Housing Liquidity, Home Ownership and Unemployment

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Abstract

The relationship between unemployment and the value of owner–occupied housing is studied in an economy with heterogeneous locations and search frictions in the markets for both labour and houses. Differences in labour market conditions between cities affect the liquidity of houses (*i.e.* the speed with which houses may be transferred between households). At the same time housing market conditions affect decisions to accept offers and the allocation of labour across cities. Our theory yields a form of unemployment that results from home-ownership, so that unemployment rates for home-owners are higher than for otherwise identical renters. However, because prices and rental rates are endogenously determined, unemployment is lower, and wages higher, in cities with greater home-ownership. At the aggregate level, home-ownership does have a positive impact on unemployment, but the effect is quantitatively small.

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1 Introduction

In this paper, we study the relationship between unemployment and the value of owner occupied housing in an economy with heterogeneous locations (cities) characterized by search frictions in the markets for both labour and houses. *Ex ante* homogeneous households choose their city of residence and whether to rent or purchase housing. Differences in productivity between cities determine unemployment in each city and in the aggregate, rental rates, and house prices. The latter, as well as housing premia (defined as the city-specific differences between the rental rate and the flow value of owning a house) depend on the speed with which households are able to sell houses if they choose to relocate, *i.e.*, on the liquidity of houses. Liquidity in turn depends on labour market conditions in the two cities.

A significant literature has considered the relationship between home ownership and labour market outcomes. In a series of articles, Oswald (1996, 1997, 1999) argues that home ownership is associated with labour immobility and thus may result in higher unemployment and lower incomes. Oswald's conjecture is based largely on a positive correlation between rates of home ownership and unemployment across OECD countries and on aggregate time series evidence for the U.S.. Cross–regional evidence for this conjecture has also been presented for various countries by Nickell (1997), Partridge and Rickman (1997) using U.S. states, Pehkohnen (1999) using Finnish regions and Cochrane and Poot (2007) using cities in New Zealand. The view that home-ownership is an important friction in labour markets has also been expressed in recent policy discussions and the popular press.[1]

More recent empirical work has, however, called into question the claim that home ownership is associated with poor labour market outcomes. Munch et al. (2006) and Rouwendal and Nijkamp (2006) review some of this work. Coulson and Fisher (2008) present evidence that for U.S. metropolitan areas a higher degree of home ownership is associated with *improved* labour market outcomes with regard to both employment and wages.[2] In particular, after controlling for other co-variates, Coulson and Fisher (2008, p.26) conclude that, "there is a negative correlation between unemployment and ownership and a positive correlation between wages and ownership across US metropolitan areas."

[1] In a recent keynote address David Blanchflower (2007) summarizes this view as follows "Unemployment rates have grown most rapidly in the nations with the fastest growth in home ownership. Workers in Michigan laid off from GM own their own homes which they can't sell and it is hard then for them to move to new jobs in other parts of the country. The large increase in European home ownership has considerable advantage over the other possible explanations for the rise in unemployment – it seems to fit the data!" See also Harford (2007).

[2] Unlike US states, it seems reasonable to think of each metropolitan area as a distinct labour market.

A number of authors have also provided related evidence based on the analysis of micro-data. Using Australian data, Flatau et al. (2003) find no evidence to support the conjecture that home-owners have a higher probability of unemployment. However, once they control for demographic and locational characteristics, Coulson and Fisher (2008) find that the marginal impact of home ownership on the likelihood of unemployment *for the individual* is positive in the US.[3] At the same time (consistent with their aggregate cross–city results) they find that the likelihood of unemployment is negatively correlated with the rate of home ownership in an individual's city of residence.

In our economy a large number of *ex ante* identical households may choose to live in either of two cities which differ in regard to the productivity of jobs. Households require housing and may either rent in a competitive market or purchase in a market characterized by a search friction. All households, whether employed or unemployed, randomly receive offers of employment in both their city of current residence and the other city. In order to take a job in the other city, a household must move and acquire housing there. A home owner that moves sells their house in competitive market to a real estate manager and initially rents in the other city. The real estate manager then re-sells the house in the decentralized housing market to another household.[4]

In this environment, the willingness of a home owner to accept a job in another city depends not only on relative wages but also on rental rates and the value of their current house. Since the latter depends on how quickly the real estate manager can find a buyer, the liquidity of housing affects employment and unemployment in the two cities. At the same time, the frequency with which households choose to relocate affects the liquidity of the housing markets. We establish the existence and uniqueness of a stationary equilibrium characterized by constant relocation and thus housing market activity.

Our model generates a form of unemployment resulting from the endogenous liquidity cost faced by unemployed home owners who receive a job offer in another location. Although all unemployed renters would accept such a job offer, a fraction of home owners turn it down because the opportunity cost of selling their houses is too high. Thus, the liquidity of the housing market affects unemployment both within a city and in the aggregate. Although, at the individual level, home-ownership is associated with an increased likelihood of unemploy-

[3]Recognizing the potential problem that ownership is endogenous, they instrument for ownership in various ways.

[4]Although we could allow households to act as their own real estate manager, this set up simplifies the analysis by ruling out the low probablity event that a migrating household returns to its original location before selling.

ment, we find that cities with higher home-ownership (*i.e.* the fraction of a city's population that owns houses) may also have lower unemployment. The reason is that some unemployed renters with no current job offer move to the city where rents are lowest, while employed renters locate in high–rent cities as long as the wage premium is sufficient. At the same time, unemployed home-owners with no job offers do not re-locate to the low-rent city because, in equilibrium, the price they can get for their house is too low.

Using a numerical example of our model that is calibrated to match key facts relating to US labour markets and mobility rates, we find that the fraction of unemployed home-owners that turn down job offers from other locations is large. However, the impact of home-ownership on the likelihood of unemployment turns out to be small relative to the effect of the rent differential on unemployed renters. Consequently, unemployment rates and home-ownership are negatively related across cities. We also consider the impact of exogenously changing the ratio of owner–occupied to rental units in the economy. Consistent with Oswald's conjecture, we find that this does yield a positive relationship between home-ownership and unemployment at the aggregate level. However the quantitative impact is very small.

In equilibrium our economy necessarily generates positive (quality–adjusted) housing premia in the two cities associated with the scarcity of rental housing. Under the assumption that both rental and owner-occupied housing yield the same direct services, housing premia in our economy are of the same sign and order of magnitude as those estimated by Campbell, Davis, Gallin, and Martin (2008). This suggests that a substantial portion of housing premia may be accounted for by the relative illiquidity of housing.

Others have developed theories of the relationship between home ownership and labour market outcomes. For example, Dohmen (2005) presents a model in which higher home ownership overall is associated with higher aggregate unemployment even though individual home owners are employed at a higher rate than renters. His theory, however, abstracts from both housing choice and transactions in the housing market. As such the mechanism at work in his model differs from ours and he does not consider the implications of the interaction of housing and labour market conditions for housing premia. Coulson and Fisher (2008) present a theory based on endogenous job creation that is consistent with their evidence on unemployment, but does less well with regard to wages. In particular, in their model home owners receive lower wages than renters as a result of their immobility. In our economy high-wage locations are characterized by a higher degree of home ownership than are low-wage ones. As a result, home owners on average may have higher incomes, in spite of the fact that *ex ante* they are identical to renters.

Rupert and Wasmer (2008) also develop a theory of the relationship between unemployment and housing market frictions. Their analysis does not distinguish between ownership and renting, and they focus on the trade-off between commuting time and locational decisions within a single labour market. In contrast, our focus is on the role of housing markets in generating frictions between labour markets. In this sense, the two papers are complementary.

There is also a substantial literature which focuses on the relationship between the length of residence spells (which tend to be higher for home owners than for renters) and investments in social capital (*e.g.* see Rossi and Weber (1996) and DiPasquale and Glaeser (1999)). Empirically, Coulson, Hwang, and Imai (2002) find that the fraction of home owners in a neighborhood is associated with higher property values. While our model is consistent with this observation (as home ownership and house prices are both higher in the high-wage city) as well as the fact that home owners remain in a city longer than renters, we abstract from all issues involving investment of any kind.

This remainder of the paper is organized as follows: Section 2 describes the environment. Section 3 defines a symmetric stationary equilibrium, establishes existence and uniqueness, and characterizes the equilibrium under various assumptions. and establishes existence and uniqueness. Section 4 considers the implications of the theory for the relationship between home ownership and unemployment at the aggregate, city-wide, and individual levels. Section 5 describes the housing premium in equilibrium and relates them to a key feature of the housing market, the supply of rental housing. Section 7 summarizes and describes future work. Proofs of propositions and longer derivations are included in an appendix.

2 The Environment

The economy is populated by a unit measure of infinitely lived, *ex ante* identical, risk-neutral *households* who discount the future at interest rate ρ. There are also a large number of *real estate managers* with similar characteristics to households. There are two locations, called *cities*, indexed by $i \in \{1, 2\}$. Households must reside in one and only one city at any point in time. They are free, however, to move between cities at any time at no direct cost. Real estate managers may be thought of as having no location, and may engage in real estate activities (described below) in either city or both at any time.

Firms in city of type i produce output y_i using labour l_i according to the production function

$$y_i = (\phi_i l_i)^\eta \quad (1)$$

where $\eta \in (0, 1)$ and ϕ_i represents a city-specific productivity parameter. Taking wages as given, firms post enough vacancies at the beginning of each period to ensure that their demand for labour during that period is satisfied. There is no cost to posting these vacancies, but a per period fixed cost F must be incurred to operate the production technology. Although in principle cities could differ in many ways, to begin with we restrict attention to cases in in which they differ only with regard to the productivity of their firms. In particular, we assume that productivity is higher in City 2 (the *high-productivity* city) than in City 1 (the *low-productivity* city). That is, $\phi_2 > \phi_1$.

As a consequence of firms' demand for labour each city has a large number of potential employment opportunities, which we refer to as *jobs*. At any point in time, each household is either employed (*i.e.* holding a job) or unemployed. A household may hold at most one job and that job must be located in their city of current residence. Employed households in City i receive flow income equal to the *wage* w_i. Unemployed households receive flow consumption b. Real estate managers cannot hold jobs.

All households, regardless of their employment status, randomly receive offers of employment both in their city of residence and in the other city. We let μ denote the Poisson rate at which households receive offers within their city of residence and μ^* denote the rate at which the receive an offer in the other city. Although rates of receiving offers are symmetric across cities we assume that $\mu > \mu^*$. A household (employed or unemployed) which receives a job offer may either accept or reject it. Employed households lose their jobs at Poisson rate δ.[5]

Households require housing at all times. Each city contains fixed stocks of two types of residential dwellings. Let R_i denote the stock of *rental* housing in City i and H_i the stock of *owner-occupied* housing. Let π^R denote the flow utility received by a household which lives as a renter in either city. Similarly, π^H is the flow utility from living in an owned house. We assume that $\pi^H \geq \pi^R$.

In each city, the entire stock of rental housing is owned by real estate managers at all times and is rented to households in a competitive market at rate r_i. We assume that maintenance costs incurred by the real estate managers are negligible, so that the only restriction on rental rates is that $r_i \geq 0$. Real estate managers can, in principle, convert housing from rental to owner-occupied and vice versa. A fixed cost, however, must be incurred to undertake either

[5] It is possible to allow for differences in offer rates and separation rates across locations.

kind of conversion. Specifically, we assume that a cost C^R must be incurred to convert to owner-occupied housing to rental and a cost C^H must be incurred to convert rental housing to owner-occupied.[6]

Owner-occupied housing may be owned by either households or real estate managers. A *household* that owns a house receives service flow, π^H. In contrast, real estate managers receive no service flow from the houses that they own and therefore hold them only for the purpose of re-sale. The market for owner-occupied housing is characterized by a one-sided matching process. Let S_i denote the stock of houses in City i owned by real estate managers (and therefore for sale). Similarly, let B_i denote the stock of potential *home buyers* in City i. This will be comprised of all households in City i who do not currently own a house (we assume that households may own only one house at any given time).

City i real estate managers (house sellers) match with potential buyers at rate γ_i, where

$$\gamma_i = \frac{\lambda B_i}{S_i} \qquad i = 1, 2. \tag{2}$$

For simplicity, we assume that a real estate manager who matches with a potential buyer makes a take-it-or-leave-it offer to the buyer, provided that the aggregate surplus from the match is positive. Let q_i^W and q_i^U denote the prices paid for houses in City i by employed and unemployed households respectively.

Home owners may sell their houses at any time to real estate managers in a competitive market. Since there are a large number of real estate managers in competition with each other, in such a transaction the seller (normally a household) receives all the surplus from the exchange. Let p_i denote the price received by a household that sells a house to a real estate manager in City i.

