

Informed Trading with Dynamic Information Dissemination*

Alex Boulatov[†] and Dmitry Livdan[‡]

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Abstract

This paper analyzes the equilibrium trading strategies of large heterogeneously informed traders who repeatedly increase their informational advantage over other agents by receiving private signals about fundamentals. In our model, which is based on rational strategic behavior, the exact timing of private information acquisition is public knowledge and all informed traders receive their signals simultaneously. In a continuous-time setting, we derive closed-form solution for the equilibrium intertemporal trading strategies of informed traders. While our solution is quite general, it can be naturally linked with a literature on market closures by assuming that new private information arrives at the beginning of each trading day, just before the stock market reopens. The key feature of the solution is that informed traders strategically anticipate future arrivals of new information. Because of this, the intertemporal pattern of optimal trading strategies is not periodic even when the timing of new information is periodic. This aperiodicity of trading is quite important since while it is a definitive feature of the data, it has been missing from the existing theoretical literature on market closures. In agreement with broad empirical evidence, we obtain a U-shape pattern of trading volume during the periods when the market is open, superimposed on a U-shaped pattern during the lifetime of the economy, before all information about the asset is revealed.

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[†]Bauer College of Business, University of Houston, Houston, TX 77204-6021. E-mail: aboulatov@uh.edu. Phone: (510) 326-9194

[‡]Walter A. Haas School of Business, University of California, Berkeley, S545 Student Services Bldg., #1900, Berkeley, CA 94720. Email: livdan@haas.berkeley.edu. Phone: 510-642-4733

1 Introduction

This paper studies the effects of dynamic acquisition of new long-lived private information by large heterogeneously informed traders on key market characteristics such as depth and trading volume. These traders care about the price impact of their trades on stock price dynamics and strategically compete for trading profits with each other and market maker.

While this is one of the most important issues in theoretical microstructure, the existing literature dealing with this problem is rather scarce. One approach taken for tractability purposes is to ignore the strategic nature of large traders and focus on pure informational effect on prices (Pearlman and Sargent (2005), Malinova and Smith (2005) and Makarov and Rytchkov (2005)). Another approach is to assume that the fundamental value process has a stationary auto-regressive structure (Taub, Bernhardt and Seiler (2005), henceforth TBS) and that the firm is liquidated at a random future time. Such assumptions imply stationarity and allow one to apply advanced computational methods based on the Fourier transform techniques such as the Weiner-Hopf method. The downside of this approach is that it requires both information acquisition and trading to have the same timing. The role of private information is no longer as clear which undermines the effect of the strategic competition between informed traders on prices and other market characteristics.

This paper solves the problem of informed trading with dynamic information acquisition by constructing a dynamic auction model based on rational strategic behavior. We focus on key differences between the effects of dynamic and static private information on trading by comparing two cases in which the same amount of private information is received by the informed traders either at a single trading date or gradually over the whole trading process. In order to explicitly capture the effects of dynamic information arrival, we construct our model so that it reduces to a known benchmark in a limit when the private information becomes static¹.

Our model has three agent types: the risk-neutral market maker (MM), insiders (informed traders), and

¹We choose the earlier work of Back, Cao and Willard (2000) as our benchmark. The main reason behind this choice is that we can “nest” the information structure from the model of Back, Cao and Willard in our information structure, while preserving analytic tractability. Note that this is nontrivial problem since a general extension of the model by Back, Cao and Willard to multiple private signals does not lead to an analytically tractable model. The main reason why the model of Back, Cao and Willard is analytically tractable is that the dynamics of agents’ information sets (based on Bayesian updating) is consistent with a special solution of the resulting Bellman equation describing the optimization for each informed trader known in technical literature as “caustic” (see, e.g. Courant and Hilbert (1989)). In our model, we are able to construct the information structure so that the “caustic” solution still works.

liquidity (noise) traders. The market maker observes aggregate demand and sets price in a regret-free way. The informed traders *strategically* compete with each other to maximize their total profits. The information structure is the key innovation of our model. In our paper informed traders, who do not know the fundamental value of the asset exactly, learn about it through *multiple* (private) signals. We formulate the problem of insiders recursively, using the backward induction method. We then show that the optimal trading strategy for each informed trader is a function of a linear combination of the current and past signals. In equilibrium, the weights of this linear combination are chosen by the informed traders in order to maximize their expected payoffs, and can be defined explicitly.

For tractability we make two key assumptions: the exact timing of private information acquisition is public knowledge and all informed traders receive their signals simultaneously. In making these assumptions we have in mind large institutional investors such as investment banks, mutual funds, or very large individual investors. Such investors have powerful in-house research departments that generate new information about their current and potential holdings. These investors also possess an accumulated expertise that allows them to assess the consequences of this new information better than small individual investors. Since the whole bulk of new information can be vast and difficult to process for the management and traders, it is therefore passed to them periodically (every day, twice a day, or even every hour) from the research department as an executive summary. Traders combine this research report with the tape to make their decisions. In this paper we concentrate our attention on the specific scenario when there is a single research report per day that arrives at the beginning of each trading day, just before the stock market reopens.² Each report contains some additional information about the asset that has become available while market has been closed and it repeatedly increases the informational advantage of informed traders over the MM.³

We also focus on a specific type of information about the asset such as future earnings or/and dividends. The key feature of such information is that while earnings or dividends do not change over rather long time interval, the content of private information that each informed trader has about them may change due to ongoing information acquisition. Therefore, it allows for different timing between the information acquisition

²We define market closures as regularly spaced in time, periodic predictable stops of trading. Although this interpretation subsumes periodic timing of new information arrival and thus is of the primary interest for us, our solution allows for more general non-periodic timing.

³The effect of increasing information asymmetry during market closures is consistent with empirical evidence (Ito, Lyons and Melvin (1998)).

and trading. The long-lived nature of such information also implies that all past private information remains relatively important to informed traders. Another important feature of such information is that it has to be publicly reported at some pre-specified future date. Therefore, this paper considers an announcement day when the fundamental value of the firm (dividend payment or earnings) becomes public thus rendering all private information useless.

Our model has three key results. First, the anticipated arrival of new information causes informed traders to use the old information more freely and, therefore, compete more fiercely against each other than in the case of single information arrival considered by Back, Cao and Willard ((2000), henceforth BCW). For example, in contrast to the results of BCW, we find that in an economy with market closures, two informed traders with uncorrelated signals typically trade *more* intensively than a monopolist trader with the same aggregate information.

Second, our model predicts a U-shape pattern in trading volume between each opening and closure. This is one of the empirical stylized facts associated with market closures.⁴ This result hinges on the interplay between new information arrival and the imperfect strategic competition among informed traders. Every time new information is received, the market briefly enters so-called “rate race” phase.⁵ During this phase, imperfectly informed insiders, who have not yet learned much from the price process about other insiders’ information, compete and trade very aggressively, trying to quickly make use of their new private information. At later times, the price reflects more of the informed traders’ private information. Each informed trader can make a better inference of other private signals. At this stage, the informed traders have incentive to trade less, in order not to reveal their own information. Therefore, trading intensity subsides during this phase. However, as the market approaches the time when new information will be received, insiders once more increase their trading intensity to make use of their current private information.

Our final finding is the key innovation of our model. While the structure of the financial market and the new information arrival pattern are both periodic by construction, we obtain a rich aperiodic temporal U-shape pattern of trading activity when the market is opened. This aperiodicity of trading is quite important since while it is a definitive feature of the data, it is missing from existing theoretical literature on market

⁴See, for example, Chan et. al. (1996) and Jain and Joh (1988).

⁵See Foster and Viswanathan (1996) and BCW.

closures. It arises quite naturally in our model due to informed traders acting strategically by anticipating the future information inflows in their strategies.

Our paper contributes to two areas of the market microstructure literature: trading in the presence of market closures and imperfect competition among informed traders.⁶ For a sake of brevity we focus our literature overview on a theoretical literature that uses both strategic⁷ and competitive settings to study the effect of market closures on trading and prices. Foster and Viswanathan (1990) examine a variation of a Kyle (1985) model with market closures. In their model, a closure allows private information to accumulate, altering the amount of adverse selection in the market and, thus, the equilibrium price process. While our model shares this feature with Foster and Viswanathan (1990), our crucial difference is that we have multiple informed traders competing for profits, while Foster and Viswanathan only consider a monopolist informed trader.

Hong and Wang (2000) study trading patterns implied by periodic market closures in a competitive market setting. In their model investors trade in stocks for two reasons. First, they hedge the risk of holding illiquid assets. Second, they speculate on future stock payoffs using their private information. When the market is closed, investors retain their closing positions from the previous trading period despite the arrival of new information. Consequently, investors optimally adjust their trading strategies during the trading period in anticipation of and following market closures, which gives rise to time variation in equilibrium returns. Hong and Wang (2000) explicitly derive investors' optimal allocations as well as the time pattern of returns. However, they find that under periodic market closures the resulting equilibrium is periodic as well, in the sense that patterns in returns and trading are the same, both qualitatively and quantitatively, for every trading day. Once again, the main difference between models using a competitive setting⁸ and our model is that we focus on the effect of the strategic imperfect competition among informed traders in the presence of market closures. By strategically timing their trades around market closures, informed traders are able to take advantage of both new and old information in our model, even when competition becomes

⁶The dynamic arrival of the new private information is what separates our paper from the work of Holden and Subrahmanyam (1992), Foster and Viswanathan (1996), and BCW on the imperfect competition among insiders.

⁷This literature was pioneered by Admati and Pfleiderer (1988, 1989) who, while not directly addressing market closures, analyze how investors' discretion in timing their liquidity trade can lead to the endogenous concentration of trades and price changes.

⁸In the partial equilibrium model Brock and Kleidon (1992) point out the link between time-variation in market activity and closures. Slezak (1994) examines the impact of closure on equilibrium returns using a noisy rational expectation equilibrium setting.

extremely intense. As a consequence, our model generates aperiodic trading patterns, even for periodic market closures. Although the optimal trading strategies remain U-shaped in every period when the market is open, the quantitative structure of this U-shape changes systematically across periods.

Methodologically, our paper is related to Taub, Bernhardt, and Seiler (2005) mentioned earlier in the introduction. They also develop a method for analyzing informed trading when agents possess long-lived private information about a firm's value and have repeated access to new information over time. In their model, insiders receive heterogeneous private information through infinite sums of AR(1) processes. Firms may be liquidated at any time with a given probability, but the true value of the firm is never revealed. Such a *stationary* structure of the economy allows TBS to map informed agents' optimization problems into the frequency domain. In the frequency domain they can use standard variational methods to find the optimal policy function. Essentially, they convert their problem, that has to be solved using conditional information, into an unconditional optimization problem. The whole problem then reduces to an Euler equation which takes the form of a Wiener-Hopf equation, which TBS solve to obtain optimal trading strategies. The key difference between our model and the TBS model is that in our model the value of the asset is fully revealed at some pre-specified future date, thus rendering all past private information useless. As a result, we solve the *non-stationary* problem for which the Wiener-Hopf method is not applicable⁹. Another difference between these two models is that we follow the traditional stream of literature represented by Kyle (1985), Foster and Viswanathan (1996), and BCW by considering *stationary* fundamentals (i.e. they do not change in time). Although the economy in TBS is essentially *stationary*, the fundamentals in their model are time-dependent by construction and, therefore, *non-stationary*. TBS motivate the arrival of new information by dividend/profit announcements that typically occur on quarterly or, at best, monthly frequency. Therefore, their model cannot be used at higher trading frequencies (i.e. daily and weekly) which are of the most interest.

The rest of the paper is organized as follows. The set-up of the model and the derivation of optimal trading strategies are presented in Section 2. The optimization problem of informed traders with market closures is formulated and solved in Section 3. The comparative dynamics and the implications of the model

⁹The Wiener-Hopf equation can be derived only for stationary problems.

on the intensity of trading and on the distribution of payoffs across the informed traders are considered in Section 4. We conclude in Section 5.

2 The model

We consider an economy in which a single risky asset is traded in the financial market over the time period $[0, T)$. There are three types of risk-neutral agents in the economy: $N \geq 1$ informed traders, the market maker (MM), and a number of uninformed liquidity (“noise”) traders. Informed traders trade continuously when the market is open. MM observes the aggregate order of informed and liquidity traders and sets the asset price, $P(t)$. The cumulative order of the liquidity traders per one insider is equal to $u(t) = \sigma_u Z(t)$, where $Z(t)$ is a standard Wiener process. The risk-free rate is set to zero.

We assume the financial market is closed periodically, and that there are a total of $K-1$ closures and K openings before the true value of the asset is revealed at $t = T$. We will use t_k^o with $k = 0, \dots, K-1$ to indicate the time of k -th opening and let $\mathbf{O} = \bigcup_{k=0}^{K-1} \mathcal{O}^k$, where $\mathcal{O}^k = \{t : t \in [t_k^o, t_k^c)\}$, denote the set of all times when market is open. Analogously, we will use t_k^c with $k = 0, \dots, K-2$ to indicate the time of k -th closure and let $\mathbf{C} = \bigcup_{k=0}^{K-2} \mathcal{C}^k$, where $\mathcal{C}^k = \{t : t \in [t_k^c, t_{k+1}^o)\}$, denote the set of all times when market is closed. We will also use a subscript k throughout the paper to indicate that a parameter has a value specific to the k -th opening and closure. Importantly, our model setting is not limited to this “equidistant” choice of trading and non-trading intervals. Since we are focusing on the trading process, we will consider all quantities as a function of the “event time” when the non-trading periods are omitted. However, we will take into account the changes of the informed traders’ information sets during the non-trading periods. This does not lead to any loss of generality and is merely a convenient notation.

We construct the private information acquisition of informed traders as follows. At $t = 0$ as well as during each period when the market is closed, $t \in \mathcal{C}^k$, each informed trader i receives a private signal, S_i^k . Informed traders receive no private signals when the market is open. The true value of the asset is revealed at T . By using this structure, we capture the important fact that there exists a potentially large inflow of information independent of the organizational structure of the financial market: investors keep receiving private information about assets when the market is closed and all trading activity is suspended.

This complete separation of periods of private information acquisition from trading periods enables us to investigate the essential effect of market closures on trading activity while allowing for closed-form analytic solutions of the optimal strategies.¹⁰ Moreover, these assumptions together with the fact that the liquidation value of the asset is fully revealed at T imply that, contrary to the case considered by TBS, we are dealing with a non-stationary problem.

