Assessing Project Risk

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When corporate planners must decide whether to go ahead with a new investment project, they should compare the discounted value of the project's expected future cash flows to the initial outlays of capital. Finding the right discount rate for the project's cash flows—or the project's cost of capital—requires a reasonably accurate estimate of the project's risk. Measuring project risk tends to be challenging, however, because it generally cannot be estimated directly but must instead be inferred from a set of comparable traded securities.

When evaluating typical, or "scale-expanding," investments by their companies, analysts often rely on the Capital Asset Pricing Model (CAPM) framework, which uses as the measure of risk a company's *beta*, or the covariance of its stock returns with the returns on some broad market index. But the risk of a contemplated project might be very different from the mix of the firm's assets which often include many growth opportunities. For example, a large pharmaceutical company with an established drug and many R&D opportunities may be considering an investment project to expand the market for the established drug. The risk of R&D opportunities is likely to be considerably larger than the risk of expanding the market for an established drug and thus the beta of the company will overestimate the appropriate project beta.

The betas of companies whose risks are judged to be comparable to those of the project are also used to refine the estimate of risk. To adjust for differences in leverage among "comparable" companies, project analysts also then typically "unlever" the equity betas to undo the effect of financial leverage and obtain estimates of "asset" (or "enterprise") betas,[1] which are then used to estimate project risk.

But if these steps are appropriate, we will argue that they are not sufficient to provide an accurate estimation of project risk. The equity returns of companies are risky not only because of their existing projects but also because of their growth opportunities. A firm's growth opportunities often include embedded "real options" such as the option to delay, abandon, or expand a project. As is well known, such real options are similar to leveraged positions in the underlying project; and as a consequence, a firm's growth opportunities are typically riskier than its existing projects.[2]

To measure project risk properly, then, analysts must also "unlever" the asset betas of the comparable public companies for the "leverage" contributed by their growth options. This means that assessing project risk requires two distinct unlevering steps: (1) reversing the effects of financial leverage on observed equity betas to determine the asset betas of comparable companies; and (2) undoing the effects of growth options leverage on these asset betas.

A related problem arises when determining the value of a firm's growth opportunities using option-based valuation models (such as the Black-Scholes Option Pricing Model). A critical input in these models is the volatility of the project return. Again, since the project does not trade, the standard approach estimates project return volatility using the equity return volatility of comparable firms (or the entire industry). For the same reasons described above, the volatility of comparable companies' equity returns must first be adjusted for financial leverage and then for growth options leverage. We show below that the same rule for unlevering asset betas can be used to unlever asset return volatility to obtain an estimate of project return volatility.

In this article, we show that standard capital budgeting methods tend to overestimate project risk and, as a result, underestimate project values. In particular, our estimates suggest that companies in several industries—notably, Healthcare, Pharmaceuticals, Communications, Medical Equipment, and Entertainment—are especially liable to underinvest in existing businesses because of their reliance on standard methods for assessing project risk.

Our analysis also suggests that standard applications of option-based valuation models use estimates of volatility that are likely to be too high for assessing the value of growth options. One implication of this finding is that the option value forgone when firms choose to undertake a project is not as high as the value implied when using standard methods.[3]

1. A firm's asset beta is the beta it would have if it had no debt outstanding at all.
2. See Jonathan Berk, Richard Green, and Vasant Naik,"Optimal Investment, Growth Options and Security Returns," *Journal of Finance* 54 (1999), pp. 1553-1608; Murray Carlson, Adlai Fischer, and Ron Giammarino, "Corporate Investment and Asset Price Dynamics: Implications for the Cross-Section of Returns," *Journal of Finance* 59 (2004), pp. 2577-2603; and Zhi Da, Re-Jin Guo, and Ravi Jagannathan, "CAPM for Estimating the Cost of Equity Capital: Interpreting the Empirical Evidence," *Journal of Financial Economics* 103 (2012), pp. 204-220.
3. Exercising an option before the end of its life is more costly when volatility is higher. If the volatility is actually low, then companies should not be so reluctant to exercise an option to invest because the value of keeping the option alive is low relative to the value created by undertaking the investment.

Table 1 **Asset Beta and Project Beta by Industry**

This table reports the average asset beta (adjusted for financial leverage), average project beta, and the ratio of these two betas across industries. The averages are reported for the sample period of 1977 to 2009.