Given the assumed structure of the markets for owner occupied housing, it takes time for houses to be transferred from one household to another. Specifically, a house must first be sold (instantaneously) to a real estate manager. The manager must then wait to match with a potential buyer. This friction results in houses being *illiquid*, as their market value depends on the speed with which an manager can find a buyer for a vacant house. Let the value of such a house in City i be denoted V_i^H. Then

$$\rho V_i^H = \gamma_i \mathrm{E}_{\{q_i^W, q_i^U\}} \left[\max\{q_i - V_i^H, 0\} \right] \qquad i = 1, 2. \tag{3}$$

In each city there are four types of households, as each may be either employed or unemployed and may either rent or own a house. The value functions for these households

[6] Zoning laws and other restrictions may effectively drive these costs to infinity.

in city i are given by

$$\begin{aligned}
\rho W_i^R &= w_i + \pi^R - r_i + \delta\left[U_i^R - W_i^R\right] + \mu^* \max\left\{W_j^R - W_i^R, 0\right\} \qquad (4)\\
&\quad +\lambda \max\left\{W_i^H - q_i^W - W_i^R, 0\right\}\\
\rho U_i^R &= b + \pi^R - r_i + \mu\left[W_i^R - U_i^R\right] + \mu^* \max\left\{W_j^R - U_i^R, 0\right\} \qquad (5)\\
&\quad +\lambda \max\left\{U_i^H - q_i^U - U_i^R, 0\right\}\\
\rho W_i^H &= w_i + \pi^H + \delta\left[U_i^H - W_i^H\right] + \mu^* \max\left\{W_j^R + p_i - W_i^H, 0\right\} \qquad (6)\\
\rho U_i^H &= b + \pi^H + \mu\left[W_i^H - U_i^H\right] + \mu^* \max\left\{W_j^R + p_i - U_i^H, 0\right\} \qquad (7)
\end{aligned}$$

where W_i^R denotes the value of being an employed renter in City i, and so on.

Real estate managers behave competitively in the rental markets in both cities and also buy and sell houses. The value of a representative manager is given by

$$\rho V^M = \sum_{i=1,2}\left[r_i R_i + [H_i - O_i]\rho V_i^H\right], \qquad (8)$$

where O_i is the measure of houses currently owned by households in City i.

3 Equilibrium with Free Entry of Firms

We restrict attention to equilibria which are stationary and symmetric in that all households of a given type behave in the same way. A *stationary symmetric equilibrium* for this economy is a collection of nine value functions, eight for the different types of households, W_i^R, W_i^H, U_i^R, and U_i^H, for $i = 1, 2$; and one for real estate managers, V^M; rental prices in each city, r_i; house prices in each city, q_i^W, q_i^U, and p_i; measures of households in each of the eight states, N_i^{WR}, N_i^{UR}, N_i^{WH}, and N_i^{UH}; and house values in each city, V_i^H, such that:

i. Given wages, firms choose employment levels l_i to maximize profits. Free entry into production implies that

$$(\phi_i l_i)^\eta - w_i l_i - F \leq 0. \qquad (9)$$

ii. Given prices and the value of houses in each city, the value functions satisfy (5)-(7), the corresponding counterparts of City 1, and (8).

iii. The rental prices, $r_i \geq 0$, clear the markets for rental housing in each city:

$$N_i^{WR} + N_i^{UR} = R_i \qquad i = 1, 2. \qquad (10)$$

iv. House purchase prices in both cities, p_i^W and p_i^U extract all of households' surplus.

v. The house sale price in each city is equal to the value of a vacant house:

$$p_i = V_i^H \qquad i = 1, 2. \tag{11}$$

vi. The distribution of households over states is consistent with the population:

$$\sum_{i=H,L} \left[N_i^{WR} + N_i^{UR} + N_i^{WH} + N_i^{UH}\right] = 1. \tag{12}$$

v. There is no conversion

$$\rho C_R > r_i - \rho p_i > -\rho C_H \tag{13}$$

We begin by assuming at least one (and possibly more) symmetric equilibrium exists, and describe several characteristics that it must necessarily have. We then finish this section with a proposition establishing existence and uniqueness.

Profit maximization by firms in City i implies that each demands labour

$$l_i = \left(\frac{\eta}{w_i}\right)^{\frac{1}{1-\eta}} \phi_i^{\frac{\eta}{1-\eta}}. \tag{14}$$

It follows that firm profits can be expressed as

$$(1-\eta)\left(\frac{\eta\phi_i}{w_i}\right)^{\frac{\eta}{1-\eta}} - F. \tag{15}$$

If there is free entry into production, the equilibrium wage in city i must satisfy

$$w_i = \eta\left(\frac{1-\eta}{F}\right)^{\frac{1-\eta}{\eta}} \phi_i. \tag{16}$$

Thus, in equilibrium with free entry of firms, the wage in each city is proportional to local productivity and is unaffected by conditions in the housing market.[7] In Section 6 we consider the implications of the model when the number of firms is fixed. Until then we will refer to the low (high) productivity city as the low (high) wage city.

Since in each city renters constitute the potential buyers in the housing market, we may write (2):

$$\gamma_i = \frac{\lambda R_i}{H_i - O_i} \qquad i = 1, 2. \tag{17}$$

[7] A special case of this would, of course, arise if $F = 0$ and $\eta = 1$.

Combining (10), (12) and (17)yields a locus of values for γ_1 and γ_2 which are consistent with both aggregation and rental market clearing:

$$\lambda \left[\frac{R_1}{\gamma_1} + \frac{R_2}{\gamma_2} \right] = R_1 + R_2 + H_1 + H_2 - 1. \tag{18}$$

We depict this locus (labeled AM) in Figure 1. Note that $\underline{\gamma}_1$ and $\underline{\gamma}_2$ are the asymptotic values below which the matching rates in each respective city cannot feasibly fall.

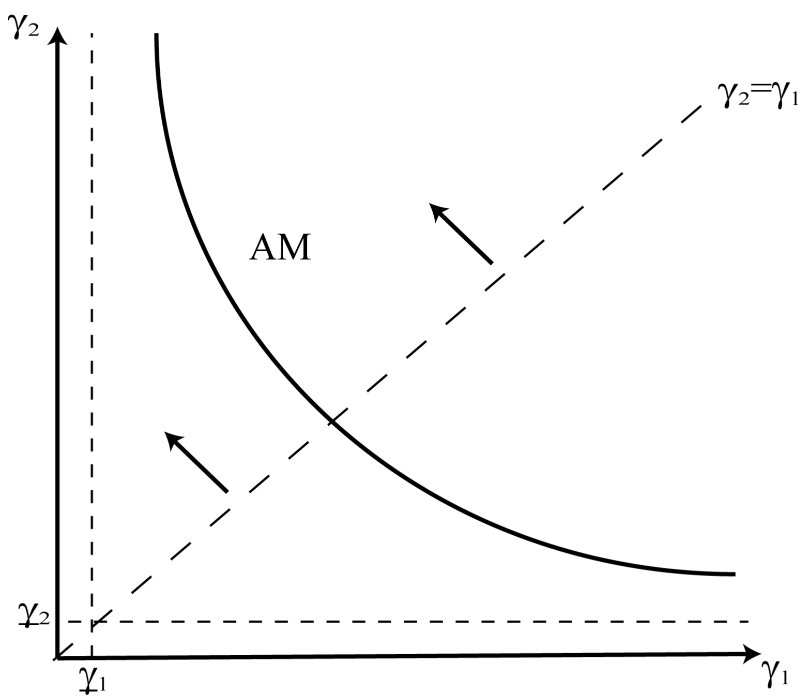

Figure 1: The AM Curve

We restrict the attention to equilibria where employed renters in the low-wage city (City 1) who are offered a job in the high-wage city choose to relocate, but not *vice versa*. That is

$$W_2^R > W_1^R. \tag{19}$$

As we demonstrate below, one consequence of this assumption is that $\gamma_2 > \gamma_1$. Thus, without loss of generality, the equilibria that we study here are all located on the the segment of the AM curve above the 45^o line.

We also restrict attention to equilibria in which all renters, whether employed or not, buy houses when they get the chance. As real estate managers make take-it-or-leave-it offers to home buyers, this will be true as long as the surplus from house sales is positive:

$$W_i^H - W_i^R \geq q_i^W > p_i \qquad U_i^H - U_i^R \geq q_i^U > p_i \qquad i = H, L. \tag{20}$$

Below we show that both of these restrictions are satisfied under only slightly more stringent assumptions than those needed to guarantee existence and uniqueness.

Because households are mobile, in any equilibrium unemployed renters must be indifferent across cities. That is,

$$U_2^R = U_1^R. \tag{21}$$

Home owners, in contrast, face effective "moving costs" associated with the illiquidity of housing. Conditions (20) and (19) together imply that employed home owners are also unwilling to move from from the high wage city to the low wage one in equilibrium:

$$W_2^H - p_2 > W_2^R > W_1^R. \tag{22}$$

Making use of the facts that unemployed renters are indifferent between locations, real estate managers extract all surplus from households who purchase houses and that employed renters do not move from the high-wage city to the low-wage one may re-write the value functions for renters:

$$\rho W_1^R = w_1 + \pi^R - r_1 + \delta(U^R - W_1^R) + \mu^*(W_2^R - W_1^R) \tag{23}$$

$$\rho U^R = b + \pi^R - r_1 + \mu(W_1^R - U^R) + \mu^*(W_2^R - U^R) \tag{24}$$

$$\rho W_2^R = w_2 + \pi^R - r_2 + \delta(U^R - W_2^R) \tag{25}$$

$$\rho U^R = b + \pi^R - r_2 + \mu(W_2^R - U^R) + \mu^*(W_1^R - U^R). \tag{26}$$

Subtracting (24) from (23) and solving in terms of U_R we have

$$W_1^R = \underbrace{\frac{w_1 - b}{\rho + \delta + \mu + \mu^*}}_{A} + U^R. \tag{27}$$

Similarly, subtracting (26) from (25) yields

$$W_2^R = \underbrace{\frac{w_2 - b - \mu^* A}{\rho + \delta + \mu}}_{B} + U^R. \tag{28}$$

Using (27) and (28) it is easily shown that

Lemma 1: *If* $w_2 > w_1$*, then* $B > A$ *and, in equilibrium,* $W_2^R > W_1^R$*.*

Note that equating (24) and (26) , and using (27) and (28) it is also apparent that

$$r_2 - r_1 = (\mu - \mu^*)(B - A) > 0 \tag{29}$$

Thus, relative rental rates in the two cities are determined by the wage differential, job offer arrival and destruction rates, the reservation utility (or unemployment benefit), and the interest rate. The *levels* of the rental rates themselves depend on the value of being an unemployed renter, U^R.

The sale price of a house in City i is given by

$$p_i = \frac{\gamma_i}{\rho + \gamma_i}\left[\alpha_i(W_i^H - W_i^R) + (1 - \alpha_i)(U_i^H - U^R)\right] \tag{30}$$

where $\alpha_i \equiv N_i^{WR}/R_i$ represents the fraction of renters in City i that are employed.

Recall that employed renters will move from the low-wage city to the high-wage one if offered a job, but not *vice versa*. Thus, employed home owners will also not move from the high-wage city to the low-wage one. Assuming that an equilibrium exits, there are two different possible cases with regard to the movement of *home owners* between cites:

I. Some fraction (possibly all) of unemployed home owners in the low-wage city move to the high-wage city if offered a job there, but employed home owners do not move.

II. *All* unemployed home owners and some fraction of the employed home owners (again, possibly all) in the low-wage city move if offered a job in the high-wage city.

In case **I**, we say that the *marginal home owner* is an unemployed one. Case **II** in contrast, is one in which the marginal home owner is employed. We will see that which case obtains depends on the magnitude of the wage differential between cities and is reflected in the relative liquidity of the housing markets in the two cities. Case **II** is associated with both a high wage differential and a relatively high matching rate between buyers and real estate managers in the high-wage city. We consider the two cases separately, with most algebraic calculations included in the appendix.

3.1 Case I – Unemployed owners turn down job offers in both cities.

Define θ_i^{UH} and θ_i^{WH} respectively as the fractions of unemployed and employed home owners in City i who move if they receive a job offer in the other city within a unit of time.

Alternatively, we may think of these as the probability which such a household accepts an offer conditional on receiving one. Case I equilibria are those in which

$$\theta_1^{UH} \in (0,1], \qquad \theta_1^{WH} = 0, \quad \text{and} \quad \theta_2^{UH} \in (0,1]. \tag{31}$$

Within this overall case, we have three possibilities:

1. $\theta_1^{UH} < 1$ and $\theta_2^{UH} < 1$: The *interior* sub-case
2. $\theta_1^{UH} = 1$ and $\theta_2^{UH} < 1$: *Corner* case X
   In this case all unemployed homeowners in the low-wage city accept jobs in the high-wage city.
3. $\theta_1^{UH} < 1$ and $\theta_2^{UH} = 1$: Corner case Y
   In this case all unemployed home owners in the high-wage city accept jobs in the low-wage city.