The next set of assumptions highlights the informational structure of our model. At initial moment $t = 0$, there is a random draw of the fundamental value V of the asset from the normal distribution with mean zero and variance σ_0^2 . We assume that the private signals at time t_k ($k = 0, \dots, K - 1$) are given by

$$S_i^k = V + \sigma_s \left(\varepsilon_i^k - \frac{1}{N} \sum_{j=1}^N \varepsilon_j^k \right), \quad (1)$$

with $\varepsilon_i^k \sim \mathcal{N}(0, 1)$ being uncorrelated for different time periods, and correlated in the cross-section

$$\text{corr}(\varepsilon_i^k, \varepsilon_j^k) = \rho_k, \quad (\forall k = 0, \dots, K - 1, i \neq j), \quad (2)$$

$$\text{corr}(\varepsilon_i^k, \varepsilon_j^{k'}) = 0, \quad (\forall i, j, \forall k = 0, \dots, K - 1, k' = 0, \dots, K - 1, k \neq k').$$

Assumption (1) implies, in the spirit of Kyle (1985) and analogous to BCW, that the arithmetic mean of the signals obtained by all insiders during every closure period, does not change with time and equals the fundamental value

$$V = \frac{1}{N} \sum_{j=1}^N S_j^k, \quad k = 0, \dots, K - 1, \quad (3)$$

and that the relation (3) is satisfied for each closure period. To make the information structure analogous to BCW, we also assume that the informed traders' as well as the market maker's valuations *before* the private signals at $t = 0$ are observed is given by

$$V \sim \mathcal{N}(0, \phi_0 \sigma_s^2), \quad (4)$$

and therefore $\sigma_0^2 = \phi_0 \sigma_s^2$, with

$$\phi_0 = \frac{1 + (N - 1) \rho_0}{N}. \quad (5)$$

From the definition (1), it follows that the conditional marginal distribution of the private signals obtained at the period k is given by

$$S_i^k | V \sim \mathcal{N}(V, (1 - \phi_k) \sigma_s^2), \quad k = 0, \dots, K - 1, \quad (6)$$

¹⁰When this assumption is relaxed, the model can still be solved numerically. However, this would not change the main conclusions.

with

$$\text{cov}(S_{i \neq j}^k, S_j^k | V) = (\rho_k - \phi_k) \sigma_s^2, \quad (7)$$

$$\text{var}(S_i^k | V) = (1 - \phi_k) \sigma_s^2,$$

as well as that, conditioned on V , private signals are i.i.d. across time

$$\text{cov}(S_i^k, S_j^{k'} | V) = 0, \quad \forall (i, j, k \neq k'). \quad (8)$$

Combining (4) and (7), we obtain for the unconditional correlations of the private signals

$$\text{corr}(S_{i \neq j}^k, S_j^k) = \rho_k, \quad (9)$$

which is also analogous to the BCW setting. The details of calculation are provided in Appendix A.

The prior distribution (4) is a common knowledge, and it is, therefore, known to the MM. Distributions (6) and (4) determine the dynamics of the Bayesian updating process of informed traders in our model. Appendix A provides full details of this Bayesian updating process.

At each trading period, the MM sets $P(t)$ to be regret-free based on her current beliefs and the observation of the total order flow which is a function of the aggregate private signal. As a result, informed traders may learn about the aggregate private signal from the price while not being able to observe it directly. Therefore, in contrast to BCW, informed traders in our model update their information set over time using two *independent* but *complementary* sources: by learning about the aggregate signal from the price $P(t)$ during periods when the market is open, and also by learning from their own private signals during the market closure periods.¹¹ This is an important feature of our model since it makes the learning process of insiders more efficient than that of the MM, who can only update her beliefs at times when informed trade occurs. The more efficient information updating by insiders causes their superior informativeness relative to that of the MM to be comparatively long-lived in our model, which, in turn, has important pricing implications.

2.1 Equilibrium Strategies

To fully capture the effect of competition among informed traders on trading as well as to obtain the results in a compact and parsimonious form, we cast the problem in a continuous time setting. According

¹¹The first feature is essentially present in the standard Kyle (1985) model. However, in Kyle (1985) and further work based on it, (e.g. Holden and Subrahmanyam (1992) and BCW), the combined impact of both sources of learning has not been addressed.

to the above discussion, we look for the dynamic equilibrium strategies when the i -th informed trader's incremental demand, $dx_i(t)$, during the time interval dt depends on her signal and the instantaneous price, $P(t)$. As it is analyzed in Foster and Viswanathan (1996), such a specification allows us to avoid the problem of "forecasting of the forecasts" which occurs when the informed traders' strategies are based on their valuations. On the other hand, the optimal informed traders' strategies are expressed in terms of the instantaneous valuations and the price at equilibrium.

Analogous to Foster and Viswanathan (1996) and BCW, the dynamic trading strategy of the i -th informed trader when her incremental demand $dx_i(t)$ is linear in her private signal as well as the instantaneous price, $P(t)$, and is given by

$$dx_i(t) = \beta(t) \left[\widehat{S}_i(t) - P(t) \right] dt, \quad (10)$$

where the effective private signal, $\widehat{S}_i(t)$, is an estimate of the fundamental, based on the private signals available at the time t ¹². In contrast to Foster and Viswanathan (1996) and BCW, informed traders in our model receive multiple private signals at times when the market is closed. Since taking into account both past and current signals increases the precision of the effective signal, the optimal trading strategy for each informed trader should involve an optimally weighted combination of the current and past signals. By assumption, the arrival of the private signals is completely separated from the trading process. For this reason, each insider constructs his effective signal independently of the trading process. Therefore, the effective signals change during the market closure periods, and remain constant when the market is open. Note that this is in a sharp contrast with the results of TBS, where the fundamentals receive shocks every period, leading to constant updating of the informed traders' information sets.

For that reason, the effective signal at period $t \in \mathcal{O}^k$, is given by

$$\widehat{S}_i(t) = \sum_{l=0}^k a_{l,k} S_i^l, \quad (11)$$

where the weights $\{a_{l,k}, l = 0, \dots, k\}$, are chosen by informed traders to maximize the precision of their effective signals and will be derived in equilibrium in the next section. It will also be shown that these weights satisfy the normalization condition $\sum_{l=0}^k a_{l,k} = 1$. Note that at each closure period $k = 0, \dots, K$, the informed traders update all the weights $\{a_{l,k}, l = 0, \dots, k\}$. Therefore, the weights are different during

¹²As we show below, (10) indeed delivers a Nash Bayesian equilibrium to our model.

different periods when the market is open.

The MM observes the total incremental demand per one informed trader

$$dy(t) = \frac{1}{N} \sum_{i=1}^N dx_i(t) + du = \beta(t) [V - P(t)] dt + \sigma_u dZ, \quad (12)$$

where we have used that

$$\frac{1}{N} \sum_{i=1}^N \widehat{S}_i(t) = V \sum_{l=0}^k a_{l,k} = V. \quad (13)$$

Let $\mathbf{F} \equiv \{\mathcal{F}(t) : t \in \mathbf{O}\}$ denote the filtration generated by $y(t)$ and set $P(t) = \mathbb{E}[V | \mathcal{F}(t)]$. We interpret \mathbf{F} as the MM's information structure. Analogous to BCW, the market maker's uncertainty $\Sigma_M(t)$ and the price are revised according to

$$d\left(\frac{1}{\Sigma_M(t)}\right) = \frac{\beta(t)^2}{\sigma_u^2} dt, \quad (14)$$

and

$$dP(t) = \lambda(t) dy(t), \quad (15)$$

where the inverse market depth parameter is given by

$$\lambda(t) = \frac{\beta(t)}{\sigma_u^2} \Sigma_M(t). \quad (16)$$

At time $t = 0$, the market maker starts with

$$\Sigma_M(0) = \phi_0 \sigma_s^2, \quad (17)$$

$$P(0) = 0.$$

Next we need to make a similar analysis of the information structure of informed traders. Let $P(t)$ denote the solution (assumed to exist) of equation (15). The informed trader i observes $P(t)$ and her effective signal $\widehat{S}_i(t)$. Let $\mathbf{F}_i \equiv \{\mathcal{F}_i(t) : t \in \mathbf{O} \cup \mathbf{C}\}$ denote the filtration generated by $P(t)$ and $\widehat{S}_i(t)$, and set $V_i(t) = \mathbb{E}[V | \mathcal{F}_i(t)]$. This is the informed trader's information structure. Analogous to BCW, we obtain that each insider's residual uncertainty $\Sigma_i(t)$ and valuation $V_i(t)$ are described by the processes

$$d\left(\frac{1}{\Sigma_i(t)}\right) = \frac{\beta(t)^2}{\sigma_u^2} dt, \quad (18)$$

and

$$dV_i = \frac{\beta(t) \Sigma_i \left[\frac{1}{\lambda(t)} dP(t) + \beta(t) (P(t) - V_i(t)) dt \right]}{\sigma_u^2}. \quad (19)$$

At time $t = 0$ informed trader i starts with

$$\Sigma_i(0) = (1 - \phi_0) \phi_0 \sigma_s^2 = (1 - \phi_0) \Sigma_M(0), \quad (20)$$

$$V_i(0) = \phi_0 S_i^0.$$

The revision processes for both MM's and the insiders' information sets follow immediately from an application of the Kalman-Bucy filter (see Lipster and Shiryaev (2000)).

Note that (20) describes the i -th informed trader's best estimate of the aggregate signal, as has been pointed out by Foster and Viswanathan (1996) and BCW. If the informed trader is perfectly informed, i.e. $\phi_0 = 1$, then $\Sigma_i(0) = 0$. The factor ϕ_k measures, therefore, the informativeness of the informed trader relative to the MM at the start of the k -th market opening. Making use of the characterizations for the informed trader's strategy and the price process given above, we obtain

$$dP(t) = \lambda(t) \{ \beta(t) [V - P(t)] dt + \sigma_u dZ(t) \}, \quad (21)$$

and

$$dV_i(t) = [1 - \delta(t)] \lambda(t) \{ \beta(t) [V - V_i(t)] dt + \sigma_u dZ(t) \}, \quad (22)$$

where the information spread parameter $\delta(t) = 1 - \frac{\Sigma_i(t)}{\Sigma_M(t)}$ characterizes the information asymmetry between the MM and the informed traders. From (20), we obtain

$$\delta(0) = \phi_0, \quad (23)$$

analogous to BCW. It follows from (14) and (18) that both price and the informed trader's valuation approach the true value V as time increases. This reflects the fact that the informed traders and the MM learn about the fundamentals over time. As we will see below, the individual rates of learning may depend on time in a complex way.

2.2 The Effective Signals

Comparing equations (14) and (18), one can see that the MM's and the informed trader's learning processes are equally efficient when the market is open and the informed traders do not receive any private signals. However, the MM's beliefs have less precision at the market re-opening periods, since the MM observes

the aggregate demand but does not receive a private signal. The MM's information set depends on the liquidity parameter $\beta(t)$, which has to be defined in a self-consistent way by optimizing the informed trader's expected payoffs. Since, by assumption, the MM does not learn during the closure periods, the market maker's uncertainty $\Sigma_M(t)$ is continuous across the closures

$$\Sigma_M(t_k^c) = \Sigma_M(t_{k+1}^o), \quad k = 0, \dots, K-2. \quad (24)$$

Since the informed trader is learning from both the price process and the private signals, one should expect that her valuation can be expressed in terms of these two quantities. These relations are formalized in Proposition 1 below.

Proposition 1 *The proposition consists of three main results.*

1. *If we define the relative valuation of the i -th informed trader as*

$$\widehat{V}_i(t) \equiv V_i(t) - (1 - \delta)P(t), \quad (25)$$

then

$$d\widehat{V}_i(t) = -k(t)\widehat{V}_i(t)dt, \quad (26)$$

with $k(t) = [1 - \delta(t)]\lambda(t)\beta(t)$.

2. *The informed trader's valuation $V_i(t)$ after the market reopens at $t = t_k^o$ is related to the price and the effective private signal by*

$$V_i(t) = [1 - \delta(t)]P(t) + \delta(t)\widehat{S}_i(t_k^o), \quad t \in \mathcal{O}^k, \quad (27)$$

where the effective signal is defined recursively by

$$\widehat{S}_i(t_{k+1}^o) = \mu_{k+1}\widehat{S}_i(t_k^o) + (1 - \mu_{k+1})S_i^{k+1}, \quad (28)$$

with

$$\mu_{k+1} = \frac{1 - \delta(t_{k+1}^o)}{1 - \delta(t_k^c)} \frac{\delta(t_k^c)}{\delta(t_{k+1}^o)}, \quad (29)$$

and the initial condition

$$\mu_0 = 0. \quad (30)$$

3. The dynamics of $\delta(t)$ are described by the ODEs

$$\begin{aligned} \frac{d}{dt} \delta(t) &= -\delta(t) [1 - \delta(t)] \lambda(t) \beta(t), \quad t \in \mathcal{O}^k, \forall k, \\ \frac{1}{1 - \delta(t_{k+1}^o)} &= \frac{1}{1 - \delta(t_k^c)} + \frac{\Sigma_M(t_k^c)}{(1 - \phi_k) \sigma_s^2}, \quad \forall k. \end{aligned} \quad (31)$$

Proof: See Appendix B.

Note that the second equation in (31) describes the dynamics of $\delta(t)$ due to the private learning process. If the private learning is absent, the dynamics of $\delta(t)$ are fully described by the first ODE in (31), and the information asymmetry is a monotonically decreasing function of time. In the presence of private learning, $\delta(t)$ increases during the market closure periods due to the private learning and thus is no longer a monotonically decreasing function of time. This makes the temporal pattern of $\delta(t)$ non-monotonic in our model. Therefore, the information asymmetry decreases when the market is opened and increases during the market closure periods. This result has a simple intuition. Namely, the informed trader's valuation is based on both the price process and her private signal. From equation (27), it is clear that the information asymmetry parameter $\delta(t)$ characterizes the relative weights of each of these two factors. During the periods when the market is open, the price process acquires more weight in the informed traders' information sets and therefore the asymmetry $\delta(t)$ should decrease. By the same argument, $\delta(t)$ should increase during the market closure periods, when the informed traders only learn from the private signals. As we will see below, this is the key reason for the increasing trading activity after the market re-openings.

After the market reopens at t_k^o , the trading strategies for the period \mathcal{O}^k are characterized by

Corollary 1 *The trading strategy of the i -th informed trader during the k -th market opening is*

$$\theta_i(t) = \beta(t) \left[\widehat{S}_i(t_k^o) - P(t) \right] = \frac{\beta(t)}{\delta(t)} [V_i(t) - P(t)]. \quad (32)$$

Proof. Immediately follows from (27).

The dynamics of the asymmetry parameter $\delta(t)$ are characterized by the following proposition.

Proposition 2 *The information asymmetry $\delta(t)$ satisfies*

$$\frac{1}{\delta(t)} = 1 + \frac{\sigma_s^2}{\Sigma_M(t)} \left(\sum_{m=0}^k \left(\frac{1}{1 - \phi_m} \right) \right)^{-1}, \quad t \in \mathcal{O}^k, \quad k = 0, \dots, K - 1. \quad (33)$$

In particular, we have

$$\frac{1}{\delta(t_k^o)} = 1 + \frac{\sigma_s^2}{\Sigma_M(t_k^o)} \left(\sum_{m=0}^k \left(\frac{1}{1 - \phi_m} \right) \right)^{-1}, \quad (34)$$

$$\frac{1}{\delta(t_k^c)} = 1 + \frac{\sigma_s^2}{\Sigma_M(t_k^c)} \left(\sum_{m=0}^k \left(\frac{1}{1 - \phi_m} \right) \right)^{-1}, \quad k = 0, \dots, K - 1.$$

Proof: See Appendix B.

With the help of Proposition 2 we can now fully characterize the weights of the effective signal, $a_{l,k}$.

Proposition 3 *The weights of the effective signal are given by*

$$a_{l,k} = \left(\sum_{n=0}^k \left(\frac{1 - \phi_l}{1 - \phi_n} \right) \right)^{-1}, \quad l = 0, \dots, k. \quad (35)$$

Proof: See Appendix B.

It follows from (35) that the insider's valuation involves a factor related to the information asymmetry which exhibits non-monotonic and non-periodic dynamics over the different market closure periods. This is in sharp contrast to the model of TBS, where a stationary time structure of the information inflow has been assumed.