Industry	Asset Beta	Project Beta	Ratio
Food Products	0.59	0.54	1.09
Candy and Soda	1.08	0.94	1.14
Beer and Liquor	0.57	0.53	1.08
Recreation	0.89	0.81	1.11
Entertainment	0.82	0.58	1.42
Printing and Publishing	0.78	0.60	1.29
Consumer Goods	0.83	0.68	1.21
Apparel	0.81	0.71	1.15
Healthcare	0.97	0.56	1.73
Medical Equipment	1.07	0.75	1.43
Pharmaceutical Products	1.25	0.73	1.70
Chemicals	0.84	0.71	1.19
Rubber and Plastic Products	0.76	0.62	1.23
Textiles	0.68	0.68	1.01
Construction Materials	0.83	0.68	1.22
Construction	0.88	0.77	1.14
Steel Works Etc.	0.90	0.80	1.13
Fabricated Products	0.83	0.86	0.96
Machinery	0.95	0.78	1.21
Electrical Equipment	1.17	0.91	1.29
Automobiles and Trucks	0.89	0.78	1.14
Aircraft	0.84	0.67	1.24
Precious Metals	0.33	0.37	0.91
Metal Mining	0.89	0.88	1.02
Petroleum and Natural Gas	0.78	0.65	1.20
Utilities	0.31	0.33	0.95
Communication	0.95	0.60	1.58
Personal Services	0.80	0.64	1.25
Business Services	1.20	0.89	1.34
Computers	1.38	1.15	1.20
Electronic Equipment	1.40	1.11	1.26
Measuring/Control Equipment	1.21	0.95	1.28
Business Supplies	0.72	0.65	1.10
Shipping Containers	0.80	0.73	1.10
Transportation	0.71	0.59	1.20
Wholesale	0.87	0.76	1.14
Retail	0.86	0.71	1.21
Restaurants, Hotels, Motels	0.73	0.57	1.27

To guard against these tendencies, we recommend two adjustments to the standard approach of project valuation: The first adjusts asset betas of comparable companies for their growth options leverage to obtain the project betas. The second similarly adjusts asset return volatility to obtain project return volatility.

Framework

A company's asset beta (β_A) is a weighted average of the beta of its existing projects (β_p) and the beta of its growth opportunities (β_G), where the weights are given by the fraction of firm value, A, that comes from existing projects (P/A) and the fraction that comes from growth opportunities (G/A).[4]

$$\beta_A = \frac{P}{A}\beta_P + \frac{G}{A}\beta_G.$$

For example, if the firm has the option to scale up an existing project at some future date, then the ratio of the beta of the growth option to the beta of the existing project (we elaborate on this relationship in the Appendix to this article) is as follows:

$$\frac{\beta_G}{\beta_P} = \eta > 1,$$

where η is known as the option elasticity. High option elasticity would mean that proportionate changes in the value of the growth options are high, given changes in the underly-

4. As mentioned, a firm's growth opportunities often include embedded "real options" that are typically riskier than its existing projects.

ing economic assumptions. The option elasticity is greater, for example, when the growth option is farther out-of-the-money.

Substituting the expression above into the prior one, we get:

$$\frac{\beta_A}{\beta_P} = \frac{P}{A} + \frac{G}{A}\eta > 1,$$

showing that the firm's asset beta overestimates underlying project risk.

We have developed a simple empirical method for estimating growth options leverage.[5] The method disentangles the beta of existing projects and the beta of growth opportunities by assuming that the fraction of the firm's value that comes from existing projects (P/A) is approximated by the ratio of the firm's book value to its market value.

In Table 1 we report for some 38 different industries our estimates of the average asset beta, the average beta of existing projects (or what might be called "assets in place"), and the ratio of the average asset beta to the average beta of existing projects. The table shows that for all but three industries—fabricated products, precious metals, and utilities—the asset beta overestimates the beta of existing projects. In some industries, such as healthcare and pharmaceutical products, the asset betas are more than 70% greater than the project betas.

Thus, to evaluate an existing project in the pharmaceutical industry, one would divide (or unlever) the average asset beta of 1.25 by the growth options unlevering factor of 1.70 to obtain an estimate of the project beta of 0.73. If we assume the real risk-free rate is 2% per annum and the market risk premium is 5% per annum, our approach implies a (real) project cost of capital of 5.65%, whereas the standard approach implies a project cost of capital of 8.25%. The difference in valuations implied by these estimates of the cost of capital is substantial.