We begin with the interior sub-case. In order for only *some* unemployed home owners to leave the low-wage city for a job in the high-wage one we require each individual to be indifferent, *i.e.*

$$W_2^R + P_1 = U_1^H. \tag{32}$$

Similarly, we require

$$W_1^R + P_2 = U_2^H \tag{33}$$

if only a fraction of unemployed home owners in the high-wage city accept jobs in the low-wage city.

The steady–state flow of workers between states in Case I is described by (10), (18) and the following equations:

$$(\delta + \mu^* + \lambda)N_1^{WR} = \mu N_1^{UR} + \mu^* \left(N_2^{UR} + M_2^{UH}\right) \tag{34}$$

$$\mu N_1^{UH} + \mu^* M_1^{UH} = \delta N_1^{WH} + \lambda N_1^{UR} \tag{35}$$

$$\delta N_1^{WH} = \lambda N_1^{WR} + \mu N_1^{UH} \tag{36}$$

$$(\delta + \lambda)\, N_2^{WR} = \mu N_2^{UR} + \mu^* \left(N_1^{UR} + M_1^{UH} + N_1^{WR}\right) \tag{37}$$

$$\mu N_2^{UH} + \mu^* M_2^{UH} = \delta N_2^{WH} + \lambda N_2^{UR} \tag{38}$$

$$\delta N_2^{WH} = \lambda N_2^{WR} + \mu N_2^{UH} \tag{39}$$

where $M_i^{UH} = \theta_i^{UH} N_i^{UH}$ denotes the measure of unemployed home-owners in city i that move if they receive an offer. Equation (34) says that the measure of agents that exit the state of being employed owners in city 1 (either by losing their job, by accepting offers in city 2 or by buying a house) equals the measure that enter that state (either unemployed renters who receive offers in either city or unemployed home-owners that receive and accept offers in city 2). Similarly, (35) says that the measure that exit the state of being unemployed home-owners in city 1 (by receiving and accepting offers in city 1 or city 2) equals the measure that enter that state (either employed home-owners who lose their jobs or unemployed renters who buy a house), and (36) says that the measure that exit the state of being employed home-owners in city 1 (by losing their jobs) equals the measure who enter that state (employed renters that buy a house and unemployed owners who accept a job offer locally). Equations (37)-(39) represent the same conditions for city 2.[8]

It is possible to show (details in the appendix) that given (32) and (33), $\theta_1^{UH} < 1$, $\theta_2^{UH} < 1$ require that the matching rates for home buyers with real estate managers in each city must exceed critical levels which we associate with the two corner cases, X and Y at which $\theta_1^{UH} = 1$ and $\theta_2^{UH} = 1$ respectively:

$$\gamma_1 > \gamma_1^X = \frac{\lambda R_1}{H_1 - (\lambda/\delta)N_1^{WR} - [(\delta+\mu)/\delta](\lambda/\mu^*)R_1} \tag{40}$$

$$\gamma_2 > \gamma_2^Y = \frac{\lambda R_2}{H_2 - (\lambda/\delta)N_2^{WR} - [(\delta+\mu)/\delta](\lambda/\mu^*)R_2}. \tag{41}$$

For each of these critical values, there are corresponding values for the matching rate in the other city implied by the AM curve (18). We denote these values γ_2^X and γ_1^Y, respectively. In addition, since for them there is no opportunity cost to moving, unemployed renters must be indifferent across locations. Combining (27) with (32) and (28) with (33) we have

$$U_1^H - P_1 = B + U^R \tag{42}$$

$$U_2^H - P_2 = A + U^R \tag{43}$$

where the expressions on the left-hand sides of (42) and (43) are the values of being an unemployed home owner in the low and high-wage cities respectively. Making use of (30) and the value functions (see appendix) we may derive expressions for the values of being an

[8]The asymmetry between (34) and (37) stems from (19).

unemployed *renter* in each of the two cities as functions of the relevant matching rate:

$$\rho U^R(\gamma_1; I) = w_1 + \pi^H - (\rho + \gamma_1)B - (\rho + \delta)A - \frac{(\rho + \delta + \alpha_1\gamma_1)\mu^* A}{\rho + \delta + \mu} \tag{44}$$

$$\rho U^R(\gamma_2) = w_2 + \pi^H - (\rho + \gamma_2)A - (\rho + \delta)B - \frac{(\rho + \delta + \alpha_2\gamma_2)\mu^* A}{\rho + \delta + \mu}. \tag{45}$$

From these expressions it may be seen that an increase in the matching rate γ_i in either city reduces the welfare of unemployed renters. The reason is that an increased matching rate in city i raises house prices, thereby lowering the opportunity costs incurred by unemployed home-owners if they move. To maintain indifference, this must be matched by an increase in rental rates in the other city j which causes U^R to fall.

Equating (44) and (45) yields a relationship between γ_1 and γ_2 which must hold in any Case I equilibrium, as it is required by the indifference of unemployed renters across locations:

$$\gamma_2 = \Omega^I + \Psi^I \gamma_1 \tag{46}$$

where Ω^I and Ψ^I are positive constants (which depend on wages; see appendix). As noted above, an increase in γ_1 drives up rental rates in City 2. To maintain the indifference of unemployed renters, this must be matched by an increase in rental rates in City 1. As a result, migration from City 2 declines and the consequent decline in houses for sale pushes up γ_2. Figure 2 depicts this relationship (labeled VVI) together with AM and illustrates an interior Case I stationary equilibrium.

We next consider the two corner sub-cases. In Corner Y, all unemployed home owners in the high-wage city strictly prefer to accept jobs (and initially rent) in the low-wage city. In this case, (41) holds with equality. Equilibrium γ_1 can then be determined by (18). Diagrammatically, we can imagine moving toward this case when the w_1 rises toward w_2. In this case VVI shifts downward and to the right (See Figure 3). Intuitively, as the difference between wages in the two cities lessens, unemployed home owners in the high-wage city have less incentive to remain and wait for a high-wage job and are thus more likely to accept an offer of employment in the low-wage city. This has the effect in equilibrium of increasing the measure of potential home buyers in the low-wage city and increasing γ_1 relative to γ_2. The corner occurs when an unemployed home owner takes an offer in the low-wage city with probability one—*i.e.* at any relative wage such that VVI lies to the right of Y.

Corner X occurs when $\theta_1^{UH} = 1$, and all unemployed home owners in the *low-wage* city accept job offers in the high-wage city. In this case (40) holds with equality, and γ_2 is determined by (18). We approach this corner as w_1 falls relative to w_2, so that the gap

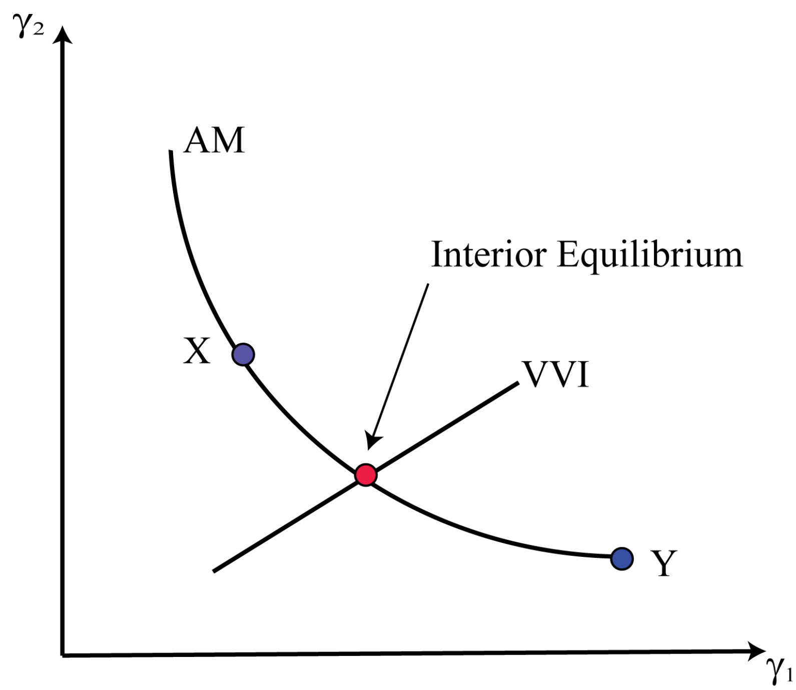

Figure 2: Case I Interior Equilibrium

between wages in the two cities rises. In this case it is more attractive for home owners in the low-wage city to become renters in the high-wage city (despite not owning a home and paying higher rent). This results in VVI shifting upward and to the left, increasing the measure of potential home buyers in the high-wage city (*i.e.* raising) γ_2 relative to γ_1 The corner occurs when an unemployed home owner in the low-wage city accepts an offer in the high-wage city with probability one. Further increases in w_2 result in the marginal low-wage city home owner being an employed one, and we turn next to that case.

3.2 Case II — Only the unemployed in the high-wage city turn down jobs.

Case II equilibria are those in which

$$\theta_1^{UH} = 1, \qquad \theta_1^{WH} \in (0, 1], \quad \text{and} \quad \theta_2^{UH} \in (0, 1). \tag{47}$$

That is, job offers in the high-wage city are accepted by *all* unemployed home owners in the low-wage city and by *employed* home owners with a positive probability. We again have two

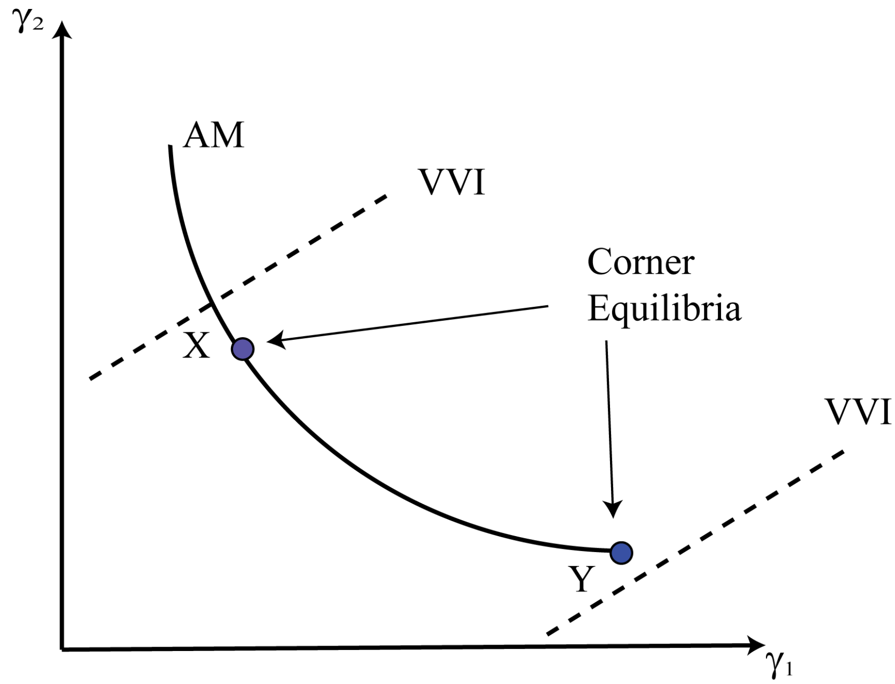

Figure 3: Case I Corner Equilibria

sub-cases:[9]

1. $\theta_1^{WH} \in (0, 1)$: The *interior* sub-case

2. $\theta_1^{WH} = 1$: Corner case Z
   In this case *all* home owners in the low-wage city, whether employed or unemployed, accept jobs in the high-wage city.

Our analysis of these cases is essentially parallel to that presented in section 3.1.

In all Case II equilibria, $\theta_2^{UH} \in (0, 1)$, implying the lower bound on γ_2 given by (41). Similarly, $\theta_1^{WH} < 1$ implies a lower bound on γ_1 given by

$$\gamma_1 > \gamma^Z = \frac{\lambda R_1}{H_1 - (\lambda/\mu^*) R_1}. \tag{48}$$

Finally, $\theta_1^{WH} > 0$ implies an *upper* bound on γ_1 given by γ_1^X as defined in (40). Thus, the matching rate for sellers in the low-wage location must satisfy

$$\gamma_1^Z < \gamma_1 < \min\left[\gamma_1^X, \gamma_1^Y\right]. \tag{49}$$

[9]Note that Corner case Y may also be considered a sub-case of Case II.

In this case (43) continues to hold as unemployed home owners in the high-wage city are indifferent to accepting job offers in the low-wage city. However, now it is employed home owners in City 1 who are indifferent to accepting a job in City 2. Thus, (42) is replaced by

$$W_1^H - P_1 = B + U^R. \tag{50}$$

Similarly, as $\theta_2^{UH} \in (0, 1)$, (33) and (45) continue to hold, but the value of being an unemployed renter in the low-wage location now changes from (44) to

$$\rho U_1^R(\gamma_1; II) = w_1 + \pi^H + (\gamma_1 - \delta) A - (\rho + \gamma_1)B. \tag{51}$$

Equating (51) and (44) we may again derive a relationship between γ_1 and γ_2 which must hold in any Case II equilibrium:

$$\gamma_2 = \Omega^{II} + \Psi^{II}\gamma_1 \tag{52}$$

where Δ_1^{II} and Δ_2^{II} are again constants which depend on labour market conditions (see appendix). Figure 4 depicts this relationship (labeled VVII) together with AM and illustrates an interior Case II stationary equilibrium. Note that VVII always lies to the right of VVI.