It immediately follows from Proposition 3 that in the specific case when correlation coefficients ρ_k are the same for all market closures, $\rho_k = \rho$, $k = 0, \dots, K - 1$ which implies $\phi_k = \phi$, $k = 0, \dots, K - 1$, the weights of the effective signal are equal to

$$a_{l,k} = \frac{1}{k + 1}, \quad l = 0, \dots, k. \quad (36)$$

Equation (36) implies that in this special case all optimal weights for the signals obtained at different times enter with *equal weights* into the effective signal. This is a consequence of the fact that in this case all private signals obtained by each insider are drawn from the same distribution, and is consistent with the result that the equally-weighted average is the optimal linear unbiased estimator of the fundamental. For the sake of brevity and without loss of generality we consider this case in all subsequent sections.

Now, we are in a position to formulate and solve the dynamic optimization problem for the informed traders.

3 Dynamic Optimization and Equilibrium

We start by defining the set of feasible trading strategies for informed traders. We define a trading strategy θ_i to be feasible for trader i if there exists a unique solution $P(t)$ to the stochastic differential equation (21) for the given λ and for the given β that characterize the other traders' strategies and if

$$\lim_{t \rightarrow T} P(t) \text{ exists a.s.}, \quad (37)$$

$$\int_0^T \theta_i(t) dt \text{ exists a.s.}, \quad (38)$$

and

$$\mathbb{E} \left[\int_0^T P^2(t) dt | V \right] < \infty. \quad (39)$$

These limits define, respectively, the price and number of shares held by trader i just before the announcement. Condition (39) is the “no doubling strategies” condition introduced in Back (1992).

Making use of Propositions 1 and 2, we can analyze the optimal trading intensity for the informed traders which characterize their optimal strategies¹³. Proposition 4 below summarizes these results.

Proposition 4 *The Bellman equation for the informed trader's value function*

$$\pi(P, t) = \max_{\theta(t)} \left\{ \mathbb{E}_t \left[\int_t^T dt' \theta(t') [V_i(t') - P(t')] \right] \right\}, \quad (40)$$

is given by

$$\max_{\theta} \left\{ \left(\frac{\partial}{\partial t} + L_D^\theta \right) \pi(P, t) + \theta [V_i - P] \right\} = 0, \quad (41)$$

where the optimization is over the set of feasible strategies and the Dynkin operator is defined by

$$L_D^\theta = \left[\lambda \beta \left(V_i - \frac{1}{N} \hat{S}_i - \frac{N-1}{N} P \right) + \frac{1}{N} \lambda \theta \right] \frac{\partial}{\partial P} + \frac{1}{2} \lambda^2 \sigma_u^2 \frac{\partial^2}{\partial P^2}.$$

Proof: See Appendix B.

Analogous to BCW, the equilibrium solution exists if

$$\frac{1}{N} \frac{\partial \pi}{\partial P} = \frac{[P - V_i]}{\lambda(t)}. \quad (42)$$

¹³Note that according to the definition of Bayesian Nash equilibrium (see e.g. Kyle (1985) and BCW), each informed trader chooses her optimal strategy assuming that the price increments (15) are characterized by a *given* inverse market depth $\lambda(t)$. In other words, in Nash equilibrium setting, the informed traders construct their “reaction function” to the inverse market depth $\lambda(t)$ (set by the MM), and do not take an impact of their strategies onto $\lambda(t)$ into account.

Making use of (42) and partially differentiating (41) with respect to P yields

$$(P - V_i)\eta + \beta \left(V_i - \frac{1}{N}\widehat{S}_i - \frac{N-1}{N}P \right) - \beta \left(\frac{N-1}{N} \right) (P - V_i) = 0, \quad (43)$$

where $\eta(t) = \frac{d}{dt} \left(\frac{1}{\lambda(t)} \right)$. Combining (43) and (27), we finally obtain the dynamics of the inverse market depth parameter $\lambda(t)$ summarized by Proposition 5 below.

Proposition 5 *When the optimal strategies are exercised, the dynamics of the market depth parameter are described by the ODE*

$$\frac{d}{dt} \left(\frac{1}{\lambda(t)} \right) = \left(2N - 1 - \frac{1}{\delta(t)} \right) \frac{\beta(t)}{N}, \quad (44)$$

and

$$\lim_{t \rightarrow T} \Sigma_M(t) = 0. \quad (45)$$

Proof: See Appendix B.

Note that the boundary condition (45) is standard (see e.g. Kyle (1985) and BCW) and states that all uncertainty about fundamentals is resolved after the public announcement at $t = T$. An equilibrium with market closures is defined as follows.

Definition 1 *An equilibrium with market closures is a set of $\{\beta_k(t), \lambda_k(t)\}_{k=0}^{K-1}$ that are continuous on $t \in [0, T)$ and continuously differentiable on $t \in \mathbf{O}$, with β_k and λ_k positive $\forall k$, and which satisfy (i) $P(t) = V_M(t)$ for all t , and (ii) the trading strategy (32) is feasible and maximizes the expected profits of the informed trader given by (40) over the set of feasible strategies.*

We would like to note that the continuity of $\{\beta_k(t)\}_{k=0}^{K-1}$ and consequently the inverse market depth $\{\lambda_k(t)\}_{k=0}^{K-1}$ across the closures is the dominating strategy for the informed traders.¹⁴ The intuition behind this is simple. According to (51) the inverse depth of the market depends on the amount of information in the order flow, which depends on the amount of information outstanding measured by Σ_M and the trading intensity of the informed traders measured by β . Suppose that the informed traders increase their trading intensity β_k after they receive new signals over the market closure period \mathcal{C}^k , $\beta_k(t_k^c) > \beta_k(t_{k-1}^c)$. This would increase λ_k and, consequently, the informed traders' expected payoffs in a short run. However, the greater intensity of trading would lead to greater learning by the market and faster declining Σ_M . Consequently, the expected profits of the informed traders would decline more in a long run than in the case of continuous

¹⁴This is not surprising given that all these parameters are obtained using the Kalman filter techniques.

β_k . Overall, the extra long-run expected profit losses from increasing β_k during the market closure outweigh the short-run expected profit gains from this strategy. Therefore, the informed traders are better off by not “tipping” their hands early and keeping β_k continuous over the market closures.

We put some extra structure on market closures by denoting the durations of the periods when the market is open or closed as τ and $\Delta\tau$, respectively (see Figure 1), so that

$$t_k^o = k(\tau + \Delta\tau), \quad (46)$$

$$t_k^c = t_k^o + \tau = k(\tau + \Delta\tau) + \tau.$$

As we have discussed above, we use the event time, which formally leads to $\Delta\tau = 0$. Our solution is obtained by solving the ODE (44) with the help of Propositions 1, 2, and 4. More specifically, our solution method involves backward induction across the market closure periods combined with solution of (44) for the periods when the market is open.

Theorem 1 *If there are multiple informed traders ($N > 1$), there is a unique linear equilibrium. Set $\Sigma_M(0) = \phi\sigma_s^2$ and define the constants $\{C_k\}_{k=0}^{K-1}$ as follows*

$$C_k = \frac{1}{\tau} \int_{r_k}^{r_{k+1}} x^{2(N-2)/N} \exp\left[-\frac{2x(1-\phi)}{N\phi(k+1)}\right] dx, \quad k = 1, \dots, K-2, \quad (47)$$

with

$$C_0 = \frac{1}{\tau} \int_1^{r_1} x^{2(N-2)/N} \exp\left[-\frac{2x(1-\phi)}{N\phi}\right] dx, \quad (48)$$

and

$$C_{K-1} = \frac{1}{\tau} \int_{r_{K-1}}^{\infty} x^{2(N-2)/N} \exp\left[-\frac{2x(1-\phi)}{N\phi K}\right] dx, \quad (49)$$

where $r_k = \Sigma_M(0)/\Sigma_M(t_k^o)$. The constants $\{C_k\}_{k=1}^{K-1}$ and $\{r_k\}_{k=0}^{K-1}$ are defined by the recursion relations

$$C_k = C_{k-1} \exp\left[\frac{2r_k(1-\phi)}{N\phi} \left(\frac{1}{k} - \frac{1}{k+1}\right)\right].$$

For each $t \in \mathcal{O}^k$, $k = 0, \dots, K-1$, define $\Sigma_M(t)$ as

$$\int_{r_k}^{r(t)} x^{2(N-2)/N} \exp\left[-\frac{2x(1-\phi)}{N\phi(k+1)}\right] dx = (t - t_k^o)C_k, \quad (50)$$

with $r(t) \equiv \Sigma_M(0)/\Sigma_M(t)$, and define $\Sigma_M(t) = \Sigma_M(t_k^c)$ for each $t \in \mathcal{C}^k$, $k = 0, \dots, K-1$. The equilibrium

is $\{\beta_k(t), \lambda_k(t)\}_{k=0}^{K-1}$, where

$$\begin{aligned}\beta_k(t) &= \sigma_u (C_k)^{1/2} \left(\frac{\Sigma_M(t)}{\Sigma_M(0)} \right)^{(N-2)/N} \exp \left[\frac{(1-\phi) \Sigma_M(0)}{N\phi(k+1) \Sigma_M(t)} \right], \\ \lambda_k(t) &= \beta_k(t) \frac{\Sigma_M(t)}{\sigma_u^2},\end{aligned}\tag{51}$$

for $t \in \mathcal{O}^k$. Finally, $\{C_k, \Sigma_M(t_k^c)\}_{k=0}^{K-1}$ is fully determined by the Proposition 5, system (50), and the continuity of $\Sigma_M(t)$ across market closures

$$\Sigma_M(t_{k+1}^o) = \Sigma_M(t_k^c), \quad k = 0, \dots, K-2.\tag{52}$$

Proof: See Appendix B.

In the case of a monopolist insider, we recover the results of Kyle (1985). The intuition is that, in the monopolist case, the insider has the exact information of the fundamentals at the very beginning of the trading process when $t = 0$, and therefore the additional signals obtained during the closure periods are irrelevant. Therefore, the problem becomes identical to the one considered in BCW with $N = 1$, which reduces to the case of Kyle (1985). This is summarized in the corollary below.

Corollary 2 (Monopolist informed trader) Consider $N = 1$ and $K - 1$ closures. The unique equilibrium for the monopolist is given by $\{\beta_k(t), \lambda_k(t)\}_{k=0}^{K-1}$, where

$$\begin{aligned}\beta_k(t) &= \frac{\sigma_u}{\sigma_s} \frac{1}{\sqrt{T}} \frac{1}{1 - \frac{t}{T}}, \\ \lambda_k(t) &= \frac{\sigma_s}{\sigma_u} \frac{1}{\sqrt{T}}, \quad \forall k = 0, \dots, K-1.\end{aligned}\tag{53}$$

The market maker's variance for each $k = 0, \dots, K$, is

$$\Sigma_M(t) = \Sigma_M(0) \left(1 - \frac{t}{T} \right),\tag{54}$$

where $T = K\tau$.

Proof: See Appendix B.

Analogous to BCW, the case of two informed traders in our model allows for an explicit analytical solution. The main reason for this is that for $N = 2$, the integrals (50) can be evaluated explicitly. The results for the duopoly case are presented in the corollary below.

Corollary 3 Consider $N = 2$. Define constants $\{A_k\}_{k=0}^K$ and $\{r_k\}_{k=0}^K$ as follows

$$\begin{aligned} A_K &= +\infty, \\ A_k &= \left(\sum_{n=k+1}^K \frac{1}{n} \right)^{-1}, \quad k = 0, \dots, K-1, \end{aligned} \tag{55}$$

and

$$\begin{aligned} r_0 &= 1, \quad r_K = +\infty, \\ r_k &= 1 + \sum_{n=1}^k \frac{n\phi}{1-\phi} \ln \left(1 + \frac{A_n}{n} \right), \quad k = 0, \dots, K-1. \end{aligned} \tag{56}$$

The unique equilibrium is given by $\{\beta_k(t), \lambda_k(t)\}_{k=0}^{K-1}$, where, in each of the intervals $t \in \mathcal{O}^k$, the coefficients $\{\beta_k(t), \lambda_k(t)\}_{k=0}^{K-1}$ are defined by (51), and the market maker's variance for each $k = 0, \dots, K-1$, is

$$\Sigma_M(t) = \Sigma_M(0) \left\{ r_k + \frac{(k+1)\phi}{1-\phi} \ln \left[\left(1 - \frac{(t-t_k^o)A_k}{k+1} \right)^{-1} \right] \right\}^{-1}, \quad t \in \mathcal{O}^k. \tag{57}$$

Proof: See Appendix B.

4 Comparative Dynamics

The result (44) looks similar to the one from BCW. As we have discussed above, the key difference is that in our case the dynamics of the asymmetry parameter $\delta(t)$ are different due to the private signals received during market closures. In particular, $\delta(t)$ undergoes jumps after the market re-openings due to the informed traders' private learning over the market closure periods. These jumps cause jumps in the aggregate trading volume.

The contribution to the trading volume of the i -th informed trader can be measured by unconditional variance

$$TV_i(t) = \text{var} \left[\frac{dx_i(t)}{dt} \right] = \beta^2(t) \text{var} \left[\widehat{S}_i(t) - P(t) \right], \tag{58}$$

where $dx_i(t)$ is the incremental demand during the time interval dt . Since the price increments in (21) depend on the aggregate signal $\frac{1}{N} \sum_{j=1}^N \widehat{S}_j(t) = V$, and the liquidity trades, they are unconditionally correlated with all individual signals $\{\widehat{S}_j(t)\}$. Therefore, (58) yields

$$TV_i(t) = \beta^2(t) \left(\text{var} \left[\widehat{S}_i(t) \right] + \text{var} [P(t)] - 2\text{cov} \left[\widehat{S}_i(t), P(t) \right] \right). \tag{59}$$

In order to evaluate (59), we will prove the following result analogous to the one obtained by BCW.

Corollary 4 *At each time $t \in [0, T]$, the price process (21) takes the form*

$$P(t) = \Sigma_M(t) \int_0^t \left\{ \frac{1}{N} \sum_{j=1}^N \widehat{S}_j(t') d \left(\frac{1}{\Sigma_M(t')} \right) + \frac{\lambda(t')}{\Sigma_M(t')} \sigma_u dZ(t') \right\}. \quad (60)$$

Proof: See Appendix B.

Note that (60) describes how the price accumulates the aggregate private signals of the informed traders and the liquidity trades over time. Clearly, the private signals enter the price linearly, but with the time-dependent weights, and the price movements are unconditionally correlated. Since the valuation of each informed trader is based on both the effective private signal and on the current price by (27), the current valuations can also be represented as a linear functional of all the available private signals with the time-dependent weights.

Define an integer function

$$n(t) = \left[\frac{t}{\tau} \right] + 1, \quad (61)$$

where $[\cdot]$ stands for the integer part of the argument. Clearly, $n(t)$ corresponds to the number of private signals obtained by each informed trader up until the time t . With this definition and making use of the above corollary, we obtain the following

Corollary 5 *The trading volume defined by (58) is given by*

$$\begin{aligned} TV_i(t) &= T_1(t) + T_2(t), \\ T_1(t) &= \beta^2(t) \sigma_s^2 \frac{(1-\phi)}{n(t)}, \\ T_2(t) &= \beta^2(t) \Sigma_M(t). \end{aligned} \quad (62)$$

Proof: See Appendix B.

