0$ we have

$$\frac{dT_2(0)}{dP_s} + \frac{dDC_2(0)}{dP_s} - \frac{\partial G(P_h^*, 0)}{\partial P_s} = \tag{71}$$

$$= \frac{dT_2(0)}{dP_s} + \frac{dDC_2(0)}{dP_s} = -\tau(1 - F_2(0)) \left[1 - \frac{dD_{02}}{dP_2} |_{P_2=0}\right] \phi$$

where the last term follows from (70), Since $X_s^d = X_s^Z = 0$ when $P_s = 0$

The derivative is negative, since $F_2(0) < 1$ and

$$\lim_{P_2 \to 0+} \frac{dD_{02}}{dP_2} < 1 \tag{72}$$

by lemma 14 in the text. The KT conditions are then violated when the subsidiary is unlevered. This concludes the proof of part i) of the theorem.

Let us consider now part (ii) of the theorem. In order to demonstrate it, we write down the KT conditions for a minimum of $T_{HS}+DC_{HS}$ under non negativity of debt for both the holding and the subsidiary, as well as under the constraint

$$P_h^* + P_s^* \geq P_1^* + P_2^* := K \tag{73}$$

They are

$$\begin{cases} \frac{\partial T_{HS}(P_h^*,P_s^*)}{\partial P_h} + \frac{\partial DC_{HS}(P_h^*,P_s^*)}{\partial P_h} = \frac{dT_1(P_h^*)}{dP_h} + \frac{dDC_1(P_h^*)}{dP_h} - \frac{\partial G(P_h^*,P_s^*)}{\partial P_h} = \mu_1 + \mu_3 (i) \\ P_h^* \geq 0 \qquad (ii) \\ \mu_1 P_h^* = 0 \qquad (iii) \\ \frac{\partial T_{HS}(P_h^*,P_s^*)}{\partial P_s} + \frac{\partial DC_{HS}(P_h^*,P_s^*)}{\partial P_s} = \frac{dT_2(P_s^*)}{dP_s} + \frac{dDC_2(P_s^*)}{dP_s} - \frac{\partial G(P_h^*,P_s^*)}{\partial P_s} = \mu_2 + \mu_3 (iv) \\ P_s^* \geq 0 \qquad (v) \\ \mu_2 P_s^* = 0 \qquad (vi) \\ P_h^* + P_s^* \geq K \qquad (vii) \\ \mu_3 (P_h^* + P_s^* - K) = 0 \qquad (viii) \\ \mu_1 \geq 0, \mu_2 \geq 0, \mu_3 \geq 0 \qquad (ix) \end{cases} \tag{74}$$

We want to demonstrate that - under the conditions in the theorem - there exists a point (P_h^*, P_s^*), with $P_h^* < P_1^*, P_s^* > K$, which solves them. All the conditions are easy to discuss. Only *(iv)* requires a little bit of caution.

Consider all the conditions except *(iv)* first. We are interested in a solution for which the constraint *(vii)* is not binding. This means that $\mu_3 = 0$. Consider a solution for which also $\mu_1 = 0$: if $\mu_3 = 0$, condition (i) is satisfied, provided that we choose $P_h^* \geq 0$ such that

$$\frac{dT_1(P_h^*)}{dP_h} + \frac{dDC_1(P_h^*)}{dP_h} - \frac{\partial G(P_h^*, P_s^*)}{\partial P_h} = 0 \tag{75}$$

It follows from well-posedness of the stand alone problem ($T_i + DC_i$ convex) that the first two terms on the left hand side are negative, if $P_h^* < P_1^*$. We also know that the third term is positive by part (a) of the above lemma and part (i) of this theorem, which rules out $P_s^* = 0$. Thus we choose $0 < P_h^* < P_1^*$ so as to satisfy (75). If such a debt does not exist, we choose $0 = P_h^* < P_1^*$ and satisfy condition (i) by letting μ_1 equal to the (positive) difference between $\frac{dT_1(P_h^*)}{dP_h} + \frac{dDC_1(P_h^*)}{dP_h}$ and $\frac{\partial G(P_h^*,P_s^*)}{\partial P_h}$. Thus P_h^*,chosen as explained, and $\mu_1 \geq 0$ satisfy conditions (i, ii, iii).

If later we choose $P_s^* > K$, also conditions (v, vi, vii) are satisfied, provided that we select $\mu_2 = 0$.

Given that we chose $\mu_1 \geq 0, \mu_2 = \mu_3 = 0$, conditions $(viii, ix)$ hold.