We now turn to Corner case Z, in which all home owners in the low-wage city who receive a job offer in the high-wage city, move there. In this case the left-hand inequality in (49) binds, determining γ_1, and γ_2 is again determined by the AM curve, (18). Intuitively, as w_2 rises relative to w_1, employed home owners in the low wage city accept job offers in the high-wage city with higher probability. This increases the measure of potential home buyers in the the high-wage city, driving up γ_2 relative to γ_1. The corner occurs when *all* households, regardless of whether they own a house or not, will accept a job offer in the high-wage city. Diagrammatically, this occurs when VVII has shifted all the way to the left of AM (See Figure 4).

Figure 5 illustrates all possible stationary equilibria for the economy, summarizing our analysis of the different cases and sub-cases. We can now identify the restrictions on parameters that are sufficient to ensure that an assertion made earlier are true in equilibrium:

Proposition 1. *If $\mu > \alpha_1\gamma_1^X$ then in a stationary equilibrium, both employed and unemployed renters buy houses when they get the chance. That is, (20) holds.*

In all of the possible equilibria other than Corner X no further restrictions are required beyond those already stated. In Corner X the restriction stated in Proposition 1 is required to ensure that unemployed renters wish to buy houses.

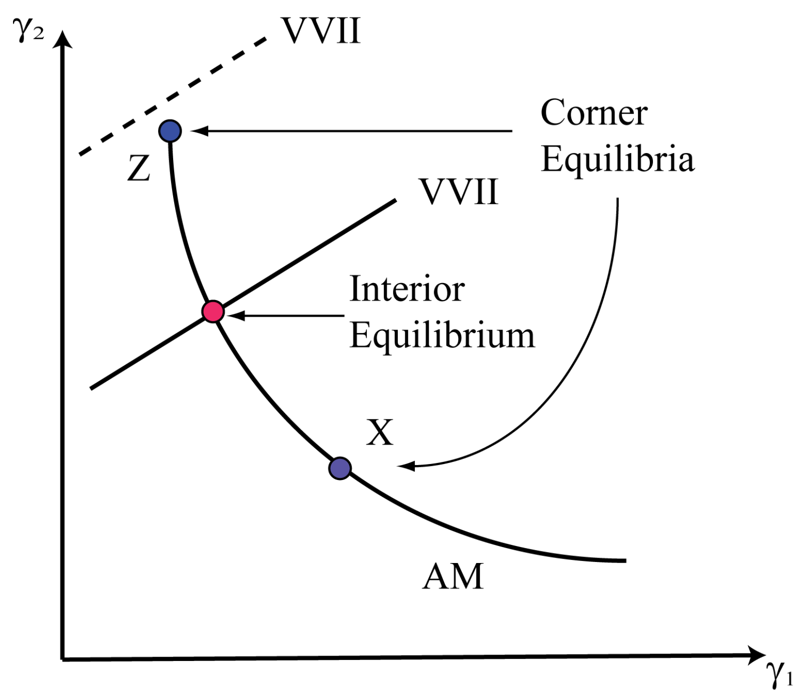

Figure 4: Case II Equilibria

Having characterized the possibilities of stationary equilibria under the assumption that at least one of them exists, we now state the following proposition which is proved in the appendix.

Proposition 2. *Subject to parameter restrictions (see appendix), there exists a unique stationary equilibrium.*

Note that stationarity of the equilibrium requires

Corollary 1. *In a stationary equilibrium, house sales take place in both cities. That is,*

$$\theta_1^{UH} > 0 \qquad \text{and} \qquad \theta_2^{UH} > 0 \tag{53}$$

4 The Relationship between Home-ownership and Unemployment

In this section we focus on the implications of the model for the relationship between home-ownership and unemployment at the individual, city and aggregate levels.

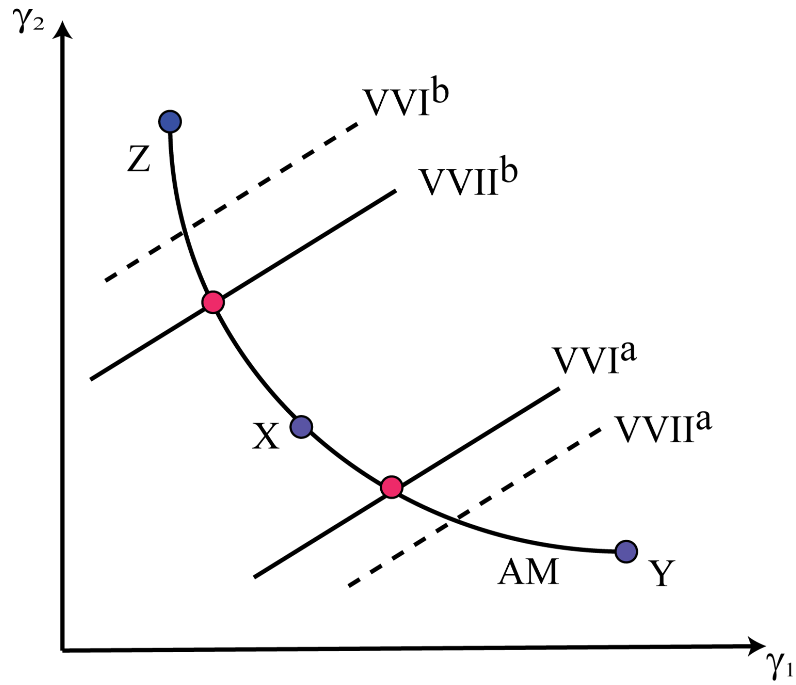

Figure 5: Stationary Equilibria

4.1 Analytical Results

Implicit in our discussion of the alternative equilibria above is the assertion that the matching rate is highest in the high wage city. We now state this implication more formally:

Proposition 3. *There exists an $\epsilon > 0$ such that if $w_2 - w_1 > \epsilon$, then the matching rate is highest in the the high-wage city: $\gamma_2 > \gamma_1$*

Note that in the numerical examples we consider below, ϵ is very small.

The home–ownership rate in city i can be expressed as a function of the matching rate:

$$h_i(\gamma_i) = \frac{\frac{H_i}{R_i} - \frac{\lambda}{\gamma_i}}{1 + \frac{H_i}{R_i} - \frac{\lambda}{\gamma_i}}. \tag{54}$$

Since this expression is increasing in γ_i it follows that

Corollary 2. *If the ratio of the stock of owned to rental houses is the same, home-ownership is greatest in the high-wage city: $h_2 > h_1$.*

Of course, the rate of home-ownership is also increasing in the ratio of the stock of owned to rental housing. If this is also higher in City 2, then the home-ownership rate would be even higher.

Owners and renters face the same rates of separation and offer rates so the only difference between them is the rate at which they accept offers. Given that owners are less likely to accept offers, the following implication should not be surprising:

Proposition 4. *The unemployment rate amongst homeowners exceeds that amongst renters.*

It should be remembered that workers in our model are ex ante identical. Although in the data unemployment rates are highest amongst renters, this reflects demographic and locational differences between them that are not modeled here. Once these characteristics are controlled for, Coulson and Fisher (1998) do find a positive marginal impact of being a home-owner on the likelihood of unemployment for US workers.

We also have the following implication

Proposition 5. *There exists a $\sigma \in (0,1)$ such that if $R_1/R_2 > \sigma$, then the fraction of renters who are employed is greatest in the high wage city: $\alpha_2 > \alpha_1$.*

In any equilibrium, the majority of households (all renters and some homeowners) from city 1 that receive job offers from there, migrate to city 2. In contrast, only unemployed renters and some unemployed homeowners from city 2 that receive offers from there, migrate to city 1. This asymmetry tends to drive up the rent in city 2 relative to city 1 which induces unemployed households with no job offer to remain in or move to city 1. Consequently, the proportion of renters who are unemployed is higher in city 1 than city 2 in equilibrium.

At the city level, the unemployment rate reflects a trade–off between these two effects. The unemployment rate in city i can conveniently be expressed as

$$\nu_i = \left(\frac{\delta}{\delta+\mu}\right) h_i + \left(1 - \left(\frac{\delta+\mu+\lambda}{\delta+\mu}\right)\alpha_i\right)(1-h_i) \tag{55}$$

where h_i is given by (54). The first term reflects the positive impact of home-ownership on the contribution to the city level unemployment rate coming from home-owners. It is obviously larger for the high wage city. The second term reflects the contribution to overall unemployment coming from renters. An increase in home-ownership implies a relatively smaller measure of renters. The smaller the fraction of these renters that is unemployed (higher α), the smaller is this contribution. Since $h_2 > h_1$ and $\alpha_2 > \alpha_1$, this term is smaller for the high wage city. Overall then the relationship between unemployment and home-ownership at the city level depends on which of these terms dominate.

Our final proposition relates to the aggregate unemployment rate across the entire economy:

Proposition 6. *Aggregate unemployment is monotonically increasing in the home–ownership rate.*

The aggregate unemployment rate, $\bar{v}$, can be derived analytically in Case I and is given by

$$\bar{\nu} = \frac{\delta\left(\delta+\mu\right)-\lambda\left(\mu+\mu^{*}+\lambda\right)}{\left(\delta+\mu\right)\left(\delta+\mu+\mu^{*}+\lambda\right)} + \left(\frac{\delta\left(\mu^{*}+\lambda\right)+\lambda\left(\mu+\mu^{*}+\lambda\right)}{\left(\delta+\mu\right)\left(\delta+\mu+\mu^{*}+\lambda\right)}\right)\bar{h} \tag{56}$$

where $\bar{h} = 1 - R_1 - R_2$ is the aggregate rate of home-ownership. An increased rate of home-ownership increases the measure of unemployed homeowners. However, it also decreases the measure of unemployed renters. Although the former effect dominates overall , it may be significantly reduced by the latter.

4.2 A Numerical Example

To illustrate our results, we consider a numerical example. We choose the parameters of the model so that the steady state matches several key averages for the US. The parameter values and the relevant targets are given in Table 1. We calibrate to monthly data and, where possible, draw estimates from the literature which reflect that frequency. In particular, target values for the discount rate, the hiring rate and the separation rate are taken from Shimer (2005), as are values for the income replacement rate and the unemployment rate. Target values for the home-ownership rate and the vacancy rate for owner–occupied homes, which pin down the total measure of rental and owner-occupied housing per capita, are taken from the most recent US Census. Note that throughout we assume that $\pi^R = \pi^H$.

Table 1 — Parameter Choices

Parameter	Value	Target	Value	Source
ρ	0.0038	Annual discount factor	0.953	Shimer (2005)
δ	0.0265	Monthly separation rate	0.027	Hall (2005)
b/w_1	0.4	Income replacement rate	0.4	Shimer (2005)
w_2/w_1	1.1	Dense/nondense metro wage premium	0.1	Glaeser and Mare (2001)
$R_1 + R_2$	0.32	Homeownership rate	0.68	US Census
$H_1 + H_2$	0.705	Homeowner vacancy rate	0.028	US Census
μ	0.43	Monthly hiring rate	0.44	Shimer(2005)
μ^*	0.0145	Unemployment rate	0.057	US post-war average
λ	0.0012	Ann. mobility (between counties)	0.061	US Census

A problematic issue, of course, is how to map the abstract two–city model we have developed into real word data. Here we consider one approach to doing this, but we do not believe that the main thrust of our results depends on this.

It is fairly well documented that an important source of wage differences across cities is the size of the population. For example, even after controlling for demographic observables (education, experience and race) and labour force composition, Glaeser and Mare (2001) estimate that living in metropolitan areas yields a significant wage premium. We therefore think of the high-wage type of city as representing "large cities" and the low-wage type of city as representing "small cities".[10] This does not necessarily mean, however, that city 2 has a bigger population than city 1. In particular, one could think of migration between a large central city and several smaller cities whose total population exceeds that of the large city.

A useful classification of US cities is discussed by Overman & Ioannides (1999), based on earlier work by Knox (1994). Cities are grouped into 4 tiers determined by function. The the top tier consists of 10 'nodal centres',[11] the second tier consists of 14 regional centres[12] and the third tier consists of 19 sub-regional centres.[13] The remaining 291 cities are allocated to the fourth tier. The total population of the top three tiers turns out to be approximately equal to the total population of the fourth tier and this distribution is fairly stable over time. We therefore choose the ratio of houses in each city $(H_2 + R_2)/(H_1 + R_1) = 1$. Glaeser and Mare (2001, Table 3) estimate a dense metropolitan wage premium for cities with more than 500,000 inhabitants of 0.24 log points and a non-dense metropolitan premium of 0.14 log points. We use the difference between these as our estimate of the wage premium between type 2 cities and type 1 cities (i.e. a 10% wage premium).[14]

Annual mobility rates (the % of the population that change address in a given year) may be found in the US Census. Although more than 15% of the US population change addresses each year, this includes people who move short distances within a county. For our purposes, a more appropriate estimate of mobility is that between labour markets. We therefore use

[10] Interestingly, Coulson and Fisher (1998) find no impact of population size on median earnings across SMAs. This may reflect the fact that they include home-ownership rates as a regressor.