The trading volume (62) has two different contributions. The first term on the right hand side of (62) reflects the amount of the trading activity when the informed traders take opposite sides of the market. By analogy with Evans and Lyons (2002), it may be referred to as the “inter-informed” trading volume component. Clearly, the “inter-informed” component originates from the heterogeneity of signals across the informed traders. The second term on the right hand side in (59) corresponds to the amount of trading when informed traders are mostly on the same side of the market (“market-informed” component) and are

proportional to the market uncertainty. In order to construct a measure of the total informed trading volume using (59), one should take into account that the “market-informed” contribution to the total trading volume from (59) has to be multiplied by the number of informed traders N , whereas the “inter-informed” component should not have such a factor. This is because the “market-informed” component corresponds to the case when most the informed traders stay on the same side of the market, as opposed to the “inter-informed” one. Therefore, the total informed trading volume is given by

$$TV(t) = N \left(\frac{1}{N} T_1(t) + T_2(t) \right). \quad (63)$$

The measure of the trading volume (63) is consistent with the results of Evans and Lyons (2002), where the *signed* order flow (the sign reflects the direction of the trade) can be considered an appropriate measure of trading activity. This implies that the aggregate order flow may be low just because different agents take opposite sides of the market, whereas the trading activity may be quite substantial (due to the heterogeneous information across the agents). Therefore, the sum of the variances of the incremental demands provides a better measure of trading activity. Analogous to (59), the total informed trading volume (63) has the “inter-informed” and the “market-informed” components.

To analyze the time profiles of all market parameters, we have to simultaneously solve the ODEs (31), (33), (14) and (44) taking into account the boundary condition (45). In particular, we obtain a closed-form analytical characterization of the equilibrium. The results of the numerical solution are presented in Figures 2 through 9.

Figure 2 presents the dynamics of the information asymmetry parameter $\delta(t)$ as a function of time for the case of five informed traders $N = 5$ and a correlation coefficient between the private signals ρ equal to 0.5. We assume that there are four trading periods and, therefore, three market closures in the economy. Clearly, the information asymmetry increases at the beginning of each trading period, since the informed traders learn from their private signals over the market closure periods. However, the magnitude of the change for the asymmetry parameter is different across the closure periods and tends to decrease towards the last trading period. The intuition is simple: the role of private learning decreases as the price becomes more informative towards the announcement date T .

In Figure 3, we plot the dynamics of the optimal informed trading parameters $\beta(t)$ for the case of the

three closures described above as a function of time and compare to the optimal $\beta(t)$ in the absence of market closures. Note that the time profile of the trading parameter in the presence of market closures has a typical U -shape over the trading period which is consistent with the results of BCW. As it was pointed out by Foster and Viswanathan (1996), in the oligopolistic economy with heterogeneous signals across informed traders, the time profiles of the optimal trading strategies can be characterized by two different phases. For sufficiently small time intervals, the informed traders do not learn much from the price process and are therefore trading very aggressively in an attempt to quickly make use of their initial private information. This is called the “rat race” phase. At later times, the informed traders learn more from the price and can make better inferences of the others’ private signals, and therefore have less incentive to trade in order not to reveal their information. The second stage is called the “waiting phase.” In contrast to Foster and Viswanathan (1996) and BCW, the information inflow in our model happens at each market closure and is therefore periodic. For this reason, one should expect that such a “two-stage” pattern occurs during each of the trading periods. However, one should also take into account that the informed traders strategically anticipate the future information inflows in their strategies and therefore the U -shaped pattern described above may have significant non-periodic distortions. This is what can be seen in Figure 3.

In Figure 4, we plot the dynamics of the trading volume defined by (63) as a function of time for the case of five informed traders with the initial correlation coefficient between the private signals $\rho = 0.5$, and compare to the optimal trading volume in the absence of market closures. From (63), it follows that the dynamics of the trading volume are directly related to the dynamics of the informed trading parameter $\beta(t)$. The trading volume in the presence of the market closures exceeds the volume obtained in the case of no closures for the whole time interval presented in Figure 4. Analogous to the dynamics of $\beta(t)$, the time profile of the trading volume in the presence of market closures has a typical U -shape over the trading period, following the “rat race” and the “waiting” phases of the optimal trading strategies described above. Also, the U -shape of the time profile of the trading volume has significant distortions since the informed traders strategically anticipate the future information inflows in their strategies.

In Figure 5, we plot the dynamics of relative trading intensity as a function of time for the case of five informed traders and three closures described above and compare to the relative trading intensity with no

closures. Following BCW, the relative trading intensity is defined as the ratio of the oligopolistic trading intensity defined in Corollary 1 as $\beta(t)/\delta(t)$, to the optimal trading intensity of the monopolist from Kyle (1985). As one can see from Figure 5, the relative trading intensity in presence of market closures exceeds the one with no closures. Note that the trading intensity drops at the beginning of each trading period, which is consistent with the results of Foster and Viswanathan (1996). The trading intensity follows monotonically increasing patterns during each trading period, but aperiodic across the different trading periods. Analogous to the above discussion, this non-periodicity arises due to the strategic nature of trading combined with the decreasing role of the private signals during the trading periods.

In Figure 6, we compare the market's residual uncertainty, showing the ratio of $\Sigma_M(t)$ for the oligopolistic and monopolistic cases, as a function of time, for the case of five informed traders and three closures described above, and comparing to the otherwise identical case with no closures. The relative residual uncertainty has a typical *U*-shape during the whole time interval consistent with the results of BCW. As one can see from Figure 6, the relative residual uncertainty in the absence of market closures exceeds the one with closures. This implies that the periodic inflow of information in the heterogeneous oligopolistic economy is informationally more efficient than the "one-shot" information inflow considered in BCW.

Figure 7 presents the dynamics of the relative market depth parameter $1/\lambda(t)$ as a function of time, for the same case of five informed traders and three market closures, compared to the case with no closures. As before, the relative market depth is defined as a ratio of the oligopolistic and the monopolistic market depth parameters. The relative market depth increases at the beginning of each trading period and decreases towards the time horizon T . The reason for the relative market depth increasing is exactly analogous to the one described in BCW. From Figure 7, one can see that the inverse market depth only slightly increases during the last trading period, implying that the liquidity problems in our model are less severe than in the case when the information inflow is non-periodic. The intuition is simple: since the informed traders strategically anticipate the future information inflows and have less incentive to wait, the adverse selection problem is less important.

In Figure 8, we plot the dynamics of the trading intensity as a function of time for the case of two informed traders and different initial correlations between the private signals and compare to the optimal

trading intensity of a monopolist (Kyle (1985)). The oligopolistic trading intensity in our model exceeds that of a monopolist over the whole range of the initial correlations presented in Figure 8. For sufficiently short time intervals (around the first trading period), the trading intensity is quite sensitive to the correlations, but it becomes less sensitive after several closure periods. The intuition is that the role of initial correlations diminishes since the informed traders learn more from the price process after several trading periods.

Figure 9 presents the dynamics of the market depth parameter $1/\lambda(t)$ as a function of time for the same case of two informed traders and different signal correlations. One should note that the market depth decreases in signal correlations for short times and increases for sufficiently long time intervals, which corresponds to the “rat race” and the “waiting” phases of the optimal strategies, respectively.

As we discussed early, an important result of our model is that contrary to the results of BCW, the trading intensity with oligopolistic heterogeneously informed traders is typically higher than the one of the monopolist. This situation is illustrated in Figure 8, where we present the ratio of the trading intensities of the two informed traders and the monopolist. The reason for this is the same mechanism of the “adverse selection problem suppression” due to the strategic anticipation of the future signals described above (see Figure 6).

Finally, we study the distribution of payoffs across all agents which is important from the perspective of welfare analysis. As usual for Kyle (1985) type models, the informed traders are winning at the expense of the uninformed ones. The payoff distribution across the informed traders can be analyzed analogous to BCW. For the informed trader i , the payoff follows the process

$$d\pi_i(t) = [V - P(t)] dx_i, \quad (64)$$

where the incremental demand $dx_i(t)$ during the time interval dt is given by (10). Making use of equation (27) and equation (10), we obtain

$$E[d\pi_i(t)] = \beta(t) \text{var}[V - P(t)|V] dt = \lambda(t) \sigma_u^2 dt. \quad (65)$$

In order to analyze the dependence of the expected payoffs on the number of informed traders, we have to re-scale $\beta(t) \rightarrow \beta(t)/N$, according to (12). Taking this into account, we obtain from (65) the following result.

Corollary 4 *In equilibrium, the expected profit of each informed trader is*

$$\frac{\sigma_u^2}{N} \sum_{k=0}^K \int_{t_k^o}^{t_k^c} \lambda_k(t) dt = \frac{\sigma_u^2}{N} \int_0^T \lambda(t) dt. \quad (66)$$

Proof: Follows immediately from (65).

The result (66) has the same form as the one of BCW, because the inverse depth $\lambda(t)$ is continuous across the market closures. One should note that the total expected gain of informed traders is the sum of the expected profits earned during the periods when the market is open.

5 Conclusions

This paper constructs a dynamic auction model based on rational strategic trading with asymmetric information across agents and *dynamic* information acquisition. The key feature of the model is that informed traders do not know the fundamentals exactly but repeatedly receive imperfect private signals about the fundamentals. Learning from prices also enables each informed trader to improve her estimation of fundamentals. Therefore, to solve for equilibrium outcomes in such setting, one must first determine how informed traders combine current and past signals together with the information in current and past prices, then determine how much they trade at each time, and finally put this all together and solve for the associated equilibrium pricing. This is a very challenging problem which we successfully solve in this paper.

In our model agents trade continuously and informed traders receive private information during times when the market is closed. We characterize analytically how information received during one market closure interacts with information from other closures. We show that it is optimal for informed traders to use a linear combination of all of their private signals obtained at different times as an effective signal in their optimal trading strategy. In a particular case when all private signals obtained by each insider are drawn from the same distribution, we demonstrate that optimal weights for the signals obtained at different times enter with *equal weights* into the effective signal. This is consistent with the result that the equally-weighted average is the optimal linear unbiased estimator of the fundamental. At the same time, we show that the informed traders' valuations also involve the factor related to the information asymmetry which exhibits non-monotonic and non-periodic dynamics over the different market closure periods. Our results indicate that the use of private information and its revelation through price never ends: each new realization of

private information leads agents to re-interpret the history of private information as well as prices.

While the model only analyzes the role of market closures from a purely information perspective, it is consistent with a wealth of empirical evidence. It predicts a *U*-shape temporal patterns for the trading activity, during the periods when the market is opened. We also obtain rich aperiodic patterns for the trading volume during each trading week, which is changing across different trading weeks. This aperiodicity property of trading activity is quite important and arises because the informed traders strategically trade during different weeks anticipating the final announcement about the fundamentals.

Finally, the anticipated arrival of new information causes informed traders to use the old information more freely and, therefore, compete more fiercely against each other than in the case of single information arrival considered by BCW. For example, in contrast to the results of BCW, we find that in the economy with market closures, two informed traders with uncorrelated signals typically trade *more* intensively than a monopolist trader with the same aggregate information. As an aftermath, our model also predicts a reduction in return volatility as compared to when new information arrives only once since by increasing the competition among insiders, the dynamic injection of new information facilitates more efficient information transmission into prices.

Appendix A: Bayesian Updating

First, let's derive the distribution in (6). By assumption, the private signals are given by (1) from which it immediately follows that $\{S_i^k\}$ are normally distributed, with

$$E[S_i^k|V] = V, \quad (\forall k = 0, \dots, K-1). \quad (\text{A1})$$

The conditional variance for $\forall k = 0, \dots, K-1$ is given by

$$\begin{aligned} \text{Var}[S_i^k|V] &= \sigma_s^2 E \left[\left(\varepsilon_i^k - \frac{1}{N} \sum_{j=1}^N \varepsilon_j^k \right)^2 \right] \\ &= \sigma_s^2 \left(1 - \frac{2}{N} [1 + (N-1)\rho_k] + \frac{1}{N^2} [N + N(N-1)\rho_k] \right) \\ &= \sigma_s^2 (1 - \phi_k), \end{aligned} \quad (\text{A2})$$

and (6) follows immediately. Analogously, we obtain for the covariances

$$\begin{aligned} \text{Cov}[S_i^k, S_{j \neq i}^k|V] &= \sigma_s^2 E \left[\left(\varepsilon_i^k - \frac{1}{N} \sum_{j=1}^N \varepsilon_j^k \right) \left(\varepsilon_j^k - \frac{1}{N} \sum_{l=1}^N \varepsilon_l^k \right) \right] \\ &= \sigma_s^2 \left(\rho_k - \frac{2}{N} [1 + (N-1)\rho_k] + \frac{1}{N^2} [N + N(N-1)\rho_k] \right) \\ &= \sigma_s^2 (\rho_k - \phi_k). \end{aligned} \quad (\text{A3})$$

The unconditional variances and covariances are

$$\begin{aligned} \text{Var}[S_i^k] &= E \left[\left(V + \sigma_s \left(\varepsilon_i^k - \frac{1}{N} \sum_{j=1}^N \varepsilon_j^k \right) \right)^2 \right] \\ &= \text{Var}[V] + \sigma_s^2 \left(1 - \frac{2}{N} [1 + (N-1)\rho_k] + \frac{1}{N^2} [N + N(N-1)\rho_k] \right) \\ &= \sigma_s^2 \phi_k + \sigma_s^2 (1 - \phi_k) = \sigma_s^2, \end{aligned} \quad (\text{A4})$$

and

$$\begin{aligned} \text{Cov}[S_i^k, S_{j \neq i}^k] &= E \left[\left(V + \sigma_s \left(\varepsilon_i^k - \frac{1}{N} \sum_{j=1}^N \varepsilon_j^k \right) \right) \left(V + \sigma_s \left(\varepsilon_j^k - \frac{1}{N} \sum_{l=1}^N \varepsilon_l^k \right) \right) \right] \\ &= \text{Var}[V] + \sigma_s^2 \left(\rho_k - \frac{2}{N} [1 + (N-1)\rho_k] + \frac{1}{N^2} [N + N(N-1)\rho_k] \right) \\ &= \sigma_s^2 \phi_k + \sigma_s^2 (\rho_k - \phi_k) = \sigma_s^2 \rho_k, \end{aligned} \quad (\text{A5})$$

respectively. Combining (A4) and (A5), we obtain (9).