Let us turn to condition (iv), which has to provide us with a choice $P_s^* > K$. In view of the other conditions, (iv) becomes

$$\frac{dT_2(P_s^*)}{dP_s} + \frac{dDC_2(P_s^*)}{dP_s} - \frac{\partial G(P_h^*, P_s^*)}{\partial P_s} = 0 \tag{76}$$

Consider its lhs as a function of P_s

$$h(P_s) = \frac{dT_2(P_s)}{dP_s} + \frac{dDC_2(P_s)}{dP_s} - \frac{\partial G(P_h^*, P_s)}{\partial P_s}$$

We know from the limit behavior of the guarantee (part (b) of the previous lemma) and from convexity of the stand alone value $(T_2 + DC_2)$ that h has a negative limit when the subsidiary debt tends to zero, and a positive limit (even non finite) when P_s diverges. It follows that there exists a positive debt level which satisfies condition (76).

We are going to show that, under the conditions posited in the statement of the theorem, $h(P_1^* + P_2^*) < 0$, which means that $P_s^* > P_1^* + P_2^*$.

$$h(P_1^* + P_2^*) = \frac{dT_2(P_1^* + P_2^*)}{dP_s} + \frac{dDC_2(P_1^* + P_2^*)}{dP_s} - \frac{\partial G(P_h^*, P_1^* + P_2^*)}{\partial P_s} =$$

$$= \phi \left\{ -\tau(1 - F_2(X_s^{Z**}))\frac{dX_s^{Z**}}{dP_s} + \right.$$

$$+\alpha X_s^{d**} f_2(X_s^{d**})\frac{dX_s^{d**}}{dP_s} +$$

$$+\frac{\alpha}{(1-\tau)}\int_0^{X_s^{Z**}} xf\left(x, \frac{P_1^* + P_2^*}{1-\tau} - \frac{x}{1-\tau}\right)dx +$$

$$\left. -\alpha\frac{dX_s^{d**}}{dP_s} \times \left[-\int_{X_s^{Z**}}^{X_s^{d**}} xf(x, X_s^{d**} - x)dx + \int_{X_h^{d*}}^{+\infty} X_s^{d**} f(X_s^{d**}, y)dy \right] \right\}$$

where X_s^{d**} and X_s^{Z**} are the default and tax shield thresholds corresponding to $P_s = P_1^* + P_2^*$, dX_s^{d**}/dP_s is given by formula (59) computed at $P_s = P_1^* + P_2^*$. Omitting ϕ, we can write the condition $h < 0$ in a more compact way as

$$-\tau(1 - F_2(X_s^{Z**}))\frac{dX_s^{Z**}}{dP_s} +$$

$$+\alpha\frac{dX_s^{d**}}{dP_s} X_s^{d**} \int_{-\infty}^{X_h^{d*}} f(X_s^{d**}, y)dy +$$

$$+\frac{\alpha}{(1-\tau)}\int_0^{X_s^{Z**}} xf(x, h(x))\,dx + \alpha\frac{dX_s^{d**}}{dP_s}\int_{X_s^{Z**}}^{X_s^{d**}} xf(x, h(x))dx < 0$$

or, recognizing that both $1/(1-\tau)$ and $\frac{dX_s^{d**}}{dP_s}$ represent $\partial h/\partial P_s$,

$$-\tau(1-F_2(X_s^{Z**}))\frac{dX_s^{Z**}}{dP_s}+$$

$$+\alpha\frac{dX_s^{d**}}{dP_s}X_s^{d**}\int_{-\infty}^{X_h^{d*}} f(X_s^{d**},y)dy+$$

$$+\alpha\frac{\partial h}{\partial P_s}\int_0^{X_s^{d**}} xf(x,h(x))dx<0$$

which can be written as

$$\alpha/\tau<\frac{\left(1-F_2(X_s^{Z**})\right)\frac{dX_s^{Z**}}{dP_s}}{X_s^{d**}\frac{dX_s^{d**}}{dP_s}\int_{-\infty}^{X_h^{d*}} f(X_s^{d**},y)dy+\frac{\partial h}{\partial P_s}\int_0^{X_s^{d**}} xf(x,h(x))dx}$$

Observe that $x\leq X_s^{d**}$. Thus the last condition holds if

$$\alpha/\tau<\frac{\left(1-F_2(X_s^{Z**})\right)\frac{dX_s^{Z**}}{dP_s}}{X_s^{d**}\left(\frac{dX_s^{d**}}{dP_s}\int_{-\infty}^{X_h^{d*}} f(X_s^{d**},y)dy+\frac{dh}{dP_s}\int_0^{X_s^{d**}} f(x,h(x))dx\right)}=$$

$$=\frac{\Pr(X_s>X_s^{Z**})\frac{dX_s^{Z**}}{dP_s}}{X_s^{d**}\left(\frac{dX_s^{d**}}{dP_s}\Pr\left(X_s=X_s^{d**},X_h<X_h^{d*}\right)+\frac{dh}{dP_s}\Pr(0<X_s<X_s^{d**},X_h=h(X_s)\right)}$$

which is the condition in the theorem statement.