For example, suppose Pfizer wanted to value its statin drug, Lipitor, in 1997 when it first came to market—and, for simplicity, let's assume that Pfizer had access to unbiased forecasts of the real cash flows generated by the drug over the period in which it had patent protection (1997-2010). If Pfizer used its estimated asset beta of 1.25 to determine the cost of capital for this project, the value of the cash flows over this period would have been estimated at $33.3 billion. But if the company instead used the estimated project beta (of 0.73) to determine the cost of capital, the value would have been correctly estimated to be $40.6 billion, an increase of $7.3 billion or approximately 22%.

With aggregate firm value in excess of one trillion dollars and aggregate annual R&D expenditures in excess of $48 billion (by publicly traded firms in 2009), the scope for valuation errors in the Pharmaceutical industry alone is substantial. Other industries with large potential misvaluations include Healthcare, Communications, Medical Equipment, and Entertainment.

Not only do standard capital budgeting methods tend to overestimate project betas and underestimate project NPVs, such methods also tend to overestimate the volatility of projects which can have the unwanted effect of discouraging the exercise of growth options (i.e. investing in new projects).[6] Option-based valuation models such as the Black-Scholes Option Pricing Model) require two critical inputs: the current project value (if undertaken for sure) and the project return volatility. Just as project NPV calculations require discount rates based on twice delevered betas, estimates of the volatility of a project's returns also must first be adjusted for financial leverage and then for growth options leverage.

The relationship between asset return volatility (σ_A) and project return volatility (σ_P) is similarly given by:

$$\frac{\sigma_A}{\sigma_P} = \frac{P}{A} + \frac{G}{A}\eta > 1,$$

implying that asset return volatility also overestimates project return volatility, and therefore the value of a firm's growth opportunities.[7] The same rule for unlevering asset betas can be used to unlever asset return volatilities to obtain a good estimate of project return volatility.

In order to evaluate the option to invest in a project at some future date in the Pharmaceutical industry, one would divide (unlever) the stock return volatility of firms in the industry first by their respective financial leverage and then by the growth options unlevering factor of 1.70 to obtain an estimate of project return volatility. This estimate can then be used to accurately assess the value of a firm's real options.

For example, let us suppose a firm in the Pharmaceutical industry has the option to invest in a drug R&D project that can be undertaken at any time in the next two years at a fixed investment cost, I, and assume that the nominal interest rate is 2%. We noted earlier that standard capital budgeting methods overestimate the project cost of capital and, therefore, underestimate the current project value (if undertaken for sure).

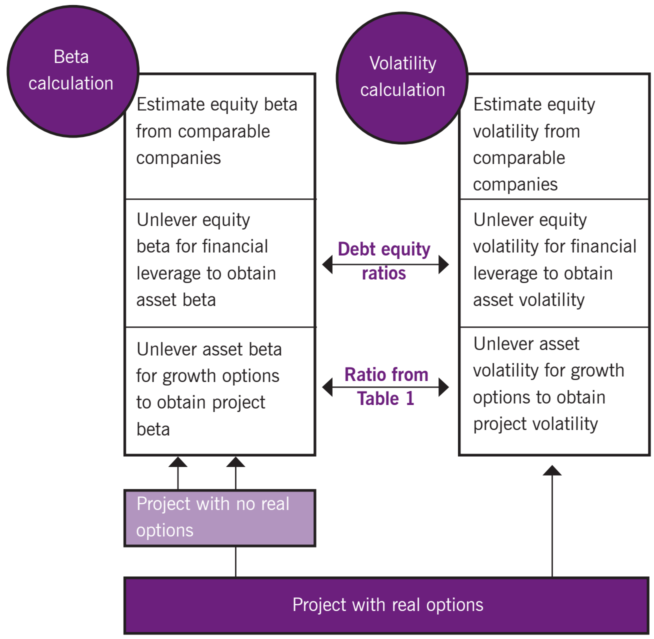
Assume that the current project value using the correct project cost of capital is equal to the investment cost, I (i.e., the growth option is at-the-money), but standard methods

5. We presented this method first in "Growth Options, Beta, and the Cost of Capital" by Antonio E. Bernardo, Bhagwan Chowdhry, and Amit Goyal in *Financial Management* 36 (2007), pp. 5-17.