Next, we derive the Bayesian updating procedure for our model. Making use of the Bayes rule, the conditional p.d.f. of V , $p_{k+1}(V|\mathcal{F}_i(t_{k+1}^o))$, is given by recursion

$$p_{k+1}(V|\mathcal{F}_i(t_{k+1}^o)) = \frac{p_k(V|\mathcal{F}_i(t_k^c)) \exp \left[-\frac{(S_i^{k+1}-V)^2}{2(1-\phi_{k+1})\sigma_s^2} \right]}{\int_{-\infty}^{+\infty} dx p_k(x|\mathcal{F}_i(t_k^c)) \exp \left[-\frac{(S_i^{k+1}-x)^2}{2(1-\phi_{k+1})\sigma_s^2} \right]}, \quad (\text{A6})$$

where S_i^{k+1} is the private signal received by the informed trader i during the k -th closure period. Taking

into account the prior distribution (4) and her own signal (6), the informed trader i 's posterior p.d.f. of V at $t = 0$ *after* she observes her private signal S_i^0 becomes

$$\begin{aligned} p_0(V|S_i^0) &= \frac{\exp\left[-\frac{V^2}{2\phi_0\sigma_s^2}\right] \exp\left[-\frac{(S_i^0-V)^2}{2(1-\phi_0)\sigma_s^2}\right]}{\int_{-\infty}^{+\infty} dx \exp\left[-\frac{x^2}{2\phi_0\sigma_s^2}\right] \exp\left[-\frac{(S_i^0-x)^2}{2(1-\phi_0)\sigma_s^2}\right]} \\ &= \frac{1}{\sqrt{2\pi\phi_0(1-\phi_0)\sigma_s^2}} \exp\left[-\frac{(V-\phi_0 S_i^0)^2}{2\phi_0(1-\phi_0)\sigma_s^2}\right]. \end{aligned} \quad (\text{A7})$$

Equation (A7) can be used as an initial condition for the recursion (A6). Now, we apply the updating rule (A6) to obtain (A12) and (A13). Since all the conditional distributions $\{p_k(V|\mathcal{F}_i(t_k^o)), k = 0, \dots, K-1\}$ are normal, we have

$$p_k(V|\mathcal{F}_i(t_k^c)) = \frac{1}{\sqrt{2\pi\Sigma_i(t_k^c)}} \exp\left[-\frac{(V_i(t_k^c) - V)^2}{2\Sigma_i(t_k^c)}\right]. \quad (\text{A8})$$

Note that all informed traders receive private signals drawn from the same distribution, and therefore have the same updating rules. The substitution of (A8) into (A6) yields the updating rule (A12) and (A13). To see that we first calculate the integral in the denominator of (A6)

$$\begin{aligned} &\int_{-\infty}^{+\infty} dx p_k(x|\mathcal{F}_i(t_k^c)) \exp\left[-\frac{(S_i^{k+1} - x)^2}{2(1-\phi_k)\sigma_s^2}\right] = \\ &= \sqrt{\frac{\widehat{\Sigma}_i(t_k^c)}{\Sigma_i(t_k^c)}} \exp\left\{-\left(\frac{V_i(t_k^c)^2}{2\Sigma_i(t_k^c)} + \frac{(S_i^{k+1})^2}{2(1-\phi_k)\sigma_s^2}\right) + \frac{\widehat{\Sigma}_i(t_k^c)}{2} \left(\frac{V_i(t_k^c)}{\Sigma_i(t_k^c)} + \frac{S_i^{k+1}}{(1-\phi_k)\sigma_s^2}\right)^2\right\}, \end{aligned} \quad (\text{A9})$$

where

$$\widehat{\Sigma}_i(t_k^c) \equiv \frac{1}{\frac{1}{\Sigma_i(t_k^c)} + \frac{1}{(1-\phi_k)\sigma_s^2}}. \quad (\text{A10})$$

Substituting (A9) and (A8) into (A6) yields

$$\begin{aligned} p_{k+1}(V|\mathcal{F}_i(t_{k+1}^o)) &= \frac{1}{\sqrt{2\pi\widehat{\Sigma}_i(t_k^c)}} \exp\left\{-\frac{\left[\widehat{\Sigma}_i(t_k^c) \left(\frac{V_i(t_k^c)}{\Sigma_i(t_k^c)} + \frac{S_i^{k+1}}{(1-\phi_k)\sigma_s^2}\right) - V\right]^2}{2\widehat{\Sigma}_i(t_k^c)}\right\} = \\ &= \frac{1}{\sqrt{2\pi\Sigma_i(t_{k+1}^o)}} \exp\left[-\frac{(V_i(t_{k+1}^o) - V)^2}{2\Sigma_i(t_{k+1}^o)}\right]. \end{aligned} \quad (\text{A11})$$

Comparing two distributions in (A11) gives the necessary updating rules for the informed trader's precision $\Sigma_i(t)$

$$\frac{1}{\Sigma_i(t_{k+1}^o)} = \frac{1}{\Sigma_i(t_k^c)} + \frac{1}{(1-\phi_k)\sigma_s^2}, \quad (\text{A12})$$

and valuation $V_i(t)$

$$\frac{V_i(t_{k+1}^o)}{\Sigma_i(t_{k+1}^o)} = \frac{V_i(t_k^c)}{\Sigma_i(t_k^c)} + \frac{S_i^{k+1}}{(1-\phi_k)\sigma_s^2}. \quad (\text{A13})$$

Appendix B: Proofs

Proof of Proposition 1

1. Making use of (14), (18) and the definition $\delta(t) = 1 - \Sigma_i(t) / \Sigma_M(t)$, we immediately obtain

$$\frac{d}{dt} \delta = -\delta(1 - \delta) \lambda \beta. \quad (\text{B1})$$

Combining (B1) and (25) yields (26) with $k(t) = [1 - \delta(t)] \lambda(t) \beta(t)$. Taking into account (B1), we obtain that $k(t) = -\frac{d}{dt} [\ln \delta(t)]$.

2 and 3. Let us prove equation (27) first. After market reopens at $t = t_k^o$ informed trader i starts with the valuation $V_i(t_k^o)$. We now have to solve an ODE for $\widehat{V}_i(t) \equiv V_i(t) - (1 - \delta(t)) P(t)$ that directly follows from (26)

$$\frac{d\widehat{V}_i(t)}{dt} = -k(t) \widehat{V}_i(t), \quad (\text{B2})$$

with initial condition $\widehat{V}_i(t_k^o)$ which will be determined later. The solution is given by

$$\widehat{V}_i(t) = \widehat{V}_i(t_k^o) \exp \left[- \int_{t_k^o}^t k(s) ds \right] = \widehat{V}_i(t_k^o) \frac{\delta(t)}{\delta(t_k^o)}, \quad (\text{B3})$$

where we made use of the relation $k(t) = -\frac{d}{dt} [\ln \delta(t)]$, and (27) follows after we introduce the effective private signal

$$\widehat{S}_i(t_k^o) = \frac{1}{\delta(t_k^o)} \widehat{V}_i(t_k^o). \quad (\text{B4})$$

Now we can prove (28). It is shown in Appendix A that after each of the market re-openings, the informed trader's precision $\Sigma_i(t)$ and valuation $V_i(t)$ are given by the updating rule (A12) and (A13) respectively. Combining (A12) and (A13), we immediately obtain

$$\frac{V_i(t_{k+1}^o)}{\Sigma_i(t_{k+1}^o)} = \frac{V_i(t_k^c)}{\Sigma_i(t_k^c)} + \left[\frac{1}{\Sigma_i(t_{k+1}^o)} - \frac{1}{\Sigma_i(t_k^c)} \right] S_i^{k+1}. \quad (\text{B5})$$

Using the definition of $\delta(t)$ and (24), we can rewrite (B5) as

$$\frac{\widehat{V}_i(t_{k+1}^o)}{1 - \delta(t_{k+1}^o)} = \frac{\widehat{V}_i(t_k^c)}{1 - \delta(t_k^c)} + \left[\frac{1}{1 - \delta(t_{k+1}^o)} - \frac{1}{1 - \delta(t_k^c)} \right] S_i^{k+1}. \quad (\text{B6})$$

In deriving the relation (B6) we have also used the definition of $\widehat{V}_i(t)$, namely

$$\frac{V_i(t_{k+1}^o)}{1 - \delta(t_{k+1}^o)} = \frac{\widehat{V}_i(t_{k+1}^o)}{1 - \delta(t_{k+1}^o)} + P(t_{k+1}^o), \quad (\text{B7})$$

and the fact that the price does not change during closure periods, $P(t_{k+1}^o) = P(t_k^c)$. Next, we multiply both sides of (B6) by $1 - \delta(t_{k+1}^o)$ and use the identity $\widehat{V}_i(t_k^c) = \widehat{V}_i(t_k^o) [\delta(t_k^c) / \delta(t_k^o)]$ to obtain

$$\widehat{S}_i(t_{k+1}^o) = \mu_{k+1} \widehat{S}_i(t_k^c) + \frac{1}{\delta(t_{k+1}^o)} \left[1 - \frac{1 - \delta(t_{k+1}^o)}{1 - \delta(t_k^c)} \right] S_i^{k+1}, \quad (\text{B8})$$

with

$$\mu_{k+1} = \frac{1 - \delta(t_{k+1}^o)}{1 - \delta(t_k^c)} \frac{\delta(t_k^c)}{\delta(t_{k+1}^o)}. \quad (\text{B9})$$

Finally, in order to establish (28) we need to show that the coefficient in front of the second term in (B8) is

equal to $1 - \mu_{k+1}$. A simple algebra exercise shows that

$$\begin{aligned}
1 - \mu_{k+1} &= 1 - \frac{1 - \delta(t_{k+1}^o)}{1 - \delta(t_k^c)} \frac{\delta(t_k^c) - 1 + 1}{\delta(t_{k+1}^o)} = \\
&= 1 + \frac{1 - \delta(t_{k+1}^o)}{\delta(t_{k+1}^o)} - \frac{1}{\delta(t_{k+1}^o)} \frac{1 - \delta(t_{k+1}^o)}{1 - \delta(t_k^c)} = \\
&= \frac{1}{\delta(t_{k+1}^o)} \left[1 - \frac{1 - \delta(t_{k+1}^o)}{1 - \delta(t_k^c)} \right].
\end{aligned} \tag{B10}$$

The recursion (B8) has initial condition

$$\widehat{S}_i(t_0^o) = \widehat{S}_i(0) = \frac{\widehat{V}_i(0)}{\delta(0)} = \frac{V_i(0)}{\delta(0)} = S_i^0, \tag{B11}$$

and can be solved forward. This is equivalent to the condition

$$\mu_0 = 0, \tag{B12}$$

if we formally set $\delta(t_{k-1}^c) = 0$. Using the definition of $\delta(t)$ and (24), we can rewrite (A12) as

$$\frac{1}{1 - \delta(t_{k+1}^o)} = \frac{1}{1 - \delta(t_k^c)} + \frac{\Sigma_M(t_{k+1}^o)}{(1 - \phi_k) \sigma_s^2}, \tag{B13}$$

which is the second of (31). Finally, the ODEs for the information asymmetry parameter follow immediately from (B1). *Q.E.D.*

Proof of Proposition 2

Combining the results of the Proposition 1, we obtain

$$\frac{d\delta(t)}{\delta(t)(1 - \delta(t))} = -\Sigma_M(t) d\left(\frac{1}{\Sigma_M(t)}\right). \tag{B14}$$

Integration on the interval $t \in \mathcal{O}^k$ with the initial conditions at $t = t_k^o$ yields

$$\delta(t) = \frac{\delta(t_k^o) \Sigma_M^{-1}(t_k^o)}{(1 - \delta(t_k^o)) \Sigma_M^{-1}(t) + \delta(t_k^o) \Sigma_M^{-1}(t_k^o)}, \tag{B15}$$

and therefore

$$\frac{1}{1 - \delta(t_k^c)} = \frac{1}{1 - \delta(t_k^o)} \left(\frac{\Sigma_M(t_k^c)}{\Sigma_M(t_k^o)} \right) + \left(1 - \frac{\Sigma_M(t_k^c)}{\Sigma_M(t_k^o)} \right). \tag{B16}$$

Substituting (B16) into the second equation of (31) and taking into account that $\Sigma_M(t_k^c) = \Sigma_M(t_{k+1}^o)$, we obtain

$$\frac{1}{1 - \delta(t_{k+1}^o)} = \frac{1}{1 - \delta(t_k^o)} \left(\frac{\Sigma_M(t_k^c)}{\Sigma_M(t_k^o)} \right) + \left(1 - \frac{\Sigma_M(t_{k+1}^o)}{\Sigma_M(t_k^o)} + \frac{\Sigma_M(t_{k+1}^o)}{(1 - \phi_k) \sigma_s^2} \right). \tag{B17}$$

Iterating the recursion (B17) with the initial conditions $\delta(0) = \phi_0$, we finally have

$$\frac{1}{1 - \delta(t_k^o)} = 1 + \frac{\Sigma_M(t_k^o)}{\sigma_s^2} \sum_{l=0}^k \frac{1}{(1 - \phi_l)}, \tag{B18}$$

with $k = 0, \dots, K - 1$. Combining (B15) and (B18), we observe that

$$\frac{1}{\delta(t)} = 1 + \frac{\sigma_s^2}{\Sigma_M(t)} \left(\sum_{l=0}^k \frac{1}{(1-\phi_l)} \right)^{-1}, \quad (\text{B19})$$

where $t \in \mathcal{O}^k, \forall k$, which is the result of Proposition 2. The result (B19) will be useful for the solution of the dynamic optimization considered below. *Q.E.D.*

Proof of Proposition 3

Combining (28) and the definition of the effective signal (11), we obtain the recursion relation for the weights of the effective signal

$$\begin{aligned} a_{k,k} &= 1 - \mu_k, \\ a_{l,k+1} &= \mu_{k+1} a_{l,k}, \quad l = 0, \dots, k. \end{aligned} \quad (\text{B20})$$

Solving the recursion (B20) with the initial condition $a_{0,0} = 1$ yields

$$a_{l,k} = (1 - \mu_l) \prod_{m=l+1}^k \mu_m. \quad (\text{B21})$$

From (B21), it follows that $\sum_{l=0}^k a_{l,k} = 1 - \prod_{m=0}^k \mu_m$. Taking into account (30), we observe that the weights satisfy the normalization condition $\sum_{l=0}^k a_{l,k} = 1$. According to the above discussion, the effective signal (11) reflects the informed traders' updating of the new signals. From Proposition 2 we have

$$\begin{aligned} \frac{1}{\delta(t_k^o)} &= 1 + \frac{\sigma_s^2}{\Sigma_M(t_k^o)} \left(\sum_{l=0}^k \frac{1}{(1-\phi_l)} \right)^{-1}, \\ \frac{1}{\delta(t_k^c)} &= 1 + \frac{\sigma_s^2}{\Sigma_M(t_k^c)} \left(\sum_{l=0}^k \frac{1}{(1-\phi_l)} \right)^{-1}, \quad k = 0, \dots, K - 1. \end{aligned} \quad (\text{B22})$$

Making use of (B22) and that the MM's residual uncertainty is continuous over the closure periods, we immediately obtain

$$\frac{1 - \delta(t_{k+1}^o)}{\delta(t_{k+1}^o)} \sum_{l=0}^{k+1} \frac{1}{(1-\phi_l)} = \frac{1 - \delta(t_k^o)}{\delta(t_k^o)} \sum_{l=0}^k \frac{1}{(1-\phi_l)}, \quad k = 0, \dots, K - 1, \quad (\text{B23})$$

and therefore

$$\begin{aligned} \mu_{k+1} &= \frac{1 - \delta(t_{k+1}^o)}{1 - \delta(t_k^c)} \frac{\delta(t_k^c)}{\delta(t_{k+1}^o)} = \frac{\sum_{l=0}^k \frac{1}{(1-\phi_l)}}{\sum_{l=0}^{k+1} \frac{1}{(1-\phi_l)}}, \\ \mu_k &= \frac{\sum_{l=0}^{k-1} \frac{1}{(1-\phi_l)}}{\sum_{l=0}^k \frac{1}{(1-\phi_l)}}, \end{aligned} \quad (\text{B24})$$

which also satisfies the initial condition (30) for $k = 0$. Combining (B24) and (B21) yields (36). *Q.E.D.*

Proof of Proposition 4

According to (21), we have that

$$dP(t) = \lambda(t) \left[\sum_{i=1}^N \theta_i(t) dt + \sigma_u dZ(t) \right]. \quad (\text{B25})$$

Assuming that all informed traders except for the i -th informed trader are following the linear strategies and taking expectations, we have

$$dP = \lambda(t) \left[\theta(t) + \beta(t) N \left(V - \frac{1}{N} \widehat{S}_i - \frac{N-1}{N} P \right) \right] dt + \lambda(t) \sigma_u dZ(t), \quad (\text{B26})$$

Making use of (B26), we immediately obtain the result of Proposition 3. *Q.E.D.*

Proof of Proposition 5

Follows directly from (43) and (27). *Q.E.D.*

Proof of Theorem 1

From (14), and (18), it follows that the equilibrium is given by β_k and λ_k satisfying the following conditions

$$\begin{aligned} \beta_k(t) &= \sigma_u \left(\frac{d(\Sigma_M^{-1}(t))}{dt} \right)^{1/2}, \\ \lambda_k(t) &= \frac{\beta_k(t) \Sigma_M(t)}{\sigma_u^2}, \end{aligned} \quad (\text{B27})$$

during the k th period when market is open.