This proves part ii) of the theorem, since all the KT conditions are satisfied. ■

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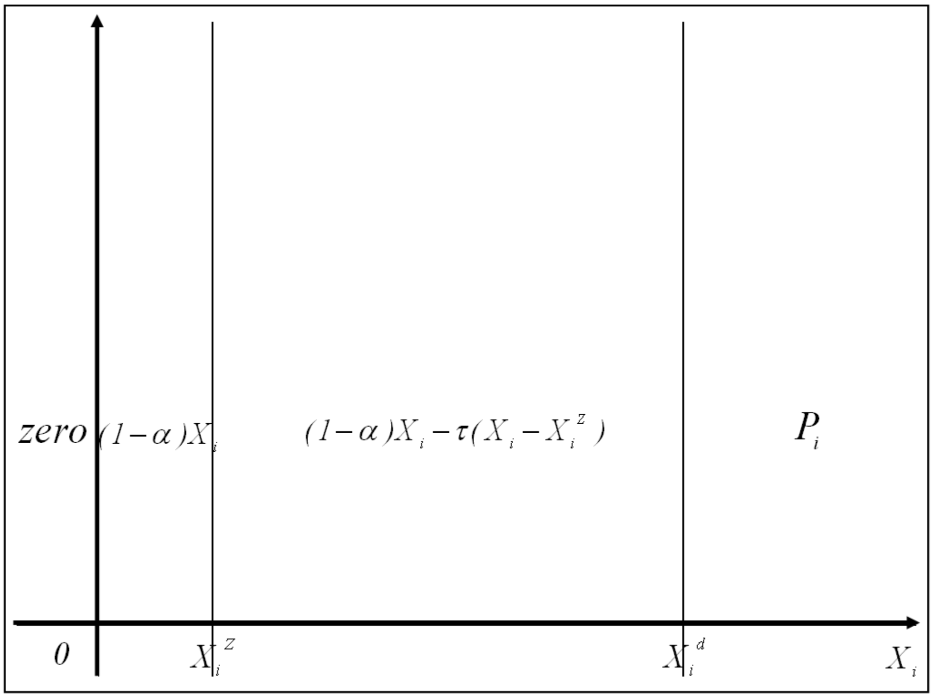

Figure 1: This figure represents the payoff to the lenders of a stand alone company. Lenders are fully reimbursed (P_i) when cash flow X_i exceeds the default threshold X_i^d. Otherwise, they receive a fraction $(1-\alpha)$ of positive cash flows. However, when the stand alone defaults and its cash flow exceeds the no tax level X_i^z, debtholders pay taxes on top of bankruptcy costs.