[11] Atlanta, Chicago, Denver, Houston, Los Angeles, New York City, Miami, San Francisco, Seattle and Washington D.C.

[12] Baltimore, Boston, Cincinnati, Cleveland, Columbus, Dallas, Indianapolis, Kansas City MO, Minneapolis, New Orleans, Philadelphia, Phoenix, Portland OR, and St. Louis.

[13] Birmingham, Charlotte, Des Moines, Detroit, Hartford, Jackson MS, Little Rock, Memphis, Milwaukee, Mobile, Nashville, Oklahoma City, Omaha, Pittsburgh, Richmond, Salt Lake City, Shreveport, Syracuse and Tampa.

[14] Most of our results in this section turn out to be quite insensitive to this value.

as a target the components of the mobility rate that is associated with people who move between counties, which is 6.1%.[15] We choose values of μ, μ^* and λ that jointly yield our target values of the monthly hiring rate, the unemployment rate and the cross-county annual mobility rate.

The equilibrium implied by this calibration turns out to be an interior Case I equilibrium. Table 2 provides a break-down of the fractions of the total population in each of the possible 8 states. As can readily be seen, city 2 has more employed renters and owners and more unemployed owners, whereas city 1 has substantially more unemployed renters. This reflects the steady–state movement of unemployed renters with no job offer towards city 1 in response to lower rents.

Table 2: Allocation of workers by job status and housing tenure

	City 1		City 2	
	Renter	Owner	Renter	Owner
Employed	0.146	0.318	0.156	0.323
Unemployed	0.014	0.019	0.004	0.020

Table 3: Labour Market Statistics

	City 1	City 2	Aggregate
Unemployment Rate			
overall	0.066	0.047	0.057
amongst renters	0.086	0.026	0.056
amongst owners	0.058	0.058	0.058
Fraction that turn down offers	0.299	0.331	0.315
Separation rate (inc. job-to-job)	0.031	0.027	0.029
Exit rate from unemployment	0.44	0.44	0.44
Home-ownership rate	0.67	0.69	0.68
Annual mobility rate			
overall	0.06	0.06	0.06
amongst renters	0.16	0.17	0.17
amongst owners	0.01	0.01	0.01

The allocation of workers is also reflected in the unemployment rates shown in Table 3. While the unemployment rate amongst owners is much the same in the two locations, the

[15] This is likely to be an upper bound on mobility.

unemployment rate amongst renters in city 1 is more than 3 times that of renters in city 2. Although a significant fraction (about 30%) of unemployed homeowners turn down offers in each city, the unemployment rate of home-owners in aggregate is not much higher than renters. This reflects the fact that offer rate from outside the city is small in order to match mobility rates. Consequently, the unemployment rate in city 1 is substantially higher than in city 2.

Table 3 also shows the break down of annual mobility rates implied by this calibration. According to the US Census, the mobility rates for renters is approximately 10% and for owners it is 2%. Thus, while we are in the right ballpark, our model overstates the mobility of renters and understates the mobility of owners. This should not be surprising, given our assumption of there being no other costs or rigidities associated with moving.

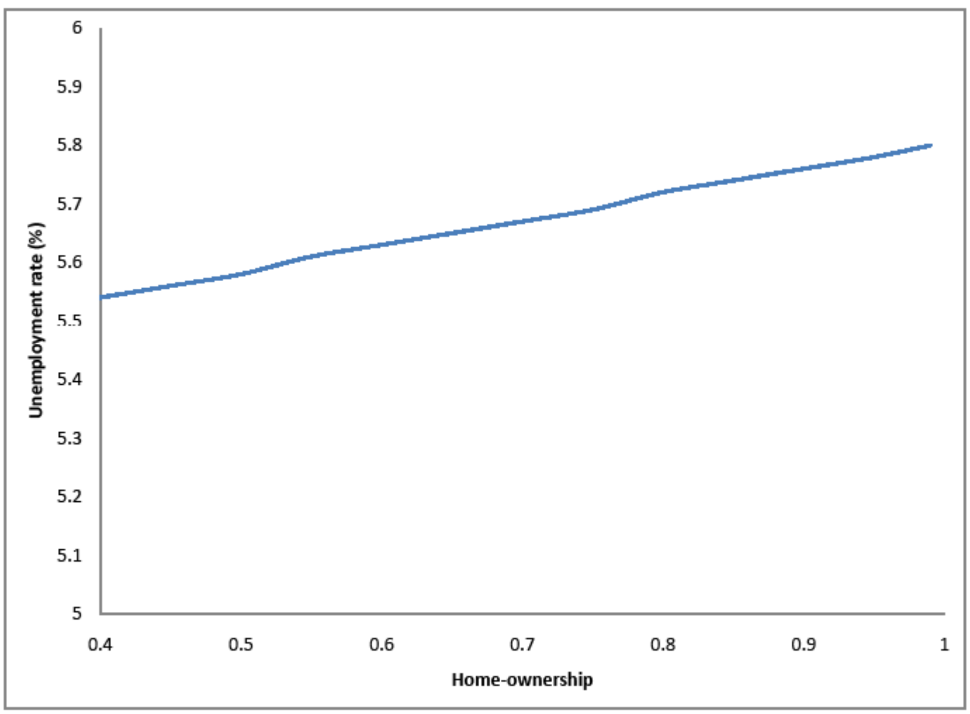

Figure 6: Impact of home-ownership on aggregate unemployment rate

Figure 6 shows the effect on aggregate unemployment of increasing the rate of home-ownership by raising $R_1 + R_2$. As can be seen, in the aggregate the "Oswald effect" is there, so that unemployment and ownership are positively related. However, for this calibration, the effect is tiny: a 10% increment in the rate of ownership results in an increase in the unemployment rate of only 0.04%.

5 Housing Premia

Our model has implications for housing premia since the relative scarcity of rental housing plays a key role in driving rents and house prices. In an interior Case I equilibrium, the housing premia in each city can be derived analytically and are given by

$$r_1 - \rho p_1 = \pi^R - \pi^O + \rho B + \mu^* (B - A) + \left(\frac{\rho + \delta}{\rho + \delta + \mu} \right) \mu^* A \tag{57}$$

$$r_2 - \rho p_2 = \pi^R - \pi^O + \rho A + \left(\frac{\rho + \delta}{\rho + \delta + \mu} \right) \mu^* A \tag{58}$$

Clearly, both premia depend on the wages and other labour market frictions. Note that since $B > A$, the premium in city 1 must exceed that in city 2. This reflects the fact that the housing market is much more liquid in City 2, so that

Table 4: Housing Market statistics

	City 1	City 2	Average
Rent (relative to wage)	0.03	0.12	0.075
Selling Price (relative to annual wage)	0.34	2.41	0.57
Annual Housing Premium ($r_i/p_i - \rho$)	0.029	0.003	0.016
Monthly matching rate	0.01	0.08	0.02

Table 4 gives the values for rents, price and premia resulting from our calibrated example. Here the housing premia are given relative to the price level, so as to be comparable to those calculated by Campbell, Davis, Gallin, and Martin (2008). These authors estimate quality–adjusted premia for US cities that vary between 1.84% and 6.45% and average 2.99%. While the housing premium in city 2 seems rather small, that for city 1 is of the right magnitude. Our results suggest that a significant fraction of the observed housing premia may be accounted for by the endogenous illiquidity of housing emphasized here.

6 Generalizations of the Basic Model

To be completed.

6.1 No Entry of Firms

In the basic model, with free entry of firms, levels of employment at each location are essentially driven by the supply–side of the labour market and wages are determined only by productivity. If there is no entry, then there is a downward sloping demand curve for labour in each city and wages are partly determined by the frictions in the housing market.

6.2 Endogenous Conversion

In the basic model, we have assumed that the conversion cost C_R is sufficiently high that real estate managers do not convert owner-occupied housing to rental. Note that if $\pi^H > \pi^R$ the housing premium could, in fact be negative, in which case conversion may go in the opposite direction. In this section we consider the implications of allowing conversion costs to be sufficiently small that the ratio of rental to owner-occupied housing becomes endogenous.

7 Conclusion

To be completed

Appendix

Case I

The solution to the 9 equation system described by (23) – (39), (10), and (18) can be expressed recursively as

$$N_2^{WR} = \frac{\mu R_2 + (\mu^* + \lambda)R_1}{\delta + \mu + \lambda} \tag{59}$$

$$N_1^{WR} = \frac{\mu R_1 + (\mu^* + \lambda)R_2 - \mu^* N_2^{WR}}{\delta + \mu + \mu^* + \lambda} \tag{60}$$

$$N_1^{UR} = R_1 - N_1^{WR} \tag{61}$$

$$N_2^{UR} = R_2 - N_2^{WR} \tag{62}$$

$$M_1^{UH} = \frac{\lambda}{\mu^*} R_1 \tag{63}$$

$$M_2^{UH} = \frac{\lambda}{\mu^*} R_2 \tag{64}$$

$$N_1^{UH}(\gamma_1) = \left(\frac{\delta}{\delta + \mu}\right)\left(H_1 - \frac{\lambda}{\delta} N_1^{WR} - \frac{\lambda}{\gamma_1} R_1\right) \tag{65}$$

$$N_1^{WH}(\gamma_1) = \left(\frac{\mu}{\delta + \mu}\right)\left(H_1 + \frac{\lambda}{\mu} N_1^{WR} - \frac{\lambda}{\gamma_1} R_1\right) \tag{66}$$

$$N_2^{UH}(\gamma_2) = \left(\frac{\delta}{\delta + \mu}\right)\left[H_2 - \frac{\lambda}{\delta} N_2^{WR} - \frac{\lambda}{\gamma_2} R_2\right] \tag{67}$$

$$N_2^{WH}(\gamma_2) = \left(\frac{\mu}{\delta + \mu}\right)\left[H_2 + \frac{\lambda}{\mu} N_2^{WR} - \frac{\lambda}{\gamma_2} R_2\right]. \tag{68}$$

The flow utilities of owners in City 1 are given by

$$\rho W_1^H = w_1 + \pi^H + \delta\left(U_1^H - W_1^H\right) \tag{69}$$

$$\rho U_1^H = b + \pi^H + \mu\left(W_1^H - U_1^H\right) + \mu^*\left(W_2^R + p_1 - U_1^H\right) \tag{70}$$

The house prices in City 1 must satisfy

$$\begin{aligned} \rho p_1 &= \gamma_1\left[\alpha_1\left(W_1^H - p_1 - W_1^R\right) + (1-\alpha_1)\left(U_1^H - p_1 - U^R\right)\right] \\ &= \gamma_1\left[\alpha_1\left(W_1^H - p_1\right) + (1-\alpha_1)\left(U_1^H - p_1\right)\right] - \gamma_1\left[\alpha_1 A + U^R\right] \end{aligned} \tag{71}$$

Subtracting (71) from (69) and rearranging yields

$$(\rho + \delta + \alpha_1\gamma_1)\left(W_1^H - p_1\right) = w_1 + \pi^H + \gamma_1\left[\alpha_1 A + U^R\right] + (\delta - (1-\alpha_1)\gamma_1)\left(U_1^H - p_1\right). \tag{72}$$

Similarly subtracting (71) from (70) and rearranging yields

$$(\rho+\mu+\mu^*+(1-\alpha_1)\gamma_1)\left(U_1^H-p_1\right)=b+\pi^H+\gamma_1\left[\alpha_1 A+U^R\right]+\mu^*\left(B+U^R\right)+(\mu-\alpha_1\gamma_1)\left(W_1^H-p_1\right) \tag{73}$$

Solving for $W_1^H-p_1$ and $U_1^H-p_1$ yields

$$W_1^H-p_1=K_{W1}^I+\beta_{W1}^I U^R \tag{74}$$

$$U_1^H-p_1=K_{U1}^I+\beta_{U1}^I U^R \tag{75}$$

where

$$K_{W1}^I=\frac{1}{\Delta_1^I}\left[(\rho+\delta+\mu+\mu^*)\left(w_1+\pi^H+\alpha_1\gamma_1 A\right)-(\delta-(1-\alpha_1)\gamma_1)\left(w_1-b-\mu^* B\right)\right] \tag{76}$$

$$\beta_{W1}^I=\frac{1}{\Delta_1^I}\left[(\rho+\mu+\mu^*)\gamma_1+(\delta+\alpha_1\gamma_1)\mu^*\right] \tag{77}$$

$$K_{U1}^I=\frac{1}{\Delta_1^I}\left[(\rho+\delta+\mu)\left(w_1+\pi^H+\alpha_1\gamma_1 A\right)-(\rho+\delta+\alpha_1\gamma_1)\left(w_1-b-\mu^* B\right)\right] \tag{78}$$

$$\beta_{U1}^I=\frac{1}{\Delta_1^I}\left[(\rho+\delta+\mu)\gamma_1+(\rho+\delta+\alpha_1\gamma_1)\mu^*\right] \tag{79}$$

and

$$\Delta_1^I=(\rho+\delta+\alpha_1\gamma_1)\mu^*+(\rho+\gamma_1)(\rho+\delta+\mu) \tag{80}$$