In order to derive the equilibrium, one has to derive a solution of the PDE (44) with the limit condition (45), with continuity of market maker beliefs across market closures, and which is such that the informed trader's strategy (10) is feasible for each informed trader. Ignoring the feasibility condition, β_k and λ_k are uniquely defined by $\Sigma_M(t)$, which is known at $t = 0$ and satisfies the limit condition (45). Therefore, if we solve for $\Sigma_M(t)$ for all time periods, we would be able to completely characterize the solution for the whole problem. Define $R(t) \equiv \Sigma_M(t)^{-1}$, and $R'(t) \equiv \frac{d}{dt} R(t)$. The necessary and sufficient conditions for the equilibrium can be written as

$$\beta_k(t) = \sigma_u \sqrt{R'(t)}, \quad (\text{B28})$$

$$\lambda_k(t) = \frac{1}{\sigma_u} \frac{\sqrt{R'(t)}}{R(t)}, \quad (\text{B29})$$

$$R''(t) + \left[\frac{2N-4}{N} \right] \frac{R'(t)^2}{R(t)} - \frac{2(1-\phi)}{N\phi(k+1)} \frac{R'(t)^2}{R(0)} = 0, \quad t \in \mathcal{O}^k, \forall k, \quad (\text{B30})$$

$$R(t) = R(t_k^c), \quad t \in \mathcal{C}^k, \forall k, \quad (\text{B31})$$

and

$$\lim_{t \rightarrow T} R(t) = +\infty. \quad (\text{B32})$$

In order to obtain equation (B30) note that $\delta(t)$ satisfies (B15) for $t \in \mathcal{O}^k$, from which it follows that

$$\delta(t)^{-1} = 1 + \frac{(1 - \delta(t_k^o))}{\delta(t_k^o)} \frac{R(t)}{R(t_k^o)} = 1 + \frac{(1 - \phi)}{(k+1)\phi} \frac{R(t)}{R(0)}, \quad (\text{B33})$$

where the last equality is obtained from (B18). We can use the expression (B29) to compute

$$\begin{aligned} \frac{d}{dt} \left(\frac{1}{\lambda_k(t)} \right) &= \frac{\sigma_u}{\sqrt{R'(t)}} \left[R'(t) - \frac{R(t)R''(t)}{2R'(t)} \right] = \\ &= \left(\frac{2N-2}{N} - \frac{(1-\phi)}{N(k+1)\phi} \frac{R(t)}{R(0)} \right) \sigma_u \sqrt{R'(t)}. \end{aligned} \quad (\text{B34})$$

A simple rearrangement of terms in (B34) immediately yields (B30). Equation (B30) is fully integrable. To see that divide it by $R'(t)$ to obtain

$$\frac{d}{dt} \left[R'(t) R(t)^{\frac{2N-4}{N}} \exp \left(-\frac{2(1-\phi)}{N(k+1)\phi} \frac{R(t)}{R(0)} \right) \right] = 0, \quad t \in \mathcal{O}^k, \forall k, \quad (\text{B35})$$

We can now integrate (B35) twice to get

$$\int_{t_k^o}^t R'(s)R(s)^{\frac{2N-4}{N}} \exp\left(-\frac{2(1-\phi)}{N(k+1)\phi} \frac{R(t)}{R(0)}\right) ds = (t - t_k^o)C_k, \quad t \in \mathcal{O}^k, \quad \forall k, \quad (\text{B36})$$

and, finally, the change of variables $x = R(s)/R(0)$ yields expression (50) with

$$r_k = \Sigma_M(0)/\Sigma_M(t_k^o). \quad (\text{B37})$$

Since $R(T) = \infty$ and $R(0) = (\Sigma_M(0))^{-1}$ we have

$$\begin{aligned} r_K &= \infty, \\ r_0 &= 1. \end{aligned} \quad (\text{B38})$$

From (50), it immediately follows that

$$\begin{aligned} C_k &= \frac{1}{\tau} \int_{r_k}^{r_{k+1}} x^{2(N-2)/N} \exp\left(-\frac{2(1-\phi)}{N(k+1)\phi} x\right) dx, \quad k = 1, \dots, K-2, \\ C_0 &= \frac{1}{\tau} \int_1^{r_1} x^{2(N-2)/N} \exp\left(-\frac{2(1-\phi)}{N\phi} x\right) dx, \\ C_{K-1} &= \frac{1}{\tau} \int_{r_{K-1}}^{\infty} x^{2(N-2)/N} \exp\left(-\frac{2(1-\phi)}{NK\phi} x\right) dx. \end{aligned} \quad (\text{B39})$$

To complete the solution for $R(t)$ we have to evaluate the constants $\{C_k\}_{k=0}^{K-1}$ and $\{r_k\}_{k=0}^K$. To accomplish that, consider a k th closure at $t = t_k^c$ with a corresponding opening at $t = t_{k+1}^o$. We have the following continuity conditions

$$\begin{aligned} R'(t_{k+1}^o) &= R'(t_k^c), \\ R(t_{k+1}^o) &= R(t_k^c), \end{aligned} \quad (\text{B40})$$

which follow from the continuity of the market uncertainty $\Sigma_M(t)$ and the inverse market depth $\lambda(t)$ over the closure periods. Making use of (B40) yields

$$C_k = C_{k-1} \exp\left[\frac{2r_k(1-\phi)}{N\phi} \left(\frac{1}{k} - \frac{1}{k+1}\right)\right], \quad (\text{B41})$$

which is a recursion condition for the constants $\{C_k\}$. The conditions (B40) combined with (B41), form a complete set of equations defining the constants $\{C_k\}_{k=0}^{K-1}$ and $\{r_k\}_{k=0}^K$.

Define the constant

$$\psi = \frac{2}{N} \left(\frac{1-\phi}{\phi}\right). \quad (\text{B42})$$

For the practical calculations, it is convenient to introduce the new variables

$$B_k = C_k \frac{1}{\tau} [\psi \Sigma_M(0)]^{(4-3N)/N}, \quad (\text{B43})$$

and

$$x_k = \psi r_k, \quad (\text{B44})$$

which is simply a change of scale. In terms of the new variables, the recursion takes the form

$$B_k = \int_{x_k}^{x_{k+1}} x^{2(N-2)/N} \exp\left(-\frac{x}{k+1}\right) dx, \quad (\text{B45})$$

$$B_k = B_{k-1} \exp\left[x_k \left(\frac{1}{k} - \frac{1}{k+1}\right)\right],$$

for $k = 1, \dots, K-1$, with the boundary conditions

$$B_0 = \int_{\psi}^{x_1} x^{2(N-2)/N} \exp(-x) dx, \quad (\text{B46})$$

$$B_{K-1} = \int_{x_{K-1}}^{\infty} x^{2(N-2)/N} \exp\left(-\frac{x}{K}\right) dx.$$

Finally, we have to prove the feasibility of the equation (10). It is feasible for all time intervals when market is open by the same argument as in BCW, and, since price does not change during the market closures, it is feasible at any $t \in [0, T]$. *Q.E.D.*

Proof of Corollary 2

Consider $N = 1$ and $K - 1$ closures. In this case, $\phi = 1$, and (B30) yields

$$R''(t) - 2\frac{R'(t)^2}{R(t)} = \frac{d}{dt} \left(\frac{R'(t)}{R^2(t)} \right) = 0, \quad (\text{B47})$$

and therefore

$$-\frac{R'(t)}{R^2(t)} = \frac{d}{dt} \Sigma_M(t) = -\gamma \equiv \text{const}, \quad (\text{B48})$$

during each period when the market is open, $t \in \mathcal{O}^k, \forall k$. Making use of (B48), the boundary conditions

$$\Sigma_M(0) = \phi\sigma_s^2, \quad \Sigma_M(T) = 0, \quad (\text{B49})$$

and the continuity condition (52), we obtain the unique solution for the residual market uncertainty in the form

$$\Sigma_M(t) = \phi\sigma_s^2 \left(1 - \frac{t}{T}\right), \quad (\text{B50})$$

and therefore the unique equilibrium for the monopolist case is given by $\{\beta_k(t), \lambda_k(t)\}_{k=0}^{K-1}$, where

$$\beta_k(t) = \frac{\sigma_u}{\sigma_s} \frac{1}{\sqrt{T}} \frac{1}{1 - \frac{t}{T}}, \quad (\text{B51})$$

$$\lambda_k(t) = \frac{\sigma_s}{\sigma_u} \frac{1}{\sqrt{T}}, \quad \forall k = 0, \dots, K.$$

The market maker's variance for each $k = 0, \dots, K$, is

$$\Sigma_M(t) = \Sigma_M(0) \left(1 - \frac{t}{T}\right), \quad (\text{B52})$$

where $T = K\tau$. *Q.E.D.*

Proof of Corollary 3

Setting $N = 2$ in (B45), and (B46) immediately leads to

$$B_k = (k+1) \left\{ \exp\left(-\frac{x_k}{k+1}\right) - \exp\left(-\frac{x_{k+1}}{k+1}\right) \right\}, \quad (\text{B53})$$

$$B_{k-1} = B_k \exp\left[-x_k \left(\frac{1}{k} - \frac{1}{k+1}\right)\right],$$

for $k = 1, \dots, K-1$, with the boundary conditions

$$\begin{aligned} B_0 &= \exp(-\psi) - \exp(-x_1), \\ B_{K-1} &= K \exp\left(-\frac{x_{K-1}}{K}\right). \end{aligned} \quad (\text{B54})$$

Now introduce the new variables A_k 's

$$B_k = A_k \exp\left(-\frac{x_k}{k+1}\right), \quad (\text{B55})$$

for $k = 0, \dots, K$. A substitution of (B55) into both of equations (B53) yields a close form recursion for the A_k 's,

$$\begin{aligned} A_{k-1} &= A_k \frac{k}{k+A_k}, \\ A_{K-1} &= K. \end{aligned} \quad (\text{B56})$$

Inverting both sides of (B56), we obtain

$$\begin{aligned} (A_{k-1})^{-1} &= (A_k)^{-1} + \frac{1}{k}, \quad k = 0, \dots, K-1, \\ A_{K-1} &= K. \end{aligned} \quad (\text{B57})$$

The recursion (B56) has the following solution

$$(A_k)^{-1} = \sum_{n=k+1}^K \frac{1}{n}, \quad (\text{B58})$$

or

$$A_k = \left(\sum_{n=k+1}^K \frac{1}{n} \right)^{-1}. \quad (\text{B59})$$

Making use of (B59), we obtain from (B55) and (B53)

$$x_k = \psi + \sum_{n=1}^k n \ln\left(1 + \frac{A_n}{n}\right), \quad (\text{B60})$$

for $k = 0, \dots, K-1$. Substituting into (B44), and taking into account that for $N = 2$, $\psi = \left(\frac{1-\phi}{\phi}\right)$, we obtain from (B60)

$$r_k = 1 + \left(\frac{\phi}{1-\phi}\right) \sum_{n=1}^k n \ln\left(1 + \frac{A_n}{n}\right), \quad k = 0, \dots, K-1. \quad (\text{B61})$$

We can also define $A_K = +\infty$, to make it consistent with the condition $r_K = +\infty$. Setting $N = 2$ in (50),

we obtain

$$r(t) = -(k+1) \ln \left[\exp \left(-\frac{r_k}{k+1} \right) - B_k \frac{t - t_k^o}{k+1} \right], \quad (\text{B62})$$

or, making use of (B55)

$$r(t) = r_k + \frac{(k+1)\phi}{1-\phi} \ln \left[\left(1 - A_k \frac{t - t_k^o}{k+1} \right)^{-1} \right], \quad (\text{B63})$$

which is equivalent to

$$\Sigma_M(t) = \Sigma_M(0) \left\{ r_k + \frac{(k+1)\phi}{1-\phi} \ln \left[\left(1 - \frac{(t - t_k^o) A_k}{k+1} \right)^{-1} \right] \right\}^{-1}, \quad t \in \mathcal{O}^k. \quad (\text{B64})$$

Q.E.D.

Proof of Corollary 4

From (21), it follows that

$$P(t) = \int_0^t \exp \left[-\int_{t'}^t d\tau \lambda(\tau) \beta(\tau) \right] \left\{ \lambda(t') \beta(t') \frac{1}{N} \sum_{j=1}^N \widehat{S}_j(t') dt' + \lambda(t') \sigma_u dZ(t') \right\}. \quad (\text{B65})$$

Taking into account (14) and (16), we observe that

$$\lambda(t) \beta(t) = \Sigma_M(t) \frac{d}{dt} \left(\frac{1}{\Sigma_M(t)} \right) = \frac{d}{dt} \left(\log \left[\frac{1}{\Sigma_M(t)} \right] \right). \quad (\text{B66})$$

Combining (??) and (B66), we obtain (60). *Q.E.D.*

Proof of Corollary 5

From the distributions (6), (4) and (36), it follows that the unconditional distribution of the effective private signals is given by

$$\widehat{S}_i^k \sim \mathcal{N} \left(0, \frac{\phi k + 1}{k+1} \sigma_s^2 \right), \quad k = 0, \dots, K-1. \quad (\text{B67})$$

Making use of (60), we obtain from (59)

$$\begin{aligned} TV_i(t) &= K_1(t) + K_2(t) + K_3(t), \\ K_1(t) &= \beta^2(t) \text{var} [\widehat{S}_i(t)] = \beta^2(t) \sigma_s^2 \frac{\phi k + 1}{k+1}, \\ K_2(t) &= -2\beta^2(t) \text{cov} [\widehat{S}_i(t), P(t)], \\ K_3(t) &= \beta^2(t) \text{var} [P(t)], \end{aligned} \quad (\text{B68})$$

with

$$K_3(t) = \beta^2(t) \left(\phi \sigma_s^2 \left(1 - \frac{\Sigma_M(t)}{\Sigma_M(0)} \right)^2 + \sigma_u^2 \Sigma_M^2(t) \int_0^t dt' \frac{\lambda^2(t')}{\Sigma_M^2(t')} \right). \quad (\text{B69})$$

Evaluating the integral in the right hand side of (B69), we obtain

$$K_3(t) = \beta^2(t) \left(\phi \sigma_s^2 \left(1 - \frac{\Sigma_M(t)}{\Sigma_M(0)} \right)^2 + \Sigma_M(t) \left(1 - \frac{\Sigma_M(t)}{\Sigma_M(0)} \right) \right). \quad (\text{B70})$$

The definition of effective signal yields

$$\frac{1}{N} \sum_{j=1}^N E [\widehat{S}_i(t), \widehat{S}_j(t')] = E [\widehat{S}_i(t), V] = \phi \sigma_s^2, \quad (\text{B71})$$

therefore

$$\text{cov} \left[\widehat{S}_i(t), P(t) \right] = \phi \sigma_s^2 \left(1 - \frac{\Sigma_M(t)}{\Sigma_M(0)} \right), \quad (\text{B72})$$

and we obtain

$$K_2(t) = -2\phi \sigma_s^2 \beta^2(t) \left(1 - \frac{\Sigma_M(t)}{\Sigma_M(0)} \right). \quad (\text{B73})$$

A substitution of (B73) and (B70) into (B68) yields

$$\begin{aligned} TV_i(t) &= \beta^2(t) \Sigma_M(t) \left(1 - \frac{\Sigma_M(t)}{\Sigma_M(0)} + \frac{\phi \sigma_s^2}{\Sigma_M(t)} \left(\frac{\Sigma_M^2(t)}{\Sigma_M^2(0)} + \frac{1 - \phi}{\phi n(t)} \right) \right) \\ &= \beta^2(t) \Sigma_M(t) \left(1 - \frac{\Sigma_M(t)}{\Sigma_M(0)} + \phi \sigma_s^2 \frac{\Sigma_M(t)}{\Sigma_M^2(0)} + \frac{1 - \phi}{n(t)} \frac{\sigma_s^2}{\Sigma_M(t)} \right). \end{aligned} \quad (\text{B74})$$

Taking the initial condition for $\Sigma_M(0) = \phi \sigma_s^2$ into account, we finally arrive at

$$\begin{aligned} TV_i(t) &= T_1(t) + T_2(t), \\ T_1(t) &= \beta^2(t) \sigma_s^2 \frac{(1 - \phi)}{n(t)}, \\ T_2(t) &= \beta^2(t) \Sigma_M(t). \end{aligned} \quad (\text{B75})$$

Q.E.D.