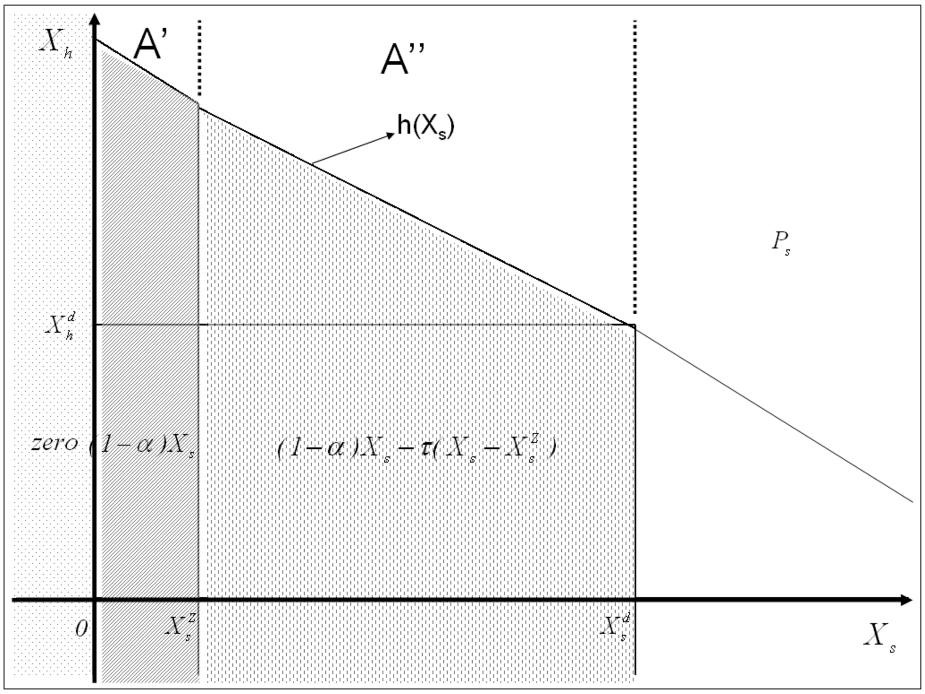

Figure 2: This figure represents the payoffs to subsidiary lenders as a function of the subsidiary cash flows (on the horizontal axis) and of the holding cash flows (on the vertical axis) for the case of infinitesimal ownership share. The figure reproduces the one for stand alone firms when X_h is lower than the holding default threshold X_h^d, as in this case the holding is unable to help its subisdiary. The area of the transfer is $A = A' \cup A''$ and is bounded by the linear function $h(X_s)$. In A'', the subsidiary does not default thanks to the transfer, but it pays taxes. In A', the subsidiary saves on both default costs and taxes thanks to the transfer. The payoff $(1-\alpha)X_s$ applies to the whole dark grey zone, while the payoff $(1-\alpha)X_s - \tau(X_s - X_s^Z)$ applies to the whole pale grey zone.

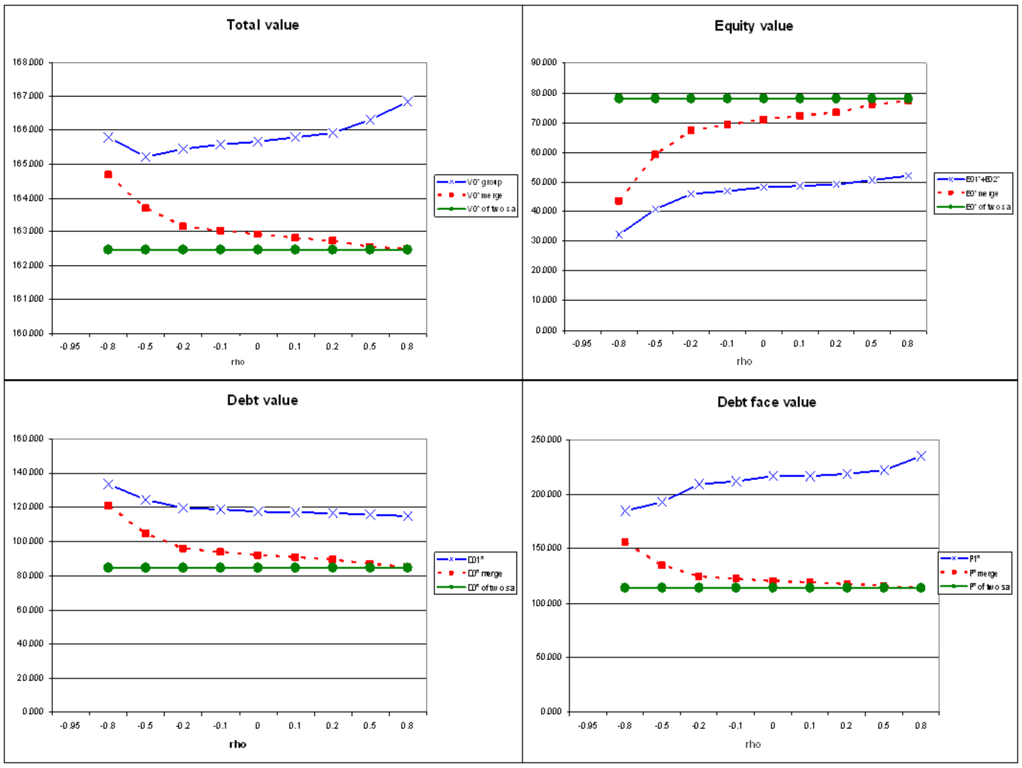

Figure 3: The upper left panel displays the value of an HS (stars), a conglomerate (big and small dots) and two stand alone firms (dotted) as the correlation coefficient between the activities cash glows varies between -0.8 and +0.8. Similarly, the upper right panel displays the value of equity, the lower left panel the market value of debt and the last one the face value of debt.