For location 2, the flow utilities of owners are given by

$$\rho W_2^H=w_2+\pi^H+\delta\left(U_2^H-W_1^H\right) \tag{81}$$

$$\rho U_2^H=b+\pi^H+\mu\left(W_2^H-U_2^H\right)+\mu^*\left(W_1^R+p_2-U_2^H\right) \tag{82}$$

Thus, following the same procedure as for city 1 we have

$$W_2^H-p_2=K_{W2}+\beta_{W2}U^R \tag{83}$$

$$U_2^H-p_2=K_{U2}+\beta_{U2}U^R \tag{84}$$

where

$$K_{W2}=\frac{1}{\Delta_2}\left[(\rho+\delta+\mu+\mu^*)\left(w_2+\pi^H+\alpha_2\gamma_2 B\right)-(\delta-(1-\alpha_2)\gamma_2)\left(w_2-b-\mu^* A\right)\right] \tag{85}$$

$$\beta_{W2}=\frac{1}{\Delta_2}\left[(\rho+\delta+\mu)\gamma_2+(\delta+\alpha_2\gamma_2)\mu^*\right] \tag{86}$$

$$K_{U2}=\frac{1}{\Delta_2}\left[(\mu-\alpha_2\gamma_2)\left(w_2+\pi^H+\alpha_2\gamma_2 B\right)+(\rho+\delta+\alpha_2\gamma_2)\left(b+\pi^H+\mu^* A+\alpha_2\gamma_2 B\right)\right] \tag{87}$$

$$\beta_{U2}=\frac{1}{\Delta_2}\left[(\rho+\delta+\mu)\gamma_2+(\rho+\delta+\alpha_2\gamma_2)\mu^*\right] \tag{88}$$

and

$$\Delta_2 = (\rho + \gamma_2)(\rho + \delta + \mu) + (\rho + \delta + \alpha_2\gamma_2)\mu^* > 0 \tag{89}$$

Interior Case: In this interior solution $\theta^X < 1$ or $N_1^{UH}(\gamma_1) > M_1^{UH}$. Using (65) and (63) this implies

$$H_1 - \frac{\lambda}{\delta} N_1^{WR} - \left(1 + \frac{\mu}{\delta}\right) \frac{\lambda}{\mu^*} R_1 > \frac{\lambda}{\gamma_1} R_1 \tag{90}$$

which implies a lower bound on γ_1 given in (40). Existence also requires that $\theta^Y < 1$ or $N_2^{UH}(\gamma_2) > Y$. Using (67) and (64) this implies

$$H_2 - \frac{\lambda}{\delta} N_2^{WR} - \left(\frac{\delta + \mu}{\delta}\right) \frac{\lambda}{\mu^*} R_2 > \frac{\lambda}{\gamma_2} R_2 \tag{91}$$

which implies a lower bound on γ_2 given in (41).

Equating (75) and (42) yields the equilibrium value of U^R as a function of City 1's matching rate

$$\rho U^R(\gamma_1; I) = w_1 + \pi^H + \gamma_1\alpha_1 A - (\rho + \gamma_1) B - \frac{(\rho + \delta + \alpha_1\gamma_1)(w_1 - b)}{\rho + \delta + \mu} \tag{92}$$

Similarly equating (84) and (43) yields the equilibrium value of $U^R(\gamma_2)$:

$$\rho U^R(\gamma_2) = w_2 + \pi^H + \gamma_2\alpha_2 B - (\rho + \gamma_2) A - \frac{(\rho + \delta + \alpha_2\gamma_2)(w_2 - b)}{\rho + \delta + \mu} \tag{93}$$

Equating $U^R(\gamma_1; I) = U^R(\gamma_2; I)$, yields the positive, linear relationship between γ_1 and γ_2 given by (46) where

$$\Omega^I = \frac{w_2 - w_1 + \rho(B - A) - \frac{(\rho+\delta)(w_2-b)}{\rho+\delta+\mu} + \frac{(\rho+\delta)(w_1-b)}{\rho+\delta+\mu}}{A - \alpha_2 B + \frac{\alpha_2(w_2-b)}{\rho+\delta+\mu}} \tag{94}$$

$$\Psi^I = \left(\frac{B - \alpha_1 A + \frac{\alpha_1(w_1-b)}{\rho+\delta+\mu}}{A - \alpha_2 B + \frac{\alpha_2(w_2-b)}{\rho+\delta+\mu}}\right) \tag{95}$$

Corner Y ($\theta^Y = 1$): In this case $\gamma_1 = \gamma_1^Y$ and $\gamma_2 = \gamma_2^Y$. That is

$$H_2 - \frac{\lambda}{\delta} N_2^{WR} - \left(\frac{\delta + \mu}{\delta}\right) \frac{\lambda}{\mu^*} R_2 = \frac{\lambda}{\gamma_2} R_2 \tag{96}$$

$$1 - R_1 - R_2 - \frac{\lambda}{\delta} N_2^{WR} - \left(\frac{\delta + \mu}{\delta}\right) \frac{\lambda}{\mu^*} R_2 = H_1 - \frac{\lambda}{\gamma_1} R_1 \tag{97}$$

Substituting into the solutions for the flow equations yields

$$N_1^{UH} = \left(\frac{\delta}{\delta+\mu}\right)\left[1 - R_1 - \left(1 + \frac{\lambda}{\mu^*} + \frac{\mu\lambda}{\delta\mu^*}\right) R_2 - \frac{\lambda}{\delta} N_1^{WR} - \frac{\lambda}{\delta} N_2^{WR}\right] \tag{98}$$

$$N_1^{WH} = \left(\frac{\mu}{\delta+\mu}\right)\left[1 - R_1 - \left(1 + \frac{\lambda}{\mu^*} + \frac{\mu\lambda}{\delta\mu^*}\right) R_2 + \frac{\lambda}{\mu} N_1^{WR} - \frac{\lambda}{\delta} N_2^{WR}\right] \tag{99}$$

$$N_2^{UH} = \frac{\lambda}{\mu^*} R_2 \tag{100}$$

$$N_2^{WH} = \frac{\lambda}{\delta} N_2^{WR} + \frac{\mu\lambda}{\delta\mu^*} R_2 \tag{101}$$

In this corner case (42) continues to hold, but (43) does not. Equating (75) and (42) yields

$$\rho U^R\left(\gamma_1^Y\right) = w_1 + \pi^H + \gamma_1^Y \alpha_1 A - \left(\rho + \gamma_1^Y\right) B - \frac{\left(\rho + \delta + \alpha_1 \gamma_1^Y\right)\left(w_1 - b\right)}{\rho + \delta + \mu} \tag{102}$$

Corner X ($\theta^X = 1$): In this case $\gamma_1 = \gamma_1^X$ and $\gamma_2 = \gamma_2^X$. That is

$$H_1 - \frac{\lambda}{\delta} N_1^{WR} - \left(\frac{\delta+\mu}{\delta}\right)\frac{\lambda}{\mu^*} R_1 = \frac{\lambda}{\gamma_1} R_1 \tag{103}$$

and

$$1 - R_1 - R_2 - \frac{\lambda}{\delta} N_1^{WR} - \left(\frac{\delta+\mu}{\delta}\right)\frac{\lambda}{\mu^*} R_1 = H_2 - \frac{\lambda}{\gamma_2} R_2 \tag{104}$$

Substituting into the solutions for the flow equations yields

$$N_1^{UH} = \frac{\lambda}{\mu^*} R_1 \tag{105}$$

$$N_1^{WH} = \frac{\lambda}{\delta} N_1^{WR} + \frac{\mu\lambda}{\delta\mu^*} R_1 \tag{106}$$

$$N_2^{WH} = \frac{\mu}{\delta+\mu}\left(1 - \left(1 + \frac{\lambda}{\mu^*} + \frac{\mu\lambda}{\delta\mu^*}\right) R_1 - R_2 - \frac{\lambda}{\delta} N_1^{WR} + \frac{\lambda}{\mu} N_2^{WR}\right) \tag{107}$$

$$N_2^{UH} = \frac{\delta}{\delta+\mu}\left(1 - \left(1 + \frac{\lambda}{\mu^*} + \frac{\mu\lambda}{\delta\mu^*}\right) R_1 - R_2 - \frac{\lambda}{\delta} N_1^{WR} - \frac{\lambda}{\delta} N_2^{WR}\right) \tag{108}$$

In this corner case (43) continues to hold, but (42) does not. Equating (84) and (43) yields

$$\rho U^R\left(\gamma_2^X\right) = w_2 + \pi^H + \gamma_2^X \alpha_2 B - \left(\rho + \gamma_2^X\right) A - \frac{\left(\rho + \delta + \alpha_2 \gamma_2^X\right)\left(w_2 - b\right)}{\rho + \delta + \mu} \tag{109}$$

Case II

Flow conditions

$$(\delta+\mu^*+\lambda)N_1^{WR} = \mu N_1^{UR}+\mu^*\left(M_2^{UH}+N_2^{UR}\right) \tag{110}$$
$$\left(\mu+\mu^*\right)N_1^{UH} = \delta N_1^{WH}+\lambda N_1^{UR} \tag{111}$$
$$\delta N_1^{WH}+\mu^* M_1^{WH} = \lambda N_1^{WR}+\mu N_1^{UH} \tag{112}$$
$$(\delta+\lambda)N_2^{WR} = \mu N_2^{UR}+\mu^*\left(N_1^{UR}+N_1^{UH}+N_1^{WR}+M_1^{WH}\right) \tag{113}$$
$$\mu N_2^{UH}+\mu^* M_2^{UH} = \delta N_2^{WH}+\lambda N_2^{UR} \tag{114}$$
$$\delta N_2^{WH} = \lambda N_2^{WR}+\mu N_2^{UH} \tag{115}$$

The solution to these flow equations can be expressed recursively as (60), (59), (61), (62), (64) and

$$M_1^{WH}(\gamma_1) = \left(\frac{\delta}{\delta+\mu+\mu^*}\right)\left[\left(\frac{\lambda}{\mu^*}+\frac{\mu\lambda}{\delta\mu^*}\right)R_1+\frac{\lambda}{\delta}N_1^{WR}-H_1+\frac{\lambda}{\gamma_1}R_1\right] \tag{116}$$
$$N_1^{UH}(\gamma_1) = \left(\frac{\delta}{\delta+\mu+\mu^*}\right)\left[H_1+\frac{\lambda}{\delta}R_1-\frac{\lambda}{\delta}N_1^{WR}-\frac{\lambda}{\gamma_1}R_1\right] \tag{117}$$
$$N_1^{WH}(\gamma_1) = \left(\frac{\mu+\mu^*}{\delta+\mu+\mu^*}\right)\left[H_1-\left(\frac{\lambda}{\mu+\mu^*}\right)R_1+\left(\frac{\lambda}{\mu+\mu^*}\right)N_1^{WR}-\frac{\lambda}{\gamma_1}R_1\right] \tag{118}$$
$$N_2^{UH}(\gamma_2) = \left(\frac{\delta}{\delta+\mu}\right)\left[H_2-\frac{\lambda}{\delta}N_2^{WR}-\frac{\lambda}{\gamma_2}R_2\right] \tag{119}$$
$$N_2^{WH}(\gamma_2) = \left(\frac{\mu}{\delta+\mu}\right)\left[H_2+\frac{\lambda}{\mu}N_2^{WR}-\frac{\lambda}{\gamma_2}R_2\right] \tag{120}$$

The flow utilities of owners in City 1 are given by

$$\rho W_1^H = w_1+\pi^H+\delta\left(U_1^H-W_1^H\right)+\mu^*\left(W_2^R+p_1-W_1^H\right) \tag{121}$$
$$\rho U_1^H = b+\pi^H+\mu\left(W_1^H-U_1^H\right)+\mu^*\left(W_2^R+p_1-U_1^H\right) \tag{122}$$

Following the same procedure as in Case I, we can derive the

$$W_1^H-p_1 = K_{W1}^{II}+\beta_{W1}^{II}U^R \tag{123}$$
$$U_1^H-p_1 = K_{U1}^{II}+\beta_{U1}^{II}U^R \tag{124}$$

where

$$K_{W1}^{II} = \frac{1}{\Delta_1^{II}} \left[(\rho + \delta + \mu + \mu^*) \left(w_1 + \pi^H + \mu^* B + \gamma_1 \alpha_1 A \right) - (\delta - (1 - \alpha_1)\gamma_1) (w_1 - b) \right] \tag{125}$$

$$\beta_{W1}^{II} = \frac{1}{\Delta_1^{II}} \left[(\rho + \delta + \mu + \mu^*) (\mu^* + \gamma_1) \right] \tag{126}$$

$$K_{U1}^{II} = \frac{1}{\Delta_1^{II}} \left[(\rho + \delta + \mu + \mu^*) \left(w_1 + \pi^H + \mu^* B + \gamma_1 \alpha_1 A \right) - (\rho + \delta + \mu^* + \alpha_1 \gamma_1) (w_1 - b) \right] \tag{127}$$

$$\beta_{U1}^{II} = \frac{1}{\Delta_1^{II}} \left[(\rho + \delta + \mu + \mu^*) (\mu^* + \gamma_1) \right] \tag{128}$$

and

$$\Delta_1^{II} = (\rho + \delta + \mu + \mu^*) (\rho + \mu^* + \gamma_1) \tag{129}$$

The Bellman equations and hence solution for City 2, and hence the (83) and (84) remain the same as in Case I.