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Figure 1: **Timing of Events**

This Figure illustrates the timing of events. Market closures are periodic so that the durations each trading and non-trading periods are equal to τ and $\Delta\tau$ respectively.

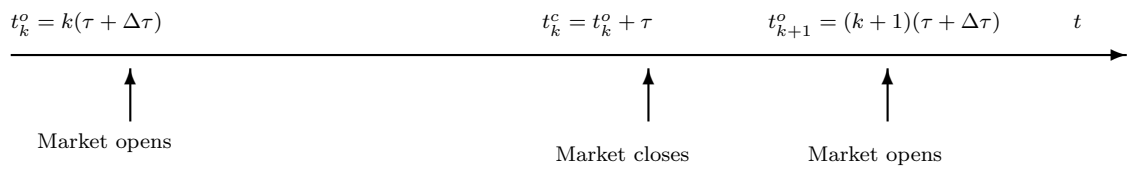


Figure 2: **Dynamics of $\delta(t)$**

The dynamics of the information asymmetry parameter $\delta(t)$ is shown as a function of time for the case of three market closures. There are five informed traders ($N = 5$) in the economy and the initial correlation coefficient between the private signals, ρ , equal to 0.5.

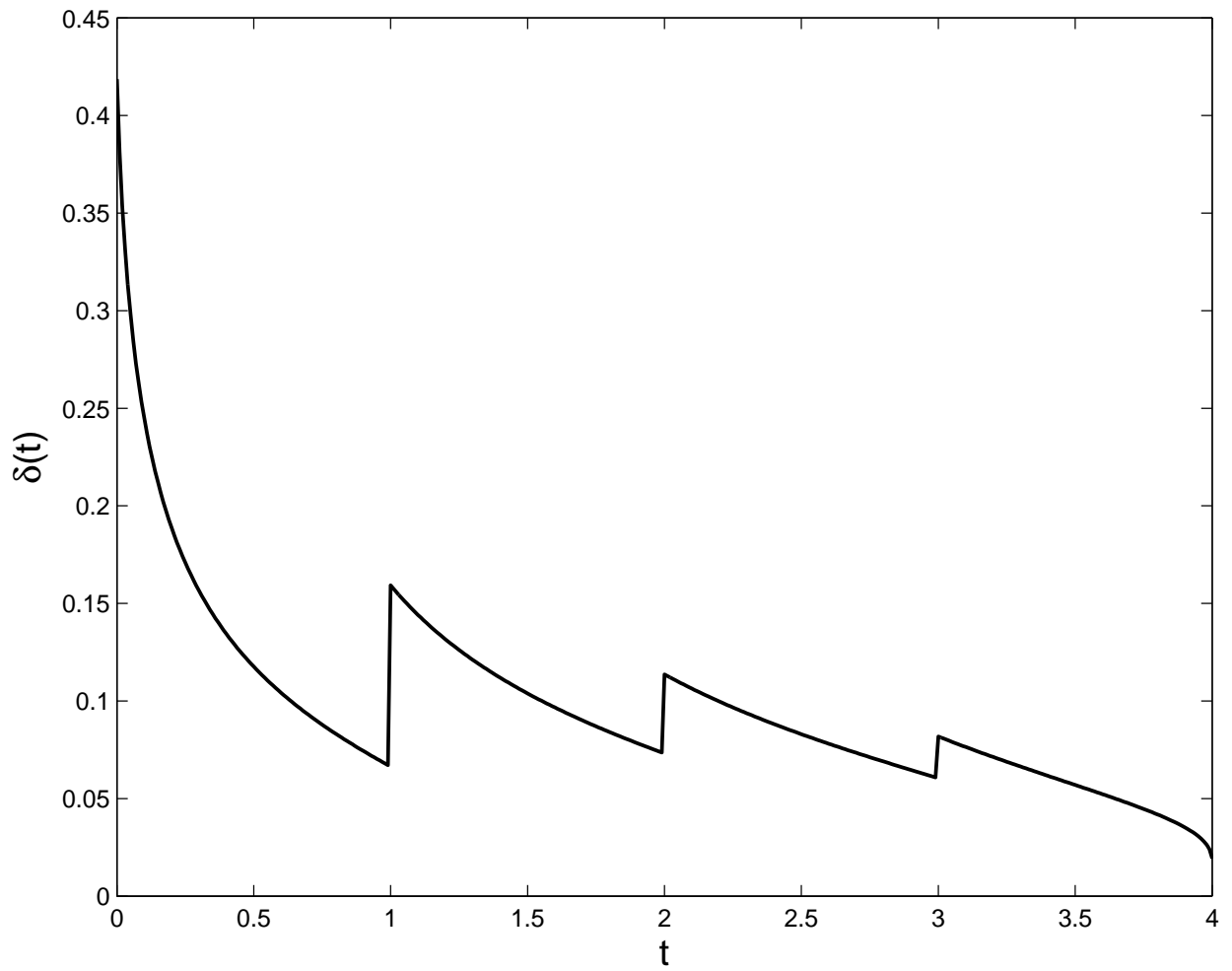


Figure 3: Dynamics of $\beta(t)$

The dynamics of the optimal informed trading parameter $\beta(t)$ is shown as a function of time for the case of the three market closures. It is compared to the intertemporal profile of the optimal $\beta(t)$ in the absence of market closures. There are five informed traders ($N = 5$) in the economy and the initial correlation coefficients between the private signals, ρ , equal to 0.5.

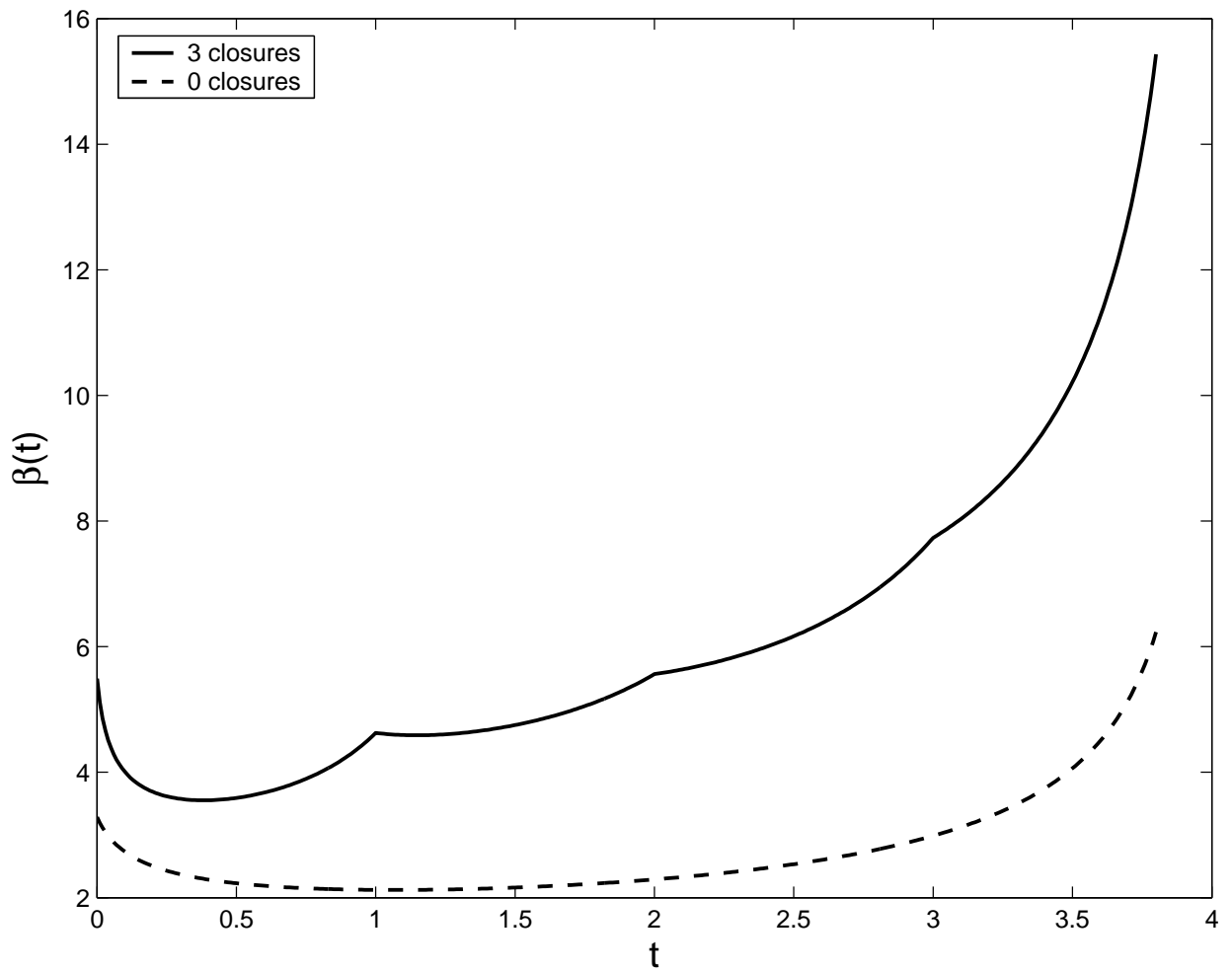


Figure 4: Dynamics of Informed Trading Volume

The dynamics of the informed trading volume, $TV(t)$, is shown as a function of time. It is compared to the optimal informed trading volume in the absence of market closures. There are five informed traders ($N = 5$) in the economy and the initial correlation coefficients between the private signals, ρ , equal to 0.5.

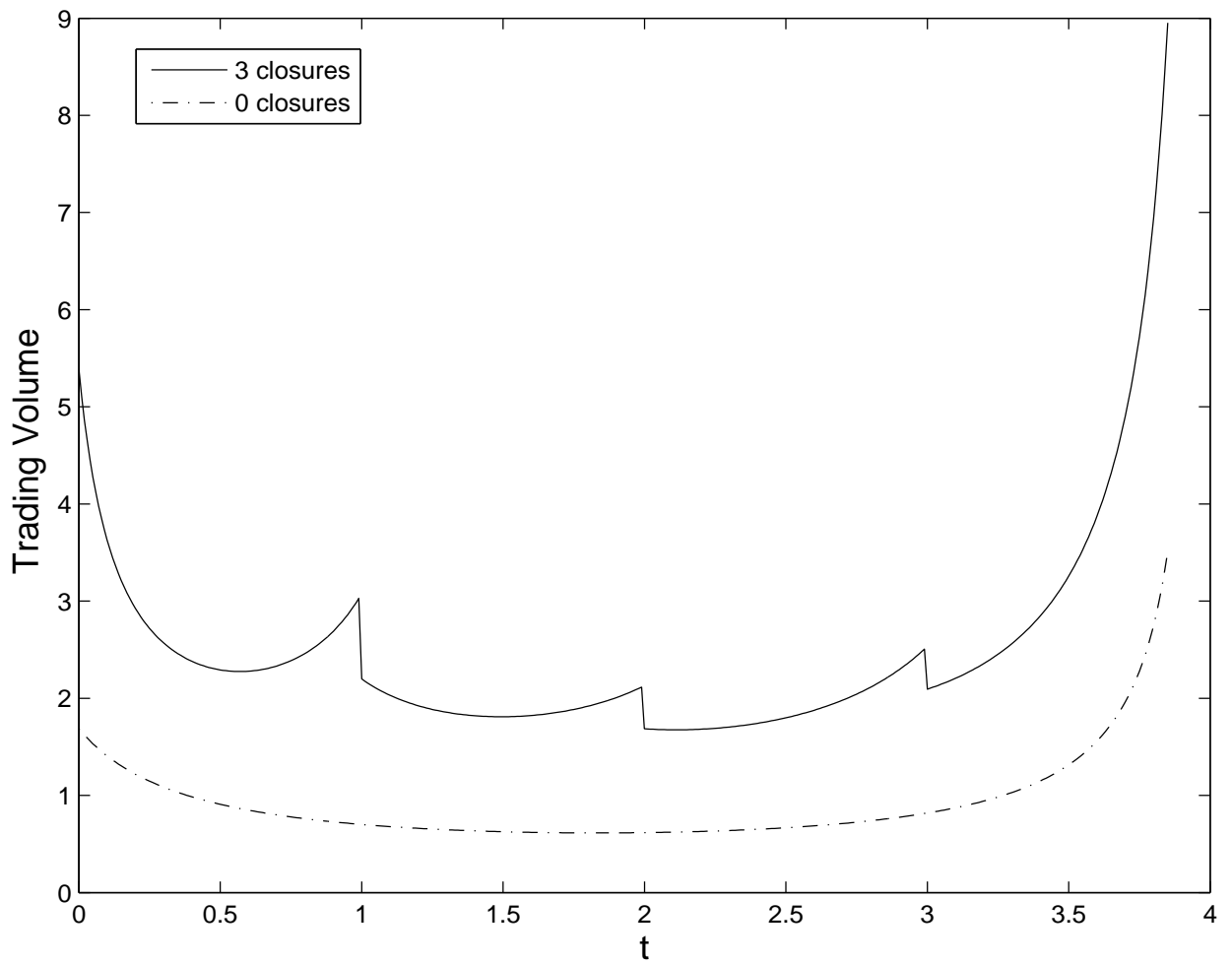


Figure 5: Dynamics of Relative Trading Intensity

The dynamics of the relative trading intensity is shown as a function of time for the case of the three market closures. It is compared the relative trading intensity in the absence of market closures. Following BCW, the relative trading intensity is defined as the ratio of the oligopolistic trading intensity defined in Corollary 1 as $\beta(t)/\delta$, to the optimal trading intensity of the monopolist informed trader from Kyle (1985). There are five informed traders ($N = 5$) in the economy and the initial correlation coefficients between the private signals, ρ , equal to 0.5.