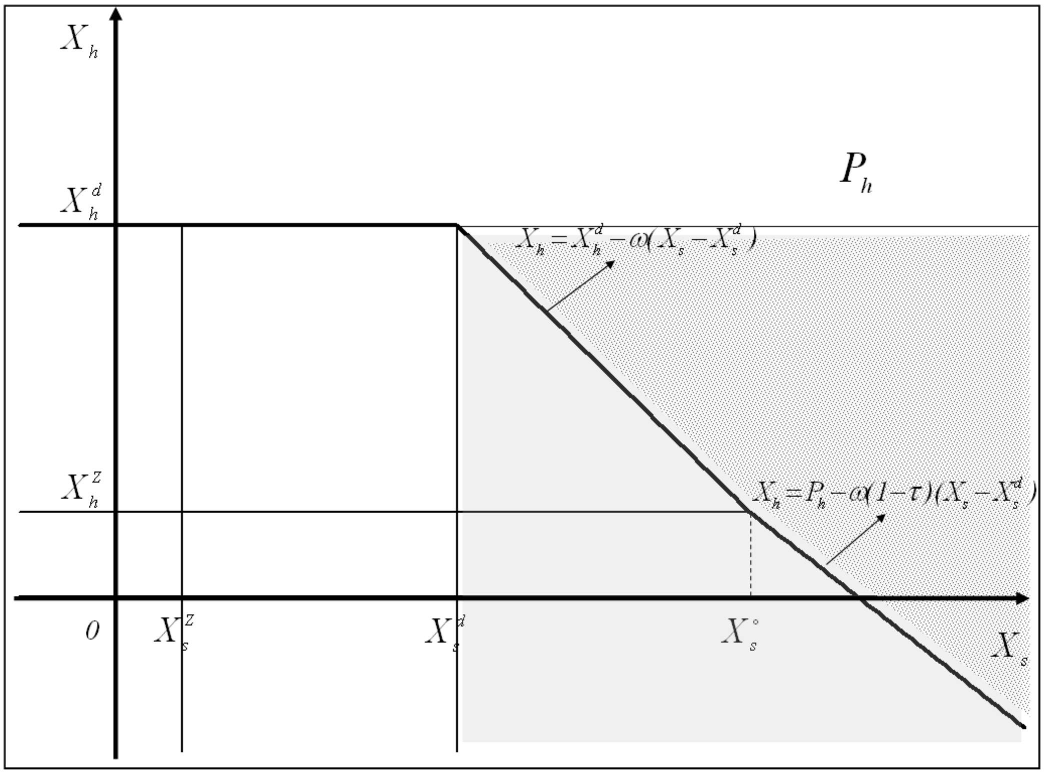

Figure 4: This figure represents the payoffs to holding lenders as a function of the subsidiary cash flows (on the horizontal axis) and of the holding cash flows (on the vertical axis) for the case of positive ownership share. Consider the shaded area where the holding would default $(X_h < X_h^d)$ and the subsidiary does not default $(X_s < X_s^d)$. The holding now receives dividends proportional to its ownership share, ω, that allow its survival in the shaded area with dots.

Table 1: Base Case Parameters

Variables	Symbols	Values
Annual Riskfree Rate	r	5.00%
Time Period/Debt Maturity (yrs)	T	5.00
T-period Riskfree Rate	$r_T = (1+r)^T - 1$	27.63%
Capitalization Factor	$Z = (1+r_T)/r_T$	4.62
Unlevered Firm Variables		
Expected Future Operational Cash Flow at T	Mu	127.63
Expected Operational Cash Flow Value (PV)	$X_0 = Mu/(1+r)^T$	100.00
Cash Flow Volatility at T	Std	49.19
Annualized operating Cash Flow Volatility	$\sigma = Std/T^{0.5}$	22.00
Tax Rate	τ	20%
Value of Unlevered Firm w/Limited Liability	V_0	80.05
Value of Limited Liability	L_0	0.057

Table 2: Optimal Capital Structure and Value

	Symbols	Values				
		Stand Alone	Holding	Subsidiary	1/2 HS	1/2 Conglom
Default Costs	α	23%	23%	23%	23%	23%
Optimal Face Value of Debt	P^*	57.20	0	220	110	58.5
Default Threshold	X^{d*}	67.75	0	249.2663	-	69.64
No Tax Profit Level	X^{Z*}	14.98	0	102.93	-	13.94
Value of Optimal Debt	D_0^*	42.22	0	117.06	58.53	44.56
Optimal Leverage Ratio	D_0^*/ν_0^*	52%	0	99.9%	70.26%	54.62%
Annual Yield Spread of Debt (%)	$(P^*/D_0^*)^{1/T} - 1 - r$	1.26%	//	8.45%	-	0.6%
Value of Optimal Equity	E_0^*	39.01	49.46	0.07	24.76	37.01
Optimal Levered Firm Value	$\nu_0^* = D_0^* + E_0^*$	81.23	49.46	117.13	83. 29	81.57
Tax Burden	T_0^*	17.62	20.01	5.39	12. 70	17.77
Tax Savings of Leverage	TS_0^*	2.33	0	14.62	7.31	2.18
Expected Default Costs	DC_0^*	0.90	0	8.13	4.07	0.61
Value of Optimal Leveraging	$\nu_0^* - V_0$	1.43	-30.59	37.08	3.24	1.57
Capitalized Value of Optimal Leverage	$Z(\nu_0^* - V_0)/V_0$	6. 81%	-1.77	2.14	20.23%	9.06%