6. Volatility means the annualized standard deviation of returns. It is a measure of project risk but distinct from beta which measures risk in the context of a diversified portfolio. Volatility, rather than beta, is the critical risk input in option models.

7. Implying that exercise of those options (investing in projects) would extinguish significant real option value.

Figure 1 **Project Valuation**



underestimate the current project value by 25% (i.e., the growth option is incorrectly deemed to be out-of-the money with current project value equal to 0.75 x I). We stated earlier that asset return volatility overestimates project return volatility. To correctly estimate project return volatility, one should divide (unlever) the stock return volatility of firms in the Pharmaceutical industry first by their respective financial leverage to obtain the asset return volatility (estimated to be 62%) and then by the growth options unlevering factor of 1.70 to obtain an estimate of project return volatility (estimated to be 0.62/1.70, or 36%). If we had instead simply applied the Black-Scholes formula to the pharmaceutical project described above using the standard method with incorrect estimates of current project value (0.75 x I instead of I) and project return volatility (62% instead of 36%), we would have underestimated the value of the growth option by 10% in this example.[8]

Optimal Exercise of Options

The implications of our framework are broader than just the value of the growth opportunity. Option models also tell option holders when to exercise their options. In general, the revised framework we provide here would incline managers to exercise their real options—that is, to actually invest in the underlying project—more quickly than they otherwise would.

In general, standard methods overestimate project volatility, thereby overvaluing the firm's growth options, but they also tend to overestimate project risks, thereby underestimating the value of developed projects. Accordingly, we recommend two adjustments to the standard approach of project valuation, and each adjustment involves a double-unlevering for financial leverage and for growth option leverage. The first deals with adjusting asset beta to obtain the true project beta. The second deals with adjusting asset return volatility to obtain project return volatility (and applies only to projects with embedded growth options). These steps are summarized above in Figure 1.

An Illustrative Example

To illustrate our approach, we offer an example very similar to the one used in the standard business school textbook by professors Richard Brealey, Stewart Myers, and Franklin Allen[9] to illustrate these concepts.

In 2011, the firm Blitzen Computers is considering investing $100 million in a project, Mark I microcomputers, which is expected to generate cash flows (in millions of 2011 dollars) for five years starting in year 2012 as shown:

8. Following the description in the example, the other parameters in the Black-Scholes formula are as follows: the option's time to expiry is 2 years, the strike price is *I*, and the interest rate is 2%.

9. See *Principles of Corporate Finance*, 10th Edition, (New York: McGraw-Hill Irwin, 2011) by Richard A. Brealey, Stewart Myers, and Franklin Allen.

Year	2011	2012	2013	2014	2015	2016
Expected cash flow	–100	10	20	40	30	20

Assume that the real risk-free rate is 2% per annum (one must use the real risk-free rate because all cash flows are estimated in 2011 dollars) and that the market risk-premium is 5% per annum. Since this project is in the Computer Industry, the textbook recommendation would be to use the asset beta for the Computer Industry, which is estimated to be 1.38 (see Table 1). Using the CAPM, the discount rate is given by:

Risk-free rate + β_A × Market-risk premium
= 2% + 1.38 × 5% = 8.90%

Discounting the cash flows above at 8.90% gives Mark I an estimated net present value of –\$8.6 million.

However, we first argue that an asset beta of 1.38 is too high because that represents an average of asset betas that have been adjusted only for financial leverage. Table 1 shows that further unlevering the betas for growth options leverage yields an estimate of 1.15 for the project beta. Using the CAPM, the correct discount rate for the project is:

Risk-free rate + β_P × Market-risk premium
= 2% + 1.15 × 5% = 7.75%

Discounting the cash flows above at 7.75% gives the Net Present Value of Mark I to be –\$5.5 million, higher than the Net Present Value computed using the standard textbook recommendation.

Now, assume further that investing in the Mark I project in 2011 gives Blitzen Computers a real option to invest in another project, Mark II, in year 2014. Mark II is just a scaled-up version of Mark I (two times the scale of Mark I) and would require an investment of \$200 million in year 2014 (in real terms, i.e., millions of 2011 dollars). As of the year 2011, the project is expected to generate cash flows (in real terms) for five years starting in year 2015 as shown below.