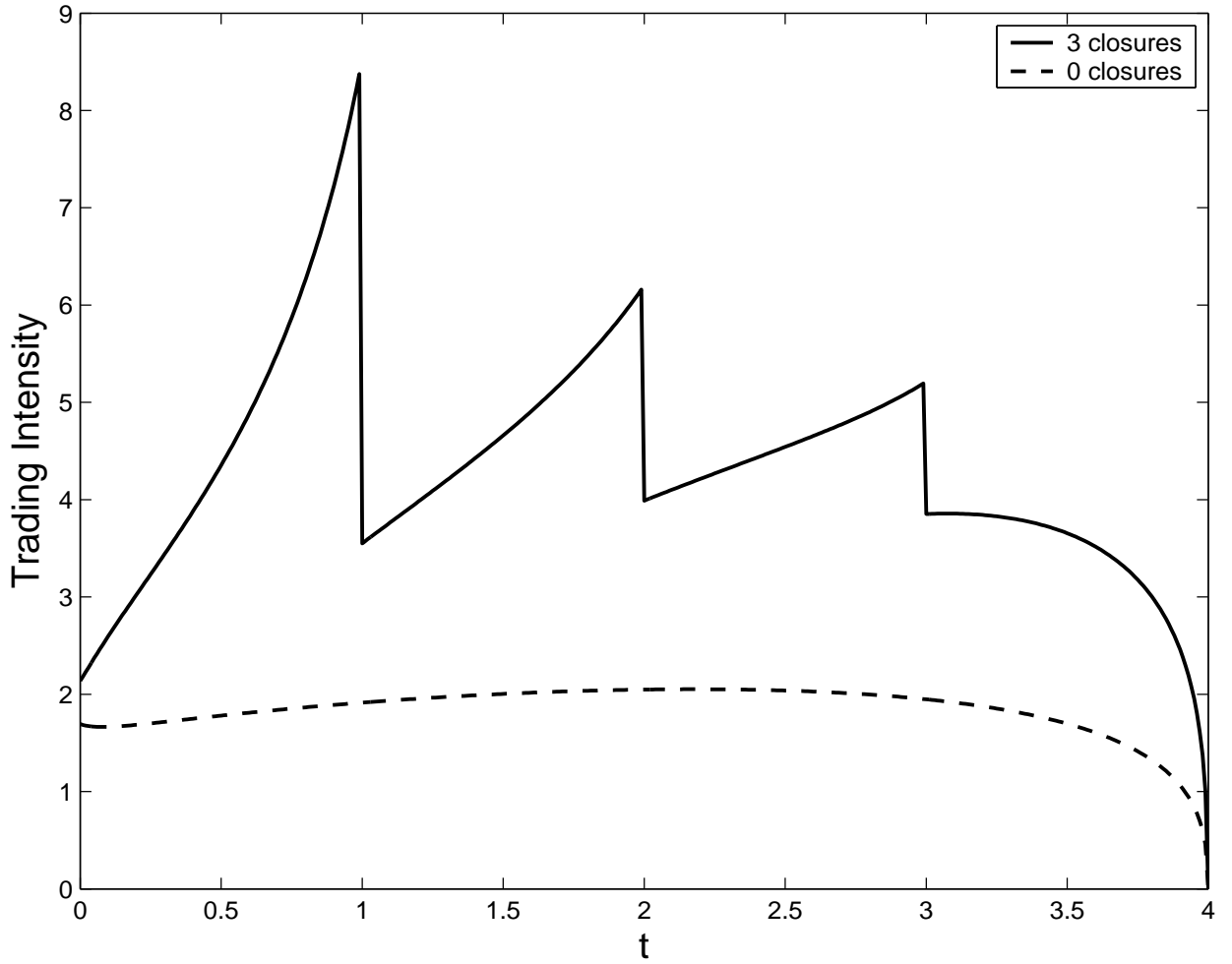


Figure 6: **Dynamics of Relative Market's Residual Uncertainty**

The relative market's residual uncertainty defined as the ratio of $\Sigma_M(t)$ for the oligopolistic and monopolistic cases is shown as a function of time for the case of three market closures. It is compared to the relative market's residual uncertainty in the absence of market closures. There are five informed traders ($N = 5$) in the economy and the initial correlation coefficients between the private signals, ρ , equal to 0.5.

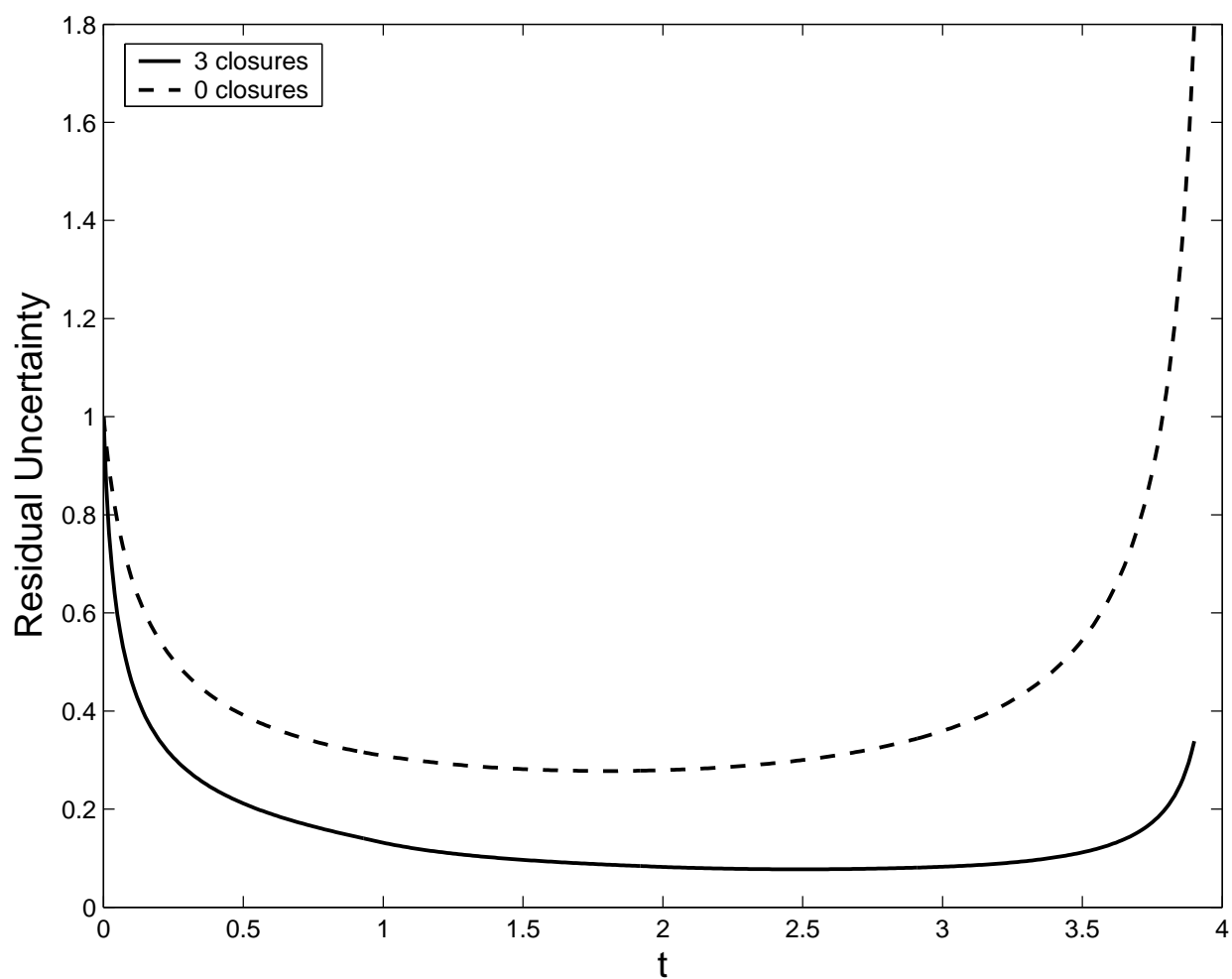


Figure 7: Dynamics of Relative Market Depth

The dynamics of the relative market depth parameter $1/\lambda(t)$ is shown as a function of time for the case of three market closures. It is compared to the relative market depth in the absence of market closures. The relative market depth is defined as a ratio of the oligopolistic and the monopolistic market depth parameters. There are five informed traders ($N = 5$) in the economy and the initial correlation coefficients between the private signals, ρ , equal to 0.5.

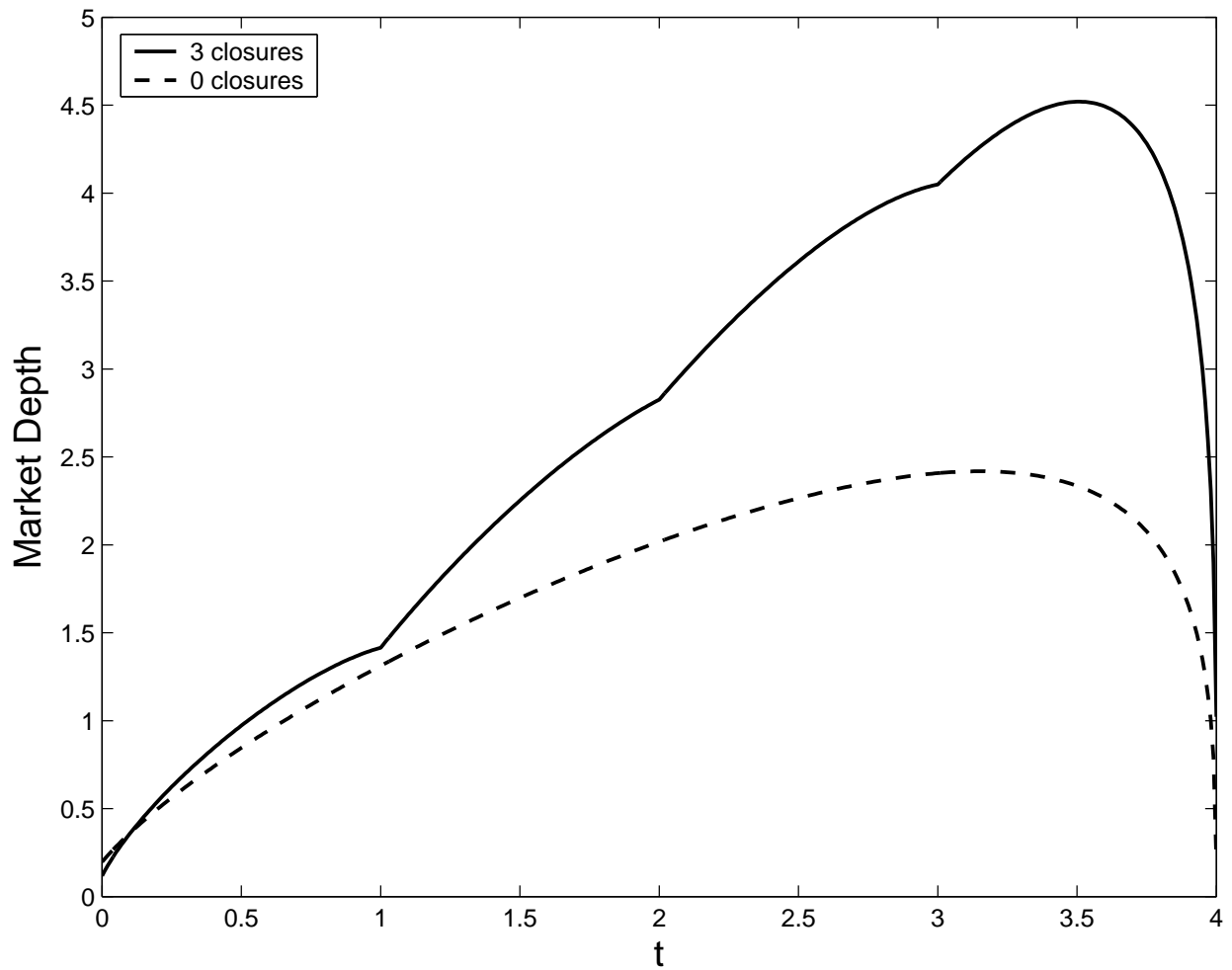


Figure 8: Dynamics of Relative Trading Intensity for Different Values of ρ

The dynamics of the relative trading intensity is shown as a function of time for several different values of the initial correlations between the private signals, ρ . There are three market closures and two informed traders ($N = 2$) in the economy.

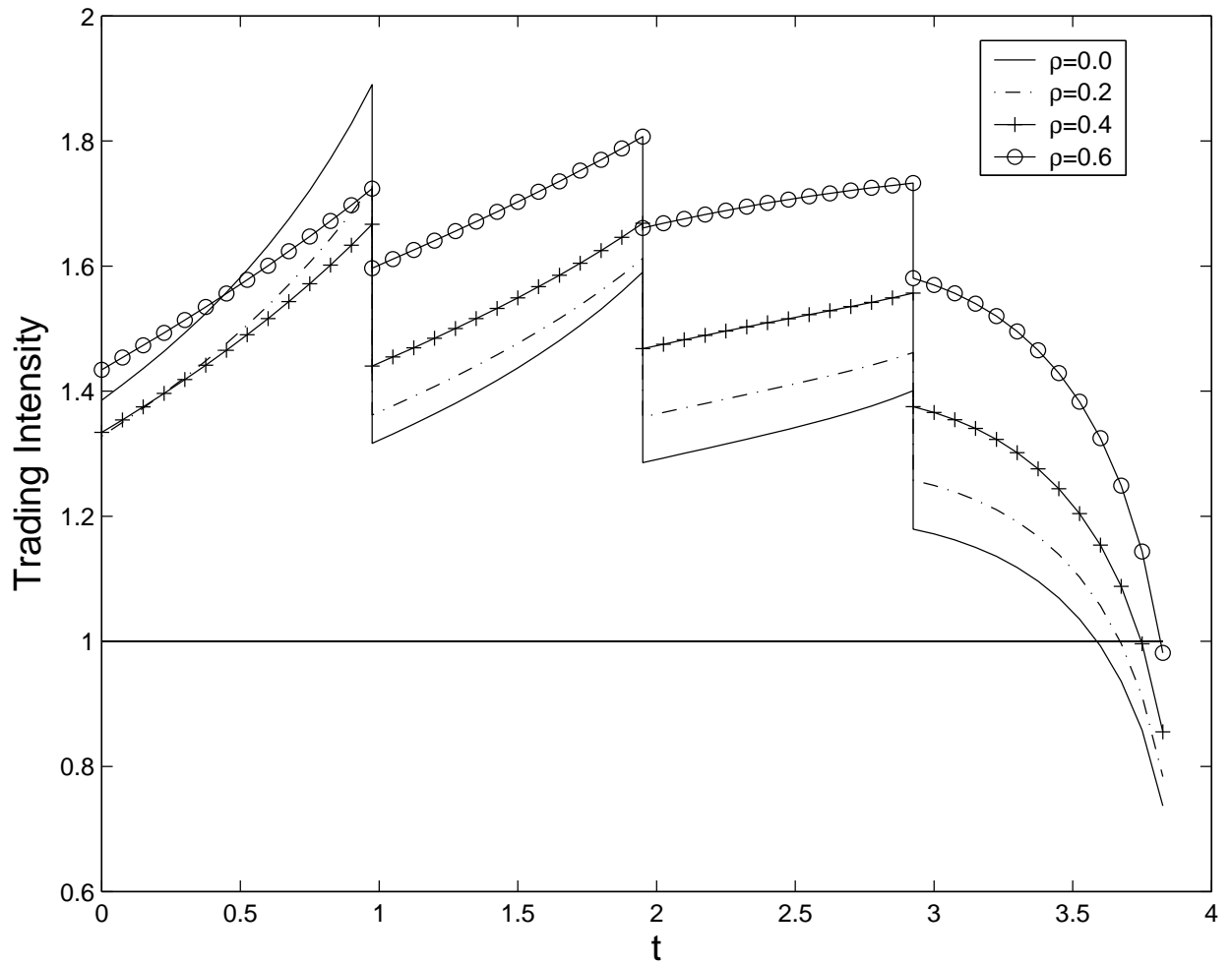


Figure 9: Dynamics of Relative Market Depth for Different Values of ρ

The dynamics of the relative market depth is shown as a function of time for several different values of the initial correlations between the private signals, ρ . There are three market closures and two informed traders ($N = 2$) in the economy.