Table 3: Asymmetric alfas: capital structure and value across organizational forms, $\rho = 0.2$

Variables	Symbols	Values					
		S. Alone	S. Alone	Holding	Subsidiary	1/2 HS	1/2 Conglomerate
Default Costs	α	75%	23%	75%	23%	23%	75%
Optimal Face Value of Debt	P^*	33	57.20	0	219	109.5	46.5
Default Threshold	X^{d*}	39.247	67.75	0	248.17	-	55. 43
No Tax Profit Level	X^{Z*}	8.01	14.98	0	102.32	-	10. 79
Value of Optimal Debt	D_0^*	24.99	42.22	0	116.68	58.34	35. 71
Value of Optimal Equity	E_0^*	55.84	39.01	49.2	0.037	24.62	45. 52
Optimal Levered Firm Value	$\nu_0^* = D_0^* + E_0^*$	80. 83	81.23	49.2	116.71	82. 95	81. 23
Optimal Leverage Ratio	D_0^*/ν_0^*	30.92%	52%	0	99.9%	70.3%	44%
Annual Yield Spread of Debt (%)	y	0.7%	1.26%	//	8.4%	-	0.4%
Tax Burden	T_0^*	18.76	17.62	19.95	5.42	12. 69	18. 31
Tax Savings of Leverage	TS_0^*	1.25	2.33	0	14.53	7.27	1. 69
Expected Default Costs	DC_0^*	0.46	0.90	0	7.98	3.99	0.455
Value of Optimal Leveraging	$\nu_0^* - V_0$	0.78	1.18	-	-	3.15	1.18
Capitalized Optimal Value of Leveraging	$Z(\nu_0^* - V_0)/V_0$	4. 50%	6. 81%			18.2 4%	6. 81%

Table 4: Asymmetric volatilities: capital structure and value across organizational forms, $\rho = 0.2, \sigma_s = 44\%, \sigma_h = 22\%$

Variables	Symbols	Values					
		S.A.($\sigma = 44\%$)	S. A.($\sigma = 22\%$)	Holding	Subsidiary	1/2 HS	1/2 Conglomerate
Default Costs	α	23%	23%	23%	23%	23%	23%
Optimal Face Value of Debt	P^*	83	57.20	0	223	111.5	59
Default Threshold	X^{d*}	95.19	67.75	0	248.169	-	34.75
No Tax Profit Level	X^{Z*}	34.25	14.98	0	102.32	-	8.50
Value of Optimal Debt	D_0^*	48.75	42.22	0	106.83	53.41	41.98
Value of Optimal Equity	E_0^*	36.10	39.01	60.29	3.01	31.65	39.64
Optimal Levered Firm Value	$\nu_0^* = D_0^* + E_0^*$	84.84	81.23	60.29	109.84	85.06	81.62
Optimal Leverage Ratio	D_0^*/ν_0^*	57.46%	52%	0	97.3%	62.8%	51.4%
Tax Burden	T_0^*	16.05	17.62	19.95	7.01	13.48	17.45
Tax Savings of Leverage	TS_0^*	4.66	2.33	0	13.59	6.80	2.60
Expected Default Costs	DC_0^*	2.64	0.90	0	5.53	2.765	1.18
Annual Yield Spread of Debt (%)	y	6.2%	1.26%	//	10.9%	-	2%
Value of Optimal Leveraging	$\nu_0^* - V_0$	4.79	1.18	-	-	5.26	1.58
Capitalized Value of Optimal Leveraging	$Z(\nu_0^* - V_0)/V_0$	27.64%	6.81%			30.45%	9.12%

Table 5: Asymmetric size: capital structure and value across organizational forms, $\rho = 0.2,\ V_{h0} = 167, V_{s0} = 33.$