Year	2014	2015	2016	2017	2018	2019
Expected cash flow	–200	20	40	80	60	40

Discounting the cash flows above at 7.75% gives Mark II a net present value, in year 2014, of –\$11.0 million. (Notice again that discounting at the incorrect 8.90% rate would make the Net Present Value lower; in this example, –\$17.2 million.)

However, Mark II represents a real option. The firm will take the project in 2014 (three years from 2011, t=3) only if it has a positive net present value when calculated in 2014. Because the project cash flows are volatile, there is a significant possibility that the project will have positive net present value in year 2014. As is standard practice, the value of this real option can be estimated using the Black-Scholes formula, as demonstrated in Brealey, Myers, and Allen (2011). The use of the Black-Scholes formula requires an estimate of the project return volatility. Brealey, Myers, and Allen suggest using the average of stock return volatility for comparable firms in the industry. However, we argue that this is incorrect. As with betas, the stock return volatility must be adjusted both for financial leverage as well as for growth options leverage, and, as we showed earlier, volatility must be unlevered by the same factor that is used for unlevering betas.

Assuming stock return volatility, (σ_A), of 62% (adjusted for financial leverage, estimated based on actual data for the period 1977–2009 for all computer firms), we would unlever this further for growth options leverage by a factor 1.38 / 1.15 = 1.20, which yields an estimate of project return volatility, σ_P, of 62% / 1.20 = 52%.

We use the Black-Scholes formula to estimate the value of Mark II in 2011 as follows:

Present Value (in 2011) of Expected Cash-Flows from years 2015 to 2019, discounted at 7.75% = \$151 million.

Present Value (in 2011) of the Exercise Price (Investment) of \$200 million in year 2014, discounted at the real risk-free rate of 2% = \$188 million.

$d_1 = \ln(151/188) / 0.52\sqrt{3} + 0.52\sqrt{3}/2 = 0.2005.$

$d_2 = d_1 - \sigma\sqrt{\tau} = 0.2005 - 0.8949 = -0.6944.$

$N(d_1) = 0.5795.$

$N(d_2) = 0.2437$

Thus, the real option value of Mark II is equal to 151 × 0.5975 – 188 × 0.2437 = \$41.6 million (this number is obtained if intermediate calculations are not rounded off). Notice that this exceeds the –\$5.5 million value of Mark I. The real-options adjusted NPV of the project is thus 41.6 – 5.5 = \$36.1 million and thus Biltzen Computers should take the project.

If we were to follow the standard textbook methodology, we would have used 8.90% to discount expected cash-flows and a volatility of 62% in the Black-Scholes formula. This would have yielded a value of –\$8.6 million for Mark I, as we saw before, and the real option value of \$46.0 million for Mark II. The real-options adjusted NPV of the project would have been 46.0 – 8.6 = \$37.4 million. In the textbook real option calculations, there are two countervailing errors: the use of the higher discount rate of 8.90% reduces the value of the project, but a higher volatility estimate of 62% increases the value of the real option. The net effect, in general, depends on the parameters and, in particular, on the period when the real option is exercised.

Conclusion

Corporate planners often rely on the Capital Asset Pricing Model (CAPM) to determine the appropriate discount rate — or cost of capital – for evaluating investment projects. Finding the appropriate discount rate requires an accurate estimate of project risk (beta). This can be challenging because project risk cannot be estimated directly but must instead be inferred from a set of traded securities, typically the equity betas of "comparable" firms in the same industry. These equity betas are then "unlevered" to undo the effect of the comparables firms' financial leverage and obtain estimates of "asset" betas which are then used to estimate project risk.

We show that asset betas estimated in this way overestimate project risk. The equity returns of companies are risky not only because of their existing projects but also because of their growth opportunities. These growth opportunities often include embedded "real options" such as the option to delay, expand, or abandon a project. As is well known, such real options are similar to leveraged positions in the underlying project; consequently, a firm's growth opportunities are typically riskier than its existing projects. Therefore, to properly assess project risk, analysts must also unlever the asset betas derived from comparable company stock returns for the leverage contributed by their growth options.

We derive a simple method for unlevering asset betas for growth options leverage in order to properly assess project risk. We then show that standard methods for assessing project risk significantly overestimate project costs of capital, by as much as 2-3% in industries such as Healthcare, Pharmaceuticals, Communications, Medical Equipment, and Entertainment. Our method should also be applied to stock return volatility to derive project volatility, an important input for determining the value of a firm's growth opportunities and the appropriate time for investing in these opportunities.