Interior Solution: In this case $N_1^{WH}(\gamma_2) > Z(\gamma_2)$. Using (118) and (116), this requires that

$$H_1 - \frac{\lambda}{\mu^*} R_1 > \frac{\lambda}{\gamma_1} R_1 \tag{130}$$

which implies a lower bound on γ_1 given by (48). We also require that $Z(\gamma_2) > 0$, which implies an upper bound on γ_1 that is equivalent to γ_1^X. Finally $N_2^{UH} \geq Y$ implies a lower bound on γ_2 which is the same as γ_2^Y.

Equating (50) and (123) yields

$$\rho U^R(\gamma_1; II) = w_1 + \pi^H + \gamma_1 \alpha_1 A - (\rho + \gamma_1) B - \frac{(\delta - (1 - \alpha_1)\gamma_1) (w_1 - b)}{\rho + \delta + \mu + \mu^*} \tag{131}$$

For City 2, $U^R(\gamma_2)$ is the same as in Case I and is given by (93). Equating $U^R(\gamma_1; II) = U^R(\gamma_2)$, yields another positive, linear relationship between γ_1 and γ_2 that must pertain in this equilibrium given by (52) where

$$\Omega^{II} = \frac{w_2 - w_1 + \rho (B - A) - \frac{(\rho+\delta)(w_2 - b)}{\rho+\delta+\mu} + \frac{\delta(w_1 - b)}{\rho+\delta+\mu+\mu^*}}{A - \alpha_2 B + \frac{\alpha_2(w_2 - b)}{\rho+\delta+\mu}} < \Omega^I \tag{132}$$

$$\Psi^{II} = \left(\frac{B - \alpha_1 A - \frac{(1-\alpha_1)(w_1 - b)}{\rho+\delta+\mu+\mu^*}}{A - \alpha_2 B + \frac{\alpha_2(w_2 - b)}{\rho+\delta+\mu}} \right) < \Psi^I. \tag{133}$$

Corner Z ($\theta^Z = 1$): In this case $\gamma_1 = \gamma_1^Z$ and $\gamma_2 = \gamma_2^Z$. That is

$$\frac{\lambda}{\gamma_1} R_1 = H_1 - \frac{\lambda}{\mu^*} R_1 \tag{134}$$

Substituting into (117) - (120) yields

$$N_1^{UH} = \frac{(\delta+\mu^*)\,\lambda}{\mu^*\,(\delta+\mu+\mu^*)} R_1 - \frac{\lambda}{\delta+\mu+\mu^*} N_1^{WR} \tag{135}$$

$$N_1^{WH} = \frac{\lambda}{\delta+\mu+\mu^*} N_1^{WR} + \frac{\mu\lambda}{\mu^*\,(\delta+\mu+\mu^*)} R_1 \tag{136}$$

$$N_2^{UH} = \left(\frac{\delta}{\delta+\mu}\right)\left[1 - \left(1+\frac{\lambda}{\mu^*}\right) R_1 - R_2 - \frac{\lambda}{\delta} N_2^{WR}\right] \tag{137}$$

$$N_2^{WH} = \left(\frac{\mu}{\delta+\mu}\right)\left[1 - \left(1+\frac{\lambda}{\mu^*}\right) R_1 - R_2 + \frac{\lambda}{\mu} N_2^{WR}\right] \tag{138}$$

In this corner case (43) continues to hold, but (42) does not. Equating (84) and (43) yields

$$\rho U^R\left(\gamma_2^Z\right) = w_2 + \pi^H + \gamma_2^Z \alpha_2 B - \left(\rho+\gamma_2^Z\right) A - \frac{\left(\rho+\delta+\alpha_2\gamma_2^Z\right)(w_2-b)}{\rho+\delta+\mu} \tag{139}$$

Lemma : *Along VVI and VVII γ_1 and γ_2 are positively related.*

Proof of Lemma 7: First note that in all cases, the function $U^R(\gamma)$ is monotonically decreasing in γ . That is:

$$\rho\frac{dU^R(\gamma_1;I)}{d\gamma_1} = -\frac{\alpha_1\,(w_1-b)}{\rho+\delta+\mu} - (B-\alpha_1 A) < 0 \tag{140}$$

$$\rho\frac{dU^R(\gamma_1;II)}{d\gamma_1} = \alpha_1 A - B + \frac{(1-\alpha_1)(w_1-b)}{\rho+\delta+\mu+\mu^*} = A - B < 0 \tag{141}$$

$$\rho\frac{dU^R(\gamma_2)}{d\gamma_2} = -\frac{\alpha_2\,(w_2-b)}{\rho+\delta+\mu} + \alpha_2 B - A = -\frac{\alpha_2\mu^*}{\rho+\delta+\mu} A - A < 0 \tag{142}$$

It follows that along the curve implicitly defined by $U^R(\gamma_1;.) = U^R(\gamma_2)$, any increase in γ_2 must be matched by an increase in γ_1.

Lemma : VVII *always lies to the right of* **VVI**.

Proof of Lemma 7: First observe that for any value of γ_1, $U^R(\gamma_1;II) > U^R(\gamma_1;I)$. From (92) and (131) it can be seen that this requires that

$$\frac{\rho+\delta+\alpha_1\gamma_1}{\rho+\delta+\mu} > \frac{\delta-(1-\alpha_1)\gamma_1}{\rho+\delta+\mu+\mu^*} \tag{143}$$

Re-arranging this yields

$$\mu^* (\rho + \delta + \alpha_1 \gamma_1) > -(\rho + \delta + \mu)(\gamma_1 + \rho), \tag{144}$$

which must be true. Now, fix a value of γ_2 and consider values γ_1^a and γ_1^b such that

$$U^R(\gamma_1^a; I) = U^R(\gamma_1^b; II) = U^R(\gamma_2) \tag{145}$$

Since from Lemma 7, $U^R(\gamma_1; I)$ and $U^R(\gamma_1; II)$ are monotonically decreasing in γ_1, it follows that $\gamma_1^b > \gamma_1^a$.

Proof of Proposition 1: We must show that the following inequalities hold in each case:

$$U_1^H - p_1 > U^R \tag{146}$$

$$W_1^H - p_1 > W_1^R \tag{147}$$

$$U_2^H - p_2 > U^R \tag{148}$$

$$W_2^H - p_2 > W_2^R \tag{149}$$

Case I:

• Inequality (146) must be true in the interior sub-case and corner Y since $U_1^H - p_1 = W_2^R > U^R$.

• In corner X, using (75), we can express (146) as

$$K_{U1}^I(\gamma_1^X) + \beta_{U1}^I(\gamma_1^X) U^R(\gamma_1^X) > U^R(\gamma_1^X).$$

Using (78), (79) and (80) and re-arranging this can be expressed as

$$w_1 + \pi^H - (\rho + \delta) A - \frac{(\rho + \delta + \alpha_1 \gamma_1)}{\rho + \delta + \mu} \mu^* (A - B) > \rho U^R(\gamma_1^X)$$

Using (27) and (28) this can be re-written as

$$w_1 + \pi^H - \delta \left(W_1^R - U^R \right) + \frac{\left(\rho + \delta + \alpha_1 \gamma_1^X \right)}{\rho + \delta + \mu} \mu^* \left(W_2^R - W_1^R \right) > \rho W_1^R.$$

But $\rho W_1^R = w_1 + \pi^R - r_1 - \delta \left(W_1^R - U^R \right) + \mu^* \left(W_2^R - W_1^R \right)$ and so

$$r_1 > \pi^R - \pi^H + \left(\frac{\mu - \alpha_1 \gamma_1^X}{\rho + \delta + \mu} \right) \mu^* (B - A).$$

• Inequality (147) must hold in all sub-cases since $W_1^H - p_1 > W_2^R > W_1^R$.
• Inequality (148) must be true in the interior sub-case and corner X since $W_1^R > U^R$. In corner Y, we need that

$$K_{U2}(\gamma_2^Y) + \beta(\gamma_2^Y)U^R > U^R \tag{150}$$

Using (87), (88) and (89) this can be expressed as

$$w_2 + \pi^H - (\rho + \delta)\, B > \rho U^R$$

Since using (28), this can be re-written as

$$w_2 + \pi^H - \delta\left(W_2^R - U^R\right) > \rho W_2^R$$

But $\rho W_2^R = w_2 + \pi^R - r_2 - \delta\left(W_2^R - U^R\right)$ and so we require that $r_2 > \pi^R - \pi^H$. Since $\pi^H \geq \pi^R$ this must be true since the rental rate must be positive in equilibrium.
• Inequality (149) must be true if (148) holds because

$$\begin{aligned} W_2^H - p_2 - W_2^R &> U_2^H - p_2 - U^R \\ W_2^H - U_2^H &> W_2^R - U^R = B \end{aligned}$$

To see this note that in the interior and corner X ($U_2^H - p_2 = W_1^R$) we have

$$W_2^H - U_2^H = \frac{w_2 - b}{\rho + \delta + \mu} > B$$

In corner Y ($U_2^H - p_2 < W_1^R$) we have

$$\begin{aligned} W_2^H - U_2^H &= \frac{w_2 - b}{\rho + \delta + \mu} - \frac{\mu^*\left(W_1^R + p_2 - U_2^H\right)}{\rho + \delta + \mu} \\ &= B + \frac{\mu^* A}{\rho + \delta + \mu} - \frac{\mu^*\left(A + U^R + p_2 - U_2^H\right)}{\rho + \delta + \mu} \\ &= B + \frac{\mu^*\left(U_2^H - p_2 - U^R\right)}{\rho + \delta + \mu} > B \end{aligned}$$

Case II: First observe that in this case it is always true that

$$W_1^H - W_1^R = U_1^H - U^R \tag{151}$$

This follows from subtracting (122) from (121).
• In the interior sub-case, (147) must be true since $W_1^H - p_1 = W_2^R > W_1^R$. From (151) it follows that (146) must also hold in this case.

• In corner Z, using (124), we can express (146) as

$$K_{U1}^{II}(\gamma_1^Z) + \beta_{U1}^{II}(\gamma_1^Z)U^R(\gamma_1^Z) > U^R(\gamma_1^Z)$$

Using (127), (128) and (129) this can be expressed as

$$w_1 + \pi^H - (\rho + \delta)\, A + \mu^* \,(B - A) > \rho U^R$$

Using (27) and (28) and re-arranging, this can be written as

$$w_1 + \pi^H - \delta \left(W_1^R - U^R\right) + \mu^* \left(W_2^R - W_1^R\right) > \rho W_1^R$$

But $\rho W_1^R = w_1 + \pi^R - r_1 - \delta \left(W_1^R - U^R\right) + \mu^* \left(W_2^R - W_1^R\right)$ and so the condition becomes $r_1 > \pi^R - \pi^H$, which must be true in equilibrium. From (151) it follows that (147) must also hold in this case.

• Inequality (148) must be true in all cases since $W_1^R > U^R$, and (149) follows by the same reasoning as for Case I.

Proof of Proposition 2:

Existence: We require that there is sufficient rental housing at each location to ensure that unemployed renters are the marginal renters overall: $R_1 > N_1^{WR}$ and $R_2 > N_2^{WR}$. Using (59) and (60) it is straightforward to show that a sufficient condition for this is the ratio of rental housing in each city lies between two bounds:

$$\frac{\delta + \lambda}{\mu^* + \lambda} > \frac{R_1}{R_2} > \frac{\mu^* + \lambda - \mu^* \frac{\mu}{\delta + \mu + \lambda}}{\delta + \lambda + \mu^* \left(1 + \frac{\mu^* + \lambda}{\delta + \mu + \lambda}\right)}. \tag{152}$$

Note that provided that $\delta > \mu^*$, the upper bound must exceed 1 and the lower bound must be less than 1.