Variables	Symbols	Values					
		S. Alone(1/3)	S. Alone(5/3)	Holding	Subsidiary	1/2 HS	1/2 Conglomerate
Default Costs	α	23%	23%	23%	23%	23%	23%
Optimal Face Value of Debt	P^*	19	95	63.33	121.33	92.33	57
Default Threshold	X^{d*}	22.50	112.54			-	67.81
No Tax Profit Level	X^{Z*}	4.98	24.85			-	13.765
Value of Optimal Debt	D_0^*	14.02	70.15	48.25	70.26	59.25	43.24
Value of Optimal Equity	E_0^*	13.04	65.24	47.15	0	23.58	38.255
Optimal Levered Firm Value	$\nu_0^* = D_0^* + E_0^*$	27.06	135.38	95.39	70.26	82.83	81.49
Optimal Leverage Ratio	D_0^*/ν_0^*	51.81%	51.81%	50.57%	100%	71.54%	53.06%
Annual Yield Spread of Debt (%)	y	1.2%	1.2%	0.6%	6.5%	-	1.1%
Tax Burden	T_0^*	5.90	29.49	30.90	0.48	15.69	17.78
Tax Savings of Leverage	TS_0^*	0.77	3.87	2.35	6.17	8.52	2.23
Expected Default Costs	DC_0^*	0.30	1.48	0.37	2.10	1.23	0.75
Value of Optimal Leveraging	$\nu_0^* - V_0$	0.38	1.96	-	-	3.03	1.44
Capitalized Value of Optimal Leveraging	$Z(\nu_0^* - V_0)/V_0$	6.58%	6.79%			17.54%	8.31%

Note: HS principal is calculated as the sum of holding and subsidiary principals.

Table 6: Optimal Capital Structure and Value, rho 0.2 different levels of π, $\omega = 0$

	Symbols	Values							
	π		0.5			1			
		Stand Alone	Holding	Subsid	1/2 HS	Holding	Subsid	1/2 HS	1/2 M
Default Costs	α	23%	23%	23%	23%	23%	23%	23%	23%
Optimal Face Value of Debt	P^*	57.20	55	69	62	0	220	110	58.5
Default Threshold	X^{d*}	67.75	65.21	81.69	-	0	249.2663	-	69.64
No Tax Profit Level	X^{Z*}	14.98	14.16	18.23	-	0	102.93	-	13.94
Value of Optimal Debt	D_0^*	42.22	40.84	50.77	45.84	0	117.06	58.53	44.56
Optimal Leverage Ratio	D_0^*/ν_0^*	52%	50.58%	61.56%	56.13%	0	99.9%	70.26%	54.62%
Annual Yield Spread of Debt (%)	$(P^*/D_0^*)^{1/T} - 1 - r$	1.26%	1.13%	1.33%	-	//	8.45%	-	0.6%
Value of Optimal Equity	E_0^*	39.01	39.91	31.70	35.80	49.46	0.07	24.76	37.01
Optimal Levered Firm Value	$\nu_0^* = D_0^* + E_0^*$	81.23	80.75	82.48	81.62	49.46	117.13	83. 29	81.57
Tax Burden	T_0^*	17.62	17.81	17.18	17.49	20.01	5.39	12. 70	17.77
Tax Savings of Leverage	TS_0^*	2.33	2.20	2.83	2.51	0	14.62	7.31	2.18
Expected Default Costs	DC_0^*	0.90	0.77	1.11	0.94	0	8.13	4.07	0.61
Value of Optimal Leveraging	$\nu_0^* - V_0$	1.43	0.70	2.33	1.57	-30.59	37.08	3.24	1.57
Cap. Value of Optimal Leverage	$Z(\nu_0^* - V_0)/V_0$	6. 81%	4.06%	14.01%	9.07%	-1.77	2.14	20.23%	9.06%

Note: both the stand alone and the conglomerate cases are invariant to π. Holding and subsidiary figures coincide with stand alone figures for $\pi = 0$.HS figures obtain by summing up the holding and the subsidiary figures.

Table 7: Capital structure and value with intercorporate dividends, different values of rho, $\pi = 0.5$