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Zhi Da, Re-Jin Guo, and Ravi Jagannathan, "CAPM for Estimating the Cost of Equity Capital: Interpreting the Empirical Evidence," *Journal of Financial Economics* 103 (2012), pp. 204-220.

Appendix

We present a simple model, based on one developed by University of British Columbia professors Murray Carlson, Adlai Fischer, and Ron Giammarino,[10] to establish the link between a firm's asset beta and volatility and its growth opportunities.

The total firm value, A, can be represented as the sum of the value of its existing projects, P, and the value of its growth options, G:

$$A = P + G$$

It then follows that

$$dA = dP + dG$$

and dividing by A, and rearranging, we get

$$\frac{dA}{A} = \frac{P}{A}\frac{dP}{P} + \frac{G}{A}\frac{dG}{G}.$$

Let $a \equiv \frac{P}{A}$,

then

$$\frac{dA}{A} = a\frac{dP}{P} + (1-a)\frac{dG}{G}.$$

It follows that

$$\frac{\mathrm{Cov}\left(dA/A, dR\right)}{\mathrm{Var}(dR)} = a\frac{\mathrm{Cov}\left(dP/P, dR\right)}{\mathrm{Var}(dR)} + (1-a)\frac{\mathrm{Cov}\left(dG/G, dR\right)}{\mathrm{Var}(dR)},$$

where dR represents the instantaneous return on the market index. Thus, we get

$$\beta_A = a\beta_P + (1-a)\beta_G \cdot$$

Dividing by β_P yields:

$$\frac{\beta_A}{\beta_P} = a + (1-a)\frac{\beta_G}{\beta_P}.$$

Assume that the instantaneous dynamics of P follows the diffusion process $dP / P = \mu_P dt + \sigma_P dz$, where μ_P is the expected growth rate of the return on existing projects, σ_P is the project return volatility, and Z is a standard Wiener process. Suppose that the firm's growth options allow it to duplicate (and perhaps scale-up) the cash flows of the existing projects for an investment of I. Assuming that this investment opportunity may be undertaken at some future date t, the value of the growth option is given by the Black-Scholes formula

$$G = N(d_1)P - N(d_2)Ie^{-rt}, \tag{1}$$

where $d_1 = \dfrac{\ln(P_0 / Ie^{-rt}) + 0.5(\sigma_P\sqrt{t})^2}{\sigma_P\sqrt{t}}$,

and $d_2 = d_1 - \sigma_P\sqrt{t}$,

and N (·) is the cumulative distribution function for the standard normal distribution.

The following results are well-known in the options literature:

$$\frac{\beta_G}{\beta_P} = \frac{\sigma_G}{\sigma_P} = \frac{dG/G}{dP/P} \equiv \eta,$$

where σ represents volatility and η is known as the option elasticity. Since the option elasticity can be rewritten as

$$\eta = \frac{dG/G}{dP/P} = \frac{dG/dP}{G/P},$$

using the well-known result $dG/dP=N(d_1)$ and from the Black-Scholes formula (1), $G/P < N(d_1)$, it follows that $\eta>1$.

Also,

$$\mathrm{Var}\left(\frac{dA}{A}\right) = a^2\mathrm{Var}\left(\frac{dP}{P}\right) + (1-a)^2\mathrm{Var}\left(\frac{dG}{G}\right) + 2a(1-a)\mathrm{Cov}\left(\frac{dP}{P},\frac{dG}{G}\right),$$

which implies that

$$\sigma_A^2 = a^2\sigma_P^2 + (1-a)^2\eta^2\sigma_P^2 + 2a(1-a)\eta\sigma_P^2 = (a + (1-a)\eta)^2\sigma_P^2,$$

and, $\dfrac{\sigma_A}{\sigma_P} = a + (1-a)\eta.$

This yields the result:

$$\frac{\beta_A}{\beta_P} = \frac{\sigma_A}{\sigma_P} = a + (1-a)\eta > 1.$$

10. See "Corporate Investment and Asset Price Dynamics: Implications for the Cross-Section of Returns," by Murray Carlson, Adlai Fischer, and Ron Giammarino in the *Journal of Finance* 59 (2004) pp. 2577-2603.