Existence of case I requires that $\gamma_1^X < \gamma_1^Y$. That is

$$R_1 + R_2 + \left(\frac{\delta + \mu}{\delta}\right) \frac{\lambda}{\mu^*} R_2 + \frac{\lambda}{\delta} N_2^{WR} + H_1 - 1 < H_1 - \frac{\lambda}{\delta} N_1^{WR} - \left(\frac{\delta + \mu}{\delta}\right) \frac{\lambda}{\mu^*} R_1 \tag{153}$$

which can be re-written as

$$\left(1 + \frac{\lambda}{\mu^*} + \frac{\mu\lambda}{\delta\mu^*}\right) R_1 + \left(1 + \frac{\lambda}{\mu^*} + \frac{\mu\lambda}{\delta\mu^*}\right) R_2 + \frac{\lambda}{\delta} N_1^{WR} + \frac{\lambda}{\delta} N_2^{WR} < 1 \tag{154}$$

That is, the total population must be sufficiently large in comparison with the stock of rental housing.

Existence of case II requires that $\gamma_1^Z < \min\left[\gamma_1^X, \gamma_1^Y\right]$. Note from (48) that it must be true that $\gamma_1^Z < \gamma_1^X$. Hence, (154) is a sufficient condition for both cases to exist. If (154) does not hold, case II may still exist if $\gamma_1^Z < \gamma_1^Y$. That is

$$R_1 + R_2 + \left(\frac{\delta + \mu}{\delta}\right)\frac{\lambda}{\mu^*}R_2 + \frac{\lambda}{\delta}N_2^{WR} + H_1 - N < H_1 - \frac{\lambda}{\mu^*}R_1$$

which can be written as

$$\left(1 + \frac{\lambda}{\mu^*}\right)R_1 + \left(1 + \frac{\lambda}{\mu^*} + \frac{\mu\lambda}{\delta\mu^*}\right)R_2 + \frac{\lambda}{\delta}N_2^{WR} < 1 \tag{155}$$

We also require that $r_1 > 0$ and $r_2 > 0$.

Uniqueness: Suppose that parameters are such that there exists an interior equilibrium (as in Case I), (γ_1^*, γ_2^*) such that $U^R(\gamma_1^*) = U^R(\gamma_2^*)$. First observe that since VVII always lies to the right of VVI, there cannot also exist an interior equilibrium as in Case II. Now consider corner case Y (γ_1^Y, γ_2^Y). Note first that from Lemma 7

$$\begin{aligned}
\gamma_1^Y > \gamma_1^* &\Rightarrow U^R(\gamma_1^Y; I) < U^R(\gamma_1^*; I) \\
\gamma_2^Y < \gamma_2^* &\Rightarrow U^R(\gamma_2^Y) > U^R(\gamma_2^*)
\end{aligned}$$

and so

$$U^R(\gamma_2^Y) > U^R(\gamma_1^Y; I)$$

If this corner case were an equilibrium then

$$W_2^R = U_1^H - p_1 \Rightarrow U^R = U^R(\gamma_1^Y; I)$$

and

$$\begin{aligned}
W_1^R &> U_2^H - p_2 \\
A + U^R &> K_{U2}(\gamma_2^Y) + \beta(\gamma_2^Y)U^R \\
U^R &> \frac{K_{U2}(\gamma_2^Y) - A}{1 - \beta(\gamma_2^Y)} = U^R(\gamma_2^Y)
\end{aligned}$$

where $\beta(\gamma_2^Y) < 1$. This implies that

$$U^R(\gamma_1^Y; I) > U^R(\gamma_2^Y).$$

Hence we have a contradiction and (γ_1^Y, γ_2^Y) cannot also be an equilibrium.

Next consider the corner case (γ_1^X, γ_2^X). Note first that

$$\gamma_1^X < \gamma_1^* \Rightarrow U^R(\gamma_1^X; I) > U^R(\gamma_1^*; I)$$
$$\gamma_2^X > \gamma_2^* \Rightarrow U^R(\gamma_2^X) < U^R(\gamma_2^*)$$

and so

$$U^R(\gamma_2^X) < U^R(\gamma_1^X; I)$$

If this corner case were an equilibrium then

$$W_1^R = U_2^H - p_2 \Rightarrow U^R = U^R(\gamma_2^{M_1^{UH}})$$

and

$$\begin{aligned} W_2^R &> U_1^H - p_1 \\ B + U^R &> K_{U1}^I(\gamma_1^X) + \beta_{U1}^I(\gamma_1^X)U^R \\ U^R &> \frac{K_{U1}^I(\gamma_1^X) - B}{1 - \beta_{U1}^I(\gamma_1^X)} = U^R(\gamma_1^X; I) \end{aligned}$$

where $\beta_{U1}^I(\gamma_1^X) < 1$. This implies that

$$U^R(\gamma_1^X; I) < U^R(\gamma_2^X).$$

Hence we have a contradiction and (γ_1^X, γ_2^X) cannot also be an equilibrium.

Similar proofs apply to the uniqueness of interior Case II.

Proof of Proposition 3:

(1) Using (46), since $\Omega^I > 0$, a sufficient condition for $\gamma_2 > \gamma_1$ is that $\Psi^I > 1$. That is

$$B - \alpha_1 A + \frac{\alpha_1 (w_1 - b)}{\rho + \delta + \mu} > A - \alpha_2 B + \frac{\alpha_2 (w_2 - b)}{\rho + \delta + \mu}$$

Using (28) to substitute out B on the right hand side and using (27) to substitute out $w_1 - b$ in the left hand side yields the condition that

$$\frac{B - A}{A} > \frac{(\alpha_2 - \alpha_1) \mu^*}{\rho + \delta + \mu}$$

Since $\alpha_2 > \alpha_1$ this holds only if the wage differential is sufficiently large.

(2) The home-ownership rate in city i is

$$h_i(\gamma_i) = \frac{N_i^{UH} + N_i^{WH}}{R_i + N_i^{UH} + N_i^{WH}} = \frac{H_i - \frac{\lambda}{\gamma_i} R_i}{R_i + H_i - \frac{\lambda}{\gamma_i} R_i}$$

which is increasing in γ_i. Since from (1) $\gamma_2 > \gamma_1$ the result follows.

Proof of Proposition 4: In aggregate the steady state flows into and out of the state of being an unemployed renter must satisfy

$$\begin{aligned}(\mu + \mu^* + \lambda)\left(N_1^{UR} + N_2^{UR}\right) &= \delta\left(N_1^{WR} + N_2^{WR}\right) \\ &= \delta\left(R_1 - N_1^{UR} + R_2 - N_2^{UR}\right)\end{aligned}$$

It follows that the unemployment rate amongst renters is given by

$$\nu^R = \frac{N_1^{UR} + N_2^{UR}}{R_1 + R_2} = \frac{\delta}{\delta + \mu + \mu^* + \lambda}.$$

In aggregate the steady-state flows into and out of being an unemployed owner must satisfy

$$\begin{aligned}\mu\left(N_1^{UH} + N_2^{UH}\right) + \mu^*\left(M_1^{UH} + M_2^{UH}\right) &= \delta\left(N_1^{WH} + N_2^{WH}\right) + \lambda\left(N_1^{UR} + N_2^{UR}\right) \\ &= \delta\left(1 - R_1 - R_2 - N_1^{UH} - N_2^{UH}\right) + \lambda\left(N_1^{UR} + N_2^{UR}\right)\end{aligned}$$

We can write this as

$$\begin{aligned}(\mu + \mu^* + \delta + \lambda)\left(N_1^{UH} + N_2^{UH}\right) &= \delta(1 - R_1 - R_2) + \mu^*\left(N_1^{UH} - M_1^{UH} + N_2^{UH} - M_2^{UH}\right) \\ &\quad + \lambda\left(N_1^{UR} + N_2^{UR} + N_1^{UH} + N_2^{UH}\right)\end{aligned}$$

Dividing by $(\mu + \mu^* + \delta + \lambda)(1 - R_1 - R_2)$ yields an expression for the rate of unemployment amongst home-owners

$$\begin{aligned}\nu^H &= \frac{N_1^{UH} + N_2^{UH}}{1 - R_1 - R_2} \\ &= \frac{\delta}{\delta + \mu + \mu^* + \lambda} + \frac{\mu^*\left(N_1^{UH} - M_1^{UH} + N_2^{UH} - M_2^{UH}\right) + \lambda\left(N_1^{UR} + N_2^{UR} + N_1^{UH} + N_2^{UH}\right)}{(\mu + \mu^* + \delta + \lambda)(1 - R_1 - R_2)}\end{aligned}$$

The second term must be positive since $M_i^{UH} \leq N_i^{UH}$, so it follows that

$$\nu^H > \nu^R.$$

Proof of Proposition 5: From (59) and (60) we can write:

$$\alpha_2 = \frac{N_2^{WR}}{R_2} = \frac{\mu + (\mu^* + \lambda)x}{\delta + \mu + \lambda} \tag{156}$$

$$\alpha_1 = \frac{N_1^{WR}}{R_1} = \frac{\mu + (\mu^* + \lambda)/x - \mu^* \alpha_2/x}{\delta + \mu + \mu^* + \lambda} \tag{157}$$

where $x = R_1/R_2$ In order for $\alpha_2 > \alpha_1$ we require that

$$\alpha_2 > \frac{\mu + (\mu^* + \lambda)/x - \mu^* \alpha_2/x}{\delta + \mu + \mu^* + \lambda}.$$

Re–arranging and substituting for α_2 using (156) yields

$$\frac{\mu + (\mu^* + \lambda)x}{\delta + \mu + \lambda} > \frac{\mu + (\mu^* + \lambda)/x}{\delta + \mu + \lambda + \mu^* \left(1 + 1/x\right)}.$$

If $x \geq 1$, this inequality must hold. It also holds for $x < 1$ provided x large enough.

Derivation of city level unemployment rate: The unemployment rate in city i is

$$\nu_i = \frac{N_i^{UH} + N_i^{UR}}{R_i + N_i^{UH} + N_i^{WH}} = \frac{R_i - N_i^{WR} + \left(\frac{\delta}{\delta+\mu}\right)\left(H_i - \frac{\lambda}{\gamma_i} R\right) - \frac{\lambda}{\delta+\mu} N_i^{WR}}{R_i + H_i - \frac{\lambda}{\gamma_i} R_i}$$

where the second equality uses (65) and (66). Dividing through by R_i and re–arranging yields

$$\nu_i = \frac{1 - \left(1 + \frac{\lambda}{\delta+\mu}\right)\alpha_i}{1 + \frac{H_i}{R_i} - \frac{\lambda}{\gamma_i}} + \left(\frac{\delta}{\delta+\mu}\right)\left(\frac{\frac{H_i}{R_i} - \frac{\lambda}{\gamma_i}}{1 + \frac{H_i}{R_i} - \frac{\lambda}{\gamma_i}}\right)$$

Re-arranging and using (54) yields (55).

Proof of Proposition 6: Using (65) and (67) the aggregate rate of unemployment is given by

$$\begin{aligned} \bar{\nu} &= N_1^{UR} + N_2^{UR} + N_1^{UH} + N_2^{UH} \\ &= R_1 + R_2 - N_1^{WR} - N_2^{WR} + \left(\frac{\delta}{\delta+\mu}\right)\left(H_1 + H_2 - \frac{\lambda}{\gamma_1} R_1 - \frac{\lambda}{\gamma_2} R_2 - \frac{\lambda}{\delta}\left(N_1^{WR} + N_2^{WR}\right)\right). \end{aligned}$$

Using (18) we can wite this as

$$\bar{\nu} = R_1 + R_2 - N_1^{WR} - N_2^{WR} + \left(\frac{\delta}{\delta+\mu}\right)\left(1 - R_1 - R_2 - \frac{\lambda}{\delta}\left(N_1^{WR} + N_2^{WR}\right)\right)$$

Using (59) and (60) it can be seen that

$$\begin{aligned} N_1^{WR} + N_2^{WR} &= \frac{\mu R_1 + (\mu^* + \lambda) R_2}{\delta + \mu + \mu^* + \lambda} + \left(1 - \frac{\mu^*}{\delta + \mu + \mu^* + \lambda}\right) \frac{\mu R_2 + (\mu^* + \lambda) R_1}{\delta + \mu + \lambda} \\ &= \left(\frac{\mu + \mu^* + \lambda}{\delta + \mu + \mu^* + \lambda}\right) (R_1 + R_2) . \end{aligned}$$

Substituting and noting that the aggregate homeownership rate is $\bar{h} = 1 - R_1 - R_2$

$$\bar{\nu} = \left(\frac{\delta}{\delta + \mu}\right) + \left(\frac{\delta}{\delta + \mu + \mu^* + \lambda}\right) (1 - \bar{h}) - \left(\frac{\delta}{\delta + \mu}\right) \left(1 + \frac{\lambda}{\delta} \left(\frac{\mu + \mu^* + \lambda}{\delta + \mu + \mu^* + \lambda}\right)\right) (1 - \bar{h}) \tag{158}$$

Thus home-ownership has two effects on unemployment. The first is negative and comes from the reduction in the number of unemployed renters. The second is positive and comes from the increase in the measure of unemployment owners. Re-arranging (158) yields (56).

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