Variables	Symbols	ρ								
		-0.8			-0.2			0		
Omega	ω	0	0.5	1	0	0.5	1	0	0.5	1
Face Value of Subs. Debt	P^*_{0s}	74	59	48	72	62	52	71	63	53
Face Value of Parent Debt	P^*_{0h}	56	82	119	55	71	88	58	70	86
Value of Subs. Debt	D^*_{0s}	54.90	44.69	36.69	53.12	46.40	39.37	52.23	46.89	39.96
Value of Parent Debt	D^*_{0h}	41.49	63.39	92.37	40.84	53.81	67.13	42.79	52.62	64.93
Levered Group Value	ν^*_{0g}	163.34	164.13	165.03	163.29	163.64	163.97	163.27	163.55	163.82
HS Lever. Ratio	D^*_{0g}/ν^*_{0g}	59.00%	65.85%	78.21%	57.54%	61.23%	64.95%	58.20%	60.84%	64.02%
Subsid. Optimal Lever. Ratio	D^*_{0s}/ν^*_{0s}	65.71%	54.14%	44.80%	64.00%	56.31%	48.06%	63.14%	56.99%	48.83%
Equity Value of Parent	E^*_h	38.31	37.86	35.96	39.45	45.44	57.47	37.76	46.35	58.92
Equity Value of Subsid.	E^*_s	28.65	37.12	45.22	29.87	35.99	42.53	30.49	35.85	41.88
Equity Value of Group	E^*_g	66.96	74.98	81.18	68.32	81.43	100.00	68.25	82.20	90.80
Default Threshold	X^d_g	154.10	168.02	199.26	150.49	158.05	166.62	152.75	157.88	165.22
No Tax Profit Level	X^Z_g	33.62	32.92	37.94	33.04	33.04	33.50	33.98	34.82	34.11
Tax Burden	T^*_0	34.80	34.91	34.14	34.89	34.92	34.82	34.85	34.82	34.72
Parent Tax Savings of Lever	TS^*_{0h}	2.26	2.89	4.13	2.20	2.67	3.24	2.36	2.70	3.27
Subsid. Tax Savings of Lever	TS^*_{0s}	2.97	2.23	1.76	2.93	2.43	1.97	2.92	2.50	2.03
Parent Exp. Default Costs	DC^*_{0h}	0.82	0.51	0.68	0.77	0.78	0.86	0.93	0.88	1.04
Subsid. Exp. Default Costs	DC^*_{0s}	1.13	0.51	0.27	1.16	0.73	0.42	1.18	0.82	0.49
Parent Yield Spread	y_h	1.18%	0.28%	0.20%	1.13%	0.70%	0.56%	1.27%	0.87%	0.78%
Subsidiary Yield Spread	y_s	1.15%	0.71%	0.52%	1.27%	1.97%	0.72%	1.33%	1.08%	0.81%
Value of Optimal Leverage	$\upsilon^*_0 - V_0$	3.25	4.04	4.94	3.20	3.55	3.88	3.18	3.46	3.72
Capital. Value of Opt. Lever	$Z(\upsilon^*_0 - V_0)/V_0$	9.37%	11.66%	14.24%	9.23%	10.25%	11.20%	9.18%	9.99%	10.75%

Table 7: Capital structure and value with intercorporate dividends, different values of rho, $\pi = 0.5$

Variables	Symbols	ρ					
		0.2			0.8		
Omega	ω	0	0.5	1	0	0.5	1
Face Value of Subs. Debt	P^*_{0s}	69	63	52	68	58	49
Face Value of Parent Debt	P^*_{0h}	55	68	83	51	66	89
Value of Subs. Debt	D^*_{0s}	50.77	46.73	39.17	38.15	42.98	36.81
Value of Parent Debt	D^*_{0h}	40.84	50.85	62.26	49.53	48.45	64.02
Levered Group Value	ν^*_{0g}	163.23	163.48	163.71	163.10	163.22	163.55
HS Lever. Ratio	D^*_{0g}/ν^*_{0g}	56.13%	59.69%	61.96%	53.75%	56.01%	61.65%
Subsid. Optimal Lever. Ratio	D^*_{0s}/ν^*_{0s}	61.56%	56.89%	47.93%	46.58%	52.65%	45.20%
Equity Value of Parent	E^*_h	39.91	48.19	62.30	43.04	52.46	62.72
Equity Value of Subsid.	E^*_s	31.70	35.41	42.56	32.38	38.65	44.62
Equity Value of Group	E^*_g	71.61	83.60	104.86	75.42	91.11	107.34
Default Threshold	X^d_g	146.90	155.40	160.36	140.92	146.86	163.21
No Tax Profit Level	X^Z_g	32.38	33.42	33.57	31.32	32.57	37.17
Tax Burden	T^*_0	34.99	34.83	34.81	35.15	34.96	34.25
Parent Tax Savings of Lever	TS^*_{0h}	2.20	2.66	3.22	1.99	2.73	3.87
Subsid. Tax Savings of Lever	TS^*_{0s}	2.83	2.53	2.00	2.86	2.34	1.90
Parent Exp. Default Costs	DC^*_{0h}	0.77	0.91	1.09	0.60	1.10	1.86
Subsid. Exp. Default Costs	DC^*_{0s}	1.11	0.87	0.49	1.25	0.83	0.51
Parent Yield Spread	y_h	1.13%	0.98%	0.92%	negative	1.38%	1.81%
Subsidiary Yield Spread	y_s	1.33%	1.16%	0.83%	7.25%	1.18%	0.89%
Value of Optimal Leverage	$\upsilon^*_0 - V_0$	3.14	3.39	3.62	3.01	3.13	3.45
Capital. Value of Opt. Lever	$Z(\upsilon^*_0 - V_0)/V_0$	9.07%	9.79%	10.46%	8.70%	9.03%	9.98%